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1. Calcular las siguientes integrales usando sustituciones
2. $\int \frac{1}{\sqrt{x}\,(1-\sqrt{x}\,)}dx = \ln\frac{c}{(1-\sqrt{x}\,)^2}$ con $c \in \mathbb{R}$
3. $\int \frac{dx}{x+\sqrt[3]{x}} = \frac{3}{2}\ln c\left(x^{\frac{2}{3}}+1\right)$ con $c \in \mathbb{R}$
4. $\int \frac{dx}{3+\sqrt{x+2}} = 2\sqrt{x+2} - 6\ln\left(3+\sqrt{x+2}\,\right) + c$
5. $\int \sin 2x dx = -\frac{1}{2}\cos 2x + c$
6. $\int \cos\frac{x}{2}dx = 2\sin\frac{x}{2} + c$

Calcular las siguientes integrales usando el metodo por partes

1. $\int \sin(\ln x)dx = \frac{x}{2}[\sin(\ln x) - \cos(\ln x)] + c$
2. $\int x\sqrt{1+x}\,dx = \frac{2}{3}x(x+1)^{\frac{3}{2}} + \frac{4}{15}(x+1)^{\frac{5}{2}} + c$
3. $\int \frac{xe^x}{(1+x)^2}dx = \frac{e^x}{1+x} + c$
4. $\int x\arccos x dx = \frac{1}{4}(2x-1)\arccos x - \frac{1}{4}x\sqrt{1-x^2} + c$
5. $\int \theta\sec\theta\tan\theta d\theta = \theta\sec\theta - \ln c|\sec\theta + \tan\theta|$

Calcular las siguientes integrales usando el metodo de cambios trigonometricos

1. $\int \frac{\cot^3 x}{\csc x}dx = -\sin x - \csc x + c$
2. $\int \left(\frac{\sec x}{\tan x}\right)^4 dx = -\frac{1}{3\tan^3 x} - \frac{1}{\tan x} + c$
3. $\int \tan^{\frac{3}{2}}x\sec^4 x dx = \frac{2}{5}\tan^{\frac{5}{2}}x + \frac{2}{9}\tan^{\frac{9}{2}}x + c$
4. $\int \tan^4 x\sec^4 x dx = \frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + c$

5. $\int \tan x \sqrt{\sec x}\, dx = 2\sqrt{\sec x} + c$

Calcular las siguientes integrales usando el metodo de sustituciones trigonometricas

1. $\int y^3 \sqrt{a^2 - y^2}\, dy = \frac{1}{5}(a^2 - y^2)^{\frac{5}{2}} - \frac{a^2}{3}(a^2 - y^2)^{\frac{3}{2}} + c$
2. $\int \dfrac{dx}{(9 + x^2)^3} = \dfrac{1}{54} \arctan \dfrac{x}{3} + \dfrac{x}{18(9 + x^2)} + c$
3. $\int \sqrt{t^2 + 4}\, dt = \frac{1}{2} t\sqrt{t^2 + 4} + 2\ln\left|\dfrac{t + \sqrt{t^2 + 4}}{2c}\right|$
4. $\int \dfrac{1}{\sqrt{x^2 - 4x + 13}} dx = \ln 3c\left|x - 2 + \sqrt{x^2 - 4x + 13}\right| + c$

Calcule las siguientes integrales usando fracciones parciales

1. $\int \dfrac{(3x^2 + 5x)}{(x - 1)(x + 1)} dx = \ln|(x + 1)|(x - 1)^2 - \dfrac{1}{1 + x} + c$
2. $\int \dfrac{z^2}{(z - 1)^3} dz = \ln|z - 1| - \dfrac{2}{z - 1} - \dfrac{1}{2(z - 1)^2} + c$
3. $\int \dfrac{(x - 18)}{4x^3 + 9x} dx = \ln \dfrac{4x^2 + 9}{x^2} + \frac{1}{6} \arctan \dfrac{2x}{3} + c$
4. $\int \dfrac{2y^3 + y^2 + 2y + 2}{y^4 + 3y^2 + 2} dy = \ln(y^2 + 2) + \arctan y + c$

