NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR
(Semester II: 2017/2018)

EE4302 – ADVANCED CONTROL SYSTEMS

April/May 2018 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. Please write your student number only. Do not write your name.
2. This question paper contains **FOUR** (4) questions and comprises **Eleven** (11) pages.
3. Answer **ALL** questions.
4. Note that the Questions do not carry equal marks.
5. This is a **CLOSED BOOK** examination. However, each student may bring ONE (1) A4 size crib sheet into the examination hall.
6. Relevant data are provided at the end of this examination paper.
7. Graphics/Programmable calculators are not allowed.

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Q.1 Consider the Figures 1a, 1b and 1c which show a particular set of notes from a typical design exercise for a state-variable control system. Here, the *augmented* state-variable signal $x_I(t)$ is generated as

$$\dot{x}_I(t) = y(t) - r(t)$$

where $y(t)$ is the measured output of the system to be controlled, and $r(t)$ is the set-point command signal, a situation which is also illustrated in the block diagram in Figure 1a.

The *augmented* state-variable signal $x_I(t)$ is incorporated in the state-variable description of the overall augmented system as shown in Figure 1b. This overall augmented system is one possible way of describing a state-feedback control system with integral control action. In addition, note that it can be stated that the computed control signal $u(t)$ in Figures 1a, 1b and 1c, is computed as:

$$u(t) = -k_I x_I(t) - k_1 x_1(t) - k_2 x_2(t)$$

and in the design calculations as shown in Figure 1c, the necessary state-feedback gain row vector K is given by

$$\mathrm{K} = [k_I \quad k_1 \quad k_2]$$

The state-variable equations in Figure 1b also includes the influence of $v(t)$, an unmeasurable additional disturbance signal.

Here, if the situation is that $v(t) = 0$ (*i.e.* zero-valued or no disturbance), and $r(t) = r_0$ (*i.e.* a constant-valued user-applied reference signal), describe in full detail (with all relevant equations and analysis) the performance of this state-feedback control system, where the necessary design calculations are as described in Figure 1c.

(12 marks)

Question 1 continues next page.

1.0a State Feedback Design including State-Augmentation

It is desired to obtain the following frequency specifications between r and y:

Closed-loop bandwidth: Not lower than 1.5 rad/s;
Resonant Peak, *Mr* : Not larger than 2 dB (or 10%);
Steady-state gain between *r* and *y*: 0dB.

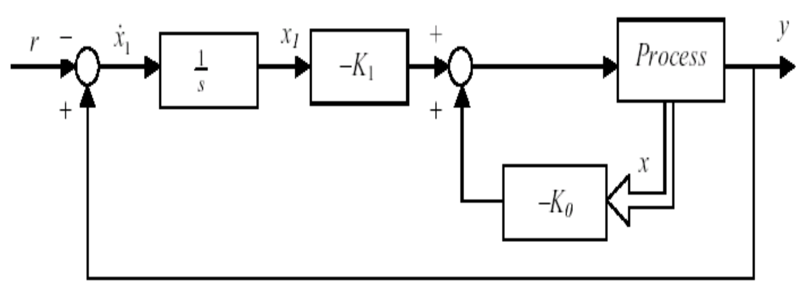

Figure 1a: A suitable state-space description of the augmented system.

Question 1 continues next page.

1.0b State Feedback Design including State-Augmentation

$$\begin{bmatrix} \dot{x}_I \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$y = x_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix}$$

Figure 1b: A suitable state-space description of the augmented system.

Question 1 continues next page.

