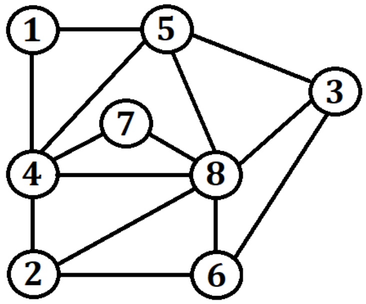
COMP 1805A SUMMER 2017 – ASSIGNMENT 4 NEMANJA ZUTKOVIC (101085982)

1. Yes, a planar representation can be created for the graph:



2.

The minimum number of colors that can be used to color this graph is 4.

To prove this, we will attempt to show that the graph can be colored with 3 colors:

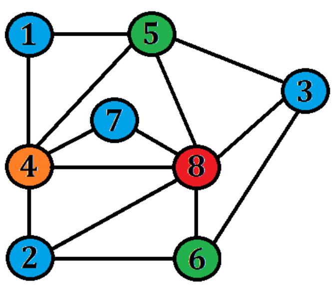
Starting with vertex 1, we choose blue (1st color).

Moving to vertex 4, we can choose any color except blue. We choose orange (2nd color).

Moving to vertex 2, we cannot use orange, but we can use any other color. We choose blue again.

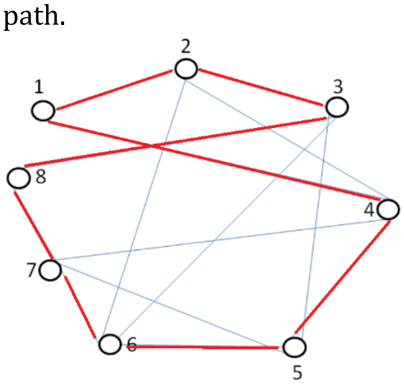
Moving to vertex 8, we cannot use blue/orange, but we can use any other color. We choose red (3rd color).

Moving to vertex 5, we cannot use blue/orange/red because vertex 5 is connected to vertex 1,4,8 and they each have their own distinct colors. This means we must use a 4th color to color in vertex 5, and because we must use a 4th color, *this contradicts our hypothesis* where we attempted to show that the graph cannot be colored in less than 3 colors.



3. This graph has a Euler cycle: {8,3}, {3,2}, {2,1}, {1,4}, {4,2}, {2,6}, {6,5}, {5,3}, {3,6}, {6,7}, {7,5}, {5,4}, {4,7}, {7,8}. Therefore, because this graph has a Euler cycle, by definition, this graph also has a Euler path.

4. This graph has a Hamiltonian cycle (see cycle below): {1,2}, {2,3}, {3,8}, {8,7}, {7,6}, {6,5}, {5,4}, {4,1}. Therefore, because this graph has a Hamiltonian cycle, by definition, this graph also has a Hamiltonian path.



5. Yes, the following graphs are isomorphic:
Left Graph (V_1, E_1)
V_1: {1,2,3,4,5,6}
E_1: {1,2}, {2,3}, {3,4}, {4,5}, {5,6}, {6,1}

Right Graph (V_2, E_2)
V_1: {a,b,c,d,e,f}
E_1: {a,c}, {a,d}, {b,e}, {b,f}, {c,e}, {d,f}

V/E Equivalencies			
V_1	**V_1 Renamed**	**E_1**	**E_2 Renamed**
1	a	{1,2}	{a,c}
2	c	{2,3}	{c,e}
3	e	{3,4}	{e,b}
4	b	{4,5}	{b,f}
5	f	{5,6}	{f,d}
6	d	{6,1}	{d,a}
∴ the graphs are isomorphic.			

6A.
$(9n-4)^2$ is $\Theta(n^2)$

$(9n-4)^2$ is $O(n^2)$ Assume $n \geq 1$
$81n^2 - 36n - 36n + 16$
$81n^2 - 72n + 16$
$81n^2 - 72n + 16 \leq 81n^2 + 16$
$81n^2 - 72n + 16 \leq 81n^2 + 16n^2$
$81n^2 - 72n + 16 \leq 97n^2$
$f(x) \leq 97n^2$, where $c = 97$ and $k = 1$

$(9n-4)^2$ is $\Omega(n^2)$ Assume $n \geq 1$
$81n^2 - 36n - 36n + 16$
$81n^2 - 72n + 16$
$81n^2 - 72n + 16 \geq 81n^2 - 72n$
$81n^2 - 72n + 16 \geq 81n^2 - 72n^2$
$81n^2 - 72n + 16 \geq 9n^2$
$f(x) \geq 9n^2$, where $c = 9$ and $k = 1$

Therefore, because $f(x)$ is $O(g(x)) \wedge f(x)$ is $\Omega(g(x))$, $f(x)$ is $\Theta(g(x))$.

