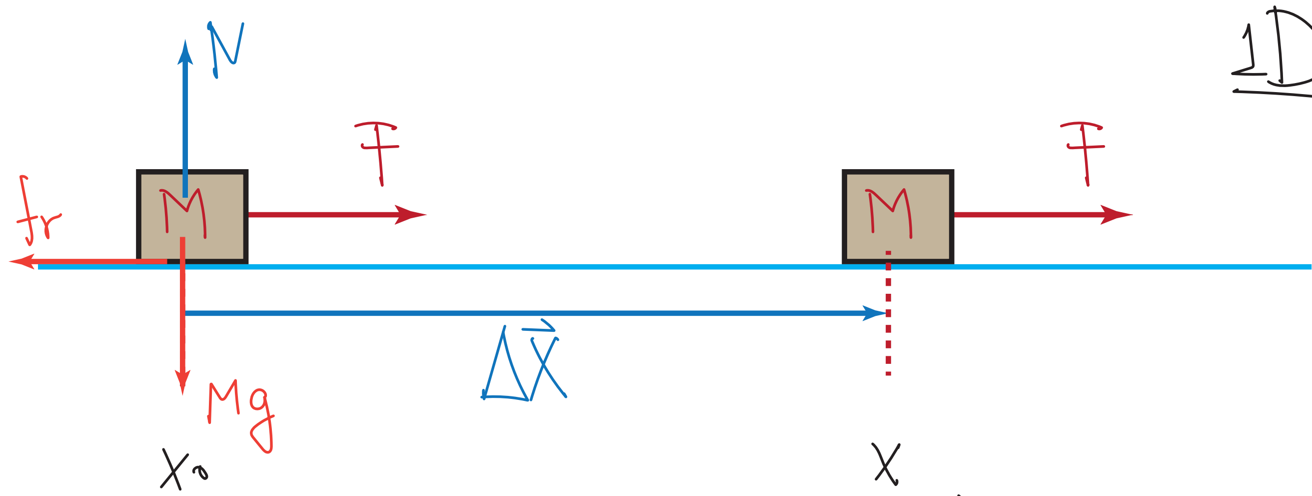
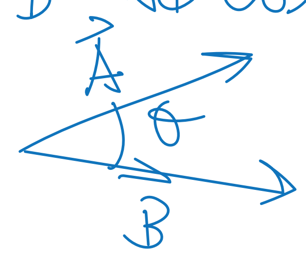
1D




