Practica 1

82) $z_n = \left(1+\frac{1}{n}\right)e^{\frac{i\pi}{n}} = \left(1+\frac{1}{n}\right)\left(\cos\left(\frac{\pi}{n}\right)+i\,sen\left(\frac{\pi}{n}\right)\right)$

$= \cos\left(\frac{\pi}{n}\right)+\left(\frac{1}{n}\right)\cdot\cos\left(\frac{\pi}{n}\right) + i\left[sen\left(\frac{\pi}{n}\right)+\left(\frac{1}{n}\right)sen\left(\frac{\pi}{n}\right)\right]$

$\rho_n = \sqrt{\left[\cos\left(\frac{\pi}{n}\right)\left(1+\frac{1}{n}\right)\right]^2+\left[Sen\left(\frac{\pi}{n}\right)\left(1+\frac{1}{n}\right)\right]^2}$

$\rho_n = \sqrt{\left[\underbrace{\cancel{sen^2\left(\frac{\pi}{n}\right)+\cos^2\left(\frac{\pi}{n}\right)}}_{1}\right]\cdot\left(1+\frac{1}{n}\right)^2}$

$\rho_n = 1+\frac{1}{n}$

$\varphi_n = \tan^{-1}\left(\frac{\cancel{\left(1+\frac{1}{n}\right)}\,Sen\left(\frac{\pi}{n}\right)}{\cancel{\left(1+\frac{1}{n}\right)}\cos\left(\frac{\pi}{n}\right)}\right) \Rightarrow \varphi_n = \tan^{-1}\left(\tan\left(\frac{\pi}{n}\right)\right) = \frac{\pi}{n}$

$\rho_0 = \lim_{n\to\infty} 1+\frac{1}{n} = 1+\frac{1}{\infty} = 1$ ✓

$\varphi_0 = \lim_{n\to\infty} \frac{\pi}{n} = \frac{\pi}{\infty} = 0$ ✓

$\lim_{n\to\infty} z_n = 1\,e^{i\cdot 0} = 1$

84) $z_n = (1+3i)^n$

$\rho_n = \left(\sqrt{1^2+3^2}\right)^n = (10)^{n/2}$

$\varphi_n =$

$$\arg(z^n) = n \arg(z)$$

$$\varphi_n = n \tan^{-1}\left(\frac{3}{1}\right)$$

$$\varphi_n = n \cdot \tan^{-1}(3)$$

$$g_0 = \lim_{n \to \infty} 10^{n/2} = 10^{\infty} = \infty$$

$$\varphi_0 = \lim_{n \to \infty} n \cdot \tan^{-1}(3) = \infty$$

$$\lim_{n \to \infty} (1+3i)^n = \infty \cdot e^{i\infty} \; = \infty$$

86) $$z_n = \frac{n+2i}{3n+7i}\left(\frac{3n-7i}{3n-7i}\right) = \frac{3n^2+14}{9n^2+49} - i\left(\frac{n}{9n^2+49}\right)$$

$$g_n = \sqrt{\left(\frac{3n^2+14}{9n^2+49}\right)^2 + \left(\frac{-n}{9n^2+49}\right)^2} = \sqrt{\frac{9n^4+84n^2+196+n^2}{(9n^2+49)^2}}$$

$$= \sqrt{\frac{(n^2-4)\cancel{(9n^2+49)}}{(9n^2+49)^{\cancel{2}}}} = \sqrt{\frac{n^2-4}{9n^2+49}}$$

$$\varphi_n = \tan^{-1}\left(\frac{\frac{-n}{9n^2+49}}{\frac{3n^2+14}{9n^2+49}}\right) = \tan^{-1}\left(\frac{-n}{3n^2+14}\right)$$

$$g_0 = \lim_{n \to \infty} \sqrt{\frac{n^2-4}{9n^2+49}} = \lim_{n \to \infty} \sqrt{\frac{\frac{n^2-4}{n^2}}{\frac{9n^2+49}{n^2}}} = \lim_{n \to \infty} \sqrt{\frac{1-\frac{4}{n^2}}{9+\frac{49}{n^2}}} = \sqrt{\frac{1-\cancel{\frac{4}{\infty}}}{9+\cancel{\frac{49}{\infty}}}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$$\varphi_0 = \lim_{n \to \infty} \tan^{-1}\left(\frac{-n}{3n^2+14}\right) = \lim_{n \to \infty} \tan^{-1}\left(\frac{\frac{-\cancel{n}}{n^{\cancel{2}}}}{\frac{3\cancel{n^2}}{\cancel{n^2}} + \frac{14}{n^2}}\right) = \tan^{-1}\left(\frac{-\overset{0}{\cancel{\frac{1}{\infty}}}}{3+\overset{0}{\cancel{\frac{14}{\infty}}}}\right)$$

$$= \tan^{-1}\left(\frac{-0}{3}\right) = 0$$

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty} \frac{n+2i}{3n+7i} = \frac{1}{3}\,\cancel{e}^{\,i\cdot\cancel{0}} = \boxed{\frac{1}{3}}$$

