Setting Controllers using Ziegler Nichols method

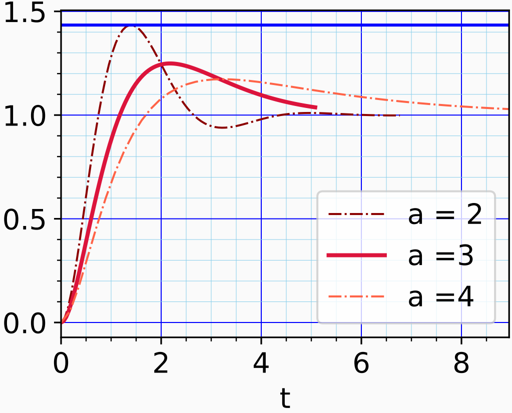
Control of Drives

Prof. Ashwin M Khambadkonne
email:eleamk@nus.edu.sg

Dept. of ECE, National University of Singapore

Control of Separately excited DC machine

Ashwin M Khambadkone
Department of Electrical and Computer Engineering, NUS



Order of the System 3 depends on number of energy storages

Order of the system

$$T_a \frac{di_a}{dt} = \frac{v_a - e_a}{r_a} - i_a \quad (1)$$

$$T_f \frac{di_f}{dt} = \frac{v_f}{r_f} - i_f \quad (2)$$

$$T_j \frac{d\omega}{dt} = m_e - m_L \quad (3)$$

Integral form - of system

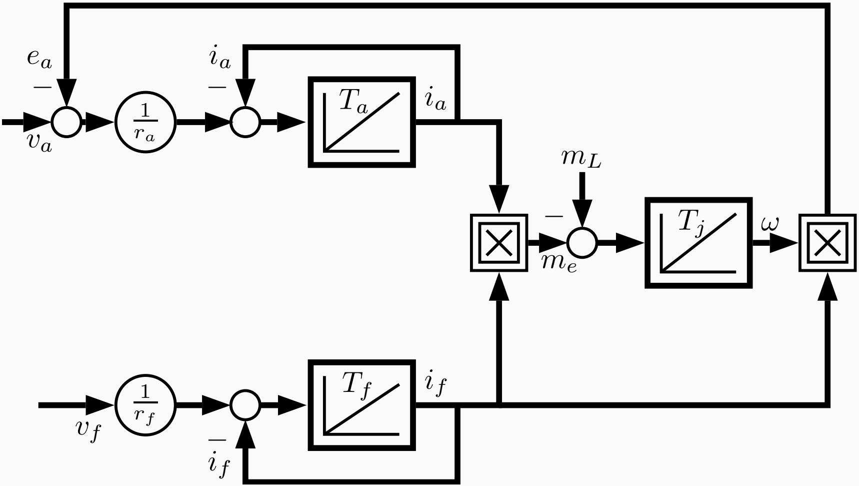
To represent the system with help of block diagrams, we can rewrite the equations in integral form as

$$i_a = \frac{1}{T_a} \int \left(\frac{v_a - e_a}{r_a} - i_a \right) dt \tag{4}$$

$$i_f = \frac{1}{T_f} \int \left(\frac{v_f}{r_f} - i_f \right) dt \tag{5}$$

$$\omega = \frac{1}{T_j} \int (m_e - m_L) dt \tag{6}$$

Block Diagram of the SE DC motor



Constant flux operation: improves the dynamics

- Field winding time constant $T_f >> T_a$ will be larger
- Fast torque change is achieved by changing I_a and
- keeping I_f is constant

Let us say the normalized value of field current is constant

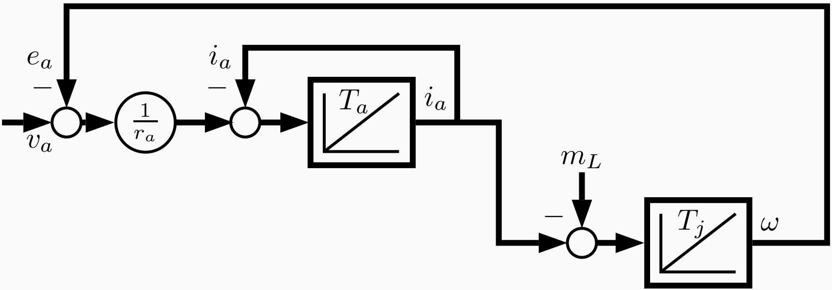
$$i_f = 1$$

The dynamics of the motor is described using

$$T_a \frac{di_a}{dt} = \frac{v_a - e_a}{r_a} - i_a \tag{7}$$

$$T_j \frac{d\omega}{dt} = m_e - m_L \tag{8}$$

Dynamics of constant flux SE DC motor



Normalized equations of the constant flux operation

To represent the system with help of block diagrams, we can rewrite the equations in integral form as

