

# GATE 2022 Assignment

## EE1205 Signals and Systems

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### Question:

Let  $x_1(t) = e^{-t}u(t)$  and  $x_2(t) = u(t) - u(t-2)$ , where  $u(\cdot)$  denotes the unit step function. If  $y(t)$  denotes the convolution of  $x_1(t)$  and  $x_2(t)$ , then  $\lim_{t \rightarrow \infty} y(t) = \underline{\hspace{2cm}}$ . (Rounded off to one decimal place)  
(GATE EC 2022 )

### Solution:

variable	value	description
$x_1(t)$	$e^{-t}u(t)$	given function 1
$x_2(t)$	$u(t) - u(t-2)$	given function 2
$y(t)$	-	convolution of $x_1(t)$ and $x_2(t)$

TABLE I

TABLE: INPUT PARAMETERS

$$y(t) = x_1(t) * x_2(t) \quad (1)$$

from Table I

$$y(t) = e^{-t}u(t) * (u(t) - u(t-2)) \quad (2)$$

By applying Laplace transform

$$Y(s) = X_1(s) \cdot X_2(s) \quad (3)$$

$$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{1+s}, \quad \text{Re}(s) > -1 \quad (4)$$

$$u(t) - u(t-2) \xleftrightarrow{\mathcal{L}} \frac{1 - e^{-2s}}{s}, \quad \text{Re}(s) > 0 \quad (5)$$

$$Y(s) = \left( \frac{1}{1+s} \right) \left( \frac{1 - e^{-2s}}{s} \right), \quad \text{Re}(s) > 0 \quad (6)$$

$$= \frac{1 - e^{-2s}}{s(s+1)} \quad (7)$$

Final value theorem

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s) \quad (8)$$

$$(9)$$

Proof:

$$\mathcal{L}x(t) = X(s) = \int_0^{\infty} x(t) e^{-st} dt \quad (10)$$

$$\mathcal{L} \frac{dx(t)}{dt} = \int_0^{\infty} \frac{d}{dt} (x(t) e^{-st}) dt \quad (11)$$

$$= sX(s) - x(0^-) \quad (12)$$

$$\lim_{s \rightarrow 0} \left( \int_0^{\infty} \frac{d}{dt} (x(t) e^{-st}) dt \right) = \lim_{s \rightarrow 0} (sX(s) - x(0^-)) \quad (13)$$

$$\Rightarrow \int_0^{\infty} \frac{dx(t)}{dt} dt = \lim_{s \rightarrow 0} (sX(s) - x(0^-)) \quad (14)$$

$$\Rightarrow [x(t)]_0^{\infty} = \lim_{s \rightarrow 0} (sX(s) - x(0^-)) \quad (15)$$

$$\Rightarrow x(\infty) - x(0^-) = \lim_{s \rightarrow 0} (sX(s) - x(0^-)) \quad (16)$$

$$\Rightarrow x(\infty) = \lim_{s \rightarrow 0} sX(s) \quad (17)$$

$$\lim_{t \rightarrow \infty} x(t) = x(\infty) = \lim_{s \rightarrow 0} sX(s) \quad (18)$$

By applying Final value theorem

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} sY(s) \quad (19)$$

$$= \lim_{s \rightarrow 0} s \left( \frac{1 - e^{-2s}}{s(s+1)} \right) \quad (20)$$

$$= \lim_{s \rightarrow 0} \left( \frac{1 - e^{-2s}}{(s+1)} \right) \quad (21)$$

$$= \left( \frac{1 - e^0}{0+1} \right) \quad (22)$$

$$\lim_{t \rightarrow \infty} y(t) = 0 \quad (23)$$