Fundamental Concepts in Electrical Power

November 9, 2023

AC circuits with load notation

If the circuit is described with load notations, the current direction is shown to enter the load. Current will flow from the positive terminal of voltage voltage source into the load positive rail and come out of negative rail of load to the negative terminal of the voltage source. Most of the time we use this notation, see Fig.1. This notation is introduced to you in EG1108. In this figure the arrow shows voltage drop. The arrow goes from higher potential to lower potential. **This notation will be used for motoring operation.**

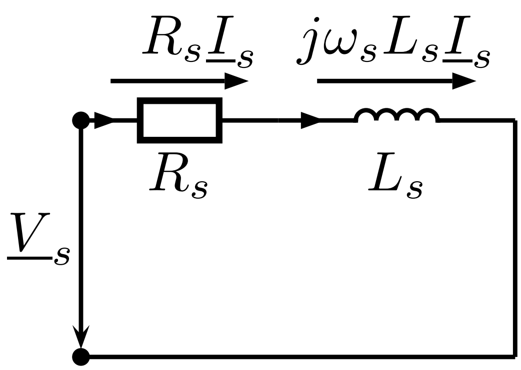

Figure 1: Circuit showing load reference notation

In this examples we can define the following phasors as

Phasors	Polar Coordinates	Rectangular coordinates
$\underline{V}_s$	$\lvert V_s\rvert e^{j\theta_V}$	$\lvert V_s\rvert cos(\theta_V) + j\lvert V_s\rvert sin(\theta_V)$
$\underline{I}_s$	$\lvert I_s\rvert e^{j\theta_I}$	$\lvert I_s\rvert cos(\theta_I) + j\lvert I_s\rvert sin(\theta_I)$
$\underline{Z}_s$	$\lvert Z_s\rvert e^{j\theta_Z}$	R+jX
Power factor angle (ϕ)	$\phi = \theta_V - \theta_I$	power factor $= cos(\phi)$
$\underline{S} = \underline{V}_s \underline{I}_s^*$	$\lvert V_s\rvert e^{j\theta_V}\lvert I_s\rvert e^{-j\theta_I} = \lvert V_s\rvert\lvert I_s\rvert e^{j\phi}$	$\lvert V_s\rvert\lvert I_s\rvert cos(\phi) + j\lvert V_s\rvert\lvert I_s\rvert sin(\phi)$
$\underline{Z}_s = \lvert Z_s\rvert e^{j\theta_z}$	$\underline{I}_s = \frac{V_s}{Z_s} e^{j(\theta_V - \theta_Z)}$	$\underline{S} = \lvert V_s\rvert \frac{\lvert V_s\rvert}{\lvert Z_s\rvert} e^{j\theta_Z}$

where

$|V_s|$: is RMS value of the voltage
$|I_s|$: is RMS value of the current
θ_V : is the phase of the complex phase $\underline{V}_s$
θ_I : is the phase of the complex phase $\underline{V}_I$

Reactive Power and Power Factor

We can defined real power, Apparent power and reactive power respectively. If we have an AC source with voltage $v(t)$ and it supplied current $i(t)$ to any load. Then the real power is average power.

$$\boxed{P = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) \cdot i(\omega t)(d\omega t)} \tag{1}$$

The root means square value (RMS) of a time varying signal is defined as

$$\boxed{X_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} x^2(\omega t)(d\omega t)}} \tag{2}$$

Apparent Power is defined as

$$\boxed{\underline{S} = \underline{V}\underline{I}^*} \tag{3}$$

The real power and reactive power can also be given in terms of complex phasors as