$$W_T = W_F + W_N + W_{Mg} + W_{fr}$$

$$W = \vec{F} \cdot \vec{\Delta X} = F \Delta X \cos\theta$$

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$




$$W_F = F \Delta X \cancelto{1}{\cos\theta_1}$$

$\theta_1 = 0$

$\cos 0 = 1$

$$W_F = F \Delta X$$

$|\vec{\Delta X}| = d$

$$W_F = Fd$$

$$W_N = N \Delta X \cancelto{0}{\cos\theta_2}$$

$\theta_2 = 90$

$\cos 90 = 0$

$$W_N = 0$$

$$W_{Mg} = (Mg) \Delta X \cos\theta_3$$

$\theta_3 = 270$

$\cos 270 = 0$

$$W_{Mg} = Mg \Delta X \cdot 0 = 0$$

$$W_{Mg} = 0$$

$$W_{fr} = f_r \Delta X \cancelto{-1}{\cos\theta_4}$$

$\theta_4 = 180$

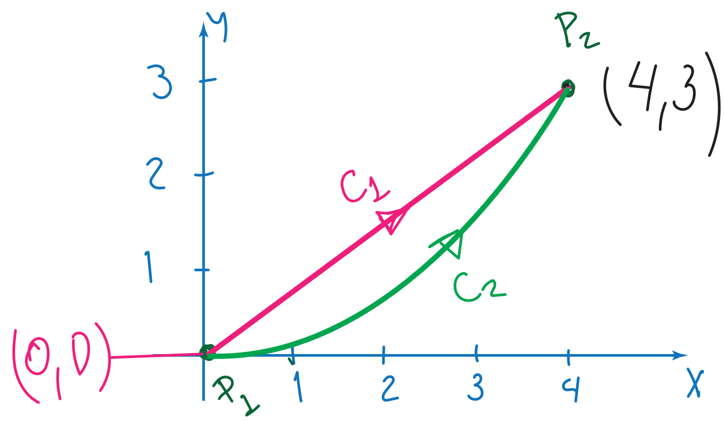
$\cos\theta_4 = \cos 180 = -1$

$$W_{fr} = f_r d$$

$$W_T = Fd + 0 + 0 + f_r d = d(F + f_r)$$

$$\boxed{W_T = d(F - f_r)}$$

Se tiene una fuerza **F** = (2y²x **i** + 2x²y **j**) N, verifique si la fuerza es o no conservativa. **R. Si**




$C_1 \Rightarrow$ Recta $(Y = A + BX)$

$C_2 \Rightarrow$ Parabola $(Y = aX^b)$

$\underline{b = 2}$

$$\vec{F} = \underbrace{2y^2x}_{F_x}\,\hat{i} + \underbrace{2x^2y}_{F_y}\,\hat{j}$$

$$\begin{cases} F_x = 2y^2x \\ F_y = 2x^2y \end{cases}$$

Caso I $\quad W_{C_1} = ?$

$$W = \int F_x\,dx + \int F_y\,dy + \int \cancel{F_z}^{\,0}\,dz$$

$$W_{C_1} = \int 2y^2x\,\underline{dx} + \int 2x^2y\,\underline{dy}$$

$$W_{C_1} = 2\int y^2x\,dx + 2\int x^2y\,dy$$

Si $\quad Y = A + BX$

$A = 0$

$$B = \frac{\Delta y}{\Delta x} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$B = \frac{3-0}{4-0} = \frac{3}{4}$$

$$\underline{y = \frac{3}{4}x} \qquad x = \frac{4}{3}y$$

$$W_{C_1} = 2\int\left(\frac{3}{4}x\right)^2 x\,dx + 2\int\left(\frac{4}{3}y\right)^2 y\,dy$$

$$W_{C_1} = 2\left(\frac{9}{16}\right)\int_0^4 x^3\,dx + 2\left(\frac{16}{9}\right)\int_0^3 y^3\,dy$$

$$W_{C_1} = 2\left(\frac{9}{4^2}\right)\frac{x^4}{4}\Big|_0^4 + 2\left(\frac{4^2}{3^2}\right)\frac{y^4}{4}\Big|_0^3$$

$$W_{C_1} = 2\left(\frac{3^2}{4^3}\right)(4^4 - 0^4) + 2\left(\frac{4}{3^2}\right)(3^4 - 0^4)$$

$$W_{C_1} = 2\left(\frac{3^2}{\cancel{4^3}}\right)4^{\cancel{4}\,1} + 2\left(\frac{4}{\cancel{3^2}}\right)\left(3^{\cancel{4}\,2}\right)$$

$$= 8\left(3^2 + 3^2\right) = 8(9 + 9)$$

$$\boxed{W_{C_1} = 144\ J}$$

$$W_{c_1} = 2\int y^2 x\,dx + 2\int x^2 y\,dy$$

$y = \frac{3}{4}x$ $\qquad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{4}x\right) = \frac{3}{4}\frac{\cancel{dx}}{\cancel{dx}}$ I

$$\frac{dy}{dx} = \frac{3}{4} \Rightarrow dy = \frac{3}{4}dx$$

$$W_{c_1} = 2\int\left(\frac{3}{4}x\right)^2 x\,dx + 2\int x^2\left(\frac{3}{4}\right)\left(\frac{3}{4}dx\right)$$

