PROJET MCMC

EM

METROPOLIS-HASTINGS

Proposal distributions

$$\theta_{new}^{(t)} \sim \mathcal{N}(\theta^{(t-1)}, \sigma_\theta)$$

$$p_{new}^{(t)} \sim \mathcal{N}(p^{(t-1)}, p_\theta)$$

Accept-Reject function

$$\rho\left(p_{new}, \theta_{new}, p^{(t-1)}, \theta^{(t-1)}\right) = \frac{\pi(p_{new}, \theta_{new} | x_{obs})}{\pi(p^{(t-1)}, \theta^{(t-1)} | x_{obs})}$$

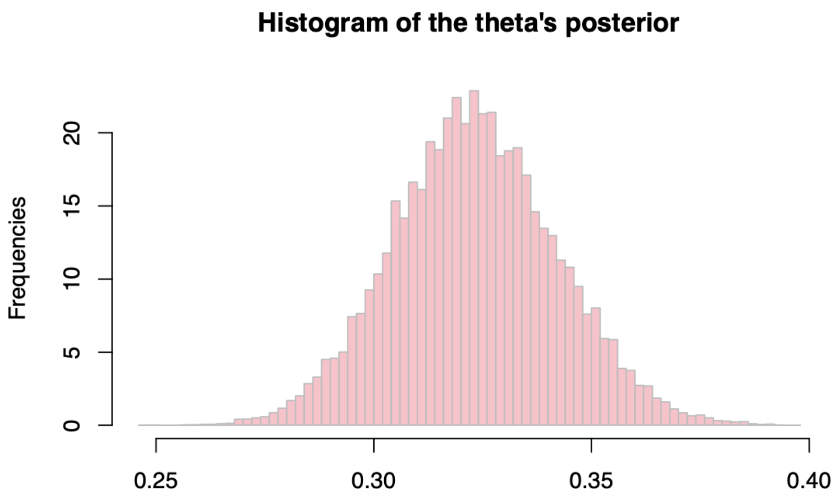
$$= \frac{\pi(p_{new}, \theta_{new})}{\pi(p^{(t-1)}, \theta^{(t-1)})} \frac{l(p_{new}, \theta_{new} | x_{obs})}{l(p^{(t-1)}, \theta^{(t-1)} | x_{obs})}$$

