## While Loops, Lists, and Charts in Python

This notebook teaches:

- While loops (repetition)
- Lists (store multiple values)
- Charts with Matplotlib
- (New) Quick NumPy intro

Keep it simple. Run cells top to bottom.

## While Loops — the basics

A while loop repeats as long as a condition is True.

Structure:

```
#while <condition>:
# <indented code>
# code continues here after loop ends
```

## Indentation in while loops

- The **colon (:)** starts the loop.
- The **indented part** is the body (what repeats).
- After the loop ends, code without indentation runs once.

```
In [214... n = 3 while n > 0:
```

```
print("Inside loop. n =", n) # repeated
    n = n - 1
print("This runs after the loop ends") # not repeated

Inside loop. n = 3
Inside loop. n = 2
Inside loop. n = 1
This runs after the loop ends
```

## Infinite loop with True and break

We can write while True: to loop forever, and use break to stop.

```
In [215...
count = 0
while True:
    print("count is", count)
    count = count + 1
    if count == 3:
        break # leave the loop
print("Loop stopped with break")

count is 0
count is 1
count is 2
Loop stopped with break
```

## Using continue

continue skips the rest of the body and goes to the next loop check.

## Counter

A **counter** is a variable that changes by a fixed step (often +1).

```
In [217...
count = 0
while count < 5:
    count = count + 1
    print("Counter now:", count)</pre>
```

```
Counter now: 1
Counter now: 2
Counter now: 3
Counter now: 4
Counter now: 5
```

#### Accumulator

An **accumulator** is a variable that adds different values each time. (No lists yet.)

```
In [218... total = 0
    total = total + 2
    total = total + 5
    total = total + 1
    print("Final accumulator:", total)
```

Final accumulator: 8

## Accumulator with while (still no lists)

We can also add a changing value each step.

```
In [219... total = 0
    k = 1
    while k <= 4:
        total = total + (k * 2)  # adds 2, 4, 6, 8
        k = k + 1
    print("Accumulator:", total)</pre>
```

Accumulator: 20

#### Lists — basics

A list stores multiple values in one variable.

- Written with square brackets [ ]
- First item is index 0
- len(list) gives the number of items
- There is **no** size() for Python lists use len(...)

```
In [220... fruits = ["apple", "banana", "cherry"]

print("Index 0:", fruits[0])
print("Index 1:", fruits[1])
print("How many items:", len(fruits))

last_index = len(fruits) - 1
print("Last item:", fruits[last_index])
```

Index 0: apple
Index 1: banana
How many items: 3
Last item: cherry

## Empty list, append, and change

- Start empty with []
- append(x) puts x at the end
- Change by index with assignment

```
In [221... grades = []  # empty list
  grades.append(90)  # [90]
  grades.append(85)  # [90, 85]
  print("After appends:", grades)

grades = [90, 85, 78]
  print("Before append:", grades)
  grades.append(100)
  print("After append:", grades)

grades[1] = 88  # change second item
  print("After change:", grades)

After appends: [90, 85]

Perform appends: [90, 85]
```

Before append: [90, 85, 78] After append: [90, 85, 78, 100] After change: [90, 88, 78, 100]

## More list operations

- insert(pos, value) → add at position
- extend([values]) → add many at once
- remove(value) → remove first match
- pop() → remove last item
- len(list) → how many items

```
In [222... nums = [10, 20, 30]
    nums.insert(1, 15)
    nums.extend([40, 50])
    print("After insert and extend:", nums)

nums.remove(20)
    print("After remove:", nums)

last = nums.pop()
    print("After pop:", nums, "popped:", last)

print("Length is", len(nums))
```

```
After insert and extend: [10, 15, 20, 30, 40, 50]
After remove: [10, 15, 30, 40, 50]
After pop: [10, 15, 30, 40] popped: 50
Length is 4
```

## List Slices (read parts of a list)

Form: list[start:end:step]

- start → index to begin (default = 0)
- end → index to stop before (default = len(list))
- **step** → how much to jump each time (default = 1)

## Negative indexes and negative step

- Negative indexes count from the end (-1 = last).
- Negative step goes backwards.

