NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR
(Semester II: 2018/2019)

EE4302 – ADVANCED CONTROL SYSTEMS

April/May 2019 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. Please write your student number only. Do not write your name.
2. This question paper contains **FOUR** (4) questions and comprises **Eleven** (11) pages.
3. Answer **ALL** questions.
4. Note that the Questions do not carry equal marks.
5. This is a **CLOSED BOOK** examination. However, each student may bring ONE (1) A4 size crib sheet into the examination hall.
6. Relevant data are provided at the end of this examination paper.
7. Graphics/Programmable calculators are not allowed.

Q1 Consider the following linear system described by the transfer function

$$\frac{Y(s)}{U(s)} = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}$$

Using the Method 1 development described in the module (also known as the Controllable Canonical Form), develop the state-variable description for this system. Show clearly all the steps in your development.

For this Method 1 (Controllable Canonical Form) state-variable realization, show that the system described in this way is always controllable. Show clearly your reasoning for this.

(15 marks)

Q2 A set of notes from a typical design exercise for a state-variable control system are shown in Figures 2a, 2b and 2c. The *augmented* state-variable signal $x_I(t)$ is generated as

$$\dot{x}_I(t) = y(t) - r(t)$$

where $y(t)$ is the measured output of the system to be controlled, and $r(t)$ is the set-point command signal, a situation which is also illustrated in the block diagram in Figure 2a.

The *augmented* state-variable signal $x_I(t)$ is incorporated in the state-variable description of the overall augmented system as shown in Figure 2b. This overall augmented system is one possible way of describing a state-feedback control system with integral control action. In addition, note that it can be stated that the computed control signal $u(t)$ in Figures 2a, 2b and 2c, is computed as:

$$u(t) = -k_I x_I(t) - k_1 x_1(t) - k_2 x_2(t)$$

and in the design calculations as shown in Figure 2c, the necessary state-feedback gain row vector K is given by

$$\mathrm{K} = [k_I \quad k_1 \quad k_2]$$

The state-variable equations in Figure 2b also includes the influence of $v(t)$, an unmeasurable additional disturbance signal.

The block diagram of Figure 2a does not yet explicitly show the additional disturbance signal $v(t)$. Based on the equations of Figure 2b,
<u>provide a re-drawing of the block diagram</u> of Figure 2a where the inclusion of the additional disturbance signal $v(t)$ is clearly shown.

Next, if the situation is that $v(t) = v_0$ (*i.e.* a constant-valued but unknown disturbance), and $r(t) = r_0$ (*i.e.* a constant-valued user-applied reference signal), <u>describe in full detail</u> (with all relevant equations and analysis) the performance of this state-feedback control system, where the necessary design calculations are as described in Figure 2c.

(20 marks)

Question 2 continues next page.

2.0a State Feedback Design including State-Augmentation

It is desired to obtain the following frequency specifications between r and y:

Closed-loop bandwidth: Not lower than 1.5 rad/s;
Resonant Peak, *Mr* : Not larger than 2 dB (or 10%);
Steady-state gain between *r* and *y*: 0dB.

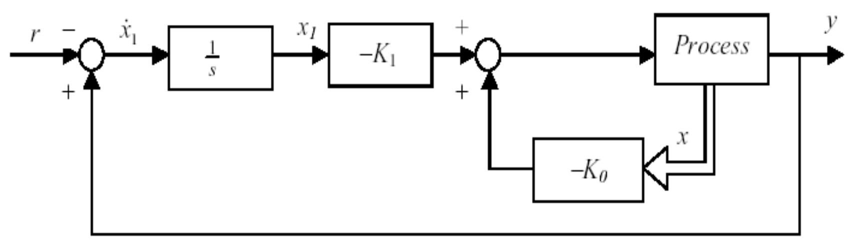

Figure 2a: A suitable state-space description of the augmented system.

Question 2 continues next page.

2.0b State Feedback Design including State-Augmentation

$$\begin{bmatrix} \dot{x}_I \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$y = x_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_I \\ x_1 \\ x_2 \end{bmatrix}$$

Figure 2b: A suitable state-space description of the augmented system.

Question 2 continues next page.

