Control of Permanent Magnet Synchronous Motors

BLDC PM, PMSM

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Control of PM SM

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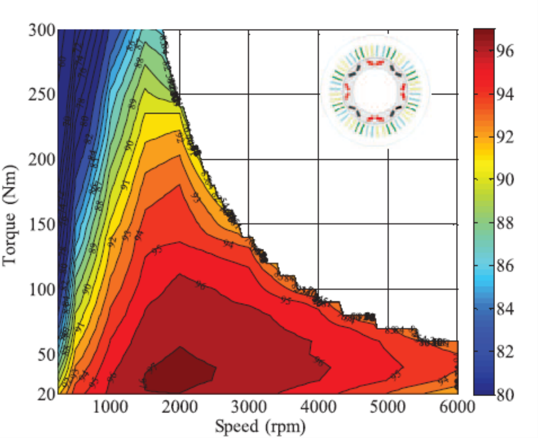
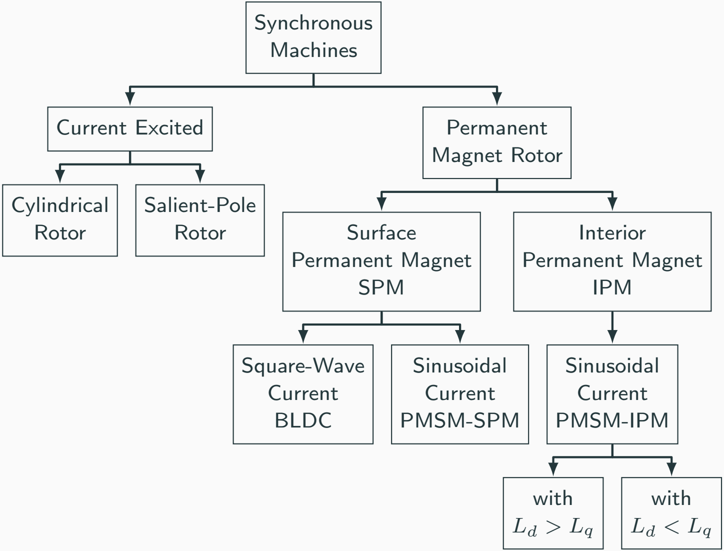
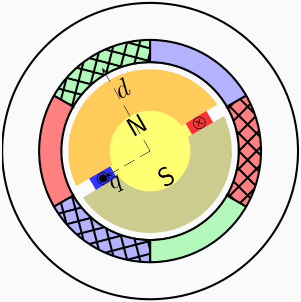


Taxonomy of Synchronous Machines



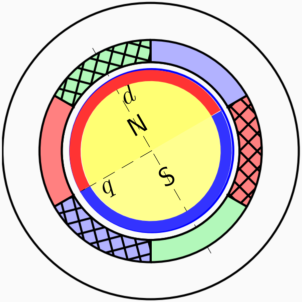
Reluctance in PMSM rotors

<u>Salient-Pole Rotor</u>



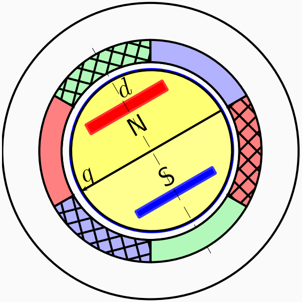
$L_d > L_q$

<u>Surface mounted PMSM</u>



$L_d = L_q$

<u>Interior mounted PMSM</u>



$L_d < L_q$

Torque in PMSM

We can write the stator voltage equation as

$$\vec{v}_s = r_s \vec{i}_s + \frac{d\vec{\psi}_s}{d\tau}$$

Splitting in d-q domain

$$\psi_{sd} = l_d i_{sd} + \psi_{r,m}$$

and

$$\psi_{sq} = l_q i_{sq}$$

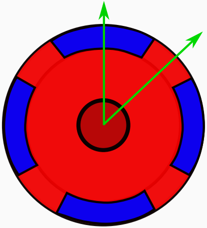
Normalized torque in p.u is given as

$$m_e = \vec{\psi}_s \times \vec{i}_s = (\psi_{sd} + j\psi_{sq}) \times (i_{sd} + j i_{sq}) = \psi_{sd} i_{sq} - \psi_{sq} i_{sd}$$

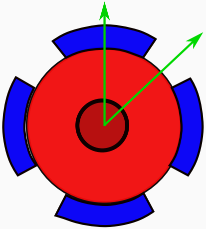
which can be expanded as

$$m_e = \underbrace{\psi_{r,m} i_{sq}}_{\text{synchronous torque}} + \underbrace{(l_d - l_q) i_{sd} i_{sq}}_{\text{reluctance torque}}$$

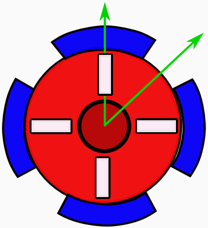
Interior Permanent Magnet Synchronous Motor



