Vector Control of Induction Motor Drives

Modeling and Control

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Vector Control of Induction Motor: Model, Control and Behaviour

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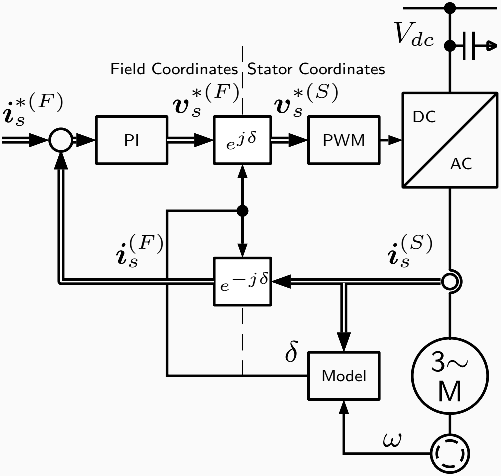


Recap:dynamics

Equations describing the AC machine

Dynamics of AC motor

The AC machine system description of given as

$$
\begin{aligned}
\vec{v}_s^s &= r_s \vec{i}_s^s + \frac{d\vec{\psi}_s^s}{d\tau} \\
0 &= r_r \vec{i}_r^s + \frac{d\vec{\psi}_r^s}{d\tau} - j\omega \vec{\psi}_r^s \\
\vec{\psi}_s &= l_s \vec{i}_s + l_h \vec{i}_r \\
\vec{\psi}_r &= l_h \vec{i}_s + l_r \vec{i}_r \\
m_e &= \vec{\psi}_s^s \times \vec{i}_s^s \\
m_e &= \Im\{\vec{\psi}_s^* \vec{i}_s\} \\
m_e &= \Im\{(\psi_{s\alpha} - j\psi_{s\beta})(i_{s\alpha} + j i_{s\beta})\} \\
p_e &= \Re\{\vec{v}_s^* \vec{i}_s\} \\
p_e &= \Re\{(v_{s\alpha} - j v_{s\beta})(i_{s\alpha} + j i_{s\beta})\} \\
\tau_m \frac{d\omega}{d\tau} &= m_e - m_L
\end{aligned}
$$

State equations of Induction machine

The state equations for the electromagnetic system of Induction machine can be written as

State equations with rotor flux and stator current as state variables

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \tag{1}$$

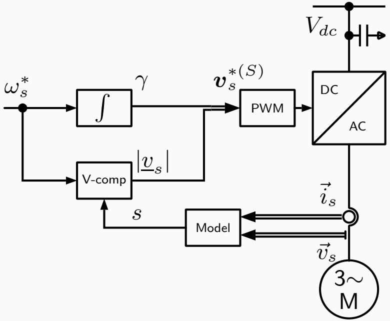
$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \tag{2}$$

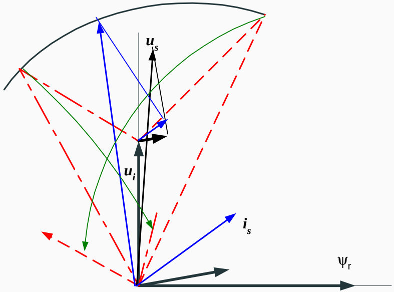
$$m_e = \vec{\psi}_s \times \vec{i}_s \tag{3}$$

$$m_e = k_r\vec{\psi}_r \times \vec{i}_s = k_r\left[\vec{\psi}_r \perp \vec{i}_r\right] \tag{4}$$

$$\frac{d\omega}{d\tau} = \frac{1}{\tau_m}\left(m_e - m_l\right) \tag{5}$$

Need to know position of Rotor Flux vector





- Even if we compensate the voltage to keep $|\vec{\psi}_r| = \text{constant}$
- We may not achieve it, as we are applying a voltage that can be any where w.r.to $\vec{\psi}_r$
- Hence, this method is called **scalar control**

Why scalar control does not work?

- If we use scalar control method, we only know the required voltage magnitude in steady state and the desired frequency.
- If we suddenly apply this voltage using the PWM set-up, what the motor sees is a sudden change in the state voltage vector. **It is no more in steady state and behaves as determined by the dynamic equations**

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}(1 - j\omega\tau_r)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \tag{6}$$

$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}(1 - j\omega\tau_r)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \tag{7}$$

- Hence it produces large un-controlled currents and transients
- **How should we control it**

First principle of Vector Control

- Torque is given by

$$m_e = k_r \left(\vec{\psi}_r \perp \vec{i}_s \right)$$

- which means if we change the current vector perpendicular to rotor flux vector, we can change torque.
- To change current vector we should satisfy

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s}$$

- Hence, we should control the **state voltage vector** with respect to **rotor flux vector**
- **Which we should know the position of all state variable space vectors with respect to each other: This is vector control**
- **But first we should know the position of rotor flux vector as we will want to keep it constant and hence we can use it as our base coordinate**

Flux estimators

We can estimate the rotor flux space vector using the stator voltage equation i

From Stator voltage equations, and flux linkage relation, we get

$$\vec{v}_s = r_s \vec{i}_s + \frac{d\vec{\psi}_s}{d\tau} \tag{8}$$

$$\vec{\psi}_s = k_r \vec{\psi}_r + \sigma l_s \vec{i}_s \tag{9}$$

$$\therefore \vec{\psi}_s = \int \vec{v}_s - r_s \vec{i}_s d\tau \tag{10}$$

$$\vec{\psi}_r = \frac{1}{k_r} \left[\vec{\psi}_s - \sigma l_s \vec{i}_s \right] \tag{11}$$

We can split this into real and imaginary components. Hence the derivative function will be

$$\frac{d\vec{\psi}_{s,\alpha}}{d\tau} = \vec{v}_{s\alpha} - r_s \vec{i}_{s\alpha} \tag{12}$$

$$\frac{d\vec{\psi}_{s\beta}}{d\tau} = \vec{v}_{s\beta} - r_s \vec{i}_{s\beta} \tag{13}$$

Stator voltage based rotor flux estimator

state voltage based rotor flux estimator

$$\frac{d\vec{\psi}_{s,\alpha}}{d\tau} = \vec{v}_{s\alpha} - r_s\vec{i}_{s\alpha} \quad (14)$$

$$\frac{d\vec{\psi}_{s\beta}}{d\tau} = \vec{v}_{s\beta} - r_s\vec{i}_{s\beta} \quad (15)$$

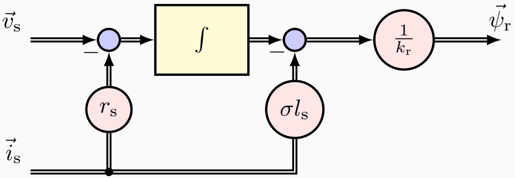
$$\vec{\psi}_{r\alpha} = \left(\vec{\psi}_{s\alpha} - \sigma l_s\vec{i}_{s\alpha}\right)\frac{1}{k_r} \quad (16)$$

$$\vec{\psi}_{r\beta} = \left(\vec{\psi}_{s\beta} - \sigma l_s\vec{i}_{s\beta}\right)\frac{1}{k_r} \quad (17)$$

