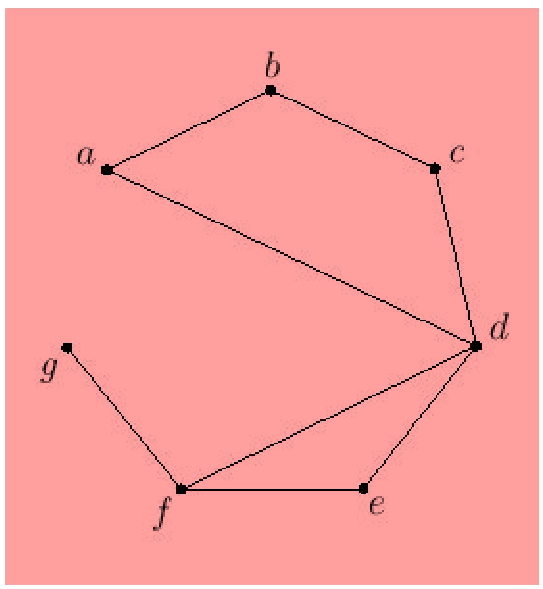
Specification for Tutorial 6

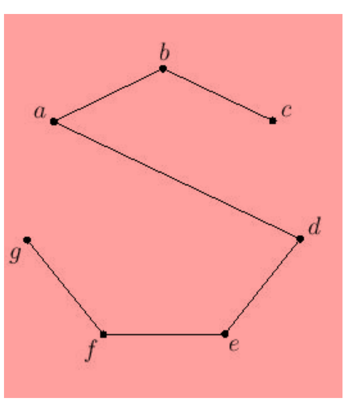
1. Draw the simple undirected graph G = (V, E) represented by the adjacency matrix below. Label the vertices using the alphabet (where a = 1, b = 2, c = 3, etc.) according to the index of the matrix entry. As a clarifying example, entry (2, 3) in the matrix would represent an edge between vertices b and c.

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

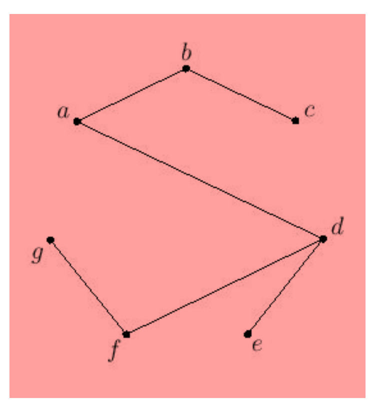
   a. Execute the depth-first search starting from vertex c and show the depth-first search tree.
   b. Execute the breadth-first search starting from vertex b and show the breadth-first search tree.



graph representation

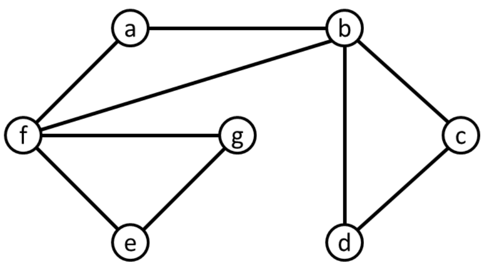


depth-first

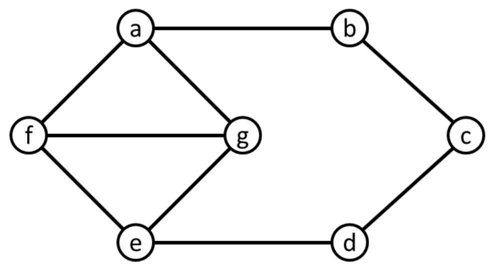


breadth-first

2. An Euler path is a path that traverses every edge of a graph once and only once and a Hamiltonian path is a path that visits every vertex once and only once. Euler cycles (or circuits) and Hamiltonian cycles (are circuits) are Euler paths and Hamiltonians paths (respectively) that start and end at the same vertex. Do the following graphs have Euler cycles? Do they have Hamiltonian cycles?



G_1



G_2

Recall that an Euler cycle is a cycle that visits each edge exactly once. A Hamiltonian cycle is a cycle that visits each vertex exactly once. The first graph has an Euler cycle but does not have a Hamiltonian cycle. An Euler cycle in the first graph is shown below. Any cycle that visits each vertex at least once must pass through vertices b and f twice since b and f are cut vertices. This means that there cannot be Hamiltonian cycle in the first graph. An undirected graph has an Euler cycle if and only if each vertex in the graph has even degree. Since vertices a, e, g and f in the second graph each have degree 3, the second graph does not contain an Euler cycle. It does contain a Hamiltonian cycle, however. A Hamiltonian cycle in G2 is shown below.