

## Analysis of the Relationship between USDX and Gold Prices in R

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### ABSTRACT

*This paper focuses on the relationship between U.S. Dollar Index (USDX) and Gold prices from 2010.01.01 to 2019.01.01. The aim of this paper is to analyze and determine the character of the co-movement between price levels. For best visualization, graphics of USDX and Gold price at monthly intervals with indicators Exponential Moving Average (EMA) and Relative Strength Index (RSI) are given. Main results of Regression Model (MSE, RMSE, MAE, RMSLE, Mean Residual Deviance, R-squared, Null Deviance and etc.) for reported on validation and training data are compared. For statistical computing, R language and environment are used. The paper also presents the basic characteristics and determinants of current price trends.*

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## 1. Introduction

The Great Recession is the period of global economic decline during the late 2000s. It was initially related to financial crisis of 2007-2008, but quickly transformed into a downturn in real activity and later into the European sovereign debt crisis. According to the National Bureau of Economic Research, which officially declares the peaks and troughs, the recession in the U.S. began in December 2007 and ended in June 2009, but in other countries the scale and timing of the recession varied. It was the largest economic downturn since the Great Depression [1].

There were multiple causes of this crisis. First of all, after the dot-com bubble burst, the Fed maintained excessively low interest rates for too long, fueling the housing bubble. Moreover, the government conducted policies intended to promote home ownership, even by people who were not able to afford it. The excessive risk-taking with toxic mortgages and derivatives, and record household debt (partially resulting from too low interest rates) also contributed to the economic imbalances and financial instability.

The Great Recession led to a rise in unemployment, an increase in bankruptcies and a rise in government debt, especially in Europe where the financial crisis morphed into sovereign debt crises as many countries decided to bail the banks out (the U.S. federal government injected \$700 billion

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to purchase distressed assets). Central banks responded aggressively to the crisis. For example, the Fed quickly started to purchase assets under its quantitative easing programs and slashed interest rates, as one can see in the chart below.

The Great Recession boosted the price of gold, although it declined in the very aftermath of the Lehman Brothers' bankruptcy, probably due to the forced sales in order to raise cash. However, it quickly rebounded and ended 2008 in positive territory.

The recession of the US economy and the eurozone ended in the second quarter of 2009, but in 2011 the second recession began in the eurozone [2], which lasted until 2013 and became the longest in its history [3].

2010s as a decade, including the years from 2010 to 2019, is characterized by a new cold war - the political conflict of the United States and its allies with Russia and China, as well as the aggravation of the contradictions between conservatism and liberalism, revolutions and civil wars in the Middle East and Ukraine, mass migration from the Third World countries and attempts to combat it.

Comparing the US Dollar Index (USDX), which tracks the value of the U.S. dollar against six other major currencies, and the value of the Dow Jones Industrial Average (DJIA), Nasdaq and S&P 500 over a 20-year period (as of 2011), the correlation coefficient is 0.35, 0.39 and 0.38, respectively. Note that all of the coefficients are positive, which means that as the value of the U.S. dollar increases, so do the stock indexes, but only by a certain amount. It's also notable that each coefficient is below 0.4, which means that only about 35% to 40% of the stock indexes' movements are associated with the movement of the U.S. dollar. USDX Index can serve as an independent trading instrument as well as a kind of indicator that allows to predict the change or continue the trends in the financial markets. This is very important, as the US dollar is the base of all world trade and it has a direct impact on the cost of a huge number of the most diverse assets that commands respect and compels all big and small investors to reckon with him.

Typically, any change in the USDX index movement allows anticipating the trend of foreign currency assets, which are directly linked to the US dollar, but it's basic or as they say in the Forex, major pairs. Some of them will also increase in value with the growth of USDX, for example, USD / JPY, USD / CAD, USD / CHF. Other pairs where the dollar is in the denominator will simultaneously fall, in particular, GBP / USD, AUD / USD, EUR / USD, NZD / USD.

Gold prices act as an indicator of the health of the economy. In times of crisis and elevated risk aversion, many investors turn to gold, the ultimate safe haven, to protect their capital. When the economy recovered, gold lost its appeal as insurance against the breakdown of the monetary system based on fiat currencies and the gold bull market ended in 2012.

