Assignment1

Your submission **must be created using Microsoft Word, Google Docs, or LaTeX**.

Your submission **must be saved as a single "pdf"** document and **have the name "a1.lastname.firstname.pdf"**

Do not compress your submission into a "zip" file.

Late assignments will not be accepted and will receive a mark of 0.

Submissions **written by hand**, **compressed into an archive**, or submitted in the **wrong format** (i.e., are not "pdf" documents) **will receive a mark of 0.**

The due date for this assignment is May 9th, 2017, by 11:30pm.

1. Let a be the proposition "It is May", b be the proposition "It is warm outside", and c be the proposition "I am a student". Translate the following expressions into English. [4 marks]

   a. $\neg c \wedge \neg a$

I am not a student and it is not May.

   b. $b \vee a$

It is warm outside or it is May, or both.
'Or both' is not necessary here, but it is ok if it is included.

   c. $b \oplus a$

It is warm outside or it is May and **not both**.

   d. $a \wedge c \rightarrow \neg\neg b$

If it is May and I am a student then it is warm outside.
Alternatively, If it is May and I am a student then it not, not warm outside.

2. Translate the following English expressions into logical statements. You must explicitly state what the atomic propositions are (e.g., "Let p be proposition ...") and then show their logical relation. [8 marks]

   a. I like math and I like studying but I don't like golf.
      Let m be the proposition "I like math"
      Let s be the proposition "I like studying"
      Let g be the proposition "I like golf"
      $m \wedge s \wedge \neg g$

b. If my alarm rings then I need to wake up.

Let a be the proposition "alarm rings"

Let w be the proposition "I need to wake up"

$a \rightarrow w$

c. Either I was born in January or I was not.

Let j be the proposition "I was born in January"

$j \vee \neg j$ (or, alternatively, $j \oplus \neg j$)

d. I am your instructor if and only if you are a Carleton University student.

Let i be the proposition "I am your instructor"

Let s be the proposition "you are a Carleton University student"

$i \leftrightarrow s$

3. Determine which of the following are True and explain why or why not. [8 marks]

a. 3 < 5 and 1 < 2 and 1 < 0

$$(3 < 5) \wedge (1 < 2) \wedge (1 < 0)$$

$$\text{True} \wedge \text{True} \wedge \text{False}$$

$$\text{True} \wedge \text{False}$$

$$\text{False}$$

(or, alternatively: "Since 1 is not less than 0, at least one of the propositions in this conjunction of three propositions will be false, and since conjunctions are only true if all the component propositions are true, this expression must evaluate to false.")

b. 1 = 2 or 2 > 1.

$$(1 = 2) \vee (2 > 1)$$

$$\text{False} \vee \text{True}$$

$$\text{True}$$

(or, alternatively: "Since 2 is greater than 1, one of the propositions in this disjunction is true, and since disjunctions are true at least one component is true, this expression must evaluate to True.")

c. If 10 = 2 then your instructor is actually Dumbledore.

$$(10 = 2) \rightarrow \text{Dumbledore}$$
$$\text{False} \rightarrow \text{Dumbledore}$$
$$\text{True}$$

(or, alternatively: "Since 10 $\neq$ 2 the antecedent of this implication statement is false and so, regardless of whether or not the instructor is Dumbledore this statement is true.")

d. If 8 = 8 then either $8 = 2^3$ or $8 = 2^2$.

$$(8 = 8) \rightarrow (8 = 2^3 \vee 8 = 2^2)$$
$$\text{True} \rightarrow (8 = 8 \vee 8 = 4)$$
$$\text{True} \rightarrow (\text{True} \vee \text{False})$$
$$\text{True} \rightarrow \text{True}$$
$$\text{True}$$

(or, alternatively: "Since $8 = 2^3 = 8$ and since the consequent is a disjunction with $8 = 2^3$, this consequent must be true. Since the consequent is true, the implication statement must also be true.")

