

GATE 2021 Assignment

EE1205 Signals and Systems

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Question: The exponential Fourier series representation of a continuous-time periodic signal $x(t)$ is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where ω_0 is the fundamental angular frequency of $x(t)$ and the coefficients of the series are a_k . The following information is given about $x(t)$ and a_k

I. $x(t)$ is real and even, having a fundamental period of 6

II. The average value of $x(t)$ is 2.

$$\text{III. } a_k = \begin{cases} k, & 1 \leq k \leq 3 \\ 0, & k > 3 \end{cases}$$

The average power of the signal $x(t)$ (rounded off to one decimal place) is ____.

Solution:

variable	value	description
T_0	6	Fundamental time period
$y(t)$	-	average power of the signal
$x(t)$	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	Input signal
a_k	$\begin{cases} k, & 1 \leq k \leq 3 \\ 0, & k > 3 \end{cases}$	coefficients of the series
a_0	2	average of $x(t)$

TABLE I

TABLE: INPUT PARAMETERS

$x(t)$ is even and real so, $a_k = a_{-k}$

Parswal's Power Theorem

Proof

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt, \quad |x(t)|^2 = x(t) x^*(t) \quad (1)$$

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) x^*(t) dt \quad (2)$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad (3)$$

$$P = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} \right) x^*(t) dt \quad (4)$$

$$P = \sum_{n=-\infty}^{\infty} C_n \left(\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^*(t) e^{jn\omega_0 t} dt \right) \quad (5)$$

$$= \sum_{n=-\infty}^{\infty} C_n C_n^* \quad (6)$$

$$\Rightarrow P = \sum_{n=-\infty}^{\infty} |C_n|^2 \quad (7)$$

By using Parsval's Power Theorem

$$\frac{1}{T} \int_0^T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2 \quad (8)$$

$$y(t) = \sum_{k=-\infty}^{\infty} |a_k|^2 \quad (9)$$

$$= 2 \sum_{k=0}^{\infty} |a_k|^2 \quad (10)$$

$$= 2 \sum_{k=0}^3 |a_k|^2 \quad (11)$$

$$= 2(1^2 + 2^2 + 3^2) + 2^2 \quad (12)$$

$$= 32 \quad (13)$$

$$(14)$$

The average power of the signal $x(t)$ (rounded off to one decimal place) is 32