PRACTICA

practica

2) P. E. (0,0)

i) $\frac{dx}{dt} = 3\text{sen}\, x + y$

$\frac{dy}{dt} = 4x + \cos y - 1$

ii) $x' = -3\,\text{sen}\, x + y$

$y' = 4x + \cos y - 1$

iii) $x' = -3\,\text{sen}\, x + y$

$y' = 4x + 3\cos y - 3$

Saco el jacobiano: en el p(0,0)

i) $\begin{bmatrix} 3\cos x & 1 \\ 4 & -\text{sen}\, y \end{bmatrix} \to \begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\frac{dx}{dt} = 3x + y$ $\quad$ $\frac{dy}{dt} = 4x$

ii) $\begin{bmatrix} -3\cos x & 1 \\ 4 & -\text{sen}\, y \end{bmatrix} \to \begin{bmatrix} -3 & 1 \\ 4 & -0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\frac{dx}{dt} = -3x + y$ $\quad$ $\frac{dy}{dt} = 4x$

iii) $\begin{bmatrix} -3\cos x & 1 \\ 4 & -3\text{sen}\, y \end{bmatrix} \to \begin{bmatrix} -3 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\frac{dx}{dt} = -3x + y$ $\quad$ $\frac{dy}{dt} = 4x$

Iggd

∴ Los sistemas ii y iii tienen el mismo retrato en (0,0)

4) $\frac{dx}{dt} = -x$ $\quad$ $\frac{dy}{dt} = -4x^3 + y$

a) Busco puntos de eq

$\left.\begin{array}{l} 0 = -x \quad ① \\ 0 = -4x^3 + y \quad ② \end{array}\right\}$

En ① $\boxed{x = 0}$

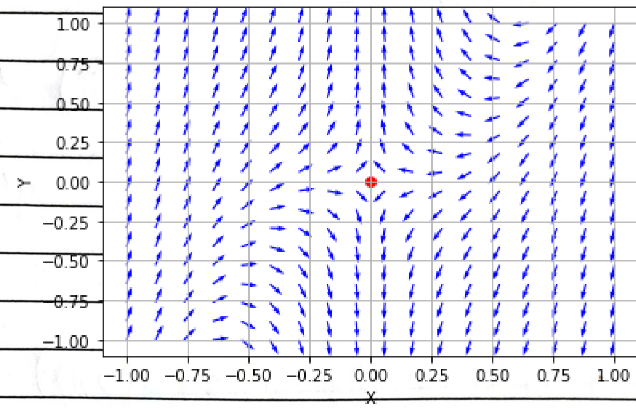
Reemplazo: $0 = -4(0)^3 + y$

$\boxed{y = 0}$

b) Jacobiano (0,0)

$$\begin{bmatrix}-1 & 0\\ -12x^2 & 1\end{bmatrix} \to \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} \to \frac{dx}{dt}=-x \qquad \frac{dy}{dt}=y$$

c) $$\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} - \lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = 0$$



$$\begin{bmatrix}-1-\lambda & 0\\ 0 & 1-\lambda\end{bmatrix} = 0$$

$$(-1-\lambda)(1-\lambda)=0$$

$\lambda_1 = -1$ $\qquad$ $\lambda_2 = 1$

$y = c_1 e^{-t} + c_2 e^{t}$

- Punto silla

6) $$\frac{dx}{dt} = 2x\left(1 - \frac{x}{2}\right) - xy$$

$$\frac{dy}{dt} = 3y\left(1 - \frac{y}{3}\right) - 2xy$$

a)

$$\left.\begin{array}{l} 0 = x(2-x-y) \\ 0 = y(3-y-2x) \end{array}\right\} \begin{array}{l} ① \\ ② \end{array}$$

Si $x=0$

$0 = y(3-y-\cancel{2(0)})$

$0 = y(3-y)$

$y=0 \qquad y=3$

$P(0,0)$ $\qquad$ $P(0,3)$

Si $x = 2-y$

$0 = y(3-y-2(2-y))$

$0 = y(-1+y)$

$y=0 \qquad y=1$

Reemplazo:

$x = 2-0 \qquad x = 2-1$

$x = 2 \qquad x = 1$

$P(2,0)$ $\qquad$ $P(1,1)$

Jacobiano:

$$\begin{bmatrix} 2-2x-y & -x \\ -2y & 3-2y-2x \end{bmatrix}$$

• $P(0,0)$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \frac{dx}{dt}=2x \quad \frac{dy}{dt}=3y$$

• $P(0,3)$

$$\begin{bmatrix} -1 & 0 \\ -6 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \frac{dx}{dt}=-x \quad \frac{dy}{dt}=-6x-3y$$

• $P(2,0)$

$$\begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \frac{dx}{dt}=-2x-2y \quad \frac{dy}{dt}=-y$$

• $P(1,1)$

$$\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \frac{dx}{dt}=-x-y \quad \frac{dy}{dt}=-2x-y$$

b) Con los autovalores

• $P(0,0)$ $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(3-\lambda)=0$$

$\lambda_1 = 2$ $\lambda_2 = 3$

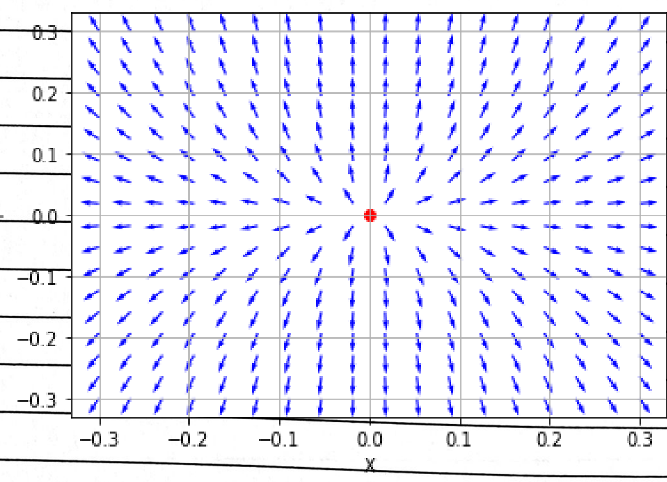
• Fuente

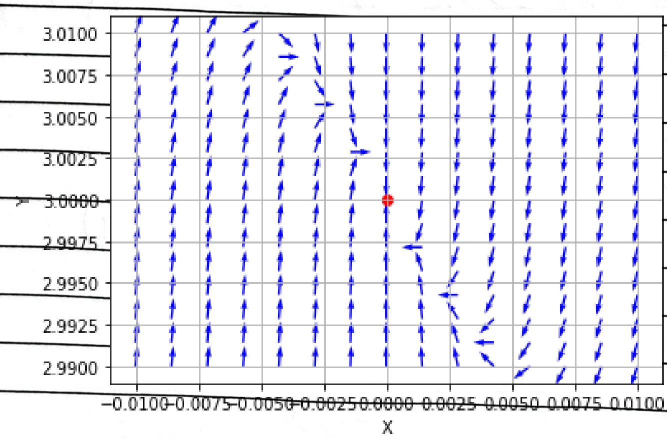
• $P(0,3)$ $\begin{bmatrix} -1 & 0 \\ -6 & -3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$