## Slice Assignment (replace parts)

Because lists are mutable, slices can be replaced.

```
• In-place sort: a.sort() (changes original)
```

Sorted copy: sorted(a) (new list)

Min/Max/Sum: min(a), max(a), sum(a)

```
In [226... | a = [3, 1, 2]
          b = [9, 8]
          print("a + b ->", a + b)
print("a * 3 ->", a * 3)
          print("2 in a ->", 2 in a)
          print("min/max ->", min(a), max(a))
          print("sum(a) ->", sum(a))
          print("sorted(a) ->", sorted(a), "; original a:", a)
          a.sort()
          print("after a.sort():", a)
         a + b \rightarrow [3, 1, 2, 9, 8]
         a * 3 \rightarrow [3, 1, 2, 3, 1, 2, 3, 1, 2]
         2 in a -> True
         min/max \rightarrow 13
         sum(a)
                   -> 6
         sorted(a) -> [1, 2, 3] ; original a: [3, 1, 2]
         after a.sort(): [1, 2, 3]
```

## **Proper Ways to Delete Items**

- By position (and return value): pop(index) (default = last)
- By position (no return): del lst[index]
- By value: remove(value) (first match)

```
In [227... vals = [10, 20, 30, 40, 50]
         last = vals.pop() # removes 50, returns it
         print("pop() ->", last, ";", vals)
         item = vals.pop(1)
                                # removes index 1 (20)
         print("pop(1) ->", item, ";", vals)
         del vals[0]
                                # remove index 0 (10)
         print("del[0] ->", vals)
         vals.remove(40)
                                # remove value 40
         print("remove(40) ->", vals)
         vals.extend([60,70,80,90])
         del vals[1:3]
                                # delete a slice
         print("del slice ->", vals)
                       # remove everything
         vals.clear()
         print("clear() ->", vals)
        pop() -> 50; [10, 20, 30, 40]
        pop(1) -> 20; [10, 30, 40]
        del[0] \rightarrow [30, 40]
        remove(40) -> [30]
        del slice -> [30, 80, 90]
        clear() -> []
```

#### Safe Remove

• By slice: del lst[a:b]

• Clear all: clear()

remove(value) raises an error if value is not found. Check first.

```
items = [1, 2, 3]
target = 4

if target in items:
    items.remove(target)
else:
    print(target, "not in list, nothing removed.")
print(items)

4 not in list, nothing removed.
```

## For loops

[1, 2, 3]

A **for** loop repeats over a sequence (like a list). Often simpler than while when you already have the items.

```
In [229... fruits = ["apple", "banana", "cherry"]
    for f in fruits:
        print("Fruit:", f)

Fruit: apple
    Fruit: banana
    Fruit: cherry
```

## For with range()

range(n) gives numbers from 0 up to n-1.

```
In [230... for i in range(5):
    print("i is:", i)

i is: 0
    i is: 1
    i is: 2
    i is: 3
    i is: 4
```

## For with list indexes (using len)

```
In [231... grades = [90, 85, 78]
    for i in range(len(grades)): # indexes: 0,1,2
        print("Index", i, "has grade", grades[i])

Index 0 has grade 90
Index 1 has grade 85
Index 2 has grade 78
```

## For with enumerate()

Gives both index and value at the same time.

```
In [232... grades = [90, 85, 78]

for index, value in enumerate(grades):
    print("Index", index, "has grade", value)

Index 0 has grade 90
Index 1 has grade 85
Index 2 has grade 78
```

## While vs For — looping through a list

Both can loop a list; style differs.

```
In [233... grades = [90, 85, 78]

# Using while
i = 0
while i < len(grades):
    print("While -> Index", i, "Grade", grades[i])
    i = i + 1

# Using for with range
for i in range(len(grades)):
    print("For -> Index", i, "Grade", grades[i])

# Using for directly
for grade in grades:
    print("For (direct) -> Grade", grade)
While -> Index 0 Grade 90
While -> Index 1 Grade 85
```

```
While -> Index 0 Grade 90 While -> Index 1 Grade 85 While -> Index 2 Grade 78 For -> Index 0 Grade 90 For -> Index 1 Grade 85 For -> Index 2 Grade 78 For (direct) -> Grade 90 For (direct) -> Grade 85 For (direct) -> Grade 85 For (direct) -> Grade 78
```

#### **Break and Continue note**

Both while and for support:

- **break** → exit the loop early.
- **continue** → skip the rest of the body and jump to the next iteration.

