Práctica N° 2

Separar la parte real e imaginaria de las funciones siguientes

59. a) $w = 2z - 1$

$w = 2(x+iy) - 1 = 2x + 2yi - 1$

$\left.\begin{matrix} u = 2x - 1 \\ v = 2y \end{matrix}\right\}$

b) $w = z + z^2$

$w = (x+iy) + (x+iy)^2 \Rightarrow (x + iy + x^2 - y^2 + 2xyi$

$\left.\begin{matrix} u = x^2 - y^2 + x \\ v = 2xy + y \end{matrix}\right\}$

c) $w = \dfrac{1}{z} = \dfrac{1}{x+iy} \cdot \dfrac{x-iy}{x-iy} \Rightarrow \dfrac{x-iy}{x^2-(iy)^2} = \dfrac{x-iy}{x^2+y^2}$

$\boxed{\dfrac{x}{x^2+y^2} - i\dfrac{y}{x^2+y^2}}$ $\therefore$ $\boxed{u = \dfrac{x}{x^2+y^2} \quad v = -\dfrac{y}{x^2+y^2}}$

60. c) $w = \operatorname{sen} z$;

$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha \cos\beta + \operatorname{sen}\beta \cos\alpha$

$= -i \operatorname{senh}(iz)$

$\Rightarrow w = \operatorname{sen}(x+iy) = \operatorname{sen} x \cos(iy) + \operatorname{sen}(iy)\cos x$

$\cos z = \operatorname{ch} iz$
$\operatorname{sen} z = -i \operatorname{sh} iz$

$\cos iy = \cosh(i(0+iy)) = \cosh(-y)$

$\Rightarrow$ Usamos

$\cosh(-y) \Rightarrow \dfrac{e^{-y}+e^{y}}{2} = \underline{\cosh(y)}$ | sol.

$\text{sen}(iy) = \text{sen}(0+iy) = -i\,\text{senh}(i(0+iy)) -$

$\Rightarrow -i\,\text{senh}\ -y$ $\qquad \Rightarrow \text{senh}(-z) = \dfrac{e^{-z} - e^{z}}{2}$

$\boxed{\Rightarrow -i\,\text{senh}\ y}$

$w = \text{sen}(z) = \text{sen}\,x\,\cosh y + (-i\,\text{senh}(y)) \cdot \cos x$

$\boxed{u = \text{sen}\,x\,\cosh y}$; $\boxed{v = -\,\text{senh}\,y\ \cos x}$

a) $w = e^{-z} = \dfrac{1}{e^{z}}$ $\qquad z = x + iy$

$\Rightarrow \dfrac{1}{e^{x+iy}} = \dfrac{1}{e^{x} \cdot e^{iy}}$ $\qquad \left.\begin{array}{l} u = \dfrac{1}{e^{x}} \\ v = \dfrac{1}{e^{iy}} \end{array}\right\}$

b) $w = e^{\bar{z}^2}$ $\qquad \Rightarrow z^2 \Rightarrow (x^2 - y^2 + 2xyi)$

$\Rightarrow w = e^{x^2 - y^2 - 2xyi}$ $\qquad \bar{z}^2 = x^2 - y^2 - 2xyi$

$w = e^{x^2 - y^2} \cdot e^{-2xyi}$

$\left.\begin{array}{l} u = e^{x^2 - y^2} \\ v = e^{-2xy} \end{array}\right\}$ solución

c) $w = \cosh(z - i)$

$w = \cosh(x + iy - i)$

$w = \cosh(x + i(y-1))$

$\text{Ch}\ z = \cos iz$

Aplicamos la función

$$\operatorname{ch} z = \frac{e^{z}+e^{-z}}{2}$$

$$\cosh(x+i(y-1)) = \frac{e^{x+i(y-1)} + e^{-(x+i(y-1))}}{2}$$

$$\Rightarrow \frac{e^{x}\cdot e^{(y-1)i} + e^{-x}\cdot e^{(-y+1)i}}{2}$$

$$\Rightarrow \frac{e^{x}}{2}\cdot\frac{e^{y-1}}{2} + \frac{e^{-x}}{2}\cdot\frac{e^{-y+1}}{2}$$

$$\left.\begin{aligned} u &= \frac{e^{x}}{2}+\frac{e^{-x}}{2} \\ v &= \frac{e^{y-1}}{2}+\frac{e^{-y+1}}{2} \end{aligned}\right\} \text{sol}$$

