R. Sandhiya

CA

## UNIT-A

## COMBINATORICS

## CHAPTER 1: PRINCIPLE OF MATHEMATICAL INDUCTION

### PRINCIPLE OF MATHEMATICAL INDUCTION :- (2m).

* Let $P(n)$ be a statement or proposition involving for all positive integers $n$.

step 1: Basis step : We verify that $P(1)$ is true

step 2: Inductive step : We show that for all positive integers $k$, if $P(k)$ is true, then $P(k+1)$ is true.

### PROBLEMS (8 OR 10 M)

1. Prove by the principle of mathematical induction, for 'n' a positive integer, $1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

[OR]

Using mathematical induction, prove that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Solution :-

* Let $P(n) : 1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}, \rightarrow (1)$

where $n$ is a positive integer

* Step 1: Basis step:

To verify that $P(1)$ is true:

when $n=1$,

$P(1) : 1^2 = \frac{1(1+1)(2(1)+1)}{6}$ (by (1))

$\Rightarrow 1 = \frac{6}{6}$

$\Rightarrow \boxed{1 = 1}$

R. Sandhiya.

CA

UNIT-A

COMBINATORICS

# CHAPTER 1: PRINCIPLE OF MATHEMATICAL INDUCTION

## PRINCIPLE OF MATHEMATICAL INDUCTION: - (2m)

* Let $P(n)$ be a statement or proposition involving for all positive integers $n$.

step 1: Basis step : We verify that $P(1)$ is true

step 2: Inductive step : We show that for all positive integers $k$, if $P(k)$ is true, then $P(k+1)$ is true.

## PROBLEMS (8 OR 10 M)

1. Prove by the principle of mathematical induction, for 'n' a positive integer, $1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$

solution: -

[OR]

Using mathematical induction, prove that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Solution:-

* Let $P(n) : 1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$, $\rightarrow$ (1)

where $n$ is a positive integer

* Step 1: Basis step:

To verify that $P(1)$ is true:

when $n=1$,

$P(1) : 1^2 = \frac{1(1+1)(2(1)+1)}{6}$ (by (1))

$\Rightarrow 1 = \frac{6}{6}$

$\rightarrow \boxed{1=1}$

$\therefore$ P(1) is true.

Step 2 : Inductive step :

Assume that $P(K) : 1^2+2^2+ \dots K^2 = \frac{K(K+1)(2K+1)}{6}$, is true $\to$ ②

To prove :- P(K+1) is true

Now, adding $(K+1)^2$ on both sides of equation ②, we have,

$$1^2+2^2+\dots K^2+(K+1)^2 = \frac{K(K+1)(2K+1)}{6}+(K+1)^2$$

$$= \frac{K(K+1)(2K+1)+6(K+1)^2}{6}$$

$$= (K+1)\left[\frac{K(2K+1)+6(K+1)}{6}\right]$$

$$= (K+1)\left[\frac{2K^2+\boxed{K+6K}+6}{6}\right]$$

$$= (K+1)\left[\frac{2K^2+\boxed{4K+3K}+6}{6}\right]$$

$$= (K+1)\left[\frac{2K(K+2)+3(K+2)}{6}\right]$$

$$= \frac{(K+1)(K+2)[(2K+3)]}{6}$$

$$1^2+2^2+\dots+K^2+(K+1)^2 = \frac{(K+1)((K+1)+1)(2(K+1)+1)}{6}$$

Therefore P(K+1) is true.

By the principle of mathematical induction,

$1^2+2^2+\dots n^2 = \frac{n(n+1)(2n+1)}{6}$ is true for any position integer n.