We need to measure stator voltage space vector and the stator current space vector

- Measure the 3 phase instantaneous voltages $v_U(t), v_V(t), v_W(t)$
- Calculate the space vector $\vec{(v)}_s(t) = v_{s\alpha}(t) + jv_{s\beta}(t)$
- Do the same with 3 phase instantaneous currents $i_U(t), i_V(t), i_W(t)$
- Use the stator voltage model to estimate Stator and Rotor flux vector
- **You will need to know the motor parameters accurately (here lies the problem**

Stator Model based Rotor Flux estimator: Signal flow graph



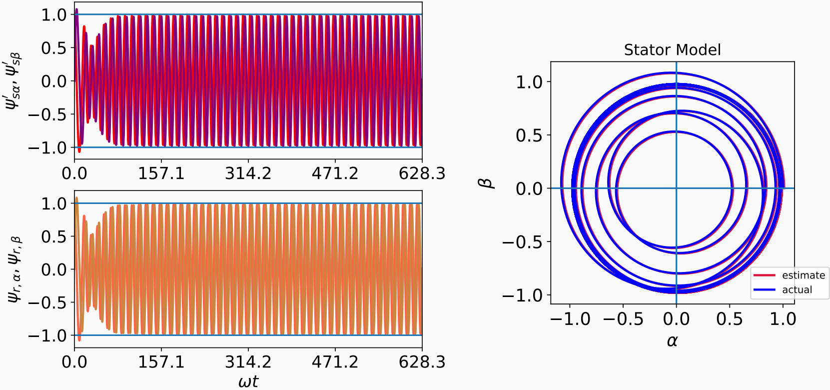
$$tan(\delta) = \frac{\psi_{r\beta}(\tau)}{\psi_{r\alpha}(\tau)}$$

$$cos(\delta) = \frac{\psi_{r\alpha}(\tau)}{\sqrt{\psi_{r\alpha}^2 + \psi_{r\beta}^2}}$$

$$\delta = \arccos\left(\frac{\psi_{r\alpha}(\tau)}{|\psi_r|}\right)$$

- Sketch the model for real and imaginary components
- Do not use arctan method to find angle as denominator goes to zero
- Use arccos as the denominator is magnitude of vector and non-zero usually

Stator Model based Rotor Flux estimator: Simulation



- Upper Curve is Estimated value and lower curve is actual value
- Right hand is the locus of rotor flux

Estimating rotor flux space vector using the rotor equation i

Similarly, Rotor voltage equations, and flux linkage relation, we get

$$0 = r_r \vec{i}_r + \frac{d\vec{\psi}_r}{d\tau} - j\omega\vec{\psi}_r \tag{18}$$

$$\vec{i}_r = \frac{1}{l_r}\left[\vec{\psi}_r - l_h\vec{i}_s\right] \tag{19}$$

$$0 = -\frac{r_r}{l_r} l_h \vec{i}_s + \frac{r_r}{l_r}\vec{\psi}_r + \frac{d\vec{\psi}_r}{d\tau} - j\omega\vec{\psi}_r \tag{20}$$

$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \tag{21}$$

The last equation can be separated in to real and imaginary part

$$\frac{d\psi_{r\alpha}}{d\tau} = -\frac{1}{\tau_r}\psi_{r\alpha} - \omega\psi_{r\beta} + \frac{l_h}{\tau_r} i_{s\alpha} \tag{22}$$

$$\frac{d\psi_{r\beta}}{d\tau} = -\frac{1}{\tau_r}\psi_{r\beta} + \omega\psi_{r\alpha} + \frac{l_h}{\tau_r} i_{s\beta} \tag{23}$$

Rotor voltage based rotor flux estimator

rotor voltage based rotor flux estimator

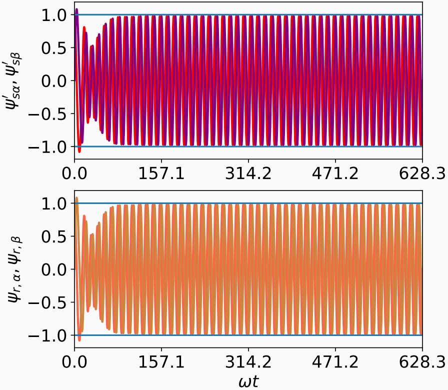
$$\frac{d\psi_{r\alpha}}{d\tau} = -\frac{1}{\tau_r}\psi_{r\alpha} - \omega\psi_{r\beta} + \frac{l_h}{\tau_r}i_{s\alpha} \tag{24}$$

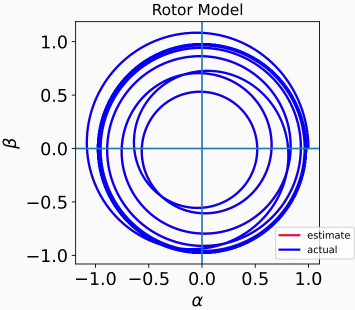
$$\frac{d\psi_{r\beta}}{d\tau} = -\frac{1}{\tau_r}\psi_{r\beta} + \omega\psi_{r\alpha} + \frac{l_h}{\tau_r}i_{s\beta} \tag{25}$$

We need to measure stator current space vector and the rotor angular velocity

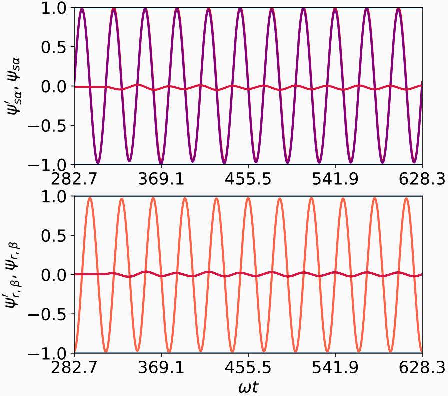
- Measure the 3 phase instantaneous currents $i_U(t), i_V(t), i_W(t)$
- Calculate the space vector $\vec{(i)}_s(t) = i_{s\alpha}(t) + ji_{s\beta}(t)$
- Measure rotor angular velocity using speed sensor or encoder
- Use the rotor voltage model to estimate rotor flux vector
- **You will need to know the motor parameters accurately (here lies the problem)**

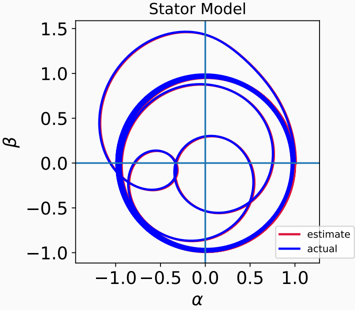
Rotor Model based Rotor Flux estimator: Simulation





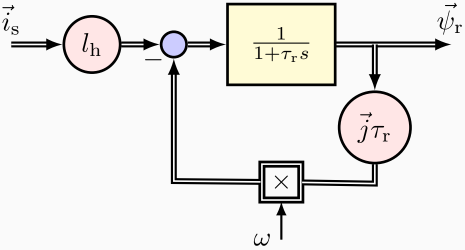
- Upper Curve is Estimated value and lower curve is actual value
- Right hand is the locus of rotor flux