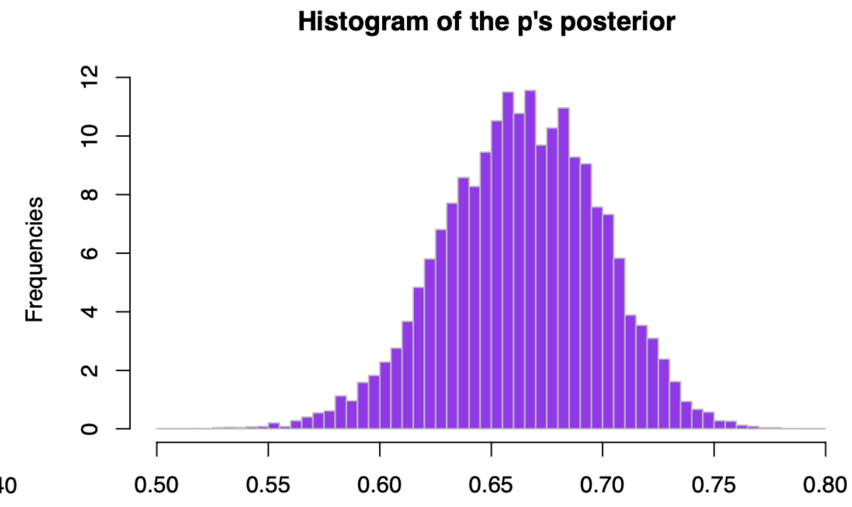
Shema ? Ou calcul des prior

METROPOLIS-HASTINGS

$$\sigma_p = \sigma_\theta = 0.2$$

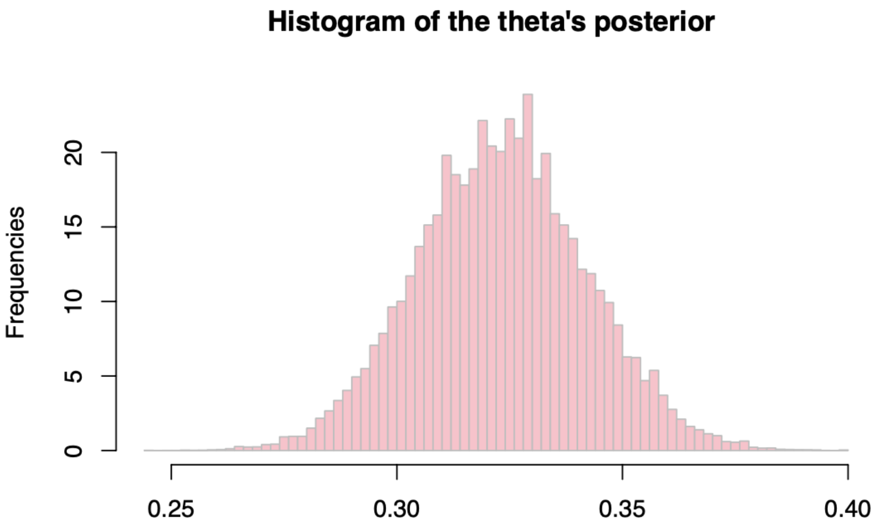
Uniform *prior*

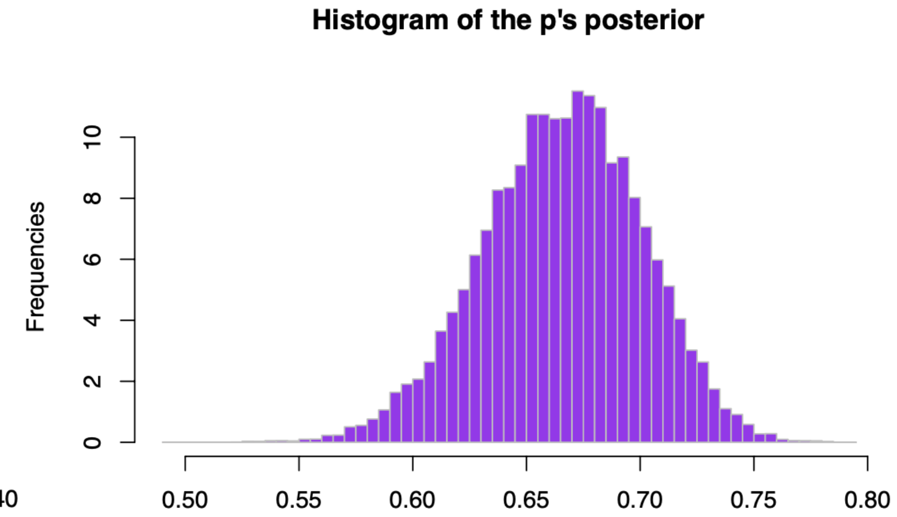




Beta *prior*

$$\alpha_p = 4, \beta_p = 2$$
$$\alpha_\theta = 2, \beta_\theta = 4$$

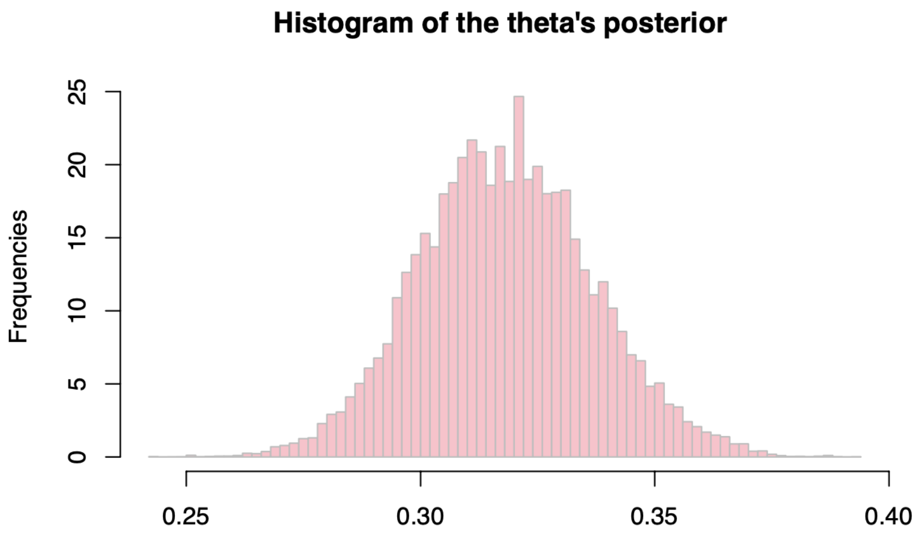


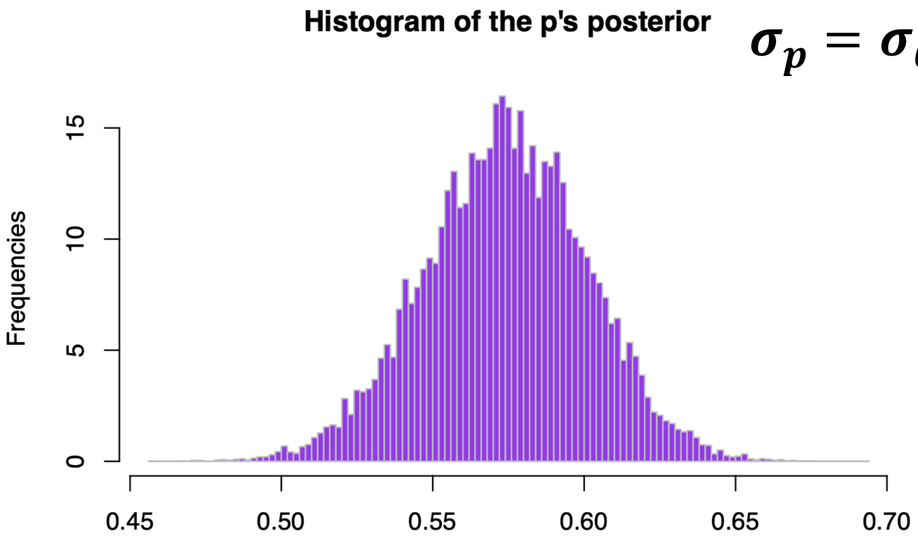


METROPOLIS-HASTINGS

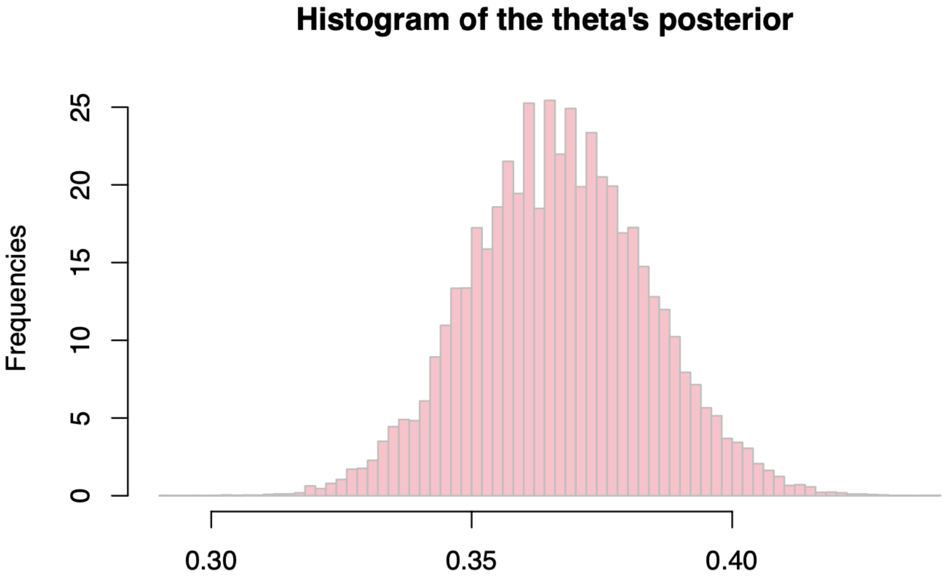
$\boldsymbol{\sigma_p = \sigma_\theta = 0.2}$

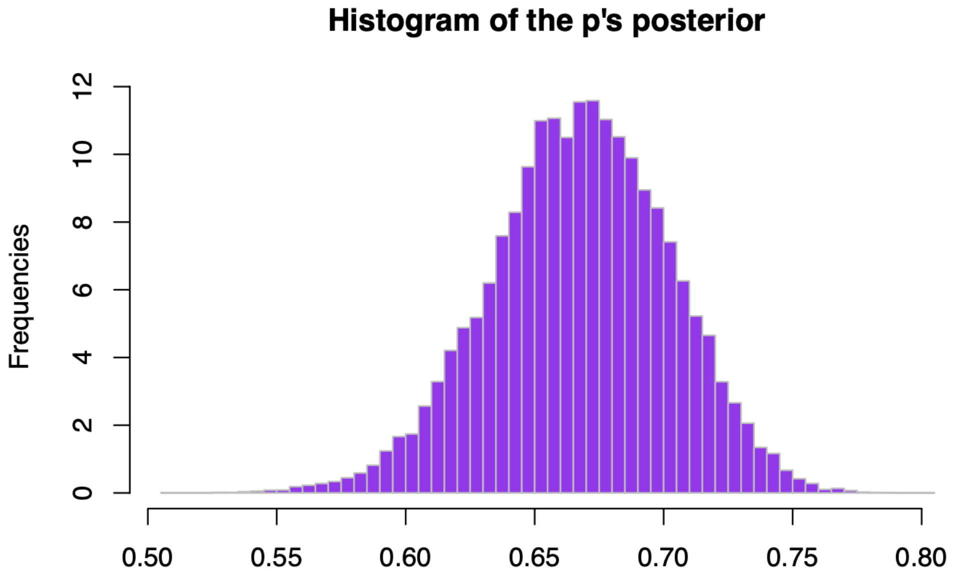
Beta *prior*
$\alpha_p = 100, \beta_p = 100$
$\alpha_\theta = 2, \beta_\theta = 3$





Beta *prior*
$\alpha_p = 2, \beta_p = 3$
$\alpha_\theta = 100, \beta_\theta = 100$

