ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2018/2019)

EE5101/ME5401 – LINEAR SYSTEMS

November 2018 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.
4. This is a **CLOSED BOOK** examination. But the candidate is permitted to bring into the examination hall a single A4 size *help sheet*. The candidate may refer to this sheet during the examination.
5. Calculators can be used in the examination, but no programmable calculator is allowed.

Q.1 (a) Given the system

$$\dot{x} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} x$$

Find the state-transition matrix e^{At} using the Caley-Hamilton Principle.

(7 marks)

(b) Given that A and $\bar{A}$ are two square matrices related via similarity transformation. Let (λ, v) and $(\bar{\lambda}, \bar{v})$ be one of their respective eigenvalues and eigenvectors. Derive the relation between (λ, v) and $(\bar{\lambda}, \bar{v})$.

(8 marks)

(c) Suppose λ is an eigenvalue of a $n \times n$ square matrix A with a corresponding eigenvector, v. Let $f(\lambda) = \sum_{i=0}^{m} \alpha_i \lambda^i$ be a polynomial of λ with real coefficients, α_i and m is some integer less than or equal to $n-1$. Show that $f(\lambda)$ is an eigenvalue of the matrix given by $f(A) = \sum_{i=0}^{m} \alpha_i A^i$. Determine the corresponding eigenvector.

(10 marks)

Q.2 Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

Determine the following of the system:

(a) Observability using the Observability Grammian,

(b) Controllability using the rank of $[\lambda I - A \quad B]$.

(13 marks)

Determine the stability of the system using the following two methods:

(c) the transfer function of the system,
(d) the Lyapunov Equation.

State clearly the type of stability for each case and explain any observation made.

(12 marks)

Q.3. (a) Find a state feedback control

$$u = -Kx + Fr$$

which decouples the plant:

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} x.$$

Derive the closed-loop transfer function matrix.

(13 marks)

(b) Design a reduced-order state observer for the plant in (a).

(12 marks)

Q.4 (a) Consider the plant,

$$\frac{dx_1}{dt} = x_2,$$

$$\frac{dx_2}{dt} = x_1 + cu,$$

$$y = x_2.$$

For $c = 1$, find the optimal state feedback control $u = -Kx$ which minimizes the following cost function:

$$J = \frac{1}{2}\int_0^\infty \{y^2 + u^2\} dt.$$

Will the optimal feedback control system remain stable if c decreases to 0.2? Why?

(12 marks)

(b) Let a plant be described by the transfer function,

$$G(s) = \frac{1}{s(s+5)}.$$

Design a unity feedback control system to meet following requirements:

(i) The dominant dynamics can be described by standard second order system with damping ratio of 0.5, and natural frequency of 2.

(ii) The feedback control system can asymptotically track a reference signal, $r = 0.2 + 3t, t \geq 0$.

(iii) The closed loop system can eliminate the effect of any step disturbance in steady state.

(13 marks)

Appendix A - Table of Laplace Transform

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s).

x(t)	**X(s)**
unit impulse δ (t)	1
unit step 1(t)	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

END OF PAPER