$$i_a = \frac{1}{T_a} \int \left(\frac{v_a - e_a}{r_a} - i_a \right) dt \tag{9}$$

$$\omega = \frac{1}{T_j} \int (m_e - m_L) dt \tag{10}$$

We should also note that

$$e_a = \frac{E_a}{E_{a0}} = \frac{k_e I_f \Omega}{k_e I_{fR} \Omega_0} = i_f \omega \tag{11}$$

$$m_e = \frac{M_e}{M_{eR}} = \frac{k_e I_f I_a}{k_e I_{fR} I_{ar}} = i_f i_a \tag{12}$$

where E_{a0} is the no load voltage of the motor and is equal to V_{aR}

The field current is held constant at its rated value $i_f = 1$. Under this condition the dynamic equation will be

$$T_a \frac{di_a}{dt} = \frac{v_a - \omega}{r_a} - i_a \tag{13}$$

$$T_j \frac{d\omega}{dt} = i_a - m_L \tag{14}$$

Taking Laplace transform, we get

Analysis of Dynamic behaviour of SE DC motor ii

$$sT_a i_a(s) + i_a(s) = \frac{v_a(s)}{r_a} - \frac{\omega(s)}{r_a} \tag{15}$$

$$sT_j \omega(s) = i_a(s) - m_L(s) \tag{16}$$

$$(sT_a + 1) i_a(s) = \frac{v_a(s)}{r_a} - \frac{\omega(s)}{r_a} \tag{17}$$

$$i_a(s) = \frac{v_a(s)}{r_a} \frac{1}{sT_a + 1} - \frac{\omega(s)}{r_a} \frac{1}{sT_a + 1} \tag{18}$$

$$\omega(s) = \frac{i_a(s)}{sT_j} - \frac{m_L(s)}{sT_j} \tag{19}$$

Substituting and solving for $i_a(s)$, we get

Analysis of Dynamic behaviour of SE DC motor iii

$$\omega(s) = \frac{i_a(s)}{sT_j} - \frac{m_l(s)}{sT_j} \tag{20}$$

$$\omega(s) = v_a(s)\frac{1}{sT_j r_a(sT_a+1)} - \frac{m_l(s) r_a(sT_a+1)}{sT_j r_a(sT_a+1)} - \omega(s)\frac{1}{sT_j r_a(sT_a+1)} \tag{21}$$

Using $T_m = T_j r_a$, we get

$$\omega(s) = \frac{i_a(s)}{sT_j} - \frac{m_l(s)}{sT_j} \tag{22}$$

$$\omega(s) = v_a(s)\frac{1}{sT_m(sT_a+1)} - \frac{m_L(s) r_a(sT_a+1)}{sT_m(sTa+1)} - \omega(s)\frac{1}{sT_m(sT_a+1)} \tag{23}$$

$$\tag{24}$$

Analysis of Dynamic behaviour of SE DC motor iv

Solving for $\omega(s)$, we get

voltage to velocity

$$\omega(s) = \frac{1}{(s^2 T_m T_a + sT_m + 1)} v_a(s) - \frac{(sT_a + 1) r_a}{(s^2 T_m T_a + sT_m + 1)} m_L(s) \quad (25)$$

Solving for $i_a(s)$, we will get

Current response

$$i_a(s) = \frac{sT_j}{(s^2 T_m T_a + sT_m + 1)} v_a(s) + \frac{1}{(s^2 T_m T_a + sT_m + 1)} m_L(s) \quad (26)$$

Response of second order system i

We can see that the system is of 2^{nd} order. The characteristic polynomial of the system is given by

$$s^2 T_m T_a + s T_m + 1 = 0 \tag{27}$$

The roots of the characteristic polynomial are given by

$$s_{1,2} = -\frac{1}{2T_a} \pm \sqrt{\frac{1}{4T_a^2} - \frac{1}{T_m T_a}} \tag{28}$$

The roots will be real if the condition

$$T_m > 4T_a \tag{29}$$

is true.

Response of second order system ii

If $T_m < 4T_a$ then the roots will be complex conjugates. The response of the system will then be oscillatory. For a general second order system, the characteristic polynomial is given as

$$S^2 + 2D\omega_n s + \omega_n^2 = 0 \tag{30}$$

where D is the damping in the system and ω_n is the natural frequency of the system.

