Note Title 5/29/2011

Incorporating Set-Point Signal

5

pre-class

$$\dot{x} = Fx + Gu$$

$$y = Hx$$

Open-loop poles are given by eigenvalues of F, or equivalently the roots of the o.l. characteristic equation

$$\alpha_{ol}(s) = \det\{sI - F\}$$

$$= s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots a_n = 0$$

Open-loop not satisfactory for precision control because e.g.

* open-loop dynamics too slow; or
* open-loop is unstable; or
* no feedback to obtain tracking; etc

Approach #1 (Set-Point)

Thus, consider the state-feedback

$$u = -\underline{k}x + $$

$\underline{k} = [k_1 \; k_2 \; \ldots \; k_n]$

r : constant set-point signal

k_s : a scaler "scaling gain"

With the state-feedback above, we have

$$\dot{x} =$$

$$u =$$

$$y =$$

i.e.

$$\dot{x} =$$

$$y = \qquad\qquad\qquad \text{——— (1)}$$

$u = -Kx + k_s r$

Note that here, closed-loop poles are given by the characteristic equation

$$\alpha_c(s) = \det\left[\qquad\qquad\qquad\right] = 0$$

$\alpha_c(s) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_n$

* choose desired $\alpha_c(s)$ using

I. Prototype Response; or

II. Symmetric Root Locus; or

III. LQR method

* after choosing $\alpha_c(s)$, calculate required K using
- Ackermann's formula
- Bass-Gura's formula

$$u = -Kx + K_s r$$

Ackermann's formula:

$$K = [0 \; 0 \; \ldots \; 0 \,\vdots\, 1] \, \mathcal{C}_G^{-1} \, \alpha_c(F)$$

$$\mathcal{C}_G = [G \;\; FG \;\; \ldots \;\; F^{n-1}G]$$

Exercise: check also Bass-Gura's formula

In equation (1) above, note that the closed-loop transfer function from r to y is now given by:

$$\frac{Y(s)}{R(s)} =$$

For steady-state set-point tracking of a constant reference signal, r, clearly we want, in addition, i.e. $k_s =$

Aside: Recall that for

$$\dot{x} = Fx + Gu$$
$$y = Hx$$

we had previously already ascertained that the transfer function from u to y is

$$\frac{Y(s)}{U(s)} = H\{sI - F\}^{-1} G$$

$\Delta\Delta\Delta$

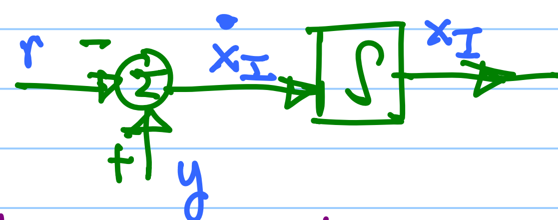
Approach #2 (Set-point)

Integral Control

$$\dot{x} = Fx + Gu$$

$$y = Hx$$

Generate a new signal, x_I, where



Consider, now, everything together,

i.e.

$$\dot{x} =$$

$$\dot{x}_I =$$

$$y =$$

and write an "augmented" system as

$$n+1\left\{ \dot{\begin{bmatrix} x \\ x_I \end{bmatrix}} \right. = \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}}_{\bar{F}} \underbrace{\begin{bmatrix} x \\ x_I \end{bmatrix}}_{\bar{x}} + \underbrace{\begin{bmatrix} \quad \\ \quad \end{bmatrix}}_{\bar{G}} u + \underbrace{\begin{bmatrix} \quad \\ \quad \end{bmatrix}}_{\bar{G}_r} r$$

$$y = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} x \\ \hdashline x_I \end{bmatrix}$$

$\bar{H}$

and now, we have the "augmented" system $\begin{bmatrix} x \\ x_I \end{bmatrix} = \bar{x}$

$$\dot{\bar{x}} =$$

$$y =$$

of order

Consider the state-feedback

$$u = -\bar{K}\bar{x}$$

$$= -\begin{bmatrix} k_1 & k_2 & \cdots & k_n & k_{n+1} \end{bmatrix} \begin{bmatrix} x \\ \\ x_I \end{bmatrix}$$

With this state-feedback, we now have

$$\underline{\dot{x}} =$$

$$u =$$

$$y =$$

i.e.

$$\underline{\dot{x}} =$$

$$y =$$

- closed-loop poles of this system are given by characteristic equation

$$\det \left\{ \qquad \right\} = 0$$

i.e. for a specified $\bar{K}$, the above gives the obtained closed-loop poles

- choose your desired closed-loop characteristic polynomial as

$$\overline{\alpha}_c(s) = s^{n+1} + \overline{\alpha}_1 s^n + \overline{\alpha}_2 s^{n-1} + \dots + \overline{\alpha}_{n+1}$$

(I) Prototype Response Tables; or

(II) Symmetric Root Locus; or

(III) LQR method.

From your chosen desired $\overline{\alpha}_c(s)$, calculate $\overline{K}$ using
- Ackermann's formula
- Bass-Gura's formula

$u = -\overline{K}\,\overline{x}$!!

Finally, note that closed-loop system is:

$$\dot{\overline{x}} = \{\overline{F} - \overline{G}\,\overline{K}\}\overline{x} + \overline{G}_r r$$

$$y = \overline{H}\,\overline{x}$$

in above, we have calculated $\overline{K}$ to obtain the desired $\overline{\alpha}_c(s)$

We have chosen a desired $\bar{\alpha}_c(s)$, suitably fast & stable closed-loop poles; and calculated the necessary $\bar{K}$ to use in

$$u = -\bar{K}\,\bar{x}$$

i.e. closed-loop system now has poles which are roots of

$$\bar{\alpha}_c(s) = 0$$

i.e. system is stable.

