

Super Froggie

Problem Description

There are N lily pads arranged in a row, numbered from 1 to N .

Super Froggie is currently on lily pad 1 and needs to reach lily pad N .

In one step, Super Froggie jumps:

- either from lily pad i to lily pad $i+1$ with probability $1 / 2^{(i-1)}$
- or from lily pad i to lily pad $i-1$ with probability $1 - 1 / 2^{(i-1)}$

Note:

- 2^x means 2 to the power of x , for example $2^3 = 8$
- When Super Froggie is at lily pad 1, it jumps to lily pad 2 with probability 1
- When Super Froggie reaches lily pad N it stops jumping

Find the **expected number of steps** it'll take Super Froggie to reach lily pad N .

For 200 points:

Your solution must be able to work for values of N between 2 and 5 inclusive.

For 500 additional points:

Your solution must be able to also work for values of N between 6 and 8 inclusive.

Example 1:

Input:

2

Output:

1

Explanation:

There are 2 lily pads. With probability 1, Super Froggie jumps from lily pad 1 to 2 and reaches its destination in one step. Therefore, the expected number of steps to reach the last lily pad is 1.

Example 2:

Input:

3

Output:

4

There are 3 lily pads. From lily pad 1, with probability 1, Super Froggie jumps to lily pad 2. From lily pad 2, with equal probability 0.5 Super Froggie can jump to lily pad 1 or to lily pad 3. Working out the math gives a result of 4.