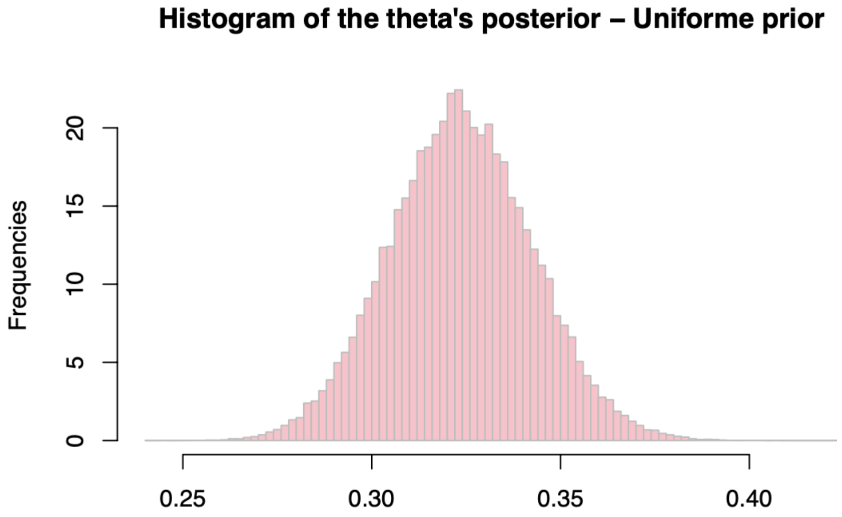


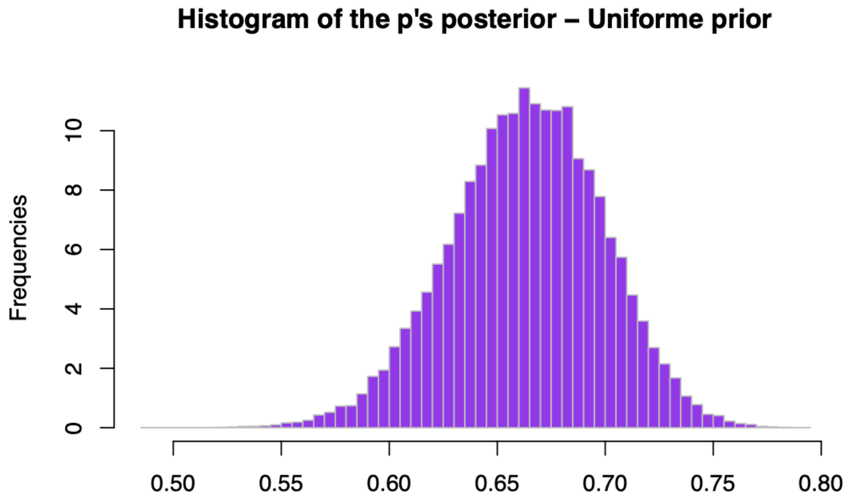


METROPOLIS-HASTINGS

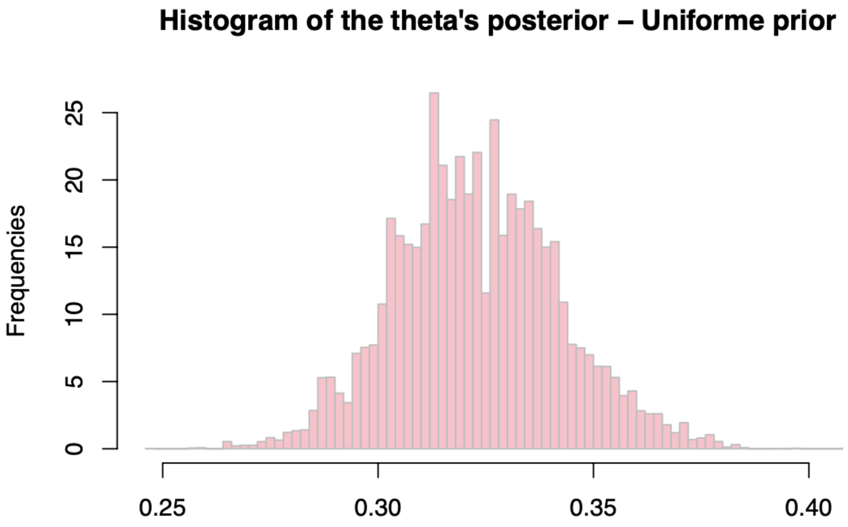
How the algorithm reacts when we change the variance of the proposal laws ?

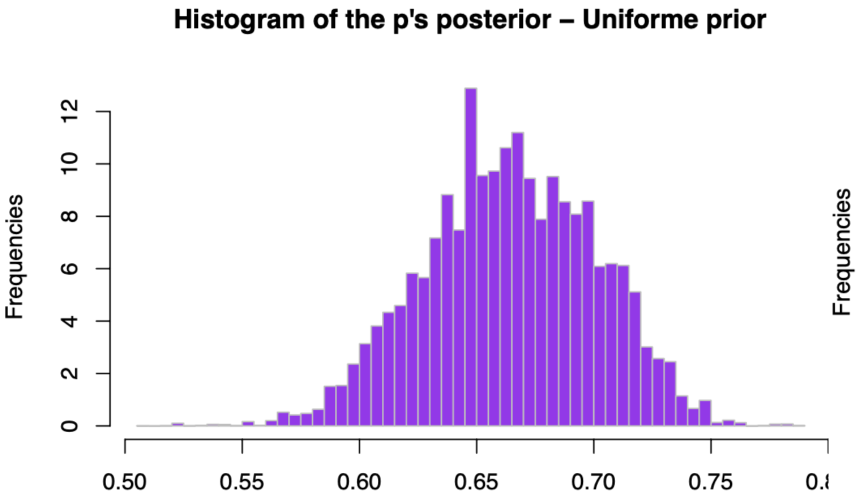
$$\sigma_p = \sigma_\theta = 0.1$$



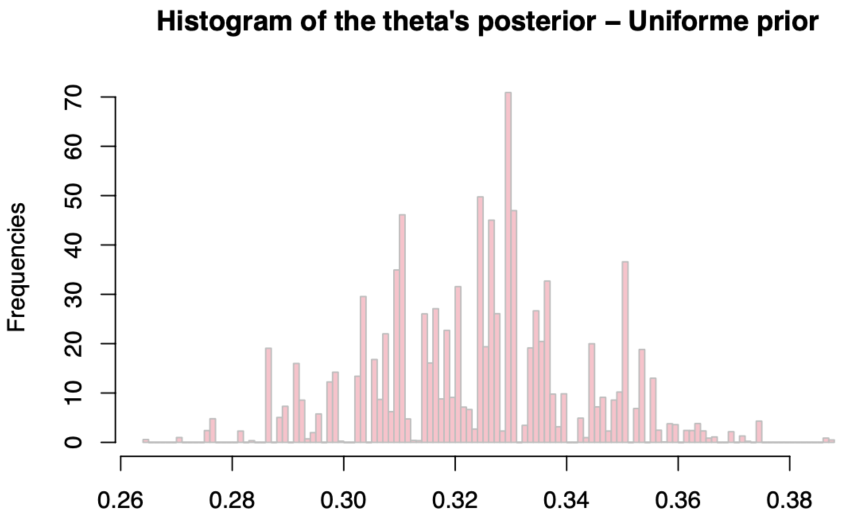


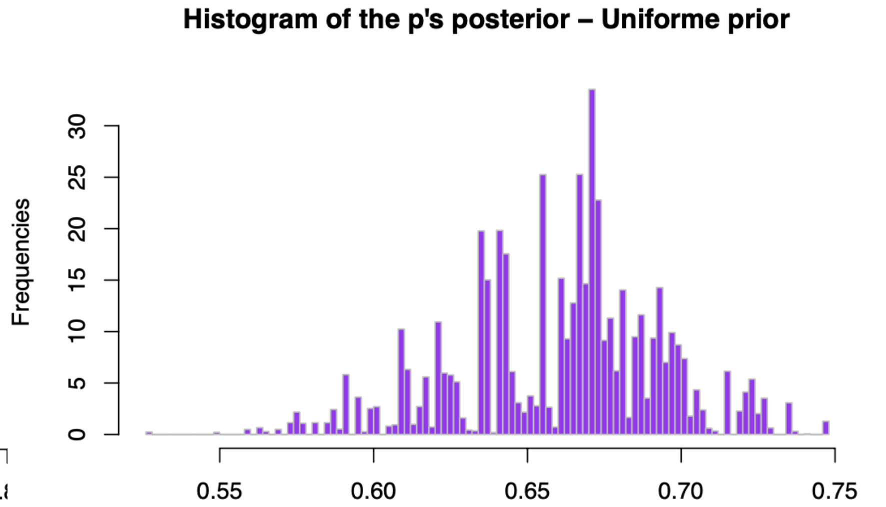
$$\sigma_p = \sigma_\theta = 0.5$$





$$\sigma_p = \sigma_\theta = 2$$





GIBBS SAMPLER

Conditional distribution of $\boldsymbol{p}, \boldsymbol{\theta} | (\boldsymbol{z}, \boldsymbol{x_{obs}})$

