Control of DC Drives

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Control of Separately excited DC machine

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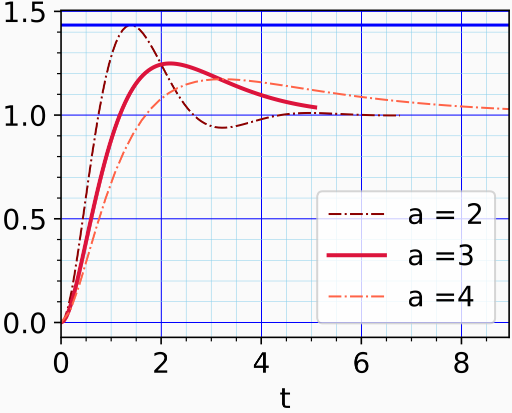
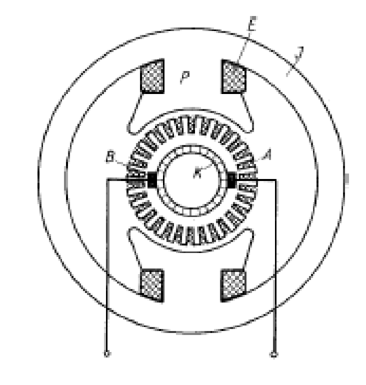


Control of Separately Excited DC Machine

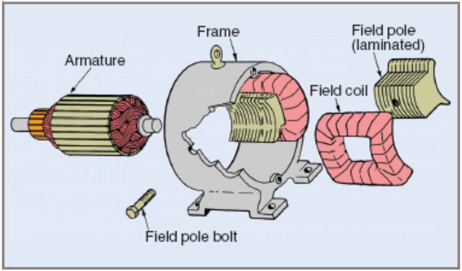
Why study Control of DC Machine

- Separately excited DC machine is a decoupled structure
- In electric motor torque is proportional to flux and current
- Flux can be controlled independently in DC motor
- For a constant Flux, the torque is controlled by controlling armature current
- Easy to study control behavior

DC motor Construction and parts



- E - Excitation coil or Field Winding
- A - Armature
- B - Brushes
- K - commutator
- P - pole



Cross fields in DC Sep Excited Motor

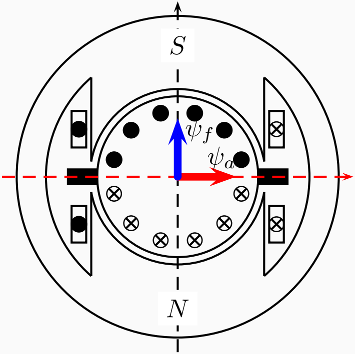
The torque produced by the two cross-fields is given as

$$M_e = \psi_f \times \psi_a \tag{1}$$

where ψ_f us the flux linkage produced by stator winding and ψ_a is the flux linkage produced by the rotor or armature winding.

$$M_e = k_e I_f I_a \tag{2}$$

where I_f is the current through field winding and I_a is the current through armature winding



Equivalent Circuit of the DC SE Machine

- Every winding can be represented by an R-L circuit
- Induced emf due to rotation in a coil make it into an R-L-E circuit

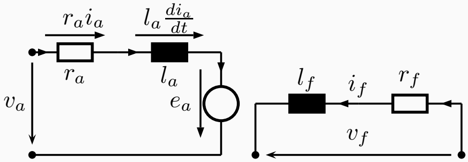
A moving coil (rotor/armature) in a stationary magnetic field will have an induced EMF as per Faraday's law

$$v_i = \frac{d\psi}{dt}$$

In case of DC SE motor, the flux is produced is stationary. Hence back-emf E_a in armature winding is given by

$$E_a = k_e I_f \Omega$$

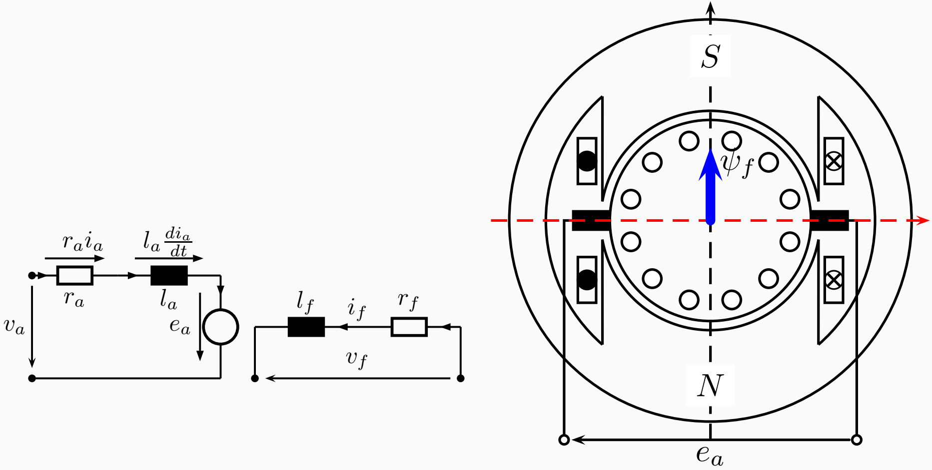
where Ω is the rotor angular velocity



$$\Omega = \frac{2\pi N}{60}$$

where N is speed in rpm

What is back-emf? where is it measured?



