ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2019/2020)

EE5103 / ME5403– COMPUTER CONTROL SYSTEMS

November 2019 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **SIX** (6) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.
4. This is a CLOSED BOOK examination. But the candidate is permitted to bring into the examination hall a single A4 size *help sheet*. The candidate may refer to this sheet during the examination.
5. Calculators can be used in the examination, but no programmable calculator is allowed.

Q.1 A system is described by

$$x(k+1) = \begin{bmatrix} 2 & 1 \\ \alpha & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

where α is a constant parameters, $x(k) = [x_1(k), x_2(k)]^T$ is the state vector, $y(k)$ is the output, $u(k)$ is the input, and $\omega(k)$ is the disturbance.

a) Find the range of α such that the system is both controllable and observable.

(2 Marks)

b) Assuming that there is no disturbance and the state variables are accessible, design a deadbeat state feedback controller.

(6 Marks)

c) Assuming that there is no disturbance and only the output $y(k)$ is available, design a deadbeat observer to estimate the state variables, and use these estimates to design an output-feedback controller.

(6 Marks)

d) Assuming that the disturbance is an unknown constant, design a deadbeat observer to estimate both the state variables and the disturbance, and use these estimates to design an output-feedback controller such that the effect of the disturbance may be completely eliminated.

(6 Marks)

e) Assuming that there are time-delays in both the state variables and the input, the corresponding model of the system is given as

$$x(k+1) = \begin{bmatrix} 2 & 1 \\ \alpha & -1 \end{bmatrix} x(k-2) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k-1).$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

What is the state of the system at time k? What is the order of the system? Justify your answers.

(5 Marks)

Q.2

a) A system is described by

$$y(k+1) = y(k) + 2u(k) + \alpha u(k-1),$$

where $y(k)$ and $u(k)$ are the output and input signals of the system, and α is a constant parameter.

Design a controller in the form of

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k)$$

such that the transfer function from the command signal, $u_c(k)$, to the system output, $y(k)$ follows the reference model, $\frac{1}{z^2}$. Discuss the condition on the parameter, α, such that perfect tracking is attainable.

(12 Marks)

b) Is it possible to use the state-space approach to design the controller to meet the same design specifications as those in part (a)? Justify your answers.

(8 Marks)

c) A nonlinear system is described by

$$y(k+1) = y(k) + y^3(k-1) + cu(k-1)$$

where c is a constant parameter.

Design a predictive controller to make the output of the system, $y(k)$, follow an arbitrary desired output, $y^*(k)$. Discuss the condition on the parameter c such that perfect tracking is attainable.

(5 Marks)

Q.3 Consider the first-order process model

$$x(k+1) = ax(k) + w(k)$$
$$y(k) = x(k) + v(k)$$

which is also the model used by the Kalman filter

$$K_f(k) = P(k|k-1)C^T(CP(k|k-1)C^T + R_2)^{-1}$$
$$K(k) = (AP(k|k-1)C^T)(CP(k|k-1)C^T + R_2)^{-1}$$
$$\hat{x}(k|k) = \hat{x}(k|k-1) + K_f(k)(y(k) - C\hat{x}(k|k-1))$$
$$\hat{x}(k+1|k) = A\hat{x}(k|k-1) + Bu(k) + K(k)\,(y(k) - C\hat{x}(k|k-1))$$
$$P(k|k) = P(k|k-1) - P(k|k-1)C^T(CP(k|k-1)C^T + R_2)^{-1}CP(k|k-1)$$
$$P(k+1|k) = AP(k|k-1)A^T - K(k)(CP(k|k-1)C^T + R_2)K^T(k) + R_1$$

where $w(k)$ and $v(k)$ are independent Gaussian noises with variances $R_1 = 0$ and $R_2 = 1$ respectively. The initial estimate $\hat{x}(0|-1) = 0$, covariance $P(0|-1) = \infty$. The measurements $y(k)$ for $k = 0$, 1, 2, and 3 are given.

(a) For $a = 1$, find the Kalman filter $\hat{x}(0|0)$, $\hat{x}(1|1)$ and $\hat{x}(2|2)$.

(7 Marks)

(b) For $a = 1$, find the least-squares estimate $\hat{x}(3)$.

(6 Marks)

(c) For $|a| < 1$ and after the Kalman filter gains have reached steady-state, find $K(k)$, $K_f(k)$, $P(k|k)$, $P(k+1|k)$ and the relationship between $\hat{x}(k|k)$ and $\hat{x}(k-1|k-1)$.

(12 Marks)

Q.4 Consider the first-order single-input and single-out process

$$x_p(k+1) = a_p x_p(k) + b_p u(k)$$
$$y(k) = x_p(k)$$

where u, y, x_p, and k are the input, output, state and sampling instance respectively. The model parameters are given as $a_p = 0.8$, and $b_p = 0.4$. The process is placed under model predictive control with prediction horizon $N_p = 3$, control horizon $N_c = 2$, and weight $r_w = 1$.

(a) Determine A, B, and C of the state-space model augmented with an integrator

$$x(k+1) = Ax(k) + B\Delta u(k)$$
$$y(k) = Cx(k)$$

where $\Delta u(k) = u(k) - u(k-1)$, $x(k) = [\Delta x_p(k) \quad y(k)]^T$, $\Delta x_p(k) = x(k) - x(k-1)$.

(5 Marks)

(b) Assuming that the states are not measurable and an observer

$$\hat{x}(k+1) = A\hat{x}(k) + B\Delta u(k) + K_{ob}\big(y(k) - C\hat{x}(k)\big)$$

is used to obtain the estimate, $\hat{x}(k)$. Obtain the observer gain, K_{ob}, for observer poles specified at $z = 0.4, 0.4$.

(8 marks)

(c) The initial conditions are given as $x_p(k) = 0$ for $k \le 0$, $\hat{x}(0) = [0.1 \quad 0.1]^T$, $u(k) = 0$ for $k < 0$ and set-point

$$r(k) = \begin{cases} 0 & k < 0 \\ 1 & k \ge 0 \end{cases}$$

Find $y(0)$, $y(1)$ and $y(2)$.

(12 marks)

Appendix A - **Table of Laplace Transform and Z Transform**

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s) and Z transforms X(z).

Entry #	Laplace Domain	Time Domain	Z Domain (t=kT)
1	1	$\delta(t)$ unit impulse	1
2	$\frac{1}{s}$	$u(t)$ unit step	$\frac{z}{z-1}$
3	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
4	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
5		$b^k \quad (b=e^{-aT})$	$\frac{z}{z-b}$
6	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
7	$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$	$\frac{z(1-e^{-aT})}{a(z-1)(z-e^{-aT})}$
8	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$\frac{z(e^{-aT}-e^{-bT})}{(z-e^{-aT})(z-e^{-bT})}$
9	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}-\frac{e^{-at}}{a(b-a)}-\frac{e^{-bt}}{b(a-b)}$	
10	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	
11	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	
12	$\frac{b}{s^2+b^2}$	$\sin(bt)$	$\frac{z\sin(bT)}{z^2-2z\cos(bT)+1}$
13	$\frac{s}{s^2+b^2}$	$\cos(bt)$	$\frac{z(z-\cos(bT))}{z^2-2z\cos(bT)+1}$
14	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin(bt)$	$\frac{ze^{-aT}\sin(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$
15	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos(bt)$	$\frac{z^2-ze^{-aT}\cos(bT)}{z^2-2ze^{-aT}\cos(bT)+e^{-2aT}}$

END OF PAPER