$$\pi(p, \theta | z, x_{obs}) = \pi(p, \theta) l(p, \theta | z, x_{obs})$$

Uniform *prior* - $\pi(p, \theta | z, x_{obs}) \propto Beta(N + 1, n_{obs} - N + 1) \times Beta\left(N_{aa}^1 + 2N_{aa}^0 + N_{aA} + 1, N_{AA}^1 + 2N_{AA}^0 + N_{aA} + 1\right)$

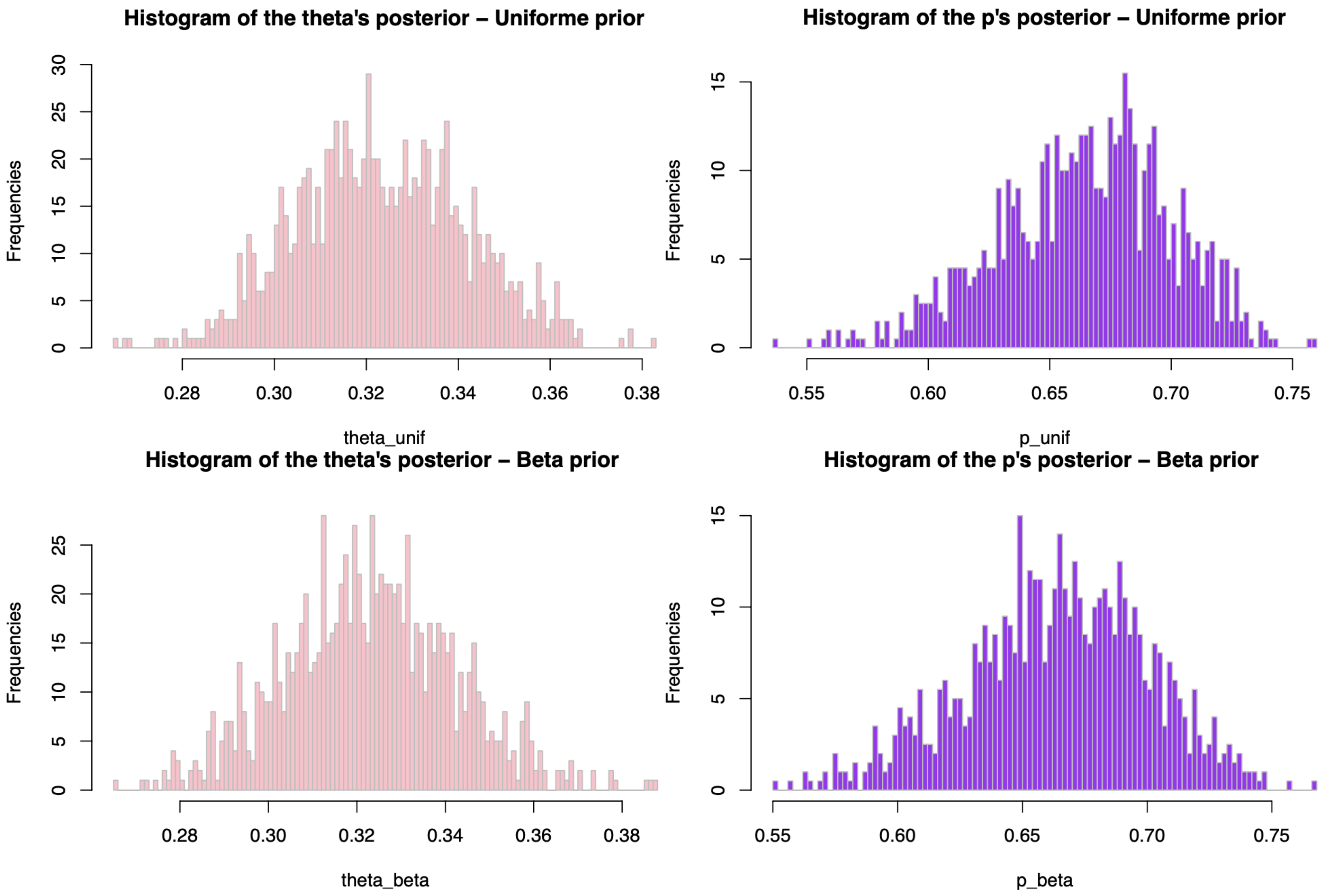
Beta *prior* - $\pi(p, \theta | z, x_{obs}) \propto Beta\left(N + \alpha_p, n_{obs} - N + \beta_p\right) \times Beta\left(N_{aa}^1 + 2N_{aa}^0 + N_{aA} + \alpha_\theta, N_{AA}^1 + 2N_{AA}^0 + N_{aA} + \beta_\theta\right)$

Conditional distribution of $\boldsymbol{z} | (\boldsymbol{p}, \boldsymbol{\theta}, \boldsymbol{x_{obs}})$

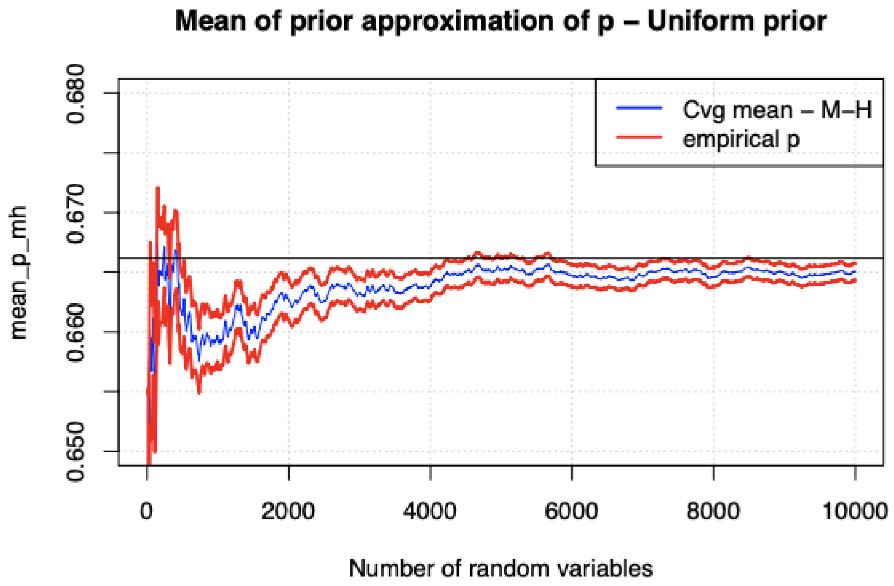
$$\begin{aligned}\mathbb{P}(Z_i = 1 | X_i = x_i, p, \theta) &= \frac{p(1-\theta)}{p(1-\theta) + (1-p)(1-\theta)^2} \quad \text{si} \quad x_i = AA \\ &= \frac{p\theta}{p\theta + (1-p)\theta^2} \quad \text{si} \quad x_i = aa \\ &= 0 \quad \text{si} \quad x_i = aA\end{aligned}$$

