EE4302 ADVANCED CONTROL SYSTEMS

EXPERIMENT 1 (Briefing Notes)

COMPUTER AIDED DESIGN OF A STATE-SPACE CONTROL SYSTEM

© Dr. K.Z.Tang & Prof. T.H.Lee

1.0 Objective

- Design of a state-feedback controller is considered.
- Desired closed-loop response is given by specifications in the frequency domain,
- A computer-aided design procedure is used to interactively achieve the specifications.

2.0 Equipment

- PCs in the Control and Simulation Laboratory, E4A level 3, ECE Department
- MATLAB software package

3.0 Introduction

- State-space approach for control systems form an integral part of modern control techniques
- Design methods explored here are Ackermann's Formula and Linear Quadratic Regulator (LQR) feedback.
- State-space techniques provide useful insights into the structure of the system and allows the control engineer great flexibility in shaping the dynamic response of systems.
- MATLAB software package is used in this experiment.

3.1 Review of State-Variables and MATLAB

Consider the following plant described in the state-space notation:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - 2x_2 + u$$
$$y = x_1$$

and where both state-variables x1 and x2 are measurable.

3.1 Review of State-Variables and MATLAB

```
%3.1 Review of State-Variables and MATLAB
  A1=[0 1;-1 -2];
  B1=[0;1];
  C1=[1 0];
  D1=0;
%frequency response plot from u to x1
%explore use of “ss”; “figure” and “bode”
%use the “help” facility as much as possible!!
  sys1=ss(A1,B1,C1,D1);
  figure(1)
  bode(sys1)
%frequency response plot from u to x2, can use:
  C2=[0 1];
%note that there is great flexibility to define systems!
%check out “help ctrldemos” for tutorials/demos!
```

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3.1 Review of State-Variables and MATLAB

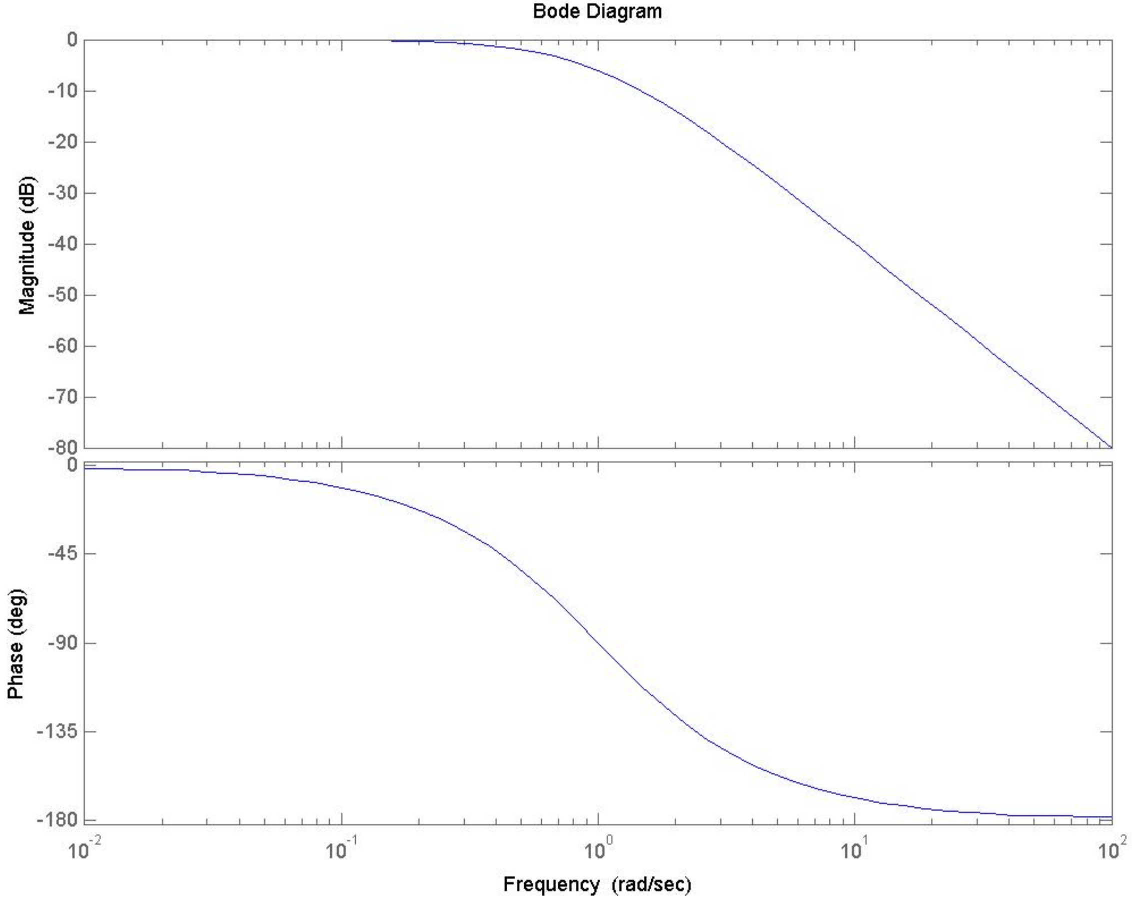


Figure 1: Frequency response from u to x_1

3.1 Review of State-Variables and MATLAB

Considering the feedback law:

$$u = -k_1 x_1 - k_2 x_2$$, for k1=0.1 and k2=0.1; for a start…

3.1 Review of State-Variables and MATLAB

```
%feedback of the closed loop system
   C3=[1 0;0 1];  %in this case the output to the control is x1 and x2 (ie. u=-k1*x1-k2*x2)
   sys3=ss(A1,B1,C3,D1);
   k1=0.1;
   k2=0.1;
   K=[k1 k2];
   A2=[0 0;0 0];
   B2=[0 0;0 0];
   C2=[0 0];
   D2=K;  %input to sys4 is x1 and x2; can you visualize sys4?
   sys4=ss(A2,B2,C2,D2);
   figure(3)
   sys5=feedback(sys3,sys4,-1);
%check out “help feedback”. draw a diagram for sys5!
   bode(sys5)   %frequency response from u to x1 and x2.
```

