EE4302 Advanced Control System CA3

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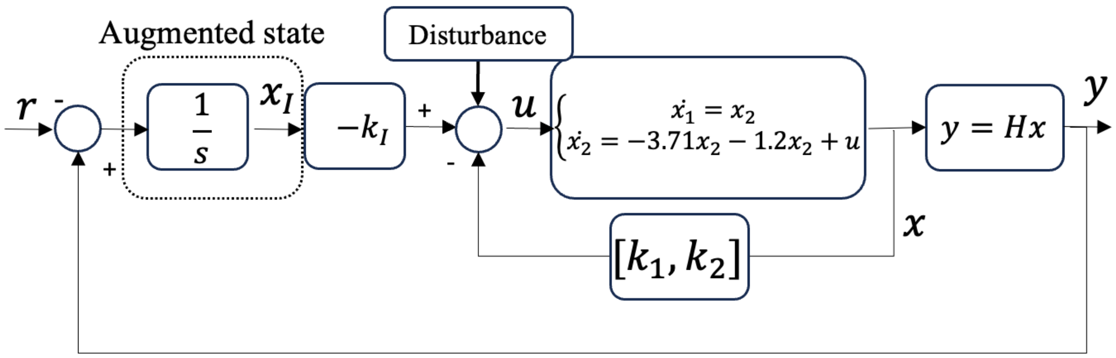
1. In the work in Section 4, the steady-state specification was met by the use of a scaling gain. **Briefly explain the disadvantage, if any, of that approach**.

Difficulty in Tuning: In practice, determining the appropriate scaling gain value can be challenging. It often requires extensive tuning and experimentation to find the right gain, and even then, it may not work well under all conditions.
Lack of Adaptability: A scaling gain doesn't adapt to changes in the system or environmental conditions. In dynamic systems with varying disturbances or changes in the plant dynamics, the fixed scaling gain may not provide the desired level of control.

2. New state-variable



Augmented state structure

Let

$$\dot{x}_I = y - r = x_1 - r$$

New augmented state space form is:

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \\ x_I \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1.2 & -3.71 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_I \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} r + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_I \end{bmatrix}$$

$$F = A = \begin{bmatrix} 0 & 1 & 0 \\ -1.2 & -3.71 & 0 \\ 1 & 0 & 0 \end{bmatrix}; \ H = C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$G_u = B_u = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; \ G_r = B_r = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}; \ G_v = B_v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

New requirement:

Closed-loop bandwidth:	Not lower than 1.5 rad/s;
Resonant Peak, M_r:	Not larger than 2dB;
Steady-state gain between r and y:	0dB.

Explain carefully why state augmentation will result in a closed-loop system meeting the specs for the two items of steady- state requirement.

Like my setting, the augmenting part is the target, we set

$$\dot{x}_I = y - r = x_1 - r$$

Expanding the state vector introduces extra degrees of freedom in crafting the control law. These additional states facilitate a more precise positioning of the closed-loop poles, a crucial factor in attaining the desired transient and steady-state response characteristics.

When the system achieves stability and converges to zero, the output closely tracks the reference signal. Examining the system structure, the integral action accumulates all the errors.

Why we don't need scaling gain when using augmentation method?

The augmented states include integral action (states representing the integral of the system error), this can inherently address steady-state errors without the need for an additional scaling gain. Integral action ensures that the system continually adjusts to eliminate any steady-state discrepancies.

3. ITAE and Ackerman formula approach

Check the ITAE table and place the desired poles to

1.5[-0.7081; -0.5210+1.068*1i; -0.5210-1.068*1i]*

The next is applying Ackerman formula. The procedure is the same as CA1.

$$\text{ans} = s^3 + 2.625\, s^2 + 4.837\, s + 3.374$$

$$W_c = [G \quad FG \quad F^2G]$$

The controllability matrix is:

```
      0    1.0000   -3.7100
 1.0000   -3.7100   12.5641
      0         0   -1.2000
```

Rank(Wc) = 3. So it's controllable.

$$\text{ans} = s^3 + 2.625\, s^2 + 4.837\, s + 3.374$$

```
Sigma = [G F*G F*F*G];
% s^3 + 1.4811s^2 + (1.8193-0.2872i)s+ 0.9006 + 0.2033i
Alpha_F = F^3 + 2.625*F^2 + 4.837*F + 3.374*eye(3)
```

```
Alpha_F = 3x3
     4.6760    7.6623         0
    -9.1948  -23.7513         0
    -4.3644    1.3020    3.3740
```

```
K = [0 0 1]*inv(Sigma)*Alpha_F
```