2.0c State Feedback Design including State-Augmentation

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%5.0 State Feedback Design including State Augmentation
% xI_dot = 0 xI +  1 x1 +  0 x2 + 0 u + -1 r + 0 v    % xI_dot= y-r = e = x1-r
% x1_dot = 0 xI +  0 x1 +  1 x2 + 0 u +  0 r + 0 v
% x2_dot = 0 xI + -1 x1 + -2 x2 + 1 u +  0 r + 1 v

   F=[0 1 0;0 0 1;0 -1 -2];
   G=[0; 0; 1];
   Gr=[-1; 0; 0];
   Gv=[0; 0; 1];
   H=[0 1 0];
   J=0;

%ITAE method in calculating state feedback gain K
  P=2*[-0.7081; -0.5210+1.068*i ; -0.5210-1.068*i];
%use w0=2 since w0=1.5 fails to satisfy the requirement
  K=acker(F,G,P);
```

Figure 2c: A suitable state-space description of the augmented system.

Q3 Consider the system in Figure 3 where

$$\begin{aligned} G_c &= 1 \\ G_p(s) &= 10\frac{1-0.1s}{s(s+1)} \end{aligned}$$

and the relay nonlinearity $f(u)$ is given by

$$v = \begin{cases} +d & \text{if } u > 0 \\ 0 & \text{if } u = 0 \\ -d & \text{if } u < 0 \end{cases}$$

The set-point $r = 0$ and the limit cycle at e can be approximated by $e = \frac{8}{\pi}\sin w_c t$.

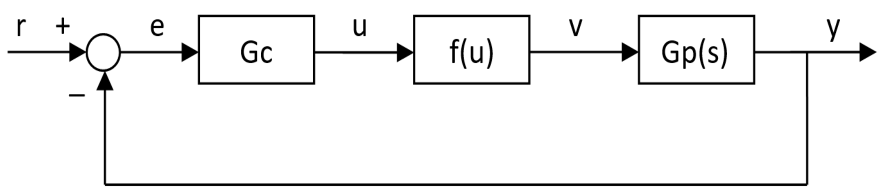

Figure 3

a) Find (i) the equivalent gain of the relay nonlinearity $f(u)$ that gives rise to the limit cycle, and (ii) the frequency, w_c, of the limit cycle.

[10 marks]

b) Find the fundamental component of the oscillation at v,

[6 marks]

c) Sketch one period of the oscillations at e and y and superimpose on them the oscillation you expect to see at v. Give the values of all the oscillation amplitudes on the sketch.

[8 marks]

d) Find the describing function of the relay nonlinearity $f(u)$ at frequency w_c.

[6 marks]

Q.4 Consider the system in Figure 3 where $G_p(s)$ and $f(u)$ are changed to

$$
\begin{aligned}
G_p(s) &= \frac{1}{s^2 + 2s + 1} \\
v &= \begin{cases} +\sqrt{u} & \text{if } u \geq 0 \\ -\sqrt{-u} & \text{if } u < 0 \end{cases}
\end{aligned}
$$

a) The equivalent gain of the square-root nonlinearity, $f(u)$, is given as $K = \frac{v}{u}$. Find K for $u = 0.5$ and 1.

[6 marks]

b) The set-point is changed as follows: $r = 0$ to 0.5, and $r = 0$ to 1. (i) Which set-point change gives a larger percentage overshoot. (ii) Explain your answers in (i) using the closed-loop poles.

[9 marks]

c) For the block G_c in Figure 3, design a proportional controller using input-output linearization to give a 10% overshoot for the set-point response. Draw the block diagram of the system to include the proportional controller and input-output linearization.

[20 marks]

END OF QUESTIONS

DATA SHEET:

1. For the matrices $\mathbf{A} \in \mathbf{R}^{n\times n}$, $\mathbf{C} \in \mathbf{R}^{1\times n}$, and $\mathbf{L} \in \mathbf{R}^{n}$, the eigenvalues of the matrix $(\mathbf{A} - \mathbf{LC})$ can be arbitrarily assigned by a suitable choice of $\mathbf{L}$ as long as

$$O(\mathbf{A},\mathbf{C}) = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{(n-1)} \end{bmatrix}$$

is non-singular.

2. For the linear system

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Fx} + \mathbf{G}u \\ y &= \mathbf{Hx} \end{aligned}$$

where $\mathbf{x} \in \mathbf{R}^n$ and $u, y \in \mathbf{R}^1$, the controllability matrix of the system is given by

$$C(\mathbf{F},\mathbf{G}) = \begin{bmatrix} \mathbf{G} & \mathbf{FG} & \ldots & \mathbf{F}^{(n-1)}\mathbf{G} \end{bmatrix}$$

If the characteristic polynomial of $\mathbf{F}$ is given by

