# PRACTICA # 2

140) $\int_C z\,\text{Im}(z^2)\,dz \quad C: |z|=1 \quad (-\pi \leq \arg z \leq 0)$

si $z = e^{i\varphi} \qquad dz = i e^{i\varphi} d\varphi$

$\underline{I} = \int_{-\pi}^{0} e^{i\varphi}\,\text{Im}(e^{2i\varphi}) \cdot i e^{i\varphi} d\varphi = \int_{-\pi}^{0} i e^{i2\varphi} \cdot \text{sen}(2\varphi)\, d\varphi$

$= \int_{-\pi}^{0} i\cos(2\varphi)\,\text{sen}(2\varphi) - \text{sen}^2(2\varphi)\, d\varphi = i\int_{-\pi}^{0} \cos(2\varphi)\,\text{sen}(2\varphi)\,d\varphi - \int_{-\pi}^{0} \text{sen}^2(2\varphi)\,d\varphi$

$i\int_{-\pi}^{0} u\,\frac{du}{2} - \int_{-\pi}^{0} \left(\frac{1 - \cos(4\varphi)}{2}\right) d\varphi$ | $u = \text{sen}(2\varphi)$, $du = 2\cos(2\varphi)\,d\varphi$

$= \frac{i u^2}{4} - \frac{\varphi}{2} + \frac{\text{sen}(4\varphi)}{8} \Big|_{-\pi}^{0}$

$= \cancel{\frac{i\,\text{sen}^2(2\cdot 0)}{4}} - \cancel{\frac{0}{2}} + \cancel{\frac{\text{sen}(4\cdot 0)}{8}} - \cancel{\frac{i\,\text{sen}^2(-2\pi)}{4}} - \frac{\pi}{2} - \cancel{\frac{\text{sen}(-4\pi)}{8}}$

$\boxed{I = -\frac{\pi}{2}}$

141) $\int_C e^{|z|^2}\,\text{Re}(z)\,dz \qquad C: \; z_1 = 0 \quad z_2 = 1 + i$

$x = y \qquad dx = dy$

$\int_C \underbrace{e^{x+y} \cdot x}_{u}\, dz = \int_0^1 (u\,dx - \cancel{v}^{0}dy) + i\int_0^1 (\cancel{v}^{0}dx) + u\,dy$

$= \int_0^1 e^x e^y x\,dx + i\int_0^1 e^x e^y x\,dy = \int_0^1 e^{2x} \cdot x\,dx + i\int_0^1 e^{2x}\,dx$

$dv = e^{2x}dx \rightarrow v = \frac{e^{2x}}{2} \qquad u = x \qquad du = dx$

$$= \frac{e^{2x}}{2}x - \int_0^1 \frac{e^{2x}}{2}dx + i\left(\frac{e^{2x}}{2}x - \int_0^1 \frac{e^{2x}}{2}dx\right)$$

$$= \frac{e^{2x}}{2}x - \frac{e^{2x}}{4} + i\left(\frac{e^{2x}}{2}x - \frac{e^{2x}}{4}\right)\Big/_0^1$$

$$= \frac{e^2}{2} - \frac{e^2}{4} + i\left(\frac{e^2}{2} - \frac{e^2}{4}\right) - \left(0 - \frac{e^0}{4} + i\left(0 - \frac{e^0}{4}\right)\right)$$

$$\boxed{= \frac{e^2}{4} - \frac{1}{4} + i\left(\frac{e^2}{4} - \frac{1}{4}\right)}$$

142) $\int_C \ln z \, dz \qquad C: |z| = 1 \; (0 \leq \arg \leq \pi)$

a) $z_0 = 1 \qquad z = e^{i\varphi} \qquad dz = ie^{i\varphi}d\varphi$

$$\int_0^\pi \ln(e^{i\varphi}) \cdot (ie^{i\varphi}d\varphi) = \int_0^\pi i\varphi \cdot ie^{i\varphi}d\varphi = -\int_0^\pi \varphi e^{i\varphi}d\varphi$$

$$u = \varphi \qquad du = d\varphi \qquad dv = e^{i\varphi}d\varphi \qquad v = \frac{e^{i\varphi}}{i}$$

$$= -\frac{\varphi e^{i\varphi}}{i} + \int_0^\pi \frac{e^{i\varphi}}{i}d\varphi = -i^3\varphi e^{i\varphi} - e^{i\varphi}\Big/_0^\pi = i\pi e^{i\pi} - e^{i\pi} - 0 + e^0$$

$$= 1 + (i\pi - 1)(\cos\pi + i\cancel{\operatorname{sen}(\pi)}^{0}) = 1 - i\pi + 1 = \boxed{2 - i\pi}$$