88) $z_n = n\,\text{sen}\left(\frac{i}{n}\right) = n\left(-i\,\text{senh}\left(\frac{i\cdot i}{n}\right)\right) = n\,i\,\text{senh}\left(\frac{1}{n}\right)$

$$\rho_n = |z_n| = \sqrt{0^2 + \left[n\,\text{senh}\left(\frac{1}{n}\right)\right]^2} = n\cdot\text{senh}\left(\frac{1}{n}\right)$$

$$\rho_0 = \lim_{n\to\infty} \rho_n = \lim_{n\to\infty} n\cdot\text{senh}\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{-i\,\text{sen}\left(\frac{i}{n}\right)}{\frac{1}{n}} \cdot \frac{\frac{i}{1}}{\frac{i}{1}}$$

$$= \lim_{n\to\infty} \frac{\text{sen}\left(\frac{i}{n}\right)}{\frac{i}{n}} = \lim_{x\to 0} \frac{\text{sen}\left(\frac{\cancel{i}\cdot x}{\cancel{i}}\right)}{\frac{\cancel{i}\cdot x}{\cancel{i}}} = \lim_{x\to 0} \frac{\text{sen}(x)}{x} = 1$$ ✓

$x = \frac{i}{n}$ si $n\to\infty$ / $x\to 0$ $\quad n = \frac{i}{x}$

$\varphi_n = \frac{\pi}{2}$

$\varphi_0 = \lim_{n\to 0} \pi = \frac{\pi}{2}$ ✓

$$\lim_{n\to 0} z_n = \lim_{n\to 0} n\,\text{sen}\left(\frac{i}{n}\right) = 1\left(e^{i\pi/2}\right) = \overset{0}{\cancel{\cos}}\left(\frac{\pi}{2}\right) + i\,\overset{1}{\cancel{\text{sen}}}\left(\frac{\pi}{2}\right) = \boxed{i}$$

90) $z_n = \frac{\text{senh}(in)}{n} = \frac{-i\,\text{sen}(i\cdot i n)}{n} = \frac{i\,\text{sen}(n)}{n}$

$$\rho_n = \sqrt{\left[\frac{\text{sen}(n)}{n}\right]^2 + 0} = \frac{\text{sen}(n)}{n}$$

$$\rho_0 = \lim_{n\to\infty} \frac{\text{sen}(n)}{n} = \lim_{x\to 0} \frac{\text{sen}\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to 0} x\,\text{sen}\left(\frac{1}{x}\right) = 0$$

$x = \frac{1}{n}$ $\quad x\to 0$ $\quad n = \frac{1}{x}$

$$\varphi_n = \frac{\pi}{2} \qquad \varphi_0 = \lim_{n\to\infty} \frac{\pi}{2} = \frac{\pi}{2}$$

$$\lim_{n\to\infty} \frac{\operatorname{senh}(in)}{n} = 0 \cdot e^{i\pi/2} = \boxed{0}$$

Diferenciación de funciones

104)

a) $w = z^2 \bar{z}$ $\qquad z = x + yi$

$$w = (x+yi)^2 (x-yi)$$

$$w = (x^2 - y^2 + 2xyi)(x - yi)$$

$$w = \underbrace{x^3 - y^2 x + 2xy^2}_{u} + i\underbrace{(2x^2 y - x^2 y + y^3)}_{v}$$

$$\frac{du}{dx} = 3x^2 + y^2 \qquad \frac{du}{dy} = 2yx$$

$$\frac{dv}{dx} = 2xy \qquad \frac{dv}{dy} = x^2 + 3y^2$$

$$\frac{du}{dx} = \frac{dv}{dy} \qquad \frac{du}{dy} = -\frac{dv}{dx}$$

$$3x^2 + y^2 = x^2 + 3y^2 \qquad 2yx = -2xy$$

$\to$ $2x^2 - 2y^2 = 0$

$$\boxed{x^2 = y^2}$$

$\therefore$ Es analítica cuando $x = y$ $\cancel{\text{}}$ X

b) $W = ze^{z}$

$$W = (x + yi)\, e^{x+yi}$$

$$w = (x + yi)\, e^{x} (\cos(y) + i\,\text{sen}(y))$$

$$w = \underbrace{e^{x}(x)\cos(y) - e^{x}(y)\,\text{sen}(y)}_{u} + i\underbrace{(e^{x}\, y\cos(y) + e^{x}\, x\,\text{sen}(y))}_{v}$$

$$\frac{du}{dx} = e^{x}x\cos y + e^{x}\cos y - e^{x}y\,\text{sen}(y)$$

$$\frac{du}{dy} = -e^{x}x\,\text{sen}(x) - e^{x}\,\text{sen}(y) - e^{x}y\cos(y)$$

$$\frac{dv}{dx} = e^{x}y\cos(y) + e^{x}x\,\text{sen}(y) + e^{x}\text{sen}(y)$$

$$\frac{dv}{dy} = e^{x}\cos(y) - e^{x}y\,\text{sen}(y) + e^{x}x\cos(y)$$

$\frac{du}{dx} = \frac{dv}{dy}$ ∴ Cumple

$\frac{du}{dy} = -\frac{dv}{dx}$ ∴ Cumple

∴ Es analitica

c) $w = |z|\,\bar{z}$

$$w = \sqrt{x^2 + y^2}\,(x + yi)$$

$$w = \underbrace{\sqrt{x^2+y^2}\cdot x}_{u} - i\underbrace{\sqrt{x^2+y^2}\cdot y}_{v}$$