4. Using only the $\rightarrow$, $\wedge$, and $\neg$ operators, find a logical expression that is equivalent to $(\boldsymbol{p} \vee \boldsymbol{q}) \leftrightarrow (\boldsymbol{q} \vee \neg \boldsymbol{r})$. Prove that the expression you found is equivalent by using logical equivalences (state their names next to the line they were applied in). Note: Your final answer **can** have $\rightarrow$**,** $\wedge$, and $\neg$ operators. [5 marks]

$(p \vee q) \leftrightarrow (q \vee \neg r)$

$((p \vee q) \rightarrow (q \vee \neg r)) \wedge ((q \vee \neg r) \rightarrow (p \vee q))$ by Biconditional Equivalence

$(\neg\neg(p \vee q) \rightarrow (q \vee \neg r)) \wedge ((q \vee \neg r) \rightarrow (p \vee q))$ by Double Negation

$(\neg\neg(p \vee q) \rightarrow \neg\neg(q \vee \neg r)) \wedge ((q \vee \neg r) \rightarrow (p \vee q))$ by Double Negation

$(\neg\neg(p \vee q) \rightarrow \neg\neg(q \vee \neg r)) \wedge (\neg\neg(q \vee \neg r) \rightarrow (p \vee q))$ by Double Negation

$(\neg\neg(p \vee q) \rightarrow \neg\neg(q \vee \neg r)) \wedge (\neg\neg(q \vee \neg r) \rightarrow \neg\neg(p \vee q))$ by Double Negation

$(\neg(\neg p \wedge \neg q) \rightarrow \neg\neg(q \vee \neg r)) \wedge (\neg\neg(q \vee \neg r) \rightarrow \neg\neg(p \vee q))$ by DeMorgan's Law

$(\neg(\neg p \wedge \neg q) \rightarrow \neg(\neg q \wedge \neg\neg r)) \wedge (\neg\neg(q \vee \neg r) \rightarrow \neg\neg(p \vee q))$ by DeMorgan's Law

$(\neg(\neg p \wedge \neg q) \rightarrow \neg(\neg q \wedge \neg\neg r)) \wedge (\neg(\neg q \wedge \neg\neg r) \rightarrow \neg\neg(p \vee q))$ by DeMorgan's Law

$(\neg(\neg p \wedge \neg q) \rightarrow \neg(\neg q \wedge \neg\neg r)) \wedge (\neg(\neg q \wedge \neg\neg r) \rightarrow \neg(\neg p \wedge \neg q))$ by DeMorgan's Law

$(\neg(\neg p \wedge \neg q) \rightarrow \neg(\neg q \wedge r)) \wedge (\neg(\neg q \wedge \neg\neg r) \rightarrow \neg(\neg p \wedge \neg q))$ by Double Negation

$(\neg(\neg p \wedge \neg q) \rightarrow \neg(\neg q \wedge r)) \wedge (\neg(\neg q \wedge r) \rightarrow \neg(\neg p \wedge \neg q))$ by Double Negation

5. Prove that the expression you found for question 4 above is equivalent to $(p \vee q) \leftrightarrow (q \vee \neg r)$ by using truth tables. Show all your work (i.e. every column). 8 marks

p	q	r	¬p	¬q	¬r	p ∨ q	q ∨ ¬r	(p ∨ q) ↔ (q ∨ ¬r)
T	T	T	F	F	F	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	F	T	F	T	F	F
T	F	F	F	T	T	T	T	T
F	T	T	T	F	F	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	F	F	T
F	F	F	T	T	T	F	T	F

¬p ∧ ¬q	¬(¬p ∧ ¬q)	¬q ∧ r	¬(¬q ∧ r)	¬(¬p ∧ ¬q) → ¬(¬q ∧ r)	¬(¬q ∧ r) → ¬(¬p ∧ ¬q)
F	T	F	T	T	T
F	T	F	T	T	T
F	T	T	F	F	T
F	T	F	T	T	T
F	T	F	T	T	T
F	T	F	T	T	T
T	F	T	F	T	T
T	F	F	T	T	F