$$(-1-\lambda)(-3-\lambda)=0$$

$\lambda_1 = -1$ $\lambda_2 = -3$ • Sumidero

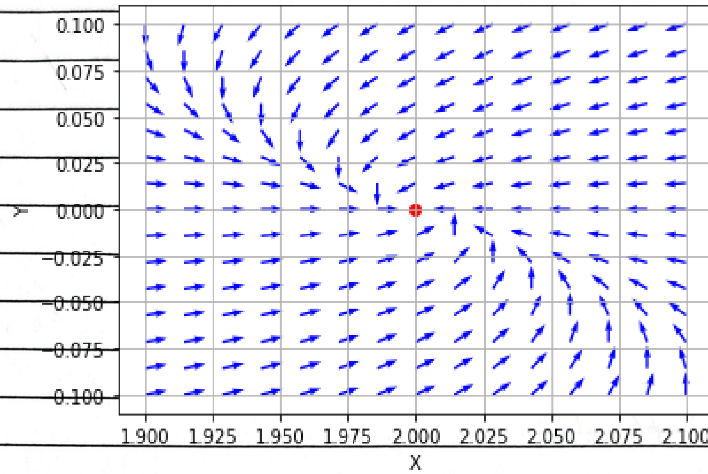
c)





$y = C_1 e^{-t} + C_2 e^{-3t}$

- $P(2,0)$



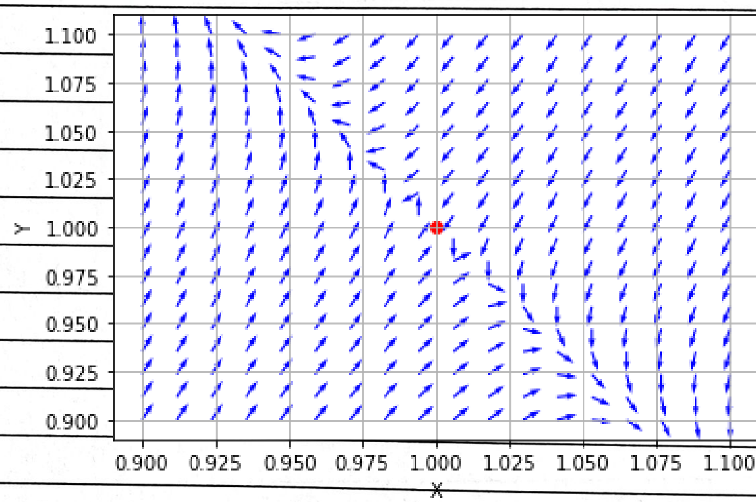
$$\begin{bmatrix} -2-\lambda & -2 \\ 0 & -1-\lambda \end{bmatrix} = 0$$

$$(-2-\lambda)(-1-\lambda) = 0$$

$$\lambda_1 = -2 \checkmark \quad \lambda_2 = -1 \checkmark$$

- Sumidero $y = C_1 e^{-2t} + C_2 e^{-t}$

- $P(1,1)$



$$\begin{bmatrix} -1-\lambda & -1 \\ -2 & -1-\lambda \end{bmatrix} = 0$$

$$1 + 2\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 + 2\lambda - 1 = 0$$

$$\lambda_1 = -1 + \sqrt{2} \checkmark \quad \lambda_2 = -1 - \sqrt{2} \checkmark$$

$$y = C_1 e^{-1+\sqrt{2}} + C_2 e^{-1-\sqrt{2}}$$

- Punto Silla

8) Solo primer cuadrante

- $\frac{dx}{dt} = x(10 - x - y) \qquad \frac{dy}{dt} = y(30 - 2x - y)$

$\left.\begin{array}{l} 0 = x(10-x-y) \\ 0 = y(30-2x-y) \end{array}\right\}$ Si $x = 0$ | Si $x = 10 - y$

$0 = y(30 - 2(0) - y) \qquad 0 = y(30 - 2(10-y) - y)$

$0 = y(30 - y) \qquad 0 = y(10 + y)$

$y = 0 \quad y = 30 \qquad y = 0 \quad y = -10$ ✗

$P(0,0)$ $P(0,30)$

Jacobiano

$$\begin{bmatrix} 10-2x-y & -x \\ -2y & 30-2x-2y \end{bmatrix}$$

$X = 10 - 0$

$X = 10$

$P(10,0)$

• $P(0,0)$

a)

$$\begin{bmatrix} 10 & 0 \\ 0 & 30 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \to \frac{dx}{dt} = 10x \qquad \frac{dy}{dt} = 30y$$

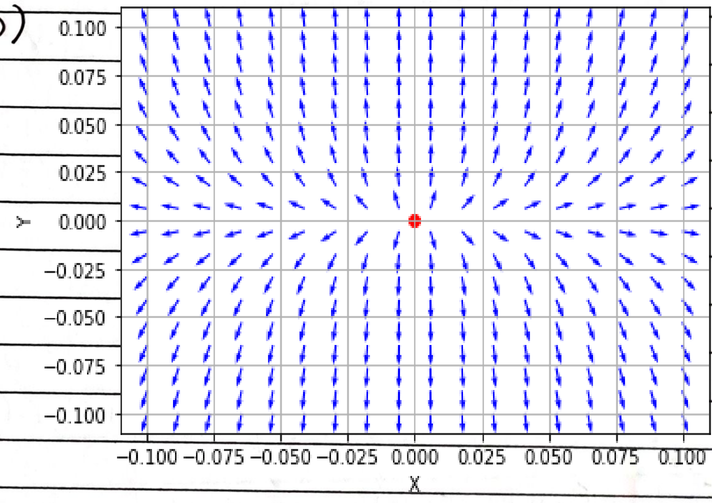
$$\begin{bmatrix} 10-\lambda & 0 \\ 0 & 30-\lambda \end{bmatrix} = 0$$