Dynamic Model of DC SE motor

The dynamic equations for a DC machines can be written as

$$L_a \frac{dI_a}{dt} = V_a - I_a R_a - E_a \tag{3}$$

$$L_f \frac{dI_f}{dt} = V_f - I_f R_f \tag{4}$$

$$J \frac{d\Omega}{dt} = M_e - M_L \tag{5}$$

We normalize it be dividing by the appropriate base values as follows

$$L_a \frac{dI_a}{dt} = V_a - I_a R_a - E_a \qquad \Big| \frac{1}{R_a} \frac{1}{I_{aR}} \tag{6}$$

$$L_f \frac{dI_f}{dt} = V_f - I_f R_f \qquad \Big| \frac{1}{R_f} \frac{1}{I_{fR}} \tag{7}$$

$$J \frac{d\Omega}{dt} = M_e - M_L \qquad \Big| \frac{1}{M_R} \tag{8}$$

where values with subscript R are rated values I_{aR}, I_{fR} etc

Normalization

$$\frac{L_a}{R_a}\frac{d}{dt}\left(\frac{I_a}{I_{aR}}\right) = \frac{V_a - E_a}{R_a I_{aR}} - \frac{I_a}{I_{aR}}$$

$$\frac{L_a}{R_a}\frac{d}{dt}\left(\frac{I_a}{I_{aR}}\right) = \frac{\frac{V_a - E_a}{V_{aR}}}{\frac{R_a I_{aR}}{V_{aR}}} - \frac{I_a}{I_{aR}}$$

$$\frac{L_a}{R_a}\frac{di_a}{dt} = \frac{v_a - e_a}{r_a} - i_a$$

where

$\frac{I_a}{I_{aR}} = i_a$
$\frac{V_a}{V_{aR}} = v_a$
$\frac{E_a}{V_{aR}} = e_a$
$\frac{R_a}{\frac{V_{aR}}{I_{aR}}} = r_a$

Apply similar method for Field winding equation use V_{fR} are rated value of field winding

Normalization

which will give us

$$\frac{L_a}{R_a}\frac{di_a}{dt} = \frac{v_a - e_a}{r_a} - i_a \tag{9}$$

$$\frac{L_f}{R_f}\frac{di_f}{dt} = \frac{v_f}{r_f} - i_f \tag{10}$$

$$\frac{J\Omega_0}{M_R}\frac{d\omega}{dt} = m_e - m_L \tag{11}$$

Time constants of the system

If we define the following time constants in [s]

$$T_a = \frac{L_a}{R_a} \tag{12}$$

$$T_f = \frac{L_f}{R_f} \tag{13}$$

$$T_j = \frac{J\Omega_0}{M_R} \tag{14}$$

Then we get the following equations

$$T_a \frac{di_a}{dt} = \frac{v_a - e_a}{r_a} - i_a \tag{15}$$

$$T_f \frac{di_f}{dt} = \frac{v_f}{r_f} - i_f \tag{16}$$

$$T_j \frac{d\omega}{dt} = m_e - m_L \tag{17}$$

Order of the System 3 depends on number of energy storages

Order of the system

$$T_a \frac{di_a}{dt} = \frac{v_a - e_a}{r_a} - i_a \tag{18}$$

$$T_f \frac{di_f}{dt} = \frac{v_f}{r_f} - i_f \tag{19}$$

$$T_j \frac{d\omega}{dt} = m_e - m_L \tag{20}$$

Integral form - of system

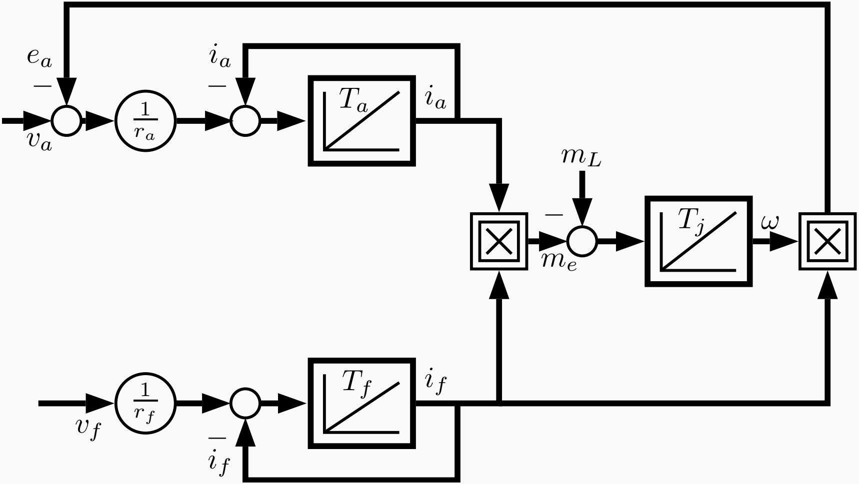
To represent the system with help of block diagrams, we can rewrite the equations in integral form as

$$i_a = \frac{1}{T_a} \int \left(\frac{v_a - e_a}{r_a} - i_a \right) dt \tag{21}$$

$$i_f = \frac{1}{T_f} \int \left(\frac{v_f}{r_f} - i_f \right) dt \tag{22}$$

$$\omega = \frac{1}{T_j} \int (m_e - m_L) dt \tag{23}$$

Block Diagram of the SE DC motor



Constant flux operation: improves the dynamics

- Field winding time constant $T_f >> T_a$ will be larger
- Fast torque change is achieved by changing I_a and
- keeping I_f is constant

Let us say the normalized value of field current is constant

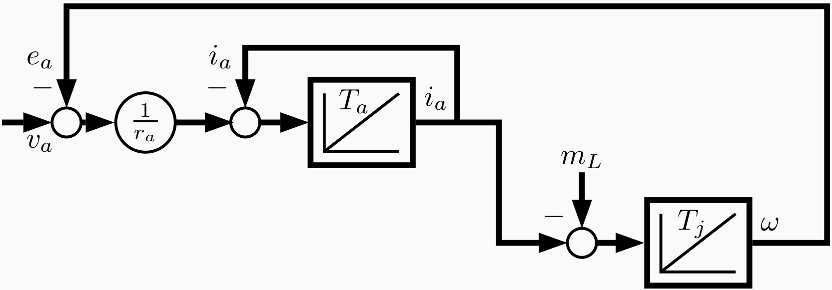
$$i_f = 1$$

The dynamics of the motor is described using

$$T_a \frac{di_a}{dt} = \frac{v_a - e_a}{r_a} - i_a \tag{24}$$

$$T_j \frac{d\omega}{dt} = m_e - m_L \tag{25}$$

Dynamics of constant flux SE DC motor



Normalized equations of the constant flux operation

To represent the system with help of block diagrams, we can rewrite the equations in integral form as

$$i_a = \frac{1}{T_a} \int \left(\frac{v_a - e_a}{r_a} - i_a \right) dt \tag{26}$$

