Clase 09/09

jueves, 9 de septiembre de 2021 02:05 p. m.

a) **Example** *Describir la función* $f(x,y) = 6 - 3x - 2y$

Example *Describir la función* $f(x,y) = \sqrt{16 - 4x^2 - y^2}$

i) $D_f = \mathbb{R}^2 = \{ (x,y) \in \mathbb{R}^2 \}$.

ii) graf del dominio de f.

$(x,y,z) \quad (x,y) \in D_f \quad z = f(x,y)$.

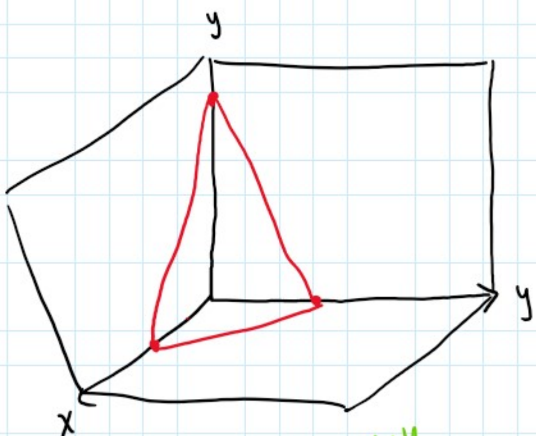
$f(x,y) = 6 - 3x - 2y$

$\rightarrow z = 6 - 3x - 2y$.. Que es?

$y=0 \quad z=0 \quad 0 = 6 - 3x \quad x = 2 \quad P(2,0,0)$

$x=0 \quad z=0 \quad 0 = 6 - 2y \quad y = 3 \quad P(0,3,0)$

$x=0 \quad y=0 \quad z = 6 \quad P(0,0,6)$.



b) **Example** *Describir la función* $f(x,y) = \sqrt{16 - 4x^2 - y^2}$

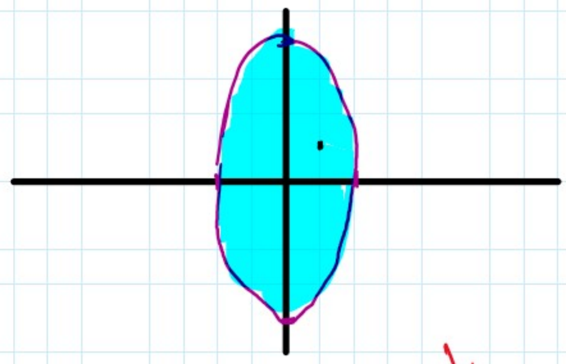
i) $D_f = \{ (x,y) \in \mathbb{R}^2 / \overbrace{16 - 4x^2 - y^2}^{} \geq 0 \}$

$\mathbb{R}^2 \qquad 16 - 4x^2 - y^2 = 0$

$4x^2 + y^2 = 16 \qquad \div 16$

$\frac{x^2}{4} + \frac{y^2}{16} = 1 \quad C(0,0)$

Si $(1,1)$

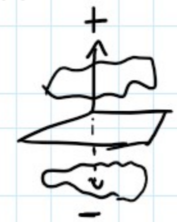
ii) 

Si $(1,1)$

$16-4-1 \geq 0$

$11 \geq 0$ V.

No olvidar!!!



iii) **Example** *Describir la función* $f(x,y) = \pm\sqrt{16-4x^2-y^2}$

z

$$z = \sqrt{16-4x^2-y^2}$$

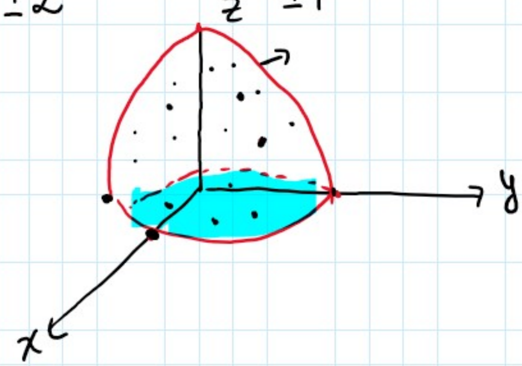
$\mathbb{R}^3$

$$z^2 = 16-4x^2-y^2$$

$$4x^2+y^2+z^2 = 16 \leftarrow \text{Elipsoide}$$

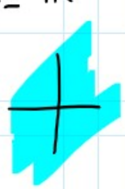
$$\rightarrow \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} = 1 \qquad C(0,0,0)$$

