COMP 4102 - Assignment 3

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(1)

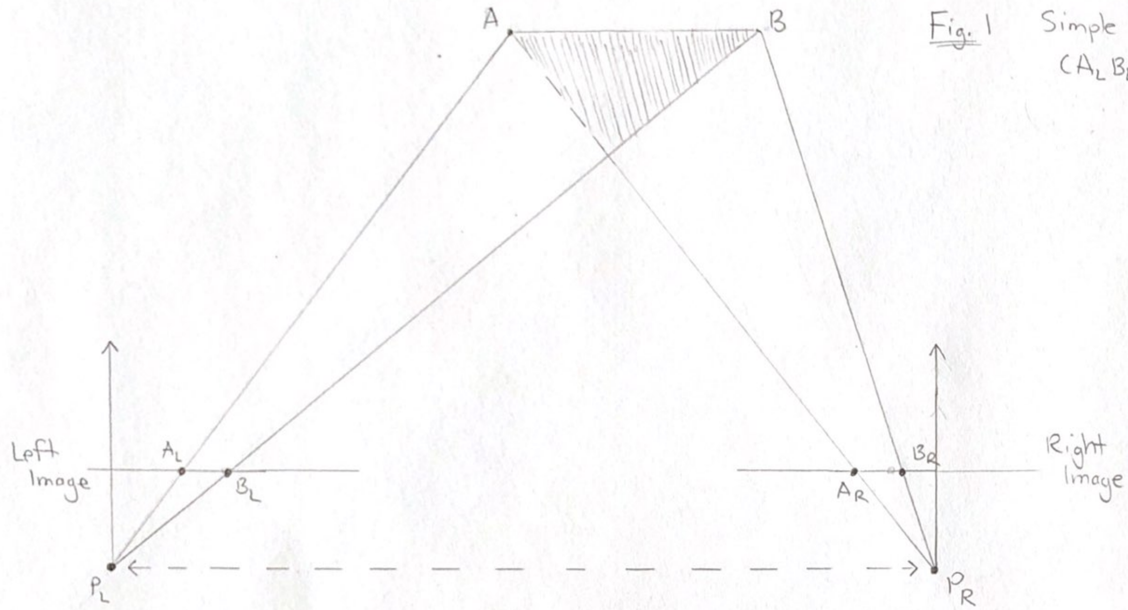


Fig. 1 Simple Stereo System ($A_L B_L$, $A_R B_R$)

