# Control of Permanent Magnet Synchronous Motors

BLDC PM, PMSM

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# Control of PM SM

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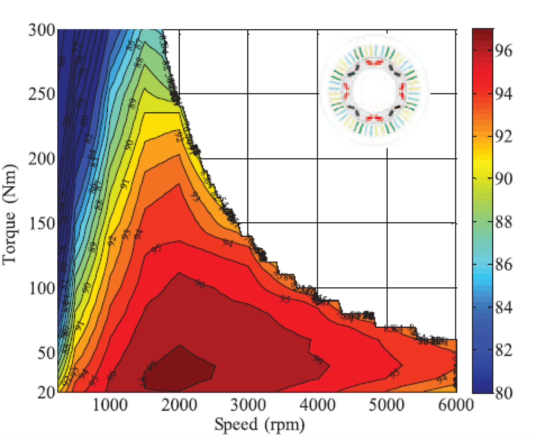
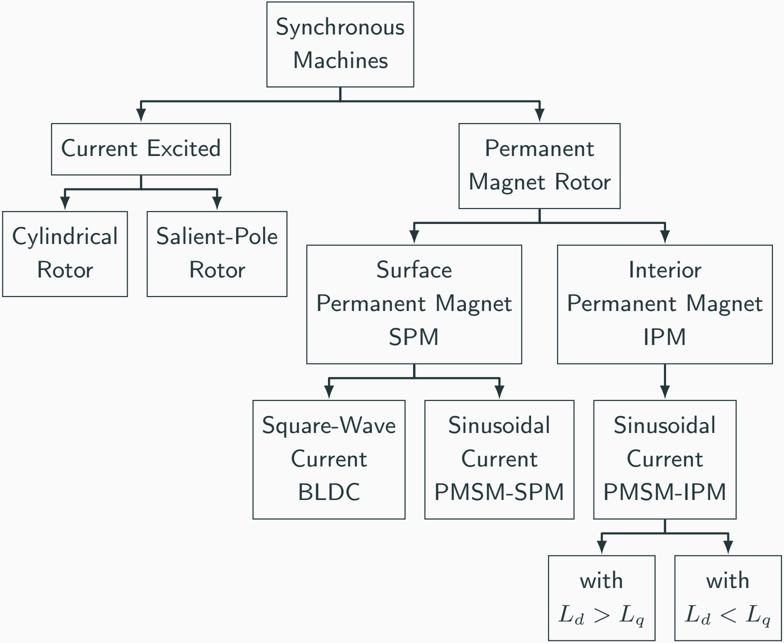
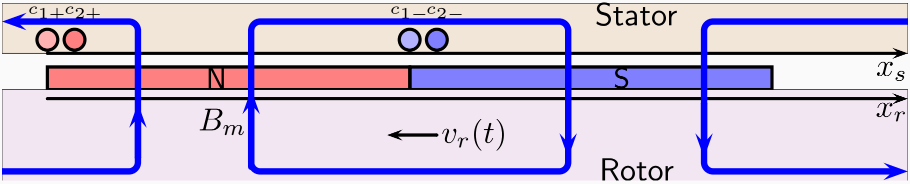


# Taxonomy of Synchronous Machines

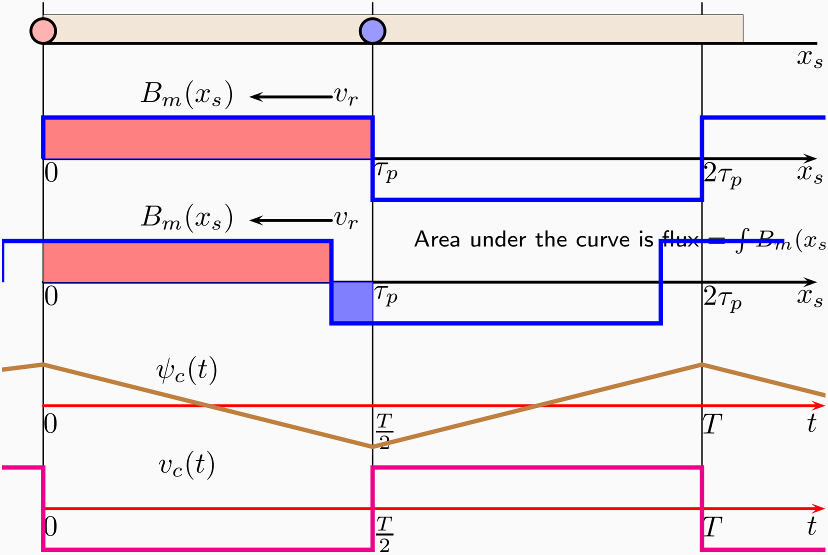


# Brushless DC Magnetic Circuit: 2 pole

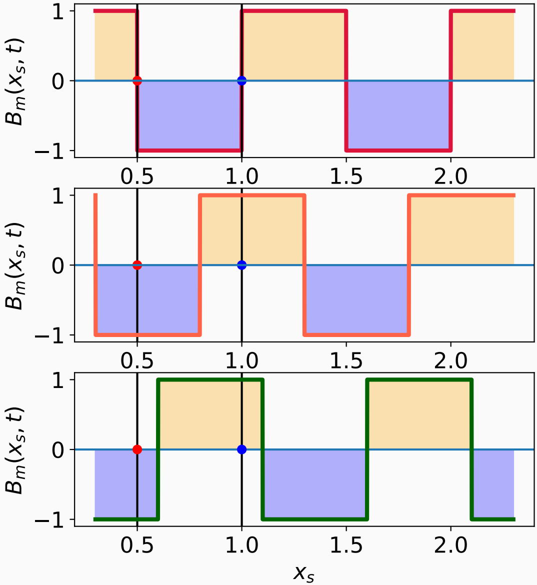



- Stator has distributed windings
- Surface mounted magnets on the rotor
- Flux links stator windings