6) b) $w = \operatorname{senh} z$

$w = \operatorname{senh} z = \dfrac{e^{z} - e^{-z}}{2}$ $\quad z = (x + iy$

$w = \operatorname{senh} z = \dfrac{1}{2}\left[e^{x} \cdot e^{iy} - e^{-x} \cdot e^{-iy} \right]$

$\frac{1}{2} e^{x} \cdot (\cos y + i \operatorname{sen} y - e^{-x} \cdot (\cos + y - i \operatorname{sen} + y)$

$\Rightarrow \frac{1}{2}\left[e^{x} \cos y + i e^{x} \operatorname{sen} y - e^{-x} \cos y + i e^{-x} \operatorname{sen} y \right)$

$\Rightarrow \frac{1}{2}\left[\cos y (e^{x} - e^{-x}) + i \operatorname{sen} y (e^{x} + e^{-x}) \right]$

$\Rightarrow \left[\cos y \cosh x + i \operatorname{sen} y \operatorname{senh} x \right]$

$\left.\begin{array}{l} u = \cos y \cosh x \\ v = \operatorname{sen} y \operatorname{senh} x \end{array}\right\}$ sol

c) $\operatorname{tg} z$ $\quad \tan(x + iy) = e^{i(x)}$

$w = \dfrac{e^{ix} \cdot e^{-y} + e^{-xi} \cdot e^{y}}{2}$

$w = \dfrac{e^{-y} \cos x + i \operatorname{sen} x - (\cos -x + i \operatorname{sen} -x)\, e^{y}}{2}$

w

$w = \left[e^{-y} \cos x + i e^{-y} \operatorname{sen} x - e^{y} \cos(-x) + i e^{y} \operatorname{sen}(x) \right] \frac{1}{2}$

$w = \cos x \dfrac{e^{y} - e^{-y}}{2} + i \operatorname{sen} x \dfrac{e^{y} - e^{-y}}{2}$

$w = \cos x \operatorname{senh} y + i \operatorname{sen} x \cosh y$

$u = \cos x \cdot \operatorname{senh} y$

$v = \operatorname{sen} x \cosh y$

6) a $W = 2^{z^2}$ $\qquad z^2 = x^2 - y^2 + 2xyi$

$a^z = e^{z \ln a}$

$\ln z = \ln z + 2k\pi i$

$W = 2^{x^2 - y^2 - 2xyi}$

$W = e^{(x^2 - y^2 - 2xyi) \ln|2|}$

$W = e^{(x^2 - y^2) \ln 2 - 2\ln 2 \, xyi}$

$W = e^{x^2 + y^2 \ln 2} \quad e^{-2\ln(2) xyi}$

Por euler $\rightarrow$ $e^{i\alpha} = \cos\alpha + i \operatorname{sen}\alpha$

$W = e^{(x^2 - y^2) \ln 2} \left(\cos(-2\ln(2) xy) + \operatorname{sen}(-2\ln 2 \, xy) \right)$

$W = e^{(x^2 - y^2) \ln(2)} \left(\cos(2xy \ln 2) - i \operatorname{sen}(2xy \ln 2) \right)$

$$\left. \begin{array}{l} \mu = e^{x^2 - y^2 \ln 2} (\cos(2xy \ln 2)) \\ v = - e^{(x^2 - y^2) \ln 2} (\operatorname{sen}(2xy \ln 2)) \end{array} \right\}$$

En los siguientes hallar el valor del modulo y el valor principal del argumento de funciones dadas en los puntos indicados