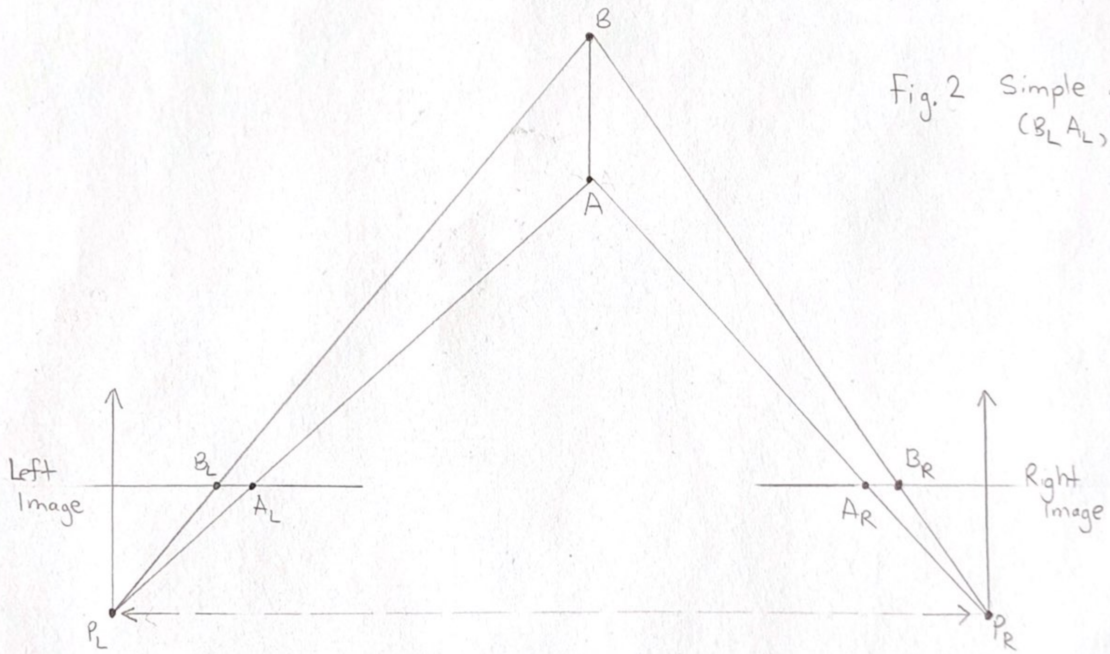


Fig. 2 Simple Stereo System ($B_L A_L$, $A_R B_R$)

(2)

$$T = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \qquad E = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

$$p^T E p = 0$$
$$p1 = [x1 \ y1 \ f]$$
$$p2 = [x2 \ y2 \ f]$$

$$T_1 = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ -b & 0 & 0 \end{bmatrix}$$

$$[x1 \ y1 \ f]\begin{bmatrix} 0 & 0 & b \\ 0 & 0 & 0 \\ -b & 0 & 0 \end{bmatrix}\begin{bmatrix} x2 \\ y2 \\ f \end{bmatrix} = 0$$

$$[x1 \ y1 \ f]\begin{bmatrix} bf \\ 0 \\ -bx2 \end{bmatrix} = 0$$

$$x1bf - fbx2 = 0 \rightarrow x1bf = fbx2 \rightarrow \boxed{x2 = x1}$$

$$T_2 = \begin{bmatrix} b \\ b \\ 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 0 & b \\ 0 & 0 & -b \\ -b & b & 0 \end{bmatrix}$$

$$[x1 \ y1 \ f]\begin{bmatrix} 0 & 0 & b \\ 0 & 0 & -b \\ -b & b & 0 \end{bmatrix}\begin{bmatrix} x2 \\ y2 \\ f \end{bmatrix} = 0$$

$$[x1 \ y1 \ f]\begin{bmatrix} bf \\ -bf \\ by2 - bx2 \end{bmatrix} = 0$$

$$x1bf - y1bf + fby2 - fbx2 = 0$$
$$bf(x1 - y1 + y2 - x2) = 0$$
$$x1 - y1 + y2 - x2 = 0 \rightarrow \boxed{x2 = y2 + x1 - y1}$$

$$T_3 = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & -b & 0 \\ b & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[x1 \, y1 \, f]\begin{bmatrix} 0 & -b & 0 \\ b & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x2 \\ y2 \\ f \end{bmatrix} = 0$$

$$[x1 \, y1 \, f]\begin{bmatrix} -by2 \\ bx2 \\ 0 \end{bmatrix} = 0$$

$$-x1by2 + y1bx2 = 0$$
$$y1bx2 = x1by2$$
$$y1x2 = x1y2$$
$$\boxed{x2 = \frac{x1y2}{y1}}$$

