Note Title 1/20/2011

4

Regarding controllability & realizations

Recall the simple example:

$$\frac{Y(s)}{U(s)} = \frac{(s+\alpha)}{(s+1)(s+2)}$$

Consider taking "partial fractions":

$$\frac{Y(s)}{U(s)} =$$

$$= \frac{1}{(s+1)} \bar{g}_1 + \frac{1}{(s+2)} \bar{g}_2$$

We can re-write this as :

$$Y(s) = \underbrace{\frac{\bar{g}_1}{(s+1)} U(s)}_{W_1(s)} + \underbrace{\frac{\bar{g}_2}{(s+2)} U(s)}_{W_2(s)}$$

$$-\lambda_1 \qquad\qquad -\lambda_2$$

$$\dot{W}_1 =$$

$$\dot{W}_2 =$$

$$y =$$

$$\frac{W_1(s)}{U(s)} = \frac{\bar{g}_1}{s - \lambda_1}$$

A rough physical meaning for controllability:

* Diagonal canonical form
or
* Modal canonical form

$$\dot{x} = Fx + Gu$$

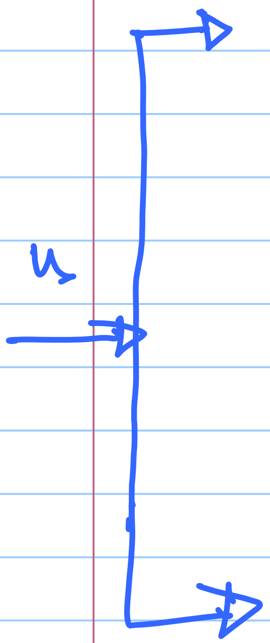
$$\Big\downarrow T$$

$$\dot{w} = \bar{F}w + \bar{G}u$$

n=2 example:

$$\begin{bmatrix} \dot{w}_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix} u$$

$$y = \begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$



u

Transformations between realisations

$$\dot{x} = F_1 x + G_1 u$$

$$p = T x$$

$$\dot{p} = F_2 p + G_2 u$$

From earlier, we have

$$F_2 = T F_1 T^{-1}$$

$$G_2 = T G_1$$

Then, note that

$$\mathcal{C}_2 = \left[G_2 \quad F_2 G_2 \quad \cdots \quad F_2^{n-1} G_2 \right]$$

Observe each term in the ~~above~~ above :

$$G_2 = T G_1$$

$$F_2 G_2 =$$

$$=$$

$$F_2^2 G_2 =$$

$$=$$

$$\vdots$$

$$F_2^r G_2 = \ldots =$$

$$\vdots$$

Thus,

$$\mathcal{C}_2 = \left[G_2 \quad F_2 G_2 \quad F_2^2 G \ \ldots \ F_2^{n-1} G_2 \right]$$

$$=$$

i.e.

$$\mathcal{C}_2 =$$

$$\boxed{T = \qquad}$$

!!!