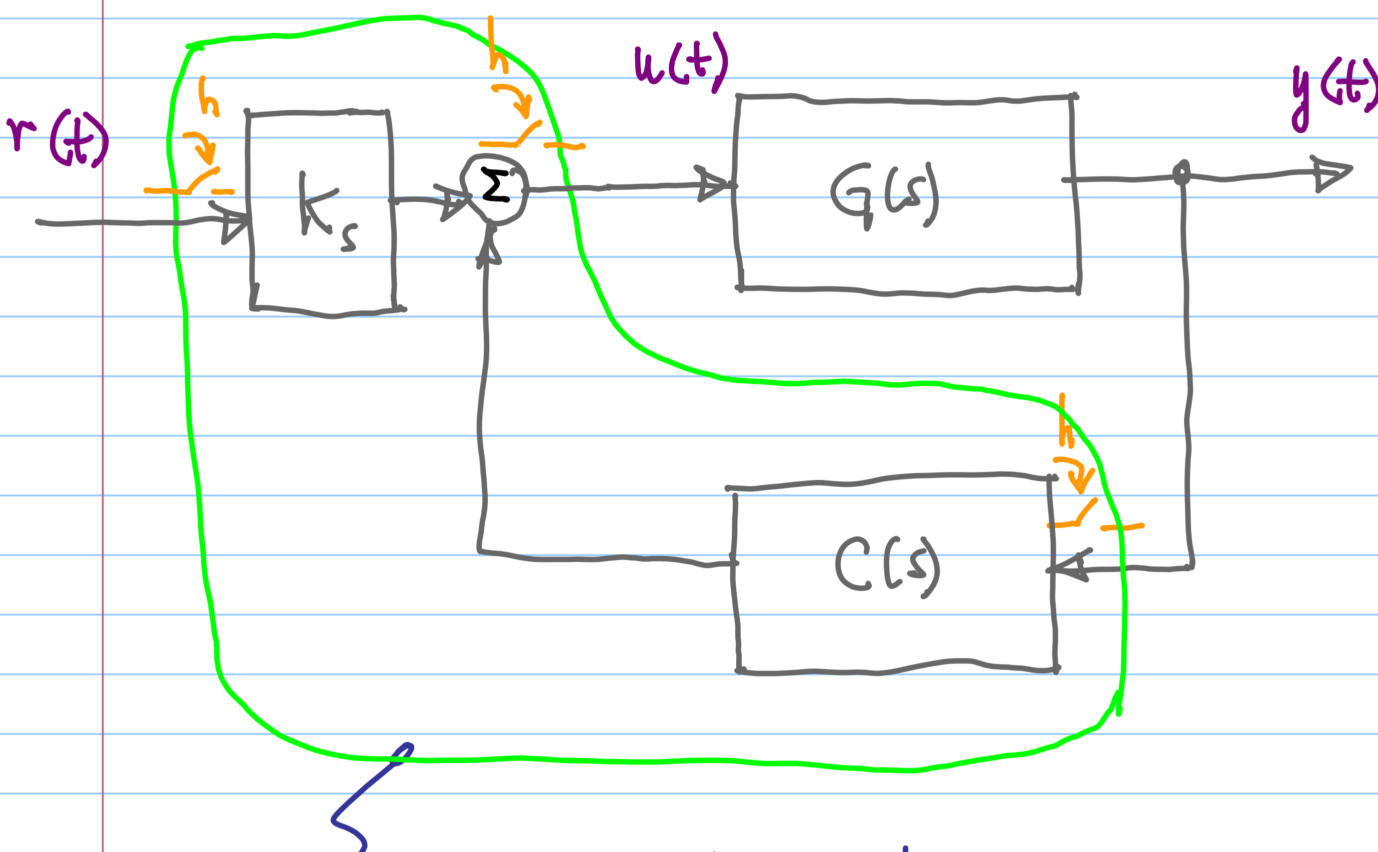
Note Title 2/27/2007

(a) Discrete-time implementation of