Rewriting the Eq.27 in that form we get,

$$s^2 + \frac{1}{T_a}s + \frac{1}{T_m T_a} = 0 \tag{31}$$

Response of second order system iii

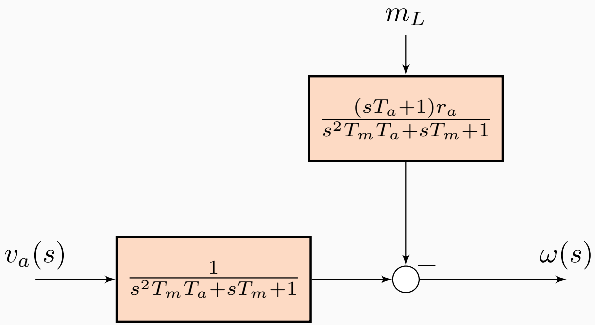
Damping and natural frequency

We can find the damping and the natural frequency as

$$D = \frac{1}{2}\sqrt{\frac{T_m}{T_a}} \tag{32}$$

$$\omega_n = \frac{1}{\sqrt{T_m T_a}} \tag{33}$$

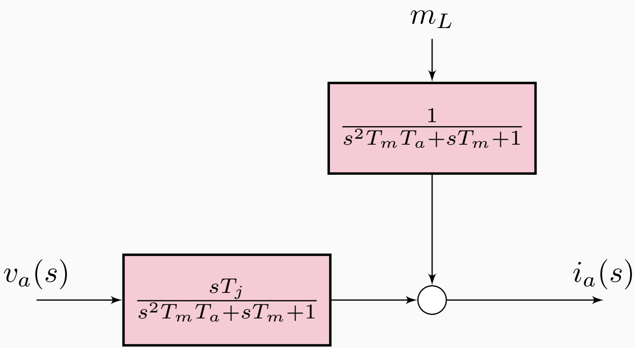
Block Diagram ang. velocity response to voltage



Ang. velocity response

- For step in armature voltage, ω response by second order transfer function $G_{\omega,va}$
- ω changes to disturbance m_L governed by disturbance path transfer function G_{ω,m_L}

Current response to change in Voltage



Current response

$$i_a(s) = \frac{sT_j}{(s^2T_mT_a + sT_m + 1)}v_a(s) + \frac{1}{(s^2T_mT_a + sT_m + 1)}m_L(s) \quad (34)$$

Example

The parameters of a separately excited DC motor, let us call it **DCM1**, are given as follows

$P_R = 22\text{kW}$ $V_{aR} = 400\text{V}$ $I_{aR} = 54\text{A}$ $n_R = 3000\text{rpm}$

$R_a = 0.2178\Omega$ $L_a = 3.4\text{mH}$ $T_j = 202\text{s}$

We can hence determine the parameters for the transfer function as

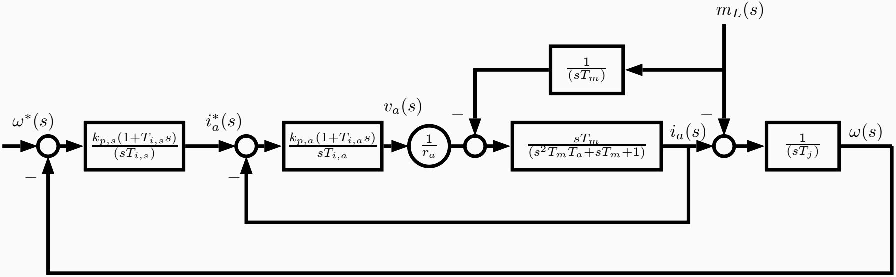
$$T_a = \frac{L_a}{R_a} = 15.61 \times 10^{-3} s$$

$$r_a = \frac{R_a I_{aR}}{V_{aR}} = 29.4 \times 10^{-3}$$

$$T_m = r_a T_j = 5.94 s$$

Determine the poles of the system.

Block diagram of cascaded control



Design of inner current control loop i

The forward transfer function of the current loop is given as

$$G_{Fiv} = \frac{i_a(s)}{v_a(s)} = \frac{sTj}{s^2 T_m T_a + sT_m + 1} \tag{35}$$

If we factorize the denominator assuming that $T_m > 4T_a$, using Eq.29, we get two negative real roots. The roots are

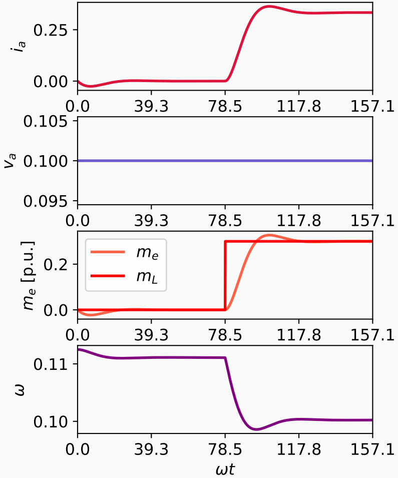
$$s_1 = -\frac{1}{2T_a} - \sqrt{\frac{1}{4T_a^2} - \frac{1}{T_m T_a}} \tag{36}$$

