Specification for Tutorial 2

1. Which of the following implication statements would evaluate to a value of true?
   a. If 1 + 11 = 12, then 2 + 2 = 5.
   The antecedent (i.e., the left-hand side or the "if" part of the conditional) is true but the consequent (i.e., the right-hand side or the "then" part of the conditional) is false. This means the statement antecedent → consequent is false.
   b. If 6 + 3 = 9, then 2 + 2 = 4.
   The antecedent is true and the consequent is true. This means the statement is true.
   c. If 12 + 4 = 7, then 2 + 2 = 4.
   The antecedent is false and the consequent is true. This means the statement is true.
   d. If 7 + 2 = 4, then 2 + 2 = 5.
   The antecedent is false and the consequent is false. This means the statement is true.

2. Translate the following sentences in English into propositions. Let T denote the proposition "Tom is a good tennis player", P denote "Tom plays tennis every day", and S denote "Tom practices his serve".
   a. Tom is a good tennis player who practices his serve but does not play every day.
   $T \wedge S \wedge \neg P$
   b. If Tom does not practice his serve then Tom is not a good tennis player.
   $\neg S \rightarrow \neg T$

3. Translate the following propositions into English. Let A denote the proposition "the computer in the lab uses Linux", B denote "a hacker breaks into the computer", and C denote "the data on the computer is lost".
   a. $(A \rightarrow \neg B) \wedge (\neg B \rightarrow \neg C)$
   If the computer in the lab uses Linux then a hacker won't break into the computer and if a hacker won't break into the computer then the data on the computer will not be lost.
   b. $C \leftrightarrow (\neg A \wedge B)$
   The data on the computer is lost if, and only if, the computer in the lab doesn't use Linux and a hacker breaks into the computer.

4. Determine whether the following propositions are tautologies, contradictions, or contingencies by using both truth tables and a sequence of logical equivalences.
   a. $\neg((p \wedge q) \rightarrow \neg(\neg q \wedge r))$

Solution:

$\neg((p \wedge q) \rightarrow \neg(\neg q \wedge r)) \equiv \neg(\neg(p \wedge q) \vee \neg(\neg q \wedge r))$	Implication Equivalence
$\equiv (p \wedge q) \wedge (\neg q \wedge r)$	De Morgan's Law
$\equiv p \wedge (q \wedge \neg q) \wedge r$	Associative Law
$\equiv p \wedge \mathbf{F} \wedge r$	Negation Law
$\equiv \mathbf{F}$	Domination

Therefore, the proposition $\neg((p \wedge q) \rightarrow \neg(\neg q \wedge r))$ is a contradiction.

   b. $(\neg(r \rightarrow \neg p) \vee (r \wedge q)) \vee \neg((p \rightarrow r) \wedge (q \rightarrow r))$

   Solution: $(\neg(r \rightarrow \neg p) \vee (r \wedge q)) \vee \neg((p \rightarrow r) \wedge (q \rightarrow r))$

$\equiv (\neg(\neg r \vee \neg p) \vee (r \wedge q)) \vee \neg((\neg p \vee r) \wedge (\neg q \vee r))$	Implication Equivalence
$\equiv ((r \wedge p) \vee (r \wedge q)) \vee \neg((\neg p \vee r) \wedge (\neg q \vee r))$	De Morgan's Law
$\equiv (r \wedge (p \vee q)) \vee \neg(r \vee (\neg p \wedge \neg q))$	Distributive Law
$\equiv (r \wedge (p \vee q)) \vee (\neg r \wedge \neg(\neg p \wedge \neg q))$	De Morgan's Law
$\equiv (r \wedge (p \vee q)) \vee (\neg r \wedge (p \vee q))$	De Morgan's Law
$\equiv (p \vee q) \wedge (r \vee \neg r)$	Distributive Law
$\equiv (p \vee q) \wedge \mathbf{T}$	Negation Law
$\equiv p \vee q$	Idempotence

Therefore, the proposition $(\neg(r \rightarrow \neg p) \vee (r \wedge q)) \vee \neg((p \rightarrow r) \wedge (q \rightarrow r))$ is a contingency.