$$W_{c_1} = 2\left(\frac{3^2}{4^2}\right)\left\{\int x^3 dx + \int x^3 dx\right\}$$

$$W_{c_1} = 2\left(\frac{3^2}{4^2}\right)\left\{2\int x^3 dx\right\} = \frac{9}{4}\int_0^4 x^3 dx$$

$$W_{c_1} = \frac{9}{4}\cdot\frac{x^4}{4}\Big|_0^4 = \frac{9}{4^2}\left(4^4 - 0^4\right)$$

$$W_{c_1} = \frac{9}{\cancel{4^2}}\cdot 4^{\cancel{4}\,2} = 9\cdot 16$$

$$\boxed{W_{c_1} = 144\ \text{J}}$$

$W_{c_2} = ?$

$y = \overset{?}{a} x^2$

$a = \frac{y}{x^2} = \frac{3}{4^2}$

$a = \frac{3}{16}$

$y = \frac{3}{16} x^2$

$W_{c_2} = 2\int y^2 x\,dx + 2\int x^2 y\,dy$

$y = \frac{3}{16} x^2 \Rightarrow x^2 = \frac{16}{3} y$

$W_{c_2} = 2\int \left(\frac{3}{16} x^2\right)^2 x\,dx + 2\int \left(\frac{16}{3} y\right) y\,dy$

$W_{c_2} = 2\left(\frac{3}{16}\right)^2 \int_0^4 x^5\,dx + \frac{32}{3}\int_0^3 y^2\,dy$

$W_{c_2} = \cancel{2}^{1}\left(\frac{3}{16}\right)^2 \frac{x^6}{\cancel{6}_{\cancel{3}}}\Big|_0^4 + \frac{32}{3}\cdot\frac{y^3}{3}\Big|_0^3 = \frac{3}{16^2}\left(4^6 - 0^6\right) + \frac{32}{\underset{3^2}{9}}\left(3^3 - 0^3\right)$

$W_{c_2} = \frac{3}{16^2}\cdot 4^6 + 32\cdot 3 \Rightarrow \boxed{W_{c_2} = 144\ \text{J}}$

$y = \frac{3}{16} x^2$ $\quad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{16} x^2\right) = \frac{3}{16}\frac{dx^2}{dx} = \frac{3}{16}(2x)$

$\frac{dy}{dx} = \frac{3}{8} x \Rightarrow dy = \frac{3}{8} x\,dx$

$W_{c_2} = 2\int y^2 x\,dx + 2\int x^2 y\,dy = 2\left\{\int \left(\frac{3}{16} x^2\right)^2 x\,dx + \int x^2\left(\frac{3}{16} x^2\right)\left(\frac{3}{8} x\,dx\right)\right\}$

$W_{c_2} = 2\left\{\frac{3^2}{16^2}\int x^5\,dx + \frac{3^2}{16\cdot 8}\int x^5\,dx\right\}$

$W_{c_2} = \frac{2\cdot 3^2}{16}\left\{\frac{1}{\underset{2\cdot 8}{16}}\int x^5\,dx + \frac{1}{8}\int x^5\,dx\right\}$

$W_{c_2} = \frac{\cancel{2}^{1}\cdot 3^2}{8\cdot \cancel{16}_{8}}\left\{\underbrace{\frac{1}{2} + 1}_{3/2}\right\}\int_0^4 x^5\,dx = \frac{3^2}{8^2}\left(\frac{3}{2}\right)\frac{x^6}{6}\Big|_0^4$

$W_{c_2} = \frac{3^3}{8^2\cdot 2\cdot 6}\left(4^6 - 0\right) = \frac{3^2}{8^2\cdot 2^2}\cdot 4^6 = \left(\frac{3}{8\cdot 2}\right)^2\cdot 4^6 = \left(\frac{3}{16}\right)^2\cdot 4^6$

