GATE 2022 Assignment
EE1205 Signals and Systems

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Question: The transfer function of a system is:

$$\frac{(s+1)(s+3)}{(s+5)(s+7)(s+9)}$$

In the state-space representation of the system, the minimum number of state variables (in integer) necessary is____.
(GATE IN 2022)
Solution:

variable	value	description
U(s)	-	input function of the system
Y(s)	-	output function of the system
H(s)	$\frac{(s+1)(s+3)}{(s+5)(s+7)(s+9)}$	transfer function of the system.
I	-	identity matrix
$\dot{\mathbf{x}}(t)$	$A\mathbf{x}(t)+Bu(t)$	derivative of State function of $\mathbf{x}(t)$

TABLE I
TABLE: INPUT PARAMETERS

From Table I

$$H(s)=\frac{(s+1)(s+3)}{(s+5)(s+7)(s+9)} \tag{1}$$

$$H(s)=\frac{P}{s+5}+\frac{Q}{s+7}+\frac{R}{s+9} \tag{2}$$

$$(s+1)(s+3)=P(s+7)(s+9)+Q(s+5)(s+9)+R(s+5)(s+7) \tag{3}$$

By solving equation (3) , we get

P = 1
Q = -6
R = 6

$$\implies H(s)=\frac{1}{s+5}-\frac{6}{s+7}+\frac{6}{s+9} \tag{4}$$

(5)

The state-space representation of the system is given by:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bu(t) \tag{6}$$

$$\mathbf{y}(t) = C\mathbf{x}(t) + Du(t) \tag{7}$$

$$H(s) = \frac{Y(s)}{U(s)} = C\left(sI - A\right)^{-1} B + D \tag{8}$$

Comparing the coefficients:

$$A = \text{coefficient of } s \text{ in } (sI - A)^{-1} \tag{9}$$

$$B = \text{coefficient of } U(s) \tag{10}$$

$$C = \text{coefficient of } Y(s) \tag{11}$$

$$D = \text{constant term} \tag{12}$$

The denominator $(s+5)(s+7)(s+9)$ suggests that the system has three poles. Thus, we'll have a third-order state-space model, and A will be a 3×3matrix.

$$(s+5)(s+7)(s+9) = s^3 + 21s^2 + 143s + 315 \tag{13}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -21 & -143 & -315 \end{bmatrix} \tag{14}$$

$$\tag{15}$$

A is a 3×3 matrix, then the characteristic polynomial will have a degree equal to the size of A, which is 3.

Therefore, the system order, and hence the minimum number of state variables, will be 3.