Clase 28/09

martes, 28 de septiembre de 2021   04:56 p. m.

... $\sin cx$. Demostrar que $w_{xx} = w_t$ para to...

38. Demostrar la relación

$$u(x,y) = \frac{xy}{x-y} \qquad x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$$

$f(x)$: . . . .

33. Si $f(u,v,w) = 3u^2 - 2v^3 + w^3 \sin v$ ademas $u = x^2 \ln y$; $v = 2xy^3 \sin y$; $w = e^{\frac{x}{y}}$ Hallar $\frac{\partial f}{\partial x}$; $\frac{\partial f}{\partial y}$

$$e^{\frac{x^2}{y}} \cdot e^{\frac{x}{y}}$$

$$\boxed{\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial f}{\partial w}\cdot\frac{\partial w}{\partial x}} \checkmark$$

$$\frac{\partial f}{\partial x} = 6u \cdot 2x \ln y + (-6v + w^3 \cos v)\cdot 2y^3 \operatorname{sen} y + 3w^2 \operatorname{sen} v \cdot \frac{1}{y} e^{\frac{x}{y}} \qquad x, y.$$

$$= 6x^2 \ln y \cdot 2x \ln y + \left[-6 \cdot 2xy^3 \operatorname{sen} y + \left(e^{\frac{x}{y}}\right)^3 \cos(2xy^3 \operatorname{sen} y)\right] 2y^3 \operatorname{sen} y + 3\left(e^{\frac{x}{y}}\right)^2 \operatorname{sen}(2xy^3 \operatorname{sen} y)\,\frac{1}{y} e^{\frac{x}{y}}$$

$$= 12x^3 \ln^2 y - 24xy^3 \operatorname{sen}^2 y + 2y^3 \cos(2xy^3 \operatorname{sen} y) \operatorname{sen} y\, e^{\frac{3x}{y}} + \frac{3}{y} e^{\frac{x^2+x}{y}} \operatorname{sen}(2xy^3 \operatorname{sen} y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} + \frac{\partial f}{\partial w}\cdot\frac{\partial w}{\partial y}$$

33. Si $f(u,v,w) = 3u^2 - 2v^3 + w^3 \sin v$ ademas $u = x^2 \ln y$; $v = 2xy^3 \sin y$; $w = e^{\frac{x}{y}}$ Hallar $\frac{\partial f}{\partial x}$; $\frac{\partial f}{\partial y}$

$$\frac{\partial f}{\partial y} = 6u\left(\frac{x^2}{y}\right) + (-6v^2 + w^3 \cos v)(6xy^2 \operatorname{sen} y + 2xy^3 \cos y) + 3w^2 \operatorname{sen} v \cdot e^{\frac{x}{y}} \cdot \left(-\frac{x}{y^2}\right)$$

29. Hallar las derivadas parciales de orden superior indicadas:

$$f(x,y) = \sin(x^2 + y^4) + \ln(1 + x^2y^4) \qquad f_{xx};\ f_{yy};\ f_{xy}$$

$$f_x(x,y) = \cos(x^2+y^4)\cdot 2x + \frac{1}{1+x^2y^4}\cdot 2xy^4 = 2x\cos(x^2+y^4) + \frac{2xy^4}{1+x^2y^4}$$

$$f_{xx}(x,y) = 2\cos(x^2+y^4) + 2x\left(-\operatorname{sen}(x^2+y^4)\cdot 2x\right) + \frac{2y^4(1+x^2y^4) - 2xy^4(2xy^4)}{(1+x^2y^4)^2}$$

$$= 2\cos(x^2+y^4) - 4x^2 \operatorname{sen}(x^2+y^4) + \frac{2y^4 + 2x^2y^8 - 4x^2y^8}{(1+x^2y^4)^2}$$

$$= 2\cos(x^2+y^4) - 4x^2 \operatorname{san}(x^2+y^4) + \frac{2y^4 - 2x^2y^8}{(1+x^2y^4)^2}$$

a^x

22. Hallar las derivadas parciales f_x; f_y; f_z de la siguiente función

$f(x,y,z) = z^{xy} + x^{y^z} - \cos\pi$

$$f_x(x,y,z) = z^{xy}\ln z\, y + y^z x^{y^z - 1}$$

$$\left(3^{2^x}\right)' = \underbrace{3^{2^x}\ln 3}\, 2^x \ln 2.$$

$$f_y(x,y,z) = z^{xy}\ln z\, x + x^{y^z}\ln x \cdot z\, y^{z-1}$$

$$f_z(x,y,z) = xy\, z^{xy-1} + x^{y^z}\ln x \cdot y^z \ln y$$

$$f(\dot{x},\dot{y},\dot{z}) =$$

Example Si $z(x,y) = e^x \sin y$ donde $x(s,t) = st^2$ y $y(s,t) = s^2t$ calcule $\frac{\partial z}{\partial s}$ y $\frac{\partial z}{\partial t}$.

