COMP 1805A SUMMER 2017 – ASSIGNMENT 2 NEMANJA ZUTKOVIC (101085982)

1A.
Let S be the premise that it is snowing.
Let H be the premise that I brought my hat to work.
$S \longrightarrow H$
H
$\therefore S$

The argument is invalid, because the first and second premises are true, but the conclusion is false. The second premise states that you brought your hat, but just because you brought your hat, does not mean it is snowing. It could be raining, very sunny, etc...

1B.
Let P(x) be the predicate “x has a PC”.
Let G(x) be the predicate “x plays computer games”.
Let C(x) be the predicate “x is a student taking COMP1501”.
$\forall\ P(x) \longrightarrow G(x)$
$\forall\ C(x) \longrightarrow G(x)$
$\therefore \forall\ C(x) \longrightarrow P(x)$

The argument is invalid, because both premises cannot be validated through quantifier negation and logical equivalencies to form the conclusion. In other words, you may be taking COMP1501, play computer games, <u>and still not have a PC</u>. Just because you are taking COMP1501, does not conclude that you have a PC, so the conclusion is false.

2.

① $\sqrt{9} + \sqrt{7}$ is an irrational number.	
② $\sqrt{9} + \sqrt{7}$ is an rational number.	**By Contradiction ①**
③ $\sqrt{9} + \sqrt{7} = (\frac{a}{b}) \land (\frac{a}{b}$ in lowest terms)	**By Definition (Rational Number) ②**
④ $\sqrt{9} + \sqrt{7} = \frac{a}{b}$	**By Simplification ③**
⑤ $3 + \sqrt{7} = \frac{a}{b}$	**By Math ④**
⑥ $\sqrt{7} = \frac{a}{b} - 3$	**By Math ⑤**

Lemma 1 (Prove 3 is rational number)

① $(\frac{3}{1}) \land (\frac{3}{1}$ in lowest terms)	**By Definition (Rational Number)**
② $\frac{3}{1}$	**By Simplification ①**

$\therefore \frac{3}{1}$ is an integer that is in lowest terms, which makes +/– 3 a rational number.

Back to Original Proof

① $\sqrt{7} = \frac{a}{b} - \frac{3}{1}$	**By Lemma 1**
② $\sqrt{7} = \frac{a}{b}$	**By Closure of Rational Numbers/Lemma 1 ①**
③ $7 = a^2 / b^2$	**By Math ②**
④ $7b^2 = a^2$	**By Math ③**
⑤ $a^2 = 7k$	**By Math ④**

Lemma 2 (Prove a/a² is divisible by 7)

① $a^2 \div 7 \longrightarrow a \div 7$	
② $\neg(a^2) \div 7 \lor a \div 7$	**By Implication Equivalence ①**
③ $\neg((\neg a^2) \div 7 \lor a \div 7)$	**By Contradiction ②**
④ $\neg\neg a^2 \div 7 \land \neg a \div 7$	**By De Morgan's Law ③**
⑤ $a^2 \div 7 \land \neg a \div 7$	**By Double Negation ④**
⑥ $a^2 = 7k \land \neg a \div 7$	**By Definition (Original Proof: ⑤)**

∴ by Contradiction we have shown a and ¬a(b in this case) are both divisible by 7.

Proof by Cases

Case 1(+ 1)

① $a^2 = 7k \land a = 7k + 1$	
② $a = 7k + 1$	**By Simplification ①**
③ $a^2 = (7k + 1)^2$	**By Math ②**
④ $a^2 = 49k^2 + 7k + 7k + 1$	**By Math ③**
⑤ $a^2 = 49k^2 + 14k + 1$	**By Math ④**
⑥ $a^2 = 7(7k^2 + 2k) + 1$	**By Math ⑤**
⑦ $a^2 = 7x + 1$	**By Math ⑥**
⑧ $a^2 = 7k \land a^2 = 7x + 1$	**By Conjunction ⑦**

∴ the **statement is false** in Case 1.