Resolver las integrales

5. $\int \dfrac{(x - 7)dx}{(x^2 - 14x + 1)} =$ $\quad sol: \dfrac{1}{2}\ln(x^2 - 14x + 1) + C$
6. $\int \dfrac{10x - 25}{\sqrt[3]{x^2 - 5x + 8}} dx =$ $\quad sol: \dfrac{15}{2}\sqrt[3]{(x^2 - 5x + 8)^2} + C$
7. $\int \sqrt[3]{3^x}\, dx =$ $\quad sol: 3\dfrac{3^{\frac{x}{3}}}{\ln 3} + C$
8. $\int \dfrac{\cos\sqrt{x}}{\sqrt{x}} dx =$ $\quad sol: 2\sin\sqrt{x} + C$
9. $\int e^{2x}\sin(e^{2x})dx =$ $\quad sol: -\dfrac{\cos(e^{2x})}{2} + C$
10. $\int \sec(6x)\tan(6x)dx =$ $\quad sol: \frac{1}{6}\sec(6x) + C$
11. $\int 2^x e^x dx =$ $\quad sol: \dfrac{2^x e^x}{1 + \ln 2} + C$
12. $\int \dfrac{5}{2x\ln x} dx =$ $\quad sol: \frac{5}{2}\ln(\ln x) + C$
13. $\int x^3 e^{\frac{x}{2}} dx =$ $\quad sol: 2e^{\frac{x}{2}}(x^3 - 6x^2 + 24x - 48) + C$
14. $\int \arccos x dx =$ $\quad sol: x\arccos x - \sqrt{1 - x^2} + C$

15. $\int \sec^4 3x \tan^3 3x dx \qquad sol: \dfrac{\tan^4 3x}{12} + \dfrac{\tan^6 3x}{18} + C$

16. $\int \sqrt{x^2-16}\,dx = \frac{1}{2}x\sqrt{x^2-16} - 8\ln\left(x+\sqrt{x^2-16}\right) + C$

17. $\int \dfrac{dx}{5-4x-x^2} = \dfrac{1}{6}\ln(x+5) - \dfrac{1}{6}\ln(x-1) + C$

18. $\int \sqrt{x^2+4x-2}\,dx = \sqrt{x^2+4x-2} - 3\ln\left(x+\sqrt{x^2+4x-2}+2\right) + \dfrac{1}{2}$
$x\sqrt{x^2+4x-2} + C$

19. $\int \dfrac{2x-3}{x^2-3x+6}dx = \ln(x^2-3x+6) + C$

20. $\int \dfrac{\ln x}{(x+2)^2}dx = -\dfrac{1}{2(x+2)}(2\ln(x+2) + x\ln(x+2) - x\ln x) + C$

21. $\int \dfrac{\sqrt{x^2+4}}{x^6}dx = -\dfrac{1}{x^5}\sqrt{x^2+4}\left(-\dfrac{1}{120}x^4 + \dfrac{1}{60}x^2 + \dfrac{1}{5}\right) + C$

22. $\int \cos(\ln x)dx = \qquad sol: \frac{1}{2}x[\cos(\ln x) + \sin(\ln x)] + C$

23. $\int (x^2+2x-1)\cos(3x)dx = \qquad sol: (x^2+2x-1)\dfrac{\sin(3x)}{3} + \dfrac{1}{9}(2x+2)\cos(3x) - \dfrac{2}{27}$

24. $\int \dfrac{x^2}{\sqrt{x^2-10}}dx = \qquad sol: \dfrac{x}{2}\sqrt{x^2-10} + 5\ln\left(\dfrac{x+\sqrt{x^2-10}}{\sqrt{10}}\right) + C$

25. $\int \dfrac{x^2}{(4+x^2)^{\frac{3}{2}}}dx = \qquad sol: \ln\left(\dfrac{\sqrt{(4+x^2)}+x}{2}\right) - \dfrac{x}{\sqrt{(4+x^2)}} + C$

26. $\int \sin^3 x \cos^4 x dx = \qquad sol: -\dfrac{\cos^5 x}{x} + \dfrac{\cos^7 x}{7} + C$

27. $\int x^2\sqrt{x^2+4}\,dx = \qquad sol: 4\left(\dfrac{\sqrt{x^2+4}}{2}\right)^3\left(\frac{x}{2}\right) - 2\left[\dfrac{\sqrt{x^2+4}}{2}\left(\frac{x}{2}\right) + \ln\left(\dfrac{\sqrt{x^2+4}}{2}\right.\right.$

28. $\int \dfrac{1}{(x+3)^2(x-4)}dx \qquad sol::$
$\dfrac{1}{49(x+3)}(3\ln(x-4) - 3\ln(x+3) - x\ln(x+3) + x\ln(x-4) + 7)$

