NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR
(Semester I: 2019/2020)

EE4302 – ADVANCED CONTROL SYSTEMS

November/December 2019 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. Please write your student number only. Do not write your name.
2. This question paper contains **FOUR** (4) questions and comprises **Twelve** (12) pages.
3. Answer **ALL** questions.
4. Note that the Questions do not carry equal marks.
5. This is a **CLOSED BOOK** examination. However, each student may bring ONE (1) A4 size crib sheet into the examination hall.
6. Relevant data are provided at the end of this examination paper.
7. Graphics/Programmable calculators are not allowed.

Q1 Consider the set of notes from a typical design exercise for a state-variable control system which are shown in Figures 1a, 1b and 1c. The *augmented* state-variable signal $x_I(t)$ is generated as

$$\dot{x}_I(t) = y(t) - r(t)$$

where $y(t)$ is the measured output of the system to be controlled, and $r(t)$ is the set-point command signal, a situation which is also illustrated in the block diagram in Figure 1a.

The *augmented* state-variable signal $x_I(t)$ is incorporated in the state-variable description of the overall augmented system as shown in Figure 1b. This overall augmented system is one possible way of describing a state-feedback control system with integral control action. In addition, note that it can be stated that the computed control signal $u(t)$ in Figures 1a, 1b and 1c, is computed as:

$$u(t) = -k_I x_I(t) - k_1 x_1(t) - k_2 x_2(t)$$

and in the design calculations as shown in Figure 1c, the necessary state-feedback gain row vector K is given by

$$\mathrm{K} = [k_I \quad k_1 \quad k_2]$$

The state-variable equations in Figure 1b also includes the influence of $v(t)$, an unmeasurable additional disturbance signal.

The block diagram of Figure 1a does not yet explicitly show the additional disturbance signal $v(t)$. Based on the equations of Figure 1b,
provide a re-drawing of the block diagram of Figure 1a where the inclusion of the additional disturbance signal $v(t)$ is clearly shown.

(5 marks)

For the overall system above, calculate the two separate transfer functions $\frac{Y(s)}{R(s)}$ and $\frac{Y(s)}{V(s)}$. (**Hint:** Since this is a linear time-invariant system, the property of linear super-position is applicable.)

Next, if the situation is that $v(t) = v_0$ (*i.e.* a constant-valued but unknown disturbance), and $r(t) = r_0$ (*i.e.* a constant-valued user-applied reference signal), describe in full detail (with all relevant equations and analysis) the performance of this state-feedback control system, where the necessary design calculations are as described in Figure 1c.

(17 marks)

Question 1 continues next page.

1.0a State Feedback Design including State-Augmentation

It is desired to obtain the following frequency specifications between r and y:

Closed-loop bandwidth: Not lower than 1.5 rad/s;
Resonant Peak, *Mr* : Not larger than 2 dB (or 10%);
Steady-state gain between *r* and *y*: 0dB.

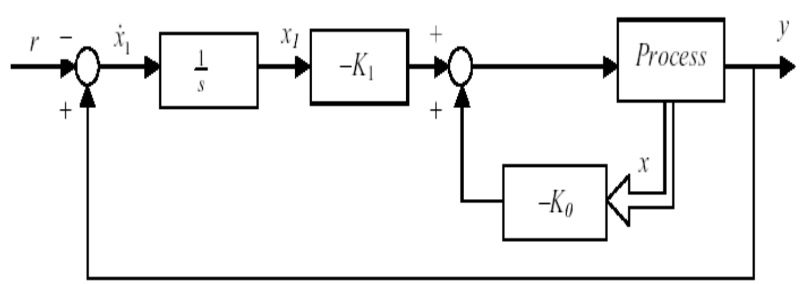

Figure 1a: A suitable state-space description of the augmented system.

Question 1 continues next page.

1.0b State Feedback Design including State-Augmentation

$$\begin{bmatrix} \dot{x}_I \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$y = x_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix}$$

Figure 1b: A suitable state-space description of the augmented system.

Question 1 continues next page.

1.0c State Feedback Design including State-Augmentation

```
%5.0 State Feedback Design including State Augmentation
% xI_dot = 0 xI +  1 x1 +  0 x2 + 0 u + -1 r + 0 v    % xI_dot= y-r = e = x1-r
% x1_dot = 0 xI +  0 x1 +  1 x2 + 0 u +  0 r + 0 v
% x2_dot = 0 xI + -1 x1 + -2 x2 + 1 u +  0 r + 1 v

   F=[0 1 0;0 0 1;0 -1 -2];
   G=[0; 0; 1];
   Gr=[-1; 0; 0];
   Gv=[0; 0; 1];
   H=[0 1 0];
   J=0;

%ITAE method in calculating state feedback gain K
  P=2*[-0.7081; -0.5210+1.068*i ; -0.5210-1.068*i];
%use w0=2 since w0=1.5 fails to satisfy the requirement
  K=acker(F,G,P);
```

Figure 1c: A suitable state-space description of the augmented system.