3.1 Review of State-Variables and MATLAB

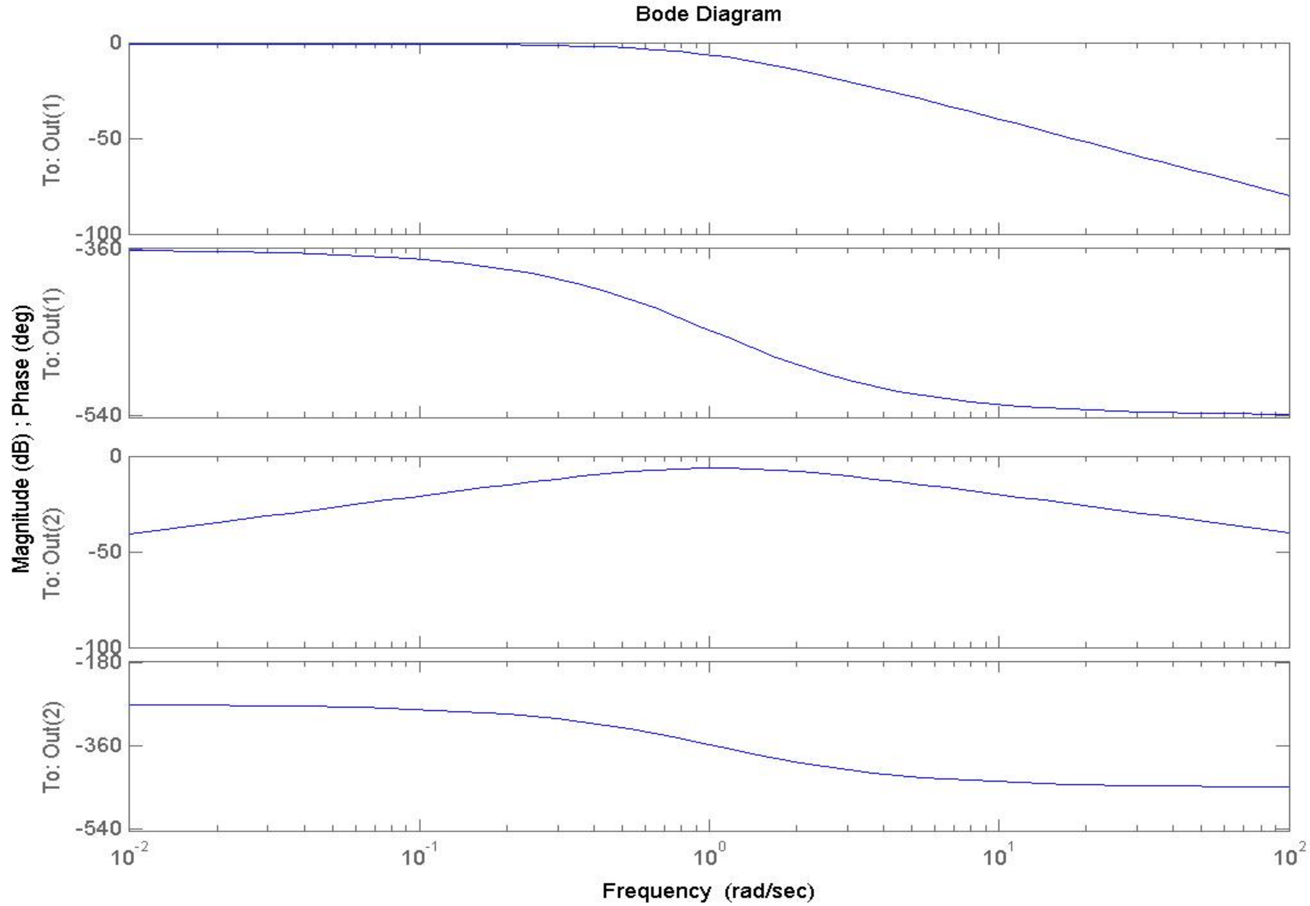
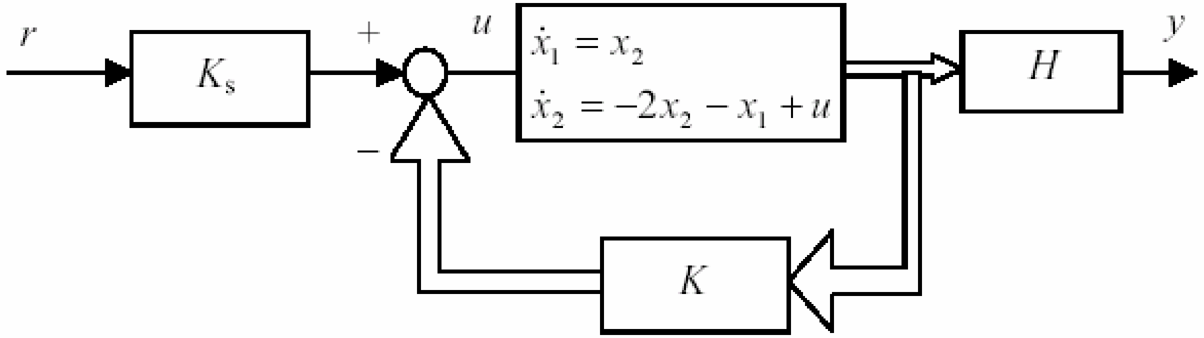


Figure 3: Frequency response for system under feedback

4.0 Simple State-Feedback Design

It is desired to design a closed-loop system such that the frequency response from the commanded signal r, to the output y, has the following specifications:

Closed-loop bandwidth:	Not lower than 3 rad/s;
Resonant Peak, *M*r :	Not larger than 2 dB (**or 10%);**
Steady-state gain between *r* and *y*:	0dB.



4.1 Design Using Ackermann's Formula

```
%4.1 Design using Ackermann's Formula
% ITAE Design.
    P=4*[-0.7071+0.7071*i ; -0.7071-0.7071*i];  %after trial of w0=3 proves to
                                                  be unsatisfactory
% Try w0=4 since when w0=3 the closed loop bandwidth is lower than 3 rad/s
   K=acker(A1,B1,P);  %pole placement using Ackermann's Formula
   D2=K;
   sys6=ss(A2,B2,C2,D2);
   sys7=feedback(sys3,sys6);
   Ks = 1/dcgain(sys7);
   figure(4)
   bode(Ks*sys7);   %Frequency Response plot of the final closed loop
                     system.
   figure(5);
   step(Ks*sys7);   %Corresponding Step response
%Explore using Bessel prototype too; or with 2nd-order dominant poles!
```

4.1 Design Using Ackermann's Formula

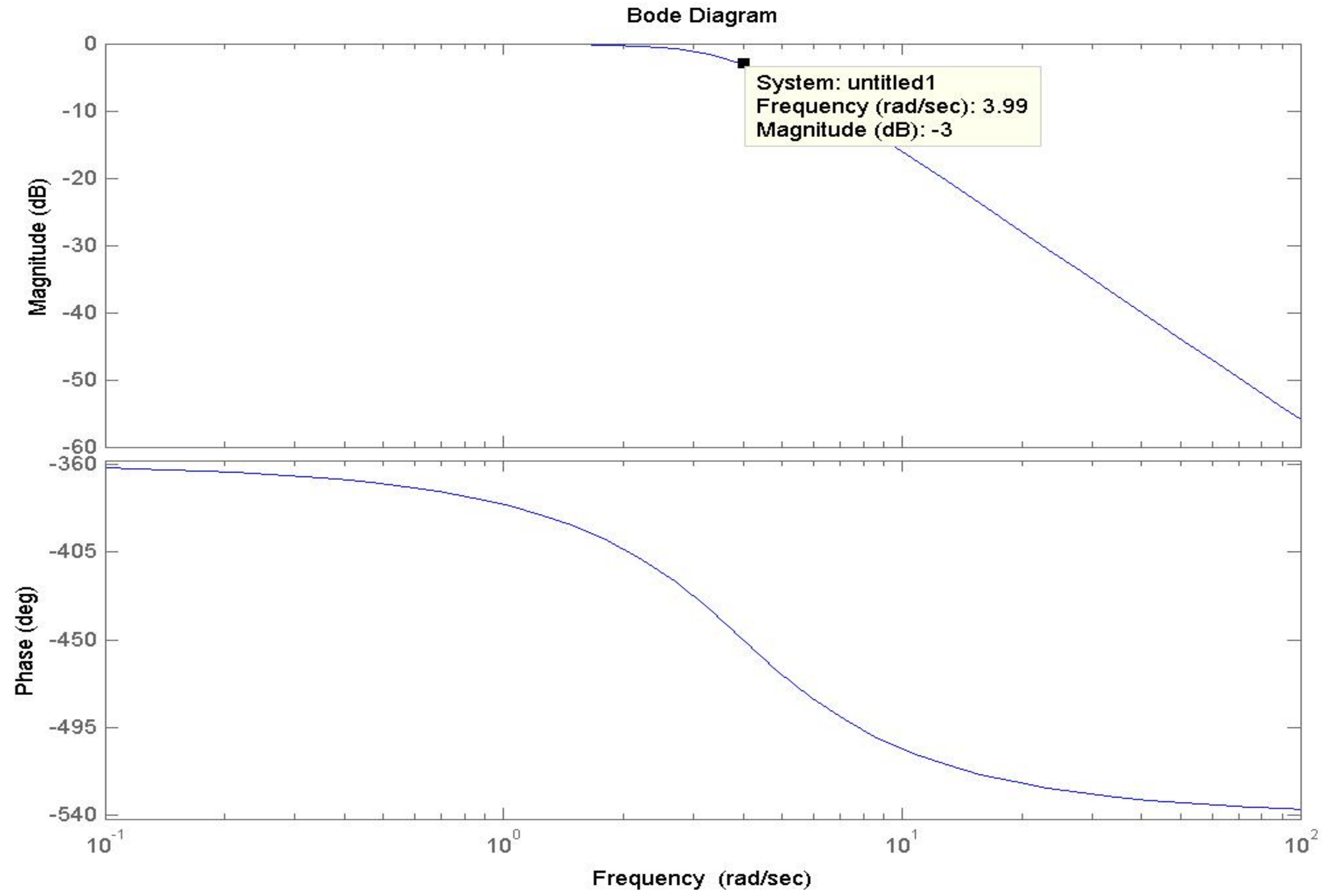


Figure 4: Frequency response for closed loop system using ITAE and Ackermann's formula

