Quiz 1 of 4 – Practice Version

Student Number	***Student Name***

This is a closed book exam. No calculators, cellphones, laptops, or other aids are permitted. Answer all questions in the space provided. Show all your work - correct answers presented without justification may receive a mark of zero.

1. Using only the rules of inference and the logical equivalences, show that the following argument is valid. You may assume that all the premises given are true. Make sure that you include both the rule and the line number(s) to which that rule is applied. You are also expected to define each predicate. Note that each of the premises and the conclusion must be translated into quantified statements. The universe of discourse is all animals. [8 marks]

Given Animals that live in the swamp can swim.
Some rats live in the swamp.
All rats have large teeth.

Show There is a rat that has large teeth and can swim.

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2. Use a proof by contradiction to show that:

$\frac{\sqrt{10}}{2}$ is an irrational number

To complete this proof you may use algebra, the logical equivalences, the inference rules, and the following additional "rules":

"the closure of rational numbers under multiplication"
the product when two rational numbers are multiplied is a rational number

"the definition of even numbers"
x is an even number if and only if x can be written as $2k$ for some integer k

"the definition of odd numbers"
x is an odd number if and only if x can be written as $2k + 1$ for some integer k

"the definition of rational numbers"
x is a rational if and only if it can be written as fraction $x = \frac{a}{b}$ in lowest form
i.e., $x = \frac{a}{b} \wedge \frac{a}{b}$ is in lowest form

"lemma 1 (from class)"
x^2 being divisible by 2 $\rightarrow x$ is also divisible by 2

"lemma 2 (from class)"
xy is an even number $\wedge$ x is an odd number $\rightarrow y$ is an even number

You may not use any rules other than those mentioned above.

You must show every step, and every step must be numbered and labelled (with whatever you "used" for that step of the proof).

Complete your proof on the following page

[10 marks]

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3. Using only the rules of inference and the logical equivalences, show that the following argument is valid. You may assume that all the premises given are true. Make sure that you include both the rule and the line number(s) to which that rule is applied. [6 marks]

Premises:

$$u \wedge t$$
$$r \to q$$
$$s \vee (p \to r)$$
$$\neg s \wedge t$$
$$\neg q \wedge u$$

Show: $\neg p$

COMP1805 (Winter 2017) – "Discrete Structures I"

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$A \wedge T = A$	Identity	$A \vee F = A$
$A \vee T = T$	Domination	$A \wedge F = F$
$A \wedge A = A$	Idempotence	$A \vee A = A$
$A \vee \neg A = T$	Negation	$A \wedge \neg A = F$
$\neg(A \wedge B) = \neg A \vee \neg B$	DeMorgan's Law	$\neg(A \vee B) = \neg A \wedge \neg B$
$A \wedge B = B \wedge A$	Commutativity	$A \vee B = B \vee A$
$(A \wedge B) \wedge C = A \wedge (B \wedge C)$	Associativity	$(A \vee B) \vee C = A \vee (B \vee C)$
$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$	Distributivity	$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$
$A \wedge (A \vee B) = A$	Absorption	$A \vee (A \wedge B) = A$
$A \rightarrow B = \neg A \vee B$	Implication Equivalence	
$A \rightarrow B = \neg B \rightarrow \neg A$	Contraposition	
$A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A)$	Biconditional Equivalence	
$\neg(\neg A) = A$	Double Negation	

p $\therefore\ p \vee q$	Addition	$p \wedge q$ $\therefore\ p$	Simplification
p q $\therefore\ p \wedge q$	Conjunction	$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	Resolution
p $p \rightarrow q$ $\therefore\ q$	Modus Ponens	$\neg q$ $p \rightarrow q$ $\therefore\ \neg p$	Modus Tollens
$\neg p$ $p \vee q$ $\therefore\ q$	Disjunctive Syllogism	$p \rightarrow q$ $q \rightarrow r$ $\therefore\ p \rightarrow r$	Hypothetical Syllogism
$\forall x\, P(x)$ $\therefore\ P(c)$	Universal Instantiation	$P(c)$ $\therefore\ \exists x\, P(x)$	Existential Generalization
$\exists x\, P(x)$ $\therefore\ P(c)$	Existential Instantiation *(for a "fresh" variable c)*	$P(c)$ $\therefore\ \forall x\, P(x)$	Universal Generalization *(with "arbitrary" variable c)*