GATE 2021 Assignment
EE1205 Signals and Systems

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Question: The exponential Fourier series representation of a continous-time periodic signal x(t) is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where ω_0 is the fundamental angular frequency of x(t) and the coefficients of the series are a_k.The following information is given about x(t) and a_k
I. x(t) is real and even,having a fundamental period of 6
II. The average value of x(t) is 2.
III. $a_k = \begin{cases} k, & 1 \le k \le 3 \\ 0, & k > 3 \end{cases}$
The average power of the signal x(t) (rounded off to one decimal place) is ____.
Solution:

variable	value	description
T_0	6	Fundamental time period
y(t)	-	average power of the signal
x(t)	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	Input signal
a_k	$\begin{cases} k, & 1 \le k \le 3 \\ 0, & k > 3 \end{cases}$	coefficients of the series

TABLE I
TABLE: INPUT PARAMETERS

x(t) is even and real so, a_k=a_{-k}
Parswal's Power Theorem

Proof

$$P = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} \left|x(t)^2\right| dt, \quad \left|x(t)^2\right| = x(t)\,x^*(t) \tag{1}$$

$$P = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} x(t)\,x^*(t)\,dt \tag{2}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \tag{3}$$

$$P = \frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} \left(\sum_{k=-\infty}^{\infty} C_n e^{jn\omega_0 t}\right) x^*(t)\,dt \tag{4}$$

$$P = \sum_{n=-\infty}^{\infty} C_n \left(\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}} x^*(t)\,e^{jn\omega_0 t} dt\right) \tag{5}$$

$$= \sum_{n=-\infty}^{\infty} C_n C_n^* \tag{6}$$

$$\Longrightarrow \quad P = \sum_{n=-\infty}^{\infty} |C_n|^2 \tag{7}$$

By using Parsval's Power Theorem

$$\frac{1}{T}\int_{0}^{T}\left|x(t)^2\right|dt = \sum_{k=-\infty}^{\infty}|a_k|^2 \tag{8}$$

$$y(t) = \sum_{k=-\infty}^{\infty}|a_k|^2 \tag{9}$$

$$= 2\sum_{k=0}^{\infty}|a_k|^2 \tag{10}$$

$$= 2\sum_{k=1}^{3}|a_k|^2 + |a_0|^2 \tag{11}$$

$$= 2\left(1^2 + 2^2 + 3^2\right) + 2^2 \tag{12}$$

$$= 32 \tag{13}$$

$$x(n) = 2\left(e^{\frac{j\pi t}{3}} + 2e^{\frac{2j\pi t}{3}} + 3e^{j\pi t}\right) + 2 \tag{14}$$

x(n) is real so, we get (15)

$$x(n) = 2\left(cos\left(\frac{\pi t}{3}\right) + 2cos\left(\frac{2\pi t}{3}\right) + 3cos(\pi t)\right) + 2 \tag{16}$$

The average power of the signal x(t) (rounded off to one decimal place) is 32

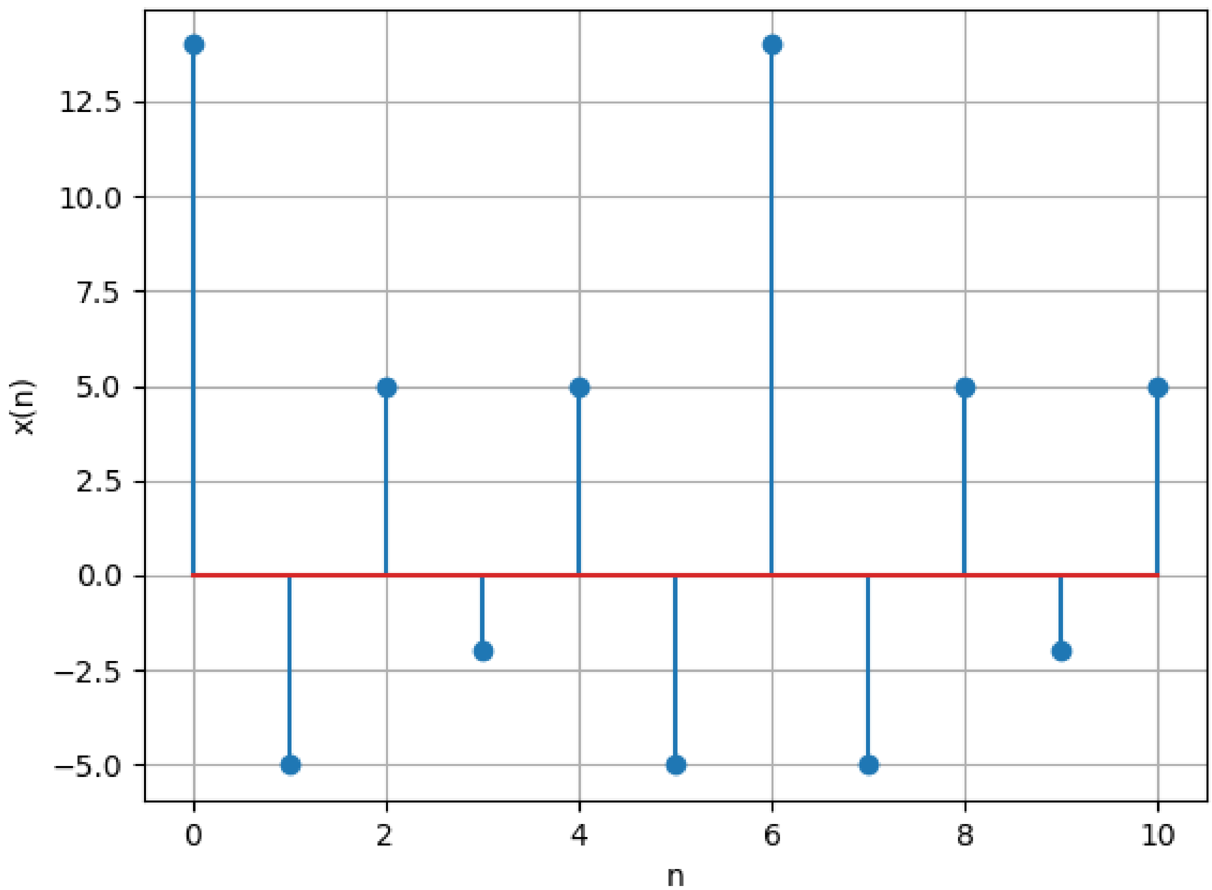

Fig. 1. STEM PLOT OF $y(n)$