Clase 23/04

viernes, 23 de abril de 2021 04:53 p. m.

. Si $y = \frac{m}{n}\sqrt{n^2 - x^2}$ demostrar que $y' = \frac{-m^2 x}{n^2 y}$

m, n ctte

$\boxed{\sqrt{x}' = \frac{1}{2\sqrt{x}}}$

$\sqrt[3]{x} = \left(x^{\boxed{\frac{1}{3}}}\right)' = \frac{1}{3} x^{-2/3}$

$y' = \frac{m}{n} \frac{1}{\cancel{2}\sqrt{n^2-x^2}} \cdot -\cancel{2}x$

$y' = -\frac{m\, x}{n\sqrt{n^2-x^2}}$

$\frac{yn}{m} = \sqrt{n^2-x^2}$

$y' = -\frac{m\,x}{n\,\frac{yn}{m}}$

$y' = -\frac{m^2 x}{n^2 y}$

b. $\begin{cases} 2x^2 + y = 75 \\ 2\log x + \log y = 2\log 2 + \log 3 \end{cases}$ →

$\log x^2 + \log y = \log 2^2 + \log 3$

$\log x^2 y = \log 12$

$x^2 y = 12.$

$\begin{cases} 2x^2 + y = 75 \Rightarrow y = 75 - 2x^2 \\ x^2 y = 12. \end{cases}$

$x^2(75 - 2x^2) = 12$

$y = 75 - 2x^2$

$75x^2 - 2x^4 = 12$

$2x^4 - 75x^2 + 12 = 0$

$x^2 = a$

$2a^2 - 75a + 12 = 0$

$a_1 = 37.34$

$a_2 = 0.14$

$\sqrt{x^2} = \sqrt{37.34}$ $x = \pm 6.11$

$\sqrt{x^2} = \sqrt{0.14}$ $x = \pm 0.4$

$x_1 = 6.11$ $y_1 = 0.33$

$x_2 = -6.11$ $y_2 = 0.33$

$x_3 = 0.4$ $y_3 = 74,68$

$x_4 = -0.4$ $y_4 = 74.68$

$(6.11, 0.33)$ $(-6.11, 0.33)$ PI

$(0.4, 74.68)$ $(-0.4, 74.68)$

Para $(6.11, 0.33)$

$\tan\alpha = m_2 - m_1$

$2x^2 + y = 75$

$y = 75 - 2x^2$

$x^2 y = 12$

y 12

$$\tan\alpha = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$2x^2 + y = 75$$
$$y = 75 - 2x^2$$
$$y' = -4x$$
$$y' = -4.(6.11)$$
$$y' = -24.44$$

$$x^2 y = 12$$
$$y = \frac{12}{x^2}$$
$$y' = \frac{-12.2\cancel{x}}{x^{\cancel{4}\,3}}$$
$$y' = -\frac{24}{x^3}$$
$$y' = -\frac{24}{(6.11)^3}$$
$$y' = -0.10$$

$$\tan\alpha = \frac{-0.10 - (24.44)}{1 + (-0.10)(-24.44)}$$
$$\tan\alpha = 7,07$$
$$\alpha = 81.95^\circ$$