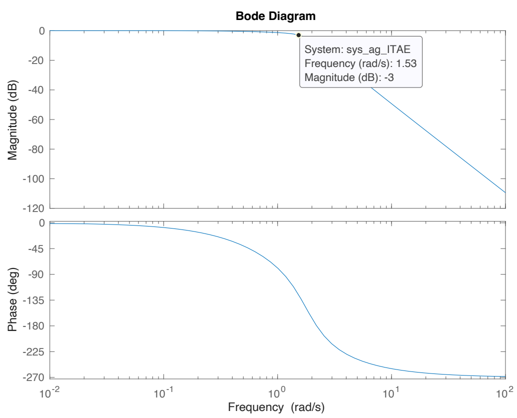
```
K = 1x3
     3.6370   -1.0850   -2.8117
```

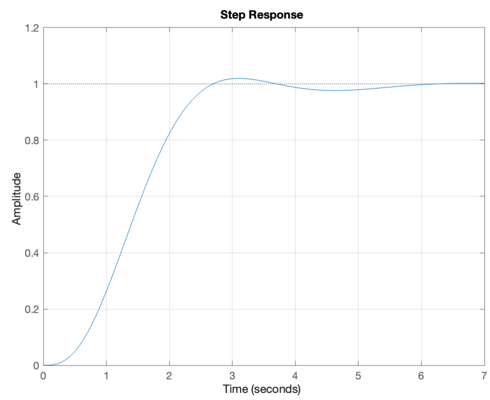
Double check the result:

```
% check ackerman formula result
K_check = acker(F, G, poles)
```

```
K_check = 1x3
     3.6373   -1.0848   -2.8122
```

We can see the result are almost the same.
The simulation results are





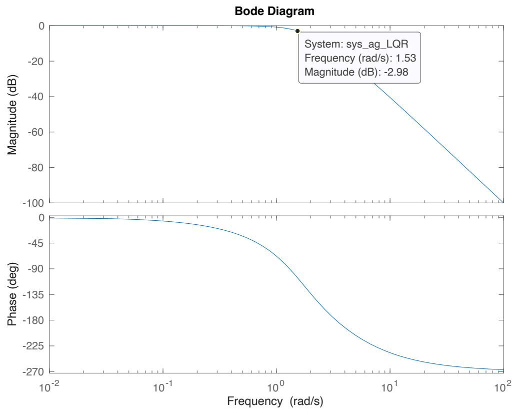
ITAE and Ackerman formula simulation results

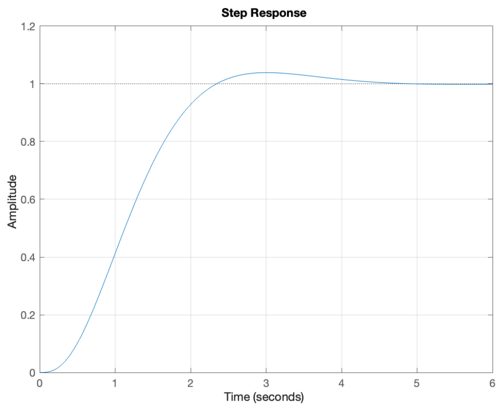
4. LQR approach

Keep R consistent as 1, change Q, the diagonal three parameters are R_1, R_2, R_3.
Change $R_{1,2,3}$ and watch the changes of frequency response I found that.
Increase R_1, R_2can make the frequency bandwidth decrease.
Increase R_3can make the frequency bandwidth increase.
After many tires I find the following parameters work:

```
Q_LQR = [5,   0,   0;
            0,   1,   0;
           0,   0, 100];
R_LQR = 1;
```

The simulation result





LQR approach simulation results

```
Q_LQR = [5,  0,  0;
        0,  1,  0;
        0,  0, 100];
R_LQR = 1;
[K_LQR,~,~] = lqr(Fbar, Gu, Q_LQR, R_LQR);
sys_ag_LQR = ss(Fbar-Gu * K_LQR, Gr, Hbar, 0);

[num, den] = ss2tf(Fbar-Gu * K_LQR, Gr, Hbar, 0);
sys_tf = tf(num,den)

figure(18)
bode(sys_ag_LQR)
figure(19)
step(sys_ag_LQR)
grid on

sys_ag_LQR_Control = ss(Fbar-Gu*K_LQR,Gr,-K_LQR,0);
% In sys_ag_LQR_Control, Hbar is changed to -K_LQR
% In this case, MATLAB would treat -K_LQR*x as the output of the system,
% which is nothing else but the control signal
% Therefore, by doing a unit step response analysis, the control signal
% response can be displayed
figure(20)
step(sys_ag_LQR_Control)
grid on
```