Complex powers

$$\underline{S} = \underline{V}\underline{I}^* \tag{4}$$

$$P = \Re\left[\underline{V}\underline{I}^*\right] \tag{5}$$

$$Q = \Im\left[\underline{V}\underline{I}^*\right] \tag{6}$$

$$\underline{S} = \left(P^2 + Q^2\right)^{1/2} \tag{7}$$

The **power factor is defined as**

$$\text{PF} = \frac{P}{|\underline{S}|} \tag{8}$$

If $\underline{Z}_s = R + jX_L = |Z_s|e^{j\theta_z}$, inductive load. Then

$$P = \Re(\underline{V}_s\underline{I}_s^*) = \Re(|V_s|\frac{|V_s|}{|Z_s|}e^{j\theta_Z}) = |V_s||I_s|cos(\theta_Z) \tag{9}$$

$$Q = \Im(\underline{V}_s\underline{I}_s^*) = \Im(|V_s|\frac{|V_s|}{|Z_s|}e^{j\theta_Z}) = |V_s||I_s|sin(\theta_Z) \tag{10}$$

Hence $Q > 0$, reactive power flows into the load. The power factor angle is $\phi = \theta_V - \theta_I = \theta_Z$ positive. The current lags the voltage. **Lagging power factor in load notation means $Q > 0$ and reactive power is consumed by the load**.

If $\underline{Z}_s = R - jX_c = |Z_s|e^{-j\theta_Z}$, capacitive load. Then

$$\begin{aligned} P = &\quad \Re(\underline{V}_s \underline{I}_s^*) \quad \Re(|V_s|\tfrac{|V_s|}{|Z_s|}e^{-j\theta_Z}) \quad |V_s||I_s|cos(-\theta_Z) \\ Q = &\quad \Im(\underline{V}_s \underline{I}_s^*) \quad \Im(|V_s|\tfrac{|V_s|}{|Z_s|}e^{-j\theta_Z}) \quad |V_s||I_s|sin(-\theta_Z) \end{aligned}$$

Hence $Q < 0$, reactive power flows out of the load. The power factor angle is $\phi = \theta_V - \theta_I = -\theta_Z$ negative. The current leads the voltage. **Leading power factor in load notation means $Q < 0$ and reactive power is generated or supplied from the load to the source**.

Generator Notation

In generator notation the current considered to flow out of the positive voltage terminal and into the negative voltage terminal see. Fig.2

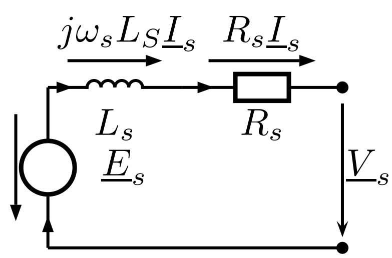

Figure 2: Circuit showing generator reference notation

Hence we can write the following in phasor notation

Phasor	Polar	Rectangular
$\underline{V}_s$	$\|V_s\|e^{j\theta_V}$	$\|V_s\|cos(\theta_V) + j\|V_s\|sin(\theta_V)$
$\underline{I}_s$	$\|I_s\|e^{j\theta_I}$	$\|I_s\|cos(\theta_I) + j\|I_s\|sin(\theta_I)$
$\underline{E}_s$	$\|E_s\|e^{j\theta_E}$	$\|E_s\|cos(\theta_E) + j\|E_s\|sin(\theta_E)$
$\underline{Z}_s$	$\|Z_s\|e^{j\theta_Z}$	R+jX
Power factor angle (ϕ)	$\phi = \theta_V - \theta_I$	power factor $= cos(\phi)$
Load angle (δ)	$\delta = \theta_E - \theta_V$	
$\underline{S} = \underline{V}_s \underline{I}_s^*$	$\|V_s\|e^{j\theta_V}\|I_s\|e^{-j\theta_I} = \|V_s\|\|I_s\|e^{j\phi}$	$\|V_s\|\|I_s\|cos(\phi) + j\|V_s\|\|I_s\|sin(\phi)$
$\underline{Z}_s = \|Z_s\|e^{j\theta_z}$	$\underline{I}_s = \frac{E_s - V_s}{Z_s}e^{j(-\theta_Z)}$	$\underline{S} = \frac{E_s V_s}{X_s}sin(\delta) + j\frac{E_s V_s}{X_s}cos(\delta) - j\frac{V_s^2}{X_s}$

where

$|V_s|$: is RMS value of the voltage at load
$|E_s|$: is RMS value of the voltage at Generating end
$|I_s|$: is RMS value of the current into load
θ_V : is the phase of the complex phase $\underline{V}_s$
θ_E : is the phase of the complex phase $\underline{E}_s$