29. $\int \dfrac{\cos^3 x}{\sqrt{\sin x}}dx = \qquad sol: \dfrac{\sin x^{\frac{1}{2}}}{\frac{1}{2}} - \dfrac{\sin x^{\frac{5}{2}}}{\frac{5}{2}} + C$

30. $\int \dfrac{\tan^3 x}{\sqrt{\sec x}}dx = \qquad sol: \dfrac{2}{3}(\sec x)^{\frac{3}{2}} + 2(\sec x)^{-\frac{1}{2}} + C$

31. $\int \dfrac{x-9}{x^3-5x^2+6x}dx \qquad sol: -\dfrac{3}{2}\ln(x) - 2\ln(x-3) + \frac{7}{2}\ln(x-2) + C$

32. $\int \dfrac{x^5-2x^4-10}{x^3-9x}dx \qquad sol: \dfrac{x^3}{3} - x^2 + 9x + \dfrac{10}{9}\ln(x) - \dfrac{415}{18}\ln(x-3) + \dfrac{71}{18}\ln(x+3) +$

33. $\int \sin 5x \cos 4x dx = \qquad sol: -\dfrac{\cos x}{2} - \dfrac{\cos 9x}{18} + C$

34. $\int \frac{dx}{\sqrt{4x^2+1}} =$ $\qquad sol: \frac{1}{2}\ln\left(\sqrt{4x^2+1}+2x\right)+C$

35. $\int \frac{x^3-3x^2+2x-3}{(x^2+1)^2}dx$ $\qquad sol: \frac{1}{2}\ln(x^2+1)-\frac{1}{2(x^2+1)}-3\arctan x+C$

36. $\int \sin^4 x\cos^3 x dx =$ $\qquad sol: \frac{3}{64}\sin x-\frac{1}{64}\sin 3x-\frac{1}{320}\sin 5x+\frac{1}{448}\sin 7x+C$

37. $\int \sec\theta\tan\theta d\theta ==$ $\qquad sol: \sec\theta+C$

38. $\int 4\arccos x dx =$ $\qquad sol: 4x\arccos x-4\sqrt{1-x^2}+C$

39. En los ejercicios siguientes calcular la integral definida

a. $\int_0^3 xe^{\frac{x}{2}}dx = 2e^{\frac{3}{2}}+4$

b. $\int_0^{\frac{\pi}{4}} x\cos 2x dx = \frac{1}{8}\pi-\frac{1}{4}$

c. $\int_0^{\frac{1}{2}} \arccos x dx = \frac{1}{6}\pi-\frac{1}{2}\sqrt{3}+1$

d. $\int_1^2 \sqrt{x}\ln x dx = \frac{4}{3}\sqrt{2}\ln 2-\frac{8}{9}\sqrt{2}+\frac{4}{9}$

e. $\int_{-\pi}^{\pi} \sin^2 x dx = \pi$

f. $\int_0^{\frac{\pi}{4}} (\sec t)^2\sqrt{\tan t}\,dt = \frac{2}{3}$

g. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\cos^3 x dx = 4$

h. $\int_{-\pi}^{\pi} \sin 5x\cos 3x dx = 0$

i. $\int_0^2 x^2e^{-2x}dx = \frac{1}{4}-\frac{13}{4}e^{-4}$

j. $\int_0^1 x\arcsin(x^2)dx,$

k. $\int_0^1 \ln(4+x^2)dx = 2\pi+\ln 5-4\arctan 2-2$

l. $\int_0^{\frac{\pi}{8}} x\sec^2 2x dx =$

m. $\int_0^{\frac{\sqrt{3}}{2}} \frac{t^2}{(1-t^2)^{\frac{3}{2}}}dt = \sqrt{3}-\frac{1}{3}\pi$

n. $\int_0^{\frac{3}{5}} \sqrt{9-25x^2}\,dx = \frac{9}{20}\pi$

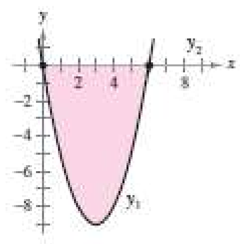
o. $\int_3^6 \frac{\sqrt{x^2-9}}{x^2}dx = \ln\left(3\sqrt{3}+6\right)-\ln 3-\frac{1}{2}\sqrt{3}$

p. $\int_3^6 \frac{x^2}{\sqrt{x^2-9}}dx = \frac{9}{2}\ln\left(6\sqrt{3}+12\right)-\frac{9}{2}\ln 6+9\sqrt{3}$