Case 2(+ 2)

① $a^2 = 7k \land a = 7k + 2$	
② $a = 7k + 2$	**By Simplification ①**
③ $a^2 = (7k + 2)^2$	**By Math ②**
④ $a^2 = 49k^2 + 14k + 14k + 4$	**By Math ③**
⑤ $a^2 = 49k^2 + 28k + 4$	**By Math ④**
⑥ $a^2 = 7(7k^2 + 4k) + 4$	**By Math ⑤**
⑦ $a^2 = 7x + 4$	**By Math ⑥**
⑧ $a^2 = 7k \land a^2 = 7x + 4$	**By Conjunction ⑦**

∴ the **statement is false** in Case 2.

Case 3(+ 3)

① $a^2 = 7k \land a = 7k + 3$	
② $a = 7k + 3$	**By Simplification ①**
③ $a^2 = (7k + 3)^2$	**By Math ②**
④ $a^2 = 49k^2 + 21k + 21k + 9$	**By Math ③**
⑤ $a^2 = 49k^2 + 42k + 9$	**By Math ④**
⑥ $a^2 = 7(7k^2 + 6k) + 9$	**By Math ⑤**
⑦ $a^2 = 7x + 9$	**By Math ⑥**
⑧ $a^2 = 7k \land a^2 = 7x + 9$	**By Conjunction ⑦**

∴ the **statement is false** in Case 3.

Case 4(+ 4)

① $a^2 = 7k \land a = 7k + 4$	
② $a = 7k + 4$	**By Simplification ①**
③ $a^2 = (7k + 4)^2$	**By Math ②**
④ $a^2 = 49k^2 + 28k + 28k + 16$	**By Math ③**
⑤ $a^2 = 49k^2 + 56k + 16$	**By Math ④**
⑥ $a^2 = 7(7k^2 + 8k) + 16$	**By Math ⑤**

⑦ $a^2 = 7x + 16$ **By Math ⑥**
⑧ $a^2 = 7k \wedge a^2 = 7x + 16$ **By Conjunction ⑦**
∴ the **statement is false** in Case 4.

Case 5(+ 5)
① $a^2 = 7k \wedge a = 7k + 5$
② $a = 7k + 5$ **By Simplification ①**
③ $a^2 = (7k + 5)^2$ **By Math ②**
④ $a^2 = 49k^2 + 35k + 35k + 25$ **By Math ③**
⑤ $a^2 = 49k^2 + 70k + 25$ **By Math ④**
⑥ $a^2 = 7(7k^2 + 10k) + 25$ **By Math ⑤**
⑦ $a^2 = 7x + 25$ **By Math ⑥**
⑧ $a^2 = 7k \wedge a^2 = 7x + 25$ **By Conjunction ⑦**
∴ the **statement is false** in Case 5.

Case 6(+ 6)
① $a^2 = 7k \wedge a = 7k + 6$
② $a = 7k + 6$ **By Simplification ①**
③ $a^2 = (7k + 6)^2$ **By Math ②**
④ $a^2 = 49k^2 + 42k + 42k + 36$ **By Math ③**
⑤ $a^2 = 49k^2 + 84k + 36$ **By Math ④**
⑥ $a^2 = 7(7k^2 + 12k) + 36$ **By Math ⑤**
⑦ $a^2 = 7x + 36$ **By Math ⑥**
⑧ $a^2 = 7k \wedge a^2 = 7x + 36$ **By Conjunction ⑦**
∴ the **statement is false** in Case 6.

Original Proof
① $7b^2 = a^2$
② $7b^2 = (7x)^2$ **By Math ①**
③ $7b^2 = 49x^2$ **By Math ②**
④ $b^2 = 7x^2$ **By Math ③**
∴ because a and b are divisible by 7 (Lemma 2), and $\frac{a}{b}$ is not in lowest terms, $\sqrt{9} + \sqrt{7}$ cannot be a rational number. $\sqrt{9} + \sqrt{7}$ is an irrational number.

