Tema:

Práctica N°2.

82.- $Z_n = \left(1 + \frac{1}{n}\right) e^{i\frac{\pi}{n}}$ $\qquad e^{i\alpha} \quad \alpha = \frac{\pi}{n}$

$Z_n = \left(1+\frac{1}{n}\right)(\cos\alpha + i\,\text{sen}\,\alpha)$

$Z_n = \left(1+\frac{1}{n}\right)\cos\alpha + i\left(1+\frac{1}{n}\right)\text{sen}\,\alpha$

$P_n = = \sqrt{\left(1+\frac{1}{n}\right)^2(\cos^2\alpha + \text{sen}^2\alpha)} = \left(1+\frac{1}{n}\right)$

$P_0 = \lim\limits_{n\to\infty} P_n = \lim\limits_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\right] = \underline{1}$

$P_0 = 1$

$\varphi_0 = \lim\limits_{n\to\infty}\left(\frac{\pi}{n}\right) = \frac{\pi}{\infty} = \underline{0}$

$\therefore \lim\limits_{n\to\infty} Z_n = 1e^{i1\alpha}$

$= 1e^{i\frac{\pi}{n}} = 1e^{i0} = \underline{1}$

$P_n = \tan^{-1}\left(\frac{\left(1+\frac{1}{n}\right)\text{sen}\,\alpha}{\left(1+\frac{1}{n}\right)\cos\alpha}\right)$

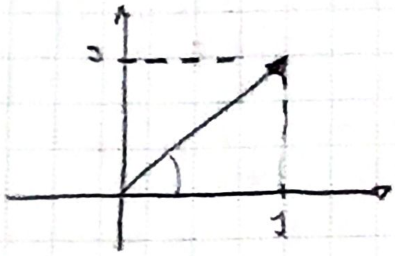
$\varphi_n = \tan^{-1}\left(\tan\,\cancel{\left(\frac{1+\frac{1}{n}}{1+\frac{1}{n}}\right)}\right)$

$P_n = \tan^{-1}(\tan\alpha)\cancel{\left(1+\frac{1}{n}\right)}$

$P_n = \alpha$

89.- $Z_n = (1+3i)^n \longrightarrow$

$r = \sqrt{1^2+3^2} = \sqrt{10}$



$\alpha = \tan^{-1}\left(\frac{3}{1}\right) = 1{,}25$

$= \sqrt{10}^n\cos(n\,1{,}25) + i\sqrt{10}^n\,\text{sen}\,(n\,1{,}25)$

$Z_n = \sqrt{10}^n\cos(1{,}25n) + i\sqrt{10}^n\,\text{sen}\,(1{,}25n)$

$P_n = \sqrt{(\sqrt{10}^n)^2\cos^2(1{,}25n) + (\sqrt{10}^n)^2\,\text{sen}^2(1{,}25n)} = \sqrt{10^n}$

$P_n = \sqrt{(\sqrt{10}^n)^2\cos^2(1{,}25n) + \text{sen}^2(1{,}25n)} = (\sqrt{10}^n)$

Tema: Fecha: / /

$$\varphi_n = \tan^{-1} \frac{\sqrt{10}^n \operatorname{sen}\alpha}{\sqrt{10}^n \cos\alpha} \qquad \text{lc} \quad \alpha = 1{,}25n$$

$$\varphi_n = \tan^{-1}(\tan\alpha)$$

$$\varphi_n = \alpha$$

$$\rho_0 = \lim_{n\to\infty} (\sqrt{10}^n)^2 = \lim_{n\to\infty} (\sqrt{10}^{2n}) = \infty$$

$$\varphi_0 = \lim_{n\to\infty} \varphi_n = \rho_0 = \lim_{n\to\infty} 1{,}25n = 1{,}25 \cdot \infty = \infty$$

$$\therefore \lim_{n\to\infty} z_n = \infty e^{i\infty} = \therefore \text{ No existe Indeterminado}$$

