Tema: Fecha:

# Práctica N°1

## Cuerpo de los Numeros Complejos.

1= Demostrar las relaciones siguientes:

a) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

$z_1 - z_2 = (x_1 + y_1 i) - (x_2 + y_2 i) = x_1 + y_1 i - x_2 - i y_2$

$\Rightarrow \overline{(x_1 - x_2) + (y_1 - y_2) i} \Rightarrow x_1 - x_2 - y_1 i + i y_2$

$\Rightarrow (x_1 - i y_1) - (x_2 - i y_2) \Rightarrow \bar{z}_1 - \bar{z}_2$

$\therefore$ si cumple $\boxed{\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2}$

b) $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$ ; $\bar{z}_1 = x_1 - y_1 i \quad z_2 = x_2 - y_2 i$

$\overline{z_1 z_2} = (x_1 + i y_1) \cdot (x_2 + i y_2) \Rightarrow x_1 x_2 + x_1 i y_2 + x_2 i y_1 + \overset{-1}{i^2} y_1 y_2$

$\Rightarrow x_1 x_2 + x_1 i y_2 + x_2 i y_1 - y_1 y_2$

$\Rightarrow x_1 x_2 - y_1 y_2 + i(x_1 y_2 + x_2 y_1)$ -

$\bar{z}_1 \cdot \bar{z}_2 = (x_1 - y_1 i)(x_2 - y_2 i)$

$\Rightarrow x_1 x_2 - x_1 y_2 i - x_2 y_1 i + \overset{-1}{i^2} y_1 y_2$

$\Rightarrow (x_1 x_2 - y_1 y_2) - i(x_1 y_2 + x_2 y_1) \quad \bar{z}_1 \cdot \bar{z}_2$

$\overline{z_1 . z_2} = (x_1 x_2 - y_1 y_2) - i(x_1 y_2 + x_2 y_1$

$\therefore$ sin son iguales $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$

c) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2} \Rightarrow \frac{(x_1 + y_1 i)}{(x_2 + y_2 i)} \cdot \frac{x_2 - i y_2}{x_2 - i y_2}$

$\Rightarrow \frac{x_1 y_2 - i x_1 y_2 + x_2 y_1 i - i^2 y_1 y_2}{x_2^2 - \cancel{i x_2 y_2} + \cancel{x_2 i y_2} - i^2 y_2^2}$

$$\Rightarrow \frac{(x_1x_2+y_1y_2)+i(x_1y_2-x_2y_1)}{x_2^2+y_2^2}$$

$$\Rightarrow \overline{\frac{(x_1x_2+y_1y_2)+i(x_1y_2-x_2y_1)}{x_2^2+y_2^2}}$$

$$\Rightarrow \overline{\left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+\frac{x_1y_2-x_2y_1}{x_2^2+y_2^2}\right)}$$

$$\Rightarrow \left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}-\frac{x_1y_2-x_2y_1}{x_2^2+y_2^2}\right)$$

$$\Rightarrow \frac{(x_1x_2+y_1y_2)-i(x_1y_2-x_2y_1)}{x_2^2+y_2^2}$$

$$\frac{\overline{z_1}}{z_2} = \overline{\frac{(x_1x_2+y_1y_2)-i(x_1y_2-x_2y_1)}{x_2^2+y_2^2}}$$

d) $\overline{\overline{z_1}+\overline{z_2}} = z_1+z_2 \therefore = (x_1+y_1i)+(x_2+iy_2)$

$\Rightarrow \overline{(x_1-iy_1)+(x_2-iy_2)}$

$\Rightarrow \overline{(x_1+x_2)-i(y_1+y_2)}$

$\Rightarrow (x_1+x_2)+i(y_1+y_2)$

$\Rightarrow (x_1+iy_1)+(x_2+iy_2) \Leftrightarrow (x_1+iy_1)+(x_2+iy_2)$

$\therefore \overline{\overline{z_1}+\overline{z_2}} = z_1+z_2$ sol.

Hallar las soluciones reales de las ecuaciones

2. $(3x-i)(2+i)+(x-iy)(1+2i)=5+6i$

$6x+i3x-i2\cancel{-i^2}^{-b}+x+i2x-iy-\cancel{i^2}2y=5+6i$

$+1+7x+i5x-i2-iy+2y=5+6i$

$(7x+2y+1)+i(5x-y-2)=5+6i$

$$\begin{cases} 7x+2y+1 = 5 \\ 5x-y-2 = 6 \quad \cdot 2 \end{cases}$$

$$\begin{cases} 7x+\cancel{2y}+1 = 5 \\ 10x-\cancel{2y}-4 = 12 \end{cases}$$

$17x \quad -3 = 17$

$17x = 17+3$

$17x = 20$

$x = \frac{20}{17}$

Reemplazamos en 2 $\quad x = \frac{20}{17}$

$y = 5x-2-6$

$y = 5\left(\frac{20}{17}\right)-2-6$

$y = 5 \times \frac{20}{17} - 8$

$\boxed{y = -\frac{36}{17}}$

3 $(x-iy)(a-ib)=i^5$ , donde a, b, son los numeros reales dados $|a| \neq |b|$

$(x-iy)(a-ib) = i^5$

$ax - ibx - iay + \overset{-1}{\cancel{i^2}}by = \cancel{i^5} \qquad i^2 \cdot i^2 \cdot i$

$(ax-by) - i(bx+ay) = i+0$

$(ax-by) + i(-bx-ay) = i+0$

$$\begin{cases} ax - by = 0 \\ -bx - ay = 1 \end{cases} \Rightarrow x = \frac{by}{a}$$

Reemplazamos en 2

$-b\left(\frac{by}{a}\right) - ay = 1$

$-\frac{b^2y}{a} - ay = 1 \quad \times a$

$-b^2y - a^2y = a$

$y(-b^2-a^2) = a$

$y = \frac{a}{(-b^2-a^2)}$

$\boxed{y = -\frac{a}{b^2+a^2}}$

Tema: Fecha: / /

Reemplazamos en (1) $\boxed{y = -\dfrac{a}{b^2+a^2}}$

$$x = \frac{by}{a}$$

$$x = \frac{b\left(-\dfrac{a}{b^2+a^2}\right)}{a}$$

$$x = -\left[\frac{\dfrac{\cancel{a}b}{b^2+a^2}}{\cancel{a}}\right] \qquad \boxed{x = -\frac{b}{b^2+a^2}}$$

$\therefore$ Los valores de $\boxed{x = -\dfrac{b}{b^2+a^2}}$ y $\boxed{y = -\dfrac{a}{b^2+a^2}}$