# Brushless DC Motor Magnetic Field

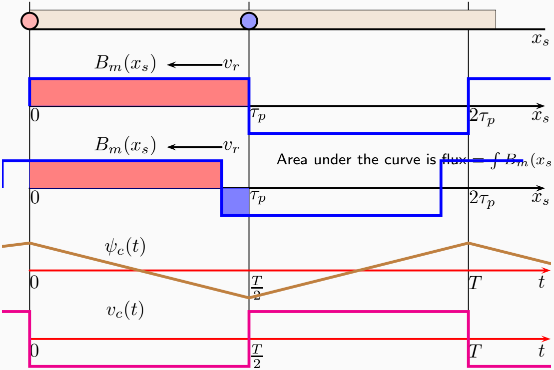


# Flux linkage changes linearly as rotor moves



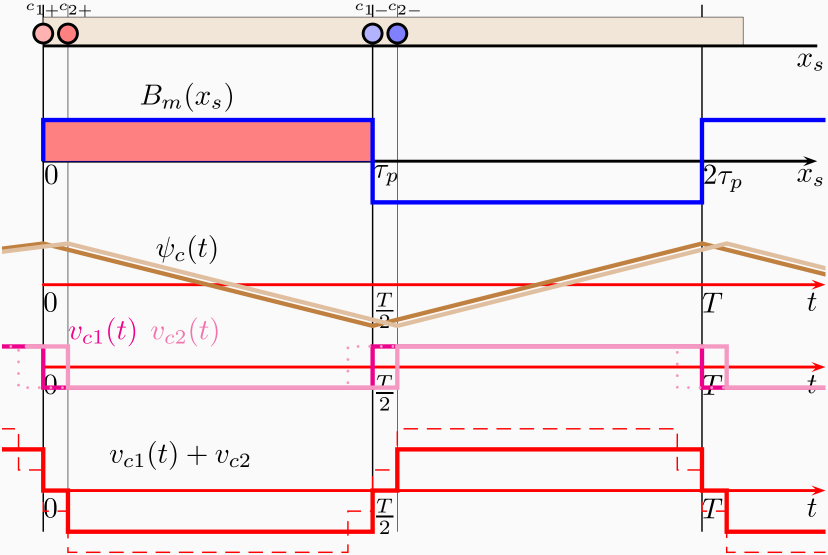

- as rotor moves
- flux linkage with coil changes linearly
- area under $B_m x_s, t$ curve between the coil span is flux

# Brushless DC Motor Magnetic Field

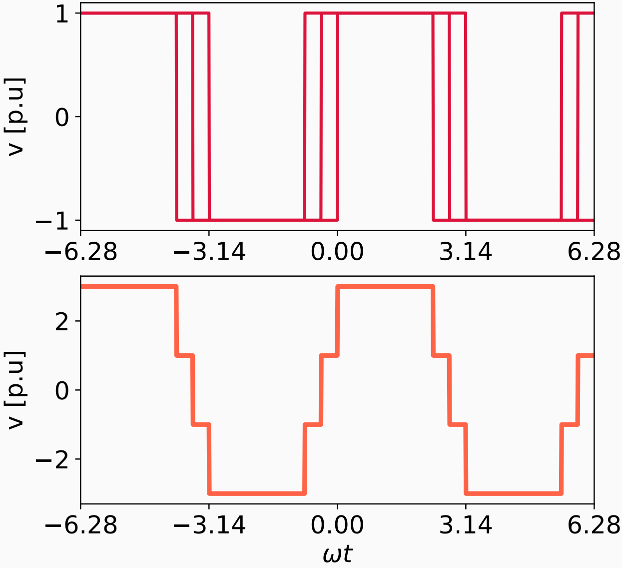



- Flux density is (square-wave shape)
- Flux-linkage changes linearly as
- rotor moves with constant velocity
- (see area under $B_m(x_s, t)$)
- $V_c = \frac{d\psi_c}{dt}$
- Induces a square wave voltage

# Brushless DC motor induced Voltage appears trapezoidal

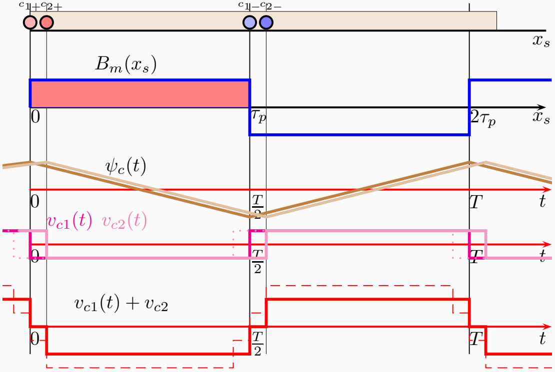


# Trapezoidal induced voltage due winding placement




- coil voltages seen phase displaced
- the phase depends on coil poisition
- Coils in series - adds voltage
- produces a stepped trapezoidal waveform

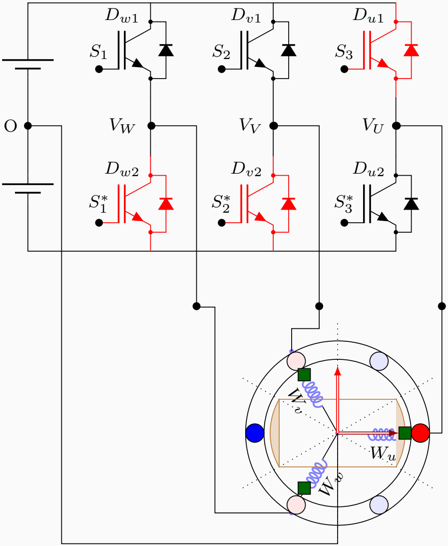
# Brushless DC motor induced Voltage appears trapezoidal

