Clase 30-3

martes, 30 de marzo de 2021 06:47 p. m.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Ej $f(x) = 4x^2 - 6x + 2$ $\quad f'(x)$ ✓, $f'(2)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f(x+h) = 4(x+h)^2 - 6(x+h) + 2$$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 - 6(x+h) + 2 - (4x^2 - 6x + 2)}{h} = \frac{0}{0} = \text{Ind}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{6x} - 6h + \cancel{2} - \cancel{4x^2} + \cancel{6x} - \cancel{2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{8xh + 4h^2 - 6h}{h} = \lim_{h \to 0} \frac{\cancel{h}(8x + 4h - 6)}{\cancel{h}} = 8x - 6$$

$$\Rightarrow f'(x) = 8x - 6 \,//$$
$$f'(2) = 8.2 - 6 = 10 \,//$$

* $f(x) = 5x^2 - x$ $\qquad f'(x) = ?$ $\qquad f'(4) = ?$

$f'(x) = 8x$ ✗

$f'(x) = 10x - 1$ ✓ $\qquad 9x - x$ ✗

$$f'(x) = \lim_{h \to 0} \frac{5(x+h)^2 - (x+h) - (5x^2 - x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{x} - h - \cancel{5x^2} + \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2 - h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(10x + 5h - 1)}{\cancel{h}} = 10x - 1 \;//$$

* $f(x) = \sqrt{4x^2+3}$ $\qquad f'(x) =$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{4(x+h)^2+3} - \sqrt{4x^2+3}}{h} = \frac{0}{0} \quad \text{Ind.}$$

$$f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{4(x+h)^2+3} - \sqrt{4x^2+3}\right)}{h} \frac{\left(\sqrt{4(x+h)^2+3} + \sqrt{4x^2+3}\right)}{\left(\sqrt{4(x+h)^2+3} + \sqrt{4x^2+3}\right)}$$

$$f'(x) = \lim_{h \to 0} \frac{4(x+h)^2 + 3 - (4x^2+3)}{h\left(\sqrt{4(x+h)^2+3} + \sqrt{4x^2+3}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 + \cancel{3} - \cancel{4x^2} - \cancel{3}}{h\left(\sqrt{4(x+h)^2+3} + \sqrt{4x^2+3}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}.(8x + 4h^2)}{\cancel{h}\left(\sqrt{4(x+h)^2+3} + \sqrt{4x^2+3}\right)}$$

$$f'(x) = \frac{8x}{\sqrt{4x^2+3} + \sqrt{4x^2+3}} = \frac{\overset{4}{\cancel{8}}x}{\cancel{2}\sqrt{4x^2+3}} = \frac{4x}{\sqrt{4x^2+3}} \;//$$

$$f(x) = \frac{8x}{\sqrt{4x^2+3}+\sqrt{4x^2+3}} = \frac{8x}{\cancel{2}\sqrt{4x^2+3}} = \frac{}{\sqrt{4x^2+3}}$$

* $f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \frac{0}{0}$$

$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$

$\lim_{x \to 0} \frac{1-\cos x}{x} = 0$

$$= \lim_{h \to 0} \frac{\cos x \cosh - \operatorname{sen} x \operatorname{sen} h - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{-\cos x (1 - \cosh) - \operatorname{sen} x \operatorname{sen} h}{h}$$

$$= \lim_{h \to 0} -\cos x \frac{(1-\cosh)}{h} - \lim_{h \to 0} \operatorname{sen} x \frac{\operatorname{sen} h}{h}$$

$$= -\cos x . \lim \frac{1-\cosh}{h} - \operatorname{sen} x \lim_{h \to 0} \frac{\operatorname{sen} h}{h} =$$

