GATE 2023 Assignment
EE1205 Signals and Systems

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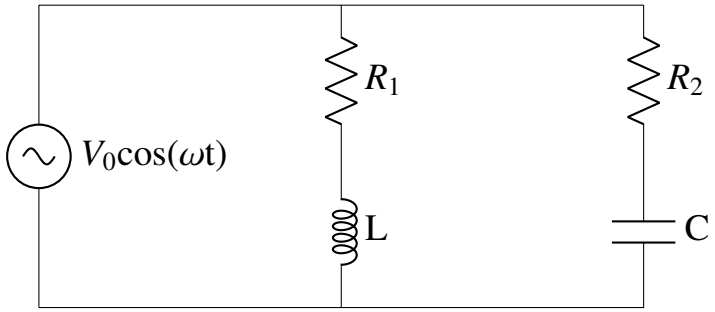
Question: In the circuit shown ,ω = 100πrads/s, R1=R2=2.2Ω and L=7mH. the capacitance C for which Y_{in} is purely real is mF



(GATE IN 2023 Q46)

Solution:

variable	value	description
Y_{in}		Admittance of circuit
X_L	$7s\Omega$	Inductive reactance
X_C	$\frac{1}{sC}\Omega$	Capacitive reactance
ω	100πrads/s	Angular frequency
V	$V_0\cos(\omega t)$	voltage of source
R_1, R_2	2.2Ω	resistance of resistors

TABLE I
TABLE: INPUT PARAMETERS

$$X_L = \mathrm{sL} \tag{1}$$

$$X_C = \frac{1}{s\mathrm{C}}\Omega \tag{2}$$

$$Y_{in} = \frac{1}{R_1 + Ls} + \frac{1}{R_2 + \frac{1}{\mathrm{sC}}} \tag{3}$$

$$Y_{in} = \frac{R_1 - Ls}{R_1^2 - (Ls)^2} + \frac{R_2 - \frac{1}{\mathrm{sC}}}{R_2^2 - \left(\frac{1}{\mathrm{sC}}\right)^2} \tag{4}$$

$$s = j\omega \tag{5}$$

$$\implies Y_{in} = \frac{R_1 - Lj\omega}{R_1^2 - (Lj\omega)^2} + \frac{R_2 - \frac{1}{j\omega\mathrm{C}}}{R_2^2 - \left(\frac{1}{j\mathrm{C}\omega}\right)^2} \tag{6}$$

$$\implies Y_{in} = \frac{R_1 - Lj\omega}{R_1^2 + (L\omega)^2} + \frac{R_2 + \frac{j}{\omega\mathrm{C}}}{R_2^2 + \left(\frac{1}{\mathrm{C}\omega}\right)^2} \tag{7}$$

According to the question given, Y_{in} is purely real , so imaginary part should be equal to zero

$$\implies \frac{-L\omega}{R_1^2 + (L\omega)^2} + \frac{\frac{1}{\omega\mathrm{C}}}{R_2^2 + \left(\frac{1}{\mathrm{C}\omega}\right)^2} = 0 \tag{8}$$

$$\implies \frac{-7(100\pi)}{(2.2)^2 + (7(100\pi))^2} + \frac{\frac{1}{(100\pi)\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{\mathrm{C}(100\pi)}\right)^2} = 0 \tag{9}$$

$$\implies \frac{-1}{4.4} + \frac{\frac{1}{\omega\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{\omega\mathrm{C}}\right)^2} = 0 \tag{10}$$

$$\implies \frac{\frac{1}{\omega\mathrm{C}}}{(2.2)^2 + \left(\frac{1}{\omega\mathrm{C}}\right)^2} = \frac{1}{4.4} \tag{11}$$

$$\implies (2.2)^2 - \frac{4.4}{\omega\mathrm{C}} + \left(\frac{1}{\omega\mathrm{C}}\right)^2 = 0 \tag{12}$$

