Discrete Assignment
EE1205 Signals and Systems

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Question 11.9.3.8: Find the sum to indicated number of term in each of the geometric progressions in $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...n$ terms
Solution: Sum of the geometric progression of $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, ...n$ terms is
Input Table:

variable	value	description
x(0)	$\sqrt{7}$	first term of the geometric progession
r	$\sqrt{3}$	common ratio of the geometeric progression
x(n)	$\sqrt{7 * 3^{(n)}}$	n^{th} term of the geometric progession
n		no of the term in the geometric progression
y(n+1)	$\frac{x(0)(r^{n+1}-1)}{r-1}$	Sum of the n+1 term of the geometric progression
U(z)	$\frac{1}{1-z^{-1}} \quad \|z^{-1}\| < 1$	z-transformation of u(n)

Z-Transformation:

$$\begin{aligned}
x(n) &= x(0)r^{(n)} && (1)\\
X(z) \overset{\mathcal{H}}{\longleftrightarrow} Z\{x(n)\} &= \sum_{n=-\infty}^{\infty} x(n)z^{-n} && (2)\\
&= \sum_{n=0}^{\infty} x(n)z^{-n} && (3)\\
&= \sum_{n=0}^{\infty} x(0)r^{n}z^{-n} && (4)\\
&= x(0)\sum_{n=0}^{\infty} r^{n}z^{-n} && (5)\\
&= x(0)(z^0r^0U(z) + r^1z^{-1}U(z) + r^2z^{-2}U(z) + r^3z^{-3}U(z) + r^4z^{-4}U(z) + ...) && (6)\\
&= x(0)(1 + r^1z^{-1} + r^2z^{-2} + r^3z^{-3} + r^4z^{-4} + r^5z^{-5} + r^6z^{-6} + ...) && (7)\\
X(Z) &= x(0)\left(\frac{1}{1-rz^{-1}}\right), \quad |rz^{-1}| < 1 && (8)\\
y(n) &= x(n)u(n) && (9)\\
Y(z) &= X(z)U(z) && (10)\\
&= x(0)\left(\frac{1}{1-rz^{-1}}\right)\left(\frac{1}{1-z^{-1}}\right) && (11)
\end{aligned}$$

Contour Integration: (12)

$$\begin{aligned}
y(n+1) &= \frac{1}{2\pi j}\oint_C Y(Z)z^n dz && (13)\\
&= \frac{1}{2\pi j}\oint_C x(0)\left(\frac{1}{1-rz^{-1}}\right)\left(\frac{1}{1-z^{-1}}\right)z^n dz && (14)\\
&= \frac{1}{2\pi j}\oint_C \sqrt{7}\left(\frac{1}{1-\sqrt{3}z^{-1}}\right)\left(\frac{1}{1-z^{-1}}\right)z^n dz && (15)\\
&= \frac{\sqrt{7}}{2\pi j}\oint_C \left(\frac{1}{1-\sqrt{3}z^{-1}}\right)\left(\frac{1}{1-z^{-1}}\right)z^n dz, \quad |z| > \sqrt{3} && (16)\\
& && (17)
\end{aligned}$$