± 2 ± 4



Ej Describir $f(x,y) = \sqrt{\underbrace{x^2+y^2}_{+}}$

i) $D_f = \mathbb{R}^2$

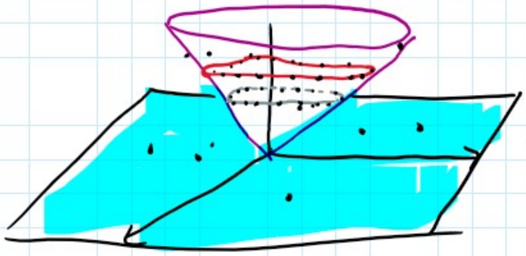
ii) 

iii) $z = \sqrt{x^2+y^2}$

$z^2 = x^2+y^2 \Rightarrow x^2+y^2-z^2 = 0$ Cono elíptico

$z^2 = x^2+y^2 \Rightarrow x^2+y^2-z^2=0$ Cono eliptico

$C(0,0,0)$



Example *Graficar la función y trazar las curvas de nivel de la función*

a) $f(x,y) = 100 - x^2 - y^2$

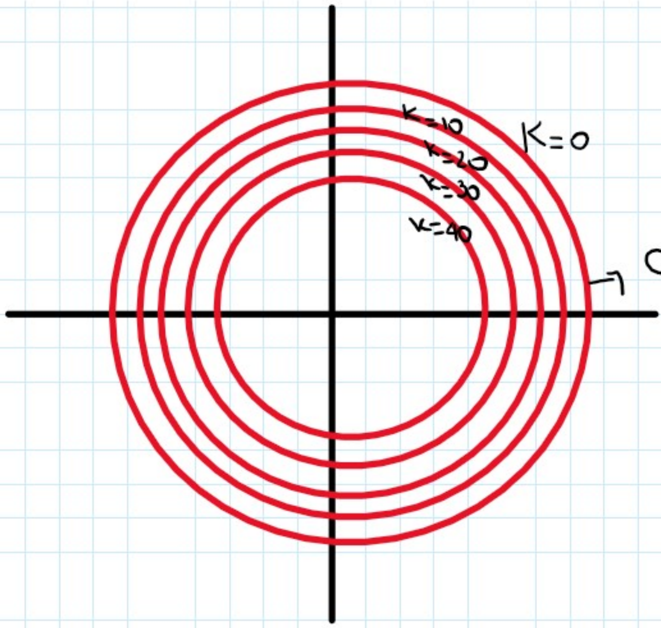
b) $f(x,y) = -\sqrt{x^2+y^2}$

a) $f(x,y) = 100 - x^2 - y^2$ K= Constante

$K = 100 - x^2 - y^2$

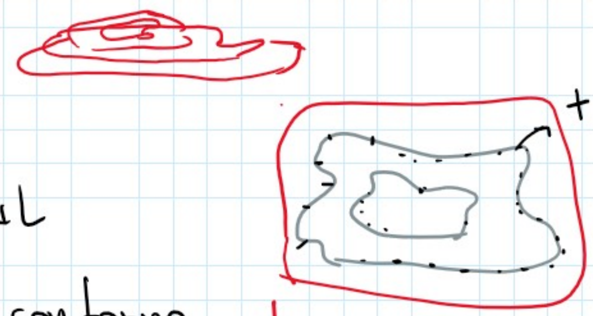
$x^2+y^2 = 100-K \rightarrow C(0,0) \quad r=\sqrt{100-K}$

