

PROJET MCMC

EM

METROPOLIS-HASTINGS

Proposal distributions

$$\theta_{new}^{(t)} \sim \mathcal{N}(\theta^{(t-1)}, \sigma_{\theta})$$

$$p_{new}^{(t)} \sim \mathcal{N}(p^{(t-1)}, p_{\theta})$$

Accept-Reject function

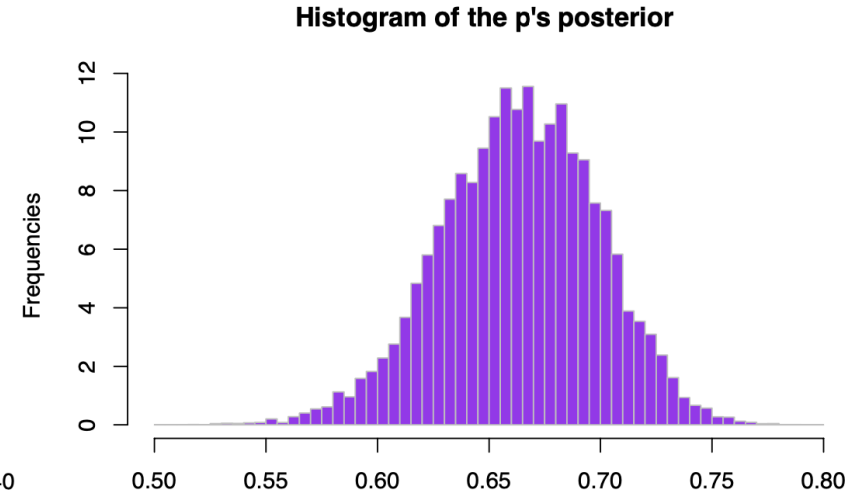
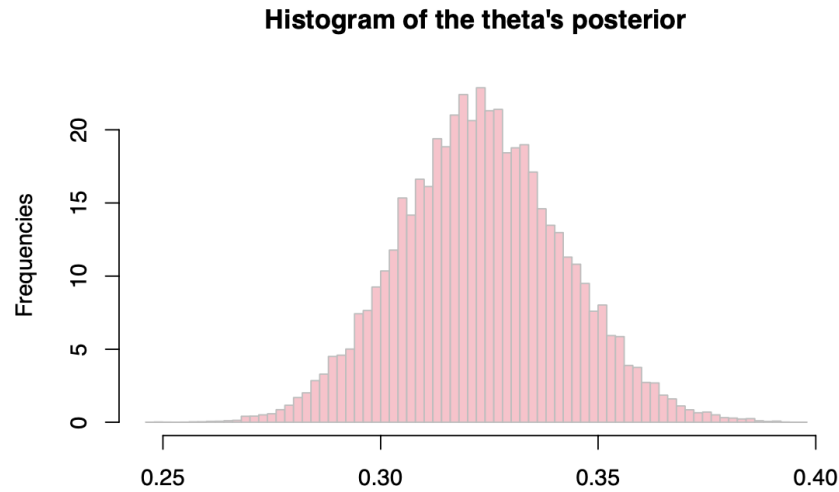
$$\begin{aligned} \rho(p_{new}, \theta_{new}, p^{(t-1)}, \theta^{(t-1)}) &= \frac{\pi(p_{new}, \theta_{new} | x_{obs})}{\pi(p^{(t-1)}, \theta^{(t-1)} | x_{obs})} \\ &= \frac{\pi(p_{new}, \theta_{new})}{\pi(p^{(t-1)}, \theta^{(t-1)})} \frac{l(p_{new}, \theta_{new} | x_{obs})}{l(p^{(t-1)}, \theta^{(t-1)} | x_{obs})} \end{aligned}$$

*Shema ? Ou
calcul des prior*

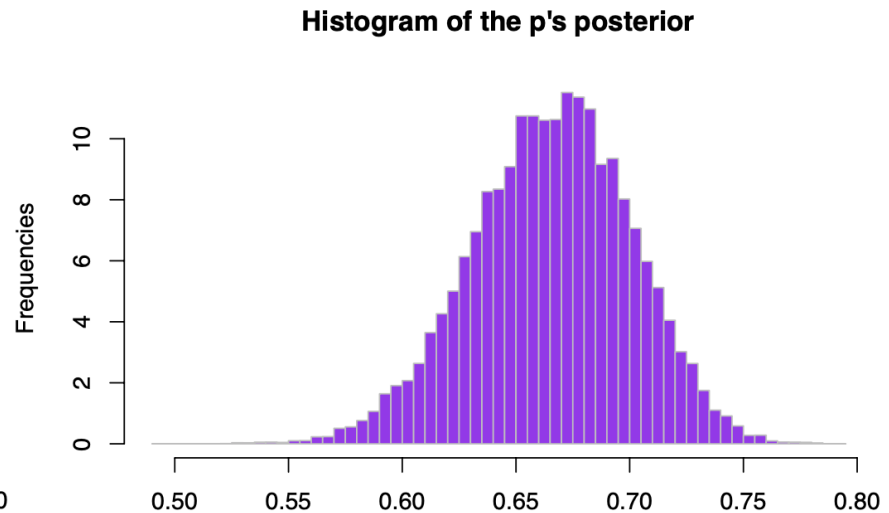
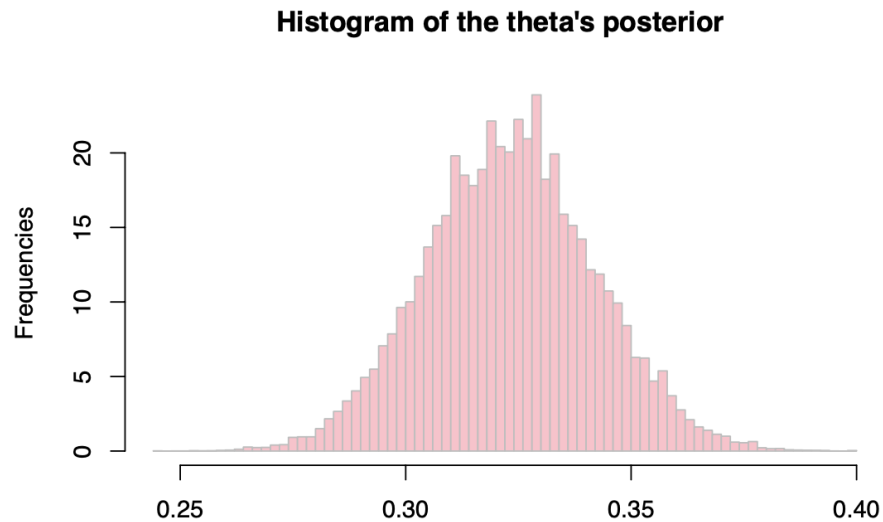
METROPOLIS-HASTINGS

$$\sigma_p = \sigma_\theta = 0.2$$

Uniform *prior*



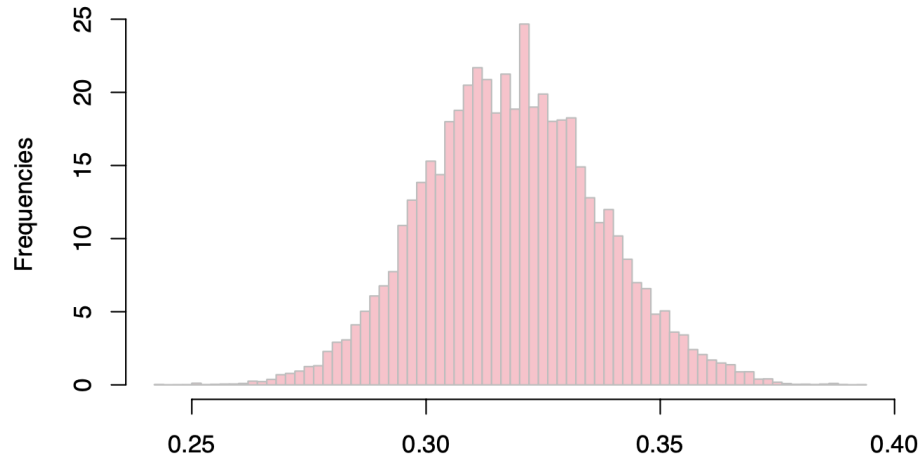
Beta *prior*
 $\alpha_p = 4, \beta_p = 2$
 $\alpha_\theta = 2, \beta_\theta = 4$