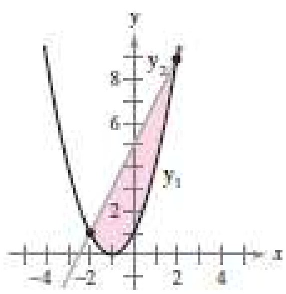
q. $\int \frac{dx}{x^3+8} = \frac{1}{12}\ln(x+2) - \frac{1}{24}\ln(x^2-2x+4) - \frac{1}{24}\sqrt{3}\,\pi + \frac{1}{12}\sqrt{3}$
$\arctan \sqrt{3}\left(\frac{1}{3}x - \frac{1}{3}\right)$

40. En los siguientes ejercicios formular la integral definida que da el area de la región

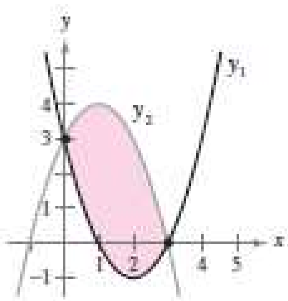
a. $y_1 = x^2 - 6x \qquad y_2 = 0$



b. $y_1 = x^2 + 2x + 1 \qquad y_2 = 2x + 5$



c. $y_1 = x^2 - 4x + 3 \qquad y_2 = -x^2 + 2x + 3$



41. Trazar la región acotada para las graficas de las funciones algebraicas y encontrar el area de la región

a. $y = x^2 - 1 \quad y = -x + 2 \quad x = 0 \quad x = 1$

b. $y = -\frac{x}{2} + \frac{e}{2} + 1$ y los ejes coordenados

c. $y = x^3 + 3 \quad y = x \quad x = -1 \quad x = 1$

d. $y = \frac{1}{2}x^3 + 2 \quad y = x + 1 \quad x = 0 \quad x = 2$

e. $y = -\frac{3}{8}(x - 8) \quad y = 10 - \frac{1}{2}x \quad x = 0$

f. $f(x) = x^2 - 4x \quad g(x) = 0$

g. $f(x) = -x^2 + 4x + 1 \quad g(x) = x + 1$

h. $f(x) = x^2 + 2x \quad g(x) = \frac{1}{2}x + 1$

i. $f(x) = -x^2 + \frac{9}{2}x + 1 \quad g(x) = \frac{1}{2}x + 1$

j. $y = x \quad y = 2 - x \quad y = 0$

k. $y = \frac{1}{x^2} \quad y = 0 \quad x = 1 \quad x = 5$

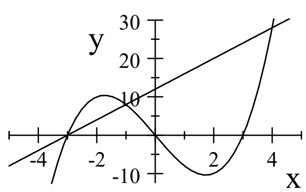
l. $f(x) = \sqrt{x} + 3 \quad g(x) = \frac{1}{2}x + 3$

m. $f(y) = y^2 \quad g(y) = y + 2$

n. $f(y) = y(2 - y) \quad g(y) = -y$

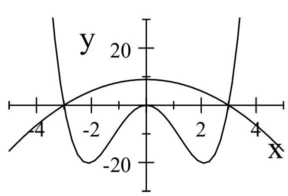
o. $f(y) = y^2 + 1 \quad g(y) = 0 \quad y = -1 \quad y = 2$

42. Calcular el área entre las curvas $h(x) = x^3 - 9x$ y $g(x) = 4x + 12$



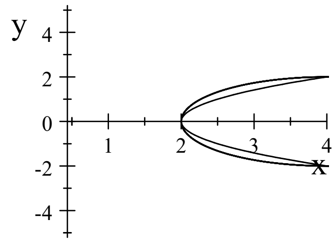
sol. $\frac{407}{4}$

43. Calcular el área entre las curvas $h(x) = x^4 - 9x^2$ y $g(x) = 9 - x^2$



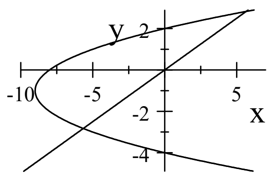
sol. $\frac{504}{5}$

44. Calcular el área entre las curvas $y^2 = -x^2 + 8x - 12$ y $y^2 = 2x - 4$



sol. $\frac{-16}{3}+2\pi$

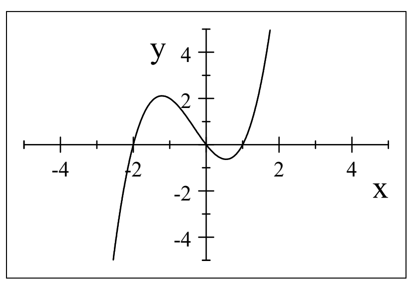
45. Calcular el área entre las curvas $x = y^2 + 2y - 8$ y $x = 2y$



