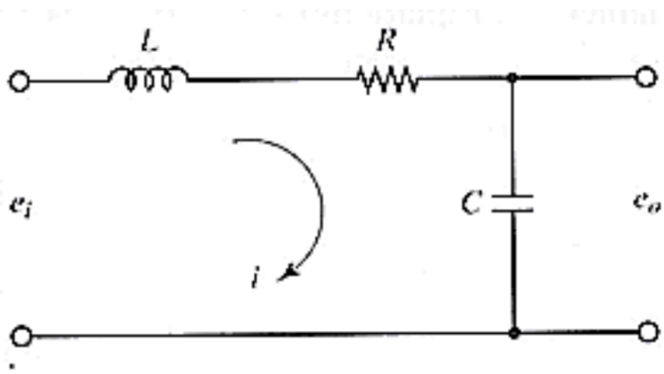
EE5103 Computer control System: Homework #1

Q1.

Consider the electrical circuit shown in the figure below. The circuit consists



of an inductance $L = 1$ henry, a resistance $R = 2$ ohm, and a capacitance $C = 0.5$ farad. Applying Kirchhoff's voltage law yields,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = e_i$$

$$\frac{1}{C}\int idt = e_o$$

a) Assuming e_i is the input u, and e_o, the output y, derive the transfer function of the system from the input u to output y.

(2 Marks)

$$i = C\dot{e}_o \rightarrow L\overset{LC\ddot{e}_o}{\frac{Cd\dot{e}_o}{dt^2}} + RC\dot{e}_o + e_o = e_i$$

$$e_i(s) = LCs^2e_o(s) + RCe_o(s) + e_o(s)$$

$$\frac{e_o(s)}{e_i(s)} = \frac{1}{LCs^2 + CRs + 1} = \frac{1}{0.5s^2 + s + 1} = \frac{2}{s^2 + 2s + 2}$$

b) Define state variables by

$$x_1 = e_o$$

$$x_2 = \dot{e}_o$$

derive the state-space representation of the system.

(2 Marks)

$x_1 = e_o \qquad x_2 = \dot{e}_o$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{LC}(e_i - RC x_2 - x_1) \end{cases} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{RC}{LC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} e_i$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$y = [1, 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c) Using zero-order-hold to sample the system, and assuming the sampling period $h = 1$, derive the state-space representation of the sampled system.

(2 Marks)

$$\begin{cases} x(k+1) = \Phi x(k) + \Gamma u(k) \\ y(k) = C x(k) \end{cases}$$

$$\Phi = e^{A} \qquad \Gamma = \int_0^1 e^{As} ds\, B$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Z transform

$$\begin{cases} zX(z) = \Phi X(z) + \Gamma U(z) \Rightarrow (zI - \Phi) X(z) = \Gamma U(z) \\ Y(z) = C X(z) \Rightarrow X(z) = C^{-1} Y(z) \end{cases}$$

$$(zI - \Phi)^{-1} \Gamma U(z) = C^{-1} Y(z)$$

$$\frac{Y(z)}{U(z)} = C(zI - \Phi)^{-1} \Gamma$$

$$\Phi = \begin{bmatrix} 0.5083 & 0.3096 \\ -0.6191 & -0.1108 \end{bmatrix} \qquad \Gamma = \begin{bmatrix} 0.4917 \\ 0.6191 \end{bmatrix}$$

d) Apply z-transform to the state-space model derived in c), and obtain the input-output model of the system.

(2 Marks)

$$G(z) = [1, 0] \left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0.5083 & 0.3096 \\ -0.6191 & -0.1108 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0.4917 \\ 0.6191 \end{bmatrix}$$

$$= [1, 0] \begin{bmatrix} z - 0.5083 & -0.3096 \\ 0.6191 & z + 0.1108 \end{bmatrix}^{-1} \begin{bmatrix} 0.4917 \\ 0.6191 \end{bmatrix}$$

$$= \frac{0.4917z + 0.2461}{z^2 - 0.3975z + 0.1353}$$

$$\Rightarrow z^2 Y(z) - 0.3975 z Y(z) + 0.1353 Y(z) = 0.4917 z U(z) + 0.2461 U(z)$$