(86) $$z_n = \frac{n+2i}{3n+7i} \cdot \frac{3n-7i}{3n-7i} = \frac{(n+2i)(3n-7i)}{(3n)^2-(7i)^2}$$

$$= \frac{3n^2 - 7in + 6ni - 14i^2}{9n^2 - 49i^2} = \frac{(3n^2+14)+i(7n)}{9n^2-49}$$

$$= \underbrace{\frac{3n^2+14}{9n^2-49}}_{R} - i\underbrace{\frac{n}{9n^2-49}}_{I}$$

$$\rho_n = \sqrt{\left(\frac{3n^2+14}{9n^2-49}\right)^2 + \left(-\frac{n}{9n^2-49}\right)^2} = \sqrt{\frac{9n^4+84n^2+196+n^2}{(9n^2-49)^2}}$$

$$= \sqrt{\frac{9n^4+85n^2+196}{(9n^2-49)^2}} = \frac{\sqrt{9n^4+85n^2+196}}{9n^2-49}$$

$$\varphi_n = \tan^{-1}\left(\frac{\left|\frac{n}{9n^2-49}\right|}{\left|\frac{3n^2+14}{9n^2-49}\right|}\right) = \tan^{-1}\left(\frac{n}{3n^2+14}\right)$$

$$\varphi_0 = \lim_{n\to\infty} \tan^{-1}\left(\frac{-n}{3n^2+14}\right) = \lim_{n\to\infty} \tan^{-1}\left(\frac{\frac{-n}{n^2}}{\frac{3n^2}{n^2}+\frac{14}{n^2}}\right)$$

Notas:

Tema:

$$= \lim_{n\to\infty} \left(\frac{-\frac{1}{n}}{3+\frac{14}{n^2}} \right) \quad \tan^{-1}\left(\frac{-\frac{1}{\infty}}{3+\frac{140}{\infty}} \right) =$$

$$= \tan^{-1}\left(\frac{0}{3} \right) = \tan^{-1}(0) \quad \varphi_0 = 0$$

$$\rho_\infty = \lim_{n\to\infty} \sqrt{\frac{n^2-4}{9n^2+49}} = \lim_{n\to\infty} \sqrt{\frac{\frac{n^2}{n^2}-\frac{4}{n^2}}{\frac{9n^2}{n^2}+\frac{49}{n^2}}}$$

$$= \sqrt{\frac{1+0}{9+0}} = \boxed{\frac{1}{3}}$$

$$\lim z_n = \frac{1}{3} \cancel{e^{0}} = \boxed{\frac{1}{3}} \quad \text{solución}$$

88. $z_n = n \operatorname{sen} \frac{i}{n} = n(\operatorname{sen} z) = n\left(-i \operatorname{senh}\left(\frac{i^2}{n}\right)\right)$

$$= n\left(-i \operatorname{senh}\left(-\frac{1}{n}\right)\right) = in \operatorname{senh}\left(\frac{1}{n}\right)$$

$$\rho_n = \sqrt{n^2 \operatorname{senh}^2\left(\frac{1}{n}\right)} = n \operatorname{senh}\left(\frac{1}{n}\right)$$

$$\theta_n = \tan^{-1}\left(n \operatorname{senh}\left(\frac{1}{n}\right) \right)$$

$$n = \frac{i}{x} \quad x = \frac{i}{n}$$

$$x \to 0$$

$$\rho_0 = \lim_{n\to\infty} n \operatorname{senh}\left(\frac{1}{n}\right)$$

$$\rho_0 = \lim_{n\to\infty} n\left(-i \operatorname{sen}\left(i\frac{1}{n}\right)\right) = \lim_{n\to\infty} -in \operatorname{sen}\left(\frac{i}{n}\right)$$

$$= \lim_{n\to\infty} \boxed{\frac{-i \operatorname{sen}\left(\frac{i}{n}\right)}{\frac{1}{n}}} \quad x = \frac{i}{n} \quad n = \frac{i}{x}$$

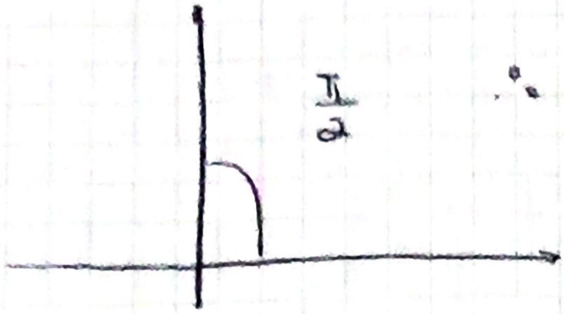
$$= \lim_{x\to 0} -i \frac{\operatorname{sen}(x)}{\frac{1}{\frac{i}{x}}} = \lim_{x\to 0} \boxed{\frac{-i \operatorname{sen}(x)}{\frac{x}{i}}}$$

$$\lim_{x\to 0} (-i^2) \frac{\operatorname{sen}(x)}{\cancel{x}} = \boxed{1}$$

Notas:

Tema: Fecha: / /

$$\varphi_0 = \lim_{n\to\infty} \tan^{-1}\left(n\,\text{senh}\left(\frac{1}{n}\right)\right) = \frac{\pi}{2}$$



$\therefore$ solo tenemos imaginario

$$\lim_{n\to\infty} Z_n = (1)\cdot e^{i\frac{\pi}{2}} = \cos\overset{0}{\frac{\pi}{2}} + i\,\text{sen}\frac{\pi}{2}\overset{1}{i} = i$$

$$\lim Z_n = e^{i\frac{\pi}{2}}$$

$$= \cos\left(\frac{\pi}{2}\right) + i\,\text{sen}\left(\frac{\pi}{2}\right)$$

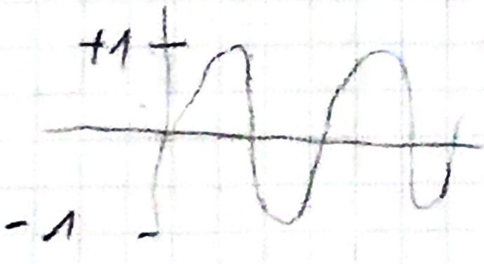
(40) $Z_n = \frac{\text{senh}\ in}{n} = \frac{-i\,\text{sen}(i^2 n)}{n} = -\frac{i}{n}\,\text{sen}(-n)$

$= \frac{i}{n}\,\text{sen}(n)$

$$P_n = \sqrt{\frac{\text{sen}^2 n}{n^2}} = \frac{\text{sen}(n)}{n}$$

$$\varphi_n = \tan^{-1}\left|\frac{\frac{\text{sen}(n)}{n}}{0}\right| = \frac{\pi}{2}$$

$$P_0 = \lim_{n\to\infty} \frac{\text{sen}\ n}{n} = \lim_{n\to\infty} \frac{1}{n}\cdot\text{sen}\,n = \cancel{\frac{1}{\infty}}^{0}\,\text{sen}(\infty) = 0$$



Tema: Fecha: / /

$\varphi_0 = \lim\limits_{n\to\infty} \varphi_n \qquad \varphi_0 = \lim\limits_{n\to\infty} \frac{\pi}{2}$

$\varphi_0 = \frac{\pi}{2}$

$\lim\limits_{n\to\infty} z_n = 0 \cdot e^{i\frac{\pi}{2}} = 0$

Utilizando la definición del limite, mostrar que

(92) $\lim\limits_{z\to 1} \frac{2z+1}{z+2} = 1 \qquad \lim\limits_{z\to 1} \frac{2+1}{1+2} = 1 \qquad \cancel{\frac{3}{3}} = 1$

(94) $\lim\limits_{z\to -i} \frac{z^2+3iz-2}{z+i} = \frac{(-i)^2+3i(-i)-2}{-i+i} = \frac{-1+3-2}{0} = \frac{0}{0}$

$= \lim\limits_{z\to i} \frac{z^2+3iz-2}{z+i} \cdot \frac{z-i}{z-i} =$

$= \lim\limits_{z\to i} \frac{(z^2+3iz-2)(z-i)}{z^2-i^2}$

$= \lim\limits_{z\to i} \frac{z^3+i3z^2-2z-iz^2+3i^2z+2i}{z^2+1}$

$= \lim\limits_{z\to i} \frac{z^3+i2z^2+z+2i}{z^2+1}$

$= \lim\limits_{z\to -i} \frac{z^3+z+i(2z^2+2)i}{z^2+1}$

Notas:

Tema: Fecha: / /

$$\lim_{z\to -i} \frac{z^3+z}{z^2+1} + i\frac{(2z^2+2)}{z^2+1} = \frac{z(z^2+1)}{z^2+1} + i\frac{2(z^2+1)}{z^2+1}$$

$$\lim_{z\to -i} z+2i = -i+2i = \underline{i} \mid \text{sol}$$

(96) $\lim_{z\to 0} \frac{\operatorname{sen} z}{\operatorname{sh} iz} = \lim_{z\to 0} \frac{-i \operatorname{senh}(iz)}{\operatorname{senh}(iz)} = \underline{-i} \mid$ sol