- Machine is hot, $r_s = 2r_s(\text{cold})$
- Error seen, creates problem at low speed ($\omega = 0.2$)

Rotor Model based Rotor Flux estimator: Signal flow graph



angle of rotor flux vector w.r.t stationary stator coordinates

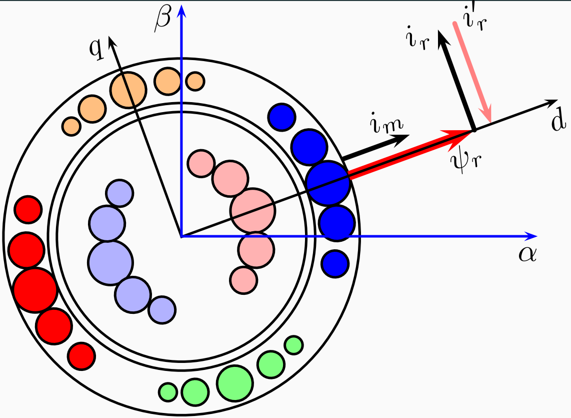
$$tan(\delta) = \frac{\psi_{r\beta}(\tau)}{\psi_{r\alpha}(\tau)}$$

$$cos(\delta) = \frac{\psi_{r\alpha}(\tau)}{\sqrt{\psi_{r\alpha}^2 + \psi_{r\beta}^2}}$$

$$\delta = \arccos\left(\frac{\psi_{r\alpha}(\tau)}{|\psi_r|}\right)$$

- Sketch the model for real and imaginary components
- redraw the model with Integrator

Rotor Field Orientation: positioning x axis of reference frame along Rotor flux



$$\vec{x}^{(S)} = \vec{x}^{(F)} e^{j\delta}$$

where δ is the angle of the rotor flux vector w.r.t α axis

- As rotor flux vector moves, $d - q$ axis will rotate
- **Carrying our coordinate transformation into $d - q$ axis is called as (Rotor) Field Orientation**

Coordinate Transform: Stator Coordinates to Field Coordinate

Let us take stator currents $\vec{i}_s^{(S)}$ in stator coordinates. These can be obtained from 3 phase instantaneous current of the motor. We, use the angle of the rotor flux space vector δ that we get from the flux estimator. We will get the stator current space vector in field coordinates $\vec{i}_s^{F} = i_{sd} + i_{sq}$ as

Coordinate transformation

$$\vec{i}_s^{(S)} = \vec{i}_s^{(F)} e^{j\delta} \tag{26}$$

$$\therefore \vec{i}_s^{(F)} = \vec{i}_s^{(S)} e^{-j\delta} \tag{27}$$

$$i_{sd} + i_{sq} = (i_{s\alpha} + j i_{s\beta})(cos(\delta) - j sin(\delta)) \tag{28}$$

$$i_{sd} = i_{s\alpha} cos(\delta) + i_{s\beta} sin(\delta) \tag{29}$$

$$i_{sq} = -i_{s\alpha} sin(\delta) + i_{s\beta} cos(\delta) \tag{30}$$

Coordinate Transform Matrix T_{SF}

The transformation matrix is often given in literature, can be written as

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \begin{bmatrix} cos\delta & sin(\delta) \\ -sin(\delta) & cos(\delta) \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \tag{31}$$

$$i_{sd} = i_{s\alpha} cos(\delta) + i_{s\beta} sin(\delta) \tag{32}$$

$$i_{sq} = -i_{s\alpha} sin(\delta) + i_{s\beta} cos(\delta) \tag{33}$$

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \begin{bmatrix} cos\delta & sin(\delta) \\ -sin(\delta) & cos(\delta) \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}$$

$$\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} = \mathbf{T}_{SF} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}$$

Reverse Coordinate Transform Matrix T_{FS}

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = \begin{bmatrix} cos\delta & -sin(\delta) \\ sin(\delta) & cos(\delta) \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} \tag{34}$$

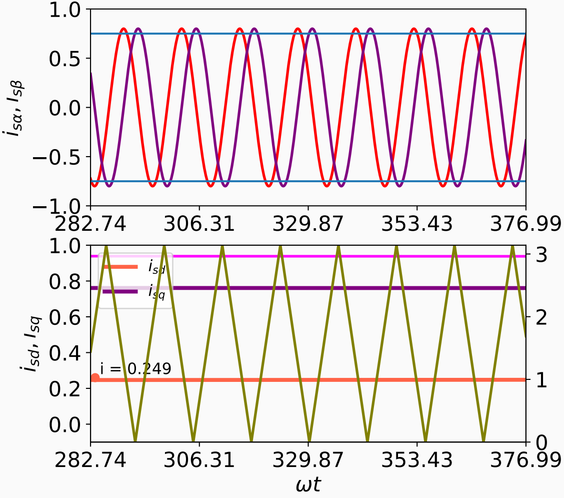
$$i_{sd} = i_{s\alpha} cos(\delta) - i_{s\beta} sin(\delta) \tag{35}$$

$$i_{sq} = i_{s\alpha} sin(\delta) + i_{s\beta} cos(\delta) \tag{36}$$

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = \begin{bmatrix} cos\delta & -sin(\delta) \\ sin(\delta) & cos(\delta) \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}$$

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = \mathbf{T}_{FS} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix}$$

Field oriented system: we are just observing the same dynamics

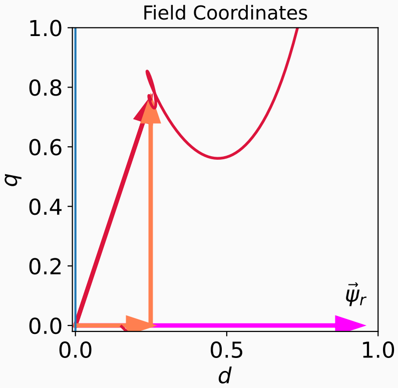


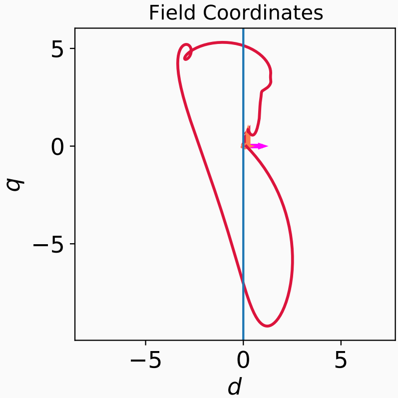
$$\vec{i}_s^{(F)} = \vec{i}_s^{(S)} e^{-j\delta}$$

where δ is the angle of the rotor flux vector w.r.t α axis

- $i_{sd} = 0.249$ in steady state
- Since there is load torque, we see $i_{sq} = 0.76$

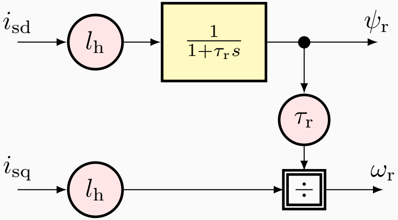
Field oriented system: we are just observing the same dynamics





