Tutorial 3

1. Let M(x) denote "x is male", F(x) denote "x is female", C(x) denote "x is a student in this class", and K(x, y) denote "x knows y". State the universe of discourse and translate the following proposition into a predicate logic formula: "Every female student in this class knows at least one male student in this class."

Solution: Let the universe of discourse be all people. Then we have

$$\forall x\, ((C(x) \wedge F(x)) \rightarrow \exists y\, (C(y) \wedge M(y) \wedge K(x,y)))$$

Alternatively, we could let the universe of discourse be all the students in this class. Then we can write the proposition as

$$\forall x\, F(x) \rightarrow \exists y\, (M(y) \wedge K(x, y))$$

2. Let the universe of discourse be all humans and let F(x, y) denote "x and y are friends".
   a. Translate the expression $\forall a\, \exists b\, F(a, b)$ into English

Solution: The statement literally translates to, "for every human a, there exists another human *b* such that the a and b are friends." This is equivalent to saying "everyone has at least one friend."

   b. Is the statement $\forall a\, \exists b\, F(a, b)$ the same as $\exists b\, \forall a\, F(a, b)$? Justify your answer.

Solution: The first statement, as we saw before, says that "everyone has at least one friend." The second statement literally translates to, "there exists a human b such that for any human *a*, *a* is friends with *b*." This is equivalent to saying "someone is friends with everyone." This is not the same thing as "everyone has at least one friend." For instance, each person may be friends with all other humans except one. In this case, it's easy to see that the first statement is true while the second one isn't. Therefore, they aren't the same.

3. Let the universe of discourse be { a, b, c }. Write out the following propositions explicitly so that they do not contain any universal or existential quantifiers:
   a. $\forall x\, \exists y\, F(x, y)$

Solution:

$$\begin{aligned}\forall x \exists y\, F(x,y) \equiv ((&F(a,a) \vee F(a,b) \vee F(a,c)) \wedge \\ (&F(b,a) \vee F(b,b) \vee F(b,c)) \wedge \\ (&F(c,a) \vee F(c,b) \vee F(c,c)))\end{aligned}$$

   b. $\exists x\, \forall y\, F(x, y)$

Solution:

$$\begin{aligned}\exists x \forall y\, F(x,y) \equiv ((&F(a,a) \wedge F(a,b) \wedge F(a,c)) \vee \\ (&F(b,a) \wedge F(b,b) \wedge F(b,c)) \vee \\ (&F(c,a) \wedge F(c,b) \wedge F(c,c)))\end{aligned}$$

   c. $\neg(\forall x\, \exists y\, F(x, y))$

Solution:

$$
\begin{aligned}
\neg(\forall x \exists y\, F(x,y)) &\equiv \neg((F(a,a) \vee F(a,b) \vee F(a,c)) \wedge \\
&\qquad (F(b,a) \vee F(b,b) \vee F(b,c)) \wedge \\
&\qquad (F(c,a) \vee F(c,b) \vee F(c,c))) \\
&\equiv (\neg(F(a,a) \vee F(a,b) \vee F(a,c)) \vee \\
&\qquad \neg(F(b,a) \vee F(b,b) \vee F(b,c)) \vee \\
&\qquad \neg(F(c,a) \vee F(c,b) \vee F(c,c))) \\
&\equiv ((\neg F(a,a) \wedge \neg F(a,b) \wedge \neg F(a,c)) \vee \\
&\qquad (\neg F(b,a) \wedge \neg F(b,b) \wedge \neg F(b,c)) \vee \\
&\qquad (\neg F(c,a) \wedge \neg F(c,b) \wedge \neg F(c,c)))
\end{aligned}
$$

4. Negate the following expression: $\exists x\ M(x) \vee N(x)$

   Solution: $\forall x\ \neg M(x) \wedge \neg N(x)$