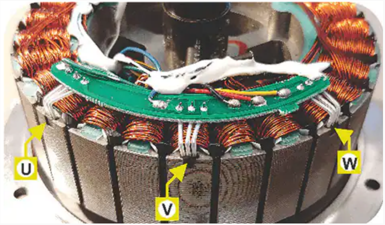



- Coils placed near by on stator
- Coil voltage phase depends on coil position
- Coils connected in series
- Total voltage adds-up
- produces a trapezoidal wave

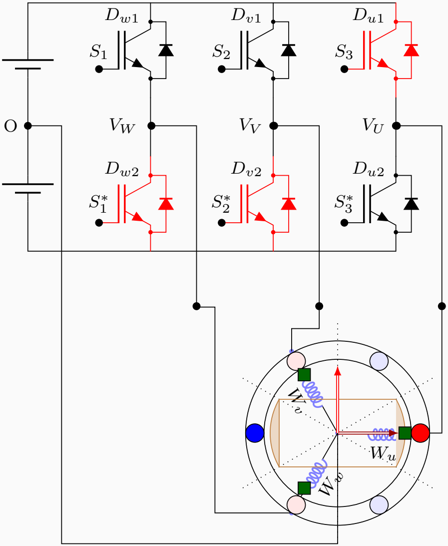
Induced voltage in coil is appear as back-emf. It will seen at stator windings **even when no electrical energy is supplied to stator**. It is caused due to permanent magnet excitation

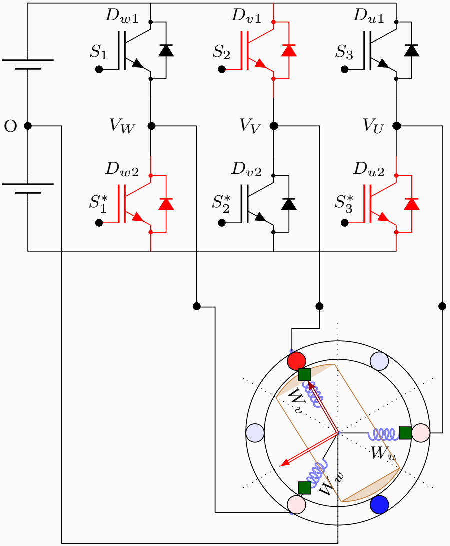
# Control of BLDC motor based on hall-effect sensor



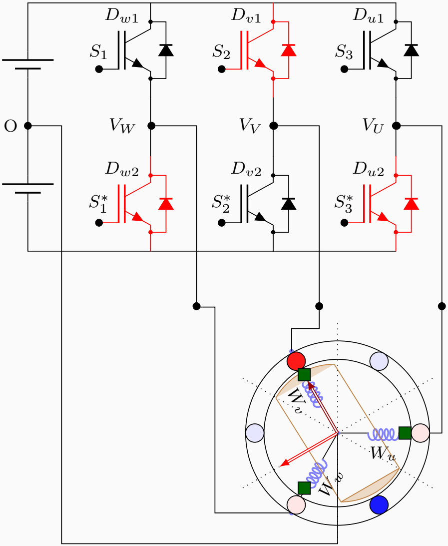



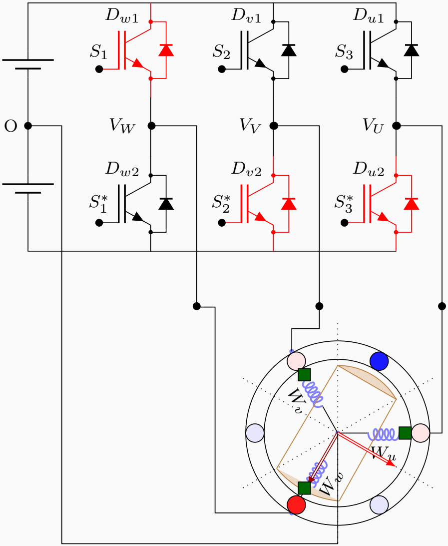
# Control of BLDC motor



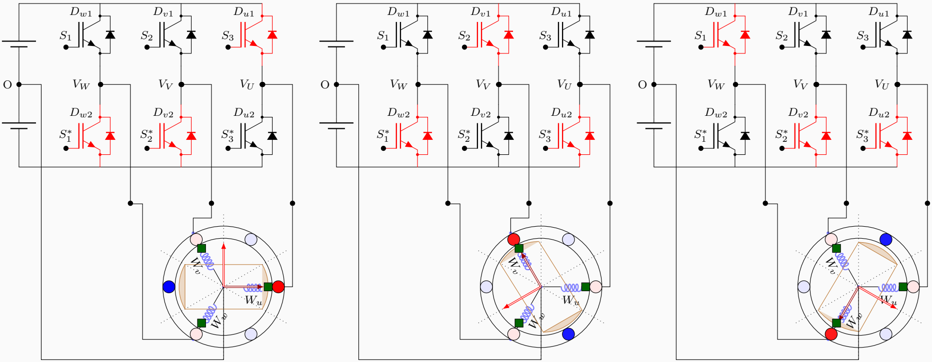



# Control of BLDC motor based



# Hall-effect sensors are place so that they generate the correct switching sequence



# Rotor Flux and stator Flux based on Hall-effect sensor switching i

Case 1: Hall effect output $(u, v, w) = (1, 0, 0)$ Since the $u$ sensor is place perpendicular to the $U$ phase axis, we can say that the rotor flux is perpendicular to $U$ phase axis. We have defined $\alpha - \beta$ axis along the $U$ Phase axis ($\alpha$ along U Phase). Hence we can write the rotor flux vector as

$$\vec{\psi}_u = 0 - j\psi_{u\beta}$$

Using the Hall=effect sensor data, a current space vector is injected into the stator of the machine along U axis for (1,0,0)