$$\omega = \frac{1}{T_j} \int (m_e - m_L) dt \tag{27}$$

We should also note that

$$e_a = \frac{E_a}{E_{a0}} = \frac{k_e I_f \Omega}{k_e I_{fR} \Omega_0} = i_f \omega \tag{28}$$

$$m_e = \frac{M_e}{M_{eR}} = \frac{k_e I_f I_a}{k_e I_{fR} I_{ar}} = i_f i_a \tag{29}$$

where E_{a0} is the no load voltage of the motor and is equal to V_{aR}

Analysis of Dynamic behaviour of SE DC motor i

The field current is held constant at its rated value $i_f = 1$. Under this condition the dynamic equation will be

$$T_a \frac{di_a}{dt} = \frac{v_a - \omega}{r_a} - i_a \tag{30}$$

$$T_j \frac{d\omega}{dt} = i_a - m_L \tag{31}$$

Taking Laplace transform, we get

Analysis of Dynamic behaviour of SE DC motor ii

$$sT_a i_a(s) + i_a(s) = \frac{v_a(s)}{r_a} - \frac{\omega(s)}{r_a} \tag{32}$$

$$sT_j \omega(s) = i_a(s) - m_L(s) \tag{33}$$

$$(sT_a + 1) i_a(s) = \frac{v_a(s)}{r_a} - \frac{\omega(s)}{r_a} \tag{34}$$

$$i_a(s) = \frac{v_a(s)}{r_a} \frac{1}{sT_a + 1} - \frac{\omega(s)}{r_a} \frac{1}{sT_a + 1} \tag{35}$$

$$\omega(s) = \frac{i_a(s)}{sT_j} - \frac{m_L(s)}{sT_j} \tag{36}$$

Substituting and solving for $i_a(s)$, we get

Analysis of Dynamic behaviour of SE DC motor iii

$$\omega(s) = \frac{i_a(s)}{sT_j} - \frac{m_l(s)}{sT_j} \tag{37}$$

$$\omega(s) = v_a(s)\frac{1}{sT_jr_a(sT_a+1)} - \frac{m_l(s)r_a(sT_a+1)}{sT_jr_a(sT_a+1)} - \omega(s)\frac{1}{sT_jr_a(sT_a+1)} \tag{38}$$

Using $T_m = T_jr_a$, we get

$$\omega(s) = \frac{i_a(s)}{sT_j} - \frac{m_l(s)}{sT_j} \tag{39}$$

$$\omega(s) = v_a(s)\frac{1}{sT_m(sT_a+1)} - \frac{m_L(s)r_a(sT_a+1)}{sT_m(sTa+1)} - \omega(s)\frac{1}{sT_m(sT_a+1)} \tag{40}$$

$$\tag{41}$$

Analysis of Dynamic behaviour of SE DC motor iv

Solving for $\omega(s)$, we get

voltage to velocity

$$\omega(s) = \frac{1}{(s^2 T_m T_a + sT_m + 1)} v_a(s) - \frac{(sT_a + 1) r_a}{(s^2 T_m T_a + sT_m + 1)} m_L(s) \quad (42)$$

Solving for $i_a(s)$, we will get

Current response

$$i_a(s) = \frac{sT_j}{(s^2 T_m T_a + sT_m + 1)} v_a(s) + \frac{1}{(s^2 T_m T_a + sT_m + 1)} m_L(s) \quad (43)$$

Response of second order system i

We can see that the system is of 2^{nd} order. The characteristic polynomial of the system is given by

$$s^2 T_m T_a + s T_m + 1 = 0 \tag{44}$$

The roots of the characteristic polynomial are given by

$$s_{1,2} = -\frac{1}{2T_a} \pm \sqrt{\frac{1}{4T_a^2} - \frac{1}{T_m T_a}} \tag{45}$$

The roots will be real if the condition

$$T_m > 4T_a \tag{46}$$

is true.

Response of second order system ii

If $T_m < 4T_a$ then the roots will be complex conjugates. The response of the system will then be oscillatory. For a general second order system, the characteristic polynomial is given as

$$S^2 + 2D\omega_n s + \omega_n^2 = 0 \tag{47}$$

where D is the damping in the system and ω_n is the natural frequency of the system.

Rewriting the Eq.44 in that form we get,

$$s^2 + \frac{1}{T_a}s + \frac{1}{T_m T_a} = 0 \tag{48}$$

Response of second order system iii

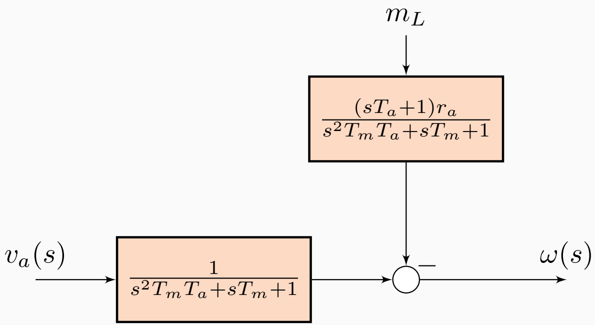
Damping and natural frequency

We can find the damping and the natural frequency as

$$D = \frac{1}{2}\sqrt{\frac{T_m}{T_a}} \tag{49}$$

$$\omega_n = \frac{1}{\sqrt{T_m T_a}} \tag{50}$$

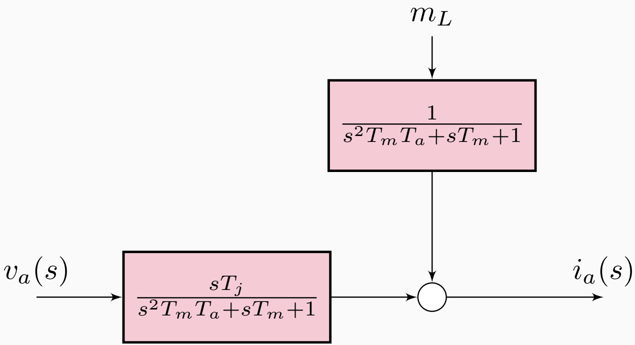
Block Diagram ang. velocity response to voltage