- See the vector position is $d - q$ coordinates in steady state
- In right figure, the large starting current dynamics is observed in filed coordinates

Rotor Model based Rotor Flux estimator: in Field Coordinates



Rotor Model based Rotor flux estimator

Transforming the rotor equation in field coordinates

$$\tau_r \frac{d\psi_r}{dt} + \psi_r = l_h i_{sd}$$

$$\omega_r \tau_r \psi_r = l_h i_{sq}$$

- Shows decoupled nature of the system like SE-DC
- i_{sd} controls the rotor flux magnitude ψ_r
- i_{sq} controls the slip-frequency or torque
- $m_e = k_r \vec{\psi}_r \times \vec{i}_s = k_r \psi_r i_{sq}$

Signal Flow Graphs of Induction machine

Let us analyse the electromagnetic system of the induction machine. The system equations can be given as

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\boldsymbol{\omega}\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \tag{37}$$

$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\boldsymbol{\omega}\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \tag{38}$$

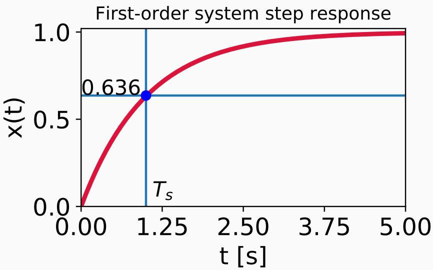
This can be re-written as

$$\underbrace{\frac{d\vec{i}_s}{d\tau} + \frac{1}{\tau_k}\vec{i}_s}_{\text{First-order system}} = \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\boldsymbol{\omega}\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \tag{39}$$

$$\underbrace{\frac{d\vec{\psi}_r}{d\tau} + \frac{1}{\tau_r}\vec{\psi}_r}_{\text{First-order system}} = j\boldsymbol{\omega}\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \tag{40}$$

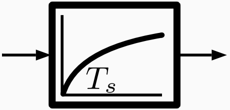
Revisiting some basics

$$T_s \frac{dx}{dt} + x = y$$
$$T_s \frac{dx}{dt} = y - x$$
$$x = \frac{1}{T_s} \int y - x dt$$
$$x = ke^{-\frac{t}{T_s}} + F(t)$$



To get the transfer function form we have to rewrite it as

$$T_s \frac{dx}{dt} + x = y$$
$$sT_sX(s) + X(s) = Y(s)$$
$$X(s) = \frac{Y(s)}{(sT_s + 1)}$$



Rewriting the Induction machine system equations

To get the First-order transfer function from, we

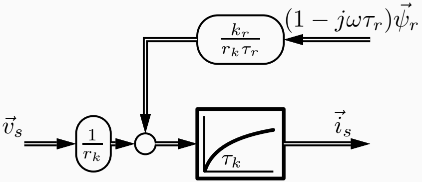
$$\underbrace{\tau_k \frac{d\vec{i}_s}{d\tau} + \vec{i}_s}_{\text{First-order system}} = \frac{k_r}{r_k \tau_r} \left(1 - j\omega\tau_r\right) \vec{\psi}_r + \frac{\vec{v}_s}{r_k} \tag{41}$$

$$\underbrace{\tau_r \frac{d\vec{\psi}_r}{d\tau} + \vec{\psi}_r}_{\text{First-order system}} = j\omega\tau_r \vec{\psi}_r + l_h \vec{i}_s \tag{42}$$

as $\tau_k = \frac{\sigma l_s}{(r_s + k_r^2 r_r)} = \frac{\sigma l_s}{r_k}$

Stator voltage equation Signal Flow Graph

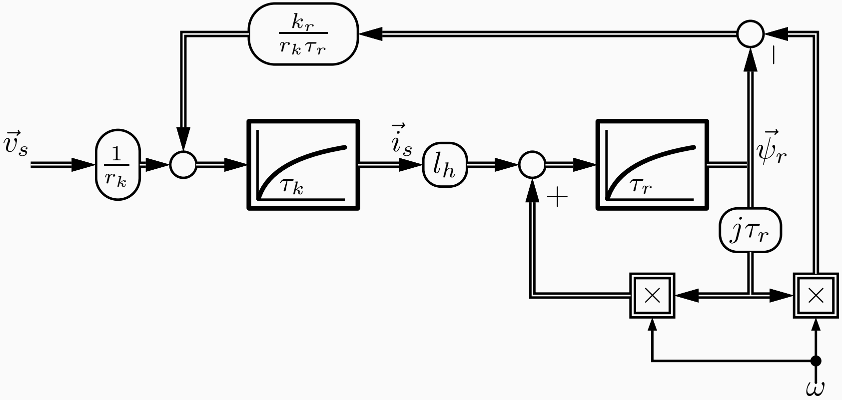
$$\underbrace{\tau_k \frac{d\vec{i}_s}{d\tau} + \vec{i}_s}_{\text{First-order system}} = \frac{k_r}{r_k \tau_r} \left(1 - j\omega\tau_r\right) \vec{\psi}_r + \frac{\vec{v}_s}{r_k}$$



Task for you

Applying the same principle. Develop the signal flow graph for the rotor equation

Stator coordinates



Convert the State Equations from stator coordinates into field coordinates

Task for you

The state equations for machine are given in stator coordinates

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s}$$

$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s$$

using coordinate transforms,

$$\vec{x}^{(S)} = \vec{x}^{(F)} e^{j\delta}$$

Write the machine equations in stator coordinates, so that we can sketch the signal flow graphs (like we did for the stator coordinates)

State equation for machine in field coordinates

$$\frac{d\vec{i}_s^F e^{j\delta}}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s^F e^{j\delta} + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r^F e^j\delta + \frac{\vec{v}_s^F e^{j\delta}}{\sigma l_s}$$

$$\frac{d\vec{i}_s^F}{d\tau} e^{j\delta} + j\omega_s \vec{i}_s^F e^{j\delta} = -\frac{1}{\tau_k}\vec{i}_s^F e^{j\delta} + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r^F e^j\delta + \frac{\vec{v}_s^F e^{j\delta}}{\sigma l_s}$$

$$\frac{d\vec{i}_s^F}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s^F - j\omega_s \vec{i}_s^F + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r^F + \frac{\vec{v}_s^F}{\sigma l_s}$$

$$\boxed{\tau_k \frac{d\vec{i}_s^F}{d\tau} + \vec{i}_s^F = -j\omega_s \tau_k \vec{i}_s^F + \frac{k_r}{r_k \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r^F + \frac{\vec{v}_s^F}{r_k}}$$

$$\frac{d\vec{\psi}_r^F e^{j\delta}}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r^F e^{j\delta} + \frac{l_h}{\tau_r}\vec{i}_s^F e^{j\delta}$$

$$\frac{d\vec{\psi}_r^F}{d\tau} e^{j\delta} + j\omega_s \vec{\psi}_r^F = -\frac{1}{\tau_r}\vec{\psi}_r^F e^{j\delta} - j\omega_r \vec{\psi}_r^F e^{j\delta} + \frac{l_h}{\tau_r}\vec{i}_s^F e^{j\delta}$$

$$\frac{d\vec{\psi}_r^F}{d\tau} = -\frac{1}{\tau_r}\vec{\psi}_r^F - j\omega_r \vec{\psi}_r^F + \frac{l_h}{\tau_r}\vec{i}_s^F$$

$$\boxed{\tau_r \frac{d\vec{\psi}_r^F}{d\tau} + \vec{\psi}_r^F = -j\omega_r \tau_r \vec{\psi}_r^F + l_h \vec{i}_s^F}$$