$$\implies \left(2.2 - \frac{1}{\omega\mathrm{C}}\right)^2 = 0 \tag{13}$$

$$\implies \frac{1}{\omega\mathrm{C}} = 2.2 \tag{14}$$

$$\implies \mathrm{C} = \frac{700}{484}\mathrm{mF} \tag{15}$$

$$\implies \mathrm{C} = 1.446281\mathrm{mF} \tag{16}$$

The capacitance of capacitor C is 1.45mF

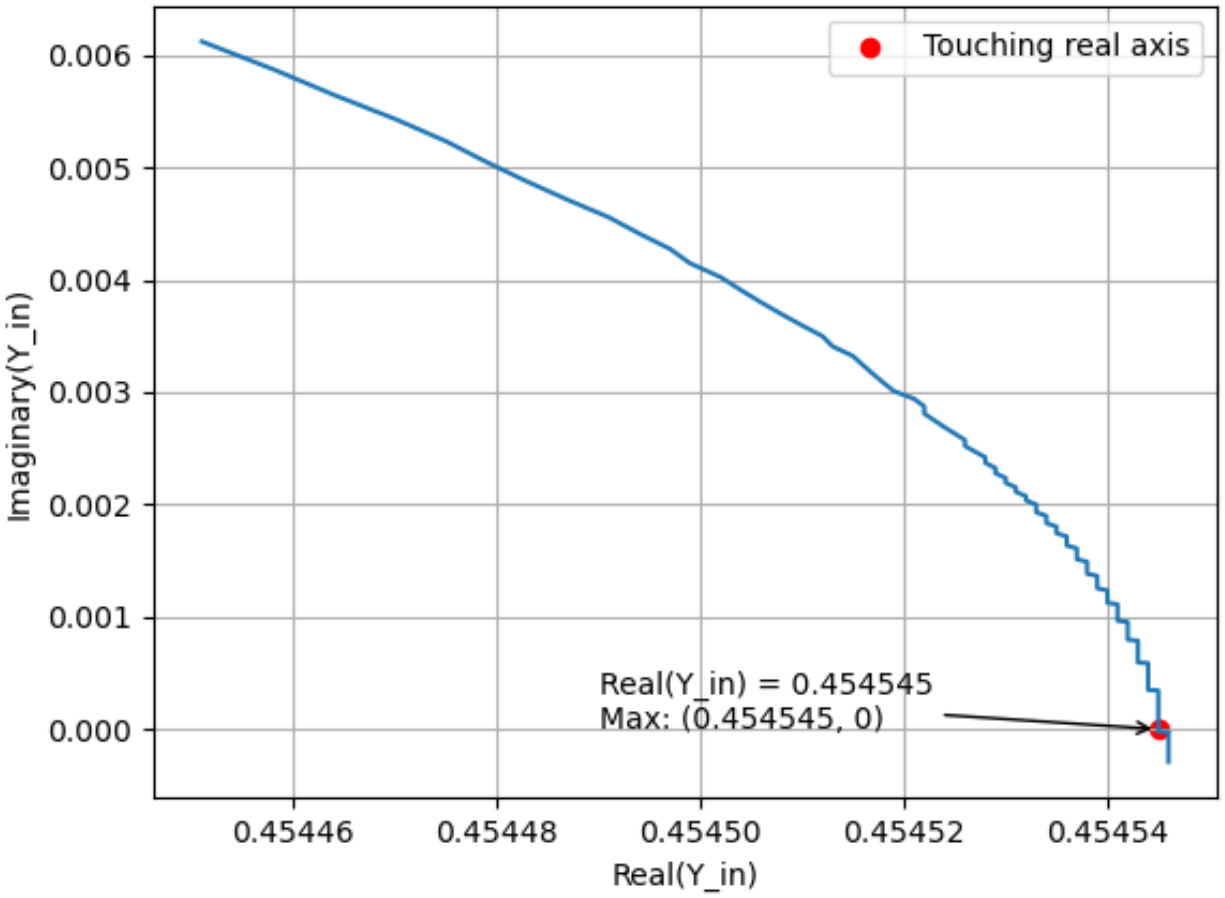


Fig. 1. the graph opf admittance(Y_{in}) amplitude