Method 2 :

2

$$\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

$$\dddot{y} + a_1 \ddot{y} + a_2 \dot{y} + a_3 y = b_1 \ddot{u} + b_2 \dot{u} + b_3 u$$

Consider another approach where we first collect ~~together~~ all terms with similar number of derivatives.

leading to :

$$y(t) = \int \left(-a_1 y + b_1 u\right) + \iint \left(-a_2 y + b_2 u\right) + \iiint \left(-a_3 y + b_3 u\right)$$

$$\text{——} \quad (2.1)$$

The state-variables will be the outputs of the integrators.

From above patching diagram, we have:

$$\dot{\bar{x}}_1 = -a_1\bar{x}_1 + 1\,\bar{x}_2 + 0\,\bar{x}_3 + b_1 u$$

$$\dot{\bar{x}}_2 = -a_2\bar{x}_1 + 0\,\bar{x}_2 + 1\,\bar{x}_3 + b_2 u$$

$$\dot{\bar{x}}_3 = -a_3\bar{x}_1 + 0\,\bar{x}_2 + 0\,\bar{x}_3 + b_3 u$$

$$y(t) = 1\,\bar{x}_1 + 0\,\bar{x}_2 + 0\,\bar{x}_3$$

1-θ

$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \\ \end{bmatrix} u$$

$$y = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix}$$

State Transformations

Suppose we have a first state-variable representation of a system as:

$$\dot{x} = Fx + Gu$$

$$y = Hx + Ju$$

and a second state-variable representation of the same system as:

$$\dot{p} = F'p + G'u$$

$$y = H'p + J'u$$

Since both state-variable representations are of the same system, then if they are related as

i.e.

$$p = Tx \quad ; \quad \dot{p} = T\dot{x}$$

then, it follows that:

$$\dot{p} = F'p + G'u$$

i.e.

$$T\dot{x} = F'(Tx) + G'u$$

i.e.

$$\dot{x} = \underbrace{\left(T^{-1}F'T\right)}_{=F}x + \underbrace{\left(T^{-1}G'\right)}_{=G}u$$

and also

$$y = H' p + J' u$$

$$= \underbrace{\left(H' T\right)}_{= H} x + \underbrace{J'}_{= J} u$$

System Transfer Functions

Consider the state-variable system:

$$\dot{x} = Fx + Gu \qquad (1.1a)$$

$$y = Hx + Ju \qquad (1.1b)$$

What is the transfer function of this state-variable system?

Recall (from EE2010 !!!) that transfer function is obtained by taking Laplace Transforms, under the situation of zero initial conditions

Taking Laplace Transforms throughout of (1.1a) gives:

Then, from (1.1b), and taking Laplace Transforms:

i.e.

