Induction Motor Drives

Modeling and Control

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Induction Motor Drives: How do I model this?

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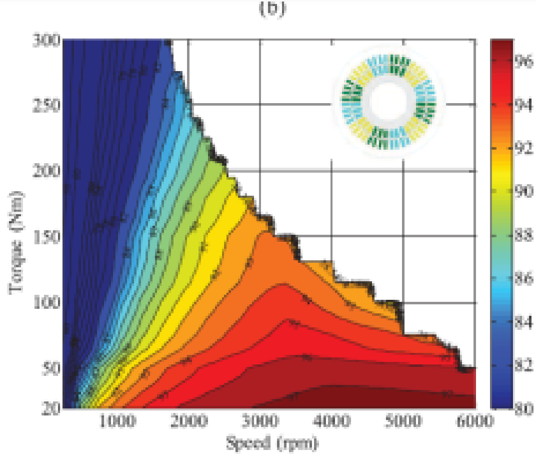


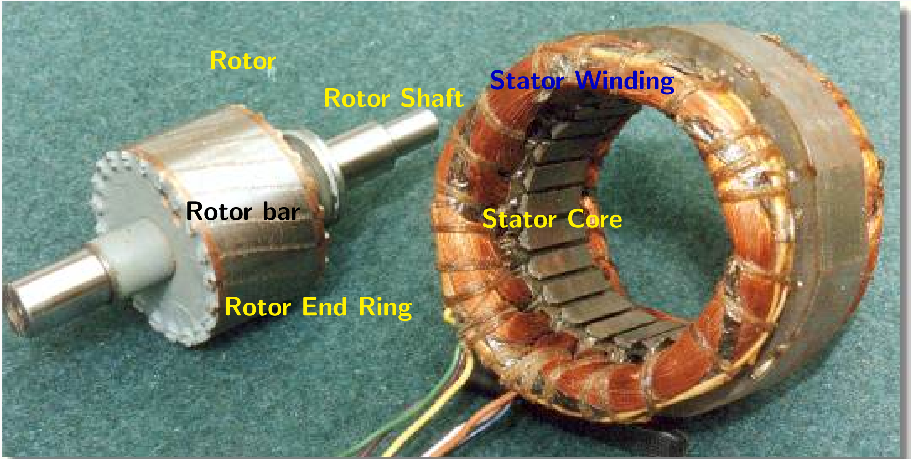
Why Induction Motors?

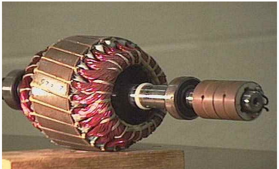
- Induction motor is the preferred AC motor in almost 80% of Industry Applications
- Pumps, fans, Blowers, Low dynamic Variable speed applications etc
- Cage Rotor Induction Motor: Easy construction and low cost
- **Tesla EV started with Induction Motor but made it more efficient, but costly**
- With digital control, IM can give as good performance as DC-SE motor but at low cost and high efficiency
- **Field-Oriented Vector Control patented by F Blaschke, from Siemens in 1972 made it happen**
- By 1992 the patent lapsed and fast development in μP technology has made Vector Controlled IM ubiquitous

Learning Objectives

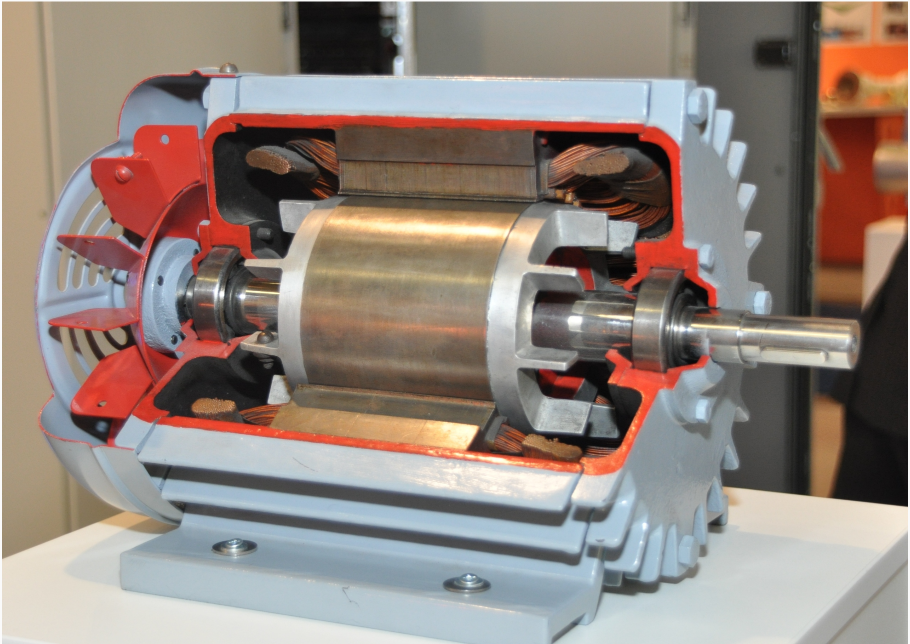
- How does the induction machine work?
- What are the different frames of observation?
- What is the relation of the stator electromagnetic quantities in these different frames of observation?
- How to represent induction machine using space vectors?
- Can we represent the space vector description using the equivalent circuit?
- How can we represent the steady state relation in induction motor using space vectors?
- How does the speed and torque of IM vary in steady state when voltage, current, frequency or resistance is changed?
- What is four quadrant operation?
- What is braking? How can we achieve it?
- How can we achieve dynamic torque control of the IM? What is the required condition?

Induction motor Disassembled: Cage Rotor





Induction Motor Assembly



Stator Produces Rotating Magnetic field

- A rotating magnetic field is produced by stator winding
- When supplied with 50 [Hz], rotates at $\omega_s = 314.15$[rad/sec]
- We have seen, how distributed winding produced rotating fields
- If Rotor is stationary, at start
- Rotor windings see a rate of change of stator flux at ω_s
- **What do you think will happen in rotor?**

Stator Field Cuts Rotor

- If rotor is stationary, you stand of rotor what will you see?
- Rotating magnetic field at ω_s
- Hence, will induce **voltage** is rotor.
- Since IM rotor is short-circuited,
- rotor current will flow
- **What is the frequency of the rotor current, if you stand and observe from rotor?**
- The rotor MMF interacts with stator field to produce torque
- The rotor starts rotating
- **Will rotor angular velocity $(\omega) = \omega_s$?**

Rotor Induced field observed from Rotor reference frame

- **Rotor angular velocity** $\omega \neq \omega_s$
- **because: if rotor moves synchronously, not rotor current can be induced**
- no rotor current, no torque -> no motion
- Hence, IM is called **Asynchronous Motor (ASM)**
- **Rotor current frequency** f_r

$$f_r = \frac{\omega_s - \omega}{2\pi}$$
$$\omega_r = \omega_s - \omega$$

slip (rated slip is in range of 0.01)

$$s = \frac{\omega_s - \omega}{\omega_s}$$

Rotor Induced field observed from Stator reference frame

- When observed from rotor, rotor MMF rotates at ω_r
- **What happens when you observe it from stator?**
- Since rotor rotates at ω w.r.t stator frame
- and on rotor, rotor MMF rotates at ω_r w.r.t rotor
- From Stator frame, rotor MMF moves at $\omega + \omega_r$
- **Rotor MMF moves at ω_s w.r.t stator frame**

Since every MMF also produces a flux linkage, Rotor flux linkage rotates at ω_s [rad/sed] when observed form stator reference frame

Stator and Rotor field observed from Stator reference frame

- If we now observe Stator field (flux linkage space vector)
- and rotor field (rotor flux linkage) from stator frame,
- Both will have same angular velocity ω_s
- But have angular displacement.
- The stator field (cause) leading the rotor field (effect)
- This is in motoring operation

Locked Rotor observed from Stator reference frame

- **For locked rotor or Starting, when** $\omega = 0$
- **slip, s = 1.0 and** $\omega_r = \omega_s$

Thought experiment: Locked Rotor

- If the rotor is locked, cannot move; $\omega = 0$
- Hence rotor MMF, on rotor will move with $\omega_r = \omega_s$
- from rotor reference frame, rotor field will have ω_s velocity
- Form stator reference frame, it will have same ω_s velocity
- **Any field/MMF observed from stator reference frame always has** ω_s **velocity**
- **From rotor frame all quantities will have relative angular velocity** ω_r
- $\omega_r = \omega_s - \omega$ **changes with rotor angular velocity**

Rotor moving faster than stator field observed from Rotor reference frame

Thought Experiment: Rotor moving faster than stator field

- Suppose the load driving the rotor, cause it to move faster than stator field
- then $\omega > \omega_s$, hence $\omega_r < 0$
- **Observing from rotor reference frame, at what ang. velocity does the stator field move?**
- it will appear to be moving in opposite direction at ω_r
- This causes power to flow from rotor to stator.

How does and Induction motor work? How do we achieve rotation? i

- The distributed windings in the stator when supplied with three phase voltages with a phase difference of 120 degrees produces a rotating magnetic field.
- This field, cuts the conductors in the rotor at an angular velocity $\omega_s = 2\pi f_s$.
- The rate of change of flux linking the rotor conductors induces an EMF which forces a current through the short circuited rotor.
- These induced current in turn produces a magnetic field which tries to catch up with the stator field by producing a torque on the rotor.
- If the rotor catches up with the stator the rate of change of flux with respect to the rotor conductors is zero and no currents are induced.
- Thus in practice the rotor can never catch up with the stator. Therefore this machine is also called as the **asynchronous machine**.

How does and Induction motor work? How do we achieve rotation? ii

- However in theory we assume that the rotor angular velocity $\omega = \omega_s$ at no-load.