6B.
$8n^2 - 9 + n$ is $O(n^2)$ Assume $n \geq 1$
$8n^2 - 9 + n \leq 8n^2 + 9n + n$
$8n^2 - 9 + n \leq 8n^2 + 9n^2 + n$
$8n^2 - 9 + n \leq 8n^2 + 9n^2 + n^2$
$8n^2 - 9 + n \leq 17n^2 + n^2$
$8n^2 - 9 + n \leq 18n^2$
$f(x) \leq 18n^2$, where $c = 18$ and $k = 1$

6C.
$\frac{20\log(n+7)}{5}$ is $O(n^2)$ Assume $n \geq 1$
$\frac{20\log(n+7)}{5} = 4\log(n+7)$
$4\log(n+7) \leq 4(n+7)$
$4\log(n+7) \leq 4n + 28$
$4\log(n+7) \leq 4n + 28n$
$4\log(n+7) \leq 32n$
$4\log(n+7) \leq 32n^2$
$f(x) \leq 32n^2$, where $c = 32$ and $k = 1$

6D.

$5n^2 - 2n$ is $\Omega(n^2)$ Assume $n \geq 1$

$5n^2 - 2n \geq 5n^2 - 2n^2$

$5n^2 - 2n \geq 3n^2$

$f(x) \geq 3n^2$, where $c = 3$ and $k = 1$

7. A linear search would locate this element more rapidly, because:

Linear Search (n), $n = 4$

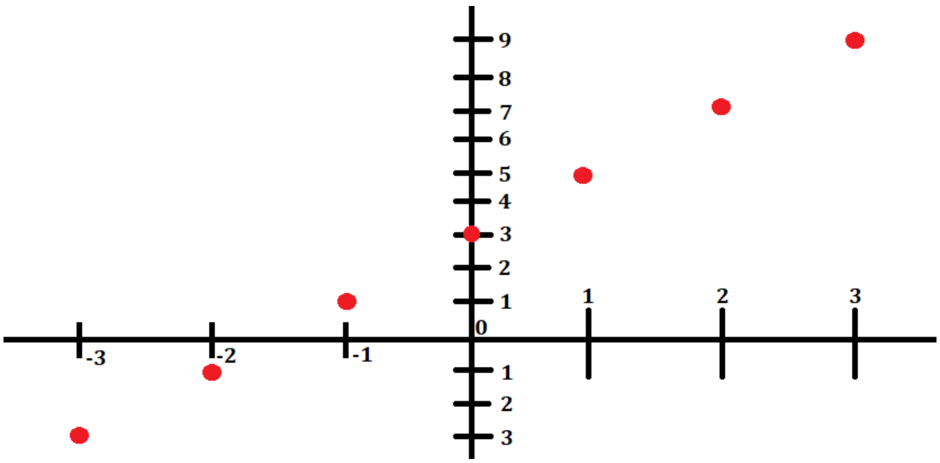
Binary Search ($\log_2 n$), $\log2(32) = 5$

8.

A) Linear Search (n), $(2n) = 2n$, which means n doubles.

B) Binary Search ($\log_2 n$), $\log_2(2n) = 1 + \log_2 n$, which means n increases by one.

9. $f(x) = 2x + 3$ where $\{x \mid \in \mathbb{Z}, -3 \leq x \leq 3\}$



10. Student #: 1010859**82**

$a = 2$

$b = 8$

A) $f \circ g$	**B) $g \circ f$**	**C) $(f \circ g) \circ g$**
$f(g(x))$ $f(c^2 - d)$ $a(c^2 - d) + c$ $a((3a)^2 - (2b)) + 3a$ $2((3(2))^2 - (2(8))) + 3(2)$ $2(6^2 - (2(8))) + 3(2)$ $2(6^2 - 16) + 3(2)$ $2(36 - 16) + 3(2)$ $2(20) + 3(2)$ $40 + 3(2)$ $40 + 6$ **$f \circ g = 46$**	$g(f(x))$ $g(c^2 - d)$ $c^2 - d$ $((3a)^2 - (2b))$ $((3(2))^2 - (2(8)))$ $(6^2 - (2(8)))$ $(6^2 - 16)$ $36 - 16$ **$g \circ f = 20$**	$f(g(g(x)))$ $f(g(c^2 - d))$ $f(c^2 - d)$ $a(c^2 - d) + c$ $a((3a)^2 - (2b)) + 3a$ $2((3(2))^2 - (2(8))) + 3(2)$ $2(6^2 - (2(8))) + 3(2)$ $2(6^2 - 16) + 3(2)$ $2(36 - 16) + 3(2)$ $2(20) + 3(2)$ $40 + 3(2)$ $40 + 6$ **$(f \circ g) \circ g = 46$**