b. $f(t) = t\sqrt{4-t}$ $\quad t < 3$

$$f'(t) = \sqrt{4-t} + t \cdot \frac{1}{2\sqrt{4-t}} \cdot (-1)$$

$\frac{5}{6}(-)$ $\quad -\frac{5}{6}$

$$f'(t) = \underbrace{\sqrt{4-t} - \frac{t}{2\sqrt{4-t}}}$$

$f'(t) = 0$ ✓

$$\sqrt{4-t} - \frac{t}{2\sqrt{4-t}} = 0$$

$t = 4$

$$\frac{2(4-t) - t}{2\sqrt{4-t}} = 0$$

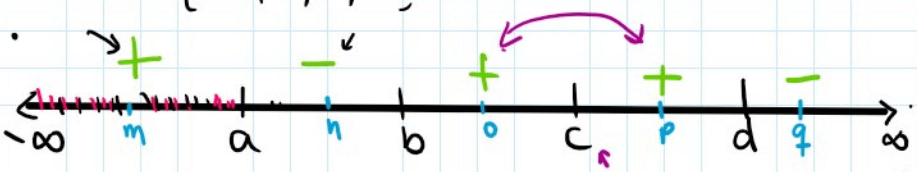
$$\frac{8 - 2t - t}{2\sqrt{4-t}} = 0$$

$PC = \{\frac{8}{3}\}$

$$8 - 3t = 0$$
$$t = \frac{8}{3}$$ ✓

1. $f'(x) = 0$
2. x donde f' NOEX
3. $f: [\ \ ]$

$PC = \{a, b, c, d\}$



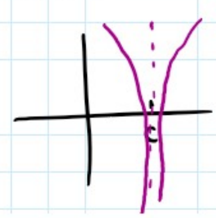
$(-\infty, a)$	crece
(a, b)	Decrece
(b, c)	crece
(c, d)	crece.
(d, ∞)	decrece.

$f'(m) = +$ $\quad f'(n) = -$ $\quad f'(o) = +$ $\quad f'(p) = +$ $\quad f'(q) =$

$(,)$

3?

$(a, f(a))$ Max $\quad + \ a\ -$

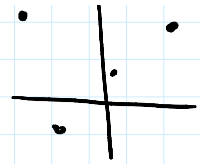


$(a, f(a))$ Max $+ \; a \; -$

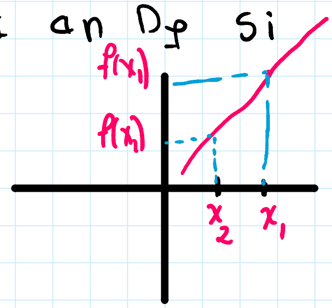
$(b, f(b))$ min $- \; a \; +$

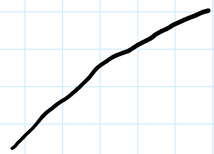
$(c, f(c))$ Punto de Inf   no hay cambio de signo

$(d, f(d))$ Max

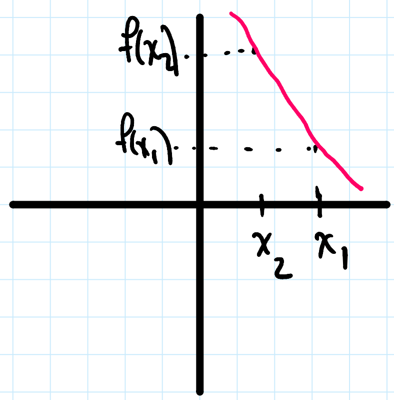


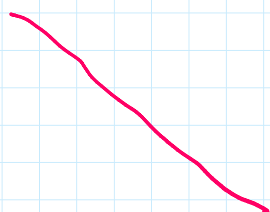
f es creciente en D_f si $\forall x_1, x_2 \quad x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