Ang. velocity response

- For step in armature voltage, ω response by second order transfer function $G_{\omega,va}$
- ω changes to disturbance m_L governed by disturbance path transfer function G_{ω,m_L}

Current response to change in Voltage



Current response

$$i_a(s) = \frac{sT_j}{(s^2T_mT_a + sT_m + 1)}v_a(s) + \frac{1}{(s^2T_mT_a + sT_m + 1)}m_L(s) \quad (51)$$

Example

The parameters of a separately excited DC motor, let us call it **DCM1**, are given as follows

$P_R = 22\text{kW}$ $\quad V_{aR} = 400\text{V}$ $\quad I_{aR} = 54\text{A}$ $\quad n_R = 3000\text{rpm}$

$R_a = 0.2178\Omega$ $\quad L_a = 3.4\text{mH}$ $\quad T_j = 202\text{s}$

We can hence determine the parameters for the transfer function as

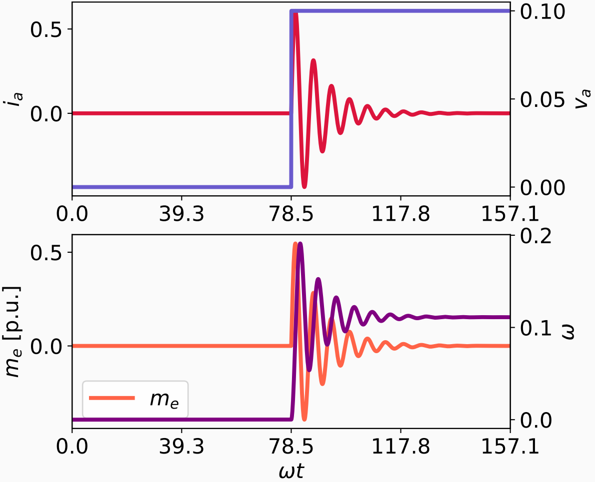
$$T_a = \frac{L_a}{R_a} = 15.61 \times 10^{-3} s$$

$$r_a = \frac{R_a I_{aR}}{V_{aR}} = 29.4 \times 10^{-3}$$

$$T_m = r_a T_j = 5.94 s$$

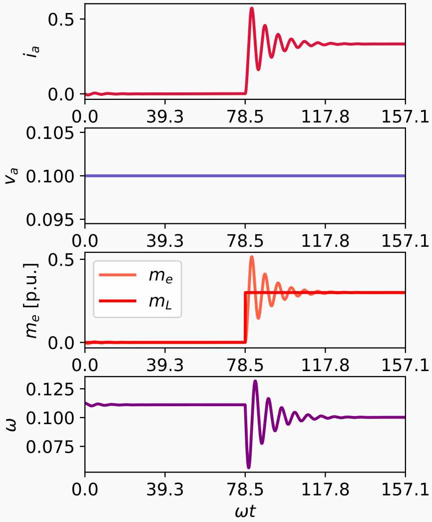
Determine the poles of the system.

Response to step change in voltage



$$i_a(s) = \frac{sT_m}{(s^2 T_m T_a + sT_m + 1)} \frac{v_a(s)}{r_a}$$

Response to step change in Load Torque



$$i_a(s) = \frac{1}{(s^2 T_m T_a + s T_m + 1)} m_L(s)$$

Cascaded Control

Why use cascaded control i

The cascaded control structure has some advantages.

- Cascaded control allows the plant to be subdivided into inner loops and outer loops. Simpler controllers can be designed for the inner loops.
- The inner loops can be assigned faster or important variables. The bandwidth of inner loop can be faster or inner loop variable can be limited by limiting the respective reference value.
- The control system can be commissioned by a step by step process staring with the inner most loop and moving out to the outer loops
- Disturbance can be isolated and eliminated in inner loops
- Effect of nonlinear or discontinuous elements can be dealt in inner loops

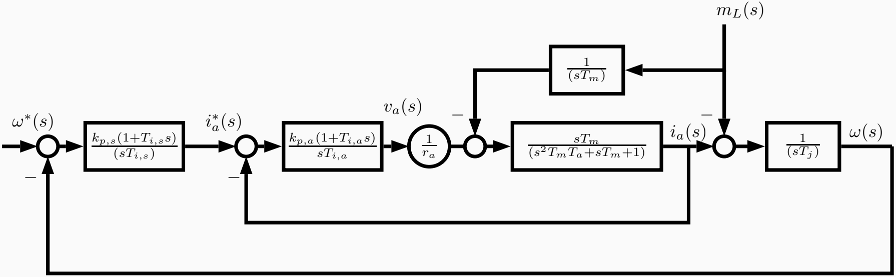
The disadvantages of this approach are

- Additional sensors and controllers are needed for each inner loop.

Why use cascaded control ii

- The bandwidth of the control loops decrease as one moves from inner loop to outer loop

Block diagram of cascaded control



Design of inner current control loop i

The forward transfer function of the current loop is given as

$$G_{Fiv} = \frac{i_a(s)}{v_a(s)} = \frac{sTj}{s^2 T_m T_a + sT_m + 1} \tag{52}$$

If we factorize the denominator assuming that $T_m > 4T_a$, using Eq.46, we get two negative real roots. The roots are

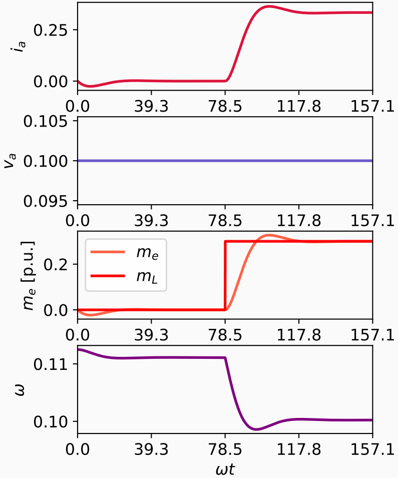
$$s_1 = -\frac{1}{2T_a} - \sqrt{\frac{1}{4T_a^2} - \frac{1}{T_m T_a}} \tag{53}$$