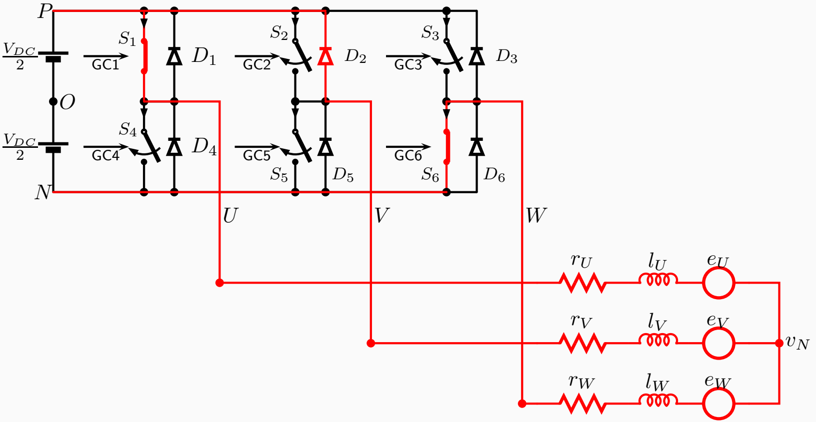
$$\begin{aligned}
\vec{i}_s &= i_{DC}\left(S_U + \vec{a}S_V + \vec{a}^2 S_W\right) \\
\vec{i}_s &= i_{DC} \\
\vec{i}_s &= i_{s\alpha} = i_{DC}
\end{aligned}$$

# Rotor Flux and stator Flux based on Hall-effect sensor switching ii

Thus the switching sequence based on the Hall-effect sensor is

| states | Hall sensor $u, v, w$ | Switching state $S_U, S_V, S_w$ |
|---|---|---|
| 1 | 1,0,0 | 1,0,0 |
| 2 | 0,1,0 | 0,1,0 |
| 3 | 0,0,1 | 0,0,1 |
| 0 | Any | 0,0,0 |

# Equivalent Circuit of BLDC operation




**Converter is necessary**

In order to operate the BLDC a 3-phase converter is necessary

# Circuit equations for the windings

Using phase variables

$$\begin{bmatrix} v_{U,N} \\ v_{V,N} \\ v_{W,N} \end{bmatrix} = \begin{bmatrix} r_U & 0 & 0 \\ 0 & r_V & 0 \\ 0 & 0 & r_W \end{bmatrix} \begin{bmatrix} i_U \\ i_V \\ i_W \end{bmatrix} + \begin{bmatrix} l_U & l_m & l_m \\ l_m & l_V & l_m \\ l_m & l_m & l_W \end{bmatrix} \begin{bmatrix} \frac{di_U}{dt} \\ \frac{di_V}{dt} \\ \frac{di_W}{dt} \end{bmatrix} + \begin{bmatrix} e_{U,N} \\ e_{V,N} \\ e_{W,N} \end{bmatrix}$$

Since

$$i_U + i_V + i_W = 0$$

and the motor is symmetrical in 3 windings, We get,

$$\begin{bmatrix} v_{U,N} \\ v_{V,N} \\ v_{W,N} \end{bmatrix} = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} i_U \\ i_V \\ i_W \end{bmatrix} + \begin{bmatrix} l - l_m & 0 & 0 \\ 0 & l - l_m & 0 \\ 0 & 0 & l - l_m \end{bmatrix} \begin{bmatrix} \frac{di_U}{dt} \\ \frac{di_V}{dt} \\ \frac{di_W}{dt} \end{bmatrix} + \begin{bmatrix} e_{U,N} \\ e_{V,N} \\ e_{W,N} \end{bmatrix}$$

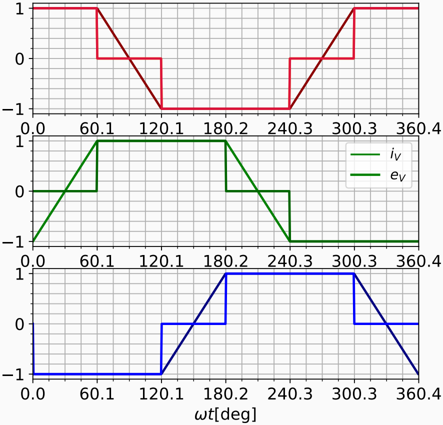
# In state space form

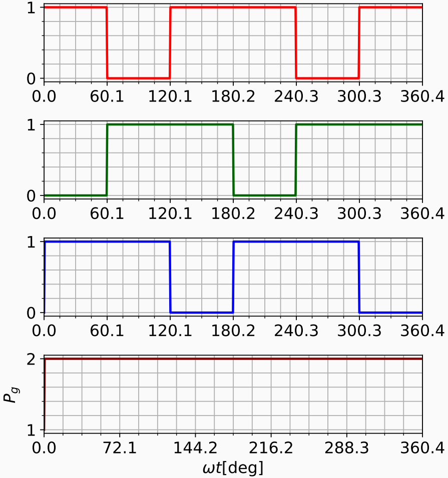
$$\frac{di_U}{dt} = -\frac{r}{l-l_m} i_U + \frac{v_{U,N} - e_{U,N}}{l-l_m}$$
$$\frac{di_V}{dt} = -\frac{r}{l-l_m} i_V + \frac{v_{V,N} - e_{V,N}}{l-l_m}$$
$$\frac{di_W}{dt} = -\frac{r}{l-l_m} i_W + \frac{v_{W,N} - e_{W,N}}{l-l_m}$$

using $l - l_m = l_s$, we can write

$$\frac{di_U}{dt} = -\frac{r}{l_s} i_U + \frac{v_{U,N} - e_{U,N}}{l_s}$$
$$\frac{di_V}{dt} = -\frac{r}{l_s} i_V + \frac{v_{V,N} - e_{V,N}}{l_s}$$
$$\frac{di_W}{dt} = -\frac{r}{l_s} i_W + \frac{v_{W,N} - e_{W,N}}{l_s}$$

