

# AC Machines Fundamentals

How is Torque Produced?

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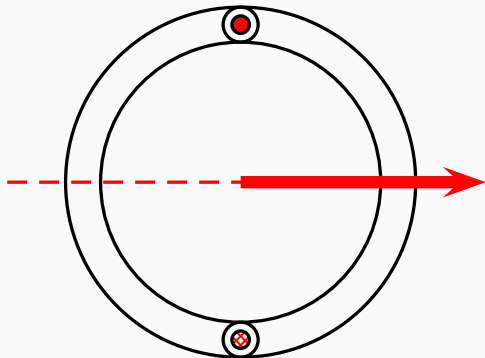
**But How to Produce Rotating  
Magnetic Field**

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## Before we go further focus on important concepts

- DC machines have stationary magnetic field produced by stator
- AC machines are supplied by AC voltage, Hence rotating field has to be produced
- Torque is produced by couple of Lorentz force
- Lorentz force is  $F = il \times B$
- Study the relation between electrical and magnetic quantities...well
  - Faradays's law  $v = \frac{d\psi}{dt}$ , where  $\psi = n\phi$
  - Ampere's law  $\oint Hdl = ni\text{MMF}$
  - Magnetic circuit properties  $B = \mu_o\mu_r H$ ,  $L = \frac{n\phi}{i}$

## Supplying DC to Stationary coils produces stationary field



- DC current supplied to coil (going in paper below)
- DC current flows and produces a magnetic field along the axis of the coil
- The field stationary

What happens when you supply AC to a stationary Coil?

Think and sketch

## When AC is supplied to a stationary winding

- Sinusoidal AC voltage supplied to the coil
- AC current flows and produces a magnetic field along the axis of the coil
- The field oscillates

## How to Produced a Rotating Field?

Take 5 mins

Think!!!!

How about using two coils?

Once you have got something  
on paper, move to next slide

## When AC is supplied to 2 orthogonal stationary winding

- 2 AC voltages
$$v_a(t) = V \cos(2\pi ft)$$
and  $v_b(t) = V \sin(2\pi ft)$ applied
- Coil a axis is orthogonal to coil b axis
- we can produce a rotating field
- Spatially displaced coils are supplied by temporally displaced currents



**Rotating field with 3 phase AC?**

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## Rotating field with 3 phase AC, title = To Do

We get 3 AC phase supply

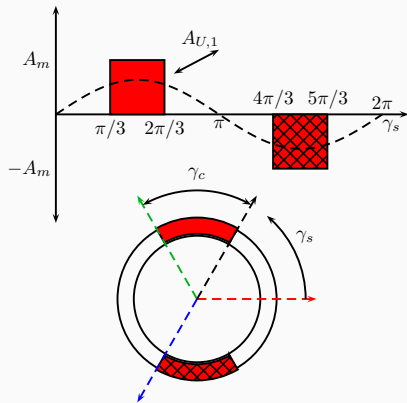
Can you think of a scheme to produce rotating field with 3 phase AC?

Think!!! apply the principles you have learned so far

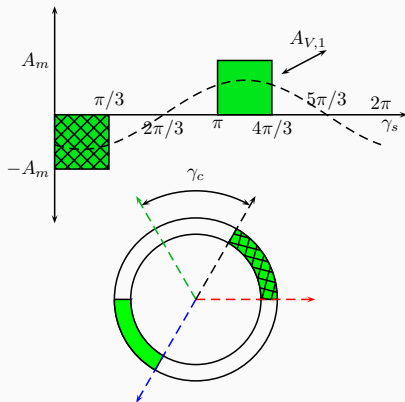
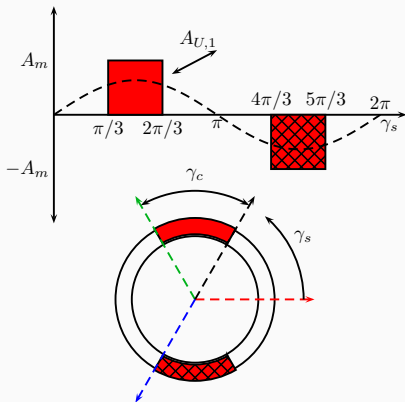
## 3 Phase AC motor lay out

