Discrete Assignment
EE1205 Signals and Systems

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Question 11.9.3.8: Find the sum to indicated number of term in each of the geometric progressions in $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...n$ terms

Solution: Sum of the geometric progression of $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...n$ terms is

Input Table:

variable	value	description
x(0)	$\sqrt{7}$	first term of the geometric progession
r	$\sqrt{3}$	common ratio of the geometeric progression
x(n)	$\sqrt{7*3^{(n)}}$	n^{th} term of the geometric progession
n		no of the term in the geometric progression
S_n		Sum of the n+1 term of the geometric progression
U(z)	$\frac{1}{1-z^{-1}} \quad \|z^{-1}\| < 1$	z-transformation of u(n)

$$r = \frac{a_2}{a_1} \tag{1}$$

$$r = \frac{\sqrt{21}}{\sqrt{7}} \tag{2}$$

$$= \sqrt{3} \tag{3}$$

$$x(0) = \sqrt{7} \tag{4}$$

$$x(n) = x(0)r^{(n)} \tag{5}$$

$$x(n) = x(0)\sqrt{3^{(n)}} \tag{6}$$

$$x(n) = \sqrt{7*3^{(n)}} \tag{7}$$

$$S_n = \frac{x(0)(r^n)}{r-1} \tag{8}$$

$$S_n = \frac{\sqrt{7}(\sqrt{3}^n)}{(\sqrt{3}-1)} \tag{9}$$

$$= \frac{\sqrt{7}(\sqrt{3}^n)}{(\sqrt{3}-1)} \tag{10}$$

$$\tag{11}$$

Z-Transformation:

$$x(n) = x(0)r^{(n)} \tag{12}$$

$$X(z) \xleftrightarrow{\mathcal{H}} Z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{13}$$

$$= \sum_{n=0}^{\infty} x(n)z^{-n} \tag{14}$$

$$= \sum_{n=0}^{\infty} x(0)r^{n}z^{-n} \tag{15}$$

$$= x(0)\sum_{n=0}^{\infty} r^{n}z^{-n} \tag{16}$$

$$= x(0)(z^0r^0U(z) + r^1z^{-1}U(z) + r^2z^{-2}U(z) + r^3z^{-3}U(z) + r^4z^{-4}U(z) + ...) \tag{17}$$

$$= x(0)(1 + r^1z^{-1} + r^2z^{-2} + r^3z^{-3} + r^4z^{-4} + r^5z^{-5} + r^6z^{-6} + ...) \tag{18}$$

$$X(Z) = x(0)\left(\frac{1}{1 - rz^{-1}}\right), \quad |rz^{-1}| < 1 \tag{19}$$

$$y(n) = x(n)u(n) \tag{20}$$

$$Y(z) = X(z)U(z) \tag{21}$$

$$= x(0)\left(\frac{1}{1 - rz^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{22}$$

Contour Intergration: (23)

$$y(n+1) = \frac{1}{2\pi j}\oint_C Y(Z)z^n dz \tag{24}$$

$$= \frac{1}{2\pi j}\oint_C x(0)\left(\frac{1}{1 - rz^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right)z^n dz \tag{25}$$

$$= \frac{1}{2\pi j}\oint_C \sqrt{7}\left(\frac{1}{1 - \sqrt{3}z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right)z^n dz \tag{26}$$