## Break and Continue inside a For loop

Grade: 90 Grade: 85

## Important Note on Break

△ Do NOT make break your normal thought process.

- break is sometimes necessary (like stopping when something goes very wrong), but it should be an **exception**, not the default way to design loops.
- continue has more natural uses (like skipping unwanted items), but it should also be used carefully.
- The normal way: design the loop condition so it ends naturally, without break .

## Charts with Matplotlib — examples

Matplotlib draws simple plots. In Colab it is already installed.

## Matplotlib style short-codes (colors / markers / line styles)

Code	Color	Code	Marker	Code	Line style
b	blue		point	-	solid
g	green	0	circle	:	dotted
r	red	X	x-mark		dashdot
С	cyan	+	plus		dashed
m	magenta	*	star		(none)
У	yellow	S	square		
k	black	d	diamond		
W	white	V	triangle down		
		^	triangle up		
		<	triangle left		
		>	triangle right		
		р	pentagon		
		h	hexagram		

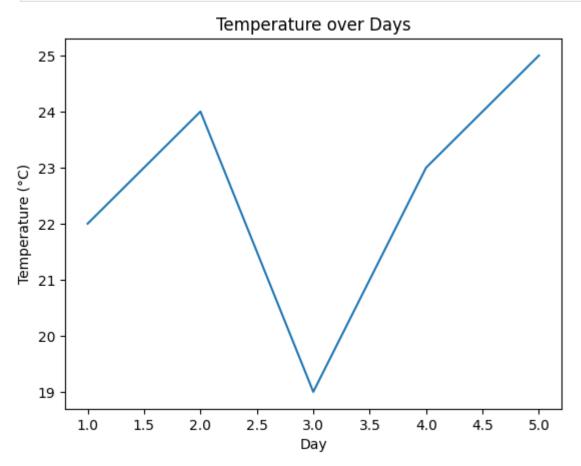
You can combine them in format strings like 'o-' (circle markers with solid line).

## 1) Line chart (simple)

```
import matplotlib.pyplot as plt
days = [1, 2, 3, 4, 5]
temperature = [22, 24, 19, 23, 25]

plt.plot(days, temperature)
plt.title("Temperature over Days")
```

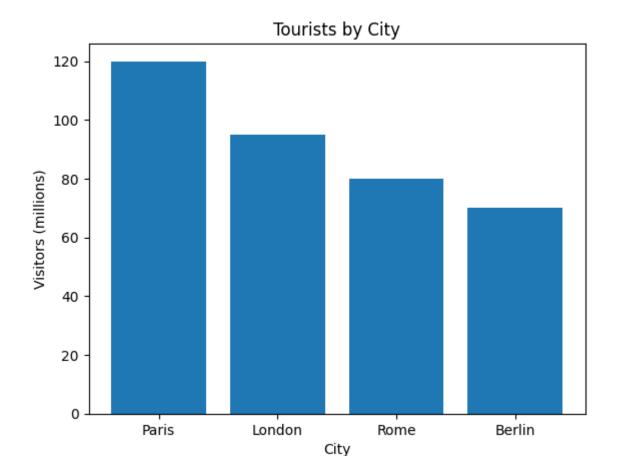
```
plt.xlabel("Day")
plt.ylabel("Temperature (°C)")
plt.show()
```



## 2) Bar chart (one series)

```
In [236... cities = ["Paris", "London", "Rome", "Berlin"]
    visitors = [120, 95, 80, 70]

    plt.bar(cities, visitors)
    plt.title("Tourists by City")
    plt.xlabel("City")
    plt.ylabel("Visitors (millions)")
    plt.show()
```

