Clase 25/05

martes, 25 de mayo de 2021 06:40 p. m.

$\int x \arccos x \, dx =$

$u = \arccos x \qquad dv = x\,dx$

$du = \dfrac{-1}{\sqrt{1-x^2}}\,dx \qquad v = \dfrac{x^2}{2}$

$$\int x \arccos x \, dx = \frac{x^2}{2}\arccos x - \int \frac{x^2}{2} \cdot \frac{-1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{x^2}{2}\arccos x + \frac{1}{2}\int \underbrace{\frac{x^2}{\sqrt{1-x^2}}}_{\text{Sust. trig.}}\,dx$$

Pausa.

2. $\int x\sqrt{1+x}\,dx =$

$u = x \qquad dv = \sqrt{1+x}\,dx$

$du = dx \qquad v = \int \sqrt{1+x}\,dx$

$t = 1+x$

$dt = dt$

$v = \int t^{1/2}\,dt = \dfrac{t^{3/2}}{\frac{3}{2}}$

$\frac{1}{2}+1$

$v = \frac{2}{3}(1+x)^{3/2}$

$v =$

$$\int x\sqrt{1+x}\,dx = x\,\frac{2}{3}(1+x)^{3/2} - \int \frac{2}{3}(1+x)^{3/2}\,dx$$

$$= \frac{2x(1+x)^{3/2}}{3} - \frac{2}{3}\int (1+x)^{3/2}\,dx$$

$t = 1+x$

$dt = dx$

$$= \frac{2x(1+x)^{3/2}}{3} - \frac{2}{3}\int t^{3/2}\,dx$$

$(1+x)^{3/2}$ $t^{5/2}$ $2x(1+x)^{3/2}$ 4 $(1+x)^{5/2} + C$

$$= \frac{2x}{3}(1+x)^{3/2} - \frac{2}{3}\,\frac{t^{5/2}}{\frac{5}{2}} = \frac{2}{3}x(1+x)^{3/2} - \frac{4}{15}(1+x)^{5/2} + C$$

$\int \frac{xe^x}{(1+x)^2}dx$:

$u = xe^x$ $\qquad$ $dv = \frac{1}{(1+x)^2}dx$

$du = (e^x + xe^x)dx$ $\qquad$ $v = \int \frac{1}{(1+x)^2}dx$

$t = 1+x$

$dt = dx$

$$\int \frac{xe^x}{(1+x)^2}dx = xe^x\left(-\frac{1}{1+x}\right) - \int -\frac{1}{1+x}\cdot(e^x + xe^x)\,dx$$

$= \int \frac{1}{t^2}\,dt$

$$= -\frac{xe^x}{1+x} + \int \frac{e^x\cancel{(1+x)}}{\cancel{(1+x)}}\,dx$$

$= \int t^{-2}\,dt$

$= \frac{t^{-1}}{-1}$

$$\frac{e^x}{1+x} + c$$

$$= -\frac{xe^x}{1+x} + e^x + C$$

$= -(1+x)^{-1}$

$v = -\frac{1}{1+x}$

$$= \frac{\cancel{-xe^x} + e^x + \cancel{xe^x}}{1+x} + C$$

$$= \frac{e^x}{1+x} + C$$

Prod. de Func. trig.

a) $\operatorname{sen}^a x \cos^b x$

i) a, b Pares $\qquad$ ii) a, b Imp $\qquad$ iii) a, b par . Imp

$$\operatorname{sen}^2 x + \cos^2 x = 1$$

$$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 2x = \frac{1-\cos 4x}{2}$$

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$$

$\rightarrow \int \cos 2x\,dx = \frac{\sin 2x}{2}$

$\rightarrow \int \cos ax\,dx = \frac{\sin ax}{a}$

$$= \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx$$

$$= \frac{1}{2}x - \frac{\sin 2x}{4} + C.$$

1). $\int \sin^2 x \cos^4 x\,dx$

$$\int \sin^2 x (\cos^2 x)^2\,dx = \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{8}\int (1-\cos 2x)(1+\cos 2x)(1+\cos 2x)\,dx$$

$$= \frac{1}{8}\int (1-\cos^2 2x)(1+\cos 2x)\,dx$$

$$= \frac{1}{8}\int (1+\cos 2x - \cos^2 2x + \cos^3 2x)\,dx$$

$$= \frac{1}{8}\int dx + \frac{1}{8}\int \cos 2x\,dx - \frac{1}{8}\int \cos^2 2x\,dx + \frac{1}{8}\int \cos^3 2x\,dx$$

$$= \frac{1}{8}x + \frac{1}{8}\frac{\sin 2x}{2} - \frac{1}{8}\int\left(\frac{1+\cos 4x}{2}\right)dx + \frac{1}{8}\int \cos 2x \cos^2 2x\,dx$$

$$= \frac{1}{8}x + \frac{1}{8}\frac{\sin 2x}{2} - \frac{1}{8}\cdot\frac{1}{2}x - \frac{1}{16}\frac{\sin 4x}{4} + \frac{1}{8}\int \cos 2x(1-\sin^2 2x)\,dx$$

$$= \frac{1}{8}x + \frac{1}{16}\sin 2x - \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{8}\int \cos 2x\,dx - \frac{1}{8}\int \cos 2x \sin^2 2x\,dx$$

$t = \sin 2x$

$dt = \cos 2x \cdot 2dx$

$\frac{dt}{2} = \cos 2x\,dx$

$$= \frac{1}{8}x + \frac{1}{16}\sin 2x - \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{8}\frac{\sin 2x}{2} - \frac{1}{8}$$