$$\frac{du}{dx} = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}(\cancel{2}x)\cdot x + \sqrt{x^2+y^2}$$

$$\frac{du}{dy} = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}(\cancel{2}y)\cdot x$$

$$\frac{du}{dx} = \left(\frac{1}{\cancel{2}}\right)(x^2+y^2)^{-1/2}(\cancel{2}x)\cdot y$$

$$\frac{du}{dy} = \left(\frac{1}{\cancel{2}}\right)(x^2+y^2)^{-1/2}(\cancel{2}y)\cdot y + \sqrt{x^2+y^2}$$

$$\frac{du}{dx} = \frac{dv}{dy}$$

$$\frac{x^2}{\sqrt{x^2+y^2}} + \sqrt{x^2+y^2} \neq \frac{y^2}{\sqrt{x^2+y^2}} + \sqrt{x^2+y^2} \quad \therefore \text{No cumple}$$

$$\frac{du}{dy} = -\frac{dv}{dx}$$

$$\frac{xy}{\sqrt{x^2+y^2}} \neq -\frac{xy}{\sqrt{x^2+y^2}} \quad \therefore \text{No cumple}$$

$\therefore$ No es analitica

d) $W = e^{z^2}$

$$W = e^{(x+yi)^2} = e^{x^2-y^2+2xyi} = e^{x^2-y^2}(\cos(2xy)+i\,\text{sen}(2xy))$$

$$W = \underbrace{\frac{e^{x^2}}{e^{y^2}}\cos(2xy)}_{u} + i\,\underbrace{\frac{e^{x^2}}{e^{y^2}}\text{sen}(2xy)}_{v}$$

$$\frac{du}{dx} = \frac{(2x)e^{x^2}}{e^{y^2}}\cdot\cos(2xy) + \frac{e^{x^2}}{e^{y^2}}\text{sen}(2xy)(2y)$$

$$\frac{du}{dy} = (-2y)\cdot\frac{e^{x^2}}{e^{y^2}}\cdot\cos(2xy) - \frac{e^{x^2}}{e^{y^2}}\text{sen}(2xy)(2x)$$

$$\frac{dv}{dx} = \frac{(2x)e^{x^2}}{e^{y^2}}\text{sen}(2xy) + \frac{e^{x^2}}{e^{y^2}}\cos(2xy)(2y)$$

$$\frac{dv}{dy} = \frac{(-2y)e^{x^2}}{e^{y^2}}\cdot\text{sen}(2xy) + \frac{e^{x^2}}{e^{y^2}}\cos(2xy)(2x)$$

$$\frac{du}{dx} = \frac{dv}{dy} \quad \therefore \text{Cumple}$$

$\therefore$ Es analitica

$$\frac{du}{dy} = -\frac{dv}{dx} \quad \therefore \text{Cumple}$$

e) $w = |z| \, Re(\bar{z})$

$$w = \underbrace{\sqrt{x^2+y^2} \cdot x}_{u} + \underbrace{i \cdot 0}_{v}$$

$$\frac{du}{dx} = \sqrt{x^2+y^2} + x\left(\frac{1}{\cancel{2}}\right)(x^2+y^2)^{-1/2}(\cancel{2}x)$$

$$\frac{du}{dy} = \frac{x}{\cancel{2}}(x^2+y^2)^{-1/2}(\cancel{2}y)$$

$$\frac{dv}{dx} = 0 \; ; \quad \frac{dv}{dy} = 0$$

$$\frac{du}{dx} = \frac{dv}{dy} \rightarrow \sqrt{x^2+y^2} + \frac{x^2}{\sqrt{x^2+y^2}} \neq 0 \quad \therefore \text{No cumple}$$

$$\frac{du}{dy} = -\frac{dv}{dx} \rightarrow \frac{xy}{\sqrt{x^2+y^2}} \neq 0 \quad \therefore \text{No cumple}$$

$\therefore$ No es analitica

f) $w = sen(3z) - i$

$$w = sen(3x + 3yi) - i$$

$$w = sen(3x)\cos(3yi) + sen(3yi)\cos(3x) - i$$

$$W = sen(3x)\cosh(3y) + i\,senh(3y)\cos(3x) - i$$

$$\frac{du}{dx} = 3 \cdot \cos(3x)\cosh(3y)$$

$$\frac{dv}{dy} = 3\cosh(3y)\cos(3x)$$

$$\therefore \frac{du}{dx} = \frac{dv}{dy} \quad \text{cumple}$$

$$\left.\begin{array}{l}\dfrac{du}{dy} = 3\,sen(3x)\,senh(3y) \\[2ex] \dfrac{dv}{dx} = -3\,senh(3y)\,sen(3x)\end{array}\right\} \therefore \dfrac{du}{dy} = -\dfrac{dv}{dx} \quad \text{Cumple}$$

