PRACTICA 1

1- Sea una variable aleatoria con función de densidad

$f(x) = \dfrac{16x^2 - 6x^3}{13}$ si $0 \leq x \leq 4$

$\int_0^4 \frac{16}{13}x^2dx \int \frac{6}{13}x^3dx \Rightarrow \left[\frac{16}{13}\frac{x^3}{3} - \frac{\cancel{6}^{3}}{13}\frac{x^4}{\cancel{4}_{2}}\right]\Big|_0^4$

$\Rightarrow \left[\frac{16}{39}x^3 - \frac{3x^4}{26}\right]\Big|_0^4 \Rightarrow \left[\frac{16}{39}\cdot 4^3 - \frac{3}{26}\cdot 4^4\right] \Rightarrow -\frac{128}{39}$

∴ No cumple

$\Rightarrow f(x) = \frac{16x^2 - 6x^3}{\cancel{13}_1} \times -\frac{\cancel{39}^{3}}{128} \Rightarrow \boxed{\frac{9}{64}x^3 - \frac{3}{8}x^2}$

$\int_0^4 \left(\frac{9}{64}x^3 - \frac{3}{8}x^2\right)dx \Rightarrow \left[\frac{9}{64}\frac{x^4}{4} - \frac{\cancel{3}}{8}\frac{x^3}{\cancel{3}}\right]\Big|_0^4$

$\left[\frac{9}{64\times}\cdot 4^3 - \frac{4^3}{8}\right] \Rightarrow 1$ ∴ Si cumple.

a) Calcular la probabilidad de $P(1 < x < 8/3)$

$\int_1^{8/3} \left(\frac{9}{64}x^3 - \frac{3}{8}x^2\right)dx \Rightarrow \left[\frac{9x^4}{64}\cdot\frac{1}{4} - \frac{\cancel{3}^{1}}{8}\frac{x^3}{\cancel{3}_1}\right]\Big|_1^{8/3}$

$\Rightarrow \left[\frac{9}{64}\cdot\frac{(8/3)^4}{4} - \frac{(8/3)^3}{8}\right] - \left[\frac{9}{64\times 4} - \frac{1}{8}\right] \Rightarrow -\frac{3475}{6912} \times 100$

$\Rightarrow 50{,}27\ \%$

b) Calcular la probabilidad de $p(-2 < x < 7/2)$

$\int_{-2}^{7/2} \left(\frac{9}{64}x^3 - \frac{3}{8}x^2\right)dx \Rightarrow \left(\frac{9}{64}\frac{x^4}{4} - \frac{3}{8}\frac{x^3}{3}\right)\Big|_{-2}^{7/2}$

$\Rightarrow \left(\frac{9}{64}\cdot\frac{(7/2)^4}{4} - \frac{3}{8}\cdot\frac{(7/2)^3}{3}\right) - \left(\frac{9}{64}\cdot\frac{(-2)^4}{4} - \frac{3}{8}\frac{(-2)^3}{3}\right)$

$\Rightarrow -0{,}084 \times 100 \Rightarrow 8.4\ \%$

c hallar el esperado de la variable

$$E(x) \Rightarrow \int_0^4 x\left(\frac{9}{64}x^3 - \frac{3}{8}x^2\right)dx$$

$$E(x) \Rightarrow \left[\frac{9}{64}\frac{x^5}{5} - \frac{3}{8}\frac{x^4}{4}\right]\Big|_0^4 = \frac{9}{64}\cdot\frac{4^5}{5} - \frac{3}{8}\frac{4^4}{4} \Rightarrow \frac{144}{5} - 24$$

$$E(x) = \frac{24}{5}$$

d. hallar el valor de la desviación estandar

$$V(x) = \sigma^2 = E(x^2) - [E(x)]^2$$

$$\int_0^4 x^2\left(\frac{9}{64}x^3 - \frac{3}{8}x^2\right)dx \Rightarrow \left[\frac{9}{64}\cdot\frac{x^6}{6} - \frac{3}{8}\frac{x^5}{5}\right]\Big|_0^4$$

$$\Rightarrow \frac{96}{5} \quad \therefore \Rightarrow \frac{96}{5} - \frac{24^2}{25} = -\frac{96}{25}$$

