Space Vector Modelling of AC Machine

How to mathematically describe AC machine behaviour?

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AC Machine Fundamentals

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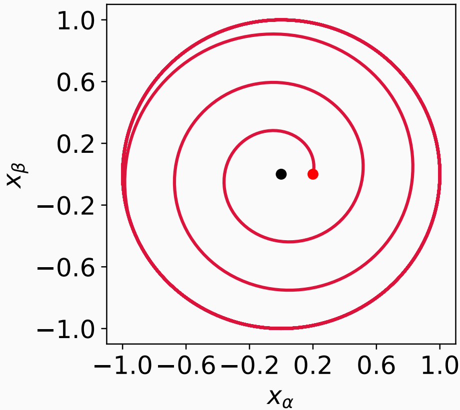
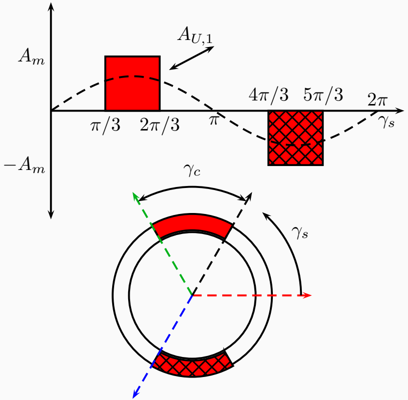
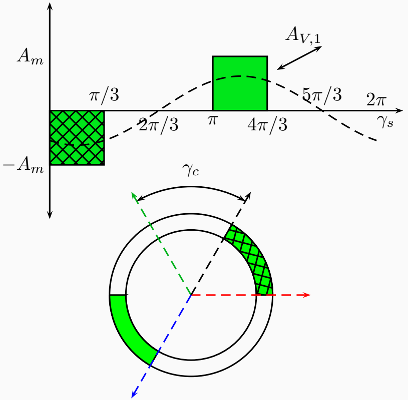


