NOTES

Financial Markets Analytics

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Abstract

The course focuses on two macro areas of topics. A first part focused on more advanced portfolio theory models, i.e. Strategic/Tactical Asset Allocation models. The focus in this first part will be more on the empirical applications of the models and the more technical data issues involved in the development of the models. In the second part the course focuses on more specific topics, related to the implementation of active investment strategies, clarifying the relationship with the market efficiency hypothesis, the relevance of data and its quality for building reliable investment strategies.

Contents

Chapter 1

Key concepts of stock/bonds valuation, liquidity and price formation

1.1 Acquisition of Capital

Corporations, big and small, need **capital** to do their business. The investors provide the capital to a corporation. A company may need a new factory to manufacture its products, or an airline a few more planes to expand into new territory. The firm acquires the money needed to build the factory or to buy the new planes from investors. The investors, of course, want a return on their investment. Therefore, we may visualize the relationship between the corporation and the investors as follows:

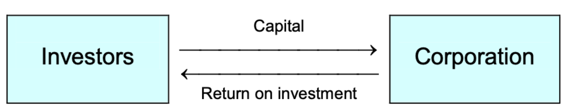


Figure 1.1: The relationship between the investors and a corporation.

Capital comes in two forms: *debt* capital and *equity* capital. To raise debt capital the companies sell **bonds** to the public, and to raise equity capital the corporation sells the **stock** of the company. Both stock and bonds are financial instruments and they have a certain intrinsic value.

Instead of selling directly to the public, a corporation usually sells its stock and bonds through an intermediary. An **investment bank** acts as an agent between the corporation and the public. Also known as **underwriters**, they raise the capital for a firm and charge a fee for their services. The underwriters may sell $100 million worth of bonds to the public, but deliver only $95 million to the issuing corporation. A corporation that is selling its bonds, or stock, for the first time may have to pay a higher percentage of the total value as underwriters' fees. Well-established companies with strong financial record can sell their stock or bonds with relative ease and so the underwriters' fees are lower. When a corporation issues its stock for the first time, it is known as an IPO, or an **initial public offering**. Later, the investors buy and sell the stock in the **secondary markets**, such as the New York Stock Exchange.

There are two types of investors, the stockholders, and the bondholders, who provide the financial capital of a company. The stockholders are the real owners of the corporation. They have an equity stake in the business. The bondholders merely lend the money to the company. They receive a set rate of return determined by the coupon rate on the bonds. So the stockholders receive dividends. However, the company does not guarantee dividends, and some companies do not pay any dividends at all. The bondholders receive regular, guaranteed interest payments. If the bondholders do not receive the interest payments on time, they have a right to sue the company and seize the assets of the firm. The bondholders also receive the face value of the bonds at maturity.

1.2 Bonds

Bonds, in particular, are a popular financial instrument used by firms to raise funds for their projects. Corporations sell bonds to borrow money from the investors. As a financial instrument, a bond represents a contractual agreement between the corporation and the bondholders. Eventually the corporation has to repay the principal to the investors and pay interest to them in the meantime. They are an attractive alternative to borrowing from traditional banking systems, which often charge high-interest rates. When a bond is *issued*, a fixed price is set, and only a fraction of the total amount of borrowed money is offered in the form of bonds. These bonds are typically sold to *institutional investors* such as insurance companies, banks, and funds. This type of bond is known as **corporate bonds** since they are issued by corporations.

Another significant category of bonds in the financial market is known as **government bonds**, which are issued by countries or states. Governments use this financial instrument to borrow money to finance their various financial structures. In Italy, for example, the public debt, which stands at 2 trillion $, is primarily composed of government bonds issued to raise money. Government bonds can be divided into two categories:

- **Short-term bonds**: include BOT, with a maturity period of 3 to 6 months, and CTZ, which have a 24-month maturity period. Both of these are *zero-coupon* bonds, meaning they do not pay any interest during their tenure.
- **Long-term bonds**: include CCT, which have a maturity period of 5 to 7 years, and BTP, which can have a maturity period ranging from 5 to 30 years. Long-term bonds generally have a *coupon payment*, which is paid out to the bondholder during the life of the bond.

Let's now take a closer look at zero-coupon bonds. Suppose that at t_0, the bond's price is 98 \$, and after six months at t_1, the price rises to 100 \$. In this case, the 2 \$ increase is referred to as *interest gained*.

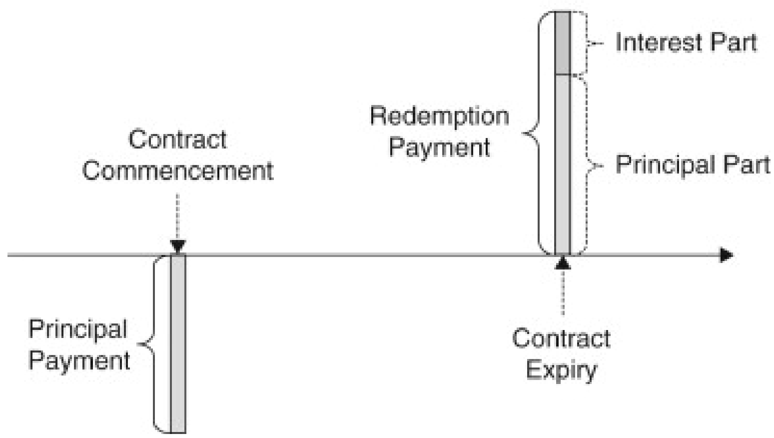


Figure 1.2: Zero-coupon bonds.

In contrast, for a long-term BTP bond, we pay 99 \$ per bond at t_0, and every six months, we receive coupons. At the end of the bond's tenure, which could be up to ten years, we will receive 100 \$ (the bond's face value). For instance, we could receive 3 \$ for each coupon payment, which is the *interest rate gained*, resulting in an *annual interest rate* of approximately 6%.

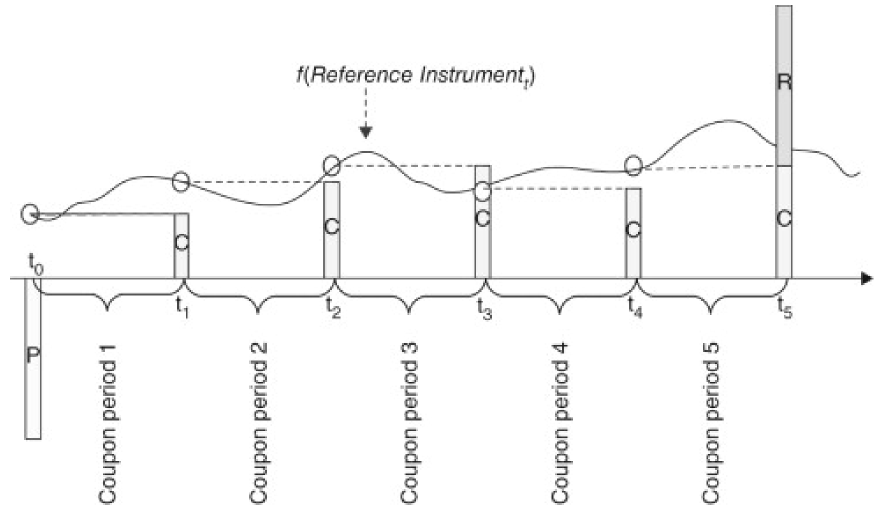


Figure 1.3: Coupon bonds.

Coupon payments can be more complex than this and are often linked to an index or may have additional elements linked to inflation (inflation-linked bonds). Corporate bonds have a structure similar to long-term government bonds.

Typically, a bond has the following features:

- *The face value*, F: The face value of a bond, or its principal, is usually \$1,000, which means that the investment in bonds is a multiple of \$1,000. The total value of the bonds issued by a company at a certain time could be millions of dollars.
- *The market value*, B: Although a bond may have a face value of \$1000, it may not sell at \$1000 in the bond market. If the issuing company is not doing well financially, its bonds may sell for less than \$1000, perhaps at \$950. If you look up their price on the Internet, or some financial newspaper, it is listed as 95. This means that the bond is selling at 95% of its face value, or \$950. The bond is selling at a discount. If the market value of the bond is more than \$1,000, and then it is selling at a *premium*. A bond with a market value less than \$1,000 is selling at a *discount*, and a bond, which is priced at its face value, is selling at *par*.
- *The time to maturity*, n: There is a definite date when a bond matures. At that time, the corporation must pay the face value of the bonds to the bondholders. This could be from as little as 5 years to as long as 100 years. The short-term bonds are also called *notes*. The companies that are starting out, do not want to carry a long-term debt burden and so they issue relatively short-term bonds. Well established companies prefer to use long-term debt in their capital, especially when the interest rates are low.
- *The coupon rate*, c: This is the stated rate of interest of the bonds. For example, a bond may be paying 8% interest to the bondholders. The dollar amount of interest C, is the product of the face amount of the bond and the coupon rate. We may write this

as

$$C = cF$$

The 8% bond is paying $.08 * 1000 = \$80$ per year to the investors. The corporations generally pay the interest semiannually, so the 8% bond really pays \$40 every six months. For example, a bond may pay interest on February 15 and August 15 in a calendar year. If an investor buys a bond between the interest payments dates, let us say on May 1, then he has to pay the accrued interest, the interest for the period February 16 to May 1, to the seller of the bond. The interest rate on a bond depends primarily on two factors.

  - First, it depends on the general level of interest rates in the economy. At the time of this writing, the interest rates are at their historical lows due to the easy-credit policy of the Federal Reserve Board. This allows companies to borrow money at lower rates enabling them to expand their business easily. At other times, the interest rates may be quite high, partly because of Fed's tight money policy. This forces all companies to borrow at a higher rate of interest.
  - Second, the company, which is issuing bonds, may not be in a strong financial condition. The sales are down, the cash flow is small, and the future prospects of the company are not too bright. It must borrow new money at a higher rate. On the other hand, well-managed companies in a strong financial position can borrow at relatively low interest rates.

- *The indenture*: The indenture is the formal contract between the bondholders and the corporation. Written in legal language, the fine print spells out the rights and responsibilities of both parties. In particular, the indenture requires the company to pay interest to the bondholders whenever it is due. The companies have to pay interest before they pay taxes or dividends on the common stock. This makes the position of the bondholders quite secure. The indenture also spells out the timetable for bond refunding. Another clause in the indenture further strengthens the position of the bondholders. This allows them to force the company into liquidation if the company fails to meet its interest obligations on time.

Now that we have defined how bonds work let's go deeper in the theory. *Refinancing* is a standard procedure for repaying bonds. It involves issuing new bonds to repay the old ones that are approaching maturity. Essentially, when a bond reaches maturity, the borrower can either repay the principal amount borrowed, including any accrued interest, or issue new bonds to raise the funds to repay the old ones. To issue new bonds, the borrower must maintain *credibility* in the market. This means that investors must perceive the borrower as trustworthy and reliable. Maintaining credibility in the market is crucial because it allows the borrower to continue refinancing their bonds and raising funds to meet their financial obligations. If a company or government is considered more risky, investors may be less willing to invest in their bonds in the future. This is because investors may perceive the borrower as more likely to *default* on their bonds, which means that the borrower would be unable to repay the principal amount borrowed (because there aren't investors for the new bonds), including any accrued interest. Defaulting on bonds can have severe consequences for the borrower, including legal action, loss of credibility in the market, and difficulty in raising funds in the future. Therefore, it is essential for borrowers to maintain credibility in the market to continue refinancing their bonds and avoid the risk of default.

We need to understand the distinction between liquid assets and non-liquid assets. We will only focus on liquid assets, which are those that can be bought immediately. On the other hand, non-liquid assets cannot be bought because there is not enough liquidity available. When we talk about **liquid assets**, we're typically referring to those that can be easily converted into cash without significantly affecting the asset's price. These types of assets are considered to be more easily traidable and therefore more desirable for investors. Examples of liquid assets include stocks and bonds that are traded on public exchanges, as well as commodities like gold and silver. **Non-liquid assets**, on the other hand, are those that cannot be easily converted into cash due to a lack of buyers or a lack of market liquidity. Examples of non-liquid assets include real estate, private equity, and certain types of bonds that are not actively traded on public exchanges. While non-liquid assets can offer attractive returns, they are typically considered riskier due to their lack of liquidity.

Institutional investors are the ones who invest in *private equity funds*, which are not liquid. Real investors cannot liquidate their position because the funds cannot be liquidated due to time rules that must be respected. These types of investments are typically only available to accredited investors due to their high minimum investment requirements and lack of market liquidity. Private equity funds can offer attractive returns, but they require investors to lock up their funds for an extended period of time, often several years or more. Although the private market is not subject to this course, it is important to know that many other financial instruments exist.

After discussing stocks and bonds, we will move on to *Portfolio Theory*. A portfolio is a group of different financial assets, such as stocks and bonds, in which we invest to reduce the risk of investing in a single asset. When it comes to building a portfolio, the goal is typically to construct a mix of assets that will provide a balance between risk and return. Modern portfolio theory takes into account factors like asset diversification, risk tolerance, and expected returns to construct a portfolio that is optimized for an investor's unique needs and preferences. By investing in a variety of assets across different sectors and asset classes, investors can reduce their overall risk and potentially achieve better returns over the long term. Modern portfolio theory is a set of rules for investing correctly by constructing an optimal portfolio, which is better than a random one.

In the market, **fixed income investments** refer specifically to bonds. One of the key drivers in the world of market movements is interest rates. **Interest rates** are primarily influenced by central banks, such as the European Central Bank (ECB), the Federal Reserve (FED), and the Bank of England (BOE). The primary goal of central banks is to maintain stable prices and keep inflation, denoted as π, low, typically below 2%. Inflation refers to the percentage change in prices within the economy. When prices begin to increase, it can create significant problems for a country. One of the primary factors that can drive prices higher is a lack of supply relative to demand. The pandemic has caused significant supply chain disruptions, leading to price increases across many sectors. Other events following the pandemic, such as war, can also contribute to this phenomenon, which has resulted in inflation levels of up to 10%, which is considered extremely high. Inflation causes a loss of purchasing power for non-invested funds, which is why central banks attempt to mitigate this phenomenon by increasing interest rates. Increasing interest rates can help to reduce inflationary pressure by discouraging borrowing. When interest rates rise, borrowing becomes more expensive, which reduces demand and, in turn, reduces prices. Essentially, increasing interest rates can help to reduce economic activity, which reduces demand and can help to curb inflation.

