

Discrete Assignment

EE1205 Signals and Systems

Kurre Vinay
EE23BTECH11036

Question 11.9.3.8: Find the sum to indicated number of term in each of the geometric progressions in $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \dots, n$ terms

Solution: Sum of the geometric progression of $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \dots, n$ terms is

Input Table:

variable	value	description
$x(0)$	$\sqrt{7}$	first term of the geometric progression
r	$\sqrt{3}$	common ratio of the geometric progression
$x(n)$	$\sqrt{7}(3^n)$	n^{th} term of the geometric progression
n		no of the term in the geometric progression
$y(n+1)$	$\frac{x(0)(r^{n+1}-1)}{r-1}$	Sum of the $n+1$ term of the geometric progression
$U(z)$	$\frac{1}{1-z^{-1}} \quad z^{-1} < 1$	z-transformation of $u(n)$

Z-Transformation:

$$x(n) = x(0)r^{(n)}u(n) \quad (1)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}u(n) \quad (2)$$

$$= \sum_{n=0}^{\infty} x(n)z^{-n} \quad (3)$$

$$= \sum_{n=0}^{\infty} x(0)r^n z^{-n} \quad (4)$$

$$= x(0) \sum_{n=0}^{\infty} r^n z^{-n} \quad (5)$$

$$= x(0)(z^0 r^0 U(z) + r^1 z^{-1} U(z) + r^2 z^{-2} U(z) + r^3 z^{-3} U(z) + r^4 z^{-4} U(z) + \dots) \quad (6)$$

$$= x(0)(1 + r^1 z^{-1} + r^2 z^{-2} + r^3 z^{-3} + r^4 z^{-4} + r^5 z^{-5} + r^6 z^{-6} + \dots) \quad (7)$$

$$X(Z) = x(0) \left(\frac{1}{1 - rz^{-1}} \right), \quad |rz^{-1}| < 1 \quad (8)$$

$$y(n) = x(n)u(n) \quad (9)$$

$$Y(z) = X(z)U(z) \quad (10)$$

$$= x(0) \left(\frac{1}{1 - rz^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (11)$$

$$= \sqrt{7} \left(\frac{1}{1 - \sqrt{3}z^{-1}} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (12)$$

$$= \left(\frac{\sqrt{7}}{\sqrt{3} - 1} \right) \left(\left(\frac{\sqrt{3}}{1 - \sqrt{3}z^{-1}} \right) - \left(\frac{1}{1 - z^{-1}} \right) \right) \quad (13)$$

$$y(n) = \frac{\sqrt{7}}{\sqrt{3} - 1} (\sqrt{3}^{n+1} - 1)u(n) \quad (14)$$

$$y(n) = \sqrt{7} \left(\frac{(\sqrt{3}^{n+1} - 1)}{\sqrt{3} - 1} \right) u(n) \quad (15)$$