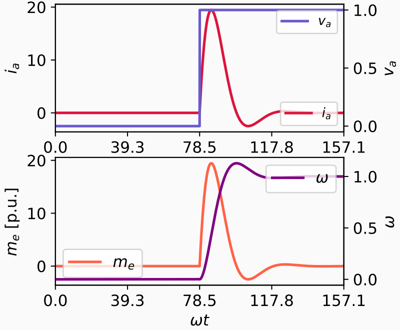
$$s_2 = -\frac{1}{2T_a} + \sqrt{\frac{1}{4T_a^2} - \frac{1}{T_m T_a}} \tag{37}$$

$$s_1 = -D\omega_n - \omega_n \sqrt{1 - D^2} \tag{38}$$

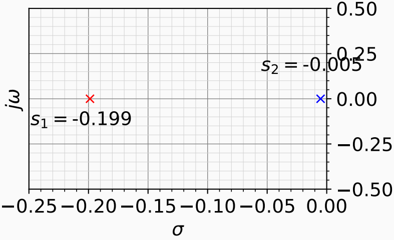
$$s_2 = -D\omega_n + \omega_n \sqrt{1 - D^2} \tag{39}$$

For case $T_m > 4T_a$





Poles of the system

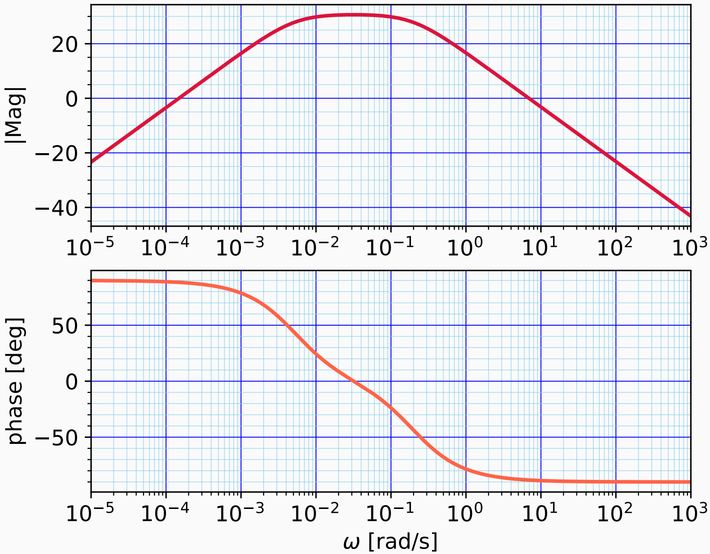


We can represent the roots by two time constants

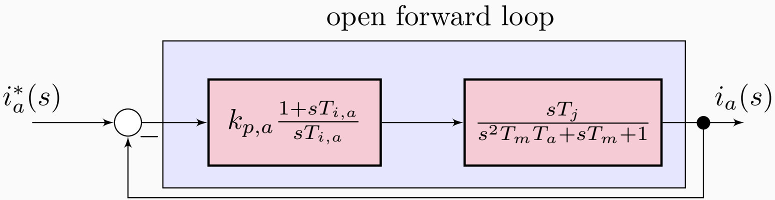
$$s_1 = -\frac{1}{T_s}$$
$$s_2 = -\frac{1}{T_l}$$

where T_s is a small time constant and T_l is the large time constant.

Bode plot of the Current forward transfer function



Current control using PI-controller



Design of inner current loop using Magnitude Optimum method i

We can represent the roots by two time constants

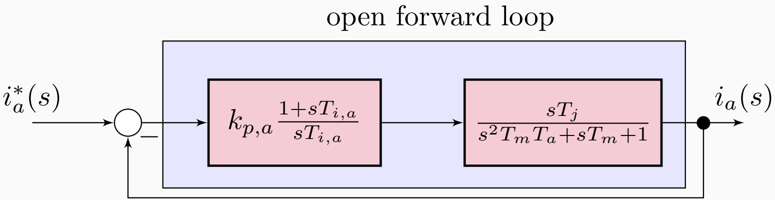
$$s_1 = -\frac{1}{T_s} \tag{40}$$

$$s_2 = -\frac{1}{T_l} \tag{41}$$

where T_s is a small time constant and T_l is the large time constant. PIController A PI controller is used, its transfer function is given as