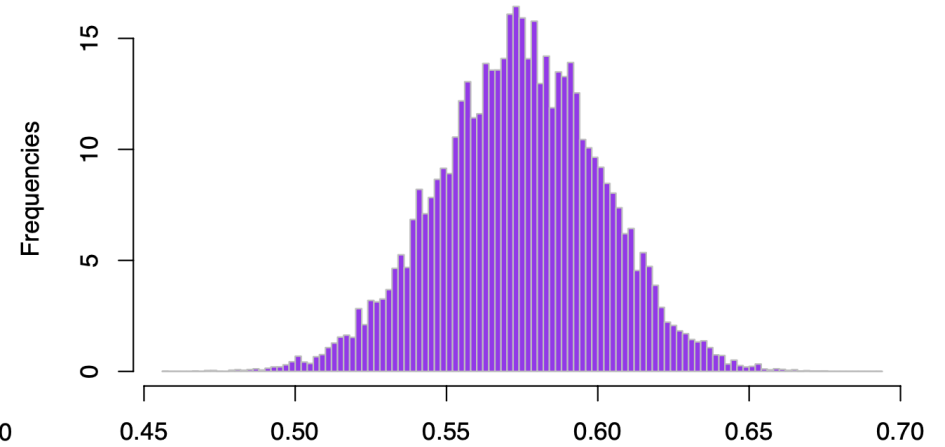
METROPOLIS-HASTINGS

Beta prior
 $\alpha_p = 100, \beta_p = 100$
 $\alpha_\theta = 2, \beta_\theta = 3$

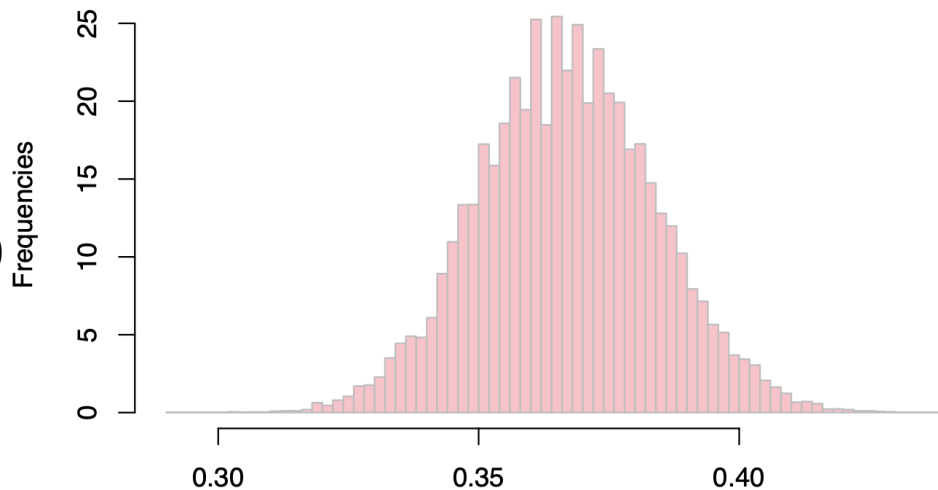
Histogram of the theta's posterior



Histogram of the p's posterior $\sigma_p = \sigma_\theta = 0.2$

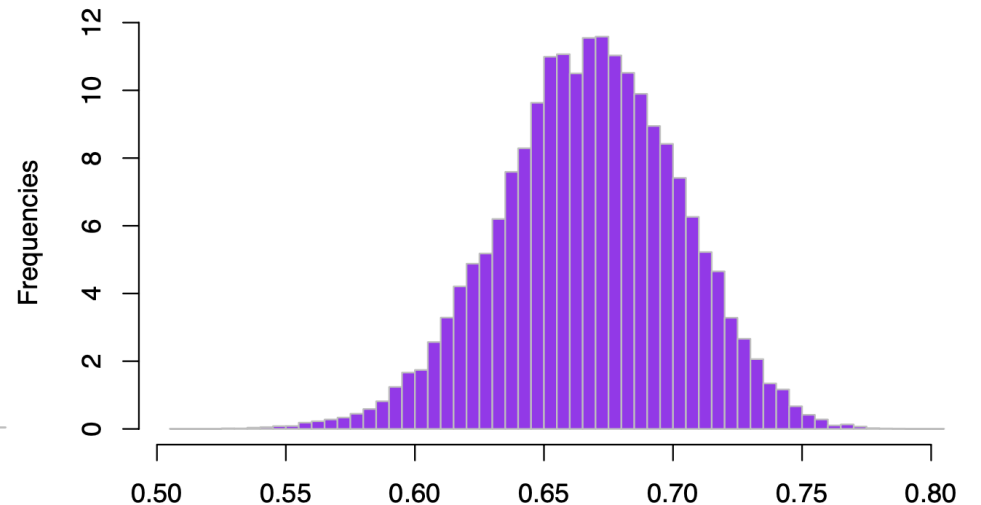


Histogram of the theta's posterior



Beta prior
 $\alpha_p = 2, \beta_p = 3$
 $\alpha_\theta = 100, \beta_\theta = 100$

Histogram of the p's posterior

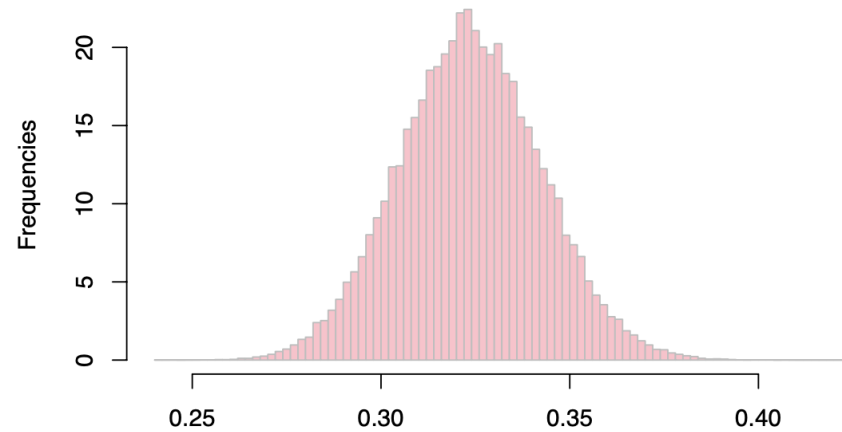


METROPOLIS-HASTINGS

How the algorithm reacts when we change the variance of the proposal laws ?