62 = $w = \cos z = \cos(x+iy)$

$\Rightarrow \cos x \cdot \cos(iy) - \operatorname{sen} x \operatorname{sen}(iy)$

$\operatorname{sen}(iy) \Rightarrow -i\operatorname{senh}(y)$

$\cos(iy) \Rightarrow \cosh(y)$

$w = \cos z = \cos x \cosh y + i \operatorname{sen} x \operatorname{senh} y$

a) $z_1 = \frac{\pi}{2} + i\ln 2$

$x = \frac{\pi}{2}$; $y = \ln 2$

$w = \cancel{\cos \frac{\pi}{2}}^{0} \cosh(\ln 2) + i \cancel{\operatorname{sen} \frac{\pi}{2}}^{1} \operatorname{senh}(\ln 2)$

$w = i\operatorname{senh}(\ln 2)$

$= i \dfrac{e^{\ln 2} - e^{-\ln 2}}{2}$

$\Rightarrow \left(\dfrac{2 - \frac{1}{2}}{2}\right) \Rightarrow \dfrac{\frac{3}{2}}{2} = \boxed{\frac{3}{4}}$ sol

$w = i\boxed{\frac{3}{4}}$

b) $z_2 = \pi + i\ln 2$

$x = \pi$ $y = \ln 2$

$w = \cancel{\cos \pi}^{-1} \cosh(\ln 2) + i \cancel{\operatorname{sen}(\pi)}^{0} \operatorname{senh}(\ln 2)$

$w = \cosh(\ln 2)$

$\cosh(\ln 2) = \dfrac{e^{\ln 2} + e^{-\ln 2}}{2} \Rightarrow \dfrac{\frac{3}{2}}{2}$

$\Rightarrow \boxed{\frac{3}{4}}$ solución

Tema: Fecha: / /

64 = $w = ze^z$, $z_0 = \pi i$ $x=0$ $y = \pi i$

$w = (x+yi) \cdot e^x \cdot e^{iy}$

$w = 0 + \pi \cancel{\cdot e^0} \cdot e^{\pi}$

$w = e^{\pi}$ sol

65 $w = ch^2 z$, $z_0 = i\ln 3$

$w = \cosh^2 i\ln 3 + 0$

$w =$

66 b. $+i$ $\quad Ln(+i) = \ln z + i \arg z + 2k\pi i$

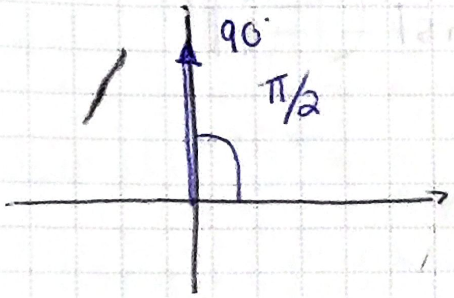
Modulo

$r = \sqrt{(+i)^2}$ $\quad \alpha = \frac{\pi}{2}$ $\quad = \ln|i| + i \arg\left(\frac{\pi}{2}\right) + 2k\pi i$

$r = \sqrt{\cancel{i^2}}$ $\quad Ln(-i) = \ln|\sqrt{-1}| + i\left(\frac{\pi}{2} + 2k\pi\right)$ sol

$r = i$

Argumento



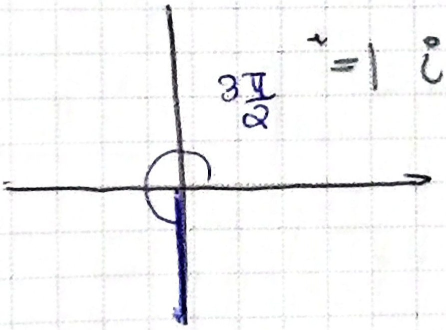
b) $-i$;

