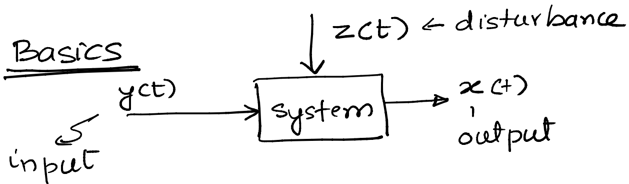
7Sep23

Thursday, September 7, 2023 5:55 PM

Todays class.

1. Control systems basics
2. Cascaded control of DC motors
   1. Setting current controllers
      Magnitude Optimum
   2. Setting speed controller
      Symmetrical optimum
3. State variable control principles

Basics



Control variable $u(t)$

$x^*(t)$ – desired value . . .

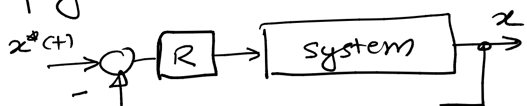
knowing/assuming system behaviour

$y(t) \longrightarrow x(t)$

$\therefore \quad x^*(t) - x(t) = e.$

This method of control is called as 1. Feedforward or open loop control. 3. Steering

If you want $e \to 0 \quad \therefore x$



Job of R ← regulator $e \to 0$

or $|e| < \epsilon$

is called 1. Feedback control
2. Closed loop control
3. Regulators

Dynamics of system using ODEs

$\frac{dx}{dt} = \dot{x}, \quad \frac{d^\nu x}{dt^\nu} = x^\nu$

General description

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 \dot{x} + a_0$

$= b_m y^m + b_{m-1} y^{m-1} + \ldots + b_1 \dot{y} + b_0 y$

$m \le n$

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 \dot{x} + a_0 x = 0$

is homogeneous system

Solution of homo. systems

$\boxed{n\text{-indepent energy storages}}$

$x_h = C \cdot e^{st}$ — soln

one energy storge

$a_n \cdot s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 = 0$

polynomial

n . . . number of roots

s

$a_n \prod_{j=1}^{n} (s - s_j) = 0$

j^{th} root

Nature of roots

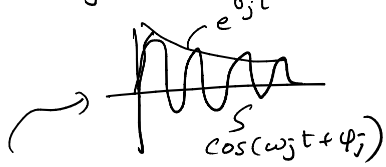
1. $s_n \in \mathbb{R}$ real roots
2. $s_K \in \mathbb{C}$ $\quad n-K$ are real.

$\frac{K}{2} \quad \frac{K}{2}$

Real roots

$x_h = \sum_{j=1}^{n} C_j e^{s_j t}$

$\sigma_j < 0$



Complex $\frac{K}{2}$ → complex

$x_h = \sum_{j=1}^{K/2} C_j e^{\sigma_j t} (\cos(\omega_j t + \varphi_j))$

$n\cancel{e}$. $s_j t$

$$x_h = \sum_{j=1} c_j e^{\sigma_j t}(\cos(\omega_j t+\varphi_j)) + \sum_{j=n-k}^{n} c_j e^{s_j t}$$

multiples $s_1 = s_2 = s_m \ldots$ restare real.

$$x_h = e^{\sigma_j t}\left(\sum_{j=1}^{m} c_j t^{j-1}\right) + \sum_{j=m+1}^{n} c_j e^{s_j t}$$

Over all response

$$x(t) = x_h(t) + x_p(t)$$

homogeneous eqn. soln

Particular soln

depends on forcing function

y ...

Laplace Transform $\frac{dx}{dt} \to s\,x(s)$

Transfer Fuction

output

$$\frac{x(s)}{y(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}$$

input $\qquad m \le n \qquad m < n$

System is stationary → all rate of changes = 0.

steady state

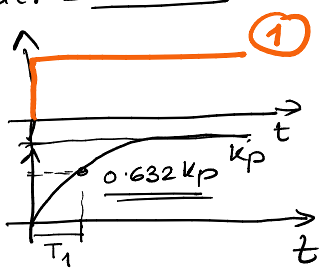
$$x_0 = \frac{b_0}{a_0}\cdot y_0 \rightsquigarrow K_{ps} = \frac{b_0}{a_0}$$

Steady state gain.

$$G_1(s) = \frac{K_p}{1+sT_1} \rightarrow \text{order} = 1$$

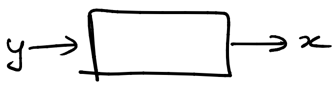
Step response

$$y = K_p(1-e^{-t/\tau})$$



System

For integrator



$$x = \int y \cdot dt$$

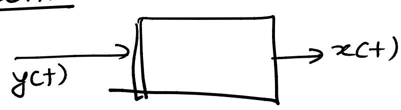
$$x(s) = \frac{y(s)}{s}$$

$$\therefore \frac{x(s)}{y(s)} = \frac{1}{s}$$

num = [Ti kp, kp]

den = [Ti, 0]

PI TF ~~Controll~~



$$x(t) = K_p\left[y(t) + \frac{1}{T_i}\int y(t)\,dt\right]$$

$$x(s) = K_p\left[y(s) + \frac{y(s)}{sT_i}\right]$$

$$\frac{x(s)}{y(s)} = K_p + \frac{K_p}{sT_i} = \frac{K_p[sT_i+1]}{sT_i} = \frac{sT_iK_p + K_p}{sT_i}$$

2 transfer function assuming $m_L = 0$.

$$\frac{i_a(s)}{v_a(s)} = \frac{sT_j}{s^2 T_m \cdot T_a + sT_m + 1}$$

→ roots of denominator are poles of the system

denom ...