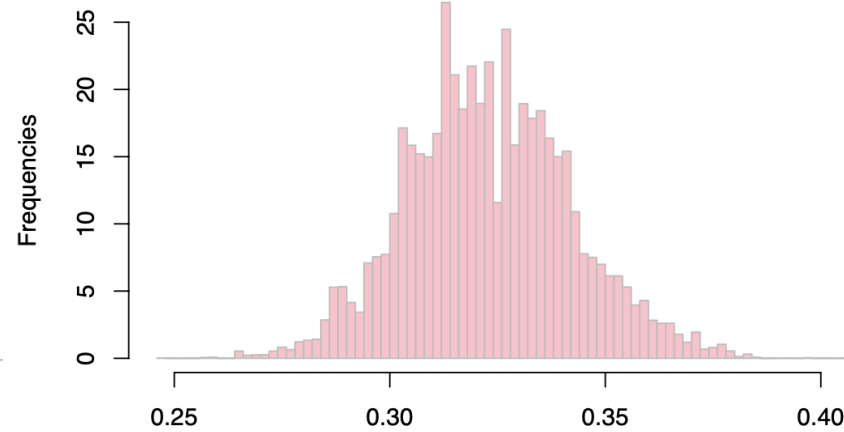
$$\sigma_p = \sigma_\theta = 0.1$$

Histogram of the theta's posterior – Uniforme prior



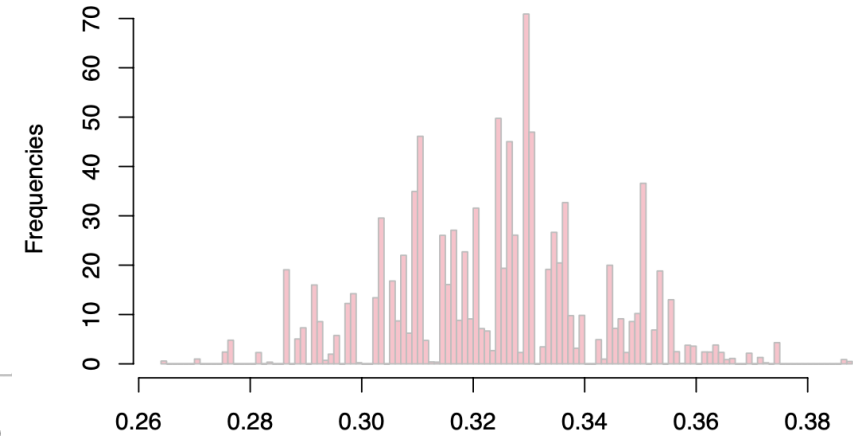
$$\sigma_p = \sigma_\theta = 0.5$$

Histogram of the theta's posterior – Uniforme prior

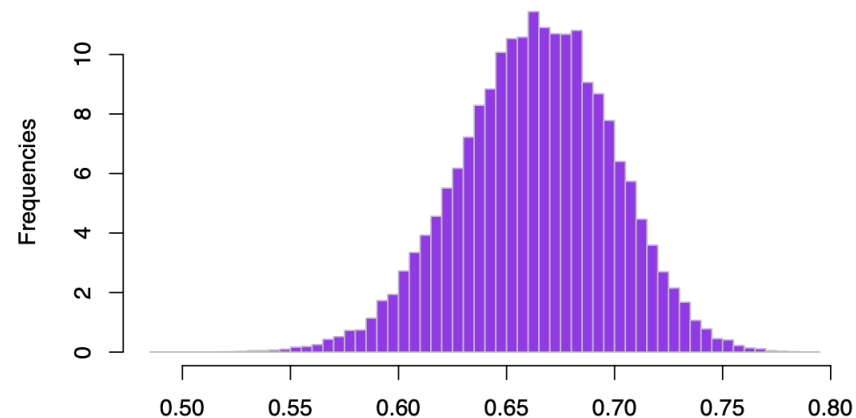


$$\sigma_p = \sigma_\theta = 2$$

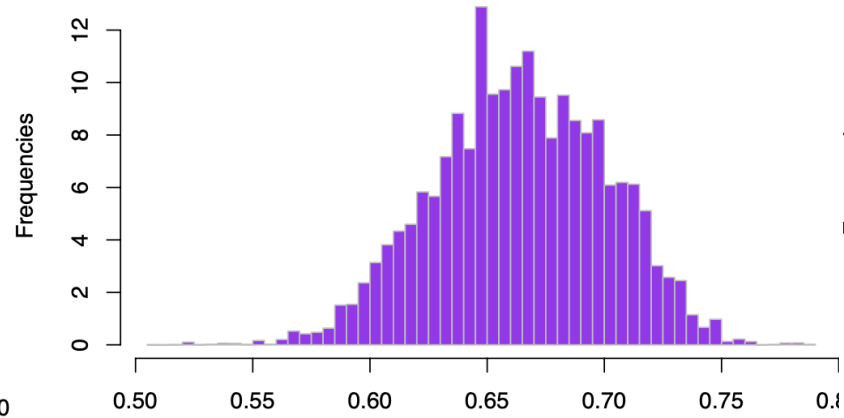
Histogram of the theta's posterior – Uniforme prior



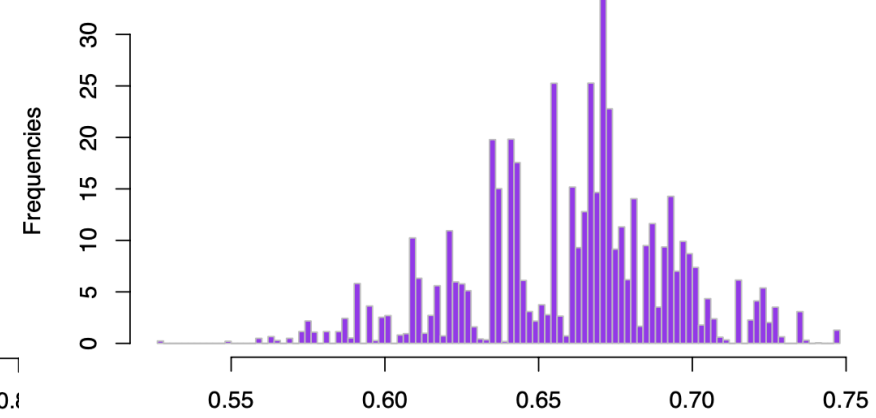
Histogram of the p's posterior – Uniforme prior



Histogram of the p's posterior – Uniforme prior



Histogram of the p's posterior – Uniforme prior



GIBBS SAMPLER

Conditional distribution of $p, \theta | (z, x_{obs})$

$$\pi(p, \theta | z, x_{obs}) = \pi(p, \theta) l(p, \theta | z, x_{obs})$$

Uniform prior - $\pi(p, \theta | z, x_{obs}) \propto \text{Beta}(N + 1, n_{obs} - N + 1) \times \text{Beta}(N_{aa}^1 + 2N_{aa}^0 + N_{aA} + 1, N_{AA}^1 + 2N_{AA}^0 + N_{aA} + 1)$

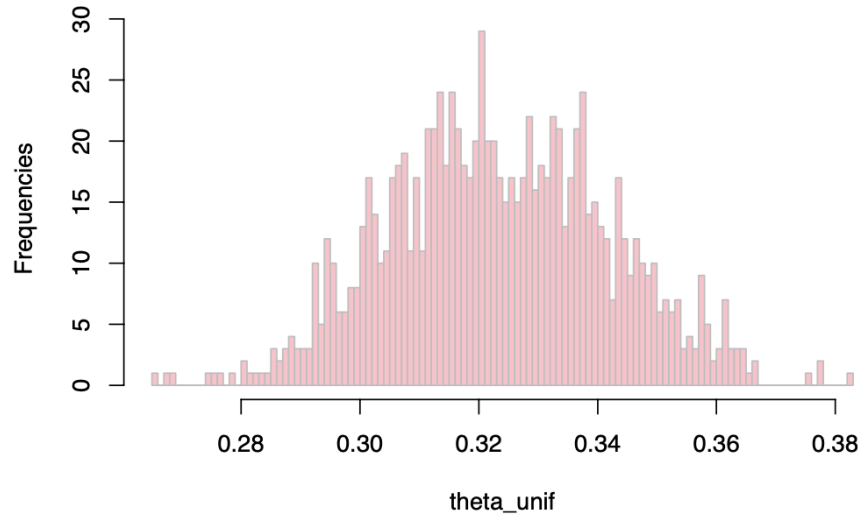
Beta prior - $\pi(p, \theta | z, x_{obs}) \propto \text{Beta}(N + \alpha_p, n_{obs} - N + \beta_p) \times \text{Beta}(N_{aa}^1 + 2N_{aa}^0 + N_{aA} + \alpha_\theta, N_{AA}^1 + 2N_{AA}^0 + N_{aA} + \beta_\theta)$