$\mathcal{Z}^{-1}$ transform:

$$y(k+2) - 0.3975 y(k+1) + 0.1353 y(k) = 0.4917 u(k+1) + 0.2461 u(k)$$

The result calculated by Matlab:

$$\frac{z_1 \left(8112963841460668169578900514406 4\, z_1{}^2 - 3225164532296458930843201149337 6\, z_1 + 1097970259372477266810764724288 1\right)}{8112963841460668169578900514406 4\, z_1{}^3 - 1133812837375712800114202713784 32\, z_1{}^2 + 4323134791668936998579739317679 9\, z_1 - 1097970259372477167016612694243 1}$$

e) Assuming the initial conditions are $y(0) = 1$, and $\dot{y}(0) = 1$. Calculate the output sequence $y(k)$, under the unit step input, $u(k) = 1$ for $k \geq 0$.

(2 Marks)

$u(k) = 1 \quad (k \geq 0) \qquad U(z) = \frac{z}{z-1}$

$x_1(0) = y(0) = 1 \qquad x_2(0) = y'(0) = 1$

$$\frac{Y(z)}{U(z)} = C(zI - \Phi)^{-1}\left[zX(0) + T\,U(z)\right]$$

$$= [1, 0] \cdot \frac{1}{z^2 - 0.3975z + 0.1353}\begin{bmatrix} z+0.1108 & 0.3096 \\ 0.6191 & z-0.5083 \end{bmatrix}\begin{bmatrix} z + 0.4917\frac{z}{z-1} \\ z + 0.6191\frac{z}{z-1} \end{bmatrix}$$

$$= \frac{1}{z^2 - \cancel{1.}\,0.3975z + 0.1353}\left[z+0.1108,\ 0.3096\right]\begin{bmatrix} \frac{z^2 - z + 0.4917z}{z-1} \\ \frac{z^2 - z + 0.6191z}{z-1} \end{bmatrix}$$

$$= \frac{z^3 - 0.0879z^2 - 0.1742z}{z^3 - 1.3975z^2 + 0.5328z - 0.1353}$$

$Y(z) = G(z) \cdot U(z)$

$\mathcal{Z}^{-1}$ transform:

$y(k+3) - 1.3975(k+2) + 0.5328\,\cancel{k}\,y(k+1) - 0.1353\,y(k) = u(k+3) - 0.0879\,u(k+2) - 0.1742\,u(k+1)$

$u(k) = 1 \quad (k \geq 0)$

C*inv(z1*I-double(phi))*(z1*X0 + double(tao)*Uz):

$$\frac{z_1\left(8112963841460668169578900514406 4\,z_1^{\,2} - 3225164532296458930843201149337 6\,z_1 + 1097970259372477266810764724288 1\right)}{8112963841460668169578900514406 4\,z_1^{\,3} - 1133812837375712800114202713784 32\,z_1^{\,2} + 4323134791668936998579739317679 9\,z_1 - 1097970259372477167016612694243 1}$$

[z1+0.1108 0.3096]*[(z1^2-z1+0.4917*z1)/(z1-1); (z1^2-z1+0.6191*z1)/(z1-1)]:

$$-\frac{z_1\left(-25000000\,z_1^{\,2} + 2197500\,z_1 + 4356157\right)}{25000000\,(z_1 - 1)}$$

Q2

Consider the system

$$G(s) = \frac{s-1}{s^2(s+1)}$$

a) Is the system stable? Does the system have a stable inverse? Justify your answers.

(3 Marks)

(a) Assume $\frac{as+b}{s^2} + \frac{c}{s+1} = \frac{s-1}{s^2(s+1)}$

$(a+c)s^2 + (a+b)s + b = s-1$

$\begin{cases} a+c=0 \\ a+b=1 \\ b=-1 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=-1 \\ c=-2 \end{cases}$

$\frac{2}{s} - \frac{1}{s^2} - \frac{2}{s+1} = \frac{s-1}{s^2(s+1)}$

$\mathcal{L}^{-1}\left[\frac{s-1}{s^2(s+1)}\right] = \mathcal{L}^{-1}\left[\frac{2}{s} - \frac{1}{s^2} - \frac{2}{s+1}\right]$

$f(t) = e^{-2t} - t - 2e^{-t}$

$t \to \infty$ $f(\infty) \to -\infty$ not stable

b) Is it possible to choose the sampling period h such that the sampled system is stable? Justify your answer.