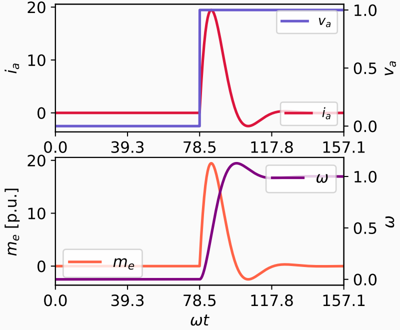
$$s_2 = -\frac{1}{2T_a} + \sqrt{\frac{1}{4T_a^2} - \frac{1}{T_m T_a}} \tag{54}$$

$$s_1 = -D\omega_n - \omega_n\sqrt{1 - D^2} \tag{55}$$

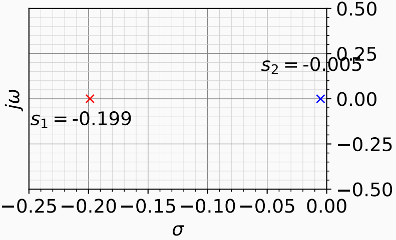
$$s_2 = -D\omega_n + \omega_n\sqrt{1 - D^2} \tag{56}$$

For case $T_m > 4T_a$





Poles of the system

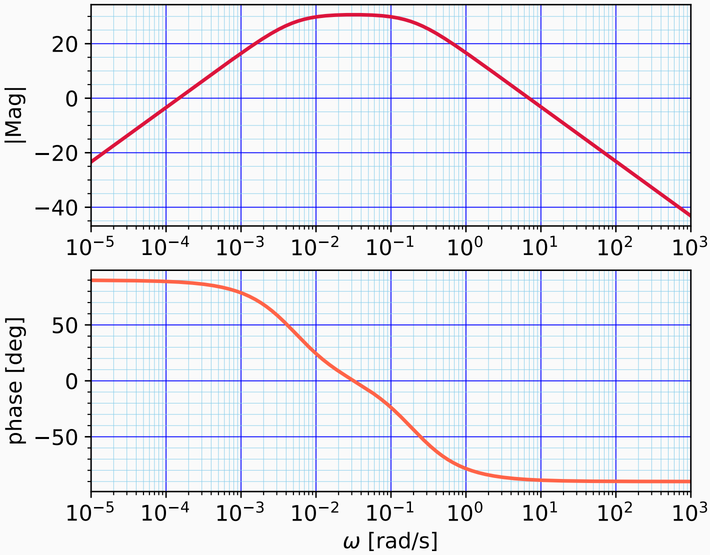


We can represent the roots by two time constants

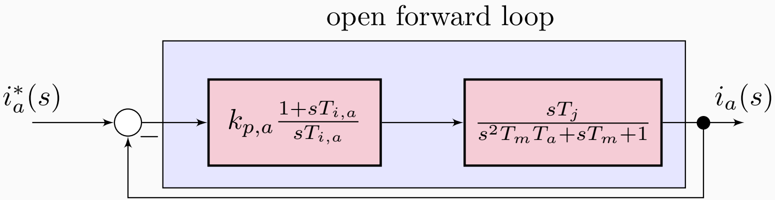
$$s_1 = -\frac{1}{T_s}$$
$$s_2 = -\frac{1}{T_l}$$

where T_s is a small time constant and T_l is the large time constant.

Bode plot of the Current forward transfer function



Current control using PI-controller



Design of inner current loop using Magnitude Optimum method i

We can represent the roots by two time constants

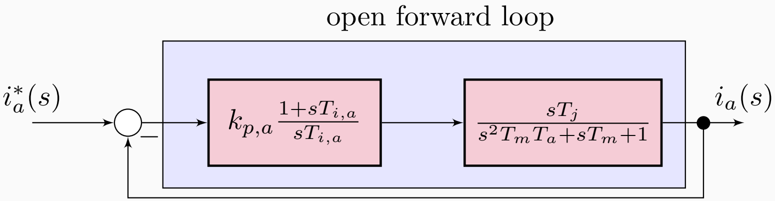
$$s_1 = -\frac{1}{T_s} \tag{57}$$

$$s_2 = -\frac{1}{T_l} \tag{58}$$

where T_s is a small time constant and T_l is the large time constant. PIController A PI controller is used, its transfer function is given as

$$G_{PI} = k_p \frac{(sT_i + 1)}{sT_i}$$

Design of inner current loop using Magnitude Optimum method ii



Hence the open loop transfer function with a PI controller for the current loop can be written as

$$G_{oiv} = \frac{k_{p,a}(1+T_{i,a}s)}{sT_{i,a}}\frac{T_lT_s}{r_a}\frac{sT_m}{(1+sT_l)(1+sT_s)} \tag{59}$$

$$G_{oiv} = \frac{k_{p,a}(1+T_{i,a}s)}{sT_{i,a}}K\frac{sT_m}{(1+sT_l)(1+sT_s)} \tag{60}$$

Design of inner current loop using Magnitude Optimum method iii

The magnitude optimum method maximizes the frequency range for which the magnitude of the transfer function is unity. We eliminate the large time constant, using the PI-controller zeros of the PI

$$T_{i,a} = T_l \tag{61}$$

The resulting transfer function becomes

$$G_{oiv} = \frac{K_{p,a}K}{sT_l}\frac{sT_m}{(1+sT_s)} \tag{62}$$

For magnitude optimum, we would like to get the damping of the inner current loop to be $D = 1/\sqrt{2}$

Design of inner current loop using Magnitude Optimum method iv

Homework exercise

- Develop the closed loop transfer function of the current loop
- Find the expression for damping in the inner current loop
- Find the value of the PI-controller gain that gives a damping of $D = 1/\sqrt{2}$ for the inner loop

The PI-Gain can be found out for optimal damping as

$$K_{p,a} = \frac{T_l}{2KT_s} \tag{63}$$

where $K = \frac{T_l T_s}{r_a}$ is the gain of G_{Fiv} and T_s and T_l are the respective constants.