# 120 deg. block operation ideal condition



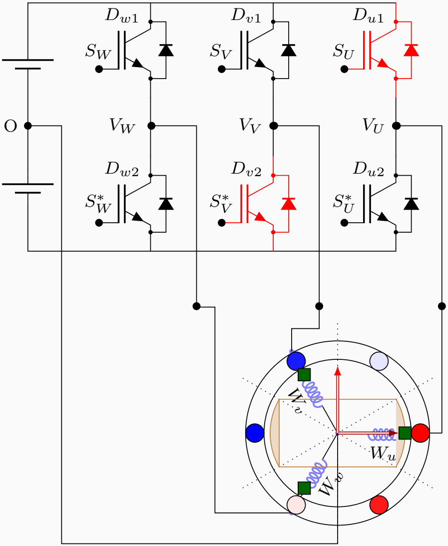



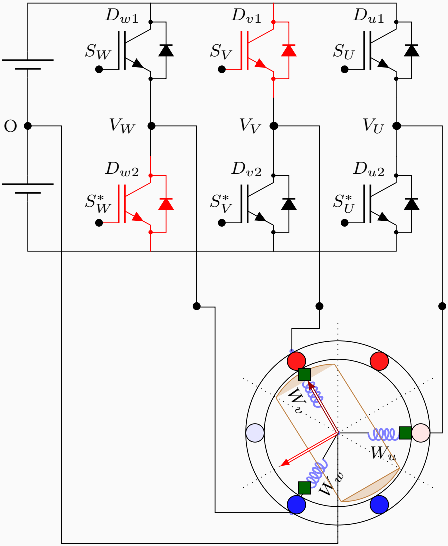

- In practice, it is not possible to get a sharp rise and fall in current ($di/dt$ has to be finite)

**At any given tie only 2 phases are conducting**

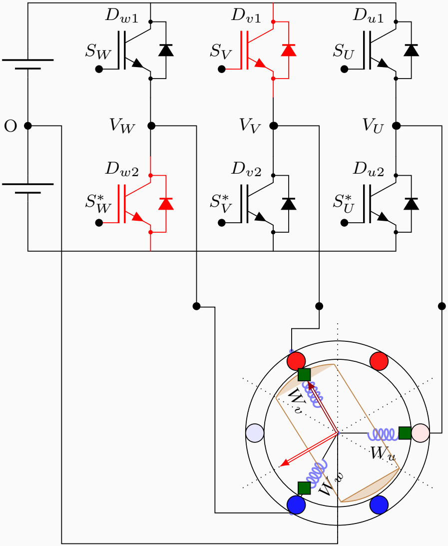
$$p_g = e_{U,N} i_U + e_{V,N} i_V + e_{W,N} i_w$$

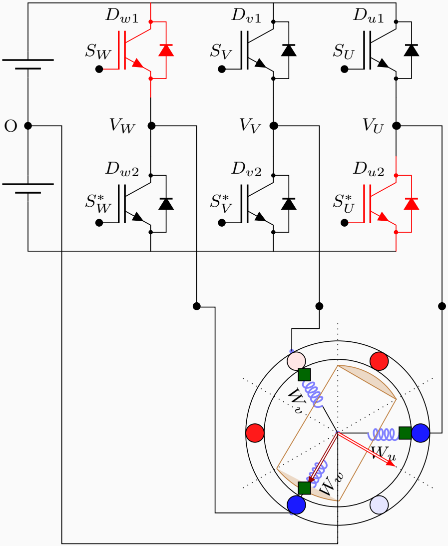
# Control of BLDC motor with 120 deg block operation



# Control of BLDC motor with 120 deg. block operation



# Power and Torque in the BLDC

The torque can be calculated from the air-gap power

$$P_g = e_U i_U + e_V i_V + e_W i_W$$

where $e_U, ..., i_U, ..,$ are the phase quantities and the electromagnetic torque is

$$m_e = \frac{P_g}{\omega}$$

$$m_e = \frac{e_U i_U + e_V i_V + e_W i_W}{\omega}$$

where $\omega$ is the angular velocity of the equivalent 2 pole machine.

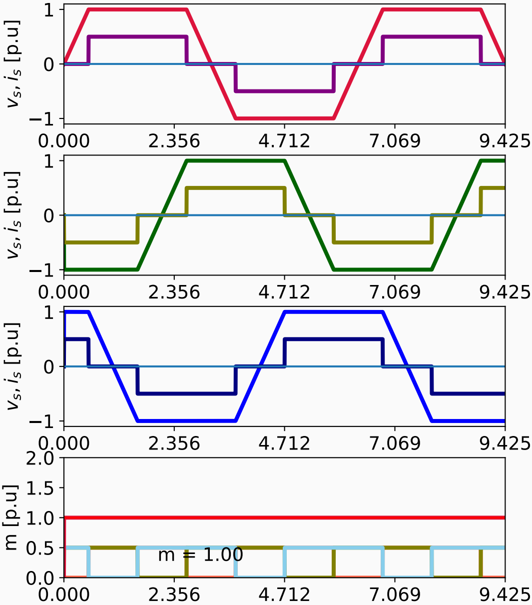
If the machine has $p$ poles than the angular velocity of the rotor will be