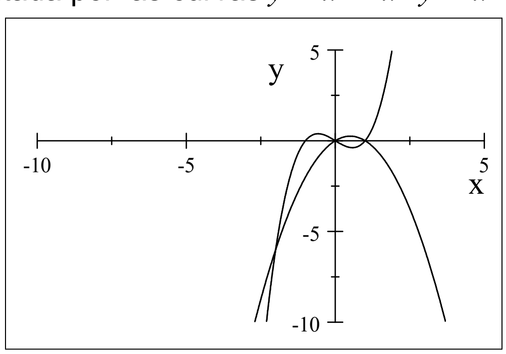
sol. $\frac{64}{3}\sqrt{2}$

46. Hallar el area encerrada por la curva $y = \sin x,$ el eje x entre $x = 0$ y $x = 2\pi \quad sol : 4$

47. Hallar el area encerrada por la curva $y = (x^2 - x)(x+2),$ y el el eje x. $sol : \frac{37}{12}$



48. Hallar el area limitada por las curvas $y = x^3 - x \quad y = x - x^2$



49. $\int \frac{(3x^2+5x)}{(x-1)(x+1)}dx = \ln|(x+1)|(x-1)^2 - \frac{1}{1+x} + c$

50. $\int \frac{z^2}{(z-1)^3} dz = \ln|z-1| - \frac{2}{z-1} - \frac{1}{2(z-1)^2} + c$

51. $\int \frac{(x-3)}{4x^3+9x} dx =$

52. $\int \frac{2y^3+y^2+2y+2}{y^4+3y^2+2} dy = \ln(y^2+2) + \arctan y + c$

53. $\int \frac{dy}{y^4+y^2} = -\frac{1}{y} - \arctan y + c$

54. $\int \frac{t^5 dt}{(t^2+4)^2} = \frac{t^2}{2} - 4\ln(t^2+4) - \frac{8}{t^2+4} + c$

55. $\int \frac{2x^3+x^2+4}{(x^2+4)^2} dx =$

56. $\int \frac{dx}{x^3-2x^2+x-2} =$

57. $\int \frac{dx}{x^3+8}$

58. Calcular las siguientes integrales

a. $\int_0^\infty e^{-x} dx \qquad sol : 1$

b. $\int_1^\infty \frac{1}{\sqrt{x}} dx \qquad sol$:divergente

c. $\int_0^\infty \frac{1}{x^2+1} dx \qquad sol : \frac{\pi}{2}$

d. $\int_\infty^1 \ln x dx \qquad sol : -1$

e. $\int_0^\infty \frac{1}{(1+x)^2} dx \qquad sol : 1$

f. $\int_0^\infty \frac{1}{\sqrt{1+x}} dx \qquad sol$:diverge

59. Calcular el area que resulta de hacer rotar el area limitada por $f(x) = \sqrt{x} + 3 \;\; x = 0 \; x = 9 \;\; y = 0 \; sol : \frac{459\pi}{2}$

60. Calcular el area que resulta de hacer rotar el area limitada por $y = \cos x \;\; y = \frac{1}{2} \;\;\; x = -\frac{\pi}{3} \;\;\; x = \frac{\pi}{3} \;\;\; sol : 2\pi\left(\frac{1}{8}\sqrt{3} + \frac{1}{12}\pi\right)$

61. Calcular el volumen determinado por rotación del área encerrada por $f(x) = x+4$ y $g(x) = 6x - x^2$ al rededor del eje $x \; sol : \frac{333\pi}{5}$

62. Calcular el volumen determinado por rotación del área encerrada por $f(x) = \frac{1}{2}(x-2)$ y las rectas $x = 9$ y $y = 0$ alrededor del eje $x.\; sol : \frac{2}{3}\pi$

63. Calcular el volumen determinado por rotación del área encerrada por

$f(x) = \dfrac{2}{x}$ y $g(x) = \dfrac{1}{x}$ al rededor del eje x $sol: \dfrac{9}{4}\pi$

$$\int \sqrt{4 - x^2}\, dx =$$