Design of inner current loop using Magnitude Optimum method v

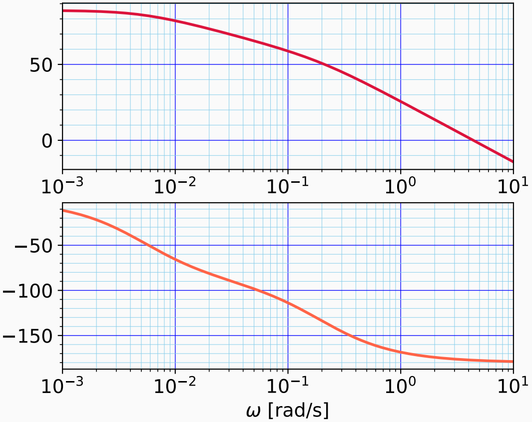
Using the rules, the open loop transfer function is given as

$$G_{oiv} = \frac{K_{p,a}(1+sT_{i,a})}{sT_{i,a}} K \frac{sT_m}{(1+sT_l)(1+sT_s)} \tag{64}$$

$$G_{oiv} = \frac{K_{p,a}K}{sT_l} \frac{sT_m}{(1+sT_s)} \tag{65}$$

$$G_{oiv} = \frac{\frac{T_m}{2T_s}}{(1+sT_s)} \tag{66}$$

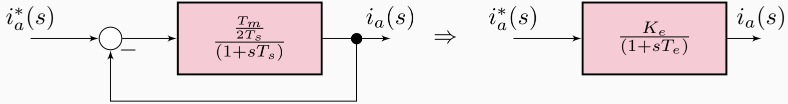
Open loop transfer function of inner current loop



Open loop transfer function

$$G_{oiv} = \frac{\frac{T_m}{2T_s}}{(1 + sT_s)}$$

Closed loop transfer function i



The closed loop transfer function of the inner current loop can be derived as

$$G_{civ} = \frac{G_{oiv}}{1 + G_{oiv}} \tag{67}$$

$$G_{oiv} = \frac{T_m/2T_s}{(1 + sT_s)} \tag{68}$$

$$G_{civ} = \frac{\frac{T_m/2T_s}{(1+sT_s)}}{1 + \frac{T_m/2T_s}{(1+sT_s)}} \tag{69}$$

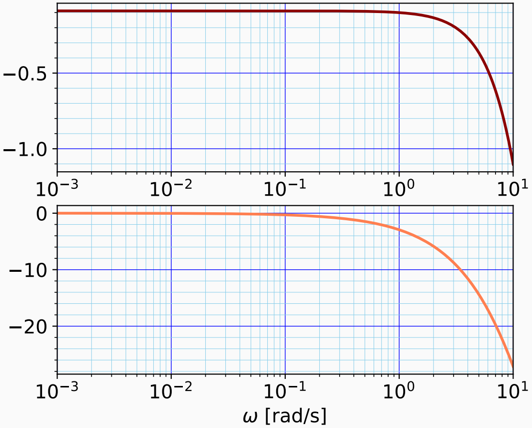
$$G_{civ} = \frac{T_m/2T_s}{1 + sT_s + T_m/2T_s} \tag{70}$$

Closed loop transfer function ii

The closed loop transfer function of the current loop can be represented by a equivalent first order system as

$$\frac{i_a(s)}{i_a^*(s)} = \frac{K_e}{sT_e + 1} = G_{civ} = \frac{T_m/2T_s}{1 + sT_s + T_m/2T_s} \tag{71}$$

Open loop transfer function of inner current loop

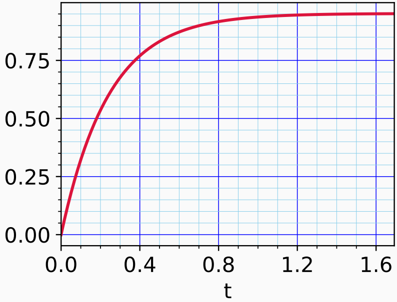


Close loop transfer function

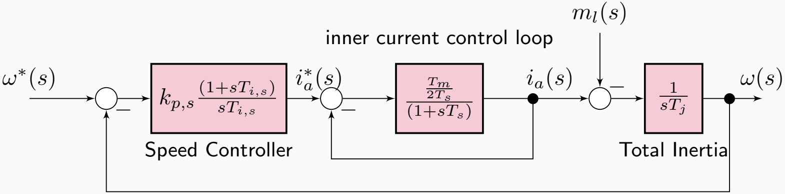
$$\frac{i_a(s)}{i_a^*(s)} = \frac{K_e}{sT_e + 1} = G_{civ} = \frac{T_m/2T_s}{1 + sT_s + T_m/2T_s}$$

Closed loop Inner current response

- the Closed loop of current is equivalent to first order system
- inner current loop has steady state error
- This is due to presence of back-emf
- It can be removed by compensating the back-emf
- feed forward compensation can be used



Design of Speed Controller using Symmetrical Optimum i



The forward transfer function of the speed loop with an inner current loop is given by

$$G_{F\omega i} = \frac{k_e}{(sTe+1)}\frac{1}{sT_j} \tag{72}$$

Design of Speed Controller using Symmetrical Optimum ii

Symmetrical optimum

The two time constant of the system are far apart

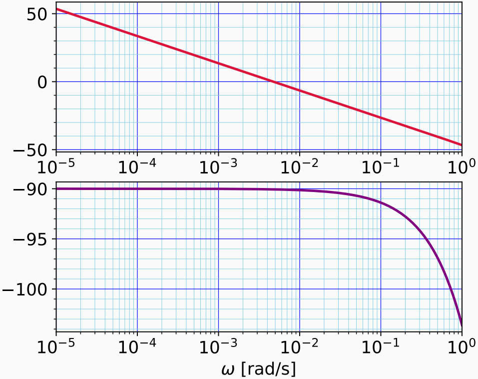
$$T_j > 10T_e$$

we use symmetrical optimum to design the controller

A PI-Controller is used to achieve closed loop speed control.