(3 Marks)

(b) Based on Table 2.1 in note book.

$G(s) = \cancel{s+s}\ \frac{s-1}{s^2(s+1)}$

$a=0$ $b=1$ $c=-1$

$\begin{cases} a_1 = -1-e^{-h} \\ a_2 = e^{-h} \end{cases}$ $\begin{cases} \cancel{b_1 = -[e^{-h} + 1(1-e^{-h}) \times (-1)]} \\ \cancel{b_2 =} \end{cases}$

$z^2 + (-1-e^{-h})z + e^{-h} = 0$

$z = \frac{1+e^{-h} \pm \sqrt{(1+e^{-h})^2 - 4e^{-h}}}{2} = \frac{1+e^{-h} \pm (1-e^{-h})}{2}$

$\begin{cases} z_1 = 1 \\ z_2 = e^{-h} \end{cases}$ $z_1 = 1$ Can't be changed by sampling.

So can't make it stable.

c) Is it possible to choose the sampling period h such that the sampled system has a stable inverse? Justify your answer.

(4 Marks)

$$G(s) = \frac{s-1}{s^2(s+1)} = \frac{1}{s(s+1)} - \frac{1}{s^2(s+1)}$$

for $\frac{1}{s(s+1)}$ $\quad b_1 = h-1+e^{-h} \quad b_2 = 1-e^{-h}-he^{-h} \quad a_1 = -(1+e^{-h}) \quad a_2 = e^{-h}$

for $\frac{1}{s^2(s+1)}$ $\quad b_1 = \frac{h^2}{2}-1 \quad b_2 = 2h \quad b_3 = -\frac{h^2}{2}-h \quad a_1 = -3 \quad a_2 = 3 \quad a_3 = -1$

Z transform is: $\dfrac{(h-1+e^{-h})z+1-e^{-h}-he^{-h}}{z^2-(1+e^{-h})z+e^{-h}} + \dfrac{\frac{h^2-2}{2}z^2+2hz-\frac{h^2}{2}-h}{z^3-3z^2+3z-1}$.

Calculated by Matlab:

$$3\,z - \frac{h-2\,h\,z-z^2\left(\frac{h^2}{2}-1\right)+\frac{h^2}{2}}{z^3} - \frac{e^{-h}-z\left(h+e^{-h}-1\right)+h\,e^{-h}-1}{z^2+\left(-e^{-h}-1\right)z+e^{-h}} - 3\,z^2 - 1$$

This form is too complicated to solve.

By analysis, just like the approvement in course, the sampling will introduce new zeros to the transfer function. The original system's zero is z = 1. It's marginal stable. So, it's possible to introduce new zeros which close to zero and make the inverse system stable.

Q3

Consider the system

$$x(k+1) = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix} x(k) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)$$

a) Is the system stable? Is the system controllable? Is the system observable? Justify your answers.

(2 Marks)

(a) $\quad \Phi\ A = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$ eigenvalues of Φ A are $(-2, 3)$

$|-2| > 1 \quad 3 > 1$ system is not stable.

~~$W_c = (0$~~ $\quad W_c = (\Gamma \;\; \Phi\Gamma) = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}$ Rank $(W_c) = 2$

It's controllable

$W_o = \begin{bmatrix} C \\ C\Phi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$ Rank $(W_o) = 2$

It's observable.

b) Use z-transform to obtain the transfer function of the system. Write down the input-output difference equation.

(2 Marks)

(b) Method 1. observable canonical form

$$z(k+1) = \begin{bmatrix} -(-\frac{1}{3}) & 1 \\ -(-2) & 0 \end{bmatrix} z(k) + \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix} u(k)$$

$$H(z) = \frac{\frac{1}{3}z+1}{z^2-\frac{1}{3}z-2} = \frac{z+3}{z^2-z-6}$$

2. $$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases}$$

z transform $$\begin{cases} zX(z) = AX(z) + BU(z) \\ Y(z) = CX(z) \end{cases}$$

$$U(z) = B^{-1}[zI - \cancel{\Phi}]X(z)$$

$$\frac{Y(z)}{U(z)} = C(zI-A)^{-1}B = \frac{z+3}{z^2-z-6} \text{ (matlab)}$$

$$z^2Y(z) - zY(z) - 6Y(z) = zU(z) + 3U(z)$$

$$z^{-1} \rightarrow y(k+2) - y(k+1) - 6y(k) = x(k+1) + 3x(k)$$

c) Assume the system is controlled by a proportional controller

$$u(k) = K(u_c(k) - y(k))$$

Derive the transfer function from the command signal $u_c(k)$ to the output $y(k)$.