5. Second-Order Dominant (SOD) approach

We need calculate the desired poles first.
The Resonant Peak calculation formula is

$$M_r = \frac{1}{\sqrt{(2\delta^2)^2 + (2\delta\sqrt{1-2\delta^2})^2}}$$

Let

$$\frac{1}{2\,\sigma\,\sqrt{1-\sigma^2}} < 2$$

The solution is

$$0.26 < \sigma < 0.97$$

Let

$$\sigma = 0.5$$

The bandwidth is

$$w_b = w_n\sqrt{1 - 2\sigma^2 + \sqrt{2 - 4\sigma^2 + 4\sigma^4}} > 2$$

Calculated

$$w_n = 1.6$$

is choose solution.
Then calculate the poles of

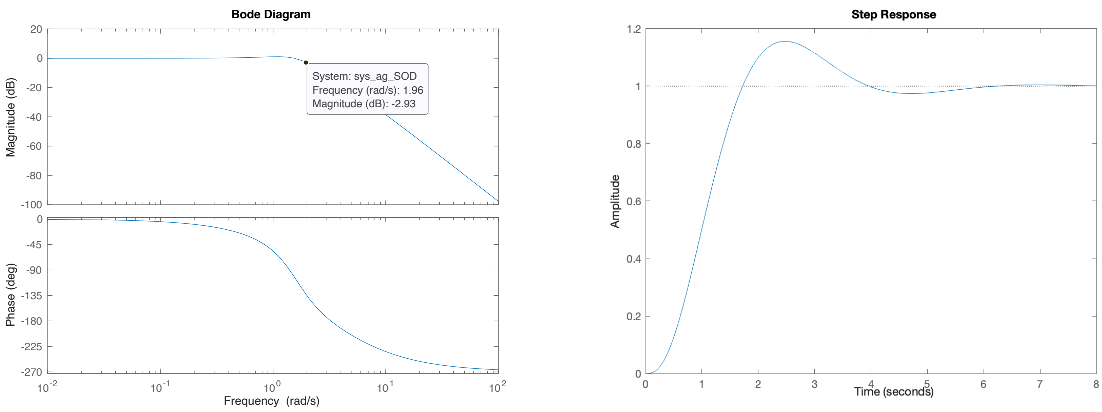
$$s^2 + 2\sigma w_n s + w_n^2 = 0$$

Get the poles:

$$\begin{pmatrix} -0.8 - 1.4\,i \\ -0.8 + 1.4\,i \end{pmatrix}$$

The third pole can be chosen far away from these two poles. I choose -5.
So, the three poles are:

$$[-5,\ -0.8 - 1.4i,\ -0.8 + 1.4i]$$

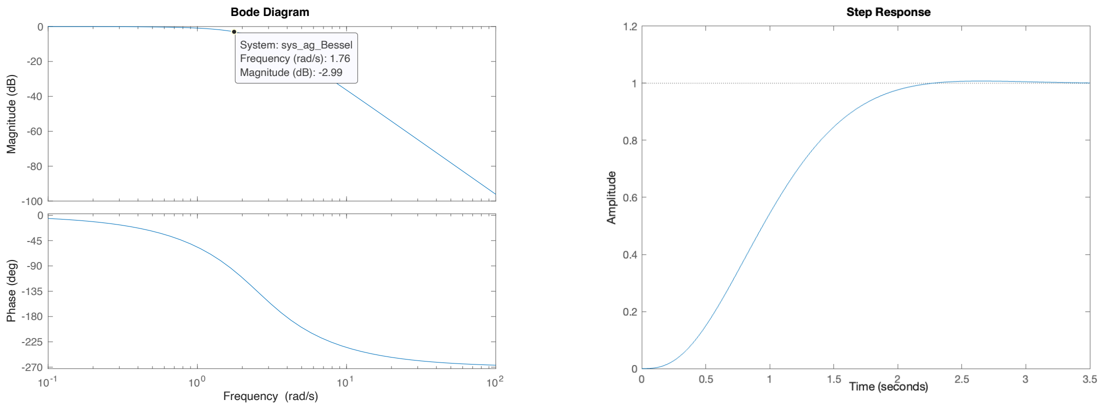


LQR approach simulation results

```
SOD_poles = [-5, -0.8-1.4i, -0.8+1.4i];
F = [0, 1 0;
   -1.2, -3.71 0;
   1 0 0]; % Process Matrix
Gu = [0; 1; 0]; % The input matrix for u
Gr = [0; 0; -1]; % The input matrix for r
Gv = [0; 1; 0]; % Disturbance input
H = [1, 0, 0]; % Output Matrix
K_SOD = acker(F, Gu, SOD_poles);
```

```
sys_ag_SOD = ss(F - Gu*K_SOD, Gr, H, 0);
bode(sys_ag_SOD)
```

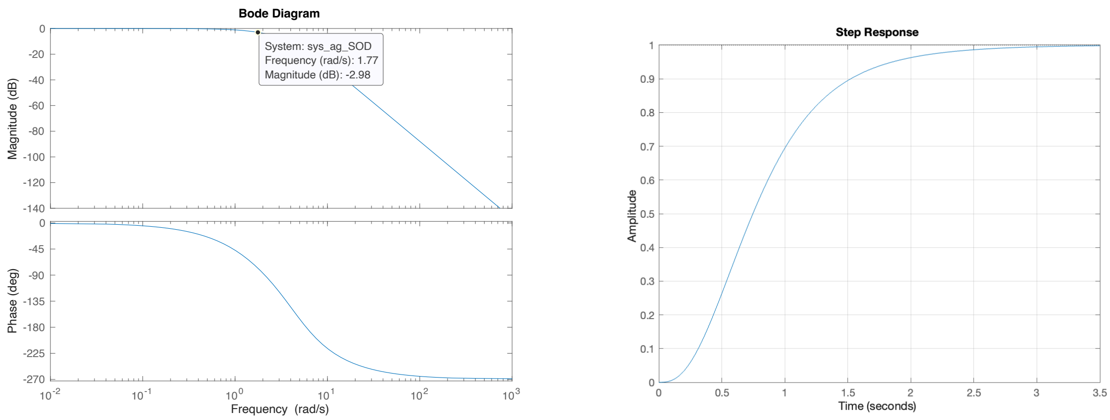
6. Bessel prototype table methodology



Bessel prototype simulation results

```
BesselPoles = 1.5*[-0.9420;-0.7455+0.7112*1i;-0.7455-0.7112*1i];
K_Bessel = acker(Fbar,Gu,BesselPoles);
sys_ag_Bessel = ss(Fbar-Gu*K_Bessel,Gr,Hbar,0);
figure(14)
bode(sys_ag_Bessel)
figure(15)
step(sys_ag_Bessel)
grid on
```

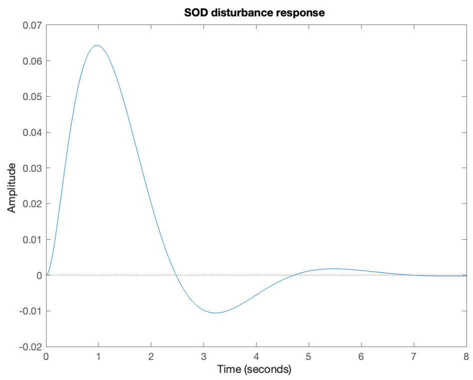
7. Second-Order Dominant Response methodology.

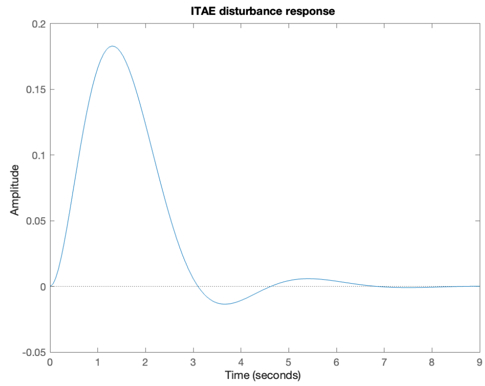


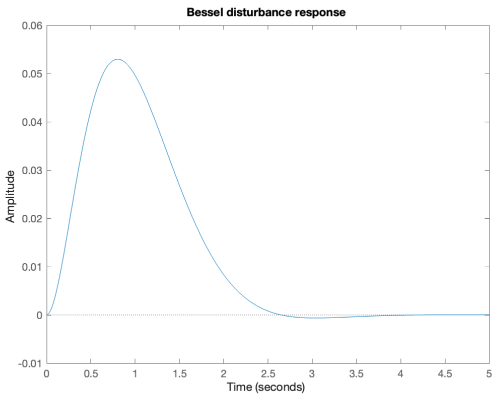
Second-Order Dominant simulation resluts