$$= -\operatorname{sen} x$$

$\Rightarrow f(x) = \cos x \qquad f'(x) = -\operatorname{sen} x.$

$f(x) = \operatorname{sen} x . \ln x . e^x \qquad f'(x) = ?$

Derivar Propiedades y tablas

1.- $f(x) = x^n \quad n \in \mathbb{R} \qquad f'(x) = n . x^{n-1}$

2.- $f(x) = \ln x \qquad f'(x) = \frac{1}{x}$

3.- $f(x) = e^x \qquad f'(x) = e^x$

3: $f(x) = e^{x}$ $\quad f'(x) = e^{x}$

4: $f(x) = K$ $\quad f'(x) = 0$

5: $f(x) = \operatorname{sen} x$ $\quad f'(x) = \cos x$

6: $f(x) = \cos x$ $\quad f'(x) = -\operatorname{sen} x$

7: $f(x) = \tan x$ $\quad f'(x) = \sec^2 x$

8: $f(x) = \sec x$ $\quad f'(x) = \sec x \tan x$

9: $f(x) = \csc x$ $\quad f'(x) = -\csc x \cot x$.

10: $f(x) = \cot x$ $\quad f'(x) = -\csc^2 x$.

Ej Demostrar $f(x) = \cot x$ es $-\csc^2 x$.

$$f'(x) = \lim_{h \to 0} \frac{\cot(x+h) - \cot x}{h} =$$

Si

$$f(x) = \frac{\cos x}{\operatorname{sen} x} \checkmark$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{\cos(x+h)}{\operatorname{sen}(x+h)} - \frac{\cos x}{\operatorname{sen} x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cos x \cosh - \operatorname{sen} x \operatorname{sen} h}{\operatorname{sen} x \cosh + \operatorname{sen} h \cos x} - \frac{\cos x}{\operatorname{sen} x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cos x \cosh \operatorname{sen} x - \operatorname{sen}^2 x \operatorname{sen} h - \cos x \operatorname{sen} x \cosh - \operatorname{sen} h \cos^2 x}{(\operatorname{sen} x \cosh + \operatorname{sen} h \cos x) \operatorname{sen} x}}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \cosh \operatorname{san} x - \cos x \operatorname{san} x \cosh - \operatorname{sanh}(\operatorname{san}^2 x + \cos^2 x)}{h \operatorname{san} x(\operatorname{san} x \cosh + \operatorname{sanh} \cos x)}$$

$$= \lim_{h\to 0} \frac{\cancel{\cos x \cosh \operatorname{san} x} - \cancel{\cos x \operatorname{san} x \cosh} - \operatorname{sanh}}{h \operatorname{san} x(\operatorname{san} x \cosh + \operatorname{sanh} \cos x)}$$

$$= \lim_{h\to 0} \frac{-\operatorname{sanh}}{h \operatorname{san} x(\operatorname{san} x \cosh + \operatorname{sanh} \cos x)}$$

$$= - \lim_{h\to 0} \frac{\operatorname{sanh}}{h} \cdot \frac{1}{\operatorname{san} x} \lim_{h\to 0} \frac{1}{\operatorname{san} x \cosh - \operatorname{sanh} \cos x}$$

$$= -\frac{1}{\operatorname{san} x} \cdot \frac{1}{\operatorname{san} x} = -\frac{1}{\operatorname{san}^2 x} = -\csc^2 x$$

Propiedades

a) $(f(x) \pm g(x))' = f'(x) \pm g'(x)$ ✓

b) $[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ ✓

c) $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

Ej a) $f(x) = \ln x + \operatorname{san} x$ $\qquad f'(x)$

Ej a) $f(x) = \ln x + \operatorname{san} x$ $\qquad f(x)$

$\Rightarrow f'(x) = \frac{1}{x} + \cos x$ »

b) $f(x) = \ln x . \operatorname{san} x$.

$f'(x) = (\ln x)' \operatorname{san} x + \ln x . (\operatorname{san} x)'$

$= \frac{1}{x} \cdot \operatorname{san} x + \ln x . \cos x$ »