$$(10-\lambda)(30-\lambda) = 0$$

$$\lambda_1 = 10 \quad \lambda_2 = 30$$

$y = C_1 e^{10t} + C_2 e^{30t}$ • Fuente

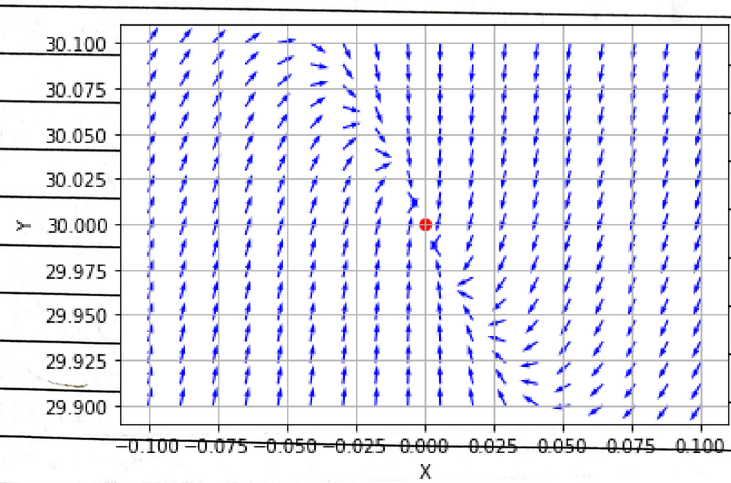
b)



• $P(0,30)$

$$\begin{bmatrix} -20 & 0 \\ -60 & -30 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{dx}{dt} = -20x \qquad \frac{dy}{dt} = -60x - 30y$$



$$\begin{bmatrix} -20-\lambda & 0 \\ -60 & -30-\lambda \end{bmatrix} = 0$$

$$(-20-\lambda)(-30-\lambda) = 0$$

$$\lambda_1 = -20 \quad \lambda_2 = -30$$

$y = C_1 e^{-20t} + C_2 e^{-30t}$ • Sumidero

• $P(10,0)$

$$\begin{bmatrix} -10 & -10 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

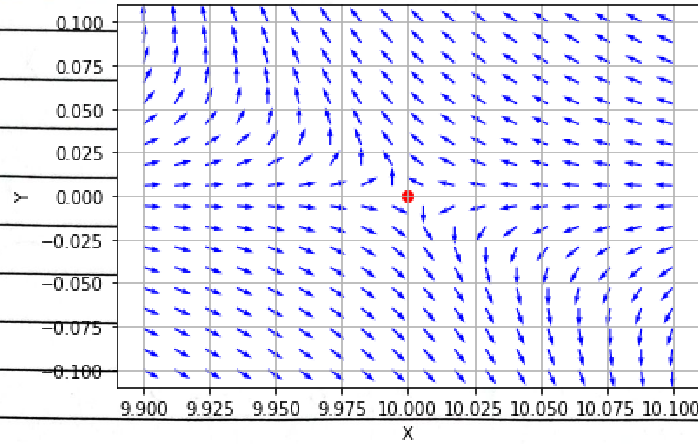
$$\frac{dx}{dt} = -10x - 10y \qquad \frac{dy}{dt} = 10y$$

$$\begin{bmatrix} -10-\lambda & -10 \\ 0 & 10-\lambda \end{bmatrix} = 0$$

$$(-10-\lambda)(10-\lambda) = 0$$

$\lambda = -10$ ✓ $\quad \lambda = 10$ ✓

$y = C_1 e^{-10t} + C_2 e^{10t}$ • Punto Silla



10)

$$\frac{dx}{dt} = x(-x-y+100) \qquad \frac{dy}{dt} = y(-x^2-y^2+2500)$$

$$\left.\begin{aligned} 0 &= -x^2 - yx + 100x \\ 0 &= -x^2y - y^3 + 2500y \end{aligned}\right\} \to 0 = x(-x-y+100)$$

Si $x = 0$	Si $x = -y+100$
$0 = -(0)^2 y - y^3 + 2500y$	$0 = -(-y+100)^2 y - y^3 + 2500y$
$0 = y(-y^2+2500)$	$0 = -y^3 + 200y^2 - 10000y - y^3 + 2500y$
$y = 0 \quad 0 = -y^2 + 2500$	$0 = -2y^3 + 200y^2 - 7500y$
$y = \pm\sqrt{2500}$	$0 = -2y(y^2 + 100y + 3750)$
$y = 50 \quad y = -50$ X	$y = 0 \quad y = 50+25\sqrt{2}i \quad y = 50-25\sqrt{2}i$
$P(0,0) \quad P(0,50)$	$x = 100 \quad x = 50-25\sqrt{2}i$ X $\quad x = 50+25\sqrt{2}i$ X
	$P(100,0)$

Jacobiano:

$$\begin{bmatrix} -2x-y+100 & -x \\ -2xy & -x^2-3y^2+2500 \end{bmatrix}$$

$P(0,0)$

$$\begin{bmatrix} 100 & 0 \\ 0 & 2500 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{dx}{dt} = 100x \qquad \frac{dy}{dt} = 2500y$$

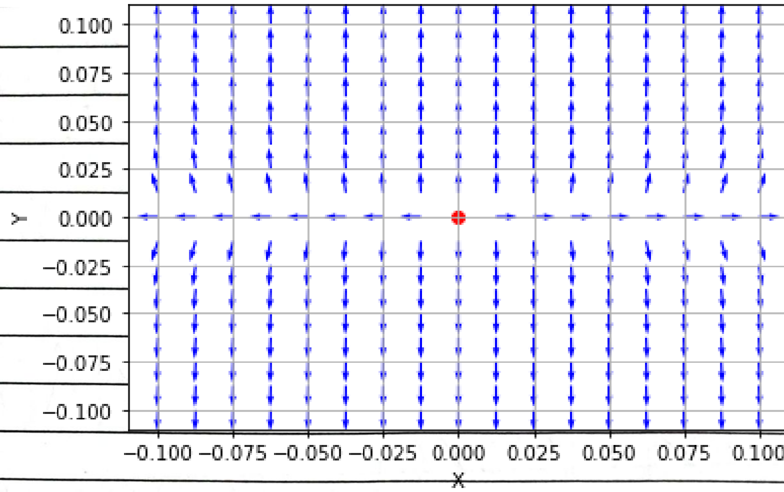
$$\begin{bmatrix} 100-\lambda & 0 \\ 0 & 2500-\lambda \end{bmatrix} = 0$$