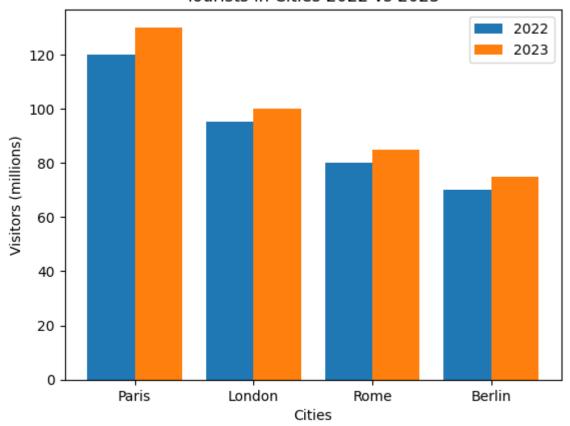


## 3) Bar chart (two series side by side)

Call plt.bar once per dataset and offset the bars.

```
cities = ["Paris", "London", "Rome", "Berlin"]
In [237...
          visitors_2022 = [120, 95, 80, 70]
          visitors_2023 = [130, 100, 85, 75]
          x = list(range(len(cities)))
          width = 0.4
          x_{\text{left}} = [xi - width/2 \text{ for } xi \text{ in } x]
          x_right = [xi + width/2 for xi in x]
          plt.bar(x_left, visitors_2022, width, label="2022")
          plt.bar(x_right, visitors_2023, width, label="2023")
          plt.xticks(x, cities)
          plt.title("Tourists in Cities 2022 vs 2023")
          plt.xlabel("Cities")
          plt.ylabel("Visitors (millions)")
          plt.legend()
          plt.show()
```

#### Tourists in Cities 2022 vs 2023



## 4) Scatter plot with triangle and pentagon markers

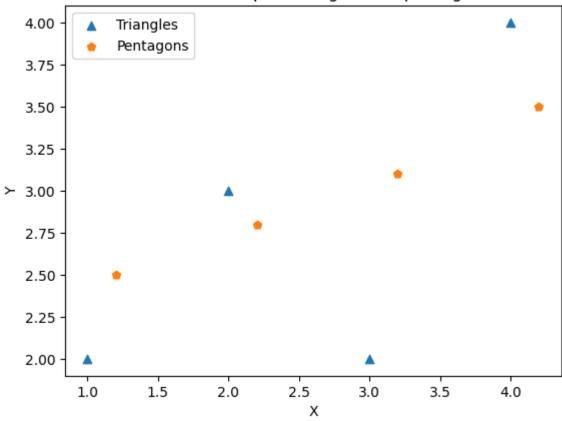
```
In [238... x1 = [1, 2, 3, 4]
    y1 = [2, 3, 2, 4]

x2 = [1.2, 2.2, 3.2, 4.2]
    y2 = [2.5, 2.8, 3.1, 3.5]

plt.scatter(x1, y1, marker='^', label='Triangles') # triangle up
    plt.scatter(x2, y2, marker='p', label='Pentagons') # pentagon

plt.title("Markers example: triangles and pentagons")
    plt.xlabel("X")
    plt.ylabel("Y")
    plt.legend()
    plt.show()
```

## Markers example: triangles and pentagons

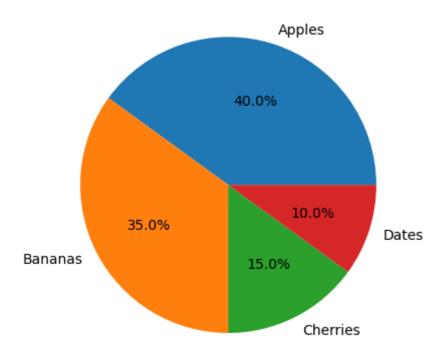


## 5) Pie chart

```
In [239... sizes = [40, 35, 15, 10]
labels = ["Apples", "Bananas", "Cherries", "Dates"]

plt.pie(sizes, labels=labels, autopct='%1.1f%%')
plt.title("Fruit Sales Share")
plt.show()
```

