Name: Nem Zutkovic **COMP 4102:** Assignment 1 February 11, 2020
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1. Time complexity of convolution with N by N sized kernel when using a direct convolution with a square 2d mask is ***$O(n^2 m^2)$***, but when using a separable kernel, the time complexity is ***$O(n^2 m)$***.

2A. False
2B. False
2C. False
2D. False

3. Initially, the image will begin to lose its' sharpness and become blurrier as the kernel (convolution filter) is applied to the source pixel and it's surrounding 8 pixels repeatedly. After a point, the image will simply remain the way it is because the filter output will become equal to the average of all the elements in the 3 by 3 matrix.

4. It is possible to get the same final result by just performing one convolution. The filter to do this is the Gaussian Filter. This is because that filter can perform multiple operations with a kernel.

5A.

$[x \quad y]^T = \begin{bmatrix} x \\ y \end{bmatrix}$ and $x' = sx$, $y' = sy$…therefore $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ sy \end{bmatrix}$

5B.

Transformation matrix for homog. coordinates (scaling): **S** = $\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} sx \\ sy \\ 1 \end{bmatrix}$

5C.

Transformation matrix for homog. coordinates (scaling/translation): **ST** = $\begin{bmatrix} s & 0 & t_x \\ 0 & s & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} sx + t_x \\ sy + t_y \\ 1 \end{bmatrix}$

5D.

Equivalent for the above (5C) matrix for 3-D vectors: $\begin{bmatrix} s & 0 & 0 & t_x \\ 0 & s & 0 & t_y \\ 0 & 0 & s & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} sx + t_x \\ sy + t_y \\ sz + t_z \\ 1 \end{bmatrix}$

6.

$\boldsymbol{Ax = b}$ where $\boldsymbol{A} = \begin{matrix}1 & 0\\ 0 & 1\\ 1 & 1\end{matrix}$ and $\boldsymbol{b} = \begin{matrix}1\\ 1\\ 0\end{matrix}$ and the Least Square solution $\boldsymbol{x = (A^TA)^{-1}\, A^Tb}$

$$x = \left[\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\\ 1 & 1\end{bmatrix}\right]^{-1}\left[\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}\right] = \begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}^{-1}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$x = \left[\frac{1}{det\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}}\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\right]\begin{bmatrix}1\\ 1\end{bmatrix}$ where the ***determinant = ad – bc*** = 2 x 2 – 1 x 1 = 4 – 1 = 3

$$x = \left[\frac{1}{3}\right]\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{2}{3} & -\frac{1}{3}\\ -\frac{1}{3} & \frac{2}{3}\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}\frac{1}{3}\\ \frac{1}{3}\end{bmatrix}$$

$\boldsymbol{e = b - Ax}$ where e is the error vector

$$e = \begin{bmatrix}1\\ 1\\ 0\end{bmatrix} - \begin{bmatrix}1 & 0\\ 0 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{3}\\ \frac{1}{3}\end{bmatrix} = \begin{bmatrix}1\\ 1\\ 0\end{bmatrix} - \begin{bmatrix}\frac{1}{3}\\ \frac{1}{3}\\ \frac{2}{3}\end{bmatrix} = \left[\begin{bmatrix}\frac{2}{3}\\ \frac{2}{3}\\ -\frac{2}{3}\end{bmatrix}\right]$$

Therefore, if $\boldsymbol{A^Te = 0}$, then the error vector is orthogonal to the columns of A.

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}\frac{2}{3}\\ \frac{2}{3}\\ -\frac{2}{3}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

Therefore, the error vector (*e)* is orthogonal to the columns of A.