Modulo

$= |i|$

$Ln(-i) = \ln z + i \operatorname{Arg} z + 2k\pi i$

$= \ln(\sqrt{-1}) + i\left(\frac{3\pi}{2} + 2k\pi\right)$ sol

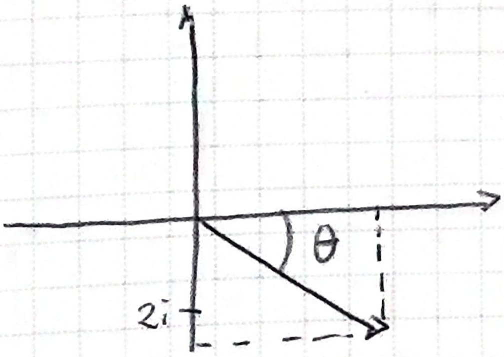


e) $3-2i$

Modulo

$\operatorname{Arg} = -\tan^{-1}\left(\frac{|-2|}{3}\right)$

$\operatorname{Arg} = -0{,}588$



Modulo $\quad r = \sqrt{3^2 + (-2)^2}$

$r = \sqrt{9+4} = \sqrt{13}$

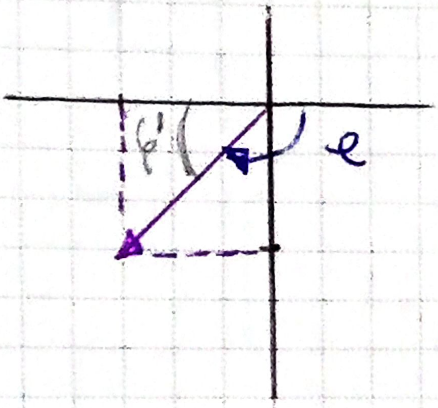
$Ln|3-2i| = \ln\sqrt{13} + i(-0{,}588 + 2k\pi)$ sol

66- Hallar logaritmos de los números siguientes:

d) $-1-i$; $\ln(-1,-i)$ $\qquad \ln z = \ln z +$

$\ln = (-1-i) = \ln(|-1-i|) + i\,(\arg(-1-i) + 2k\pi)$

$|-1-i| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$



$\text{Arg}(-1-i) = -(\pi - \theta')$

$\text{Arg}(-1-i) = -\pi + \tan^{-1}\left(\frac{|-1|}{|-1|}\right)$

$\text{Arg}\ -1-i = -\pi + \frac{\pi}{4}$

$\text{Arg} = -\frac{3}{4}\pi$

$\ln(-1-i) = \ln(\sqrt{2}) + i\left(-\frac{3}{4}\pi + 2k\pi\right)$

a) e ;

$\ln e = \ln|z| + i\,\text{Arg}\, z = \ln|z| + i\arg|z + 2k$

$= \ln|e| + i\,\text{Arg}\,|\tan^{-1}|e| + + 2k\pi$

$= \ln|\sqrt{e^2}| + i\left(\text{Arg}\,|\tan^{-1}|e| + 2k\pi\right)$ Modulo

$= \ln\sqrt{e^2} + \text{Arg}\,|\tan^{-1}|e| + 2k\pi)$ $\quad r = \sqrt{e^2 + 0}$

sol

$2 \quad \pi$

67. hallar (a) (b) (e) (f) y (g) $\boxed{a^z = e^{z \ln a}}$

b. $i^{1/i}$

$\ln z = \ln|z| + i\operatorname{Arg} z$

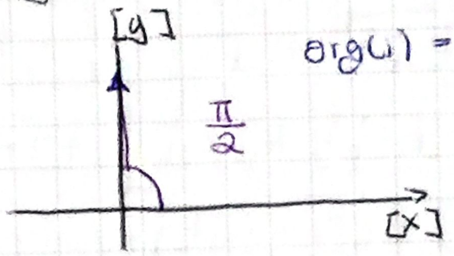
$\frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i = i^{-i}$; $\boxed{a = i}$; $\boxed{z = -i}$