4.1 Design Using Ackermann's Formula

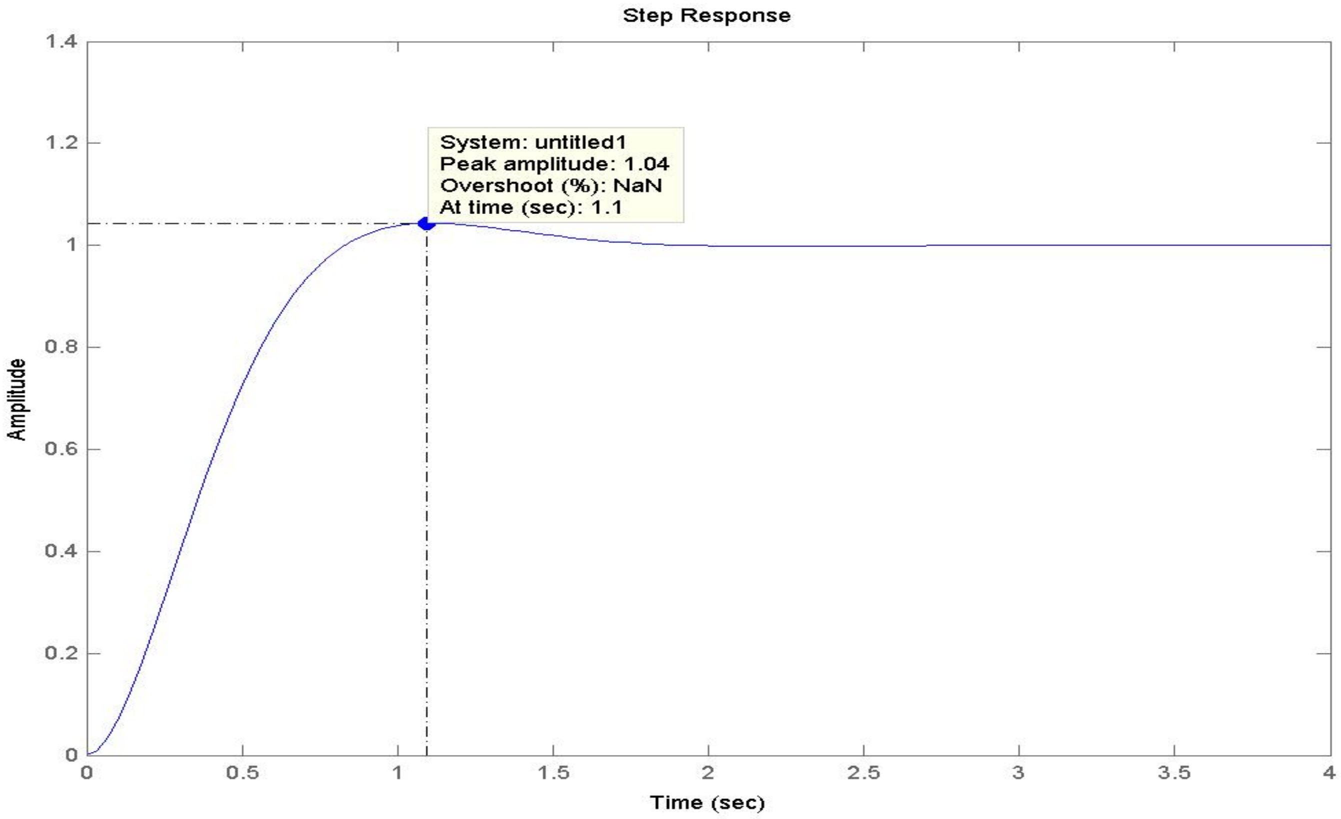


Figure 5: Step response for the closed loop system

4.2 Design Using Linear Quadratic Regulator Weightings

- The Linear Quadratic Regulator (LQR) method adopts a different approach which focuses on calculating a set of gains to minimize the criterion.

$$J = \int_0^{\infty} (x^T Q x + r u^2) dt$$

4.2 Design Using Linear Quadratic Regulator Weightings

%4.2 Design using Linear Quadratic Regulator Weightings

```
R=1;    %set this to a default value of 1 and vary the weighting of the Q matrix
Q=[150 0; 0 1];   % trial and error... from [1 0; 0 1], where the specifications are
                    not met, we increase the weighting on the x1 state since it is
                    more important to us, finally we arrive at a value of [150 0;0 1]
                    for the Q matrix; this is not unique, explore different choices…
[K,S,E] = LQR(A1,B1,Q,R);
D2=K;     % the set of gains is in the K matrix
sys8=ss(A2,B2,C2,D2);    %recall that A2, B2, C2 (as defined earlier) are all
                          empty (0) matrices of appropriate dimensions.
sys9=feedback(sys3,sys8);   %feedback system
Ks = 1/dcgain(sys9);  %calculation of the scaling gain so that the final system has
                       a 0dB steady state gain
figure(6)
bode(Ks*sys9)
figure(7)
step(Ks*sys9)
```

4.2 Design Using Linear Quadratic Regulator Weightings

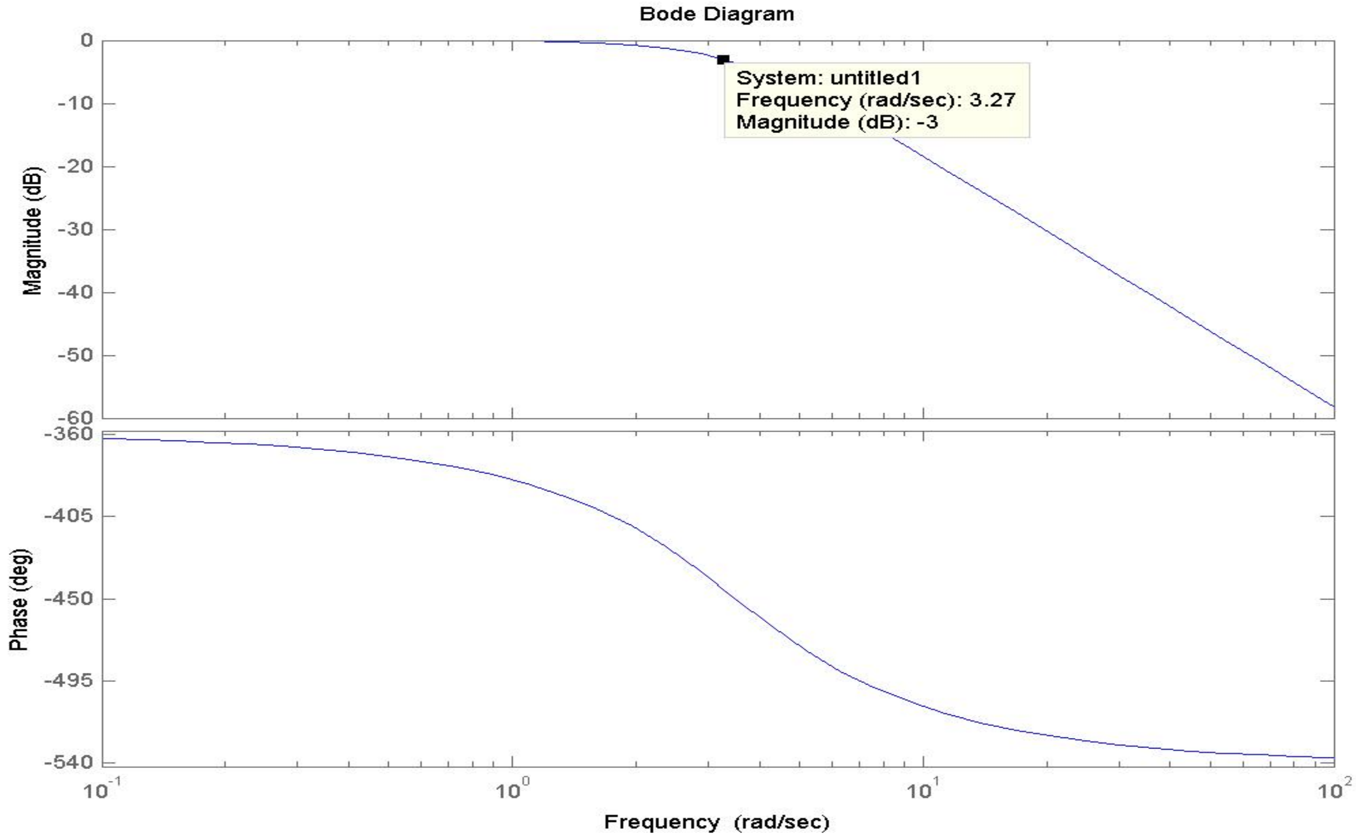


Figure 6: Frequency response for Q=[150 0; 0 1]