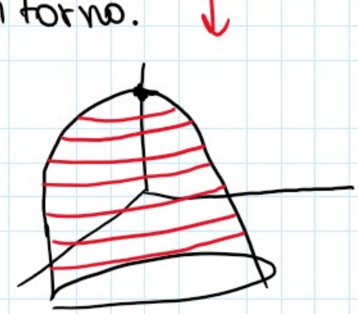
$K=0$	$x^2+y^2=100$	$C(0,0)$	$r=10$
$K=10$	$x^2+y^2=90$	"	$r=\sqrt{90}$
$K=20$	$x^2+y^2=80$	"	$r=\sqrt{80}$
$K=30$	$x^2+y^2=70$	"	$r=\sqrt{70}$
$K=40$	$x^2+y^2=60$	"	$r=\sqrt{60}$
$K=-20$	$x^2+y^2=120$	"	$r=\sqrt{120}$



Curva de nivel

mapa de contorno.





a) $f(x,y) = 100 - x^2 - y^2$

$z = 100 - x^2 - y^2$

$z - 100 = -x^2 - y^2$ Paraboide

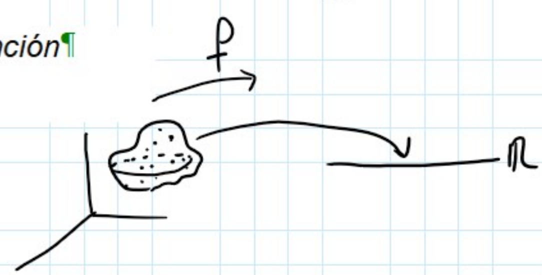
$V(0,0,100)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = \frac{z-p}{c^2}$$

$V(h,k,p)$

$\mathbb{R}^3 — \mathbb{R}$

Example *Describa las superficies de nivel de la función*
$f(x,y,z) = \sqrt{x^2+y^2+z^2}$



$\mathbb{R}^4$

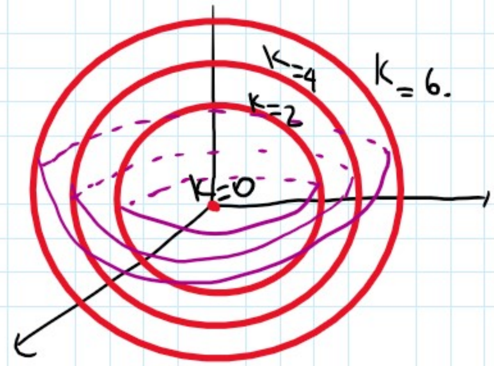
$K = \sqrt{x^2+y^2+z^2}$

$K^2 = x^2+y^2+z^2$ Esf. $C(0,0,0)$ $r = K$.

$K=0$ $x^2+y^2+z^2=0$

$K=2$ $x^2+y^2+z^2=4$

$K=4$ $x^2+y^2+z^2=16$



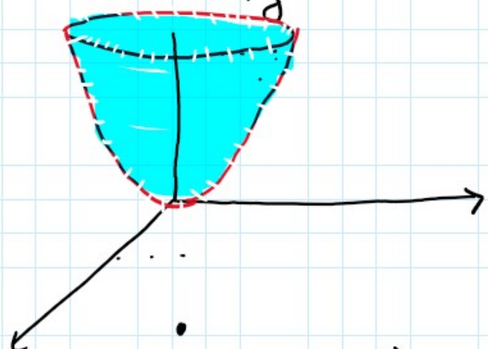
7. Hallar el dominio y graficar las superficies de nivel de la función:

5.

$f(x,y,z) = \log(z - x^2 - y^2)$

$D_f = \{ (x,y,z) \in \mathbb{R}^3 \;/\; z - x^2 - y^2 > 0 \}$

$z = x^2 + y^2$ Paraboloida. $V(0,0,0)$



si $(2, 3, -2)$

$-2 - (2)^2 - 3^2 > 0$

$-2 - 4 - 9 > 0$ F

$K = \log(z - x^2 - y^2)$

$10^K = z - x^2 - y^2$

$x^2 + y^2 = z - 10^K \rightarrow$ Parb.