1.2.1 Bonds Indicators

Rated by Fitch or other agencies, the letters AAA, AA, and A represent the quality of bonds. The highest quality, or least risky, bonds are designated by AAA, and so on. We notice two things. First, the longer maturity bonds of the same quality rating have a higher yield. For instance, for bonds with A rating, the yield for 2-year maturity is 5.13%; and for 20 years, it is 5.82%. Second, the yield is higher for riskier bonds. Consider 5-year bonds. The yield rises from 5.06% to 5.20% when the rating drops from AAA to A.

Normally, when an investor buys a bond he has to pay the accrued interest on the bond. This is the interest earned by the bond since the last interest payment date. Occasionally some bonds trade without the accrued interest and they are thus dealt in flat. Due to poor financial condition of the company, such bonds sell at a deep discount from their face value.

An investor buys a bond for its future cash flows. To evaluate a bond, therefore, we have to find the present value of the cash flows. We use a very fundamental concept in finance: *The present value of a bond is simply the present value of all future cash flows from the bond, discounted at the risk-adjusted discount rate.*

We may use this concept to find the value of any financial instrument, whether it is a stock, a bond, or a call option. For a bond, we need to find the present value of all the interest payments and the present value of the final payment, namely, the face amount of the bond. We may write it mathematically as:

$$B = \sum_{i=1}^{n} \frac{C}{(1+r)^i} + \frac{F}{(1+r)^n}$$

In the above equation, we define:

B = the present value, or the market value of the bond
C = cash flow from the interest of the bond, and for semiannual interest payments, it should be one-half of the annual interest paid by the bond
n = the number of semiannual payments received
F = face amount of the bond
r = risk-adjusted discount rate for the bond. For riskier bonds, the discount rate is higher.

This equation is unsolvable for r, typically we use some iterative procedure to find the risk.

We can do the summation by using:

$$\sum_{i=1}^{n} \frac{C}{(1+r)^i} = \frac{C[1-(1+r)^{-n}]}{r}$$

Thus, we can find the value of a **bond value** by:

$$\text{Bond value} = B = \frac{C[1-(1+r)^{-n}]}{r} + \frac{F}{(1+r)^n}$$

Consider a bond that is never going to mature, that is, it is a perpetual bond. An investor will buy such a bond and earn interest on it. The bond will pay a steady income forever. If he no longer needs an income, he can simply sell the bond to another investor. The bond represents a perpetual income stream and we can evaluate it by using:

$$\sum_{i=1}^{\infty} \frac{C}{(1+r)^i} = \frac{C}{r}$$

For perpetual bonds it become:

$$B = \frac{C}{r}$$

It is also possible to get it by setting $n = \infty$ in the previous one.

Another type of a bond is a zero-coupon bond. Such a bond does not pay any interest but it does pay the principal at maturity. An investor who does not need a steady income, but requires \$1000 at a future time, may buy such a bond. The value of a zero-coupon bond is found by letting $C = 0$ in bond value formula. The result is:

$$B = \frac{F}{(1+R)^n}$$

The holder of a **convertible bond** is entitled to convert it into a fixed number of shares of the stock of the issuing corporation at any time before maturity. As the stock price rises, the value of the bond also rises. Occasionally, convertible bonds sell well above the par value. The convertible bonds are quite difficult to evaluate. An investor buys a bond for its *yield*, which is the annual return on the investment. We may define the *current yield*, y, of a bond as the annual interest C in dollars, divided by the market price of the bond B in dollars. In symbols,

$$y = C/B$$

This represents the return on investment provided one holds the bond for a short time. For instance, you buy a 5% coupon bond at 60. Then the annual interest received is \$50, and the market price of the bond \$600. Dividing one by the other, we get the current yield as $y = 50/600 = 8.33\%$ Suppose a bondholder wants to hold the bond all the way to its maturity. Then he may be interested to find its yield-to-maturity.

- **yield-to-maturity**, Y: The yield-to-maturity of a bond is that particular value of r that will equate the market value of a bond to its calculated value by using the formula of bond value.

In practice, one can calculate the yield to maturity accurately by using Excel, WolframAlpha, or Maple.

When you hold a bond to maturity, you receive money in the form of interest payments, plus there is a change in the value of the bond. If you have bought the bond at a discount, it will rise in value reaching its face value at maturity. On the other hand, the bond may drop in price if you have bought it at a premium. In any case, it should be selling for its face value at maturity. The total price change for the bond is $(F - B)$ which may be positive or negative depending upon whether F is more or less than B. On the average, the price change per year is $(F - B)/n$. The average price of the bond for the holding period is $(F + B)/2$. We may calculate the yield to maturity of a bond, approximately, by dividing the average annual return by the average price. We write it as follows:

$$Y \approx \frac{\text{annual interest received} + \text{annual price change}}{\text{average price of the bond for the entire holding period}}$$

Or,

$$Y \approx \frac{C + (f - B)/n}{(F + B)/2}$$

Consider a bond with coupon rate 8% and 10 years to maturity. If the discount rate is 8%, then the bond is selling at par. Its value will remain \$1000 with the passage of time. This is shown as the straight horizontal line in the middle of Fig. 1.4.

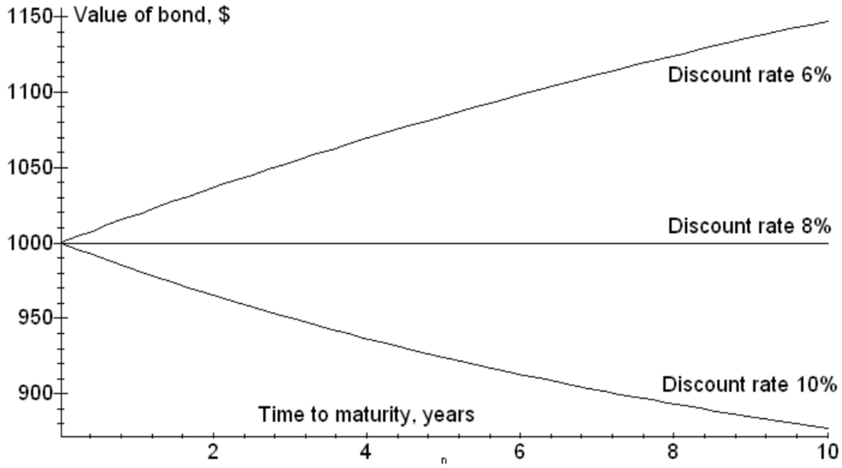


Figure 1.4: The value of a bond with coupon rate 8%, discounted at different discount rates. As the bond approaches maturity, time to maturity becomes zero, and its value approaches \$1000. Before maturity, its value is more than \$1000 if the discount rate is less than 8%. Similarly, the value is less than \$1000 for a discount rate higher than 8%.

If the discount rate is 10%, the bond will sell at a price less than \$1000. Its calculated value is \$875.38. This is shown as the bottom curve in Fig. 3.3. With the passage of time the bond actually rises in value, and at maturity, it becomes \$1000. Assuming that the company is financially strong, it will redeem the bonds at \$1000 at maturity.

Typically on the y axes we have interest rate and time on the x, the typically curve increase, and this graph help to know the interest rate after a specific period of time. But during crises or difficult time, the shape of (terms fraction) change, to flat shape or downward case during recession period. This situation are typically caused by expectation of investors, central banks only act changing the level on only short maturity, the shape in the long terms is changed by the supplier demand by investors. Using the formula we have to use a number of different r equal to the number of cash flows. If we know B, lets assume it is 97 \$ per bond, and lets also assume $C = 3$ and $F = 100$, whit different trials (because the equation is not directly solvable) we can calculate IRR that is the rate named "yield to maturity" of the bond. It is a synthetic indicator of the profitability of investors. There is a negative relationship between interest rate and price of bonds, so a reduction of r increase the price and vice versa. The dynamic of price in bond market derive form the dynamic of interest rate, which is not the ones made by central bank.

It is important to keep in mind that interest rates play a key role in the movements of fixed income assets. For example, suppose that today I buy a bond with a maturity of five years for 100 \$. This bond pays a coupon of three \$ per year, and at the end of the five years, the bond will be worth 100 \$ plus the sum of the coupons received, which is also equal to 115 \$. This means that the interest rate on this bond is $i = 3\%$. However, let's suppose that inflation arrives and the ECB decides to increase interest rates up to $i = 4\%$ after six months. This would cause a decrease in the value of the initial investment, as the coupon payments would increase from 3 to 4 \$. As a result, the bond's market price could drop to 90 \$, which could be of interest to a new investor. The capital gain for someone who pays 90 \$ for this bond would be 0.

In reality, the market interest rate changes every day based on investor sentiment, news, and other market factors, and not just based on decisions made by central banks. This leads to fluctuations in prices and a highly dynamic market environment.

1.3 Stocks

Stocks are considered as the most frequently used financial instrument in the financial markets. They are used to share the ownership of firms with the investors. When investors possess stocks, they become the *owners* of the firms and they share the business risks with the firms. As compared to bonds, stocks are more risky as the value of stocks depends on the performance of the firm in the market.

There are two ways of getting money from stocks. The first way is through **capital gain**, which means that investors gain in terms of the value of their stocks. For example, if someone buys stocks at 10 \$ and sells them at 13 \$, their capital gain will be 3 \$. The second way is through **dividends**, which are annual payments made by the firms to the shareholders. Dividends are the sharing of the earnings of the firm. Every year, firms calculate their earnings in the balance sheet and decide whether they want to share their earnings with equity investors or not. However, they may also choose to retain the earnings for future investments. In some cases, firms may not pay dividends if they do not have sufficient earnings to share or if they want to use the money for future investment.

The stockholders are taking on more *risk* because their dividends are dependent on uncertain cash flows. To conserve cash a company may resort to eliminating cash dividends. The bondholders' position is much safer. The company must pay the interest before it pays the income tax or dividends. The risk associated with dividends is completely different from that of coupons. Taxes on dividends are usually around 26%. Investing in stocks has an expected return of approximately 10%, but with a volatility of 20%. Volatility refers to the range of variation in return, and it is an important factor to consider while investing in stocks. It is also important to keep in mind that the stock market is highly dynamic, and the value of stocks can change rapidly based on news and other factors affecting the market.

The stockholders participate in the growth of the company. They also bear the losses when the times are tough. The bondholders cannot participate in the growth of the company. At the most, they can receive the interest payments and the face value of the bonds. The stockholders have the right to elect the board of directors of the corporation. The board is responsible for the implementation of major decisions at the company, such as the appointment of the president. In this way, the stockholders can participate in the management of the firm. The bondholders cannot participate in the running of the company. The cash flows for a stock are quite random. The firm faces economic uncertainties. There is the possibility of labor strife, or shortage of raw materials, or unexpected action by the competitors. Each turn of events can make the earnings of the company unpredictable. The sudden changes in the stock price are essentially due to the changes in the financial condition of a firm. A look at the stock pages in a newspaper, with up and down movements of the stock prices, makes this idea quite clear. The stockholders are sharing this risk of the company.

The valuation of the equity of a firm is a much more difficult process due to the inherent uncertainty of the cash flows. Equation for P_0 gives a general formula for the stock valuation. However, we derive this formula under severe restrictions:

1. The growth of the company is uniform from year to year, that is, the growth rate g is constant. This is not true for actual firms. The company may grow by 10% and it may drop in value by 5% the following year.
2. The growth will continue forever. This assumption is also unrealistic, because the companies go through a supernormal growth period for a while. Then the competition gets in and slows the growth. Mature companies show little growth, and they may even shrink in value.
3. The dividends paid out by the firms will also grow at the same rate as the overall growth of the company. In real life, the companies set their dividend policy based on their investment needs, their cash flow projections, and their capital structure.
4. The required rate of return of the stockholders is greater than the growth rate of the company. This is strictly a mathematical requirement to make sure that the formula will work properly.

Since no firm can meet all these conditions, the formula is only approximately true.

To develop the formula, let us suppose that the dividend paid during the current year has been D0. If the dividends are growing uniformly, the dividend next year is $(1+g)$ times the dividend this year, that is:

$$D_1 = D_0(1+g)$$

g is commutable only if the price of dividends is made explicit by firms, and it is **growth rate**. The dividend available two years from now will be: $D_2 = D_1(1+g)$ and so on. The dividend available after three years should be: $D_3 = D_2(1+g) = D_1(1+g)^2$

It is possible (like in Stellantis case) that the firms also communicate a buy back strategy within the dividends payment."Buy back" means that the firm buy his own stocks, this is an aggressive way to give a reword to shareholders. This will increase the dividends value and potentially also the stocks value. This buy back is made with specific strategy in way that the investors does not keep their stocks blocked by the news of the buy back, it happened 'secretly' without communicating the date and by the fact that in the 'pay book' the names of buyers and sellers are unknown.

When we buy the stock we expect to receive dividends D1, D2, D3, ... after one, two, three, ... years. The sum of the future dividends, properly discounted, is just equal to the current price of the stock. Thus:

$$P_0 = \frac{D_1}{1+R} + \frac{D_1(1+g)}{(1+R)^2} + \frac{D_1(1+g)^2}{(1+R)^3} + ...\infty$$

We can find the summation using WolframAlpha. This is an infinite geometric series with first term $\alpha = \frac{D_1}{1+R}$, and ratio between the terms, $x = \frac{1+g}{1+R}$. And we get:

$$P_0 = \frac{\frac{D_1}{1+R}}{1 - \frac{1+g}{1+R}}$$

After some simplification, we get the final result as:

$$P_0 = \frac{D_1}{R - g}$$

In the above equation, R is the *risk-adjusted discount rate*, it is a fundamental parameter risk (discount factor), knowing the different rate make possible to calculate the price, but obviously we didn't know it for the future. The stocks are much riskier and thus we must use a much higher discount rate. This discount rate is the *required rate* of return, required by the investor who is putting his money at risk. Usually called **Gordon's growth model**, after **Myron J. Gordon** who initially developed the above equation in 1959 at University of Toronto.