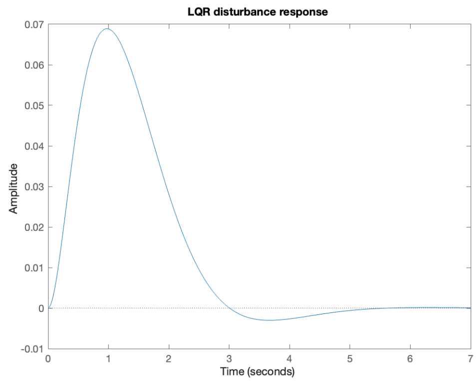
```
SODPoles = [-2; -3.6+2.7*1i; -3.6-2.7*1i];
K_SOD = acker(Fbar,Gu,SODPoles);
sys_ag_SOD = ss(Fbar-Gu*K_SOD,Gr,Hbar,0);
figure(16)
bode(sys_ag_SOD)
figure(17)
step(sys_ag_SOD)
grid on
```

8. Disturbance analysis





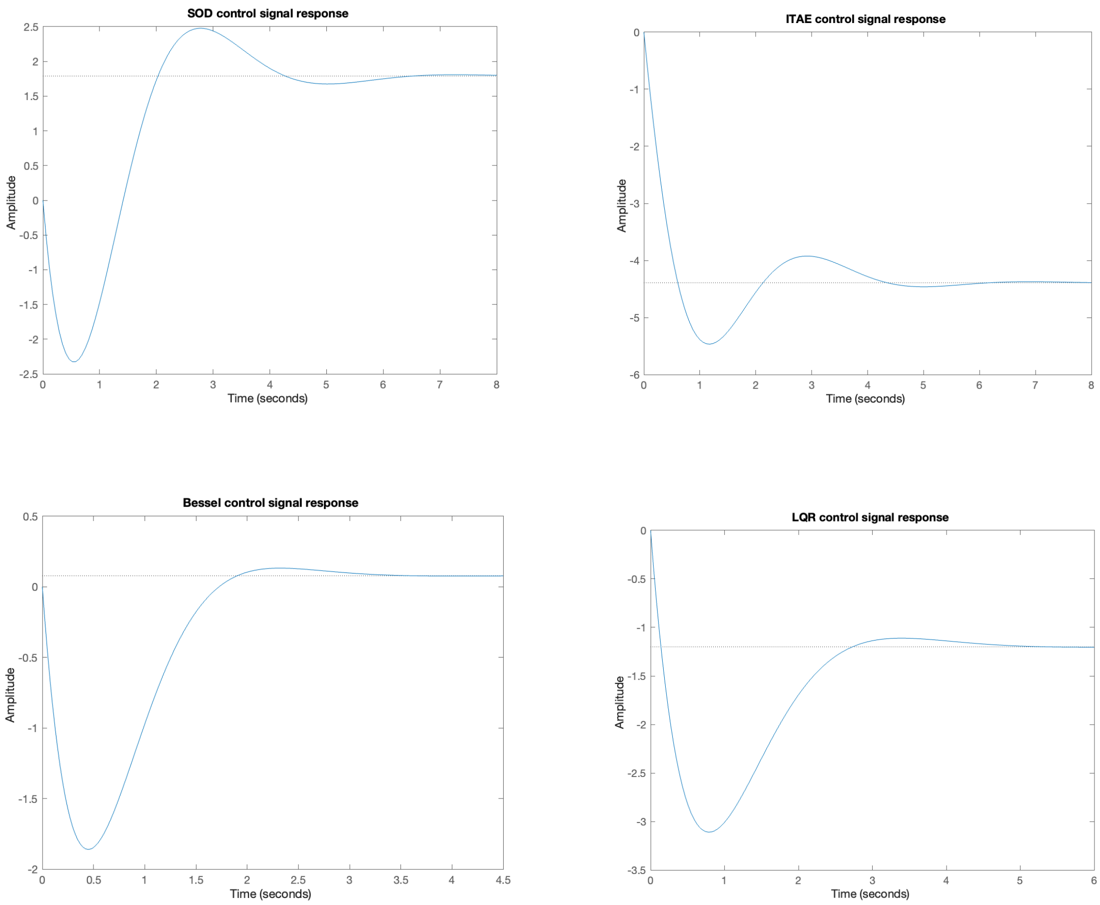




Disturbance comparation and analysis

Analysing the simulation outcomes reveals variations in amplitude and settling time, just for the output response, the ITAE and Bessel approaches exhibiting superior performance compared to other methods. The selection of poles significantly influences the results. Despite methodological differences, all four approaches prove effective, as evidenced by the prompt convergence of the disturbance response to zero.