4 = $\dfrac{1}{z-i} + \dfrac{z+i}{1+i} = \sqrt{2}$, donde $z = x+iy$

$$\frac{1}{z-i} + \frac{z+i}{1+i} = \sqrt{2}$$

$$\frac{(1+i) + (z+i)(z-i)}{(z-i)\,(1+i)} = \sqrt{2}$$

$$\frac{(1+i) + z^2 \cancel{-zi} \cancel{+iz} - i^2}{(z-i)\,(1+i)} = \sqrt{2}$$

$$(2z+2) + i(z-1) = \sqrt{2}\,(z+iz-i+1)$$

$$(2z+2) + i(z-1) = \sqrt{2}z + \sqrt{2}iz - i\sqrt{2} + \sqrt{2}$$

$$2z + 2 - \sqrt{2}z - \sqrt{2} = -iz + i + \sqrt{2}iz - i\sqrt{2}$$

$$z(2-\sqrt{2}) + (2-\sqrt{2}) + i\left(z(1-\sqrt{2}) - (1-\sqrt{2})\right)$$

$$(x+iy)(2-\sqrt{2}) + (2-\sqrt{2}) + i\left((x+iy)(1-\sqrt{2}) - (1-\sqrt{2})\right) = 0$$

$$2x - \sqrt{2}x + 2iy - \sqrt{2}iy + (2-\sqrt{2}) + i\left[x - \sqrt{2}x + iy - \sqrt{2}iy - 1 + \sqrt{2} = 0\right]$$

Tema: Fecha: / /

$x(2-\sqrt{2}) + iy(2-\sqrt{2}) + (2-\sqrt{2}) + ix - \sqrt{2}xi - y + \sqrt{2}y$
$-i + \sqrt{2}i = 0$

$$\begin{cases} (2-\sqrt{2})x + (\sqrt{2}-1)y + (2-\sqrt{2}) = 0 \\ (1-\sqrt{2})x + (2-\sqrt{2})y + (-1+\sqrt{2}) = 0 \quad \cdot(-2-\sqrt{1}) \end{cases}$$

$$\begin{cases} \cancel{(2-\sqrt{2})x} + (-1+\sqrt{2})y + (2-\sqrt{2}) = 0 \\ \cancel{(-2+\sqrt{2})x} + (-4+\sqrt{2})y + (2-\sqrt{2}) = 0 \end{cases}$$

$0 \qquad -5+2\sqrt{2}y + (4-2\sqrt{2}) = 0$

$(-5+2\sqrt{2})y = -4+2\sqrt{2}$

$y = \dfrac{-4+2\sqrt{2}}{-5+2\sqrt{2}}$

$$\boxed{y = \frac{12-2\sqrt{2}}{17}}$$

Reemplazo en (1)

$(2-\sqrt{2})x + = -2\cancel{+\sqrt{2}} + (1\cancel{-\sqrt{2}})\left(\dfrac{12-2\sqrt{2}}{17}\right)$

$$x = \frac{-1\left(\frac{12-2\sqrt{2}}{17}\right)}{(2-\sqrt{2})}$$

$$x = -\frac{16+4\sqrt{2}}{17} \quad \text{sol}$$

Tema: Fecha: / /

5= Presentar el numero complejo $\frac{1}{(a+ib)^2} + \frac{1}{(a-ib)^2}$ en la forma algebraica.

$$\frac{1}{a^2+2abi+\cancel{ib^2}} + \frac{1}{a^2-2abi+i^2b^2} = 0$$

$$\frac{1}{a^2+2abi-b^2} + \frac{1}{a^2-2abi-b^2}$$

$$\frac{(a^2-\cancel{2abi}-b^2)+(a^2+\cancel{2abi}-b^2)}{(a^2+2abi-b^2)\cdot(a^2-2abi-b^2)}$$

$$\Rightarrow \frac{2a^2-2b^2}{(a^2+2abi-b^2)(a^2-2abi-b^2)}$$

$$\Rightarrow \frac{2a^2-2b^2}{(a^2-b^2)^2+(2ab)^2}$$

$$\Rightarrow \frac{2a^2-2b^2}{a^4-2a^2b^2+b^4+4a^2b^2}$$

$$\Rightarrow \frac{2a^2-2b^2}{a^4+2a^2b^2+b^4}$$

$$\Rightarrow \frac{2a^2-2b^2}{(a^2+b^2)^2} \quad \rfloor \text{ sol.}$$

6 Demostrar que $\frac{\sqrt{1+x^2}+ix}{x-i\sqrt{1+x^2}} = i$ (x es real)

$$\frac{\sqrt{1+x^2}+ix}{x-i\sqrt{1+x^2}} \cdot \frac{x+i\sqrt{1+x^2}}{x+i\sqrt{1+x^2}}$$

$$\frac{x\sqrt{1+x^2}+i(\sqrt{1+x^2})^2+ixx+i^2x\sqrt{1+x^2}}{x^2-(i^2\sqrt{1+x^2})}$$

$$\Rightarrow \frac{\cancel{x\sqrt{1+x^2}}+i(1+x^2)+ix^2\cancel{-1x\sqrt{1+x^2}}}{x^2+\sqrt{1+x^2}}$$

$$\frac{i + ix^2 + ix^2}{x^2 + (\sqrt{1+x^2})^2} = i$$

$$\frac{i + 2x^2}{x^2 + (\sqrt{1+x^2})^2} = i$$

$$\frac{i(1+2x^2)}{x^2 + (\sqrt{1+x^2})^2} = i$$

$$i(1+2x^2) = i(x^2+1+x^2)$$

$$i\cancel{(1+2x^2)} = i\cancel{(1+2x^2)}$$

$$i = i$$

$\therefore$ son iguales

8- Hallar todos los numeros complejos que satisfacen la condicion $\bar{z} = z^2$

$z = (x+iy) \rightarrow \boxed{\bar{z} = x-iy}$

$(x+iy)^2 = z^2$

$x^2 + 2xyi + \cancel{i^2} y^2$

$(x^2 - y^2) + 2xyi = (x-iy)$

$$\begin{cases} x^2 - y^2 = x \\ 2xy = -y \end{cases} \rightarrow 2xy + y = 0$$

$y(2x+1) = 0$

$\boxed{y=0}$ $\boxed{x = -1/2}$

$x = -1/2$

$x^2 - x = y^2$

$\left(-\frac{1}{2}\right)^2 + \frac{1}{2} = y^2$

$y^2 = \frac{1}{4} + \frac{1}{2}$

$y = \pm\sqrt{\frac{3}{4}}$

$x = -\frac{1}{2}$ $y = +\sqrt{\frac{3}{4}}$

$x = -\frac{1}{2}$ $y = -\sqrt{\frac{3}{4}}$

Para $y = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ $y = 0$

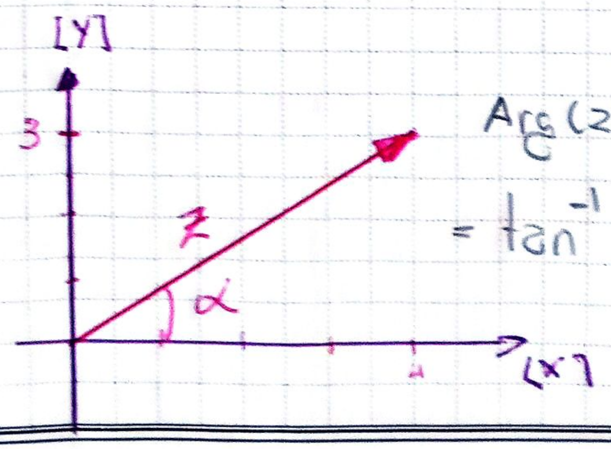
$x = 1$ $y = 0$

9- En los problemas siguientes hallar el modulo y el valor principal del argumento de los numeros complejos.