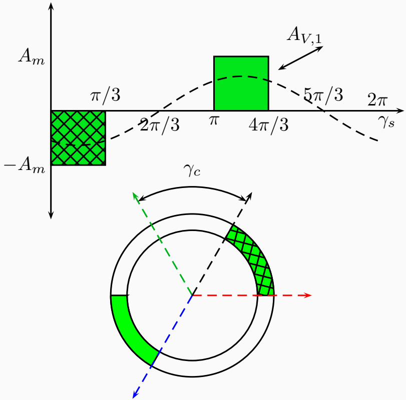
Understanding Space vectors

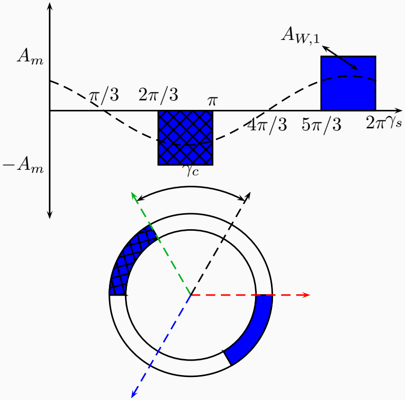
MMF of the three windings



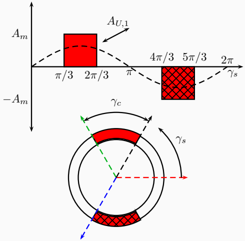


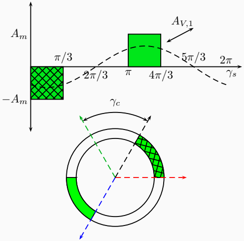
MMF of the three windings

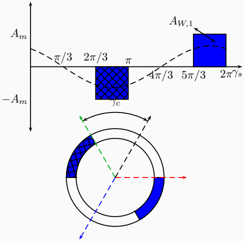


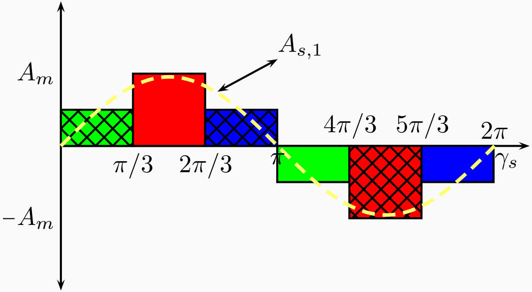


MMF of the three windings

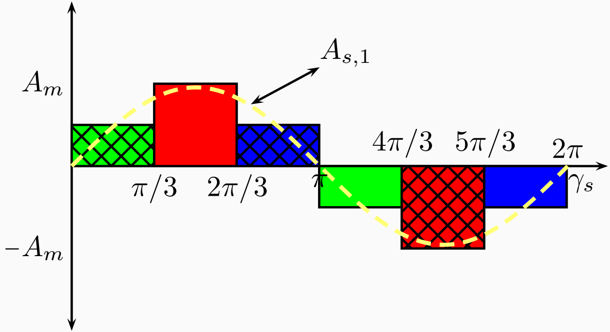








Resulting MMF due to 3 phase distributed winding i



$$A_{s,1}(\gamma_s, t) = \hat{A}_s sin(\gamma_s - \omega_s t) \quad (1)$$

Resulting MMF due to 3 phase distributed winding ii

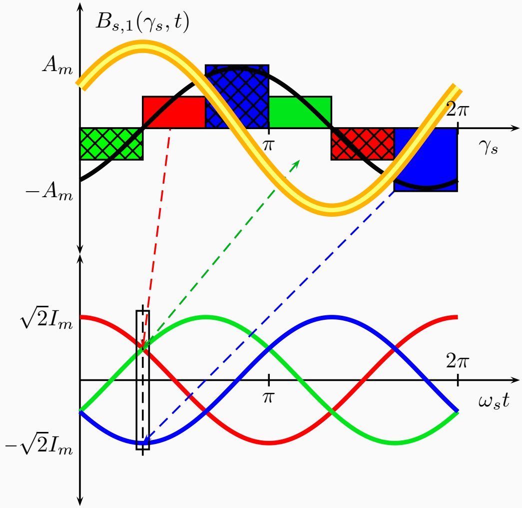
We can obtain the fundamental component of the current resulting current density by adding the individual densities, as

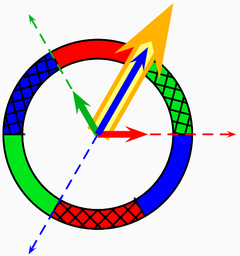
$$\begin{aligned} A_{s,1}(\gamma_s, t) &= \hat{A}_{m,1}[sin(\gamma_s)cos(\omega_s t) + sin(\gamma_s - 2\pi/3)cos(\omega_s t - 2\pi/3) \\ &\quad + sin(\gamma_s - 4\pi/3)cos(\omega_s t - 4\pi/3)] \\ &= \hat{A}_{m,1}\left[\frac{3}{2}sin(\gamma_s - \omega_s t)\right] \\ A_{s,1}(\gamma_s, t) &= \frac{3}{2}\hat{A}_{m,1}\left[sin(\gamma_s - \omega_s t)\right] \end{aligned}$$

$$A_{s,1}(\gamma_s, t) = \hat{A}_s sin(\gamma_s - \omega_s t)$$

Flux density wave produced by stator winding MMF

Hence the flux density wave leads the current density wave by 90^o.





Interaction of Stator and Rotor field produces Torque i

Let us consider a motor with distributed windings on the stator and a permanent magnet on the rotor. The distributed windings on the stator are supplied by three phase voltages that are 120 degrees phase shifted in time. The currents in the distributed three phase windings will result in a current density wave given by Eq.(1). At a particular time instant, this wave can be expressed as

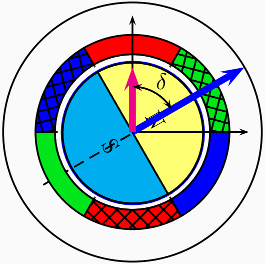
$$A_{s,1}(\gamma_s) = \hat{A}_s sin(\gamma_s) \tag{2}$$

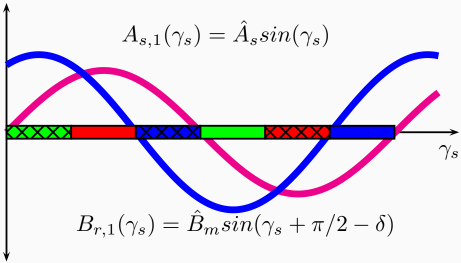
The permanent magnet rotor produces a flux density wave that will travel along the air gap as the rotor moves. At a particular time instant, the flux density wave is given as

$$B_{r,1}(\gamma_S) = B_m cos(\gamma_s - \delta) \tag{3}$$

Interaction of Stator and Rotor field produces Torque ii

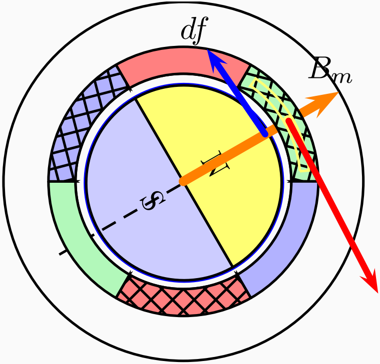
The peak flux density lags the peak current density by δ. If we place a vector along the peak value of the two waves, the current density vector leads the flux density vector by δ as shown in





Torque production

A portion of the stator, $d\gamma_s$ will have a current $A_{s,1}(\gamma_s) r l d\gamma_s$ coming out of the plane of the paper, where r is the radius of the stator and l its depth. This forms like a current sheet along the inner radius of the stator.