$\therefore$ Es analitica

105) a) $w = \bar{z}\,R(z)$

$$w = \underbrace{x^2}_{u} - \underbrace{i\,xy}_{v}$$

$$\left.\begin{array}{l}\dfrac{du}{dx} = 2x \\[2ex] \dfrac{dv}{dy} = -x\end{array}\right\} \therefore \dfrac{du}{dx} \neq \dfrac{dv}{dy} \quad \text{no cumple}$$

$$\left.\begin{array}{l}\dfrac{du}{dy} = 0 \\[2ex] \dfrac{dv}{dx} = -y\end{array}\right\} \therefore \dfrac{du}{dy} \neq -\dfrac{dv}{dx} \quad \text{no cumple}$$

$\therefore$ Solo es analitica en el p(0,0)

b) $w = \bar{z}\cdot Im(z) = \underbrace{xy}_{u} + \underbrace{y^2 i}_{v}$

$$\left.\begin{array}{l}\dfrac{du}{dx} = y \\[2ex] \dfrac{dv}{dy} = -2y\end{array}\right\} \therefore \dfrac{du}{dx} \neq \dfrac{dv}{dy} \quad \text{no cumple}$$

$$\left.\begin{array}{l}\dfrac{du}{dy} = x \\[2ex] \dfrac{dv}{dx} = 0\end{array}\right\} \therefore \dfrac{du}{dy} \neq \dfrac{dv}{dx} \quad \text{no cumple}$$

$\therefore$ Solo es analitica en el p(0,0)

c) $w = |z|\,Im(z) = \underbrace{y\sqrt{x^2+y^2}}_{u} + i\cdot 0$

$$\frac{du}{dx} = y\left(\frac{1}{\cancel{2}}\right)(x^2+y^2)^{-1/2}(\cancel{2}x) = \frac{xy}{\sqrt{x^2+y^2}} \quad ; \frac{dv}{dy} = 0$$

$$\frac{du}{dx} \neq \frac{dv}{dy} \quad \text{no cumple}$$

$$\frac{xy}{\sqrt{x^2+y^2}} = 0 \rightarrow xy = 0$$

$$\frac{du}{dy} = \sqrt{x^2+y^2} + \frac{y}{\cancel{2}}\left(x^2+y^2\right)^{-1/2}(\cancel{2}y)$$

$$\frac{dv}{dx} = 0 \; ; \quad \frac{du}{dy} \neq \frac{dv}{dx} \; \therefore \text{no cumple}$$

$$\sqrt{x^2+y^2} + \frac{y^2}{\sqrt{x^2+y^2}} = 0 \rightarrow \frac{x^2+y^2+y^2}{\sqrt{x^2+y^2}} = 0 \rightarrow x^2+2y^2=0$$

∴ Solo es analitica en el origen

d) $w = \cosh(z) = \cosh(x+iy)$

$= \cos(xi - y) = \cos(xi)\cos(y) + \operatorname{sen}(xi)\operatorname{sen}(y)$

$= \underbrace{\cos(y)\cosh(x)}_{u} + i\underbrace{\operatorname{sen}(y)\operatorname{senh}(x)}_{v}$

$$\left.\begin{aligned} \frac{du}{dx} &= \cos(y)\operatorname{senh}(x) \\ \frac{dv}{dy} &= \cos(y)\operatorname{senh}(x) \end{aligned}\right\} \quad \frac{du}{dx} = \frac{dv}{dy} \quad \therefore \text{Cumple}$$

$$\left.\begin{aligned} \frac{du}{dy} &= -\operatorname{sen}(y)\cosh(x) \\ \frac{dv}{dx} &= \operatorname{sen}(y)\cosh(x) \end{aligned}\right\} \quad \frac{du}{dy} = -\frac{dv}{dx} \quad \therefore \text{Cumple}$$

∴ Es analitica

106) $\operatorname{Re} z > 0$

$w = \ln z$

107) $(z^n)' = n \cdot z^{n-1}$

$$\frac{dw}{dx} = \frac{df}{dz} = \frac{df}{dz} = -i\frac{dw}{dy}$$

$$f(z) = z^n = (x+iy)^n = w$$

$$\frac{df}{dz} = \frac{dw}{dx} = n(x+iy)^{n-1}(1) = nz^{n-1}$$

$$\frac{df}{dz} = -i\frac{dw}{dy} = -in(x+iy)^{n-1}(i) = nz^{n-1}$$

108) $f'(z) = \varphi'(z)$ $\quad f(z) = \varphi(z) + \text{const}$

La derivada de una constante es 0 entonces no afecta a la derivada

107) $(z^n)' = n \cdot z^{n-1}$ si $z = x + yi$

Sabemos: $\frac{dw}{dx} = \frac{df}{dz}$ $\frac{df}{dz} = -i \frac{dw}{dy}$

Entonces:

$$f(z) = z^n = (x+yi)^n = w$$

$$\frac{df}{dz} = \frac{d(x+yi)^n}{dx} = n(x+yi)^{n-1}(1)$$

$$\frac{df}{dz} = -i\frac{d(x+yi)^n}{dy} = (-i)(n)(x+yi)^{n-1}(i) = n(x+yi)^{n-1}(1)$$

109) $\frac{du}{d\rho} = \frac{1}{\rho}\frac{dv}{d\varphi}$ $\frac{dv}{d\rho} = -\frac{1}{\rho}\frac{du}{d\varphi}$