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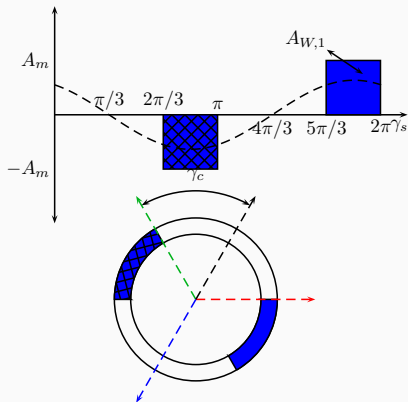
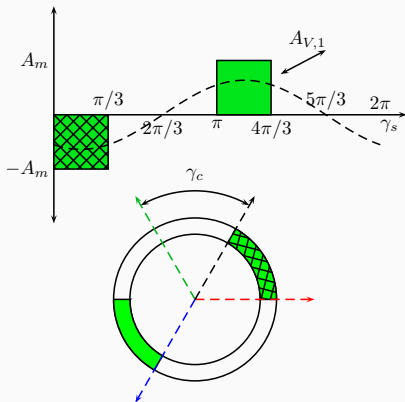
## Distributed winding as current sheet: Phase U i



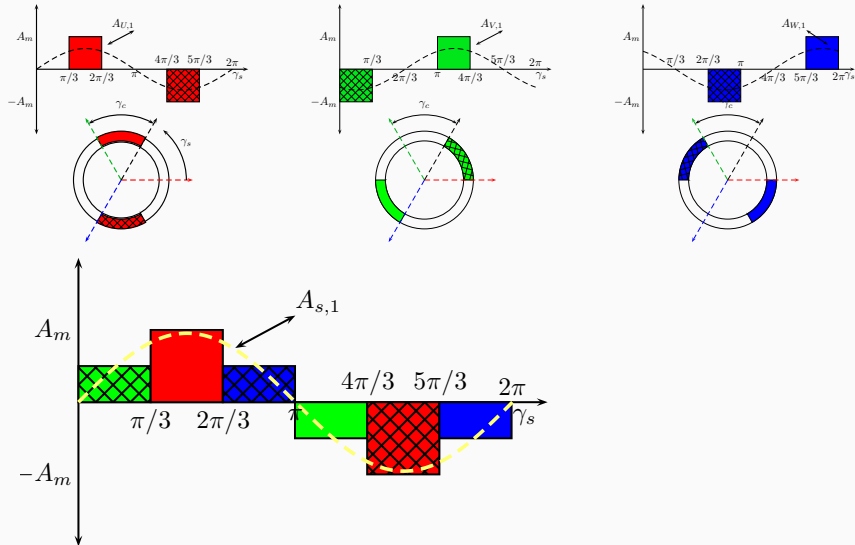
# MMF of the three windings



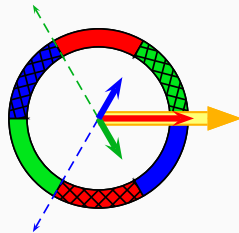
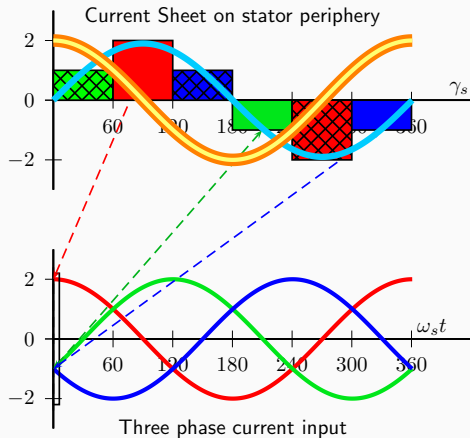
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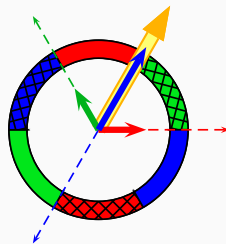
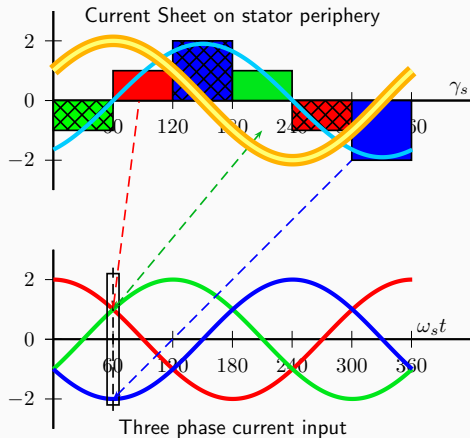