$\boxed{p1 = (0, 1, f)}$

(1A) $x2 = x1$

$x2 = 0$

(2A) $x2 = y2 + x1 - y1$

$x2 = y2 + 0 - 1$

$x2 = y2 - 1$

(3A) $x2 = \frac{x1y2}{y1}$

$x2 = \frac{0 \times y2}{y1}$

$x2 = 0$

$\boxed{p1 = (1, 1, f)}$

(1B) $x2 = x1$

$x2 = 1$

(2B) $x2 = y2 + x1 - y1$

$x2 = y2 + 1 - 1$

$x2 = y2$

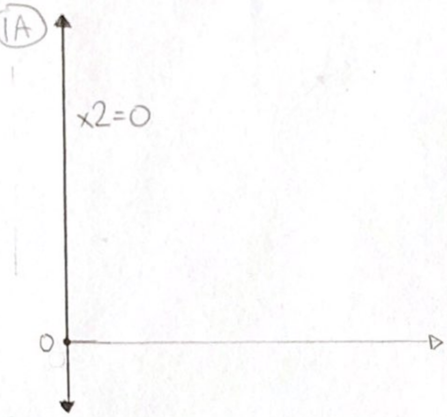
(3B) $x2 = \frac{x1y2}{y1}$

$x2 = \frac{1 \times y2}{1}$

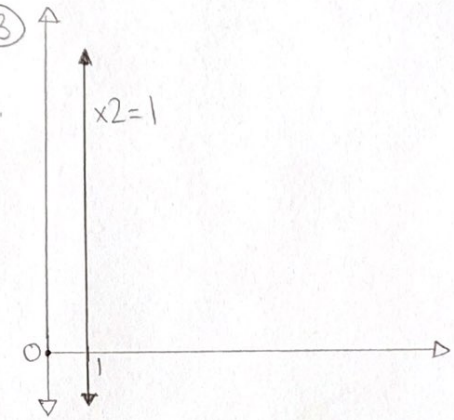
$x2 = y2$

Graphs

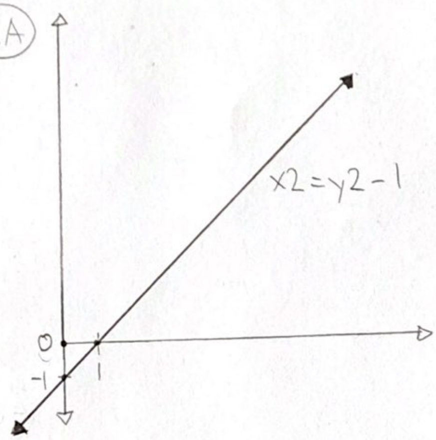
(1A)



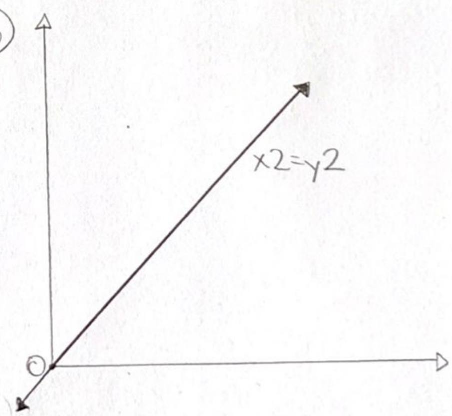
(1B)



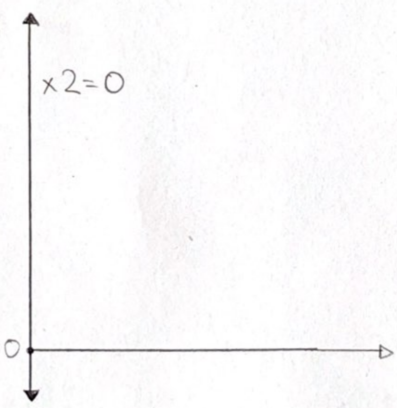
(2A)



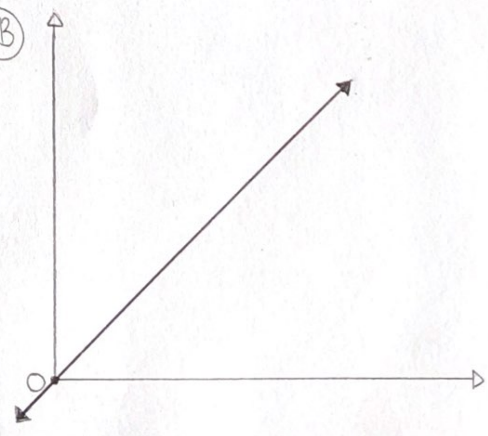
(2B)



