Practica 5

5.1. Encontrar la diferencial dw

a. $w = x^3 - x^2y + 3y^2$

$\frac{dw}{dx} = 3x^2 - 2xy + 0$

$dw = 3x^2 - 2xy$ / sol.

$\frac{dw}{dy} = (6y - x^2)$

$dw = (6y - x^2)\,dy$ / sol

b. $w = x^2 \sin y + 2y^{3/2}$

$\frac{dw}{dx} = 2x \cdot \sin y + 0 \Rightarrow \boxed{dw = (2x \sin y)\,dx}$ / sol.

$\frac{dw}{dy} = x^2 \cos y + 2 \Rightarrow dw = x^2 \cos y + 3y^{1/2}$

$\boxed{dw = x^2 \cos y + 3\sqrt{y}\ dy}$ / sol

c. $w = x^2 \ln(y^2 + z^2)$

$\frac{dw}{dx} = 2x \ln(y^2 + z^2)$ $\quad dw = 2x \ln(y^2 + z^2)\,dx$ / sol.

$\frac{dw}{dy} = x^2 \frac{1}{y^2+z^2} \cdot 2y + 0$ $\quad \boxed{dw = \frac{2x^2 y}{y^2+z^2}\,dy}$

$\frac{dw}{dz} = x^2 \frac{1}{y^2+z^2} \cdot 2z + 0 \Rightarrow \boxed{dw = \frac{2x^2 z}{y^2+z^2}\,dz}$ sol.

5.2.- Use la regla de la cadena para encontrar $\frac{dw}{dr}$, $\frac{\partial w}{\partial \theta}$

a) $w = \frac{yz}{x}$ $\quad x = \theta^2$ $\quad y = r + \theta$ $\quad z = r - \theta$

$\frac{dw}{dr} = \frac{dw}{dx} \cdot \frac{dx}{dr} + \frac{dw}{dy} \cdot \frac{dz}{dr}$

$\frac{dw}{dr} = yz\ 0 + \frac{z}{x} \cdot 1 + \frac{y}{x} \cdot 1$

$\frac{dw}{dr} = \frac{z}{x} + \frac{y}{x} = \frac{z+y}{x} = \frac{r - \theta + r\theta}{\theta^2} = \frac{dw}{dr} = \frac{2r}{\theta^2}$

b) $w = \arctan \frac{y}{x}$ $\qquad x = \cos\theta \qquad y = r \sin\theta$

$$\frac{dw}{dr} = \frac{r\,0\,\operatorname{sen}\theta}{r\cos\theta} \qquad \frac{dw}{d\theta} = \tan^{-1}\left(\frac{\operatorname{sen}\theta}{\cos\theta}\right) \qquad \frac{dw}{dr} = 0$$

$$\frac{dw}{d\theta} = \frac{1}{1+\left(\frac{\operatorname{sen}}{\cos}\right)^2} \cdot \left(\frac{\cos\cos\theta - \operatorname{sen}(-\operatorname{sen}\theta)}{\cos^2\theta}\right) = \frac{1/1}{\frac{\cos^2\theta + \operatorname{sen}^2\theta}{\cos^2\theta}} \cdot \frac{\cos^2 + \operatorname{sen}}{\cos^2\theta}$$

$$\frac{dw}{d\theta} = 1 \text{ sol}$$

53.- hallar dy / dx por derivación implícita

a. $\ln\sqrt{x^2+y^2} + x + y = 4$

$$\frac{1}{x^2+y^2} \cdot 2x + 2y \cdot y' + 1 + 1 = 0$$

$$2y\frac{dy}{dx} = -\frac{2y+2}{x^2+y^2}$$

$$\frac{dy}{dx} = -\frac{2y+2}{x^2+y^2+2y} \text{ sol.}$$

55.- Encontrar el gradiente de f en el punto indicado. deriva direccional F

a. $f(x,y) = e^x \sin y \qquad u = i \qquad P\left(1, \frac{\pi}{2}\right)$

$f_x = e^x \sin y$

$f_x(P) = e^x \sin(\pi/2) = e \qquad i$

$f_y = e^x \cos y$

$f_y(P) = e\cos(\pi/2) = e\,0 = 0 \qquad (e) \cdot i \Rightarrow -e$

~~56.- hallar la gradiente~~ b) $f(x,y) = \sqrt{x^2+y^2} \qquad u = 3i - 4j \qquad P(3,4)$