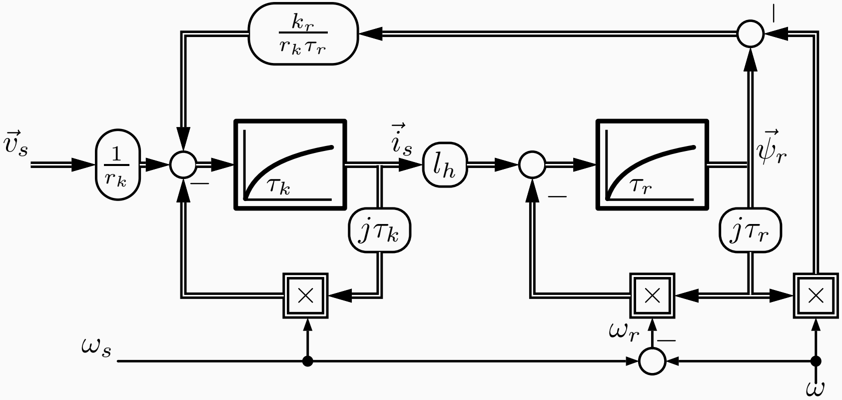
State Space equation of IM in field coordinates

$$
\begin{aligned}
\frac{d\vec{i}_s^F}{d\tau} &= -\frac{1}{\tau_k}\vec{i}_s^F - j\omega_s\vec{i}_s^F + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r^F + \frac{\vec{v}_s^F}{\sigma l_s} \\
\frac{d\vec{\psi}_r^F}{d\tau} &= -\frac{1}{\tau_r}\left(1 - j\omega_r\tau_r\right)\vec{\psi}_r^F + \frac{l_h}{\tau_r}\vec{i}_s^F
\end{aligned}
$$

We can rewrite them in terms of d-q component as

$$
\begin{aligned}
\frac{di_{sd}}{d\tau} &= -\frac{1}{\tau_k}i_{sd} + \omega_s i_{sq} + \frac{k_r}{\sigma l_s \tau_r}\psi_r + \frac{v_{sd}}{\sigma l_s} \\
\frac{di_{sq}}{d\tau} &= -\frac{1}{\tau_k}i_{sq} - \omega_s i_{sd} - \frac{k_r}{\sigma l_s}\omega\psi_r + \frac{v_{sq}}{\sigma l_s} \\
\frac{d\psi_r}{d\tau} &= -\frac{1}{\tau_r}\psi_r + \frac{l_h}{\tau_r}i_{sd} \\
0 &= -\omega_r\psi_r + \frac{l_h}{\tau_r}i_{sq} \\
\omega_r &= \frac{l_h}{\tau_r\psi_r}i_{sq}
\end{aligned}
$$

Rotor Field coordinates



Task for you

How will the signal flow graph change, if we use a very fast current control? (Assuming that $i_{sd} = i_{sd}^*$ and $i_{sq} = i_{sq}^*$

Rotor equations in field coordinates

Rotor equations in field coordinates

$$\tau_r \frac{d\psi_r}{d\tau_r} + \psi_r = l_h i_{sd}$$

$$\omega_r \tau_r \psi_r = l_h i_{sq}$$

Rotor equations in steady state in field coordinates

$$\psi_r = l_h i_{sd}$$

$$\omega_r \tau_r \psi_r = l_h i_{sq}$$

$$i_{sd} = \frac{\psi_r}{l_h}$$

$$i_{sq} = \frac{\omega_r \tau_r}{l_h}\psi_r = \frac{m_e}{k_r \psi_r}$$

i_{sd} produces rotor flux

i_{sq} produces torque

Signal Flow Graphs

Steps to develop signal flow graphs

- Develop the dynamic model of the AC machine in stator coordinates using the desired control variables
- Convert the equation into the desired reference coordinate system using the coordinate transforms

$$\vec{x}^s = x^c e^{j\epsilon} \quad \text{and} \quad \vec{x}^c = \vec{x}^s e^{-j\epsilon}$$

  where ϵ is the angle of rotation of axis c with respect to stator coordinate s
- Separate the equations into first order ODE with differentiation of one state variable on left hand side (LHS)

$$\frac{dx}{dt} = f(x, y, u)$$

- Develop the signal flow graph with integrators with the RHS of each equation as the input to the integrator and the single state variable as the output of the integrator
- Alternatively, develop a first order form on LHS of each equation of single state variable as

$$\frac{dx}{dt} + x = f(y, u)$$

- Develop the signal flow graph starting with first order system using the RHS as input

Current control in field coordinates

The stator current dynamics is given by

$$\frac{di_{sd}}{d\tau} = -\frac{1}{\tau_k} i_{sd} + \omega_s i_{sq} + \frac{k_r}{\sigma l_s \tau_r} \psi_r + \frac{v_{sd}}{\sigma l_s}$$
$$\frac{di_{sq}}{d\tau} = -\frac{1}{\tau_k} i_{sq} - \omega_s i_{sd} - \frac{k_r}{\sigma l_s} \omega \psi_r + \frac{v_{sq}}{\sigma l_s}$$

PI based current control can be implemented as

$$v^*_{sd} = k_p(e_{sd}) + k_i \int (e_{sd}) d\tau \tag{43}$$

$$v^*_{sq} = k_p(e_{sq}) + k_i \int (e_{sq}) d\tau \tag{44}$$

where $$e_{sd} = i^*_{sd} - i_{sd} \tag{45}$$

$$e_{sq} = i^*_{sq} - i_{sq} \tag{46}$$

PI controller: expression in discrete form for simulation

The PI controller can be described using

$$y(\tau) = k_p \left(x_e(\tau) + \frac{1}{T_i} \int x_e(\tau) d\tau \right)$$

where k_p is the proportional gain and $\frac{1}{T_i}$ is the integration time constant. We can write it in discrete time form as

