

# A Tableau algorithm for $\mathcal{ALCSCC}$

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## 1 Introduction

## 2 Tableau for *ALCSCC*

# Tableau Algorithm

Main Idea:

$$x : A \sqcap B \sqcap \exists r.C$$

$x$  must be in  $A$

$x$  must be in  $B$

$x$  must have  
an  $r$ -successor  
in  $C$

# *ALCSCC*: successors

~~$\exists r.C$~~

~~$\forall r.C$~~

~~$\leq m r.C$~~

~~$\geq m r.C$~~

$$\boxed{\text{succ}(c)}$$

$c$ : **set constraint** or a **cardinality constraint**

# *ALCSCC*: constraints

## set constraint:

- $r \subseteq s$
- $C \cap r \subseteq D$
- $\text{succ}(C \cap r) \subseteq \text{succ}(D)$

## cardinality constraint

- $2 \text{ dvd } |r|$
- $|C \cap r| \leq |D|$
- $|\text{succ}(C \cap r)| \leq |\text{succ}(D)|$

# Problem with successors constraints

$$x : succ(|s| > 1) \sqcap succ(|r| = |s|) \sqcap succ(|r| > |s|)$$

*s*-successors



*r*-successors



# Problem with successors constraints

$$x : \underline{\text{succ}(|s| > 1)} \sqcap \text{succ}(|r| = |s|) \sqcap \text{succ}(|r| > |s|)$$

*s*-successors



*r*-successors



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# Problem with blocking

$$x : \text{succ}(2 \cdot |r| \leq 5 \cdot |s|) \sqcap \text{succ}(5 \cdot |s| \leq 2 \cdot |r|) \sqcap \text{succ}(|r| > 1)$$

*s*-successors



*r*-successors



# Problem with blocking

$$x : succ(2 \cdot |r| \leq 5 \cdot |s|) \sqcap succ(5 \cdot |s| \leq 2 \cdot |r|) \sqcap \underline{succ(|r| > 1)}$$

*s*-successors



*r*-successors



# Problem with blocking

$$x : \underline{\text{succ}(2 \cdot |r| \leq 5 \cdot |s|)} \sqcap \text{succ}(5 \cdot |s| \leq 2 \cdot |r|) \sqcap \text{succ}(|r| > 1)$$

*s*-successors



*r*-successors



## QFBAPA formula and solver

$$x : \text{succ}(2 \cdot |r| \leq 5 \cdot |s|) \sqcap \text{succ}(5 \cdot |s| \leq 2 \cdot |r|) \sqcap \text{succ}(|r| > 1)$$

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$$2 \cdot |X_r| \leq 5 \cdot |X_s| \wedge 5 \cdot |X_s| \leq 2 \cdot |X_r| \wedge |X_r| > 1$$

## QFBAPA formula and solver

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$$2 \cdot |X_r| \leq 5 \cdot |X_s| \wedge 5 \cdot |X_s| \leq 2 \cdot |X_r| \wedge |X_r| > 1$$



$$|X_r| = 5 \text{ and } |X_s| = 2$$