$w = f(z)$

$z = x + iy = \rho(\cos(\varphi) + i\,\text{sen}(\varphi))$

$x = \cos(\varphi)\cdot\rho$ $y = \text{sen}(\varphi)\cdot\rho$

$$\begin{cases} \frac{dx}{d\varphi} = -\rho\cdot\text{sen}(\varphi) & \frac{dy}{d\varphi} = \rho\cos(\varphi) \\ \frac{dx}{d\rho} = \cos(\varphi) & \frac{dy}{d\rho} = \text{sen}(\varphi) \end{cases}$$

$$\frac{dw}{d\rho} = \frac{df}{dz}\left(\overset{1}{\cancel{\frac{dz}{dx}}}\right)\left(\frac{dx}{d\rho}\right) + \frac{df}{dz}\left(\overset{i}{\cancel{\frac{dz}{dy}}}\right)\left(\frac{dy}{d\rho}\right)$$

$$\frac{dw}{d\rho} = \frac{df}{dz}\left(\cos(\varphi)\right) + i\frac{df}{dz}\,\text{sen}(\varphi)$$

$$\frac{dw}{d\rho} = \frac{df}{dz}\left(\cos(\varphi) + i\,\text{sen}(\varphi)\right)$$

$$\frac{dw}{d\varphi} = \frac{df}{dz}\left(\overset{1}{\cancel{\frac{dz}{dx}}}\right)\left(\frac{dx}{d\varphi}\right) + \frac{df}{dz}\left(\overset{i}{\cancel{\frac{dz}{dy}}}\right)\left(\frac{dy}{d\varphi}\right)$$

$$\frac{dw}{d\varphi} = \frac{df}{dz}\left(-\rho\cdot\text{sen}(\varphi)\right) + i\frac{df}{dz}\left(\rho\cos(\varphi)\right)$$

$$\frac{dw}{d\varphi} = \frac{df}{dz}\cdot\rho\left(-\text{sen}(\varphi) + i\cos(\varphi)\right) = i\frac{df}{dz}\cdot\rho\left(\cos(\varphi) + i\,\text{sen}(\varphi)\right)$$

Entonces: $\frac{dw}{d\varphi} = i \cdot \rho \cdot \frac{dw}{d\rho}$

Ahora:

$w = u(\rho,\varphi) + iv(\rho,\varphi) \rightarrow w = u + iv$

Reemplazo:

$\left.\begin{aligned} \frac{dw}{d\rho} &= \frac{du}{d\rho} + \frac{dv}{d\rho} i \\ \frac{dw}{d\varphi} &= \frac{du}{d\varphi} + \frac{dv}{d\varphi} i \end{aligned}\right\}$ $\quad \frac{du}{d\varphi} + i\frac{dv}{d\varphi} = i\rho\left(\frac{du}{d\rho} + \frac{dv}{d\rho}i\right)$

$\frac{du}{d\varphi} + i\frac{dv}{d\varphi} = -\rho\frac{dv}{d\rho} + i\rho\frac{du}{d\rho}$

Separo $\mathbb{R}$ e $\mathbb{I}$: $\quad \frac{du}{d\varphi} = -\rho\frac{dv}{d\rho} \qquad \frac{dv}{d\varphi} = \rho\frac{du}{d\rho}$

$\boxed{\frac{dv}{d\rho} = -\left(\frac{1}{\rho}\right)\frac{du}{d\varphi}} \qquad \boxed{\frac{du}{d\rho} = \left(\frac{1}{\rho}\right)\frac{dv}{d\varphi}}$

condicione de Cauchy-Riemman para cordenadas polares ρ

110) $w = f(z) = u(x,y) + v(x,y)i \quad ; \quad z = x + yi$

Si el dominio es real entonces $u = a \quad v = 0$

a = Un numero cualquiera $\in \mathbb{R}$

Entonces comprobamos si es analitica

$\frac{du}{dx} = 1 \; ; \; \frac{du}{dy} = 1 \; ; \; \frac{dv}{dx} = 0 \; ; \; \frac{dv}{dy} = 0$

$\frac{du}{dx} = \frac{dv}{dy}$

113) $w = f(z) = R(x,y)\, e^{i\varphi(x,y)}$

$$w = f(z) = \underbrace{R\cos\varphi}_{u} + i\underbrace{R\,sen\,\varphi}_{v}$$

$$\frac{dw}{dx} = \frac{du}{dx} + i\frac{dv}{dx}$$

$$\frac{dw}{dy} = \frac{du}{dy} + i\frac{dv}{dy}$$

$$\frac{du}{dx} = \frac{du}{dR}\cdot\frac{dR}{dx} + \frac{du}{d\varphi}\cdot\frac{d\varphi}{dx}$$

