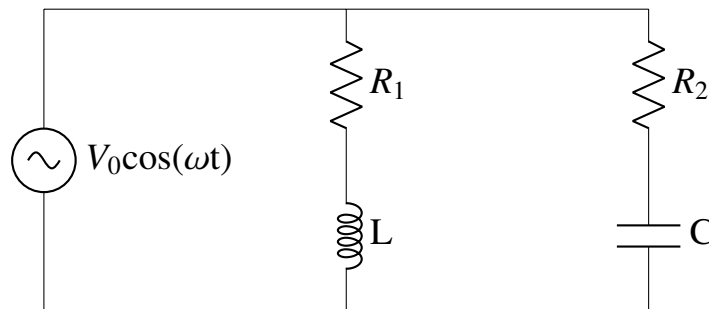


GATE 2023 Assignment

EE1205 Signals and Systems

Kurre Vinay
EE23BTECH11036

Question: In the circuit shown, $\omega = 100\pi \text{ rad/s}$, $R_1 = R_2 = 2.2\Omega$ and $L = 7 \text{ mH}$. the capacitance C for which Y_{in} is purely real is _____ mF



(GATE IN 2023)

Solution:

From *Table I*

variable	value	description	formulae
Y_{in}	??	Admittance of circuit	$\frac{R_1 - Ls}{R_1^2 - (Ls)^2} + \frac{R_2 - \frac{1}{sC}}{R_2^2 - (\frac{1}{sC})^2}$
X_L	$7s\Omega$	Inductive reactance	sL
X_C	$\frac{1}{sC}\Omega$	Capacitive reactance	$\frac{1}{sC}$
s	$100\pi j$	Laplace complex frequency	$j\omega$
ω	$100\pi \text{ rad/s}$	Angular frequency	-
V	$V_0 \cos(\omega t)$	voltage of source	-
R_1, R_2	2.2Ω	resistance of resistors	-

TABLE I

TABLE: INPUT PARAMETERS

$$Y_{in} = \frac{R_1 - Ls}{R_1^2 - (Ls)^2} + \frac{R_2 - \frac{1}{sC}}{R_2^2 - \left(\frac{1}{sC}\right)^2} \quad (1)$$

$$\text{Im}(Y_{in}) = \frac{-Ls}{R_1^2 - (Ls)^2} + \frac{-\frac{1}{sC}}{R_2^2 - \left(\frac{1}{sC}\right)^2} \quad (2)$$

According to the question given, Y_{in} is purely real, so imaginary part should be equal to zero

$$\frac{-L\omega}{R_1^2 + (L\omega)^2} + \frac{\frac{1}{\omega C}}{R_2^2 + \left(\frac{1}{C\omega}\right)^2} = 0 \quad (3)$$

$$\text{Take the values from Table I} \quad (4)$$

$$\frac{-7(100\pi)}{(2.2)^2 + (7(100\pi))^2} + \frac{\frac{1}{(100\pi)C}}{(2.2)^2 + \left(\frac{1}{C(100\pi)}\right)^2} = 0 \quad (5)$$

$$\frac{-1}{4.4} + \frac{\frac{1}{(100\pi)C}}{(2.2)^2 + \left(\frac{1}{(100\pi)C}\right)^2} = 0 \quad (6)$$

$$\frac{\frac{1}{(100\pi)C}}{(2.2)^2 + \left(\frac{1}{(100\pi)C}\right)^2} = \frac{1}{4.4} \quad (7)$$

$$(2.2)^2 - \frac{4.4}{(100\pi)C} + \left(\frac{1}{(100\pi)C}\right)^2 = 0 \quad (8)$$

$$\left(2.2 - \frac{1}{(100\pi)C}\right)^2 = 0 \quad (9)$$

$$\frac{1}{(100\pi)C} = 2.2 \quad (10)$$

$$C = \frac{700}{484} \text{mF} \quad (11)$$

$$C = 1.446281 \text{mF} \quad (12)$$

The capacitance of capacitor C is 1.45mF

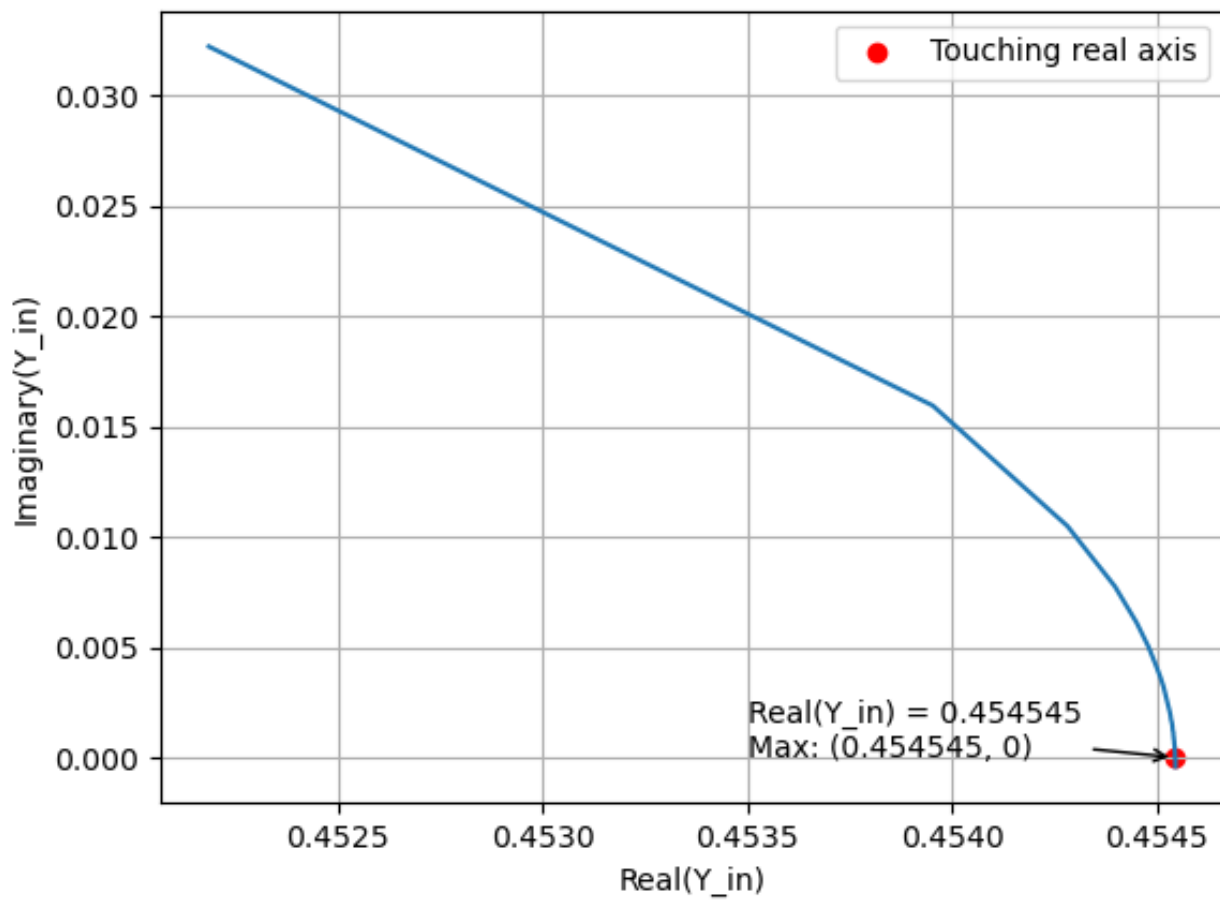


Fig. 1. the graph of admittance(Y_{in}) amplitude