$$G_{PI} = k_p \frac{(sT_i + 1)}{sT_i}$$

Design of inner current loop using Magnitude Optimum method ii



Hence the open loop transfer function with a PI controller for the current loop can be written as

$$G_{oiv} = \frac{k_{p,a}(1+T_{i,a}s)}{sT_{i,a}}\frac{T_lT_s}{r_a}\frac{sT_m}{(1+sT_l)(1+sT_s)} \tag{42}$$

$$G_{oiv} = \frac{k_{p,a}(1+T_{i,a}s)}{sT_{i,a}}K\frac{sT_m}{(1+sT_l)(1+sT_s)} \tag{43}$$

Design of inner current loop using Magnitude Optimum method iii

The magnitude optimum method maximizes the frequency range for which the magnitude of the transfer function is unity. We eliminate the large time constant, using the PI-controller zeros of the PI

$$T_{i,a} = T_l \tag{44}$$

The resulting transfer function becomes

$$G_{oiv} = \frac{K_{p,a}K}{sT_l}\frac{sT_m}{(1+sT_s)} \tag{45}$$

For magnitude optimum, we would like to get the damping of the inner current loop to be $D = 1/\sqrt{2}$

Design of inner current loop using Magnitude Optimum method iv

Homework exercise

- Develop the closed loop transfer function of the current loop
- Find the expression for damping in the inner current loop
- Find the value of the PI-controller gain that gives a damping of $D = 1/\sqrt{2}$ for the inner loop

The PI-Gain can be found out for optimal damping as

$$K_{p,a} = \frac{T_l}{2KT_s} \tag{46}$$

where $K = \frac{T_l T_s}{r_a}$ is the gain of G_{Fiv} and T_s and T_l are the respective constants.

Design of inner current loop using Magnitude Optimum method v

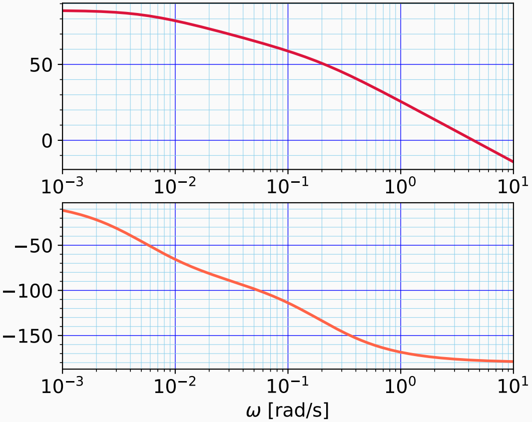
Using the rules, the open loop transfer function is given as

$$G_{oiv} = \frac{K_{p,a}(1 + sT_{i,a})}{sT_{i,a}} K \frac{sT_m}{(1 + sT_l)(1 + sT_s)} \tag{47}$$

$$G_{oiv} = \frac{K_{p,a}K}{sT_l} \frac{sT_m}{(1 + sT_s)} \tag{48}$$

$$G_{oiv} = \frac{\frac{T_m}{2T_s}}{(1 + sT_s)} \tag{49}$$

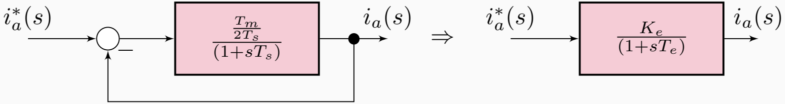
Open loop transfer function of inner current loop



Open loop transfer function

$$G_{oiv} = \frac{\frac{T_m}{2T_s}}{(1 + sT_s)}$$

Closed loop transfer function i



The closed loop transfer function of the inner current loop can be derived as

$$G_{civ} = \frac{G_{oiv}}{1 + G_{oiv}} \tag{50}$$

$$G_{oiv} = \frac{T_m/2T_s}{(1 + sT_s)} \tag{51}$$

$$G_{civ} = \frac{\frac{T_m/2T_s}{(1+sT_s)}}{1 + \frac{T_m/2T_s}{(1+sT_s)}} \tag{52}$$

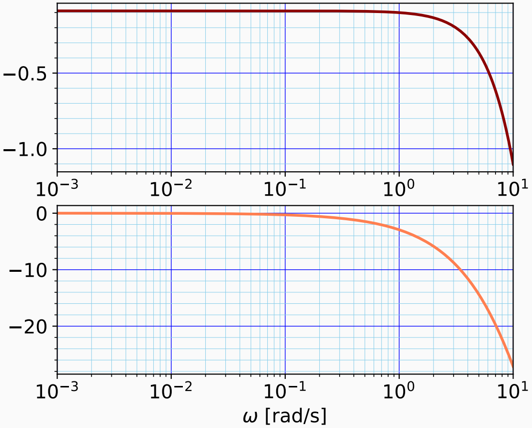
$$G_{civ} = \frac{T_m/2T_s}{1 + sT_s + T_m/2T_s} \tag{53}$$