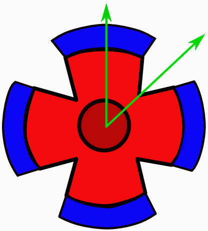
$L_d < L_q$



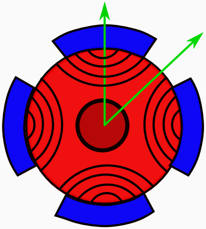
$L_d = L_q$



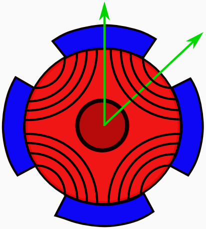
$L_d > L_q$



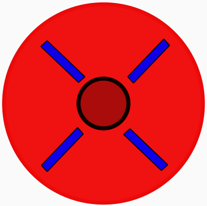
$L_d > L_q$



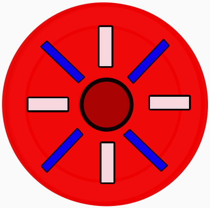
$L_d < L_q$



$L_d > L_q$

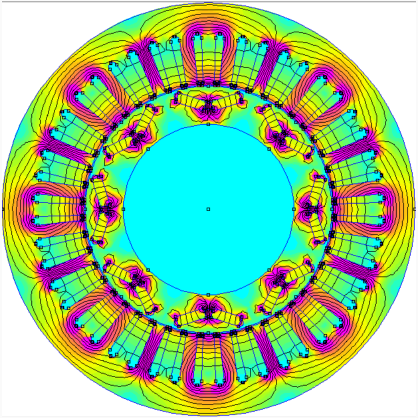


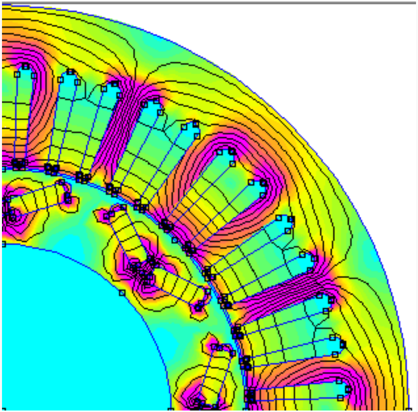
$L_d < L_q$



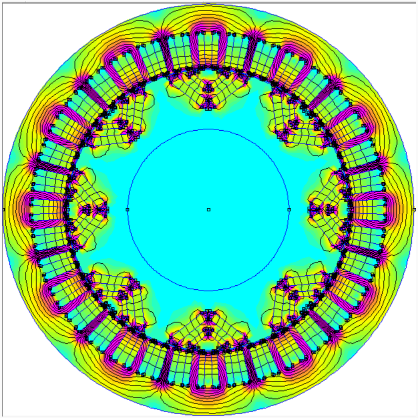
$L_d > L_q$

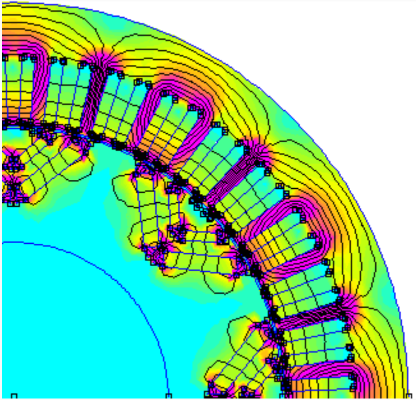
Toyota Prius IPM-SM design





Chevy Bolt IPM-SM design





Dynamics of PMS motors

Dynamic state equations of the PMSM i

The rotor does not have any electromagnetic dynamics, as the flux is produced using permanent magnets. Hence we will have only the stator voltage equation to described the dynamics In d-q frame due to saliency the flux-linkages can be written as

$$\psi_{sd} + j\psi_{sq} = (l_d i_{sd} + \psi_{r,m}) + j l_q i_{sq} \tag{1}$$

$$\psi_{sd} = l_d i_{sd} + \psi_{r,m} \tag{2}$$

$$\psi_{sq} = l_q i_{sq} \tag{3}$$

$$\tag{4}$$

In stator coordinates, stator voltage is

$$\vec{v}_s^{(S)} = r_s \vec{i}_s^{(S)} + \frac{d\vec{\psi}_s^{(S)}}{d\tau} \tag{5}$$

Converting into d-q coordinate frame using $\vec{v}_s^S = \vec{v}_s^{(F)} e^{j\omega_s \tau}$ we get

Dynamic state equations of the PMSM ii

Dynamics of PMSM in rotor field coordinates

$$\vec{v}_s^{(F)} = r_s \vec{i}_s^{(F)} + j\omega_s \psi_s^{(F)} + \frac{d\vec{\psi}_s^{(F)}}{d\tau} \tag{6}$$

$$v_{sd} = r_s i_{sd} - \omega_s \psi_{sq} + \frac{d\psi_{sd}}{d\tau} \tag{7}$$

$$v_{sq} = r_s i_{sq} + \omega_s \psi_{sd} + \frac{d\psi_{sq}}{d\tau} \tag{8}$$

$$v_{sd} = r_s i_{sd} - \omega_s l_q i_{sq} + l_d \frac{di_{sd}}{d\tau} \tag{9}$$

$$v_{sq} = r_s i_{sq} + \omega_s l_d i_{sd} + \omega_s \psi_{r,m} + l_q \frac{di_{sq}}{d\tau} \tag{10}$$

as $\frac{d\psi_{r,m}}{d\tau} = 0$

Dynamic differential equations for PMSM

$$\frac{di_{sd}}{d\tau} = -\frac{r_s}{l_d} i_{sd} + \omega_s \frac{l_q}{l_d} i_{sq} + \frac{v_{sd}}{l_d} \tag{11}$$

$$\frac{di_{sq}}{d\tau} = -\frac{r_s}{l_q} i_{sq} - \omega_s \frac{l_d}{l_q} i_{sd} - \omega_s \frac{\psi_{r,m}}{l_q} + \frac{v_{sq}}{l_q} \tag{12}$$