a) $z = 4+3i$ $\rho = \sqrt{x^2+y^2} = \sqrt{16+9} = 5$

Modulo = 5 sol

Argumento




$\operatorname{Arg}(z) = \alpha$

$= \tan^{-1}\left(\frac{|3|}{|4|}\right)$

$Arg(z) = 0,6435 \text{ (rad)} \times \frac{180}{\pi} = 36,87$

b) $z = -2 + 2\sqrt{3}i$ ;

$\rho = \sqrt{x^2+y^2} = \sqrt{2^2 + (2\sqrt{3})^2} = 4$

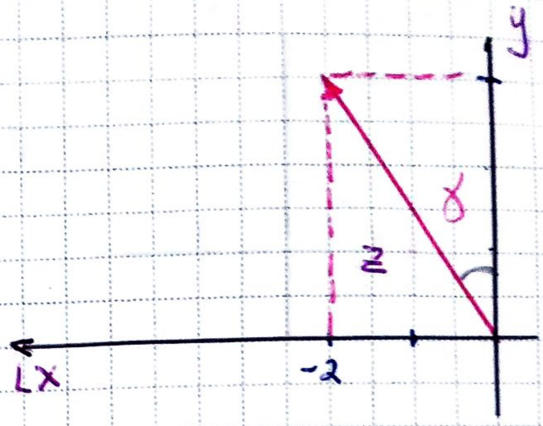
Modulo = 4

Argumento

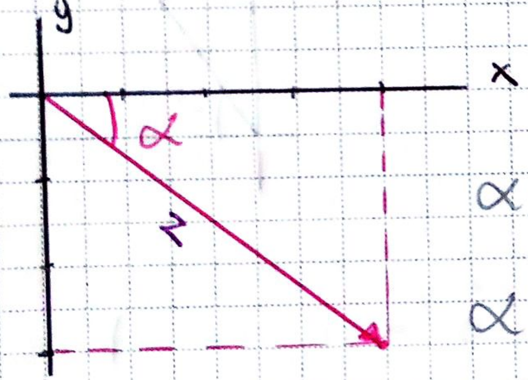
$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

$\alpha = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$

$\alpha = \frac{1}{3}\pi$




e.) $z = 4 - 3i$ Modulo $\rho = \sqrt{4^2+3^2} = \sqrt{16+9} = 5$




$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

$\alpha = -\tan^{-1}\left(\frac{3}{4}\right)$

$\alpha = -0,6435 \text{ (rad)}$

f) $z = \cos\alpha - i\,\text{sen}\,\alpha \quad (\pi < \alpha < \frac{3}{2}\pi)$

$\rho = \sqrt{(\cos\alpha)^2 + (\text{sen}^2\alpha)}$

$\rho = \sqrt{\cos^2\alpha + \text{sen}^2\alpha}$

$\rho = \sqrt{1} = 1$ modulo

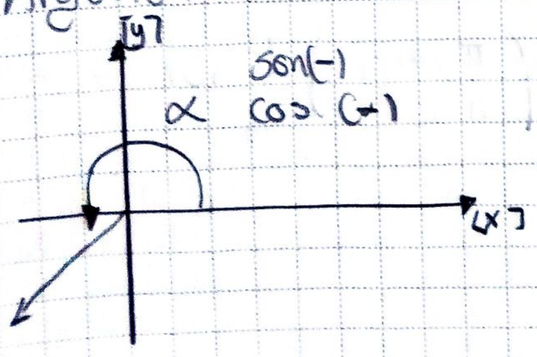
$z = -A - i(-B) = -A + iB$

$\alpha = Arg\,z = \pi - \theta'$

$\alpha = \pi - \tan^{-1}\left(\frac{|-\text{sen}\,\alpha|}{|-\cos\alpha|}\right)$

$= \pi - \tan^{-1}(\tan\alpha)$

Argumento




$\alpha = \pi - \alpha$

$\boxed{\theta = Arg\,z = \pi - \alpha}$

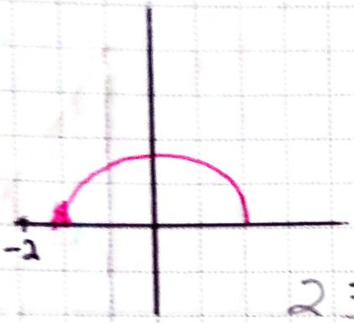
Fecha: / /

10.- Expresar los siguientes numeros complejos en la forma trigonometrica

a) $-2$: $z = x + iy \rightarrow z = \rho(\cos\alpha + i\,\text{sen}\,\alpha)$

Trigonometrica

$z = -2 + 0i = \sqrt{4} = \underline{2}$ modulo

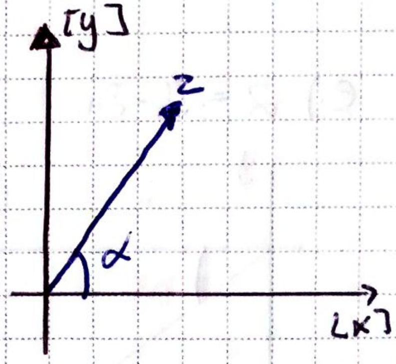


$\text{Arg}(z) = \pi$ (rad

$\alpha = \pi$ rad

$z = 2(\cos(\pi) + i\,\text{sen}(\pi))$ ∴ forma trigono

d. $\underbrace{1-\text{sen}\,\alpha}_{\text{Real}} + \underbrace{i\cos\alpha}_{\text{Ima}}$ $(0 < \alpha < \frac{\pi}{2})$:



$r = \sqrt{(1-\text{sen}\,\alpha)^2 + (\cos\alpha)^2}$

$r = \sqrt{1^2 - 2\,\text{sen}\,\alpha + \underbrace{\text{sen}^2\alpha + \cos^2\alpha}_{1}}$

$r = \sqrt{2 - 2\,\text{sen}\,\alpha}$

$\text{Arg}\ z = \tan^{-1}\left(\frac{\cos\alpha}{1-\text{sen}\,\alpha}\right)$ $\qquad z = r(\cos\alpha + i\,\text{sen}\,\alpha)$

$z = \sqrt{2-2\,\text{sen}\,\alpha}\left[\cos\left(\arctan\frac{|\cos\alpha|}{|1-\text{sen}\,\alpha|}\right) + i\,\text{sen}\,\frac{|\cos\alpha|}{|1-\text{sen}\,\alpha|}\right]$

∴ forma trigonometrica

f) $-2$ $\boxed{z = x+iy}$ $\boxed{z = \rho e^{i\alpha}}$ Igualdad de Euler

$r = 2$ Algebraica Exponencial $e^{i\alpha} = \cos\alpha + i\,\text{sen}\,\alpha$