It interacts with the perpendicular flux density at that position on the stator, to produce a force.This will produces a tangential force on the stator and by reaction, an equal and opposite force on the rotor, given as

$$df = r l_e A_{s,1} \times \vec{B}_r d\gamma_s \tag{4}$$

To get the torque, we take the moment of the force which gives us $dM_e = rdf$

$$dM_e = rdf \tag{5}$$

$$dM_e = r^2 l_e B_{r,1} A_{s,1} d\gamma_s \tag{6}$$

Space Vectors

Two phase quadrature system

if we have coils a, and b, in quadrature and they are fed with currents

$$i_a(t) = I_m cos(\omega_s t)$$
$$i_b(t) = I_m sin(\omega_s t)$$

we get two flux linkages along the axis of the respective coils. The resultant is a **Space Vector of Flux linkage**, given as

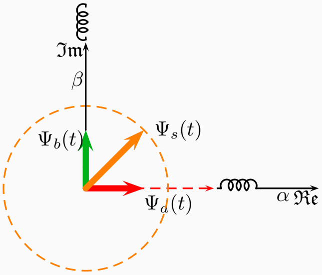
$$\vec{\Psi}_s(t) = \Psi_a(t) + j\Psi_b(t) \quad (7)$$
$$\vec{\Psi}_s(t) = \Psi_m cos(\omega_s t) + j\Psi_m sin(\omega_s t) \quad (8)$$
$$\vec{\Psi}_s(t) = \Psi_m e^{j\omega_s t} \quad (9)$$

where

$$\Psi = LI$$
$$L_a = L_b = L_m$$

and

$$\Psi_m = L_m I_m$$



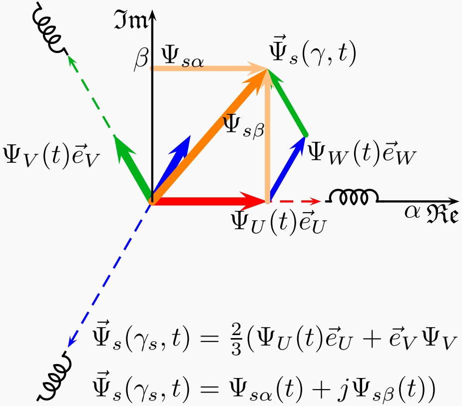
$$\vec{\Psi}_s(\gamma_s, t) = \Psi_{s\alpha}(t) + j\Psi_{s,\beta}(t))$$

Why a quadrature system?

- It is decoupled system (ideally) and mathematically orthogonal
- Minimum number of windings required to model a circular rotating field
- Ideally no flux linkage between the two coils

We have a 3 phase supply, how to create a system like the quadrature 2 coil system

Create a system with 3 phase windings that produces a same flux linkage as a quadrature 2 phase system



$$\vec{\Psi}_s(\gamma_s, t) = \frac{2}{3}(\Psi_U(t)\vec{e}_U + \vec{e}_V\Psi_V(t) + \vec{e}_V\Psi_W(t)$$

$$\vec{\Psi}_s(\gamma_s, t) = \Psi_{s\alpha}(t) + j\Psi_{s\beta}(t))$$

Space Vector: Flux Linkages and Mathematical Model

The vector sum of the individual phase flux linkages can be added vectorially. Given the unit vectors along each axis is

$$\vec{e}_U = 1e^{j0} \quad \vec{e}_V = 1e^{j\frac{2\pi}{3}} \quad \vec{e}_W = 1e^{\frac{4\pi}{3}} \tag{10}$$

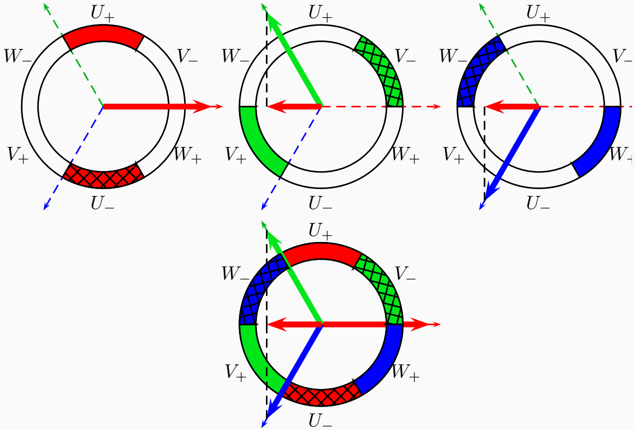
The stator flux linkages produced by the sinusoidal resultant mmf can be expressed as a space vector resulting from the individual phase flux linkages as

$$\vec{\Psi}_s = \frac{2}{3}\big(\Psi_U(t) + \vec{a}\Psi_V(t) + \vec{a}^2\Psi_W(t)\big) \tag{11}$$

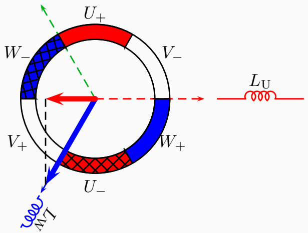
where $\vec{a} = e^{j\frac{2\pi}{3}}$

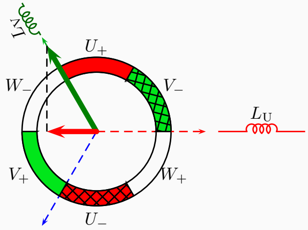
Flux linkages and magnetizing inductance

Flux coupling phase U.