$i^{-i} = e^{-i\ln(i)}$; $\ln(i) = \ln(i) + i\arg(i) + 2k\pi i$

Modulo

$|i| = \sqrt{1^2} = 1$

Argumento i



$\arg(i) = \frac{\pi}{2}$

$\ln(i) = \cancel{\ln(1)} + i\frac{\pi}{2} + 2k\pi i$

$= \left(\frac{\pi}{2} + 2k\pi\right) i$

$i^{-i} \Rightarrow e^{-i\left(\frac{\pi}{2} + 2k\pi\right)i} = \underline{e^{\frac{\pi}{2} + 2k\pi}}$ resolución

$\boxed{a^z_z = e^{z \ln a}}$

e) $\left(\frac{1+i}{\sqrt{2}}\right)^{2i}$ $a = \frac{1+i}{\sqrt{2}}$ $z = 2i$

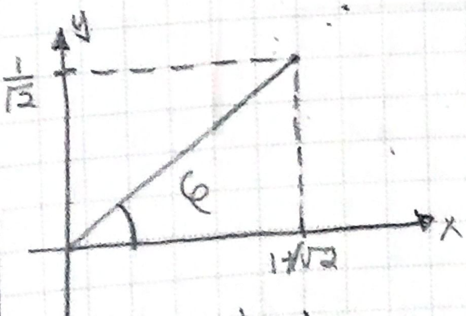
$= e^{2i \ln\left(\frac{1+i}{\sqrt{2}}\right)} \Rightarrow \ln\left(\frac{1+i}{\sqrt{2}}\right) = \ln\left|\frac{1+i}{\sqrt{2}}\right| + i\arg\left(\frac{1+i}{\sqrt{2}}\right) + 2k\pi i$

Modulo

$\left|\frac{1+i}{\sqrt{2}}\right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{2}{2}} \Rightarrow 1$

Argumento

$\operatorname{Arg}\left(\frac{1+i}{\sqrt{2}}\right) = \theta = \tan^{-1}\left(\frac{\left|\frac{1}{\sqrt{2}}\right|}{\left|\frac{1}{\sqrt{2}}\right|}\right)$



$\alpha = \tan^{-1}(1)$

$\boxed{\alpha = \frac{\pi}{4}}$

Reemplazamos

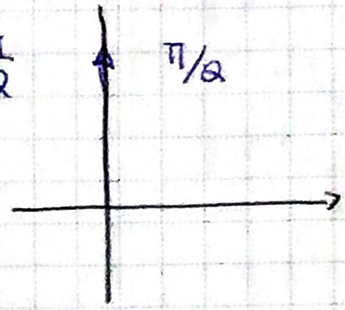
$\ln\left(\frac{1+i}{\sqrt{2}}\right) = \cancel{\ln(1)}^{0} + i\frac{\pi}{4} + 2k\pi i = \left(\frac{\pi}{4} + 2k\pi\right)i$

$\left(\frac{1+i}{\sqrt{2}}\right)^{2i} = e^{2i\left(\frac{\pi}{4}+2k\pi\right)i} = e^{-\left(\frac{\pi}{2}+4k\pi\right)}$ sol.

a) i^i $\qquad a^z = e^{z\ln a}$

$e^{i\ln(i)}$

Arg $\frac{\pi}{2}$



$|i| = \sqrt{0+1^2} = 1$

$\ln|i| = \cancel{\ln(1)}^{0} = +i\frac{\pi}{2} + 2k\pi i$

$i^i = e^{i\left(\frac{\pi}{2}+2k\pi\right)i}$

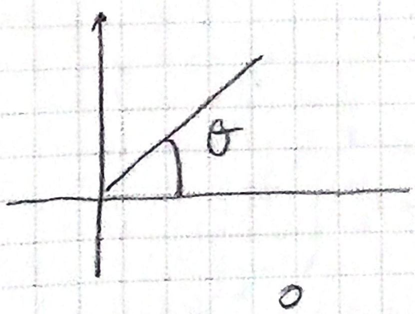
$i^i = e^{-\frac{\pi}{2}-2k\pi}$ sol

f) $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{1+i}$ $\qquad a^z = e^{z\ln|a|}$