Definition

The difference between the stator angular velocity and the rotor angular velocity is often expressed as slip

$$s = \frac{\omega_s - \omega}{\omega_s} \tag{1}$$

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$$s = \frac{\omega_s - \omega}{\omega_s} \tag{2}$$

Frames of reference and relative frequency

Important concept to note

- The stator is fed with a voltage of synchronous frequency. This voltage produces a rotating magnetic field which rotates at synchronous angular velocity ω_s.
- Once the rotor accelerates and rotates at an asynchronous angular velocity $\omega < \omega_s$.
- If you stand on the rotor, which is rotating at ω, what is the frequency at which the stator field is rotating with respect to you?
- It should be $\omega_s - \omega = s\omega_s = s2\pi f_s$. **It is called as slip frequency**
- The frame of reference when you stand on rotor is called as **Rotor reference frame or Rotor coordinate system**

Rotor coordinate system

Definition

Rotor coordinate system or rotor reference system is when the observer stands on the rotor, rotating at angular velocity ω. The induced rotor currents due a stator rotating field, rotating at ω_s will have frequency

$$\omega_r = \omega_s - \omega = 2\pi f_r = s 2\pi f_s \tag{3}$$

All electromagnetic state variables when observed from rotor coordinates will have same frequency $f_r = s f_s$

Stator coordinate system or Stator reference frame

Standing on stator

- Stator is supplied with voltage of angular frequency ω_s
- Rotor is rotating at angular velocity ω
- The induced rotor current, when observers from rotor has ang. freq $\omega_r = \omega_s - \omega$
- What is the frequency of the rotor currents (or magnetic field produced by them), when observed from stator?
- Ans: The rotor field produced by rotor currents when seen from rotor has an angular velocity of $\omega_r = \omega_s - \omega$
- Since the rotor is rotating with ω angular velocity
- The relative angular velocity of the rotor field when seen from stator will be

$$\omega_s - \omega + \omega = \omega_s$$

- **All electromagnetic variables of the Induction machine, when observed from the stator will have angular velocity ω_s**

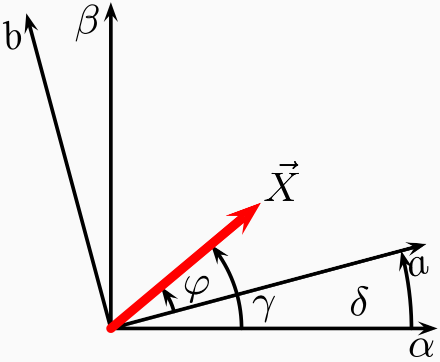
Example

Example

Suppose an induction machine is supplied by a 50 Hz AC supply and the slip is equal to 0.05. What is the angular velocity of the stator and the rotor currents when observed from the stator and the rotor?

- The stator is supplied from 50 Hz AC hence the angular velocity of the stator current as observed from the stator is $2\pi 50$.
- Rotor angular velocity: As the slip is 0.05 the rotor moves at an electrical angular velocity of $\omega = \omega_s - s\omega_s$
- $\omega = (1 - 0.05)2\pi 50$
- Rotor currents observed from the rotor will have frequency $sf_s = 0.05 \times 50$ Hz
- Rotor currents observed from stator will have frequency 50 Hz
- Stator currents observed from stator will have frequency 50 Hz
- Stator current observed from rotor will have frequency 0.05×50 Hz

Coordinate Transformation: Important concept



We have 2 coordinate systems. s system is given by axes $\alpha - \beta$ and r system is given by axes $a - b$. A space vector $\vec{X}$ can be defined by any of the coordinate system using its magnitude and angle. We can say that the all angles are time varying. Then we can write

$$\vec{X}^s = |X|e^{j\gamma} \tag{4}$$

$$\vec{X}^r = |X|e^{j\varphi} \tag{5}$$

$$\vec{X}^s = |X|e^{j\gamma} \tag{6}$$

$$\vec{X}^s = |X|e^{(j\varphi + j\delta)} \tag{7}$$

$$\vec{X}^s = \vec{X}^r e^{j\delta} \tag{8}$$

$$\vec{X}^r = \vec{X}^s e^{-j\delta} \tag{9}$$

Space Vector representation of Induction Machine

System equation for AC machine

We have derived the normalized space vector equation for an AC machine

$$\vec{v}_s^s = r_s \vec{i}_s^s + \frac{d\vec{\psi}_s^s}{d\tau} \tag{10}$$

$$\vec{v}_r^r = r_r \vec{i}_r^r + \frac{d\vec{\psi}_r^r}{d\tau} \tag{11}$$

$$m_e = \vec{\psi}_s \times \vec{i}_s \tag{12}$$

$$m_e = \Im\{\vec{\psi}_s^* \vec{i}_s\} \tag{13}$$

$$m_e = \Im\{(\psi_{s\alpha} - j\psi_{s\beta})(i_{s\alpha} + j i_{s\beta})\} \tag{14}$$

$$p_e = \Re\{\vec{v}_s^* \vec{i}_s\} \tag{15}$$

$$p_e = \Re\{(u_{s\alpha} - j u_{s\beta})(i_{s\alpha} + j i_{s\beta})\} \tag{16}$$

$$J\frac{d\omega}{d\tau} = m_e - m_L \tag{17}$$

for Induction motor, since the rotor is short-circuited, $\vec{v}_r^s = 0$

Space vector representation of Induction Machine

$$\vec{v}_s^s = r_s \vec{i}_s^s + \frac{d\vec{\psi}_s^s}{d\tau} \tag{18}$$

$$0 = r_r \vec{i}_r^s + \frac{d\vec{\psi}_r^s}{d\tau} - j\omega\vec{\psi}_r^s \tag{19}$$

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$$J\frac{d\omega}{d\tau} = m_e - m_L \tag{25}$$

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- $$v_{s\alpha} + jv_{s\beta} = \frac{2}{3}\left(v_U(t) + \vec{a}v_V(t) + \vec{a}^2 v_W(t)\right)$$

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- Write space vector equation for stator and rotor flux linkages
- $$\vec{\psi}_s = l_s \vec{i}_s + l_h \vec{i}_r$$
  $$\vec{\psi}_r = l_h \vec{i}_s + l_r \vec{i}_r$$

Revision: Rotor equation

- In rotor coordinate, the rotor voltage equation, using Faraday's and Ohm's law is given as

$$\vec{v}_r^r = r_r \vec{i}_r^r + \frac{d\vec{\psi}_r^r}{d\tau}$$

  Derive the rotor equation for the induction machine in stator coordinates to be

$$0 = r_r \vec{i}_r^s + \frac{d\vec{\psi}_r^s}{d\tau} - j\omega\vec{\psi}_r^s$$

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$$\vec{x}^s = \vec{x}^r e^{j\omega t}$$

- Hint: A space vector can be expressed in polar form as

$$\vec{x} = x e^{j\omega_s \tau}$$

  Hence, it's differential will be

$$\frac{d\vec{x}}{d\tau} = \frac{dx}{d\tau} e^{j\omega_s \tau} + j\omega_s x e^{j\omega_s \tau}$$

Rotor Voltage equation in rotor coordinate system

- Rotor voltage equation when observed from the stator can be written as

$$\vec{v}_r^r = r_r \vec{i}_r^r + \frac{d\vec{\psi}_r^r}{d\tau}$$
$$\vec{v}_r^s e^{-j\omega\tau} = r_r \vec{i}_r^s e^{-j\omega\tau} + \frac{d\vec{\psi}_r^s e^{-j\omega\tau}}{d\tau}$$

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- Hence, we can write

$$\vec{v}_r^s e^{-j\omega\tau} = r_r \vec{i}_r^s e^{-j\omega\tau} + \frac{d\vec{\psi}_r^s}{d\tau} e^{-j\omega\tau} - j\omega \vec{\psi}_r^s e^{-j\omega\tau}$$

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Before we go forward, Retrieve:

- Suppose an IM is operating at $\omega = 1.0$ and $m_L = 0.5$
- For the same torque I would like to operate at $\omega = 0.5$
- How do achieve this?

Steady State operation of Induction Machine

What is steady state?

Definition

Steady State of the machine is when the **magnitude of the space vector is constant w.r.t time** $\frac{d|x|}{dt} = 0$, the angle of the space vector changes as constant angular velocity ω_s which depends on the supply frequency

The flux linkage space vector can be defined as $\vec{\psi} = \psi e^{j\omega t}$ the induced emf in steady state is

$$\vec{v}_i = \frac{d\psi_s e^{j\omega_s t}}{dt} \tag{27}$$

$$\vec{v}_i = \frac{d\psi_s}{dt} e^{j\omega_s t} + j\omega_s \psi_s e^{j\omega_s t} \tag{28}$$

$$\vec{v}_i = j\omega_s \vec{\psi}_s \tag{29}$$

Steady state description

Magnitude constant, angle changes

$$\vec{v}_i = \underline{\vec{v}}_i e^{j\omega_s \tau}$$

Stator Voltage equation in steady state

The stator voltage equation in steady state can be written as

$$\vec{v}_s = r_s\vec{i}_s + j\omega_s\vec{\psi}_s \tag{30}$$

We can write the above equation in a form

$$\underline{v}_s e^{j\gamma + j\omega_s\tau} = r_s\underline{i}_s e^{j\varphi + j\omega_s\tau} + j\omega_s\underline{\psi}_s e^{j\omega_s\tau} \tag{31}$$

$$\underline{v}_s e^{j\gamma} = r_s\underline{i}_s e^{j\varphi} + j\omega_s\underline{\psi}_s \tag{32}$$

Thus we can write a steady state equation where the phase relation and magnitude between the space vectors remain constant.