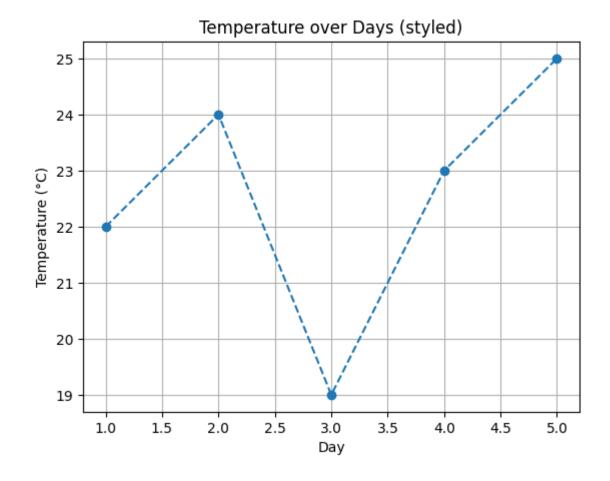
#### Fruit Sales Share



## 6) Customizing charts

```
In [240... days = [1, 2, 3, 4, 5]
  temperature = [22, 24, 19, 23, 25]

# Example of markers and line style together; color left to default.
  plt.plot(days, temperature, marker='o', linestyle='--')
  plt.title("Temperature over Days (styled)")
  plt.xlabel("Day")
  plt.ylabel("Temperature (°C)")
  plt.grid(True)
  plt.show()
```



#### Basic statistics on lists

multimode: [85]

```
In [241... import statistics as stats
         grades = [90, 85, 78, 92, 88, 76, 95, 89, 85]
                         :", stats.mean(grades))
         print("mean
         print("median
                         :", stats.median(grades))
                        :", stats.mode(grades))
         print("mode
                                                  # may raise if no unique mode
         print("multimode:", stats.multimode(grades)) # returns list of all modes
        mean
                 : 86.4444444444444
                 : 88
        median
        mode
                 : 85
```

## Manual average (good to understand)

```
In [242... values = [2, 4, 6, 8]
    avg = sum(values) / len(values)
    print("manual average:", avg)

manual average: 5.0
```

## **List Comprehensions (optional)**

```
In [243... # Squares of 0..5
    squares = [x*x for x in range(6)]
    print(squares)

# Keep only even numbers
    evens = [x for x in range(10) if x % 2 == 0]
    print(evens)

[0, 1, 4, 9, 16, 25]
[0, 2, 4, 6, 8]
```

## Very quick NumPy intro

NumPy arrays are like lists but optimized for **fast math**.

- Create from a list: np.array([1,2,3])
- Vectorized math: add, multiply, etc. works **elementwise**
- Useful creators: np.arange , np.linspace
- Basic stats: .mean(), .sum(), .min(), .max()

```
In [244... import numpy as np
         # Create arrays
         a = np.array([1, 2, 3, 4])
         b = np.array([10, 20, 30, 40])
         print("a:", a)
         print("b:", b)
         # Elementwise operations
         print("a + b ->", a + b)
         print("a * 2 ->", a * 2)
print("b - a ->", b - a)
                                           # broadcasting with a scalar
         print("a ** 2 ->", a ** 2)
                                           # power elementwise
         # Quick creators
         c = np.arange(0, 10, 2)
                                           # 0,2,4,6,8
         d = np.linspace(0, 1, 5)
                                        # 5 points between 0 and 1
         print("arange:", c)
         print("linspace:", d)
         # Basic stats
         print("mean(a):", a.mean())
         print("sum(b):", b.sum())
         print("min(b):", b.min(), "max(b):", b.max())
         # Shape and dtype
         print("shape of a:", a.shape, "dtype:", a.dtype)
```

```
a + b     -> [11 22 33 44]
a * 2     -> [2 4 6 8]
b - a     -> [ 9 18 27 36]
a ** 2     -> [ 1 4 9 16]
arange: [0 2 4 6 8]
linspace: [0.     0.25 0.5  0.75 1. ]
mean(a): 2.5
sum(b): 100
min(b): 10 max(b): 40
shape of a: (4,) dtype: int64
d = np.linspace(0, 1, 5) # 5 points between 0 and 1
```

## What does linspace do???

np.linspace(start, stop, num) generates num evenly spaced numbers
 between start and stop, inclusive.