$V(0,0,10^K)$

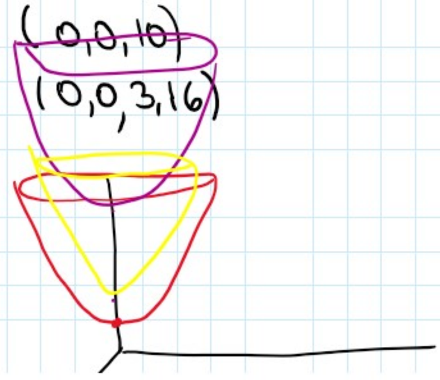
$K = 0 \quad x^2 + y^2 = z - 1 \qquad (0,0,1)$

$K = 1 \quad x^2 + y^2 = z - 10 \qquad (0,0,10)$

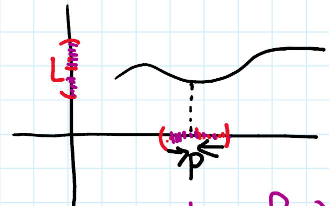
$K = 1/2 \quad x^2 + y^2 = z - 3{,}16 \qquad (0,0,3{,}16)$

$K = 1/4$

$K =$



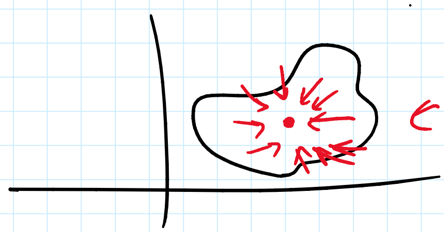
$f(x).$



$$\lim_{x \to p} f(x) = L$$

$$\lim_{x \to p^+} f(x) = \lim_{x \to p^-} f(x)$$

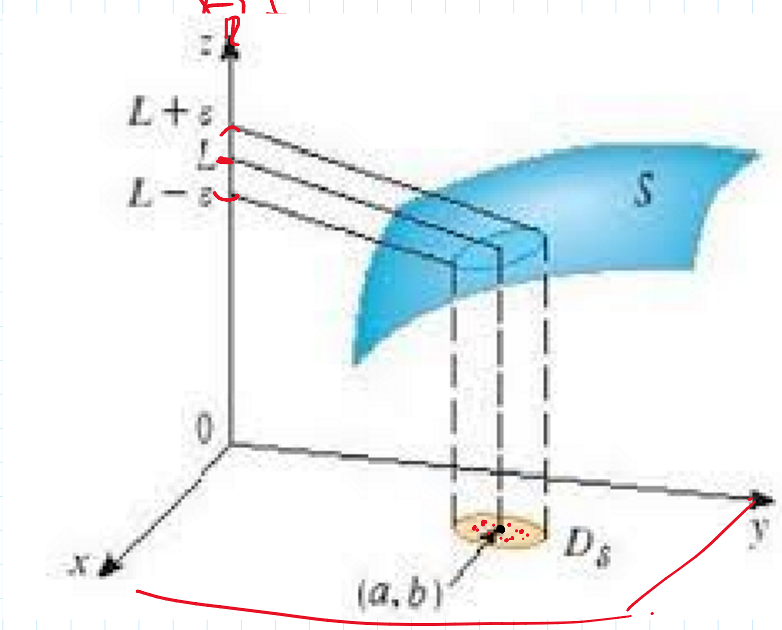
$\Rightarrow$ Lim Exists



$$\lim_{x \to 1} \frac{\sqrt{x-1}}{} =$$

$f(x,y) \to L$ $\quad f$

g



$$\lim_{(x,y) \to (a,b)} f(x,y) = L$$

$$\lim \; x^2 - 4 = 0 \quad ??$$

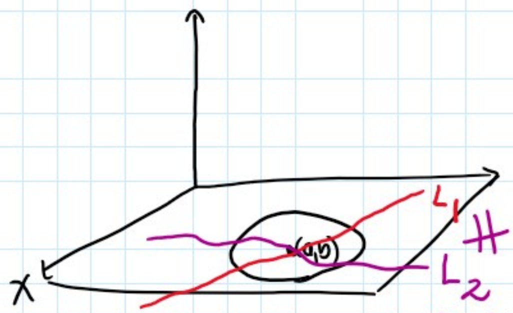
$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$$

L.

$$\lim_{(x,y)\to(1,1)} 3x^2+2y^2 = 5$$

→ . . No Ex.