$$Y(s) = \underbrace{\left[H\{sI - F\}^{-1} G + J \right]}_{= G(s)} U(s)$$

defines the transfer function for

$$\dot{x} = Fx + Gu$$

$$y = Hx + Ju$$

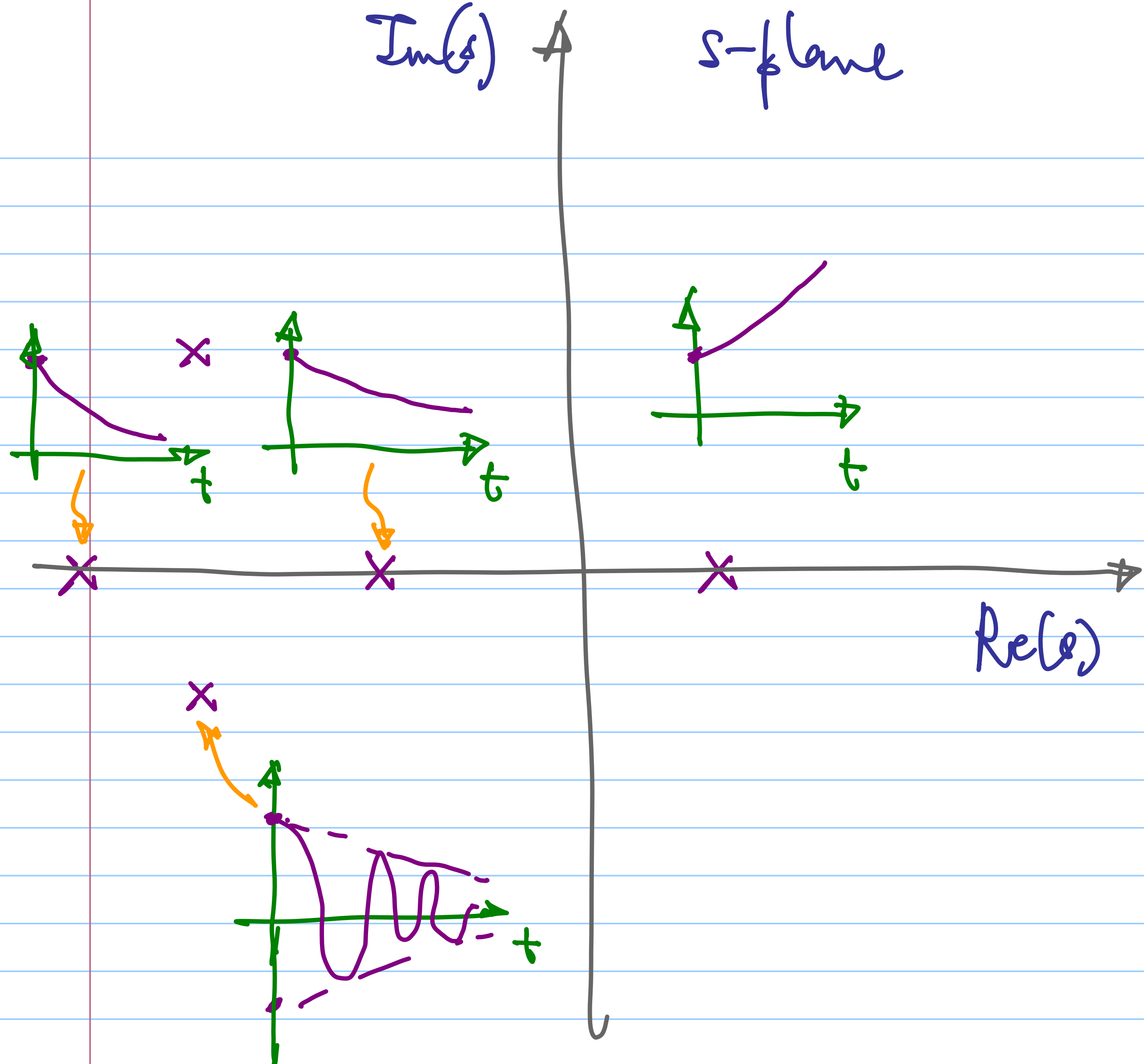
Poles of a system

$$\dot{x} = Fx + Gu \qquad (3.1a)$$

$$y = Hx + Ju \qquad (3.1b)$$

What are the poles of this system?

Poles of the system are charaterized by the zero-input response of the system. (From EE 2010)



i.e. for the state-variable system (3.1a) and (3.1b), i.e. we are looking at the response of the system with

$$u(t) \equiv 0$$

and

$$x(t) = x_0 e^{\lambda_i t} \qquad \text{--- (3.3)}$$

where λ_i is a pole of the system

Thus, from (3.1a), we have

$$\dot{x} = Fx + Gu \qquad \text{--- (3.4)}$$

i.e.

L.H.S. of (3.4) $= \dot{x} =$

from (3.3)

R.H.S. of (3.4) $=$

from (3.3)

i.e., we must have

$$x_0 \lambda_i e^{\lambda_i t} = F x_0 e^{\lambda_i t}$$

$$e^{\lambda_i t} \lambda_i I_{n \times n} x_0 = e^{\lambda_i t} F x_0$$

i.e.,

$$e^{\lambda_i t} \{\lambda_i I - F\} x_0 = 0 \quad \longrightarrow (3.5)$$

The expression (3.5) is only possible when

$$\det\{\lambda_i I - F\} = 0$$

i.e., λ_i is an eigenvalue of F

i.e. For the system

$$\dot{x} = Fx + Gu \qquad (3.1)$$

$$y = Hx + Ju$$

the poles of the system are the eigenvalues of F

i.e. the poles of the system are given by the "characteristic equation" of F, i.e.