$$m_e = \vec{\psi}_s \times \vec{i}_s \tag{13}$$

$$= \mathfrak{Im}\left[\vec{\psi}_s^* \vec{i}_s\right] \tag{14}$$

$$= \mathfrak{Im}\left[(\psi_{sd} - j\psi_{sq})(i_{sd} + j i_{sq})\right] \tag{15}$$

$$= \psi_{sd} i_{sq} - \psi_{sq} i_{sd} \tag{16}$$

$$= (\psi_{r,m} + l_d i_{sd}) i_{sq} - l_q i_{sq} i_{sd} \tag{17}$$

$$= \psi_{r,m} i_{sq} + (l_d - l_q) i_{sd} i_{sq} \tag{18}$$

$$\frac{d\omega_s}{d\tau} = \frac{m_e - m_L}{\tau_m} \tag{19}$$

Dynamic differential equations for Symmetrical PMSM

Since $l_d = l_q = l_s$, the stator flux linkage can be written as

$$\psi_{sd} = l_s i_{sd} + \psi_{r,m} \tag{20}$$

$$\psi_{sq} = l_s i_{sq} \tag{21}$$

$$\frac{di_{sd}}{d\tau} = -\frac{r_s}{l_s} i_{sd} + \omega_s i_{sq} + \frac{v_{sd}}{l_s} \tag{22}$$

$$\frac{di_{sq}}{d\tau} = -\frac{r_s}{l_s} i_{sq} - \omega_s i_{sd} - \omega_s \frac{\psi_{r,m}}{l_s} + \frac{v_{sq}}{l_s} \tag{23}$$

$$m_e = \psi_s \times \vec{i}_s \tag{24}$$

$$= \psi_{sd} i_{sq} - \psi_{sq} i_{sd} = \psi_{r,m} i_{sq} + l_s i_{sd} i_{sq} - l_s i_{sq} i_{sd} \tag{25}$$

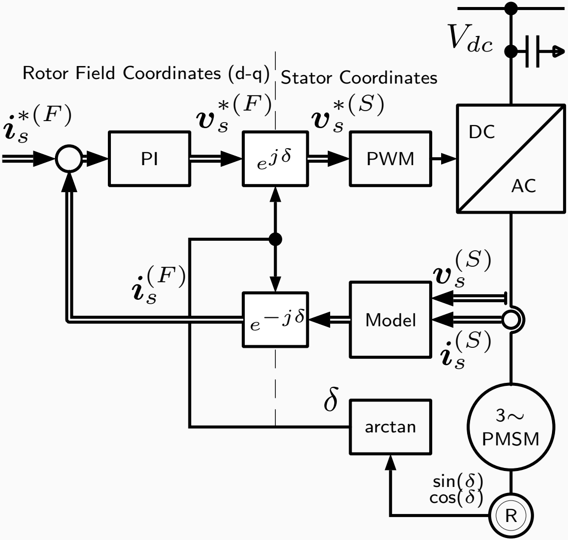
$$= \psi_{r,m} i_{sq} \tag{26}$$

$$\frac{d\omega_s}{d\tau} = \frac{m_e - m_L}{\tau_m} \tag{27}$$

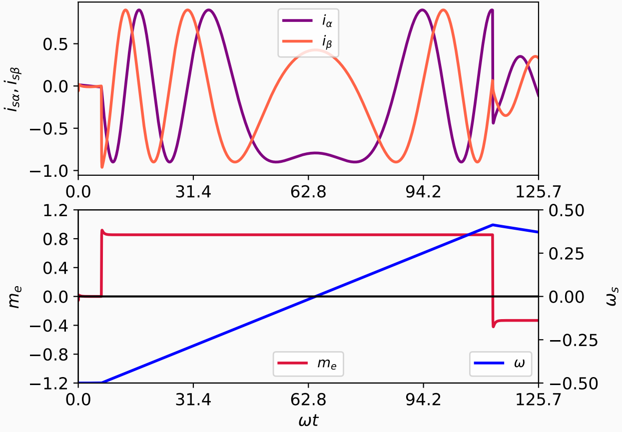
In Symmetrical PMSM rotor, we can make $i_{sd} = 0$ and produce full torque. Hence the ohmic losses will be lower

Current control in d-q coordinates

- rotor position obtained using resolver
- voltage and current measured
- Converted to d-q coordinates
- current control in d-q coordinates
- voltage reference reconverted to stator coordinates

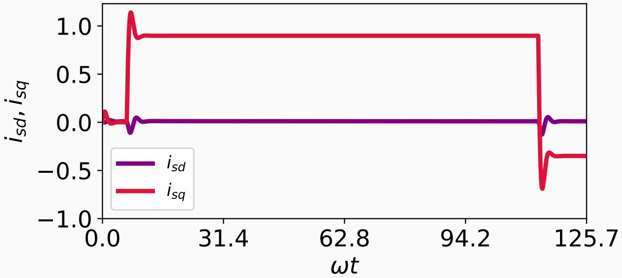


Current Control of PMSM with symmetrical rotor ($l_d = l_q$)



1. Initial State
   $i_{sd} = i^*_{sd} = 0$
2. Step change
   $i^*_{sq} = i_{sq} = 0.8$
3. Set change
   $i^*_{sq} = i_{sq} = -0.35$
4. Motor reversal can be seen
5. No initial current dynamics seen
6. Motor currents are controlled

Current Control of PMSM with symmetrical rotor ($l_d = l_q$)



