5Oct23

Thursday, October 5, 2023 6:05 PM

Induction Motor – Dynamic Model

1. How does IM motor work?
2. Frames of reference →
3. Dynamic Model of IM →
4. Steady State . . .

Space vector Model of AC Machine.

$\vec{v}_s = \vec{i}_s \cdot r_s + \dfrac{d\vec{\Psi}_s}{d\tau}$ → stator coordinates

$\vec{v}_r = \vec{i}_r r_r + \dfrac{d\vec{\Psi}_r}{d\tau}$ → rotor coordinates

$\vec{\Psi}_s = l_s \vec{i}_s + l_h \vec{i}_r$

$\vec{\Psi}_r = l_h \vec{i}_s + l_r \vec{i}_r$

electromagnetic

$m_e = \vec{\Psi}_s \times \vec{i}_s$

$$\boxed{m_e - m_L = \tau_m \cdot \frac{d\omega}{d\tau}}$$

mechanical system.

O.D.E

$\dfrac{dx}{dt} = f(x, u, d)$

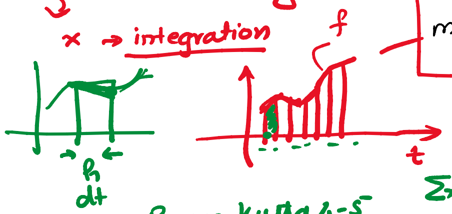
State, Forcing, dist.

matlab – m, octave – ODEint

ODEint ↳ scipy.integrate

x → integration

f



Σx

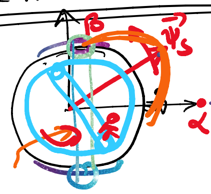
$\Sigma x \cdot dt$

Runge kutta 4-5

Runge kutta-2-3

ABf

How the induction motor works?



3phase windings & 3phase voltage.

Angular velocity of resultant stator flux vector

$\omega_s = 2\pi f_1 \rightarrow f_1 = 50\ Hz$

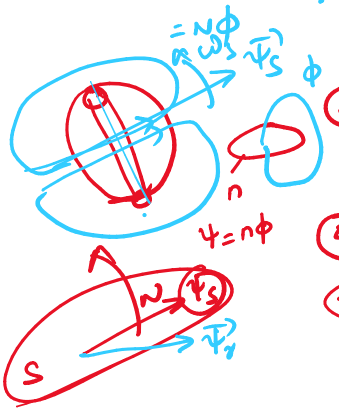
2. Initially rotor is stationary – when standing on rotor, there is a rate of change flux linkage.

3. Faraday's Voltage will be induced → because it is a caged rotor . . . currents will be induced.

4. Lenz law Induced current → Ψ_r ~>

$\vec{\Psi}_r$ will try to align with $\vec{\Psi}_s$

Torque



5. Rotor starts moving . . . angular ω

6. will $\omega = \omega_s$?

7. If $\omega = \omega_s$ → no rate of flux is seen on rotor. ∴ no voltage is induced → no rotor current no torque – – – –

8. ∴ $\omega \neq \omega_s$. IM is also called as Asynchronous Machine.

is $\omega < \omega_s$ or $\omega > \omega_s$

For above condition – $\omega < \omega_s$

Motoring

→ rotor angular

for above condition ...

Motoring

(9) $\omega < \omega_s$ $\boxed{\omega_r = \omega_s - \omega}$ → rotor angular velocity → frequency

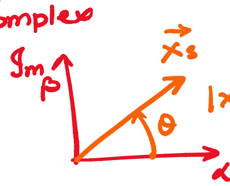
Angular velocity of stator flux linkage

(10) A steady state condition - rotor angular velocity

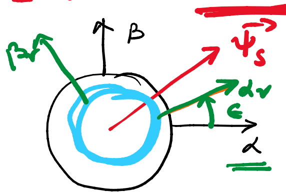
$\omega < \omega_s$ $\omega_s = 2\pi(50) = 100\pi = 314.15$

$\boxed{\omega_r = \omega_s - \omega}$

Space vectors → Complex



$|x_s| \cdot e^{j\theta}$ $\theta = \omega_s t$



$\vec{\psi}_s = |\psi_s| e^{j\omega_s t}$

$\epsilon = \omega t$

Frames of reference

Standing on rotor ... ω_s, ω

1. what is the angular frequency of $\vec{\psi}_s$ when observed from rotor = $\omega_r = \omega_s - \omega$ ← rotor angular velocity → relative angular velocity dependent on f_1

2. what is the frequency of induced rotor currents

Hz — $\boxed{f_r = \frac{\omega_r}{2\pi}}$

Stator f_1 (50Hz) → $\omega_s = 2\pi f_1$

rotor f_r → $\omega' = 2\pi f_r = \omega_r$

3. rotor currents produce the $\vec{\psi}_r'$ fluxlinkage...