Steady State description of Machine using space vectors i

We will first write the non-normalized equation for voltage space vector in terms of its phase voltage

$$\vec{V}_s = \frac{2}{3}\left(V_U(t) + \vec{a}V_V(t) + \vec{a}^2 V_W(t)\right) \tag{33}$$

The cosine quantities of the phase voltage can be written as

$$V_U(t) = \hat{V}cos(\omega_s t + \gamma) = \frac{\hat{V}}{2}[\underline{c} + \underline{c}^*] \tag{34}$$

$$V_V(t) = \hat{V}cos(\omega_s t + \gamma - \frac{2\pi}{3}) = \frac{\hat{V}}{2}[\underline{c}\vec{a}^2 + \underline{c}^*\vec{a}] \tag{35}$$

$$V_W(t) = \hat{V}cost(\omega_s t + \gamma - \frac{4\pi}{3} = \frac{\hat{V}}{2}[\underline{c}\vec{a} + \underline{c}^* \vec{a^2}] \tag{36}$$

Steady State description of Machine using space vectors ii

where $\underline{c} = e^{j\omega_s t+\gamma}$ and $\underline{c}^*$ is complex conjugate of $\underline{c}$ Hence we can write

$$\vec{V}_s = \frac{2}{3}\left(\frac{\hat{V}}{2}[\underline{c}+\underline{c}^*] + \vec{a}\frac{\hat{V}}{2}[\underline{c}\vec{a}^2 + \underline{c}^*\vec{a}] + \vec{a}^2\frac{\hat{V}}{2}[\underline{c}\vec{a}^* + \underline{c^*\vec{a}}]\right) \tag{37}$$

$$\vec{V}_s = \frac{2}{3}\left(\frac{\hat{V}}{2}[\underline{c}+\underline{c}^*] + \frac{\hat{V}}{2}[\underline{c}\vec{a}^3 + \underline{c}^*\vec{a}^2] + \frac{\hat{V}}{2}[\underline{c}\vec{a}^3 + \underline{c}^*\vec{a}^4]\right) \tag{38}$$

Space Vector in Steady State i

Since$\vec{a}^2 = a^*$, $\vec{a}^3 = 1$ and $\vec{a}^4 = \vec{a}$, we can write

$$\vec{V}_s = \frac{2}{3}\left(\frac{\hat{V}}{2}[\underline{c} + \underline{c}^*] + \frac{\hat{V}}{2}[\underline{c} + \underline{c}^*\vec{a}^2] + \frac{\hat{V}}{2}[\underline{c} + \underline{c}^*\vec{a}]\right) \tag{39}$$

$$\vec{V}_s = \frac{2}{3}\left(\frac{3}{2}\hat{V}[\underline{c}] + \frac{\hat{V}}{2}[\underline{c}^*(1 + \vec{a} + \vec{a}^2)]\right) \tag{40}$$

$$\vec{V}_s = \hat{V}\underline{c} \tag{41}$$

$$\vec{V}_s = \hat{V}e^{(j\omega_s t + \gamma)} \tag{42}$$

Since $(1 + \vec{a} + \vec{a}^2) = 0$

Note: Steady State space vector magnitude is same as the peak value of the phase voltage

$$\vec{V}_s = \hat{V}e^{(j\omega_s t + \gamma)}$$

Space Vector in Steady State ii

Normalizing, with base as the peak value of phase voltage we get

Definition

Steady state space vector has same magnitude as the phase voltage

$$\vec{v}_s = v_{ph} e^{j(\omega_s t + \gamma)} = 1 e^{(j\omega_s t + \gamma)}$$

Steady State space vectors in synchronous coordinate frame = Time phasors

If we now observe, the voltage in a synchronous reference frame moving at angular velocity ω_s, we get **Time phasor**

Definition

Steady state description of space vector normalized

$$\vec{X}^e = \vec{X}^s e^{-j\omega_s t} \tag{43}$$

$$\vec{v}_s e^{-j\omega_s t} = v e^{(j\omega_s t + \gamma)} e^{-j\omega_s t} \tag{44}$$

$$\underline{v} = v^{j\gamma} \tag{45}$$

Steady State equations of Induction machine using phasors

In steady state the stator voltage equation is given by

$$\vec{v}_s = r_s \vec{i}_s + j\omega_s \vec{\psi}_s$$

becomes

$$\underline{v}_s = r_s \underline{i}_s + j\omega_s \underline{\psi}_s \tag{46}$$

$$\underline{v}_s = r_s \underline{i}_s + j\omega_s l_s \underline{i}_s + j\omega_s l_h \underline{i}_r \tag{47}$$

$$\underline{v}_s = r_s \underline{i}_s + j\omega_s \sigma_s l_h \underline{i}_s + j\omega_s l_h (\underline{i}_s + \underline{i}_r) \tag{48}$$

where $l_s = (1 + \sigma_s) l_h$ and $l_r = (1 + \sigma_s) l_h$

Rotor Voltage Equation is steady state

Similarly, rotor voltage equations in terms of normalized phasors can be derived as

$$0 = r_r \vec{i}_r^s + \frac{d\vec{\psi}_r^s}{d\tau} - j\omega \vec{\psi}_r^s \tag{49}$$

$$0 = r_r \vec{i}_r^s + j\omega_s \vec{\psi}_r^s - j\omega \vec{\psi}_r^s \tag{50}$$

$$\tag{51}$$

Since we are referring to stator frame, we will drop the super-script s

$$0 = r_r \vec{i}_r + j(\omega_s - \omega)\vec{\psi}_r \tag{52}$$

$$0 = r_r \underline{i}_r + j(\omega_s - \omega)\underline{\psi}_r \tag{53}$$

$$0 = r_r \underline{i}_r + j\omega_r \left(l_h \underline{i}_s + l_r \underline{i}_r\right) \tag{54}$$

$$0 = r_r \underline{i}_r + js\omega_s \left(l_h \underline{i}_s + l_r \underline{i}_r\right) \tag{55}$$

$$0 = \frac{r_r}{s}\underline{i}_r + j\omega_s \sigma_r l_h \underline{i}_r + j\omega_s l_h(\underline{i}_s + \underline{i}_r) \tag{56}$$

Steady State: magnetising current

We introduce

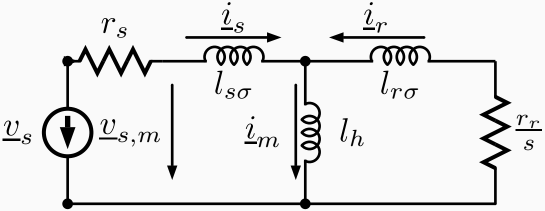
$$\underline{i}_m = \underline{i}_s + \underline{i}_r \tag{57}$$

Hence we can re-write the equations as

$$\underline{v}_s = r_s\underline{i}_s + j\omega_s\sigma_s l_h\underline{i}_s + j\omega_s l_h\underline{i}_m \tag{58}$$

$$0 = \frac{r_r}{s}\underline{i}_r + j\omega_s\sigma_r l_h\underline{i}_r + j\omega_s l_h\underline{i}_m \tag{59}$$

based on the Eq.58,we can develop the equivalent circuit of the Induction motor



in steady-state.

Steady State condition - no-load $\omega = \omega_s$

Definition

When no-load is attached to rotor shaft, the slip $s\omega_s = \omega_s - \omega$, will be very small. **At no-load condition, slip is very small and will be assumed to be zero**

$$s_0 \approxeq 0 \tag{60}$$

As at no load $\omega = \omega_s$. At no-load condition, the equivalent rotor resistance - open circuit

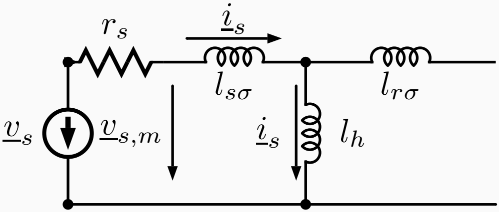
$$\frac{r_r}{s} = \frac{r_r}{0} \tag{61}$$
$$\frac{r_r}{s} \approx \infty \tag{62}$$

Hence, **At no-load condition**

$$\underline{i}_r(s_0) = 0 \tag{63}$$

Equivalent Circuit during no-load

Since rotor circuit is open and $\underline{i}_r = 0$, the equivalent circuit becomes



The steady state behaviour is described by following equations

$$\underline{v}_s = r_s \underline{i}_s + j\omega_s l_s \underline{i}_s \tag{64}$$

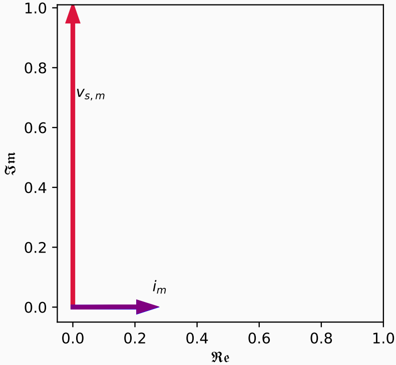
$$\underline{v}_{s,m} = j\omega_s l_s \underline{i}_s = j\omega_s l_s \underline{i}_m \tag{65}$$

Magnetising current at no load

The no-load stator current is equal to the magnetising current. The magnetising current, will be kept constant, so that we have a constant flux

$$\underline{v}_s = r_s \underline{i}_s + j\omega_s l_s \underline{i}_s \tag{66}$$

$$\underline{v}_{s,m} = j\omega_s l_s \underline{i}_s = j\omega_s l_s \underline{i}_m \tag{67}$$



Steady State characteristics of Induction motor

We can write the machine equation in terms of state voltage $\underline{v}_s$, stator current $\underline{i}_s$ and slip s

$$\underline{v}_s = r_s\underline{i}_s + j\omega_s l_s\underline{i}_s + j\omega_s l_h\underline{i}_r \tag{68}$$

$$0 = \frac{r_r}{s}\underline{i}_r + j\omega_s l_r\underline{i}_r + j\omega_s l_h\underline{i}_s \tag{69}$$

$$\underline{i}_r = -\frac{j\omega_s l_h}{\frac{r_r}{s} + j\omega_s l_r}\underline{i}_s \tag{70}$$