Conditional distribution of $z | (p, \theta, x_{obs})$

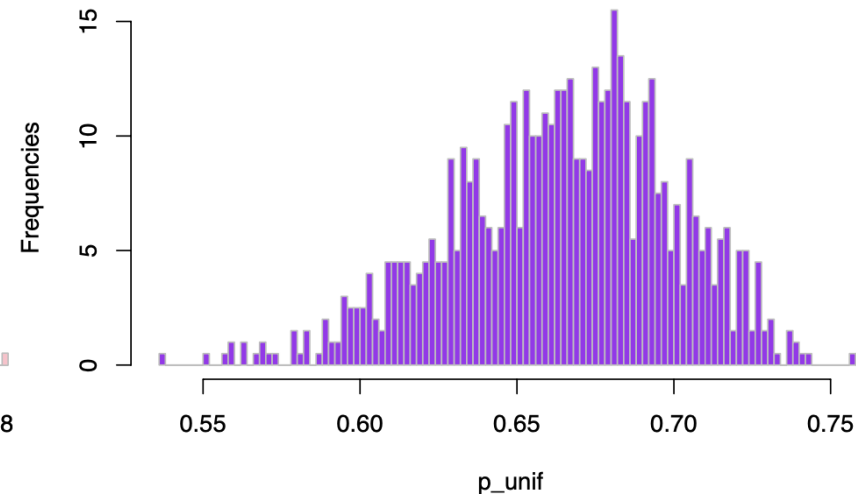
$$\begin{aligned} \mathbb{P}(Z_i = 1 | X_i = x_i, p, \theta) &= \frac{p(1 - \theta)}{p(1 - \theta) + (1 - p)(1 - \theta)^2} \quad \text{si } x_i = AA & \mathbb{P}(Z_i = 0 | X_i = x_i, p, \theta) &= \frac{(1 - p)(1 - \theta)^2}{p(1 - \theta) + (1 - p)(1 - \theta)^2} \quad \text{si } x_i = AA \\ &= \frac{p\theta}{p\theta + (1 - p)\theta^2} \quad \text{si } x_i = aa & &= \frac{(1 - p)\theta^2}{p\theta + (1 - p)\theta^2} \quad \text{si } x_i = aa \\ &= 0 \quad \text{si } x_i = aA & &= 1 \quad \text{si } x_i = aA \end{aligned}$$

GIBBS SAMPLER

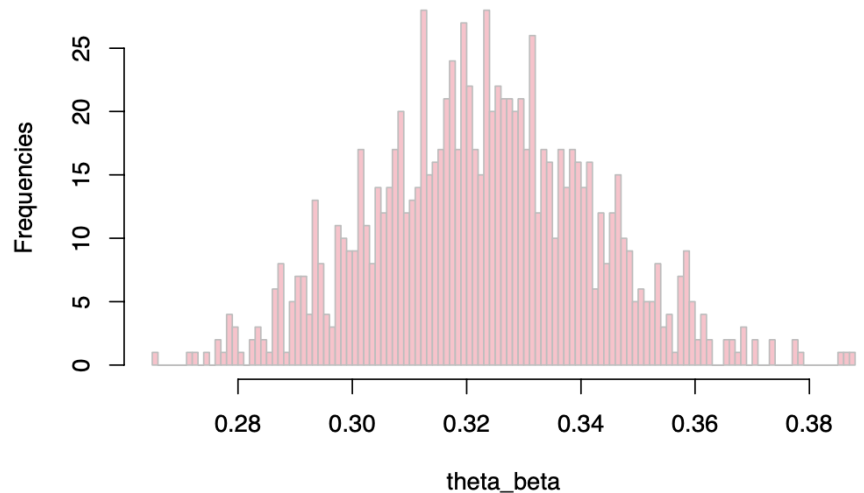
Histogram of the theta's posterior – Uniforme prior



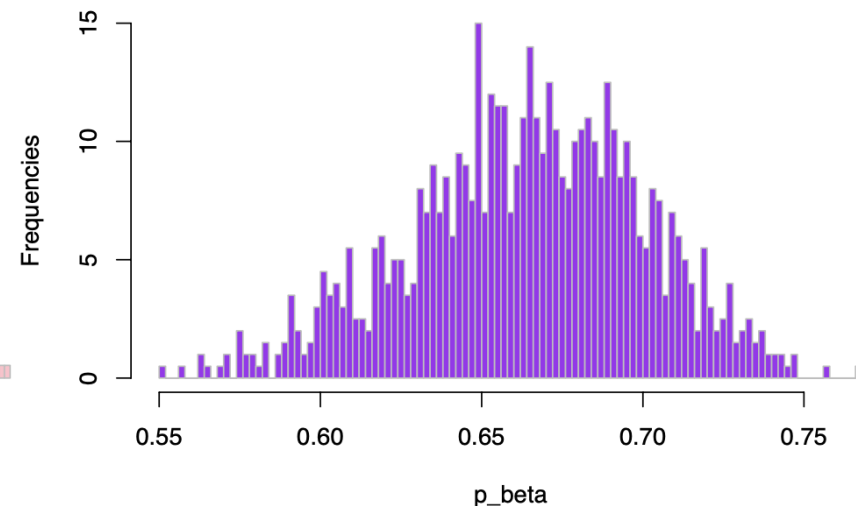
Histogram of the p's posterior – Uniforme prior