So here:

a: [1 2 3 4] b: [10 20 30 40]

- start = 0
- stop = 1
- num = 5

#### Step calculation

It divides the interval [0, 1] into 4 equal steps (because 5 points create 4 gaps).

Step size =

$$\frac{stop - start}{num - 1} = \frac{1 - 0}{5 - 1} = \frac{1}{4} = 0.25$$

#### **Actual values**

So the points are:

## Why use it?

- When you want **evenly spaced samples** from a range.
- Useful in plotting, interpolation, or numerical simulations.
- Unlike np.arange, linspace ensures that both endpoints are included (unless you specify endpoint=False).

#### Extra options

• endpoint=False → excludes the stop value. Example:

```
np.linspace(0, 1, 5, endpoint=False)
# [0. 0.2 0.4 0.6 0.8]
• retstep=True → also returns the step size:

np.linspace(0, 1, 5, retstep=True)
# (array([0. , 0.25, 0.5 , 0.75, 1. ]), 0.25)
```

## Notice the difference between a list array and a numpy array!

# NumPy: Counting, Summing, and Frequencies

```
In [247... import numpy as np # I'm only putting it here to show we need it, it doesn't
years = np.array([2010, 2015, 2020, 2018, 2020, 2012, 2019, 2021, 2015, 2020
```

#### Count matches (booleans: True = 1, False = 0)

```
In [248... # Count years of
    count_2020_plus = np.count_nonzero(years >= 2020)
    print("Frequency of 2020+:", count_2020_plus)

# Count prices >= 2015 but below 2020
    count_15_upto_20 = np.count_nonzero((years >= 2015) & (years < 2020))
    print("Prices >=2015 (but not 2020+):", count_15_upto_20)

Frequency of 2020+: 4
    Prices >=2015 (but not 2020+): 6

Note: np.sum(grades) adds the numeric values.
    np.sum(grades == 100) adds booleans → a count.
```

#### Sum the values that match a condition (masking)

```
In [249... prices = np.array([150, 200, 99, 300, 150, 220, 180, 99, 250, 150, 175, 99])
# Sum of prices >= 180 but not 300
sum_180_up = np.sum(prices[(prices >= 180) & (prices < 250)])
print("Sum of 180-249:", sum_180_up)

Sum of 180-249: 600
Masking rule: array[condition] keeps only the elements where condition is
True.</pre>
```

#### Frequencies: unique values and counts

```
In [250... values, counts = np.unique(years, return_counts=True) # This returns a tuple
    print("values:", values) # sorted unique grades
    print("counts:", counts) # frequency of each value

# Optional: dict form (value -> count)
    freq = dict(zip(values, counts))
    print(freq)

values: [2010 2012 2015 2017 2018 2019 2020 2021]
    counts: [1 1 3 1 1 1 3 1]
    {2010: 1, 2012: 1, 2015: 3, 2017: 1, 2018: 1, 2019: 1, 2020: 3, 2021: 1}

If you just print(np.unique(grades, return_counts=True)), you'll see a tuple:
    (array_of_values, array_of_counts).
```

## Optional: percentages of groups

## Recap

- While loop: repeats while condition is True; indentation defines the body.
- Counter = fixed step; Accumulator = adds varying values.
- Lists: [ ] , index starts at 0; use len() for length; slicing uses [start:end:step].

- Modify lists: append, index assign, insert, extend, remove, pop, del, clear.
- For loops: iterate items directly, or with range(len(...)), or enumerate.
- break / continue exist, but avoid break by default.
- Matplotlib: line, bar, scatter, pie; markers/linestyles table above.
- NumPy arrays: fast math, vectorized operations, simple creators/stats.
- Count matches: np.count\_nonzero(condition) (clear), or (condition).sum()
- **Sum matches:** np.sum(arr[condition])
- **Unique + counts:** np.unique(arr, return\_counts=True)
- Combine conditions: & (and), | (or), ~ (not)