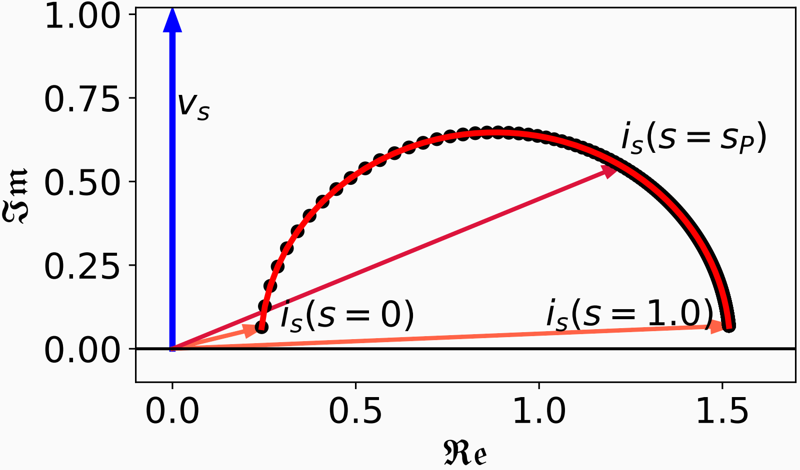
$$\therefore \underline{v}_s = r_s\underline{i}_s + j\omega_s l_s\underline{i}_s + \frac{\omega_s^2 l_h^2}{\frac{r_r}{s} + j\omega_s l_r}\underline{i}_s \tag{71}$$

$$\underline{v}_s = \underline{z}_s(s)\underline{i}_s \tag{72}$$

$$\underline{i}_s(s) = \frac{\underline{v}_s}{\underline{z}(s)} \tag{73}$$

Stator Current locus for slip operation from $s = 0$ to $s = 1.0$

If we plot the stator current and stator voltage complex phasor in the complex plane, We see that the locus of current phasor for varying values of $s \in 0, 1.0$ describes a circle. This is called as the circle diagram or "Osanna's circle diagram"



Power Delivered to air-gap, power delivered to shaft i

The power delivered to air-gap is same as power delivered to rotor. The effective resistance of rotor is r_r/s hence

$$p_e = \underline{i}_r^2 \frac{r_r}{s} \tag{74}$$

$$p_e = \underline{i}_r^2 r_r + \underline{i}_r^2 \frac{(1-s)}{s} \tag{75}$$

$$p_e = p_{r,loss} + p_{sh} \tag{76}$$

(77)

Power Delivered to air-gap, power delivered to shaft ii

Rations of Power and losses

The ratio of the powers is give as

$$
\begin{gathered}
p_e : p_{cu,loss} : p_{sh} \\
1 : s : (1-s)
\end{gathered}
\tag{78}
$$

Torque produced in steady state i

Since effective power delivered to shaft is known, we can find the torque as

$$m_{sh} = \frac{p_{sh}}{\omega} \tag{79}$$

$$m_{sh} = \frac{p_e(1-s)}{\omega_s(1-s)} \tag{80}$$

$$m_{sh} = \frac{p_e}{\omega_s} = \frac{1}{\omega_s}\underline{i}_r^2\frac{r_r}{s} = m_e \tag{81}$$

To find the rotor current, we refer to the main equivalent circuit.

$$\underline{v}_s = r_s\underline{i}_s + j\omega_s\sigma_s l_h\underline{i}_s + j\omega_s l_h\underline{i}_m \tag{82}$$

$$0 = \frac{r_r}{s}\underline{i}_r + j\omega_s\sigma_r l_h\underline{i}_r + j\omega_s l_h\underline{i}_m \tag{83}$$

$$\underline{z}_h = j\omega_s l_h \tag{84}$$

$$\underline{z}_s^{'} = r_s + j\omega_s\sigma_s l_h \tag{85}$$

Torque produced in steady state ii

We will find the Thevenin's equivalent circuit, looking from the rotor side, we can get

$$r_e + j\omega_s l_e = \frac{\underline{z}_h \underline{z}_s^{'}}{\underline{z}_h + \underline{z}_{'}} \tag{86}$$

$$\underline{v}_e = \frac{\underline{z}_h}{\underline{z}_h + \underline{z}_s^{'}} \underline{v}_s \tag{87}$$

Hence the rotor current will be given as

$$\underline{i}_r = \frac{\underline{v}_e}{(r_e + \frac{r_r}{s}) + j\omega_s(l_e + \sigma_r l_r)} \tag{88}$$

Hence Torque is given as

$$m_e = \frac{1}{\omega_s} \underline{i}_r^2 \frac{r_r}{s} \tag{89}$$

$$m_e = \frac{1}{\omega_s} \frac{\underline{v}_e^s}{(r_e + \frac{r_r}{s})^2 + (\omega_s(l_e + \sigma_r l_r))^2} \frac{r_r}{s} \tag{90}$$

Definition

Maximum Torque will be produced, when maximum power transferred to rotor resistance. We can use the maximum power transfer theorem to find the slip at which this occurs. This slip value is called **pull-out slip** and is given by

$$s_P = \pm \frac{r_r}{\sqrt{r_e^2 + \left(\omega_s l_e + \omega_s \sigma_r l_h\right)^2}} \tag{91}$$

Pull-out Torque or maximum torque ii

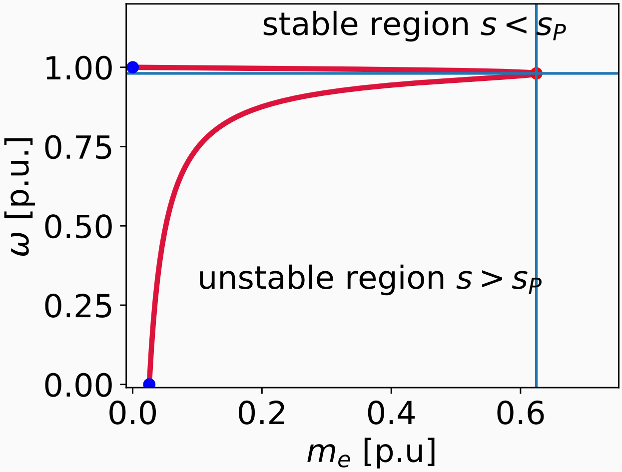
Hence we can write pull-out torque as for $s > 0$ motoring

$$m_{e,P} = \frac{1}{\omega_s} \frac{\underline{v}_e^s}{\left(r_e + \sqrt{r_e^2 + \left(\omega_s l_e + \omega_s \sigma_r l_h\right)^2}\right)^2 + (\omega_s l_e + \omega_s \sigma_r l_r)^2} \sqrt{r_e^2 + (\omega_s l_e + \sigma_r l} \tag{92}$$

$$m_{e,P} = \frac{\underline{v}_e^2}{\omega_s} \frac{\sqrt{r_e^2 + \left(\omega_s l_e + \omega_s \sigma_r l_h\right)^2}}{2r_e^2 + 2(\omega_s l_e + \omega_s \sigma_r l_h)^2 + 2r_e \sqrt{r_e^2 + \left(\omega_s l_e + \omega_s \sigma_r l_h\right)^2}} \tag{93}$$

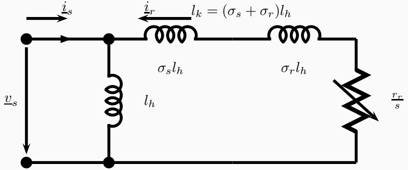
$$m_e = \frac{\underline{v}_e^2}{2\omega_s} \frac{1}{r_e + \sqrt{r_e^2 + \left(\omega_s l_e + \omega_s \sigma_r l_h\right)^2}} \tag{94}$$

Speed-Torque characteristics of full equivalent circuit



Equivalent Circuit Approximation 2 $x_h >> x_{s\sigma}$ and $rs = 0$

Usually, the magnetising current i_m needed to produce the required flux in the air-gap will be kept low. This reduces the ohmics losses in the magentising circuit. Hence the impedance of magnetising branch $\omega_s l_h >> \omega_s \sigma_s l_h$. Hence the current we could move the magnetising branch outside as shown in the approximate equivalent circuit



Why approximate circuit?

- They allow fast to calculate the good enough range of the performance of the machine
- They allows us to see the dependency of torque speed characteristics on motor parameters
- They help us to apply first principles and assess the performance of the machine with good enough accuracy

Torque and Torque speed characteristics of an Induction machine i

Using the 2nd approximate model of Induction machine, we get

$$\underline{i}_r = \frac{-\underline{v}_s}{\frac{r_r}{s} + j\omega_s(\sigma_s + \sigma_r)lh} \tag{95}$$

We define the short-circuit impedance of the induction machine as

$$\boxed{x_k = \omega_s(\sigma_s + \sigma_r)l_h = \omega_s l_k \tag{96}}$$

Torque and Torque speed characteristics of an Induction machine ii

Hence the torque, depending on the active power supplied to effective rotor resistance by the synchronous angular velocity is given as

$$m_e = \frac{\mathfrak{Re}\left[\underline{v}_s \underline{i}_s^*\right]}{\omega_s} \tag{97}$$

$$m_e = \frac{1}{\omega_s} i_r^2 \frac{r_r}{s} \tag{98}$$

$$m_e = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r}{s}\right)^2 + (\omega_s l_k)^2} \frac{r_r}{s} \tag{99}$$

Speed torque characteristics of the machine i

We can see that the torque is a function of slip s, we can find 3 distinct points of operation of the machine