1.0c State Feedback Design including State-Augmentation

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%5.0 State Feedback Design including State Augmentation
% xI_dot = 0 xI +  1 x1 +  0 x2 + 0 u + -1 r + 0 v    % xI_dot= y-r = e = x1-r
% x1_dot = 0 xI +  0 x1 +  1 x2 + 0 u +  0 r + 0 v
% x2_dot = 0 xI + -1 x1 + -2 x2 + 1 u +  0 r + 1 v

   F=[0 1 0;0 0 1;0 -1 -2];
   G=[0; 0; 1];
   Gr=[-1; 0; 0];
   Gv=[0; 0; 1];
   H=[0 1 0];
   J=0;

%ITAE method in calculating state feedback gain K
  P=2*[-0.7081; -0.5210+1.068*i ; -0.5210-1.068*i];
%use w0=2 since w0=1.5 fails to satisfy the requirement
  K=acker(F,G,P);
```

Figure 1c: A suitable state-space description of the augmented system.

Q.2 The block diagram of Figure 1a does not yet explicitly show the additional disturbance signal $v(t)$. Based on the equations of Figure 1b,
provide a re-drawing of the block diagram of Figure 1a where the inclusion of the additional disturbance signal $v(t)$ is clearly shown.

Next, for a general additional disturbance signal $v(t)$, and a general user-applied reference signal $r(t)$, develop and describe in full detail (with all relevant equations and analysis) the two transfer functions $\frac{Y(s)}{R(s)}$ and $\frac{Y(s)}{V(s)}$.

Further, using these transfer functions developed above, and if the situation is now that $v(t) = v_0$ (*i.e.* a constant-valued but unknown disturbance), and $r(t) = r_0$ (*i.e.* a constant-valued user-applied reference signal), describe in full detail (with all relevant equations and analysis) the performance of this state-feedback control system, where the necessary design calculations are as described in Figure 1c.

Also for the same system, what would be a suitable set of poles from using the methodology of Second-Order Dominant Response? State these carefully, explaining also suitable reasons for your choice.

(18 marks)

Q.3 Consider the Van der Pol equation

$$\ddot{x} + 0.2(x^2 - 1)\dot{x} + x = 0$$

a) Give the equation for the isocline of slope α. Let $x_1(t) = x(t)$ and $x_2(t) = \dot{x}(t)$.

[10 marks]

b) Sketch the isoclines for $\alpha = -1$, 0, 1. Use x axis for $x_1(t)$ and y axis for $x_2(t)$ with both x and y axes between -1 and 1.

[10 marks]

c) Using the isocline in Part (b), sketch the trajectory that starts from $x_1(0) = -0.2$, $x_2(0) = 0.25$ and ends when it reaches $x_2(t) = 0.25$ again.

[10 marks]

d) Divide the trajectory in Part (c) into segments of $\Delta x_1 = 0.1$. Sketch $x_1(t)$ versus time, t, for $-0.2 \leq x_1(t) \leq 0.3$.

[5 marks]

Q.4 The model of a first-order system is given by

$$\begin{aligned} \dot{x}(t) &= 0.5x(t) + u(t) \\ y(t) &= x(t) \end{aligned}$$

The sliding variable and sliding controller are given by $\sigma = -x(t)$ and $u(t) = M\text{sign}(\sigma)$ respectively.

a) Determine $y(t)$ analytically given $x(0) = 1$.

[12 marks]

b) Find M such that it takes $t = 2$ to reach $\sigma = 0$.

[13 marks]

c) Using the value of M found in Part (b), sketch $u(t)$ and $y(t)$ for $0 \leq t \leq 3$

[10 marks]

END OF QUESTIONS

DATA SHEET:

1. For the matrices $\mathbf{A} \in \mathbf{R}^{n\times n}$, $\mathbf{C} \in \mathbf{R}^{1\times n}$, and $\mathbf{L} \in \mathbf{R}^{n}$, the eigenvalues of the matrix $(\mathbf{A} - \mathbf{LC})$ can be arbitrarily assigned by a suitable choice of $\mathbf{L}$ as long as

$$O(\mathbf{A}, \mathbf{C}) = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{(n-1)} \end{bmatrix}$$

is non-singular.

2. For the linear system