3.
① $\sqrt{25} + \sqrt{5}$ is an irrational number.
② $\sqrt{25} + \sqrt{5}$ is an rational number. **By Contradiction ①**
③ $\sqrt{25} + \sqrt{5} = (\frac{a}{b}) \wedge (\frac{a}{b}$ in lowest terms$)$ **By Definition (Rational Number) ②**
④ $\sqrt{25} + \sqrt{5} = \frac{a}{b}$ **By Simplification ③**
⑤ $5 + \sqrt{5} = \frac{a}{b}$ **By Math ④**
⑥ $\sqrt{5} = \frac{a}{b} - 5$ **By Math ⑤**
⑦ $\sqrt{5} = \frac{a}{b}$ **By Closure of Rational Numbers ⑥**
⑧ $5 = a^2 / b^2$ **By Math ⑦**
⑨ $5b^2 = a^2$ **By Math ⑧**

Lemma 1 (Prime Factorization Proof)
① $a = f_1 \times f_2 \ldots f_n$ **By Definition (Prime Number)**
② $a^2 = (f_1 \times f_2 \ldots f_n)^2$ **By Math ①**

③ $a^2 = (f_1 \times f_2 \ldots f_n)\ (f_1 \times f_2 \ldots f_n)$ **By Math ②**
④ $a^2 = (f_1 \times f_1 \times f_2 \times f_2 \ldots f_n \times f_n)$ **By Math ③**

Back to Original Proof
① $5b^2 = a^2$
② a^2 and a is a multiple of 5 **By Lemma 1 (Prime Factorization)**
③ $5x = a$ **By Math ①**
④ $5b^2 = (5x)^2$ **By Math ③**
⑤ $5b^2 = 25x^2$ **By Math ④**
⑥ $b^2 = 5x^2$ **By Math ⑤**
⑦ b^2 and b is a multiple of 5. **By Lemma 1 (Prime Factorization)**
∴ $\frac{a}{b}$ is not in lowest terms because both a and b have multiples/factors of 5, which shows $\sqrt{25} + \sqrt{5} = \frac{a}{b}$ is not true, meaning $\sqrt{25} + \sqrt{5}$ is irrational.

4.
① $n^3 + 5$ is odd ⟶ n is even
② n is odd ⟶ $n^3 + 5$ is even **By Indirect Proof (Contrapositive) ①**
③ $n^3 + 5 = (2k + 1)^3 + 5$ **By Definition (Odd Number: 2k + 1) ②**
④ $n^3 + 5 = (2k + 1)\ (2k + 1)\ (2k + 1) + 5$ **By Math ③**
⑤ $n^3 + 5 = (4k^2 + 2k + 2k + 1)\ (2k + 1) + 5$ **By Math ④**
⑥ $n^3 + 5 = (4k^2 + 4k + 1)\ (2k + 1) + 5$ **By Math ⑤**
⑦ $n^3 + 5 = 8k^3 + 4k^2 + 8k^2 + 4k + 2k + 1 + 5$ **By Math ⑥**
⑧ $n^3 + 5 = 8k^3 + 12k^2 + 6k + 6$ **By Math ⑦**
⑨ $n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$ **By Math ⑧**
⑩ $n^3 + 5 = 2(k)$ **By Definition (Even Number: 2k) ⑨**
∴ $n^3 + 5$ is even and since "n is odd ⟶ $n^3 + 5$ is even" is true, the contrapositive/original statement "$n^3 + 5$ is odd ⟶ n is even" is also true.

5.
The error in the proof is that **no basis case where n = 1 has been shown**. This is needed to progress to the inductive hypothesis.