1. Starting condition $s = 1$

At starting condition, rotor is at stand-still $\omega = 0$, hence $s = (\omega_s - 0)/\omega_s = 1$. Hence **Starting torque or stand-still torque is given by**

$$m_{e,1}(s=1) = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{r_r^2 + (\omega_s l_k)^2} r_r \tag{100}$$

2. No Load condition Torque

At no-load, since the rotor rotates almost synchronously to stator field, $s = 0$ as $\omega \approxeq \omega_s$.

$$m_{e,0}(s=0) = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r}{s}\right)^2 + (\omega_s l_k)^2} \frac{r_r}{s} \tag{101}$$

$$m_{e,0}(s=0) = \frac{1}{\omega_s} \frac{s\underline{v}_s^2}{r_r^2 + s^2(\omega_s l_k)^2} r_r \tag{102}$$

$$m_{e,0}(s=0) = 0 \tag{103}$$

3. Pull-out Torque or Maximum Torque condition

Maximum torque is produced, when maximum power is transferred to the effective rotor resistance. The slip at which this occurs, is obtained by maximum power transfer theorem and for approximate eq. ckt. is given by

$$s_P = \pm \frac{r_r}{\omega_s l_k} \tag{104}$$

The **maximum of pull-out torque is given by**

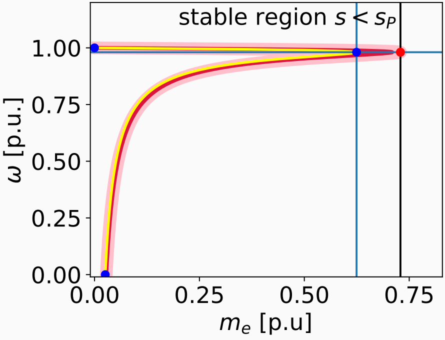
$$m_{e,P}(s = s_P) = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r}{s_P}\right)^2 + (\omega_s l_k)^2} \frac{r_r}{s_P} \tag{105}$$

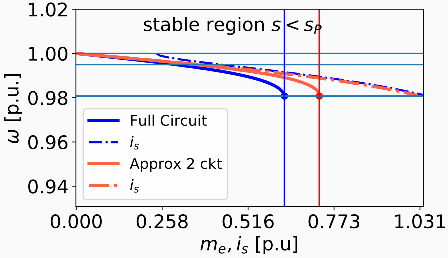
$$m_{e,P}(s = s_P) = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r(\omega_s l_k)}{r_r}\right)^2 + (\omega_s l_k)^2} \frac{r_r(\omega_s l_k)}{r_r} \tag{106}$$

$$m_{e,P}(s = s_P) = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{2(\omega_s l_k)^2} \omega_s l_k \tag{107}$$

$$m_{e,P}(s = s_P) = \frac{1}{2 l_k} \left(\frac{\underline{v}_s}{\omega_s}\right)^2 \tag{108}$$

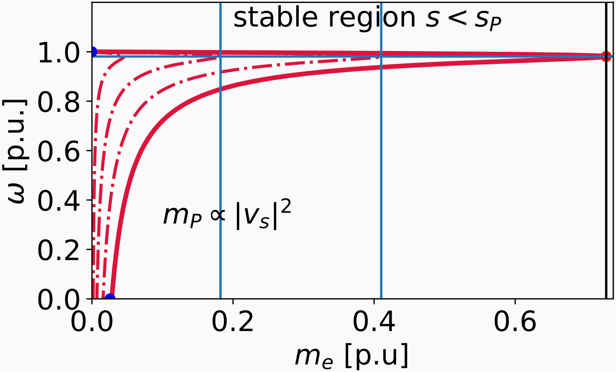
Speed Torque curve, approximate and full equivalent circuit





The pull-out torque calculated with approximate equivalent circuit is around 15% higher than the actual pull-out torque. This is because, when we neglect the stator resistance, we see more than the actual voltage across the rotor. But the approximate speed torque characteristics behaviour is very close to the actual characteristics. This helps us understand the impact of parameters on the speed-torque characteristics

Effect of reducing voltage on speed -torque characteristics



If we reduces the voltage, without changing the stator frequency, then the torque reduces proportional to square of voltage

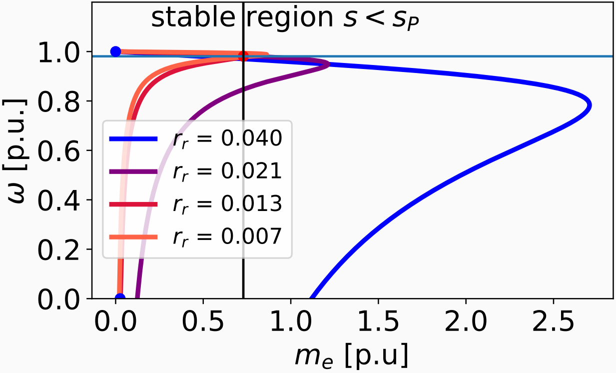
$$m_{e,P} \propto \underline{v}_s^2$$

Since, Torque is given by

$$m_e = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r}{s}\right)^2 + (\omega_s l_k)^2} \frac{r_r}{s}$$

$$m_{e,P} = \frac{1}{2l_k} \left(\frac{\underline{v}_s}{\omega_s}\right)^2$$

Speed-torque characteristics as Machine rotor resistance increases



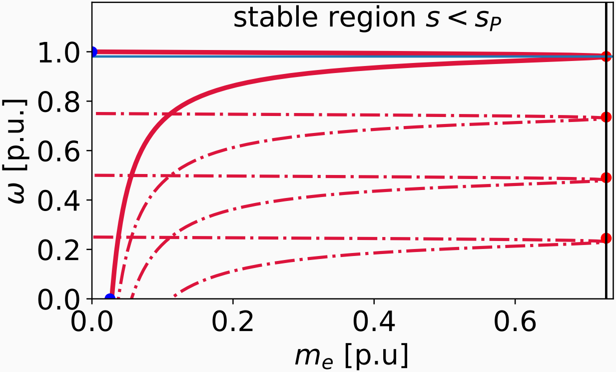
Rotor resistances increases staring torque

Higher rotor resistance, produces higher starting torque

Starting torque is given as

$$m_{e,1}(s=1) = \frac{1}{\omega_s}\frac{\underline{v}_s^2}{r_r^2 + (\omega_s l_k)^2} r_r \tag{109}$$

Speed-torque characteristics keeping vs/f constant



v/f = constant

keeping ratio

$$\frac{\underline{v}_s}{\omega_s} = \text{cosnt}$$

Is best way to control speed
Keeps pull-out torque
constant at all speeds

Since, Torque is given by

$$m_e = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r}{s}\right)^2 + (\omega_s l_k)^2} \frac{r_r}{s}$$

$$m_{e,P} = \frac{1}{2l_k} \left(\frac{\underline{v}_s}{\omega_s}\right)^2$$

Simplified relation for Torque - Kloss's equation

Substituting the pull-out slip expression, in torque equation, we get

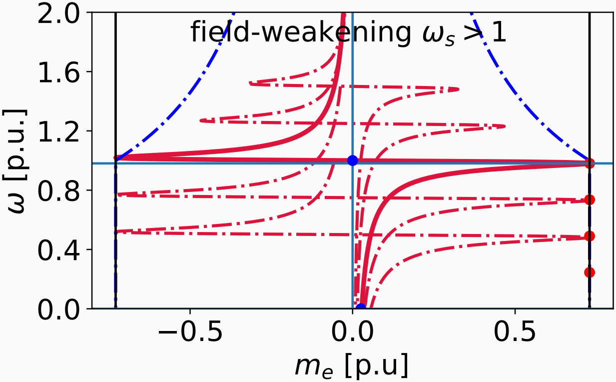
Kloss's equation important approximated model for IM

$$s_P = \pm \frac{r_r}{\omega_s l_k} \tag{110}$$

$$\begin{aligned} m_e &= \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r}{s}\right)^2 + (\omega_s l_k)^2} \frac{r_r}{s} \\ &= \frac{\underline{v}_s^2}{\omega_s} \frac{1}{\frac{s_P^2}{s^2} + 1} \frac{s_P}{s} \frac{1}{\omega_s l_k} \end{aligned}$$

$$\frac{m_e}{m_{e.P}} = \frac{2}{\frac{s_P}{s} + \frac{s}{s_P}} \tag{111}$$

Speed-torque characteristics in field weakening keeping vs/f constant



$v = 1$ for $f_s > 1.0$ Field weakening range

Voltage cannot exceed the rated value, due to insulation constraints. Hence, making $\omega_s > 1$ causes field weakening. **pull-out torque reduces above rated angular velocity**

Since, Torque is given by

$$m_e = \frac{1}{\omega_s} \frac{\underline{v}_s^2}{\left(\frac{r_r}{s}\right)^2 + (\omega_s l_k)^2} \frac{r_r}{s}$$

$$m_{e,P} = \frac{1}{2l_k} \left(\frac{\underline{v}_s}{\omega_s}\right)^2$$

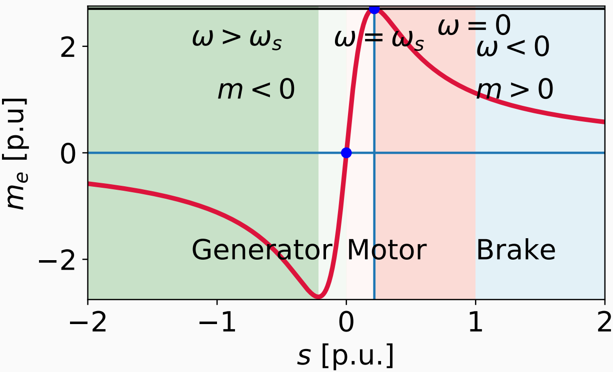
4 quadrant operation of the Induction machine