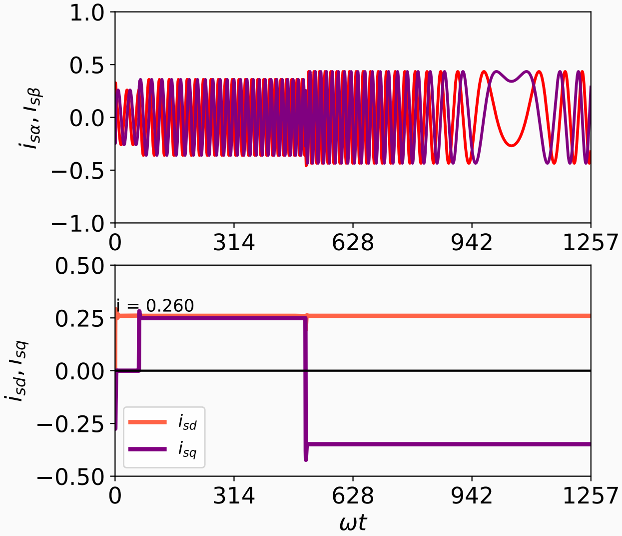
$$y = k_p \left(x_e + \frac{1}{T_i} \int x_e d\tau \right) \tag{47}$$

$$y[n] = k_p x_e[n] + \frac{k_p}{T_i} \sum_{k=1}^{n} x_e[k] \tag{48}$$

$$y[n-1] = k_p x_e[n-1] + \frac{k_p}{T_i} \sum_{k=1}^{n-1} x_e[k] \tag{49}$$

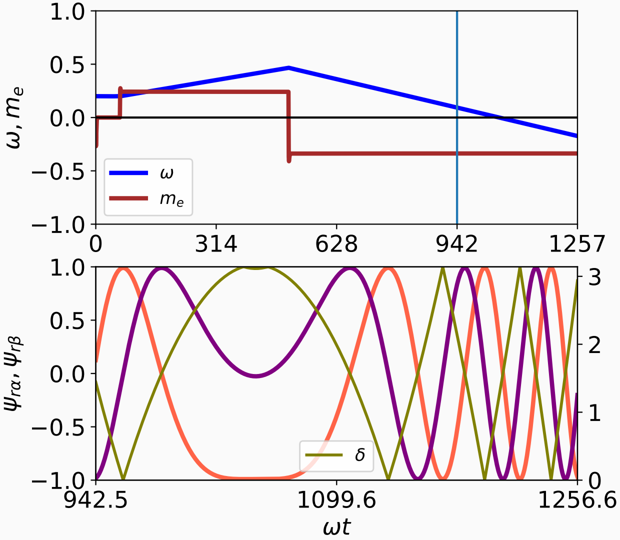
$$y[n] = k_p x_e[n] + \frac{k_p}{T_i} \sum_{k=1}^{n-1} x_e[k] + \frac{k_p}{T_i} x_e[n] \tag{50}$$

$$y[n] = y[n-1] + k_p(x_e[n] - x_e[n-1]) + \frac{k_p}{T_i} x_e[n] \tag{51}$$



1. Initial State
   $i_{sd} = i^*_{sd} = 0.26$
2. Step change
   $i^*_{sq} = i_{sq} = 0.25$
3. Set change
   $I^*_{sq} = i_{sq} = -0.35$
4. Motor reversal can be seen

Field-oriented current control using PI Controller - Simulation(speed)



❶ Initial State

$i_{sd} = i_{sd}^* = 0.26$

❷ Step change

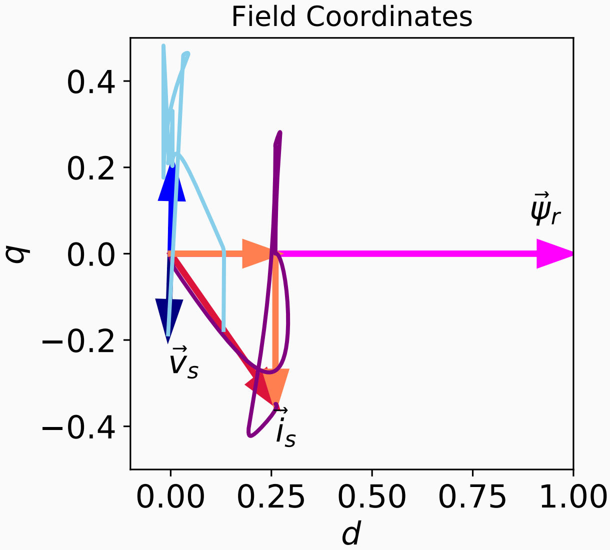
$i_{sq}^* = i_{sq} = 0.25$

❸ Set change

$I_{sq}^* = i_{sq} = -0.35$

❹ Motor reversal can be seen

Field-oriented current control using PI Controller - Simulation(d-q)



1. Initial State
   $i_{sd} = i_{sd}^* = 0.26$
2. Step change
   $i_{sq}^* = i_{sq} = 0.25$
3. Set change
   $I_{sq}^* = i_{sq} = -0.35$
4. Motor reversal can be seen
5. No initial current dynamics seen
6. Motor currents are controlled

Current control in field coordinates with compensation

The stator current dynamics is given by

$$\frac{di_{sd}}{d\tau} = -\frac{1}{\tau_k} i_{sd} + \omega_s i_{sq} + \frac{k_r}{\sigma l_s \tau_r}\psi_r + \frac{v_{sd}}{\sigma l_s}$$
$$\frac{di_{sq}}{d\tau} = -\frac{1}{\tau_k} i_{sq} - \omega_s i_{sd} - \frac{k_r}{\sigma l_s}\omega\psi_r + \frac{v_{sq}}{\sigma l_s}$$

PI based current control can be implemented using decoupling compensation

$$v_{sd}^* = \underbrace{k_p(e_{sd}) + k_i \int (e_{sd}) d\tau}_{\text{PI Controller}} \underbrace{-\omega_s \sigma l_s i_{sq}}_{\text{Compensating cross couping terms}} \tag{52}$$

$$v_{sq}^* = k_p(e_{sq}) + k_i \int (e_{sq}) d\tau + \omega_s \sigma l_s i_{sd} \tag{53}$$

where $$e_{sd} = i_{sd}^* - i_{sd} \tag{54}$$

$$e_{sq} = i_{sq}^* - i_{sq} \tag{55}$$

Signal Flow Graph with Current control

- Actual current follow the reference value
- What will happen to stator voltage equation?

Signal Flow Graph with Current control

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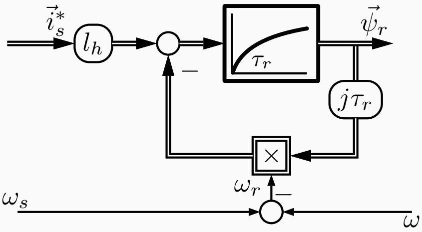
- What will happen to stator voltage equation?



