58. En forma directa usando la gradiente, calcular la derivada direccional, dirección $\vec{u}$

$f = 6x^2 - 3yz$ $\qquad u = \frac{\langle 2,2,1\rangle}{3}$ $\qquad P$

$\vec{u}$ = unitario $\qquad \|u\| = 1$

$\nabla f(x,y,z) = \langle 12x, -3z, -3y \rangle$

$\|u\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$

$D_u f(x,y,z) = \vec{\nabla f} \cdot \vec{u}$

$= \sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}} = 1$

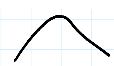
$D_u f(x,y,z) = \langle 12x, -3z, -3y \rangle \cdot \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$

$= 12x \cdot \frac{2}{3} - 3z \cdot \frac{2}{3} - 3y\left(\frac{1}{3}\right) =$

$= 8x - 2z - y$

62. Encontrar el (los) punto (s) sobre la superficie en la cual el plano tangente es horizontal.

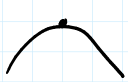
a. $z = x^2 - xy + y^2 - 2x - 2y$

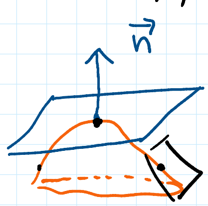


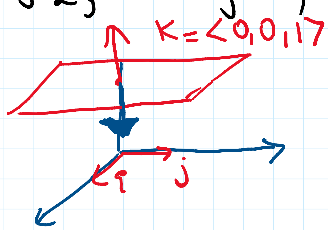
$\vec{n} = \nabla z = \langle$

$F(x,y,z) = x^2 - xy + y^2 - 2x - 2y - z$

$\nabla F(x,y,z) = \langle 2x - y - 2, -x + 2y - 2, -1 \rangle = \vec{n}$

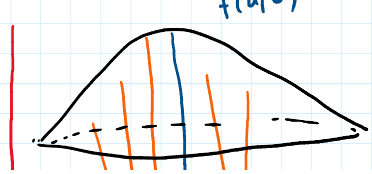






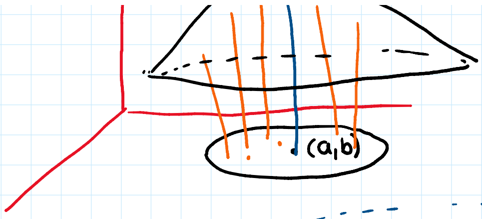
$$\begin{cases} 2x - y - 2 = 0 \\ -x + 2y - 2 = 0 \end{cases} \qquad \begin{matrix} x = 2 \\ y = 2. \end{matrix}$$

MAXIMOS y MINIMOS (PUNTOS EXTREMOS).

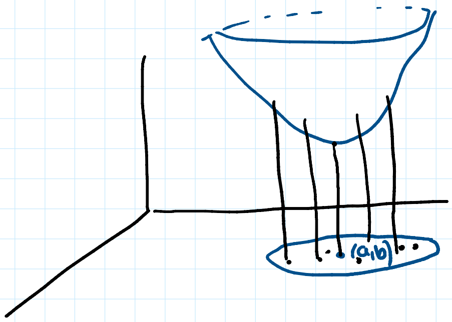


si $f(a,b) \geq f(x,y) \quad \forall (x,y) \in D_{(a,b)}$

$\Rightarrow (a,b,f(a,b))$ Es Maximo.

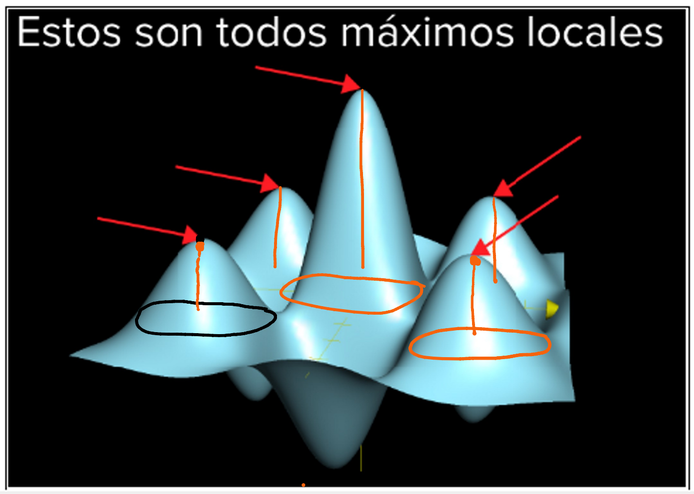


$\Rightarrow (a,b,f(a,b))$ Es Maximo.



