Vineet

Ques 1 -)

Size $\to n$

Mean $\to \theta 1$

Variance $\to \theta 2$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}.\sigma} \cdot e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$P(x_1) = \frac{1}{\sqrt{2\pi}.\sigma} \times e^{-1/2\left(\frac{x_1-\mu}{\sigma}\right)^2}$$

$$P(x_2) = \frac{1}{\sqrt{2\pi}.\sigma} \times e^{-1/2\left(\frac{x_2-\mu}{\sigma}\right)^2}$$

$\vdots$

$$P(x_n) = \frac{1}{\sqrt{2\pi}.\sigma} \times e^{-1/2\left(\frac{x_n-\mu}{\sigma}\right)^2}$$

$$L(x_1, x_2, x_3 \ldots x_n) = \Pi(f(x))$$

$$= \left[\frac{1}{\sqrt{2\pi}.\sigma}\right]^n \times e^{-1/2\left[\left(\frac{x_1-\mu}{\sigma}\right)^2 + \left(\frac{x_2-\mu}{\sigma}\right)^2 \cdots \left(\frac{x_n-\mu}{\sigma}\right)^2\right]}$$

$$= \left[\frac{1}{\sqrt{2\pi}.\sigma}\right]^n \times e^{-1/2\left[\frac{\sum_{i=1}^{N} x_i^2 - 2\mu\left(\sum_{i=1}^{n} x_i\right) + \mu^2 \times n}{\sigma^2}\right]} \quad \text{— (1)}$$

Taking ~~natural~~ derivative w.r.t μ

$$\frac{dL}{d\mu} = \frac{-1}{2}\left(\frac{-2 \times \Sigma x_i + 2\mu \cdot n}{\sigma^2}\right) = 0$$

$$\cancel{2}\Sigma x_i = \cancel{2}\mu \cdot n \Rightarrow \boxed{\mu = \frac{\Sigma x_i}{n}} \quad \text{— (2)}$$

Now Taking derivative w.r.t σ^2 ②

$$\ln(L) = n\left[\log 1 - \log(\sqrt{2\pi}\cdot\sigma)\right] - \frac{1}{2\sigma^2}(\Sigma x_i - \mu)^2$$

$$\Rightarrow -\frac{n}{2}\cdot\log(\sigma^2\cdot 2\pi) - \frac{1}{2\sigma^2}(\Sigma x_1 - \mu)^2$$

$$\Rightarrow -\frac{n}{2}\left[\log(\sigma^2) + \log 2\pi\right] - \frac{1}{2\sigma^2}(\Sigma x_1 - \mu)^2$$

Now Taking derivative w.r.t σ^2 – ②

$$\frac{dL}{d\sigma^2} \Rightarrow -\frac{n}{2} \times \left[\frac{1}{\sigma \cancel{x}} \times 2\cancel{\sigma}\right] + \frac{1}{2\sigma^3}\left[\Sigma x_1 - \mu\right]^2 = 0$$

$$\Rightarrow \frac{n}{\sigma} = \frac{(\Sigma x_1 - \mu)^2}{2\sigma^3}$$

$$\boxed{\sigma^2 = \frac{\Sigma x_1 - \mu}{n}}$$

Ques 2

PMF of Binomial Distribution $= {}^{n}C_{x} \cdot p^{x} \cdot (1-p)^{n-x}$

$P(x_1) = {}^{n}C_{x} \cdot p^{x_1} \cdot (1-p)^{n-x_1}$

$P(x_2) = {}^{n}C_{x} \cdot p^{x_2} \cdot (1-p)^{n-x_2}$

$\vdots$

$P(x_n) = {}^{n}C_{x} \cdot p^{x_n} \cdot (1-p)^{n-x_n}$

Joint Prob^n

$L(x_1, x_2, \ldots x_N) = ({}^{n}C_{x})^{n} \cdot p^{\Sigma x_i} \cdot (1-p)^{n^2 - \Sigma x_i}$

taking log on both sides.

$\ln(L) = n \cdot \ln[{}^{n}C_{x}] + \Sigma x_i \ln p + (n^2 - \Sigma x_i) \cdot \ln(1-p)$

$$\frac{dL}{dp} = \frac{\Sigma x_i}{p} - \frac{(n^2 - \Sigma x_i)}{(1-p)} = 0$$

$$\frac{\Sigma x_i (1-p) - (n^2 - \Sigma x_i) \cdot p}{p(1-p)} = 0$$

$$\Sigma x_i - \cancel{\Sigma x_i \times p} - n^2 p + \cancel{\Sigma x_i \cdot p} = 0$$

$$\boxed{p = \frac{\Sigma x_i}{n^2}}$$