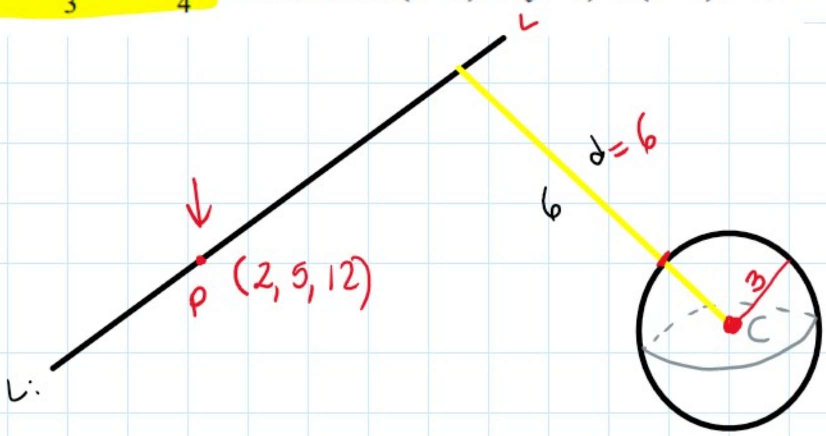
53. Calcular los puntos de minimo y maxima distancia entre la recta

$L: \frac{x-2}{2} = \frac{y-5}{3} = \frac{z-12}{4}$ con la esfera $(x-2)^2 + (y-6)^2 + (z-4)^2 = 9$



C(2, 6, 4)
radio = 3

$d_{min} = 3$
$d_{MAX} = 9$

P : un punto de recta
C : centro de la esfera
$\vec{v}$: vector dirección de la recta.

$$d(C,L) = \frac{\|\vec{PC} \times \vec{v}\|}{\|\vec{v}\|}$$

$\vec{v} = \langle 2,3,4 \rangle \quad C(2,6,4) \quad P(2,5,12)$

$\vec{PC} = \langle 0,1,-8 \rangle$

$$\vec{PC} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & 1 & -8 \\ 2 & 3 & 4 \end{vmatrix} = 28i - 16j - 2k.$$

$$\|\vec{PC} \times \vec{v}\| = \sqrt{28^2 + 16^2 + 2^2} = 6\sqrt{29}$$

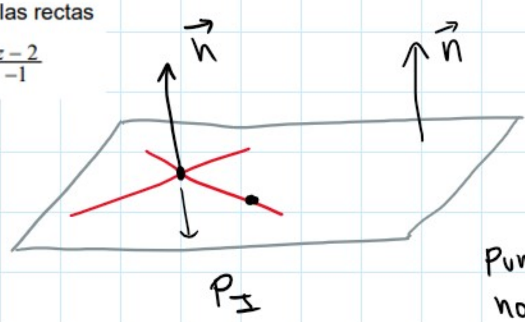
$$\|\vec{v}\| = \sqrt{4+9+16} = \sqrt{29}.$$

$$d(L,C) = \frac{6\sqrt{29}}{\sqrt{29}} = 6$$

RESP: $D_{min} = 3 \quad D_{max} = 9 \quad (0,2,8)(1,4,6)(3,8,2)$

54. Hallar una ecuación del plano que contiene las rectas

$L_1 \quad \frac{x-1}{-2} = y-4 = z \quad$ y $L_2: \frac{x-2}{-3} = \frac{y-1}{4} = \frac{z-2}{-1}$



PL. P $\vec{n}$.

Punto
normal

P_I

Punto normal

$\vec{n} = \vec{v}_1 \times \vec{v}_2$

$L_1 \cap L_2$

$L_1: \begin{cases} x = 1 - 2t \\ y = 4 + t \\ z = t \end{cases}$ $\quad L_2 \begin{cases} x = 2 - 3m \\ y = 1 + 4m \\ z = 2 - m \end{cases}$

$$\begin{cases} 1 - 2t = 2 - 3m & (1) \\ 4 + t = 1 + 4m & (2) \\ t = 2 - m \end{cases} \qquad \boxed{\begin{matrix} t = 1 \\ m = 1 \end{matrix}}$$

$1 = 2 - 1$ ✓ v.