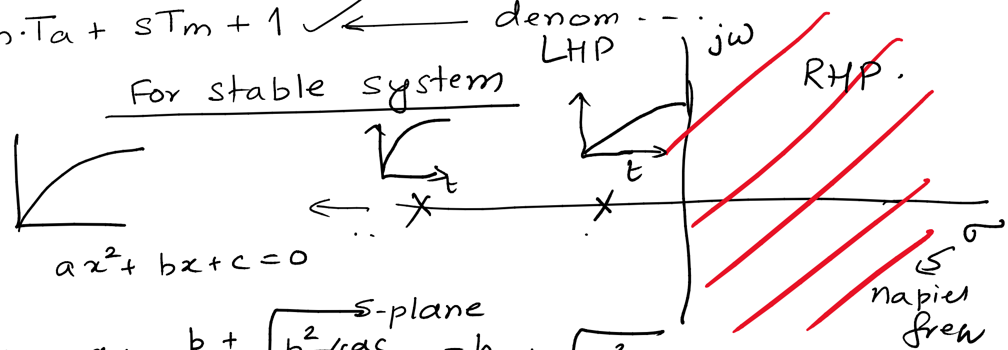
LHP ... jw

$$\frac{i_a(s)}{V_a(s)} = \frac{sT_j}{s^2 T_m \cdot T_a + sT_m + 1}$$

$\leftarrow$ denom ... are poles of the system

$T_m = r_a \cdot T_j$

For stable system



$ax^2 + bx + c = 0$

$s^2 + 2D\omega_n s + \omega_n^2 = 0$

$$x_{1,\ldots} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

$$s^2 T_m \cdot T_a + sT_m + 1 = 0 \rightarrow S_{1,2} = -\frac{1}{2T_a} \pm \sqrt{\frac{1}{4T_a^2} - \frac{1}{T_m T_a}}$$

$$s^2 + s \cdot \frac{1}{T_a} + \frac{1}{T_m \cdot T_a} = 0$$

For roots real negative

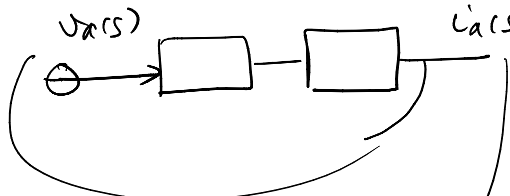
$T_m > 4T_a$

$$\frac{1}{4T_a^2} > \frac{1}{T_m T_a}.$$

$$\therefore \omega_n = \frac{1}{\sqrt{T_m \cdot T_a}} \qquad 2D\omega_n = \frac{1}{T_a}$$

$$D = \frac{1}{2} \cdot \sqrt{\frac{T_m}{T_a}}. \qquad 2 \cdot D \cdot \frac{1}{\sqrt{T_m \cdot T_a}}$$

$$= \frac{sT_j}{s^2 T_m T_a + sT_m + 1} = \frac{T_L T_s \cdot sT_j}{(s + \frac{1}{T_L})(s + \frac{1}{T_s})}$$



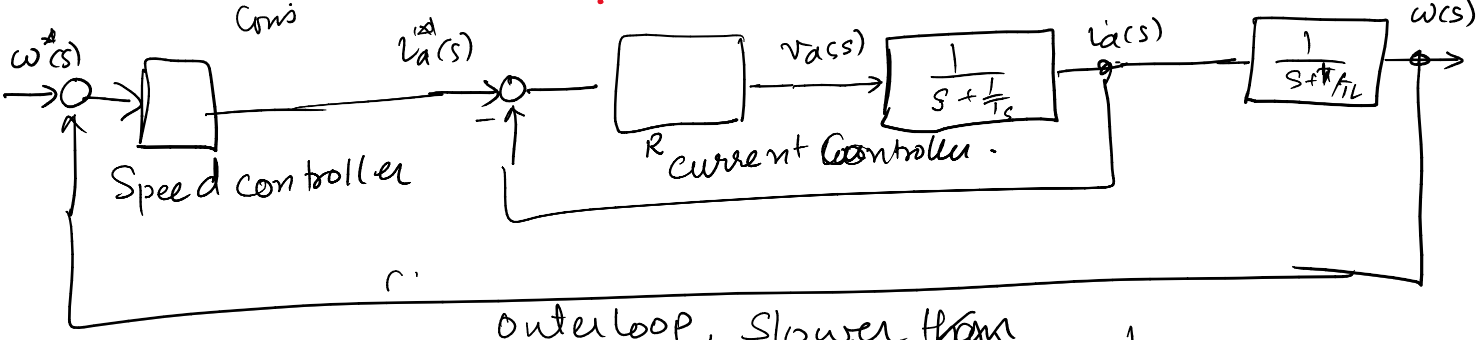
$(s + \frac{1}{T_L})$

$T_m > 4T_a$ → armature time const

mechanical time Cons

Cascaded Control.

Faster. inner current Loop



$K_p T_i s + K_p$

$$\frac{1}{(T_1 s + 1)} \qquad \frac{1}{(T_1 s + 1)} \cdot \frac{1}{(T_2 s + 1)}$$