Gold had served as money for thousands of years until 1971 when the gold standard was abandoned for a fiat currency system. Since that time, gold has been used as an investment. Gold is often classified as a commodity; however, it behaves more like a currency. The yellow metal is very weakly correlated with other commodities and is less used in the industry. Unlike national currencies, the yellow metal is not tied to any particular country. Gold is a global monetary asset and its price reflects the global sentiment, however, it is mostly influenced by the U.S. macroeconomic conditions [4].

[5] includes a quantitative analysis of the variables, such as Granger causality test, Johansen cointegration test and Vector Error Correction model. This paper reveals the existence of a long-term relationship between the analysis of relationship between oil and gold prices from January 1970 to December 2010. In [6] the relationship between oil prices and the price of gold and silver was analyzed by BDS test, non-linear ARDL approach and two non-linear Granger causality methods for 1973:1-2012:11 period in Turkey. [7] focuses on the relationship between Brent Crude Oil and gold prices from 2010.01.01 to 2016.03.01 and the aim of this work is to analyze and determine the character of the co-movement between price levels. [8] used VAR model to discuss

macroeconomics effect of structural oil shocks in four major Asian oil-consuming economics.

The aim of [9] is to examine the role of gold in the global financial system. We test the hypothesis that gold represents a safe haven against stocks of major emerging and developing countries. A descriptive and econometric analysis for a sample spanning a 30Â year period from 1979 to 2009 shows that gold is both a hedge and a safe haven for major European stock markets and the US but not for Australia, Canada, Japan and large emerging markets such as the BRIC countries.

The price movements in gold market are considered to detect non-linear dependencies with stock market in the Saudi Arabian context. Both the univariate and multivariate models of generalized autoregressive conditional heteroskedasticity (GARCH) class are employed in [10].

The United States (US) dollar-based gold price and the exchange rate between the Australian dollar and the US dollar (AUD/USD) have a combined and significant impact on both the trend of the Australian minerals industry and the overall Australian economy. The trend in this case refers to the economic sustainability and growth of the minerals industry as well as the physical extraction and production rates of the collective industry. To estimate gold mining project values, it is necessary to determine the correlation between the gold price, stated in US dollars per ounce (US\$/oz), and the relevant exchange rate, being the Australian dollar against the US dollar (AUD/USD). In [11] study, the aim is to investigate the long-run relationship between the US quoted gold price and the relevant periodic exchange rates (AUD/USD), and to determine the correlation coefficient between them using weekly average data over the period 1996 to 2014. In addition, the continuously compounded percentage return series for gold prices and AUD/USD exchange rates has been calculated, the results of which have exposed an interesting finding in that a bi-directional causality exists between the continuously compounded percentage return of periodic gold prices and the returns of the commensurate AUD/USD exchange rates. The authors of [12] are of the opinion that gold and oil serve as substitutes to investments in US dollar value as they are safer.

[13] introduced a new R package, BeSS, for solving the best subset selection problem in linear, logistic and Cox's proportional hazard (CoxPH) models. It utilizes a highly efficient active set algorithm based on primal and dual variables, and supports sequential and golden search strategies for best subset selection.

Unlike the above studies, this paper analyzes relationship between USDX and Gold prices from 2010.01.01 to 2019.01.01. For statistical computing, R language and environment are used. R is a language and environment for statistical computing and graphics. It provides a wide variety of statistical (linear and nonlinear modelling, classical statistical tests, time-series analysis, classification, clustering and so on) and graphical techniques, and is highly extensible.

## 2. Primary discussions and formulation of the problem

USDX futures contract is a leading benchmark for the international value of the US dollar and the world's most widely-recognized traded currency index. In a single transaction the USDX enables market participants to monitor moves in the value of the US dollar relative to a basket of world currencies, as well as hedge their portfolios against the risk of a move in the dollar. Following the descriptive statistics of USDX prices and the graphic was used in the article from 2010.01.01 to 2019.01.01 are given:

```
> summary(USDX)
Min. 1st Qu. Median Mean 3rd Qu. Max.
78.79 80.28 83.15 87.34 95.62 100.20
Highest: Lowest: Difference: Change
103.81 72.86 30.95 %: 14.38
```



**Fig.1.** Graph of USDX at monthly intervals with indicators EMA and RSI [14]

The relative strength index is a momentum indicator that measures the magnitude of recent price changes to evaluate overbought or oversold conditions in the price of a stock or other asset. The RSI is displayed as an oscillator (a line graph that moves between two extremes) and can have a reading from 0 to 100.