Besides common stock, a firm may also issue preferred shares of stock. The **preferred stock** lies somewhere between the common stock and the bonds of a company in terms of priority of claims on the assets of the firm. The preferred stockholders get constant dividends and they are not entitled to participate in the growth of the company. Thus substituting $g = 0$, we get the value of a preferred share as:

$$P_0 = \frac{D}{R}$$

Comparing the formula, we may think of preferred stock to be like a perpetual bond with dividends D annually, and discount rate R. Sometimes the preferred stock is callable, that is, the company has the right to buy it back at a certain future date for a certain price. In this case, the preferred stock is similar to an ordinary bond.

We may write the main result, the Gordon's model formula, as:

$$R = \frac{D_1}{P_0} + g$$

The interpretation of this equation is quite simple: the required rate of return from a stock R is the sum of its two components, the dividend yield D_1/P_0, and the growth rate of the dividends, g.

The Internet has become one of the best sources of financial information. The information is available easily. It is accurate, thorough, and timely. It is also possible to trade stocks or pay bills on-line. A direct consequence of this information explosion is that the investors are much better informed. The trading costs have become very reasonable. Further, it is possible to trade in your account from anywhere in the world if you have access to the Internet.

1.3.1 The Gordon's growth model

The Gordon Model is used to estimate the intrinsic value of a stock based on its expected dividends and future growth rate. The formula for the Gordon Model can be expressed as:

$$P_0 = \frac{D_1}{R - g}$$

where:

- P_0 is the current price of the stock
- D_1 is the expected dividend payment in the next period (usually one year)
- g is the expected growth rate of the dividend payments

This formula assumes that the dividends are expected to grow at a constant rate indefinitely. It also assumes that the required rate of return for the stock is greater than the growth rate of the dividends.

The Gordon model, also known as the Gordon's growth model, is a simple yet powerful tool for valuing stocks. The model calculates the fair price of a stock as a ratio between the most recent known dividend payment and the *risk-adjusted discount rate* R minus the expected growth rate of future dividends. R represents the required rate of return, which investors demand to compensate for the risk of investing in the stock market. The higher the risk of the stock, the higher the discount rate used in the calculation. Despite its simplicity, the Gordon model remains a widely used and valuable tool for investors looking to estimate the fair value of a stock.

Calculating the discounting factors for the Gordon model can be a challenging task for investors. R is particularly difficult to calculate as it requires a thorough understanding of the underlying risk of the stock. The required rate of return is influenced by various factors such as macroeconomic conditions, market trends, and company-specific risks. This makes R a volatile factor that can change rapidly based on new information or changes in the market environment.

The *expected growth rate of future dividends* g is another factor that is difficult to calculate. The growth rate of a company's dividend payments is dependent on many factors such as the company's financial performance, market conditions, and industry trends. These factors can be unpredictable, making it challenging to estimate the future growth rate of a company's dividends accurately. This volatility makes g a risky factor in the Gordon model, which can change rapidly based on new information.

Furthermore, changes in R and g can have a significant impact on the fair price of a stock. A sudden increase in R, for example, can cause a significant drop in the fair price of a stock, reflecting the higher risk associated with the stock. Similarly, changes in the expected growth rate of future dividends g can have a significant impact on the fair price of a stock, reflecting the changing expectations of investors for the company's future performance.

In summary, while the Gordon model is a useful tool for valuing stocks, the discounting factors (R and g) can be difficult to calculate accurately and are subject to significant volatility. As such, investors need to remain vigilant and constantly reassess the underlying risks associated with the stocks they hold.

1.3.2 Comparables

While the Gordon model is one type of dividend discount model (DDM) used to estimate the intrinsic value of a stock based on its expected dividends and growth rate, there are other DDMs that can be used as well. In addition to DDMs, there are other methods of valuing stocks such as using *multiples*. Multiples are a way of understanding whether a stock price is cheap or expensive by comparing it to similar companies or **comparables**. This is done by taking a certain financial metric, such as earnings or sales, and dividing it by the stock price to get a ratio or multiple. This multiple is then compared to the multiples of similar companies to determine if the stock is undervalued or overvalued relative to its peers.

Comparables are a key tool used in financial analysis to evaluate the performance and valuation of companies. Here are some examples of commonly used comparables and their use:

1. *Price-to-earnings ratio*, P/E: This ratio compares the current market price of a company's shares to its earnings per share (EPS). A high P/E ratio typically indicates

that investors expect the company to grow its earnings in the future, while a low P/E ratio suggests that the company may be undervalued relative to its earnings potential. However, a high P/E ratio may also indicate that the stock is overvalued.

2. *Price-to-book ratio*, P/B: This ratio compares a company's market price per share to its book value per share. Book value is the value of a company's assets minus its liabilities, and is often used as a proxy for the company's intrinsic value. A low P/B ratio may indicate that the company is undervalued relative to its assets, while a high P/B ratio may indicate that the company is overvalued.
3. *Price-to-cash flow ratio*, P/CF: This ratio compares a company's market price per share to its cash flow per share. Cash flow is an important metric because it represents the actual cash generated by a company's operations, rather than its accounting earnings. A low P/CF ratio may indicate that the company is undervalued relative to its cash flow potential, while a high P/CF ratio may indicate that the company is overvalued.
4. *Debt-to-equity ratio*, D/E: This ratio compares a company's total debt to its equity. A high D/E ratio may indicate that the company is highly leveraged and therefore more risky, while a low D/E ratio may indicate that the company is less risky. Investors typically look for a D/E ratio that is appropriate for the industry in which the company operates, and that balances the need for financing with the need for financial stability.

Overall, comparables provide a useful way to evaluate the performance and valuation of companies, and can be used in combination with other financial metrics and analysis to make informed investment decisions.

Peers, or comparable firms, are companies that operate in the same industry or sector as the company being analyzed. These firms can be used to benchmark the performance of the company being analyzed and to assess its relative valuation. One way to use peers is to calculate financial ratios, such as the P/E ratio, for each company and then take an average of those ratios to establish a benchmark or multiple for the industry. This benchmark can then be used to value a non-public company in the same industry by applying the benchmark multiple to the company's earnings or other financial metrics.

For example, suppose Generali wants to acquire a small bank that is not publicly traded. The small bank is asking for a purchase price of \$300 million, but Generali believes that the bank's true value is closer to \$180 million. To support its valuation, Generali could use peers to establish a benchmark multiple for the industry, such as the average P/E ratio of three comparable banks. Suppose that the average P/E ratio of the comparable banks is 8, and the small bank has earnings of \$30 million. Generali could then apply the P/E multiple of 8 to the small bank's earnings to arrive at a valuation of \$240 million ($30M\$ \times 8$). Based on this valuation, Generali may offer to acquire the small bank for a price between \$180 million and \$240 million.

It's important to note that using peers to value a company requires careful consideration of the comparability of the firms being analyzed. For example, a large, publicly-traded bank may not be directly comparable to a small, privately-held bank due to differences in scale, business model, and other factors. Adjustments may need to be made to account for these differences and ensure that the benchmark multiple accurately reflects the value of the company being analyzed. Other comparables indicators, such as P/B ratio, P/CF ratio, and D/E ratio, may also be used in conjunction with P/E ratio to provide a more complete picture of a company's valuation.

Chapter 2

The characteristics of returns and of portfolios, Modern Portfolio Theory

2.1 Modern Portfloio Theory

Modern Portfolio Theory (MPT) is a popular approach to investing that is focused on constructing a portfolio that balances the risks and rewards of different investments. When discussing MPT, one of the primary questions that arises is, "how can we build a good portfolio?" Unlike a single investment, which is limited to one asset or security, a portfolio is a collection of multiple investments, such as stocks, bonds, and mutual funds. The idea behind diversification is to spread investments across a range of asset classes and sectors to reduce the risk of any one investment significantly impacting the overall performance of the portfolio. **Diversification** can be an effective technique to reduce risk, but it's essential to recognize that there's a **tradeoff between risk and reward**. While a well-diversified portfolio can help minimize the potential losses from a single investment, it may also result in lower returns overall. However, MPT offers a way to balance this tradeoff by optimizing the risk and reward of the portfolio. The goal is to find the optimal combination of investments that provide the best returns for a given level of risk. The theory suggests that by diversifying investments across different asset classes, investors can reduce overall risk while still achieving their desired returns. Therefore, when building a good portfolio, investors should consider diversification, asset allocation, and risk management techniques to help them achieve their investment goals. By balancing risk and return, MPT provides a framework for constructing a portfolio that can help investors achieve their long-term financial objectives.

A management fee of 2.5% is very high. Also Insurance Investments, unit liked has a 3.5 or 4 percent of management fee so it is basically impossible to have a return, but they are untouchable by governments so they are secure in terms of life events.

While MPT can be an effective approach to portfolio management, it's essential to keep in mind the costs associated with investing. One such cost is the **management fee** charged by investment firms or insurance companies, which can be as high as 2.5% or more. For instance, unit-linked insurance investments may have a management fee of around 3.5% to 4%, making it difficult to achieve even an actual return. However, despite the high management fees charged by some investment products, they offer security in terms of life events, which may be important to some investors. On the other hand, managing your own portfolio is becoming an increasingly popular option, particularly with the rise of **virtual portfolio platforms**. Moreover, there are various types of virtual portfolios available, such as government bond indices and corporate bond indices, as well as equity indices. Investors can even copy virtual portfolios created by other investors.

When investing in corporate bonds, the key factor to consider is the **probability of default** (PD) in the future. Credit rating agencies assign credit ratings to different bonds based on their creditworthiness. Bonds with higher credit ratings such as AAA, AA, A, and BBB are considered investment-grade bonds, while those with lower ratings like BB, B, C, and D are classified as speculative-grade bonds. Investment-grade bonds are typically less risky and have a lower probability of default, while speculative-grade bonds carry a higher risk of default and offer higher potential gains. High-yield bonds, also known as junk bonds, are a type of speculative-grade bond that offers higher yields to compensate for the increased risk.

Emerging Markets Bonds are considered riskier investments as they are issued by countries that are still developing and have less stable economies compared to established markets. These countries may face economic or political instability, currency devaluation, or default risks, which could lead to significant losses for bond investors.

The *MSCI World Index* is a market-capitalization-weighted equity index that represents the performance of global stock markets. The index is produced by Morgan Stanley and is used by investors and financial analysts as a benchmark for measuring the performance of their own investments. By tracking the performance of the MSCI World Index, investors can gain insights into how the global equity markets are performing.

Financial providers like *Bloomberg* and *FactSet* are well-known for providing financial information and data to investors and analysts. Bloomberg is widely used by professionals in the finance industry, and FactSet provides financial data and analytics to both institutional and individual investors. The University of Bicocca offers access to FactSet data to its account holders for free, while Bloomberg terminals are available at a fee in the university's U7 building.

When it comes to measuring the performance of financial investments, we use arithmetic and logarithmic returns.

- Arithmetic return $\frac{P_t - P_{t-1}}{P_{t-1}}$
- Logarithmic Return $ln(P_t) - ln(P_{t-1}) = ln(\frac{P_t}{P_{t-1}})$

These two measures of return are commonly used by investors and financial analysts to assess the performance of investments over time.

In financial markets, it is nearly **impossible to forecast prices**. Instead, investors often focus on predicting returns, as returns do not have any inherent trends and are easier to analyze. When building a portfolio, it is not necessary to try and forecast the market or wait for the best time to invest. Instead, investors can focus on creating a well-diversified portfolio with a mix of different assets to manage risk.

Correlation is a crucial concept in finance, as it helps investors understand how different assets are related to one another. A high negative correlation between assets means that they tend to move in opposite directions, which can provide better diversification and help to reduce overall portfolio risk. A relatively good correlation value in finance is less than 0.30, while linearly independent assets are considered even better. However, it is nearly impossible to find assets that are perfectly correlated in finance, as this would eliminate the benefits of diversification. While lower risk may mean lower returns in some cases, this is not always true in portfolio management, as diversification can help investors to achieve better risk-adjusted returns.

In finance, *diversification is often considered the only "free lunch" available*. This is because combining multiple assets in a portfolio can help to reduce overall risk without sacrificing returns. However, diversification is only effective if the assets in the portfolio are not highly correlated with one another. This is why it is crucial for investors to analyze the correlation between assets when building a portfolio. By finding assets with low or negative correlation, investors can reduce their overall portfolio risk without sacrificing potential returns.

2.2 Markowitz Model

2.2.1 Hypothesis of the Markowitz Model

The Markowitz model is based on the following assumptions:

1. The investment time horizon is *single-period*.
2. Investors are risk averse.
3. Investors select portfolios based on three parameters:

- the **expected average return** ($E(r)$)
- the **expected risk** measured by the standard deviation of returns or mean square deviation (σ)
- the **correlation between returns** measured by the coefficient (ρ).

The first assumption states that the investment time horizon is single-period. After calculating an optimal portfolio based on a time horizon T (for example, 3 years), it must represent the only form of investment for the entire T period. This means that the model does not consider tactical choices that may modify the initial investment strategy during the T period.

The second assumption is that investors are risk-averse, meaning that risk is a negative aspect. An investor is willing to assume more risk only if it corresponds to a higher return.

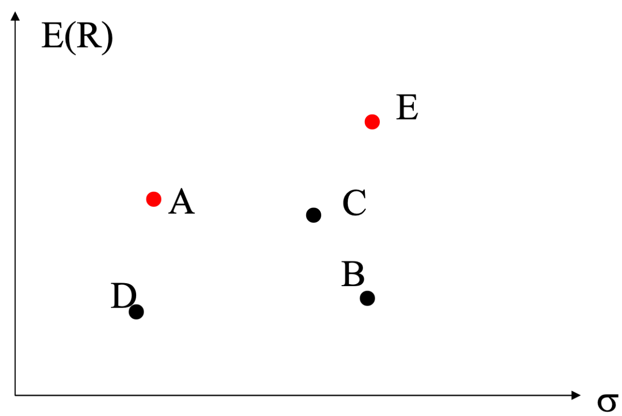


Figure 2.1: B,C and D are inefficient portfolio because they are dominated.