Standing on the rotor, what is the angular velocity $\vec{\psi}_r'$ = ω_r

4. What is the angular velocity of the induced rotor flux linkage. when observed from stator = ω_s.

Thought experiment

$\omega + \omega_r = \omega_s$ $v + v_r = v_s$

(5). If standing on stator all quantities appear to be moving at ω_s & on rotor = ω_r.

$\boxed{\text{slip} \quad s = \frac{\omega_s - \omega}{\omega_s}} = \frac{\omega_r}{\omega_s}$ $\boxed{\omega_r = s \cdot \omega_s}$

ω_s → Stator angular velocity

$\omega_s = 2\pi f_1$ → 1 - fundamental component of stator current.

electrical angular velocity of rotor ω

ω_r → frequency of rotor induced current $\boxed{f_r = \frac{\omega_r}{2\pi}}$

$\omega_{s1} = 100\pi$

$\omega_1 = 100\pi$

E.g. An induction (4 pole induction) is supplied by 50Hz 3phase AC operates at

1. 1500 rpm at no-load.

2. Operates at 1480 rpm at some load.

$\therefore s = 0$

of stator current.

$\omega_{s1} = 100\pi$
$\omega_1 = 100\pi$

$\therefore s = 0$

$|i_r| = 0$

2. Operates at 1480 rpm at some load.

s, ω_r

RPM - - - -

RPS = $\frac{RPM}{60}$

2. Ans

$S_2 = \frac{100\pi - \omega_2}{100\pi} = \boxed{0.013}$

$\omega_2 = \frac{4}{2} \times \frac{1480}{60} \times 2\pi$

= ______

$\omega_{s_2} = 2\pi(50) = 100\pi$

Ans 1 $\quad s = \frac{\omega_s - \omega}{\omega_s}$, 100π

$\omega = \frac{p}{2} \cdot \Omega$ (no. of pole)

electrical angular velocity of rotor

Angular freq.

$\Omega = 2\pi \cdot RPS$

mechanical angular velocity of rotor.

$= 2\pi \cdot RPS$

$= \frac{2\pi \times 1500}{60}$ (50, 3)

$= 50\pi$

$= \frac{4}{2} \times 50\pi = 100\pi$

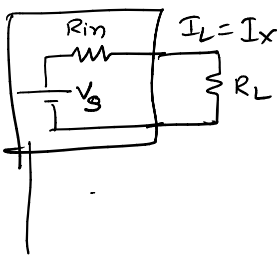
Q for case 2 what is the freq of induced rotor current?

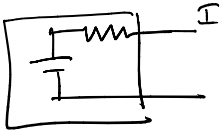
① $\omega r_2 = S_2 \cdot \omega_s = 0.013(50)100\pi$

$f_{r_2} = \frac{\omega r_2}{2\pi} = \frac{0.013 \times 100\pi}{2\pi} = 0.013(50)$ Hz

no-load current.

$I_L = I_x \rightarrow$ eq. no-load condition.

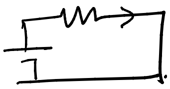




$I_L = 0$.
$R_L = \infty$

Short circuit condit

I_{sc}.