# Rotating Magnetic field in 3 phase AC Machine

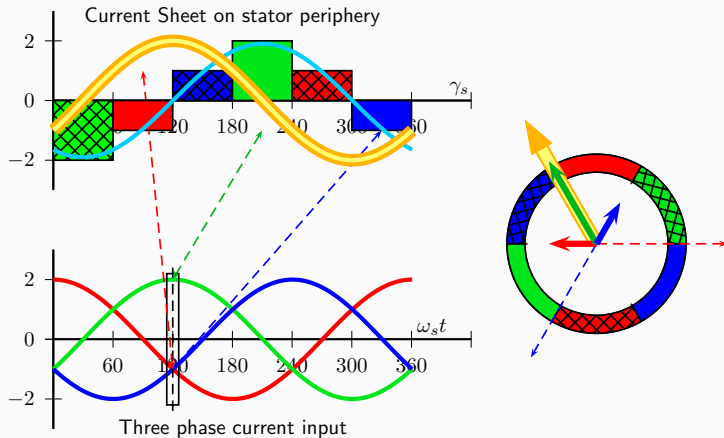




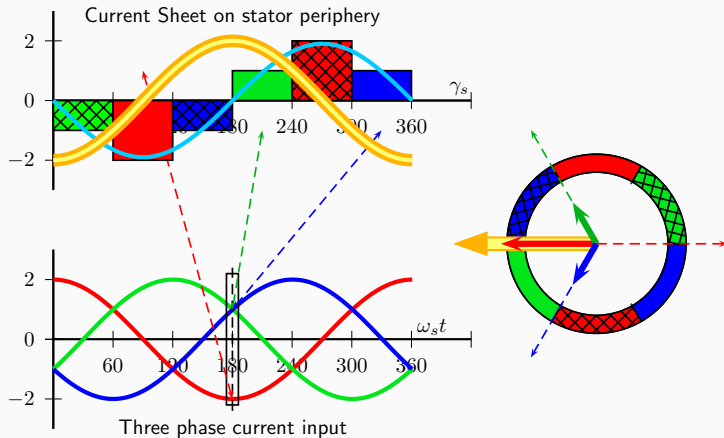
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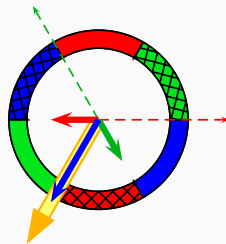
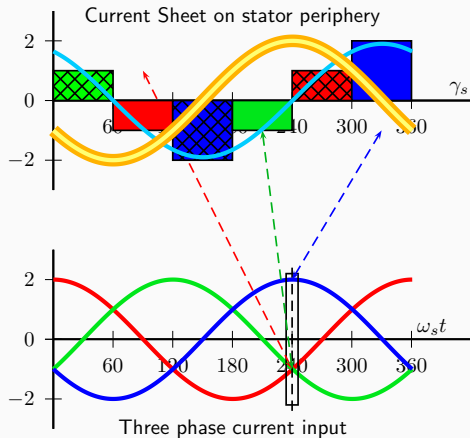
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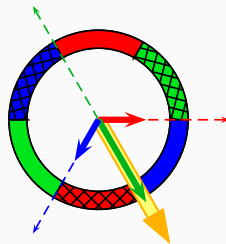
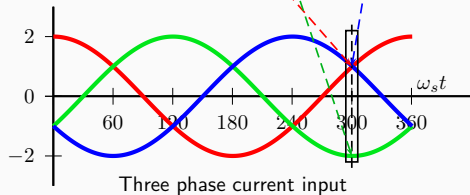
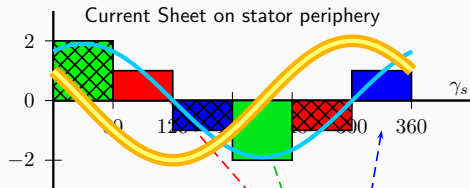
# Rotating Magnetic field in 3 phase AC Machine



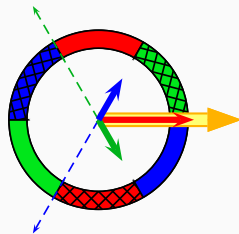
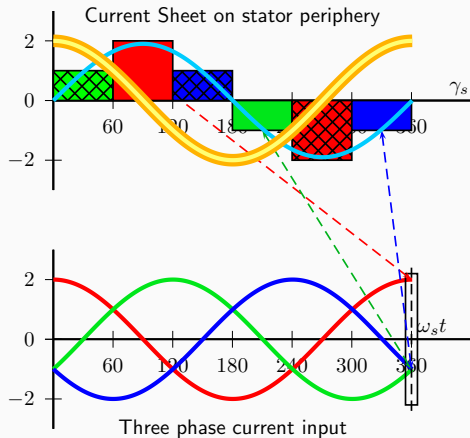
# Rotating Magnetic field in 3 phase AC Machine



# Rotating Magnetic field in 3 phase AC Machine



# Rotating Magnetic field in 3 phase AC Machine



# The "Mexican Wave" principle

- A simple example is the creation of the "Mexican wave" in a stadium.
- Each individual spectator goes up and down at his position with a time lag between the subsequent spectators
- Though no one is actually moving from his place we get the illusion of the "Mexican wave" moving around the stadium.

Similarly as each individual winding produces an oscillating flux along its axis with a time lag between the subsequent phases, we get a resultant flux rotating along the air gap.