(2 Marks)

(c)

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases}$$

$$u(k) = K[u_c(k) - y(k)]$$

$$\begin{cases} x(k+1) = Ax(k) + BK[u_c(k) - y(k)] \\ y(k) = Cx(k) \end{cases}$$

$\Downarrow$ z transform

$$\begin{cases} zX(z) = AX(z) + BK[U_c(z) - Y(z)] \\ Y(z) = CX(z) \end{cases}$$

$$Y(z) = C(zI - A)^{-1}BK[U_c(z) - Y(z)]$$

Use $T = C(zI-A)^{-1}B$.

① $$\frac{Y(z)}{U(z)} = \frac{KT}{1+KT} = \frac{kz+3K}{z^2+(k-1)z+3k-6}$$

If directly use the form of feed back control

② $$G(z) = \frac{KT(z)}{1+T(z)K} \quad * \quad \cancel{z} \quad T(z) = \frac{z+3}{z^2-z-6}$$

① ② are the same.

$$G(z) = \frac{k(z+3)}{z^2+(k-1)z+3k-6}$$

d) Apply Jury's stability criterion to determine the range of controller gain, K, such that the closed-loop system is stable.

(2 Marks)

(d)

	$a_1 = 1$	$a_2 = k-1$	$a_3 = 3k-6$	
Reverse	$3k-6$	$k-1$	1	$\times(3k-6)$
	$(3k-6)^2$	$(k-1)(3k-6)$	$3k-6$	
	$1-(3k-6)^2$	$k-1-(k-1)(3k-6)$		
Reverse	$(k-1)(7-3k)$	$1-(3k-6)^2$		$\times \frac{(k-1)(7-3k)}{1-(3k-6)^2}$
	$1-(3k-6)^2 - \frac{(k-1)^2(7-3k)^2}{1-(3k-6)^2}$			

① $1-(3k-6)^2 > 0 \Rightarrow \frac{5}{3} < k < \frac{7}{3}$

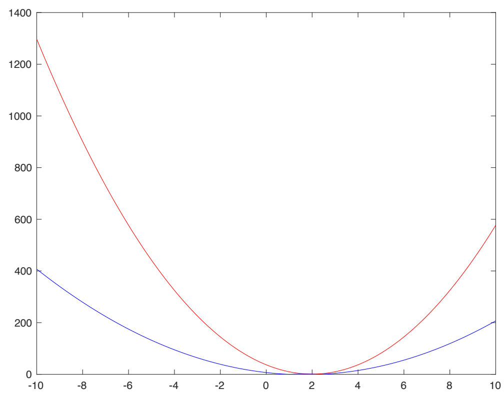
② $1-(3k-6)^2 - \frac{(k-1)^2(7-3k)^2}{1-(3k-6)^2} > 0 \Rightarrow [1-(3k-6)^2]^2 > [(k-1)(7-3k)]^2$

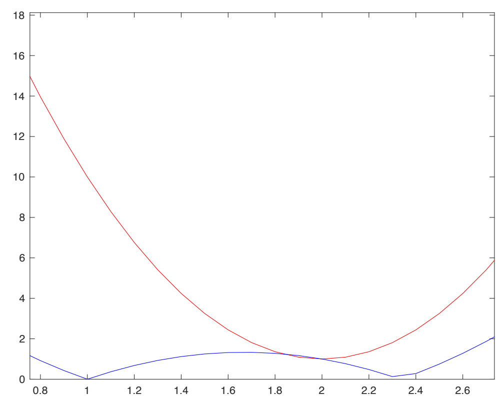
$|1-(3k-6)^2| > |(k-1)(7-3k)| \Rightarrow |9k^2-36k+37| > |-3k^2+10k-7|$

