Q_1:

a) Initial Condition: $H(z) = \dfrac{z+0.9}{z^2-1.8z+0.81}$ $A_m(z) = z^2-1.5z+0.7$

To cancle zero, $R(z)$ should contain $(z+0.9)$, let's set $R(z) = z+0.9$

set $S(z) = s_0 z + s_1$

$\therefore AR+BS = (z^2-1.8z+0.81)(z+0.9) + (z+0.9)(s_0 z + s_1)$

$= (z^2 + (-1.8+s_0)z + 0.81+s_1)(z+0.9)$

we can find that $A_m(z)$ don't contain $(z+0.9)$, so we have to set $A_0(z) = z+0.9$, so $A_{cl}(z) = A_m(z)A_0(z) = (z^2-1.5z+0.7)(z+0.9)$

$\because AR+BS = A_{cl} \Rightarrow z^2 + (-1.8+s_0)z + 0.81+s_1 = z^2-1.5z+0.7$

$\Rightarrow s_0 = 0.3,\ s_1 = -0.11$

Now design $T(z)$, just set $T(z) = t_0 \cdot A_0(z) = t_0 \cdot (z+0.9)$

$\therefore \dfrac{Y(z)}{U_c(z)} = \dfrac{T(z)\cdot B(z)}{A_{cl}(z)} = \dfrac{t_0\cdot(z+0.9)}{z^2-1.5z+0.7}$

$\because$ gain is 1. $\Rightarrow \left.\dfrac{Y(z)}{U_c(z)}\right|_{z=1} = \dfrac{t_0\cdot 1.9}{0.2} = 1 \Rightarrow t_0 = 0.11$

$\therefore$ controller: $U(z) = \dfrac{0.11(z+0.9)}{z+0.9}\cdot U_c(z) - \dfrac{0.3z-0.11}{z+0.9}Y(z)$

b) Set $R(z) = z+r$, $S(z) = s_0 z + s_1$

$\therefore AR+BS = (z^2-1.8z+0.81)(z+r) + (z+0.9)(s_0 z + s_1)$

$A_{cl}(z) = A_m(z)\cdot A_0(z) = (z^2-1.5z+0.7)\cdot A_0(z)$

set $A_0(z) = z$

$A_{cl}(z) = z^3 - 1.5z^2 + 0.7z$

$\because AR+BS = A_{cl} \Rightarrow \begin{cases} r-1.8+s_0 = -1.5 \\ -1.8r+0.81+0.9s_0+s_1 = 0.7 \\ 0.81r+0.9s_1 = 0 \end{cases} \Rightarrow \begin{cases} r = 0.11 \\ s_0 = 0.19 \\ s_1 = -0.01 \end{cases}$

Now design $T(z)$, set $T(z) = t_0\cdot A_0(z) = t_0\cdot z$

$\therefore \dfrac{Y(z)}{U_c(z)} = \dfrac{T(z)\cdot B(z)}{A_{cl}(z)} = \dfrac{t_0\cdot z\cdot(z+0.9)}{z^3-1.5z^2+0.7z} \Rightarrow$ when $z=1$, $\dfrac{t_0\cdot 1.9}{0.2} = 1$

$t_0 = 0.11$

$\therefore\ U(z) = \frac{0.11 \cdot z}{z+0.11} \cdot U_c(z) - \frac{0.19z - 0.01}{z+0.11} Y(z)$

Q_2:

a) $\begin{cases} x_1(k+1) = 0.5x_1(k) + x_2(k) + 0.2u(k) + v(k) \\ x_2(k+1) = 0.5x_1(k) + 0.7x_2(k) + 0.1u(k) \end{cases}$ $\quad y(k) = x_1(k)$

$\Rightarrow \begin{cases} z \cdot X_1(z) = 0.5X_1(z) + X_2(z) + 0.2U(z) + V(z) \\ z \cdot X_2(z) = 0.5X_1(z) + 0.7X_2(z) + 0.1U(z) \end{cases}$ $\quad \because Y(z) = X_1(z)$

$\Rightarrow Y(z) = \frac{0.2z - 0.04}{z^2 - 1.2z - 0.15} U(z) + \frac{z - 0.7}{z^2 - 1.2z - 0.15} V(z)$

Consider v, $\frac{Y(z)}{V(z)} = \frac{z-0.7}{z^2-1.2z-0.15}$

Use feed back controller $\frac{S(z)}{R(z)}$, the T.F. changes into

$$T.F. = \frac{\frac{Y(z)}{V(z)}}{1 + \frac{Y(z)}{V(z)} \cdot \frac{S(z)}{R(z)}}$$

$\because v(k)$ can be eliminated $\Rightarrow R(z)$ should contain $(z-1)$

set $R(z) = z-1$, $S(z) = s_0$

$\therefore$ DC gain is $0 \Rightarrow \left. \frac{BR}{AR+BS} \right|_{z=1} = 0$

$\Rightarrow \frac{(z-0.7)(z-1)}{(z^2-1.2z-0.15)(z-1) + s_0(z-0.7)} = 0 \Rightarrow$ set $s_0 = 0.5$

$\therefore$ controller: $\frac{S(z)}{R(z)} = \frac{0.5}{z-1}$

b) T.F. way is simpler because we don't need to add any observers.

