EE4302 ADVANCED CONTROL SYSTEM TUTORIAL

1. Consider the pendulum equation

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\sin x_1 - c x_2 + u\end{aligned}$$

(a) Let $u = 0$, $c = 0.5$ and find all the equilibrium points.

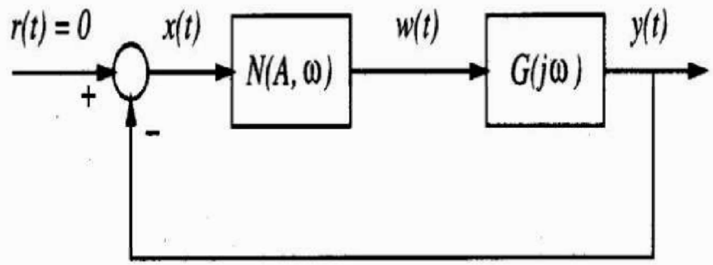
(b) Let $y = x_1$, $c = 0.5$ and find the linearized transfer function $\frac{Y(s)}{U(s)}$ about any equilibrium point.

(c) For $-\pi \leq x_1 \leq \pi$ and $c = 0$ sketch the isoclines for tangent slopes of $\alpha = \pm 1/8, \pm 1/4, \pm 1/3, \pm 1/2, \pm 1, \infty$. Limit the plot to $-4 \leq x_2 \leq 4$.

(d) Using the isoclines, sketch the phase plane trajectory. Start from the initial states of $x_1 = 1$, $x_2 = 0$ and stop when the trajectory reaches the state $x_2 = 0$ again.

(e) By dividing the trajectory into a number of segments of $\Delta x_1 = 0.5$, draw the x_1 versus t (time) plot.

2. Consider the nonlinear system in the Figure.



(a) Find the describing function, $N(A, \omega)$, for

$$w = x^5$$

Given: $\sin^6 \theta = \left[\frac{1}{2}(1 - \cos 2\theta)\right]^3$; $\cos^2 2\theta = \frac{1}{2}(1 + \cos 4\theta)$.

(b) Sketch $-1/N(A, \omega)$ in the complex plane.

(c) Let

$$G(j\omega) = \frac{1 - j\omega}{j\omega(j\omega + 1)}$$

Using the describing function method, investigate the possibility of existence of periodic solutions and the possible frequency and amplitude of oscillation.

4. The nonlinearity and the transfer function of the plant in the Figure are given as

$$u = \mathrm{sgn}(e)$$

and

$$G_p(s) = \frac{3}{s(2s+1)}$$

respectively.

(a) Determine the isocline equation for the states defined as $x_1 = e$ and $x_2 = \dot{e}$.

(b) On a phase-plane of $-10 \leq x_1 \leq 10$ and $-10 \leq x_2 \leq 10$, sketch the isoclines for slopes of $\alpha = -2,\ -1,\ -\frac{3}{4},\ 0,\ 1,\ \infty$.

(c) Using the isoclines, sketch the trajectory of the states. Start from

$x_1 = 0^-$, $x_2 = -6$ and end when $x_1 = 0^-$ again.

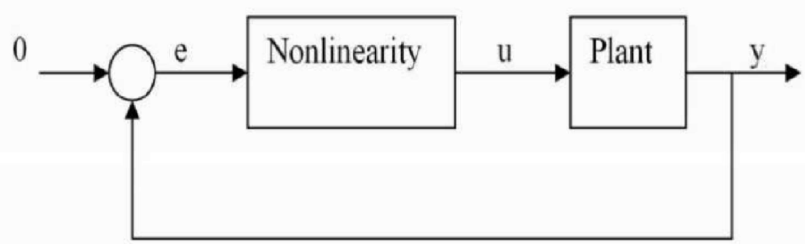
5. The Describing Function of the nonlinearity and the transfer function of the plant in the Figure are given as

$$N = \frac{1}{2} + \frac{1}{\pi}\left(\sin^{-1}\frac{2}{a} + \frac{2}{a}\sqrt{1 - \frac{4}{a^2}}\right)$$

and

$$G_p(j\omega) = \frac{50}{j\omega(5 + j\omega)(1 + j\omega)}$$

respectively. By considering the frequency ω from 2 rad/s to 4 rad/s in steps of 0.5 rad/s and amplitude a from 2 to 22 in steps of 5, find the limit cycle graphically.



6. The relay nonlinearity and the transfer function of the plant in the Figure are given as

$$u = \begin{cases} 1 & e > 0 \\ -1 & e < 0 \end{cases}$$

and

$$G_p(s) = \frac{1-s}{s(s+1)}$$

respectively.

(a) Plot K versus e where K is the effective gain of the nonlinearity for $0.5 \leq |e| \leq 2$.