continuous-time controller

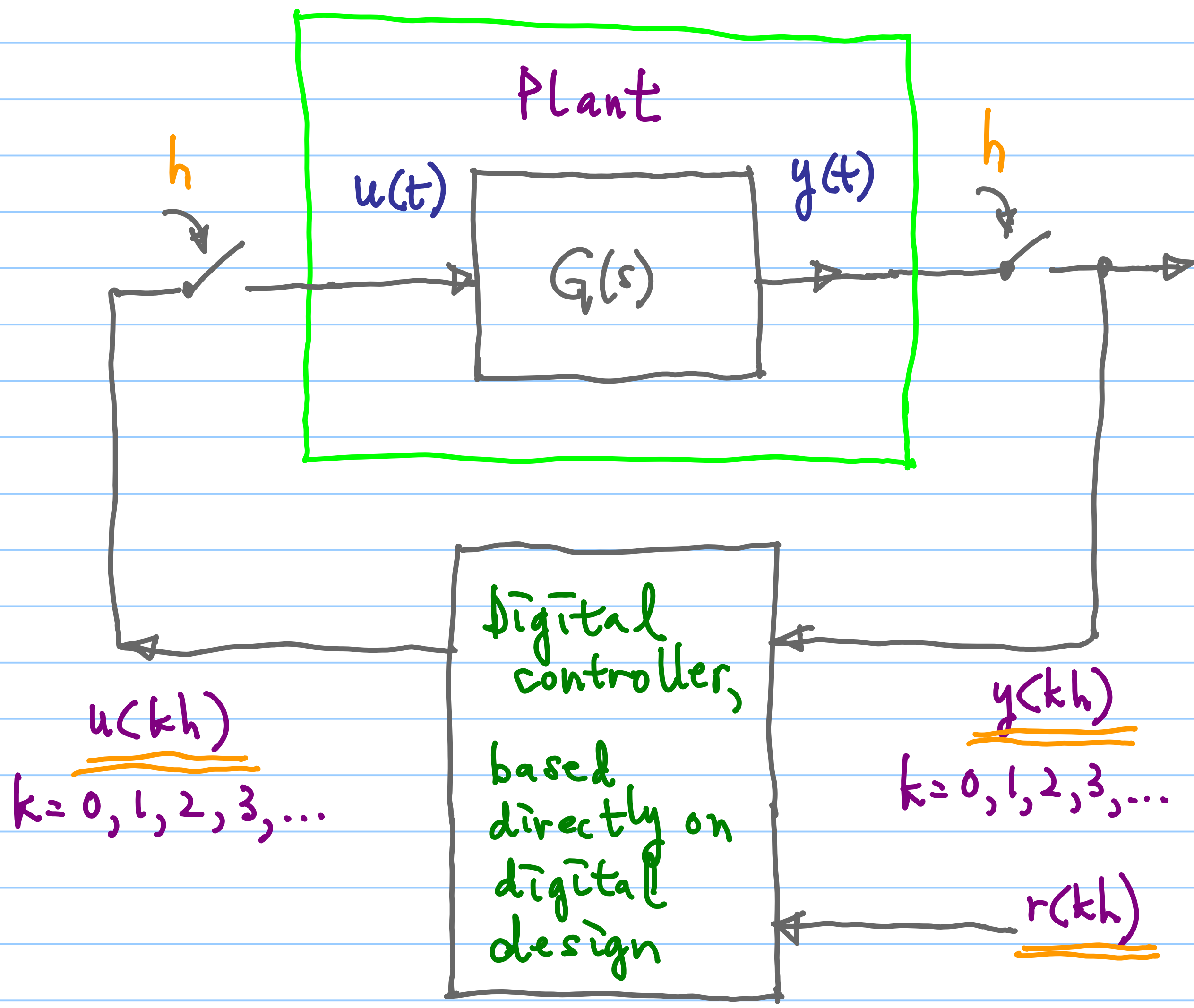
i.e. • designed in "continuous-time" domain

