GATE 2023 Assignment
EE1205 Signals and Systems

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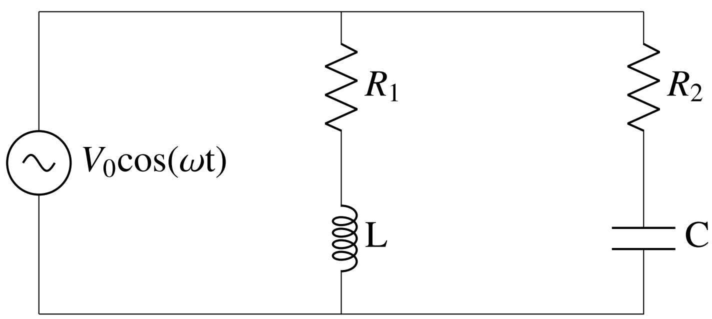
Question: In the circuit shown ,$\omega = 100\pi$rads/s, R_1=R_2=2.2Ω and L=7mH. the capacitance C for which Y_{in} is purely real is ____ mF



(GATE IN 2023)

Solution:

From *Table I*

variable	value	description	formulae
Y_{in}	-	Admittance of circuit	$\frac{R_1-Ls}{R_1^2-(Ls)^2}+\frac{R_2-\frac{1}{sC}}{R_2^2-\left(\frac{1}{sC}\right)^2}$
X_L	$7s\Omega$	Inductive reactance	sL
X_C	$\frac{1}{sC}\Omega$	Capacitive reactance	$\frac{1}{sC}$
s	100πj	Laplace complex frequency	$j\omega$
ω	100πrads/s	Angular frequency	-
V	$V_0\cos(\omega t)$	voltage of source	-
R_1, R_2	2.2Ω	resistance of resistors	-

TABLE I
TABLE: INPUT PARAMETERS

$$Y_{in} = \frac{R_1 - Ls}{R_1^2 - (Ls)^2} + \frac{R_2 - \frac{1}{sC}}{R_2^2 - \left(\frac{1}{sC}\right)^2} \tag{1}$$

$$Im\left(Y_{in}\right) = \frac{-Ls}{R_1^2 - (Ls)^2} + \frac{-\frac{1}{sC}}{R_2^2 - \left(\frac{1}{sC}\right)^2} \tag{2}$$

According to the question given, Y_{in} is purely real , so imaginary part should be equal to zero

Take the values from *Table I*

$$\frac{-1}{4.4} + \frac{\frac{1}{(100\pi)\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{(100\pi)\mathrm{C}}\right)^2} = 0 \tag{3}$$

$$\frac{\frac{1}{(100\pi)\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{(100\pi)\mathrm{C}}\right)^2} = \frac{1}{4.4} \tag{4}$$

$$(2.2)^2 - \frac{4.4}{(100\pi)\mathrm{C}} + \left(\frac{1}{(100\pi)\mathrm{C}}\right)^2 = 0 \tag{5}$$

$$\left(2.2 - \frac{1}{(100\pi)\mathrm{C}}\right)^2 = 0 \tag{6}$$

$$\frac{1}{(100\pi)\mathrm{C}} = 2.2 \tag{7}$$

$$\mathrm{C} = \frac{700}{484}\mathrm{mF} \tag{8}$$

$$\mathrm{C} = 1.446281\mathrm{mF} \tag{9}$$

The capacitance of capacitor C is 1.45mF

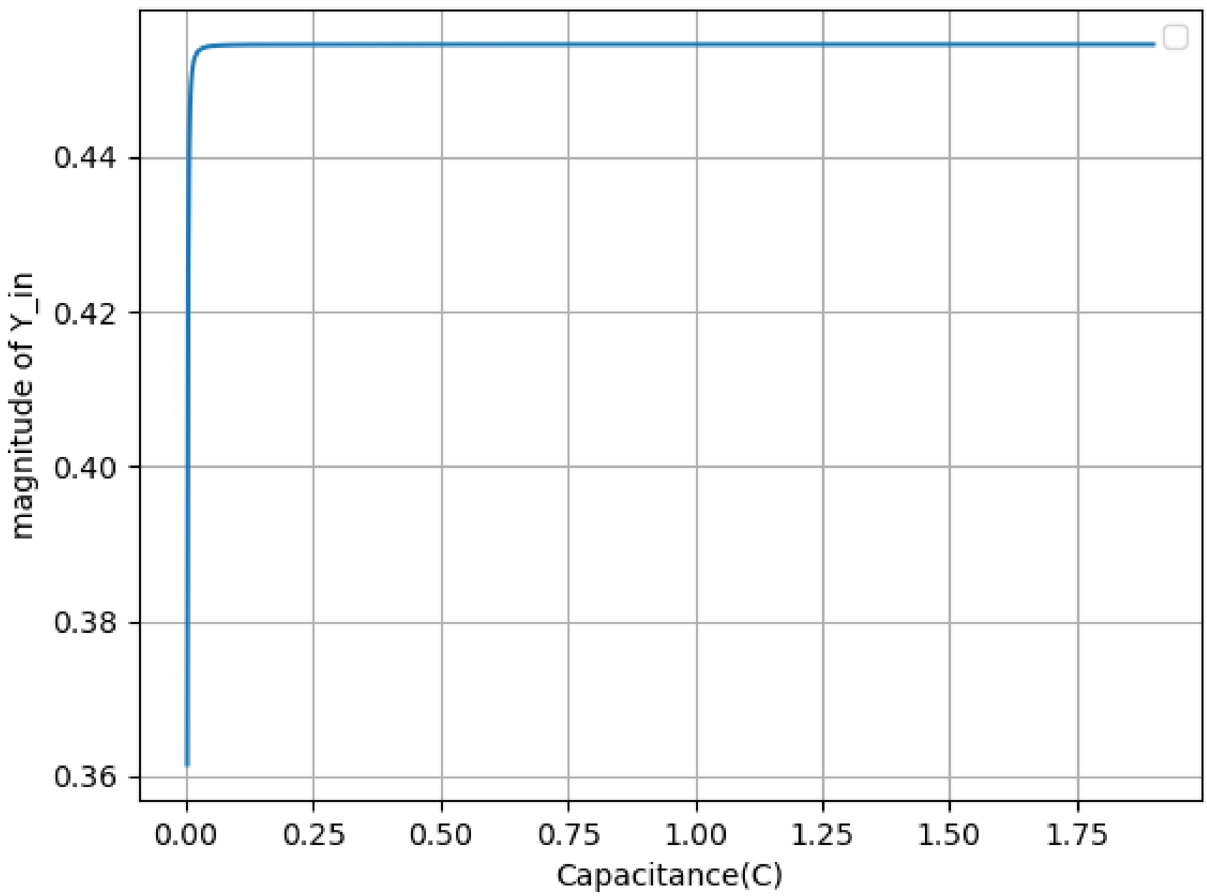


Fig. 1. the polt of capacitance vs magnitude of Y_{in}