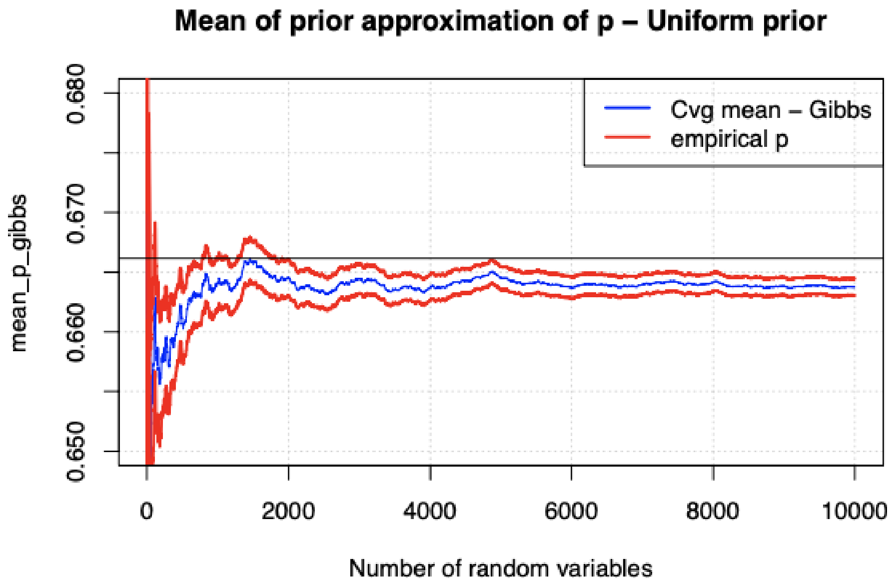
$$\begin{aligned}\mathbb{P}(Z_i = 0 | X_i = x_i, p, \theta) &= \frac{(1-p)(1-\theta)^2}{p(1-\theta) + (1-p)(1-\theta)2} \quad \text{si} \quad x_i = AA \\ &= \frac{(1-p)\theta^2}{p\theta + (1-p)\theta^2} \quad \text{si} \quad x_i = aa \\ &= 1 \quad \text{si} \quad x_i = aA\end{aligned}$$