Since system is stable, and since r is a constant set-point, we will have

$$\lim_{t \to \infty}$$

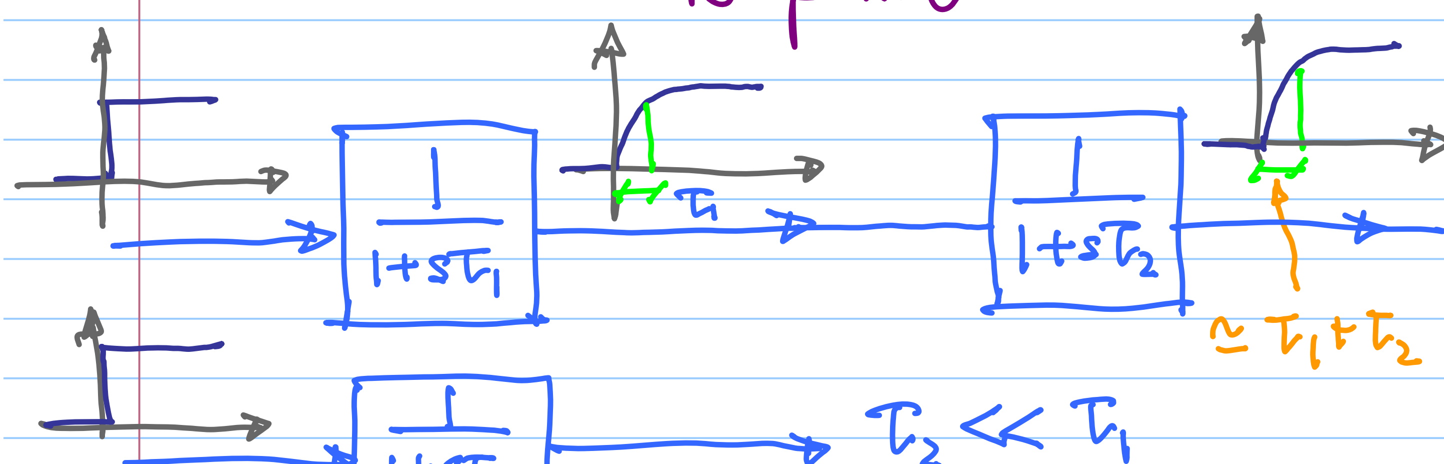
Appendix/Aside

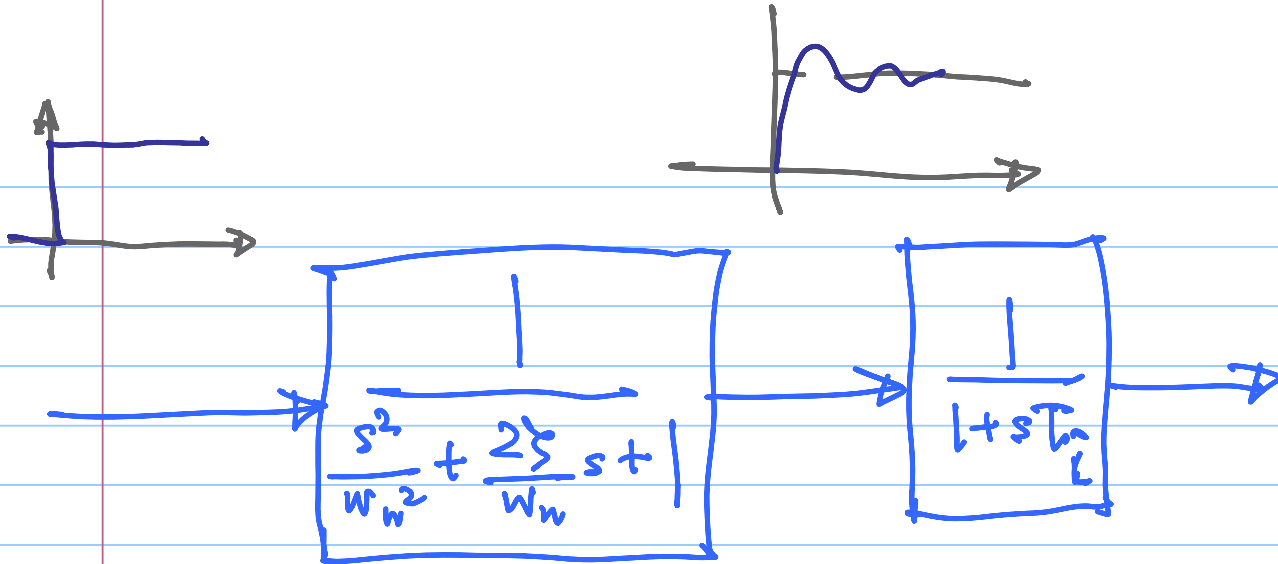
Approach

Recall ways to choose $\alpha_c(s)$

1. Prototype Response Tables
2. Symmetric Root Locus
3. LQR method

Approach 1.15 "Second Order Dominant Response"





$$T_i \ll \frac{1}{\omega_n}$$

Consider the characteristic equation

$T_i \ll \frac{1}{\omega_n}$, then the response of the system looks largely the same as the 2nd-order one with characteristic equation i.e. the "Dominant Second-Order Response" method says that you should choose your desired $\alpha_c(s)$ as

$$\alpha_c(s) = (s^2 + 2\zeta\omega_n s + \omega_n^2)(s+p_1)\cdots(s+p_q)$$

where $p_i = \frac{1}{T_i}$

i.e. $p_i \gg \omega_n$

typically at least 3 times ω_n