Q_3: a) $\because$ we want to reject constant disturbance

$\therefore R(z)$ should contain $(z-1)$

Then, to ~~canele~~ cancel process zeros because $B(z) = z+0.5$ is stable, it can be ~~cancled~~ cancelled, we set $R(z) = (z-1)(z+0.5)$, $S(z) = s_0z^2 + s_1z + s_2$

$\therefore AR+BS = (z+0.5)\left[z^3+(s_0-3)z^2+(s_1+2.6)z+s_2-0.6\right]$

$A_{cl}(z) = A_m(z)\cdot A_o(z) = (z^2-1.8z+0.9)\cdot A_o(z)$

set $A_o(z) = z\cdot(z+0.5)$

so $AR+BS = A_{cl} \Rightarrow \begin{cases} -3+s_0=-1.8 \\ 2+0.6+s_1=0.9 \\ -0.6+s_2=0 \end{cases} \Rightarrow \begin{cases} s_0=1.2 \\ s_1=-1.7 \\ s_2=0.6 \end{cases}$

$$\therefore \frac{Y(z)}{U_c(z)} = \frac{T\cdot B}{A_{cl}} = \frac{T\cdot(z+0.5)}{A_m(z)\cdot z\cdot(z+0.5)} = \frac{B_m(z)}{A_m(z)} \Rightarrow T = z\cdot(z-0.9)$$

$$\therefore \text{controller}: \ U(z) = \frac{z^2-0.9z}{z^2-0.5z-0.5}U_c(z) - \frac{1.2z^2-1.7z+0.6}{z^2-0.5z-0.5}Y(z)$$

b) To ~~just~~ reject disturbance, $R(z)$ should contain $(z-1)$

set $R(z)=z-1$, $S(z)=s_0z+s_1$

$\therefore AR+BS = (z^2-2z+0.6)(z-1)+(z+0.5)(s_0z+s_1)$

$A_{cl}(z) = A_m(z)\cdot A_o(z) = (z^2-1.8z+0.9)\cdot A_o(z)$

set $A_o(z) = z-a_0$

$\therefore AR+BS = A_{cl} \Rightarrow \begin{cases} -3+s_0=-1.8-a_0 \\ 2.6+0.5s_0+s_1=0.9+1.8a_0 \\ -0.6+0.5s_1=-0.9a_0 \end{cases} \Rightarrow \begin{cases} a_0=0.85 \\ s_0=0.35 \\ s_1=-0.34 \end{cases}$

To match $\frac{B_m(z)}{A_m(z)}$, $\Rightarrow H_{ff}(z) = \frac{A_o\cdot B_m}{B\cdot R} = \frac{(z-0.85)(z-0.9)}{(z+0.5)(z-1)} = \frac{z^2-1.75z+0.77}{z^2-0.5z-0.5}$

$$\therefore \text{controller}: \ U(z) = -\frac{0.35z-0.34}{z-1}Y(z) + \frac{z^2-1.75z+0.77}{z^2-0.5z-0.5}U_c(z)$$

Q_4: a) $y(k+1) = \sin(y(k)) + u(k-1) + c\cdot u(k-2)$

$\Rightarrow y(k+2) = \cancel{\sin(y(k))}\ \sin(y(k+1)) + u(k) + c\cdot u(k-1)$

set $y(k+2)=r(k+2) \Rightarrow \cancel{r(k+2)}\ u(k) = r(k+2) - \sin(y(k+1)) - c\cdot u(k-1)$

$\Rightarrow u(k) = r(k+2) - \sin\left[\sin(y(k)) + u(k-1) + c\cdot u(k-2)\right] - c\cdot u(k-1)$

$y(k+2) = r(k+2)$

b) $y(k) \to r(k) \Rightarrow r(k+1) = \sin(r(k)) + u(k-1) + c \cdot u(k-2)$ ※set $\sin(r(k))$ coefficient is $A(z)$

$$\Rightarrow z \cdot R(z) + A(z) \cdot R(z) = z^{-1} \cdot U(z) + c \cdot z^{-2} \cdot U(z)$$

$$\Rightarrow \frac{U(z)}{R(z)} = \frac{z + A(z)}{z^{-1} + c \cdot z^{-2}} = \frac{z^2(z + A(z))}{z + c}$$

let $(z+c)$ is stable so that perfect tracking is attainable

$\therefore |c| < 1 \Rightarrow -1 < c < 1$