Utilizando las Condiciones de Cauchy - Rieman aclarar cuales funciones de las siguientes son analiticas por lo menos en un punto y cuales son:

$z = x+iy$

(104) a $w = z^2\bar{z} =$

$$w = (x+iy)^2(x-iy) = (x^2+2xy\underline{i}-y^2)(x-iy)$$

$$= x^3+2x^2yi - y^2x - x^2yi - i^2 2xy^2 + y^3 i$$

$$= \underbrace{(x^3 - y^2x + 2xy^2)}_{u} + i\underbrace{(2x^2y - x^2y + y^3)}_{v}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad ; \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = 3x^2 - y^2 + 2y^2 \qquad \frac{\partial v}{\partial y} = 2x^2 - x^2 + 3y^2$$

$$\frac{\partial u}{\partial y} = 0 - 2yx + 4xy \qquad \frac{\partial v}{\partial x} = -(4xy - 2xy)$$

(1) $3x^2 + y^2 = x^2 + 3y^2$ ∴ No cumple

(2) $4xy - 2xy = -4xy + 2xy$ ∴ No cumple

∴ No son analiticas tanto punto (1) y (2)

Notas:

Materia: Fecha: DÍA MES AÑO

Tema:

b) $w = ze^z \rightarrow (x+iy)\, e^{x+iy} = e^{x+iy}x + i e^{x+iy} y$

$e^x x e^{iy} + i e^x . e^{iy} y = xe^x(\cos y + i\,\text{sen}\, y) + ye^x(\cos y + i\,\text{sen}\, y)$

$\Rightarrow xe^x \cos y + i xe^x \text{sen}\, y + ye^x \cos y + i ye^x \text{sen}(y)$

$= \underbrace{xe^x \cos y + ye^x \cos y}_{u} + i\underbrace{(xe^x \text{sen}\, y + ye^x \text{sen}\, y)}_{v}$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$\frac{\partial u}{\partial x} = \cos y (1)' e^x + \cos y\, x e^{x'} + y \cos y\, e^{x'}$

$\frac{\partial v}{\partial y} = xe^x \cos(y) + e^x (1)' \text{sen}\, y + e^x y \cos(y)'$

$= e^x \cos y + xe^x \cos y + e^x y \cos y = e^x \text{sen}(y)$

$+ xe^x \cos(y) + e^x y \cos(y)$

$\therefore$ No cumple

$\frac{\partial u}{\partial y} = -xe^x \text{sen}(y) + e^x (1)' \cos(y) - e^x y\, \text{sen}(y)$

$\frac{\partial v}{\partial x} = \text{sen}(y)(1)' e^x + \text{sen}\, y\, x e^{x'} + y\, \text{sen}\, y\, e^{x'}$

$= -xe^x \text{sen}\, y + e^x \cos y - e^x y\, \text{sen}\, y = -xe^x \text{sen}\, y - e^x \text{sen}(y)$

$- e^x y\, \text{sen}\, y$

$\therefore$ No cumplen no son

analíticos

c) $w = |z|\bar{z} \qquad [(x^2) + y^2]^{1/2} . (x - iy)$

$= \underbrace{x\sqrt{x^2+y^2}}_{u} - i\underbrace{\sqrt{x^2+y^2}}_{v}$

Materia:

Tema:

$u = x(x^2+y^2)^{1/2}$ $\quad v = -y\,(x^2+y^2)^{1/2}$

$\frac{\partial u}{\partial x} = (1)(x^2+y^2)^{1/2} + x\left(+\frac{1}{2}\right)(x^2+y^2)^{-1/2}\cdot(2x)$

$= (x^2+y^2)^{1/2} + x^2(x^2+y^2)^{-1/2}$

$\frac{\partial v}{\partial y} = -(1)(x^2+y^2)^{1/2} + -y\left(\frac{1}{2}\right)(x^2+y^2)^{-1/2}\cdot(2y)$

$= -(x^2+y^2)^{1/2} - y^2(x^2+y^2)^{-1/2}$

① $(x^2+y^2) + x^2(x^2+y^2)^{-1/2} = -(x^2+y^2)^{1/2} - y^2(x^2+y^2)^{-1/2}$

$\therefore$ No es analítica

$\frac{\partial u}{\partial y} = x\left(\frac{1}{2}\right)(x^2+y^2)^{-1/2}\cdot(2y) = xy\,(x^2+y^2)^{-\frac{1}{2}}$

