Cilindro

En la Tabla 2.4 escribir las mediciones de la altura H y el diámetro D del cilindro.

n	H [cm]	D [cm]
1	3,912	1,232
2	4,012	1,181
3	4,022	1,251
4	3,910	1,240
5	4,014	1,273
6	3,986	1,202

Tabla 2.4: Medidas de la longitud y el diámetro del cilindro

Longitud	Diámetro
$\bar{H}$ = 3,976 cm	$\bar{D}$ = 1,22983 cm
σ_H = 0,02 cm	σ_D = 0,01 cm
P = 0,002 cm	P_D = 0,001 cm
e_H = 0,02 cm	e_D = 0,01 cm

Tabla 2.5: Valores medios, errores de la media aritmética, presiciones y errores de las mediciones del cilindro

Resultados de la medición
H = (3,98 ± 0,02)[cm]; 0,5%
D = (1,23 ± 0,01)[cm]; 0,8%
m = (34,56 ± 0,01)[g]; 0,03%

Tabla 2.6: Resultados de la medición de la altura, el diámetro y la masa del cilindro

$\bar{H} = 3{,}97|6\ cm \approx 3{,}98\ cm$

$P_H = 0{,}02\ mm = 0{,}002\ cm \Rightarrow$ Presición del Instro

$$\sigma_{\bar{H}} = \frac{\sigma_{n-1}}{\sqrt{n}} = \sqrt{\frac{\sum d_i^2}{n(n-1)}}$$

$$\sigma_{\bar{H}} = \frac{0{,}05178}{\sqrt{6}} = 0{,}02|11\ cm \approx 0{,}02\ cm$$

$\sigma_{\bar{H}} = 0{,}02\ cm$

Si $\sigma_{\bar{H}} > P_H$

$\hookrightarrow e_H = \sigma_{\bar{H}}$

$e_H = 0{,}02\ cm$

$$\bar{H} = (3{,}98 \pm 0{,}02)[cm];\ 0{,}503\%$$

Diametro

$\bar{D} = 1{,}22|983\ cm \approx 1{,}23\ cm$

$P_D = 0{,}01\ mm = 0{,}001\ cm$

$$\sigma_{\bar{D}} = \frac{\sigma_{n-1}}{\sqrt{n}} = \frac{0{,}03340}{\sqrt{6}} = 0{,}01|363\ cm \approx 0{,}01\ cm$$

$\sigma_{\bar{D}} > P_D$

$$\bar{D} = (1{,}23 \pm 0{,}01)[cm];\ 0{,}8\%$$

mediciones indirectas y propagacion de errores

* no hay un instrumento de medicion directa

* las medidas indirectas se realcionan mediante ecuciones matematicas o algebraicas con las medidas directas

ejemplos de medidas indirectas:

M. Directa

$$V_{esfera} = \frac{\pi}{6} D^3$$

M. Indirecta

$$A_{circulo} = \frac{\pi}{4} D^2$$

$$R_{circulo} = \frac{D}{2}$$

$$\rho_{esfera} = \frac{m_{esfera}}{V_{esfera}}$$

como calculamos el error de esta medida indirecta

$$f = f(x, y, z.....)$$

donde consideraremos que las variables x, y ,z ,,,,, son variables medidas y tienen sus erroresv

$$x = (x_{rep} \pm e_x)[U]; E\%$$

$$y = (y_{rep} \pm e_y)[U]; E\%$$

$$z = (z_{rep} \pm e_z)[U]; E\%$$

$$\boxed{e_f = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 ....}}$$

$$\Delta x = \left| \frac{\partial f}{\partial x} \right| * e_x$$

$$\Delta y = \left| \frac{\partial f}{\partial y} \right| * e_y$$

$$\Delta z = \left| \frac{\partial f}{\partial z} \right| * e_z$$

3.5.1. Cilindro

Datos

$H = (3{,}98 \pm 0{,}02)[cm];\ 0{,}5\%$

$D = (1{,}23 \pm 0{,}01)[cm];\ 0{,}8\%$

$m = (34{,}56 \pm 0{,}01)[g];\ 0{,}03\%$

Cálculo del volumen y su error:

$$V = \frac{\pi D^2 H}{4} = \frac{\pi}{4}(1{,}23)^2(3{,}98)$$

$$V = 4{,}72915[cm^3]$$

$$V = V(D,H) \quad e_v = \sqrt{\Delta D^2 + \Delta H^2}$$

$$\Delta D = \left|\frac{\partial V}{\partial D}\right| \cdot e_D \qquad \frac{dx^n}{dx} = nx^{n-1}$$

