Discrete Assignment
EE1205 Signals and Systems

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Question 11.9.3.8: Find the sum to indicated number of term in each of the geometric progressions in $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...n$ terms

Solution: Sum of the geometric progression of $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...n$ terms is

Input Table:

variable	value	description
x(0)	$\sqrt{7}$	first term of the geometric progession
r	$\sqrt{3}$	common ratio of the geometeric progression
x(n)	$\sqrt{7}(3^n)$	n^{th} term of the geometric progession
n		no of the term in the geometric progression
y(n+1)	$\frac{x(0)(r^{n+1}-1)}{r-1}$	Sum of the n+1 term of the geometric progression
U(z)	$\frac{1}{1-z^{-1}} \quad \lvert z^{-1}\rvert < 1$	z-transformation of u(n)

Z-Transformation:

$$X(Z) = x(0)\left(\frac{1}{1 - rz^{-1}}\right), \quad |rz^{-1}| < 1 \tag{1}$$

$$y(n) = x(n)u(n) \tag{2}$$

$$Y(z) = X(z) * U(z) \tag{3}$$

$$= \sqrt{7}\left(\frac{1}{1 - \sqrt{3}z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{4}$$

$$= \left(\frac{\sqrt{7}}{\sqrt{3} - 1}\right)\left(\left(\frac{\sqrt{3}}{1 - \sqrt{3}z^{-1}}\right) - \left(\frac{1}{1 - z^{-1}}\right)\right) \tag{5}$$

Using Contour Integration to find the inverse Z-transform

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^n dz \tag{6}$$

$$= \frac{1}{2\pi j} \oint_C \frac{\sqrt{7}}{\sqrt{3}-1} \left(\left(\frac{\sqrt{3}}{1-\sqrt{3}z^{-1}} \right) - \left(\frac{1}{1-z^{-1}} \right) \right) z^{n-1} dz \tag{7}$$

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z) \right) \tag{8}$$

$$R = R_1 + R_2 \tag{9}$$

$$R_1 = \frac{1}{(0)!} \lim_{z \to \sqrt{3}} \frac{d^0}{dz^0} (z - \sqrt{3}) \left(\frac{\sqrt{7}}{\sqrt{3}-1} \right) \left(\frac{\sqrt{3} z^n}{z - \sqrt{3}} \right) \tag{10}$$

$$= \frac{\sqrt{7}\sqrt{3}}{\sqrt{3}-1} \lim_{z \to \sqrt{3}} z^n \tag{11}$$

$$= \frac{\sqrt{7}\sqrt{3}^{n+1}}{\sqrt{3}-1} \tag{12}$$

$$R_2 = \frac{1}{(0)!} \lim_{z \to \sqrt{3}} \frac{d^0}{dz^0} (z-1) \left(\frac{\sqrt{7}}{\sqrt{3}-1} \right) \left(\frac{-z^n}{z-1} \right) \tag{13}$$

$$= \frac{\sqrt{7}}{\sqrt{3}-1} \lim_{z \to 1} -z^n \tag{14}$$

$$= \frac{-\sqrt{7}}{\sqrt{3}-1} \tag{15}$$

$$R = R_1 + R_2 \tag{16}$$

$$= \frac{\sqrt{7}\sqrt{3}^{n+1}}{\sqrt{3}-1} - \frac{\sqrt{7}}{\sqrt{3}-1} \tag{17}$$

$$= \sqrt{7} \left(\frac{\sqrt{3}^{n+1} - 1}{\sqrt{3}-1} \right) \tag{18}$$

$$y(n) = \sqrt{7} \left(\frac{\sqrt{3}^{n+1} - 1}{\sqrt{3}-1} \right) \tag{19}$$

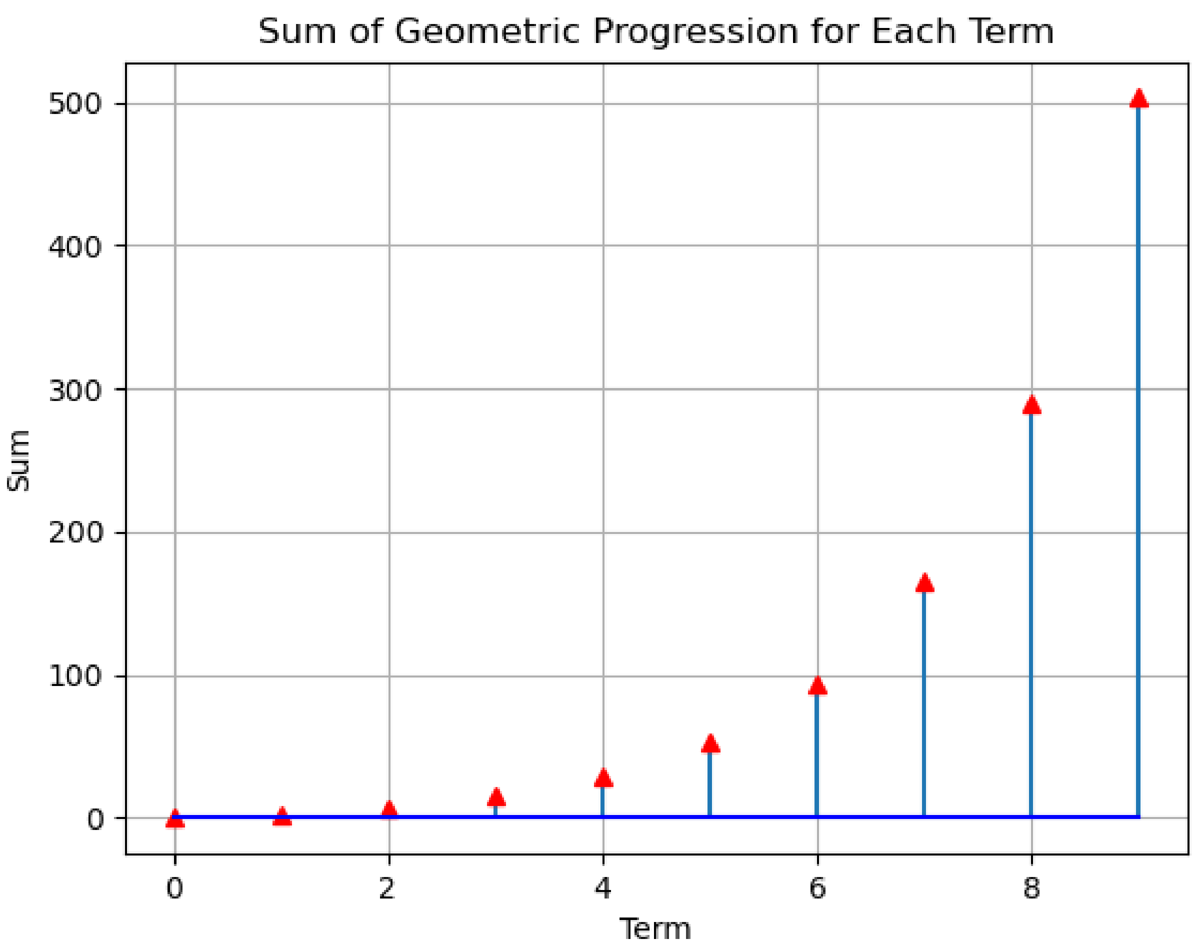

Fig. 0. **sum of the geometric progression after adding each term**