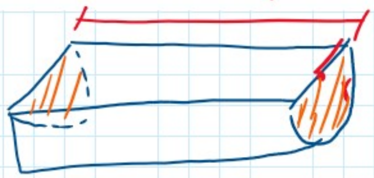
95. Se desea construir una bañera semicilíndrica. Si el costo por metro cuadrado de la parte cilíndrica es el triple del costo por metro cuadrado de las partes semicirculares, determine sus dimensiones para un costo mínimo si debe almacenar un volumen conocido V.

→ RESP: $r = 3\sqrt[3]{\frac{V}{9\pi}}$. $h = 2\sqrt[3]{\frac{V}{9\pi}}$



C_T:

$V = V$ fun dato   fun Obj.

$\boxed{\text{Precio} = P}$

$C_L = \text{Precio} \cdot \text{Cantidad}$

$\to C_L = 3P \cdot \pi r h$ ✓

$C_B = P \cdot \pi r^2$

$A_L = 2\pi r h$

$C_T = C_L + C_B$

$C_T = 3P\pi r h + P\pi r^2$ fun obj

$V = \frac{\pi r^2 h}{2}$ fun dato.

r, h variables

P, V constantes.

$$\begin{cases} 3P\pi h + 2P\pi r = \lambda(\pi r h) & (1) \\ 3P\pi r = \lambda\left(\frac{\pi r^2}{2}\right) & (2) \\ \frac{\pi r^2 h}{2} = V & (3) \end{cases}$$

$$\frac{3\cancel{P}\cancel{\pi}h + 2\cancel{P}\cancel{\pi}r}{3\cancel{P}\cancel{\pi}r} = \frac{\cancel{\lambda}\cancel{\pi}rh}{\frac{\cancel{\lambda}\cancel{\pi}r^2}{2}}$$

$$\frac{3h + 2r}{3\cancel{r}} = \frac{2h}{\cancel{r}} \qquad r \neq 0$$

$3h + 2r = 6h$

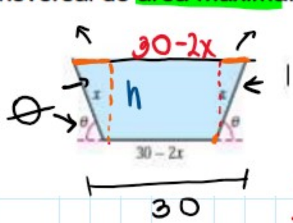
$2r = 3h \qquad h = \frac{2}{3}r$

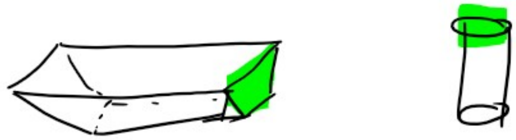
$$\frac{\pi r^2 \cdot \frac{2}{3}r}{2} = V$$

$$\frac{\cancel{2}\pi r^3}{\cancel{6}_{3}} = V \Rightarrow r^3 = \frac{3V}{\pi}$$

$$r = \sqrt[3]{\frac{3V}{\pi}} \quad //$$

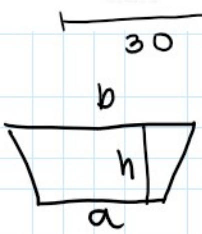
79. Un comedero de secciones transversales en forma de trapecio se forma doblando los extremos de una lámina de aluminio de 30 pulgadas de ancho (ver la figura). Hallar la sección transversal de área máxima.





x, h

fun obj $\quad A = \frac{\left[(30-2x) + (30 - 2x + 2\sqrt{x^2-h^2})\right] h}{2}$



fun obj

$$A = \frac{\left[(30-2x) + (30-2x+2\sqrt{x^2-h^2})\right]h}{2}$$

$$A = \frac{(a+b)h}{2}$$

$$A = (30-2x)h + \frac{2h\sqrt{x^2-h^2}}{2}$$

Copia x = Mayo!

$$x^2 = a^2 + h^2$$

$$a = \sqrt{x^2-h^2}$$

$$A = \frac{\left(60-4x+2\sqrt{x^2-h^2}\right)h}{2}$$

$$A = \left(30-2x+\sqrt{x^2-h^2}\right)h \quad \text{C.Q.D}$$

P.C.