Rewrite machine dynamics in terms of current components

We have,

$$\tau_r \frac{d\vec{\psi}_r}{d\tau} + \vec{\psi}_r = j\omega_r \tau_r \vec{\psi}_r + l_h \vec{i}_s$$

Since
$\vec{\psi}_r = \psi_{rd} + j\psi_{rq}$

Rotor Flux orientation

Since Field coordinates are aligned with rotor flux vector $\psi_{rq} = 0$ As torque

$$m_e = k_r \vec{\psi}_r \times \vec{i}_s$$

$$m_e = k_r \psi_r i_{sq}$$

Complete the following equations

$$i_{sd} =$$

$$i_{sq} =$$

Rewrite machine dynamics in terms of current components

We have,

$$\tau_r \frac{d\vec{\psi}_r}{d\tau} + \vec{\psi}_r = j\omega_r \tau_r \vec{\psi}_r + l_h \vec{i}_s$$

Since
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Rotor Flux orientation

Since Field coordinates are aligned with rotor flux vector $\psi_{rq} = 0$ As torque

$$m_e = k_r \vec{\psi}_r \times \vec{i}_s$$

$$m_e = k_r \psi_r i_{sq}$$

Complete the following equations

$$i_{sd} = \frac{1}{l_h}\left[\tau_r \frac{d\psi_r}{d\tau} + \psi_r\right]$$

$$i_{sq} =$$

Rewrite machine dynamics in terms of current components

We have,

$$\tau_r \frac{d\vec{\psi}_r}{d\tau} + \vec{\psi}_r = j\omega_r \tau_r \vec{\psi}_r + l_h \vec{i}_s$$

Since
$\vec{\psi}_r = \psi_{rd} + j\psi_{rq}$

Rotor Flux orientation

Since Field coordinates are aligned with rotor flux vector $\psi_{rq} = 0$ As torque

$$m_e = k_r \vec{\psi}_r \times \vec{i}_s$$

$$m_e = k_r \psi_r i_{sq}$$

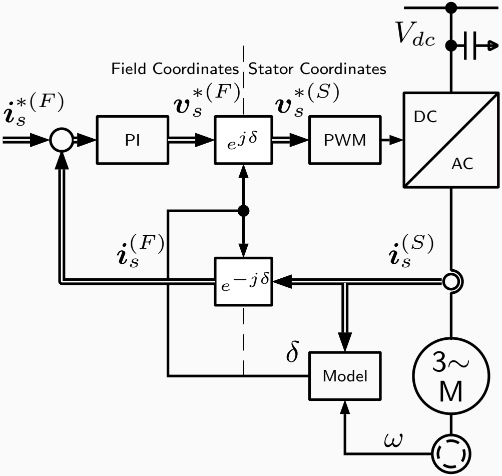
Complete the following equations

$$i_{sd} = \frac{1}{l_h}\left[\tau_r \frac{d\psi_r}{d\tau} + \psi_r\right]$$

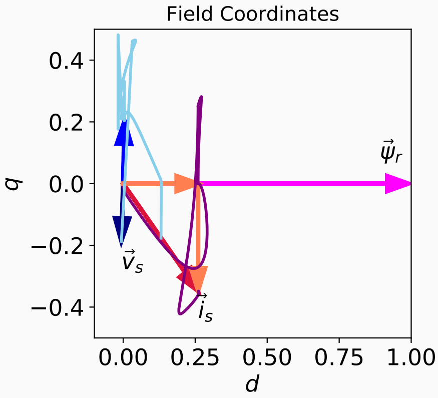
$$i_{sq} = \frac{\omega_r \tau_r}{l_h}\psi_r$$

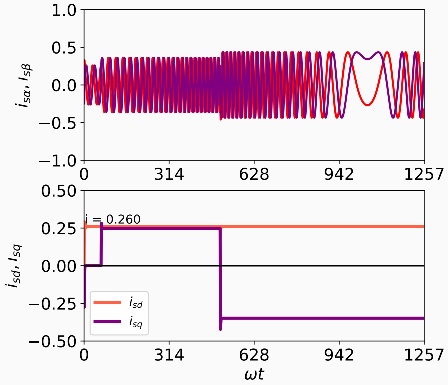
Current Control in Field Coordinates to have faster dynamics

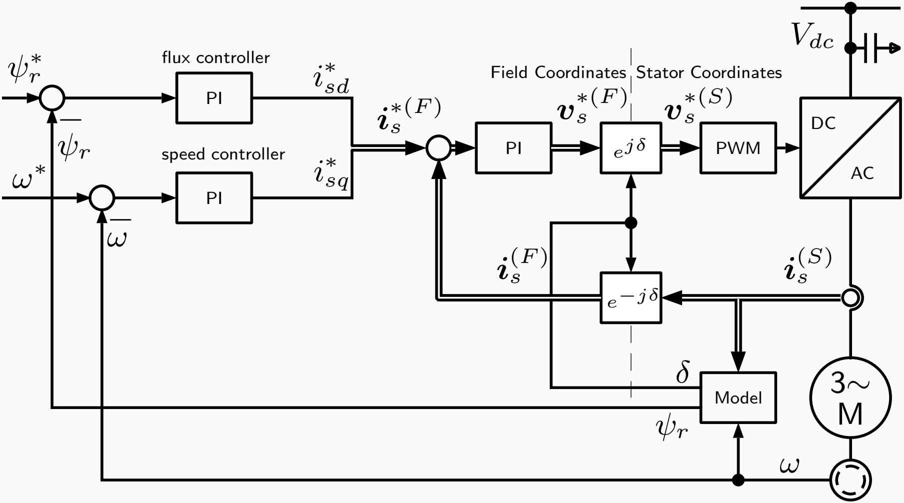
- In field coordinates, we see currents as DC (zero frequency)
- This is due to coordinate transform in a synchronous frequency reference frame
- It is called Field-oriented Current control as the reference frame is oriented along rotor flux space vector



Fast dynamics by keeping rotor flux constant and changing only i_{sq}

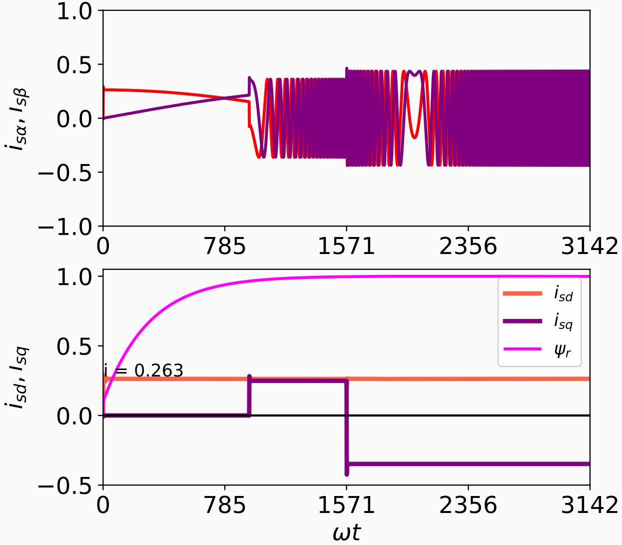






ψ_r^*
flux controller
PI
i_{sd}^*
ψ_r
ω^*
speed controller
PI
i_{sq}^*
ω
$\boldsymbol{i}_s^{*(F)}$
Field Coordinates
Stator Coordinates
PI
$\boldsymbol{v}_s^{*(F)}$
$e^{j\delta}$
$\boldsymbol{v}_s^{*(S)}$
PWM
V_{dc}
DC
AC
$\boldsymbol{i}_s^{(F)}$
$e^{-j\delta}$
$\boldsymbol{i}_s^{(S)}$
δ
Model
ψ_r
3~
M
ω