$$\frac{\partial V}{\partial D} = \frac{\partial}{\partial D}\left(\frac{\pi D^2 H}{4}\right) = \frac{\pi}{4} H \frac{\partial D^2}{\partial D} = \frac{\pi}{\cancel{4}_2} H (\cancel{2}^1 D)$$

$$\frac{\partial V}{\partial D} = \left|\frac{\pi H D}{2}\right|$$

$$\Rightarrow \Delta D = \frac{\pi H D}{2} \cdot e_D$$

$$\Delta D^2 = \left(\frac{\pi(3{,}98)(1{,}23)}{2} \cdot 0{,}01\right)^2$$

$$\Delta D^2 = 5{,}9131 \times 10^{-3}$$

Cálculo de la densidad y su error:

$$\rho = \frac{m}{V}$$

$$\rho = \frac{34{,}56}{4{,}73}$$

$$\rho = 7{,}3065 \left[\frac{g}{cm^3}\right]$$

Resultado de la medición del volumen

$V = (4{,}73 \pm 0{,}08)[cm^3];\ 1{,}69\%$

Resultado de la medición de la densidad

$\rho = (7{,}3 \pm 0{,}1)[g/cm^3];\ 1{,}37\%$

$$\Delta H = \left|\frac{\partial V}{\partial H}\right| e_H$$

$$\frac{\partial V}{\partial H} = \frac{\partial}{\partial H}\left(\frac{\pi}{4} H D^2\right) = \frac{\pi D^2}{4} \cancelto{1}{\frac{\partial H}{\partial H}} = \frac{\pi D^2}{4}$$

$$\Delta H = \frac{\pi D^2}{4} e_H$$

$$\Delta H^2 = \left(\overbrace{\frac{\pi (1{,}23)^2}{4}} \cdot 0{,}02\right)^2$$

$$\Delta H^2 = 5{,}6475 \times 10^{-4}$$

$$\hookrightarrow e_V = \sqrt{5{,}9131 \times 10^{-3} + 5{,}6475 \times 10^{-4}}$$

$$e_V = 0{,}0804 \approx 0{,}08 \, [cm^3]$$

$$V = 4{,}72915 \, [cm^3] \approx 4{,}73 \, [cm^3]$$

$$V = (4{,}73 \pm 0{,}08) \, [cm^3]; \; 1{,}69\%$$

$$e_\rho = \sqrt{\Delta m^2 + \Delta V^2}$$

$$\rho = \frac{m}{V}$$

$$\rho = f(m, V)$$

$$\Delta m = \left|\frac{\partial \rho}{\partial m}\right| \cdot e_m$$

$$\frac{\partial \rho}{\partial m} = \frac{\partial (m/V)}{\partial m} = \frac{1}{V}\,\overset{1}{\cancel{\frac{\partial m}{\partial m}}} = \frac{1}{V}$$

$$\Delta m = \frac{e_m}{V} \Rightarrow \Delta m^2 = \left(\frac{0{,}01}{4{,}73}\right)^2$$

$$\Delta m^2 = 4{,}4696 \times 10^{-6}$$

$$\Delta V = \left|\frac{\partial \rho}{\partial V}\right| \cdot e_V$$

$$\frac{1}{a^n} = a^{-n}$$

$$\frac{\partial \rho}{\partial V} = \frac{\partial \left(\frac{m}{V}\right)}{\partial V} = m\,\frac{\partial\left(\frac{1}{V}\right)}{\partial V} = m\,\frac{\partial V^{-1}}{\partial V} = m\left((-1)V^{-2}\right)$$

$$\left|\frac{\partial \rho}{\partial V}\right| = \left|-\frac{m}{V^2}\right| = \frac{m}{V^2}$$

$$\Delta V = \frac{m}{V^2} e_V \Rightarrow \Delta V^2 = \left(\frac{34{,}56}{(4{,}73)^2} \cdot 0{,}08\right)^2$$

$$\Delta V^2 = 0{,}0152$$

$$e_\rho = \sqrt{4{,}4696 \times 10^{-6} + 0{,}0152}$$

$$e_\rho = 0{,}123 \approx 0{,}1 \left[g/cm^3\right]$$

$$e_\rho = 0,1\ [g/cm^3]$$

$$\rho = 7,3|0\overline{65}\ \left[\frac{g}{cm^3}\right] \approx 7,3\ [g/cm^3]$$

$$\rho = (7,3 \pm 0,1)\,[g/cm^3];\ 1,37\%$$