Practica 2

Parte 5

40: Hallar las diferenciales de orden superior indicada

$$f(x,y) = e^x \cos y \qquad (d^3 f)$$

$$(df)^3 = \left(\frac{df}{dx} \cdot dy + \frac{df}{dy} dy\right)^3$$

$$d^3 f = \frac{d^3 f}{dx^3} dx^3 + \frac{3d^3 f dx^3 dy}{dx\,dy} + \frac{3d^3 f dx\,dy^2}{dx\,dy^2} + \frac{d^3 f\,dy^3}{dy^3}$$

$$\frac{df}{dx} = \cos y\, e^x \quad \frac{df}{dx\,dy} = -e^x \operatorname{sen} y \quad \frac{d^3 f}{dx\,dy^2} = -e^x \cos y$$

$$\frac{d^2 f}{dx^2} = \cos y\, e^x \Rightarrow \frac{d^3 f}{dx^2 dy} = -e^x \operatorname{sen} y$$

$$\frac{d^3 f}{dx^3} = \cos y\, e^x$$

$$\frac{df}{dy} = -e^x \operatorname{sen} y \quad \frac{d^2 f}{dy^2} = -e^x \cos y \quad \frac{d^3 f}{dy^3} = e^x \operatorname{sen} y$$

$$d^3 f = e^x \cos y\, dx + 3(-e^x \operatorname{sen} y)\, dx^2 dy + 3(-e^x \cos y)\, dx\, dy^2 + e^x \operatorname{sen} y\, dy^3$$

$$d^3 f = e^x(\cos y\, dx^3 - 3\operatorname{sen} y\, dx^3 dy - 3\cos y\, dx\,dy^2 + \operatorname{sen} y\, dy^3)$$

41: Determinar si el diferencial es exacto:

$$\underbrace{x \cos y}_{M}\, dx + \underbrace{x \operatorname{sen} y}_{N}\, dy$$

$$\frac{dM}{dy} = -x \operatorname{sen} y \qquad \frac{dN}{dy} = \operatorname{sen} y$$

$$\frac{dM}{dy} \neq \frac{dN}{dy} \qquad \text{No es exacto.}$$

42. Determinar si el diferencial es exacto

$$\underbrace{2xy\ \text{sen}(x^2 y)}_{M}\ dx + \underbrace{x^2\ \text{sen}(x^2 y)}_{N}\ dy$$

$\frac{dM}{dy} = (2x)\ \text{sen}(x^2 y) + (2xy)\, x^2 \cos(x^2 y)$

$\frac{dN}{dy} = 2x\ \text{sen}(x^2 y) + (2xy)\, x^2 \cos(x^2 y)$

$\frac{dM}{dy} = \frac{dN}{dy}$ es exacto.

43. Demostrar la relación entre derivadas parciales:

$f = f(xy)$

$x f_{xx} + f_x = y f_{xy}$

$f_x = y$

$y = y(1)$

$f_{xx} = 0$

$y = y$

$f_{xy} = 1$

44. Demostrar la relación entre derivadas parciales:

$f = f(xy)^2$

$y f_{xy} = 2f_x + 2x f_{xx}$

$f = y^2$

$y(2y) = 2(y^2)$

$f_{xx} = 0$

$2y^2 = 2y^2$

$f_{xy} = 2y$

45. Demostrar la relación entre derivadas parciales

$z = yf(x^2 - y^2)$

$\frac{z_x}{x} + \frac{z_y}{y} = \frac{z}{y^2}$

$z_x = y \cdot zx$

$\frac{z_{xy}}{x} + \frac{x^2 - 3y^2}{y} = \frac{yf(x^2 - y^2)}{y^2}$

$z_y = 1(x^2 - y^2) + y(-2y)$

$= x^2 - y^2 - 2y^2$

$\frac{2xy^2 + (x^3 - 3xy^2)}{xy} = \frac{f(x^2 - y^2)}{y}$

$= x^2 - 3y^2$

$$\frac{2xy^3+x^3-3xy^2}{xy}=\frac{f(x^2-y^2)}{y}$$

$$\frac{x(2y^2+x^2-3y^2)}{xy}=\frac{f(x^2-y^2)}{y}$$

$$\frac{(x^2-y^2)}{y}=\frac{(x^2-y^2)}{y}$$

46: Demostrar la relación entre derivadas parciales:

$$z=\frac{1}{x}\left(f(x-y)+g(x-ay)\right)$$

- $z_x=-\frac{1}{x^2}\left(f(x-y)+g(x-ay)\right)+\frac{1}{x}(-y+1)$

$$z_x=-\frac{1}{x^2}\left(f(x-y)+g(x-ay)\right)-\left(\frac{y+1}{x}\right)$$

- $z_{xx}=\frac{2}{x^3}\left(f(x-y)+g(x-ay)\right)-\frac{1}{x^2}(-y+1)-\left(-\frac{y+1}{x^2}\right)$

$$z_{xx}=\frac{2}{x^3}\left(f(x-y)+g(x-ay)\right)-\left(-\frac{y+1}{x^2}\right)+\left(\frac{y+1}{x^2}\right)$$

- $z_y=\frac{1}{x}(-1-a)$ • $z_{yy}=0$

$$a^2 z_{xx}=z_{yy}$$

$$a^2\left(\frac{2}{x^3}\left(f(x-y)+g(x-ay)\right)+\left(-\frac{y+1}{x^2}+\frac{y+1}{x^2}\right)\right)=0$$

$$a^2\left(\frac{2x^3-2x^2y+2x^3-2x^2ay+x^3y-x^2+x^3y-x^3}{x^3\,x^2}\right)=0$$

$$a^2\left(\frac{-2y+2x-2ay+2xy}{x^3}\right)$$

$$\frac{-2a^2y+2a^2x-2a^3y+2a^2xy}{x^3}=0$$

//

47.- Probar que la función $z = \arctan\left(\frac{-y}{x}\right)$ satisface la ecuación de laplace $z_{xx} + z_{yy} = 0$

- $z_x = \dfrac{\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2} = \dfrac{\frac{-y}{x^2}}{\frac{x^2+y^2}{x^2}} = \dfrac{-y}{x^2+y^2}$

$z_{xx} = \dfrac{2xy}{(x^2+y^2)^2}$

- $z_y = \dfrac{\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2} = \dfrac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \dfrac{\frac{1}{x}}{\frac{x^2+y^2}{x^2}} = \dfrac{x}{x^2+y^2}$

$z_{yy} = \dfrac{-2xy}{(x^2+y^2)^2}$ $\qquad \therefore \dfrac{2xy}{(x^2+y^2)} - \dfrac{2xy}{(x^2+y^2)^2} = 0$

48.- Demostrar la relación:

$u(x,y) = \cos(x+2y) + \ln(x-2y)$

- $U_x = -\operatorname{Sen}(x+2y)(1) + \dfrac{1}{x-2y}(1)$

$U_x = -\operatorname{Sen}(x+2y) + \dfrac{1}{x-2y}$

$U_{xx} = -\cos(x+2y) - \dfrac{1}{(x-2y)^2}$

- $U_y = -\operatorname{Sen}(x+2y)(2) + \dfrac{1}{x-2y}(-2)$

$U_y = -2\operatorname{Sen}(x+2y) - \dfrac{2}{x-2y}$

$U_{yy} = -2\cos(x+2y)(2) - \dfrac{4}{(x-2y)^2}$

$= -4\cos(x+2y) - \dfrac{4}{(x-2y)^2}$

$$4u_{xx} = u_{yy}$$

$$4\left(-\cos(x+2y) - \frac{1}{(x-2y)^2}\right) = 4\cos(x+2y) - \frac{4}{(x-2y)^2}$$

$$-4\cos(x+2y) - \frac{4}{(x-2y)^2} = -4\cos(x+2y) - \frac{4}{(x-2y)^2} \quad //$$