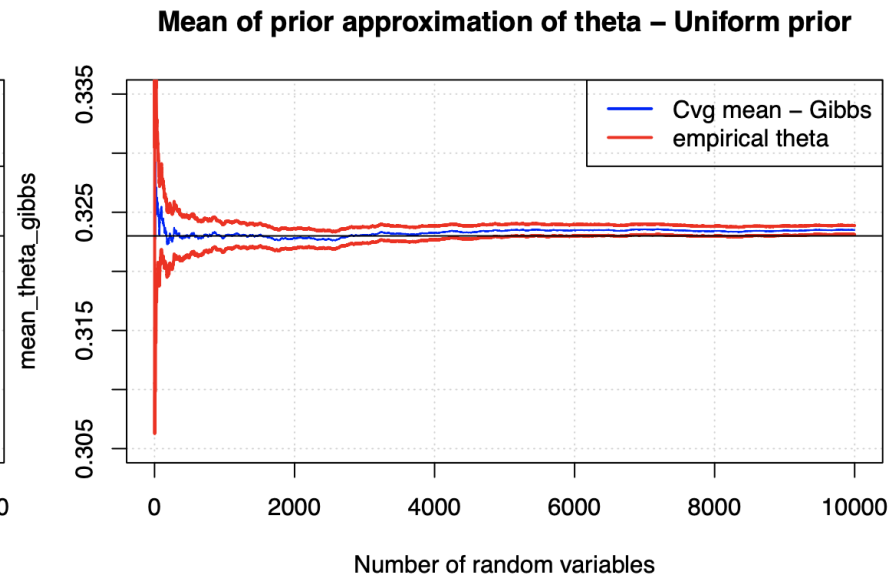
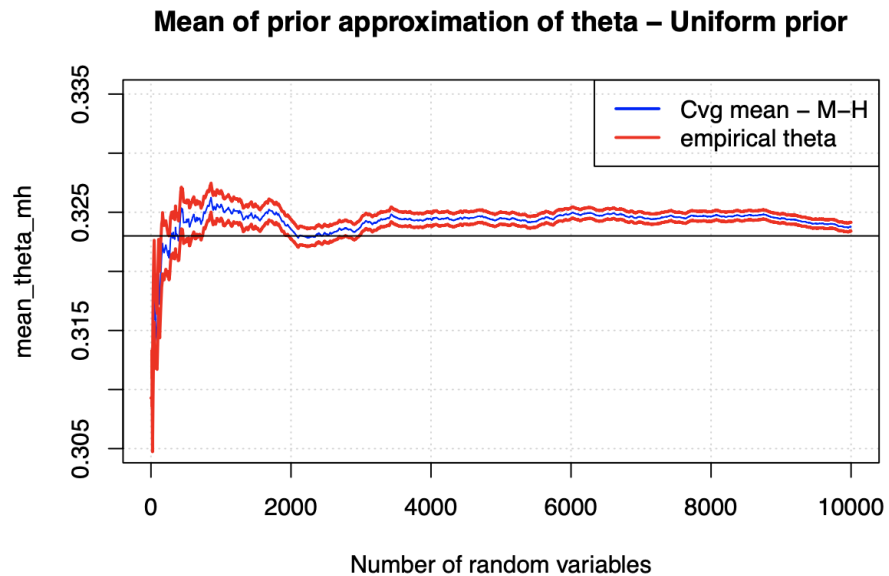
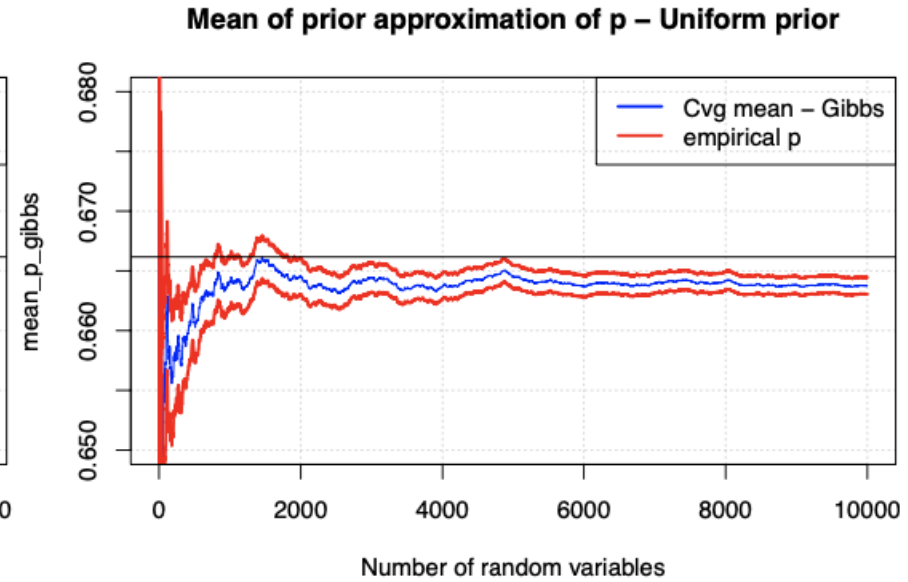
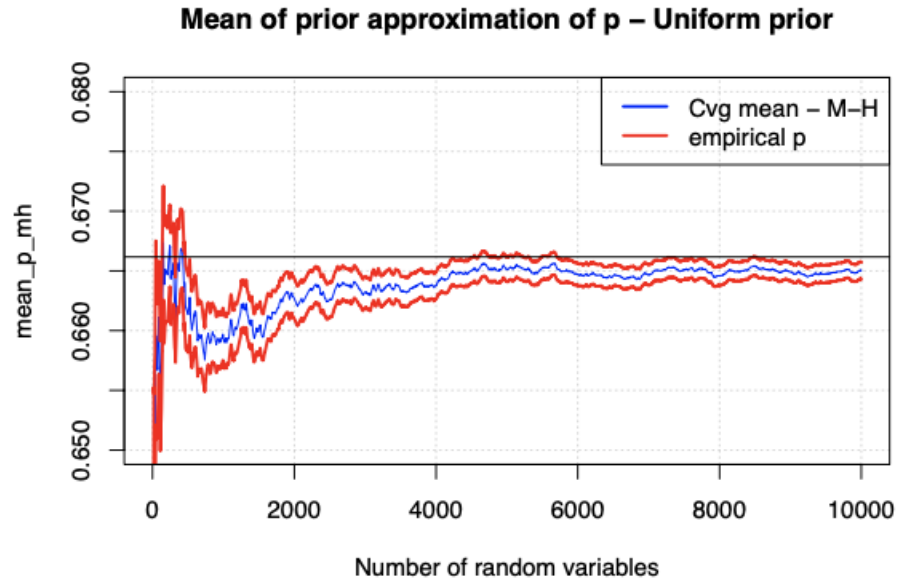
Histogram of the theta's posterior – Beta prior



Histogram of the p's posterior – Beta prior



GIBBS SAMPLER



ABC: Approximate bayesian computation

Summary

$$\begin{aligned} l(x|p, \theta) &= \ln(p_{AA}) \sum_{i=1}^n \mathbb{1}_{x_i=AA} + \ln(p_{aa}) \sum_{i=1}^n \mathbb{1}_{x_i=aa} + \ln(p_{aA}) \sum_{i=1}^n \mathbb{1}_{x_i=aA} \\ &= \ln(p_{AA})N_{AA} + \ln(p_{aa})N_{aa} + \ln(p_{aA})(n - N_{AA} - N_{aa}) \end{aligned}$$

$$\begin{cases} N_{AA} &= \sum_{i=1}^n \mathbb{1}_{x_i=AA} \\ N_{aa} &= \sum_{i=1}^n \mathbb{1}_{x_i=aa} \end{cases}$$