9. Control signal analysis



Control signal comparation and analysis

```
%% Response to Control signal
% SOD Output response to control signal
sys_ag_SOD_ctr = ss(F -Gu * K_SOD, Gr, K_LQR, 0);
% Bode plot and step plot without scaling gain
step(sys_ag_SOD_ctr)
title('SOD control signal response')
% ITAE Output response to control signal
sys_ag_ITAE_ctr = ss(F -Gu * K_ITAE, Gr, K_LQR, 0);
% Bode plot and step plot without scaling gain
step(sys_ag_ITAE_ctr)
title('ITAE control signal response')

% Bessel Output response to control signal
sys_ag_Bessel_ctr = ss(F -Gu * K_Bessel, Gr, K_LQR, 0);
% Bode plot and step plot without scaling gain
step(sys_ag_Bessel_ctr)
title('Bessel control signal response')

% LQR Output response to control signal
sys_ag_LQR_ctr = ss(F -Gu * K_LQR, Gr, K_LQR, 0);
% Bode plot and step plot without scaling gain
step(sys_ag_LQR_ctr)
```

```
title('LQR control signal response')
```

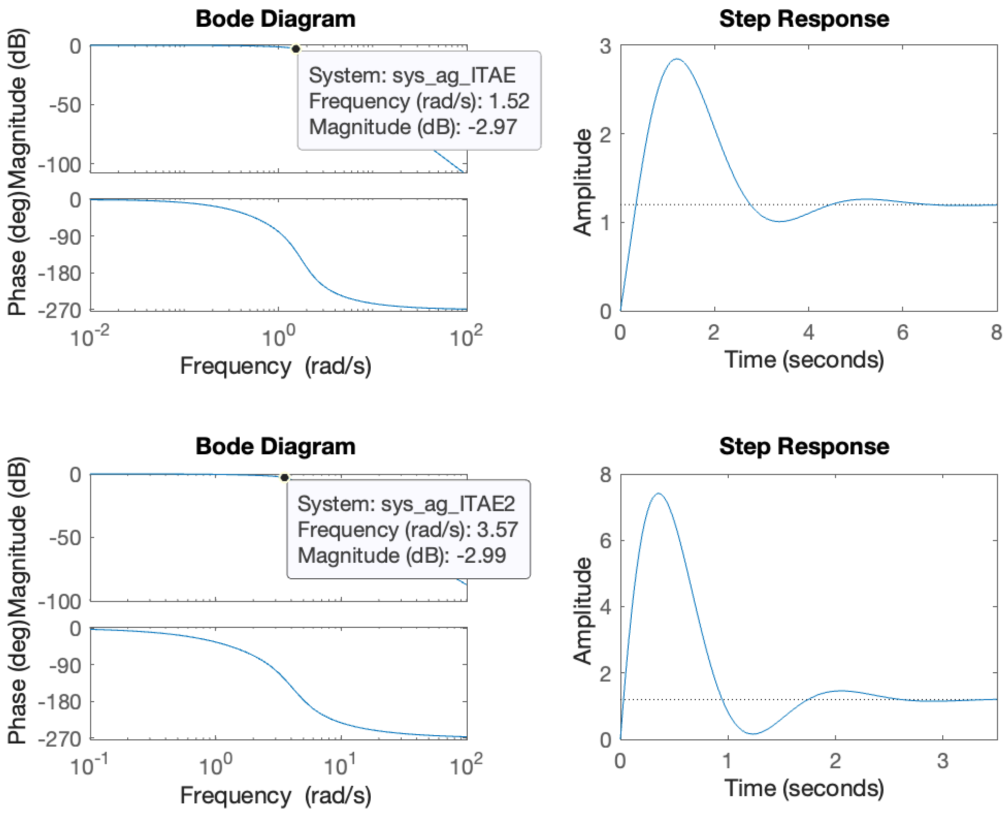
10. Transfer function comparation

```
sys_tf_SOD =

                13
  ----------------------------
  s^3 + 6.6 s^2 + 10.6 s + 13


sys_tf_ITAE =

                3.375
  ---------------------------------
  s^3 + 2.625 s^2 + 4.837 s + 3.375


sys_tf_Bessel =

                15.63
  --------------------------------
  s^3 + 6.082 s^2 + 15.41 s + 15.63


sys_tf_LQR =

                10
  ------------------------------
  s^3 + 5.886 s^2 + 11.14 s + 10
```

```
[num, den] = ss2tf(F-Gu * K_SOD, Gr, H, 0);
sys_tf_SOD = tf(num,den)
[num, den] = ss2tf(F-Gu * K_ITAE, Gr, H, 0);
sys_tf_ITAE = tf(num,den)
[num, den] = ss2tf(F-Gu * K_Bessel, Gr, H, 0);
sys_tf_Bessel = tf(num,den)
[num, den] = ss2tf(F-Gu * K_LQR, Gr, H, 0);
sys_tf_LQR = tf(num,den)
```