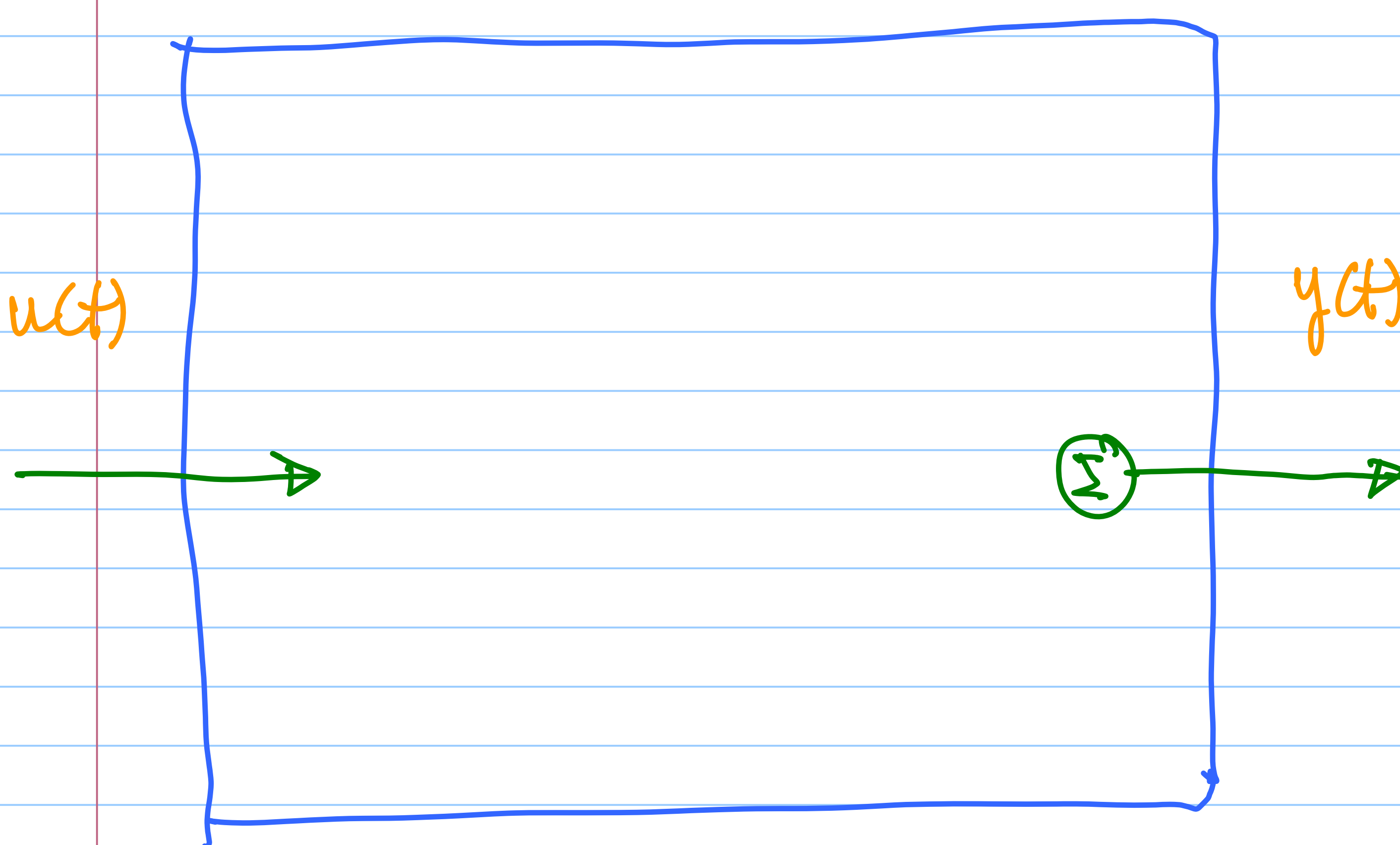
$$\det\{\lambda_i I - F\} = 0$$

Zeros of the System

$$\dot{x} = Fx + Gu$$
$$y = Hx + Ju \quad (3.1)$$

What are the zeros of this system?

Go back to EE2010 .....



The "zeros" of the system is when for no initial-condition in the system, and with an generalized sinusoidal input at frequency s for the input $u(t)$, i.e.

$$u(t) = U_0 e^{st} \quad \text{——} \quad (3.21)$$

and the output

$$y(t) \equiv 0 \quad \text{——} \quad (3.22)$$

Thus, we must have

$$x(t) = X_a e^{st} \quad \text{——} \quad (3.33)$$

Then, in (3.1), we must have

$$\overset{\circ}{x}(t) = F x(t) + G u(t)$$

$$e^{st}\left\{ \left[sI - F \right] X_a - G U_0 \right\} = 0 \quad \text{--- (3.31)}$$

And also

$$y(t) = H x(t) + J u(t)$$

i.e.

i.e.

$$e^{st}\left\{ H X_A + J U_0 \right\} = 0$$

$$\text{--- (3.42)}$$

Because $e^{st} \neq 0$, we must thus have

$$\left[\begin{array}{c|c} sI - F & -G \\ \hline H & J \end{array}\right] \begin{bmatrix} X_A \\ U_0 \end{bmatrix} = 0$$

i.e. only possible when

$$\det \begin{bmatrix} sI - F & -G \\ H & J \end{bmatrix} = 0$$

$\dot{x} = Fx + Gu$

$y = Hx + Ju$

i.e. zero "s" of the system (3.1)

is given by above equation.

Additional (Exercise)

Consider the system

$$\frac{Y(s)}{U(s)} = \frac{(s+\beta_1)}{(s+\alpha_1)(s+\alpha_2)}$$

(a) Write out a state-variable description for this system in the form of

$$\dot{x} = Fx + Gu$$

$$y = Hx + Ju$$

showing clearly the entries in $\{F, G, H, J\}$

(b) For the state-variable system in part (a),

(b1) Calculate the "poles" of the system; and

(b2) Calculate the "zeros" of the system.