$\frac{du}{dR} = \cos\varphi \; ; \quad \frac{du}{d\varphi} = -R\,sen\,\varphi$

$\frac{dv}{dR} = sen\,\varphi ; \quad \frac{dv}{d\varphi} = R\cos\varphi$

$$\frac{du}{dx} = \cos\varphi\frac{dR}{dx} + (-R\,sen\varphi)\frac{d\varphi}{dx}$$

$$\frac{du}{dy} = \cos\varphi\frac{dR}{dy} - R\,sen\,\varphi\frac{d\varphi}{dy}$$

$$\frac{dv}{dx} = \frac{dv}{dR}\cdot\frac{dR}{dx} + \frac{dv}{d\varphi}\cdot\frac{d\varphi}{dx} = sen\,\varphi\cdot\frac{dR}{dx} + R\cos\varphi\frac{d\varphi}{dx}$$

$$\frac{dv}{dy} = sen\,\varphi\frac{dR}{dy} + R\cos\varphi\frac{d\varphi}{dy}$$

- $\frac{du}{dx} = \frac{dv}{dy}$

$$\cos\varphi\frac{dR}{dx} - R\,sen\,\varphi\frac{d\varphi}{dx} = sen\,\varphi\frac{dR}{dy} + R\cos\varphi\frac{d\varphi}{dy}$$

$$\boxed{\frac{dR}{dx} = R\frac{d\varphi}{dy}} \qquad \boxed{-R\frac{d\varphi}{dx} = \frac{dR}{dy}}$$

- $\frac{du}{dy} = -\frac{dv}{dx} \Rightarrow \cos\varphi\frac{dR}{dy} - R\,sen\,\varphi\frac{d\varphi}{dy} = -sen\,\varphi\frac{dR}{dx} - R\cos\varphi\frac{d\varphi}{dx}$

$$\boxed{\frac{dR}{dy} = -R\frac{d\varphi}{dx}} \qquad -R\frac{d\varphi}{dy} = -\frac{dR}{dx} \Rightarrow \boxed{R\frac{d\varphi}{dy} = \frac{dR}{dx}}$$

126) $u = f(ax+by)$ $\qquad t = ax+by$

$u = f(t)$

$$\frac{du}{dx} = \frac{df}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2u}{dx^2} = \frac{d^2f}{dt^2} \cdot \frac{dt}{dx} \cdot \frac{dt}{dx} + \frac{df}{dt} \cdot \frac{d^2t}{dx^2}$$

- Si $\frac{dt}{dx} = a$ Entonces: $\frac{d^2u}{dx^2} = f'' \cdot a^2 + \cancel{f' \cdot 0}$

$\frac{d^2t}{dx} = 0$

$$\frac{d^2u}{dx} = f'' \cdot a^2 \checkmark$$

$$\frac{du}{dy} = \frac{df}{dt} \cdot \frac{dt}{dy} \to \frac{d^2u}{dy} = f'' \cdot \left(\frac{dt}{dy}\right)^2 + f' \frac{d^2t}{dy}$$

- Si $\frac{dt}{dy} = b$ Entonces: $\frac{d^2u}{dy^2} = f'' \cdot b^2 + f' \cdot 0$

$\frac{d^2t}{dy^2} = 0$

$$\frac{d^2u}{dy^2} = f'' \cdot b^2 \checkmark$$

Reemplazo:

$$\frac{d^2u}{dx^2} + \frac{d^2v}{dy^2} = 0 \to a^2 f'' + b^2 f'' = 0$$

$$f''(a^2+b^2) = 0$$

Usando ecuaciones diferenciales de orden superior homogenea

$\lambda^2 (a+b) = 0$

$$f(t) = C_1 \cdot \cancel{e^{0t}} + C_2 \cancel{e^{0t}} \cdot t$$

$\lambda_1 = 0 \quad \lambda_2 = 0$

$$u = f(t) = C_1 + C_2 t$$

Reemplazo t y entonces:

$$\boxed{u = C_1 + C_2(ax+by)}$$

127) $u = f(xy)$

$u = f(t)$

$t = xy$

$\frac{dt}{dx} = y \; ; \; \frac{d^2t}{dx^2} = 0$

$\frac{dt}{dy} = x \; ; \; \frac{d^2t}{dy^2} = 0$

- $\frac{du}{dx} = \frac{df}{dt} \cdot \frac{dt}{dx}$

$\frac{d^2u}{dx^2} = \frac{d^2f}{dt^2} \cdot \frac{dt}{dx} \cdot \frac{dt}{dx} + \frac{df}{dt} \cdot \frac{d^2t}{dx^2}$

$\frac{d^2u}{dx^2} = f''(t)\, y^2 + 0$

- $\frac{du}{dy} = \frac{df}{dt} \cdot \frac{dt}{dy}$

$\frac{du}{dy} = f''(t) \left(\frac{dt}{dy}\right)^2 + f'(t)\, \frac{d^2t}{dy^2}$

$\frac{du}{dy} = f''(t)\, x^2 + 0$

- $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$

$f''(t)\, y^2 + f''(t)\, x^2 = 0$

$f''(t) \cdot (y^2 + x^2) = 0$

$f''(t) = 0$

$\lambda^2 = 0$

$\boxed{f = C_1 + C_2 \cdot t} \rightarrow \boxed{f(xy) = C_1 + C_2 (xy)}$

128) $u = f\left(\frac{y}{x}\right)$

$u = f(t)$

$t = \frac{y}{x}$

$\frac{dt}{dx} = -\frac{y}{x^2}$ $\quad \frac{d^2t}{dx^2} = \frac{2y}{x^3}$

$\frac{dt}{dy} = \frac{1}{x}$ $\quad \frac{d^2t}{dy^2} = 0$

- $\frac{du}{dx} = \frac{df}{dt} \cdot \frac{dt}{dx}$

$$\frac{d^2u}{dx^2} = f''(t)\left(\frac{dt}{dx}\right)^2 + f'(t)\frac{d^2t}{dx^2} = f''(t)\frac{y^2}{x^4} + f'(t)\frac{2y}{x^3}$$