If $|9k^2-36k+37| = |-3k^2+10k-7|$ $\cancel{k_1 = \frac{5}{3}}$ $\cancel{k_2 = \frac{7}{3}}$

solution: $k_1 = 2$ $k_2 = \frac{11}{6}$ So $k > 2$ or $k < \frac{11}{6}$

① and ② $\boxed{2 < k < \frac{7}{3} \text{ or } \frac{5}{3} < k < \frac{11}{6}}$





e) Determine the steady-state error, $u_c - y$, when u_c is a unit step.

(2 Marks)

e) state gain

$$G = \frac{k(\cancel{z}1+3)}{1+k-1+3k-6} = \frac{\cancel{z}4k}{4k-6}$$

steady state error

$$E = \frac{G}{1+G} = \frac{\frac{4k}{4k-6}}{1+\frac{2k}{2k-3}} = \frac{2k}{4k-3}$$

$y(k+2)+3y(k+1)+2y(k) = u(k+1)+u(k)$

Q4

Consider the system described by the following difference equation

$$y(k+1) = 3y(k) - 2y(k-1) + u(k-1) + 2u(k-2)$$

a) What is the transfer function? Is the system stable? Does the system have a stable inverse?

(2 Marks)

a) $y(k+3) = 3y(k+2) - 2y(k+1) + u(k+1) + 2u(k)$

$\mathcal{Z}$ transform: $z^3Y(z) - 3z^2Y(z) + 2zY(z) = zU(z) + 2U(z)$

$$\frac{Y(z)}{U(z)} = \frac{z+2}{z(z-1)(z-2)}$$

Zero is $z=-2$ $\Downarrow$ Inverse is not stable.

Poles: $z_1=0$ $z_2=1$ $\underline{z_3=2}$ $\Downarrow$ Not stable

```
z = sym('z')
A = [3 -2 0; 1 0 0; 0 1 0];
B = [1, 0, 0]';
C = [0, 1, 2];
I = eye(3,3);
new_T = simplify(C * inv(z*I - A)* B)

z =

z


new_T =

(z + 2)/(z*(z^2 - 3*z + 2))
```

b) Is it possible to realize the system such that it is observable but not controllable? If yes, write down the corresponding state-space equation. If no, justify your answer.

(3 Marks)

b) $y(k+1) = 3y(k) - 2y(k-1) + u(k-1) + 2u(k-2)$

let : $x_1(k) = y(k)$

$x_1(k+1) = 3x_1(k) - 2y(k-1) + u(k-1) + 2u(k-2)$

let : $x_2(k) = -2y(k-1) + u(k-1) + 2u(k-2)$

~~let~~ $x_1(k+1) = 3x_1(k) + x_2(k)$

$x_2(k+1) = -2x_1(k) + u(k) + 2u(k-1)$

let : $x_3(k) = 2u(k-1)$

$x_2(k+1) = -2x_1(k) + x_3(k) + u(k)$

$x_3(k+1) = 2u(k)$

$$\begin{cases} x_1(k+1) = 3x_1(k) + x_2(k) \\ x_2(k+1) = -2x_1(k) + x_3(k) + u(k) \\ x_3(k+1) = 2u(k) \end{cases} \qquad \begin{cases} x_1(k) = y(k) \\ x_2(k) = -2y(k-1) + u(k-1) + 2u(k- \\ x_3(k) = 2u(k-1) \end{cases}$$

error

$$x(k+1) = \underbrace{\begin{pmatrix} 3 & 1 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_{\Phi} x(k) + \underbrace{\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}}_{\Gamma} u(k)$$

$$y(k) = \underbrace{(1, 0, 0)}_{C}\, x(k)$$

$$W_c = (\Gamma \ \Phi\Gamma \ \Phi^2\Gamma) = \begin{pmatrix} 0 & 1 & 5 \\ 1 & 2 & -2 \\ 2 & 0 & 0 \end{pmatrix} \quad \text{rank}(W_c) = 3 \qquad \text{Controllable}$$

$$W_o = \begin{pmatrix} C \\ C\Phi \\ C\Phi^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 7 & -3 & 1 \end{pmatrix} \quad \text{rank}(W_o) = 3 \qquad \text{Observable.}$$