$$(100-\lambda)(2500-\lambda) = 0$$

$\lambda = 100$ ✓ $\lambda = 2500$ ✓

$y = C_1 e^{100t} + C_2 e^{2500t}$

- Fuente



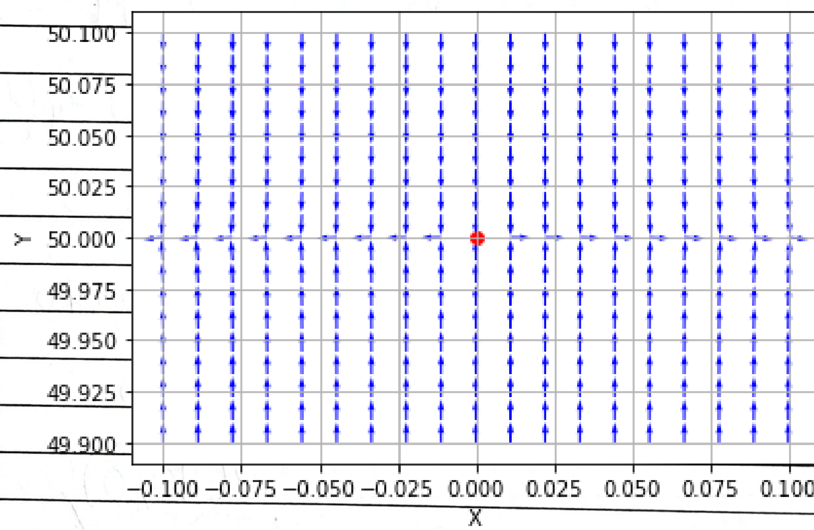
- $P(0,50)$

$$\begin{bmatrix} 50 & 0 \\ 0 & -5000 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\frac{dx}{dt} = 50x - \quad \frac{dy}{dt} = -5000y$

$$\begin{bmatrix} 50-\lambda & 0 \\ 0 & -5000-\lambda \end{bmatrix} = 0$$

$$(50-\lambda)(-5000-\lambda) = 0$$



$\lambda = 50$ ✓ $\lambda = -5000$ ✓

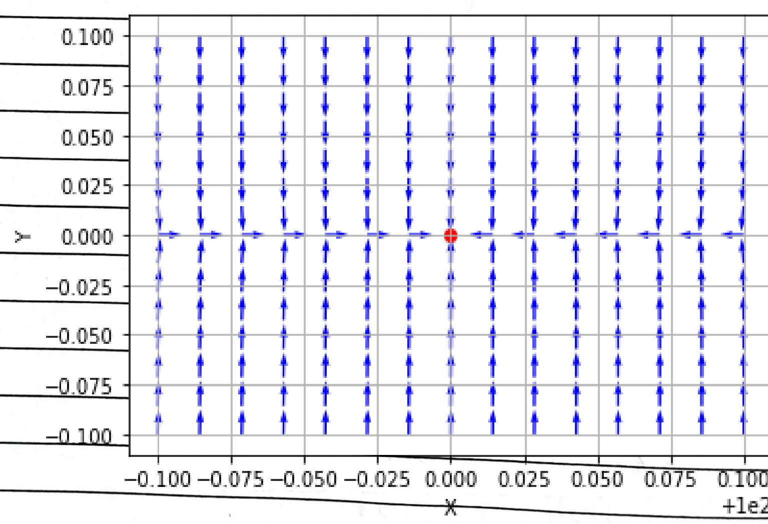
$y = C_1 e^{50t} + C_2 e^{-5000t}$

- Punto silla

- $P(100,0)$

$$\begin{bmatrix} -100 & -100 \\ 0 & -7500 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\frac{dx}{dt} = -100x - 100y \quad \frac{dy}{dt} = -7500y$



$$\begin{bmatrix} -100-\lambda & -100 \\ 0 & -7500-\lambda \end{bmatrix} = 0$$

$$(-100-\lambda)(-7500-\lambda) = 0$$

$\lambda = -100$ ✓ $\lambda = -7500$ ✓

$y = C_1 e^{-100t} + C_2 e^{-7500t}$ • Sumidero

12) $\frac{dx}{dt} = x(-4x - y + 160)$ $\qquad$ $\frac{dy}{dt} = y(-x^2 - y^2 + 2500)$

$$\left.\begin{aligned} 0 &= -4x^2 - yx + 160x \\ 0 &= -x^2y - y^3 + 2500y \end{aligned}\right\}$$

Si $x = 0$

$0 = -\cancel{0^2y} - y^3 + 2500y$

$0 = y(-y^2 + 2500)$

$y = 0$ $\qquad$ $y = \pm 50$

$P(0,0)$ $\qquad$ $P(0,50)$

Si $x = -\frac{y}{4} + 40$

$0 = -\left(-\frac{y}{4} + 40\right)^2 y - y^3 + 2500y$

$(-16)\ 0 = -\frac{y^3}{16} + 20y^2 - 1600y - y^3 + 2500y$

$0 = 17y^3 - 320y^2 - 14400y$

$0 = y(17y^3 - 320y - 14400)$

$y = 0$ $\qquad$ $y = 40$ $\qquad$ $y = -\frac{360}{17}$ X

$x = 0 + 40$ $\qquad$ $x = -\frac{40}{4} + 40$

$x = 40$ $\qquad$ $x = 30$

$P(40,0)$ $\qquad$ $P(30,40)$

Jacobiano:

$$\begin{bmatrix} -8x - y + 160 & -x \\ -2xy & -x^2 - 3y^2 + 2500 \end{bmatrix}$$

$P(0,0)$

$$\begin{bmatrix} 160 & 0 \\ 0 & 2500 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$\frac{dx}{dt} = 160x$ $\qquad$ $\frac{dy}{dt} = 2500y$

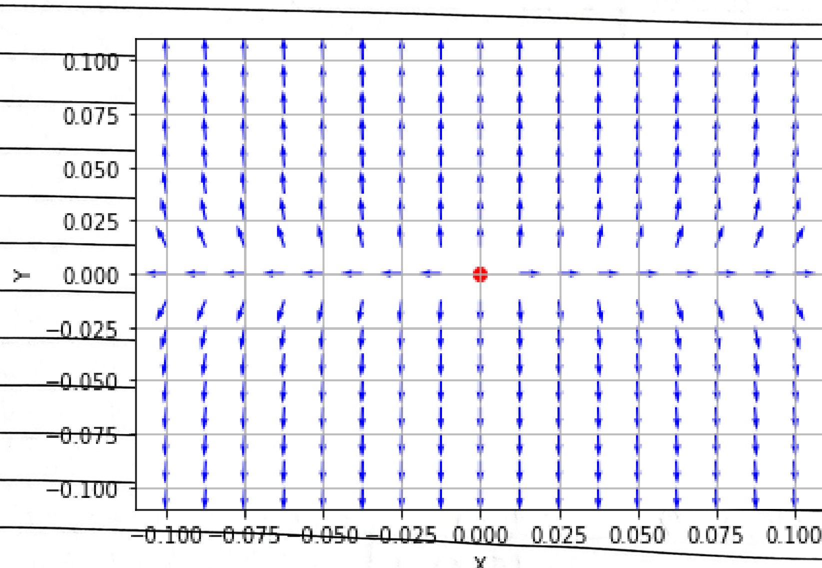
$$\begin{vmatrix} 160-\lambda & 0 \\ 0 & 2500-\lambda \end{vmatrix} = 0$$