Modulo $r = \sqrt{1^2+1^2} = \sqrt{2}$

$\ln|1| = \ln|z| + i\arg z + 2k\pi i$ $\qquad |a| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$\ln|1| = \ln 1 + i$ $\qquad a = |1|$



$\text{Arg} = \tan^{-1}\left(\frac{\left|\frac{1}{2}\right|}{\left|\frac{\sqrt{3}}{2}\right|}\right)$

$\text{Arg } z = \frac{1}{6}\pi$

$\ln|1| = \cancel{\ln|1|}^{0} + \frac{\pi}{6} + 2k\pi i$

$\Rightarrow e^{(1+i)\cdot i\left(\frac{\pi}{6}+2k\pi\right)}$

$\Rightarrow e^{i\left(\frac{\pi}{6}+2k\pi\right) - \frac{\pi}{6} - 2k\pi} \Rightarrow e^{-\frac{\pi}{6}-2k\pi+\left(2k\pi+\frac{\pi}{6}\right)i}$

sol.

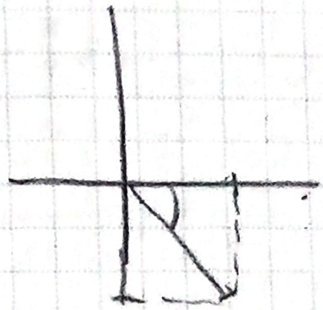
$a\ \ (1-i)^{3-3i}$ $\qquad a^z = e^{z \ln a}$ $\qquad a=(1-i)$, $z = 3-3i$

$$= e^{(3-3i)\ln|a|}$$

$$\ln(1-i) = \ln|1-i| + i(\text{Arg } a + 2k\pi)$$

Modulo $\qquad\qquad$ Argumento

$r = \sqrt{1^2+(-1)^2} = \sqrt{2}$ $\qquad \alpha = -\tan^{-1}\left(\frac{|-1|}{|+1|}\right)$



$$\alpha = -\frac{\pi}{4}$$

$$\ln(1-i) = \ln|\sqrt{2}| + i\left(-\frac{\pi}{4} + 2k\pi\right)$$

$$\Rightarrow e^{3-3i \cdot \ln\sqrt{2} + i\cdot\left(-\frac{\pi}{4}+2k\pi\right)}$$

$$\Rightarrow e^{3\ln\sqrt{2} - \frac{3\pi}{4} + 6k\pi i - 3\ln\sqrt{2}\, i - \frac{3\pi}{4} + 6k\pi}$$

$\frac{\cancel{6}^{3}}{\cancel{4}_{2}}$

$$\Rightarrow e^{\ln\sqrt{2}^3 - \frac{3}{2}\pi + 6k\pi + i\left(6k\pi - \ln\sqrt{2}^3\right)}$$ sol.

$$\Rightarrow \underbrace{e^{\ln(\sqrt{2})^3 - \frac{3}{2}\pi + 6k\pi}}_{\text{Real}} \cdot \underbrace{e^{i\, 6k\pi - \ln\sqrt{2}^3}}_{\text{Imaginario}}$$

68- hallar el modulo ρ y el argumento φ de los numeros complejos: a) th πi b) $|0|$ c) 3^{2-i}

a) $\tanh(\pi i)$ $\boxed{\operatorname{tgh} = -i \operatorname{tg} iz}$

$= i\tan(-\pi) = 0 - i\tan(-\pi)$

Modulo $r = \sqrt{0^2 + (-\operatorname{tg}(-\pi))^2} = \underline{\tan(-\pi)}$

Argumento

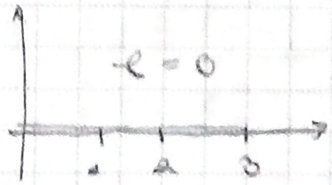
$\alpha = \tanh i = \underline{\frac{\pi}{2}(50)}$

c) $3^{2-i} = e^{(2-i)\ln(3)}$

$\ln(3) = \ln(3) + i\arg(3) + 2k\pi i$

$|3| = \sqrt{3^2 + 0} \to \underline{3}$

Argumento



$\ln(3) = \ln(3) + i(0) + 2k\pi i$

$\boxed{\begin{array}{l} e^{x+iy} \\ e^x \cdot e^{iy} \\ e^x(\cos y + i\operatorname{sen} y) \end{array}}$