49.- Si $z = y + f(x^2 - y^2)$ donde f es diferenciable, demuestre que $y\frac{dz}{dx} + x\frac{dz}{dy} = x$

$$\frac{dz}{dx} = 2x$$

$$\frac{dz}{dy} = 1 + (-2y)$$

$$\frac{dz}{dy} = 1 - 2y$$

$$y(2x) + x(1-2y) = x$$

$$2xy + x - 2xy = x$$

$$x = x \quad //$$

50.- Demostrar la relación entre derivadas parciales:

$$z = [f(x+ay) + g(x-ay)] \qquad a^2 z_{xx} = z_{yy}$$

- $z_x = 1 + 1 = 2 \qquad z_{xx} = 0 \qquad a^2(0) = 0$
- $z_y = a - a = 0 \qquad z_{yy} = 0 \qquad 0 = 0$

51.- Encontrar la diferencial dw

a) $w = x^3 - x^2y + 3y^2$

$$w_{(x,y)} = x^3 - x^2y + 3y^2$$

$$\frac{dw}{dx} = 3x^2 - y \cdot 2x + 0 = 3x^2 - 2xy$$

$$\frac{dw}{dy} = 0 - x^2 \cdot 1 + 3 \cdot 2y = -x^2 + 6y = 6y - x^2$$

b) $\omega = x^2 \sin y + 2y^{3/2}$

$\frac{dw}{dx} = 2x \cdot \sin y + 0 = 2x \sin y$

$\frac{dw}{dy} = x^2 \cos y + 2 \cdot \frac{3}{2} y^{1/2} = x^2 \cos y + 3y^{1/2}$

$= x^2 \cos y + \sqrt{y}$

c) $\omega = x^2 \ \ln(y^2 + z^2)$

$\frac{dw}{x} = 2x \ln(y^2 + z^2)$

$\frac{dw}{y} = x^2 \cdot \frac{2y}{y^2+z^2} = \frac{2x^2 y}{y^2+z^2}$

$\frac{dw}{z} = x^2 \cdot \frac{2z}{y^2+z^2} = \frac{2x^2 z}{y^2+z^2}$

$dw = 2x \ln(y^2+z^2)\, dx + \left(\frac{2x^2 y}{y^2+z^2}\right) dy + \left(\frac{2x^2 z}{y^2+z^2}\right) dz$

52: Use la regla de la cadena para encontrar $\frac{dw}{dr}$; $\frac{dw}{d\theta}$

a) $w = \frac{yz}{x}$ $\quad x = \theta^2 \quad y = r + \theta \quad z = r - \theta$

$\frac{dw}{dr} = \frac{dw}{dx} \cdot \frac{dx}{dr} + \frac{dw}{dy} \cdot \frac{dy}{dr} + \frac{dw}{dz} \cdot \frac{dz}{dr}$

$\frac{dw}{dr} = yz \cdot 0 + \frac{z}{x} \cdot 1 + \frac{y}{x} \cdot 1$

$\frac{dw}{dr} = \frac{z}{x} + \frac{y}{x} = \frac{z+y}{x} = \frac{r - \cancel{\theta} + r + \cancel{\theta}}{\theta^2}$ $\quad \boxed{\frac{dw}{dr} \Rightarrow \frac{2r}{\theta^2}}$ //

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b) $\omega = \arctan \frac{y}{x}$ $\quad x = r\cos\theta \quad y = r\sin\theta$

$\frac{dw}{dr} = \frac{r\theta \operatorname{sen}\theta}{r\cos\theta}$ $\quad \frac{dw}{d\theta} = \tan^{-1}\left(\frac{\operatorname{sen}\theta}{\cos\theta}\right)$