Flux linking phase U i





Total flux linkage of winding U is

$$\Psi_U = w_U[\phi_{UU} + \phi_{UV} + \phi_{UW}]$$

$$\Psi_U = w_U \left[\frac{L_m I_U(t)}{w_U} - \frac{1}{2} \frac{L_m I_V(t)}{w_V} - \frac{1}{2} \frac{L_m I_W(t)}{w_W} \right]$$

Since $w_U = w_V = w_W$, and $I_U(t) = I_m cos(\omega_s t)$,
$I_V(t) = I_m cos(\omega_s t - 120)$ and $I_W(t) = I_m cos(\omega_s t - 240)$

Flux linking phase U ii

we get

$$
\begin{aligned}
\Psi_U =& L_m[I_m cos(\omega_s t) - \frac{1}{2} I_m cos(\omega_s t - 120) - \frac{1}{2} I_m cos(\omega_s t - 240)] \\
\Psi_U =& L_m I_m[cos(\omega_s t) - \frac{1}{2}(cos(\omega_s t)(-\frac{1}{2}) - sin(\omega_s t)(0.866) + \\
& cos(\omega_s t)(-\frac{1}{2}) - sin(\omega_s t)(-0.866))] \\
\Psi_U =& \frac{3}{2} L_m I_m cos(\omega_s t)
\end{aligned}
$$

Hence the total inductance of winding U is

$$
\begin{aligned}
L_U &= \frac{\Psi_U}{I_m cos(\omega_s t)} \\
L_U &= \frac{3}{2} L_m \qquad (12)
\end{aligned}
$$

Flux linking phase U iii

We call this **three phase mutual inductance** of a symmetrical 3-phase machine

Three phase mutual inductance

$$\begin{aligned} L_U &= \frac{3}{2}L_m \\ L_V &= \frac{3}{2}L_m \\ L_W &= \frac{3}{2}L_m \end{aligned}$$

Space Vectors: Describes rotating field quantities produced by time-varying currents i

AC machine consist of electromagnetic quantities that are distributed in space (means the value of the quantity varies with the angular position on the air gap periphery), or varies in time (means the value of the quantity at any position on the air gap periphery varies with time). Both these quantities interact to produce torque. Hence, in order to describe this interaction, we need a method of modelling that incorporates both the space dependent and time dependent quantities. Hence, we use space vectors for the purpose.

Note

What are space vectors? Space vectors are vector quantities (i.e. they have various attributes and not just the magnitude). A space vector quantity is a function of angular position and time.

We know about time phasors from basic complex analysis of AC circuits and using them to draw phasor diagrams. Suppose an AC sinusoidal voltage is given by

$$V = \sqrt(2)V cos(\omega t - \phi) \tag{13}$$

$$V = \frac{\sqrt{2}}{2}[Ve^{-j\phi}e^{j\omega t} + Ve^{j\phi}e^{-j\omega t}] \tag{14}$$

$$V = \frac{\sqrt{2}}{2}[\underline{V}e^{j\omega t} + \underline{V}^* e^{-j\omega t}] \tag{15}$$

Then a time phasor of this voltage can be represented by a vector of magnitude V and angle ϕ with respect to reference phasor say

$$r = 1cos(\omega t) \tag{16}$$

which is a unit vector along the x-axis.

Definition

Time phasors are used to represent steady state sinusoidal quantities and their relation with respect to each other in time.

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Space Vector Definition i

Definition

Space Vector Transformation transforms the electromagnetic quantities of a multi-phase rotating field machine into a quadrature axis 2 phase model. The Flux linkage vector produced in a multiphase machine is made equal to a the flux linkage vector produced by a 2 phase quadrature axis machine

Since the flux linkage in an electromagnetic system is related to the current and the effective inductance, given as

$$\vec{\Psi}_s = L_s \vec{I}_s \tag{17}$$

we can define a current space vector

Space Vector Definition ii

A 3 phase winding excited by 3 phase current system will produce a resultant sinusoidal mmf in the machine. The cause of this mmf can be attributed to a current space vector defined as

$$\vec{I}_s = \frac{2}{3}\left(I_U(t) + \vec{a}I_V(t) + \vec{a}^2 I_W(t)\right) \tag{18}$$

$$\vec{I}_{s,0} = \frac{1}{3}\left(I_U(t) + I_V(t) + I_W(t)\right) \tag{19}$$

where

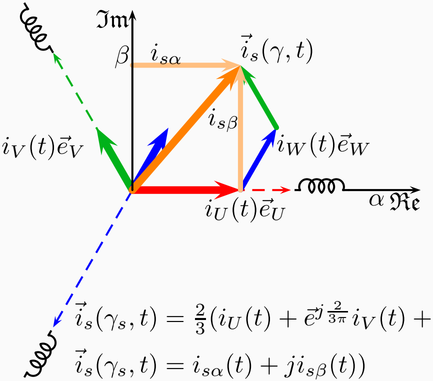
$$I_U(t) = \hat{I}_U(t)cos(\omega_s t) \tag{20}$$

$$I_V(t) = \hat{I}_V(t)cos(\omega_s t - 2\pi/3) \tag{21}$$

$$I_W(t) = \hat{I}_W(t)cos(\omega_s t - 4\pi/3) \tag{22}$$

$$\vec{a} = e^{j\frac{2\pi}{3}} \tag{23}$$

Space Vector Definition iii



$$\vec{i}_s(\gamma_s, t) = \frac{2}{3}(i_U(t) + \vec{e}^{j\frac{2}{3\pi}} i_V(t) + \vec{e}^{j\frac{4}{3\pi}} i_W(t))$$

$$\vec{i}_s(\gamma_s, t) = i_{s\alpha}(t) + j i_{s\beta}(t))$$

It is seen that a space vector is a complex quantity by definition. Therefore the trajectory of the space vector describes the instantaneous value of the electromagnetic variable in a complex plane.