# Applying the principle to multiple-phases

## What about multiple phases

- If the same principle can be applied with multiple phases,
  - if Yes, then why use 3 phase?
  - if No, then why not?
- Why does our Grid system use 3 phase AC ?

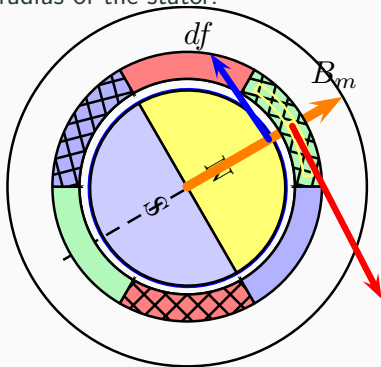


# How Torque is Produced?

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# Torque production

A portion of the stator,  $d\gamma_s$  will have a current  $A_{s,1}(\gamma_s)rl d\gamma_s$  coming out of the plane of the paper, where  $r$  is the radius of the stator and  $l$  its depth. This forms like a current sheet along the inner radius of the stator.



It interacts with the perpendicular flux density at that position on the stator, to produce a force. This will produce a tangential force on the stator and by reaction, an equal and opposite force on the rotor, given as

$$df = rl_e A_{s,1} \times \vec{B}_r d\gamma_s \quad (1)$$

To get the torque, we take the moment of the force which gives us  $dM_e = r df$

$$dM_e = r df \quad (2)$$

$$dM_e = r^2 l_e B_{r,1} A_{s,1} d\gamma_s \quad (3)$$

# Torque production

To calculate the total torque produced by the total current along the stator, we integrate Eq.(3) over the complete circumference. Hence

$$M_e = \int_0^{2\pi} dM_e = \int_0^{2\pi} r^2 l_e B_{r,1} A_{s,1} d\gamma_s \quad (4)$$

$$= r^2 l_e \int_0^{2\pi} B_m \cos(\gamma_s - \delta) \hat{A}_s \sin(\gamma_s) d\gamma_s \quad (5)$$

$$= \frac{r^2 l_e B_m \hat{A}_s}{2} \left[ \int_0^{2\pi} \sin(\delta) d\gamma_s + \int_0^{2\pi} \cos(2\gamma_s - \delta) d\gamma_s \right] \quad (6)$$

$$= \frac{r^2 l_e B_m \hat{A}_s}{2} \left[ \sin(\delta) \gamma_s \Big|_0^{2\pi} + \sin(2\gamma_s - \delta) / 2 \Big|_0^{2\pi} \right] \quad (7)$$

$$= \frac{r^2 l_e B_m \hat{A}_s}{2} [2\pi \sin(\delta) + 0] \quad (8)$$

$$= \pi r^2 l_e B_m \hat{A}_s \sin(\delta) \quad (9)$$

$$= \pi r^2 l_e \vec{\hat{A}}_s \times \vec{B}_m \quad (10)$$

## Torque from Mechanics equation i

We can also get torque from Mechanics of the system. The rotor is accelerated due to the torque. The torque on the rotor can be expressed in terms of power supplied to the rotor. The power available at the shaft of the motor is

$$P_{sh} = M_{sh}\Omega \quad (11)$$

$$P_{sh} = P_{\text{air-gap}} - P_{\text{rotor-loss}} \quad (12)$$

$$P_{sh} = P_{\text{stator}} - P_{\text{stator-loss}} - P_{\text{rotor-loss}} \quad (13)$$

$$(14)$$

## Torque from Mechanics equation ii

Let us assume that the losses are negligible (**this is purely a thought experiment for understanding**), Then

$$P_{\text{stator}} = P_e = \Re [\underline{VI}^*] \quad (15)$$

$$P_e = M_e \omega_s \quad (16)$$

$$\omega_s = p\Omega \quad (17)$$

where  $p$  is number of **pole-pairs** of the machine.

### Poles and pole pairs

A 4 pole machine has 2 pole pairs. One pole pair is a set of north-south poles

# Shaft Torque and Electromagnetic Torque

Hence

Electromagnetic torque is given as

$$M_e = \frac{P_e}{\omega_s} = \frac{\Re[VI^*]}{\omega_s} \quad (18)$$

The Shaft Torque is given as

$$M_{sh} = \frac{P_{\text{shaft-power}}}{\Omega} = \frac{P_e - \sum P_{\text{losses}}}{\Omega} \quad (19)$$

The power equation we used is valid for steady-state balanced 3 phase

If we have to study the dynamics of the AC machine, we cannot use the steady-state equations. We need to study how electrical and magnetic variables of the AC machine vary with time. As time progresses, the spatial position and the magnitude of these variables change. **What tools can be used to model the dynamics of the machine**