θ_I : is the phase of the complex phase $\underline{V}_I$

If $\theta_E > \theta_I$, then current lags the voltage and $\phi_G = \theta_E - \theta_I > 0$. Hence

$$P = \Re(\underline{E}_s \underline{I}_s^*) = \Re(|E_s||I_s|e^{(\theta_E - \theta_I)}$$

Motor Power Flow direction

If we consider an electric motor with rotor voltage = 0, we can sketch the power flow from stator to load as

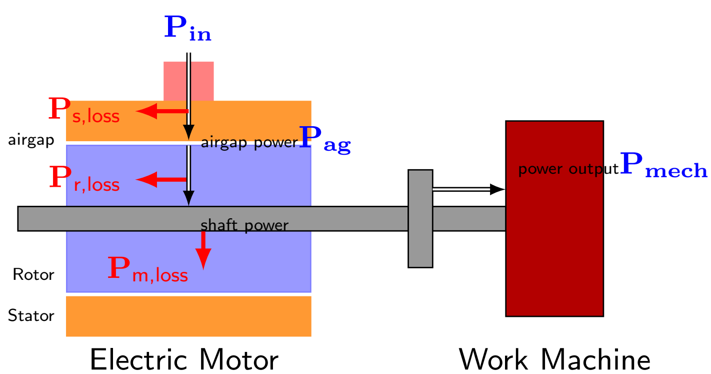

Figure 3: Motor power flow with rotor voltage

If we add a rotor source, and use motoring we can sketch a diagram as If we say that the system acts as a generator where the

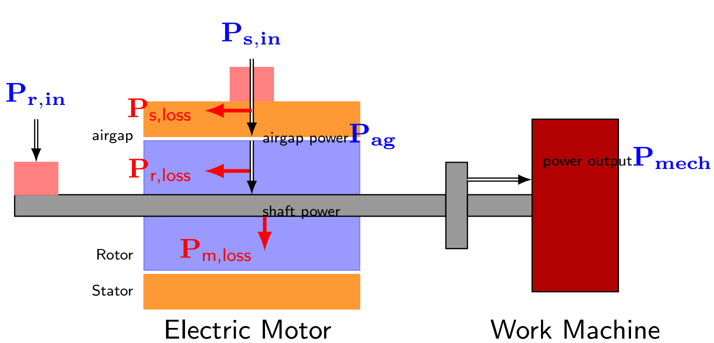

Figure 4: Motor power flow with rotor voltage

mechanical power is converted to stator and rotor electric power, we get The electric power flow for an equivalent circuit we will consider

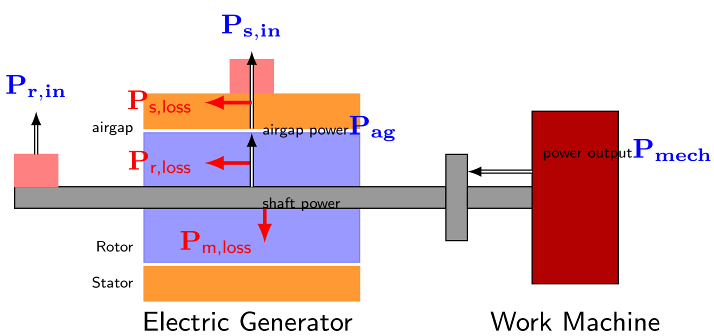

Figure 5: Generator power flow with rotor voltage

the following notation

- For all voltage source if $p > 0$, means source is supplying power. The current comes out of the phase positive terminal of the source.
- For all passive loads if $p > 0$, power is absorbed by the load. The current direction goes into the load.
- For all active loads, there will be a phase positive terminal. If $p > 0$ the power is absorbed by the load and the current goes into the phase positive terminal of the load.