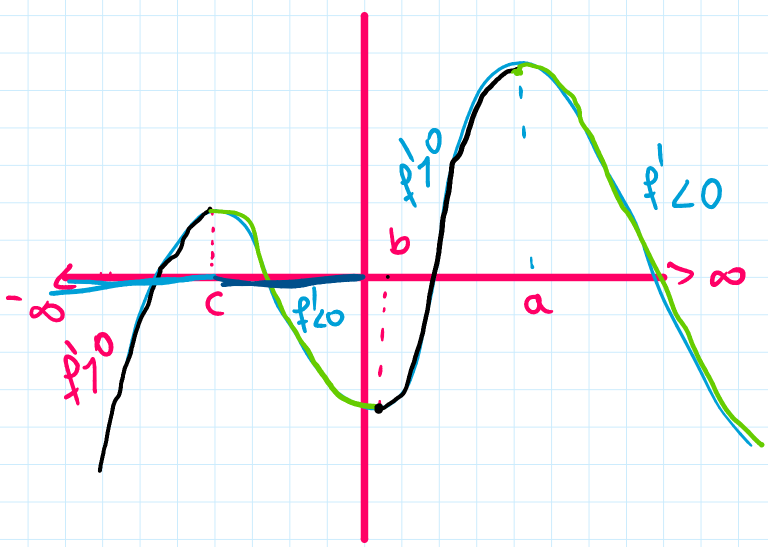




f es decreciente en D_f si $x_1 > x_2 \Rightarrow f(x_2) > f(x_1)$







$(-\infty, c)$	Crece
(c, b)	Decrece
(b, a)	Crece
(a, ∞)	Decrece

Para los siguientes ejercicios se pide encontrar a) puntos críticos, b) máximos y/o mínimos locales, c) puntos de inflexión, d) intervalos de crecimiento y decrecimiento e) raices de la función y f) graficar la función:

10. $f(x) = x^4 - 6x^3 + 11x^2 - 6x$

11. $f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$ • I IC

12. $f(x) = \frac{1}{5}x^5 + x^4 - \frac{4}{3}x^3 + 2x^2 - 5x$

13. $f(x) = x^2 - \frac{54}{x}$

14. $f(x) = \frac{1}{4}x^4 - \frac{5}{36}x^3 - \frac{1}{16}x^2 + \frac{1}{24}x$

15. $f(x) = (4x^2 + 3x)e^{2x}$

16. $f(x) = x\ln x$

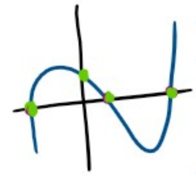
17. $f(x) = x^2(x^2 - 4)$

18. $f(x) = x^3 - 4x$

19. $f(x) = |x - 3|$

20. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$

21. $f(x) = \frac{x^2}{x-2}$



$f(x) = x^4 - 6x^3 + 11x^2 - 6x$

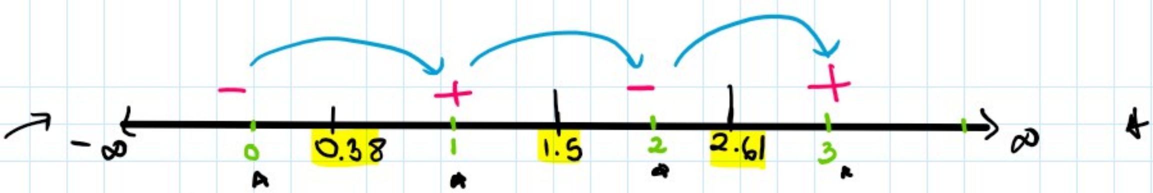
$f'(x) = 4x^3 - 18x^2 + 22x - 6$

1) ✓
2) N
3 N

$4x^3 - 18x^2 + 22x - 6 = 0$

$x_1 = 0.38 \qquad x_2 = 2.61 \qquad x_3 = 1.5.$

a) $PC = \{0.38, 2.61, 1.5\}$



$f'(0) = -$

$f'(1) = +$

$f'(2) = -$

$f'(3) = +$

b) $(0.38, -0.99)$ Min $\qquad f(0.38) =$

$(1.5, \ 0.56)$ Max $\qquad f(1.5)$

$(2.61, -0.99)$ Min $\qquad f(2.61)$

c) No se tiene puntos de inflexión.

d) $(-\infty, 0.38)$ Decrece $\qquad (2.61, \infty)$ crece.