## Some interesting behaviours

```
valid = False
number = 1
while not valid:
    value = input("Enter a number: ")
    if value.replace(".", "", 1).isdigit():
        #if you don't even want floats just take out '.replace(".", "", 1)'
        number = int(value) # or float(value) depending what you want
        valid = True
    else:
        print(f"Invalid input '{value}', please enter a valid number")

print("valid number: ", number)
```

valid number: 4

## Step-by-step explanation of string operation - The more you know!

- replace(old, new, count) creates a **new string** it does not modify the original one, because strings are immutable in Python.
- For example:

```
s = "12.3"
s.replace(".", "", 1)
print(s) # still "12.3"
```

• To actually change it, you must reassign:

```
s = s.replace(".", "", 1)
print(s) # now "123"
```

## Why use 1 as the count?

• With 1 → only one dot is removed:

```
"12.3.4" replace(".", "", 1) # "123.4"

This still has a dot, so .isdigit() returns False . The invalid input "12.3.4" is rejected.
```

• With 2 → two dots are removed:

```
"12.3.4" replace(".", "", 2) # "1234" .isdigit() returns True . "12.3.4" would be incorrectly accepted as valid.
```

Using 1 enforces the rule of **at most one decimal point**, which is how proper numbers are written.

#### Why negatives are rejected

```
• Input: "-12.3"
```

- After .replace(".", "", 1) → "-123"
- .isdigit() sees the and returns False.
- Result: negative numbers are not accepted.

## Allowing negatives with lstrip("-")

To allow a **leading minus**, we can combine .lstrip("-") with .replace:

```
if value.lstrip("-").replace(".", "", 1).isdigit():
    number = float(value)
```

• .lstrip("-") only removes a minus sign from the start of the string.

```
"-123".lstrip("-") # "123"
"12-3".lstrip("-") # "12-3" (unchanged)
```

This prevents invalid inputs like "12–3" from being accepted.

- Because .lstrip returns a new string, we can **chain** it directly with .replace(".", "", 1).
- There's no need for a temporary variable, since both methods already produce new strings each time.

This way the validation works for:

- "42" → valid
- "3.14" → valid
- "-42" → valid with lstrip
- "-3.14"  $\rightarrow$  valid with lstrip
- "12.3.4" → rejected
- "12-3" → rejected
- "abc" → rejected

## np.linspace(start, stop, num) confusion vs arange

- Generates exactly num points evenly spaced between start and stop.
- By default, it **includes** the stop value.
- Example:

```
np.linspace(0, 1, 5)
Output:

[0. 0.25 0.5 0.75 1. ]

→ 5 points from 0 to 1, inclusive.
```

• If you set endpoint=False, the stop is excluded:

```
np.linspace(0, 1, 5, endpoint=False)
Output:

[0. 0.2 0.4 0.6 0.8]

→ 5 points, but the last one is just before 1.
```

## np.arange(start, stop, step)

- Generates numbers starting at start, increasing by step, stopping **just before** stop (like Python's built-in range).
- The number of points depends on step size, not a fixed count.
- Example:

```
np.arange(0, 1.01, 0.25)
Output:

[0. 0.25 0.5 0.75 1. ]

\rightarrow Same sequence as linspace(0, 1, 5), but here you control the step (0.25), not the number of points.
```

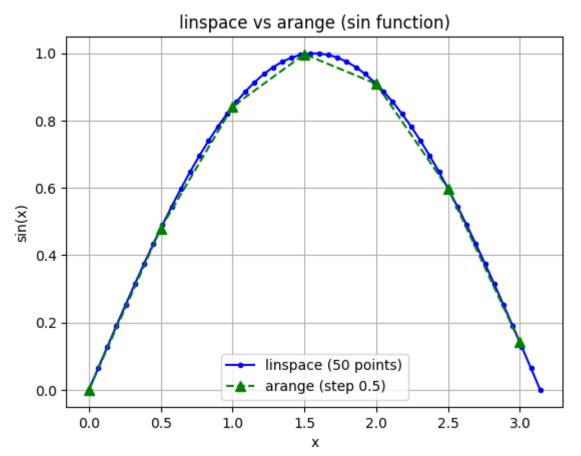
• If step doesn't divide evenly, the stop might not be hit exactly:

```
np.arange(0, 1, 0.3)
Output:
[0. 0.3 0.6 0.9]
→ Notice it stopped before 1.2 (would exceed 1).
```

