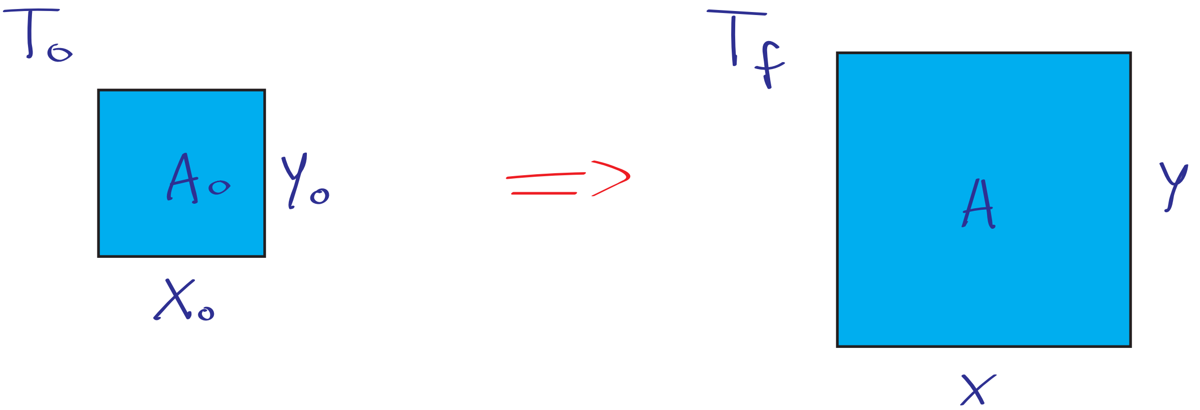
Demostrar

$$\Delta A = 2\alpha A_0 \Delta T$$



$$A_0 = X_0 Y_0 \qquad\qquad A = X Y$$

$$\Delta A = A - A_0$$

$$\Delta A = XY - X_0 Y_0 \qquad \text{ec } 1$$

Sabemos

$$L = L_0 + \alpha L_0 \Delta T$$

Tenemos

$$x = X_0 + \alpha_1 X_0 \Delta T$$

$$y = y_0 + \alpha_2 y_0 \Delta T$$

Remplazando

$$\Delta A = XY - X_0 Y_0$$

$$\Delta A = (X_0 + \alpha_1 X_0 \Delta T)(y_0 + \alpha_2 y_0 \Delta T) - X_0 Y_0$$

$$\Delta A = \cancel{X_o y_o} + \alpha_2 X_o y_o \Delta T + \alpha_1 X_o y_o \Delta T$$

$$+ \alpha_1 \alpha_2 X_o y_o \Delta T^2 - \cancel{X_o Y_o}$$

$$\Delta A = (\alpha_2 + \alpha_1) X_o Y_o \Delta T + \alpha_1 \alpha_2 X_o Y_o \cancel{\Delta T^2}^{\approx 0}$$

$$\Delta A = (\alpha_1 + \alpha_2) X_o Y_o \Delta T$$

Sabemos

$$\alpha_1 = \alpha_2 = \alpha$$

$$\Delta A = 2\alpha A_o \Delta T$$

Usando Calculo diferencial

$$A = XY$$

$$\frac{dA}{dT} = \frac{d}{dT}(XY) = \frac{dx}{dT} Y_o + X_o \frac{dY}{dT}$$

$$\frac{dA}{dT} = \frac{dx}{dT} Y_o \frac{X_o}{X_o} + X_o \frac{Y_o}{Y_o} \frac{dY}{dT}$$

$$\frac{dA}{dT} = A_o \frac{1}{X_o} \frac{dX}{dT} + A_o \frac{1}{Y_o} \frac{dY}{dT}$$

Sabemos

$$\Delta L = \alpha L_0 \Delta T$$

$$dL = \alpha L_0 dT$$

$$\alpha = \frac{1}{L_0} \frac{dL}{dT}$$

Remplazando

$$\frac{dA}{dT} = A_0 \frac{1}{X_0} \frac{dx}{dT} + A_0 \frac{1}{Y_0} \frac{dY}{dT}$$

$$\frac{dA}{dT} = A_0 \alpha_1 + A_0 \alpha_1$$

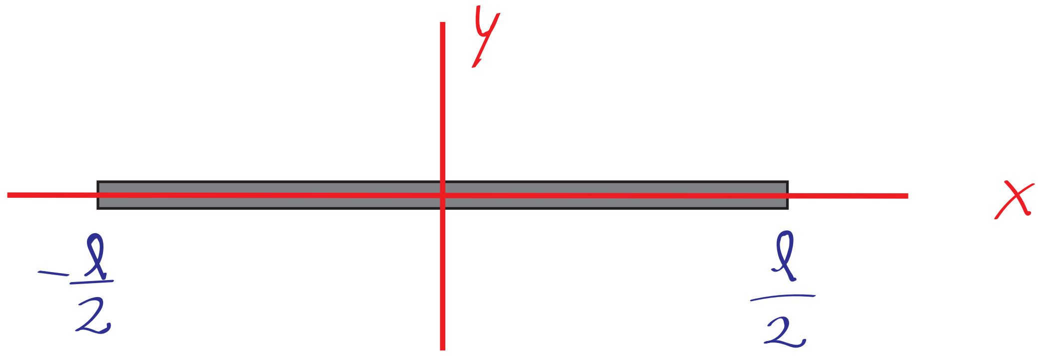
$$\alpha_1 = \alpha_2 = \alpha$$

$$\frac{dA}{dT} = 2 A_0 \alpha$$

$$dA = 2 A_0 \alpha dT$$

$$\Delta A = 2 A_0 \alpha \Delta T$$

4. Demostrar que la variación del momento de inercia con la temperatura, para la varilla está dada por la siguiente ecuación:



Sabemos

$$I = \frac{1}{12} m l^2 \quad \text{Momento Inercia}$$

$$I_o = \frac{1}{12} m l_o^2$$

$$I = \frac{1}{12} m l^2$$

Tenemos $\quad I = f(T)$

$$\Delta I = I - I_o$$

$$\Delta I = \frac{1}{12} m l^2 - \frac{1}{12} m l_o^2$$

$$\Delta I = \frac{1}{12} m \left(l^2 - l_o^2 \right)$$

Sabemos

$$l = l_0 (1 + \alpha \Delta T)$$

Remplazando

$$\Delta I = \frac{1}{12} m \left(l_0^2 (1 + \alpha \Delta T)^2 - l_0^2 \right)$$

$$\Delta I = \frac{1}{12} m \left[l_0^2 \left((1 + \alpha \Delta T)^2 - 1 \right) \right]$$

$$\Delta I = \frac{1}{12} m l_0^2 \left(\cancel{1} + 2 \alpha \Delta T + \alpha^2 \Delta T^2 \cancel{- 1} \right)$$

$$\Delta I = \frac{1}{12} m l_0^2 \left(2 \alpha \Delta T + \cancel{\alpha^2 \Delta T^2} \right) \approx 0$$

$$\Delta T << \qquad 0.1^2 = 0.01 \approx 0$$

$$0.01^2 = 0.0001 \approx 0$$

$$\Delta I = \left(\frac{1}{12} m l_0^2 \right) 2 \alpha \Delta T$$

$$\Delta I = I_0 \, 2 \alpha \Delta T$$

Variacion de la Inercia con la Temperatura

Usando Calculo diferencial

$$I = \frac{1}{12} m \ell^2 \rightarrow \text{Inercia para cualquier } T$$

$$\frac{dI}{dT} = \frac{d}{dT}\left(\frac{1}{12} m \ell^2\right)$$

$$= \frac{1}{12} m \frac{d}{dT} \ell^2 \frac{d\ell}{d\ell}$$

$$= \frac{1}{12} m \frac{d\ell^2}{d\ell} \frac{d\ell}{dT}$$

$$= \frac{1}{12} m (2\ell) \frac{\ell}{\ell} \frac{d\ell}{dT}$$

$$= \frac{1}{12} m 2 \ell^2 \underbrace{\left(\frac{1}{\ell} \frac{d\ell}{dT}\right)}_{\alpha}$$

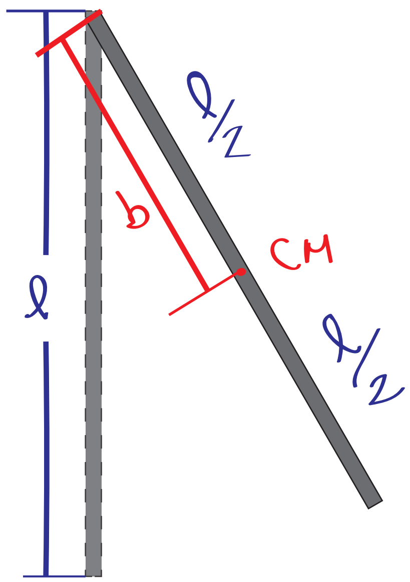
$$\frac{dI}{dT} = \left(\frac{1}{12} m \ell^2\right) 2\alpha$$

$$\frac{dI}{dT} = I_0 \, 2\alpha$$

$$dI = I_0\, 2\alpha\, dT$$

$$\Delta I = I_0\, 2\alpha\, \Delta T$$

Hallar ΔP $\quad$ $P \to$ Periodo



Sabemos

$$P = 2\pi\sqrt{\frac{I_{eje}}{mgb}}$$

$$I_{eje} = I_{CM} + mx^2$$

$$I_{eje} = I_{CM} + mb^2$$

$$I_{eje} = \frac{1}{12}m\ell^2 + mb^2$$

$$I_{eje} = \frac{1}{12} m \ell^2 + m\left(\frac{\ell}{2}\right)^2$$

$$I_{eje} = \frac{1}{12} m \ell^2 + \frac{1}{4} m \ell^2$$

$$I_{eje} = \frac{4+12}{48} m \ell^2$$

$$I_{eje} = \frac{1}{3} m \ell^2$$

Remplazando

$$P = 2\pi \sqrt{\frac{I_{eje}}{mgb}}$$

$$P = 2\pi \sqrt{\frac{\frac{1}{3} \cancel{m} \ell^2}{\cancel{m} g \frac{\ell}{2}}}$$

$$P = 2\pi \sqrt{\frac{1 \; \ell^{\cancel{2}}}{3g \frac{\cancel{\ell}}{2}}}$$

$$P = 2\pi \sqrt{\frac{2\ell}{3g}}$$

Periodo de Oscilación

Hallar $\Delta P = f(\Delta T)$

$$\frac{dP}{dT}$$