4.2 Design Using Linear Quadratic Regulator Weightings

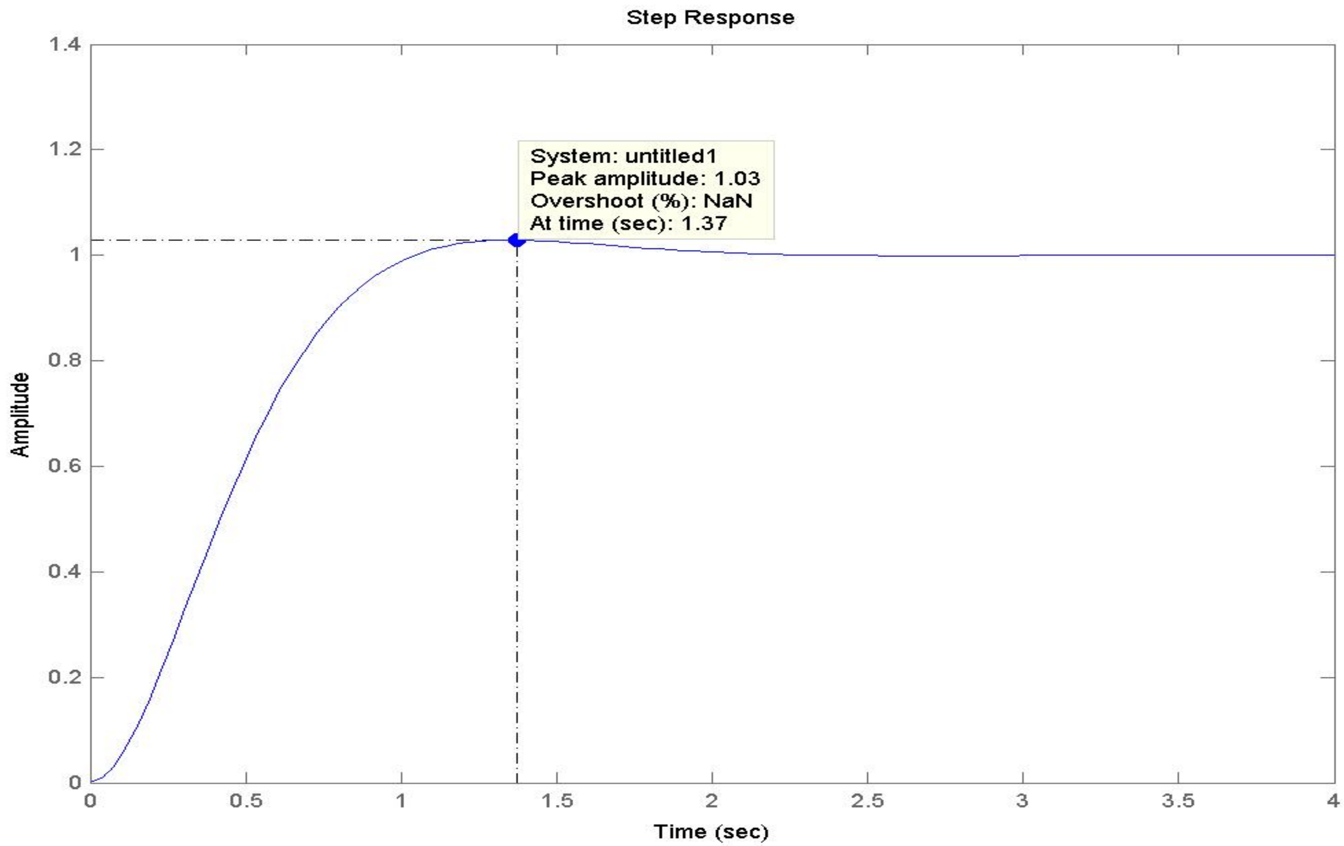


Figure 7: Step response for Q=[150 0; 0 1]

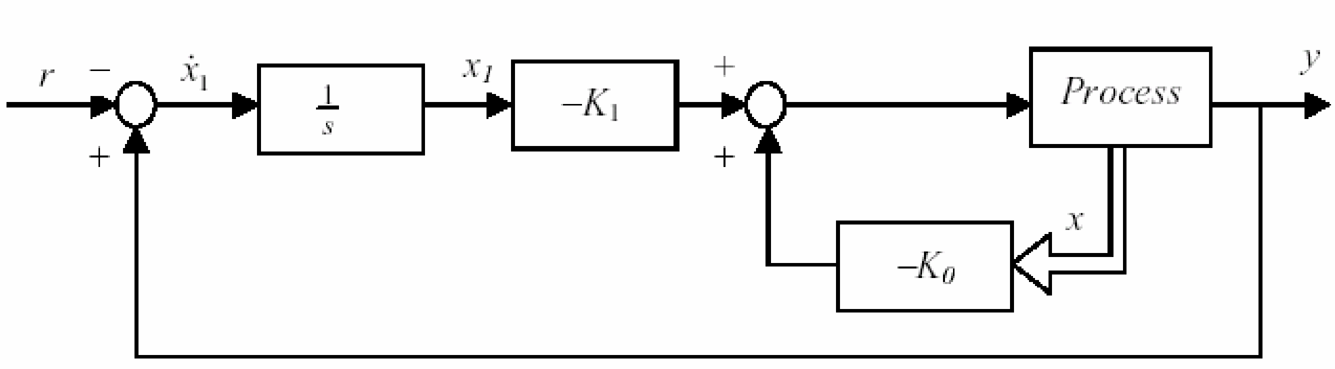
5.0 State Feedback Design including State-Augmentation (Optional)

- The disadvantage of using the scaling gain method is that this method does not regulate against persistent disturbances. Also, if there are differences between what the plant actually is and what we assume it to be, then our calculation of the scaling gain might not be accurate enough. For the method to work we need good knowledge of the plant parameters, and any change in them will cause the error to be nonzero.
- In comparison, integral control via state augmentation will automatically ensure that the steady state error will be zero and reject a persistent disturbance.

5.0 State Feedback Design including State-Augmentation (Optional)

It is desired to obtain the following frequency specifications between r and y:

Closed-loop bandwidth:	Not lower than 1.5 rad/s;
Resonant Peak, *M*r :	Not larger than 2 dB (or 10%);
Steady-state gain between *r* and *y*:	0dB.



5.0 State Feedback Design including State-Augmentation (Optional)

$$\begin{bmatrix} \dot{x}_I \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$y = x_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix}$$

A suitable state-space description of the augmented system. Check(!) that this is OK.

5.0 State Feedback Design including State-Augmentation (Optional)

```
%5.0 State Feedback Design including State Augmentation
% xI_dot = 0 xI +  1 x1 +  0 x2 + 0 u + -1 r + 0 v    % xI_dot= y-r = e = x1-r
% x1_dot = 0 xI +  0 x1 +  1 x2 + 0 u +  0 r + 0 v
% x2_dot = 0 xI + -1 x1 + -2 x2 + 1 u +  0 r + 1 v

   F=[0 1 0;0 0 1;0 -1 -2];
   G=[0; 0; 1];
   Gr=[-1; 0; 0];
   Gv=[0; 0; 1];
   H=[0 1 0];
   J=0;

%ITAE method in calculating state feedback gain K
  P=2*[-0.7081; -0.5210+1.068*i ; -0.5210-1.068*i]; %use w0=2 since w0=1.5
                                                     fails to satisfy the
                                                     requirement

  K=acker(F,G,P);
```

5.0 State Feedback Design including State-Augmentation (Optional)

```
%Response of output y to a unit step in r
  sys=ss(F-G*K,Gr,H,J);   %Gr used here, assuming v=0.
  figure(8)
  bode(sys)
  figure(9)
  step(sys)

%Response of output y to a unit step in the unmeasurable disturbance v
  sys=ss(F-G*K,Gv,H,J);   %Gv used here, assuming r=0.
  figure(10)
  step(sys)
%Response of control input u to a unit step in the unmeasurable disturbance v
   H=-K;                    %u=-Kx. Here u is regarded as output(!).
   sys=ss(F-G*K,Gv,H,J);   %Gv is used here because initiating factor is v
   figure(11)
   step(sys)
```