$r = \sqrt{2^2} = 2$ módulo $\text{Arg}\, z = \pi = -\pi$

$z = 2 \cdot e^{i\pi}$ forma Exponencial

h) $-i$ $z = r e^{i\alpha}$

$r = \sqrt{(-1)^2} = 1$ $\text{Argu}(z) = \pi$

$z = e^{i\pi}$ forma exponencial

k) $5+3i = r = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

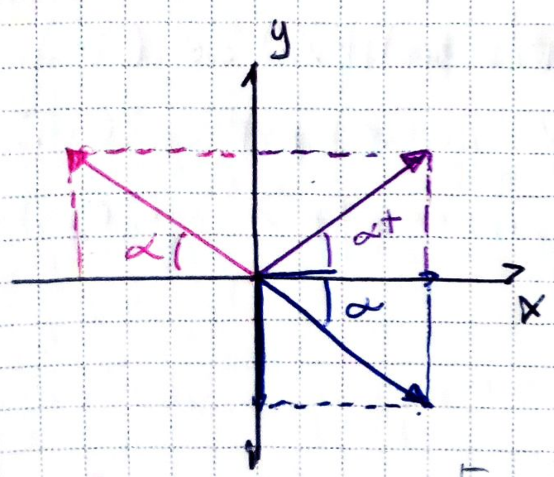
$z = r e^{i\alpha}$

$z = \sqrt{34}\, e^{i\pi}$ forma exponencial

j) $\text{sen}\,\alpha - i\cos\alpha$ $\left(\frac{\pi}{2} < \alpha < \pi\right)$;

$x = \text{sen}\,\alpha$ ; $y = -\cos\alpha$

$r = \sqrt{(\text{sen}\,\alpha)^2 + (-\cos\alpha)^2} = \sqrt{\text{sen}^2\alpha + \cos^2\alpha} = 1$




$\frac{\pi}{2} < \alpha < \pi$ Angulo $2^{do}$ C

$\text{sen}\,\alpha = +$

$\cos\alpha = -$

$\varphi = \text{Arg}(z) = \tan^{-1}\left(\frac{|-\cos\alpha|}{|\text{sen}\,\alpha|}\right)$

$\varphi = -\tan^{-1}(\cot g\alpha)$

$\text{sen}\,\alpha + i\cos\alpha = 1\left[\cos(-\tan^{-1}\cot g\alpha) + i\,\text{sen}(-\tan^{-1}(\cot g\alpha))\right]$

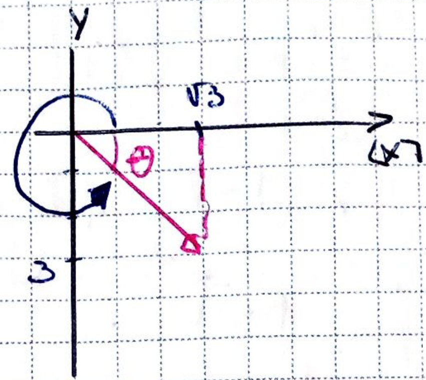
Tema: c, d    Fecha: / /

c $(\sqrt{3}-3i)^6$ => Usamos el teorema de Moivre

1• hallamos el modulo

$r = \sqrt{(\sqrt{3})^2+(-3)^2} = 2\sqrt{3}$ sol

2 = hallamos el Argumento




$Arg(z) = \tan^{-1}\left(\frac{|-3|}{\sqrt{3}}\right)$

$Arg(z) = +2\pi - \frac{1}{3}\pi$

$Arg(z) = \frac{5}{3}\pi$

3 = Transformamos a modo Polar

$z^6 = (2\sqrt{3})^6 \left(\cos\frac{5}{3}\pi + i\,sen\frac{5}{3}\pi\right)^6$

$z^6 = (2\sqrt{3})^6 \cos\frac{5}{\cancel{3}_1}\pi \cdot \cancel{6}^2 + i\,sen\frac{5}{3}\pi \times 6$

$z^6 = 1728\left[\cos 10\pi + i\,sen\,10\pi\right]$ sol.

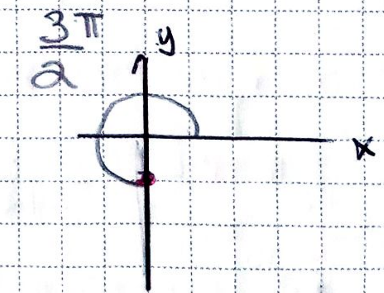
d. $\left(\frac{1-i}{1+i}\right)^8$

=> 1 simplificamos

$\frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{1-i-i+i^2}{1^2-i^2} \Rightarrow \frac{\cancel{1}-2i-\cancel{1}}{1+1} = \frac{-\cancel{2}i}{\cancel{2}} = -i$

2 = Modulo = 1

3 = Argumento




$Arg(z) = \tan^{-1}(-1)$

$Arg(z) = \frac{3\pi}{2}$

$z^8 = 1^8\left(\cos\frac{3\pi}{\cancel{2}_1}\cdot\cancel{8}^4 + i\,sen\frac{3}{2}\pi\times 8\right)$

$z^8 \left(\cos 12\pi + i\,sen\,12\pi\right)$ sol.

13 - Demostrar que

$$\left(\frac{1+i\,tg\,\alpha}{1-i\,tg\,\alpha}\right)^n = \frac{1+i\,tg\,n\alpha}{1-i\,tg\,n\alpha}$$

sabemos $(\cos\alpha + i\,sen\,\alpha)^n = \cos n\alpha + i\,sen\,n\alpha$

Entonces $\left(\dfrac{1+i\dfrac{sen\,\alpha}{\cos\alpha}}{1-i\dfrac{sen\,\alpha}{\cos\alpha}}\right)^n \Rightarrow \left[\dfrac{\dfrac{\cos\alpha+i\,sen\,\alpha}{\cancel{\cos\alpha}}}{\dfrac{\cos\alpha-i\,sen\,\alpha}{\cancel{\cos\alpha}}}\right]^n$

$\Rightarrow \left(\dfrac{\cos\alpha + i\,sen\,\alpha}{\cos\alpha - i\,sen\,\alpha}\right)^n \Rightarrow \dfrac{\cos n\alpha + i\,sen\,\alpha n}{\cos n\alpha + i\,sen\,\alpha n}$

$\Rightarrow \dfrac{\dfrac{\cos n\alpha + i\,sen\,\alpha n}{\cos n\alpha}}{\dfrac{\cos n\alpha - i\,sen\,n\alpha}{\cos n\alpha}} \Rightarrow \dfrac{1+i\,tag\,n\alpha}{1-i\,tag\,n\alpha}$

∴ Si cumple la siguiente expresión

14 - Demostrar que si $(\cos\alpha + i\,sen\,\alpha)^n = 1$, entonces $(\cos\alpha - i\,sen\,\alpha)^n = 1$

$(\cos\alpha - i\,sen\,\alpha)^n = 1$

$(\cos n\alpha - i\,sen\,n\alpha)^2 = (1)^2$

$(\cos^2 n\alpha - i\cos n\alpha \cdot sen\,\alpha - sen^2\alpha) = 1$

$(\underbrace{\cos^2 n\alpha - sen^2\alpha}_{1} - i\cos n\alpha \cdot sen\,n\alpha) = 1$

$1 - i\cos n\alpha \cdot sen\,n\alpha) = 1 + 0$

$\begin{cases} 1 = 1 \\ i\cos n\alpha \cdot sen\,n\alpha) = 0 \end{cases}$

$1 = 1$ ∴ Demostrado

15- Utilizando la formula de Moivre, expresar mediante las potencias sen φ y cos φ las funciones siguientes de los angulos multiples.