$\boxed{\frac{dw}{dr} = \theta}$

$$\frac{dw}{d\theta} = \frac{1}{1+\left(\frac{\operatorname{sen}\theta}{\cos\theta}\right)^2} \cdot \left(\frac{\cos\theta \cdot \cos\theta - \operatorname{sen}\theta(-\operatorname{sen}\theta)}{\cos^2\theta}\right)$$

$$\frac{dw}{d\theta} = \frac{1/1}{\frac{\cos^2\theta + \operatorname{sen}^2\theta}{\cos^2\theta}} \cdot \left(\frac{\cos^2\theta + \operatorname{sen}^2\theta}{\cos^2\theta}\right)$$

$$\frac{dw}{d\theta} = \cos^2\theta \cdot \frac{1}{\cos^2\theta} \Rightarrow \frac{dw}{d\theta} = 1$$

59.- Hallar dy/dx por derivación implícita

a) $\ln\sqrt{x^2+y^2} + x + y = 4$

$$\frac{d}{dx}\left(\ln(\sqrt{x^2+y^2}) + x + y\right) = \frac{d}{dx}$$

$$\frac{d}{dx}\left(\ln(\sqrt{x^2+y^2})\right) + \frac{d}{dx}(x) + \frac{d}{dx}$$

$$\frac{1}{u} \frac{d}{}\left(\ln(u)\right) \frac{d}{dx}\left(\sqrt{x^2+y}\right)$$

$$= \frac{1}{u} \frac{d}{dx}\left(\sqrt{x^2+y^2}\right)$$

$$= \frac{1}{\sqrt{x^2+y^2}} \frac{d}{dx}\left(\sqrt{x^2+y^2}\right)$$

$$= \frac{d}{du}(\sqrt{u}) \frac{d}{dx}(x^2+y^2)$$

$$= \frac{d}{du}(u^{1/2}) = \frac{1}{2} u^{1/2 - 1} = \frac{1}{2\sqrt{u}}$$

$$= \frac{1}{2\sqrt{u}} \frac{d}{dx}(x^2+y^2)$$

54.- Encontrar el gradiente de f en el punto indicado

a) $f(x,y) = \sqrt{x^2+y^2}$ $P(-4,3)$ $f_x(P) = -4/5$

$f_y(P) = 3/5$

$f = (x^2+y^2)^{1/2}$

$f_x = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f_x(P) = \frac{-4}{\sqrt{(-4)^2+3^2}} = -\frac{4}{\sqrt{25}} = -\frac{4}{5}$

$f_y = \frac{1}{2}(x^2+y^2)^{-1/2} \, 2y = \frac{y}{\sqrt{x^2+y^2}} = \frac{3}{\sqrt{(-4)^2+3^2}} = \frac{3}{5}$

b) $f(x,y) = e^{3x}\tan y$ $P\left(0, \frac{\pi}{4}\right)$

$f_x = 3e^{3x}\tan y$

$f_x(P) = 3e^{3(0)} + \tan(\pi/4) = 3 \cdot 1 = 3$

$f_y = e^{3y}\sec^2 y$

c) $f(x,y,z) = yz^3 - 2x^2$ $P(2,-3,1)$

$P = -8, 1, -9$

$f_x = -4x$

$f_x(P) = -4(2) = -8$

$f_y = z^3 = (1)^3 = 1$

$f_z = y\,3z^2 = 3(-3 \cdot 1^2) = -9$

55.- Calcular la derivada direccional de f en el punto indicado

a) $f(x,y) = e^x \operatorname{sen} y$ $u = -i$ $P\left(1, \frac{\pi}{2}\right)$

• $f_x = e^x \operatorname{sen} y$

$f_x(P) = e^1 \operatorname{sen}(\pi/2) = e \cdot 1 = e$

• $f_y = e^x \cos y$

$f_y(P) = e^1 \cos(\pi/2) = e \cdot 0 = 0$ $(e) \cdot -1 \Rightarrow -e$

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b) $f(x,y) = \sqrt{x^2+y^2}$ $\quad u = 3i - 4j \quad$ $P(3,4)$