$\frac{\partial v}{\partial x} = -\left[-y\,\frac{1}{2}(x^2+y^2)^{-1/2}\cdot(2x)\right] = +xy\,(x^2+y^2)^{-1/2}$

$\therefore$ No cumple.

d) $w = e^{z^2} = e^{(x+iy)^2} = e^{(x^2-y^2)+i(2xy)} = e^{x^2-y^2}\cdot e^{i2xy}$

$e^{x^2-y^2}\cdot \cos(2xy) + i\,\text{sen}(2xy)$

$\underbrace{\frac{e^{x^2}}{e^{y^2}}\cdot\cos 2xy}_{u} + i\,\underbrace{\frac{e^{x^2}}{e^{y^2}}\,\text{sen}(2xy)}_{v}$

$\frac{\partial u}{\partial x} = \frac{1}{e^{y^2}}\cdot e^{x^2}\cdot 2x\cos 2xy + \frac{e^{x^2}}{e^{y^2}}\cdot -\text{sen}(2xy)\cdot(2y)$

$= \frac{e^{x^2}}{e^{y^2}}\cdot 2x\cos 2xy - \frac{e^{x^2}}{e^{y^2}}\cdot 2y\,\text{sen}(2xy$

$\frac{\partial v}{\partial y} = e^{x^2}\cdot\frac{0-1\cdot\cancel{e^{y^2}}\cdot 2y}{(e^{y^2})^{\cancel{2}}}\,\text{sen}(2xy) + \frac{e^{x^2}}{e^{y^2}}\cdot\cos(2xy)\cdot(2x)$

Materia:

Tema:

① $\frac{e^{x^2}}{e^{y^2}} \cdot 2x\cos(2xy) - \frac{e^{x^2}}{e^{y^2}} \cdot 2y\,\text{sen}(2xy)$

$= \frac{e^{x^2}}{e^{y^2}} 2x\cos(2xy) - \frac{e^{x^2}}{e^{y^2}} 2y\,\text{sen}(2xy$

∴ son iguales

$\frac{\partial u}{\partial y} = e^{x^2} \cdot (e^{-y^2})(-2y)\cos(2xy) + \frac{e^{x^2}}{e^{y^2}} \cdot -\text{sen}(2xy) \cdot 2x$

$-\frac{e^{x^2}}{e^{y^2}}(2y)\cos(2xy) - \frac{e^{x^2}}{e^{y^2}}\text{sen}(2xy)(2x)$

$\frac{\partial v}{\partial x} = \frac{1}{e^{y^2}} \cdot e^{x^2} \cdot (2x)\,\text{sen}(2xy) + \frac{e^{x^2}}{e^{y^2}} \cdot \cos(2xy)(2y)$

② $-\frac{e^{x^2}}{e^{y^2}} 2y\cos(2xy) - \frac{e^{x^2}}{e^{y^2}} 2x\,\text{sen}(2xy) = -$

$-\frac{e^{x^2}}{e^{y^2}} 2y\cos(2xy) - \frac{e^{x^2}}{e^{y^2}} \cdot 2x\,\text{sen}(2xy)$

∴ son analiticas si cumplen

e) $w = |z|\,\text{Re}\,\bar{z}$; $= (\sqrt{x^2+y^2} \cdot x^2)\}$ Real

$\frac{\partial u}{\partial x} = 2x(x^2+y^2)^{1/2} + x^2 \cdot \frac{1}{2}(x^2+y^2)^{-1/2} \cdot (2x)$

$\frac{\partial v}{\partial y} = 0$

$\frac{\partial u}{\partial y} = x^2 \frac{1}{2}(x^2+y^2)^{-1/2} \cdot (2y)$ $\quad \frac{\partial v}{\partial y} = 0$

① $x^3(x^2+y^2)^{-1/2} + 2x(x^2+y^2)^{1/2} = 0$

② $x^2(x^2+y^2)^{-1/2}$

∴ No son analiticas.