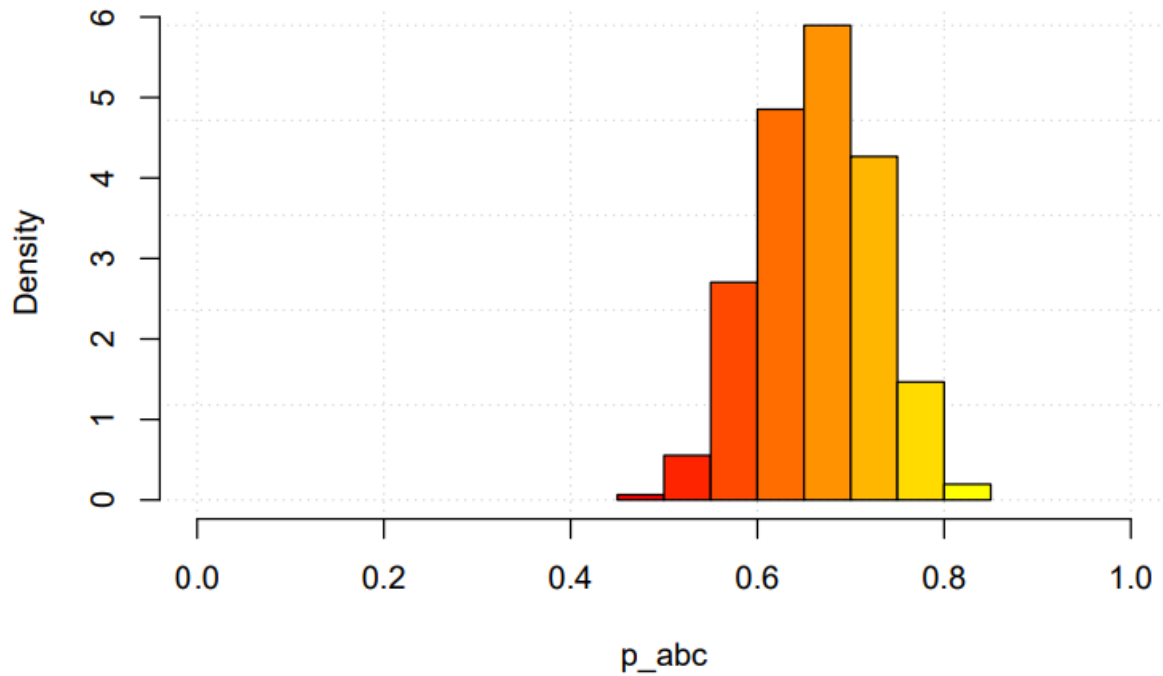
Algorithm 2: ABC i-th iteration

input: $\pi, (X_k^{obs})_{1 \leq k \leq n}, \epsilon$

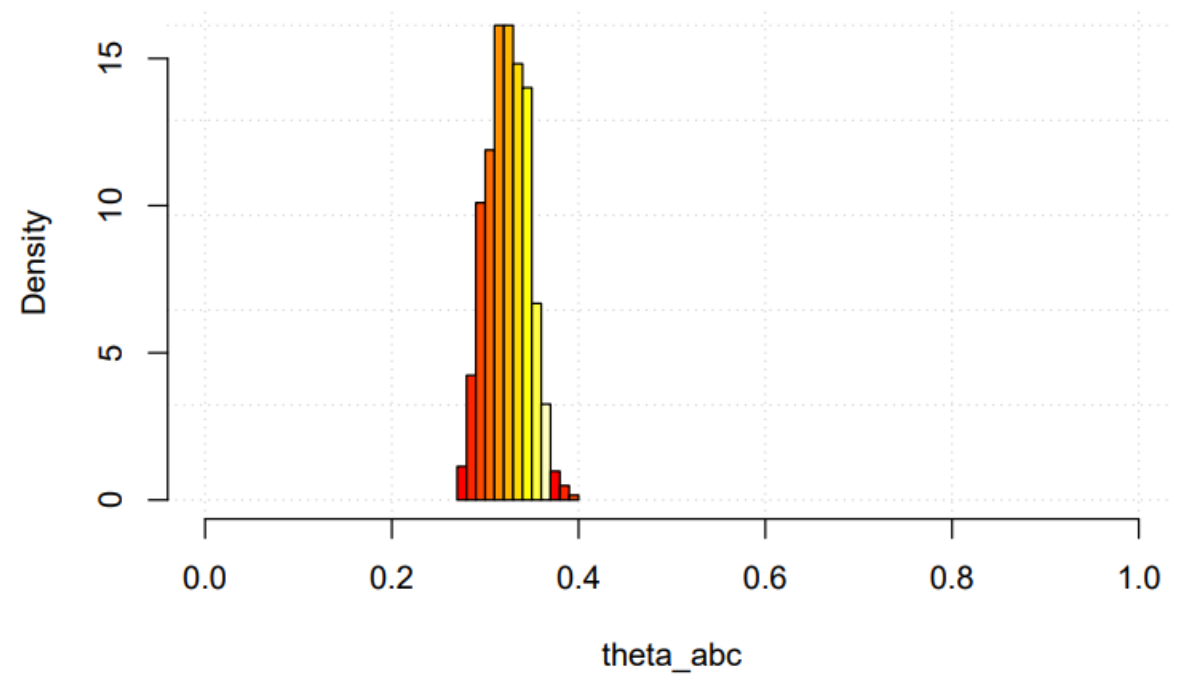
- 1 $(p_i, \theta_i) \sim \pi$
- 2 $p_{AA,i} = p_i(1 - \theta_i) + (1 - p_i)(1 - \theta_i)^2$
- 3 $p_{aa,i} = p_i\theta_i + (1 - p_i)\theta_i^2$
- 4 $p_{aA,i} = 1 - p_{AA,i} - p_{aa,i}$
- 5 $(X_{i,k}^{new})_{1 \leq k \leq n} \sim \mathcal{L}(p_{AA,i}, p_{aa,i}, p_{aA,i})$
- 6 $N_i^{new} := (N_{AA,i}^{new}, N_{aa,i}^{new})$
- 7 **if** $d(N_i^{new}, N^{obs}) \leq \epsilon$ **then**
- 8 **return** (p_i, θ_i) ;
- 9 **else**
- 10 **return** \emptyset ;

ABC: First results (eps=15)

Simulation of p with ABC

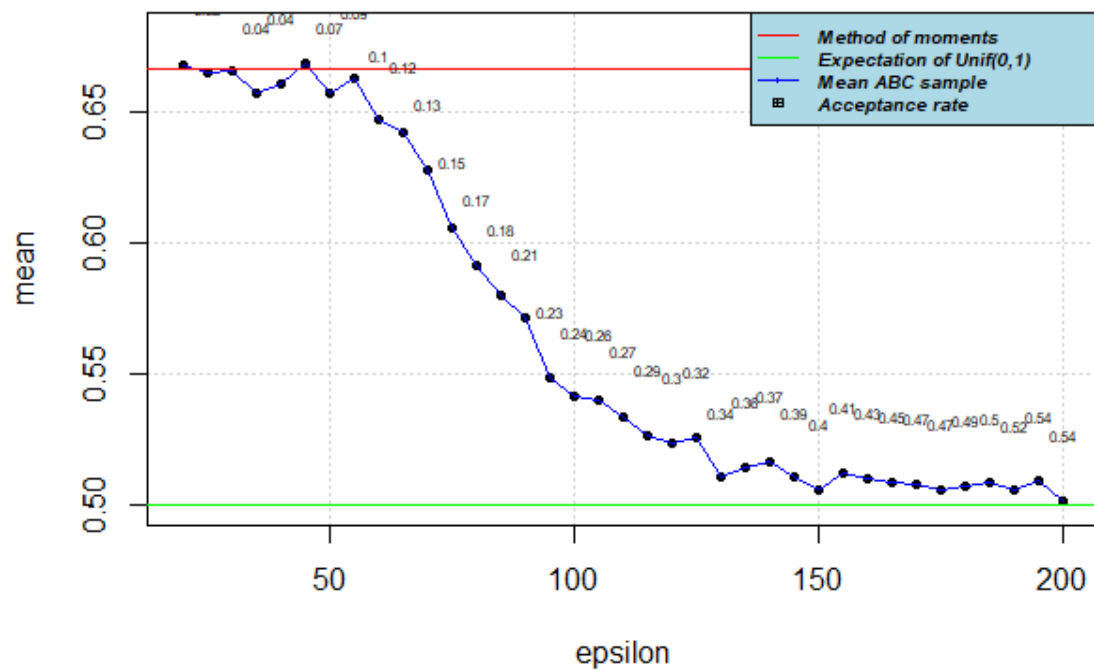


Simulation of θ with ABC

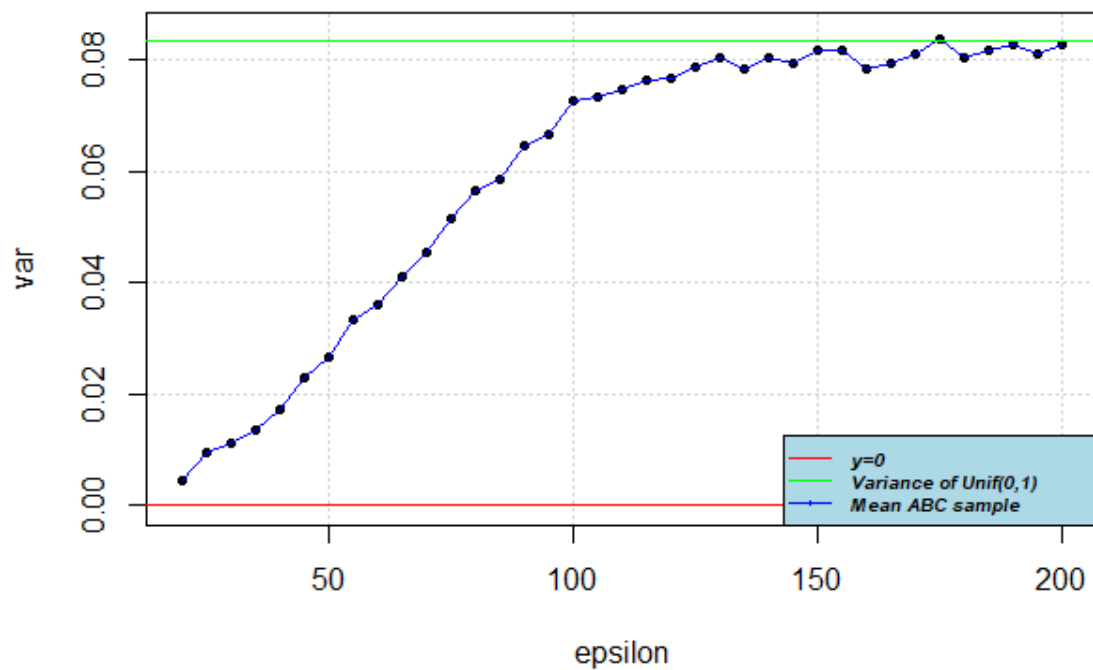


ABC: Parameter Epsilon

posterior's mean in function of epsilon (N= 10000)