$$\omega_m = \frac{p}{2}\omega$$

Hence the shaft torque can be also written as

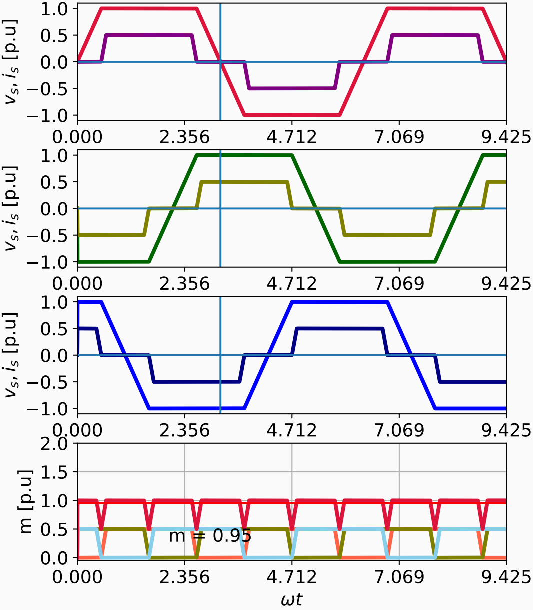
$$m_m = \frac{P_g - P_{w,loss}}{\omega_m}$$

# Feeding BLDC with square wave currents - see current and back-emf are constant



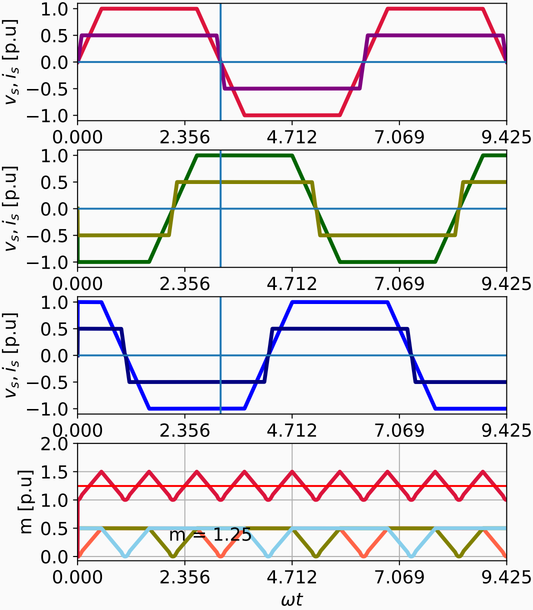

- constant current is fed for same duration as
- when induced voltage is constant
- Torque is produced
- $m_e = \frac{e_{U,N} i_U + e_{V,N} i_V + e_{W,N} i_W}{\omega}$

# Practical Block currents will cause torque ripple



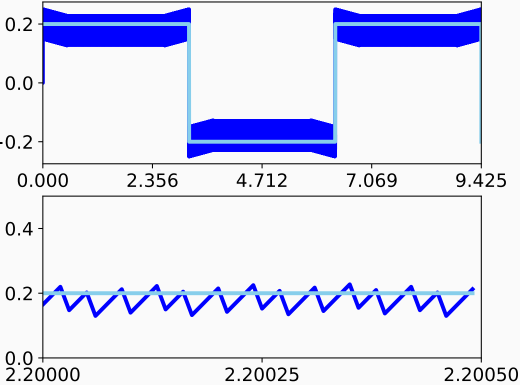

- actual current has finite rise-time and fall-time
- Effective current is constant for less than 120 degree
- Produces torque ripple
- Torque is produced
- $m_e = \frac{e_{U,N} i_U + e_{V,N} i_V + e_{W,N} i_W}{\omega}$

# Practical Block currents are active for more than 120 deg




- actual current has finite rise-time and fall-time
- Effective current is now constant for more than 120 degree
- reduces torque ripple
- Torque is produced
- $m_e = \frac{e_{U,N} i_U + e_{V,N} i_V + e_{W,N} i_W}{\omega}$

# We need current control to inject the square wave currents

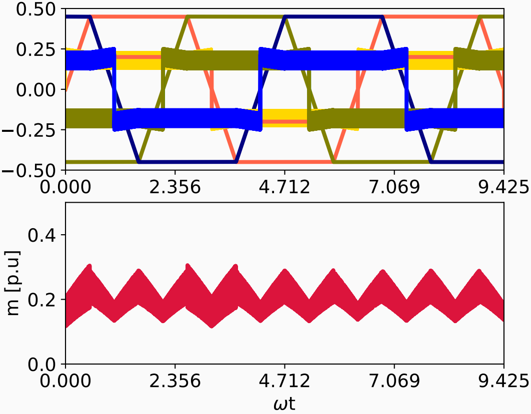



- Hysteresis control can be used
- square wave current reference
- compared with actual current

**input** : $ei[k] = i^*[k] - i[k]$
**output:** vs[k]

**if** $|ei[k]| < eilim$ **then**
    vs[k] = vs[k-1];
**else**
    **if** *(ei[i] >= eilim)* **then**
        vs[k] = -1.0;
    **end**
    **else if** *(ei[k] <= - eilim)* **then**
        vs[k] = 1.0
    **end**
**end**

# Torque with current control

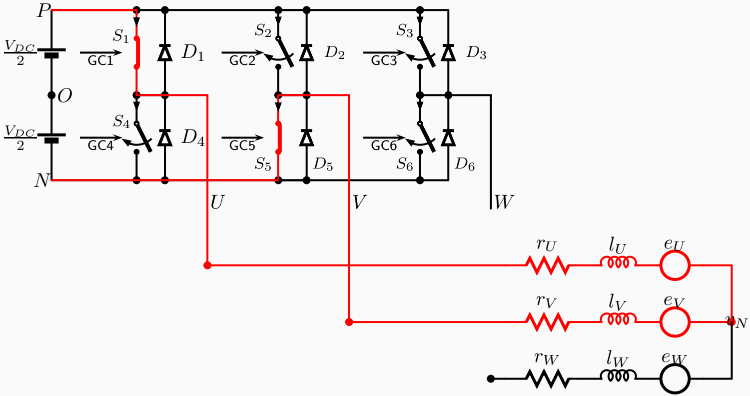



- Torque at $\omega_s = 0.5$
- torque ripple at 6 times fundamental frequency
- Not good for high performance servos