Materia:

Tema:

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① $w = sen(3z) - i = sen(3x + 3iy) - i$

$= sen(3x)\cos(i3y) + sen(i3y)\cdot\cos(3x) - i$

$\cos(i3y) = \cosh(i^2 3y) = \cosh(-3y)$

$sen(i3y) = -i\, senh(i^2 3y) = -i\, senh(-3y)$

$= \underbrace{sen(3x)\cosh(3y)}_{u} - i\underbrace{(senh(-3y)\cos(3x) + 1)}_{v}$

$\frac{\partial u}{\partial x} = \cos(3x)\cdot 3\cosh(3y) = 3\cos(3x)\cdot\cosh(3y)$

$\frac{\partial v}{\partial y} = -\cosh(-3y)(-3)\cos(3x) + 0 = 3\cosh(3y)\cos(3x)$

① $= 3\cos(3x\cdot\cosh(3y) = 3\cos(3x)\cdot\cosh(3y)$

∴ Si cumple

$\frac{\partial u}{\partial y} = sen(3x)\cdot - senh(3y)\cdot + 3$

$=$

$\frac{\partial v}{\partial x} = + senh(3y) \cdot - sen(3x)\cdot 3$

$= -3\, sen(3x)\, senh(3y - 3\, sen(3x)\cosh(3y) =$

$-3\, sen(3x)\, senh(3y) - 3\, sen(3x)\, senh(3y)$

∴ Si cumplen que son analíticas

(105) a) $\Rightarrow w = \bar{z}\, Re\, z = (x - iy)(x) = x^2 - iyx$

$u = x^2 \qquad v = -yx$

$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial v}{\partial y} = -x \qquad \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial v}{\partial x} = +y$

① $2x = -x$ ② $0 = +y$

∴ No son analíticas.

Materia:

Tema:

Fecha: DIA MES AÑO

b) $w = \bar{z}\,\mathrm{Im}\,z = (x-iy)(iy) = \underbrace{ixy}_{v} + \underbrace{y^2}_{u}$

$\frac{\partial u}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = x \qquad \frac{\partial u}{\partial y} = 2y \quad \frac{\partial v}{\partial x} = y$

① $0 = x$ ② $2y = y$

∴ No son analíticas

c) $w = |z|\,\mathrm{Im}\,z = (x^2+y^2)^{1/2} \cdot (iy)^2 \quad (x^2+y^2 \cdot (i^2 y^2)^2$

$(x^2+y^2) \cdot (-y^4))^{1/2} = (-x^2y - y^4)^{1/2}$

$\frac{\partial u}{\partial x} = \frac{1}{2}(-x^2y-y^4)^{-1/2} \cdot (-2xy) - 0$

$= -(-x^2y-y^4)^{-1/2} \cdot xy$

$\frac{\partial v}{\partial y} = 0$ ① $-xy(-x^2$

$\frac{\partial u}{\partial y} = \frac{1}{2}(-x^2y-y^4)^{-1/2} \cdot (-x^2-4y^3)$

$= \frac{1}{2}(-x^2y-y^4)^{-1/2} \cdot (-x^2-4y^3) = 0$ ∴ No son analític

d) $w = \cosh(z) = \frac{e^z + e^{-z}}{2} = \frac{1}{2}\left[e^x \cdot e^{iy} + e^{-x} \cdot e^{-iy}\right]$

$= \frac{1}{2}\left[e^x \cdot \cos y + i \,\mathrm{sen}\, y + e^{-x} \cdot (\cos y - i\,\mathrm{sen}\, y\right]$

$= \frac{1}{2}(e^x \cos y + i e^x \mathrm{sen}\, y + e^{-x}\cos y - e^{-x} i\,\mathrm{sen}\, y]$

$= \frac{1}{2}\left[\cos y\,(e^x + e^{-x}) + i\,\mathrm{sen}\, y\,(e^x - e^{-x})\right]$

$= \cos y \left(\frac{e^x+e^{-x}}{2}\right) + i\,\mathrm{sen}\, y \left(\frac{e^x - e^{-x}}{2}\right)$

$= \underbrace{\cos y \cosh(x)}_{u} \underbrace{- i\,\mathrm{sen}\, y\,(\mathrm{senh}(x))}_{v}$

Tema:

107 = Mostrar, utilizando el calculo directo para n natural

$$(z^n)' = n z^{n-1}$$

$$\left.\begin{aligned} \frac{\partial w}{\partial x} &= \frac{\partial f}{\partial z} \\ \frac{\partial f}{\partial z} &= \frac{\partial w}{\partial y} \end{aligned}\right\}$$

$$f(z) = z^n = (x+iy)^n = w$$

$$\frac{\partial f}{\partial z} = \frac{\partial w}{\partial x} = n(x+iy)^{n-1}(1)$$

$$= n(x+iy)^{n-1}$$

$$\frac{\partial f}{\partial z} = n(z^{n-1})$$

$$f(z) = z^n = (x+iy)^n$$

$$\frac{\partial f}{\partial z} = -i\frac{\partial w}{\partial y} = -i\,n(x+iy)(i) = \left|n(x+iy)\right|^{n-1}$$

$$\boxed{= n(z)^{n-1}}$$

109 Mostrar que pasando las coordenadas cartesianas (x,y) a coordenadas polares (ρ, φ)

$$\frac{\partial u}{\partial R} = \frac{1}{R}\frac{\partial v}{\partial \varphi}, \quad \frac{\partial v}{\partial R} = -\frac{1}{R}\frac{\partial u}{\partial \varphi}$$

$w = f(z)$ $\qquad z = x+iy$

$x = R\cos\varphi$

$y = R\operatorname{sen}\varphi$

$$\frac{\partial x}{\partial R} = \cos\alpha$$

$$\frac{\partial y}{\partial R} = +\operatorname{sen}\alpha$$

$$\frac{\partial x}{\partial \alpha} = -R\operatorname{sen}\alpha$$

$$\frac{\partial y}{\partial \alpha} = R\cos\alpha$$

$$\frac{\partial w}{\partial R} = \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial R} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial R}$$

$$= \frac{\partial f}{\partial z}(1)(\cos\alpha) + \left(\frac{\partial f}{\partial z}\right)(i)\operatorname{sen}\alpha$$

$$\frac{\partial w}{\partial R} = \frac{\partial f}{\partial z}(\cos\alpha + i\operatorname{sen}\alpha)$$

$$\frac{\partial u}{\partial \alpha} = \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \alpha} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial \alpha}$$

$$= \frac{\partial f}{\partial z}(1)(-R\operatorname{sen}\varphi) + \frac{\partial f}{\partial z}(i)(R\cos\alpha)$$

Notas:

Materia:

Tema:

113. Mostrar el modulo y el argumento de la función analítica

$$f(z) = R\,e^{i\Phi} \quad \begin{matrix} \nearrow u \\ \searrow v \end{matrix} \qquad w = f(z)$$

$$f(z) = R(x,y)\cos(\Phi) + \operatorname{sen}(\Phi)$$

$$f(z) = \underbrace{R(\cos\Phi)}_{u} + \underbrace{iR\operatorname{sen}(\Phi)}_{v}$$

$$\boxed{\frac{\partial w}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}} \; ① \qquad \boxed{\frac{\partial w}{\partial y} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}} \; ②$$

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial u}{\partial R}\cdot\frac{\partial R}{\partial x} + \frac{\partial u}{\partial \Phi}\cdot\frac{\partial \Phi}{\partial x} \\ \frac{\partial u}{\partial y} &= \frac{\partial u}{\partial R}\cdot\frac{\partial R}{\partial y} + \frac{\partial u}{\partial \Phi}\cdot\frac{\partial \Phi}{\partial y} \end{aligned}\right\} ①$$

$$\left.\begin{aligned} \frac{\partial v}{\partial x} &= \frac{\partial v}{\partial R}\cdot\frac{\partial R}{\partial x} + \frac{\partial v}{\partial \Phi}\cdot\frac{\partial \Phi}{\partial y} \\ \frac{\partial v}{\partial y} &= \frac{\partial v}{\partial R}\cdot\frac{\partial R}{\partial y} + \frac{\partial v}{\partial \Phi}\cdot\frac{\partial \Phi}{\partial y} \end{aligned}\right\} ②$$

$$\frac{\partial u}{\partial x} = \cos(\Phi)\cdot\frac{\partial R}{\partial x} + R(-\operatorname{sen}(\Phi))\,\frac{\partial \Phi}{\partial x} =$$

$$\frac{\partial u}{\partial y} = (1)\cos(\Phi)\,\frac{\partial R}{\partial y} + R(-\operatorname{sen}(\Phi))\cdot\frac{\partial \Phi}{\partial y}$$

$$\frac{\partial v}{\partial x} = \operatorname{sen}(\Phi)\,\frac{\partial R}{\partial x} + R\cos(\Phi)\cdot\frac{\partial \Phi}{\partial y}$$

$$\frac{\partial v}{\partial y} = \operatorname{sen}(\Phi)\,\frac{\partial R}{\partial y} + R\cos(\Phi)\cdot\frac{\partial \Phi}{\partial y}$$