si $f(a,b) \le f(x,y)\ \forall (x,y) \in D_{(a,b)}$

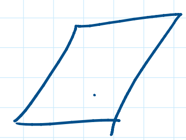
$\Rightarrow (a,b,f(a,b))$ es minimo



Example *Hallar los puntos criticos de*

1) $f(x,y) = 2x^2 + y^2 + 8x - 6y + 20$

2) $f(x,y) = 1 - (x^2 + y^2)^{\frac{1}{3}}$



1) $f_x(x,y) = 4x + 8$ $\leftarrow$ $f_y(x,y) = 2y - 6$

$$\begin{cases} 4x + 8 = 0 \\ 2y - 6 = 0 \end{cases} \qquad \begin{aligned} x &= -2 \\ y &= 3 \end{aligned} \qquad PC = \{(-2,3)\}$$

2) $f_x(x,y) = -\frac{1}{3}(x^2+y^2)^{-2/3} \cdot 2x$ $\qquad f_y(x,y) = -\frac{1}{3}(x^2+y^2)^{-2/3} \cdot 2y$

$$f_x(x,y) = \frac{-2x}{3(x^2+y^2)^{2/3}} \qquad f_y(x,y) = \frac{-2y}{3(x^2+y^2)^{2/3}}$$

$f_x(x,y) = \frac{-2x}{3(x^2+y^2)^{2/3}}$ $\qquad f_y(x,y) = \frac{-2y}{3(x^2+y^2)^{2/3}}$

$x=0$
$y=0$
b $\boxed{(0,0)}$

$$\begin{cases} \frac{-2x}{3(x^2+y^2)^{2/3}} = 0 \\ \frac{-2y}{3(x^2+y^2)^{2/3}} = 0 \end{cases}$$

$\frac{a}{b} = 0 \Rightarrow a = 0$

$-2x = 0 \Rightarrow x = 0$

$-2y = 0 \Rightarrow y = 0$

$PC = \{(0,0)\}$

$f(x,y) = -x^3 + 4xy - 2y^2 + 1$

$f_x(x,y) = -3x^2 + 4y$ $\qquad f_y(x,y) = 4x - 4y.$

$$\begin{cases} -3x^2 + 4y = 0 & (1) \\ 4x - 4y = 0 & (2) \end{cases}$$

$4x = 4y \Rightarrow x = y$

$-3y^2 + 4y = 0 \qquad y(-3y+4) = 0$

$y_1 = 0 \quad \cup \quad y_2 = \frac{4}{3}$

$x_1 = 0 \qquad x_2 = \frac{4}{3}$

$$\boxed{PC = \{(0,0)\left(\tfrac{4}{3}, \tfrac{4}{3}\right)\}}$$

$f(x,y) = -x^3 + 4xy - 2y^2 + 1$

1. PC.
2.

$f_x(x,y) = -3x^2 + 4y$ $\qquad f_y(x,y) = 4x - 4y.$

$f_{xx}(x,y) = -6x$ $\qquad f_{yy}(x,y) = -4$

$f_{xy}(x,y) = 4 \longleftrightarrow f_{yx} = 4$

$$\rightarrow d(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} -6x & 4 \\ 4 & -4 \end{vmatrix}$$

Para $(0,0)$

$$d(0,0) = \begin{vmatrix} 0 & 4 \\ 4 & -4 \end{vmatrix} = -16 < 0$$

1) Si $d(a,b) > 0$ y $f_{xx}(a,b) > 0$ entonces f tiene un ***mínimo relativo*** en $(a,b,f(a,b))$

2) Si $d(a,b) > 0$ y $f_{xx}(a,b) < 0$ entonces f tiene un ***máximo relativo*** en $(a,b,f(a,b))$

→ 3) Si $d(a,b) < 0$ entonces $(a,b,f(a,b))$ es un ***punto silla***

4) Si $d(a,b) = 0$ el criterio no lleva a ninguna conclusión

$(0,0,1)$ ES PUNTO SILLA

$$f(x,y) = -x^3 + 4xy - 2y^2 + 1 \qquad f(0,0)=1$$

Para $\left(\frac{4}{3}, \frac{4}{3}\right)$

$$\rightarrow d(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} -6x & 4 \\ 4 & -4 \end{vmatrix}$$

$$d\left(\frac{4}{3}, \frac{4}{3}\right) = \begin{vmatrix} -6(4/3) & 4 \\ 4 & -4 \end{vmatrix} = \begin{vmatrix} -8 & 4 \\ 4 & -4 \end{vmatrix} = 16 > 0$$

→ 1) Si $d(a,b) > 0$ y $f_{xx}(a,b) > 0$ entonces f tiene un ***mínimo relativo*** en $(a,b,f(a,b))$

→ 2) Si $d(a,b) > 0$ y $f_{xx}(a,b) < 0$ entonces f tiene un ***máximo relativo*** en $(a,b,f(a,b))$