Space Vector Definition: General

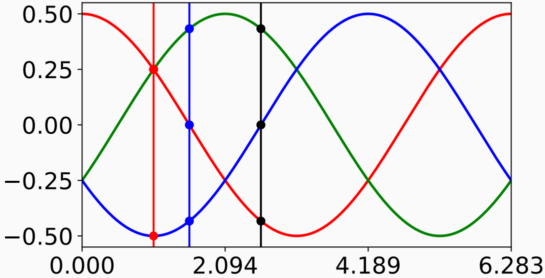
A 3 phase time varying system represented by $x_u(t), x_v(t), x_w(t)$ can be converted into a complex vector. The complex plane is defined by 2 stationary axis α(Real) and β(Imaginary). The space vector is given by

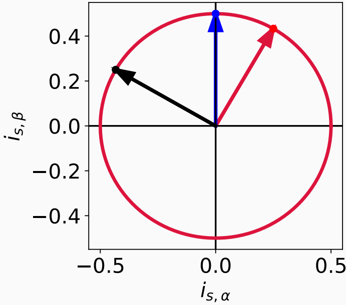
$$\boxed{\vec{X}_s = X_\alpha + jX_\beta = \frac{2}{3}\left(x_u(t) + \vec{a}x_v(t) + \vec{a}^2 x_w(t)\right)} \qquad (24)$$

where

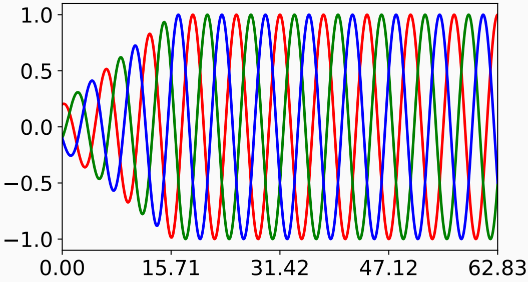
$$\vec{a} = e^{\frac{2\pi}{3}}$$

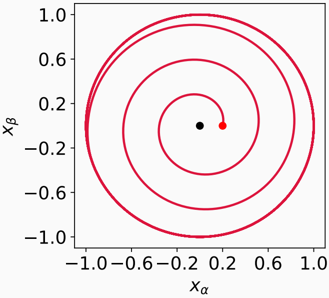
Defining Space vectors: Visually





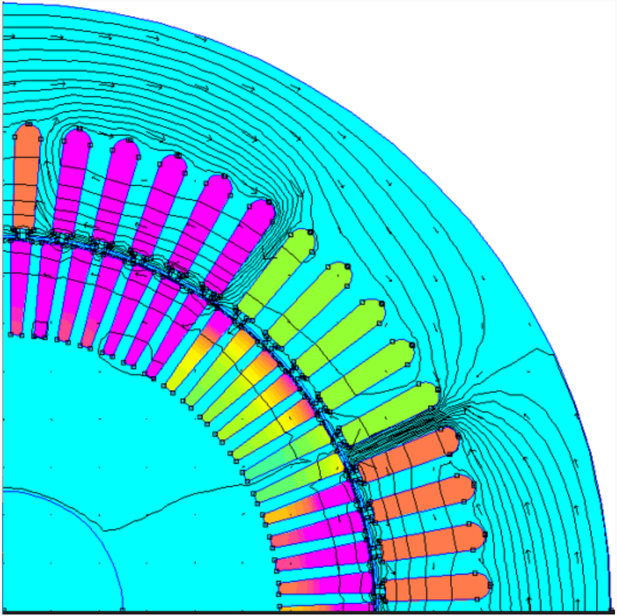
Space Vectors can describe dynamics due to time-varying phase variables





Curve is the trajectory followed by the tip of the space vector

Stray Flux and main Flux



Flux linkage for rotating field

The **three phase mutual inductance is** $L_h = \frac{3}{2} L_m$. Every winding has some leakage flux and this flux is associated only with that winding and not with any other winding. Hence the leakage flux space vector is easily found as

$$\vec{\Psi}_{s\sigma} = L_{s\sigma} \vec{I}_s \tag{25}$$

Hence the flux linkage space vector is the sum of the two. Therefore the **three phase self inductance** is defined as