5.0 State Feedback Design including State-Augmentation (Optional)

```
%LQR method in calculating state feedback gain K
  H=[0 1 0];
  R=1;

%increase weighting in xI because it is of primary interest to us.
  Q=[100 0 0;0 1 0;0 0 1];

% trial and error... from [1 0 0 ;0 1 0;0 0 1] to [100 0 0; 0 1 0 ;0 0 1]
   [K,S,E] = LQR(F,G,Q,R);

%Response of output y to a unit step in r
  sys=ss(F-G*K,Gr,H,J);
  figure(12)
  bode(sys)
  figure(13)
  step(sys)
```

5.0 State Feedback Design including State-Augmentation (Optional)

```
%Response of output y to a unit step in the unmeasurable disturbance v
  sys=ss(F-G*K,Gv,H,J);
  figure(14)
  step(sys)

%Response of control input u to a unit step in the unmeasurable disturbance v
  H=-K;
  sys=ss(F-G*K,Gv,H,J);
  figure(15)
  step(sys)
```

5.0 State Feedback Design including State-Augmentation (Optional)

```
Q=[1 0 0;0 100 0;0 0 1];   %to see the effect of weighting more heavily on x1 instead
[K,S,E] = LQR(F,G,Q,R);

%Response of output y to a unit step in r
sys=ss(F-G*K,Gr,H,J);
figure(x1)
bode(sys)
Q=[1 0 0;0 1 0;0 0 100];   %to see the effect of weighting more heavily on x2 instead
                            [K,S,E] = LQR(F,G,Q,R);

%Response of output y to a unit step in r
sys=ss(F-G*K,Gr,H,J);
figure(x2)
bode(sys)

%---------------------- THE END -------------------------------------------------%
```

Design using ITAE criterion and Ackermann's Formula

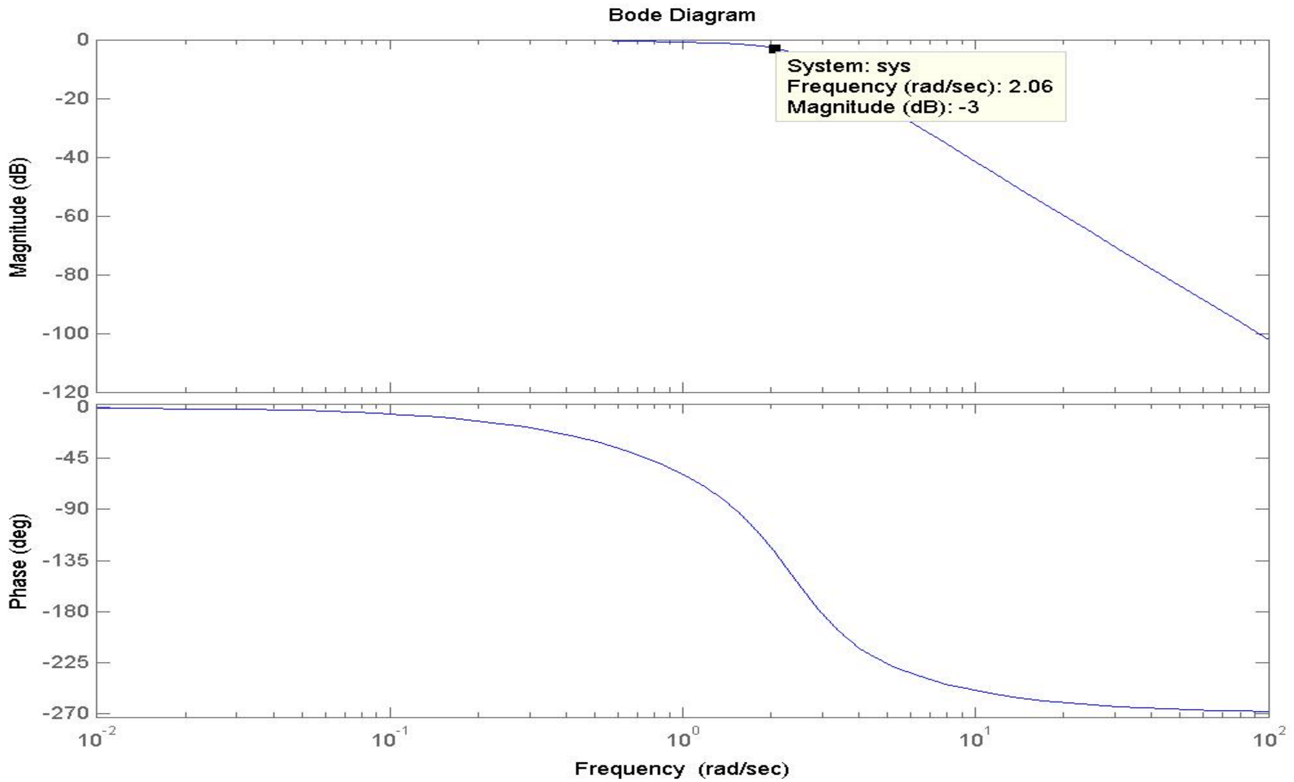


Figure 8: Frequency response of y due to r for ITAE method, 3rd order system, **w0=2**

Design using ITAE criterion and Ackermann's Formula

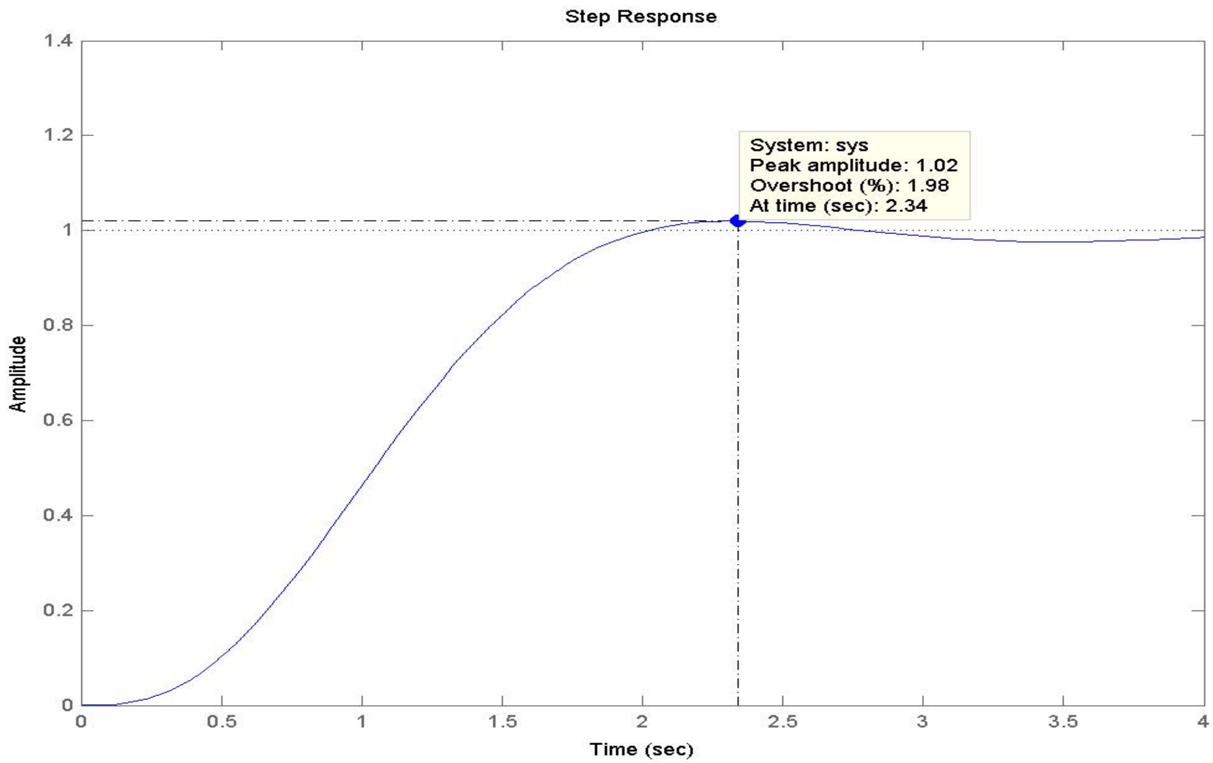


Figure 9: Step response of y due to r for ITAE method, 3rd order system, **w0=2**