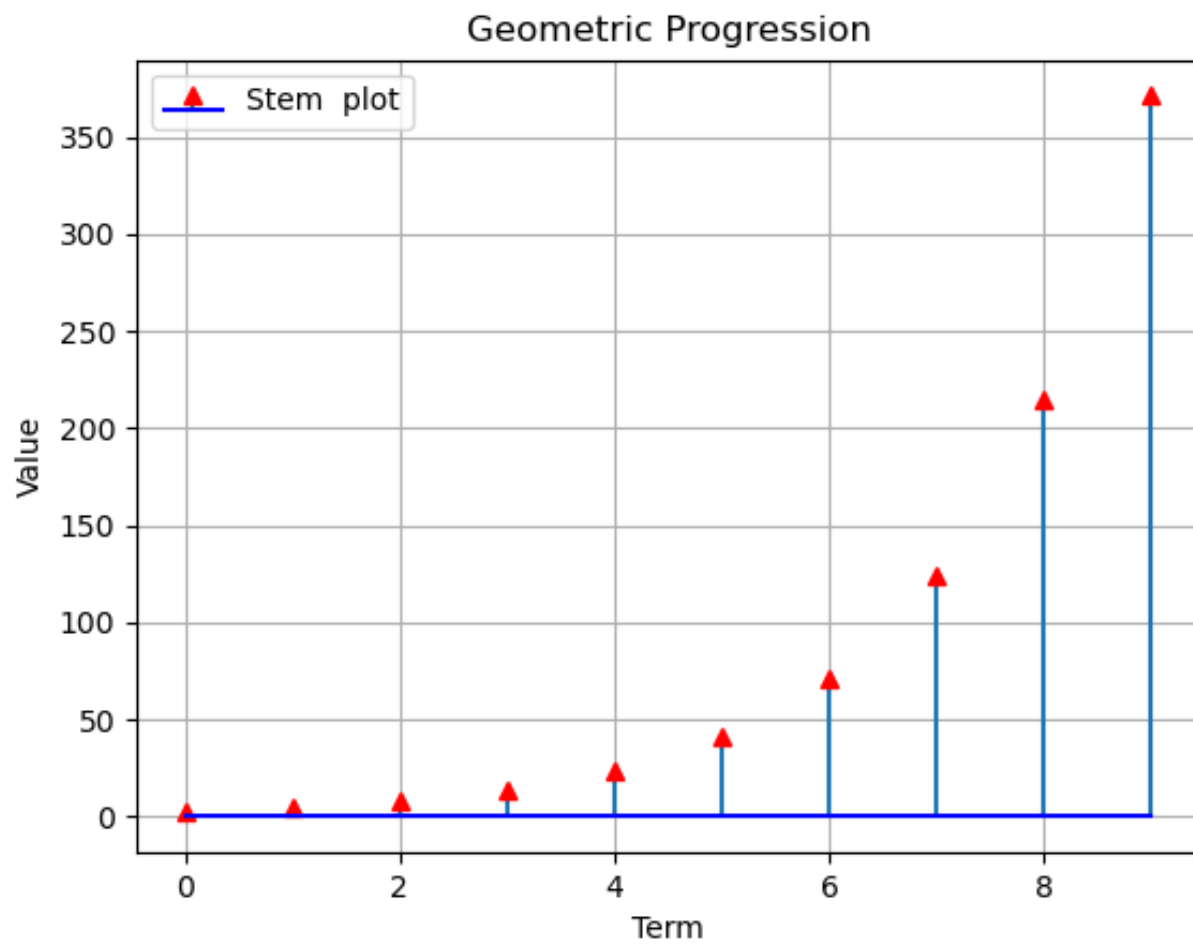


Fig. 0. values of geometric progression

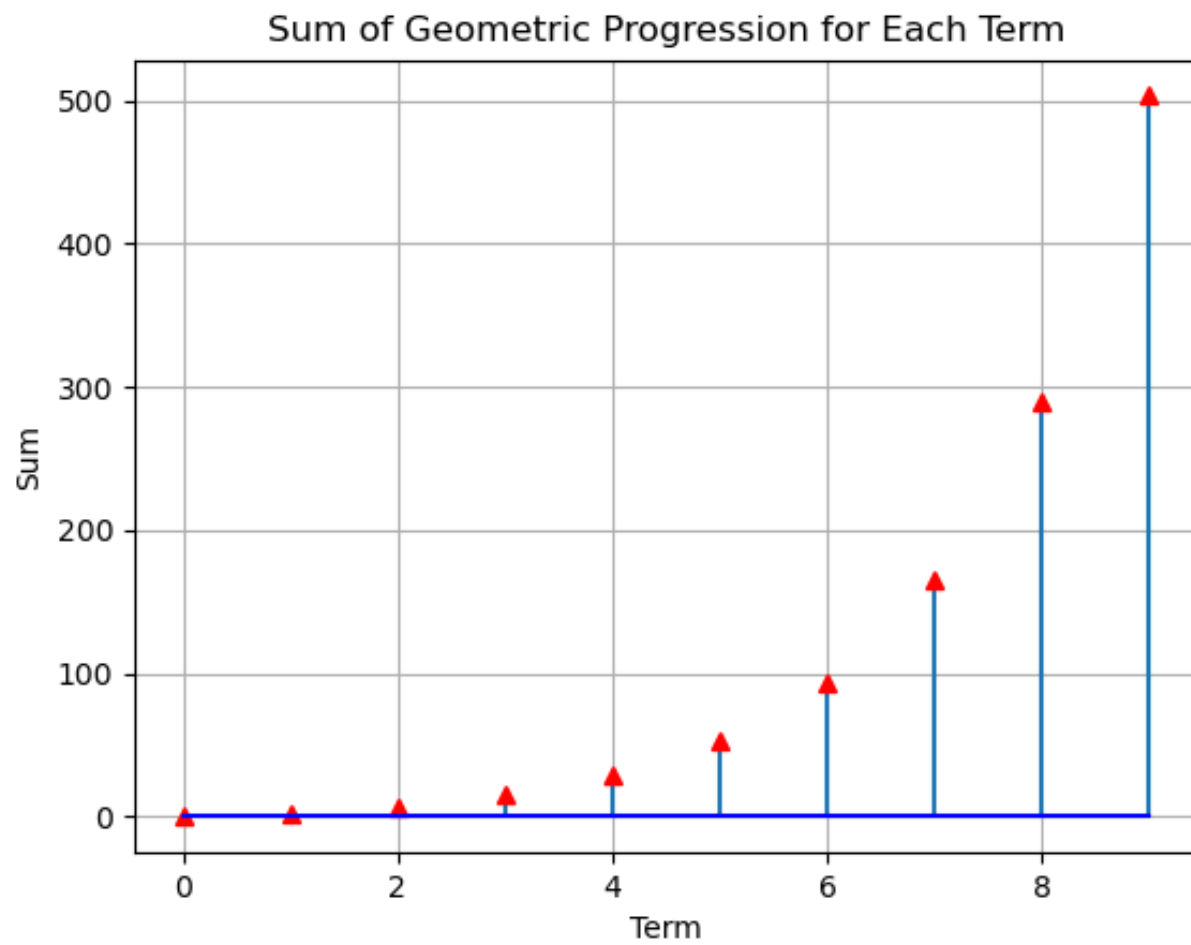


Fig. 0. **sum of the geometric progression after adding each term**