f) $\cos 5\varphi$ $(\cos\varphi + i\,\text{sen}\,\varphi)^5 = \cos 5\varphi + i\,\text{sen}\,5\varphi$

$\Rightarrow \cos^5\varphi + 5\cos^4\varphi(i\,\text{sen}\,\varphi) + 10\cos^3\varphi(i\,\text{sen}\,\varphi)^2$
$+ 10\cos^2\varphi(i\,\text{sen}\,\varphi)^3 + 5\cos\varphi(i\,\text{sen}\,\varphi)^4$
$+ (i\,\text{sen}\,\varphi)^5$

$\Rightarrow \cos^5\varphi + i(5\cos^4\varphi \cdot \text{sen}\,\varphi) - 10(\cos^3\varphi\,\text{sen}^2\varphi$
$- i(10\cos^2\varphi\,\text{sen}^3\varphi + 5\cos\varphi\,\text{sen}^4\varphi + i\,\text{sen}^5\varphi$

$\cos 5\varphi \Rightarrow \cos^5\varphi - 10\cos^3\varphi\,\text{sen}^2\varphi + 5\cos\varphi\,\text{sen}^4\varphi$

$\text{sen}\,5\varphi \Rightarrow 5\cos^4\varphi \cdot \text{sen}\,\varphi - 10\cos^2\varphi\,\text{sen}^3\varphi + \text{sen}^5\varphi$

a) $\text{sen}\,3\varphi$ $(\cos\varphi + i\,\text{sen}\,\varphi)^3 = \cos 3\varphi + i\,\text{sen}\,3\varphi$

$\Rightarrow \cos^3\varphi + 3\cos^2\varphi(i\,\text{sen}\,\varphi) + 3\cos\varphi(i\,\text{sen}\,\varphi)^2$
$+ (i\,\text{sen}\,\varphi)^3$

$\Rightarrow \cos^3\varphi + i(3\cos^2\varphi\,\text{sen}\,\varphi) - 3\cos\varphi\,\text{sen}^2\varphi - i\,\text{sen}^3\varphi$

$$\boxed{\begin{aligned}\cos 3\varphi &\Rightarrow \cos^3\varphi - 3\cos\varphi\,\text{sen}^2\varphi \\ \text{sen}\,3\varphi &\Rightarrow 3\cos^2\varphi\,\text{sen}\,\varphi - \text{sen}^3\varphi\end{aligned}}$$

∴ a y b

c) $\text{sen}\,4\varphi$ $(\cos\varphi + i\,\text{sen}\,\varphi)^4 = \cos 4\varphi + i\,\text{sen}\,4\varphi$

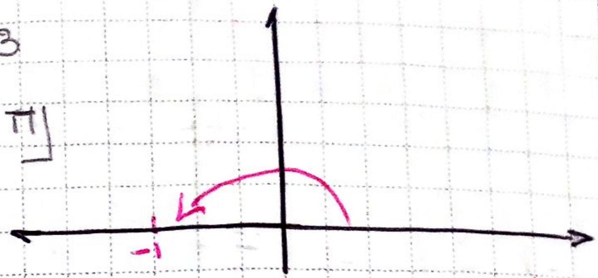
$\Rightarrow \cos^4\varphi + 4\cos^3\varphi(i\,\text{sen}\,\varphi) + 6\cos^2\varphi(i\,\text{sen}\,\varphi)^2$
$+ 4\cos\varphi(i\,\text{sen}\,\varphi)^3 + (i\,\text{sen}\,\varphi)^4$

$\cos 4\varphi \Rightarrow \cos^4\varphi - 6\cos^2\varphi\,\text{sen}^2\varphi + \text{sen}^4\varphi$

$\text{sen}\,4\varphi \Rightarrow 4\cos^3\varphi \cdot \text{sen}\,\varphi - 4\cos\varphi\,\text{sen}^3\varphi$ sol.

En los siguientes hallar todos los valores de raíz

16) a $\sqrt[4]{-1}$ $\quad n=4 \quad k=0,1,2,3$




Modulo $= z=\sqrt{1^2} = 1$ $\quad$ Arg $\Rightarrow \pi$

$k=0 \quad \sqrt[4]{-1} = 1\left(\cos \frac{\pi}{4} + i \operatorname{sen} \frac{\pi}{4}\right)$

$k=1 \quad \sqrt[4]{-1} = 1\left(\cos \frac{3\pi}{4} + i \operatorname{sen} \frac{3\pi}{4}\right)$

$k=2 \quad \sqrt[4]{-1} = 1\left(\cos \frac{5\pi}{4} + i \operatorname{sen} \frac{5\pi}{4}\right)$

$k=3 \quad \sqrt[4]{-1} = 1\left(\cos \frac{7\pi}{4} + i \operatorname{sen} \frac{7}{4}\pi\right)$

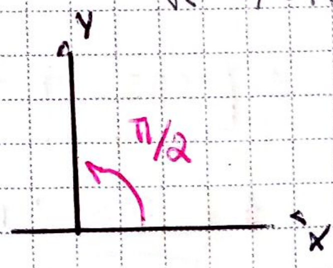
$k=4 \quad \sqrt[4]{-1} = 1\left(\cos \frac{9\pi}{4} + i \operatorname{sen} \frac{9\pi}{4}\right)$ solución

c $\sqrt[3]{i}$ $\quad \sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\varphi + 2k\pi}{n}\right) + i \operatorname{sen}\left(\frac{\varphi + 2k\pi}{n}\right)\right)$

$k = 0, 1, 2, n-1$

Datos

$n=3, \; k=0,1,2$ $\quad$ Arg $(z) = \frac{\pi}{2}$




$z = \sqrt{0^2+1^2} = 1$

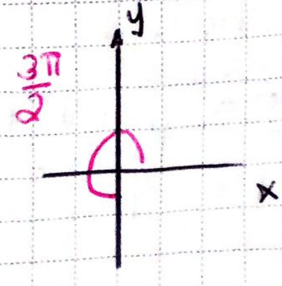
$k=0 \quad \sqrt[3]{i} = 1\left(\cos\left(\frac{\pi}{6}\right) + i \operatorname{sen}\left(\frac{\pi}{6}\right)\right)$

$k=1 \quad \sqrt[3]{i} = 1\left(\cos\left(\frac{\pi/2 + 2\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi/2 + 2\pi}{3}\right)\right)$

$\Rightarrow 1\left(\cos\left(\frac{5}{6}\pi\right) + i \operatorname{sen}\left(\frac{5}{6}\pi\right)\right)$ sol.

d) $\sqrt[4]{-i}$

Datos




Argumento $= \frac{3\pi}{2}$ sol

$n=4 \quad k=0,1,2,3$

$z = 1$

$k=0$ , $\sqrt[4]{-i} = 1\left(\cos\left(\frac{\pi}{2}\right) + i\,\text{sen}\,\frac{\pi}{2}\right)$