Add poles and zeros:

$$\frac{z(z+2)}{z(z^3 - 3z^2 + 2z)} = \frac{z^2 + 2z}{z^4 - 3z^3 + 2z^2}.$$

~~The same~~ $a_1 = -3 \quad a_2 = 2 \quad a_3 = 0 \quad a_4 = 0$

$b_1 = 0 \quad b_2 = 1 \quad b_3 = 2 \quad b_4 = 0$

In observable canonical form

$$z(k+1) = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} z(k) + \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [1, 0\ 0\ 0]\, z(k)$$

$$W_c = [\Gamma \ \Phi\Gamma \ \Phi^2\Gamma \ \Phi^3\Gamma] = \begin{bmatrix} 0 & 1 & 5 & 13 \\ 1 & 2 & -2 & -10 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\text{Rank}(W_c) = 3 < 4$

So It's observable but not controllable.

```
phi=[3 -2 0 0; 1 0 0 0; 0 1 0 0; 0 0 1 0];
C = [0 1 2 0];
Wo = [C; C*phi; C*phi*phi; C*phi*phi*phi];
rank(Wo)

ans =

     3

>> Wo

Wo =

     0     1     2     0
     1     2     0     0
     5    -2     0     0
    13   -10     0     0
```

c) Is it possible to realize the system such that it is controllable but not observable? If yes, write down the corresponding state-space equation. If no, justify your answer.

(3 Marks)

$$G(z) = \frac{z+2}{z^3-3z^2+2z} \quad \text{add zeros and poles}$$

$$\frac{z(z+2)}{z(z^3-3z^2+2z)} = \frac{z^2+2z}{z^4-3z^3+2z^2}$$

$a_1=-3 \quad a_2=2 \quad a_3=0 \quad a_4=0$

$b_1=0 \quad b_2=1 \quad b_3=2 \quad b_4=0$

In controllable form

$$z(k+1) = \begin{bmatrix} 3 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} z(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k)$$

~~y(k+1)~~ $y(k) = \begin{bmatrix} 0 & 1 & 2 & 0 \end{bmatrix} z(k)$

$$W_o = \begin{bmatrix} C \\ C\Phi \\ C\Phi^2 \\ C\Phi^3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 2 & 0 & 0 \\ 5 & -2 & 0 & 0 \\ 13 & -10 & 0 & 0 \end{bmatrix} \quad \mathrm{Rank}(W_o) = 3 < 4$$

So It's controllable but not observable

```
>> phi=[3 1 0 0; -2 0 1 0; 0 0 0 1; 0 0 0 0];
tao = [0 1 2 0]';
Wc = [tao phi*tao phi*phi*tao phi*phi*phi*tao ];
rank(Wc)

ans =

     3

>> Wc

Wc =

     0     1     5    13
     1     2    -2   -10
     2     0     0     0
     0     0     0     0
```

d) Is it possible to realize the system such that it is both controllable and observable? If yes, write down the corresponding state-space equation. If no, justify your answer.

(2 Marks)

In question b), the state-space is the controllable and observable.
Directly based on the transformation, apply the controllable canonical form can get the same result, the realization is controllable and observable.

Assuming it's controllable

$a_1 = -3 \quad a_2 = 2 \quad a_3 = 0 \qquad b_1 = 0 \quad b_2 = 1 \quad b_3 = 2$

$$x(k+1) = \underbrace{\begin{bmatrix} 3 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{A} x(k) + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{B} u(k)$$

$$y(k) = \underbrace{[0, 1, 2]}_{C}\, x(k)$$

$$W_o = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 5 & -2 & 0 \end{bmatrix} \qquad Rank(W_o) = 3$$

Do reverse to check the result:

```
z = sym('z')
A = [3 -2 0; 1 0 0; 0 1 0];
B = [1, 0, 0]';
C = [0, 1, 2];
I = eye(3,3);
new_T = simplify(C * inv(z*I - A)* B)

z =

z


new_T =

(z + 2)/(z*(z^2 - 3*z + 2))
```

The transfer function is the same as the a), this realization satisfy the requirement.