Closed loop transfer function ii

The closed loop transfer function of the current loop can be represented by a equivalent first order system as

$$\frac{i_a(s)}{i_a^*(s)} = \frac{K_e}{sT_e + 1} = G_{civ} = \frac{T_m/2T_s}{1 + sT_s + T_m/2T_s} \tag{54}$$

Open loop transfer function of inner current loop

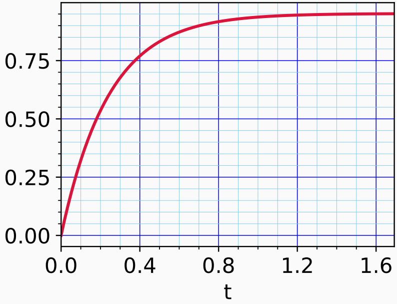


Close loop transfer function

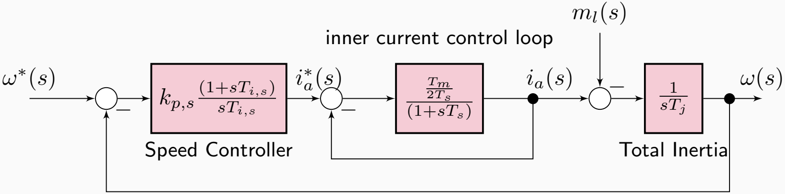
$$\frac{i_a(s)}{i_a^*(s)} = \frac{K_e}{sT_e + 1} = G_{civ} = \frac{T_m/2T_s}{1 + sT_s + T_m/2T_s}$$

Closed loop Inner current response

- the Closed loop of current is equivalent to first order system
- inner current loop has steady state error
- This is due to presence of back-emf
- It can be removed by compensating the back-emf
- feed forward compensation can be used



Design of Speed Controller using Symmetrical Optimum i



The forward transfer function of the speed loop with an inner current loop is given by

$$G_{F\omega i} = \frac{k_e}{(sTe+1)}\frac{1}{sT_j} \tag{55}$$

Design of Speed Controller using Symmetrical Optimum ii

Symmetrical optimum

The two time constant of the system are far apart

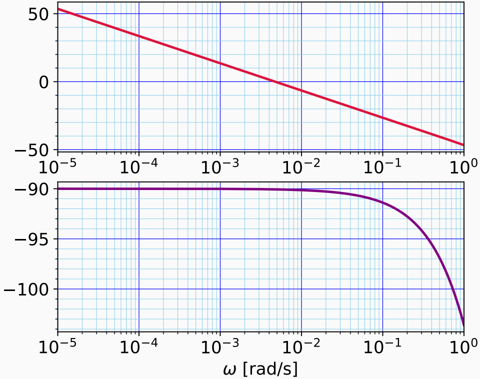
$$T_j > 10T_e$$

we use symmetrical optimum to design the controller

A PI-Controller is used to achieve closed loop speed control.

Forward Gain of speed loop

- $T_j \approx 200$
- $T_e \approx 0.2$
- The integrator effect of inertia dominant
- use Symmetrical optimum



Symmetrical Optimum

Symmetrical Optimum

- If we eliminate the smaller time constant with PI zero, the system will be a double integrator
- In symmetrical optimum, we lift the phase plot near the cross over frequency to provide sufficient phase margin
- The gain plot will be symmetrical around crossover frequency
- It will have a slope of 20 [db/decade].
- We choose the PI integral time constant as

