

GATE 2022 Assignment

EE1205 Signals and Systems

Kurre Vinay
EE23BTECH11036

Question: The transfer function of a system is:

$$\frac{(s+1)(s+3)}{(s+5)(s+7)(s+9)}$$

In the state-space representation of the system, the minimum number of state variables (in integer) necessary is ____.

(GATE IN 2022)

Solution:

variable	value	description
U(s)	-	input function of the system
Y(s)	-	output function of the system
H(s)	$\frac{(s+1)(s+3)}{(s+5)(s+7)(s+9)}$	transfer function of the system.
I	-	identity matrix

TABLE I

TABLE: INPUT PARAMETERS

From Table I

$$H(s) = \frac{(s+1)(s+3)}{(s+5)(s+7)(s+9)} \quad (1)$$

$$H(s) = \frac{P}{s+5} + \frac{Q}{s+7} + \frac{R}{s+9} \quad (2)$$

$$(s+1)(s+3) = P(s+7)(s+9) + Q(s+5)(s+9) + R(s+5)(s+7) \quad (3)$$

By solving equation (3), we get

$$P = 1$$

$$Q = -6$$

$$R = 6$$

$$\Rightarrow H(s) = \frac{1}{s+5} - \frac{6}{s+7} + \frac{6}{s+9} \quad (4)$$

$$(5)$$

The state-space representation of the system is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (6)$$

$$y(t) = Cx(t) + Du(t) \quad (7)$$

$$H(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \quad (8)$$

Comparing the coefficients:

$$A = \text{coefficient of } s \text{ in } (sI - A)^{-1} \quad (9)$$

$$B = \text{coefficient of } U(s) \quad (10)$$

$$C = \text{coefficient of } Y(s) \quad (11)$$

$$D = \text{constant term} \quad (12)$$

The denominator $(s + 5)(s + 7)(s + 9)$ suggests that the system has three poles. Thus, we'll have a third-order state-space model, and A will be a 3×3 matrix.

$$(s + 5)(s + 7)(s + 9) = s^3 + 21s^2 + 143s + 315 \quad (13)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -21 & -143 & -315 \end{bmatrix} \quad (14)$$

$$(15)$$

A is a 3×3 matrix, then the characteristic polynomial will have a degree equal to the size of A, which is 3.

Therefore, the system order, and hence the minimum number of state variables, will be 3.