Finally, the third assumption is that investors select portfolios based on two parameters, the expected average return ($E(r)$) and the standard deviation of returns (σ). The standard deviation measures the average magnitude of the differences between individual period returns and the average return.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{1} 2(R_i - \overline{R})^2}{n}}$$

But is the standard deviation the best measure of volatility? Markowitz himself expressed doubts about the usefulness of the standard deviation as a measure of risk, although he believed that using **semi-variance** was more accurate. However, he still opted for σ due to its ease of use. The question is: when is the use of the standard deviation an inadequate measure of risk?

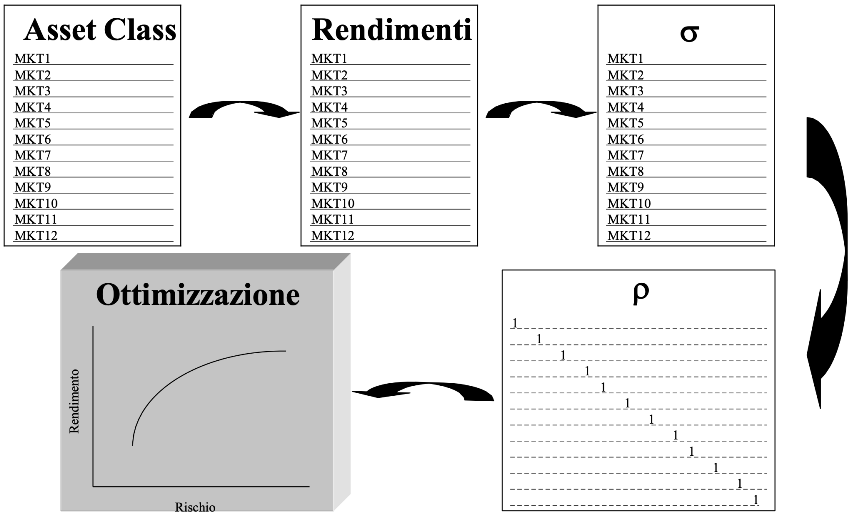


Figure 2.2: The necessary steps to construct the efficient frontier according to Markowitz.

2.2.2 Markowitz's Portfolio Optimization

Markowitz's portfolio optimization involves the use of three parameters: returns, risk, and correlation. By utilizing quadratic programming, we can determine the optimal portfolio on the efficient frontier, which is a line that represents the relationship between the level of risk (x) and expected returns (y). The efficient frontier identifies the optimal portfolio, while all other portfolios below the line are suboptimal.

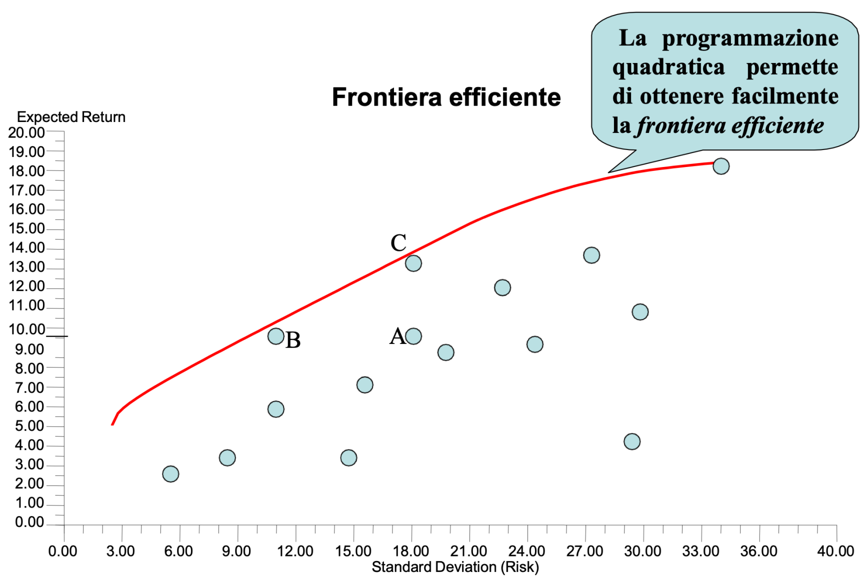


Figure 2.3: Efficient frontier.

The number of necessary calculations for a portfolio with n assets is given by $\frac{n^2-n}{2}$. When the Markowitz method was first introduced, there was a problem with computational complexity. Later on, William Sharp developed the Capital Asset Pricing Model (CAPM) with the goal of simplifying the Markowitz model. However, it was later discovered that the two models are different in composition and solution.

In situations where two assets are perfectly correlated, the Markowitz model's risk measure simplifies. The risk is simply the sum of the two weighted risks, but this scenario is not possible in reality. However, there are cases where assets have a high correlation, which can increase the risk of a portfolio. Therefore, it's desirable to have a low level of correlation between assets. However, it's challenging to find a level of correlation below 0.3 in reality. The formula used in Markowitz's Portfolio Optimization for calculating the **portfolio's risk** is:

$$\sigma_p = \sqrt{(\sigma_1 \times w_1)^2 + (\sigma_2 \times w_2)^2 + 2 \times \sigma_1 \times \sigma_2 \times w_1 \times w_2 \times \rho_{12}}$$

Where σ_1 and σ_2 are the volatility for market 1 and market 2, w_1 and w_2 are the percentage weight of market 1 and market 2, and ρ_{12} is the correlation coefficient between market 1 and market 2. This formula utilizes the variance-covariance matrix and the portfolio weights to calculate the portfolio's risk. Correlation is more interpretable than covariance because the latter is a measure of the joint variability between two random variables and does not have a scale. On the other hand, correlation is a standardized measure that ranges from -1 to 1. In the first version of the Markowitz model, variance was used as the measure of risk, and the average of historical returns was used as the measure of expected returns. This version is known as the "mean-variance graph."

When computing the risk in a portfolio composed by more then two markets the formula is simply an extension of the one used before. Usually matrix algebra is useful for risk computation of a portfolio made by various markets. The formula is the following one:

$$\sigma_p = \sqrt{w \Sigma w^T}$$

Where Σ is the variance-covariance matrix.

In the world of investments, annual return is the conventional metric used to compare different investment options. However, to calculate annual return, we often need to convert monthly returns to annual returns. Similarly, we convert monthly standard deviation to annual standard deviation to make the metrics comparable. But, what is the relationship between monthly risk and annual risk? Unfortunately, the answer is not straightforward. If we have a time series of monthly returns, we may want to convert the monthly risk measure, denoted by σ_m, to an annual risk measure, denoted by σ_a. One statistical approach is to use past annual returns to calculate the annual risk. However, this approach is not always feasible as we need at least 30 years of data to have a robust measure of volatility, and it is rare to have time series that long. Additionally, the oldest data may not be relevant in today's market. Due to the challenges of calculating annual risk, we often use an approximation formula: $\sigma_a = \sigma_m\sqrt{12}$. However, this formula assumes that the monthly returns are independent and identically distributed (i.i.d.), meaning that t_i and t_{i-1} are independent and both come from the same probability distribution. However, in reality, returns are not always i.i.d. Over time, they can be correlated due to the principle of "auto-correlation of time series." However, this assumption may not hold in practice, as we can verify through correlation analysis. In reality, auto-correlation exists but is generally small and does not significantly affect investment decisions.

As mentioned earlier, the assumption of independence among monthly returns makes it difficult to use technical analysis (TA) rules or algorithms based on charts. Therefore, investors often turn to weekly or daily time series as an alternative ($\sigma_a = \sigma_w\sqrt{52}$ or $\sigma_a = \sigma_a = \sigma_d\sqrt{262}$). However, using these shorter time series introduces greater errors compared to using monthly time series.

Despite the limitations, the best approximation for annual risk is still done using monthly time series, as it is the most accurate available method. Once we have calculated the monthly risk measure, we can calculate the correlation. It's important to note that the correlation calculated from monthly returns is generally more stable with less variability compared to correlations calculated from daily or weekly returns. Therefore, the monthly correlation is the preferred measure in most cases.

Markowitz's Optimization Algorithm Markowitz's portfolio optimization is realized through an algorithm whose structure takes into account both all the inputs indicated and the fact that the "utility" that an investor derives from their investment choices is directly linked to returns but inversely connected to risk.

The optimization algorithm is a minimization problem, minimizing the portfolio risk, with three constraints, and it is characterized as it follows:

$$\begin{array}{ll} \text{MIN:} & \sigma_{\text{portfolio}} \\ \text{Subject to:} & \text{Expected Return} = R_p \\ & w_1 + ... + w_i + ... + w_n = 1 \\ & w_i >= 0 \end{array}$$

In alternative we can characterize it as a maximization, maximizing the expected return, problem as it follows:

MAX:	$R_{\text{portfolio}}$
Subject to:	Portfolio Risk $= \sigma_p$
	$w_1 + ... + w_i + ... + w_n = 1$
	$w_i >= 0$

The decision variable is w_i so it is the result of the optimization problem. Let's now focus on the meaning of the constraints:

1. Portfolio Risk $= \sigma_p$, this imply a specific risk level
2. $w_1 + .. + w_i + ... + w_n = 1$
3. $w_i >= 0$ Not short selling permitted (bet against an asset)

Short selling is generally viewed unfavorably from an ethical standpoint and during certain crises, regulators may even prohibit it for a period of time. This is due to the potentially high risk involved in short selling, primarily because of the use of leverage and the possibility of a short squeeze. In reality, short selling can be a highly risky activity for investors.

Now let's see an example of how it works. Lets first consider the following inputs:

	RENDIMENTI ATTESI	RISCHI ATTESI
Mercato 1	3,58%	0,31%
Mercato 2	5,91%	3,42%
Mercato 3	6,85%	9,69%
Mercato 4	7,00%	11,59%
Mercato 5	7,20%	20,45%
Mercato 6	8,51%	19,44%
Mercato 7	8,81%	16,97%

Matrice di correlazione							
	Mercato 1	Mercato 2	Mercato 3	Mercato 4	Mercato 5	Mercato 6	Mercato 7
Mercato 1	1	0,25	0,18	-0,15	-0,21	-0,01	-0,06
Mercato 2	0,25	1	0,33	-0,08	-0,05	-0,1	-0,17
Mercato 3	0,18	0,33	1	0,6	0,32	0,48	0,24
Mercato 4	-0,15	-0,08	0,6	1	0,53	0,68	0,56
Mercato 5	-0,21	-0,05	0,32	0,53	1	0,66	0,6
Mercato 6	-0,01	-0,1	0,48	0,68	0,66	1	0,85
Mercato 7	-0,06	-0,17	0,24	0,56	0,6	0,85	1

We wonder if the portfolio with the following structure is the optimal one.

Mercato 1	30,00%
Mercato 2	10,00%
Mercato 3	20,00%
Mercato 4	0,00%
Mercato 5	20,00%
Mercato 6	20,00%
Mercato 7	0,00%

Resulting in:

$$r_{\text{portfolio}} = 6.7790\%$$

$$\sigma_{\text{portfolio}} = 9.04\%$$

Since, in an attempt to maintain the yield and lower the risk, we arrive at the following structure:

Mercato 1	0,00%
Mercato 2	62,24%
Mercato 3	11,54%
Mercato 4	0,00%
Mercato 5	0,00%
Mercato 6	0,00%
Mercato 7	26,23%

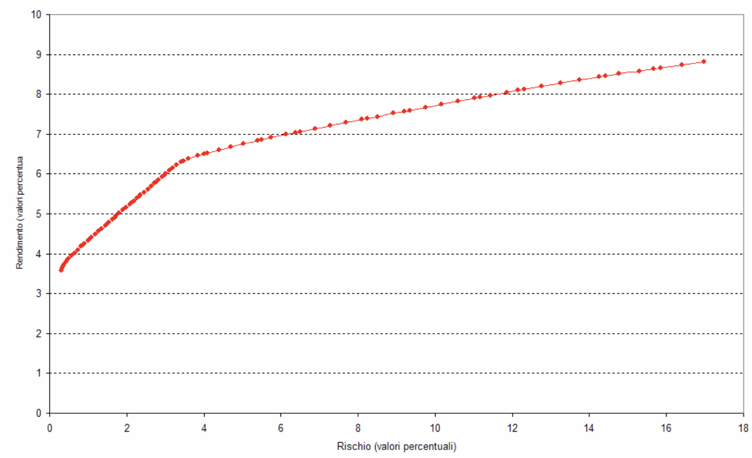
Resulting in:

$$r_{\text{portfolio}} = 6.7790\%$$

$$\sigma_{\text{portfolio}} = 5,12\%$$

The first portfolio must be considered dominated; the second portfolio is an optimal portfolio because it represents the composition of markets that, given the inputs, allows for the achievement of a prospective target return (6.7790%) with the lowest possible risk (alternatively, it could be said that for a risk of 5.13%, a return of 6.7790% is the maximum level that can be aimed for).

The application of the Markowitz algorithm, in the two versions seen previously, for progressive risk constraints or for progressive target returns, leads to identifying the location of the optimal (dominant) portfolios, defined as: **the efficient frontier**



2.3 CAPM

The Capital Asset Pricing Model (CAPM) was introduced by William Sharpe in 1969 as a possible solution to the problem posed by Harry Markowitz's portfolio theory. Markowitz's theory aimed to optimize returns on a portfolio of assets while minimizing risk, but it was hindered by the problem of calculating the covariance matrix. Sharpe's idea was to simplify the theory by assuming that all investors follow Markowitz's model and that there is an underlying equilibrium in the financial market.

The CAPM describes the functioning of a financial market by assuming that all investors hold the market portfolio, which is a weighted average of all assets in the market. The market portfolio represents the collective sentiment of all investors and is considered the optimal portfolio because it maximizes returns for a given level of risk. The CAPM also assumes that there is a risk-free asset that investors can lend or borrow at a known rate.

The creation of each asset in the market is based on the supply and demand of investors. The price of an asset is determined by the market's perception of the asset's risk and return compared to the market portfolio. If an asset has a higher risk than the market portfolio, its price will be lower to compensate for the additional risk. Conversely, if an asset has a lower risk, its price will be higher to reflect the lower risk.

