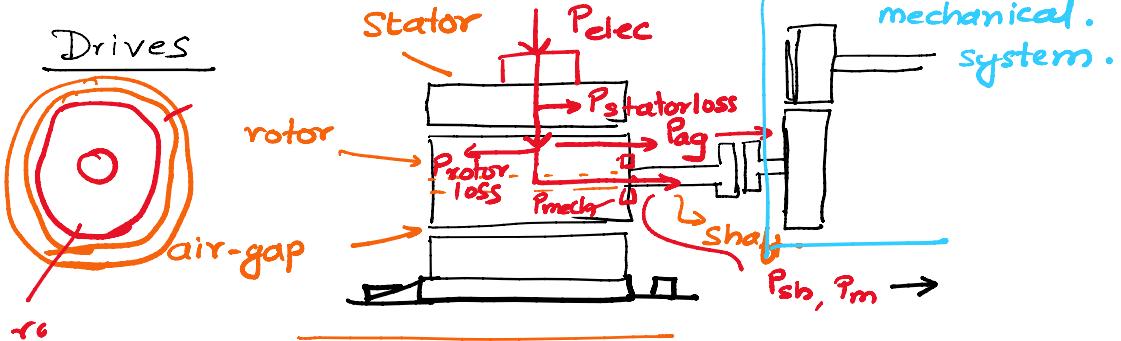


Today's class.

1. Modelling Mechanics with Drives.
2. Dynamics & stability at Equilibrium point
what is Equilibrium point?

3. Linearization ... why?



electrical → circuit models
electro-mechanical.

Cross-sectional view.

Mechanics of Drive System

2 types of motion

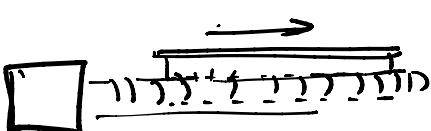
1. Linear motion

2. Angular motion.

Linear motion x - distance \xrightarrow{x}

$$\frac{dx}{dt} = v \quad \text{velocity}$$

$$\frac{d^2x}{dt^2} = a \quad \text{acceleration.}$$



$$\theta \text{- angle, angular distance} \rightarrow x = r\theta$$

 $\omega / \Omega \dots \text{angular velocity}$

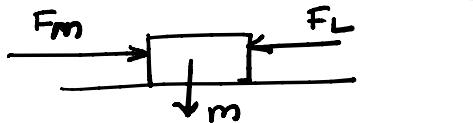
$$\frac{d\theta}{dt} = \omega \text{ or } \Omega$$

$$\frac{d^2\theta}{dt^2} = \Gamma \rightarrow \text{angular acceleration.}$$

$$M = F \cdot r$$

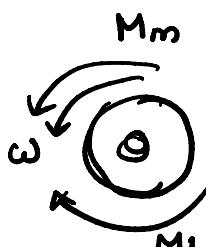
$$a = r \cdot \Gamma$$

$$\begin{aligned} \text{Torque} \\ \text{Moment} \\ M_A = \bar{M}_m - M_L \\ = J \cdot \frac{d\omega}{dt} \end{aligned}$$

Newton's law of motion

$$F_A = F_m - F_L = m \cdot a = m \cdot \frac{dv}{dt}$$

mass → inertia



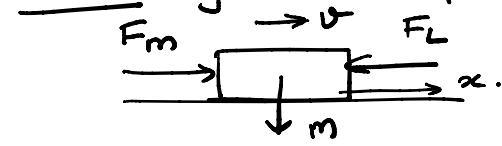
$$\begin{aligned} \text{Moment of Inertia} \\ = \underline{\underline{I}} \cdot r^2 \end{aligned}$$

Model dynamics of linear motion

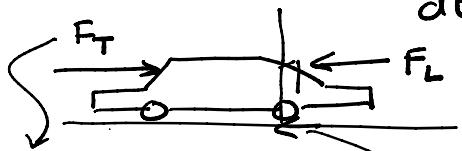
$$F_m \rightarrow v \quad F_L$$

Dynamics

Model dynamics of linear motion



$$F_A = F_m - F_L = m \cdot \frac{dv}{dt}$$



Const. velocity
 $F_L = A + Bv + Cv^2$

Tractive Force.

$$F_A = 0 \dots \frac{dv}{dt} = 0 \rightarrow \text{Rolling resistance.}$$

$$F_D \propto v^2 \\ = \frac{1}{2} C_D P \cdot v^2$$

Coast down test

① 120 kmph const. over flat road.