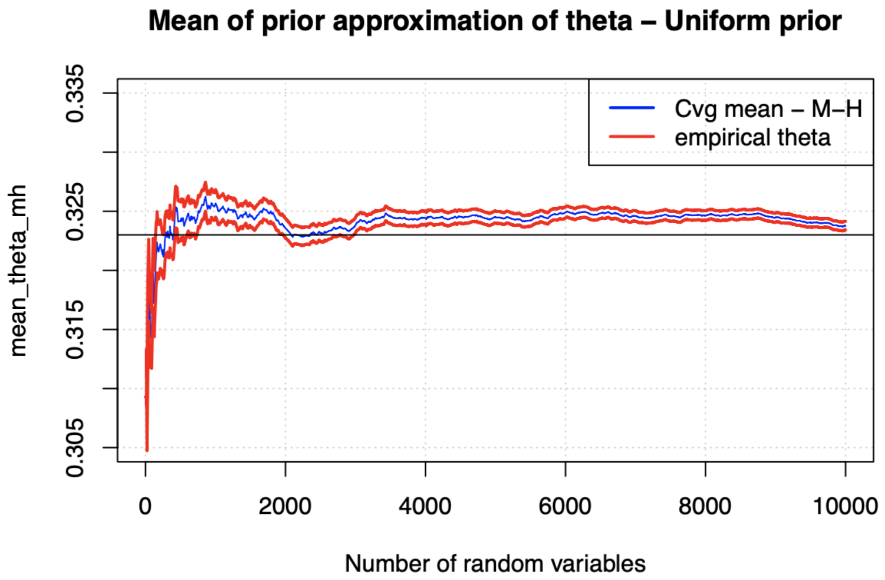
GIBBS SAMPLER

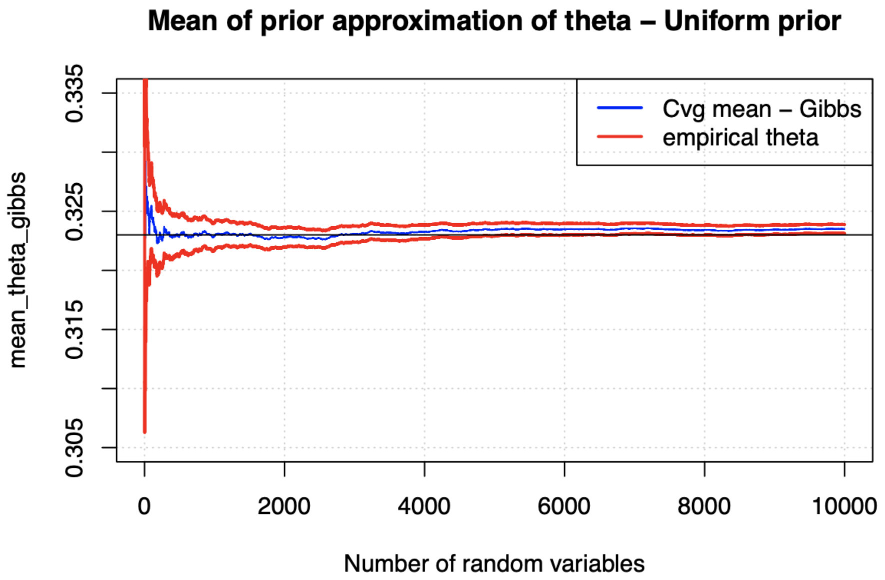


GIBBS SAMPLER









ABC: Approximate bayesian computation

Summary

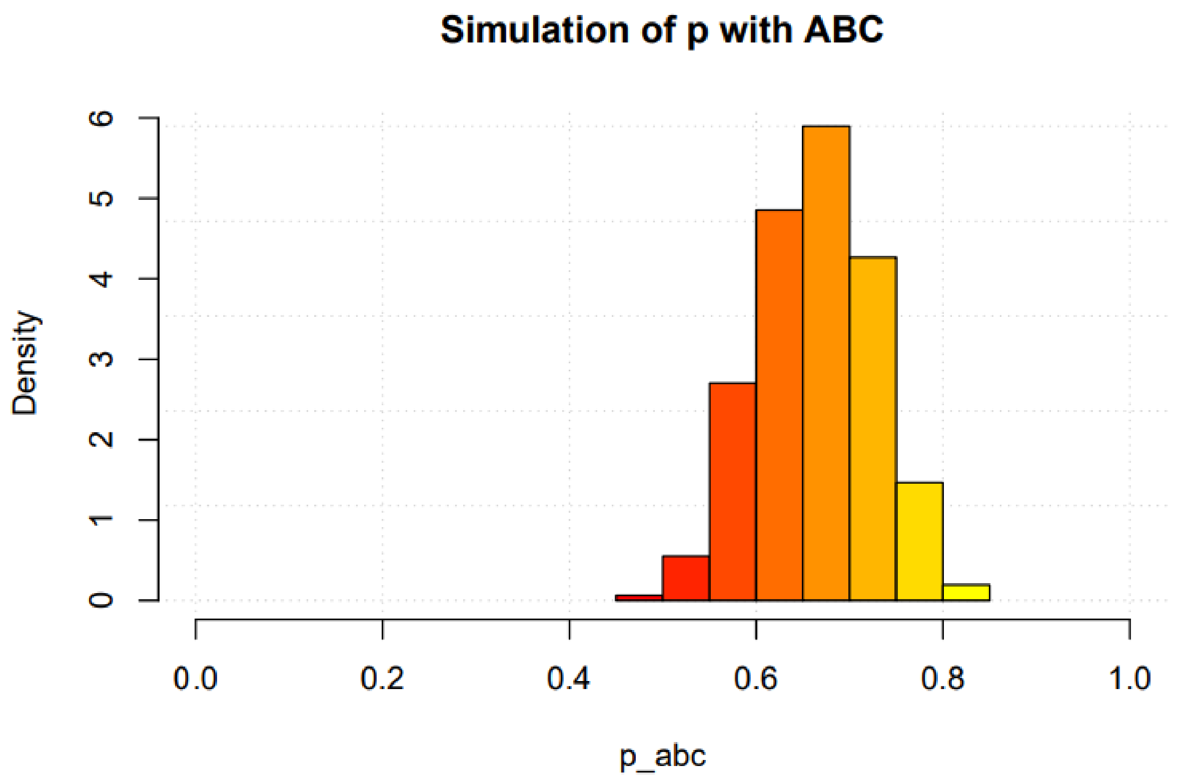
$$
\begin{aligned}
l(x|p,\theta) &= ln(p_{AA})\sum_{i=1}^{n}\mathbb{1}_{x_i=AA} + ln(p_{aa})\sum_{i=1}^{n}\mathbb{1}_{x_i=aa} + ln(p_{aA})\sum_{i=1}^{n}\mathbb{1}_{x_i=aA} \\
&= ln(p_{AA})N_{AA} + ln(p_{aa})N_{aa} + ln(p_{aA})(n - N_{AA} - N_{aa})
\end{aligned}
$$

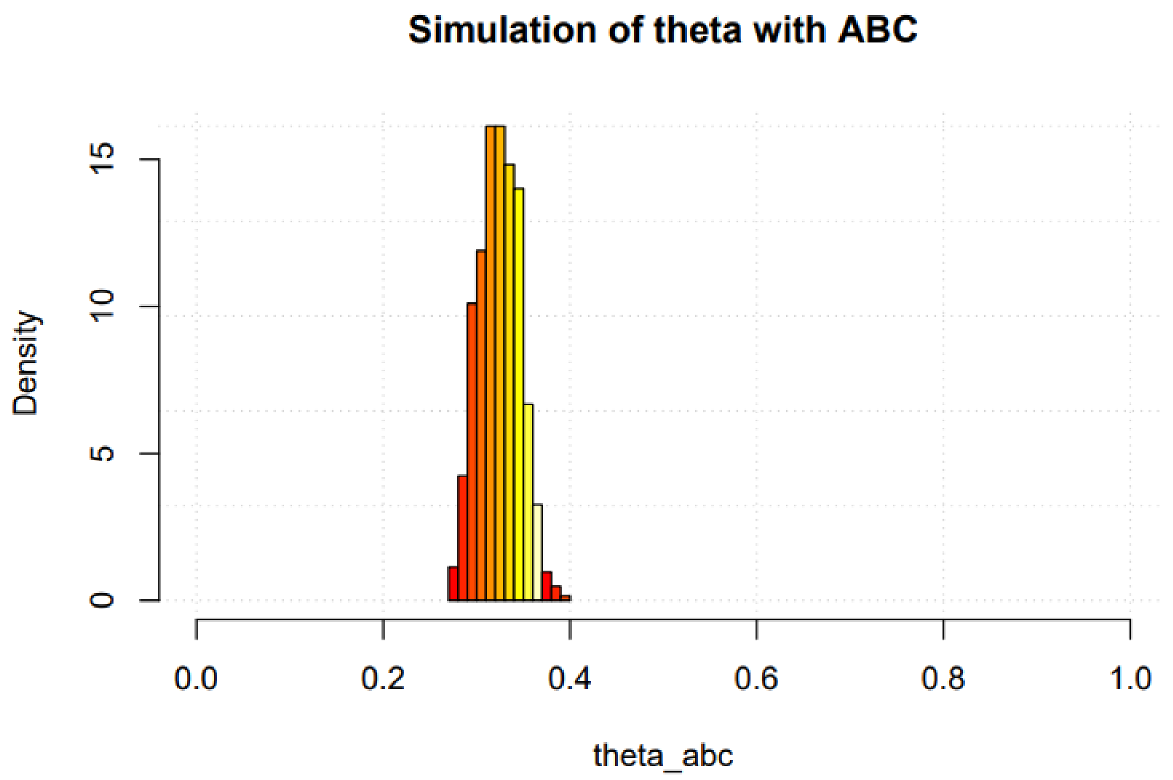
$$
\begin{cases} N_{AA} &= \sum_{i=1}^{n}\mathbb{1}_{x_i=AA} \\ N_{aa} &= \sum_{i=1}^{n}\mathbb{1}_{x_i=aa} \end{cases}
$$

Algorithm 2: ABC i-th iteration

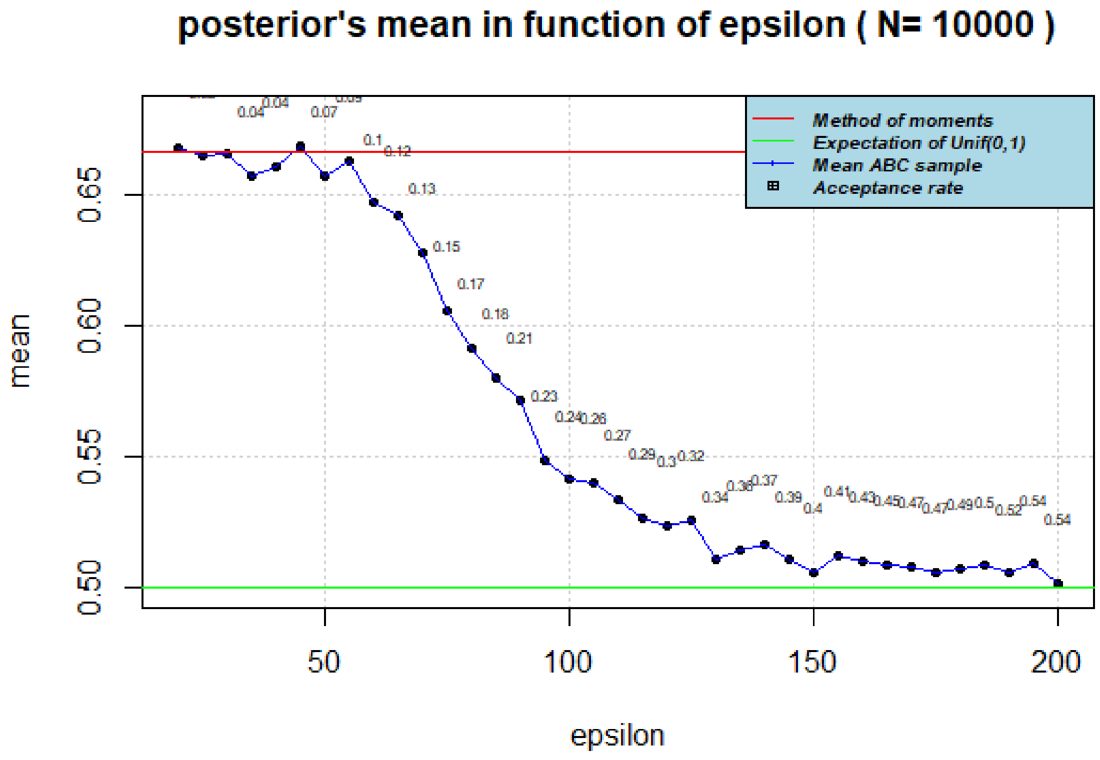
```
input: π, (X_k^obs)_{1≤k≤n}, ε
(p_i, θ_i) ~ π
p_{AA,i} = p_i(1 − θ_i) + (1 − p_i)(1 − θ_i)^2
p_{aa,i} = p_i θ_i + (1 − p_i)θ_i^2
p_{aA,i} = 1 − p_{AA,i} − p_{aa,i}
(X_{i,k}^new)_{1≤k≤n} ~ L(p_{AA,i}, p_{aa,i}, p_{aA,i})
N_i^new := (N_{AA,i}^new, N_{aa,i}^new)
if d(N_i^new, N^obs) ≤ ε then
    return (p_i, θ_i) ;
else
    return ∅ ;
```