The CAPM has several consequences for practical investments. First, it suggests that investors should only hold the market portfolio and a risk-free asset. Any deviation from the market portfolio will result in a suboptimal portfolio. Second, it suggests that the expected return of an asset is proportional to its beta, which measures the asset's sensitivity to changes in the market portfolio. A high beta asset will have a higher expected return than a low beta asset, assuming the same level of risk. Finally, the CAPM implies that investors cannot beat the market through active trading, as any information they use to make investment decisions is already reflected in the market price of the asset.

The hypotheses for the CAPM are the following:

- 1. There are no transaction costs.
- 2. Assets are infinitely divisible.
- 3. There are no income taxes.
- 4. No one can affect the price of securities with their operations.
- 5. Investors make their choices solely on the basis of expected returns and risk.
- 6. Short positions can be taken on securities.
- 7. One can borrow and invest in the risk-free asset without any limit.
- 8. All investors have the same time horizon.
- 9. All investors have the same set of inputs (returns, volatilities, and correlations).
- 10. All assets are traded in the market and have a price.

It is important to note that while the assumptions underlying the CAPM are not all true, they serve as simplifications that allow for the model to be used in practice. Furthermore, the results of the CAPM remain valid even when some of these assumptions are relaxed.

In order to account for the existence of a risk-free asset, Sharp introduced the concept of a risk-free financial asset, which in reality does not exist. Short-term government bonds are often considered to be the closest approximation to a risk-free asset. During the financial crisis of 2008, German short-term bonds (BUNDs) were considered the safest place to invest money, leading to negative interest rates.

However, it is important to note that while short-term bonds are generally considered risk-free during normal periods, they may not be during financial crises. The BUNDs were an exception during the 2008 financial crisis, as they were considered to be the safest asset available at the time. A risk-free financial asset is one for which the standard deviation of returns, σ_{rf}, is equal to zero. While zero-coupon bonds (ZCBs) carry the risk of default by the issuing country, in general, if an investor invests 100 in an asset with an interest rate of 1%, it is guaranteed that after six months, they will receive 101. It is also important to note that the assumption that the interest rate for borrowing money is higher than the interest rate for lending money is not always true. This is known as an inverted yield curve and can occur in certain economic situations.

2.3.1 CAPM Hypothesis

The working hypotheses of the CAPM, a model developed by Sharpe, are broader than those of the Markowitz approach.

- Hypotheses common to both methods:
  - Investors are risk averse and maximize expected utility
  - Investors select portfolios based on expected returns and return variances
  - The horizon is one period
- Additional hypotheses specific to the CAPM:
  - There is no restriction on borrowing or lending at the risk-free rate
  - There are no taxes or other market imperfections
  - Investors have homogeneous expectations about the expected values, variances, and covariances of security returns.

Based on the CAPM assumptions about homogeneous expectations of market participants, the portfolio M that appeared on the efficient frontier obtained for the case of I risky securities and a risk-free security (analyzed previously) coincides with the "market portfolio," that is, the portfolio composed of all risky securities present on the market that appear in the same proportions as they are traded on the stock exchange.

Consider the possibility of investing in a risk-free security (F) and in an efficient portfolio characterized by risky assets (A). Let X be the fraction invested in the portfolio. $(1 - X)$ will be the fraction invested in the risk-free security. The expected return of the combination (C) will be as follows: $R_C = (1 - X) \cdot R_F + X \cdot R_A$.

The fifth hypothesis say that the portfolio volatility will be affected only by the shared in risky assets, so the equation for the risk is really simple. The expected return of the combination (C) will be as follows:

$$R_C = (1 - X) \cdot R_F + X \cdot R_A$$

The risk of the combination will instead be given by:

$$O_C = [(1 - X)^2 \cdot \sigma_F^2 + X^2 \cdot \sigma_A^2 + 2 \cdot X \cdot (1 - X) \cdot \sigma_F \cdot \sigma_A \cdot \rho_{FA}]^{1/2}$$

Since the basic assumption is that the risk-free asset has zero volatility.

$$O_C = [X^2 \cdot \sigma_A^2]^{1/2} = X \cdot \sigma_A$$

$$X = \frac{\sigma_C}{O_A}$$

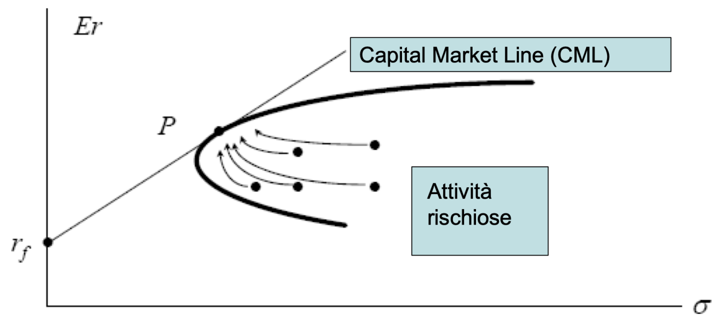
The sixth hypothesis show how if we substitute the expression of X into the expression for expected return, we obtain the following.

$$R_C = R_F + (\frac{R_A - R_F}{\sigma_A}) \cdot \sigma_C$$

show the link between return and risk, very important when invest in a risky and a risk free asset. In this context, this linear relationship is called the **Capital Market Line**, where:

- the intercept is the price of time, and
- the slope is the price per risk unit.

Therefore, we can write: $E(r) = (Priceoftime) + (Priceofrisk) \times (Amountofrisk)$ The relationship between return and risk is not a 1:1 relationship, but it is imbalance in the direction of the risk. More some thing like $\frac{Return}{Risk} = 0.4$.



The Capital Market Line (CML) represents a significant improvement over Markowitz's efficient frontier. The CML allows us to identify the tangent portfolio, which is the best risky portfolio to select, making all others sub-optimal. The new frontiers are simple straight lines, and the CML is the best among them. The market portfolio is considered the best portfolio, as it has several properties and specific characteristics.

This result has far-reaching practical implications. The area above the market portfolio on the CML represents a space where investors can borrow money at the risk-free rate and invest it in the market portfolio. However, this option is only available to institutional investors, who have the ability to engage in leverage. What's interesting about this concept is that unlike the efficient frontier of Markowitz, increasing the risk does not necessarily result in a linear increase in returns. Therefore, engaging in leverage can be advantageous, particularly for hedge funds and speculative funds.

The market portfolio is defined by the relative weights of its constituent securities, denoted by $w_1, ..., w_n$. These weights are computed as the fraction of the market value of each security relative to the total market value of all securities. Specifically, for each security i, we have:

- p_i denotes the price of the i-th asset
- n_i denotes the number of shares of the i-th asset
- $v_i = n_i p_i$ denotes the total value of the i-th asset in the market

The market portfolio includes all the assets in the market, each with a corresponding weight. These weights are proportional to the market capitalization of each security, which reflects its relative success in the market. Total market value is the following sum of total values: $V = v_1 + v_2 + ... + v_n$

In reality, we have indexes that represent virtual portfolios and are subsets of the entire equity market. Within each index, the rule of composition is based on market capitalization. The vast majority of indexes are cap-weighted, meaning that the weight of each asset is proportional to its market capitalization. However, there are some exceptions, such as the Dow Jones Industrial Average, which is price-weighted, with each stock's weight based on its price relative to the sum of all stock prices in the index. While the DJIA is informative, it is not considered a benchmark for real investments because price alone is not economically informative. A benchmark is an index that someone follows to invest in a similar way, allowing them to compare the performance of their investments to the overall market.

The "buy and hold" strategy is a passive investment approach that involves replicating the performance of an index, with the portfolio automatically adjusting to market drops as long as the overall value remains the same. This concept is known as **passive management** and originated with the CAPM and CM portfolio with value-weighted indices, allowing investors to stay on the capital market line. Interestingly, a whopping 78 percent of global investment assets are now managed passively. Basically passive management have the following features:

- It is also called indexed management
- Market weights are copied
- No market timing is done
- The investor can approximate the market portfolio by copying the market index

Exchange Traded Funds (ETFs) are the most cost-effective form of passive investment and are highly liquid. Prior to 2000, there were many passive funds available, but they typically charged around 1.5% of implied costs. With the arrival of ETFs, the fees for passive investments have dropped to 0.2%, making them highly competitive. Over the last 15 years, ETFs have captured a significant share of the passive management market. The few funds that survived the competition were only able to do so because they were already established and could afford to drop their fees to compete with ETFs.

The CAPM is based on the following two theorems:

- Every portfolio lying on the CML is perfectly correlated with the "market portfolio"
- For any portfolio with expected return given by $E(r)$ and risk σ, it follows that:

$$E(R_p) = R_f + \frac{E(R_M) - R_f}{\sigma_M^2} \cdot Cov(R_p, R_M)$$

2.3.2 Security Market Line (SML)

The correlation between a portfolio P (made with the Market Portfolio and the Risk Free Asset) and the Market Portfolio is 1. The Market Portfolio becomes the center of gravity.

So the equation of the CAPM becomes simpler:

$$E(R_p) = R_f + \frac{E(R_M) - R_f}{\sigma_M^2} \cdot Cov(R_p, R_M)$$

to:

$$E(R_p) = y + \frac{E(R_M) - R_f}{\sigma_M} \cdot \sigma_p$$

where

- y is the price of time
- σ_M is premium/price of risk
- σ_p Level of risk

For the generic k-th asset, due to theorem 1 and 2 we can write $E(R_k) = R_f + \frac{E(R_M)-R_f}{\sigma_M} \cdot \rho_{k,M} \cdot \sigma_k \cdot \sigma_M$ In conclusion, for every single stock we will be only rewarded for the part of the risk that is related to the market portfolio. SML e CAPM come from the same equation, but becomes very different if you group $Cov(R_k, R_M)$ with σ_M^2. The fraction between the two is denoted with β_k. This new variable compares the Total Market Risk σ_M with the relevant share of the single stock risk $(\rho_{k,M} \cdot \sigma_k)$. This coefficient is very used.

SML is a tool used to express the relationship between the returns of a stock and its systematic risk, as measured by its beta (β). Beta is a measure of a stock's risk in relation to the market movements, also known as **systematic risk**. It is calculated by dividing the covariance of the stock's returns with the market returns by the variance of the market returns. Beta values can range from negative to positive, with a value of zero indicating that the stock has a weak link with the movement of the market, while a value of one indicates that the stock is similar to the market portfolio. Beta values greater than one are associated with aggressive stocks, while beta values less than one are associated with defensive stocks. It is very rare to find a stock with a negative beta, which would indicate that the stock moves in the opposite direction of the market. The beta of a portfolio is calculated as the weighted mean of all the beta values of the stocks in the portfolio. Beta is an amplifier of the market movement, so a portfolio with a beta value of 1.2, for example, would gain 1.2% if the market goes up by 1%. Lowering the beta of a portfolio can be achieved by selling high-beta stocks and buying more low-beta stocks.

About Beta, we can note that:

- the stock has a very weak link with the movement of the market. Very rare. ($\beta = 0$)
- Market portfolio ($\beta = 1$)
- $\beta > 1$ Aggressive stock
- $\beta < 1$ Defensive stock
- $\beta < 0$ can happen for a small period of time. To get a negative beta you have to have a negative correlation coefficient, meaning that the stock has to go the opposite

direction of the index systematically. Of course is very difficult to find a stock with this characteristic.

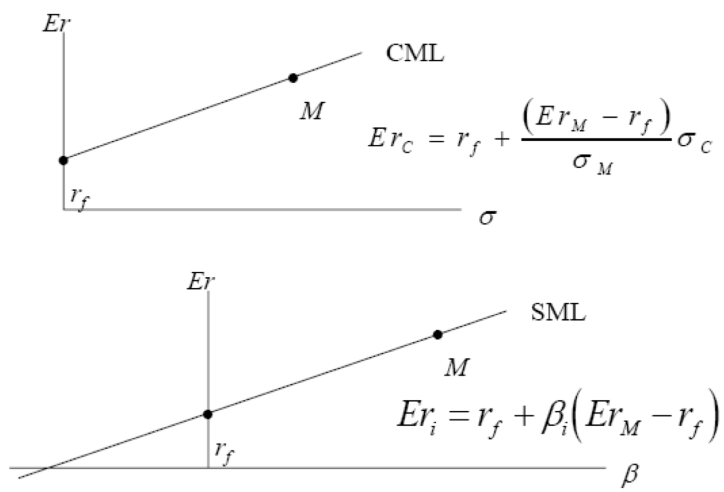
The beta of a portfolio can also be used for market timing. If an up-trend in the market is forecasted, the beta of the portfolio should be increased, while if a down-trend is predicted, the beta of the portfolio should be decreased. To compute beta, a regression can be performed using the time series of returns. This regression formula is: $R_i = \alpha + \beta \cdot R_M + \epsilon$. The result of this regression will be the same as using the formula mentioned earlier. If a stock performs better than predicted with beta, it could be overvalued, and this can be used for stock picking or selection. In this case, overvalued stocks can be sold or short-sold, while undervalued stocks can be purchased to generate extra returns over the index. The extra return generated over the index is denoted by α.

The **systematic risk** exists because individual stocks are affected by market-wide factors that are beyond their control. While the performance of a particular firm is affected by its own specific factors, such as its management, financial performance, and strategic decisions, these factors are not the only ones that affect its value. Negative market movements are often related to bad economic news that affects a large number of firms, not just one. For example, a recession or an economic downturn can have a widespread impact on the stock market, causing many stocks to decline in value. In addition, the decisions of asset managers can also impact the market as a whole, and therefore affect the value of individual stocks. For example, if there is negative news about a particular industry or sector, asset managers may decide to sell their holdings in ETFs that contain stocks from that industry or sector. This selling pressure can lead to a market impact that affects the value of many stocks within the ETF, even those of firms that are otherwise performing well.

The rest of the total risk that affects a firm's value is specific to the firm and is known as **specific risk**. This risk is not paid for by the market and is unique to the firm. Specific risk arises from the firm's own internal factors such as company management, financials, and operational efficiency. Investing in a single stock can be risky if you do not have enough experience or information about the firm and its future prospects. This is why diversification across a range of stocks is often recommended to minimize specific risk and maximize overall returns.