$dt = \cos 2x \cdot 2dx$

$\frac{dt}{2} = \cos 2x \, dx$

$$= \frac{1}{8}x + \frac{1}{16}\sin 2x - \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{8}\frac{\sin 2x}{2} - \frac{1}{8}$$

$$\int \frac{t^2}{2}\,dt = \frac{t^3}{6}$$

$$= \frac{\sin^3 2x}{6}$$

$$\frac{1}{8}x + \frac{1}{16}\sin 2x - \frac{1}{16}x - \frac{1}{64}\sin 4x + \frac{1}{8}\frac{\sin 2x}{2} - \frac{1}{8}\frac{\sin^3 2x}{6} + C.$$

$\cos x' = -\sin x$

2) $\int \sin^3 x \cos^5 x \, dx$

$$\int \sin^2 x \, \sin x \, \cos^5 x \, dx = \int (1-\cos^2 x)\cos^5 x \sin x \, dx$$

$$= \int \cos^5 x \sin x \, dx - \int \cos^7 x \sin x \, dx$$

$t = \cos x$

$dt = -\sin x \, dx \Rightarrow -dt = \sin x \, dx$

$$= \int t^5(-dt) - \int t^7(-dt)$$

$$= -\int t^5 dt + \int t^7 dt$$

$$= -\frac{t^6}{6} + \frac{t^8}{8} + c \Rightarrow -\frac{\cos^6 x}{6} + \frac{\cos^8 x}{8} + C$$

3) $\int \sin^3 x \cos^4 x \, dx = \int \sin^2 x \sin x \cos^4 x \, dx$

$$= \int (1-\cos^2 x)\cos^4 x \sin x \, dx$$

$$= \int \cos^4 x \sin x \, dx - \int \cos^6 x \sin x \, dx$$

$t = \cos x \qquad dt = -\sin x \, dx \Rightarrow -dt = \sin x \, dx$

$$= \int t^4(-dt) - \int t^6(-dt)$$

$t = \cos x \qquad dt = -\sin x \, dx$

$$= \int t^4 (-dt) - \int t^6 (-dt)$$

$$= -\frac{t^5}{5} + \frac{t^7}{7} = -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$$

Prod. $\tan^n x \sec^m x$

$\tan x' = \sec^2 x$

$\sec x' = \sec x \tan x$

* $1 + \tan^2 x = \sec^2 x$.

a) $\int \tan^2 x \sec^4 x \, dx$

$$\int \tan^2 x \sec^2 x \sec^2 x \, dx$$

$$\int \tan^2 x (1 + \tan^2 x) \sec^2 x \, dx$$

$$\int \tan^2 x \sec^2 x \, dx + \int \tan^4 x \sec^2 x \, dx$$

$t = \tan x$

$dt = \sec^2 x \, dx$

$$\int t^2 \, dt + \int t^4 \, dt$$

$$\frac{t^3}{3} + \frac{t^5}{5} \Rightarrow \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$$

2) $\int \tan^4 x \sec^3 x \, dx = \int (\tan^2 x)^2 \sec^3 x \, dx$

$$= \int (\sec^2 x - 1)^2 \sec^3 x \, dx$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \sec^3 x \, dx$$

$$= \underbrace{\int \sec^7 x \, dx}_{I} - 2\underbrace{\int \sec^5 x \, dx}_{II} + \underbrace{\int \sec^3 x \, dx}_{III}$$

$\int \sec^7 x \, dx =$ Puntos Extras ?

$\int \sec^5 x \, dx =$

3)

$$\int \tan^3 x \sec^5 x \, dx = \int \tan^2 x \tan x \sec^4 x \sec x \, dx$$

$$= \int \tan^2 x \sec^4 x \tan x \sec x \, dx$$

$$= \int (\sec^2 x - 1) \sec^4 x \tan x \sec x \, dx$$

$$= \int \sec^6 x \tan x \sec x \, dx - \int \sec^4 x \tan x \sec x \, dx$$

$$t = \sec x$$
$$dt = \sec x \tan x \, dx$$

$$= \int t^6 \, dt - \int t^4 \, dt$$

$$= \frac{t^7}{7} - \frac{t^5}{5}$$

$$= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

$$\int \frac{\cot^2 x \cdot \sec^3 x}{\csc^5 x} \, dx$$

$\cot x = \frac{\cos x}{\sin x}$ $\csc x = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$

$\int \cancel{\cos^2 x} \cdot 1$

$$\int \frac{\frac{\cancel{\cos^2 x}}{\cancel{\sin^2 x}} \cdot \frac{1}{\cos^3 x}}{\frac{1}{\sin^{\cancel{5}\,3} x}}\,dx = \int \frac{\frac{1}{\cos x}}{\frac{1}{\sin^3 x}}\,dx = \int \frac{\sin^3 x}{\cos x}\,dx$$

$$\int \frac{\sin^2 x \sin x}{\cos x}\,dx = \int \frac{(1-\cos^2 x)\sin x}{\cos x}\,dx$$

$$\int \left(\frac{\sin x}{\cos x} - \frac{\cos^2 x \sin x}{\cos x}\right) dx$$

$$\int \frac{\sin x}{\cos x}\,dx - \int \cos x \sin x\,dx$$

$t = \cos x$

$dt = -\sin x\,dx$

$$\int \frac{-dx}{t} - \int t(-dt) = -\ln t + \frac{t^2}{2}$$

$$= -\ln|\cos t| + \frac{\cos^2 t}{2}.$$