$P_I = (-1, 5, 1)$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 1 \\ -3 & 4 & -1 \end{vmatrix} = -5i - 5j - 5k$$

$\vec{n} = \langle -5, -5, -5 \rangle$ $\quad \vec{n} = 5\langle -1, -1, -1 \rangle$ $\quad \vec{n} = \langle 1, 1, 1 \rangle$

$-5 + 25 + 5$

Ec Plano: $-5(x+1) - 5(y-5) - 5(z-1) = 0$

$-5x - 5y - 5z + 25 = 0$ //

$\vec{n} = \langle \frac{1}{5}, \frac{2}{5}, \frac{3}{5} \rangle$ ¿

$n = \langle 1, 2, 3 \rangle$ ✓

43. ¿Para que valores de a son paralelas las rectas

$$r: \begin{cases} 4x + 5y + 2z - 3 = 0 \\ x + 3y + 4z - 5 = 0 \end{cases} \qquad s: \begin{cases} 5x + y + 2az - 7 = 0 \\ 10x + 9y + \frac{1}{2}z + 9 = 0 \end{cases}$$

RESP: $a = -4$

$a = ?$

$r \parallel s$

$\vec{v}_1 \qquad \vec{v}_2 \qquad \vec{v}_1 = K\vec{v}_2$

$$\begin{cases} 4x + 5y + 2z = 3 \\ x + 3y + 4z = 5 \quad (-4) \end{cases}$$

$$\begin{array}{r} 4x + 5y + 2z = 3 \\ -4x - 12y - 16z = -20 \\ \hline -7y - 14z = -17 \end{array}$$

$+\frac{17}{7} - \frac{14z}{7} = y.$

$x + 3\left(\frac{17}{7} - 2t\right) + 4t = 5$

$x + \frac{51}{7} - 6t + 4t = 5$

$x = -\frac{16}{7} + 2t$

$y = \frac{17}{7} - 2t$

$$\frac{+17}{7} - \frac{14}{7}z = y.$$

$$y = \frac{17}{7} - 2t$$

$$\langle -1, 1, -2 \rangle$$

$$z = t$$

$$y = \frac{17}{7} - 2t$$

$$z = \quad t$$

$$\vec{v} = \langle 2, -2, 1 \rangle$$

$$: \begin{cases} 5x + y + 2az - 7 = 0 \\ 10x + 9y + \frac{1}{2}z + 9 = 0 \end{cases} \quad (-2)$$

$$-10x - 2y - 4az + 14 = 0$$

$$7y + \left(\frac{1}{2} - 4a\right)z + 23 = 0$$

$$z = \frac{-23 - 7y}{\left(\frac{1}{2} - 4a\right)} \qquad y = t$$

$$z = \frac{-23 - 7t}{\frac{1-8a}{2}} = \frac{-46 - 14t}{1 - 8a}$$

$$5x + t + 2a\left(\frac{-46 - 14t}{1 - 8a}\right) - 7 = 0$$

$$5x = 7 + t - 2a\left(\frac{-46 - 14t}{1 - 8a}\right)$$

$$5x = \frac{7 - 56a + t - 8at + 92a + 28at}{1 - 8a}$$

92
56
36

$$x = \frac{7 + 36a + 20at + t}{5 - 40a}$$

$$x = \frac{7 + 36a}{5 - 40a} + \left(\frac{20a + 1}{5 - 40a}\right)t$$

$$y = \quad t$$

$$z = \frac{-46}{1 - 8a} - \frac{14}{1 - 8a}t$$

$$\vec{V} = \left(\left(\frac{20a + 1}{5 - 40a}\right), 1, -\frac{14}{1 - 8a}\right)$$

$\vec{V}_1 = K\vec{V}_2 \qquad \exists K.$

?!

