NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2017/2018)

EE5101/ME5401 – LINEAR SYSTEMS

November 2017 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.
4. This is a **CLOSED BOOK** examination. But the candidate is permitted to bring into the examination hall a single A4 size *help sheet*. The candidate may refer to this sheet during the examination.
5. Calculators can be used in the examination, but no programmable calculator is allowed.

Q.1 (a) Given the system

$$\dot{x} = \begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix} x$$

(i) Find the state-transition matrix e^{At} using the Caley-Hamilton Principle.

(ii) Find the two sets of initial conditions such that only one eigenmode is present in each of the responses. Draw the trajectories of the system starting from these initial states, together with one trajectory from $x(0)=[1 \quad 1]^T$, in the same state space.

(15 marks)

(b) If A is a constant, square and invertible matrix, show that

$$\int_0^t e^{A\tau} d\tau = A^{-1}\left[e^{At} - I\right].$$

Hence, or otherwise, obtain an expression of A^{-1} when A is asymptotically stable in the sense of Lyapunov.

(10 marks)

Q.2 (a) Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ \alpha^2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

Determine the controllability of the system. Can the system become uncontrollable for some value of α? If so, indicate these values. If not, explain your answer.

(5 marks)

(b) Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

Determine the controllability of this system using the method involving the Controllability Grammian. Suppose one wishes to move the state from the origin to $x(t) = [2 \quad 0]^T$ when t =1. Is that possible? If yes, explain (but not solve) how to get the control input to do so. If not, explain your answer.

(15 marks)

(c) A linear-time-invariant system was given to a student for stability determination via Lyapunov equation. He chose Q as the identity matrix and solving $A^TP+P^TA=-Q$, found that P was positive semi-definite. How should he conclude? Explain your answer.

(5 marks)

Q.3. Consider a process given by

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x$$

Design Specification. It is desired that the poles are placed at -1, -2 and -3.

(a) Assume that all the state variables are accessible, design a state feedback controller to place the poles to the desired ones. There are many ways to solve the pole placement problem for multiple input system. For this problem, only **FULL RANK** method can be used. In other words, the state transformation matrix T has to be constructed to transform the state space model into its controllable canonical form and the pole placement problem can be solved with the aid of its canonical form.

(12 marks)

(b) The solution to the above pole placement problem for multi-input system in part (a) is not unique. In the design process, there are multiple ways to specify the desired closed loop matrix A_d which is the same as the canonical form except the non-trivial rows. In addition to the one used in part (a), list three different ways to design the desired closed loop matrix A_d. You only need to write down the three matrices which have the same structure of the canonical form and share the same eigenvalues, -1, -2 and -3. No need to complete the pole placement design for each matrix.

(5 marks)

(c) If only the output can be measured while the state variables are not accessible, is it still possible to meet the design specification? Justify your answer.

(8 marks)

Q.4 (a) Design a controller in unity output feedback configuration, which decouples and stabilizes the MIMO plant:

$$G(s)=\begin{bmatrix} \frac{1}{(s-1)(s+2)} & \frac{1}{s-1} \\ \frac{1}{s-2} & \frac{-1}{s-2} \end{bmatrix}.$$

(10 marks)

(b) Let a plant be described by the transfer function,

$$G(s)=\frac{1}{s^2}.$$

Design a unity feedback control system to meet following requirements:

(i) The dominant dynamics can be described by standard second order system with damping ratio of 0.5, and natural frequency of 1.

(ii) The feedback control system can asymptotically track a reference signal, $r=c+a\sin(t), t\geq 0$, where a and c can be any positive constants.

(iii) The closed loop system can eliminate the effect of any step disturbance in steady state.

After you design the controller, please explain how the three requirements are met.

(15 marks)

Appendix A - Table of Laplace Transform

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s).

x(t)	**X(s)**
unit impulse δ (t)	1
unit step 1(t)	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

END OF PAPER