Condiciones $\quad \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$

Tema:

$$\cos(\Phi)\cdot\frac{\partial R}{\partial x} - R\,\text{sen}(\Phi)\cdot\frac{\partial \Phi}{\partial x} = \text{sen}(\Phi)\frac{\partial R}{\partial y} + R\cos(\Phi)\cdot\frac{\partial \Phi}{\partial y}$$

$$F f_1(x) + G f_2(x) = H f_1(x) + I f_2(x)$$

$$\cancel{\cos(\Phi)}\frac{\partial R}{\partial x} = R\cancel{\cos(\Phi)}\cdot\frac{\partial \Phi}{\partial y}$$

$$\boxed{\frac{\partial R}{\partial x} = R\frac{\partial \Phi}{\partial y}}$$

$$-R\cancel{\text{sen}(\Phi)}\frac{\partial \Phi}{\partial x} = \cancel{\text{sen}(\Phi)}\frac{\partial R}{\partial y}$$

$$\boxed{-R\frac{\partial \Phi}{\partial x} = \frac{\partial R}{\partial y}}$$

(2) $$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\cos(\Phi)\cdot\frac{\partial R}{\partial y} - R\,\text{sen}(\Phi)\cdot\frac{\partial \Phi}{\partial y} = -\left[\text{sen}(\Phi)\cdot\frac{\partial R}{\partial x} + R\cos(\Phi)\frac{\partial \Phi}{\partial y}\right.$$

$$\cos(\Phi)\cdot\frac{\partial R}{\partial y} - R\,\text{sen}(\Phi)\cdot\frac{\partial \Phi}{\partial y} = -\text{sen}(\Phi)\frac{\partial R}{\partial x} - R\cos(\Phi)\frac{\partial \Phi}{\partial y}$$

$$\cancel{\cos(\Phi)}\frac{\partial R}{\partial y} = -R\cancel{\cos(\Phi)}\frac{\partial \Phi}{\partial y}$$

$$\boxed{\frac{\partial R}{\partial y} = -R\frac{\partial \Phi}{\partial y}}$$

$$\cancel{-}R\cancel{\text{sen}(\Phi)}\frac{\partial \Phi}{\partial y} = \cancel{-\text{sen}(\Phi)}\frac{\partial R}{\partial x}$$

$$\boxed{R\frac{\partial \Phi}{\partial y} = \frac{\partial R}{\partial x}}$$

Materia: Funciones Armonicas

Tema:

Fecha: DIA MES AÑO

(126) $\alpha = f(ax+by)$, a y b son ctte.

$\frac{\partial u}{\partial x} = a + 0$ $\frac{\partial^2 u}{\partial x^2} = 0$.

$\frac{\partial u}{\partial y} = b$ $\frac{\partial^2 u}{\partial y^2} = 0$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ $0 + 0 = 0$

∴ si es una función Armonica

$\boxed{X = C_1(ax+by) + C_2}$

127- $u = f(xy)$.

$\frac{\partial u}{\partial x} = y$ $\frac{\partial^2 u}{\partial x^2} = 0$ $\frac{\partial u}{\partial y} = x$ $\frac{\partial^2 u}{\partial y^2} = 0$

∴ si cumple

(1) $t = x+y$ $u = f(t)$

$\frac{\partial u}{\partial x} = \frac{\partial^2 f}{\partial t^2} \cdot \frac{\partial t}{\partial x} \cdot \frac{\partial t}{\partial x} + \frac{\partial f}{\partial t} \cdot \frac{\partial^2 t}{\partial x^2}$ $C_1 xy + C_2$

$\frac{\partial t}{\partial x} = y$ $\frac{\partial^2 t}{\partial x^2} = 0$ $\frac{\partial^2 u}{\partial x^2} = yx\, f''(t) + 0\, f'(t)$

$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 f}{\partial t^2} \cdot \frac{\partial t}{\partial y} \cdot \frac{\partial t}{\partial y} + \frac{\partial f}{\partial t} \cdot \frac{\partial^2 t}{\partial y^2}$

$t = xy \rightarrow x \cdot f'' + 0 \cdot f'$

Igualación

$xy\, f''(t) + x \cdot f''(t) = 0$

$2xy\, f''(t) = 0$

$h(t) = 0$ $t = h'(t) = 0$

$h'(t) = f''(t)$ $t = \frac{dh}{dt}$ $t\, dt = dh$

$\int t\, dt = \int dh$