• implemented in discrete-time



implemented digitally; updated every sampling interval, $\underline{\underline{h}}$

(b) Direct design of digital controllers



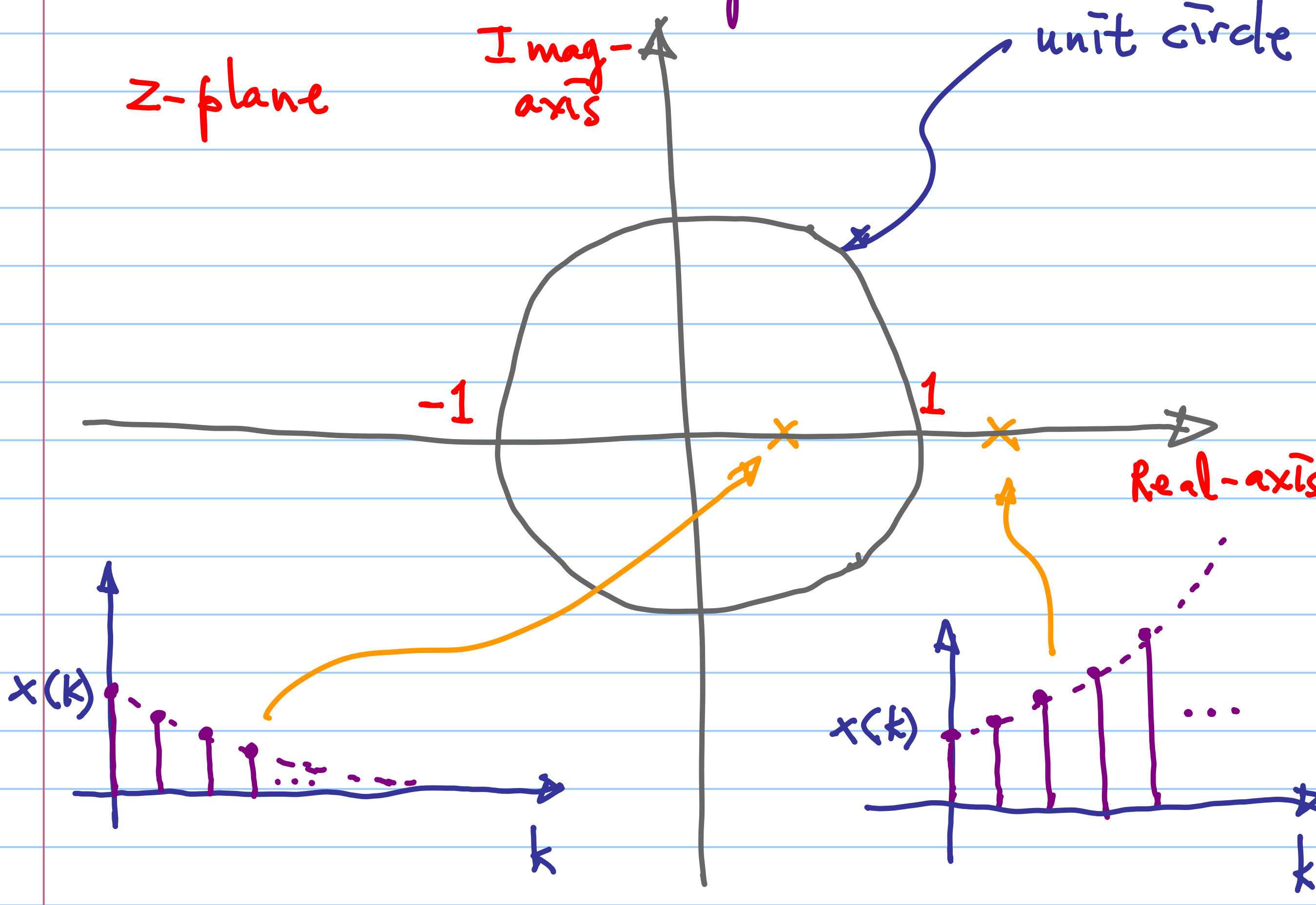
Discrete-time state-variable description:

$$x(k+1) = \Phi\, x(k) + \Gamma\, u(k)$$

$$y(k) = H\, x(k)$$

$$k = 0, 1, 2, 3, \ldots$$

Recall the z-plane diagram, & poles of a discrete-time system:



Compare:

Continuous-time

$$\dot{x} = Fx + Gu$$
$$y = Hx$$

State-feedback

$$u = -Kx$$

leading to closed-loop

$$\dot{x} = (F - GK)\,x$$
$$y = Hx$$

Closed-loop poles are freely assignable iff

$\mathcal{C}(F, G)$ is of full rank.

Recall:

$$\mathcal{C}(F,G) \triangleq \left[G \,\vdots\, FG \,\vdots\, F^2G \,\vdots\, \cdots \,\vdots\, F^{n-1}G \right]$$

Discrete-time

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$

$$y(k) = H x(k)$$

State-feedback

$$u(k) = -K x(k)$$

leading to closed-loop

$$x(k+1) = \left(\Phi - \Gamma K\right) x(k)$$

$$y(k) = H x(k)$$

Recall the properties of the poles of a discrete-time state-variable system

$\therefore$, here, closed-loop state-fb poles are:

$$\alpha_c(z) = \det\left[sI - \left(\Phi - \Gamma K\right)\right]$$

and closed-loop poles are freely assignable iff $\mathcal{C}(\Phi, \Gamma)$ is of full rank.

Continuous-time

Estimator

$$\dot{\hat{x}} = F\hat{x} + Gu + L(y - H\hat{x})$$

leading to estimator closed-loop

$$\tilde{x} \triangleq \hat{x} - x$$

$$\dot{\tilde{x}} = (F - LH)\tilde{x}$$

Estimator closed-loop poles are freely assignable iff

$\mathcal{O}(H, F)$ is of full rank.

Recall:

$$\mathcal{O}(H,F) \triangleq \begin{bmatrix} H \\ \hdashline HF \\ \hdashline HF^2 \\ \hdashline \vdots \\ \hdashline HF^{n-1} \end{bmatrix}$$

Discrete-time

Estimator

$$\hat{x}(k+1) = \Phi \hat{x}(k) + \Gamma u(k) + L\left\{y(k) - H\hat{x}(k)\right\}$$

leading to estimator closed-loop

$$\tilde{x}(k) \triangleq \hat{x}(k) - x(k)$$

$$\tilde{x}(k+1) = (\Phi - LH)\tilde{x}(k)$$

And, noting the identical equation structure, it follows that here, the estimator closed-loop poles are likewise freely assignable iff

$$\Theta(H, \Phi) \quad \text{is of full rank.}$$

Summary

Continuous-time

$$\dot{x} = Fx + Gu$$
$$y = Hx$$

Complete estimator-controller:

$$\dot{\hat{x}} = (F - GK - LH)\hat{x} + Ly$$
$$u = -K\hat{x}$$

- State-fb gain K chosen so that

$$\alpha_c(s) = \det\left[sI - (F - GK)\right]$$

are the closed-loop state-fb poles.

- Estimator gain L chosen so that

$$\alpha_e(s) = \det\left[sI - (F - LH)\right]$$

are the closed-loop estimator poles.

Discrete-time

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$

$$y(k) = H x(k)$$

Complete estimator-controller

$$u(k) = -K \hat{x}(k)$$

$$\hat{x}(k+1) = \Phi x(k) + \Gamma u(k) + L \left\{ y(k) - H\hat{x}(k) \right\}$$

$$= \left(\Phi - \Gamma K - LH \right) \hat{x}(k) + L y(k)$$

- State-fb gain K chosen so that

$$\alpha_c(z) = \det \left[zI - (\Phi - \Gamma K) \right]$$

are the closed-loop state-fb poles.

- Estimator gain L chosen so that

$$\alpha_e(z) = \det \left[zI - (\Phi - LH) \right]$$

are the closed-loop estimator poles.