$\boxed{W_{c_2} = 144\ \text{J}}$

$W_{C'_4} = 0\,J$

$W_{C''_4} = 2\left\{\int_0^4 9x\,dx\right\}$

$W_{C''_4} = \cancel{2}\cdot 3^2\,\frac{x^2}{\cancel{2}}\Big|_0^4 = 3^2\cdot 4^2 = (3\cdot 4)^2 = 12^2$

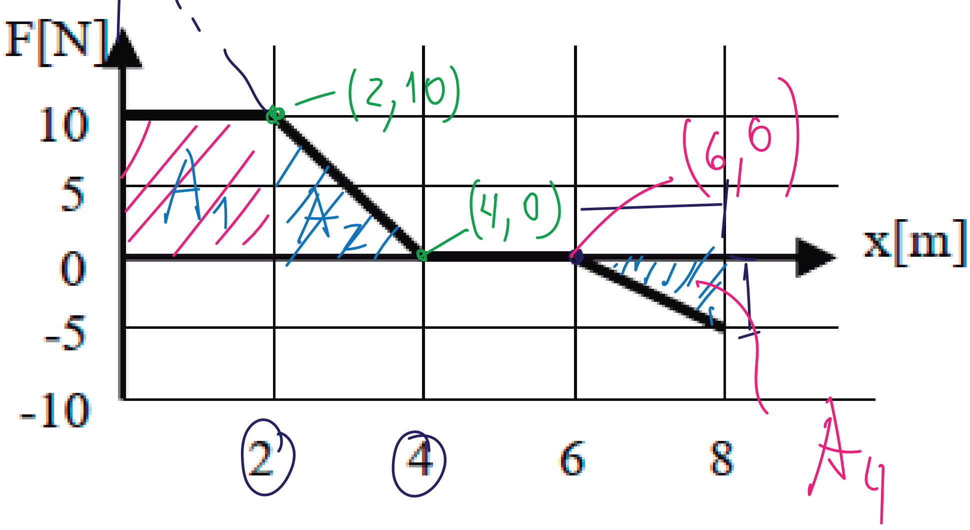
$W_{C''_4} = 144\,J$

$W_{C_4} = \overset{0}{\cancel{W_{C'_4}}} + \overset{144}{\cancel{W_{C''_4}}}$

$$\boxed{W_{C_4} = 144\,J}$$

$\vec{F} \Rightarrow$ Es conservativa

7.- Un bloque de 5.0 kg se mueve en línea recta sobre una superficie horizontal sin fricción bajo la influencia de una fuerza que varía con la posición, como se muestra en la figura. ¿Cuánto trabajo efectúa la fuerza cuando el bloque se mueve desde el origen hasta x=8.0 m? R. 25J