Self-inductance of a 3 phase machine

$$L_s = L_h + L_{s\sigma} \tag{26}$$

Voltage space vector from 3 phase flux linkage

Since the induced voltage is given by (Faraday's law)

$$V_i = \frac{d\Psi}{dt} \tag{27}$$

we can also talk about a voltage space vector as

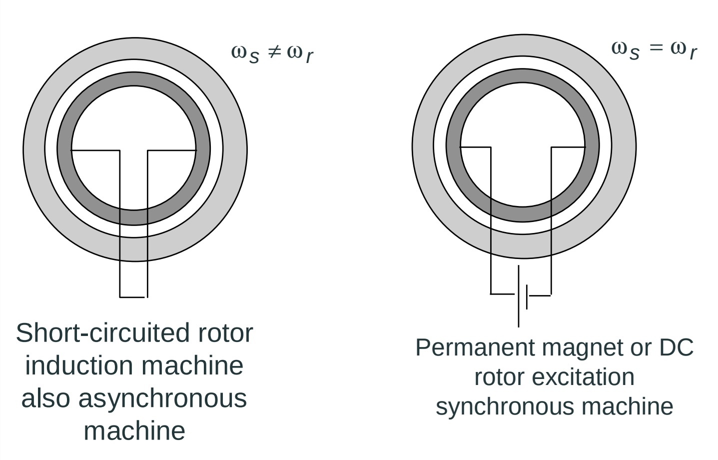
$$\vec{V}_s = \frac{2}{3}\left(V_a(t) + \vec{a}V_b(t) + \vec{a}^2 V_c(t)\right) \tag{28}$$

For a practical stator winding, considering resistive drop, we can write

Model of a Stator winding

$$\vec{V}_s = R_s \vec{I}_s + \frac{d\vec{\Psi}_s}{dt} \tag{29}$$

Types of AC machines



For both cases we can model a rotor flux and voltage space vector as

$$\vec{V}_r = R_r \vec{I}_r + \frac{d\vec{\Psi}_r}{dt} \qquad (30)$$

Torque in terms of Space Vectors

The instantaneous power is given as

$$P = \frac{3}{2}\mathfrak{Re}\{\vec{V}_s^* \vec{I}_s\} \tag{31}$$

where $\vec{V}_s^*$ is the complex conjugate of the voltage space vector. To prove this we will write

$$P = \frac{3}{2}\Re\left\{\frac{2}{3}\left[V_U + \vec{a}^* V_V + \vec{a}^{*2} V_W\right]\frac{2}{3}[I_U + \vec{a} I_b + \vec{a}^2 I_c\right]\right\} \tag{32}$$

$$P = \frac{2}{3}\Re\{V_U I_U + V_V I_V + V_W I_W + \tag{33}$$

$$\ldots \vec{a}^2(V_U I_W + V_V I_U + V_W I_V) + \vec{a}(V_U I_V + V_V I_W + V_W I_U)\} \tag{34}$$

Since

$$\vec{a}^* = \vec{a}^{-1} = \vec{a}^2$$

$$\vec{a}^{*2} = \vec{a}^{-2} = \vec{a}$$

$$\Re\{\vec{a}\} = \Re\{\vec{a}^2\} = -\frac{1}{2}$$

Hence

$$P = \frac{2}{3}\left\{V_U I_U + V_V I_V + V_W I_W - \frac{1}{2}\left[V_U(I_V + I_W) + V_V(I_U + I_W) + V_W(I_U +\right.\right.$$

Since for a three phase three wire system $I_U + I_V + I_W = 0$ we get, $I_V + I_W = -I_U$ and so on, hence

$$P = \frac{2}{3}\left\{V_U I_U + V_V I_V + V_W I_W + \frac{1}{2}\left[V_U I_U + V_V I_V + V_W I_W\right]\right\}$$

We get

Power using Space vectors

$$P = V_U I_U + V_V I_V + V_W I_W \tag{35}$$

Torque definition

Electromagnetic Torque

For now we will define the torque with a working definition. The instantaneous electromagnetic torque is given by

$$M_e = \frac{3}{2}\mathfrak{Im}\left\{\vec{\Psi}_s^* \vec{I}_s\right\} \tag{36}$$

It can be also written as

$$\boxed{M_e = \frac{3}{2}\left(\vec{\Psi}_s \times \vec{I}_s\right)} \tag{37}$$

We will see a complete derivation of the equation in context of induction machine in the next chapter

Normalization: Why use normalization? i

- Instead of using the actual values we will use per unit values.
- The per unit values have no dimensions (cannot be written as xx volts or yy Amps).
- In so doing we can compare machines of different power ratings.

Example

If we say a motor requires 10 amperes when starting and want to compare with another motor of higher power rating. It is possible that the higher power motor requires 200 amperes, then what can we say about the starting characteristics. In order to compare it we look at the rated current of the motor in the low power motor the rated current may be just 5 A whereas as the rated current in higher power motor may be 50 A. In which case we can say that the higher power motor requires four times the rated current at starting as compared to the low power motor that takes only twice the rated current.