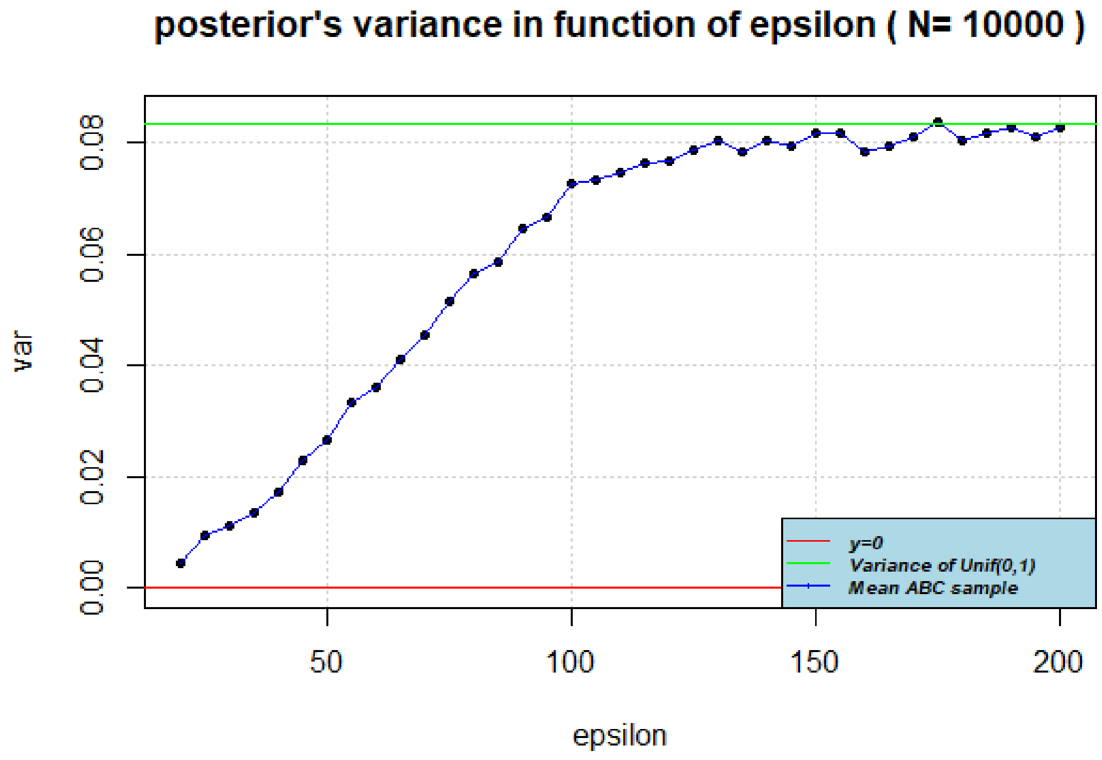
ABC: First results (eps=15)