Prob. 7

$W = \vec{F} \cdot \Delta\vec{X}$

$W = F \Delta X \cos\theta$

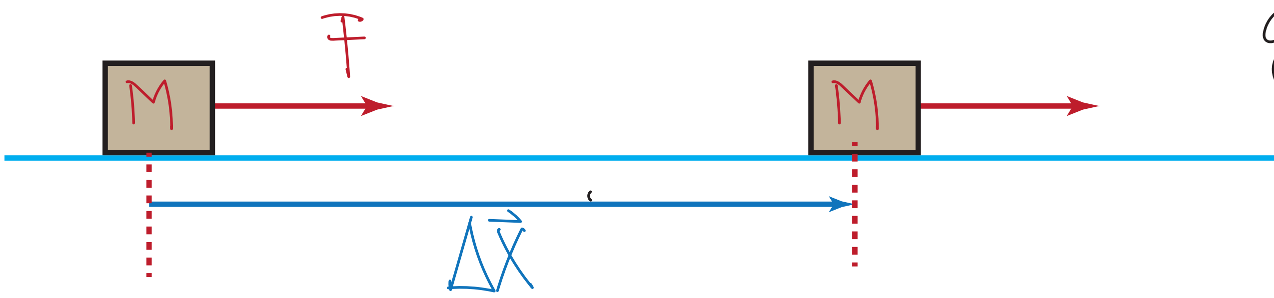
$\theta = 0 \quad \cos 0 = 1$

$\Delta X = d$

$W = F d$

$W = A$

gra $F = f(x)$




$W_1 = F_1 \Delta X \qquad 0 < X < 2$

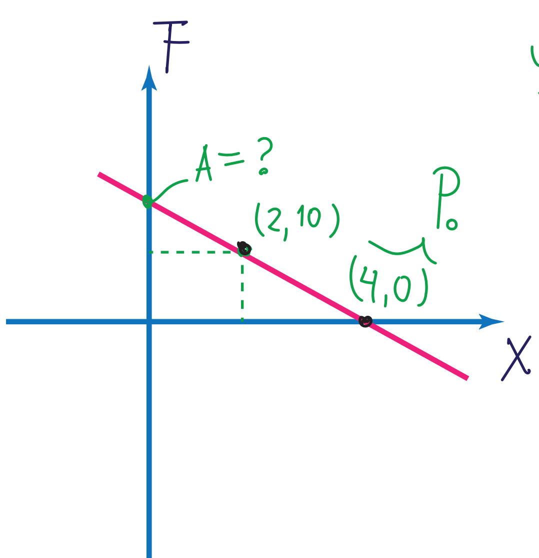
$F_1 = 10\,N \qquad \Delta X = 2 - 0 = 2\,m$

$W_1 = 10 \cdot 2 \Rightarrow W_1 = 20\,J$

$W_1 = A_1 = b \cdot h = 2 \cdot 10 = 20\,J$

$2 < X < 4$

$F \neq ctt \qquad W = \int F_x\,dx + \cancel{\int F_y\,dy} + \cancel{\int F_z\,dz}$




$y = (y_0 - BX_0) + BX$

$A = y_0 - BX_0$

$(X_0 ; Y_0) \rightarrow$ Punto conocido

$(4 ; 0)$

$A = 0 - (-5)(4) = 20$

$B = \dfrac{0 - 10}{4 - 2} = -\dfrac{10}{2}$

$B = -5$

$F = 20 - 5X$

$$W_4 = A_4 = \frac{1}{2} b h = \frac{1}{\cancel{2}} (\cancel{2} \cdot (-5))$$

$$W_4 = A_4 = -5 \text{ J}$$

$$W_T = W_1 + W_2 + \cancel{W_3}^{0} + W_4$$

$$W_T = 20 + 10 + (-5)$$

$$\boxed{W_T = 25 \text{ J}}$$

sol.