② Turn off the Engine. $m \frac{dv}{dt} = -F_D$
 $F_T \rightarrow 0 \quad F_D$

③ Measure the velocity as the vehicle comes to a stop.

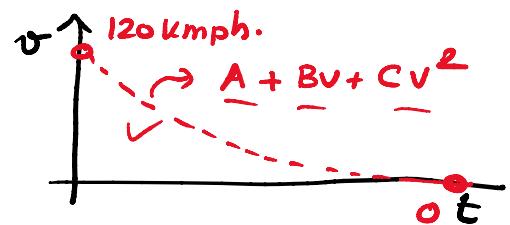
Dynamics

$$m \cdot \frac{dv}{dt} = F_A = F_m - F_L =$$

$$\frac{dv}{dt} = F_x - Bv$$

$$m \cdot \frac{dv}{dt} + Bv = F_x$$

O.D.E. First order



Static condition Steady state condition

$$\frac{d}{dt} \rightarrow \frac{dv}{dt} = 0 \dots v - \text{constant.} \Rightarrow F_A = 0$$

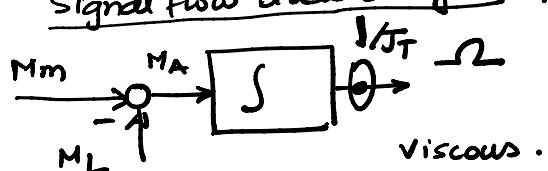
Rotational Motion. \rightarrow Motor torque

$$J \frac{d\omega}{dt} = M_A = M_m - M_L$$

↓ Accelerating torque.

Load torque.

ODE \rightarrow integration
Signal flow block diagram.



Steady state condition

$$M_A = 0 \therefore \frac{d\omega}{dt} = 0.$$

$M_m = M_L \approx \text{equilibrium}$

Q

$$M_L = M_{L0} + K\omega^2$$

Viscous Friction

$$\therefore J_T \frac{d\omega}{dt} = M_m - M_{L0} - K\omega^2$$

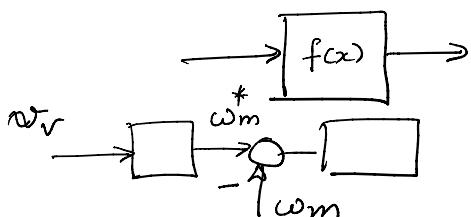
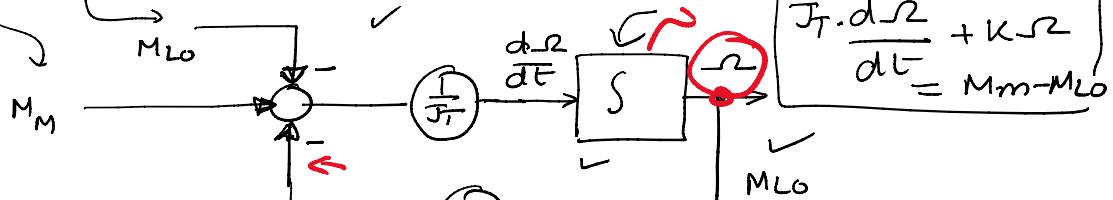
① A signal flow block diagram of the system.

② What does the system represent.

$$\therefore J_T \cdot \frac{d\Omega}{dt} = M_m - M_{lo} - k \cdot \Omega.$$

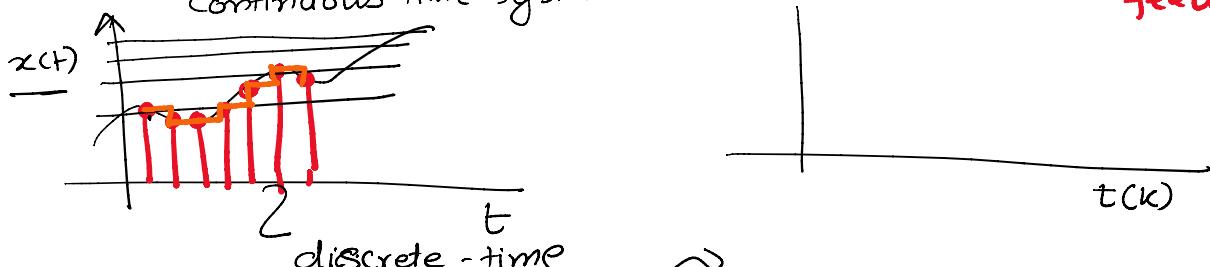
② What does the system represent.
--> guess/approximate
how s_2 will vary with time.