Using base quantities for normalization

Voltage	peak value of rated phase voltage	$\hat{V}_{Un}$
Current	peak value of rated phase current	$\hat{I}_U$
Impedance	ratio of base voltage and base current	$Z_B = \frac{\hat{V}_{an}}{\hat{I}_a}$
angular velocity	rated ang. velocity.	$\omega_R = 2\pi f_R$
mechanical angular velocity	–	$\Omega_B = \frac{\omega_R}{p}$
Power	–	$S_B = \frac{3}{2}\hat{V}_{Un}\hat{I}_U$
Torque	Base power/Base speed	$M_B = \frac{3p\hat{V}_{Un}\hat{I}_a}{2\omega_R} = \frac{3\hat{V}_{Un}\hat{I}_U}{2\Omega_B}$
time	–	$t_B = \frac{1}{\omega_R}$
inductance	–	$L_B = \frac{Z_B}{\omega_R}$
flux linkage	–	$\Psi_B = \frac{\hat{V}_{Un}}{\omega_R}$

We can express the stator voltage equation as

$$\vec{V}_s = R_s \vec{I}_s + \frac{d\vec{\Psi}_s}{dt} \tag{38}$$

$$\vec{v}_s \hat{V}_{Un} = R_s \vec{i}_s \hat{I}_U + \frac{d(\vec{\psi}_s \Psi_B)}{d\tau t_B} \tag{39}$$

$$\vec{v}_s = \frac{R_s \hat{I}_U}{\hat{V}_{Un}} \vec{i}_s + \frac{\hat{V}_{Un}\omega_R}{\omega_R \hat{V}_{Un}} \frac{d\vec{\psi}_s}{d\tau} \tag{40}$$

$$\vec{v}_s = r_s \vec{i}_s + \frac{d\vec{\psi}_s}{d\tau} \tag{41}$$

where $r_s = \frac{R_s \hat{I}_U}{\hat{V}_{Un}}$

Process of Normalization ii

Definition

Normalized space vector voltage equation

$$\vec{v}_s = r_s \vec{i}_s + \frac{d\vec{\psi}_s}{d\tau}$$

Normalization of Flux linkages and inductance i

$$\vec{\Psi}_s = L_s \vec{I}_s + L_h \vec{I}_r$$
$$\vec{\Psi}_r = l_h \vec{I}_s + L_r \vec{I}_r$$

We can now write it as

$$\vec{\psi}_s \Psi_B = L_s \vec{i}_s \hat{I}_U + L_h \vec{i}_r \hat{I}_U \tag{42}$$
$$\vec{\psi}_s = \frac{L_s \hat{I}_U}{\Psi_B} \vec{i}_s + \frac{L_h \hat{I}_U}{\Psi_B} \vec{i}_r \tag{43}$$
$$\vec{\psi}_s = \frac{\omega_R L_s \hat{I}_U}{\hat{V}_{Un}} \vec{i}_s + \frac{\omega_R L_h \hat{I}_U}{\hat{V}_{Un}} \vec{i}_r \tag{44}$$
$$\vec{\psi}_s = l_s \vec{i}_s + l_h \vec{i}_r \tag{45}$$

Normalization of Flux linkages and inductance ii

Definition

Normalized inductance and reactance

$$l_s = \frac{\hat{I}_U}{\Psi_B} L_s = \omega_R L_s \frac{\hat{I}_U}{\hat{V}_{Un}} = x_s$$

Normalized Power and Torque i

We can normalize power as

$$\frac{P}{S_B} = \frac{\frac{3}{2}\Re\{\vec{V}_s^*\vec{I}_s\}}{\frac{3}{2}\hat{V}_{Un}\hat{I}_U}$$

$$p_e = \Re\{\frac{\vec{V}_s^*}{\hat{V}_{Un}}\frac{\vec{I}_s}{\hat{I}_{Un}}\}$$

$$p_e = \Re\{\vec{v}_s^*\vec{i}_s\}$$

$$p_e = \Re\{(v_{s\alpha} - jv_{s\beta})(i_{s\alpha} + ji_{s\beta})\}$$

$$p_e = v_{s\alpha}i_{s\alpha} + v_{s\beta}i_{s\beta}$$

Normalized Power and Torque ii

Definition

Normalized Power

$$p_e = v_{s\alpha} i_{s\alpha} + v_{s\beta} i_{s\beta} \tag{46}$$

Normalized Power and Torque iii

and Torque as

$$
\begin{aligned}
\frac{M_e}{M_B} &= \frac{\Omega_B \frac{3p}{2} \Im\{\vec{\Psi}_s^* \vec{I}_s\}}{\frac{3}{2}\hat{V}_{Un}\hat{I}_U} \\
m_e &= \Im\{\frac{\vec{\Psi}_s^* \Omega_B p}{\hat{V}_{Un}} \frac{\vec{I}_s}{\hat{I}_{Un}}\} \\
m_e &= \Im\{\frac{\vec{\Psi}_s^* \omega_R}{\hat{V}_{Un}} \frac{\vec{I}_s}{\hat{I}_{Un}}\} \\
m_e &= \Im\{\vec{\psi}_s^* \vec{i}_s\} \\
m_e &= \Im\{(\psi_{s\alpha} - j\psi_{s\beta})(i_{s\alpha} + j i_{s\beta})\} \\
m_e &= \psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha}
\end{aligned}
$$

Normalized Power and Torque iv

Definition

Normalized Torque

$$m_e = \psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha} \tag{47}$$

Space Vector representation of an AC Machine

From the basic electromagnetic relation we can write the equation for the voltage on the stator winding as

$$\vec{v}_s^s = r_s \vec{i}_s^s + \frac{d\vec{\psi}_s^s}{d\tau} \tag{48}$$

where the superscript stands for the stator coordinate system which is fixed to the stator frame. We define a complex coordinate system fixed to the stator with the real axis along the phase a and an orthogonally placed imaginary axis.