$3^{2-i} = e^{(2-i)(\ln 3 + 2k\pi i)}$

$(2-i)\ln(3) + 2k\pi i = 2\ln(3) + 4k\pi i - i\ln(3) - 2k\pi i^2$

$= 2\ln(3) + 4k\pi i - i\ln(3) + 2k\pi$

$= 2\ln(3) + 2k\pi + (4k\pi - \ln(3))i$

$= e^{2\ln(3) + 2k\pi} \cdot e^{(4k\pi - \ln(3))}$

$= e^{2\ln(3) + 2k\pi} \cdot (\cos(4k\pi - \ln(3)) + i\operatorname{sen}(4k\pi - \ln(3)))$

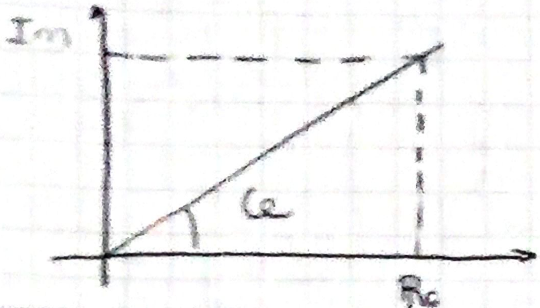
$= \underbrace{e^{2\ln 3 + 2k\pi} \cdot \cos(4k\pi - \ln(3))}_{\text{Real}} + i\underbrace{e^{2\ln(3) + 2k\pi} \cdot \operatorname{sen}(4k\pi - \ln(3))}_{\text{Imag}}$

$$r = \sqrt{\left[e^{2\ln(3)+2k\pi}\cos(4k\pi - \ln(3))\right]^2 + \left[e^{2\ln(3)+2k\pi}\operatorname{sen}(4k\pi - \ln(3))\right]^2}$$

$$r = \left(\sqrt{e^{2\ln(3)+2k\pi}}\right)^2 \sqrt{\underbrace{\left[\cos^2(4k\pi - \ln(3)) + \operatorname{sen}^2(4k\pi - \ln(3))\right]}_{1}}$$

$$\boxed{r = e^{2\ln(3)+2k\pi}}$$ solución

$\cos^2 x + \operatorname{sen}^2 x = 1$



$$\operatorname{Arg}(z) = \tan^{-1}\left(\frac{e^{2\ln(3)+2k\pi}\operatorname{sen}(4k\pi - \ln(3))}{e^{2\ln(3)+2k\pi}\cdot\cos(4k\pi - \ln(3))}\right)$$

$$\operatorname{Arg}(z) = \tan^{-1}\left(\operatorname{tag}(4k\pi - \ln(3))\right)$$

$\boxed{\operatorname{tg}^{-1}(\tan\alpha) = \alpha}$

$$\boxed{\operatorname{Arg}(z) = 4k\pi - \ln(3)}$$ solución

b) 10^i $\qquad a^z = e^{z\ln a}$ $\qquad a = 10$, $z = i$

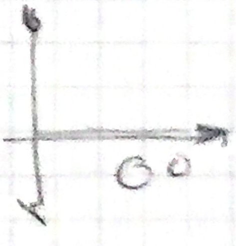
$e^{i\ln(a)}$

$\operatorname{Ln}|10| = \ln|10| + i(0 + 2k\pi i$

$= \ln|10| + i2k\pi$

$\Rightarrow e^{i\ln 10 + i^2 2k\pi}$

$$\Rightarrow \underbrace{e^{-2k\pi}}_{\text{Real}} \cdot \underbrace{e^{\ln|10| i}}_{\text{Imagin}}$$