143) $\int_C z \, R(z) \, dz \qquad C: |z| = 1 \qquad (-\pi \leq \arg z \leq 0)$

$$z = e^{i\varphi} \Rightarrow dz = ie^{i\varphi}d\varphi$$
$$= \cos\varphi + i\operatorname{sen}\varphi$$

$$= \int_{-\pi}^0 e^{i\varphi}\cos\varphi \, ie^{i\varphi}d\varphi = \int_{-\pi}^0 i\, e^{2i\varphi}\cos\varphi \, d\varphi = \int_{-\pi}^0 [i\cos(2\varphi) - \operatorname{sen}(2\varphi)]\cos\varphi \, d\varphi$$

$$\int_{-\pi}^0 i(\cos^3\varphi - \operatorname{sen}^2\varphi\cos\varphi) - 2\cos^2\varphi\operatorname{sen}\varphi \, d\varphi$$

$$= \operatorname{sen}\varphi - \frac{1}{3}\operatorname{sen}^3\varphi - \frac{\operatorname{sen}^3\varphi}{3} + 2\frac{\cos^3\varphi}{3}\Big/_{-\pi}^0 = \cancel{\operatorname{sen} 0} - \cancel{\frac{2}{3}\operatorname{sen}^3 0} + \frac{2}{3}\cos^3\varphi - \cancel{\operatorname{sen} -\pi} + \frac{2}{3}\cancel{\cos^3 -\pi}$$

$$= \frac{2}{3} - \frac{2}{3} = \boxed{0}$$

144) $\int_c z\bar{z}\,dz$ $\quad c: |z|=1 \quad (-\pi \leq \arg z \leq 0)$

$$z = e^{i\varphi} \qquad \bar{z} = e^{-i\varphi}$$

$$= \int_{-\pi}^{0} \cancel{e^{i\varphi}} \cdot \cancel{e^{(i(-\varphi))}} \cdot ie^{\varphi i}\,d\varphi = \int_{-\pi}^{0} ie^{\varphi i}\,d\varphi = e^{\varphi i}\Big/_{-\pi}^{0}$$

$$= e^{0} - (\cos(-\pi) + i\cancel{\operatorname{sen}(-\pi)}^{0}) = 1-1 = \boxed{0}$$

145) $\int_1^i ze^z\,dz =$

$$u = z \qquad dz = du \qquad dv = e^z dz \qquad v = e^z$$

Entoncés:

$$= ze^z - \int_1^i e^z\,dz = ze^z - e^z\Big/_1^i = ie^i - e^i - \cancel{e} + \cancel{e}$$

$$= ie^i - e^i = \boxed{e^i(i-1)}$$

146) $\int_c \operatorname{Re}(z)\,dz$

a) $c: z = (2+i)t \quad (0 \leq t \leq 1) \quad 2x = y \quad dy = 2dx$

$$I = \int_0^2 x\,dx - \cancel{0}\,dy + i\int_0^1 \cancel{0}\,dx + x \cdot 2dx$$

$$I = \frac{x^2}{2}\Big/_0^2 + i\,x^2\Big/_0^1 = \boxed{2+i}$$

b) $z_1 = 2 \qquad z_2 = 2+i \qquad x = 2 \qquad dx = 0$

151) $\int_C \cos z\,dz$ $\quad C: z_1 = \frac{\pi}{2} \quad z_2 = \pi + i$

- $I = \int_{\pi/2}^{\pi+i} \cos z\,dz = \operatorname{sen} z \Big|_{\pi/2}^{\pi+i} = \operatorname{sen}(\pi+i) - \operatorname{sen}(\pi/2) =$

$I = \cancelto{0}{\operatorname{sen}(\pi)}\cos(i) + \operatorname{sen}(i)\cos(\pi) - 1 = i\operatorname{senh}(1)(-1) - 1 =$

$\boxed{I = -1 - i\operatorname{senh}(1)}$

154) $\int_{1+i}^{2i} (z^3 - z)e^{\frac{z^2}{2}}\,dz$ $\qquad u = \frac{z^2}{2} \quad du = z\,dz$