(¬(¬p ∧ ¬q) → ¬(¬q ∧ r)) ∧ (¬(¬q ∧ r) → ¬(¬p ∧ ¬q))
T
T
F
T
T
T
T
F

6. Determine if the following expressions are tautologies, contradictions, or contingencies by using truth tables. Show all your work. [4, 8, 8 marks]

   a. $\neg\left(\neg\left(p \wedge \neg(\neg q \wedge q)\right) \wedge \neg\neg p\right)$

p	q	$\neg p$	$\neg q$	$\neg q \wedge q$	$\neg(\neg q \wedge q)$	$p \wedge \neg(\neg q \wedge q)$	$\neg(p \wedge \neg(\neg q \wedge q))$	$\neg\neg p$	$\neg(p \wedge \neg(\neg q \wedge q)) \wedge \neg\neg p$
T	T	F	F	F	T	T	F	T	F
T	F	F	T	F	T	T	F	T	F
F	T	T	F	F	T	F	T	F	F
F	F	T	T	F	T	F	T	F	F

$\neg(\neg(p \wedge \neg(\neg q \wedge q))) \wedge \neg\neg p)$
T
T
T
T

Thus, it is a tautology.

   b. $\left(q \leftrightarrow \left((p \vee r) \wedge r\right)\right) \wedge \neg(p \rightarrow p)$

p	q	r	$p \vee r$	$(p \vee r) \wedge r$	$q \leftrightarrow ((p \vee r) \wedge r)$	$p \rightarrow p$	$\neg(p \rightarrow p)$	$(q \leftrightarrow ((p \vee r) \wedge r)) \wedge \neg(p \rightarrow p)$
T	T	T	T	T	T	T	F	F
T	T	F	T	F	F	T	F	F
T	F	T	T	T	F	T	F	F
T	F	F	T	F	T	T	F	F
F	T	T	T	T	T	T	F	F
F	T	F	F	F	F	T	F	F
F	F	T	T	T	F	T	F	F
F	F	F	F	F	T	T	F	F

Thus, it is a contradiction.

c. $\left(\left((p \wedge r) \vee q \vee (p \wedge \neg r)\right) \wedge (r \vee p)\right)$

p	q	r	p ∧ r	(p ∧ r) ∨ q	¬r	(p ∧ ¬r)	(p ∧ r) ∨ q ∨ (p ∧ ¬r)	(r ∨ p)	((p ∧ r) ∨ q ∨ (p ∧ ¬r)) ∧ (r ∨ p)
T	T	T	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T	T	T
T	F	T	T	T	F	F	T	T	T
T	F	F	F	F	T	T	T	T	T
F	T	T	F	T	F	F	T	T	T
F	T	F	F	T	T	F	T	F	F
F	F	T	F	F	F	F	F	T	F
F	F	F	F	F	T	F	F	F	F

Thus, it is a contingency.

7. Determine if the following expressions are tautologies, contradictions, or contingencies by using logical equivalences. Show all your work. [4, 4, 4 marks]

a. $\neg\left(\neg\left(p \wedge \neg(\neg q \wedge q)\right) \wedge \neg\neg p\right)$

¬(¬(p ∧ ¬(q ∧ ¬q)) ∧ ¬¬p)	by Commutativity
¬(¬(p ∧ ¬(False)) ∧ ¬¬p)	by Negation
¬(¬(p ∧ True) ∧ ¬¬p)	–
¬(¬p ∧ ¬¬p)	by Identity
¬(¬p ∧ p)	by Double Negation
¬(False)	by Negation
True	

b. $\left(q \leftrightarrow \left((p \vee r) \wedge r\right)\right) \wedge \neg(p \rightarrow p)$

