Indian Institute of Information Technology Vadodara
End-semester Examination-Autumn 2020-21
MA101: Matrices and Linear Algebra

March 23, 2021

Maximum Marks: 30 $\hfill$ Time: 60 minutes

- Start new question on new page.

- Write down name, id and sign on each page of your answersheet.

- Each question carries 5 marks.

1. Find the inverse of following matrix using Gaussian elimination, if it exists. If no then give reason. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}$

2. Find all real solutions of following linear system using LU decomposition. What can you say about no of solutions of a linear system with no. of equations= no. of variables using LU decomposition.
$x + 2y + 4z = 1$
$3x + 8y + 14z = 2$
$2x + 6y + 13z = 3$

3. Give an example of an inner product $<,>$ on $\mathbb{R}^3$ which is different from standard inner product. Prove that it satisfies all properties of an inner product. What is the $< u, u >$, where $u = \begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix}$ and i is the last digit of your id modulo 2.

4. Find the minimal polynomial of following matrix

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

   What can you say about diagonalizability of A?

5. Find SVD decomposition $(A = U\Sigma V^T)$ of the following matrix A. What is the relation between U and V?

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

   What do you observe with SVD decomposition of A? Can you generalize the observation?

6. Describe Power Method and QR algorithm for calculating eigenvectors and eigenvalues. What are the conditions required for convergence of each method? What are the differences between two?

7. Let $v = \begin{bmatrix} 1 \\ 1 \\ i \end{bmatrix}$ where i is the last digit of your student id. Define a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(v) = 0$ and $v \in T(\mathbb{R}^3)$, image of T.