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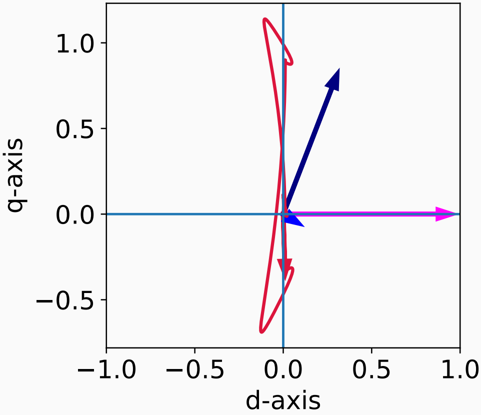


❶ Initial State $i_{sd} = i^*_{sd} = 0$

❷ Step change $i^*_{sq} = i_{sq} = 0.8$

❸ Set change $i^*_{sq} = i_{sq} = -0.35$

❹ Motor reversal can be seen

❺ No initial current dynamics seen

❻ Motor currents are controlled

❼ d-q plane currents

❽ d axis control parameters not

❾ same as q-axis control parameters

Dynamic differential equations for PMSM

For $l_d \neq l_q$ and $l_d < l_q$, $i_{sd} \cdot i_{sq} < 0$ will produce a positive reluctance torque

$$\frac{di_{sd}}{d\tau} = -\frac{r_s}{l_d} i_{sd} + \omega_s \frac{l_q}{l_d} i_{sq} + \frac{v_{sd}}{l_d} \tag{28}$$

$$\frac{di_{sq}}{d\tau} = -\frac{r_s}{l_q} i_{sq} - \omega_s \frac{l_d}{l_q} i_{sd} - \omega_s \frac{\psi_{r,m}}{l_q} + \frac{v_{sq}}{l_q} \tag{29}$$

$$m_e = \vec{\psi}_s \times \vec{i}_s \tag{30}$$

$$= \psi_{sd} i_{sq} - \psi_{sq} i_{sd} \tag{31}$$

$$= \psi_{r,m} i_{sq} + (l_d - l_q) i_{sd} i_{sq} \tag{32}$$

$$\frac{d\omega_s}{d\tau} = \frac{m_e - m_L}{\tau_m} \tag{33}$$

Task for you

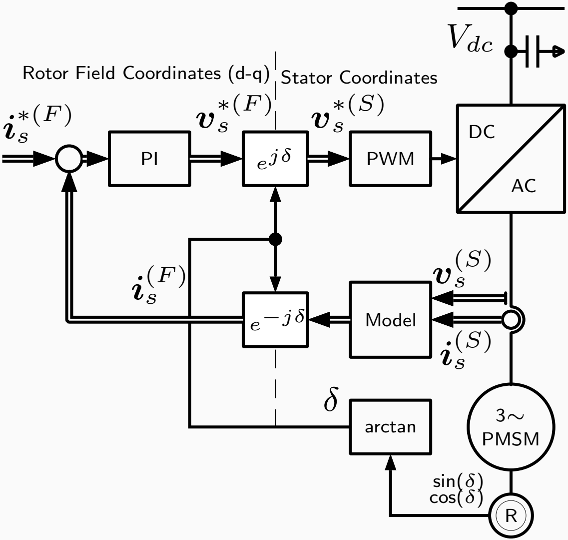
Sketch the signal flow diagram

Using the principles used in Induction machine chapter, sketch the signal flow diagram the describes the dynamics of a permanent magnet synchronous motor. Do this for cases

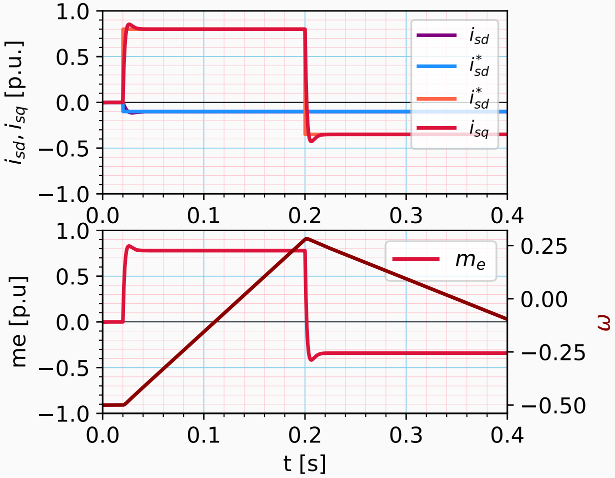
- Synchronous rotor
- IPMSM where $l_d \neq l_q$

Current control in d-q coordinates

- rotor position obtained using resolver
- voltage and current measured
- Converted to d-q coordinates
- current control in d-q coordinates
- voltage reference reconverted to stator coordinates

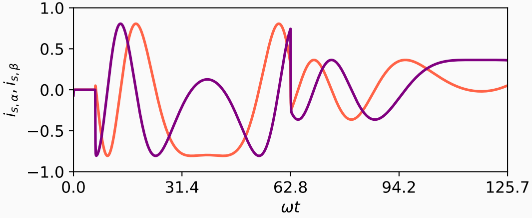


Current Control of PMSM with asymmetrical rotor ($l_d < l_q$)



1. Initial State
   $i_{sd} = i^*_{sd} = 0$
2. Step change
   $i^*_{sq} = i_{sq} = 0.7,$
   $i_{sd} = -0.1$
3. Set change
   $i^*_{sq} = i_{sq} = -0.35,$
   $i_{sd} = 0.1$
4. Motor reversal can be seen
5. No initial current dynamics seen
6. Motor currents are controlled