$$\sigma^2 = \sqrt{-\frac{96}{25}} \qquad \boxed{\sigma = 1,96} \text{ sol.}$$

2- Considere la siguiente función de densidad de probabilidad

$f(x) = 2Kx \qquad 0<x<2 \qquad f(x) = \frac{K}{2}(4-x)$

$$\int_0^2 2Kx\,dx + \int_2^4 \frac{K}{2}(4-x)\,dx$$

$$\Rightarrow \left[\cancel{2}K\frac{x^2}{\cancel{2}}\right]\Big|_0^2 + \left[\frac{4Kx}{2} - \frac{x^2K}{2\cdot2}\right]\Big|_2^4 \Rightarrow 4K + \left[\frac{4K\cdot4}{2} - \frac{4^2K}{\cancel{4}}\right]$$

$$-\left[\frac{4K\cdot\cancel{2}}{\cancel{2}} - \frac{\cancel{2^2}K}{\cancel{4}}\right] \Rightarrow \cancel{4K} + 8K - \cancel{4K} - 4K + K = 0$$

$$5K = 1 \qquad \boxed{K = 1/5}$$

a)

b) Encuentre la esperanza y la Varianza de x

$$E(x) = \int_0^2 x \, \frac{2}{5} x \, dx + \int_2^4 x \left(\frac{2}{5} - \frac{1}{10} x \right) dx$$

$$= \left[\frac{2}{5} x^3 \right] \Big|_0^2 + \left[\frac{\cancel{2}}{5} \frac{x^2}{\cancel{2}} - \frac{1}{10} \frac{x^3}{3} \right] \Big|_2^4$$

$$\Rightarrow \left[\frac{2}{5} \cdot \frac{2^3}{3} \right] + \left[\frac{4^2}{5} - \frac{4^3}{30} \right] - \left[\frac{2^2}{5} - \frac{2^3}{30} \right]$$

$$\Rightarrow \frac{16}{15} + -\frac{8}{15} = \frac{8}{15} \Big| \text{ sol}$$

$$V(x) = E(x^2) - [E(x)]^2 = -\left(\frac{8}{15}\right)^2$$

$$\int_0^2 x^2 \, \frac{2}{5} x \, dx + \int_2^4 x^2 \left(\frac{2}{5} - \frac{1}{10} x \right) dx$$

$$\left[\frac{2x^3}{5} \cdot \frac{1}{3} \right] \Big|_0^2 + \left[\frac{2}{5} \frac{x^3}{3} - \frac{x^4}{4 \times 10} \right] \Big|_2^4$$

$$\left[\frac{2 \times 2^3}{5} \times \frac{1}{3} \right] + \left[\frac{2}{15} \times 4^3 - \frac{4^4}{40} \right] - \left[\frac{2}{15} \times 2^3 - \frac{2^4}{40} \right]$$

$$= \frac{8}{5} + \frac{22}{15} = \frac{46}{15}$$

$$V(x) = \frac{46}{15} - \left(\frac{8}{15}\right)^2 \Rightarrow \frac{626}{225}$$

$$\sigma^2 = \frac{626}{225} = \sigma = \sqrt{\frac{626}{225}} \qquad \sigma = 1{,}67$$

3- La función de densidad de una variable aleatoria x está dada por la función

$$F(x) = \frac{e^{-\frac{x}{\alpha}}}{\alpha} \qquad x > 0$$

$$\mu = -\frac{x}{\alpha} \qquad du = -\frac{dx}{\alpha}$$

$$\int \frac{e^{-x/\alpha}}{\alpha} dx \Rightarrow$$

$$-\int e^{u} du \Rightarrow -e^{u} = -e^{-x/\alpha}\Big] \quad x>0 \quad = -e^{-0/\alpha} = \underline{-1}\Big].$$

a. $E(x)$

$$E(x) \Rightarrow \int x \frac{e^{-x/\alpha}}{\alpha} dx = -\int \frac{x e^{-x/\alpha}}{\alpha} dx \qquad \left\{ \begin{array}{l} u = -\frac{x}{\alpha} \\ du = -\frac{dx}{\alpha} \end{array} \right.$$

$$= \int -u e^{u} - du \Rightarrow \qquad -du = \frac{dx}{\alpha}$$

expon ILATE

$$\Rightarrow \int u e^{u} du \Rightarrow u e^{u} - \int e^{u} du = \underline{u e^{u} - e^{u}}\Big]$$