$$\begin{cases} A_x = 0 \\ A_h = 0 \end{cases}$$

$$\begin{cases} -2h + \dfrac{xh}{\sqrt{x^2-h^2}} = 0 \implies x = \dfrac{2h}{\sqrt{3}} \quad x^2 = \dfrac{4h^2}{3} \\ 30-2x+\sqrt{x^2-h^2} - \dfrac{h^2}{\sqrt{x^2-h^2}} = 0 \end{cases}$$

$$\sqrt{\frac{4h^2}{3} - h^2} = \sqrt{\frac{1}{3}h^2} = \frac{h}{\sqrt{3}}$$

$$30 - 2\cdot\frac{2h}{\sqrt{3}} + \frac{h}{\sqrt{3}} - \frac{h^2}{\frac{h}{\sqrt{3}}} = 0 \implies 30 - \frac{4h}{\sqrt{3}} + \frac{h}{\sqrt{3}} - h\sqrt{3} = 0$$

$$30 - \frac{3}{\sqrt{3}}h - h\sqrt{3} = 0$$

$$30 = \left(\frac{3}{\sqrt{3}} + \sqrt{3}\right)h$$

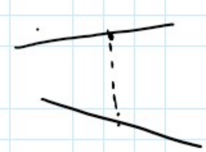
$A =$

$$h = \frac{30}{\frac{3}{\sqrt{3}} + \sqrt{3}} = 5\sqrt{3}$$

$x = 10$ pul.

$$A = \left(30-2x+\sqrt{x^2-h^2}\right)h$$

$$A = 75\sqrt{3}\,[\text{pul}^2] \approx 130 \text{ pul}^2$$



81. Determine los puntos más cercanos y la mínima distancia entre las rectas L_1 y L_2 si:

$L_1: \begin{cases} x = 1 + 4t \\ y = -2 + t \end{cases} \quad L_2: \begin{cases} x = 3 + 2s \\ y = 1 + 3s \end{cases}$

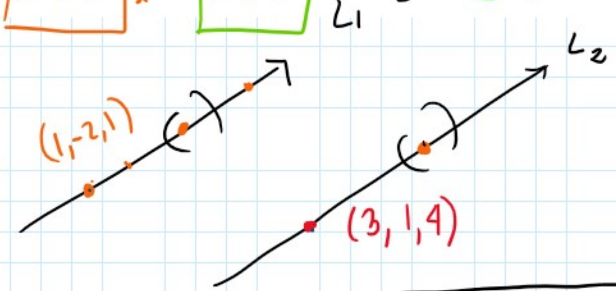
$v_1 = \langle 4, 1, -3 \rangle$

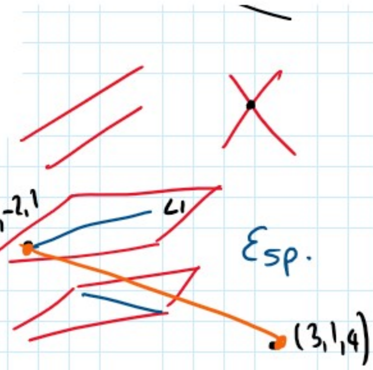
$v_2 = \langle 2, 3, -1 \rangle$

$L_1: \begin{cases} x = 1 + 4t \\ y = -2 + t \\ z = 1 - 3t \end{cases}$ $\quad L_2: \begin{cases} x = 3 + 2s \\ y = 1 + 3s \\ z = 4 - s \end{cases}$

$v_1 = \langle 4, 1, -3 \rangle$

$v_2 = \langle 2, 3, -1 \rangle$





Fun Ob. $\rightarrow$ $d = \sqrt{(1+4t-3-2s)^2 + (-2+t-1-3s)^2 + (1-3t-4+s)^2}$

$D = (1+4t-3-2s)^2 + (-2+t-1-3s)^2 + (1-3t-4+s)^2$

$D_s = 28s - 28t + 20 = 0$

$D_t = 52t - 28s - 4 = 0$

$t = -\frac{2}{3} \qquad s = -\frac{29}{21}$

$$\begin{array}{r} 28s - 28t = -20 \\ -28s + 52t = 4 \\ \hline +24t = -16 \end{array}$$

$t = \frac{-16}{24} = -\frac{2}{3}$

$L_1: \begin{cases} x = 1 + 4t \\ y = -2 + t \\ z = 1 - 3t \end{cases}$

$x = 1 + 4\left(-\frac{2}{3}\right) = 1 - \frac{8}{3} = -\frac{5}{3}$

$y = -2 - \frac{2}{3} = -\frac{8}{3}$

$z = 1 - 3\cdot\left(-\frac{2}{3}\right) = 3$

$P\left(-\frac{5}{3}, -\frac{8}{3}, 3\right)$.

$L_2: \begin{cases} x = 3 + 2s \\ y = 1 + 3s \\ z = 4 - s \end{cases}$