$(0.38, 1.5)$ crece

$(1.5, 2.61$ Decrece

e) Int con eje 'x' reemplazar $y=0$ $f(x)=0$

$f(x) = x^4 - 6x^3 + 11x^2 - 6x$

$0 = x^4 - 6x^3 + 11x^2 - 6x$

$0 = x(x^3 - 6x^2 + 11x - 6)$

$x = 0 \qquad x^3 - 6x^2 + 11x - 6 = 0$

$0 = x\,(x^3 - 6x^2 + 11x - 6)$

$x=0$ $\qquad x^3 - 6x^2 + 11x - 6 = 0$

$x_1 = 1 \qquad x_2 = 3 \qquad x_3 = 2$

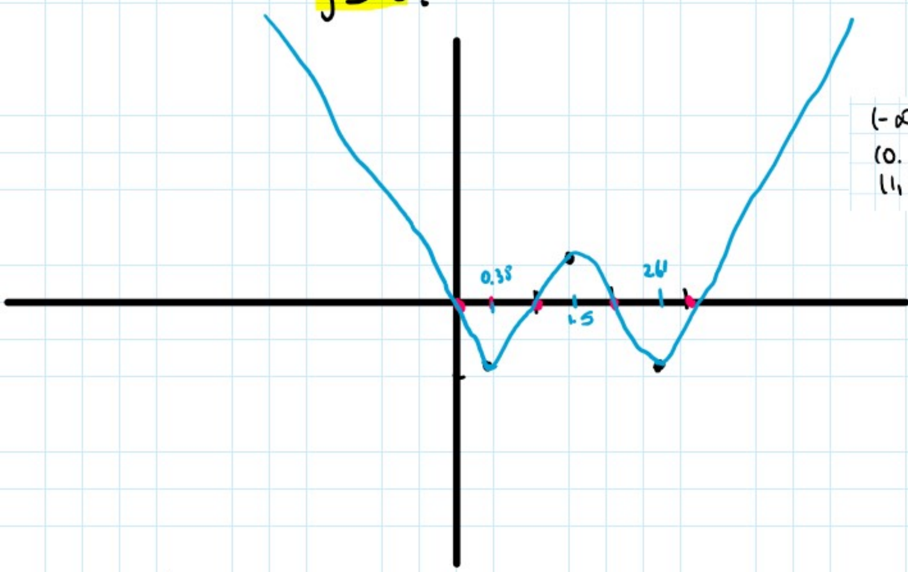
Int con eje 'y' reemplazar $x=0$

$f(x) = x^4 - 6x^3 + 11x^2 - 6x$

$y = 0.$

(0.38, -0.99) Min
(1.5, 0.56) Max
(2.61, -0.99) Min

(-∞, 0.38) Decrece
(0.38, 1.5) Crece
(1.5, 2.61) Decrece
(2.61, ∞) Crece



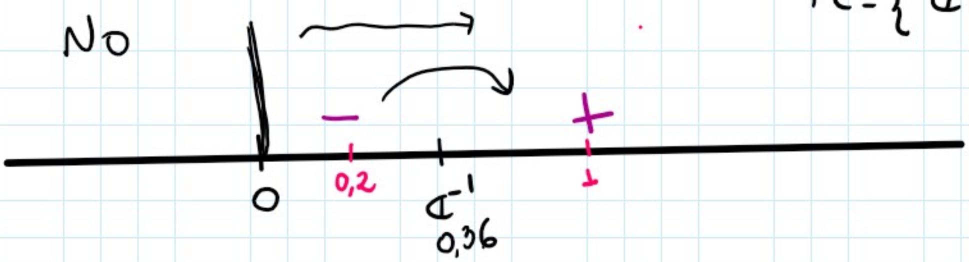
$f(x) = x \ln x$ $\qquad$ Df: $x > 0.$

$f'(x) = \ln x + x \cdot \frac{1}{x}$ $\qquad \ln x + 1 = 0$ $\qquad e^{-1} \approx 0{,}36$