Reemplazamos en el actual

$$\alpha^2 \int u e^{u} du \Rightarrow \alpha^2 u e^{u} - e^{u} \alpha^2$$

$$\cancel{\alpha^2} x - \frac{x}{\cancel{\alpha}} e^{-\frac{x}{\alpha}} - \alpha^2 e^{-\frac{x}{\alpha}} = \underline{-e^{-x/\alpha}(x+\alpha) + C}\Big] \text{ sol}$$

b. $Var(x)$

$$Var(x) = \sigma^2 = E(x)^2 - [E(x)]^2$$

$$\int x^2 \frac{e^{-\frac{x}{\alpha}}}{\alpha} dx \Rightarrow \frac{1}{\alpha} \int x^2 e^{-x/\alpha} dx \qquad \mu = -\frac{x}{\alpha}$$

$$du = -\frac{dx}{\alpha}$$

$$\Rightarrow \int \mu^2 \alpha^2 e^{u} \cdot -du \qquad -du = +\frac{dx}{\alpha}$$

$$(-\alpha \mu)^2 = x^2$$

$$\mu^2 \alpha^2 = x^2$$

$\Rightarrow -\alpha^2 \int \underbrace{u^2 e^u}\, du$ $\quad u^2 = 2u \quad \int e^u = e^u$

$= -\alpha^2 u^2 e^u + 2\alpha^2 u e^u - 2\alpha^2 e^u$ $\quad = u^2 e^u - \int \underbrace{2u e^u}\, du$

$= -\cancel{\alpha^2} \frac{x^2}{\cancel{\alpha^2}} e^{-x/\alpha} + 2\cancel{\alpha^2} \frac{x^2}{\cancel{\alpha^2}} e^{-x/\alpha} - 2\alpha^2 e^{-x/\alpha}$ $\quad = u^2 e^u - 2u e^u + 2e^u$

$= -x^2 e^{-x/\alpha} + 2x^2 e^{-x/\alpha} - 2\alpha^2 e^{-x/\alpha} + c$

$\boxed{\Rightarrow -x^2 e^{-x/\alpha} + 2x^2 e^{-x/\alpha} - 2\alpha^2 e^{-x/\alpha} + c}$ so!

4.- Sea x una variable aleatoria con función dada por

$f(x) = \frac{1}{30} x \qquad 0 < x < 30$

Comprobamos

$\int_0^{30} \frac{1}{30} x \Rightarrow \left[\frac{x^2}{60}\right]\Big|_0^{30} \Rightarrow \frac{900}{60} = 15 \therefore$ No cumple

$\int_0^{30} kx = 1 \Rightarrow \frac{kx^2}{30} = 1 \times 2 \quad \frac{k\, 30^{\cancel{2}}}{\cancel{30} \times 2} = 1 \quad k = \frac{1}{\cancel{2}} \; \frac{}{\cancel{30}\, 15}$

$\int \frac{1}{30} x \times \frac{1}{15}\, dx \Rightarrow \left[\frac{x^2}{450} \times \frac{1}{2}\right]_0^{30} \Rightarrow \frac{30^2}{900} = 1$ cumple

hallar

a) El valor de la esperanza media

$E(x) \Rightarrow \int_0^{30} x \left(\frac{x}{450}\right) dx \Rightarrow \left[\frac{x^3}{3 \times 450}\right]\Big|_0^{30} \Rightarrow \frac{30^3}{1350} \Rightarrow 20$

b) VAR (x)

$E(x^2) \Rightarrow \int_0^{30} x^2 \left(\frac{x}{450}\right) dx \Rightarrow \left[\frac{x^4}{4} \times \frac{1}{450}\right]\Big|_0^{30} \Rightarrow \frac{30^4}{1800} = 450$

$VA(x) = \sigma^2 = 450 - 20^2 \Rightarrow 50$

$\sigma = \sqrt{50} \Rightarrow \underline{\sigma = 7{,}1}$ sol.

5: $f(x) = 2e^{-2x}$ $\quad 0 > x > 10$

$\int_0^{10} 2e^{-2x}\,dx \Rightarrow \cancel{2}\frac{e^{-2x}}{-\cancel{2}} \Rightarrow \left[-e^{-2x}\right]\Big|_0^{10} = 0{,}999$

∴ Si cumple

a) $E(x)$ $\int_0^{10} 2x\, e^{-2x}\,dx$ $\quad u = -2x$

$du = -2dx$

$\int \frac{u}{2} e^{u} \Rightarrow \boxed{-xe^{-2x} - \frac{e^{-2x}}{2} + C}\Big|_0^{10}$ $\quad -\frac{du}{2} = dx$

$E(x) = 0{,}4999$

b)