Note Title 1/20/2011

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$$\dot{x} = Fx + Gu$$

$$y = Hx + Ju$$

Poles of this system, $\lambda_i$, are given by:

$$\det\left[\lambda_i I - F\right] = 0$$

"characteristic equation"

or equivalently, by

$$\det\left[sI - F\right] = 0$$

polynomial in s, "characteristic polynomial"

# State-Feedback

Thus, given the system:

$$\dot{x} = Fx + Gu \quad \text{—— (3.1a)}$$

$$y = Hx \quad \text{—— (3.1b)}$$

the "open-loop" poles are given the roots of:

$$a(s) = \det\{sI - F\} = 0$$

—— (3.2)

$=$

We consider the state-feedback

$$u = -k_1 x_1 - k_2 x_2 \ldots - k_n x_n$$

$$= -\begin{bmatrix} k_1 & k_2 & \ldots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= -k\,x \longrightarrow (3.5)$$

This changes the system (3.1) to

$$\overset{\circ}{x} =$$

$$=$$

$$= \longrightarrow (3.11a)$$

$$y = \longrightarrow (3.11b)$$

and the "closed-loop" poles are obtained [changed by the state-feedback (3.5)] are given by:

$$\alpha_c(s) = \det\left[\quad\quad\quad\quad\right] = 0 \quad (3.12)$$

We will specify that we want our desired closed-loop poles to be given by:

$$\alpha_c(s) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_n \quad (3.13)$$

Then, the formulaic solution (through all the "intermediate steps" of the Method 1 realization) is given by

Ackermann's formula:

$$K = [\underbrace{0\ 0\ \ldots\ 0}_{(n-1)\ \text{entries}} \vdots\ 1]\ \mathcal{C}^{-1}\,\alpha_c(F) \quad \text{for } K \text{ in } (3.2)$$

denoted also as $\mathcal{C}[F, G]$

where

$$\mathcal{C} \triangleq [G \vdots FG \vdots F^2 G \vdots \ldots \vdots F^{n-1} G]$$

$$\alpha_c(F) = F^n + a_1 F^{n-1} + \ldots + a_n I$$

Here, clearly $K$ cannot be calculated if we cannot compute $\mathcal{G}^{-1}$ !!!

if $\mathcal{G}$ is a singular matrix

equivalently, $\det\{\mathcal{G}\} = 0$

Thus, for the Linear Time-Invariant System:

$$\dot{x} = Fx + Gu$$

$$y = Hx \qquad \text{—— (3.1)}$$

$$u = -Kx \qquad \text{—— (3.5)}$$

the following 3 statements
are equivalent:

- The system (3.1) can have its "closed-loop" poles placed anywhere by the state-feedback (3.5).
- $\mathcal{C}[F, G]$ is non-singular.
- The system (3.1) is "controllable".