(3A)



(3B)



(3) $Z = \frac{fB}{d} \rightarrow d = \frac{fB}{Z}$ $\quad Z_{high} = \frac{fB}{d-1}$ $\quad Z_{low} = \frac{fB}{d+1}$ $\quad Z_{diff} = Z_{high} - Z_{low}$

$B = 0.5m = 500mm$
$Z = 2.0m = 2000mm$ $\quad d = \frac{50 \times 500}{2000} = 12.5mm$
$f = 50mm$

$Z = 1.0m = 1000mm$ $\quad d = \frac{50 \times 500}{1000} = 25mm$

$Z = 0.5m = 500mm$ $\quad d = \frac{50 \times 500}{500} = 50mm$

$\boxed{\text{When } d = 12.5mm}$

$Z_{high} = \frac{50 \times 500}{11.5} = 2173.91$

$Z_{low} = \frac{50 \times 500}{13.5} = 1851.85$

$Z_{diff} = 2173.91 - 1851.85 = 321.96$

$\boxed{\text{When } d = 25mm}$

$Z_{high} = \frac{50 \times 500}{24} = 1041.67$

$Z_{low} = \frac{50 \times 500}{26} = 961.54$

$Z_{diff} = 1041.67 - 961.54 = 80.13$

$\boxed{\text{When } d = 50mm}$

$Z_{high} = \frac{50 \times 500}{49} = 510.20$

$Z_{low} = \frac{50 \times 500}{51} = 490.20$

$Z_{diff} = 510.20 - 490.20 = 20.00$

$\boxed{Z_{diff}(2.0m) \text{ to } Z_{diff}(1.0m) \text{ Ratio}}$

$\frac{321.96}{80.13} = 4.02 \approx 4$ or 1:4 Ratio

$\boxed{Z_{diff}(1.0m) \text{ to } Z_{diff}(0.5m) \text{ Ratio}}$

$\frac{80.13}{20} = 4.01 \approx 4$ or 1:4 Ratio

$\therefore$ when we cut our depth in half, the depth resolution improves by a factor of $\underline{\underline{4}}$.

④ Prove $|\partial_L| = |\partial_R|$ Ⓐ $\partial_L = x_o - x_L$ Ⓑ $\partial_R = x_R - x_o$

$$\tan C_R = \frac{f}{x_R} \longrightarrow x_R = \frac{f}{\tan C_R}$$

$$\tan C_o = \frac{f}{x_o} \longrightarrow x_o = \frac{f}{\tan C_o}$$

$$\tan C_L = \frac{f}{x_L} \longrightarrow x_L = \frac{f}{\tan C_L}$$

$$\partial_L = x_o - x_L = \frac{f}{\tan C_o} - \frac{f}{\tan C_L} \qquad \partial_R = x_R - x_o = \frac{f}{\tan C_R} - \frac{f}{\tan C_o}$$

These both represent the mathematical distance between C_L / C_o and C_R / C_o.

$$|\partial_L| = |\partial_R| \rightarrow \frac{f}{\tan C_o} - \frac{f}{\tan C_L} = \frac{f}{\tan C_R} - \frac{f}{\tan C_o}$$

$$\frac{f}{\tan C_o} + \frac{f}{\tan C_o} = \frac{f}{\tan C_R} - \frac{f}{\tan C_L}$$

$$\frac{f}{\frac{f}{x_o}} + \frac{f}{\frac{f}{x_o}} = \frac{f}{\frac{f}{x_R}} - \frac{f}{\frac{f}{x_L}}$$

$$x_o + x_o = x_R - x_L$$

$$x_o - x_L = x_R - x_o$$

$$|\partial_L| = |\partial_R|.$$

∴ because camera C_L and C_R are both equal distances apart from C_o, $|\partial_L| = |\partial_R|$.