Tutorial 8

1. Let f: A → B be a function with f(1) = b, f(2) = a, and f(3) = b.
    a. What is the domain of f?

    the set A

    b. What is the codomain of f?

    the set B

    c. What is the range of f?

    {a, b}

2. Determine whether the following functions from the real numbers to the real numbers are injective, surjective, or bijective.
    a. $f(x) = |x|$

    For injectivity, we need to check if f(x) = f(y) → x = y. Consider f(2) and f(-2). Since f(2) = |2| = 2 = |-2| = f(-2), it is not the case that f(x) = f(y) → x = y, so the function is not injective. For surjectivity, we need to check if all possible real numbers can result from applying f. Since f is the absolute value function, it must be the case that f(x) > 0 for all x. Therefore, there is no x such that f(x) = -1. Therefore, the function is not surjective. Since it is not injective (and not surjective, although one is enough), f is not bijective.

    b. $f(x) = 3.14$

    We have f(1) = 3:14 = f(2), so the function is not injective. Similarly, since f(x) = 3:14 for all x, there is no x such that f(x) = 1, for instance. Therefore, the function is not surjective either. Since it is not injective (and not surjective, although one is enough), f is not bijective.

    c. $f(x) = 2x^2$

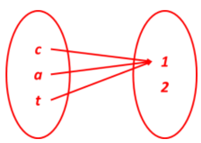
    We have f(-1) = 2 = f(1), so the function is not injective. To determine if the function is surjective, we try to find an x such that f(x) = -1. We have -1 = $2x^2$, so x = $\sqrt{-1/2}$. x is therefore not a real number, so the function is not surjective. Since it is not injective (and not surjective, although one is enough), f is not bijective.

d. $f(x) = 3x^3 + 2$

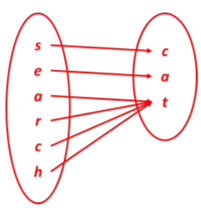
To determine if the function is injective, we need to determine if $f(x) = f(y) \rightarrow x = y$. Assume $f(x) = f(y)$. So $3x^3 + 2 = 3y^3 + 2$. Subtract 2 from both sides to get $3x^3 = 3y^3$. Divide both sides by 3 to get $x^3 = y^3$. Take the cube root of both sides to get $x = y$. Thus, $f(x) = f(y) \rightarrow x = y$ and so the function is injective. To determine if the function is surjective, suppose we want to find the value x such that $f(x) = y$ for some y. Then $y = 3x^3 + 2$. Solving for x, we get $x = \sqrt[3]{(1/3)y - 2}$, which is always a real number when y is a real number. Therefore, we can produce any real number as a result of applying f, and so f is surjective. Since f is both injective and surjective, f is also bijective.

3. Let A = {c, a, t}, B = {d, o, g}, C = {s, e, a, r, c, h}, D = {1, 2}. Using these sets as domain and co-domain, define functions having the following properties and explain why the properties hold, or explain why it is impossible to have such a function.

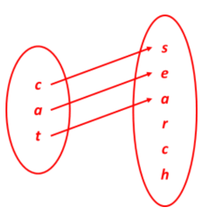
a. A function that is not injective or surjective.



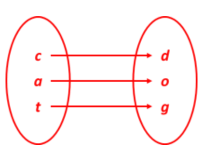
b. A function that is surjective but not injective.



c. A function that is injective but not surjective.

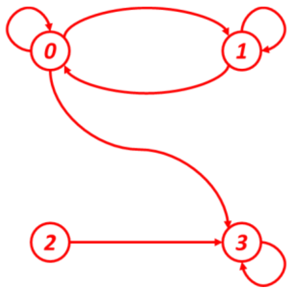


d. A function that is bijective.



4. Consider the relation R_1= { (0, 0), (0, 1), (0, 3), (1, 1), (1, 0), (2, 3), (3, 3) } defined on the set A = {0, 1, 2, 3}.

    a. Draw the directed graph.

    b. Determine whether or not the relation is reflexive.

    This relation is not reflexive because $2 \not{R}_1 2$.

    c. Determine whether or not the relation is symmetric.

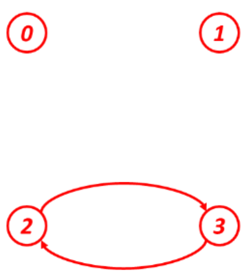
    This relation is not symmetric because $2\ R_1\ 3$ but $3 \not{R}_1 2$.

    d. Determine whether or not the relation is transitive.

    This relation is not transitive because $1\ R_1\ 0$ and $0\ R_1\ 3$ but $1 \not{R}_1 3$.

5. Consider the relation R_1= { (2, 3), (3, 2) } defined on the set A = {0, 1, 2, 3}.

    a. Draw the directed graph.

    b. Determine whether or not the relation is reflexive.

    This relation is not reflexive because $0 \not{R}_2 0$.

    c. Determine whether or not the relation is symmetric.

    This relation is symmetric.

    d. Determine whether or not the relation is transitive.

    This relation is not transitive because $2\ R_2\ 3$ and $3\ R_2\ 2$ but $2 \not{R}_2 2$.