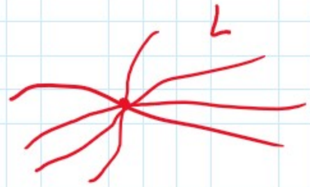
$$= \frac{0}{0} \, ?$$



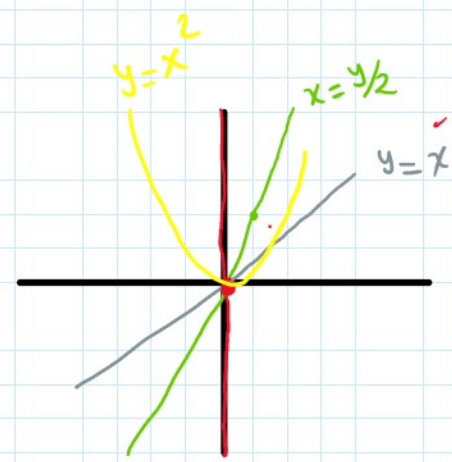
$$\sqrt{(x-a)^2+(y-b)^2}$$

⇒ Limf No Ex.

P(x,y) P(a,b)



(1,1)



Example *Dada la función* g *definida por* $g(x,y) = \frac{3xy}{x^2+y^2}$

Calcule $\lim_{(x,y)\to(0,0)} g(x,y)$ *en cada caso*

a) A lo largo de la recta de ecuación $x = 0$.

b) A lo largo de la recta de ecuación $y = x$.

c) A lo largo de la recta de ecuación $x = \frac{y}{2}$

d) A lo largo de la parábola de ecuación $y = x^2$

e) Concluya acerca de la existencia del límite entorno al orige

a) t_1 $x=0$ ✓

$$\lim_{(0,y)\to(0,0)} \frac{3xy}{x^2+y^2} = \lim_{y\to 0} \frac{0}{y^2} = 0$$

t_2 $\vec{y}=x$

$$\lim_{(x,x)\to(0,0)} \frac{3xy}{x^2+y^2} = \lim_{x\to 0} \frac{3.x.x}{x^2+x^2} = \lim_{x\to 0} \frac{3\cancel{x^2}}{2\cancel{x^2}} = \frac{3}{2}$$

t_3 $x = y/2$

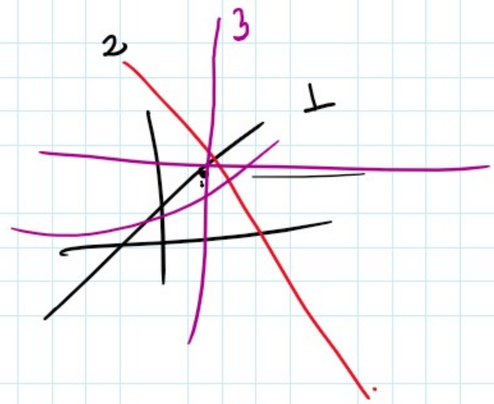
$$\lim_{(\frac{y}{2},y)\to(0,0)} \frac{3xy}{x^2+y^2} = \lim_{y\to 0} \frac{3\,\frac{y}{2}\cdot y}{\frac{y^2}{2}+y^2} = \lim_{y\to 0} \frac{\cancel{\frac{3}{2}y^2}}{\cancel{\frac{3}{2}y^2}} = 1$$

$t_4 \quad y = x^2$

$$\lim_{(x,x^2)\to(0,0)} \frac{3xy}{x^2+y^2} = \lim_{x\to 0} \frac{3xx^2}{x^2+x^4} = \lim_{x\to 0} \frac{3x^3}{x^2+x^4}$$

$$= \lim_{x\to 0} \frac{3x}{1+x^2} = \frac{0}{1} = 0$$

$$\therefore \lim_{(x,y)\to(0,0)} f(x,y) = \text{No Exista.}$$



$$\lim_{(x,y)\to(1,1)} \frac{3x}{x^2-y^2}$$

$-$

2

2 ✓

$L_1 \neq L_2 \implies \lim$ No Ex.