Forward Gain of speed loop

- $T_j \approx 200$
- $T_e \approx 0.2$
- The integrator effect of inertia dominant
- use Symmetrical optimum



Symmetrical Optimum

Symmetrical Optimum

- If we eliminate the smaller time constant with PI zero, the system will be a double integrator
- In symmetrical optimum, we lift the phase plot near the cross over frequency to provide sufficient phase margin
- The gain plot will be symmetrical around crossover frequency
- It will have a slope of 20 [db/decade].
- We choose the PI integral time constant as

$$T_{i,s} = a^2 Te \tag{73}$$

- $a > 1$ is length of symmetrical region

Design of Speed controller with Symmetrical Optimum

We have, PI time constant

$$T_{i,s} = a^2 T_e \tag{74}$$

where, the equivalent inner loop is

$$G_{civ} = \frac{k_e}{(sTe+1)}$$

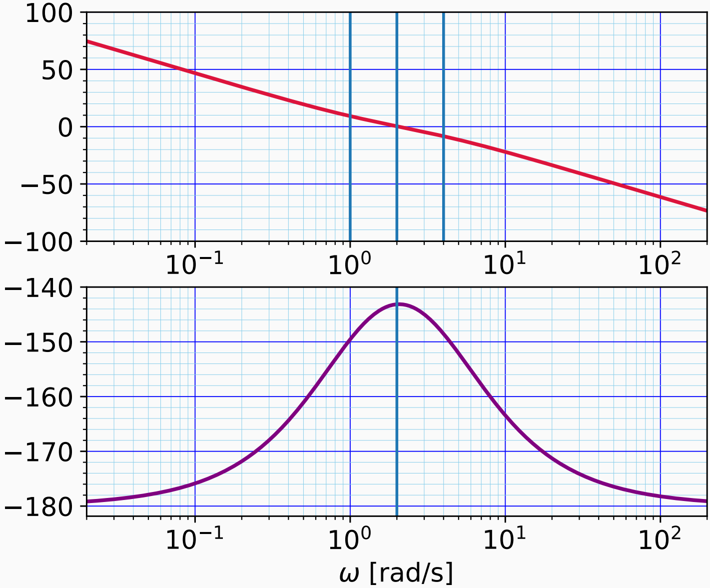
The PI gain is selected as

$$k_{p,s} = \frac{T_j}{a k_e T_e} \tag{75}$$

Hence the open loop transfer function with PI is

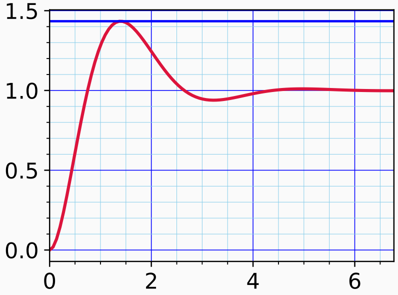
$$G_{osi} = \frac{T_j}{a k_e T_e} \frac{1 + s a^2 T_e}{s a^2 T_e} \frac{k_e}{(sTe+1)} \frac{1}{sT_j} \tag{76}$$

Symmetrical optimum bode plot for $a = 2$



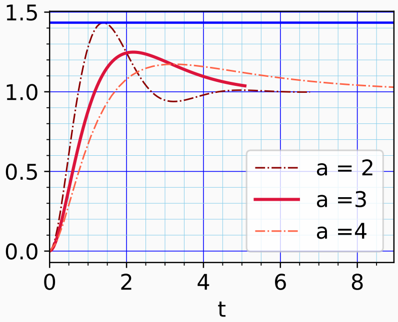
Step response with $a = 2$ produces 43.4% overshoot

- Step response for the closed loop speed control
- Produces a 43.4% overshoot
- A large overshoot in speed in not desired
- Many applications need tight speed control
- increasing $a > 2$ reduces overshoot but slows response



Step response with changing a

- Step response for the closed loop speed control
- Produces a 43.4% overshoot
- A large overshoot in speed in not desired
- Many applications need tight speed control
- increasing $a > 2$ reduces overshoot but slows response



State Variable Control of the Sep. EXcited DC Motor

The DC motor dynamics can be described by the equations

$$T_a \frac{di_a}{dt} = -i_a - \frac{1}{r_a} e_a + \frac{1}{r_a} v_a$$
$$T_j \frac{d\omega}{dt} = m_e - m_L$$

For the separately excited DC machine, when the flux is constant, we get

$$e_a = k_e \omega \tag{77}$$
$$m_e = k_t i_a \tag{78}$$

We can write the dynamics as

$$T_a \frac{di_a}{dt} = -i_a - \frac{1}{r_a} k_e \omega + \frac{1}{r_a} v_a$$
$$T_j \frac{d\omega}{dt} = k_t i_a - m_L$$

State Variable Control of the Sep. EXcited DC Motor

We can re arrange the for to $\dot{x} = Ax + Bu$ as

$$\frac{di_a}{dt} = -\frac{1}{T_a} i_a - \frac{1}{l_a} k_e \omega + \frac{1}{l_a} v_a \tag{79}$$

$$\frac{d\omega}{dt} = \frac{k_t}{T_J} i_a - m_L \frac{1}{T_J} \tag{80}$$

If we take the load torque as consisting of viscous friction then $m_L = b\omega$, and the equations will be

$$\frac{di_a}{dt} = -\frac{1}{T_a} i_a - \frac{1}{l_a} k_e \omega + \frac{1}{l_a} v_a \tag{81}$$

$$\frac{d\omega}{dt} = \frac{k_t}{T_J} i_a - \frac{b}{T_J} \omega \tag{82}$$

State Variable Equations in matrix form for SE DC motor

Representing it in standard matrix form, we get

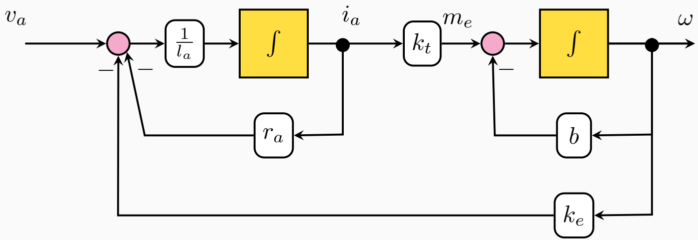
$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{l_a} \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{l_a} \\ 0 \end{bmatrix} v_a \tag{83}$$

As the system now is represented in

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

A state-variable block diagram can be drawn based on the equations which gives us a better insight into the physical system.