Current Control of PMSM with asymmetrical rotor ($l_d < l_q$)



1. Initial State $i_{sd} = i^*_{sd} = 0$
2. Step change $i^*_{sq} = i_{sq} = 0.7$, $i_{sd} = -0.1$
3. Set change $i^*_{sq} = i_{sq} = -0.35$, $i_{sd} = 0.1$
4. Motor reversal can be seen
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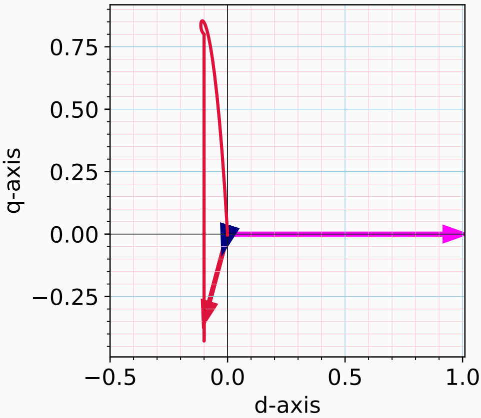
Current Control of PMSM with asymmetrical rotor ($l_d < l_q$)



❶ Initial State $i_{sd} = i^*_{sd} = 0$

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❸ Set change $i^*_{sq} = i_{sq} = -0.35$, $i_{sd} = 0.1$

❹ Motor reversal can be seen

❺ No initial current dynamics seen

❻ Motor currents are controlled

❼ d-q plane currents

Maximum Torque Per Ampere (MTPA)

Torque in IMP=PMSM

Due to $ld \neq lq$, torque in PMSM has 2 components

$$m_e = \underbrace{\psi_{r,m} i_{sq}}_{\text{Synchronous torque}} + \underbrace{(l_d - l_q) i_{sd} i_{sq}}_{\text{reluctance torque}}$$

Is maximum torque produced when $i_{sd} = 0$? The goal is to find what is the maximum torque produced for the minimum stator current. The stator current is given by

$$i_{sm} = \sqrt{i_{sd}^2 + i_{sq}^2}$$

Maximum Torque per Ampere (MTPA)

Since we are able to inject both i_{sd} and i_{sq}, and use the reluctance torque. We would like to produce the maximum torque for the given inverter current. The total inverter current can be expressed as

$$i_{sm}^2 = i_{sd}^2 + i_{sq}^2$$

which can be represented as family of circles in the $i_{sd} - i_{sq}$ plane. if we write

$$i_{sd} = i_{sm} cos\beta$$
$$i_{sq} = i_{sm} sin\beta$$

Then the maximum torque per ampere for a given inverter current can be found as

$$\begin{aligned} m_e &= \left[\psi_m i_{sm} sin\beta + \frac{1}{2}(l_d - l_q) i_{sm}^2 sin2\beta\right] \\ \frac{dm_e}{d\beta} &= \left[\psi_m i_{sm} cos\beta + (l_d - l_q) i_{sm}^2 cos2\beta\right] = 0 \end{aligned}$$

Maximum torque per Ampere

Solving for maximum torque for a given i_{sm}, we get, a quadratic equation

$$\psi_m i_{sd} + (l_d - l_q) i_{sm}^2 (2cos^2\beta - 1) = 0 \tag{34}$$

$$2(l_d - l_q) i_{sd}^2 + \psi_m i_{sd} - (l_d - l_q) i_{sm}^2 = 0 \tag{35}$$

$$\underbrace{1}_{a} i_{sd}^2 + \underbrace{\frac{\psi_m}{2(l_d - l_q)}}_{b} i_{sd} - \underbrace{\frac{1}{2} i_{sm}^2}_{-c} = 0 \tag{36}$$

$$ax^2 + bx + c = 0 \tag{37}$$

i_{sd} minimum for given i_{sm}

$$i_{sdm} = -\frac{\psi_m}{4(l_d - l_q)} \pm \sqrt{\frac{\psi_m^2}{16(l_d - l_q)^2} + \frac{1}{2} i_{sm}^2} \tag{38}$$

MTPA with i_{sq} assumption

Since

$$i_{sm}^2 = i_{sd}^2 + i_{sq}^2$$

we can substitute and get

$$(l_d - l_q)i_{sd}^2 + \psi_m i_{sd} - (l_d - l_q)i_{sq}^2 = 0 \tag{39}$$

$$i_{sd}^2 + \frac{\psi_m}{(l_d - l_q))} i_{sd} - i_{sq}^2 = 0 \tag{40}$$

We get 2 roots of the quadratic equation

$$i_{sdm} = -\frac{\psi_m}{2(l_d - l_q)} + \sqrt{\frac{\psi_m^2}{4(l_d - l_q)^2} + i_{sq}^2}$$

$$i_{sdm} = -\frac{\psi_m}{2(l_d - l_q)} - \sqrt{\frac{\psi_m^2}{4(l_d - l_q)^2} + i_{sq}^2}$$

Since the square root term magnitude is greater that the firstb term and $i_{sd} < 0$, the minimum current is given by

$$\boxed{i_{sdm} = -\frac{\psi_m}{2(l_d - l_q)} - \sqrt{\frac{\psi_m^2}{4(l_d - l_q)^2} + i_{sq}^2}} \tag{41}$$