6.
For x: $-2 \leq x \leq 2$, Prove $y < 0$: $y = x^4 - 36x^2 + 9x - 5$

CASE 1 (-2)
① $y = (-2)^4 - 36(-2)^2 + 9(-2) - 5$ **By Math**
② $y = 16 - 36(-2)^2 + 9(-2) - 5$ **By Math ①**
③ $y = 16 - 36(4) + 9(-2) - 5$ **By Math ②**
④ $y = 16 - 144 + 9(-2) - 5$ **By Math ③**
⑤ $y = 16 - 144 - 18 - 5$ **By Math ④**
⑥ $y = -128 - 18 - 5$ **By Math ⑤**
⑦ $y = -146 - 5$ **By Math ⑥**
⑧ $\mathbf{y = -151}$ **By Math ⑦**

CASE 2 (-1)
① $y = (-1)^4 - 36(-1)^2 + 9(-1) - 5$ **By Math**
② $y = 1 - 36(-1)^2 + 9(-1) - 5$ **By Math ①**
③ $y = 1 - 36(1) + 9(-1) - 5$ **By Math ②**

④ $y = 1 - 36 + 9(-1) - 5$ **By Math ③**
⑤ $y = 1 - 36 - 9 - 5$ **By Math ④**
⑥ $y = -35 - 9 - 5$ **By Math ⑤**
⑦ $y = -44 - 5$ **By Math ⑥**
⑧ **$y = -49$** **By Math ⑦**

CASE 3 (0)
① $y = (0)^4 - 36(0)^2 + 9(0) - 5$ **By Math ①**
② $y = - 36(0)^2 + 9(0) - 5$ **By Math ②**
③ $y = 9(0) - 5$ **By Math ③**
④ **$y = - 5$** **By Math ④**

CASE 4 (1)
① $y = (1)^4 - 36(1)^2 + 9(1) - 5$ **By Math**
② $y = 1 - 36(1)^2 + 9(1) - 5$ **By Math ①**
③ $y = 1 - 36(1) + 9(1) - 5$ **By Math ②**
④ $y = 1 - 36 + 9(1) - 5$ **By Math ③**
⑤ $y = 1 - 36 + 9 - 5$ **By Math ④**
⑥ $y = -35 + 9 - 5$ **By Math ⑤**
⑦ $y = -26 - 5$ **By Math ⑥**
⑧ **$y = -31$** **By Math ⑦**

CASE 5 (2)
① $y = (2)^4 - 36(2)^2 + 9(2) - 5$ **By Math**
② $y = 16 - 36(2)^2 + 9(2) - 5$ **By Math ①**
③ $y = 16 - 36(4) + 9(2) - 5$ **By Math ②**
④ $y = 16 - 144 + 9(2) - 5$ **By Math ③**
⑤ $y = 16 - 144 + 18 - 5$ **By Math ④**
⑥ $y = -128 + 18 - 5$ **By Math ⑤**
⑦ $y = -110 - 5$ **By Math ⑥**
⑧ **$y = -115$** **By Math ⑦**
$\therefore$ $y < 0$ **has been proved in all cases** for x: $-2 \leq x \leq 2$ and **holds true**.

7.
Prove by Induction that $1 + 3 + 5 + \ldots + (2k - 1) = k^2$, for all positive integers n.
Basis Step: (n=1)
① $(2(1) - 1) = (1)^2$ **By Math**
② $(2 - 1) = (1)^2$ **By Math ①**
③ $1 = (1)^2$ **By Math ②**
④ $1 = 1$ **By Math ③**

Inductive Hypothesis: Assume n = k: $1 + 3 + 5 \ldots + (2k-1) = k^2$, where k is a positive integer $k \geq 1$.
Inductive Step: $(n = k + 1)$
① $1 + 3 + 5 + \ldots + (2k - 1) + (2(k + 1) - 1) = (k + 1)^2$ **By Math**
② $1 + 3 + 5 + \ldots + (2k - 1) + 2k + 2 - 1 = (k + 1)^2$ **By Math ①**
③ $1 + 3 + 5 + \ldots + (2k - 1) + 2k + 1 = (k + 1)^2$ **By Math ②**
④ $k^2 + 2k + 1 = (k + 1)^2$ **By Recursion ③**
$\therefore$ through Proof by Induction we have shown $1 + 3 + 5 + \ldots + (2k - 1) = k^2$ is true for all positive integers n.