Definition

This is the basic definition that we get for any space vector.

$$\vec{x}_s = \vec{x}_\alpha + j\vec{x}_\beta \tag{49}$$

$$\vec{x}_\alpha = \vec{1}e^{(0)} \tag{50}$$

$$\vec{x}_\beta = \vec{1}e^{j\pi/2} \tag{51}$$

Applying Faraday's and Ohm's law to the rotor electromagnetic circuit, we can write

$$\vec{v}_r^r = r_r\vec{i}_r^r + \frac{d\vec{\psi}_r^r}{d\tau} \tag{52}$$

The equation is written for space vector quantities as seen from the rotor.

What does as seen from rotor mean?

The super-script ^{s} stands from stator reference frame and ^{r} stands for rotor reference frame. We will see this in detail in the next chapter on Induction motor. It will be easier to grasp the concept there.

The coupling between the stator and the rotor is given by

$$\vec{\psi}_s^s = l_s \vec{i}_s^s + l_h \vec{i}_r^s \tag{53}$$

$$\vec{\psi}_r^s = l_h \vec{i}_s^s + l_r \vec{i}_r^s \tag{54}$$

Space Vector representation of AC Machine iv

where $\vec{\psi}_s$ and $\vec{\psi}_r$ are the stator and the rotor flux linkage respectively. The inductance are expressed as

$$l_s = l_{s\sigma} + l_h \tag{55}$$

$$l_r = l_{r\sigma} + l_h \tag{56}$$

$$l_{s\sigma} = \sigma_s l_h \tag{57}$$

$$l_{r\sigma} = \sigma_r l_h \tag{58}$$

$$l_s = l_h(1 + \sigma_s) \tag{59}$$

$$l_r = l_h(1 + \sigma_r) \tag{60}$$

Space Vector representation of AC Machine v

The leakage factor or also called as the stray factor is given as

$$\sigma = 1 - \frac{1}{(1+\sigma_s)}\frac{1}{(1+\sigma_r)} \tag{61}$$

$$\sigma = 1 - \frac{l_h}{l_s}\frac{l_h}{l_r} \tag{62}$$

$$\sigma = 1 - \frac{l_h^2}{l_s l_r} \tag{63}$$

Torque in AC Machines

The torque is expressed as a vector product and is given by

$$m_e = \vec{\psi}_s^s \times \vec{i}_s^s \tag{64}$$

The torque energizes the mechanical system, the dynamics of which is given by

$$\tau_m \frac{d\omega}{d\tau} = m_e - m_L \tag{65}$$

where m_L is the load torque and τ_m is the mechanical time constant of rotor and the load.

To note

The earlier part explains how and why we got here. The normalized equations of AC machine is where we really start modeling and controlling the machine.

Equations describing the AC machine

Equations you should learn well!

The AC machine system description of given as

$$\vec{v}_s^s = r_s \vec{i}_s^s + \frac{d\vec{\psi}_s^s}{d\tau} \tag{66}$$

$$0 = r_r \vec{i}_r^s + \frac{d\vec{\psi}_r^s}{d\tau} - j\omega\psi_r^s \tag{67}$$

$$m_e = \vec{\psi}_s^s \times \vec{i}_s^s \tag{68}$$

$$m_e = \Im\{\vec{\psi}_s^* \vec{i}_s\} \tag{69}$$

$$m_e = \Im\{(\psi_{s\alpha} - j\psi_{s\beta})(i_{s\alpha} + j i_{s\beta})\} \tag{70}$$

$$p_e = \Re\{\vec{v}_s^* \vec{i}_s\} \tag{71}$$

$$p_e = \Re\{(v_{s\alpha} - j v_{s\beta})(i_{s\alpha} + j i_{s\beta})\} \tag{72}$$

$$\tau_m \frac{d\omega}{d\tau} = m_e - m_L \tag{73}$$

Revision: What you should be able to do i

- Please do not learn to derive all equation...they are meant only for completion
- You should derive a space vector is obtained from 3 phase time-varying quantities
- You should able to use the flux linkage relation of stator and rotor
- You should be able to write the stator and rotor voltage equations in terms of space vectors
- You should be able to write and explain the torque equation in terms of space vector
- You should be able to normalize the machine parameters and explain the behaviour using normalized parameters
- Though there are too many equations in this section, they will not be tested upon.

Revision: What you should be able to do ii

- How a torque is produce? How the size of the machine depends on Power and speed are things you should know all your life as an engineer.
- Focus on space vectors...you should be able to explain space vectors in terms of rotating field theory
- you should be able to describe the AC machine in terms of space vectors: doing so you can model any AC machine and also can simulate it