State Variable Block Diagram of SE DC motor

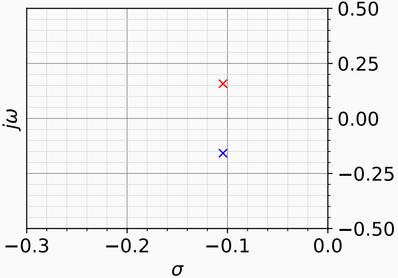


From the diagram we can see that

- The r_a is gain for the feedback of the physical state i_a
- k_e is the gain for the physical state feedback of ω
- b is the gain for physical state feedback ω

The poles of the system are given by the eigenvalues of $\mathbf{A}$ and depend on the gains of the physical feedback.

State Variable Block Diagram of SE DC motor



where

r_a : 0.0294
l_a : 0.1442
t_j : 199.755
b : 0.5
$k_e = k_t$: 1

The poles of the system are given by the eigenvalues of $\mathbf{A}$ and depend on the gains of the physical feedback.

Shaping the response using state-variable feedback

We can at a feedback from the state variables to change the response of the system. The state feedback vector is $\mathbf{k}$ where

$$\mathbf{k} = \begin{bmatrix} -k_1 & -k_2 \\ 0 & 0 \end{bmatrix} \tag{84}$$

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a} & -\frac{k_e}{l_a} \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \frac{k_g}{l_a} \begin{bmatrix} -k_1 & -k_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{k_g}{l_a} \\ 0 \end{bmatrix} v_a \tag{85}$$

The system matrix with feedback can be calculated using

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & -\frac{k_e}{l_a} - \frac{k_g k_2}{l_a} \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{k_g}{l_a} \\ 0 \end{bmatrix} v_a \tag{86}$$

Insight into feedback and pole placement

Given the new $\mathbf{A}$ matrix

$$\begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & -\frac{k_e}{l_a} - \frac{k_g k_2}{l_a} \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \tag{87}$$

we can use the feedback gain to do the following

- augment the physical state feedback gain or,
- cancel the impact of physical feedback
- or reverse the polarity of the physical feedback gain

We can see that the back-emf state feedback acts as a disturbance to the current path, we can decouple by making $a_{1,2} = 0$

Decoupling back-emf effect

by choosing

$$k_2 = -\frac{k_e}{k_g}$$

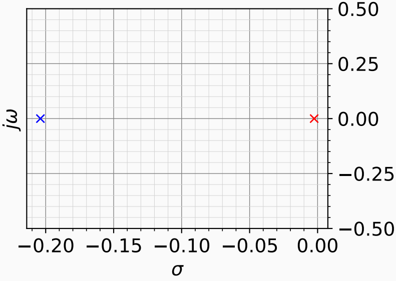
we get

$$\begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & -\frac{k_e}{l_a} - \frac{k_g}{l_a}\frac{-k_e}{k_g} \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \tag{88}$$

We get

$$\begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & 0 \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \tag{89}$$

Pole plot for decoupling the back-emf



- We have 2 real poles, one close to origin and another at (-0.204,0)
- The current response will depend on the faster time constant given by pole at (-0.204,0), by choosing k_1 we can shift this pole to left to produce faster response

where

r_a : 0.0294

l_a : 0.1442

t_j : 199.755

b : 0.5

$k_e = k_t$: 1 $k_2 = -\frac{k_e}{k_g}$

$k_1 = 0$

$$\begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & 0 \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix}$$

Decoupling back-emf effect and shaping response

by choosing

$$k_2 = -\frac{k_e}{k_g}$$

and

$$k_1 = k_g k_1 \quad \text{with} \quad k_1 > r_a$$

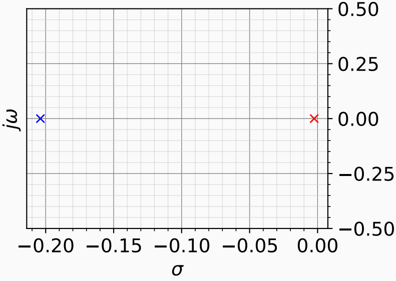
we get

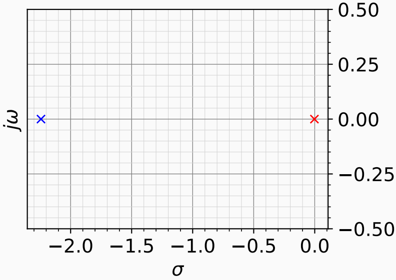
$$\begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & -\frac{k_e}{l_a} - \frac{k_g}{l_a}\frac{-k_e}{k_g} \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \tag{90}$$

We get

$$\begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & 0 \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix} \tag{91}$$

Pole plot for decoupling the back-emf and pole shifting





- We have 2 real poles, one close to origin and another at (-0.204,0)
- The current response will depend on the faster time constant given by pole at (-0.204,0), by choosing k_1 we can shift this pole to left to produce faster response

$$k_1 = 10r_a$$

$$\begin{bmatrix} -\frac{1}{T_a} - \frac{k_g k_1}{l_a} & 0 \\ \frac{k_t}{T_j} & -\frac{b}{T_J} \end{bmatrix}$$

The left hand has pole has moved from (-0.204,0) to (-2.242,0). A faster response is produced

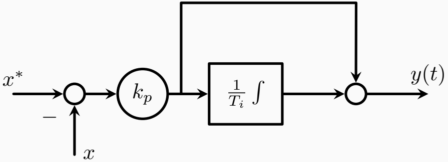
Appendix: Proportional Integral Controller PI-Control

The PI- controller works on the error input

$$y(t) = k_p \left(e(t) + \frac{1}{T_i} \int e(t) dt \right)$$

$$Y(s) = \left(k_p + \frac{k_p}{sT_i} \right) E(s)$$

$$\frac{Y(s)}{E(s)} = k_p \frac{sT_i + 1}{sT_i}$$



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