76) $\operatorname{sen} h\, iz = -i \quad * -i$

$-i \operatorname{sen} h\,(iz) = i^2$

$\operatorname{sen} z = -1$

$\dfrac{e^{iz} - e^{-iz}}{2i} = -1$

$e^{iz} - e^{-iz} = -2i$

$\cancel{\cos z} + i \operatorname{sen} z - \cancel{\cos z} + i \operatorname{sen} z = 2i$

$+\cancel{2i} \operatorname{sen} z = \cancel{2i}\ 1$

$\operatorname{sen}(x+iy) = 1$

$x + xy = \dfrac{\pi}{2} + 2\pi k$

$x(1+y) = \dfrac{\pi}{2} + 2\pi k$

$x = \dfrac{\pi}{2(1+y)} + \dfrac{2\pi k}{1+y}$ sol

74) $e^z + i = 0$; $z = x + iy$ $x = ?$ $y = ?$

$e^{x+iy} + i = 0 \Rightarrow e^x e^{iy} + i = 0 \Rightarrow e^x(\cos y + \text{sen } y) + i = 0$

$e^x \cos y + i\, e^x \text{sen } y + i = 0$

$\underbrace{e^x \cos y} + i\underbrace{(e^x \text{sen } y + 1)} = 0 + 0i$

$\begin{cases} e^x \cos y = 0 \\ e^x \text{sen } y + 1 = 0 \end{cases}$ $\begin{cases} e^x \text{sen } y = -1 \\ e^x \cos y = 0 \end{cases}$ $\dfrac{e^x \text{sen } y}{e^x \cos y} = \dfrac{-1}{0}$

$\tan y\ \infty$

$e^x \text{sen}\left(\frac{\pi}{2}\right) - 1 = e^x = 1$ $\boxed{e^x = -1} = ?$

$\begin{cases} x = \ln(-1) \\ x = 0 \end{cases}$ $e^{i\alpha} = \cos \alpha + i \text{sen } \alpha -$ $\alpha = -\pi = -180°$

$e^{-i\pi} = \cos(-\pi) + i \text{ sen}(-\pi) = -1 + 0i$

$e^{-i\pi} = \cos(-\pi) + i \text{sen}(-\pi) = -1 + 0i$

78) $e^{ix} = \cos \pi x$ ~~$+ i \text{ sen}(-\pi)$~~ $x = \in \mathbb{R}$

$\cos x + i \text{ sen } x = \cos(\pi x) + 0i$

$\begin{cases} \cos x = \cos(\pi x) \ \% \\ \text{sen } x = 0 \end{cases}$ $x = \tan^{-1}(0)$

$\dfrac{\text{sen } x}{\cos x} = \dfrac{0}{\cos \pi x}$; $\tan x = 0$ $x = 0$

$x = \tan^{-1}(0)$

$x = 0$

80 $\cosh z = i$

$$\frac{e^{z}+e^{-z}}{2} = i$$

$$e^{z}+e^{-z} = 2i$$

$$e^{z}+\frac{1}{e^{z}} = 2i$$

$$\frac{e^{2z}+1}{e^{z}} = 2i$$

$$e^{2z}+1 = 2ie^{z}$$

$$\begin{cases} e^{2z} = -1 \\ e^{z} = 0 \end{cases}$$

$$\frac{e^{2z}}{e^{z}} = \frac{-1}{0}$$

$$e^{z} = -1$$

$$\begin{cases} e^{2z}-2ie^{z} = -1 \end{cases}$$

$$\begin{cases} e^{2z} = -1 \\ \cancel{-2}e^{z} = 0 \end{cases}$$

$$e^{x}\cdot e^{iy} = -1$$

$$e^{x} = -1$$

$$\cancel{-1\cdot e^{iy} = -1}$$

$$\boxed{e^{iy} = 0}$$