COMP 1805A SUMMER 2017 – ASSIGNMENT 1 **NEMANJA ZUTKOVIC (101085982)**

1.

A) I am not a student and it is not May.
B) It is warm outside or it is May, or both.
C) It is warm outside or it is May, but not both.
D) If it is May and I am a student, then it is warm outside.

2.

A) Let p stand for “I like math”.
Let q stand for “I like studying”.
Let r stand for “I like golf”.
Logical Statement: $p \land q \land \neg r$

B) Let p stand for “my alarm rings”.
Let q stand for “I need to wake up”.
Logical Statement: $p \longrightarrow q$

C) Let p stand for “I was born in January”.
Logical Statement: $p \oplus \neg p$

D) Let p stand for “I am your instructor”.
Let q stand for “You are a Carleton University student”.
Logical Statement: $p \leftrightarrow q$

3.

A) 3 < 5 and 1 < 2 and 1 < 0
$3 < 5 \land 1 < 2 \land 1 < 0$
T $\land$ **T** $\land$ **F**
T $\land$ F
F

B) 1 = 2 or 2 > 1
$1 = 2 \lor 2 > 1$
F $\lor$ **T**
T

C) Let d stand for “your instructor is actually Dumbledore”.
$10 = 2 \longrightarrow d$
F $\longrightarrow$ d
T

D) If 8=8, then $8=2^3$ or $8=2^2$
$8=8 \longrightarrow 8=2^3 \lor 8=2^2$
$8=8 \longrightarrow 8=\mathbf{8} \lor 8=\mathbf{4}$
T $\longrightarrow$ **T** $\lor$ **F**
T $\longrightarrow$ **T**
T

4.

$(p \lor q) \leftrightarrow (q \lor \neg r)$

$(\neg p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg r)$ **Implication Equivalence**

$((\neg p \rightarrow q) \rightarrow (\neg q \rightarrow \neg r)) \land ((\neg q \rightarrow \neg r) \rightarrow (\neg p \rightarrow q))$ **Biconditional Equivalence**

5.

TRUTH TABLE: $(p \lor q) \leftrightarrow (q \lor \neg r)$

p	q	r	¬r	p ∨ q	q ∨ ¬r	**(p ∨ q) ↔ (q ∨ ¬r)**
T	T	T	F	T	T	**T**
T	T	F	T	T	T	**T**
T	F	T	F	T	F	**F**
T	F	F	T	T	T	**T**
F	T	T	F	T	T	**T**
F	T	F	T	T	T	**T**
F	F	T	F	F	F	**T**
F	F	F	T	F	T	**F**

TRUTH TABLE: $((\neg p \rightarrow q) \rightarrow (\neg q \rightarrow \neg r)) \land ((\neg q \rightarrow \neg r) \rightarrow (\neg p \rightarrow q))$

p	q	r	¬p	¬q	¬r	X: (¬p → q)	Y: (¬q → ¬r)	Z1: X→Y	Z2: Y→X	**Z1 ∧ Z2**
T	T	T	F	F	F	T	T	T	T	**T**
T	T	F	F	F	T	T	T	T	T	**T**
T	F	T	F	T	F	T	F	F	T	**F**
T	F	F	F	T	T	T	T	T	T	**T**
F	T	T	T	F	F	T	T	T	T	**T**
F	T	F	T	F	T	T	T	T	T	**T**
F	F	T	T	T	F	F	F	T	T	**T**
F	F	F	T	T	T	F	T	T	F	**F**

6.

A) TRUTH TABLE: $\neg(\neg(p \land \neg(\neg q \land q)) \land \neg\neg p)$

p	q	¬p	¬q	¬q ∧ q	X: p ∧ ¬(¬q ∧ q)	Y: ¬(X)	**¬ (Y ∧ ¬¬p)**
T	T	F	F	F	T	F	**T**
T	F	F	T	F	T	F	**T**
F	T	T	F	F	F	T	**T**
F	F	T	T	F	F	T	**T**

Therefore, the following expression is a **tautology**.

B) TRUTH TABLE: $(q \leftrightarrow ((p \lor r) \land r)) \land \neg(p \rightarrow p)$

p	q	r	p ∨ r	X: (p ∨ r) ∧ r	Y: (q ↔ X)	¬(p → p)	**Y ∧ ¬(p → p)**
T	T	T	T	T	T	F	**F**
T	T	F	T	F	F	F	**F**
T	F	T	T	T	T	F	**F**
T	F	F	T	F	F	F	**F**
F	T	T	T	T	T	F	**F**
F	T	F	F	F	T	F	**F**
F	F	T	T	T	T	F	**F**
F	F	F	F	F	T	F	**F**

Therefore, the following expression is a **contradiction**.

C) **TRUTH TABLE: $((p \land r) \lor q \lor (p \land \neg r)) \land (r \lor p)$**

p	q	r	¬r	X: (p ∧ r)	Y: (p ∧ ¬r)	Z: X ∨ q ∨ Y	(r ∨ p)	**(Z) ∧ (r ∨ p))**
T	T	T	F	T	F	T	T	**T**
T	T	F	T	F	T	T	T	**T**
T	F	T	F	T	F	T	T	**T**
T	F	F	T	F	T	T	T	**T**
F	T	T	F	F	F	T	T	**T**
F	T	F	T	F	F	T	F	**F**
F	F	T	F	F	F	F	T	**F**
F	F	F	T	F	F	F	F	**F**

Therefore, the following expression is a **contingency**.

