Specification for Tutorial 5 of 8

1. List the members of the following sets:

   a. { x | x is a negative integer and $x^2 < 50$}

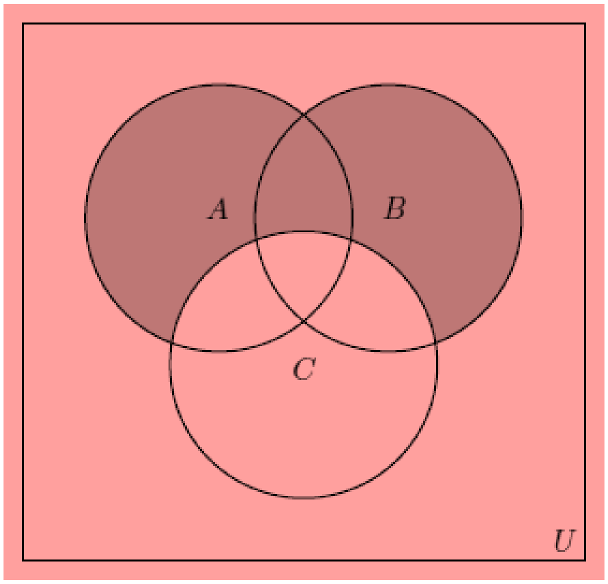
   {-1, -2, -3, -4, -5, -6, -7}

   b. { x | x is the cube of an integer and $0 \leq x < 101$}

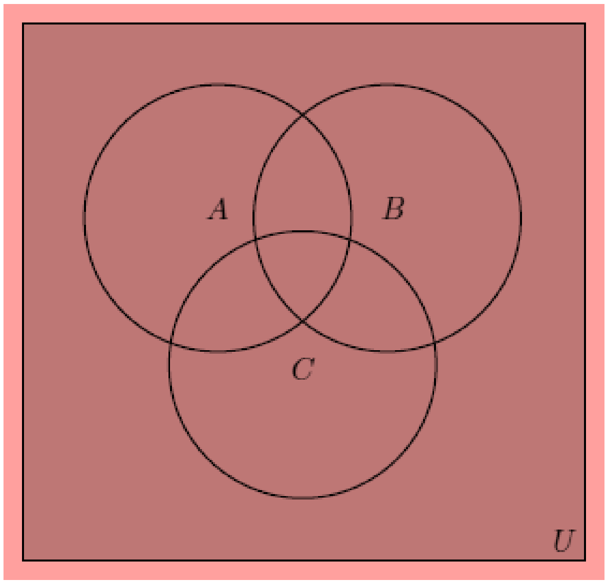
   {0, 1, 8, 27, 64}

2. Draw Venn diagrams for $(A \cup B) \cap \bar{C}$ and $\overline{(A \cap B)} \cup (B \cup \bar{C})$.

$(\boldsymbol{A} \cup \boldsymbol{B}) \cap \bar{\boldsymbol{C}}$



$\overline{(\boldsymbol{A} \cap \boldsymbol{B})} \cup (\boldsymbol{B} \cup \bar{\boldsymbol{C}})$



3. Prove or disprove the equivalence of the sets $\bar{B} \cap (A \cap C)$ and $\overline{(B \cup \bar{A})} \cap \overline{(B \cup \bar{C})}$ using...

   a. ...membership tables.

A	B	C	$\bar{B}$	$(A \cap C)$	$\bar{B} \cap (A \cap C)$
0	0	0	1	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	1	0

A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	$B \cup \bar{A}$	$\overline{(B \cup \bar{A})}$	$(B \cup \bar{C})$	$\overline{(B \cup \bar{C})}$	$\overline{(B \cup \bar{A})} \cap \overline{(B \cup \bar{C})}$
0	0	0	1	1	1	1	0	1	0	0
0	0	1	1	1	0	1	0	0	1	0
0	1	0	1	0	1	1	0	1	0	0
0	1	1	1	0	0	1	0	1	0	0
1	0	0	0	1	1	0	1	1	0	0
1	0	1	0	1	0	0	1	0	1	1
1	1	0	0	0	1	1	0	1	0	0
1	1	1	0	0	0	1	0	1	0	0

   b. ...a sequence of equivalences.

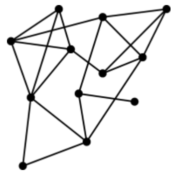
$\overline{(B \cup \bar{A})} \cap \overline{(B \cup \bar{C})}$

$= (\bar{B} \cap A) \cap (\bar{B} \cap C)$ **by DeMorgan's (and Double Negation)**

$= (\bar{B} \cap \bar{B}) \cap (A \cap C)$ **by Associativity and Commutativity**

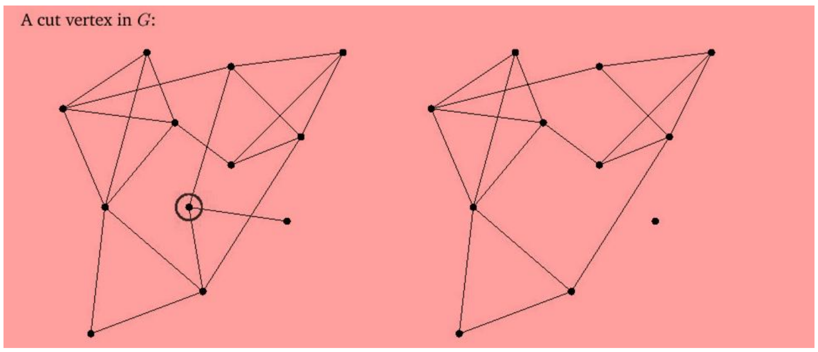
$= \bar{B} \cap (A \cap C)$ **by Idempotence**

4. For the following questions, use the graph below.

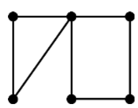


   a. How many edges are in the graph? 20

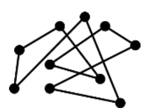
   b. Are there any cut vertices? If yes, which?



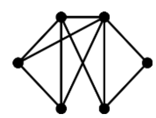
5. Find the chromatic number of each of the following graphs.



G_1



G_2



G_3

A three-colouring exists for G_1 and is included below. Since a two-colouring cannot exist without a graph colouring violation in the K_3 subgraph (i.e., clique of size 3 from the green, blue, and red nodes), the chromatic number of G_1 is 3.

Since G_2 is a cycle with an even number of vertices, the chromatic number of G_2 is 2.

A four-colouring exists for G_3 and is included below. Since a three-colouring cannot exist without a graph colouring violation in the K_4 subgraph (i.e., clique of size 4 from the green, blue, red, and orange nodes), the chromatic number of G_3 is 4.