3) Si $d(a,b) < 0$ entonces $(a,b,f(a,b))$ es un ***punto silla***

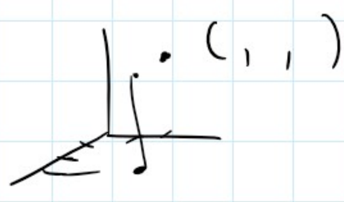
4) Si $d(a,b) = 0$ el criterio no lleva a ninguna conclusión

$$f_{xx}\left(\frac{4}{3}, \frac{4}{3}\right) = -8 < 0$$

$\left(\frac{4}{3}, \frac{4}{3}, \frac{59}{27}\right)$ Es un MÁXIMO.

$$f(x,y) = -x^3 + 4xy - 2y^2 + 1 \qquad f\left(\frac{4}{3}, \frac{4}{3}\right) = \frac{59}{27}$$

$(3, 2, \ )$



Example *Identificar los extremos relativos de*

$$f(x,y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4$$

$$f_x(x,y) = 20xy - 10x - 4x^3 \qquad f_y = 10x^2 - 8y - 8y^3$$

$$\begin{cases} 20xy - 10x - 4x^3 = 0 & ① \\ 10x^2 - 8y - 8y^3 = 0 & ② \end{cases}$$

$PC = \{(0,0),$

$2x(10y - 10 - 4x^2) = 0$

$\boxed{a \cdot b = 0 \quad a = 0 \vee b = 0}$

$2x = 0 \quad \vee$ ② $10y - 5 - 2x^2 = 0$ ①

① $x = 0$

$-8y - 8y^3 = 0$

$8y(-1 - y^2) = 0$

$y = 0 \quad \vee \quad \cancel{y^2 = -1}$

$(0,0)$

$\boxed{x^2 = \dfrac{10y - 5}{2}}$

$\overset{5}{\cancel{10}}\left(\dfrac{10y - 5}{\cancel{2}}\right) - 8y - 8y^3 = 0$ ④

$50y - 25 - 8y - 8y^3 = 0$

$0 = 8y^3 - 42y + 25$

$\cancel{y_1 = -2.54} \quad y_2 = 1,89 \quad y_3 = 0,65$

Para -2.54

$\cancel{x^2 = \dfrac{10(-2.54) - 5}{2}}$

Para $y = 1.89$

$x^2 = \dfrac{10(1.89) - 5}{2} = 6.95$

$x_1 = \pm 2.64$

$(2.64, 1.89) \; (-2.64, 1.89)$

Para $y = 0.65$

$x^2 = \dfrac{10(0.65) - 5}{2} = 0.75$

$x_1 = \pm 0.86$

$(0.86, 0.65) \; (-0.86, 0.65)$

$x^2 + 5x + 6 = 0$

$x \quad 3$

$x \quad 2$

$(x+3)(x+2) = 0$

$x = -3 \vee x = -2$ ✓

$PC = \{(0.86, 0.65)\; (-0.86, 0.65)\; (2.64, 1.89)\; (-2.64, 1.89)\; (0,0)\}$.

$f_x(x,y) = 20xy - 10x - 4x^3 \qquad f_y = 10x^2 - 8y - 8y^3$

$f_{xx}(x,y) = 20y - 10 - 12x^2$ $\qquad$ $f_{yy} = -8 - 24y^2$

$f_{xy}(x,y) = 20x$ $\quad \leftrightarrow \quad$ $f_{yx}(x,y) = 20x$

$$d(x,y) = \begin{vmatrix} 20y - 10 - 12x^2 & 20x \\ 20x & -8 - 24y^2 \end{vmatrix}$$

Para $(0.86, 0.65)$

$$d(0.86, 0.65) = \begin{vmatrix} -5.87 & 17.2 \\ 17.2 & -18.14 \end{vmatrix} = 106.48 - 295.84 < 0$$

$\downarrow$

$(0.86, 0.65, \quad)$ Es Punto Silla.

Para $(-0.86, 0.65)$

$$d(-0.86, 0.65) = \begin{vmatrix} -5.87 & -17.2 \\ -17.2 & -18.14 \end{vmatrix} = < 0$$

$$d(x,y) = \begin{vmatrix} 20y - 10 - 12x^2 & 20x \\ 20x & -8 - 24y^2 \end{vmatrix}$$

$(-0.86, 0.65, \quad)$ Es Punto Silla.

Para $(2.64, 1.89)$

$$d(2.64, 1.89) = \begin{vmatrix} \boxed{-55.83} & 52.8 \\ 52.8 & \boxed{-93,73} \end{vmatrix} = 5232,95 - 2787.84 = > 0$$

$f_{xx}(2.64, 1.89) = < 0 \Rightarrow$

$(2.64, 1.89, \quad)$ ES UN MAXIMO

Para $(-2.64, 1.89)$