The relative strength index (RSI) is computed with a two-part calculation that starts with the following formula [15]:

$$RSI_{step\ one} = 100 - \left[ \frac{100}{1 + \frac{\text{Average gain}}{\text{Average lost}}} \right]$$

The average gain or loss used in the calculation is the average percentage gain or losses during a look-back period. The formula uses positive values for the average losses.

An exponential moving average (EMA) is a type of moving average (MA) that places a greater weight and significance on the most recent data points. The exponential moving average is also referred to as the exponentially weighted moving average. An exponentially weighted moving average reacts more significantly to recent price changes than a simple moving average (SMA), which applies an equal weight to all observations in the period.

All the moving averages commonly used in technical analysis are, by their very nature, lagging indicators. Consequently, the conclusions drawn from applying a moving average to a particular market chart should be to confirm a market move or to indicate its strength. Very often, by the time a moving average indicator line has made a change to reflect a significant move in the market, the optimal point of market entry has already passed. An EMA does serve to alleviate this dilemma to some extent. Because the EMA calculation places more weight on the latest data, it “hugs” the price action a bit more tightly and therefore reacts more quickly. This is desirable when an EMA is used to derive a trading entry signal [16].

Like all moving average indicators, they are much better suited for trending markets. When the market is in a strong and sustained uptrend, the EMA indicator line will also show an uptrend and vice-versa for a down trend. A vigilant trader will not only pay attention to the direction of the EMA line but also the relation of the rate of change from one bar to the next. For example, as the price action of a strong uptrend begins to flatten and reverse, the EMA’s rate of change from one bar to the next will begin to diminish until such time that the indicator line flattens and the rate of

change is zero.

Because of the lagging effect by this point, or even a few bars before, the price action should have already reversed. It follows, therefore, that observing a consistent diminishing in the rate of change of the EMA could itself be used as an indicator that could further counter the dilemma caused by the lagging effect of moving averages.

EMAs are commonly used in conjunction with other indicators to confirm significant market moves and to gauge their validity. For traders who trade intraday and fast-moving markets, the EMA is more applicable. Quite often, traders use EMAs to determine a trading bias. For example, if an EMA on a daily chart shows a strong upward trend, an intraday trader's strategy may be to trade only from the long side on an intraday chart.

Gold is still very popular in the world because of its "conversion" in any commodity or currency. This is important for the gold in the world-historical fact. The volume of gold reserves of a particular state is not the last feature of the domestic economy and the stability of the country. Similarly as the oil price of gold on the world market or supports the American currency or vice versa exerts pressure on it. Following the descriptive statistics of Gold prices and graphic was used in the article from 2010.01.01 to 2019.01.01 are given:

> summary(gold)

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1060	1204	1288	1357	1540	1771

Highest:	Lowest:	Difference:	Change
1,911.60	1,046.20	865.40	%: -3.93



**Fig.2.** Graph of Gold price at monthly intervals with indicators EMA and RSI [17]

The expected model for the data is

$$\text{GOLD} = \beta_0 + \beta_1 \times \text{USDX}$$

where  $\beta_0$  is the theoretical y-intercept and  $\beta_1$  is the theoretical slope. The goal of a linear regression is to find the best estimates for  $\beta_0$  and  $\beta_1$  by minimizing the residual error between the experimental and predicted gold. The final model is

$$\text{GOLD} = b_0 + b_1 \times \text{USDX} + e$$

where  $b_0$  and  $b_1$  are the estimates for  $\beta_0$  and  $\beta_1$  and  $e$  is the residual error.

Logarithmically transforming variables in a regression model is a very common way to handle

situations where a non-linear relationship exists between the independent and dependent variables.<sup>3</sup> Using the logarithm of one or more variables instead of the un-logged form makes the effective relationship non-linear, while still preserving the linear model. Logarithmic transformations are also a convenient means of transforming a highly skewed variable into one that is more approximately normal. (In fact, there is a distribution called the log-normal distribution defined as a distribution whose logarithm is normally distributed – but whose untransformed scale is skewed.)

$$\text{LOG}(GOLD) = b_0 + b_1 \times \text{LOG}(USDX) + e$$

In instances where both the dependent variable and independent variable(s) are log-transformed variables, the interpretation is a combination of the linear-log and log-linear cases above. In other words, the interpretation is given as an expected percentage change in Y when X increases by some percentage. Such relationships, where both Y and X are log-transformed, are commonly referred to as elastic in econometrics, and the coefficient of log X is referred to as an elasticity [18].

