Clase 07/10

jueves, 7 de octubre de 2021 04:52 p. m.

2) $f(x,y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$ ←

$f_x(x,y) = 20xy - 10x - 4x^3$ $f_y(x,y) = 10x^2 - 8y - 8y^3$

$$\begin{cases} 20xy - 10x - 4x^3 = 0 \Rightarrow x(20y - 10 - 4x^2 = 0 \\ 10x^2 - 8y - 8y^3 = 0 \leftarrow \end{cases}$$

$x = 0$ (1) $\quad 20y - 10 - 4x^2 = 0$ (2)

si $x = 0$

$-8y - 8y^3 = 0$

$-8y(1 + y^2) = 0$

$y = 0$ $\quad \cancel{1 + y^2 = 0}$

$(0,0)$

si $20y - 10 - 4x^2 = 0$

$\frac{20y - 10}{4} = x^2$ $\quad x^2 = \frac{10y - 5}{2}$

$10x^2 - 8y - 8y^3 = 0$

$\cancel{10}^{5}\left(\frac{10y - 5}{\cancel{2}_1}\right) - 8y - 8y^3 = 0$

$50y - 25 - 8y - 8y^3 = 0$

$0 = 8y^3 - 42y + 25$.

$\cancel{y_1 = -2.59}$ $\quad y_2 = 1.9$ $\quad y_3 = 0.65$

si $y_1 = -2.59$

$\cancel{x^2 = \frac{10(-2.59) - 5}{2}}$

si $y = 1.9$

$x^2 = \frac{10(1.9) - 5}{2}$. $\quad x^2 = 7 \Rightarrow x = \pm\sqrt{7}$

$(\sqrt{7}, 1.9)(-\sqrt{7}, 1.9)$.

Si $y = 0.65$

$x^2 = \frac{10(0.65)-5}{2}$ $x^2 = \frac{3}{4}$ $x = \pm\frac{\sqrt{3}}{2}$

$\left(\frac{\sqrt{3}}{2}, 0.65\right)\left(-\frac{\sqrt{3}}{2}, 0.65\right)$

$PC = \left\{(0,0)\left(\frac{\sqrt{3}}{2}, 0.65\right)\left(-\frac{\sqrt{3}}{2}, 0.65\right)\left(\sqrt{7}, 1.9\right)\left(-\sqrt{7}, 1.9\right)\right\}$

Example *Hallar el valor mínimo de* $f(x,y,z) = 2x^2 + y^2 + 3z^2$ *sujeto a restricción o ligadura* $2x - 3y - 4z = 49.$

$f(x,y,z) = 2x^2+y^2+3z^2$ fun. Obj.

$g(x,y,z) = 2x-3y-4z = 49$ func R. o Lig.

$\nabla f(x,y,z) = \langle 4x, 2y, 6z \rangle$

$\nabla g(x,y,z) = \langle 2, -3, -4 \rangle$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = C. \end{cases}$$

$$\begin{cases} 4x = \lambda 2 & (1) \\ 2y = \lambda(-3) & (2) \\ 6z = \lambda(-4) & (3) \\ 2x-3y-4z = 49 & (4) \end{cases}$$

λ ?

$\frac{(1)}{(2)}$

$\frac{4x}{2y} = \frac{2\cancel{\lambda}}{-3\cancel{\lambda}}$ $4x = \frac{2 \cdot 2y}{-3}$