- $$\frac{du}{dy} = f'(t) \cdot \frac{dt}{dy}$$

$$\frac{d^2u}{dy^2} = f''(t)\left(\frac{dt}{dy}\right)^2 + f'(t)\frac{d^2t}{dy^2} = \frac{f''(t)}{x^2}$$

- $$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$

$$f''(t)\frac{y^2}{x^4} + f'(t) \cdot \frac{2y}{x^3} + \frac{f''(t)}{x^2} = 0$$

$$f''(t) \cdot \frac{t^2}{x^2} + f'(t) \cdot \frac{2t}{x^2} + \frac{f''(t)}{x^2} = 0$$

$f'(t) = h$

$f''(t) = h'$

$$f''(t)\, t^2 + 2t f'(t) + f''(t) = 0$$

$$h'(t^2+1) = -2t\, h$$

$s = t^2+1$

$ds = 2t\, dt$

$$\int \frac{dh}{h} = \int \frac{-2t}{(t^2+1)}\, dt$$

$$\ln(h) = -\int \frac{ds}{s}$$

$$\ln(h) = -\ln(s) + C_1$$

$$h = e^{\ln\left(\frac{C_1}{s}\right)}$$

$$h = \frac{C_1}{s}$$

$$h = \frac{C_1}{t^2+1} \rightarrow f'(t) = \frac{C_1}{t^2+1} \rightarrow f(t) = \int \frac{C_1}{t^2+1}\, dt$$

$$f(t) = C_1 \tan^{-1}(t) + C_2$$

$$\boxed{f\left(\frac{y}{x}\right) = C_1 \tan^{-1}\left(\frac{y}{x}\right) + C_2}$$

129) $u = f(x^2 - y^2)$

$t = x^2 - y^2$

$$\frac{dt}{dx} = 2x \qquad \frac{d^2t}{dx^2} = 2$$

$$\frac{dt}{dy} = -2y \qquad \frac{d^2t}{dx^2} = -2$$

- $$\frac{du}{dx} = f'(t) \cdot \frac{dt}{dx}$$

$$\frac{d^2u}{dx^2} = f''(t)\left(\frac{dt}{dx}\right)^2 + f'(t)\left(\frac{d^2t}{dx^2}\right)$$

$$\frac{d^2u}{dx^2} = f''(t) \cdot 4x^2 + f'(t) \cdot 2$$

- $$\frac{du}{dy} = f'(t) \; \frac{dt}{dy}$$

$$\frac{d^2u}{dy^2} = f''(t)\left(\frac{dt}{dy}\right)^2 + f'(t) \cdot \frac{d^2t}{dy^2} = f''(t)\, 4y^2 - 2f'(t)$$

- $$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$

$$4x^2 f''(t) + \cancel{2f'(t)} + f''(t) \cdot 4y^2 - \cancel{2f'(t)} = 0$$

$$(4x^2 + 4y^2)\, f''(t) = 0$$

$$f''(t) = 0$$

$$\lambda^2 = 0 \quad \rightarrow \quad \lambda_1 = 0 \qquad \lambda_2 = 0$$

$$f(t) = C_1 + C_2 t$$

$$\boxed{f(x^2 - y^2) = C_1 + C_2(x^2 - y^2)}$$

130) $u = f(x + \sqrt{x^2+y^2})$

$u = f(t)$

$\frac{du}{dx} = \frac{df}{dt}\frac{dt}{dx}$

$\frac{d^2u}{dx^2} = f''\left(\frac{dt}{dx}\right)^2 + f'\frac{d^2t}{dx^2}$

$\frac{d^2u}{dx^2} = f''\left(\frac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}}\right)^2 + f'\left(\frac{y^2}{(x^2+y^2)^{3/2}}\right)$

$\frac{du}{dy} = f' \cdot \frac{dt}{dy}$

$\frac{d^2u}{dy^2} = f''\left(\frac{dt}{dy}\right)^2 + f'\frac{d^2t}{dy^2}$

$= \frac{f'' \cdot y^2}{x^2+y^2} + f'\left(\frac{x^2}{(x^2+y^2)^{3/2}}\right)$

$t = x + \sqrt{x^2+y^2} \to t^2 = x^2 + 2x\sqrt{x^2+y^2} + x^2 + y^2$

$\frac{dt}{dx} = 1 + \frac{x}{\sqrt{x^2+y^2}}$

$-y^2 = 2x^2 + 2x \cdot \sqrt{x^2+y^2} - t^2$

$y^2 = t^2 - 2xt$

$\frac{d^2t}{dx^2} = \frac{\sqrt{x^2+y^2} - \left(\frac{x \cdot x}{\sqrt{x^2+y^2}}\right)}{x^2+y^2}$