### 3. Main results

Regression analysis is almost certainly the most important tool at the econometrician's disposal. In very general terms, regression is concerned with describing and evaluating the relationship between a given variable and one or more other variables. More specifically, regression is an attempt to explain movements in a variable by reference to movements in one or more other variables.

The correlation between two variables measures the degree of linear association between them. If it is stated that y and x are correlated, it means that y and x are being treated in a completely symmetrical way. Thus, it is not implied that changes in x cause changes in y, or indeed that changes in y cause changes in x. Rather, it is simply stated that there is evidence for a linear relationship between the two variables, and that movements in the two are on average related to an extent given by the correlation coefficient. In regression, the dependent variable (y) and the independent variable (x) is treated very differently. The y variable is assumed to be random or 'stochastic' in some way, i.e. to have a probability distribution. The x variables are, however, assumed to have fixed ('non-stochastic') values in repeated samples. Regression as a tool is more flexible and more powerful than correlation [19].

Syntax for a regression analysis in R is [20]

**lm(Y~model)**

where Y is the object containing the dependent variable to be predicted and *model* is the formula for the chosen mathematical model. The command **lm()** provides the model's coefficients but no further statistical information; thus

`lm.r=lm(log(GOLD)~log(USDX))`

Call:

`lm(formula = log(GOLD) ~ log(USDX))`

Coefficients:

(Intercept)	log(USDX)
12.328	-1.149

Some functions allow to display the resulting R model including **summary()** – displays a certain set statistical parameters (statistical tests ...), **residuals()** – indicates the remains of a regression, **predict()** – predicted values, and **coef()** – the vector parameter estimates. In the system R for calculating the sampling of the correlation coefficient use function **cor()**.

`> summary(lm.r)`

Call:

```
lm(formula = log(GOLD) ~ log(USDX))
```

Residuals:

Min	1Q	Median	3Q	Max
-0.199902	-0.098527	-0.000012	0.103364	0.185501

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	12.3283	0.7300	16.888	< 2e-16 ***
log(usdx)	-1.1488	0.1634	-7.029	3.09e-09 ***

Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.107 on 56 degrees of freedom

Multiple R-squared: 0.4688, Adjusted R-squared: 0.4593

F-statistic: 49.41 on 1 and 56 DF, p-value: 3.094e-09

```
cor (log(USDX),log(GOLD))
- 0.7
```

The section of output labeled ‘Residuals’ gives the difference between the experimental and predicted gold price. Estimates for the model’s coefficients are provided along with their standard deviations (‘Std Error’), and a t-value and probability for a null hypothesis that the coefficients have values of zero. In this case, for example, we see that there is no evidence that the intercept ( $\beta_0$ ) is different from zero and strong evidence that the slope ( $\beta_1$ ) is significantly different than zero. At the bottom of the table we find the standard deviation about the regression (sr or residual standard error), the correlation coefficient and an F-test result on the null hypothesis that the MSreg/MSres is 1.

From the analysis of the results we obtain the following:

Residual standard error (0.1075) – estimate of the standard deviation of residuals. It is assumed that the residuals are normally distributed with mean a value of 0 and a standard deviation  $\sigma$ . The analysis results of this line is exactly assesses the value of  $\sigma$ .

Multiple R-squared (0.4688) and Adjusted R-squared (0.4593) – the coefficient of determination and the coefficient of determination adjusted for the number of model parameters respectively.