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Fx} + \mathbf{G}u \\ y &= \mathbf{Hx} \end{aligned}$$

where $\mathbf{x} \in \mathbf{R}^n$ and $u, y \in \mathbf{R}^1$, the controllability matrix of the system is given by

$$C(\mathbf{F}, \mathbf{G}) = \begin{bmatrix} \mathbf{G} & \mathbf{FG} & ... & \mathbf{F}^{(n-1)}\mathbf{G} \end{bmatrix}$$

If the characteristic polynomial of $\mathbf{F}$ is given by

$$\alpha(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + ... + a_n$$

then the state-feedback $u = -\mathbf{Kx}$ which yields the closed-loop characteristic polynomial

$$\alpha_c(s) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + ... + \alpha_n$$

can be calculated using the Bass-Gura's formula

$$\mathbf{K} = \begin{bmatrix} \alpha_1 - a_1 & \alpha_2 - a_2 & ... & \alpha_n - a_n \end{bmatrix} \{C(\mathbf{F}, \mathbf{G})W\}^{-1}$$

where

$$W = \begin{bmatrix} 1 & a_1 & a_2 & ... & a_{n-1} \\ 0 & 1 & a_1 & ... & a_{n-2} \\ \vdots & \vdots & \ddots & ... & \vdots \\ 0 & 0 & 0 & ... & 1 \end{bmatrix}$$

3. For the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= -a_n x_1 - a_{n-1} x_2 - ... - a_1 x_n + b_0 u \\ y &= x_1 \end{aligned}$$

the characteristic polynomial is

$$\alpha_o(s) = s^n + a_1 s^{n-1} + ... + a_n$$

and the transfer function is

$$\frac{Y(s)}{U(s)} = \frac{b_0}{s^n + a_1 s^{n-1} + ... + a_n}$$

4. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top [sI - A_m]^{-1} b_m = \frac{-a_3}{s^3 - a_2 s^2 - a_1 s - a_3}$$

5. In the development of a reduced-order observer, note that a state-variable system with state vector $\mathbf{x}$ of order n, where the first n_1 state-variables, in a vector $\mathbf{x}_1$ are essentially measurable, can be written as:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{F}_{11}\mathbf{x}_1 + \mathbf{F}_{12}\mathbf{x}_2 + \mathbf{G}_1 u \\ \dot{\mathbf{x}}_2 &= \mathbf{F}_{21}\mathbf{x}_1 + \mathbf{F}_{22}\mathbf{x}_2 + \mathbf{G}_2 u \end{aligned}$$

with the remaining n_2 state-variables, in a vector $\mathbf{x}_2$ to be estimated (or observed). Here, $\mathbf{F}_{11}$, $\mathbf{F}_{12}$, $\mathbf{F}_{21}$ and $\mathbf{F}_{22}$ are all known system matrices (obtained by calibration data, for example), and the measurement is typically given by

$$\mathbf{y}_m = \mathbf{H}_1 \mathbf{x}_1$$

where $\mathbf{H}_1$ is also a known $(n_1 \times n_1)$ system matrix. Under these circumstances, a suitable form for the estimator for $\mathbf{x}_2(t)$ is

$$\begin{aligned} \hat{\mathbf{x}}_2 &= \mathbf{L}\mathbf{y}_m + \mathbf{z} \\ \dot{\mathbf{z}} &= \bar{\mathbf{F}}\mathbf{z} + \bar{\mathbf{G}}\mathbf{y}_m + \bar{\mathbf{H}} u \end{aligned}$$

where a suitable set of matrices defining the reduced-order observer are given by:

$$\begin{aligned}
\bar{\mathbf{F}} &= \mathbf{F}_{22} - \mathbf{L}\mathbf{H}_1\mathbf{F}_{12} \\
\bar{\mathbf{G}} &= \left\{\mathbf{F}_{21} - \mathbf{L}\mathbf{H}_1\mathbf{F}_{11} + \bar{\mathbf{F}}\mathbf{L}\mathbf{H}_1\right\}\mathbf{H}_1^{-1} \\
\bar{\mathbf{H}} &= \mathbf{G}_2 - \mathbf{L}\mathbf{H}_1\mathbf{G}_1
\end{aligned}$$

6. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s$[a]
ITAE	1	$s+1$
	2	$s+0.7071 \pm 0.7071j$[b]
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s+0.8660 \pm 0.5000j$[b]
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a \pm bj)$ to conserve space.

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$u(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{a}{s(s+a)}$	$1-e^{-at}$
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$

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