$I = \int_{1+i}^{2i} z^3 e^{z^2/2} - z e^{z^2/2}\,dz$ $\qquad t = u \quad ds = e^u\,du$

$I = \int_{1+i}^{2i} 2u\cdot e^u\,du - \int_{1+i}^{2i} e^u\,du$ $\qquad dt = du \quad s = e^u$

- $I = 2\left(u e^u - \int_{1+i}^{2i} e^u\,du\right) - e^u \Big|_{1+i}^{2i}$

$I = 2\left(u e^u - e^u\right)\Big|_{1+i}^{2i} - e^{z^2/2}\Big|_{1+i}^{2i}$

$I = e^{z^2/2}\cancel{2}\left(\frac{z^2}{\cancel{2}}\right) - 2e^{z^2/2} - e^{z^2/2}\Big|_{1+i}^{2i}$

$I = \left\{\left[(2i)^2 - (1+i)^2\right] - 3\right\}\left[e^{(2i)^2/2} - e^{(1+i)^2/2}\right]$

$I = \left\{\left(-4 - \cancel{1} - 2i + \cancel{1}\right) - 3\right\}\left[e^{-2} - \cancel{e^{1/2}}\cdot e^{i}\cdot \cancel{e^{-1/2}}\right]$

$\boxed{I = (-7 - 2i)(e^{-2} - e^{i})}$

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156) $\int_1^i z \operatorname{sen} z \, dz$

$z = u \qquad dv = \operatorname{sen} z \, dz$

$du = dz \qquad v = -\cos(z)$

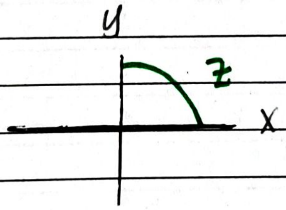
$I = -z\cos(z) - \int_1^i -\cos(z)\, dz$

$I = -z\cos(z) + \operatorname{sen}(z) \Big|_1^i$

$I = -i\cos(i) + \operatorname{sen} i + 1\cos(1) - \operatorname{sen}(1)$

$\boxed{I = \cos(1) - \operatorname{sen}(1) - ie^{i^2}}$

158) $\int_1^i \frac{\ln(z+1)}{z+1}\, dz \qquad |z| = 1 \qquad I(z) \geq 0 \qquad R(z) \geq 0$




$I = \int_1^i u\, du$

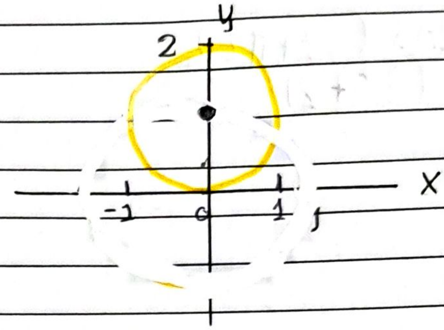
$u = \ln(z+1)$

$du = \frac{dz}{z+1}$

$I = \frac{u^2}{2}\Big|_1^i = \frac{[\ln(z+1)]^2}{2}\Big|_1^i$

$\boxed{I = \frac{1}{2}\left[(\ln(i+1))^2 - (\ln(2))^2\right]}$

168) $\oint_{|z-i|=1} \frac{e^{iz}}{z^2+1} \, dz$




$x^2 + (y-1)^2 = 1$

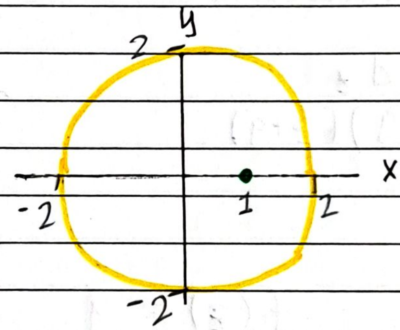
$C(0,1) \quad r=1$

$\frac{e^{iz}}{z^2+1} = \frac{e^{iz}}{(z+i)(z-i)} \qquad f(z) = \frac{e^{iz}}{z+i} \qquad z_0 - i = 0$

$z_0 = i$

$I = \oint \frac{e^{iz}}{z^2+1} = 2\pi i \cdot \left(\frac{e^{i \cdot i}}{i+i}\right) = \cancel{2}\pi \cancel{i}\left(\frac{e^{-1}}{\cancel{2i}}\right) = \boxed{\frac{\pi}{e}}$