$\rightarrow \langle 2,-2,1\rangle = K\left\langle \frac{20a+1}{5-40a}, 1, -\frac{14}{1-8a} \right\rangle$

① $2 = K\left(\frac{20a+1}{5-40a}\right)$

② $\rightarrow \boxed{-2 = K}$

③ $\rightarrow 1 = -\frac{14K}{1-8a}$

$1 = \frac{28}{1-8a}$

$1-8a = 28$

$-27 = 8a$

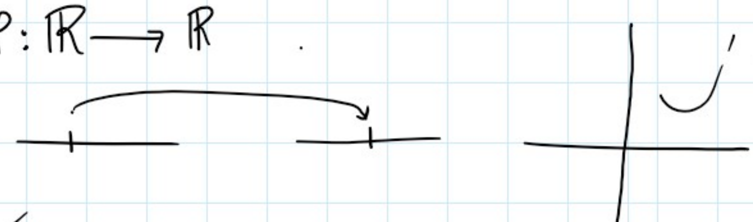
$\boxed{a = \frac{-27}{8}}$ ✓

$2 = -2\left(\frac{20\cdot\left(-\frac{27}{8}\right)+1}{5-40\cdot\left(-\frac{27}{8}\right)}\right)$

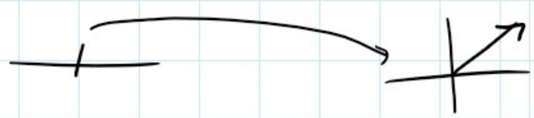
$2 \neq \frac{19}{20}$

∴ No Exista 'a' para que r y S sean ∥s.

$\rightarrow f: \mathbb{R} \longrightarrow \mathbb{R}$



$\rightarrow \vec{r}: \mathbb{R} \longrightarrow \mathbb{R}^2, \mathbb{R}^3$

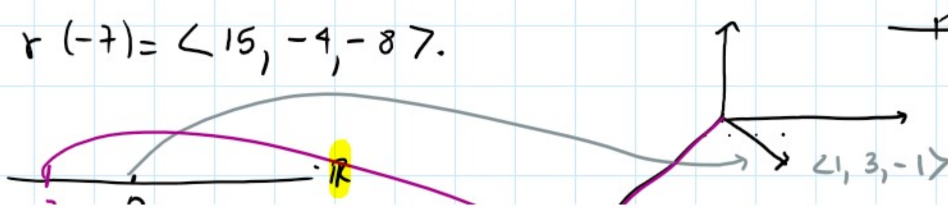


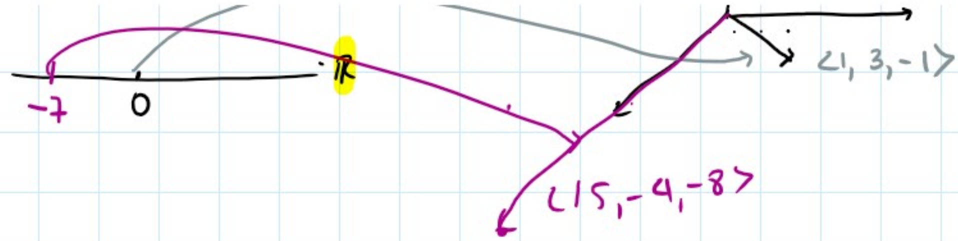
Example Sea $\vec{r}: \mathbb{R} \to \mathbb{R}^3$ tal que $r(t) = \langle 1-2t, 3+t, -1+t\rangle$ Hallar $r(t)$ para $t = 0, -7, \frac{1}{4}$