Maximum torque per Ampere Characteristics i

Since,

$$i_{sm}^2 = i_{sdm}^2 + i_{sq}^2 \tag{42}$$

and the peak torque is given by

$$\boxed{m_{em} = \psi_m i_{sq} + (l_d - l_q) i_{sdm} i_{sq}}$$

If i_{sm} is given then

$$i_{sq} = \sqrt{i_{sm}^2 - i_{sdm}^2}$$

$$\boxed{m_{em} = \psi_m (i_{sm}^2 - i_{sdm}^2)^{\frac{1}{2}} + (l_d - l_q) i_{sdm} (i_{sm}^2 - i_{sdm}^2)^{\frac{1}{2}}}$$

Pointer for first approximation estimate of MTPA

- if $l_d < l_q$ and $\frac{l_q}{l_d}$ close to 1
- We can estimate $i_{sq} = i_{sm}$ from m_{em} neglecting reluctance torque
- Using i_{sm} and parameters find i_{sdm}

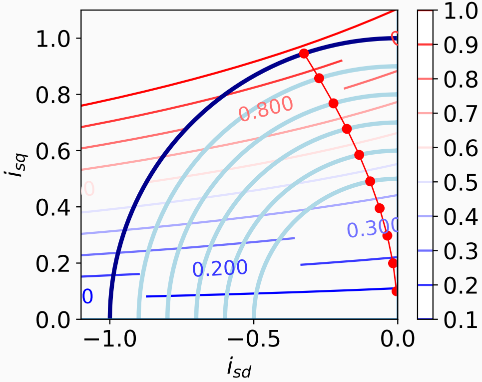
Maximum torque per Ampere Characteristics ii

- Using i_{sdm}, i_{sq} recalculate torque
- Change i_{sq} to get the correct torque

Important points to note

- In a normalized machine the rated current will be 1 [p.u.]
- We may want to operate at any of the current locus for maximum operating current < 1 [p.u]
- Maximum operating current is the current required to operate at a given point it is not the maximum possible current
- If the required torque cannot be generated at maximum operating current, we will have to increase the maximum operating current. As long as it is $<$ 1[p.u]

Constant Current Locus of PMSM



Locus describes a family of circle.

$$i_{sm}^2 = i_{sdm}^2 + i_{sq}^2$$

As the current limit increases, the radius of the circle also increases

In Field Weakening Voltage is limited, Maximum Torque per Voltage used

We can write the total stator flux as

$$\psi_{sd} = l_d i_{sd} + \psi_m \tag{43}$$

$$\psi_{sq} = l_q i_{sq} \tag{44}$$

$$\psi_s^2 = (l_d i_{sd} + \psi_m)^2 + (l_q i_{sq})^2 \tag{45}$$

$$\left(\frac{v_s}{\omega_s}\right)^2 = (l_d i_{sd} + \psi_m)^2 + (l_q i_{sq})^2 \tag{46}$$

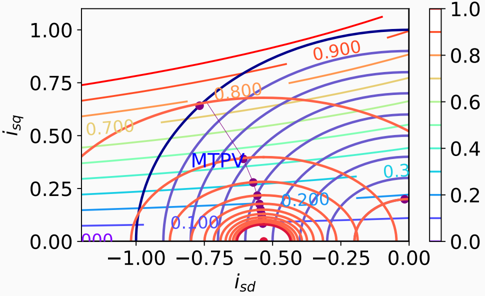
(47)

Solving for i_{sd}, we get

$$i_{sd} = -\frac{\psi_m}{l_d} \pm \frac{1}{l_d}\sqrt{\left(\frac{v_s}{\omega_s}\right)^2 - (l_q i_{sq})^2}$$

The actual minimum derivation is more complex. We stick to the simple explanation

Constant voltage locus with increasing speed

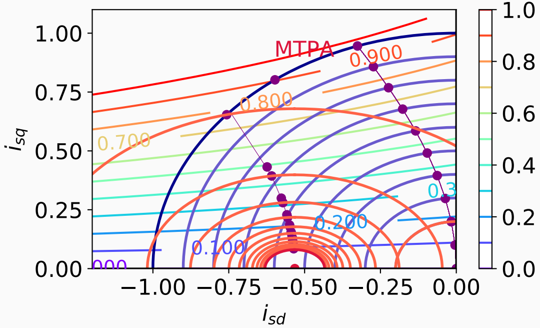


$$\left(\frac{v_s}{\omega_s}\right)^2 = (l_d i_{sd} + \psi_m)^2 + (l_q i_{sq})^2$$

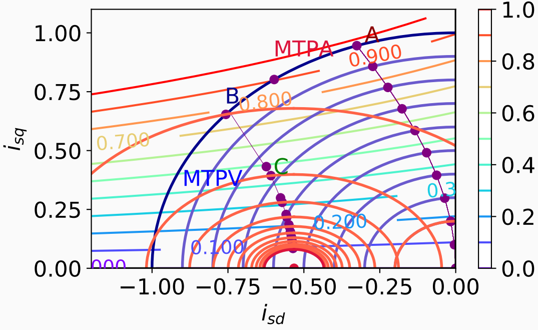
describes an ellipse in $i_{sd} - i_{sq}$ plane
As $\omega_s > 1.0$ increases the ellipse becomes smaller.
When $i_{sq} = 0$ and voltage reserve is very small $\frac{v_s}{\omega_s} \to 0$, then

$$i_{sd} = -\frac{\psi_m}{l_d}$$

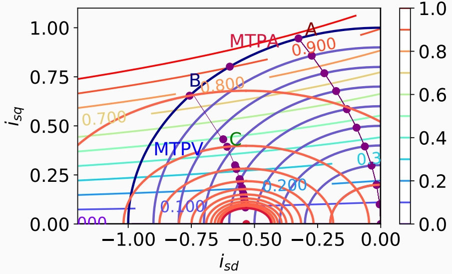
Once the speed reaches the rated speed, the voltage of motor reaches the rated voltage. The voltage limit then decides the torque that can be produced. We can define the Maximum Torque per Voltage.