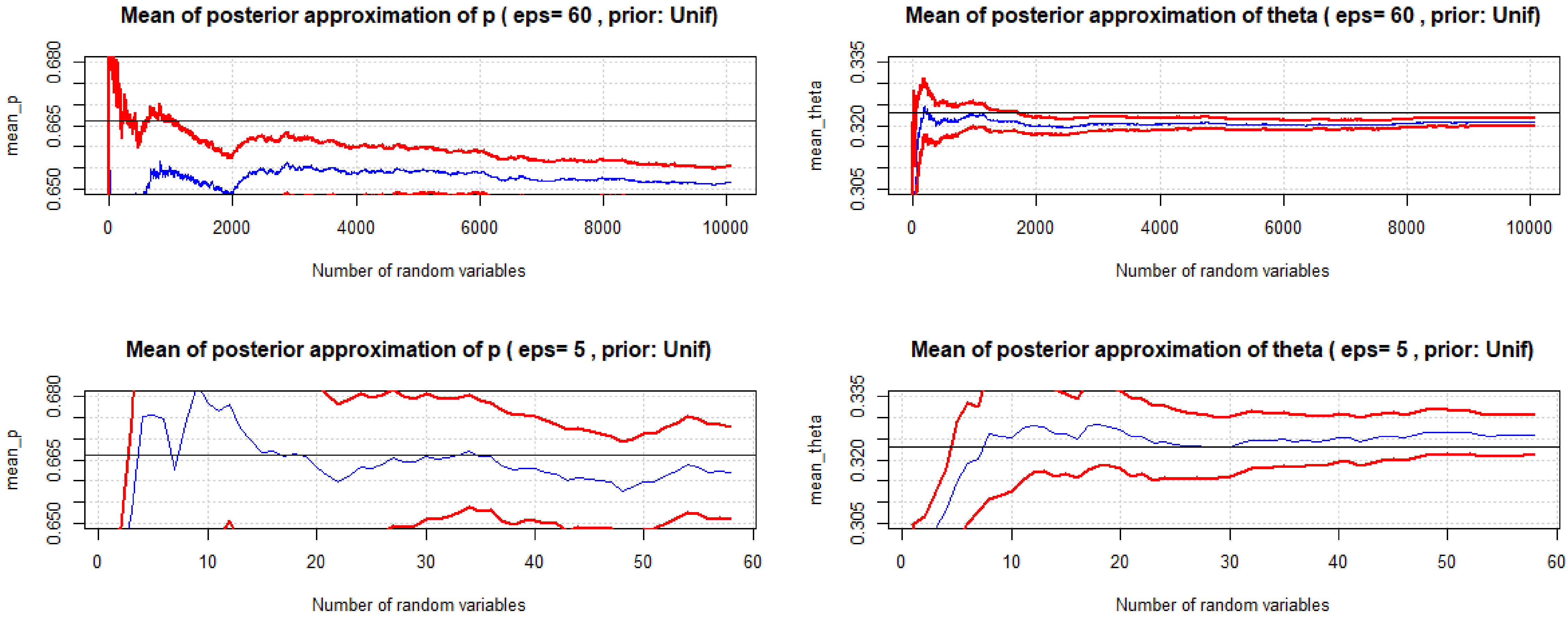


ABC: Parameter Epsilon

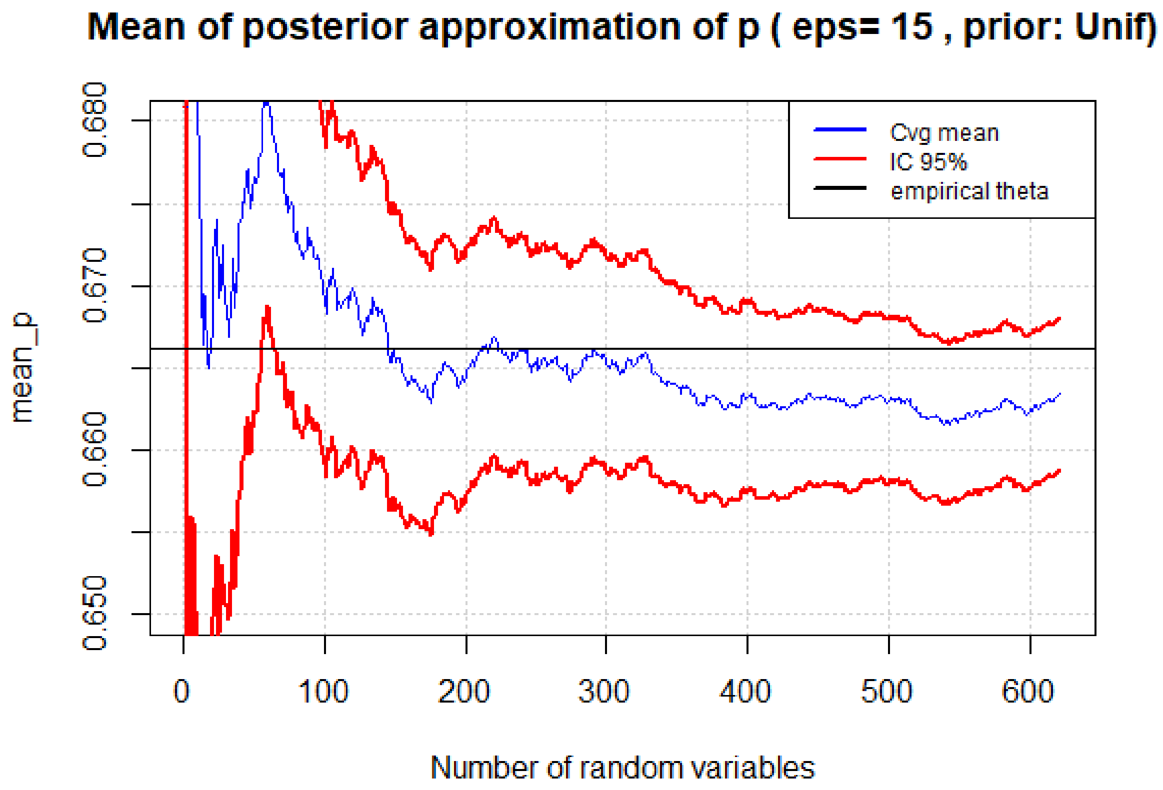


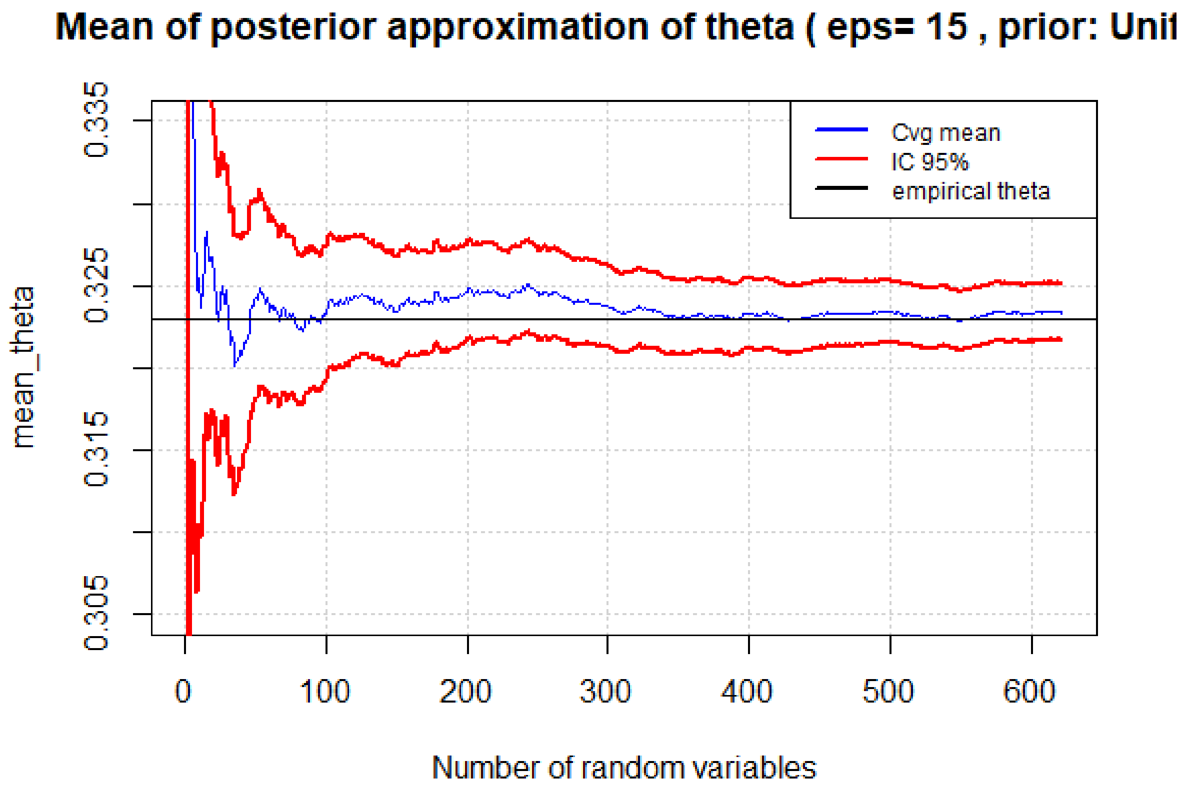


ABC: Poseterior Means



ABC: Poseterior Means





ABC: Model choice (1)

Algorithm 3: ABC model selection i-th iteration

input: $(X_k^{obs})_{1 \leq k \leq n}$, ϵ

$M_i \sim \mathcal{U}(\{1, 2\})$

if $M_i = 1$ **then**

  $\theta_i \sim \mathcal{U}([0, 1])$

  $p_{AA,i} = (1 - \theta_i)^2$

  $p_{aa,i} = \theta_i^2$

  $p_{aA,i} = 1 - p_{AA,i} - p_{aa,i}$

else

  $(p_i, \theta_i) \sim \mathcal{U}^2([0, 1])$

  $p_{AA,i} = p_i(1 - \theta_i) + (1 - p_i)(1 - \theta_i)^2$

  $p_{aa,i} = p_i\theta_i + (1 - p_i)\theta_i^2$

  $p_{aA,i} = 1 - p_{AA,i} - p_{aa,i}$

$(X_{i,k}^{new})_{1 \leq k \leq n} \sim \mathcal{L}(p_{AA,i}, p_{aa,i}, p_{aA,i})$

$N_i^{new} := (N_{AA,i}^{new}, N_{aa,i}^{new})$

if $d(N_i^{new}, N^{obs}) \leq \epsilon$ **then**

  return M_i ;

else

  return $\emptyset$;

ABC: Model choice (2)