Design using ITAE criterion and Ackermann’s Formula

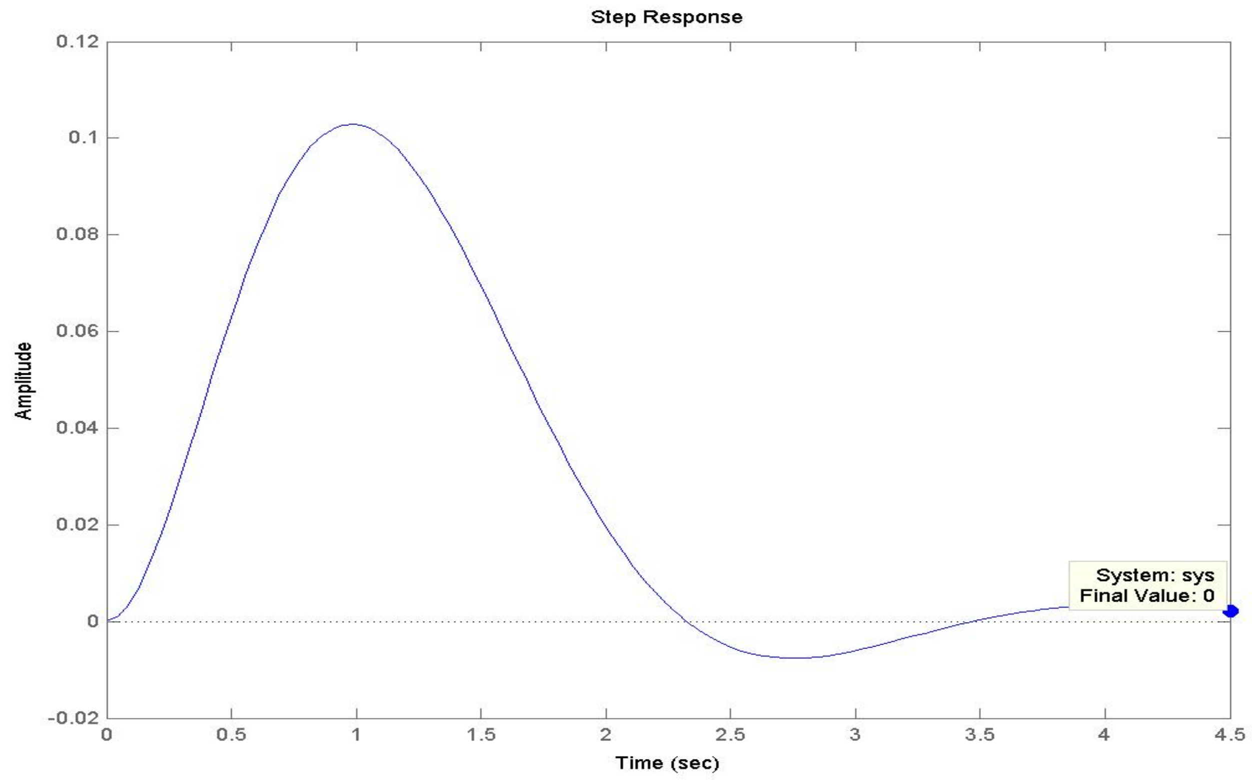


Figure 10: Step response of y to a unit step in the disturbance v

Design using ITAE criterion and Ackermann's Formula

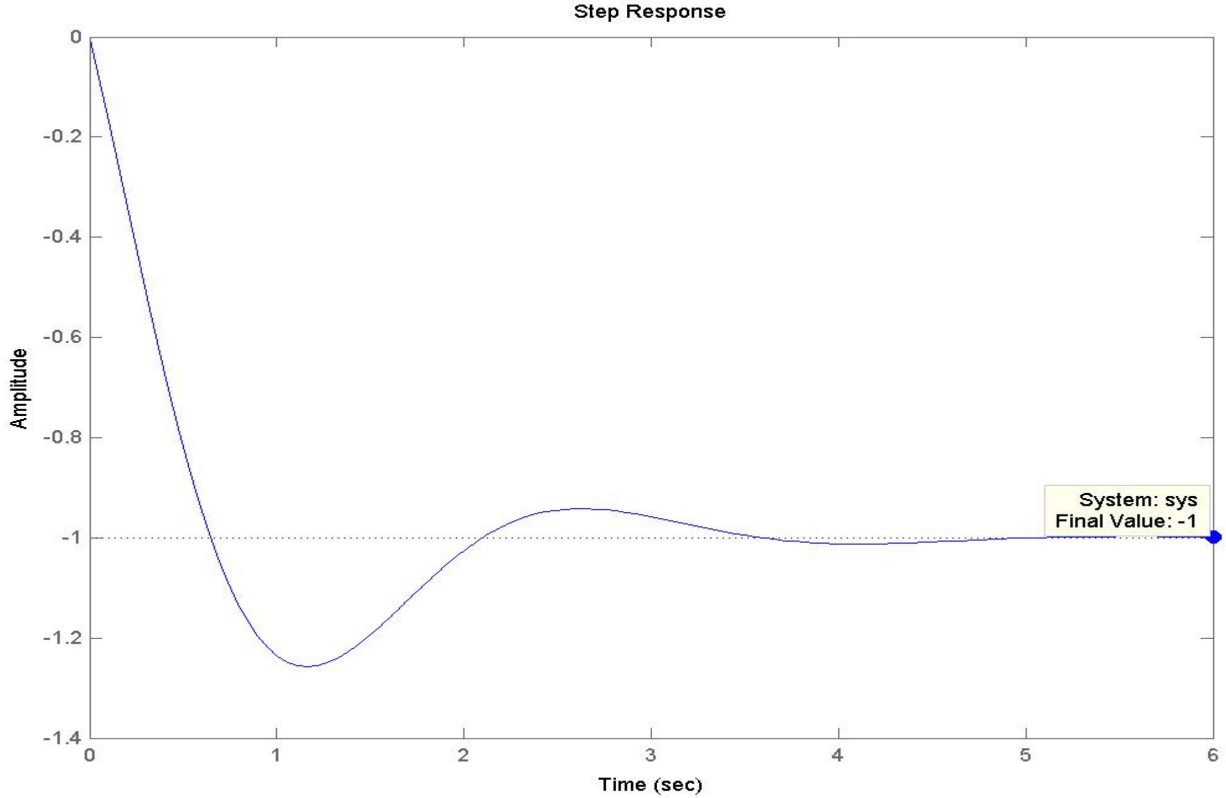


Figure 11: Step response of control signal u to a unit step in the disturbance v

Design using LQR design method

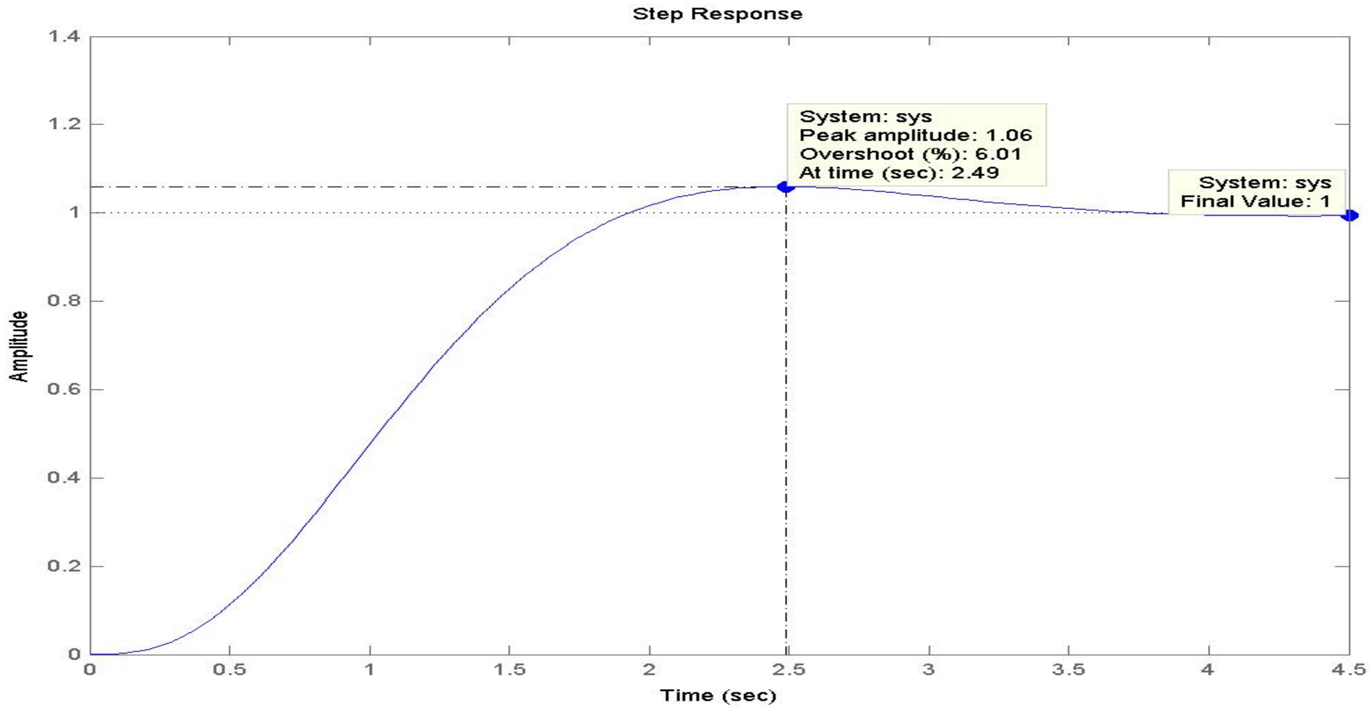


Figure 13: Step response of y due to r for LQR method, R=1, **Q=[100 0 0;0 1 0;0 0 1]**