$$T_{i,s} = a^2 Te \tag{56}$$

- $a > 1$ is length of symmetrical region

Design of Speed controller with Symmetrical Optimum

We have, PI time constant

$$T_{i,s} = a^2 T_e \tag{57}$$

where, the equivalent inner loop is

$$G_{civ} = \frac{k_e}{(sTe + 1)}$$

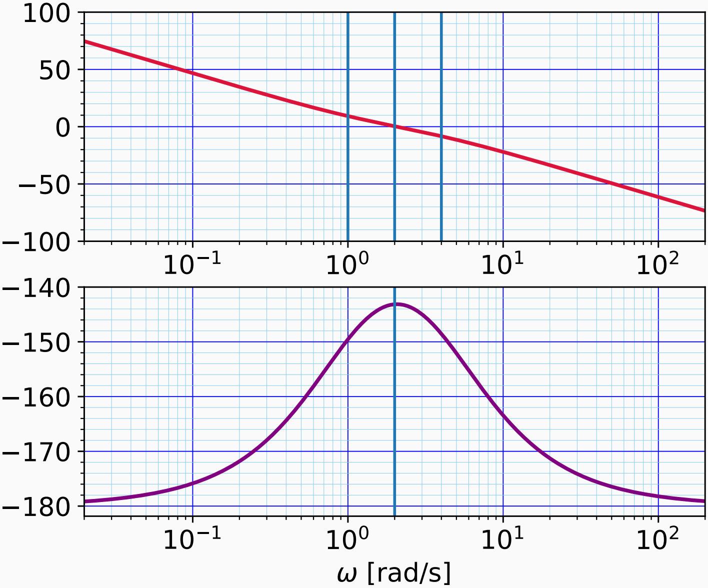
The PI gain is selected as

$$k_{p,s} = \frac{T_j}{a k_e T_e} \tag{58}$$

Hence the open loop transfer function with PI is

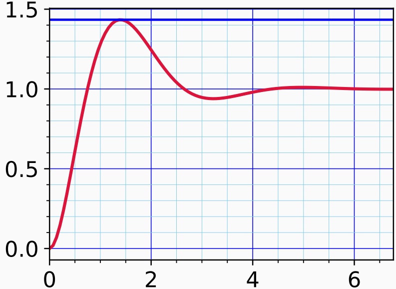
$$G_{osi} = \frac{T_j}{a k_e T_e} \frac{1 + s a^2 T_e}{s a^2 T_e} \frac{k_e}{(sTe + 1)} \frac{1}{sT_j} \tag{59}$$

Symmetrical optimum bode plot for $a = 2$



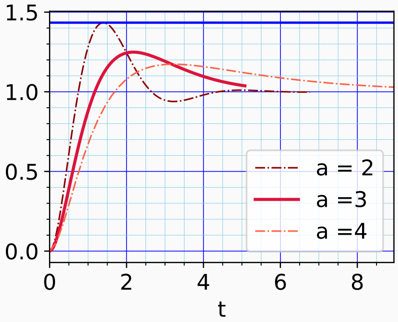
Step response with $a = 2$ produces 43.4% overshoot

- Step response for the closed loop speed control
- Produces a 43.4% overshoot
- A large overshoot in speed in not desired
- Many applications need tight speed control
- increasing $a > 2$ reduces overshoot but slows response



Step response with changing a

- Step response for the closed loop speed control
- Produces a 43.4% overshoot
- A large overshoot in speed in not desired
- Many applications need tight speed control
- increasing $a > 2$ reduces overshoot but slows response



State Variable Control of the Sep. EXcited DC Motor

The DC motor dynamics can be described by the equations

$$
\begin{aligned}
T_a \frac{di_a}{dt} &= -i_a - \frac{1}{r_a} e_a + \frac{1}{r_a} v_a \\
T_j \frac{d\omega}{dt} &= m_e - m_L
\end{aligned}
$$

For the separately excited DC machine, when the flux is constant, we get

$$
e_a = k_e \omega \tag{60}
$$

$$
m_e = k_t i_a \tag{61}
$$

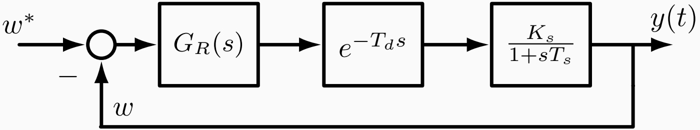
We can write the dynamics as

$$
\begin{aligned}
T_a \frac{di_a}{dt} &= -i_a - \frac{1}{r_a} k_e \omega + \frac{1}{r_a} v_a \\
T_j \frac{d\omega}{dt} &= k_t i_a - m_L
\end{aligned}
$$

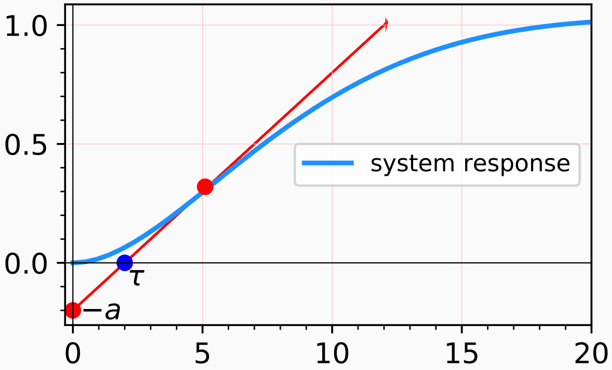
Ziegler Nichols

Setting Controllers using Ziegler Nichols method

A number of processes in practical systems have a dead-time in control. In DC motor control it could be dead time of the converter. If we have a system with a dead time T_d and a first order system with time constant T_s