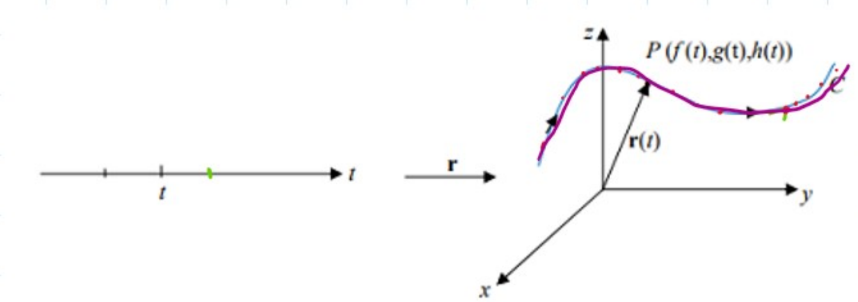
$r(0) = \langle 1-2(0), 3+0, -1+0\rangle = \langle 1, 3, -1\rangle$

$r(-7) = \langle 15, -4, -8\rangle.$

$r: \mathbb{R} \to \mathbb{R}^2$







Example *Calcular el dominio de la función* $r(t) = \ln t\,i + \sqrt{1-t}\,j + tk$

$r: \mathbb{R} \longrightarrow \mathbb{R}^3$

$D_r = D_f \cap D_g \cap D_h$

$f(t) = \ln t$ $\qquad g(t) = \sqrt{1-t}$ $\qquad h(t) = t$ $\qquad$ func Comp.

a) $f(t) = \ln t$ $\qquad t > 0$ $\qquad D_f = (0, \infty)$

b) $g(t) = \sqrt{1-t}$ $\qquad 1 - t \geq 0$ $\qquad -t \geq -1$ $\qquad t \leq 1$ $\qquad D_g = (-\infty, 1]$

c) $h(t) = t$ $\qquad t \in \mathbb{R}$ $\qquad D_h = \mathbb{R}$

$D_r = (0, 1]$ $\qquad\qquad f(x) = \sqrt{1+x^2}$

Example *Dibujar la curva plana representada por la función vectorial*

$r(t) = 2\cos t\,i - 3\sin t\,j$ $\qquad 0 \leq t \leq 2\pi.$ $\qquad r: \mathbb{R} \longrightarrow \mathbb{R}^2$

Example *Dibujar la curva plana representada por la función vectorial*

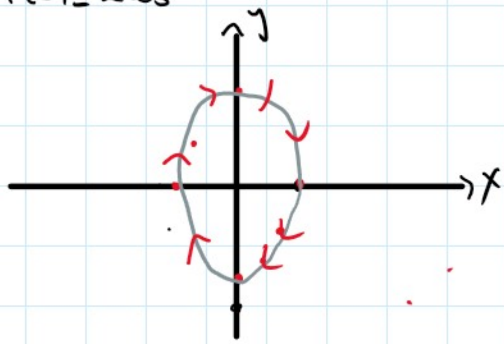
$r(t) = \underbrace{2\cos t}_{x}\, i - \underbrace{3\sin t}_{y}\, j \qquad 0 \le t \le 2\pi.$

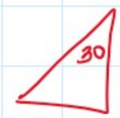
$r: \mathbb{R} \longrightarrow \mathbb{R}^2$

$t \longmapsto \langle 2\cos t, -3\sin t \rangle$

$x = f(t) = 2\cos t$

t	x	y
0	2	0
$\frac{\pi}{6}$	$\sqrt{3}$	$-3/2$
$\frac{\pi}{4}$	$\sqrt{2}$	$-\frac{3\sqrt{2}}{2}$
$\frac{\pi}{2}$	0	-3
π	-2	0
$\frac{5\pi}{4}$	$-\sqrt{2}$	$\frac{3\sqrt{2}}{2}$
$\frac{3\pi}{2}$	0	3
2π	2	-3





$225^\circ \frac{\pi}{180^\circ} =$

1 Forma.

Example *Dibujar la curva plana representada por la función vectorial*

$r(t) = 2\cos t\, i - 3\sin t\, j \qquad 0 \le t \le 2\pi.$

$x = 2\cos t$

$y = -3\sin t$

$\frac{x}{2} = \cos t$

$\frac{y}{-3} = \sin t$

$\frac{x^2}{4} = \cos^2 t$ +

$\frac{y^2}{9} = \sin^2 t$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$ Elipse

orient