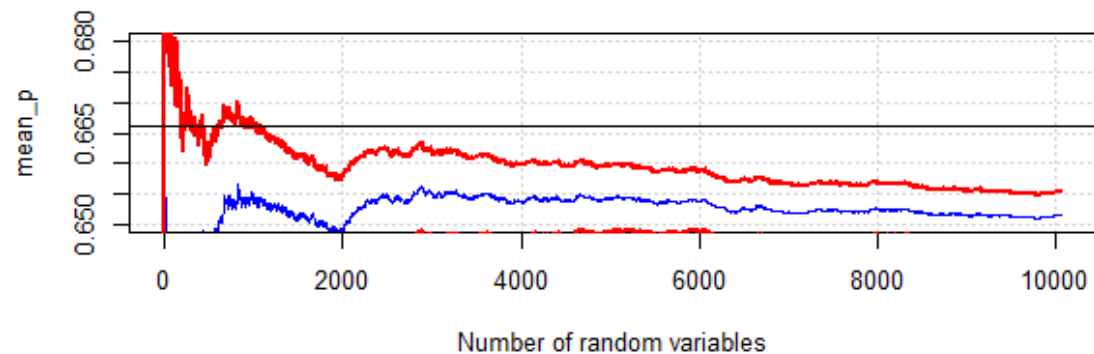


posterior's variance in function of epsilon (N= 10000)

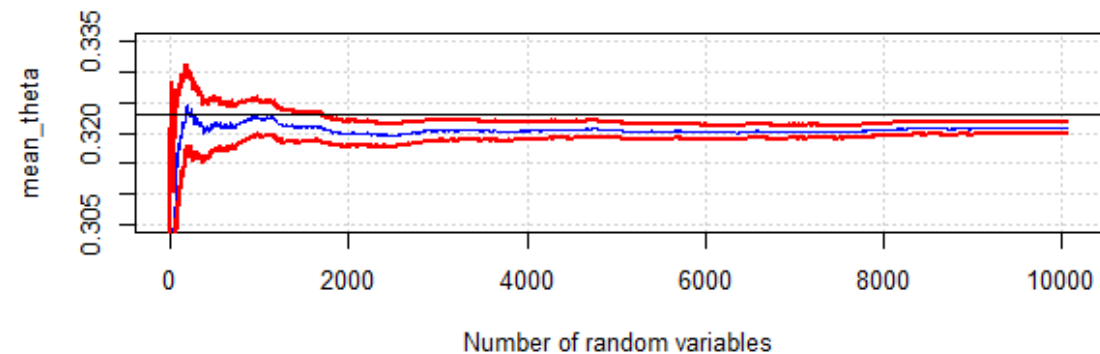


ABC: Poseterior Means

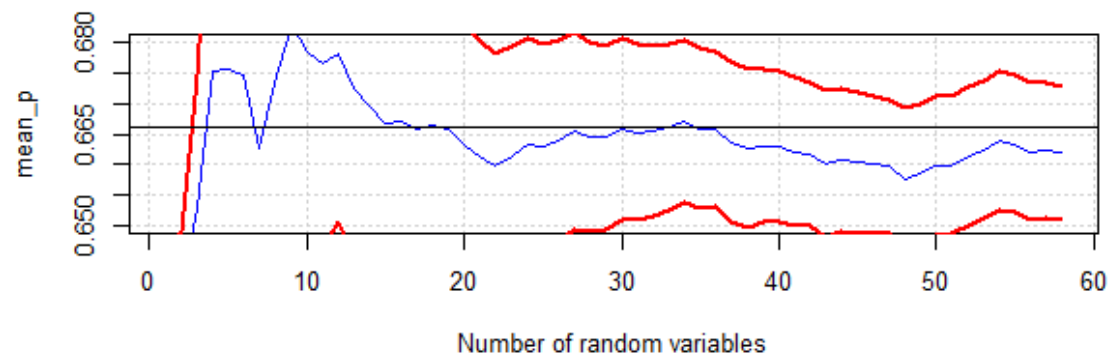
Mean of posterior approximation of p ($\text{eps}=60$, prior: Unif)



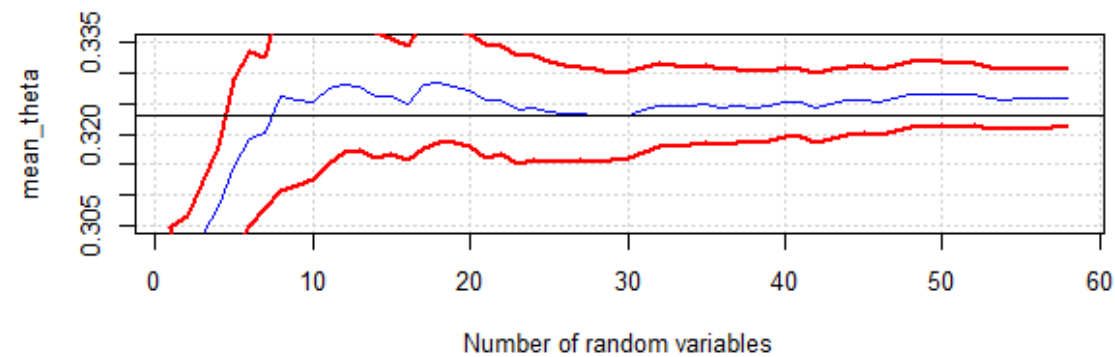
Mean of posterior approximation of θ ($\text{eps}=60$, prior: Unif)



Mean of posterior approximation of p ($\text{eps}=5$, prior: Unif)

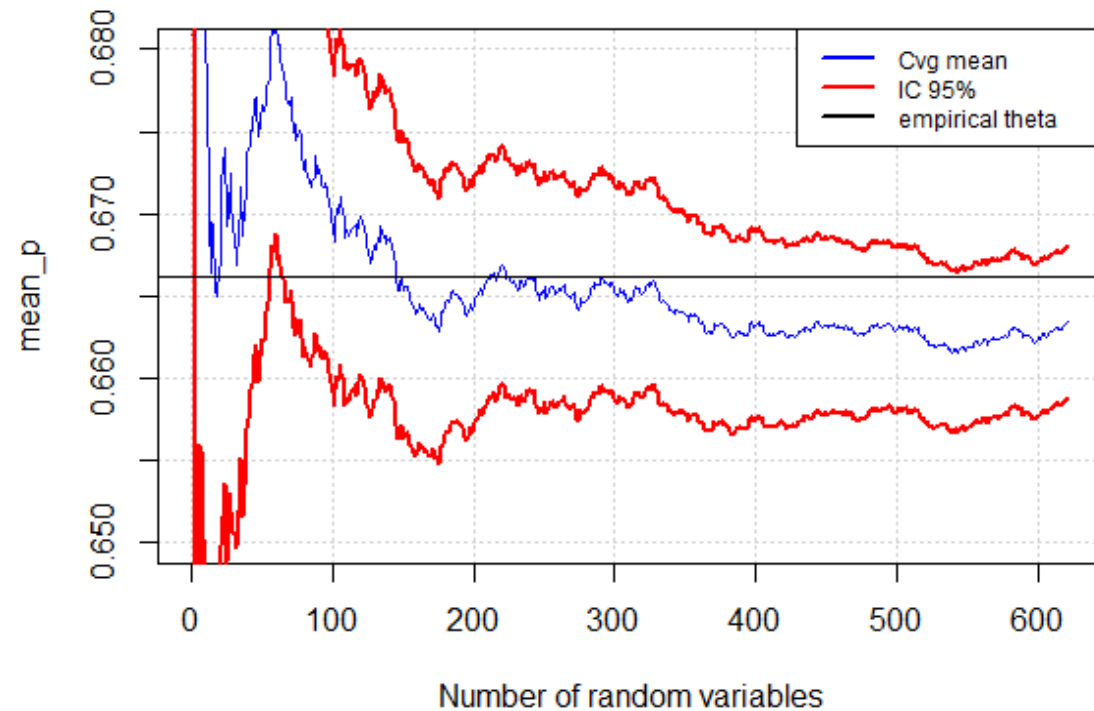


Mean of posterior approximation of θ ($\text{eps}=5$, prior: Unif)