Alpha is a measure of the excess return generated by an investment, over and above the market's return. It is calculated by subtracting the expected return on the investment based on its beta from its actual return. If the result is positive, it indicates that the investment has outperformed the market.

Alpha can be generated through stock picking or market timing. Stock picking involves selecting individual stocks that are expected to outperform the market, based on factors such as their financial performance, management team, or growth prospects. Market timing involves making strategic moves in and out of the market to take advantage of expected market movements. However, generating alpha is not an easy task. It requires skillful and knowledgeable investment decisions and is difficult to achieve consistently. Moreover, high alpha funds are rare, and even when they exist, they may not be stable over time. This means that an investment that generates alpha in one period may not do so in another. In addition, funds that generate alpha tend to have higher management fees, as the fund managers need to be compensated for their skills and expertise. These fees can eat into the excess return generated by the investment, reducing the overall benefit to investors. Finally, it is important to assess whether any observed alpha is statistically significant. Statistical significance means that the alpha is unlikely to have occurred by chance and is therefore a true indicator of investment skill. A statistically significant alpha suggests that the investment strategy is repeatable and not just a random outcome.



While the Capital Market Line is drawn in a plot with σ in the x axis and Expected return in the y axis, in the SML we have β in the x axis.

The choice of a proxy for the CAPM is an interesting topic of discussion, as there are various factors to consider. The Bolomber method automatically proposes the SP 500 Index as a benchmark, but it is changeable, and the period of time (weekly/monthly/annual, weekly in this case) and the range of time (for example, the last two years) can also be adjusted. One important question to consider is whether the number of points obtained from the range is enough to guarantee robustness and if the period is appropriate.

In finance, we do not have access to as much data from different sources as in other fields, so it's crucial to keep in mind that we work with time series, which have a rate of change over time. As a result, beta changes over time, and the sample on which we base our calculation must be carefully chosen to ensure stability and robustness. We also want to use beta in forecasting, so it's essential that it is stable both statistically and financially.

When calculating beta over a long period of time, weekly data may be the right choice, and we must keep in mind the concept of stability. If we only want to calculate for short terms (e.g., for risk justification), daily data may be appropriate, and the period could also be shorter than six months to capture all the variability in the data. Typically, a stock's beta will change over time with a trend, and this trend is a convergence trend of all beta over 1, known as Market Portfolio Beta. Thus, stocks with betas of 2 and 0.5 will both converge to the same beta, meaning that the aggressive and conservative ones will become the same. This is known as "Blume's evidence," and it can be kept in mind in many applications to adjust beta over time.

To adjust for this convergence trend, we can use **adjusted Beta**, which is calculated as follows: $\beta_{adj} = \frac{2}{3}\beta_{raw} - \frac{1}{3}1$, where 1 is the Market Portfolio Beta. However, there are many unpredictable scenarios that can push beta to a value that is very different from 1, such as mergers and acquisitions, splits of a firm, or a complete change in the business model of a firm. These types of events are not always disclosed and can have a significant impact on beta.

Chapter 3

Improving Asset Allocation: Re-Sampling and Black&Litterman Model

In this discussion, we will be exploring the topic of **Improving Asset Allocation**, specifically by examining two methods: the *re-sampling of the efficient frontier* and the *Black and Litterman model*. The original Markowitz model is often criticized for its lack of diversification and instability, and as such, re-sampling has emerged as a possible solution to address these issues.

The concept of **re-sampling** involves recalculating a new frontier that is more diversified and stable than the original Markowitz frontier. This technique can help investors identify a more optimal asset allocation strategy by incorporating a wider range of asset classes and reducing the risk associated with concentrated portfolios.

Another approach to improving asset allocation is the **Black and Litterman model**, which seeks to combine the views of a portfolio manager with the returns generated by the Capital Asset Pricing Model (CAPM). This model is based on the idea that to build a portfolio, one should use the manager's beliefs and expectations about future market conditions in conjunction with equilibrium returns derived from the market portfolio. However, despite the potential benefits of this model, it is not widely used by fund managers in practice due to its complexity and difficulty in implementation.

In contrast, re-sampling is a more straightforward approach that many investors find easier to apply in practice. While the Black and Litterman model may be more challenging to implement, a well-executed implementation can help investors compute alpha more accurately and lead to better investment outcomes. Ultimately, the choice between these two methods will depend on an investor's specific needs, preferences, and level of expertise in asset allocation.

Chapter 4

Multi-Factor Models

4.1 Intro to Multi-Factor Models

In finance, multi-factor models are an important tool for understanding and predicting returns on investments. At their core, these models use a linear regression framework to identify the key factors that drive returns, with the goal of improving our ability to forecast future performance.

$$R_i = \alpha + \beta F_k + \epsilon$$

where $E(\sigma) = 0$

One commonly used model is the market model, which explains returns in terms of a single risk factor, typically the performance of a market index. This approach assumes that all returns can be explained by a constant term and a beta coefficient that captures the relationship between the asset and the market index. However, it is clear that there are many other factors that can impact returns, and so researchers have developed more complex models to account for these additional sources of risk.

One such model is the **Arbitrage Pricing Theory** (APT), which is based on the idea that short-selling can help to identify the factors that drive returns. By examining the returns on assets that are similar in all respects except for the presence or absence of a particular factor, it is possible to identify the impact of that factor on returns.

One challenge in using multi-factor models is that different sources of risk may not be directly comparable. In order to address this issue, researchers use risk-adjusted performance measures, which take into account both the level of returns and the associated risk. However, it is important to be aware that even well-designed models can be subject to temporary anomalies, such as when an asset is temporarily mispriced due to market conditions.

In contrast to pure arbitrage, statistical arbitrage involves taking advantage of temporary price discrepancies between two related assets that have historically moved together. This approach involves creating a long/short portfolio by buying the good asset and simultaneously selling short the bad asset. This strategy aims to profit from the spread between the two assets as they move back into alignment.

Unlike **pure arbitrage**, which is risk-free, **statistical arbitrage** involves some level of risk. The profit potential of this approach is based on the assumption that past pricing errors will be corrected in the future, but there is no guarantee that this will occur. Nevertheless, many investors find this approach appealing because it allows them to take advantage of market inefficiencies without incurring excessive risk.

One important question when constructing a multi-factor model is which factors to include. Researchers have proposed many different factors over the years, including macroeconomic variables such as interest rates and inflation, as well as industry-specific factors such as earnings growth and dividend yield. The specific factors included in a model will depend on the goals of the investor and the characteristics of the assets being analyzed.

In practice, there is no single "correct" multi-factor model. Rather, there are many different models that can be used, each with its own strengths and weaknesses. Ultimately, the choice of which model to use will depend on a variety of factors, including the investor's risk tolerance, investment objectives, and the characteristics of the assets being analyzed.

4.2 QEPM

The aim is to gain a deeper understanding of the framework in which multi-factor models prove useful in investments. Equity Portfolio Management involves constructing a portfolio, and Quantitative methods are specifically geared towards utilizing multi-factor models. These models differ from CAPM and can be considered as an improvement upon it.

Furthermore, there exist seven principles of the QEPM.

1. Markets are mostly efficient.
2. Pure arbitrage opportunities do not exist.
3. Quantitative analysis creates opportunities for statistical arbitrage.
4. Quantitative analysis efficiently combines all available information. When you build a quantitative model you are able to combine all the information in an efficient way.
5. Quantitative models should be based on solid economic theory. Markowitz, CAPM, diversification remain valid also in quantitative model.

6. Quantitative models should reflect stable and persistent evidence.
7. Deviations of a portfolio from a benchmark are justified only if the degree of uncertainty is relatively low.

The first principle of QEPM is Market Efficiency, which was first introduced by Fama and French in the 1970s to explain the relationship between financial markets and information. According to the efficiency theory, financial markets can be classified into three categories:

- Weak form efficiency: all past information is already reflected in the time series data, and any investment strategies based on technical analysis or momentum strategies are unlikely to yield profits. This means that if the market is truly efficient in a weak form, there is no chance of making a profit with an investment strategy.
- Semi-strong form efficiency: all public information, including historic data and macroeconomic news, are already incorporated in the price. This type of information can change the behavior of a good, and the market reacts immediately to new information, making it difficult for investors to earn extra profits. However, there are exceptions to the semi-strong form efficiency, such as when market anomalies arise due to mergers and acquisitions news. In such cases, hedge funds and active asset managers can make money by exploiting these anomalies. The market portfolio is not beatable unless such anomalies exist.
- Strong efficiency: a theoretical form in which even internal information is known by everyone.

The QEPM hypothesis is that the market is not efficient in either strong or semi-strong form. In summary, the market is efficient in a weak form, and technical analysis is not relevant, but it may not be efficient in strong or semi-strong form, leaving room for opportunities to earn profits.

Acquiring information can be costly, and not everyone is willing to pay for it. Even public information takes time to be fully incorporated into the market. Moreover, not all investors have the ability to process large amounts of data, especially when it comes to quantitative information. However, by filtering through public information, one can create valuable private information. It's worth noting that some investors base their investment decisions on sentiment, rather than a logical interpretation of the information available to them. This can create inefficiencies in the market, as some investors may attempt to exploit the alleged irrationality of others. Additionally, individuals may not be quick to adapt to technological changes, and transaction costs can create a gap between economic models and reality. Taxes can also distort markets, and the market surveillance framework may not always reflect reality.

In the past, empirical research has shown a lot of anomalies in the market. Building a multifactor model can provide an advantage over competitors if you are able to process a large amount of data and transform public information into private information. Moreover, the presence of irrationality in the market can also be exploited.

The total risk of a stock can be divided into two parts: systematic risk (factor exposure) and specific risk (factor risk). The specific risk is independent of the systematic part, and it's important to avoid multicollinearity when building a multi-factor model.

4.2.1 Fundamental Law of Asset Management

In response to principles 3 and 4 of QEPM, the idea of statistical arbitrage is aimed at efficiently combining information. This concept can be better understood in light of the fundamental law of G. and K., which states that the information ratio (IR) is a product of two elements: the information coefficient (IC) and the square root of Breadth (BR):

$$IR = IC * \sqrt{BR}$$

$$\frac{\alpha^*}{\omega} = IC * \sqrt{BR}$$

The IC can be improved by identifying major factors with informative content. On the other hand, the breadth (BR) can be increased by identifying new factors that are uncorrelated with those already present in the model. However, adding new factors only increases the BR if they are uncorrelated.

We hope that alpha is positive and statistically significant, as it represents the return, while omega is the risk, also known as tracking error. It refers to the risk that an asset manager takes to produce alpha. Omega is the standard deviation of the time series of the difference in returns between the funds and the benchmark.

It's important to note that risk factors at time t can explain risk factors at time t, meaning that past performance can have an impact on future performance.

The Fundamental Law is a fundamental concept in quantitative equity portfolio management (QEPM). Although it is not used directly in the process of QEPM, it is highly valuable for interpreting the results generated by this type of model. The Basic Law provides insight into the use of QEPM models in the following ways:

- The first insight provided by the Basic Law is that the information ratio squared (IR^2) is approximately equal to the R2 of the equation that predicts future returns. This means that the better the model is at predicting future returns, the higher the information ratio will be.
- The breadth (BR) of the model is related to the number of factors included in the model. Therefore, including more factors will increase the breadth of the model.

- The information coefficient squared (IC^2) is the average contribution of each explanatory variable in increasing the R2 of the model. This means that the higher the IC^2, the more informative each explanatory variable is in the model.
- The Basic Law decomposes the statistical goodness of the model into the number of explanatory factors and their average informative contribution. This means that it provides a way to assess the overall performance of the model and identify areas for improvement.
- Finally, if a benchmark is excluded and returns are net of the risk-free rate, then the IR is nothing more than the maximum achievable Sharpe ratio. This index will be a function of the number of factors and their average informative contribution. This means that the IR can be used as a measure of the model's ability to generate excess returns over and above what could be achieved by simply holding the benchmark.

The fundamental law of active management is a powerful tool, but it is only valid if the manager uses all the information available in an efficient manner to create an optimal portfolio. For example, consider a single factor model that estimates the performance of a particular security using the equation:

$$r_{i,t+1} = \alpha_i + \beta_{i,t} f + \epsilon_{i,t+1}$$

Here, the variable β represents an improvement in ranking, while f is the link between performance and ranking improvement. If the manager were to equally weight all securities with a beta of 1 in a given month, they would not be maximizing the information ratio, and would therefore not achieve the maximum Sharpe ratio (i.e., maximum return per unit of risk). The equation not only provides an indication of yield, but also of risk, which is often overlooked. Ignoring risk may result in an overweighted stock, violating the information criterion.

To quantify the loss of information, one can calculate the difference between the maximum possible information ratio and the actual information ratio achieved, or equivalently, the loss of return per unit of risk (Sharpe ratio).

By applying the variance operator to the factor model, we can easily compute the risk associated with each stock. It is important to note that the specific risk will differ for each stock, while the risk associated with ϵ will remain fixed. In order to construct an optimal portfolio, a suitable weighting scheme must be chosen. One possible approach is to weight all the stocks according to the inverse of their volatility. This ensures that stocks with higher levels of risk are given lower weights in the portfolio, while less risky stocks are assigned higher weights. This strategy can help to mitigate the overall risk of the portfolio and ensure that it is properly diversified.

4.2.2 Basic Models and Portfolio Construction Procedures

Factor models provide a framework that expresses the relationship between the average return on stocks and the exposure to specific risks, known as factors, as well as the associated risk premium. The general form of a factor model equation is:

$$R_{it} = \alpha_i + \sum_{i=k}^{n} \beta_{ik} \cdot F_{kt} + \epsilon_{it}$$

Let's explore two types of factor models:

- **Economic Factor Models**: These models incorporate macroeconomic factors, such as GDP growth, inflation, and other relevant economic variables. They were the first to be introduced and focus on economic risk factors. For example, factors can include variations in GDP, unemployment rates, inflation rates, and the difference between long-term and short-term interest rates. Additionally, factors like the price of gold, which is considered a proxy for market risk aversion, and oil prices, which reflect energy market dynamics, can be included. It is common to use unexpected variations in these factors rather than their actual values, as market prices are influenced by deviations from expectations.
- **Fundamental Factor Models**: These models utilize fundamental factors related to the characteristics of individual securities. Factors can include metrics such as price-to-earnings ratio (P/E), market capitalization, leverage, and other balance sheet values. In the fundamental model, each asset has specific variables associated with it, reflecting its unique characteristics.