Brake

- $m_e > 0$
- $\omega < 0$

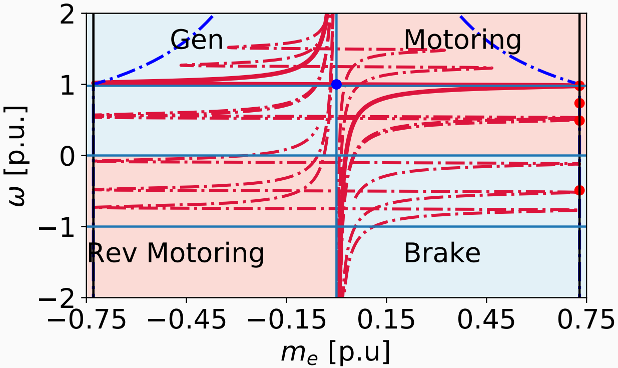
Generation

- $m_e < 0$
- $\omega > \omega_s$



- If induction motor is fed by constant frequency supply
- The load is regenerative
- It operates as generator, when the load torque drives the rotor faster than the stator field
- If the load drives the rotor to reverse, the torque will be opposing the rotor causing braking

4 quadrant operation with v/f control the Induction machine



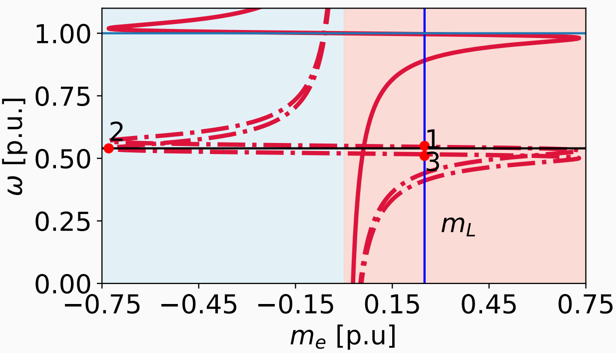
Generation

- $\omega > \omega_s$
- $m_e < 0$

Regenerative Braking

- $\omega > 0$
- $\omega > \omega_s$
- $m_e < 0$

Regenerative braking in IM with v/f control



Point 1

- $\omega(1) < \omega_s(1)$
- $m_e(1) > 0$

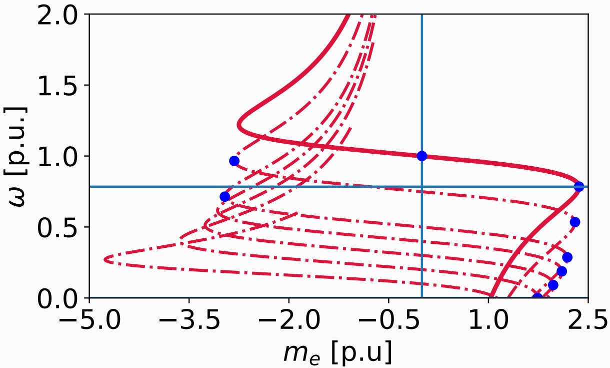
Point 2

- $\omega(1) = \omega(2) > \omega_s(2)$
- $m_e(2) < 0$
- $\frac{d\omega}{d\tau} \propto -m_e - m_L$
- ω reduces along the second curve

Point 3

- $m_e(3) > 0$
- $\omega(3) < \omega_s(2)$
- $m_e(3) = m_L$
- operating point becomes $\omega(3)$

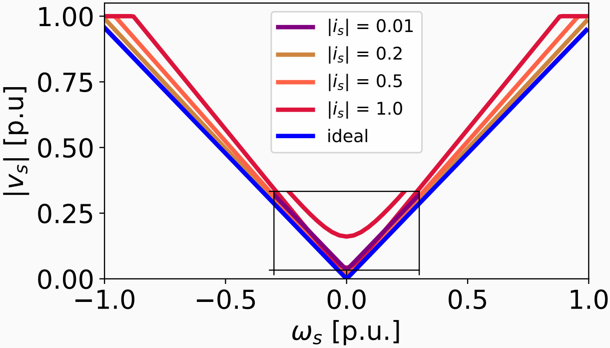
V/f = constant does not keep flux constant in actual machine



- See what happens to pull-out torque
- Why?
- Can you explain it using the torque equation?

- At low speeds, the effective voltage across magnetising branch reduces due to voltage drop across $r_s + j\omega_s \sigma_s l_h$
- Hence, air-gap flux reduces, causes decrease in peak torque at low speeds
- We need a method to keep the flux constant

Voltage supplied should be compensated for the drops to keep rotor flux constant



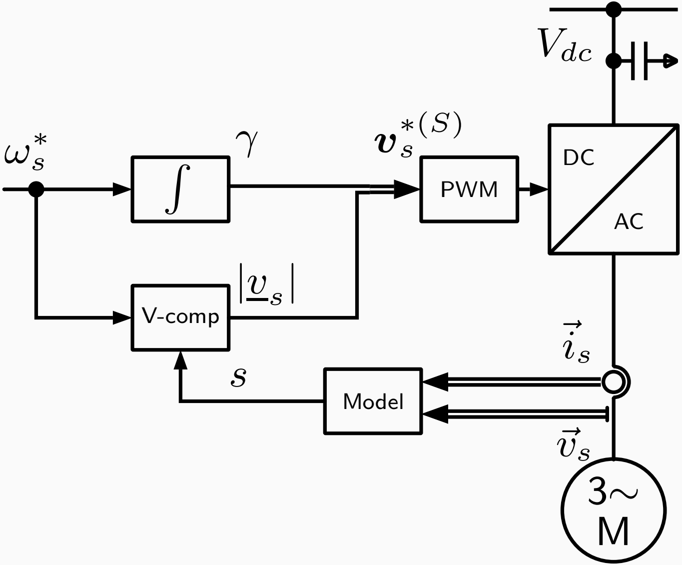
$$\vec{\psi}_s = l_s \vec{i}_s + l_h \vec{i}_r$$
$$\vec{\psi}_r = l_h \vec{i}_s + l_r \vec{i}_r$$
$$\therefore \vec{\psi}_s = \sigma l_s \vec{i}_s + k_r \vec{\psi}_r$$

V-Comp

$$\underline{v}_s = r_s \underline{i}_s + j\omega_s \underline{\psi}_r \tag{112}$$

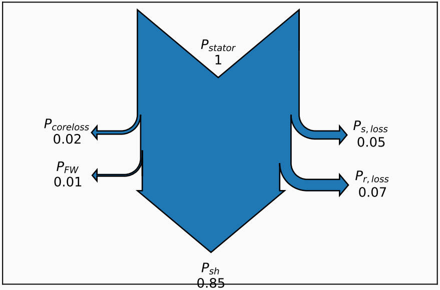
$$\underline{v}_s = r_s \underline{\mathrm{i}}_s + j\omega_s \sigma l_s \underline{i}_s + j\omega_s k_r \underline{\psi}_r \tag{113}$$

Compensated v/f feedforward control



Modeling Losses in Induction Machine (extra in Steady state analysis

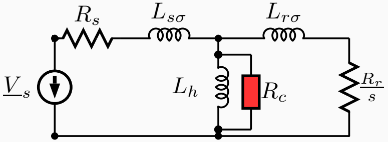
Losses in Induction machine



Losses in Induction Machine

- Stator winding loss: i^2R loss in stator winding
- Core Loss in stator and rotor
  - Hysteresis loss
  - Eddy Current Loss
- Rotor winding loss
- Friction and windage loss

Loss Modeling in electrical circuits: resistance



- Using equivalent circuit (that is not normalized)
- Core losses modeled as a resistance R_c
- Core loss $= \frac{\underline{V}_h^2}{R_c}$
- this consist of two type of losses... (How to model them?)

Hysteresis Loss and Eddy current Loss

We need the Magnetization characteristics

$$H(B) = B\left(k_1 e^{k_2(\hat{B}_m)^2} + k_3\right)$$

We get the area enclosed in the curve as the energy density

$$\begin{aligned} w_{fe} &= \int_0^{\hat{B}_m} H(B)dB \\ &= \frac{1}{2}\left[f\frac{k_1}{k_2}(e^{k_2(\hat{B}_m)^2} - 1) + f^2 k_3 \hat{B}_m^2\right] \end{aligned}$$

using the volume, we get W_{fe} and

$$R_c = \frac{V_s^2}{W_{Fe}} \tag{114}$$

Usually Ferromagentic Iron used in core. Manufacture give loss density [W/kg] as

$$\begin{aligned} w_{Fe} = &\underbrace{k_h f B^\alpha}_{\text{Hysteresis Loss}} \\ &+ \underbrace{k_e f^2 B^2}_{\text{Eddy current Loss}} \\ &+ \underbrace{k_a f^{3/2} B^{3/2}}_{\text{anomalous Loss}} \end{aligned}$$