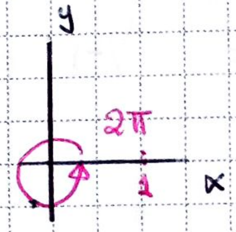
$k=1$ $\sqrt[4]{-i} = 1\left(\cos\left(\frac{\frac{3\pi}{2}+2\cdot 1\pi}{4}\right) + i\,\text{sen}\left(\frac{\frac{3\pi}{2}+2\pi}{4}\right)\right)$

$\Rightarrow 1\left(\cos\left(\frac{7}{8}\pi\right) + i\,\text{sen}\left(\frac{7}{8}\pi\right)\right)$ solución

$k=2$ $\sqrt[4]{-i} \Rightarrow 1\left(\cos\left(\frac{11}{8}\pi\right) + i\,\text{sen}\left(\frac{11}{8}\pi\right)\right)$

$k=3$ $\sqrt[4]{-i} \Rightarrow 1\left(\cos\left(\frac{15}{8}\pi\right) + i\,\text{sen}\left(\frac{15}{8}\pi\right)\right)$

17- a $\sqrt[4]{1}$ $\Rightarrow$ Modulo $= 1$ Argumento $2\pi$




$n=4$ $k=0,1,2,3$

$\sqrt[4]{-i} = 1\left(\cos\left(\frac{\varphi+2k\pi}{n}\right) + i\,\text{sen}\left(\frac{\varphi+2k\pi}{n}\right)\right)$

$k=0$ $1\left(\cos\left(\frac{\pi}{2}\right) + i\,\text{sen}\left(\frac{\pi}{2}\right)\right)$

$k=1$ $1\left(\cos\pi + i\,\text{sen}\,\pi\right)$

$k=2$ $1\left(\cos\frac{3}{2}\pi + i\,\text{sen}\,\frac{3}{2}\pi\right)$

$k=3$ $1\left(\cos 2\pi + i\,\text{sen}\,2\pi\right)$

c) $\sqrt{2-2\sqrt{3}i}$ $n=2$ $=$ $k=0,1$

Modulo

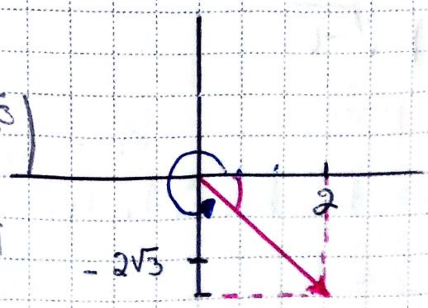
$r = \sqrt{(2)^2+(-2\sqrt{3})^2}$

$r = 4$

Argumento

$\alpha = 2\pi - \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$

$\alpha = 2\pi - \frac{1}{3}\pi = \frac{5}{3}\pi$



$k=0$   $\sqrt{2-2\sqrt{3}i} = 4\left(\cos\frac{5}{6}\pi + i\,\text{sen}\left(\frac{5}{6}\pi\right)\right)$

$k=1$   $\sqrt{2-2\sqrt{3}i} = 4\left(\cos\frac{\pi}{6} + i\,\text{sen}\frac{\pi}{6}\right)$

Indicar las lineas se determinen por las ecuaciones siguientes

31 a) $\text{im}\, z^2 = 2$;   $z = x+iy$   $z^2 = (x+iy)^2$

$\text{Im}(z^2) =$   $\Rightarrow x^2+2ixy-y^2$

$\text{Im}(x^2-y^2+\underbrace{i2xy}) = 2$   $\Rightarrow \underbrace{x^2-y^2}_{R} + \underbrace{i(2xy)}_{I}$

$\cancel{2}xy = \cancel{2}$ ; $\boxed{xy=1}$

$\boxed{y=\frac{1}{x}}$ ecuacion


32 b) $\text{Im}\left(\overline{z^2-\bar{z}}\right) = 2-\text{Im}\,z$

$\text{Im}(x^2-y^2+i2xy-x-iy) = 2-iy)$

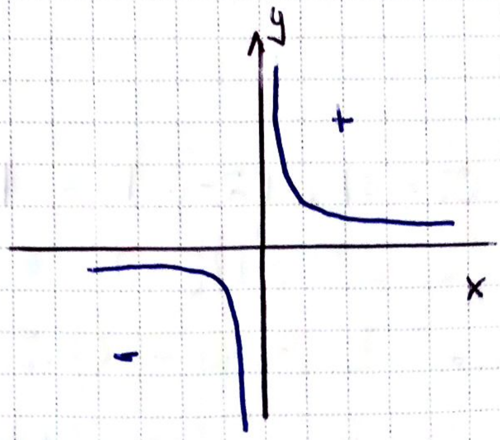
$(-i2xy-iy) = 2-iy$

$-(2xy+y) = 2-y$

$\cancel{2}xy\cancel{+y} = -\cancel{2}\cancel{+y}$

$xy = -1$   Hiperbola.

$y = -1/x$



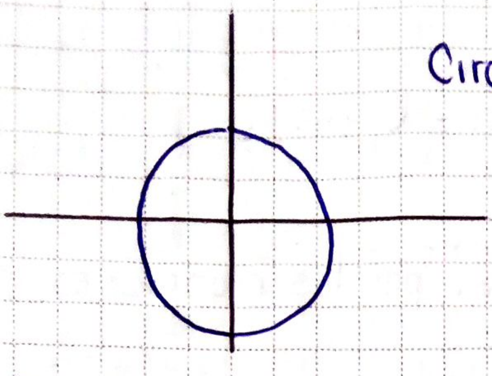

33 $z^2+\bar{z}^2 = 1$

$(x^2+\cancel{2xyi}-y^2)+(x^2-y^2-\cancel{2xyi}) = 1$

$2x^2-2y^2 = 1$   $(x^2-y^2) = \frac{1}{2}$ 1 sol.

$2(x^2-y^2 = 1$   Hiperbola

Circunferencia



34 $2z\bar{z} + (2+i)z + (2-i)\bar{z} = 2$

$2(x+iy)(x-iy) + (2+i)(x+iy) + (2-i)(x-iy) = 2$

$\cancel{2x^2} - \cancel{2xiy} + 2iyx + 2i^2y^2 + \cancel{2x} + \cancel{2iy} + \cancel{ix} + i^2y$

$+ \cancel{2x} - 2iy - \cancel{ix} + i^2y = 2$

$2x^2 + 4x - 2y^2 - 2y + 2iyx - 2iy = 2$

$(2x^2+4x) - (2y^2+2y) + yi(2x-2) = 2$

$$\begin{cases} 2x^2+4x-2y^2+2y = 2 & ① \div 2 \\ 2yx - 2 = 0 \longrightarrow & ② \div 2 \end{cases}$$

$yx = 1$

$\boxed{y = \frac{1}{x}}$ hiperbola

$\left(x^2+2x+\frac{4^2}{4}\right) - \left(y^2+y+\frac{1^2}{4}\right) = 0$

$(x+1)^2 - (y+\frac{1}{2}) = 0$

37 a) $|z-2| = |1-2\bar{z}|$

$z-2 = x+iy-2 = x-2+iy$ ①

$1-2\bar{z} = 1-2(x-iy) = 1-2x+2yi$ ②

Modulo $\left(\sqrt{(x-2)^2+y^2}\right)^2 = \left(\sqrt{(1-2x)^2+(2y)^2}\right)^{\cancel{2}}$