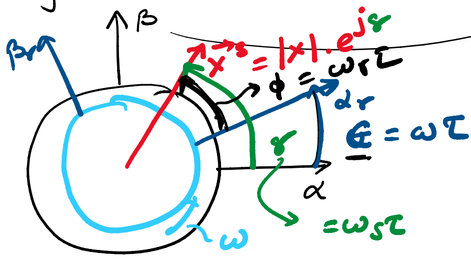
$R_L = 0$

Every thing on stator - moves at ω_s

Everything on rotor - appears to move at ω_r.

reference frame. Stator coordinate

$\vec{v}_s^{(s)} = \vec{i}_s^{(s)} \cdot r_s + \frac{d\psi_s^{(s)}}{dt}$



$\vec{x}_s^s = |x_s| \cdot e^{j\gamma}$

$= |x_s| e^{j(\epsilon + \phi)}$

$\gamma = \epsilon + \phi$

$= |x_s| e^{j\phi} \cdot e^{j\epsilon}$

$\boxed{\vec{x}_s^r = \vec{x}_s^s \cdot e^{-j\omega t}}$

Coordinate Transformations

$= \vec{x}_s^r \cdot e^{j\epsilon}$

$\vec{x}_s^s = \vec{x}^r \cdot e^{j\omega t}$ any velocity coordinate.

Stationary Coordinates, moving coordinates

$\vec{x}_s^s = x_{s\alpha} + jx_{s\beta} = (x_{s\alpha_r} + jx_{s\beta_r})(\cos(\omega t) + j\sin(\omega t))$

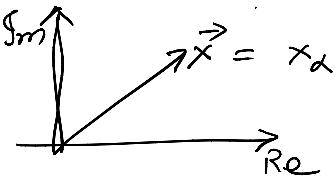
$x_{s\alpha} = x_{s\alpha_r} \cdot \cos(\omega t) - x_{s\beta r} \cdot \sin(\omega t)$

$$x_{s\alpha} = x_{s\alpha r}\cdot\cos(\omega\tau) - x_{s\beta r}\cdot\sin(\omega\tau)$$

$$x_{s\beta} = x_{s\alpha r}\cdot\sin(\omega\tau) + x_{s\beta r}\cdot\cos(\omega\tau)$$

Stator coordinate System ← rotor coordinate system

$$\begin{bmatrix} x_{s\alpha} \\ x_{s\beta} \end{bmatrix} = \begin{bmatrix} \cos(\omega\tau) & -\sin(\omega\tau) \\ \sin(\omega\tau) & \cos(\omega\tau) \end{bmatrix} \begin{bmatrix} x_{s\alpha r} \\ x_{s\beta r} \end{bmatrix} \begin{matrix} \leftarrow \text{real} \\ \leftarrow \text{imag} \end{matrix}$$



$\vec{x} = x_\alpha + j x_\beta$.

Dynamic Equation of Induction machine

Stator coordinates

$$\vec{v}_s^{\,s} = r_s \vec{i}_s^{\,s} + \frac{d\vec{\Psi}_s^{\,s}}{d\tau} \quad \text{- - - -}$$

Rotor coordinates

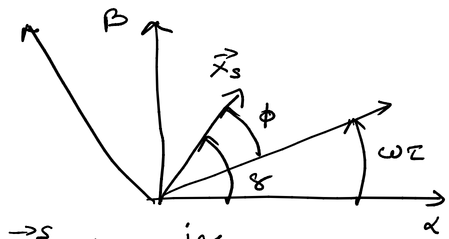
$$\vec{v}_r^{\,r} = r_r\cdot\vec{i}_r^{\,(r)} + \frac{d\vec{\Psi}_r^{\,r}}{d\tau} \quad \text{- - -}$$

Convert rotor eqn in Stator coordinates.

$\vec{\Psi}_r^{\,r} \;\circ\!\!-\!\!\bullet\; \vec{\Psi}_r^{\,(s)}$

Substituting $\vec{\Psi}_r^{\,r} = \vec{\Psi}_r^{\,s}\cdot e^{-j\omega\tau}$

$$\vec{v}_r^{\,s}\cdot e^{-j\omega\tau} = r_r\cdot\vec{i}_r^{\,s}\cdot e^{-j\omega\tau} + \frac{d(\vec{\Psi}_r^{\,s}\cdot e^{-j\omega\tau})}{d\tau}$$



$\vec{x}_s^{\,s} = |x_s|\cdot e^{j\gamma}$

$= |x_s| e^{j\phi}\cdot e^{j\omega\tau}$

$\vec{x}_s^{\,s} = \vec{x}_s^{\,r}\cdot e^{j\omega\tau}$

$\vec{x}_s^{\,r} = \vec{x}_s^{\,s}\cdot e^{-j\omega\tau}$

$$\boxed{\vec{v}_r^{\,s} = r_r\cdot\vec{i}_r^{\,(s)} - j\omega\,\vec{\Psi}_r^{\,s} + \frac{d\vec{\Psi}_r^{\,s}}{d\tau}.}$$ – rotor eqn in Stator coordinates