$$\frac{d\mathcal{L}}{dt} = \frac{1}{J_T} \left[M_m - M_{LO} - k \cdot r^2 \right]$$



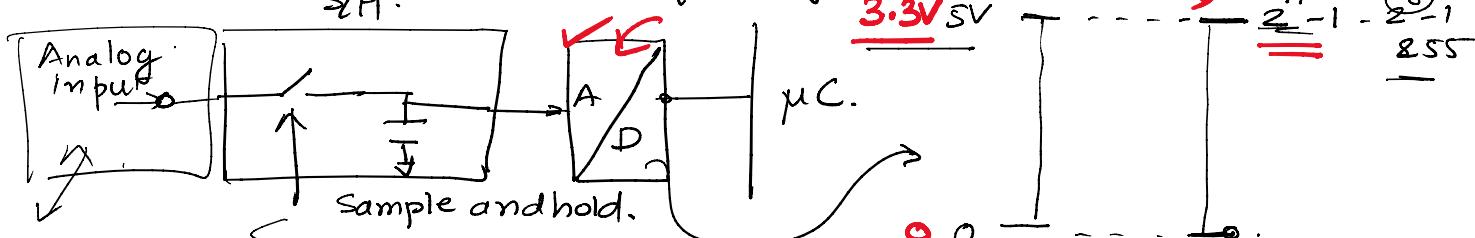
First order systems
= \int with negative feedback

Continuous time system.



Sampling

digital control \rightarrow discrete-time control
S/H. + quantification.

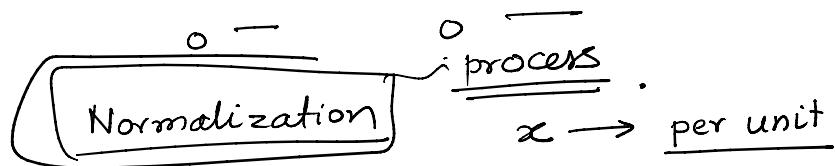


Sampling period.

8-bit - 8051C. -

32-bit STM-32

EV - 500-700 V DC



$$= \frac{x}{x_B} = \frac{2\phi A}{10\phi A} = \underline{\underline{0.2 \text{ P.u}}}$$

Motor current ... rated current → 24

Maximum - RMS- current the motor 80A

can draw -- when operating at rated voltage

& rated Power - $\frac{24}{17} \times 365 \text{ days}$

Continuous rating

100A.

Base value → Maximum value of the variable that I want to measure.

300 kmph.

33V → 300.

$$\frac{V_{\text{actual}}}{V_B} = \frac{12\phi}{300} = \underline{\underline{0.4}}$$

- 50 kmph.

↔ Units based system → per unit system.

$$J_T \cdot \frac{d\omega}{dt} = M_A$$

$[N \cdot m]$

$$[kg \cdot m^2] \frac{[rad/s]}{[s]} \quad kg \cdot \frac{m}{s^2} \cdot m$$

M_B ← Base value of Torque.

ω_B ← Base value..

$$s = 7.8$$

$$T_m \cdot \frac{d\omega}{dt} \frac{1}{M_B} = \frac{M_m}{M_B} - \frac{M_L}{M_B}$$