Both types of factor models follow the same equation structure, but they differ in terms of the data used and the constraints applied. In fundamental factor models, the exposure to factors is directly observable, such as price-to-book value ratios. On the other hand, in economic factor models, exposure to factors is obtained through time-series regressions performed on individual stocks. Similarly, the estimation of risk premiums for factors varies between the two models. In fundamental factor models, cross-sectional regressions are typically used to determine risk premiums, while economic factor models often rely on directly observable time-series data, such as inflation rates.

Back testing is a crucial step in evaluating the effectiveness of an investment strategy. It involves testing the strategy on historical data to gauge its performance. When conducting a back test, it is important to consider two key factors: the length of the time series and the quality of the data.

One valuable resource for conducting back tests is the CRSP database. This database contains information on all US stocks since 1929, and each stock has been thoroughly reviewed and adjusted. However, access to the database comes with a hefty price tag of €100k per year.

When comparing two models to assess the linear relationship between stock returns and factor exposure, we can make use of either a fundamental or an economic model. In the fundamental model, we can directly use the P/E ratio to evaluate the risk factor. On the other hand, in the economic model, we can calculate the risk factor by comparing the returns of a portfolio comprised of stocks with very high P/E ratios to that of a portfolio with stocks having very low P/E ratios.

4.2.3 Choice of the Correct Model

Factor selection and data determines the true success of a QEPM model, assuming that the various models available are used by managers. Fundamental factor models restrict the choice of factors to only company-specific factors, while economic models offer greater flexibility, with the possibility of including both fundamental and economic factors. The data used are related to both cross-section and time series profiles. The former refers to elements such as the number of securities (universe of NYSE SP500 stocks) and affects both the final portfolio and estimation difficulties. The latter instead pertains to data frequency (daily, weekly, monthly) and depth, with effects on the stability and potential uncertainty of the estimated parameters.

In the case of economic models, the factors are known (e.g. real GDP growth). The β of the model that represent the exposure of the return to the factor are estimated through a multivariate time series regression. Factors such as inflation (e.g. 3%) are not directly the factor premium, but can be estimated by constructing an appropriate zero-exposure portfolio.

$$r_i = \alpha_i + \sum_{j=k}^{n} \beta_{ij} \cdot F_j + \epsilon_i$$

Fundamental models present the opposite situation, in which the exposure to factors is already known a priori (P/E..etc..), but the risk premium associated with that factor (f) needs to be estimated. The estimation is carried out through a cross-section regression, in which information from many securities is used in a single time period, or a panel regression in which multiple securities are involved, but with observations at different time periods.

Both versions of multifactor models allow for a precise understanding of which factors are most explanatory of returns and the exposure to those factors.

The same reasoning applies to the risk front, the estimation allows to break down the risk into two components, diversifiable and non-diversifiable: where $\alpha_i + \sum_{j=k}^{n} \beta_{ij} \cdot F_j$ is the non-diversifiable systematic risk and ϵ_i is the diversifiable specific risk.

Finally we can se how to perform the choiche of the correct model. The fifth principle of an effective QEPM states that models should be based on solid theoretical foundations. This means that expected high returns are the result of exposure to higher risks, and that different returns on small-cap and value stocks are related to different levels of risk. It's better to have an economic model that links returns to sources of risk, rather than just looking at size and P/E as potential risk factors. If the relationship with factor exposure is linear, the two models coincide, but what if it's not? The fundamental model would lack theoretical justification and lead to an econometrically incorrect cross-sectional regression, i.e., an incorrect estimation of the equation.

Other relevant factors in the choice of model include the *combination of different types of factors*. Both models can include fundamental factors (P/E, P/B, size, etc.) or technical factors such as momentum or trading volume. For other factors, such as macroeconomic factors for which exposure is unknown but the factor is known, only economic models can be used.

Ease of implementation is another important consideration. Although less flexible in the choice of factors, fundamental models only require estimation of the factor premium, while economic models require estimation of both exposure to factors and prediction of factors for estimating future returns.

Data requirements are also a key factor. The fundamental model doesn't require a deep historical dataset, whereas the economic model requires deep historical series of factors and returns. However, in both cases, increasing the dataset (periods in the fundamental case) and stocks in the economic case improves the stability of parameters and estimation error.

4.2.4 Fama & French Economic Model

The Fama French Economic Model is a well-known and widely used asset pricing model that was developed by Eugene Fama and Kenneth French in the early 1990s. The model attempts to explain stock returns by identifying factors that influence them. Specifically, the model posits that stock returns are related to three main factors: the return of the market, the size of the stock, and the book-to-market ratio of the stock.

The Kenneth R. French Data Library is a web page that provides access to the data used in the Fama & French models. The data includes historical prices and other financial metrics for a variety of stocks and portfolios.

The first factor in the Fama & French model is the return of the market minus the risk-free rate (Rm-Rf). This factor represents the excess return that investors can expect to earn by investing in the stock market, after adjusting for the risk-free rate of return.

The second factor in the model is the size of the stock (SMB). Fama & French argue that the size of a firm is relevant in explaining the stock's variability. They hypothesize that small firms have higher risk and higher expected returns than large firms. For example, in the last semester, small stocks outperformed large stocks, which is consistent with the SMB factor.

The third factor in the model is the high minus low (HML) factor. This factor is based on the fundamental factor of book-to-market ratio (B/M) but translated into economic terms. It represents the differential return of a long-short portfolio of high book-to-market stocks minus low book-to-market stocks. Fama & French argue that high B/M stocks have higher expected returns than low B/M stocks because they are riskier.

Overall, the Fama & French model has been influential in the field of finance and has provided a framework for understanding the relationship between risk and return in the stock market. By identifying the factors that influence stock returns, the model has helped investors and analysts make more informed investment decisions.

4.3 Fundamental vs Economic Multifactor Models

4.3.1 Fundamental Models

The **fundamental models**, also known as *cross-sectional models*, represent a class of models that have been recently applied compared to economic models inspired by Ross's arbitrage pricing theories of '76.

$$R_i = \alpha_i + \sum_{k=1}^{n} \beta_{ik} \cdot F_k + \epsilon_i$$

Where β_{ik} is the historic exposure and F_k is the estimated factor return. The model decomposes non-diversifiable risk as: factor exposure^2 $*$ factor risk (f).

$$V(r_i) = V(\alpha_i + \beta_{i1} f_1 + \cdots + \beta_{i_k} f_k) + V(\epsilon)$$

Where $V(f)$ is the $(K+1)$ dimensional Variance-Covariance matrix of premium factors.

The steps for the fundamental model are the following ones:

- Preliminary work:
  1. Chose the factors for the model
  2. Determine the treatment of the risk-free rate
  3. Define the investment universe
  4. Decide on the time interval and time period of the data
- Main steps for modeling total stock return (for nonbenchmark managers):

1. Stock returns are estimated with the following equation: $r_i = \alpha_i + \beta_{i1} + \cdots + \beta_{iK} f_k + \epsilon_i$
2. Collect stock-return and factor-exposure data for the time period at each time interval
3. Estimate the factor premium from a panel regression of the stock return on the factor exposure using either the OLS, MAD, or the GLS method
4. Check the robustness of the factor premium estimation by splitting the data into subsets and comparing the estimates for each subset. If the estimates are similar across subsets, the estimation is robust
5. If the estimation is not robust, try a different estimation method
6. Calculate each stock's average return as the product of the factor premium an the vector containing the stock's factor exposures
7. Decompose the risk of the stock return into its diversifiable and dondiverifable components
8. Calculate the correlation between the returns of the stocks in the investment universe

- Main steps for modelling residual stock return (for benchmark managers):
  1. Stock returns are estimated with the following equation: $r_i = \tilde{\alpha} + \tilde{\beta}_i r_B + \tilde{\epsilon}_i$
  2. Collect stock return and benchmark return data for the given time period at the given time intervals
  3. After estimating $\tilde{\alpha}$ and $\tilde{\beta}$, calculate the residual return as $\tilde{r}_i \equiv \tilde{\alpha}_i + \tilde{\epsilon}_i$
  4. Estimate residual return with the following equation: $\tilde{r}_i = \alpha + \beta_{i1} f_1 + \cdots + \beta_{iK} f_K + \epsilon_i$
  5. Follow steps 2 through 8 of the main steps for modeling total stock return, substituting the residual return for the total return.

Let's now see the the possible factors:

- *Fundamental valuation factors*: Market capitalization (size), price-to-book (P/B) ratio, price-to-earnings (P/E) ratio, price-to-sales (P/S) ratio
- *Fundamental financial risk and solvency factors*: Debt-to-equity (D/E) ratio, current (CUR) ratio, interest coverage ratio (ICR)
- *Fundamental growth potential factors*: Capital-expenditure-to-sales ratio, R&D-expenditure-to-sales ratio, advertising-expenditure-to-sales ratio
- *Technical factors*: Momentum, trading volume, short interest shares shorted
- *Analyst factors*: Analyst rating changes, earnings revisions

And Risk free, securities universe and the dataset.

- Risk-free: While it may be challenging to pinpoint it precisely (such as the return of US T-Bills or MMFs), the returns often refer to those exceeding the risk-free rate.

- Securities universe: External restrictions and precision limitations come into play when dealing with a larger number of securities.
- Dataset: The frequency of data collection should align with the rebalancing process (monthly, annually), and it's preferable to have a shorter time span for greater stability. Typically, the dataset covers around 36-60 time intervals.

The presence of a benchmark in portfolio management implies the need to incorporate it into the model. There are two alternatives to achieve this:

1. Consider only the excess returns (both positive and negative) of securities relative to the benchmark as the dependent variable in the model, using a preliminary regression analysis.
$$r_i = \tilde{\alpha}_i + \tilde{\beta}_i r_b + \tilde{\epsilon}_i$$
$$\tilde{r}_i \equiv \tilde{\alpha}_i + \tilde{\epsilon}_i$$
2. Include the benchmark as an explanatory factor in the model, which is more typical in economic models.
3. Take it into account at a later stage, during portfolio construction, by controlling for the benchmark.

Finally we can decompose the risk in various effect:

- Systematic risk: $\beta_{iT}'\hat{V}(\hat{f})\beta_{iT}$
- Specific risk: $\hat{\sigma}_i^2 = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \beta_{it}'\hat{f})^2$
- Total risk: $\beta_{iT}'\hat{V}(\hat{f})\beta_{iT} + \hat{\sigma}_i^2$
- Assets' correlation: $C(r_{it}, r_{jt}) = \beta_{iT}'V(f)\beta_{it} + C(\epsilon_{it}, \epsilon_{jt})$, where $C(\epsilon_{it}, \epsilon_{jt})$ is null because not relevant for the investor

4.3.2 Economic Models

The structure of the model remains unchanged compared to fundamental models, so similar considerations apply to the securities universe, the risk-free rate, and the dataset.

$$r_i = \alpha + \beta_{i1} f_1 + \cdots + \beta_{iK} f_K + \epsilon_i$$

The factors could be interpreted as deviations from the factor mean (surprises), but in any case, the intercept would encompass the phenomenon. The risk is defined as: $V(r_i) = \beta_i' V(f)\beta_i + V(\epsilon_i)$

Factors in the economic model can be divided in three, the first one are more simple to include, the second one are more complex, while the third ones are very complex.

1. Economic/behavioral/market factors: Gross domestic product (GDP), inflation, unemployment, interest rates, and other macroeconomic variables; consumer sentiment

index, business confidence index, investor sentiment index, or other survey-based indexes; returns on broad market indexes such as the Standard Poor's (SP) 500 Index or returns on other market group/industry indexes

2. Fundamental/technical/analyst factors: Size, P/B ratio, P/E ratio, D/E ratio, and other firm characteristics available through financial statements; momentum, trading volume, and other information reflected in trading data; analyst rating changes, earnings revision, or other information provided by analysts
3. Statistical factors: Factors obtained from principal-component analysis applied to historical returns

In this case as well, it is possible to consider the excess returns of securities relative to the benchmark. However, in this scenario, including the benchmark as a factor is also consistent with the Capital Asset Pricing Model (CAPM).

$$r_i = \alpha + \beta_{i1} f_1 + \cdots + \beta_{i,K-1} f_{K-1} + \beta_{iK} r_B \epsilon_i$$
$$\alpha^B = \alpha + \beta_{i1} f_1 + \cdots + \beta_{i,K-1} f_{K-1}$$

Where α^B is the direct measure of performances wtr the benchmark.

In the case of economic models, the factor premiums change over time, while the exposure factors remain stable. However, it's not always possible to directly observe the factors. Calculations are relatively straightforward for macro variables but become more complex when dealing with fundamental factors and even more so in statistical cases. For example, the percentage change in the University of Michigan's Sentiment Index or a portfolio of stocks from NYSE, AMEX, and NASDAQ in excess of the risk-free rate (1-month US Treasury bill) can be included directly in the model. Although these are not the actual factor premiums, they can be incorporated directly into the model. However, when dealing with fundamental, technical, or analyst forecast factors, the factor premiums are not directly observable and require the construction of zero-investment portfolios. Calculating the return differential between top and bottom portfolios relative to the factor needs to be determined for each period. If data is missing, the portfolios from the previous period are typically retained. In the case of statistical factors, principal component analysis (PCA) is used.