$f_x = \frac{1}{2} \cdot (x^2+y^2)^{-1/2} \cdot 2x = x(x^2+y^2)^{-1/2} = 3(3^2+4^2) = \frac{3}{5}$

$f_y = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y = y(x^2+y^2)^{-1/2} \cdot 4(3^2+4^2) = \frac{4}{5}$ ✓

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$3\left(\frac{3}{5}\right)i - 4\left(\frac{4}{5}\right)j = -\frac{7}{25}$ sol.

56= Hallar el gradiente de la función y el valor maximo de la deriv. direccional en el punto dado.

a. $f(x,y) = ye^{-x}$ $P(0,5)$

$f_{(x,y)} = \langle -e^{x}y, e^{-x}\rangle = e^{-x}\langle -y, 1\rangle = e^{-x}(-yi + j)$ sol

$f(0,5) = \sqrt{26}$

b $f(x,y,z) = xy^2z^2$ $P(2,1,1)$

$\nabla f(x,y,z) = (z^2y^2, 2yxz^2, 2zxy^2)$

$= yz\,(yzi + 2xzj + 2xyk)$ sol

$\nabla f_{x,y,z} = \sqrt{33}$ sol.

57 = En forma directa usando la gradiente, calcular la derivada direccional, dirección $\vec{u}$

$f(x,y) = e^{xy} + 5\ln y$ $u = \frac{\langle 4,8\rangle}{10}$

$\partial f = ye^{xy} + xe^{xy} + \frac{5}{y}$

$\partial f = ye^{xy}\frac{2}{5} + \left(xe^{xy} + \frac{5}{y}\right)\cdot\frac{4}{5}$ mcm 5

$\partial f = \frac{e^{xy}\left(2y + 4x + \frac{5}{y}\right)}{5}$

$\partial f = \frac{e^{xy}(3y + 4x) + \frac{4}{y}}{5}$ sol.

58= En forma directa usando la gradiente, calcular la derivada direccional dirección $\vec{u}$

$f = 6x^2 - 3yz$

$\nabla f(x,y,z) = (12x, -3z, -3y)$

$\nabla f(x,y,z) = \langle 12x, -3z, -3y\rangle \cdot \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle$

Fecha.

$= 8x - 2z - y$, sol.

(59) a. $z = x^2 - y^2$ $(3,2,5)$

$\nabla f(x,y,z) = \langle 2x, -2y, -1 \rangle$

$\nabla f(3,2,5) = (6,4,-1)$ $\Rightarrow n$ $P(3,2,5)$

$6(x-3) - 4(y-2) - 1(z-5) = 0$

$6x - 4y - z - 5 = 0$ Ec. plano tan.

$\left.\begin{array}{l} x = 3 + 6t \\ y = 2 - 4t \\ z = 5 - t \end{array}\right\}$ Ec Recta Normal $\dfrac{x-3}{6} = \dfrac{y-2}{4} = \dfrac{z-5}{-1}$

b. $xyz = 10$ $(1,2,5)$

$\Delta f(x,y,z) = xyz - 10$

$\Delta f(x,y,z) = \langle yz, xz, xy \rangle$

$\Delta f(1,2,5) = \langle 10, 5, 2 \rangle$

$10(x-1) + 5(y-2) + 2(z-5)$

$10x + 5y + 2z - 30 = 0$ $\Rightarrow$ $10x + 5y + 2z = 30$ Ec. plano

60: hallar la ecuación de los planos tangentes a las superficies dadas

$x^2 + y^2 = z = 0$ $P_0(4,3,2,5)$

$\nabla f(x,y,z) = (2x, 2y, -1)$

$\nabla f(4,3,2,5) = \langle 8, 6, -1 \rangle = \bar{n}$

$8(x-4) + 6(y-3) + (-1)(z-2) = 0$

$8x + 6y - z = 48$, s