Torque ripple for high performance applications such as servos in CNC machines and Robotics has to be low

$$\frac{\Delta m}{m_N} = \frac{1}{100} = 0.01[p.u.]$$

# 120 deg. Conduction method with current commutation




- $e_U = -e_V$
- $i_U = -i_V = i$
- $\frac{di_U}{dt} = -\frac{di_V}{dt} = \frac{di}{dt}$

# Equivalent model for 120 deg conduction - $i = i_U - i_V$

Based on the circuit, we can write the following KVL for each phase

$$v_{U,O} = \frac{1}{2}V_{DC} = r_U i_U + l_U \frac{di_U}{dt} + e_{U,N} + v_{N,O} \tag{1}$$

$$v_{V,O} = \frac{1}{2}V_{DC} = r_V i_V + l_V \frac{di_V}{dt} + e_{V,N} + v_{N,O} \tag{2}$$

$$v_{W,O} = e_{U,N} + v_{N,O} \tag{3}$$

$$v_{N,O} = \frac{1}{2}(-e_{U,N} - e_{V,N}) \tag{4}$$

We can write it as

$$v_{U,N} = \frac{1}{2}V_{DC} - v_{N,O} = r_U i_U + l_U \frac{di_U}{dt} + e_{U,N} \tag{5}$$

$$v_{V,O} = \frac{1}{2}V_{DC} - v_{N,O} = r_V i_V + l_V \frac{di_V}{dt} + e_{V,n} \tag{6}$$

$$v_{W,N} = e_{W,N} \tag{7}$$

As $i_U + i_V + i_W = 0$, we get

$$i = i_U = -i_V$$

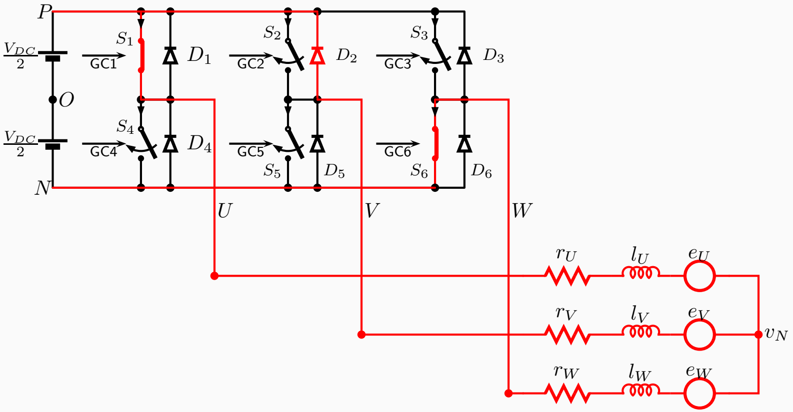
and $e_U = -e_V = e$ and Torque will be

$$m = \frac{e_U i_U + e_V i_V}{\omega} = \frac{2ei}{\omega}$$

# Phase V stops conduction and Phase W to start

$S_6$ is turned on and $S_5$ is turned off, however due to inductance, $i_V$ continues to flow in same direction. Hence, diode $D_2$ turns on. Now $i_V$ will start to decrease and $i_W$ will start to increase. so at node N, we get

$$i_U = -(i_V + i_W)$$



# Commutation interval between $i_V$ to $i_W$

We can write the KVL for the phases as

$$v_{U,O} = \frac{1}{2}V_{DC} = r_U i_U + l_U \frac{di_U}{dt} + e_{U,N} + v_{N,O} \tag{8}$$

$$v_{V,O} = \frac{1}{2}V_{DC} = r_V i_V + l_V \frac{di_V}{dt} + e_{V,N} + v_{N,O} \tag{9}$$

$$v_{W,O} = -\frac{1}{2}V_{DC} = r_W i_W + l_V \frac{di_W}{dt} + e_{W,N} + v_{N,O} \tag{10}$$

$$v_{N,O} = \frac{1}{3}(\frac{V_{dc}}{2} - (e_U + e_V + e_W)) \tag{11}$$

and can be written as

$$v_{U,N} = \frac{1}{2}V_{DC} - v_{N,O} = r_U i_U + l_U \frac{di_U}{dt} + e_{U,N} \tag{12}$$

$$v_{V,O} = \frac{1}{2}V_{DC} - v_{N,O} = r_V i_V + l_V \frac{di_V}{dt} + e_{V,N} \tag{13}$$

$$v_{W,O} = -\frac{1}{2}V_{DC} - v_{N,O} = r_W i_W + l_V \frac{di_W}{dt} + e_{W,N} \tag{14}$$

$$v_{N,O} = \frac{1}{3}(\frac{V_{dc}}{2} - (e_U + e_V + e_W)) \tag{15}$$

# Commutation ripple i

we had $i_V < 0$ and due to the voltage in phase V, we get

$$\frac{di_V}{dt} > 0$$

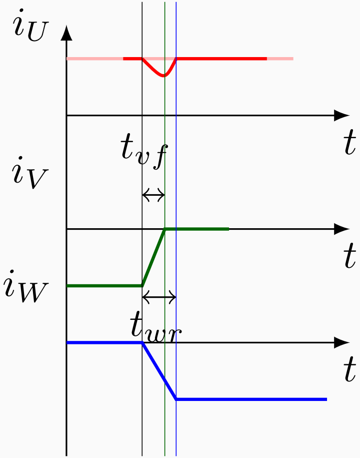
Hence, $i_v \to 0$. Similarly, $i_W = 0$ at the beginning of the commutation and now due to the voltage in phase W, we get