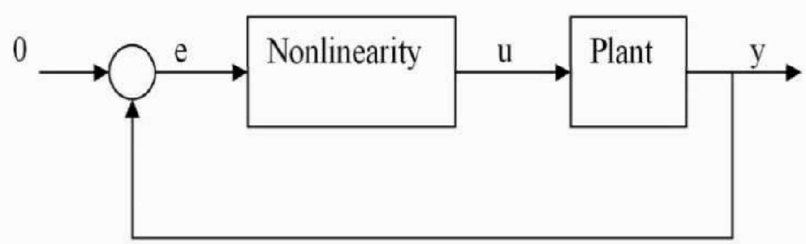
(b) Draw the locus of closed-loop poles (root-locus) for the range of effective gain computed in part (a).

(c) Determine the ultimate gain and ultimate period of the system from the root-locus.

(d) Derive the describing function of the relay nonlinearity.

(e) Using the describing function, determine the ultimate gain and ultimate period of the system.

(f) Find the describing function of the relay with amplitude M and hysteresis h.



9. The model of a first-order system is given by

$$\begin{aligned}\dot{x}(t) &= ax(t) + bu(t) + d(t)\\ y(t) &= x(t)\end{aligned}$$

where b is a positive constant, $a^- \leq a \leq a^+$, $|x| \leq x^+$ are within known bounds, $u(t)$ is the control signal and disturbance, $d(t)$, is bounded as $|d(t)| \leq d^+$.

Define the sliding variable as

$$\sigma = -x(t),$$

the sliding controller as

$$u(t) = M\text{sign}(\sigma)$$

and the Lyapunov function as

$$V = \frac{\sigma^2}{2b}$$

The goal is to design a controller such that $x(t) = 0$ is an asymptotically stable solution.

(a) Determine $y(t)$ analytically, given $a^- = a^+ = a = 0.5$, $b = 1$, $d^+ = d = 0$, $M = 2$ and $x(0) = 1$.

(b) If $M > g^+$ then $\dot{V} < 0$. Determine g^+ in terms of a^+, x^+ and d^+.

10. Consider the unstable system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u = Ax + Bu$$

which has the transfer function

$$\frac{Y(s)}{U(s)} = \frac{1}{s(s-1)}$$

(a) Design a variable-structure controller. Choose the switching line as

$$\sigma = x_1 + x_2$$

and the amplitude of the "sign" function as 0.5.

(b) Draw the states as a function of time. The initial conditions are $x_1(0) = 1.5$ and $x_2(0) = 0$.

(c) Draw the phase plane trajectory.

11. Consider the system

$$\ddot{y} = -1.5\dot{y}^2 \cos 3y + u$$

a) Design a sliding mode controller, given that the switching line is

$$\sigma = \dot{y} + 20y$$

and the amplitude of the "sign" function is 0.1.

b) Calculate the time required to reach the switching line from the initial states of $y = \dot{y} = 1$.

12. Consider the tank system in the Figure where h in m/s, A and a in m^2 are the liquid level, cross-sectional area of the tank and outlet respectively. The difference between the rate of inflow, u and outflow $a \times v$ in m^3/s gives the rate of change in the volume.

$$A\dot{h} = u - a \times v$$

where v is the velocity in m/s of the liquid leaving the outlet. To obtain v, we can equate the potential energy of a small mass m of liquid at height h with its kinetic energy at the outlet.

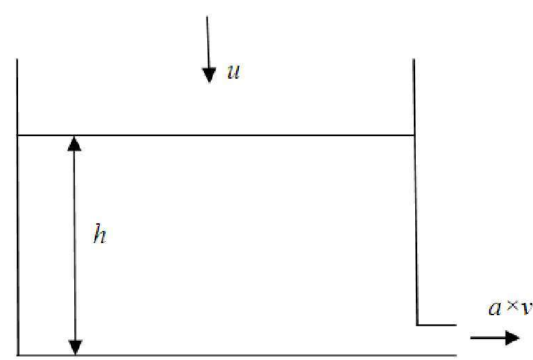
$$mgh = \frac{1}{2}mv^2$$

(a) Find the differential equation relating the rate of change in liquid height, $\dot{h}$, to the rate of inflow u.

(b) By solving the equation in (a) find the expression for time, T, to empty the tank given $u = 0$ and the initial liquid level of h_0.

(c) Find the expression for the steady-state input, $\bar{u}$, when the output is at steady-state $\bar{h}$.

(d) Derive the linearized transfer function $\frac{\Delta H(s)}{\Delta U(s)}$ about the operating point $\bar{u}$ and $\bar{h}$.



13. Consider the tank system in Q.12 where u and h may be considered as the input and output of the system respectively. For the characteristic equation specified as

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

where $\zeta = 0.6$ and $\omega_n = 1$, design a controller using

(a) gain-scheduling, and

(b) input-output linearization.