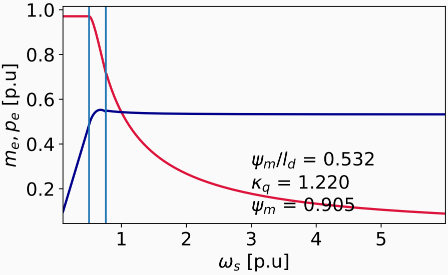


From MTPA to MTPV transition in FW region



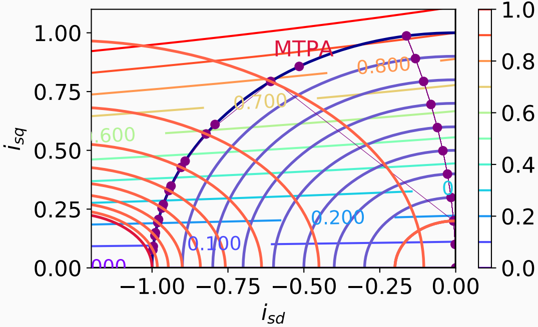
- Constant Torque range - move along MTPA
- Reach point A to get maximum Torque
- If $(\omega_s >= 1)$ then:
- move to point B (as $i_{sm} = i_{max}$ and $|v_s| = 1.0$)
- If ω_s is to be increased further
- move along MTPV to C





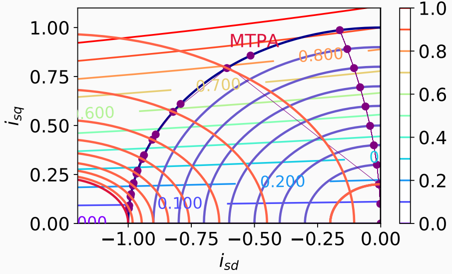
For

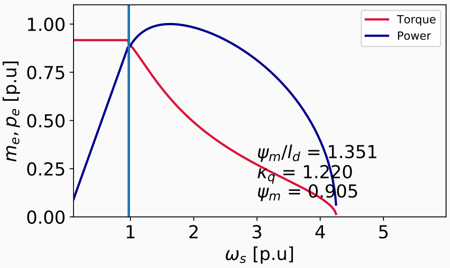
$$\boxed{i_{sd} = -\frac{\psi_m}{l_d}}$$



- Constant Torque range - move along MTPA
- Reach point A to get maximum Torque
- If $(\omega_s >= 1)$ then:
- move to point B2 (where the maximum voltage ellipse intersect the maximum current curve)
- If
$$i_{sd} = \left| \frac{\psi_m}{l_d} \right| > i_{sm}$$
- Then no MTPV curves can be used. Just Voltage limit comes into play, with max current.

Limited FW region

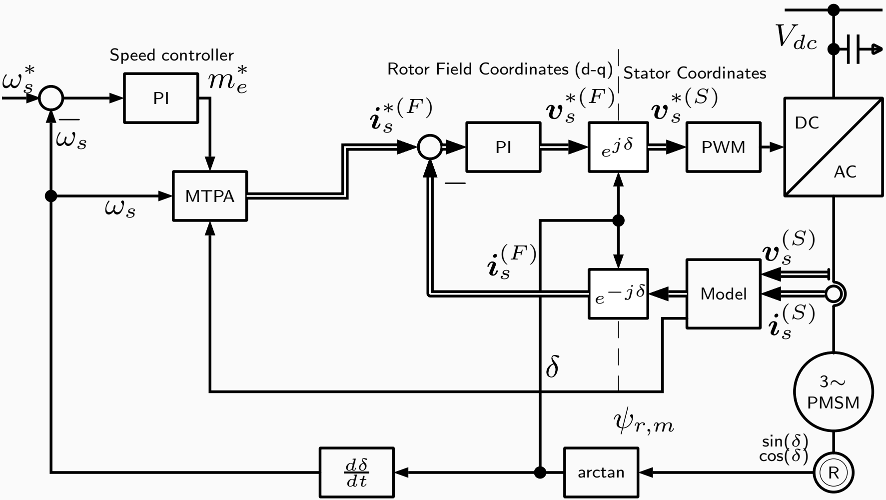




For

$$\boxed{|i_{sd}| = \left|\frac{\psi_m}{l_d}\right| > i_{sm}}$$

Full Vector Control of PMSM



PMSM in field weakening range

To achieve field weakening, the effective flux linkage along d-axis needs to be reduced, we will use a $i_{sd} < 0$ Since Torque is

$$\boxed{m_e = \psi_m i_{sq} + (l_d - l_q)\, i_{sd} i_{sq}}$$

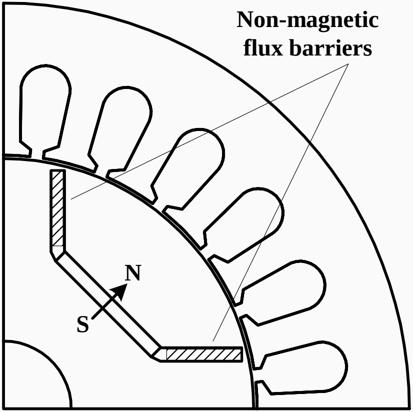
To get a positive contribution of reluctance torque, we will need