Dynamic Behaviour of Induction Machine

Non-linear differential equations describing dynamics of Induction machine i

Induction Motor dynamics

$$\vec{v}_s^s = r_s \vec{i}_s^s + \frac{d\vec{\psi}_s^s}{d\tau} \tag{115}$$

$$\vec{v}_r^r = r_r \vec{i}_r^r + \frac{d\vec{\psi}_r^r}{d\tau} \tag{116}$$

$$m_e = \vec{\psi}_s \times \vec{i}_s \tag{117}$$

$$\frac{d\omega}{d\tau} = \frac{1}{\tau_m}(m - m_L) \tag{118}$$

We will describe the dynamics of the machine in terms of 2 state variables

- stator current space vector $\vec{i}_s$

Non-linear differential equations describing dynamics of Induction machine ii

- rotor flux linkage space vector $\vec{\psi}_r$

To this end we will use

$$\vec{\psi}_s = l_s \vec{i}_s + l_h \vec{i}_r \tag{119}$$

$$\vec{\psi}_r = l_h \vec{i}_s + l_r \vec{i}_r \tag{120}$$

$$\vec{i}_r = \frac{1}{l_r}\left(\vec{\psi}_r - l_h \vec{i}_s\right) \tag{121}$$

$$\vec{\psi}_s = \left(1 - \frac{l_h^2}{l_s l_r}\right) l_s \vec{i}_s + \frac{l_h}{l_r}\vec{\psi}_r \tag{122}$$

$$\vec{\psi}_s = \sigma l_s \vec{i}_s + k_r \vec{\psi}_r \tag{123}$$

Non-linear differential equations describing dynamics of Induction machine iii

Substituting in the stator voltage equation and rotor voltage equation, we get

$$\vec{v}_s = r_s \vec{i}_s + \frac{d\left(\sigma l_s \vec{i}_s + k_r \vec{\psi}_r\right)}{d\tau} \tag{124}$$

$$\vec{v}_s = r_s \vec{i}_s + \sigma l_s \frac{d\vec{i}_s}{d\tau} + k_r \frac{d\vec{\psi}_r}{d\tau} \tag{125}$$

$$0 = r_r \frac{1}{l_r}\left(\vec{\psi}_r - l_h \vec{i}_s\right) + \frac{d\vec{\psi}_r}{d\tau} - j\omega\psi_r \tag{126}$$

$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \tag{127}$$

We can write the stator equation, using the rotor equation as

Non-linear differential equations describing dynamics of Induction machine iv

$$\vec{v}_s = r_s\vec{i}_s + \sigma l_s \frac{d\vec{i}_s}{d\tau} + k_r \left[-\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \right] \tag{128}$$

$$\vec{v}_s = r_s\vec{i}_s + \frac{k_r l_h}{\tau_r}\vec{i}_s + \sigma l_s \frac{d\vec{i}_s}{d\tau} - \frac{k_r}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r \tag{129}$$

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\sigma l_s}(r_s + k_r^2 r_r)\vec{i} + s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \tag{130}$$

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \tag{131}$$

State equations of Induction machine

The state equations for the electromagnetic system of Induction machine can be written as

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \quad (132)$$

$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \quad (133)$$

$$m_e = \vec{\psi}_s \times \vec{i}_s \quad (134)$$

$$m_e = k_r\vec{\psi}_r \times \vec{i}_s = k_r\left[\vec{\psi}_r \perp \vec{i}_r\right] \quad (135)$$

$$\frac{d\omega}{d\tau} = \frac{1}{\tau_m}\left(m_e - m_l\right) \quad (136)$$

Order of the system is 3. Why?

Induction motor is a system of order 3. There are 3 energy storages in the induction motor

Study the equations to understand the nature

$$\frac{d\vec{i}_s}{d\tau} = -\frac{1}{\tau_k}\vec{i}_s + \frac{k_r}{\sigma l_s \tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{\vec{v}_s}{\sigma l_s} \tag{137}$$

$$\frac{d\vec{\psi}_r}{d\tau} = -\frac{1}{\tau_r}\left(1 - j\omega\tau_r\right)\vec{\psi}_r + \frac{l_h}{\tau_r}\vec{i}_s \tag{138}$$

$$m_e = k_r\vec{\psi}_r \times \vec{i}_s = k_r\left[\vec{\psi}_r \perp \vec{i}_r\right] \tag{139}$$

$$\frac{d\omega}{d\tau} = \frac{1}{\tau_m}\left(m_e - m_l\right) \tag{140}$$

- It is nonlinear as state depends on state variable ω
- It is time varying, as the voltage can change during the dynamics
- **To linearize the system, we have to analyze it around a stable operation point given by** $(\omega_o, \vec{i}_{s,o}, \vec{\psi}_{r,o})$

Solving the system dynamics using software

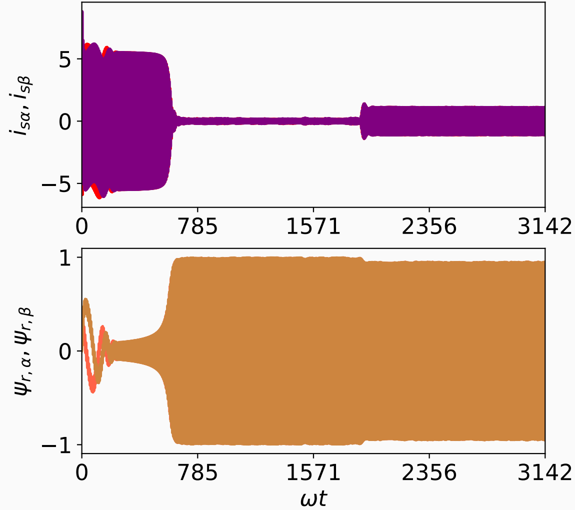
To solve the non-linear differential equations, we can writ

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$$

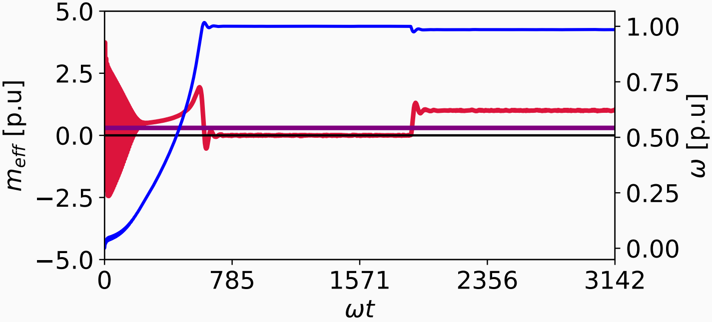
We can then use any software package that will integrate the function. Mostly Runge-Kutte 4,5 method will be used for integration. We can do this using Python, MatLab, Octave or C.

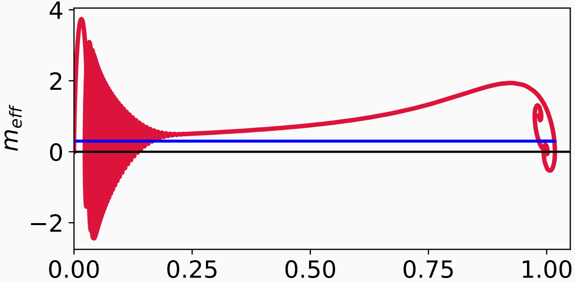
Dynamics of the Induction Motor at rated voltage and frequency

An induction machine is a non-linear dynamic response system. Starting the induction motor on the AC power supply produces dynamics



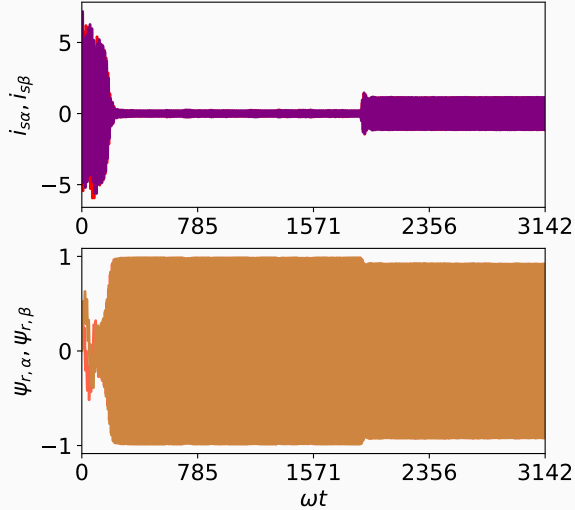
Dynamics of Torque speed at $|v_s| = \omega_s = 1.0$



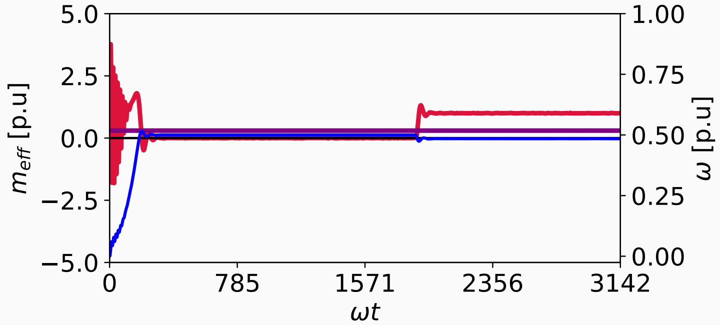


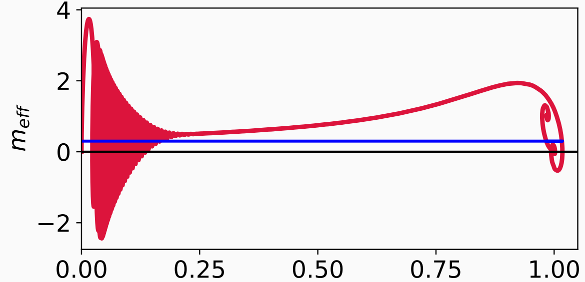
Dynamics of the Induction Motor at half the rated voltage and frequency

An induction machine is a non-linear dynamic response system. Starting the induction motor on the AC power supply produces dynamics



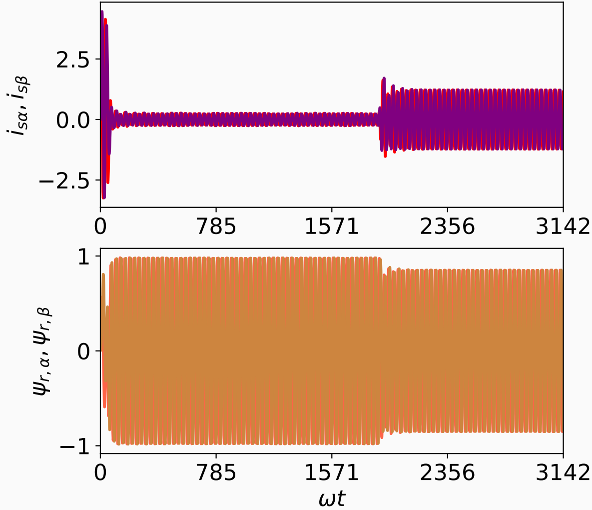
Dynamics of Torque speed at $|v_s| = \omega_s = 0.5$