8. A = {1, 6, 8] ∴ P(A) = {∅, {1}, {6}, {8}, {1, 6}, {1, 8}, {6, 8}, {1, 6, 8}}

9.
A) {one, two, three, eight, ten, eleven, twelve, thirteen, eighteen, twenty}
B) {enzo}

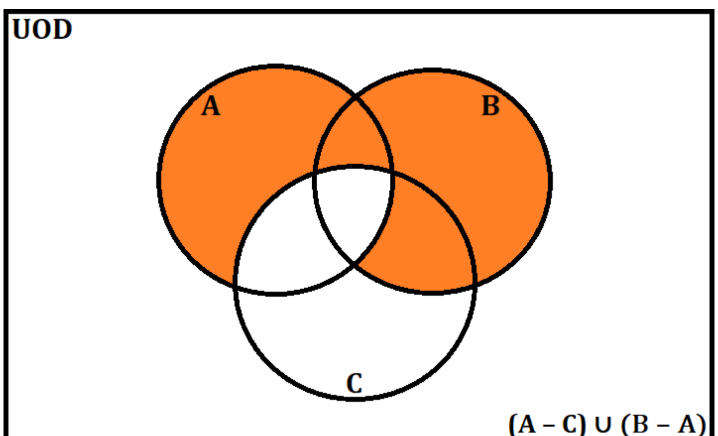
10.
A) They both have the same cardinality.
B) S ∩ T = {4}
C) The cardinality of the union S ∪ T is 9.

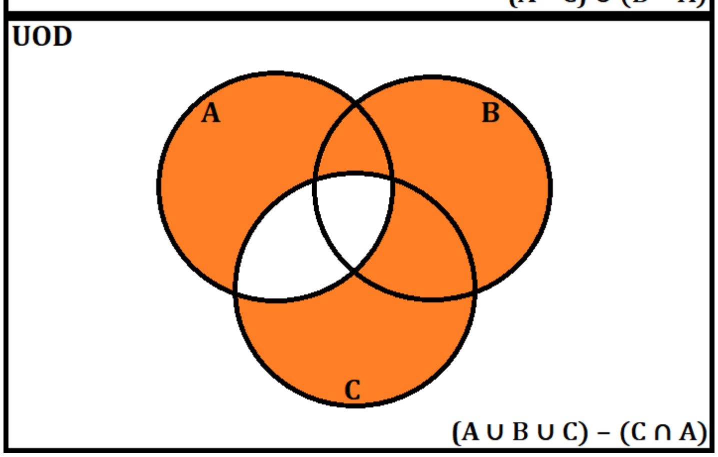
11.

A	B	C	A – C	B – A	**(A – C) ∪ (B – A)**	(A ∪ B ∪ C)	(C ∩ A)	**(A ∪ B ∪ C) – (C ∩ A)**
1	1	1	0	0	0	1	1	0
1	1	0	1	0	1	1	0	1
1	0	1	0	0	0	1	1	0
1	0	0	1	0	1	1	0	1
0	1	1	0	1	1	1	0	1
0	1	0	0	1	1	1	0	1
0	0	1	0	0	**0**	1	0	**1**
0	0	0	0	0	0	0	0	0

∴ Therefore the following sets **are not valid.**

12.





13.
Student Number #: **101085982**
Set B (Digits in student #) = {1, 0, 1, 0, 8, 5, 9, 8, 2}
Set C (Set of all even numbers) = {0, 2, 4, 6, 8}
Set A ∩ (B ∩ C) (Intersection) = {0, 2, 8}