(q ↔ ((p ∨ r) ∧ r)) ∧ ¬ (p → p)	
(q ↔ ((p ∨ r) ∧ r)) ∧ ¬ (¬p ∨ p)	by Implication Equivalence
(q ↔ ((p ∨ r) ∧ r)) ∧ ¬ (p ∨ ¬p)	by Commutativity
(q ↔ ((p ∨ r) ∧ r)) ∧ ¬ (True)	by Negation
(q ↔ ((p ∨ r) ∧ r)) ∧ False	-
False	by Domination

c. $((p \wedge r) \vee q \vee (p \wedge \neg r)) \wedge (r \vee p)$

$((p \wedge r) \vee q \vee (p \wedge \neg r)) \wedge (r \vee p)$

$((p \wedge r) \vee (p \wedge \neg r) \vee q) \wedge (r \vee p)$ by Commutativity

$((p \wedge (r \vee \neg r)) \vee q) \wedge (r \vee p)$ by Distributivity

$((p \wedge (\text{True})) \vee q) \wedge (r \vee p)$ by Negation

$(p \vee q) \wedge (r \vee p)$ by Identity

$(p \vee q) \wedge (p \vee r)$ by Commutativity

$p \vee (q \wedge r)$ by Distributivity

8. Let D(x) be the predicate "x is a dog", M(x) be the predicate "x is a mammal", and E(x) be the predicate "x eats meat". Translate the following expressions into English. The universe of discourse is all animals. [3 marks]

a. $\exists a\, (D(a) \leftrightarrow E(a))$
There is at least one animal that is a dog if and only if it eats meat.

b. $\exists b\, (D(b) \wedge \neg E(b))$
There is at least one animal that is a dog but does not eat meat.

c. $\forall c\, (M(c) \rightarrow (D(c) \vee \neg D(c)))$
Every animal that is a mammal is either a dog or it is not a dog.

9. Negate the following predicate logic statements using the quantifier negation rules discussed in class. Show all your work and ensure that no negation operations appear before any of the quantifiers in the expression you create. Let the universe of discourse be all animals. [8 marks]

a. Every cat purrs and is furry.

Let C(x) be the predicate "x is a cat"

Let P(x) be the predicate "x is purrs"

Let F(x) be the predicate "x furry"

$\forall x\, C(x) \rightarrow P(x) \wedge F(x)$

... this is the expression but now it must be negated...

$\neg\forall x\ (C(x) \rightarrow P(x) \wedge F(x))$ Negated expression

$\exists x \neg\ (C(x) \rightarrow P(x) \wedge F(x))$ Negated Quantifier

$\exists x \neg\ (\neg C(x) \vee (P(x) \wedge F(x)))$ by Implication Equivalence

$\exists x \neg \neg C(x) \wedge \neg\ (P(x) \wedge F(x))$ by DeMorgan's Law

$\exists x\ C(x) \wedge \neg\ (P(x) \wedge F(x))$ by Double Negation

$\exists x\ C(x) \wedge (\neg P(x) \vee \neg F(x))$ by DeMorgan's Law

$\exists x\ C(x) \wedge (\neg P(x) \vee \neg F(x))$ by Double Negation

b. There is at least one cat who doesn't eat mice and can catch fish.

Let C(x) be the predicate "x is a cat"

Let M(x) be the predicate "x is eats mice"

Let F(x) be the predicate "x can catch fish"

$\exists x\ C(x) \wedge \neg M(x) \wedge F(x)$

... this is the expression but now it must be negated...

$\neg\exists x\ C(x) \wedge \neg M(x) \wedge F(x)$ Negated expression

$\forall x\ \neg(C(x) \wedge \neg M(x) \wedge F(x))$ Negated Quantifier

$\forall x\ \neg((C(x) \wedge \neg M(x)) \wedge F(x))$ by Associativity

$\forall x\ (\neg\ (C(x) \wedge \neg M(x)) \vee \neg F(x))$ by DeMorgan's Law

$\forall x\ ((\neg C(x) \vee \neg\neg M(x)) \vee \neg F(x))$ by DeMorgan's Law

$\forall x\ ((\neg C(x) \vee M(x)) \vee \neg F(x))$ by Double Negation