$\boxed{x = -\frac{y}{3}}$

$\frac{(2)}{(3)}$

$\frac{2y}{6z} = \frac{-3\cancel{\lambda}}{-4\cancel{\lambda}}$

$2y = \frac{3 \cdot 6z}{4}$

$$2y = \frac{3 \cdot 6 z}{4}$$

$$y = \frac{9}{4} z \Rightarrow \boxed{z = \frac{4}{9} y}$$

$$2x - 3y - 4z = 49$$

$$2\left(-\frac{y}{3}\right) - 3y - 4\left(\frac{4}{9} y\right) = 49$$

$$-\frac{2}{3} y - 3y - \frac{16}{9} y = 49 \quad * 9 \qquad 8$$

$$\begin{matrix} 22 \\ 27 \end{matrix}$$

$$-6y - 27y - 16y = 441$$

$$-\cancel{49} y = \cancel{49} \cdot 9$$

$$\boxed{y = -9} \qquad z = \frac{4}{9}(-9) \qquad \boxed{z = -4}$$

$$x = -\frac{y}{3}$$

$$x = -\left(\frac{-9}{3}\right) \Rightarrow \boxed{x = 3}$$

$x = 3 \quad y = -9 \quad z = -4$ $\qquad$ $x = 2 \quad y = 5 \quad z = 3$

$f(3, -9, -4) = 10$ $\qquad$ $f(2, 5, 3) = 15$

Example *La función de producción de Cobb-Douglas para un fabricante de software está dada por $f(x,y) = 100x^{\frac{3}{4}}y^{\frac{3}{4}}$ donde x representa las unidades de trabajo (a \$150 por unidad) y y representa las unidades de capital (a \$250 por unidad). El costo total de trabajo y capital está limitado a \$50000. Hallar el nivel máximo de producción de este fabricante.*

$$f(x,y) = 100x^{3/4} y^{3/4} \leftarrow \text{func Obj}$$

x: Un. de trabajo $\quad$ 150 \$ c.u

y: Un. de Capital $\quad$ 250 \$ c.u.

$$\rightarrow g(x,y) = 150x + 250y = 50000 \qquad \text{Res. Lig, Cond:}$$

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \end{cases} \qquad \begin{cases} \frac{3}{4} 100 x^{-1/4} y^{3/4} = \lambda 150 & (1) \\ \frac{3}{4} \cdot 100 x^{3/4} y^{-1/4} = \lambda 250 & (2) \end{cases}$$

$$\begin{cases} f_y = \lambda g_y \\ g(x,y) = 5000 \end{cases} \qquad \begin{cases} \frac{3}{4} \cdot 100 x^{3/4} y^{-1/4} = \lambda 250 \quad (2) \\ 150x + 250y = 50000 \end{cases}$$

$$\frac{\cancel{\frac{3}{4}}\,\cancel{100}\, x^{-1/4} y^{3/4}}{\cancel{\frac{3}{4}}\,\cancel{100}\, x^{3/4} y^{-1/4}} = \frac{\cancel{\lambda} 1\cancel{50}^{\,3}}{\cancel{\lambda} 2\cancel{50}_{\,5}}$$

$-\frac{1}{4} - \frac{3}{4}$

$\frac{3}{4} - \left(-\frac{1}{4}\right) = 1$

$$x^{-1} y = \frac{3}{5} \Rightarrow \frac{y}{x} = \frac{3}{5} \Rightarrow \boxed{y = \frac{3}{5} x}$$

$$150x + \cancel{250}^{\,50} \cdot \frac{3}{\cancel{5}} x = 50000$$

$$150x + 150x = 50000$$

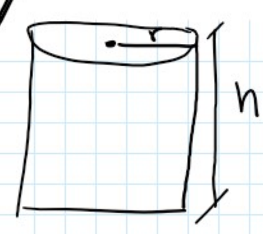
$$300x = 50000$$

$$x = \frac{5000\cancel{0}}{30\cancel{0}} \qquad x = \frac{500}{3} \approx 166.6 \approx 167$$

$$y = \frac{\cancel{3}}{\cancel{5}} \cdot \frac{\cancel{500}^{\,100}}{\cancel{3}} \qquad y = 100.$$

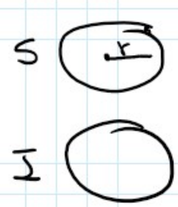
87. Utilizar multiplicadores de Lagrange para encontrar las dimensiones de un cilindro circular recto con volumen de V_0 unidades cúbicas y superficie mínima.