Design using LQR design method

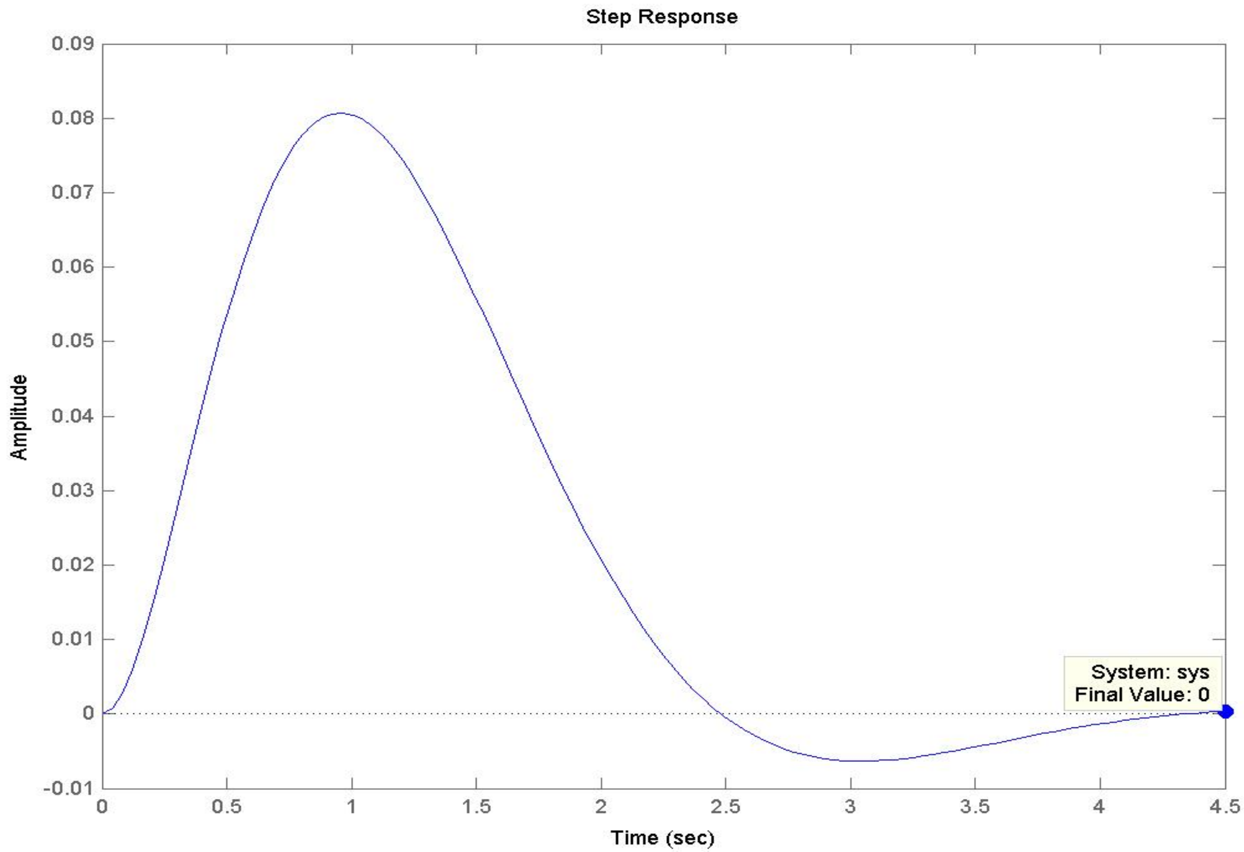


Figure 14: Step response of y to a unit step in the disturbance v

Design using LQR design method

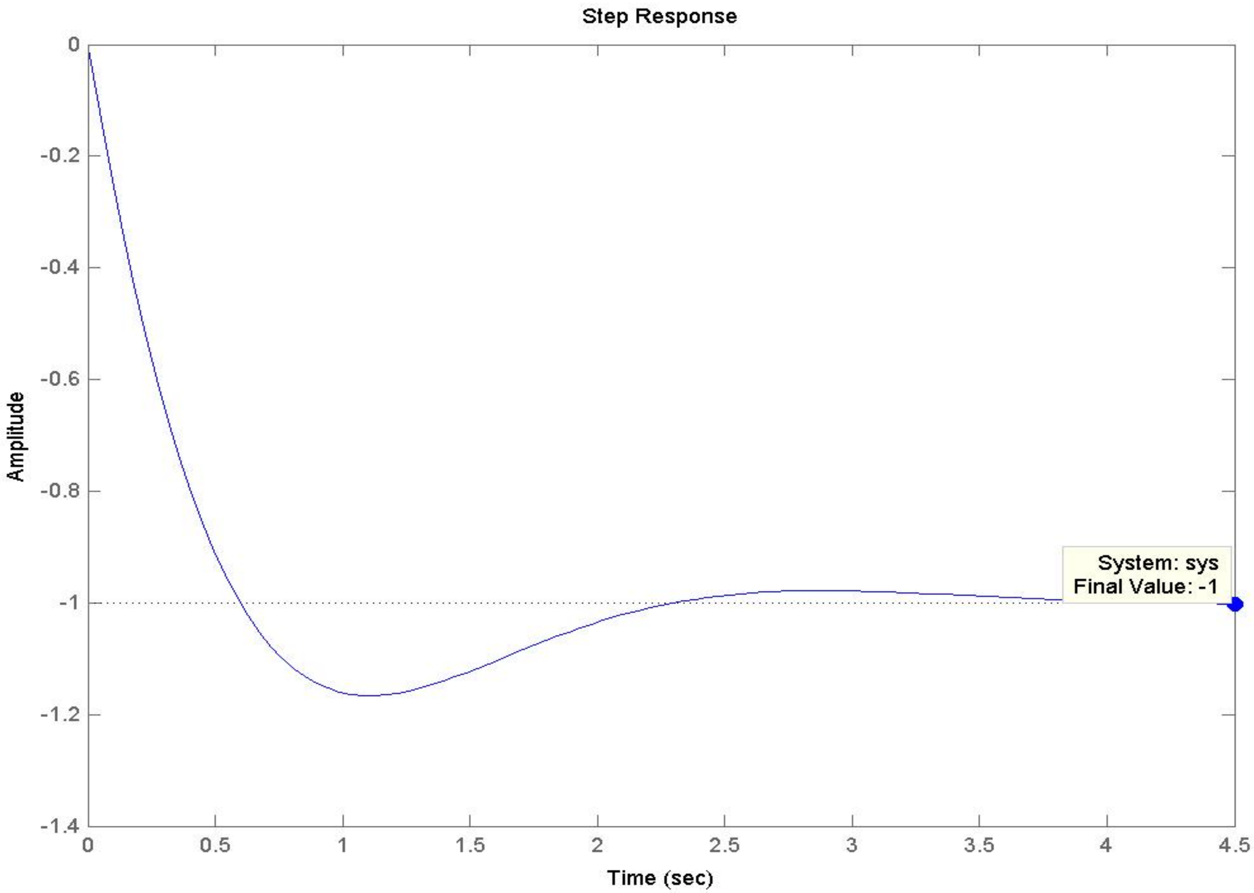


Figure 15: Step response of control signal u to a unit step in the disturbance v

Design using LQR design method

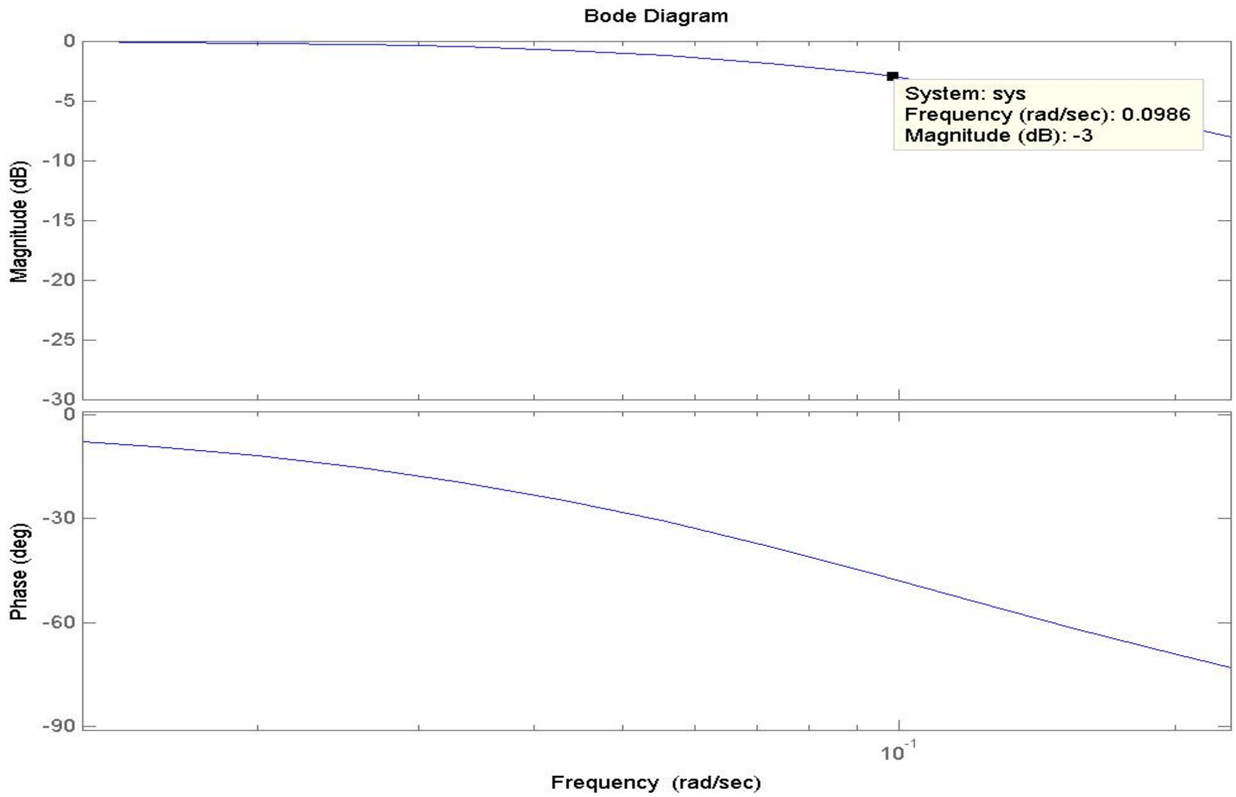


Figure x1: Frequency response of y due to r for LQR method, R=1, Q=[1 0 0;0 100 0;0 0 1]

Design using LQR design method

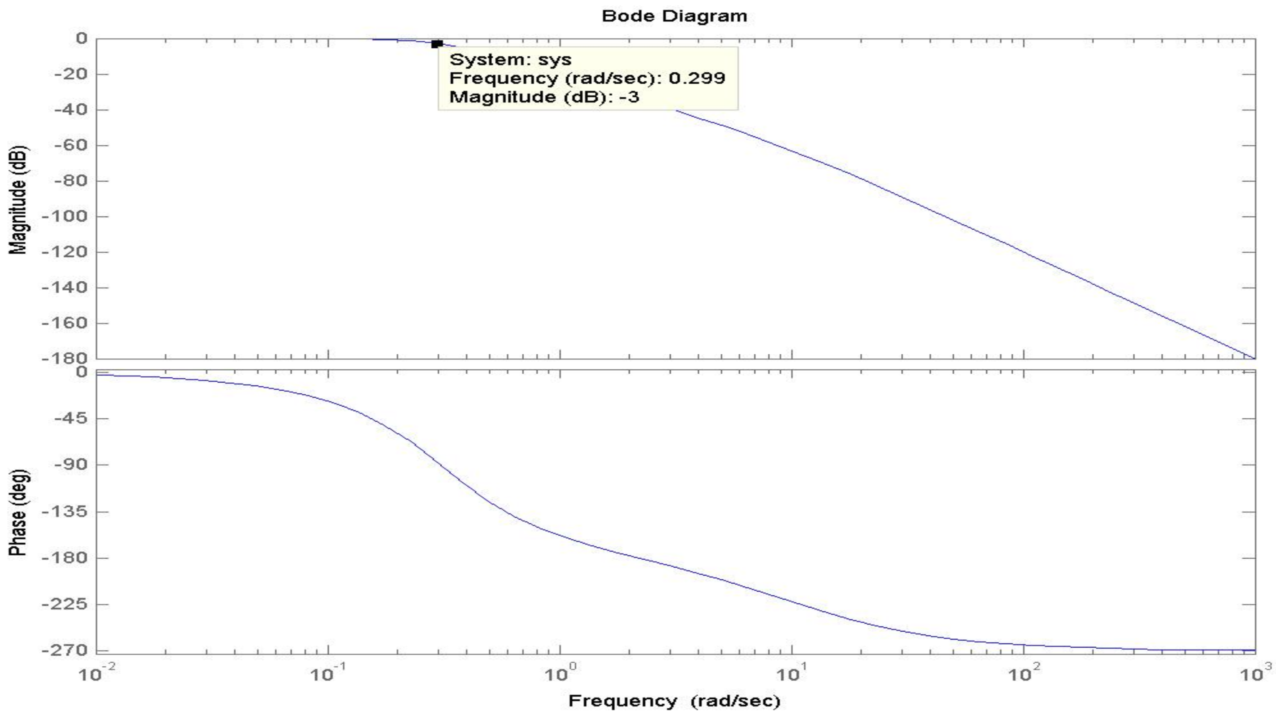


Figure x2: Frequency response of y due to r for LQR method, R=1, **Q=[1 0 0;0 1 0;0 0 100]**

Closing thoughts…

- Given some closed-loop specifications that is required of the system, the required closed-loop characteristics can be analyzed and evaluated using several approaches, with the aid of computer-based design software.

- In this briefing, the ITAE and LQR methods have been utilized. Both can yield the required results in this experiment. The ITAE method has the advantage of simplicity. The LQR method offers additional flexibility in the design procedure as it allows us to decide on the relative importance of the state-variables and control effort involved.

- In addition, integral control can be introduced to obtain steady-state tracking of a step reference-signal by augmenting the plant state. This design adds the capability of rejecting persistent constant disturbances.

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Concluding Notes

- Experiment 1 report is due on:
  - Date: to be decided, but will be as late as possible!!
  - Day: Thursday
  - Time: 12.00 noon
  - Venue to submit report: Control and Simulation Lab
  - Person-in-charge: Mr Zhang Hengwei