$(x-2)^2+y^2 = (1-2x)^2+4y^2$

$x^2 \cancel{-4x} + 4 + y^2 = 1 \cancel{-4x} + 4x^2 + 4y^2$

$3x^2 + 3y^2 - 3 = 0$ $\qquad x^2+y^2 = 1$

$\cancel{3}x^2 + \cancel{3}y^2 = \cancel{3}$

Circunferencia

b $|z - z_1| = |z - z_2|$:

$$(x+iy) - (x_1 + iy_1) = (x+iy) - (x_2 + iy_2)$$

$$|(x - x_1) + i(y - y_1)| = |(x - x_2) + i(y - y_2)|$$

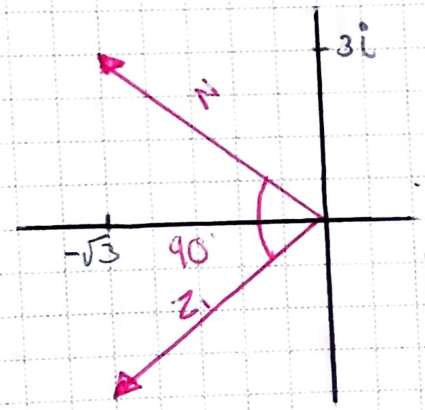
$$\left(\sqrt{(x-x_1)^2 + (y-y_1)^2}\right)^2 = \left(\sqrt{(x-x_2)^2 + (y-y_2)^2}\right)^2$$

$$\cancel{x^2} - 2xx_1 + x_1^2 + \cancel{y^2} - 2yy_1 + y_1^2 = \cancel{x^2} - 2xx_2 + x_2^2 + \cancel{y^2} - 2yy_2 + y_2^2$$

$$\left[x_1^2 - 2xx_1 - 2yy_1 + y_1^2\right] = \left[x_2^2 - 2xx_2 - 2yy_2 + y_2^2\right]$$

$$x_1(x_1 - 2x) + y_1(y_1 - 2y) = x_2(x_2 - 2x) + y_2(y_2 - 2y)$$

44 = A que vector se convierte $c - \sqrt{3} + 3i$ al girarlo en el angulo 90°?




$$z = -\sqrt{3} + 3i$$

$$z_1 = z_R z$$

$$z_R = \cancel{\cos\alpha + i\,\text{sen}\,\alpha}$$

$$z_R = \cancel{\cos 90 + i\,\text{sen}\,90}$$

$$z_1 = (-\sqrt{3} + 3i) \times 1i$$

$$z_1 = i(-\sqrt{3} + 3i)$$

$$\boxed{z_1 = -\sqrt{3}i - 3}$$

46 = Hallar el angulo en el cual es necesario girar el vector $4 - 3i$ para recibir el vector $-\frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}}i$

$$z = 4 - 3i \qquad z_1 = -\frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}}i$$

$$z_R = \cos\alpha + i\,\text{sen}\,\alpha$$

$$\alpha = ?$$

$$z_1 = z_R z$$

Modulo

$$r = \sqrt{4^2 + (-3)^2} = 5$$

$$r_{z_1} = \sqrt{\left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} \Rightarrow 5$$

$z_1 = z_R z$

$-\frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}}i = (\cos\alpha + i\,\text{sen})(4-3i)$

$= 4\cos\alpha - 3i\cos\alpha + 4i\,\text{sen}\,\alpha + 3\,\text{sen}\,\alpha$

$\Rightarrow 4\cos\alpha + 3\,\text{sen}\,\alpha + i(4\,\text{sen}\,\alpha - 3\cos\alpha)$

$$\begin{cases} -\frac{5}{\sqrt{2}} = 4\cos\alpha + 3\,\text{sen}\,\alpha & \cdot 3 \\ \frac{5}{\sqrt{2}} = -3\cos\alpha + 4\,\text{sen}\,\alpha & \cdot 4 \end{cases}$$

$$\begin{cases} -\frac{15}{\sqrt{2}} \Rightarrow \cancel{12\cos\alpha} + 9\,\text{sen}\,\alpha \\ \frac{20}{\sqrt{2}} \Rightarrow \cancel{-12\cos\alpha} + 16\,\text{sen}\,\alpha \end{cases}$$

$\frac{5}{\sqrt{2}} = 25\,\text{sen}\,\alpha$

$\text{sen}\,\alpha = \frac{5}{25\sqrt{2}} = \frac{1}{5\sqrt{2}}$ $\quad \alpha = \text{sen}^{-1}\left(\frac{1}{5\sqrt{2}}\right)$

$\text{sen}\,\alpha = \frac{1}{5\sqrt{2}}$

$\boxed{\alpha = 0{,}1419\,[rad]}$

---

$\frac{-5}{\sqrt{2}} = 4\cos + 3\,\text{sen}\,\alpha \quad \cdot -4$

$\frac{5}{\sqrt{2}} = -3\cos\alpha + 16\,\text{sen}\,\alpha \quad \cdot 3$

$\frac{20}{\sqrt{2}} = -16\cos\alpha - \cancel{12\,\text{sen}\,\alpha}$

$\frac{15}{\sqrt{2}} = -9\cos\alpha + \cancel{12\,\text{sen}\,\alpha}$

$\frac{35}{\sqrt{2}} = -25\cdot\cos\alpha$

$\boxed{\cos = -\frac{7}{5\sqrt{2}}}$

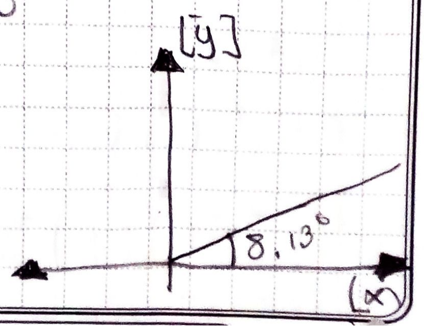
$$\left.\begin{array}{l} \text{sen}\,\alpha = \frac{1}{5\sqrt{2}} \\ \cos\alpha = -\frac{7}{\sqrt{2}} \end{array}\right\} \div$$

$\frac{\text{sen}\,\alpha}{\cos\alpha} \quad \frac{-\frac{1}{5\sqrt{2}}}{\frac{7}{5\sqrt{2}}} = -\frac{1}{7}$

$\boxed{\tan g = -\frac{1}{7}}$

$\alpha = -8{,}13$

∴ el vector gira



Tema: Fecha: / /

47. Hallar el angulo, en el cual es necesario girar el vector $3\sqrt{2} + i2\sqrt{2}$ para recibir $-5+i$

$z = 3\sqrt{2} + i2\sqrt{2}$

$\boxed{z_1 = z_R z}$

$z_1 = -5 + i$

Modulos

$z = \sqrt{(3\sqrt{2})^2 + (2\sqrt{2})^2} \Rightarrow z = \sqrt{26}$

$z_1 = \sqrt{(-5)^2 + (1)^2} = z_1 = \sqrt{26}$

$z_1 = z_R z$

$-5 + i = (\cos\alpha + i\,\text{sen}\,\alpha)(3\sqrt{2} + i2\sqrt{2})$

$= 3\sqrt{2}\cos\alpha + i3\sqrt{2}\,\text{sen}\,\alpha + i2\sqrt{2}\cos\alpha - 2\sqrt{2}\,\text{sen}\,\alpha$