$(160 - \lambda)(2500 - \lambda) = 0$

$\lambda = 160$ ✓ $\qquad$ $\lambda = 2500$ ✓

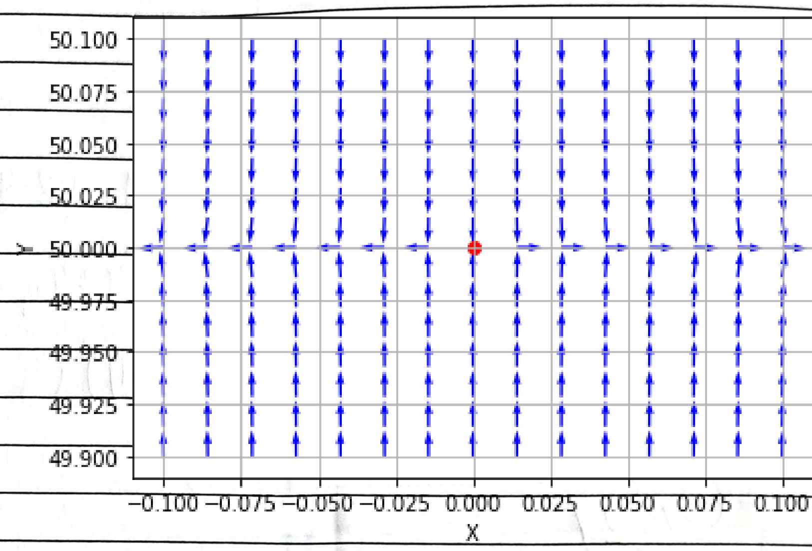
$y = C_1 e^{160t} + C_2 e^{2500t}$

- Fuente



• P(0,50)

$$\begin{bmatrix} 110 & 0 \\ 0 & -5000 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$$\frac{dx}{dt} = 110x \qquad \frac{dy}{dt} = -5000y$$

$$\begin{bmatrix} 110-\lambda & 0 \\ 0 & -5000-\lambda \end{bmatrix} = 0$$

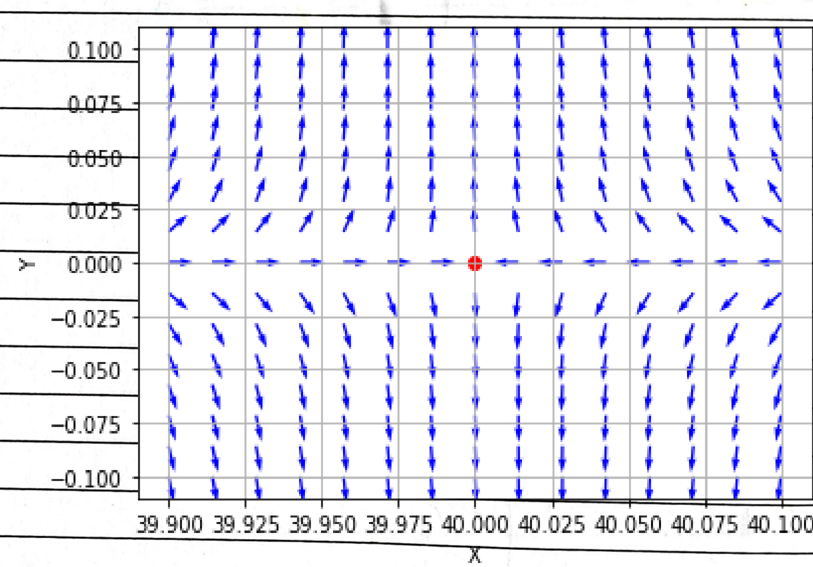
$$(110-\lambda)(-5000-\lambda)=0$$

$$y = C_1 e^{110t} + C_2 e^{-5000t}$$

• Punto Silla

• P(40,0)

$$\begin{bmatrix} -160 & -40 \\ 0 & 900 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



$$\frac{dx}{dt} = -160 - 40y \qquad \frac{dy}{dt} = 900y$$

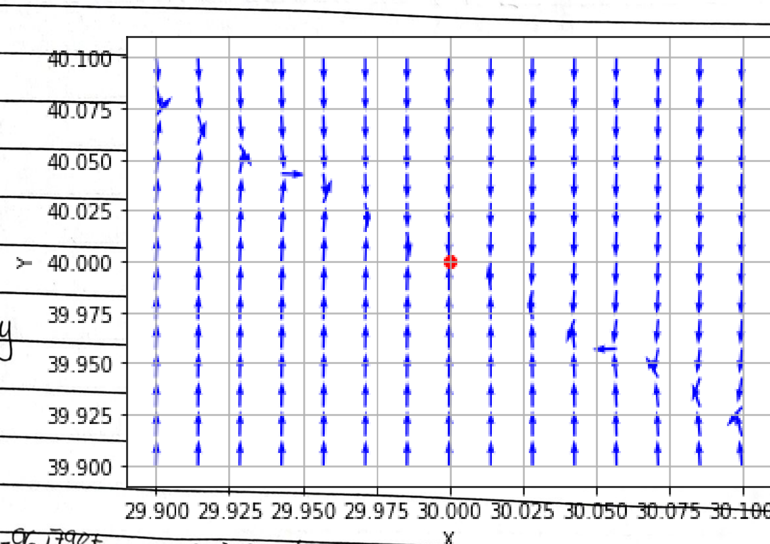
$$\begin{bmatrix} -160-\lambda & -40 \\ & 900-\lambda \end{bmatrix} = 0$$

$\lambda = -160$ $\lambda = 900$

$$y = C_1 e^{-160t} + C_2 e^{900t}$$

• Punto Silla

• P(30,40)



$$\begin{bmatrix} -120 & -30 \\ -2400 & -3200 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{dx}{dt} = -120x - 30y \qquad \frac{dy}{dt} = -2400x - 3200y$$

$$(-120-\lambda)(-3200-\lambda) - 72000 = 0$$

$\lambda = -96{,}798$ $\lambda = -3223{,}202$ $y = C_1 e^{-96{,}798t} + C_2 e^{-3223{,}202t}$ • Sumidero