170) $\oint_{|z|=2} \frac{sen(iz)}{z^2-4z+3}$




$x^2+y^2=2^2$

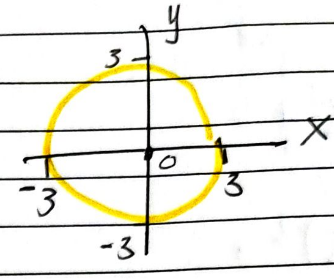
$C(0,0); r=2$

$\frac{Sen\,(iz)}{(z-3)(z-1)} = \frac{\frac{sen\,(iz)}{(z-3)}}{(z-1)} \to f(z) = \frac{sen\,(iz)}{z-3}; \quad z_0 = 1$

$I = \oint \frac{sen(iz)}{z^2-4z+3} = 2\pi i \left(\frac{-i\, senh(-1)}{1-3}\right) = -\cancel{2}\pi i^2 \left(\frac{senh\, -1}{-\cancel{2}}\right) =$

$= -\pi\, senh(-1) = \boxed{\pi\, senh(1)}$

172) $\displaystyle\int_{|z|=3} \frac{\cos(z+\pi i)}{z(e^z+2)}$




$x^2+y^2=3^2$

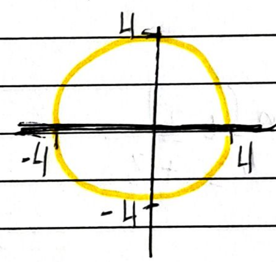
$\dfrac{\cos(z+\pi i)}{z(e^z+2)} \to f(z) = \dfrac{\cos(z)\cos(\pi i) - \operatorname{sen}(z)\operatorname{sen}(\pi i)}{e^z+2}$

$z_0 = 0$

$I = 2\pi i \left( \dfrac{\cos(0)\cos(\pi i) - \cancel{\operatorname{sen}(0)\operatorname{sen}(\pi i)}}{e^0+2} \right)$

$\boxed{I = \dfrac{2\cdot\pi\cdot i\cdot\cosh(\pi)}{3}}$

174) $\displaystyle\int_{|z|=4} \frac{dz}{(z^2+9)(z+9)}$




$x^2+y^2=4^2$

$\dfrac{\frac{1}{z+9}}{(z-3i)(z+3i)}$ $\qquad f(z)=\dfrac{1}{z+9}$ ; $z_0=3i$ $\quad z_0=-3i$

$\dfrac{1}{z^2+9} = \dfrac{A}{(z-3i)} + \dfrac{B}{(z+3i)} \Rightarrow A(z+3i)+B(z-3i)=1$

Si $z=-3i$ $\qquad -6iB=1 \to B=\dfrac{1i}{6}$

Si $z=3i$ $\qquad 6iA=1 \to A=-\dfrac{i}{6}$

$I = -\dfrac{i}{6}\displaystyle\int \frac{dz}{(z-3i)(z+9)} + \frac{i}{6}\int \frac{dz}{(z+3i)(z+9)}$

$I = -\dfrac{i}{6}\left[2\pi i\left(\dfrac{1}{+3i+9}\right)\right] + \dfrac{i}{6}\left[2\pi i\left(\dfrac{1}{-3i+9}\right)\right]$

$$I = \left(\frac{1\pi}{9i+27}\right) - \left(\frac{\pi}{-9i+27}\right)$$

$$I = \frac{\pi(+9i+\cancel{27}+9i-\cancel{27})}{81+27\cdot27} = \frac{-\pi 18i}{810}$$

$$\boxed{I = \frac{-\pi i}{45}}$$

176) $\displaystyle\oint_{|z|=2} \frac{\operatorname{sen} z \operatorname{sen}(z-1)}{z^3-z}\,dz$

$x^2+y^2=2^2$

$$\frac{\operatorname{sen}(z)\operatorname{sen}(z-1)}{z(z-1)(z+1)} \qquad f(z) = \operatorname{sen} z \operatorname{sen}(z-1)$$

$$\frac{A}{z} + \frac{B}{z-1} + \frac{C}{z+1} = 1 \Rightarrow A(z-1)(z+1) + B(z)(z+1) + C(z)(z-1) = 1$$