$\bullet\ f_x = \frac{1}{2} \cdot (x^2+y^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}} = \frac{3}{\sqrt{3^2+(4)^2}} = \frac{3}{5}$

$\bullet\ f_y = \frac{1}{2} \cdot (x^2+y^2)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}} = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$

$3\left(\frac{3}{5}\right)i - 4\left(\frac{4}{5}\right)j = \frac{7}{5}$ //

56. Hallar el gradiente de la función y el valor máximo de la derivada direccional en el punto dado.

a) $f(x,y) = y e^{-x}$ $\quad P(0,5)$

$f_{(x,y)} = \langle -e^{x} y,\ e^{-x} \rangle$

$= e^{-x}(-y, 1) = e^{-x}(-yi + 1j)$

$\boxed{f_{(0,5)} = \sqrt{26}}$

b) $f(x,y,z) = xy^2z^2$ $\quad P(2,1,1)$

$\nabla f_{(x,y,z)} = (z^2y^2,\ 2yxz^2,\ 2zxy^2)$

$= yz\,(yzi + 2xzj + 2xyk)$

$\nabla f(x,y,z) = \sqrt{33}$

57.- En forma directa usando la gradiente, calcular la derivada direccional, dirección $\vec{u}$

$f(x,y) = e^{xy} + 5 \ln y$ $\qquad u = \frac{\langle 4, 8 \rangle}{10}$

$\nabla = y e^{xy}, \quad x e^{xy} + \frac{5}{y}$

$df = y e^{xy} \cdot \frac{2}{5} + \left(x e^{xy} + \frac{5}{y}\right) \cdot \frac{4}{5}$ $\qquad$ m.c.m 5

$df = \frac{e^{xy} \left(2y + 4x + \frac{5}{y}\right)}{5}$

$df = \frac{e^{xy} (3y + 4x) + \frac{4}{y})}{5}$

58.- En forma directa usando la gradiente, calcular la derivada direccional, dirección $\vec{u}$

$f = 6x^2 - 3yz$ $\qquad u = \frac{\langle 2, 2, 1 \rangle}{3}$

$\nabla f(x,y,z) = \langle 12x, -3z, -3y \rangle$

$D_u f_{(x,y,z)} = \langle 12x, -3z, -3y \rangle \cdot \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

$= 8x - 2z - y$ //

59.- Hallar una ecuación del plano tangente y hallar ecuaciones simétricas para la recta normal a la superficie en el punto dado

a) $z = x^2 - y^2$ $(3,2,5)$

$\nabla f(x,y,z) = \langle 2x, -2y, -1 \rangle$

$\nabla f(3,2,5) = \langle 6, 4, -1 \rangle = \vec{n}$ $P(3,2,5)$

$6(x-3) - 4(y-2) - 1(z-5) = 0$

$6x - 4y - z - 5 = 0 \longrightarrow$ Ec. plano tang.

$\left.\begin{array}{l} x = 3 + 6t \\ y = 2 - 4t \\ z = 5 - t \end{array}\right\}$ Ec. Recta Normal $\quad \dfrac{x-3}{6} = \dfrac{y-2}{4} = \dfrac{z-5}{-1}$ //

b) $xyz = 10$ $(1,2,5)$

$\nabla f(x,y,z) = xyz - 10$

$\nabla f(xyz) = \langle yz, xz, xy \rangle$

$\nabla f(1,2,5) = \langle 10, 5, 2 \rangle$

$10(x-1) + 5(y-2) + (2(z-5)$

$10x + 5y + 2z - 30 = 0 \Rightarrow 10x + 5y + 2z = 30$ Ec. Plano

$\left.\begin{array}{l} x = 1 + 10t \\ y = 2 + 5t \\ z = 5 + 2t \end{array}\right\}$ Ec Recta Normal $\quad \dfrac{x-1}{10} = \dfrac{y-2}{5} = \dfrac{z-5}{2}$ //