#### When to use which?

- Use **linspace** when you want a specific number of evenly spaced samples (e.g., 100 points for plotting a curve).
- Use **arange** when you care about step size (e.g., count by 0.25 seconds, generate multiples of 2).

```
In [253... import numpy as np
         import matplotlib.pyplot as plt
         # Generate data with linspace (smooth curve)
         x_{lin} = np.linspace(0, np.pi, 50)
         y_{lin} = np.sin(x_{lin})
         # Generate data with arange (uneven step, 0.3 to make points less aligned)
         x_ar = np.arange(0, np.pi, 0.5)
         y_ar = np.sin(x_ar)
         # Plot
         plt.plot(x_lin, y_lin, label="linspace (50 points)", color="blue", linestyle
         plt.plot(x_ar, y_ar, label="arange (step 0.5)", color="green", linestyle="--
         plt.title("linspace vs arange (sin function)")
         plt.xlabel("x")
         plt.ylabel("sin(x)")
         plt.legend()
         plt.grid(True)
         plt.show()
```



```
In [254... \#x ar = np.arange(0, np.pi , 0.5)
         #y ar = np.sin(x ar)
         print("x arange values (0, np.pi , 0.5): ")
         print(x ar)
         print("y arange values : ")
         print(y_ar)
        x arange values (0, np.pi , 0.5):
        [0. 0.5 1. 1.5 2. 2.5 3.]
        y arange values :
                    0.47942554 0.84147098 0.99749499 0.90929743 0.59847214
        [0.
         0.14112001]
In [255...] #x lin = np.linspace(0, np.pi, 50)
         #y lin = np.sin(x lin)
         print("x linspace values np.linspace(0, np.pi, 50): ")
         print(x lin)
         print("y linspace values : ")
         print(y_lin)
        x linspace values np.linspace(0, np.pi, 50):
                    0.06411414 0.12822827 0.19234241 0.25645654 0.32057068
         0.38468481 0.44879895 0.51291309 0.57702722 0.64114136 0.70525549
         0.76936963 0.83348377 0.8975979 0.96171204 1.02582617 1.08994031
         1.15405444 1.21816858 1.28228272 1.34639685 1.41051099 1.47462512
         1.53873926 1.60285339 1.66696753 1.73108167 1.7951958 1.85930994
         1,92342407 1,98753821 2,05165235 2,11576648 2,17988062 2,24399475
         2.30810889 2.37222302 2.43633716 2.5004513 2.56456543 2.62867957
         2.6927937 2.75690784 2.82102197 2.88513611 2.94925025 3.01336438
         3.07747852 3.14159265]
        y linspace values :
        [0.00000000e+00 6.40702200e-02 1.27877162e-01 1.91158629e-01
         2.53654584e-01 3.15108218e-01 3.75267005e-01 4.33883739e-01
         4.90717552e-01 5.45534901e-01 5.98110530e-01 6.48228395e-01
         6.95682551e-01 7.40277997e-01 7.81831482e-01 8.20172255e-01
         8.55142763e-01 8.86599306e-01 9.14412623e-01 9.38468422e-01
         9.58667853e-01 9.74927912e-01 9.87181783e-01 9.95379113e-01
         9.99486216e-01 9.99486216e-01 9.95379113e-01 9.87181783e-01
         9.74927912e-01 9.58667853e-01 9.38468422e-01 9.14412623e-01
         8.86599306e-01 8.55142763e-01 8.20172255e-01 7.81831482e-01
         7.40277997e-01 6.95682551e-01 6.48228395e-01 5.98110530e-01
         5.45534901e-01 4.90717552e-01 4.33883739e-01 3.75267005e-01
         3.15108218e-01 2.53654584e-01 1.91158629e-01 1.27877162e-01
         6.40702200e-02 1.22464680e-16]
```