7.A) $\neg(\neg(p \land \neg(\neg q \land q)) \land \neg\neg p)$

$\neg(\neg(p \land \neg(\neg q \land q)) \land \mathbf{p})$	**Double Negation**
$\neg(\neg(p \land \mathbf{\neg\neg q \lor \neg q}) \land p)$	**De Morgan's Law**
$\neg(\neg(p \land \mathbf{q} \lor \neg q) \land p)$	**Double Negation**
$\neg(\mathbf{\neg p \lor \neg q \land \neg\neg q} \land p)$	**De Morgan's Law**
$\neg(\neg p \lor \neg q \land \mathbf{q} \land p)$	**Double Negation**
$\mathbf{\neg\neg p \land \neg\neg q \lor \neg q \lor \neg p}$	**De Morgan's Law**
$\mathbf{p} \land \mathbf{q} \lor \neg q \lor \neg p$	**Double Negation**
$p \land \mathbf{T} \lor \neg p$	**Negation**
$\mathbf{p} \lor \neg p$	**Identity**
$\mathbf{T}$	**Negation**

Therefore, using logical equivalencies we proved 6A is a **Tautology**.

B) $(q \leftrightarrow ((p \lor r) \land r)) \land \neg(p \rightarrow p)$

$(q \leftrightarrow ((p \lor r) \land r)) \land \neg(\mathbf{\neg p \lor} p)$	**Implication Equivalence**
$(q \leftrightarrow ((p \lor r) \land r)) \land (\mathbf{\neg\neg p \land \neg p})$	**De Morgan's Law**
$(q \leftrightarrow ((p \lor r) \land r)) \land (\mathbf{p} \land \neg p)$	**Double Negation**
$(q \leftrightarrow \mathbf{(r \land (r \lor p)})) \land \mathbf{F}$	**Negation**
$\mathbf{F}$	**Domination**

Therefore, using logical equivalencies we proved 6B is a **Contradiction**.

C) $((p \land r) \lor q \lor (p \land \neg r)) \land (r \lor p)$

$((p \land r) \lor \mathbf{(p \land \neg r) \lor q}) \land (r \lor p)$	**Commutative Law**
$(\mathbf{p \land (r \lor \neg r)} \lor q) \land (r \lor p)$	**Distributive Law**
$(p \land \mathbf{T} \lor q) \land (r \lor p)$	**Negation**
$(p \land \mathbf{T}) \land (r \lor p)$	**Domination**
$\mathbf{p} \land (r \lor p)$	**Identity**
$p \land \mathbf{(p \lor r)}$	**Commutative Law**
$\mathbf{p}$	**Absorption**

Therefore, using logical equivalencies we proved 6C is a **Contingency.**

8.
A) There exists at least one animal that is a dog which only eats meat.
B) There exists at least one animal that is a dog and does not eat meat.
C) If an animal is a mammal then it is a dog or it is not a dog.

9. UOD: All Animals

A)
Let C(x) be the predicate “x is a cat”.
Let P(x) be the predicate “x purrs”.
Let F(x) be the predicate “x is furry”.
Every cat purrs and is furry.

$\forall(x)(C(x) \longrightarrow (P(x) \land F(x)))$	
$\neg\forall(x)(C(x) \longrightarrow (P(x) \land F(x)))$	
$\exists(x)\ \neg(C(x) \longrightarrow (P(x) \land F(x)))$	**Quantifier Negation**
$\exists(x)\ (\neg(\neg C(x) \lor (P(x) \land F(x)))$	**Implication Equivalence**
$\exists(x)\ (\neg\neg C(x) \land \neg(P(x) \land F(x)))$	**De Morgan’s Law**
$\exists(x)\ (\neg\neg C(x) \land \neg P(x) \lor \neg F(x))$	**De Morgan’s Law**
$\exists(x)\ (C(x) \land \neg P(x) \lor \neg F(x))$	**Double Negation**
$\exists(x)\ (C(x) \land P(x) \longrightarrow \neg F(x))$	**Implication Equivalence Inverse**

B)
Let C(x) be the predicate “x is a cat”.
Let M(x) be the predicate “x eats mice”.
Let F(x) be the predicate “x can catch fish”.
There is at least one cat who doesn’t eat mice and can catch fish.

$\exists(x)(C(x) \land (\neg\ M(x) \land F(x))$	
$\neg\exists(x)(C(x) \land (\neg\ M(x) \land F(x))$	
$\forall(x)\ \neg(C(x) \land (\neg\ M(x) \land F(x))$	**Quantifier Negation**
$\forall(x)\ (\neg C(x) \lor \neg\ (\neg\ M(x) \land F(x))$	**De Morgan’s Law**
$\forall(x)\ (\neg C(x) \lor \neg\neg\ M(x) \lor \neg F(x))$	**De Morgan’s Law**
$\forall(x)\ (\neg C(x) \lor M(x) \lor \neg F(x))$	**Double Negation**
$\forall(x)\ (C(x) \longrightarrow M(x) \lor \neg F(x))$	**Implication Equivalence Inverse**