c) $z = \arctan \dfrac{y}{x}$ $\left(1, 1, \dfrac{\pi}{4}\right)$

$f(x,y,z) = \arctan \dfrac{y}{x} - z$

$\nabla f(x,y,z) = \left\langle \dfrac{y}{x^2+y^2}, \dfrac{-x}{x^2+y^2}, -1 \right\rangle$

$\nabla f\left(1,1,\frac{\pi}{4}\right)\left\langle \frac{1}{2}, \frac{-1}{2}, -1\right\rangle$

$\frac{1}{2}(x-1)-\frac{1}{2}(y-1)-1\left(z-\frac{\pi}{4}\right)$

$\frac{1}{2}x-\frac{1}{2}y-z+\frac{\pi}{4}=0 \;.\;(2)$

$x-y-2z+\frac{\pi}{2}=0 \implies x+y-2z=\frac{\pi}{2}$

$\cdot\left\langle \frac{1}{2}, \frac{-1}{2}, -1\right\rangle \cdot (2)$

$\left.\begin{array}{l} x=1+t \\ y=1-t \\ z=\frac{\pi}{4}-2t \end{array}\right\}$ Ec. Recta Normal

$\frac{x-1}{1}=\frac{y-1}{-1}=\frac{z-\pi/4}{-2}$

60.- Hallar la ecuación de los planos tangentes a las superficies dadas por:

$x^2+y^2-z=0$ $\qquad P_0(4,3,2,5)$

$\nabla f(x,y,z)=\langle 2x, 2y, -1\rangle$

$\nabla f_{(4,3,2,5)}=\langle 8, 6, -1\rangle=\vec{n}$

$8(x-4)+6(y-3)+(-1)(z-2)=0$

$8x+6y-z-48=0 \implies 8x+6y-z=48$ //

61.- Determine el plano tangente y la recta normal a las superficies

a) $\sqrt{x^2-2y^3+3}+z^2=12$ en el punto $(2,-1,3)$

$\nabla f_{(x,y,z)}\left\langle \frac{x}{\sqrt{x^2-2y^3+3}}, -\frac{3y^2}{\sqrt{x^2 2y^3+3}}, 2z\right\rangle$

$\nabla f_{(2,-1,3)}=\left\langle \frac{2}{3}, -1, 6\right\rangle$

$$\frac{2}{3}(x-2)+(-1)(y+1)+6(z-3)=61$$

$$2x-3y+18z=61$$

$$\left.\begin{aligned} x &= 2+2t \\ y &= 1-3t \\ z &= 3+18t \end{aligned}\right\} \text{Recta normal}$$

63.- Demostrar que las intersecciones

a) $z=2xy^2 \qquad 8x^2-5y^2-8z=-13 \qquad (1,1,2)$

$\nabla f = 2y^2, 4xy, -1$

$\nabla f\,(1,1,2) = 2, 4, -1 \rightarrow n$

$2(x-1)+4(y-1)-1(z-2)=0$

$2x-2+4y-4-z+2=0$

$2x+4y-z=4$

$0=8x^2-5y^2-8z+13$

$\nabla f = 16x, -10y, -8 \implies \nabla f\,(1,1,2)=16,-10,-8) \neq$ No es

b) $x^2+y^2+z^2+2x-4y-4z-12=0$

$\nabla f = 2x+2;\ 2y-4,\ 2z-4$

$\nabla f\,(1,-2,1) = 4,-8,-2$

$4x^2+y^2+16z^2=2y$

$0=4x^2+y^2+16z^2-26$

$\nabla f = 8x, 2y, 32z$

$\nabla f = (1,-2,1) = 8,-16,-64$

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