14) $\dfrac{dx}{dt} = x(2-x-y)$ $\qquad$ $\dfrac{dy}{dt} = y(y-x^2)$

$0 = x(2-x-y)$

$0 = y(y-x^2)$

Si $x=0$

$0 = y(y-0)$

$y=0$

$P(0,0)$

Si $x = 2-y$

$0 = y(y-(2-y)^2)$

$0 = y(y-4+4y-y^2)$

$0 = y(y-4)(y-1)$

$y=0 \qquad y=4 \qquad y=1$

$x = 2-0 \qquad x=2-4 \qquad x=2-1$

$x=2 \qquad x=-2 \qquad x=1$

$P(2,0) \qquad$ X $\qquad P(1,1)$

Jacobiano:

$$\begin{bmatrix} 2-2x-y & -x \\ -2xy & 2y-x^2 \end{bmatrix}$$

$P(0,0)$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

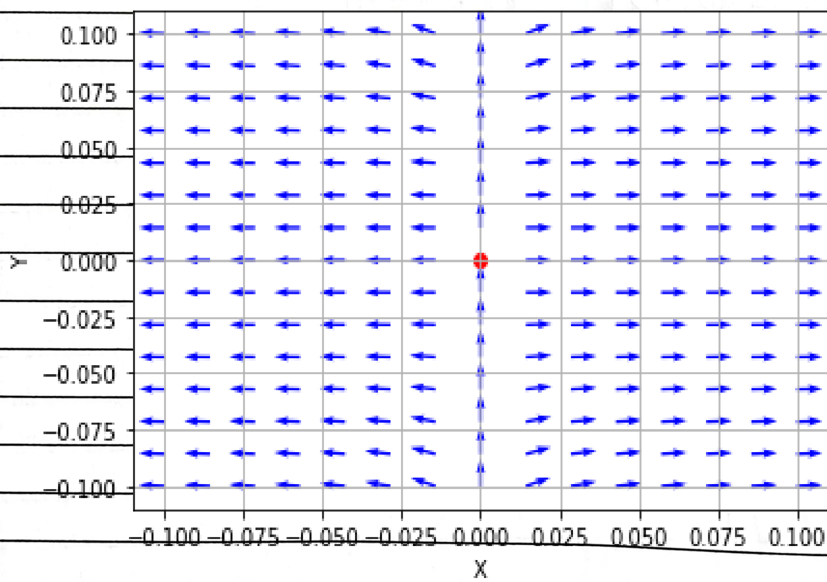
$\dfrac{dx}{dt} = 2x \qquad \dfrac{dy}{dt} = 0$

$$\begin{bmatrix} +2-\lambda & 0 \\ 0 & -\lambda \end{bmatrix} = 0$$

$-\lambda(2-\lambda)=0$

$\lambda=0 \qquad \lambda=2$

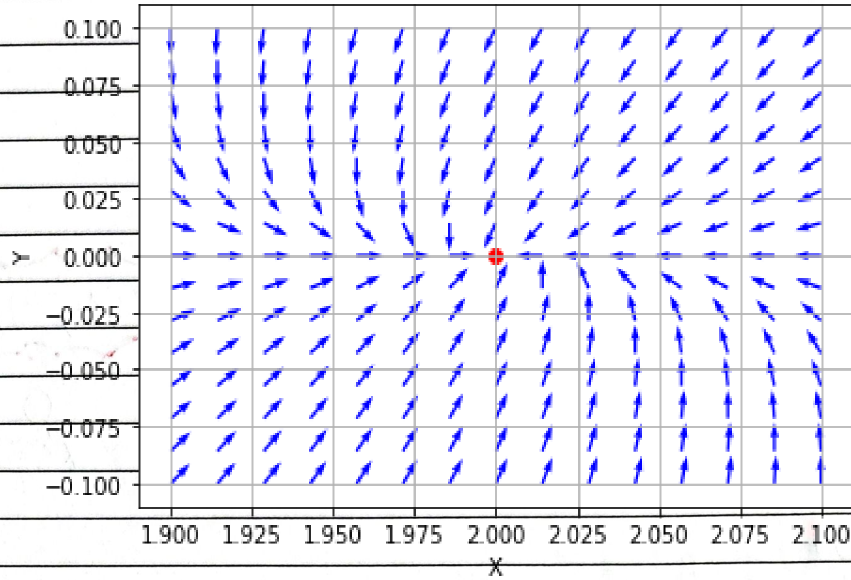
$y = C_1 e^{2t} + C_2$



• Fuente ?