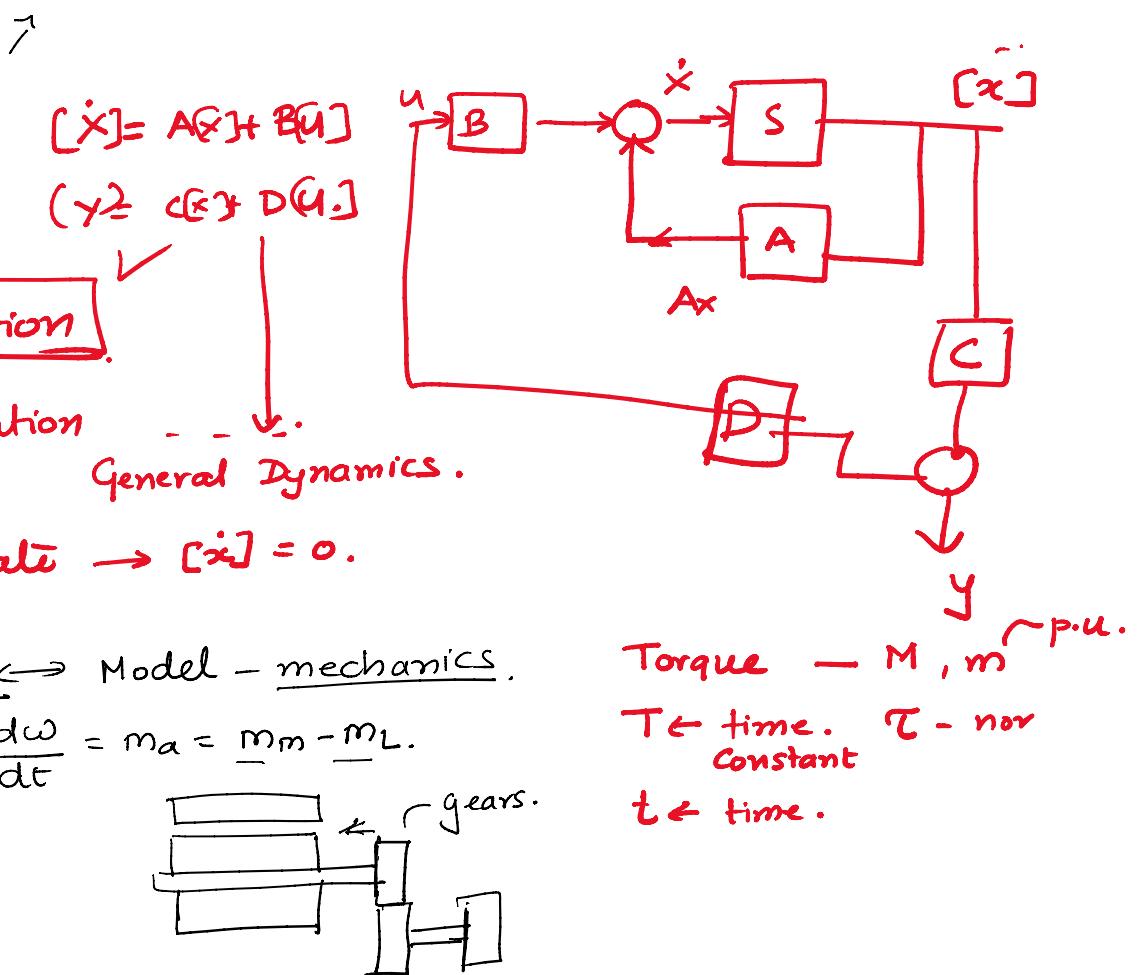
$$\frac{J_T \cdot \omega_B}{M_B} \cdot \frac{d}{dt} \left(\frac{\omega}{\omega_B} \right) = m_m - m_L$$

$$\frac{kg \cdot m^2 \cdot rad}{s} \cdot \frac{d\omega}{dt} = m_m - m_L$$

mechanical time constant.

$$T_m \cdot \frac{d\omega}{dt} = m_m - m_L$$

— u — \times — [x]



Gear System

$n_g = \frac{\omega_1}{\omega_2}$ gear ratio

$\frac{\omega_1}{\omega_2} = \frac{n_i}{n_o} = n_g$

Conversion ratios of gear

lossless gear

$\frac{1}{2} J_{2,1} \cdot \omega_1^2 = \frac{1}{2} J_2 \cdot \omega_2^2$

$J_{2,1} = \left(\frac{\omega_2}{\omega_1}\right)^2 \cdot J_2$

$J_{2,1} = \left(\frac{1}{n_g}\right)^2 \cdot J_2$

$w_i \xrightarrow{P_i} \boxed{\quad} \xrightarrow{P_o} w_o$

$w_i = \frac{w_o}{n_g}$

Equivalent moments of inertia

$J_{eq} = J_p + J_{m,L}$

$\frac{1}{2} J_{m,L} \omega_2^2 = \frac{1}{2} m \cdot v^2$

$v = R_p \cdot \omega_2$

$J_{m,L} \omega_2^2 = m \cdot R_p^2 \cdot \omega_2^2$

$J_{m,L} = m R_p^2$

$J_{eq} = J_p + J_{m,L}$

$$w_i = \frac{w_0}{\gamma g}$$

$$\begin{aligned} J_{eq} &= J_p + J_m, L \\ &= J_p + m R_p^2 \end{aligned}$$