RESP: $r = \sqrt[3]{\frac{V_0}{2\pi}} \quad h = 2\sqrt[3]{\frac{V_0}{2\pi}}$



$A_S = 2A_B + A_L$ fun Ob.

$V_0 =$ fun Rest.



$A = \pi r^2$

$A_L = 2\pi r h$.

$V_0 = cte$

$$A_s = 2\pi r^2 + 2\pi r h \quad \text{fun. Ob.}$$

$A_s = 2\pi r + 2\pi r h$ fun. ob.

$g = \pi r^2 h = V_0 \Rightarrow h$ fun rest.

$$\begin{cases} 4\cancel{\pi} r + 2\cancel{\pi} h = \lambda\, 2\cancel{\pi} r h & (1) \\ 2\cancel{\pi}\cancel{r} = \lambda \cancel{\pi}\cancel{r}\, r & (2) \Rightarrow 2 = \lambda r \Rightarrow \frac{2}{r} = \lambda \\ \pi r^2 h = V_0 & (3) \end{cases}$$

$4r + 2h = \lambda\, 2 r h \quad \div 2$

(1) $2r + h = \lambda r h$

$2r + h = \frac{2}{\cancel{r}} \cancel{r} h$

$2r = h$

$\pi r^2 \cdot 2r = V_0$

$2\pi r^3 = V_0 \Rightarrow r^3 = \frac{V_0}{2\pi}$

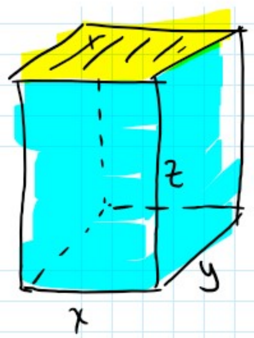
$r = \sqrt[3]{\frac{V_0}{2\pi}}$

$h = 2\sqrt[3]{\frac{V_0}{2\pi}}$

92. Determine cuales deben ser las dimensiones de un envase para leche de caras rectangulares y volumen de $512 cm^3$ y costo mínimo, si el material de los lados de la caja cuestan 10 \$ el centímetro cuadrado y el material de la tapa y el fondo cuestan 20 \$ el centímetro cuadrado. Hállese también el costo mínimo.

$C = 40xy + 40xz$

$g = xyz = 512$



$C_L = 10 \cdot (2xz + 2yz)$

$C_t = 20(2xy)$

$C_t = 20xz + 20yz + 40xy$ Obj.

$g = xyz = 512$

$y = \frac{512}{xz}$

$$C_T = 20xz + 20 \cdot \frac{512}{x\cancel{z}}\cancel{z} + 40x \cdot \frac{512}{xz}$$

$$C_T = 20xz + \frac{10240}{x} + \frac{20480}{z}$$

$$C_x = 20z - \frac{10240}{x^2}$$

$$C_z = 20x - \frac{20480}{z^2}$$

$$\begin{cases} 20z - \dfrac{10240}{x^2} = 0 \\ 20x - \dfrac{20480}{z^2} = 0 \end{cases}$$

$$\begin{cases} z - \dfrac{512}{x^2} = 0 \\ x - \dfrac{1024}{z^2} = 0 \end{cases} \Rightarrow z = \frac{512}{x^2}$$

$$x - \frac{1024}{\left(\frac{512}{x^2}\right)^2} = 0$$

$$x - \frac{1024x^4}{262144} = 0$$

$$262144x - 1024x^4 = 0$$

$$1024x(256 - x^3) = 0$$

$$\cancel{1024x = 0} \qquad x^3 = 256. \qquad x = 16$$

$$z = \frac{512}{x^2}$$

$$z = \frac{512}{16^2}$$

$$z = \frac{512}{16^2}$$

$PC = \{ (16, 2) \}$ ← Valor minimo

$$z = 2$$

$$y = \frac{512}{xz} = \frac{512}{16.2} = 16$$