In the case of IPOs or merger transactions within the sample (which are often quite frequent), adjustments are necessary. Beta adjustment refers to a weighted merger of the beta values of the two companies based on their market capitalization.

$$\hat{\beta} = \frac{s_A}{s_A + s - B}\hat{\beta}_A + \frac{s_B}{s_A + s_B}\hat{\beta}_B$$

For newly issued securities where sufficient data is lacking, a comparable company approach will be used. This procedure involves utilizing a Z-score model in the first case (characteristic matching). Alternatively, beta values can be substituted with the industry average.

$$\hat{\beta_C} = \frac{1}{M}(\hat{\beta}_1 + \cdots + \hat{\beta}_M$$

Finally also in this case we can decompose risk in several effects:

- Systematic risk: $\hat{V}(f) = \frac{1}{T}\sum_{t=1}^{T}(f_t - \overline{f})(f_t - \overline{f})'$
- Specific risk: $\hat{V}(\epsilon_i) = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \hat{\beta}_i' f_t)^2$
- Total risk: $V(r_i) = \beta_i' V(f)\beta_i + V(\epsilon_i)$
- Assets' correlation: $C(r_i, r_j) = \beta_i' V(f)\beta_j + C(\epsilon_i, \epsilon_j)$ where $C(\epsilon_i, \epsilon_j)$ also in this case is null

4.3.3 Economic and Fundamental Models Equivalence

The equivalence between the economic and fundamental models, based on the same information, holds if we rewrite the multi-factor model using the premium d_k.

Debt-to-equity ratio (D/E) is a measure of financial structure, where equity represents the shareholders' capital, and the remaining portion comes from liabilities. D/E is a metric that reflects a firm's leverage. In a balance sheet, assets consist of equity and liabilities.

When the return is fixed and interest rates are not significantly influential, reducing equity increases the returns on equity, thus improving capital allocation. Every investor aims to maximize returns on equity.

If the D/E ratio is 2, it means that the debt is twice the amount of risk capital, indicating a normal level of leverage. Increasing the D/E ratio further, resulting in high leverage, may be positive for shareholders in terms of returns, but it also introduces the risk of financial instability. This risk is evident in debt service (DS), which includes interest payments and capital repayments.

Is it possible to assume a linear relationship between returns and the debt-to-equity ratio? It could be true to a certain extent, but beyond a certain point, it introduces excessive financial risk.

Leverage plays a crucial role in finance. For example, banks tend to have high leverage (around 10) as they rely heavily on deposits and other financial instruments, utilizing only a small portion of equity. In banks, equity capital serves as a cushion to absorb unexpected losses. If the equity is too small and a significant unexpected loss occurs, the bank may default.

Regulatory bodies closely regulate the quantity of equity capital in banks, particularly after the 2007 financial crisis, to ensure banks increase their equity capital. However, banks often resist such regulations as they aim to protect returns for shareholders.

In reality, there is no direct equivalence between the two models. Fundamental models are relatively easier to implement but lack strong theoretical foundations and explanations.

They are often used with managerial discretion. On the other hand, economic models are challenging to implement as they require robust theoretical foundations and extensive data.

4.4 Screening Models

4.4.1 Sequential and Simultaneous Screening

Screening models are a popular version of fundamental models that have less theoretical rigor behind them. These models can be implemented in two ways: **sequential screening** and **simultaneous screening**. The main objective is to identify relevant factors and classify stocks based on their high or low exposure to these factors.

In **sequential screening**, the initial universe of stocks is narrowed down by selecting those with the highest level of exposure to the relevant factors. However, the drawback of this approach is that the resulting portfolio may not be well-diversified due to the rapid reduction of the universe.

To address the issue of potentially losing out on valuable assets, **simultaneous screening** can be employed. This approach involves applying multiple screening criteria simultaneously and combining them using a Z-score. The final score obtained is then used to select the best-performing stocks.

However, a challenge in simultaneous screening is the scaling problem and the need for standardization. In finance, standardization must be done with great care. The question arises: which mean and standard deviation should be used? Should standardization be performed among different stocks or over time for the same stock? The answer depends on the type of portfolio being constructed.

Standardizing over time for the same stock can lead to data mining errors and invalidate backtesting. This is because future data cannot be taken into account. For example, if you start from t-10 (ten time periods ago), you cannot consider data from time periods beyond that point.

4.4.2 Z-Score

The concept behind the Z-score model is to derive a comprehensive score by combining different factors. It's important to note that screening models are a type of fundamental model.

The individual scores are aggregated to calculate the final score for each stock. However, careful consideration should be given to the weighting scheme applied to the factors. Let's assume equal weighting, where the average z-score for a stock is calculated as:

$$\overline{z}_i = \left(\frac{1}{K}\right)(z_{i1} + \cdots + z_{iK})$$

An alternative approach, although subjective, is to assign weights to the factors based on their perceived importance, as determined by the manager. A more sensible approach would be to start with a univariate analysis based on historical information ratio (IR) (no correlation) or the contribution of individual factors to profitability.

The idea is to build a strategy for each screening process.

In performance measurement, the information ratio (IR) is used to assess fund performance. When examining returns and risk, investors consider the return adjusted for risk. While every investor desires higher returns, they also consider the level of risk. There are both absolute and relative metrics used in this regard.

Absolute metrics, such as the Sharpe ratio, Treynor ratio, and Sortino ratio, are indicators that are not compared to a benchmark. The Sharpe ratio is calculated as $S = \frac{R_p - r_f}{\sigma_p}$, where R_p is the return of the portfolio, r_f is the risk-free rate, and σ_p is the standard deviation of the portfolio. A higher Sharpe ratio is preferable. However, the only external data incorporated in this ratio is the risk-free rate.

The Treynor ratio is calculated as $T = \frac{R_p - r_f}{\beta_p}$, where the beta of the portfolio (β_p) is a measure of systematic risk. The main difference between the Sharpe and Treynor ratios lies in the use of total risk (σ) in the latter.

The Sortino ratio employs a more specific risk measure by not considering symmetric risk. In other words, it accounts for the fact that risk can be both positive and negative, treating both types of risk equally. The Sortino ratio is given by $So = \frac{R_p - r_f}{SemySD}$, where SemySD is a symmetric version of the standard deviation σ but only considers observations that are less than or equal to the average return $\overline{r}$ and less than or equal to zero.

These metrics are referred to as absolute because they are not relative to any benchmark. Some investors even exclude the risk-free rate (r_f) from the formulas. By using all three indicators, different investment alternatives can be compared. It is important that they are calculated over the same time period and frequency of data.

The information ratio (IR) is a performance measure based on a benchmark. It is calculated as $IR = \frac{TE}{TEV}$, where TE (tracking error) represents the excess return of the fund over the benchmark. TE can be calculated in various ways, such as $R_p - R_b$. It is important to note that beating a benchmark is not an easy task. TE is akin to active risk, which refers to the risk taken by an asset manager to outperform the benchmark through different portfolio choices. Active risk should be justified by generating positive α or strong returns that compensate for the additional risk. TEV (tracking error volatility) is the standard deviation of the time series of tracking errors. TEV accounts for the fact that a portfolio with

higher TE might also have a higher TEV, thereby reducing the information ratio. Thus, the second portfolio would be preferred.

In the Z-score model, the IR is interpreted more as a risk-adjusted performance measure, similar to the Sharpe ratio $\frac{R_p}{\sigma}$. If a benchmark is present, the calculation of risk-adjusted returns remains the same. Here, IR is used to understand the relative relevance of each screening process in terms of performance.

There is another group of factors known as technical factors, which will be discussed in more detail later.

If you regress the aggregated z-scores of each stock, you can assess the "value relevance" of your efforts by examining the impact of these numbers on future returns. If the impact is null or even negative, it implies that the effort has not been useful.

Same procedure, but assigning equal weights to groups of factors: valuation, profitability, etc...

- Score on single group: $\overline{z}_{im} = \sum_{k \in S_m} w_k^m z_{ik}$
- Total score : $\overline{z}_i = \sum_{m=1}^{M} w_m \overline{z_{im}}$

If we assume that there is an inverse proportionality between factor premiums and the standard deviation of factor exposures in a fundamental model, then the two models coincide.

It is possible to use the Z-score as an alpha in a multi-factor model or as alpha for benchmark optimization. However, the issue is that the scores are not directly excess returns, which can lead to an excessive weight on securities with high Z-scores, even if limited to a maximum of $\pm 3\%$. A more complex solution involves linking the score to the alpha of the factor model in order to forecast it. The risk in this case often lies in the incorrect specification of the model.

4.4.3 Momentum

The concept of momentum is based on the intuition that if a stock has performed well in the past six months, with a return of 45%, the question to ask is, "Will this stock continue to rise in price?" In other words, is a positive trend from the past still relevant in predicting future performance?

The idea behind momentum is to follow the market's momentum, meaning that a stock that has performed well in the past is expected to continue performing well in the next period. The strategy involves screening the market for stocks with the best momentum and then constructing a portfolio comprising the top-performing stocks (for example, 10%). After a holding period, the screening process is repeated, and a new portfolio of the best-performing stocks is created. This approach has shown strong empirical evidence across various markets and has the potential to generate returns higher than the market index. However, momentum is considered a significant anomaly because it lacks a clear economic explanation.

The momentum portfolio is often regarded as the preferred long-term investment strategy, despite the empirical evidence suggesting higher returns (along with increased risk). Suppose the market index has a return of R1 and a risk of σ_1. In that case, the relative momentum portfolio offers a higher return of $R_1 + Kbps$ and a higher risk of $\sigma_1 + Jpps$, allowing investors to choose whether to accept the additional risk for potential additional gains.

Behavioral finance theory has discovered that investors are not perfectly rational and are subject to various cognitive biases that influence their decision-making (e.g., confirmation bias). These biases may partially explain the existence and persistence of momentum effects.

Removing the last month of return from the analysis helps clean the indicator and improves the signal by minimizing the influence of short-term momentum and focusing on longer-term trends.

To illustrate an example, let's consider the case of Citigroup Smith Barney's SP 500 Industry Group Rotation Model. Equity lines represent the hypothetical growth of an investment starting with one dollar, constructed based on historical returns. They created a global score (Z-score) using various factors and applied a ranking system based on the aggregated scores. The industries were then divided into three groups, with the top-performing third considered as the outperformers. Portfolios were constructed by investing in the sectors that outperformed the benchmark, based on their Z-scores.

The premium achieved by the outperforming sectors is almost 10% higher than the underperforming sectors. The annual standard deviation of the benchmark plays a crucial role because while past performance often leads to higher returns than the benchmark, it is also associated with higher levels of risk (standard deviation of returns). However, in this particular case, the higher returns are achieved without a significant increase in risk, making it an interesting finding.

Momentum Approaches

There are two approaches to exploit the concept of momentum: the **time-series approach** (TS) and the **cross-sectional approach** (CS).

Momentum was first introduced by Jagadeesh, and the initial introduction focused on the CS approach. In CS, a universe of stocks is observed, and the stocks with the best momentum relative to that universe are chosen, while the stocks with the weakest momentum are shorted.

On the other hand, TS focuses on individual stocks and does not consider relative performance among stocks like CS does. In TS, each stock is analyzed individually, considering its historical performance and using technical analysis tools such as moving averages to identify trends.

The main difference between CS and TS is that CS uses a ranking system and does not consider the individual performance of each stock, whereas TS focuses on a single stock at a time.

To make use of both CS and TS, it is possible to compute a score for each individual TS and use that score to select the best performers. Additionally, the new score can be used as weights, moving away from the equal weighting scheme.

CS strategies are always dollar-neutral, meaning that the short positions finance the long positions. In the case of TS, a long-short strategy does not necessarily result in a dollar-neutral position.

There are various choices to be made when implementing a momentum strategy. The formation period refers to the past period considered for calculating momentum, typically ranging from 3 to 12 months or even (6-1) months. This choice can significantly impact the results. The holding period, which is the duration of time for which the portfolio is held (e.g., 1 week, 1 month, 3 months), also affects transaction costs and has an impact on the type of momentum strategy being implemented.

TS momentum with a medium formation period is generally an outperforming approach. However, it may not perform as well as CS when using a time-varying scheme of weights similar to TS. This advantage still holds for short-term look-back periods.

It is important to note that the higher the returns gained from a momentum strategy, the higher the risk of price reversals. This is a challenge associated with momentum strategies.

According to Robeco, residual momentum is a less risky and more sustainable approach. However, they do not disclose specific information on how to implement their strategy, which may raise doubts about this approach.

Chapter 5

Pairs Trading

Pairs trading is a stationary arbitrage strategy that is closely connected to the Arbitrage Pricing Theory (APT) and is often implemented as a long-short strategy. The fundamental idea behind pairs trading is to identify instances where two stocks or assets exhibit a temporary mispricing or misalignment. Pairs trading can also be linked to economic factor models.

In pairs trading, two assets are considered "similar" when they share certain characteristics that make them alternative choices for investors. The challenge lies in defining the criteria for assessing the similarity between two assets. Once a suitable evaluation is established, the two similar assets are considered a pair.

The core concept of pairs trading revolves around monitoring the price spread between the two assets and taking investment positions when the distance between them becomes significant. The strategy is based on the belief that the prices of the two assets will eventually converge, resulting in a profitable outcome.

The strategy involves taking a long position on the undervalued asset, creating upward pressure on its price. This pressure helps eliminate the undervaluation, as the increased demand drives up the price. Conversely, a short position is taken on the overvalued asset, exerting downward pressure on its price. This pressure causes the overvaluation to diminish. Eventually, the price difference between the two assets narrows, and they realign.

Determining the similarity between assets is crucial from a quantitative perspective. In finance, common approaches for evaluating similarity include correlation and co-integration.

Co-integration is particularly important from an econometric standpoint, especially when dealing with time series data. It examines the underlying long-term trend between two assets. Co-integration can be statistically tested, often using methods like the Augmented Dickey-Fuller test. This test checks for the absence of a trend in the difference between the two time series. Another related concept is the "half-life," which represents the time it takes for the deviation to revert.

It is worth noting that if the price difference between the two assets is too small, the pairs trading strategy may not generate sufficient profitability to cover transaction costs. Moreover, pairs trading strategies come with inherent risks, such as the potential for divergence in the spread.

To implement a successful pairs trading strategy, it is crucial to ensure that co-integration exists not only in historical data but also in future market conditions. Additionally, incorporating time-varying parameters or employing techniques like the Kalman filter can be potential avenues for improving the strategy's performance and adaptability.