$$\frac{di_W}{dt} < 0$$

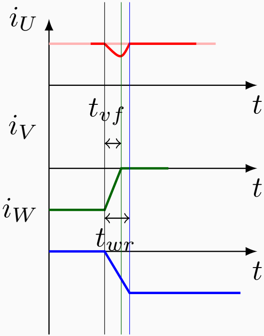
, hence $i_W \to -i$. So that

$$i_U = -(i_V + i_W) = i$$

# Commutation ripple ii



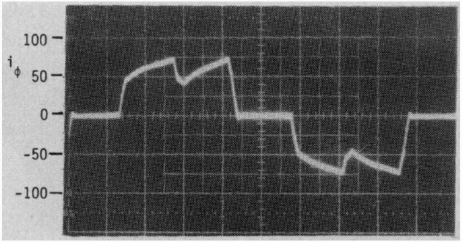
# Commutation ripple iii




If time duration for $i_V$ to reach zero is $t_1$ and the time duration for $i_w$ to reach $-i$ is $t_2$, we can have 3 conditions

1. $t_{wr} > t_{vf}$: $i_U$ decreases till $t_{vf}$ and then increase between $t_{vf} - t_{wr}$
2. $t_{wr} = t_{vf}$: $i_U$ us constant over $t_{wr} = t_{vf}$
3. $t_{wr} < t_{vf}$: $i_U$ increase until $t_{wr}$ and decreases between $t_{vf} - t_{wr}$

# Current ripple in $i_U$ durin commutation between phase V and W




major disadvantage in this machine cause due to commutation ripple. At high speeds the current does not dip, but creates a positive bump

# Comparing PMSM (sinusoidal) with BLDC 120 degree i

The peak value of PMSM phase current is $\hat{I}_{SM}$ and the peak value of BLDC (120 degrees block DC current is) $\hat{I}_{BLDC}$. We can get the RMS value as for sinewave as

$$I_{SM} = \frac{\hat{I}_{SM}}{\sqrt{2}}$$

and for a 120 degree conducting quasi-square which has a conduction ratio of $120/180 = 2/3$

$$I_{BLDC} = \sqrt{\frac{2}{3}} \hat{I}_{BLDC}$$

Assuming the losses in both machines are equal, we can write the loss using RMS values of currents

$$\underbrace{3 \cdot I_{SM}^2 R_s}_{\text{PMSM loss}} = \underbrace{3 \cdot I_{BLDC}^2 R_s}_{\text{BLDC loss}}$$

# Comparing PMSM (sinusoidal) with BLDC 120 degree ii

Hence the relation between the two peak current for same losses is

$$\hat{I}_{BLDC} = \frac{\sqrt{3}}{2}\hat{I}_{SM}$$

We can calulate the power ratio for the two machines as

$$\begin{aligned}
\frac{\text{Power output BLDC}}{\text{Power output PMSM}} &= \frac{2\hat{E}_U\hat{I}_{BLDC}}{3\frac{\hat{E}_U}{\sqrt{2}}\frac{\hat{I}_{SM}}{\sqrt{2}}} \\
&= \frac{4\hat{I}_{BLDC}}{3\hat{I}_{SM}} \\
&= \frac{4\frac{\sqrt{3}}{2}\hat{I}_{SM}}{3\hat{I}_{SM}} \\
&= \frac{2}{\sqrt{3}} = 1.155
\end{aligned}$$

# Comparing PMSM (sinusoidal) with BLDC 120 degree iii

**BLDC has power density**

For the same losses, the BLDC power output is 15% higher than that of PMSM. This is because the RMS value to peak value in BLDC is higher than sinusoidal PSMSM

# Why we need the PM AC synchronous motor

## Why use PMAC

- BLDC motor produces torque ripple due to current blocks and commutation
- Such torque ripple is not good for servo applications where a smooth torque is needed
- A sinusoidal flux-density based PM machine is an AC machine
- Produces rotating magnetic field and is fed with sinusoidal currents
- Torque ripple is minimized in PMAC
- However, as BLDC uses block currents, it can produce more torque for the same RMS current as the AC machine

# Control of BLDC

# Dynamic model of BLDC i

Due to 120 deg. conduction, at any given time 2 phases will be conducting. Suppose we have $i_u = -i_v$ Then we can describe the current dynamics as

$$V_{UV} = 2r_U i + 2l_U \frac{di}{dt} + 2e_U$$
$$V_{DC} = r_s i + l_s \frac{di}{dt} + k_e \omega$$

where $V_{DC}$ is the DC link voltage, $k_e$ back-emf coefficient. The torque produced will be given by

$$m_e = k_t i \tag{16}$$

and the speed dynamics will be given as

$$T_j \frac{d\omega}{dt} = k_t i - m_L$$
$$sT_j \omega(s) = k_t i(s) - m_L(s)$$

# Dynamic model of BLDC ii

and the current transfer function is

$$
\begin{aligned}
(1 + sT_s)i(s) &= \frac{v(s)}{r_s} - \frac{k_e\omega(s)}{r_s} \\
i(s) &= \frac{v(s)}{r_s(1 + sT_s)} - \frac{k_e}{r_s(1 + sT_s)}\omega(s) \\
i(s) &= \frac{v(s)}{r_s(1 + sT_s)} - \frac{k_e}{r_s(1 + sT_s)}\left(\frac{k_t}{sTj}i(s) - \frac{1}{sT_j}m_L(s)\right)
\end{aligned}
$$

**Homework Assigment**

- Derive the transfer function for $\frac{i(s)}{v(s)}$, assume $k_e = k_t = 1$
- Derive the transfer function for $\frac{\omega(s)}{v(s)}$
- Compare it with the Separately excited DC machine and comment on it