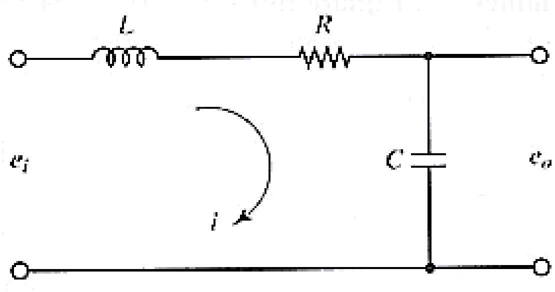
EE5103 Computer Control Systems: Homework #1
(Due date: 24/09/2023)

Q1. (10 Marks)
Consider the electrical circuit shown in the figure below. The circuit consists



of an inductance $L = 1$ henry, a resistance $R = 2$ ohm, and a capacitance $C = 0.5$ farad. Applying Kirchhoff's voltage law yields,

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = e_i$$
$$\frac{1}{C}\int idt = e_o$$

a) Assuming e_i is the input u, and e_o, the output y, derive the transfer function of the system from the input u to output y.

(2 Marks)

b) Define state variables by

$$x_1 = e_o$$
$$x_2 = \dot{e}_o$$

derive the state-space representation of the system.

(2 Marks)

c) Using zero-order-hold to sample the system, and assuming the sampling period $h = 1$, derive the state-space representation of the sampled system.

(2 Marks)

d) Apply z-transform to the state-space model derived in c), and obtain the input-output model of the system.

(2 Marks)

e) Assuming the initial conditions are $y(0) = 1$, and $\dot{y}(0) = 1$. Calculate the output sequence $y(k)$, under the unit step input, $u(k) = 1$ for $k \geq 0$.

(2 Marks)

Q2. (10 Marks)
Consider the system

$$G(s) = \frac{s-1}{s^2(s+1)}$$

a) Is the system stable? Does the system have a stable inverse? Justify your answers.

(3 Marks)

b) Is it possible to choose the sampling period h such that the sampled system is stable? Justify your answer.

(3 Marks)

c) Is it possible to choose the sampling period h such that the sampled system has a stable inverse? Justify your answer.

(4 Marks)

Q3. (10 Marks)
Consider the system

$$x\left(k+1\right) = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix} x\left(k\right) + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u\left(k\right)$$

$$y\left(k\right) = \begin{pmatrix} 1 & 0 \end{pmatrix} x\left(k\right)$$

a) Is the system stable? Is the system controllable? Is the system observable? Justify your answers.

(2 Marks)

b) Use z-transform to obtain the transfer function of the system. Write down the input-output difference equation.

(2 Marks)

c) Assume the system is controlled by a proportional controller

$$u(k) = K(u_c(k) - y(k))$$

Derive the transfer function from the command signal $u_c(k)$ to the output $y(k)$.

(2 Marks)

d) Apply Jury's stability criterion to determine the range of controller gain, K, such that the closed-loop system is stable.

(2 Marks)

e) Determine the steady-state error, $u_c - y$, when u_c is a unit step.

(2 Marks)

Q4. (10 Marks)

Consider the system described by the following difference equation

$$y(k+1) = 3y(k) - 2y(k-1) + u(k-1) + 2u(k-2)$$

a) What is the transfer function? Is the system stable? Does the system have a stable inverse?

(2 Marks)

b) Is it possible to realize the system such that it is observable but not controllable? If yes, write down the corresponding state-space equation. If no, justify your answer.

(3 Marks)

c) Is it possible to realize the system such that it is controllable but not observable? If yes, write down the corresponding state-space equation. If no, justify your answer.

(3 Marks)

d) Is it possible to realize the system such that it is both controllable and observable? If yes, write down the corresponding state-space equation. If no, justify your answer.

(2 Marks)