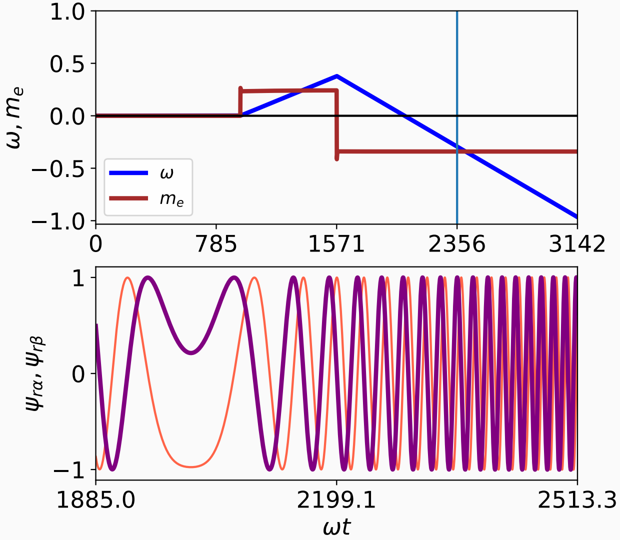
Field-oriented current control using PI Controller +Flux Control



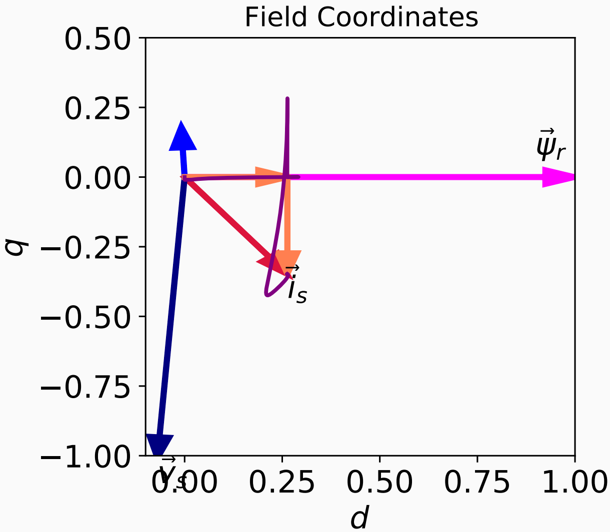
❶ Initial State

$i_{sd} = i^*_{sd} = 0.26$

Field-oriented current control using PI Controller + Flux control)

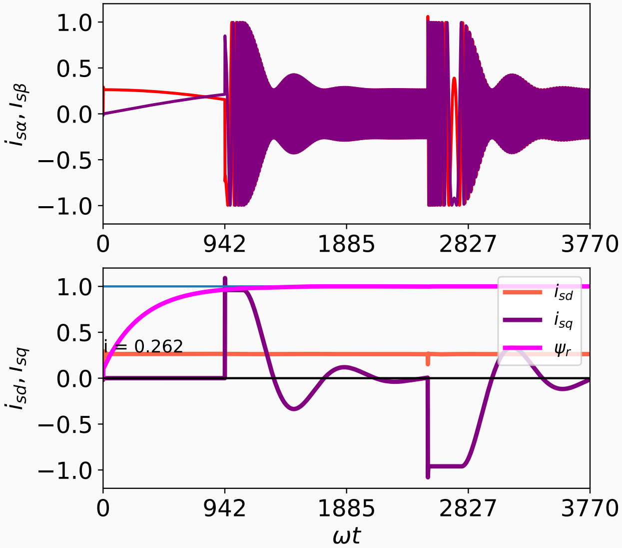


1. Initial State
   $i_{sd} = i_{sd}^* = 0.26$
2. Step change
   $i_{sq}^* = i_{sq} = 0.25$
3. Set change
   $I_{sq}^* = i_{sq} = -0.35$
4. Motor reversal can be seen



1. Initial State
   $i_{sd} = i^*_{sd} = 0.26$
2. Step change
   $i^*_{sq} = i_{sq} = 0.25$
3. Set change
   $I^*_{sq} = i_{sq} = -0.35$
4. Motor reversal can be seen
5. No initial current dynamics seen
6. Motor currents are controlled

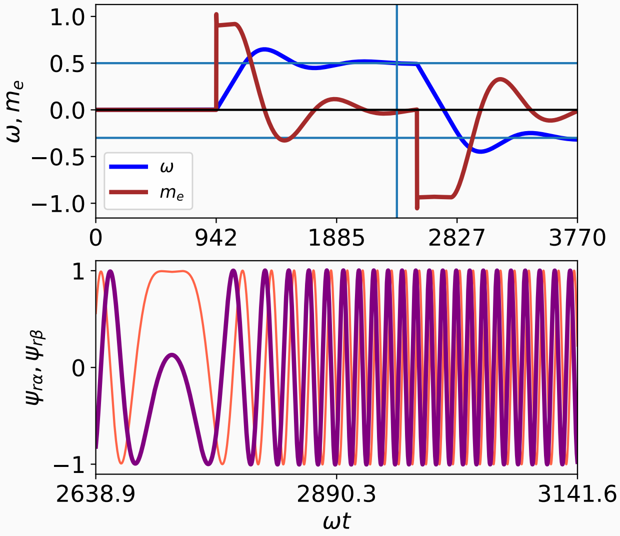
Field-oriented current control using PI Controller + Speed Control



❶ Initial State

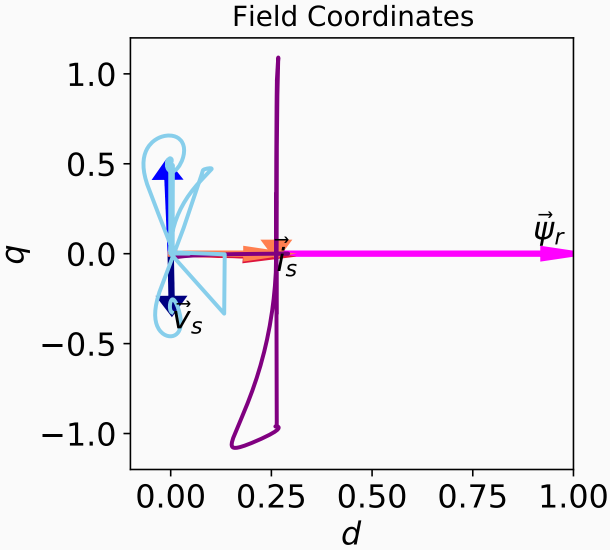
$$i_{sd} = i_{sd}^{*} = 0.26$$

Field-oriented current control using PI Controller + Speed control)



1. Initial State $\omega^* = 0.001$
2. Step change $\omega^* = 0.5$
3. Set change $\omega^* = -0.3$
4. Motor reversal can be seen
5. No initial current dynamics seen
6. Motor currents are controlled

Field-oriented current control using PI Controller - with Speed control)



1. Initial State $\omega^* = 0.001$
2. Step change $\omega^* = 0.5$
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4. Motor reversal can be seen
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6. Motor currents are controlled

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