$$W = \int_2^4 (20 - 5x)\,dx = 20\int_2^4 dx - 5\int_2^4 x\,dx$$

$$W = 20\,x\Big|_2^4 - 5\,\frac{x^2}{2}\Big|_2^4$$

$$W = 20(4-2) - \frac{5}{2}(4^2 - 2^2)$$

$$W = 20(2) + \frac{5}{\cancel{2}}(\underbrace{16-4}_{\cancel{12}\ 6})$$

$$W_2 = 40 - 30 = 10\text{ J}$$

$$A_2 = \frac{1}{2}bh = \frac{1}{2}(2 \cdot 10)$$

$$\boxed{A_2 = 10\text{ J}}$$

$4 < x < 6\text{ m} \qquad F = 0$

$$W_3 = 0$$

$A = y_0 - B x_0 \qquad B = -\frac{5}{2} \qquad (6,0) = P_0$

$$A = 0 - \left(-\frac{5}{\cancel{2}} \cdot \cancel{6}^{\,3}\right)$$

$$A = 15 \Rightarrow y = 15 - \frac{5}{2}x$$

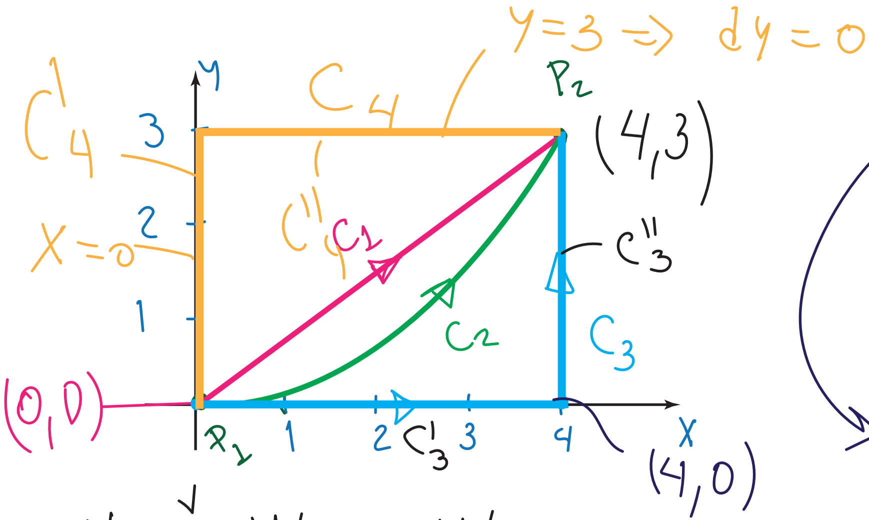
$$W_4 = \int \left(15 - \frac{5}{2}x\right)dx = 15\int_6^8 dx - \frac{5}{2}\int_6^8 x\,dx$$

$$W_4 = 15x\Big|_6^8 - \frac{5}{2}\,\frac{x^2}{2}\Big|_6^8 = 15(8-6) - \frac{5}{4}(8^2 - 6^2)$$

$64 - 36$

$28$

$$W_4 = 30 - 35 = -5\text{ J}$$




$$W_{C_3} = W_{C'_3} + W_{C''_3}$$

$$W_{C'_3} = 2\left\{\int y^2 x\,dx + \int x^2 y\,dy\right\}$$

$$y = 0 \Rightarrow dy = 0$$

$$W_{C'_3} = 2\left\{\cancel{\int 0^2 x\,dx}^{\,0} + \cancel{\int x^2 \cdot 0 \cdot dy}^{\,0}\right\}$$

$$W_{C'_3} = 0\ J$$

$$W_{C''_3} = 2\left\{\int y^2 x\,dx + \int x^2 y\,dy\right\}$$

$$x = 4 \qquad dx = 0$$

$$W_{C''_3} = 2\left\{\cancel{\int y^2 x \cdot 0}^{\,0} + \int x^2 y\,dy\right\}$$

Si $x = 4$

$$W_{C''_3} = 2\int 4^2 y\,dy = 2 \cdot 4^2 \int_0^3 y\,dy = \cancel{2} \cdot 4^2 \cdot \frac{y^2}{\cancel{2}}\Big|_0^3$$

$$W_{C''_3} = 4^2(3^2 - 0^2) = 4^2 \cdot 3^2 = (4 \cdot 3)^2 = 12^2$$

$$W_{C''_3} = 144\ J$$

$$W_{C_3} = \cancel{W_{C'_3}}^{\,0} + \cancel{W_{C''_3}}^{\,144} \Rightarrow \boxed{W_{C_3} = 144\ J}$$

$$y - y_0 = B(x - x_0)$$

$$B = \frac{y - y_0}{x - x_0}$$

$$y = \underbrace{y_0 - Bx_0}_{A} + Bx$$

$$y = A + Bx$$

$$A = 0$$

$$B = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 0}{4 - 0} = \frac{0}{4}$$

$$B = 0$$

$$\boxed{y = 0}$$

$$B = \frac{3 - 0}{4 - 4} = \frac{3}{0} = \infty$$