Example Si $z(x,y) = x^2y + 3xy^4$, donde $x(t) = \sin 2t$ y $y(t) = \cos t$, determine $\frac{dz}{dt}$ cuanto $t = 0$

$$\boxed{\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}}$$

$$= e^x \operatorname{san} y \cdot t^2 + e^x \cos y \cdot 2st. \qquad s,t,x,y$$

$$\frac{\partial z}{\partial s} = e^{st^2}\operatorname{san}(s^2t)\,t^2 + e^{st^2}\cos(s^2t)\,2st \Leftarrow \qquad s,t$$

$$\boxed{\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}} = e^x \operatorname{san} y \cdot 2st + e^x \cos y\, s^2 = e^{st^2}\operatorname{san}(s^2t)\,2st + e^{st^2}\cos(s^2t)\,s^2$$

$$z = f(m,n,o,p) \qquad m(a,b,c) \quad n(a,b,c) \quad o(a,b,c) \quad p(a,b,c)$$

$$\frac{\partial z}{\partial a}, \frac{\partial z}{\partial b}, \frac{\partial z}{\partial c}$$

$$\frac{\partial z}{\partial a} = \frac{\partial z}{\partial m}\cdot\frac{\partial m}{\partial a} + \frac{\partial z}{\partial n}\cdot\frac{\partial n}{\partial a} + \frac{\partial z}{\partial o}\cdot\frac{\partial o}{\partial a} + \frac{\partial z}{\partial p}\cdot\frac{\partial p}{\partial a}$$

$\partial a \quad \partial m \quad \partial a \quad \partial n \quad \partial a \quad \partial o \quad \partial a \quad \partial p \quad \partial a$

$$\frac{\partial z}{\partial b} = \frac{\partial z}{\partial m}\cdot\frac{\partial m}{\partial b} + \frac{\partial z}{\partial n}\cdot\frac{\partial n}{\partial b} + \frac{\partial z}{\partial o}\cdot\frac{\partial o}{\partial b} + \frac{\partial z}{\partial p}\cdot\frac{\partial p}{\partial b}$$

$$\frac{\partial z}{\partial c} = \frac{\partial z}{\partial m}\cdot\frac{\partial m}{\partial c} + \frac{\partial z}{\partial n}\,\frac{\partial n}{\partial c} + \frac{\partial z}{\partial o}\cdot\frac{\partial o}{\partial c} + \frac{\partial z}{\partial p}\,\frac{\partial p}{\partial c}$$

Example *Sea* $z = f(x,y) = y\varphi(y^2 - x^2, x^2 - y^2)$ *donde* φ *es una función real de dos variables diferenciable. Pruebe que*

$$\boxed{\frac{y}{x}\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{z}{y}}$$

$\varphi(y^2 - x^2, x^2 - y^2)$

$\mu = y^2 - x^2$

$v = x^2 - y^2$

$$\boxed{z = y\cdot\varphi(\mu, v)}$$

$$\frac{\partial z}{\partial x} = y\left[\frac{\partial \varphi}{\partial \mu}\,\frac{\partial \mu}{\partial x} + \frac{\partial \varphi}{\partial v}\cdot\frac{\partial v}{\partial x}\right].$$

$$\frac{\partial z}{\partial x} = y\left[\frac{\partial \varphi}{\partial \mu}(-2x) + \frac{\partial \varphi}{\partial v}\cdot(2x)\right] = -2xy\frac{\partial \varphi}{\partial \mu} + 2xy\frac{\partial \varphi}{\partial v}.$$

$$\frac{\partial z}{\partial y} = \varphi(\mu, v) + y\cdot\left[\frac{\partial \varphi}{\partial \mu}\cdot\frac{\partial \mu}{\partial y} + \frac{\partial \varphi}{\partial v}\cdot\frac{\partial v}{\partial y}\right]$$

$$= \varphi(\mu, v) + y\frac{\partial \varphi}{\partial \mu}\cdot 2y + y\frac{\partial \varphi}{\partial v}\cdot(-2y)$$

$$\frac{\partial z}{\partial y} = \varphi(\mu, v) + 2y^2\frac{\partial \varphi}{\partial \mu} - 2y^2\frac{\partial \varphi}{\partial v}.$$

$$\boxed{\frac{y}{x}\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{z}{y}}$$

$$\frac{y}{x}\left(-2xy\frac{\partial \varphi}{\partial \mu} + 2xy\frac{\partial \varphi}{\partial v}\right) + \varphi(\mu, v) + 2y^2\frac{\partial \varphi}{\partial \mu} - 2y^2\frac{\partial \varphi}{\partial v} = \frac{\cancel{y}\,\varphi(\mu, v)}{\cancel{y}}$$

$$-\cancel{2y^2\frac{\partial \varphi}{\partial \mu}} + \cancel{2y^2\frac{\partial \varphi}{\partial v}} + \varphi(\mu, v) + \cancel{2y^2\frac{\partial \varphi}{\partial \mu}} - \cancel{2y^2\frac{\partial \varphi}{\partial v}} = \varphi(\mu, v)$$

$$\varphi(\mu, v) = \varphi(\mu, v) \quad \blacksquare$$

Derivación Implícita

$z = f(x,y) = x^2 + xy$ Fun Explicita.