Si $z=0$ $\quad -A = 1 \Rightarrow A = -1$

$z=1$ $\quad 2B = 1 \Rightarrow B = 1/2$

$z=-1$ $\quad 2C = 1 \Rightarrow C = 1/2$

$$I = -\oint \frac{\operatorname{sen}(z)\operatorname{sen}(z-1)}{z}\,dz + \frac{1}{2}\oint \frac{\operatorname{sen} z \operatorname{sen} z-1}{z-1} + \frac{1}{2}\oint \frac{\operatorname{sen} z \operatorname{sen} z-1}{z+1}$$

$$I = \cancel{-2\pi i \operatorname{sen}(0)\operatorname{sen}(-1)} + \cancel{\frac{1}{2}2\pi i \operatorname{sen}(1)\operatorname{sen}(1-1)} + \frac{1}{\cancel{2}}\cancel{2}\pi i \operatorname{sen}(-1)\operatorname{sen}(-2)$$

$$\boxed{I = \pi i \operatorname{sen}(-1)\operatorname{sen}(-2)}$$

178) $\displaystyle\oint_{|z|=1} \frac{\operatorname{Senh}^2 z}{z^3}\, dz$

$x^2 + y^2 = 1^2$

$\dfrac{\operatorname{senh}^2 z}{z^3} \rightarrow f(z) = \operatorname{senh}^2 z \;;\; z = 0 \qquad n+1 = 3$
$n = 2$

$f' = 2 \operatorname{senh}^2 z \cosh z$

$f'' = 2\cosh^2(z) + 2\operatorname{Sen}^2(z)$

$f''(0) = 2$

$\displaystyle\oint \frac{\operatorname{senh}^2(z)}{z^3}\, dz = \frac{2\pi i}{2!}(2) = \boxed{2\pi i}$

180) $\displaystyle\oint_{|z|=2} \frac{z \operatorname{sen} h\, z}{(z^2-1)^2}\, dz$

$|z| = 2 \rightarrow x^2 + y^2 = 2^2$

$\dfrac{z \operatorname{Sen} h\, z}{(z-1)^2 (z+1)^2} \rightarrow f(z) = z \operatorname{sen} h\, z \qquad z_0 = 1 \;,\; z_0 = -1$

$n+1 = 2$
$n = 1$

$\dfrac{A}{z-1} + \dfrac{B}{z+1} = \dfrac{1}{(z-1)(z+1)}$

$A(z+1) + B(z-1) = 1$

Si $z = -1 \qquad -2B = 1 \rightarrow B = -1/2$

$z = 1 \qquad 2A = 1 \rightarrow A = 1/2$

$f' = \operatorname{sen} h z + z \cosh z$

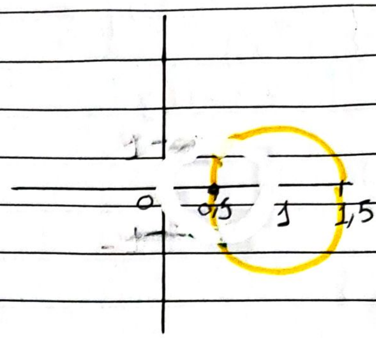
$f'' = 2\cosh z + z \operatorname{sen} h z$

$I = \displaystyle\oint \frac{z \operatorname{senh} z}{(z-1)^2} + \oint \frac{z \operatorname{sen} h z}{(z+1)^2}$

$I = \dfrac{2\pi i}{1!}\left(2\cosh(1) + \operatorname{senh}(1) + 2\cosh(-1) - \operatorname{senh}(-1)\right)$

$\boxed{I = 2\pi i\,(4\cosh(1) + 2\operatorname{senh}(1))}$

$$186) \int_{|z-1|=\frac{1}{2}} \frac{e^{iz}}{(z^2-1)^2}$$




$$(x-1)^2 + y^2 = \left(\frac{1}{2}\right)^2$$

$$C(1,0) \quad r = 0,5$$

$$\frac{e^{iz}}{(z^2-1)^2} = \frac{e^{iz}}{(z-1)^2(z+1)^2} \qquad f(z) = \frac{e^{iz}}{(z+1)^2} \;;\; z = 1$$

$$f'(z) = \frac{ie^{iz}(z+1)^2 - 2e^{iz}(z+1)}{(z+1)^4}$$

$$I = \int \frac{e^{iz}}{(z^2-1)^2} = \frac{2\pi i}{1!}\left(\frac{ie^{i}(1+1)^2 - 2e^{i}(1+1)}{(1+1)^4}\right) = 2\pi i e^{i}\left(\frac{4i-4}{16}\right)$$

$$\boxed{I = \frac{\pi e^{i}}{2}\left(-1-i\right)}$$