Q2 For the same system, what would be a suitable set of closed-loop poles from using the methodology of Second-Order Dominant Response? State these carefully, explaining also suitable reasons for your choice.

Discuss also, in as much detail as possible, the various possible reasons and situations which would influence your choice of usage of either the ITAE Prototype Response Table, or the methodology of Second-Order Dominant Response.

(8 marks)

Q3 Consider the system in Figure 3a where the process

$$G_p(s) \quad = \quad \frac{1-s}{(s+1)^2}$$

the nonlinearity

$$u = f(e) = \begin{cases} m \times e + d & \text{if } e > 0 \\ 0 & \text{if } e = 0 \\ m \times e - d & \text{if } e < 0 \end{cases}$$

$m = 0$, $d = 1$.

a) The phase portrait in Figure 3b gives the states x_1 and x_2 defined as e and $\dot{e}$ respectively. Find the time taken to go from $x_1 = 0.6$ to $x_1 = 0.4$.

[10 marks]

b) If the nonlinearity $f(e)$ is approximated by an equivalent gain K, find the equivalent gain for limit-cycle to take place, the frequency of the limit-cycle and the closed-loop poles.

[15 marks]

c) For the limit cycle found in Part (b), sketch the u versus t curve and superimpose on it the e versus t curve. Indicate the amplitude of the oscillations clearly given that a square wave of amplitude b can be approximated by a sine wave of amplitude $\frac{4b}{\pi}$.

[10 marks]

Question 3 continues next page.

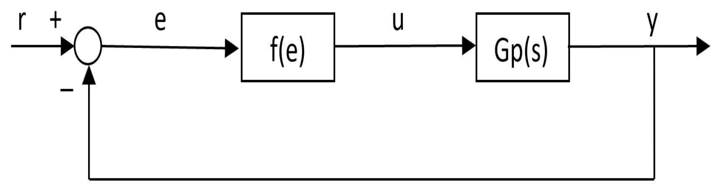

Figure 3a

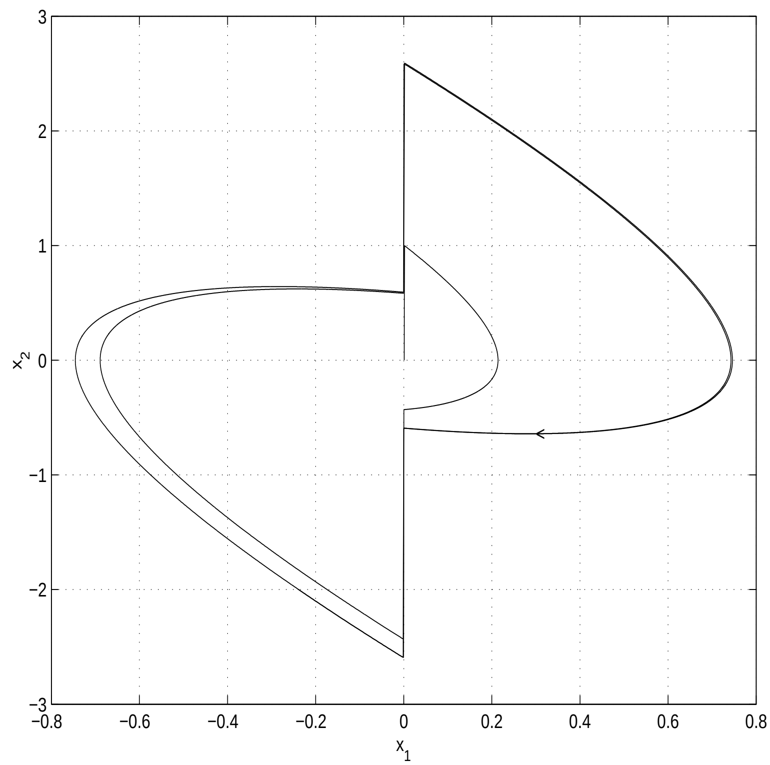

Figure 3b

Q4 Consider the process $G_p(s)$ and nonlinearity $f(e)$ in Figure 3a and Question Q3. Let $m = d = 1$.

a) Sketch the nonlinearity $f(e)$ i.e. graph of u versus e for $-1 \leq e \leq 1$.

[5 marks]

b) Find the describing function of nonlinearity $f(e)$.

[15 marks]

c) Find the limit cycle $A \sin wt$.

[10 marks]

d) The amplitude A of the limit cycle varies with m. For $d = 1$, find the value of m that gives $A = \infty$.