Dynamic equation of IM in Stator coordinates

$\vec{v}_s = r_s\vec{i}_s + \frac{d\vec{\Psi}_s}{d\tau}$ ← Ψ_r, $\vec{i}_s$

$0 = r_r\cdot\vec{i}_r - j\omega\vec{\Psi}_r + \frac{d\vec{\Psi}_r}{d\tau}$ – rotor is short-circuited $\therefore \vec{v}_r = 0$.

$\vec{\Psi}_s = l_s\cdot\vec{i}_s + l_h\vec{i}_r$

$\vec{\Psi}_r = l_h\cdot\vec{i}_s + l_r\vec{i}_r$

$m_e = \vec{\Psi}_s \times \vec{i}_s$

$\tau_m \frac{d\omega}{d\tau} = m_e - m_L$

$$\tau_m \frac{d\omega}{dt} = m_e - m_L$$

Model induction in 2 space vector state variable $\vec{\psi}_r, \vec{i}_s$

$$\vec{i}_r = \frac{1}{l_r}\left[\vec{\psi}_r - l_h \vec{i}_s\right]$$

$$\therefore \quad \vec{\psi}_s = l_s \cdot \vec{i}_s + l_h \cdot \frac{1}{l_r}\left[\vec{\psi}_r - l_h \vec{i}_s\right]$$

$$= \frac{l_h}{l_r} \cdot \vec{\psi}_r + l_s \vec{i}_s - \frac{l_h^2}{l_r} \cdot \vec{i}_s$$

$$= k_r \cdot \vec{\psi}_r + l_s \left[1 - \frac{l_h^2}{l_s l_r}\right] \vec{i}_s$$

mutual inductance

$$l_s = l_h + \underbrace{\sigma_s \cdot l_h}_{l_{s\sigma}} \qquad l_r = l_h + \underbrace{\sigma_r \cdot l_h}_{l_{r\sigma}}$$

$$\sigma = 1 - \frac{l_h^2}{l_s \cdot l_r} = 1 - \frac{\cancel{l_h}}{\cancel{l_h}(1+\sigma_s)} \cdot \frac{\cancel{l_h}}{\cancel{l_h}(1+\sigma_r)}$$

$$= 1 - \frac{1}{(1+\sigma_s)} \cdot \frac{1}{(1+\sigma_r)}$$

$$\vec{\psi}_s = k_r \cdot \vec{\psi}_r + \sigma l_s \cdot \vec{i}_s$$

$$\vec{v}_s = \vec{i}_s \cdot r_s + \frac{d}{dt}\left(k_r \vec{\psi}_r + \sigma l_s \vec{i}_s\right)$$

$$\vec{v}_s = \vec{i}_s \cdot r_s + k_r \cdot \frac{d\vec{\psi}_r}{dt} + \sigma l_s \cdot \frac{d\vec{i}_s}{dt}$$

rotor eqn

$$0 = r_r \cdot \frac{1}{l_r}\left[\vec{\psi}_r - l_h \cdot \vec{i}_s\right] + \frac{d\vec{\psi}_r}{dt} - j\omega \vec{\psi}_r$$

$\tau_r = \frac{l_r}{r_r}$

rotor time constant

$$\boxed{0 = \frac{1}{\tau_r} \cdot \vec{\psi}_r - \frac{l_h}{\tau_r} \cdot \vec{i}_s + \frac{d\vec{\psi}_r}{dt} - j\omega \vec{\psi}_r}$$

$$m_e = \vec{\psi}_s \times \vec{i}_s$$

$$= \left[k_r \vec{\psi}_r + \sigma l_s \vec{i}_s\right] \times \vec{i}_s$$

$$= [k_r \psi_r' + \sigma l_s \bar{i}_s] \times i_s$$

$$= k_r \cdot \vec{\psi}_r \times \vec{i}_s + 0$$

$$\boxed{m_e = k_r \vec{\psi}_r \times \vec{i}_s}$$

$$\boxed{T_m \frac{d\omega}{dt} = m_e - m_L}$$