$xy^z - \ln\left(\frac{4}{z+3}\right) = \cos^2(xyz)$ Fun. Implicita..

1. Hallar $\frac{dy}{dx}$ dada la ecuacion $y^3 + y^2 - 5y - x^3 + 4 = 4x^2 - 5y^7$

$F(x,y) = 0$

2. Hallar $\frac{\partial z}{\partial x}$ y $\frac{\partial z}{\partial y}$ dada la ecuación $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = xyz - z^2$

$$y^3 + y^2 - 5y - x^3 + 4 - 4x^2 + 5y^7 = 0$$

$$F(x,y) = y^3 + y^2 - 5y - x^3 + 4 - 4x^2 + 5y^7$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{-3x^2 - 8x}{3y^2 + 2y - 5 + 35y^6}$$

$$0 = 4x^2 - 5y^7 - y^3 - y^2 + 5y + x^3 - 4$$

$$F(x,y) = 4x^2 - 5y^7 - y^3 - y^2 + 5y + x^3 - 4$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{8x + 3x^2}{-35y^6 - 3y^2 - 2y + 5}$$

$$= -\frac{-(-3x^2 - 8x)}{-(35y^6 + 3y^2 + 2y - 5)} = -\frac{-3x^2 - 8x}{35y^6 + 3y^2 + 2y - 5}$$

2. Hallar $\frac{\partial z}{\partial x}$ y $\frac{\partial z}{\partial y}$ dada la ecuación $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = xyz - z^2$

$$F(x,y,z) = 3x^2z - x^2y^2 + 2z^3 + 3yz - 5 - xyz + z^2$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{6xz - 2xy^2 - yz}{3x^2 + 6z^2 + 3y - xy + 2z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-2x^2y + 3z - xz}{3x^2 + 6z^2 + 3y - xy + 2z}$$

$$\boxed{D_u f(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+ha, y_0+hb) - f(x_0,y_0)}{h}}$$

$\underline{\vec{u} = \langle a, b \rangle}$ unitario

$$f_x = \lim_{h\to 0} \frac{f(x+h, y) - f(x,y)}{h} \qquad \vec{u} = \underline{i} = \langle 1, 0 \rangle$$

$$\underline{j} = \langle 0, 1 \rangle$$

Ej $\boxed{f(x,y) = 3x^2y + x}$ $\qquad \vec{u} = \langle 5, 6 \rangle \quad D_u f(x,y) = ? \quad D_u f(4,2) = ?$ Def

$$D_u f(x,y) = \lim_{h\to 0} \frac{f(x+ha, y+hb) - f(x,y)}{h}$$

$$f(x+ha, y+hb) = 3(x+ha)^2(y+hb) + (x+ha)$$

$$D_u f(x,y) = \lim_{h\to 0} \frac{3(x+ha)^2(y+hb) + (x+ha) - (3x^2y + x)}{h} = \frac{0}{0}$$

$$= \lim_{h\to 0} \frac{(3y+3hb)(x^2 + 2xha + h^2a^2) + \cancel{x} + ha - 3x^2y - \cancel{x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3yx^2} + 6xyha + 3yh^2a^2 + 3hbx^2 + 6h^2xab + 3h^3ba^2 + ha - \cancel{3x^2y}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6xya + 3yha^2 + 3bx^2 + 6hxab + 3h^2ba^2 + a)}{\cancel{h}}$$

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$$D_u f(x,y) = 6xya + 3bx^2 + a. \qquad \vec{u} = \langle 5, 6 \rangle \quad \|u\| = \sqrt{5^2 + 6^2} = \sqrt{61}$$

$$u = \langle \underbrace{\frac{5}{\sqrt{61}}}_{a}, \underbrace{\frac{6}{\sqrt{61}}}_{b} \rangle$$

$$D_u f(x,y) = 6xy\frac{5}{\sqrt{61}} + 3\frac{6}{\sqrt{61}}x^2 + \frac{5}{\sqrt{61}}$$

$$D_u f(4,2) = 6.4.2.\frac{5}{\ } + \frac{18.4^2}{\ } + \frac{5}{\ } =$$

$D_u f(4,2) = 6.4 2 . \frac{5}{\sqrt{61}} + \frac{18.4^2}{\sqrt{61}} + \frac{5}{\sqrt{61}} =$

taorama:

$\frac{\partial f(x,y)}{\partial x} = \frac{x^2}{x+1}$ $x=1$

$\boxed{f(x,y) = 3x^2y + x}$ $\vec{u} = \langle 5, 6 \rangle$

$u = \langle \frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}} \rangle$

$D_u f(x,y) = f_x a + f_y b \leftarrow$

$D_u f(x,y) = (6xy + 1) . \frac{5}{\sqrt{61}} + 3x^2 . \frac{6}{\sqrt{61}}$

$D_u f(x,y) = 6xy \frac{5}{\sqrt{61}} + 3 \frac{6}{\sqrt{61}} x^2 + \frac{5}{\sqrt{61}} \leftarrow$ lim