Clase 12/10

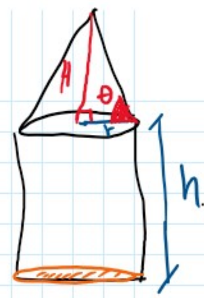
martes, 12 de octubre de 2021 04:52 p. m.

72. Un cilindro circular recto esta coronado por un cono, la superficie total de este cuerpo es de $S = 10cm^2$. De manera que el volumen sea máximo, hallar el ángulo entre la cara lateral del cono, con la cara Basal.

RESP.: $41.81°$

$\tan\theta = \frac{H}{r}$

$\theta =$.

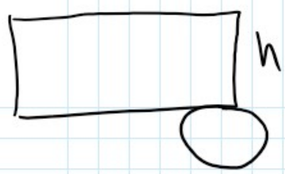


$S_T = 10\ cm^2 \Rightarrow$ fun. Cond

$\to V = \quad \Rightarrow$ fun Ob.

r, h, H

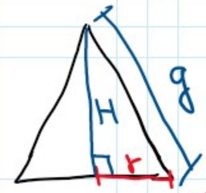
$S_T = S_{\square} + S_{\Delta}$



$S_{\square} = S_B + S_L$

$S_{\square} = \pi r^2 + 2\pi r h$

$S_{\Delta} =$



$g = \sqrt{H^2 + r^2}$

$S_{\Delta} = \pi g r$

$= \pi r \sqrt{H^2 + r^2}$

fun Cond $\Rightarrow S_T = \pi r^2 + 2\pi r h + \pi r \sqrt{H^2 + r^2} = 10$

fun Obj $\Rightarrow$

$V_T = V_{\square} + V_{\Delta} = \pi h r^2 + \frac{\pi H r^2}{3}$ $\quad r, H, h.$

$$
\begin{cases}
2\pi h r + \frac{2}{3}\pi H r = \lambda\left(2\pi r + 2\pi h + \pi\sqrt{H^2+r^2} + \pi r \frac{1}{2\sqrt{H^2+r^2}} \cdot 2r\right) & (1) \\
\pi r^2 = \lambda(2\pi r) & (2) \\
\frac{\pi r^2}{3} = \lambda\left(\pi r \frac{1}{2\sqrt{H^2+r^2}} \cdot 2H\right) & (3) \\
\pi r^2 + 2\pi r h + \pi r\sqrt{H^2+r^2} = 10 & (4)
\end{cases}
$$

$\to r = 2\lambda \Rightarrow \lambda = \frac{r}{2}$

(2) / (3)

$$\frac{\cancel{\pi r^2}}{\frac{\cancel{\pi r^2}}{3}} = \frac{2\cancel{\lambda}\cancel{\pi r}}{\frac{\cancel{\lambda}\cancel{\pi r} H}{\sqrt{H^2+r^2}}} \Rightarrow 3 = \frac{2\sqrt{H^2+r^2}}{H} \Rightarrow \sqrt{H^2+r^2} = \frac{3H}{2} \ *$$

$H^2 + r^2 = \frac{9}{4}H^2$

$2\pi h r + \frac{2}{3}\pi H r = \lambda\left(2\pi r + 2\pi h + \pi\sqrt{H^2+r^2} + \pi r \frac{1}{2\sqrt{H^2+r^2}} \cdot 2r\right)$

$4H^2 + 4r^2 - 9H^2$

$$2\pi hr + \frac{2}{3}\pi Hr = \lambda\left(2\pi r + 2\pi h + \pi\sqrt{H^2+r^2} + \pi r\frac{1}{2\sqrt{H^2+r^2}}\cdot 2r\right)$$

$$2hr + \frac{2}{3}Hr = \frac{r}{2}\left(2r + 2h + \sqrt{H^2+r^2} + \frac{r^2}{\sqrt{H^2+r^2}}\right)$$

$$2hr + \frac{2}{3}Hr = r^2 + rh + \frac{r}{2}\sqrt{H^2+r^2} + \frac{r^3}{2\sqrt{H^2+r^2}}$$

$$hr + \frac{2}{3}Hr = r^2 + \frac{r}{2}\,\frac{3H}{2} + \frac{r^3}{\cancel{2}\,\frac{3H}{\cancel{2}}}$$