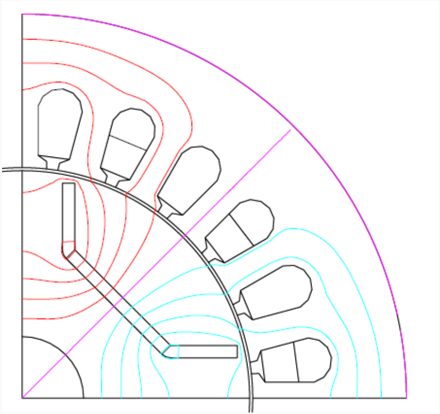
$$\boxed{l_q > l_d}$$

or

$$\boxed{\frac{l_q}{l_d} > 1}$$

IPMSM designed for high Constant Power Speed Range (CPSR)

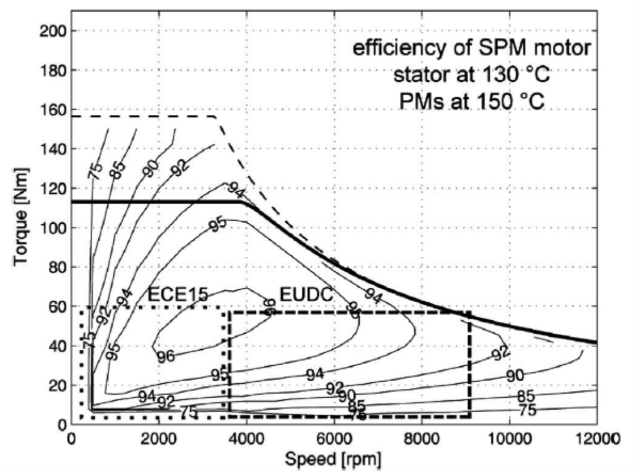




Comparison SPM

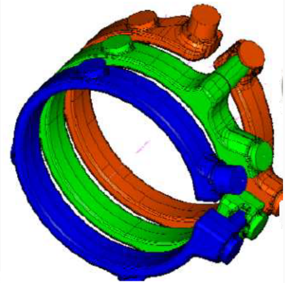
- No power overload due to MTPV clamping
- Efficiency as good as IPM Better in lower speed range
- drops in higher range

Comparison [?]

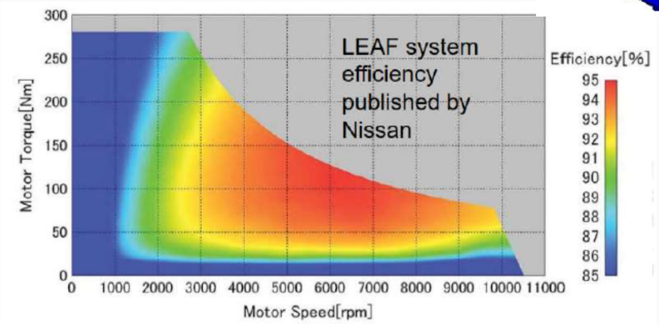


Some examples: Nissan Leaf Motor

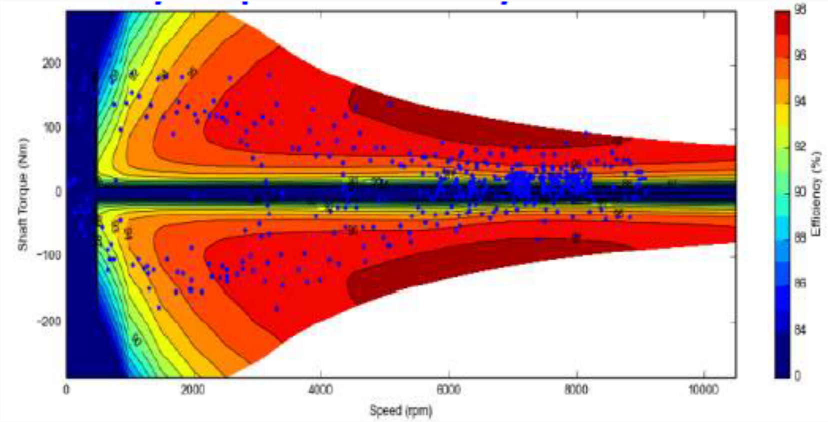
Rotor and Stator



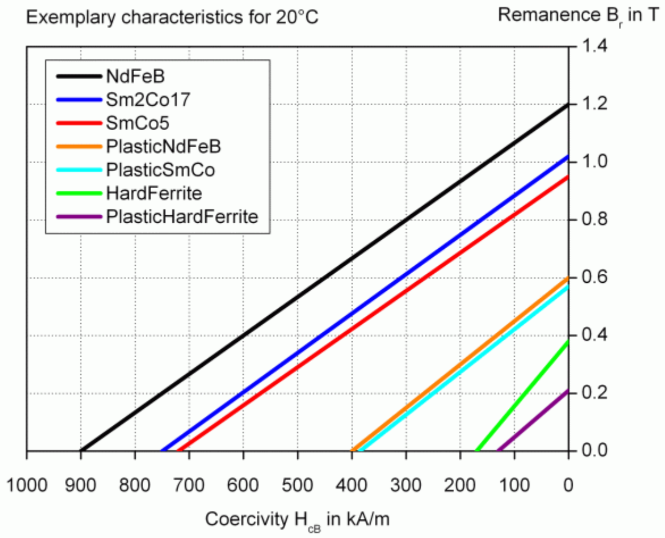
Water Jacket



Nissan leaf performance



Permanent Magnets needed



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