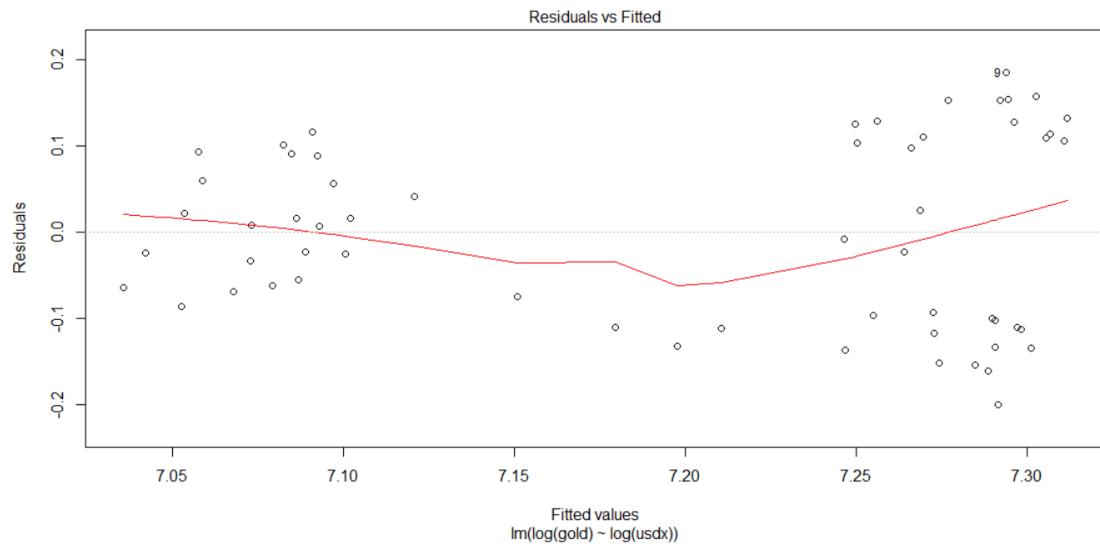
F-statistic (49.41) – the value of Fisher's criterion, by which is verified the null hypothesis that all the coefficients of the true model (in this case,  $\beta_1$  and  $\beta_2$ ) are equal to 0.

The value of F-test for linear models is calculated as the ratio of the variance in the data, "explained" the parameters of the model, to the residue of the dispersion (i.e., of the total variance, that model "Doesn't explain").

Obtained LM regression equation is

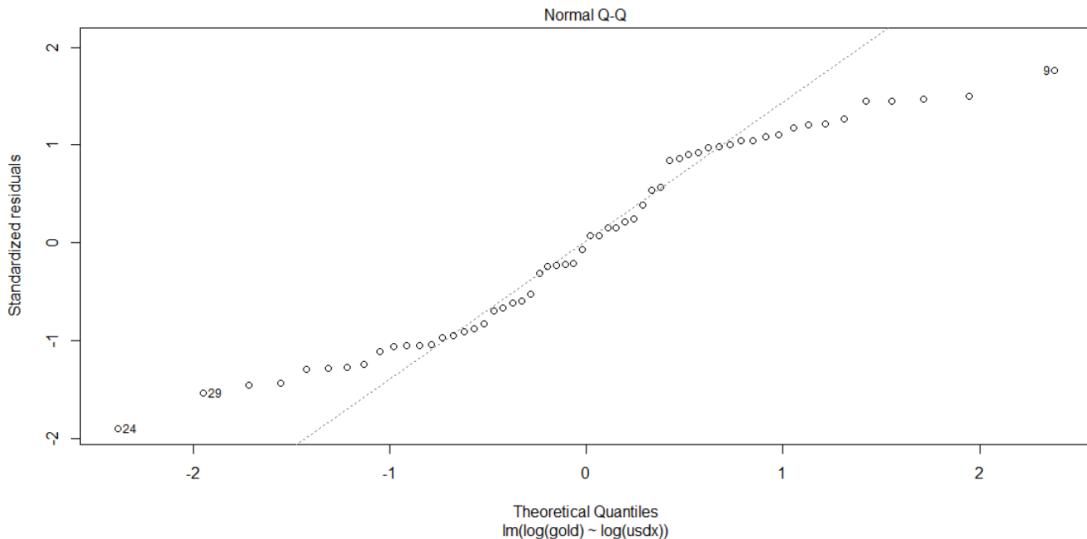
$$\text{LOG}(GOLD) = 12.3 - 1.149 * \text{LOG}(USDX)$$

which means that for every one point increase in USDX price, the gold price decrease by 1.149 point. The correlation coefficient – 0,7 reveals the inverse relationship between the indicators consequently from 2010.01.01 to 2019.01.01.



**Fig.3.** The residual errors plotted versus their fitted values

In Fig.3 the residuals should be randomly distributed around the horizontal line representing a residual error of zero; that is, there should not be a distinct trend in the distribution of points.



**Fig.4.** The standard Q-Q plot

Fig.4 shows a plot which should suggest that the residual errors are normally distributed. The scale-location plot in the upper right shows the square root of the standardized residuals (sort of a square root of relative error) as a function of the fitted values.