$$hr + \frac{2}{3}Hr = r^2 + \frac{3rH}{4} + \frac{r^3}{3H}$$

$$h + \frac{2}{3}H = r + \frac{3}{4}H + \frac{r^2}{3H}$$

$$h + \frac{2}{3}\cdot\frac{2}{\sqrt{5}}r = r + \frac{3}{4}\,\frac{2}{\sqrt{5}}r + \frac{r^{\cancel{2}}}{3\,\frac{2}{\sqrt{5}}\cancel{r}}$$

$$h = -\frac{4}{3\sqrt{5}}r + r + \frac{3}{2\sqrt{5}}r + \frac{\sqrt{5}}{6}r$$

$$h = \left(\frac{5+\sqrt{5}}{5}\right)r$$

$$4H^2 + 4r^2 = 9H^2$$

$$\Rightarrow 4r^2 = 5H^2$$

$$* \quad H^2 = \frac{4}{5}r^2$$

$$\downarrow$$

$$\frac{4}{5}r^2 + r^2$$

$$H^2 = \frac{4}{5}r^2$$

$$H = \frac{2}{\sqrt{5}}r$$

$$\pi r^2 + 2\pi rh + \pi r\sqrt{H^2+r^2} = 10$$

$$\pi r^2 + 2\pi r\left(\frac{5+\sqrt{5}}{5}\right)r + \pi r\sqrt{\frac{4}{5}r^2 + r^2} = 10$$

$$\pi r^2 + \frac{2}{5}\pi r^2(5+\sqrt{5}) + \pi r\cdot\frac{3}{\sqrt{5}}r = 10$$

$$\pi r^2 + \frac{2}{5}\pi r^2(5+\sqrt{5}) + \frac{3\pi r^2}{\sqrt{5}} = 10$$

$$\sqrt{\frac{9}{5}r^2} = \frac{3}{\sqrt{5}}r$$

$$r^2\left(\pi + \frac{2}{5}\pi(5+\sqrt{5}) + \frac{3\pi}{\sqrt{5}}\right) = 10$$

$$r^2(16.0824) = 10$$

$$r^2 = \frac{10}{16.0824}$$

$$r = 0.79.$$

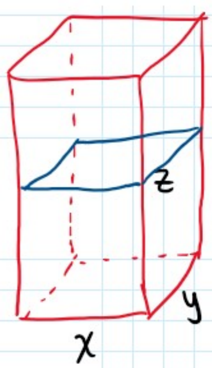
$$h = \left(\frac{5+\sqrt{5}}{5}\right)r \qquad h = 1.14.$$

$$H = \frac{2}{\sqrt{5}}r \qquad H = 0,6976$$

$$\theta = \tan^{-1}\left(\frac{0.6976}{0.79}\right)$$

$$\theta = 41.45^\circ$$

71. Hallar el máximo volumen de una caja rectangular, si su altura más su fleje (f) es $254cm$. Fleje es la longitud alrededor de la caja, medida ~~mente~~ perpendicular a su lado más largo, da su altura.



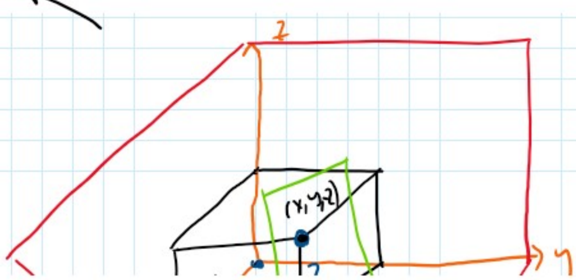
$V \Rightarrow$ func Obj.

$$z + \overbrace{2x + 2y}^{f} = 254 \quad \text{Cond.}$$

$$V = xyz. \quad \text{fob}$$

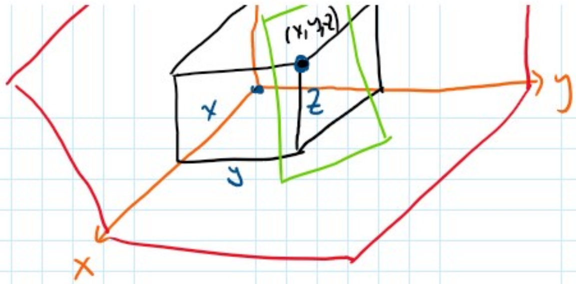
74. Hallar las dimensiones del paralelepípedo de volumen máximo, si tres de sus caras están sobre los planos coordenadas y un vértice se encuentra sobre el plano:

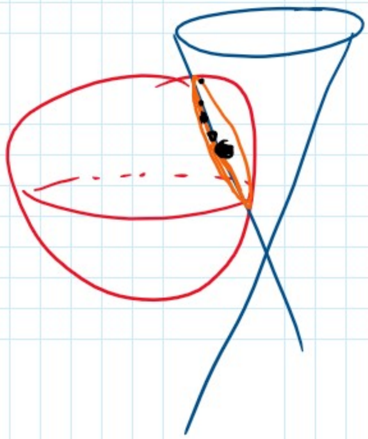
$$3x + 2y + z - 6 = 0$$



$V = xyz \quad$ f. obj.

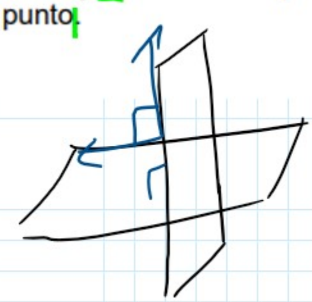
$3x+2y+z-6=0 \quad$ func Restric.





63. a) Demostrar que las superficies intersecan en el punto dado y b) demostrar que las superficies tienen planos tangentes perpendiculares en este punto.

a. $z = 2xy^2 \qquad 8x^2 - 5y^2 - 8z = -13 \qquad (1,1,2)$



$2 = 2.1.1^2$

$2 = 2$ ✓

$8.1^2 - 5.1^2 - 8.2 = -13$ ✓

$\nabla F = \vec{n}_1 \qquad \nabla G = \vec{n}_2$

$\nabla F \cdot \nabla G = 0 \Rightarrow$

c. $f(x,y) = 2x \quad + xy \quad + 3x \quad + y$

d. $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

$f_x = 3x^2 + 3y^2 - 6x \qquad f_y = 6xy - 6y$

$$\begin{cases} 3x^2 + 3y^2 - 6x = 0 \\ 6xy - 6y = 0 \Rightarrow 6y(x-1) = 0 \end{cases}$$

$y = 0 \qquad x = 1$

Part.

Si $y = 0$

$3x^2 - 6x = 0$

Si $x = 1$

$3 + 3y^2 - 6 = 0$

Si $y=0$

$3x^2-6x=0$

$3x(x-2)=0$

$x=0 \quad x=2$

Si $x=1$

$3+3y^2-6=0$

$3y^2=3$

$y=\pm 1$

PC $\boxed{(0,0) \quad (2,0) \qquad (1,1) \quad (1,-1)}$

. $f(x,y) = xy - \ln(x^2+y^2)$

$f_x = y - \frac{1}{x^2+y^2} \cdot 2x \qquad f_y = x - \frac{1}{x^2+y^2} 2y$

PC $\{(0,0)$

$(1,1) \quad (-1,-1)$

$y=x$

$$\begin{cases} y - \frac{2x}{x^2+y^2} = 0 \\ x - \frac{2y}{x^2+y^2} = 0 \end{cases}$$

$$\begin{cases} y(x^2+y^2) - 2x = 0 & -x \\ x(x^2+y^2) - 2y = 0 & y \end{cases}$$

$-yx(x^2+y^2) + 2x^2 = 0$

$-yx(x^2+y^2) - 2y^2 = 0$

$+2x^2 - 2y^2 = 0$

$x^2 - y^2 = 0$

$(x-y)(x+y) = 0$

$y(x^2+y^2) - 2x = 0$

$x=y \quad x=-y$

Si $x=y.$

$y(y^2+y^2) - 2y = 0$

$2y^3 - 2y = 0 \qquad 2y(y-1) = 0 \qquad y=0 \quad y=1$

$x=0 \quad x=1$

$(0,0) \quad (1,1)$

si $x=-y$

$-y(y^2+y^2)+2y=0$

$-2y^3+2y=0$ $\quad$ $y=0$ $\quad$ $y=1$

$2y(-y+1)=0$ $\quad$ $x=0$ $\quad$ $x=1$

$y=-1$

$d(0,0)=0$

$d>0$ Ext

$d<0$ S

$d=0$ ✓