If the parameters of the system are not known, the controller is set using the following process



Parameters of Controllers ii

1. The step response of the system in open loop is generated. As shown above
2. Get the parameters a and τ form the step response as shown in fig above
3. τ is the delay in the system and $\frac{a}{\tau}$ is the steepest slope.
4. we can use this parameter to set the controller using values in table Table below

when the time constants of system are known, a method of optimal parameter control for P and PI controller can be set as proposed by ZIEGLER and NICHOLS

Controller Type	k_p	T_i	T_d
P-Controller	$\frac{1}{a}$		
PI-Controller	$\frac{0.9}{a}$	3τ	
PID-Controller	$\frac{1.2}{a}$	2τ	0.5τ

Ziegler Nichols method with frequency response

If we know the system, then the frequency response can also be used to set the parameters. We first find the critical frequency using the frequency plot. ω_c is the frequency at which the Nyquist curve of the system transfer function first intersects the negative real axis.

We have to find the gain of the system for which the system has a phase lag of 180 degrees.

Controller Type	k_p	T_i	T_d
P-Controller	$0.5k_c$		
PI-Controller	$0.45k_c$	$0.8T_c$	
PID -controller	$0.6k_c$	$0.5T_c$	$0.125T_c$

Table 1: ZIEGLER and NICHOLS proposed optimal parameters

where

$$T_c = \frac{2\pi}{\omega_c}$$

Setting Higher order system controllers using step response

For a general higher order system, a step response is obtained as shown in Fig.1. We may not know the actual transfer function of the system.

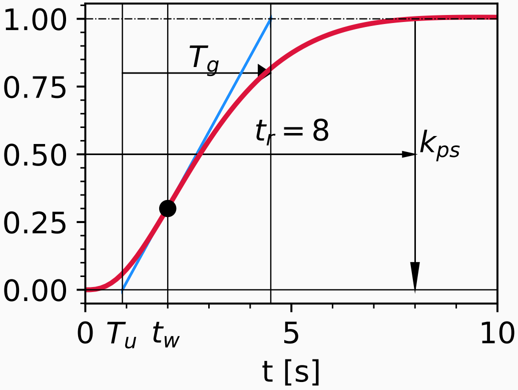


Figure 1: Step response of a higher order system

Parameters of Step response

It can be characterized by following parameters

T_g : Total Time constant of the system
T_u : Delay time
k_{ps} : System gain

Setting controller using optimal performance

For a condition

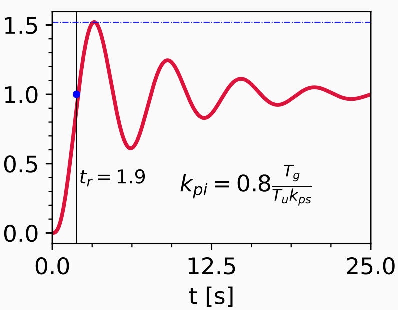
$$\frac{T_u}{T_g} < 0.3$$

we can set optimal parameters of the various controllers as shown in table T.2.

Controller Type	k_p	T_i	T_d
P-Controller	$\frac{T_g}{k_{ps}T_u}$		
PI-Controller	$0.8\frac{T_g}{T_u k_{ps}}$	$3.0T_u$	
PD - Controller	$1.2\frac{T_g}{T_u k_{ps}}$		$(0.25 - 0.5))T_u$
PID -controller	$1.2\frac{T_g}{T_u k_{ps}}$	$2.0T_u$	$0.42T_u$

Table 2: Step response based proposed optimal parameters

Resulting step response



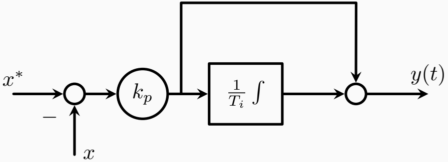
(a) Step response with gain $k_p i = 0.8\frac{T_g}{T_u k_{ps}}$ **(b)** Step response with gain $k_p i = 0.4\frac{T_g}{T_u k_{ps}}$

Figure 2: Step response using PI-controller

Appendix: Proportional Integral Controller PI-Control

The PI- controller works on the error input

$$y(t) = k_p \left(e(t) + \frac{1}{T_i} \int e(t) dt \right)$$
$$Y(s) = \left(k_p + \frac{k_p}{sT_i} \right) E(s)$$
$$\frac{Y(s)}{E(s)} = k_p \frac{sT_i + 1}{sT_i}$$



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