If we apply GLM without considering the dataset as a time-series case, we will get R-squared around 46 on validation data. In addition, R-squared for train set is considerably higher 54.7.

Obtained GLM regression equation is

$$\text{GOLD} = 2693.3 - 4.1 * \text{USDX} - 3.96 * \text{OPEN} - 3.69 * \text{HIGH} - 3.44 * \text{Low} + 0.004 * \text{Vol.} - 3.05 * \text{CHANGE\%}.$$

In the following table given Main results of Regression Model for Reported on validation and training data:

Main results of Regression Model	Reported on validation data	Reported on training data
MSE:	22625.25	16252.74
RMSE:	150.4169	127.4863
MAE:	133.1464	98.55586
RMSLE:	0.1060192	0.0888658
Mean Residual Deviance:	22625.25	16252.74
R-squared:	0.4688	0.5473
Null Deviance:	840241.5	2620741
Null D.o.F. :	23	72
Residual Deviance :	543006.1	1186450
Residual D.o.F.:	17	66
AIC:	324.7528	930.9743

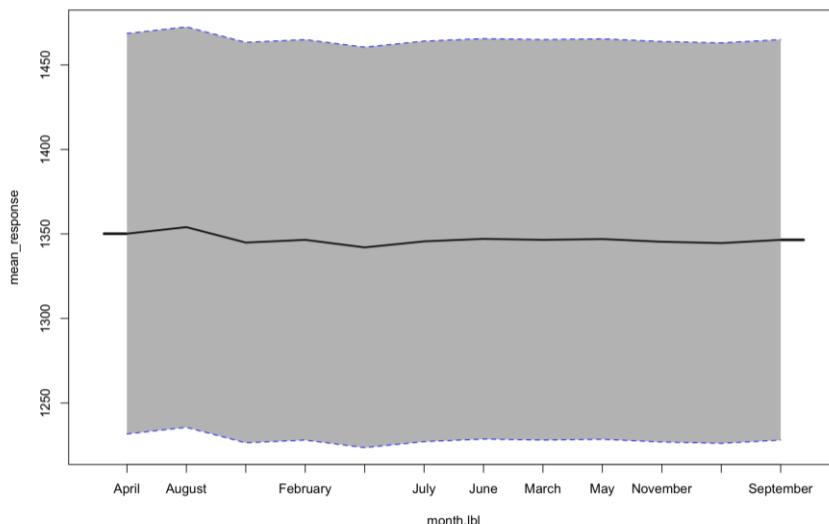
After the model specified, its performance characteristics should be verified or validated by comparison of its forecast with historical data for the process it was designed to forecast. We can use the error measures such as MAPE (Mean absolute percentage error), MSE (Mean square error) [21] or RMSE (Root mean square error) [22].

Selection of an error measure has an important effect on the conclusions about which of a set of forecasting methods is most accurate.

Usually the MSE or RMSE can be used as the criterion for selecting an appropriate smoothing constant. For instance, by assigning  $\alpha$  values from 0.1 to 0.99, we select the value that produces the smallest MSE or RMSE.

Higher R-squared for training data tells us that we could get better R-squared if we would apply time-series technique on the train dataset.

According to GLM model, we can observe the price of gold over month, the interesting fact that for August month the price becomes relatively higher:



**Fig.5.** The price of gold over month according to GLM model

#### 4. Conclusion

This paper analyzes the relationship between USDX and Gold prices from 2010.01.01 to 2019.01.01 in R codes. If we apply GLM without considering the dataset as a time-series case, we will get R-squared around 46 on validation data. In addition, R-squared for train set is considerably higher 54.7. From the analysis above, it can be concluded that USDX and Gold prices have a negative correlation (-0.7). The obtained plots show that the residual errors plotted versus their fitted values, the residual errors are normally distributed (a standard Q-Q), the each points leverage, which is a measure of its importance in determining the regression result. For best visualization graphics of USDX and Gold price at monthly intervals with indicators Exponential Moving Average (EMA) and Relative Strength Index (RSI) are given. The main results of Regression Model (MSE, RMSE, MAE, RMSLE, Mean Residual Deviance, R-squared, Null Deviance and etc.) for reported on validation and training data are compared. According to GLM model, we can observe the price of gold over month, the interesting fact that for August month the price becomes relatively higher.

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