GATE 2023 Assignment
EE1205 Signals and Systems

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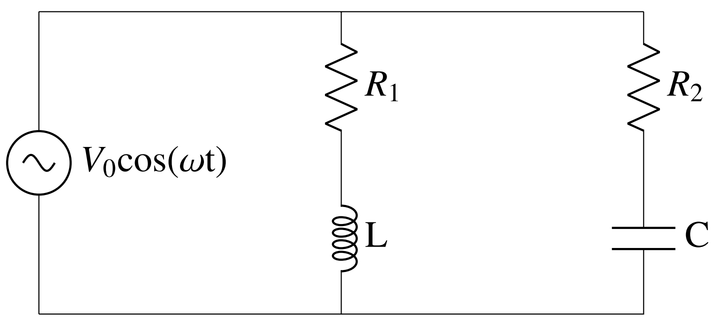
Question: In the circuit shown ,ω = 100πrads/s, R1=R2=2.2Ω and L=7mH. the capacitance C for which Y_{in} is purely real is ____ mF



(GATE IN 2023)

Solution:

variable	value	description	formulae
Y_{in}	??	Admittance of circuit	$\frac{R_1-Ls}{R_1^2-(Ls)^2} + \frac{R_2-\frac{1}{sC}}{R_2^2-(\frac{1}{sC})^2}$
X_L	$7s\Omega$	Inductive reactance	sL
X_C	$\frac{1}{sC}\Omega$	Capacitive reactance	$\frac{1}{sC}$
s	100πj	Laplace complex frequency	$j\omega$
ω	100πrads/s	Angular frequency	-
V	V_0cos(ωt)	voltage of source	-
R_1, R_2	2.2Ω	resistance of resistors	-

TABLE I
Table: Input Parameters

According to the question given, Y_{in} is purely real , so imaginary part should be equal to zero
as shown in table

$$Y_{in}(\text{imaginary part}) = \frac{-Ls}{R_1^2 - (Ls)^2} + \frac{-\frac{1}{\mathrm{sC}}}{R_2^2 - \left(\frac{1}{\mathrm{sC}}\right)^2} \tag{1}$$

$$\frac{-L\omega}{R_1^2 + (L\omega)^2} + \frac{\frac{1}{\omega \mathrm{C}}}{R_2^2 + \left(\frac{1}{\mathrm{C}\omega}\right)^2} = 0 \tag{2}$$

$$\frac{-7(100\pi)}{(2.2)^2 + (7(100\pi))^2} + \frac{\frac{1}{(100\pi)\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{\mathrm{C}(100\pi)}\right)^2} = 0 \tag{3}$$

$$\frac{-1}{4.4} + \frac{\frac{1}{(100\pi)\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{(100\pi)\mathrm{C}}\right)^2} = 0 \tag{4}$$

$$\frac{\frac{1}{(100\pi)\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{(100\pi)\mathrm{C}}\right)^2} = \frac{1}{4.4} \tag{5}$$

$$(2.2)^2 - \frac{4.4}{(100\pi)\mathrm{C}} + \left(\frac{1}{(100\pi)\mathrm{C}}\right)^2 = 0 \tag{6}$$

$$\left(2.2 - \frac{1}{(100\pi)\mathrm{C}}\right)^2 = 0 \tag{7}$$

$$\frac{1}{(100\pi)\mathrm{C}} = 2.2 \tag{8}$$

$$\mathrm{C} = \frac{700}{484}\mathrm{mF} \tag{9}$$

$$\mathrm{C} = 1.446281\mathrm{mF} \tag{10}$$

The capacitance of capacitor C is 1.45mF

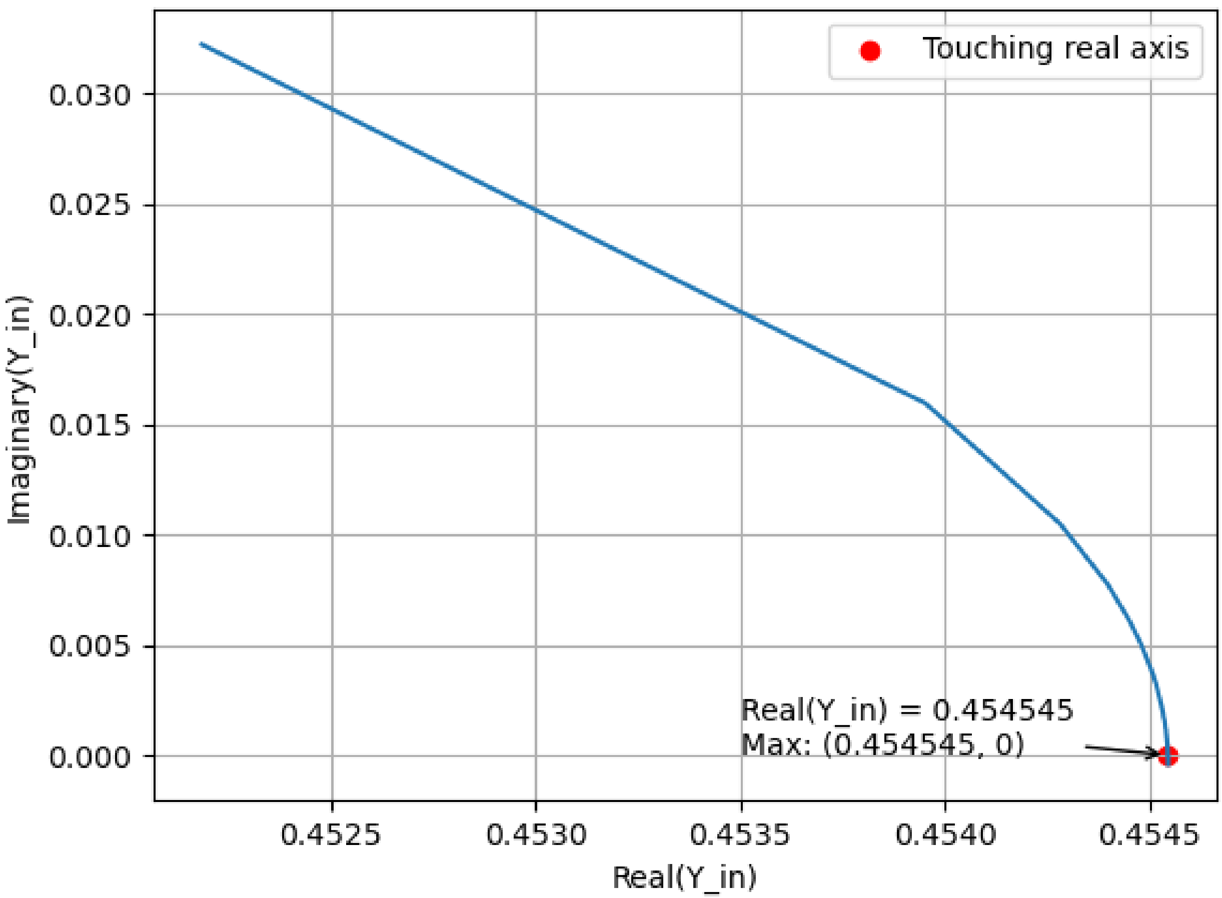


Fig. 1. the graph opf admittance(Y_{in}) amplitude