GATE 2022 Assignment
EE1205 Signals and Systems

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Question:

Let $x_1(t) = e^{-t}u(t)$ and $x_2(t) = u(t) - u(t-2)$, where $u(.)$ denotes the unit step function. If $y(t)$ denotes the convolution of $x_1(t)$ and $x_2(t)$,then $\lim_{t\to\infty} y(t) =$ _____. (Rounded off to one decimal place)

(GATE EC 2022)

Solution:

variable	value	description
$x_1(t)$	$e^{-t}u(t)$	given function 1
$x_2(t)$	$u(t) - u(t-2)$	given function 2
$y(t)$	-	convolution of $x_1(t)$ and $x_2(t)$

TABLE I

TABLE: INPUT PARAMETERS

$$y(t) = x_1(t) * x_2(t) \tag{1}$$

from Table Table I

$$y(t) = e^{-t}u(t) * (u(t) - u(t-2)) \tag{2}$$

By applying Laplace transform

$$Y(s) = X_1(s).X_2(s) \tag{3}$$

$$e^{-t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{1+s}, \quad \text{Re}(s) > -1 \tag{4}$$

$$u(t) - u(t-2) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1 - e^{-2s}}{s}, \quad \text{Re}(s) > 0 \tag{5}$$

$$Y(s) = \left(\frac{1}{1+s}\right)\left(\frac{1 - e^{-2s}}{s}\right) \tag{6}$$

$$= \frac{1 - e^{-2s}}{s(s+1)} \tag{7}$$

By applying Final value theorem

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{8}$$

$$= \lim_{s\to 0} s\left(\frac{1 - e^{-2s}}{s(s+1)}\right) \tag{9}$$

$$= \lim_{s\to 0} \left(\frac{1 - e^{-2s}}{(s+1)}\right) \tag{10}$$

$$= \left(\frac{1 - e^0}{0+1}\right)\lim_{t\to\infty} y(t) \qquad = 0 \tag{11}$$