It's interesting to see that all the four transfer functions are similar with each other but the control results are different. It also demonstrates that the poles placement play a very important role to control the system to desired station. Overall, the Bessel method seems better, but we also need consider the control signal and other factors to judge them. Anyway, it's really interesting to see that the small changes of transfer function can bring so much difference.

11. Why does it make sense (assuming that we are operating using an identical "hardware set" in real-life) to require a lower closed-loop bandwidth ($1.5\ \mathrm{rad/s}$) here? Experiment, explore and discuss.



Higher and lower bandwidth comparation

Examining the simulation results, it becomes evident that a higher bandwidth is associated with increased overshoot and greater control energy. In contrast, systems with narrower bandwidths necessitate a lower closed-loop bandwidth. This choice holds relevance in specific scenarios, particularly when working with an identical hardware setup in practical applications. Such decisions often involve trade-offs between performance and stability.

Opting for a lower closed-loop bandwidth can enhance the system's capacity to reject noise and disturbances. This adjustment contributes to improved robustness in the control system by reducing sensitivity to variations in plant dynamics or uncertainties in parameters. Ultimately, lowering the bandwidth can lead to a more stable closed-loop system.

12. Additional exploration

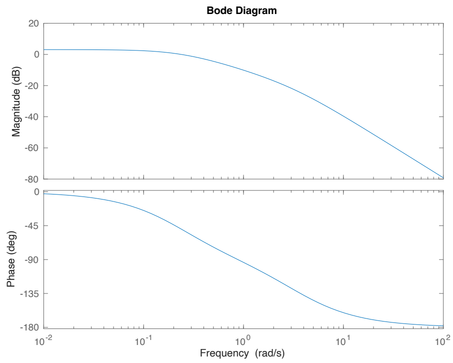
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -3.51x_2 - 0.77x_1 + 1.10u$$
$$y = x_1$$

Explore, discuss, analyze with all suitable simulations, analysis and descriptions in this type of situation on the use of the methods of:

- Using a "Scaling Gain"
- Using the augmented state variable $\dot{x}_I = y - r$

Provide a suitably comprehensive exploration / discussion.

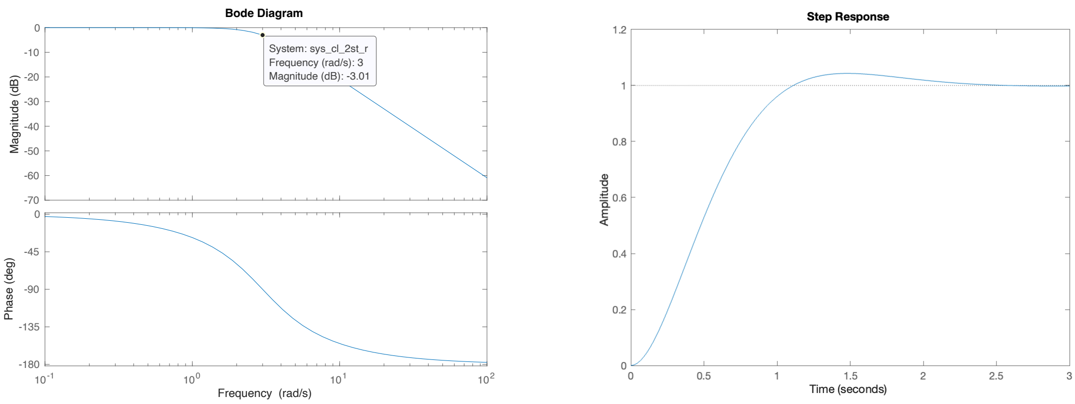
Open loop Bode diagram:



Addition system open loop bode diagram

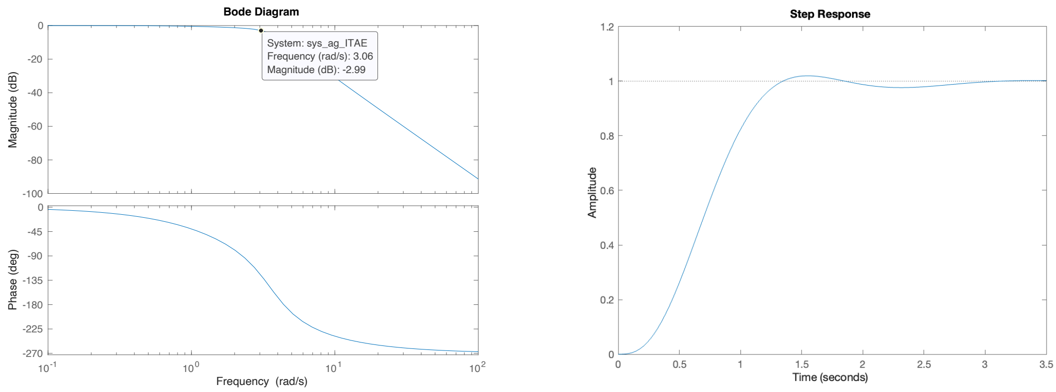
Suppose set the target bandwidth is bigger than 3dB

Scaling gain approach:



Additional system scaling gain control results

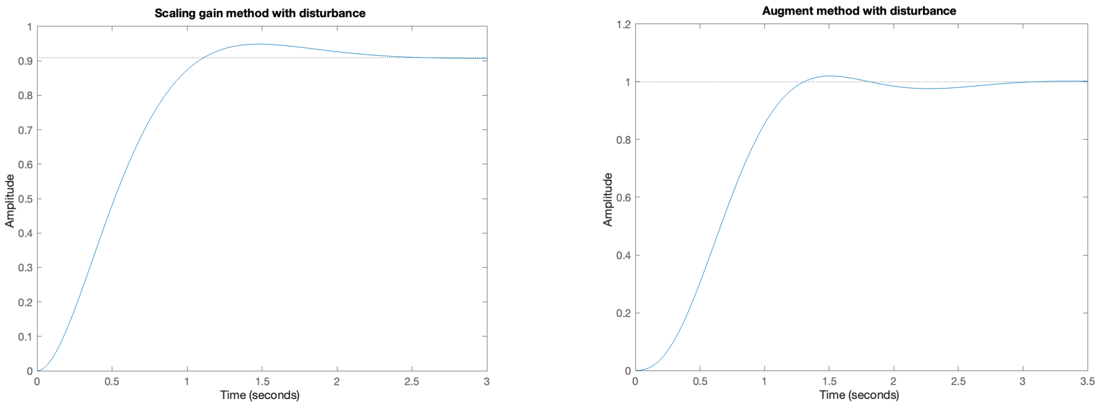
Augment state variable approach:



Additional system Augment state variable control results

13. Add disturbance

Add a small disturbance to these two systems, we can find the difference below.



Disturbance analysis on additional system

14. Comparation

Simulation results indicate that the scaling gain approach exhibits a faster and more precise response, albeit limited to steady state. Its structure and calculations are simpler, enabling quick implementation. However, determining the scaling gain in practice may be complex, requiring numerous experiments. On the other hand, the augmented state variable approach performs exceptionally well in the presence of disturbances, excelling in dynamic performance. Nevertheless, it introduces complexity, potential for instability, and increased computational load. The structure of the augmented state variable approach is inherently more complex.

15. Conclusion

This CA delved into the scaling gain and augmented state variable approaches, providing a clear understanding of their respective advantages and disadvantages. The augmented state variable method emerged as a powerful tool, effectively meeting steady-state requirements without the need for scaling gain. The exploration highlighted the importance of selecting the right control approach based on specific system requirements and achieving a balance between precision, adaptability, and robustness. Overall, the investigation serves as a valuable practical amendment to the theoretical concepts learned in class.