[5 marks]

END OF QUESTIONS

DATA SHEET:

1. For the matrices $\mathbf{A} \in \mathbf{R}^{n \times n}$, $\mathbf{C} \in \mathbf{R}^{1 \times n}$, and $\mathbf{L} \in \mathbf{R}^{n}$, the eigenvalues of the matrix $(\mathbf{A} - \mathbf{LC})$ can be arbitrarily assigned by a suitable choice of $\mathbf{L}$ as long as

$$O(\mathbf{A}, \mathbf{C}) = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{(n-1)} \end{bmatrix}$$

is non-singular.

2. For the linear system

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Fx} + \mathbf{G}u \\ y &= \mathbf{Hx} \end{aligned}$$

where $\mathbf{x} \in \mathbf{R}^n$ and $u, y \in \mathbf{R}^1$, the controllability matrix of the system is given by

$$C(\mathbf{F}, \mathbf{G}) = \begin{bmatrix} \mathbf{G} & \mathbf{FG} & \ldots & \mathbf{F}^{(n-1)}\mathbf{G} \end{bmatrix}$$

If the characteristic polynomial of $\mathbf{F}$ is given by

$$\alpha(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n$$

then the state-feedback $u = -\mathbf{Kx}$ which yields the closed-loop characteristic polynomial

$$\alpha_c(s) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_n$$

can be calculated using the Bass-Gura's formula

$$\mathbf{K} = \begin{bmatrix} \alpha_1 - a_1 & \alpha_2 - a_2 & \ldots & \alpha_n - a_n \end{bmatrix} \{C(\mathbf{F}, \mathbf{G})W\}^{-1}$$

where

$$W = \begin{bmatrix} 1 & a_1 & a_2 & \ldots & a_{n-1} \\ 0 & 1 & a_1 & \ldots & a_{n-2} \\ \vdots & \vdots & \ddots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}$$

3. For the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= -a_n x_1 - a_{n-1} x_2 - \ldots - a_1 x_n + b_0 u \\ y &= x_1 \end{aligned}$$

the characteristic polynomial is

$$\alpha_o(s) = s^n + a_1 s^{n-1} + ... + a_n$$

and the transfer function is

$$\frac{Y(s)}{U(s)} = \frac{b_0}{s^n + a_1 s^{n-1} + ... + a_n}$$

4. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top [sI - A_m]^{-1} b_m = \frac{-a_3}{s^3 - a_2 s^2 - a_1 s - a_3}$$

5. In the development of a reduced-order observer, note that a state-variable system with state vector $\mathbf{x}$ of order n, where the first n_1 state-variables, in a vector $\mathbf{x}_1$ are essentially measurable, can be written as:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{F}_{11}\mathbf{x}_1 + \mathbf{F}_{12}\mathbf{x}_2 + \mathbf{G}_1 u \\ \dot{\mathbf{x}}_2 &= \mathbf{F}_{21}\mathbf{x}_1 + \mathbf{F}_{22}\mathbf{x}_2 + \mathbf{G}_2 u \end{aligned}$$

with the remaining n_2 state-variables, in a vector $\mathbf{x}_2$ to be estimated (or observed). Here, $\mathbf{F}_{11}$, $\mathbf{F}_{12}$, $\mathbf{F}_{21}$ and $\mathbf{F}_{22}$ are all known system matrices (obtained by calibration data, for example), and the measurement is typically given by

$$\mathbf{y}_m = \mathbf{H}_1 \mathbf{x}_1$$

where $\mathbf{H}_1$ is also a known $(n_1 \times n_1)$ system matrix. Under these circumstances, a suitable form for the estimator for $\mathbf{x}_2(t)$ is

$$\begin{aligned} \hat{\mathbf{x}}_2 &= \mathbf{L}\mathbf{y}_m + \mathbf{z} \\ \dot{\mathbf{z}} &= \bar{\mathbf{F}}\mathbf{z} + \bar{\mathbf{G}}\mathbf{y}_m + \bar{\mathbf{H}}u \end{aligned}$$

where a suitable set of matrices defining the reduced-order observer are given by:

$$\begin{aligned} \bar{\mathbf{F}} &= \mathbf{F}_{22} - \mathbf{L}\mathbf{H}_1\mathbf{F}_{12} \\ \bar{\mathbf{G}} &= \left\{ \mathbf{F}_{21} - \mathbf{L}\mathbf{H}_1\mathbf{F}_{11} + \bar{\mathbf{F}}\mathbf{L}\mathbf{H}_1 \right\} \mathbf{H}_1^{-1} \\ \bar{\mathbf{H}} &= \mathbf{G}_2 - \mathbf{L}\mathbf{H}_1\mathbf{G}_1 \end{aligned}$$

6. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s^a$
ITAE	1	$s+1$
	2	$s+0.7071 \pm 0.7071j^b$
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s+0.8660 \pm 0.5000j^b$
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a \pm bj)$ to conserve space.

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$u(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{a}{s(s+a)}$	$1-e^{-at}$
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$

END OF PAPER