- $P(2,0)$

$$\begin{bmatrix} -2 & -2 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$



$$\frac{dx}{dt} = -2x - 2y \qquad \frac{dy}{dt} = -4y$$

$$\begin{bmatrix} -2-\lambda & -2 \\ 0 & -4-\lambda \end{bmatrix} = 0$$

$$(-2-\lambda)(-4-\lambda) = 0$$

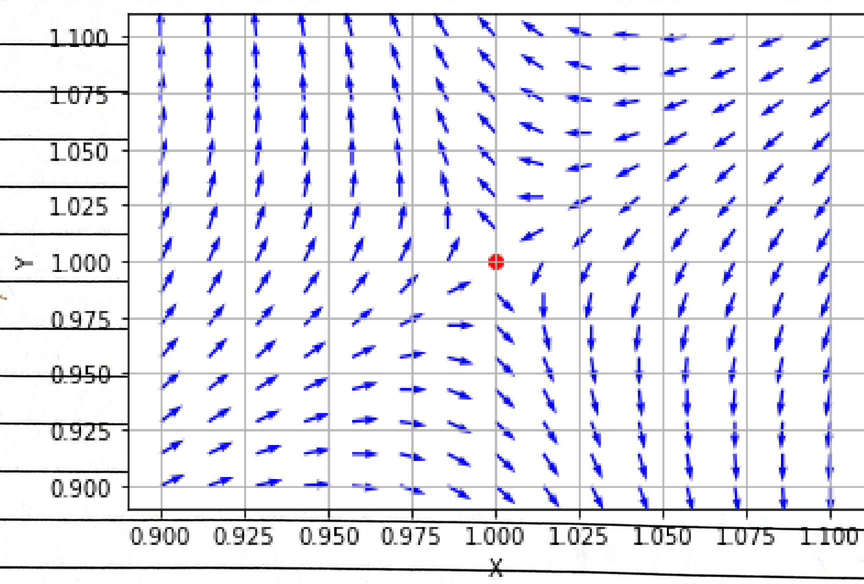
$$\lambda = -2 \qquad \lambda = -4$$

$$y = C_1 e^{-2t} + C_2 e^{-4t}$$

- Sumidero

- $P(1, 1)$

$$\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$



$$\frac{dx}{dt} = -y \qquad \frac{dy}{dt} = -y$$

$$\begin{bmatrix} -\lambda-1 & -1 \\ 0 & -\lambda-1 \end{bmatrix} = 0$$

$$(-\lambda-1)(-\lambda-1) = 0$$

$$\lambda = -1 \qquad \lambda = -1$$

$$y = C_1 e^{-t} t + C_2 e^{-t}$$

- Sumidero

16) $\frac{dx}{dt} = x(x-1)$ $\qquad$ $\frac{dy}{dt} = y(x^2-y)$

$0 = x(x-1)$

$0 = y(x^2-y)$

Si $x=0$

$0 = y(0-y)$

$y=0$

$\boxed{P(0,0)}$

Si $x=1$

$0 = y(1-y)$

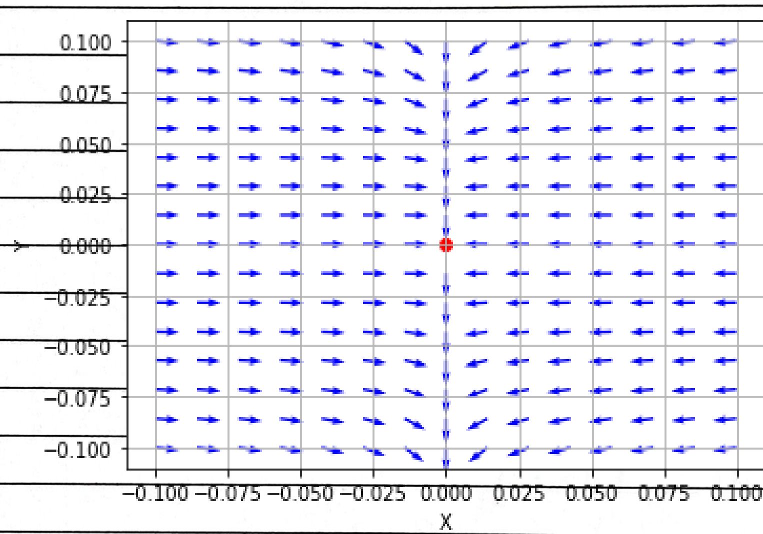
$y=0$ $\qquad$ $y=1$

$\boxed{P(1,0)}$ $\qquad$ $\boxed{P(1,1)}$

Jacobiano: $\begin{bmatrix} 2x-1 & 0 \\ 2xy & x^2-2y \end{bmatrix}$

• $P(0,0)$

$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$



$\frac{dx}{dt} = -x$ $\qquad$ $\frac{dy}{dt} = 0$

$\begin{bmatrix} -1-\lambda & 0 \\ 0 & -\lambda \end{bmatrix} = 0$

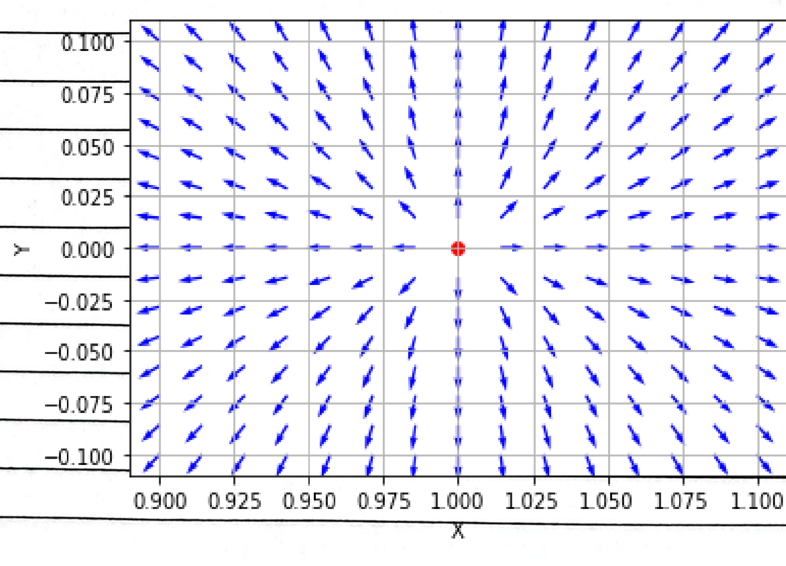
$(-1-\lambda)(-\lambda) = 0$

$\lambda = 0$ ✓ $\quad \lambda = -1$ ✓

$y = C_1 + C e^{-t}$ • Sumidero

• $P(1,0)$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$



$\frac{dx}{dt} = x$ $\qquad$ $\frac{dy}{dt} = y$

$\begin{bmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} = 0$

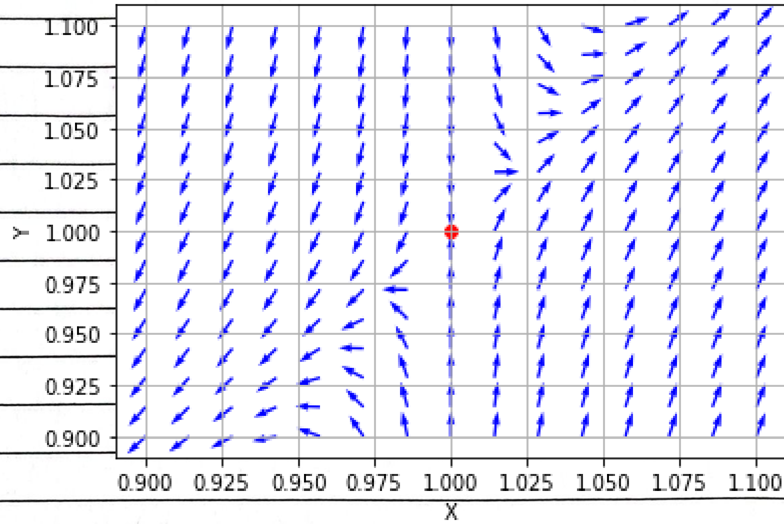
$(1-\lambda)^2 = 0$

$\lambda_{1,2} = 1$ ✓

$y = C_1 e^t \cdot t + C_2 e^t$ • Fuente

• $P(1,1)$

$$\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$



$\frac{dx}{dt} = x \qquad \frac{dy}{dt} = 2x - y$

$$\begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} = 0$$

$(1-\lambda)(-1-\lambda) = 0$

$\lambda = 1 \qquad \lambda = -1$

$y = C_1 e^{t} + C_2 e^{-t}$ • Punto Silla

18) $\frac{dx}{dt} = x^2 - a \qquad \frac{dy}{dt} = -y(x^2+1)$

a) si $a < 0$

$0 = x^2 - a \rightarrow x^2 = a \rightarrow x = \pm\sqrt{a}$ si "a" es (-) entonces no es punto de eq

b) $x = \pm\sqrt{a}$ con "a" (+)