$f'(x) = \ln x + 1$ $\qquad \ln x = -1 \qquad e^{-1} = x$

$PC = \{ e^{-1} \}$

No



$f'(0{,}2) = -$

$f'(1) = +$

b) $(e^{-1}; -e^{-1})$ minimo $\qquad f(e^{-1}) = -e^{-1} \approx -0.37$

c) No Existe P.I

d) $(0, e^{-1})$ Decrece
(e^{-1}, ∞) Crece

$f(x) = x \ln x$

$f(e^{-1}) = e^{-1} \ln e^{-1}$

$= -e^{-1} \ln e$

e) Int con eje 'x' $y=0$

(e^{-1}, ∞) Crece

$f(e^{-1}) = e^{-1}\ln e^{-1}$

$= -e^{-1}\ln e$

$= -e^{-1}$

d) Int con eje 'x' $y=0$

$\rightarrow f(x) = x\ln x$

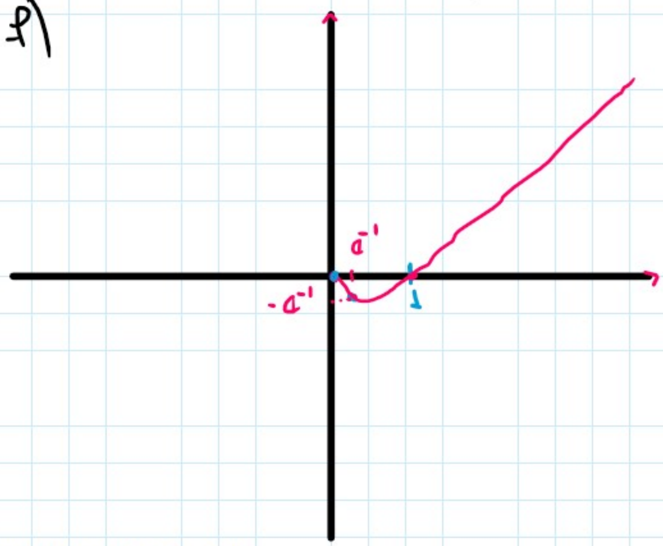
$0 = x\ln x$

$x=0$ $\quad \ln x = 0 \quad e^0 = x \Rightarrow x=1$

Int con eje 'y' $x=0$

No hay int. con y.

f)



$(e^{-1}, -e^{-1})$ minimo

0,36 -0.36

$(0, e^{-1})$ Decrece

(e^{-1}, ∞) Crece

$x=0$

intervalos de crecimiento y de...

$f(x) = x^4 - 6x^3 + 11x^2 - 6x$

$f'(x) = 4x^3 - 18x^2 + 22x - 6 = 0$ $\quad PC = \{0.38, 2.61, 1.5\}$

$+ > 0$

$- < 0$

f'

$f''(x) = 12x^2 - 36x + 22$

$f''(0.38) = +$ $\quad f''(2.61) = +$ $\quad f''(1.5) = -$

$\Rightarrow (0.38, \quad)$ minimo

$(2.61, \quad)$ minimo

$(1.5, \quad)$ Maximo

$f''(3) = 0$

3 P.I

$f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

$PC = \{1,38, 3.61, 2.5\}$

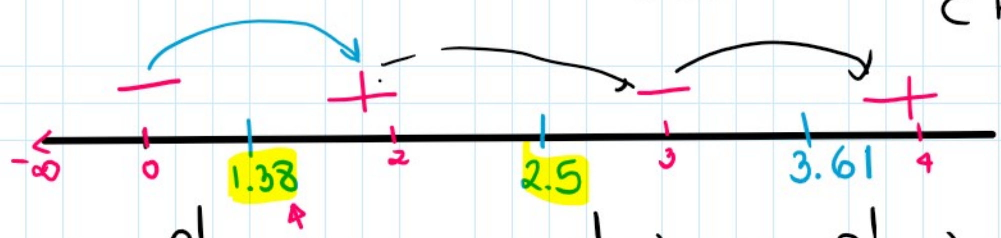
$f'(x) = 4x^3 - 30x^2 + 70x - 50 = 0$

f

C Pri. Der

$f'(x) = 4x^3 - 30x^2 + 70x - 50 = 0$

C Pri. Der



$f'(0) = -$ $\quad f'(2) = +$ $\quad f'(3) = -$ $\quad f'(4) = +$

(1,38,) Min

(2,5,) Max

(3.61,) Min

Criterio de la Primera derivada.

$f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

$PC = \{1{,}38,\ 3.61,\ 2.5\}$

$f'(x) = 4x^3 - 30x^2 + 70x - 50 = 0$

C Pri. Der

$f''(x) = 12x^2 - 60x + 70$

$f''(1.38) = + \Rightarrow (1.38, \quad)$ minimo

$f''(3.61) = + \Rightarrow (3.61, \quad)$ minimo

$f''(2.5) = - \Rightarrow (2.5, \quad)$ Maximo

Criterio de la Seg. Derivada.