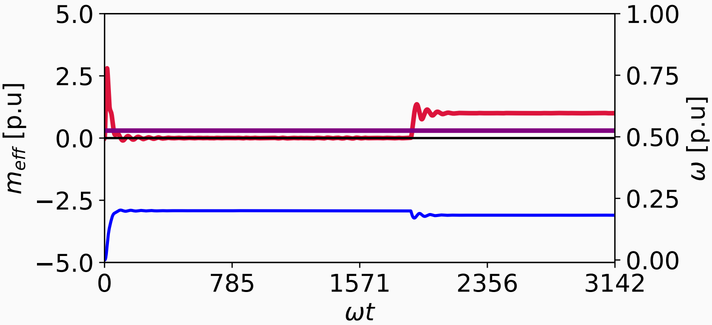


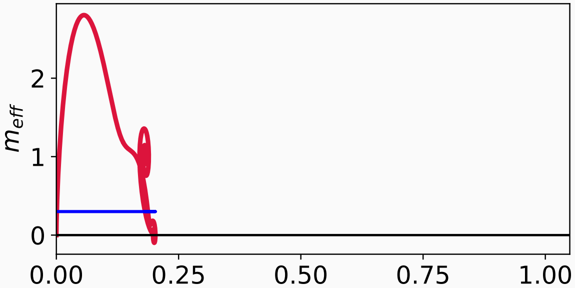
Dynamics of the Induction Motor at 0.2 times rated voltage and frequency

An induction machine is a non-linear dynamic response system. Starting the induction motor on the AC power supply produces dynamics

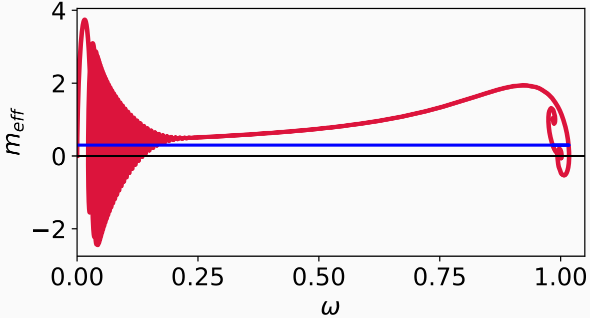


Dynamics of Torque speed $|v_s| = \omega_s = 0.2$

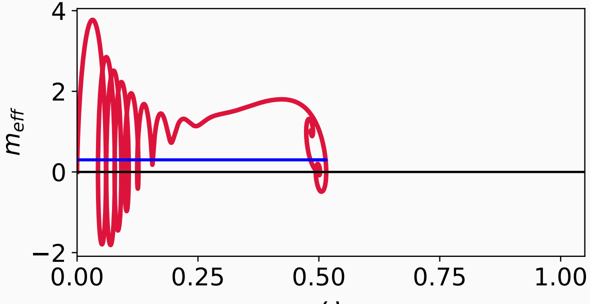




Dynamics shown as Torque versus speed: Comparison

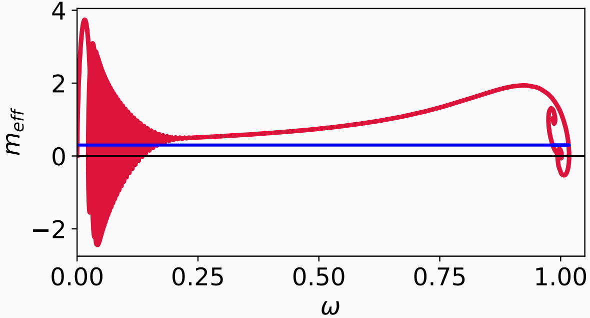


Starting at $\underline{v}_s = 1.0$ and $\omega_s = 1.0$

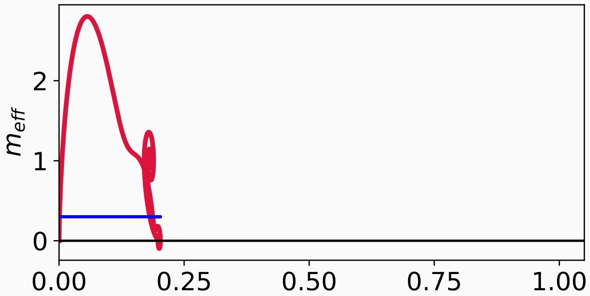


Starting at $\underline{v}_s = 0.5$ and $\omega_s = 0.5$

Dynamics shown as Torque versus speed: Comparison

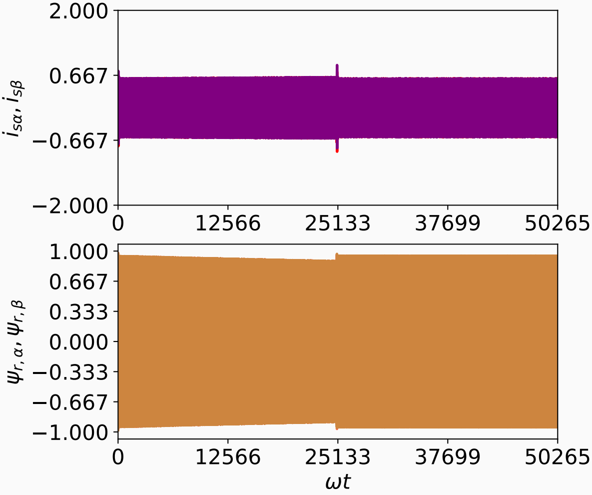


Starting at $\underline{v}_s = 1.0$ and $\omega_s = 1.0$



Starting at $\underline{v}_s = 0.2$ and $\omega_s = 0.2$

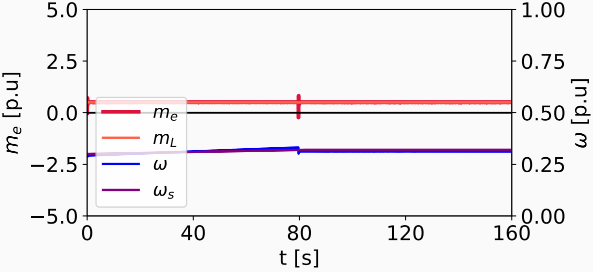
The rate of v/f has to be very slow…this can take seconds to accelerate

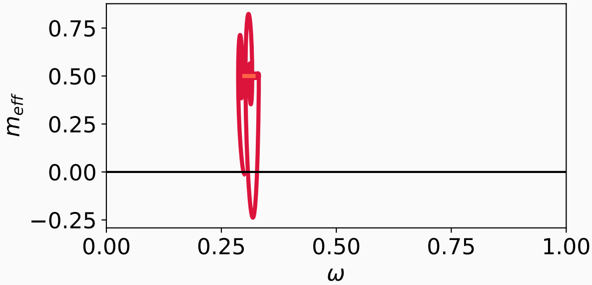


- The rate at which frequency is changed has to be slow to avoid large current dynamics
- Large currents (2 x rated current) can cause the power supply to trip due to over-current protection

- Hence the method is good only if steady state operation at particular speed is preferred

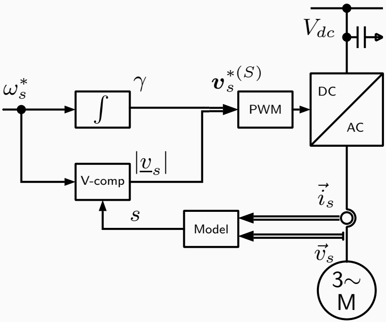
Both voltage and frequency has to vary at a very low rate

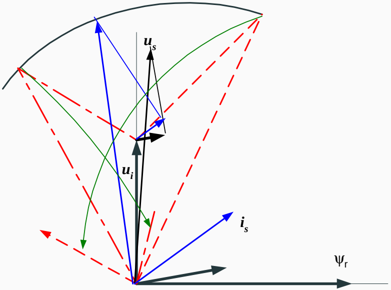




Very small dynamic seen (the simulation is not giving a compensated voltage)

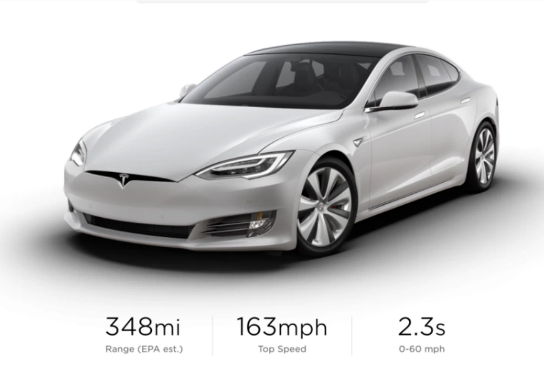
Why is there a dynamic?



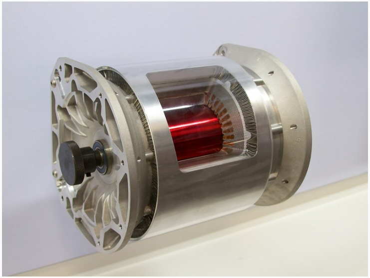


- Even if we compensate the voltage to keep $|\vec{\psi}_r| =$ constant
- We may not achieve it, as we are applying a voltage that can be any where w.r.to $\vec{\psi}_r$
- Hence, this method is called **scalar control**

So how come Tesla achieve acceleration 0-60 mph 2.3 sec



Source: **Fred Lambert**



Source: **Windell Oskay - cc by 2.0**

Tesla Car uses Induction Motor (caged rotor)

Tesla uses Induction motor drive. The rotor winding is made of copper cage.

Welcome to Vector Control of AC Machines