$$\alpha(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n$$

then the state-feedback $u = -\mathbf{Kx}$ which yields the closed-loop characteristic polynomial

$$\alpha_c(s) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_n$$

can be calculated using the Bass-Gura's formula

$$\mathbf{K} = \begin{bmatrix} \alpha_1 - a_1 & \alpha_2 - a_2 & \ldots & \alpha_n - a_n \end{bmatrix} \{C(\mathbf{F},\mathbf{G})W\}^{-1}$$

where

$$W = \begin{bmatrix} 1 & a_1 & a_2 & \ldots & a_{n-1} \\ 0 & 1 & a_1 & \ldots & a_{n-2} \\ \vdots & \vdots & \ddots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}$$

3. For the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= -a_n x_1 - a_{n-1} x_2 - \ldots - a_1 x_n + b_0 u \\ y &= x_1 \end{aligned}$$

the characteristic polynomial is

$$\alpha_o(s) = s^n + a_1 s^{n-1} + \ldots + a_n$$

and the transfer function is

$$\frac{Y(s)}{U(s)} = \frac{b_0}{s^n + a_1 s^{n-1} + \ldots + a_n}$$

4. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top [sI - A_m]^{-1} b_m = \frac{-a_3}{s^3 - a_2 s^2 - a_1 s - a_3}$$

5. In the development of a reduced-order observer, note that a state-variable system with state vector $\mathbf{x}$ of order n, where the first n_1 state-variables, in a vector $\mathbf{x}_1$ are essentially measurable, can be written as:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{F}_{11}\mathbf{x}_1 + \mathbf{F}_{12}\mathbf{x}_2 + \mathbf{G}_1 u \\ \dot{\mathbf{x}}_2 &= \mathbf{F}_{21}\mathbf{x}_1 + \mathbf{F}_{22}\mathbf{x}_2 + \mathbf{G}_2 u \end{aligned}$$

with the remaining n_2 state-variables, in a vector $\mathbf{x}_2$ to be estimated (or observed). Here, $\mathbf{F}_{11}$, $\mathbf{F}_{12}$, $\mathbf{F}_{21}$ and $\mathbf{F}_{22}$ are all known system matrices (obtained by calibration data, for example), and the measurement is typically given by

$$\mathbf{y}_m = \mathbf{H}_1 \mathbf{x}_1$$

where $\mathbf{H}_1$ is also a known $(n_1 \times n_1)$ system matrix. Under these circumstances, a suitable form for the estimator for $\mathbf{x}_2(t)$ is

$$\begin{aligned} \hat{\mathbf{x}}_2 &= \mathbf{L}\mathbf{y}_m + \mathbf{z} \\ \dot{\mathbf{z}} &= \bar{\mathbf{F}}\mathbf{z} + \bar{\mathbf{G}}\mathbf{y}_m + \bar{\mathbf{H}}u \end{aligned}$$

where a suitable set of matrices defining the reduced-order observer are given by:

$$\begin{aligned} \bar{\mathbf{F}} &= \mathbf{F}_{22} - \mathbf{L}\mathbf{H}_1\mathbf{F}_{12} \\ \bar{\mathbf{G}} &= \left\{ \mathbf{F}_{21} - \mathbf{L}\mathbf{H}_1\mathbf{F}_{11} + \bar{\mathbf{F}}\mathbf{L}\mathbf{H}_1 \right\} \mathbf{H}_1^{-1} \\ \bar{\mathbf{H}} &= \mathbf{G}_2 - \mathbf{L}\mathbf{H}_1\mathbf{G}_1 \end{aligned}$$

6. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s^a$
ITAE	1	$s+1$
	2	$s+0.7071 \pm 0.7071j^b$
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s+0.8660 \pm 0.5000j^b$
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a \pm bj)$ to conserve space.

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$u(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{a}{s(s+a)}$	$1 - e^{-at}$
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at}(1 + at)$
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at} \sin \omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at} \cos \omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t} \sin \omega_n\sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$

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