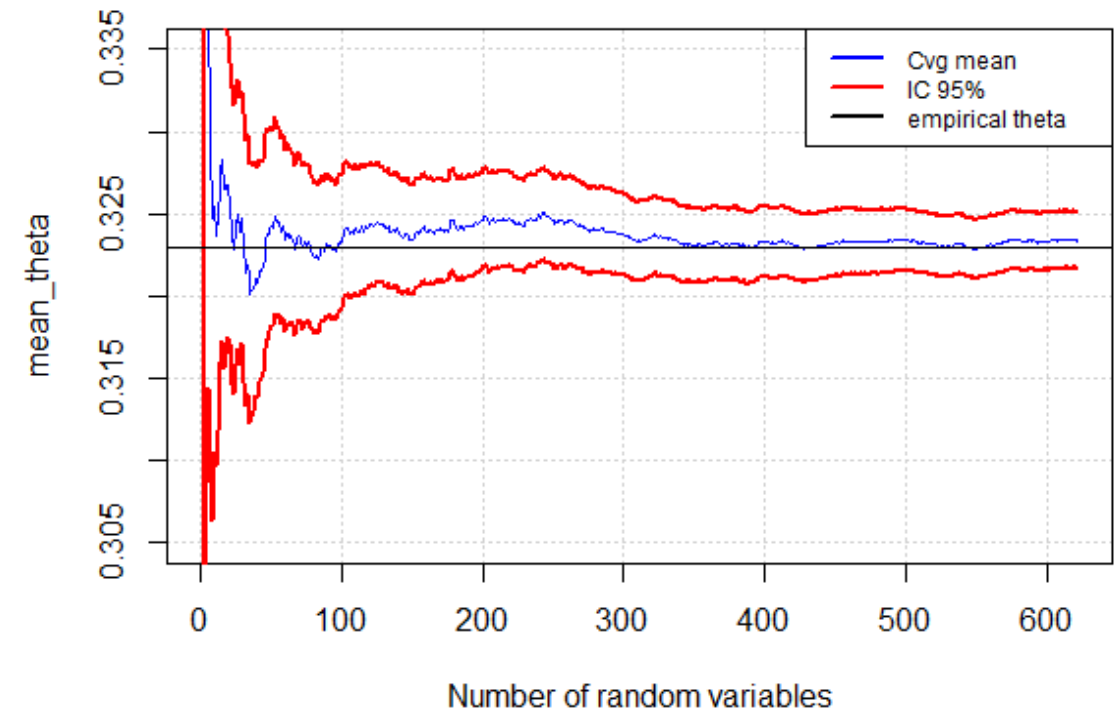


ABC: Posterior Means

Mean of posterior approximation of p ($\text{eps}=15$, prior: Unif)



Mean of posterior approximation of θ ($\text{eps}=15$, prior: Unif)



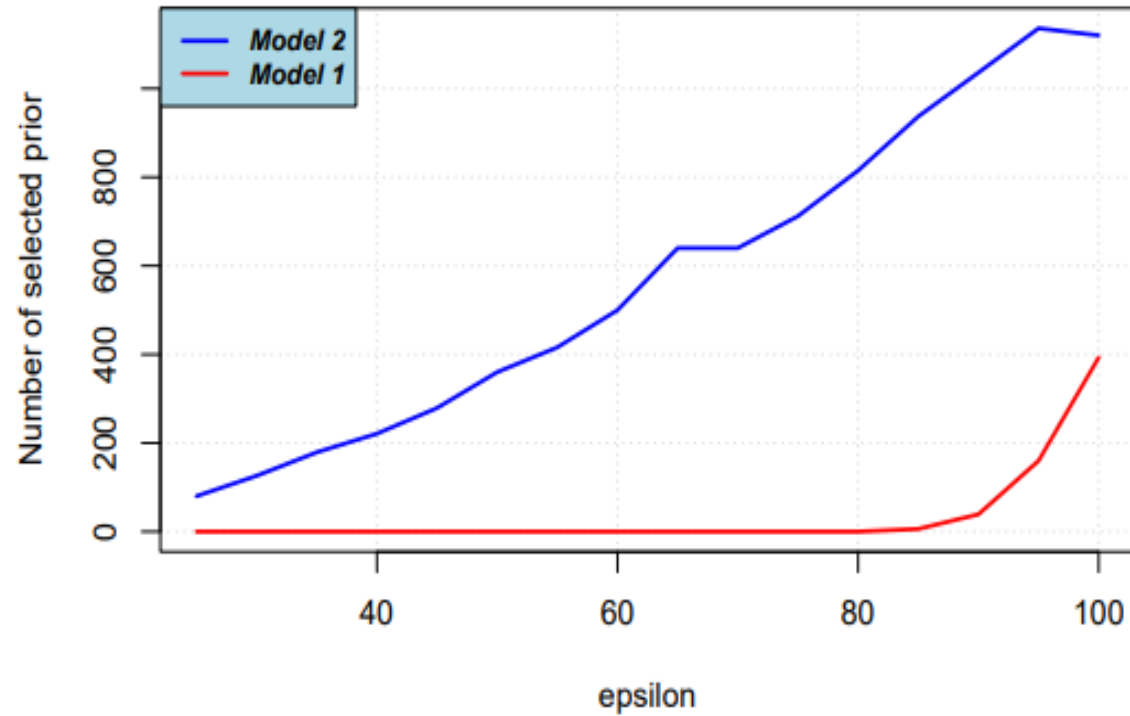
ABC: Model choice (1)

Algorithm 3: ABC model selection i-th iteration

```
input:  $(X_k^{obs})_{1 \leq k \leq n}$ ,  $\epsilon$ 
1  $M_i \sim \mathcal{U}(\{1, 2\})$ 
2 if  $M_i = 1$  then
3    $\theta_i \sim \mathcal{U}([0, 1])$ 
4    $p_{AA,i} = (1 - \theta_i)^2$ 
5    $p_{aa,i} = \theta_i^2$ 
6    $p_{aA,i} = 1 - p_{AA,i} - p_{aa,i}$ 
7 else
8    $(p_i, \theta_i) \sim \mathcal{U}^2([0, 1])$ 
9    $p_{AA,i} = p_i(1 - \theta_i) + (1 - p_i)(1 - \theta_i)^2$ 
10   $p_{aa,i} = p_i\theta_i + (1 - p_i)\theta_i^2$ 
11   $p_{aA,i} = 1 - p_{AA,i} - p_{aa,i}$ 
12  $(X_{i,k}^{new})_{1 \leq k \leq n} \sim \mathcal{L}(p_{AA,i}, p_{aa,i}, p_{aA,i})$ 
13  $N_i^{new} := (N_{AA,i}^{new}, N_{aa,i}^{new})$ 
14 if  $d(N_i^{new}, N^{obs}) \leq \epsilon$  then
15   return  $M_i$  ;
16 else
17   return  $\emptyset$  ;
```

ABC: Model choice (2)

Number of 1 and 2 variables models selected



Model 1 Variable VS Model 2 Variables