$= 3\sqrt{2}\cos\alpha - 2\sqrt{2}\,\text{sen}\,\alpha + i(3\sqrt{2}\,\text{sen} + 2\sqrt{2}\cos\alpha)$

$$\begin{cases} -5 = 3\sqrt{2}\cos\alpha - 2\sqrt{2}\,\text{sen}\,\alpha & \times 2\sqrt{2} \\ 1 = 3\sqrt{2}\,\text{sen}\,\alpha + 2\sqrt{2}\cos\alpha & \times -3\sqrt{2} \end{cases}$$

$$\begin{cases} -10\sqrt{2} = \cancel{12\cos\alpha} - 8\,\text{sen}\,\alpha \\ -3\sqrt{2} = \cancel{-12\cos\alpha} - 18\,\text{sen}\,\alpha \end{cases}$$

$-13\sqrt{2} = -26\,\text{sen}\,\alpha \Rightarrow \text{sen} = \dfrac{-13\sqrt{2}}{-26}$

$\boxed{\text{sen} = \dfrac{\sqrt{2}}{2}}$

$$\begin{cases} -5 = 3\sqrt{2}\cos\alpha - 2\sqrt{2}\,\text{sen}\,\alpha & \times 3\sqrt{2} \\ 1 = 2\sqrt{2}\cos\alpha + 3\sqrt{2}\,\text{sen}\,\alpha & \times 2\sqrt{2} \end{cases}$$

$$\begin{cases} -10\sqrt{2} = 18\cos\alpha - \cancel{12\,\text{sen}\,\alpha} \\ 2\sqrt{2} = 8\cos\alpha + \cancel{12\,\text{sen}\,\alpha} \end{cases}$$

$-8\sqrt{2} = 26\cos\alpha$

$\cos\alpha = -\dfrac{8\sqrt{2}}{26}$

$\cos\alpha = -\dfrac{4\sqrt{2}}{13}$

$\left.\begin{aligned} \text{sen}\ & \frac{\sqrt{2}}{2} \\ \cos & = -4\sqrt{2} \end{aligned}\right\}$ ✓

$\dfrac{\text{sen}\,\alpha}{\cos\alpha} = \dfrac{\frac{\sqrt{2}}{2}}{-\frac{4\sqrt{2}}{13}}$

$\tan\alpha = -\dfrac{13}{8}$

$\alpha = \tan^{-1} -\dfrac{13}{8}$

$\boxed{\alpha = -58{,}39}$

∴ gira el vector

49 - $\cos x + i \operatorname{sen} x = \operatorname{sen} \alpha + i \cos x$.

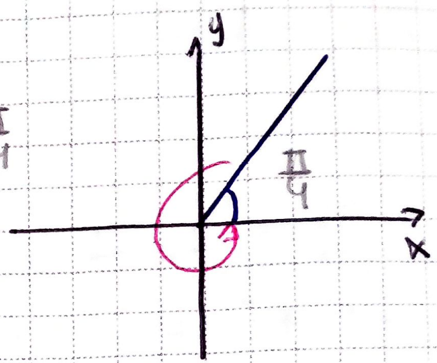
$$\begin{cases} \cos x = \operatorname{sen} x \ \text{①} \\ \operatorname{sen} x = \cos x \ \text{②} \end{cases}$$

$1 \cos x = \operatorname{sen} x$

$1 = \frac{\operatorname{sen}}{\cos x}$

$\tan x = 1 \qquad x = \tan^{-1}(1) \qquad x = \frac{\pi}{4}$

$x = \frac{\pi}{4} \Big]$ sol




50 - Haller el vector que se obtendrá el girar en 45° y duplicar el vector $z = 3+4i$

$z = 3+4i \qquad \alpha = 45° \qquad$ duplicar su modulo $\quad z_1 = ?$

$z_1 = z_R \, z \quad ; \boxed{z_R = 2(\cos 45 + i \operatorname{sen} 45)}$

$r = \sqrt{3^2+4^2} = 5$

$z_1 = 2(\cos \frac{\pi}{4} + i \operatorname{sen} \frac{\pi}{4}) \times (3+4i)$

$z_1 = 6 \cos \frac{\pi}{4} + 8i \cos \frac{\pi}{4} + 6i \operatorname{sen} \frac{\pi}{4} + 8i^2 \operatorname{sen} \frac{\pi}{4}$

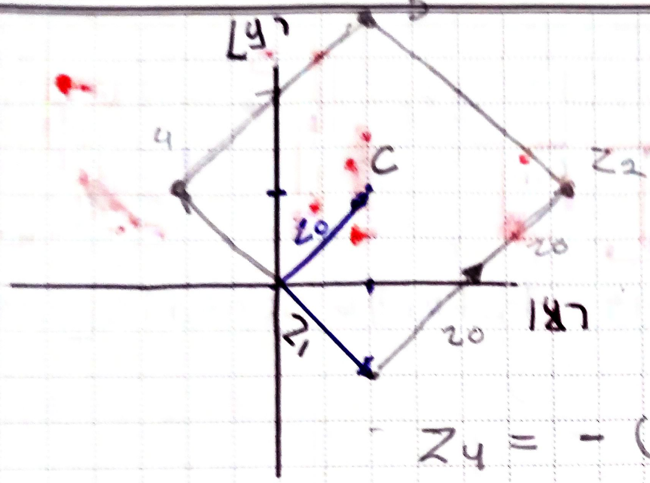
$z_1 = \cancel{6}^{3} \frac{\sqrt{2}}{\cancel{2}} + \cancel{8} i \frac{\sqrt{2}}{\cancel{2}} + \cancel{6}^{3} i \frac{\sqrt{2}}{\cancel{2}} - \cancel{8} \frac{\sqrt{2}}{\cancel{2}}$

$z_1 = 3\sqrt{2} + i\,4\sqrt{2} + i\,3\sqrt{2} - 4\sqrt{2}$

$z_1 = -\sqrt{2} + i\,7\sqrt{2} \Big]$

$r_{z_1} = \sqrt{(-\sqrt{2})^2 + (7\sqrt{2})^2} = \sqrt{100} = 10 \Big]$ sol

51 El centro de un cuadrado se encuentra en el punto $z_0 = 1+i$, y uno de los vertices se encuentra en el punto $z_1 = 1-i$ ¿En que puntos se encuentran los demas vertices del cuadrado?




$Z_2 = 2Z_0 + Z_1$

$Z_2 = 2(1+i) + (1-i)$

$Z_2 = 2 + 2i + 1 - i = \underline{3+i}$

$Z_4 = -(Z_1)$

$\boxed{Z_4 = -1 + i}$

$Z_3 = 2Z_0 + Z_4$

$Z_3 = 2(1+i) + (-1+i)$

$Z_3 = 2 + 2i - 1 + i$

$\boxed{Z_3 = 1 + 3i}$