$0 = -y(a+1)$

$y = 0$ entonces $P(\sqrt{a}, 0)$ $P(-\sqrt{a}, 0)$

c)

$x = \pm\sqrt{a}$ si "a" $= 0 \rightarrow x = 0$

Reemplazo: $0 = -y(0+1)$

$y = 0 \qquad P(0,0)$

Jacobiano:

$$\begin{bmatrix} 2x & 0 \\ -2xy & -1 \end{bmatrix}$$

- $P(0,0)$

$$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \frac{dx}{dt}=0 \qquad \frac{dy}{dt}=-y$$

$$\left|\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} -\lambda & 0 \\ 0 & -\lambda-1 \end{vmatrix} = 0$$

$$-\lambda(-\lambda-1)=0$$

$$\lambda=0 \qquad \lambda=-1$$

20) $\frac{dx}{dt} = y-x^2 \qquad \frac{dy}{dt} = y-a$

a) $\left.\begin{aligned} 0 &= y-x^2 \\ 0 &= y-a \end{aligned}\right\} \Rightarrow y=a$ Remplazo: $0=a-x^2$

$x=\pm\sqrt{a}$

$P(\sqrt{a}, a)$ $P(-\sqrt{a}, a)$ b) $a=0$ valor de bifurcacion

c) Si $a<0$ no hay PE, si $a=0$ hay 1 PE, si $a>0$ hay 2 P.E

22) $\frac{dx}{dt} = y-x^2$

$\frac{dy}{dt} = y-x-a$

a) $\left.\begin{aligned} 0 &= y-x^2 \\ 0 &= y-x-a \end{aligned}\right\} \Rightarrow y=x+a$ Reemplazo: $0=x+a-x^2$

$x^2-x-a=0$

$x = \frac{1\pm\sqrt{(-1)^2-4(-1)a}}{2}$

$x = \frac{1\pm\sqrt{1+a}}{2}$

$P\left(\frac{1+\sqrt{1+a}}{2}, \frac{1+\sqrt{1+a}}{2}+a\right)$

$P\left(\frac{1-\sqrt{1+a}}{2}, \frac{1-\sqrt{1+a}}{2}+a\right)$

b) $a=-1$ es un valor de bifurcacion del Parametro

c) Si $a<-1$ entonces no hay PE, si $a=-1$ hay 1 PE, si $a>-1$ hay 2 PE

24) $\frac{dx}{dt} = y - x^2 + a \qquad \frac{dy}{dt} = y + x^2 - a$

$0 = y - x^2 + a$ ①
$0 = y + x^2 - a$ ②

Si 1 + 2 : $y = 0$

Reemplazo en ①

$0 = 0 - x^2 + a \Rightarrow x = \pm\sqrt{a}$

$P(\sqrt{a}, 0)$ $\quad$ $P(-\sqrt{a}, 0)$

b) $a=0$ es un valor de bifurcacion

c) Si $a<0$ no hay PE, si $a=0$ hay un PE, si $a>0$ hay 2 PE

26)

$\frac{dx}{dt} = x(-x-y+70) \qquad \frac{dy}{dt} = y(-2x-y+a)$

$0 = x(-x-y+70)$
$0 = y(-2x-y+a)$

Si $x = 0$

$0 = y(0-y+a)$

$y=0 \qquad y=a$

$P(0,0)$ $\quad$ $P(0,a)$

Si $x = -y+70$

$0 = y(-2(-y+70)-y+a)$

$0 = y(y-140+a)$

$y=0 \qquad y = 140-a$

$x = 70 \qquad x = -70+a$

$P(70,0)$ $\quad$ $P(-70+a, 140-a)$

No hay bifurcaciones

28)

Suponiendo: $\frac{dx}{dt} = f(x,y)$

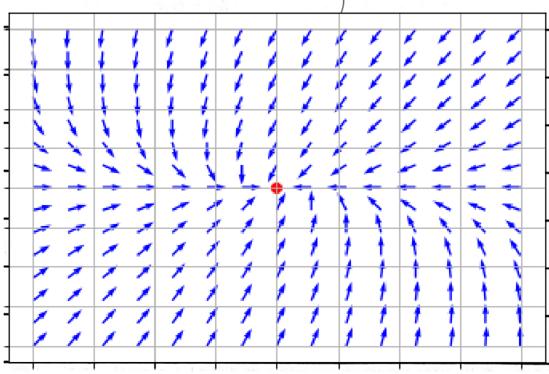
$\frac{dy}{dt} = G(x,y)$

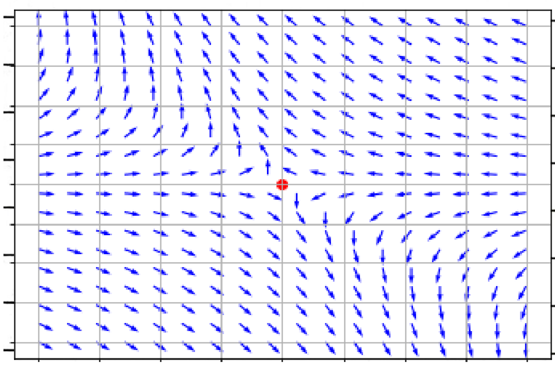
a) Dadas las condiciones se podria decir que al reproducirse lentamente y tener una fuerte competencia, ambas especies estan disminuyendo y en el punto (0,0) una o ambas especies se acerca, por lo tanto seria un punto silla o un atolladero

b) Yo diria que nuestro campo de pendientes tiende al P(0,0) acercandose y entonces seria un sumidero.

c) Sumidero o Punto silla

d)





30)

$\frac{dy}{dt} = h(y,z)$

$\frac{dz}{dt} = K(y,z)$

a) Son la velocidad en aumentar el armamento en el pais Y y Z; estando estas relacionadas

b) Seria algo asi: $0 = \frac{dz}{dt} \to \frac{dy}{dt} = a - b \to \frac{dy}{dt} = 0$

donde a y b son iguales cuado $dz/dt = 0$

c) Podria haber muchos puntos de equilibrio pero uno seguro es en $P(0,0)$ este segun las funciones podria ser sumidero y atolladero

En la jacobiana seria:

$$\begin{bmatrix} (+) & 0 \\ 0 & (+) \end{bmatrix} \text{ ó } \begin{bmatrix} (-) & 0 \\ 0 & (-) \end{bmatrix}$$