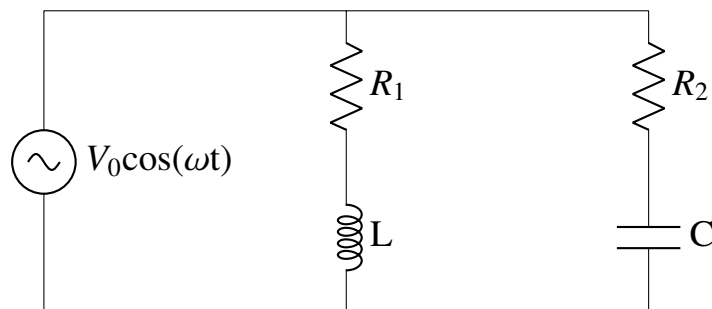


GATE 2023 Assignment

EE1205 Signals and Systems

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Question: In the circuit shown, $\omega = 100\pi \text{ rad/s}$, $R_1 = R_2 = 2.2\Omega$ and $L = 7 \text{ mH}$. the capacitance C for which Y_{in} is purely real is mF



(GATE IN 2023 Q46)

Solution:

variable	value	description
Y_{in}		Admittance of circuit
X_L	$7s\Omega$	Inductive reactance
X_C	$\frac{1}{sC}\Omega$	Capacitive reactance
ω	$100\pi \text{ rad/s}$	Angular frequency
V	$V_0 \cos(\omega t)$	voltage of source

TABLE I

TABLE: INPUT PARAMETERS

$$X_L = sL \quad (1)$$

$$= 7s\Omega \quad (2)$$

$$X_C = \frac{1}{sC}\Omega \quad (3)$$

$$Y_{in} = \frac{1}{2.2 + 7s} + \frac{1}{2.2 + \frac{1}{sC}} \quad (4)$$

$$s = j\omega \quad (5)$$

$$\Rightarrow Y_{in} = \frac{1}{2.2 + 7j\omega} + \frac{1}{2.2 + \frac{1}{j\omega C}} \quad (6)$$

$$\Rightarrow Y_{in} = \frac{1 - j}{4.4} + \frac{2.2 + \frac{j}{\omega C}}{(2.2)^2 + \left(\frac{1}{\omega C}\right)^2} \quad (7)$$

According to the question given, Y_{in} is purely real , so imaginary part should be equal to zero

$$\Rightarrow \frac{-1}{4.4} + \frac{\frac{1}{\omega C}}{(2.2)^2 + \left(\frac{1}{\omega C}\right)^2} = 0 \quad (8)$$

$$\Rightarrow \frac{\frac{1}{\omega C}}{(2.2)^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{1}{4.4} \quad (9)$$

$$\Rightarrow (2.2)^2 - \frac{4.4}{\omega C} + \left(\frac{1}{\omega C}\right)^2 = 0 \quad (10)$$

$$\Rightarrow \left(2.2 - \frac{1}{\omega C}\right)^2 = 0 \quad (11)$$

$$\Rightarrow \frac{1}{\omega C} = 2.2 \quad (12)$$

$$\Rightarrow C = \frac{700}{484} \text{mF} \quad (13)$$

$$\Rightarrow C = 1.446281 \text{mF} \quad (14)$$

The capacitance of capacitor C is 1.45mF