$\frac{d^2t}{dx^2} = \frac{\cancel{x^2}+y^2-\cancel{x^2}}{(x^2+y^2)^{3/2}} = \frac{y^2}{(x^2+y^2)^{3/2}}$ ✓

$\frac{dt}{dy} = \frac{y}{\sqrt{x^2+y^2}}$

$\frac{d^2t}{dy^2} = \frac{\sqrt{x^2+y^2} - \frac{y^2}{\sqrt{x^2+y^2}}}{x^2+y^2} = \frac{x^2}{(x^2+y^2)^{3/2}}$ ✓

$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$

$\frac{f''(\sqrt{x^2+y^2}+x)^2}{x^2+y^2} + \frac{f'\,y^2}{(x^2+y^2)(x^2+y^2)^{1/2}} + \frac{f''\,y^2}{x^2+y^2} + f'\left(\frac{x^2}{(x^2+y^2)(x^2+y^2)^{1/2}}\right) = 0$

$\frac{f''(t^2+y^2) + f'\left(\frac{y^2+x^2}{\sqrt{x^2+y^2}}\right)}{(x^2+y^2)} = 0$

$f''(t^2+y^2) + f'(y^2+x^2)^{1/2} = 0$

$f''(t^2+y^2) + f'(t^2-2xt+x^2)^{1/2} = 0$

$f''(t^2+y^2) + f'(t-x) = 0$

$f''(t^2+t^2-2xt) + f'(t-x) = 0$

$$f''(2t)(t-x) + f'(t-x) = 0$$

$$f''(2t) + f' = 0$$

$h = f'$ $\quad h' = f''$

$$h'(2t) = -h$$

$$\int \frac{dh}{h} = \int -\frac{dt}{2t}$$

$$\ln(h) = -\frac{1}{2}\ln(t) + C_1$$

$$\ln h = \ln\left(\frac{C_1}{t^{1/2}}\right)$$

$$h = \frac{C_1}{t^{1/2}}$$

$$f' = \frac{C_1}{t^{1/2}}$$

$$\int df = \int \frac{C_1}{t^{1/2}}\, dt$$

$$f = 2C_1 t^{1/2} + C_2$$

$$f = C_1\sqrt{t} + C_2$$

$$\boxed{f = C_1\sqrt{x + \sqrt{x^2+y^2}} + C_2}$$

131) $u = f\left(\frac{x^2+y^2}{x}\right)$

$t = \frac{x^2+y^2}{x} = x + y^2x^{-1} \Rightarrow y^2 = xt - x^2$

$\frac{dt}{dx} = 1 - \frac{y^2}{x^2}$; $\quad \frac{d^2t}{dx^2} = \frac{2y^2}{x^3}$ ✓

$\frac{dt}{dy} = \frac{2y}{x}$; $\quad \frac{d^2t}{dy^2} = \frac{2}{x}$ ✓

- $\frac{du}{dx} = f' \frac{dt}{dx}$

$$\frac{d^2t}{dx^2} = f''\left(\frac{dt}{dx}\right)^2 + f' \frac{d^2t}{dx^2}$$

$$\frac{d^2t}{dx^2} = \left(\frac{x^2-y^2}{x^2}\right)^2 f'' + \frac{2y^2}{x^3} f'$$

- $\frac{du}{dy} = f' \frac{dt}{dy}$

$$\frac{d^2u}{dy^2} = f''\left(\frac{dt}{dy}\right)^2 + f' \frac{d^2t}{dy^2} = \frac{4y^2}{x^2} f'' + \frac{2}{x} f'$$

- $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$

$$\left(\frac{x^2-y^2}{x^2}\right)^2 f'' + \frac{2y^2}{x^3} f' + \frac{4y^2}{x^2} f'' + \frac{2f'}{x} = 0$$

$$f''\left(\frac{(x^2-y^2)^2}{x^4}+\frac{4y^2}{x^2}\right)+f'\left(\frac{2y^2}{x^3}+\frac{2}{x}\right)=0$$

$$f''\left(\frac{x^4-2x^2y^2+y^4+4y^2x^2}{x^4}\right)+f'\left(\frac{2y^2+2x^2}{x^3}\right)=0$$

$$f''\left(\frac{x^4+y^4+2x^2y^2}{x^4}\right)+f'\left(\frac{2t}{x^2}\right)=0$$

$$f''\frac{(x^2+y^2)^2}{x^4}+f'\left(\frac{2t}{x^2}\right)=0$$

$$f''\frac{t^2}{x^2}+f'\frac{(2t)}{x^2}=0$$

$$f''t^2+f'(2t)=0$$

$h=f'$ $\quad h'=f''$

$$h't^2=-h\,2t$$

$$\int\frac{dh}{h}=\int-\frac{2t}{t^2}\,dt$$

$$\ln h=-2\ln t+c_1$$

$$h=\frac{C_1}{t^2}$$

$$f'=\frac{C_1}{t^2}$$

$$\int df=\int\frac{C_1}{t^2}\,dt$$

$$f=-\frac{C_1}{t}+C_2$$

$$\boxed{f=-C_1\left(\frac{x}{x^2+y^2}\right)+C_2}$$