

Cosmological Gravitational Waves

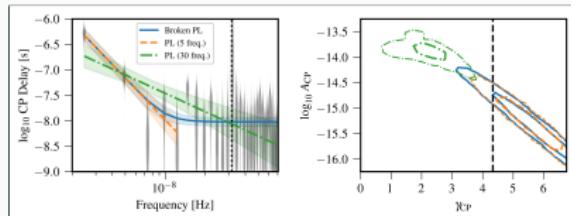
VIPER PTA Summer School | July 21, 2022

Kai Schmitz, assistant professor
University of Münster, Germany

First glimpse of a SGWB signal?

Strong evidence for a new stochastic *red* process in latest pulsar timing data

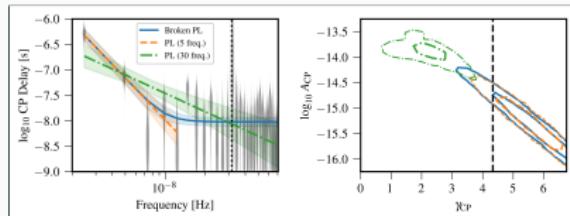
NANOGrav [2009.04496]



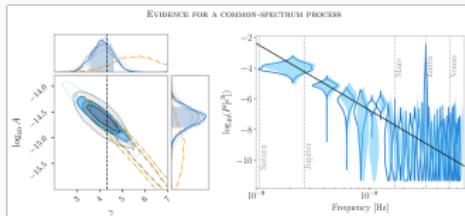
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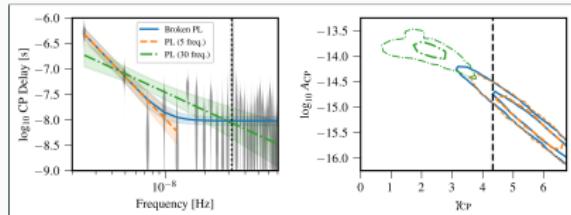
PPTA [2107.12112]



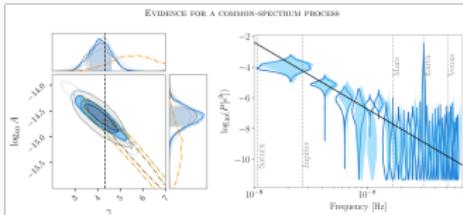
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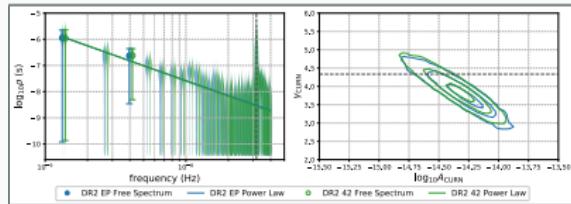
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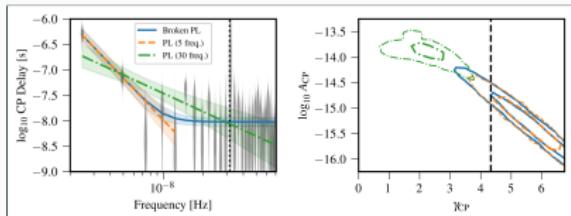
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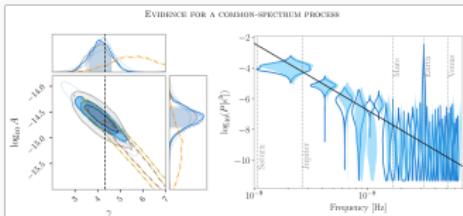
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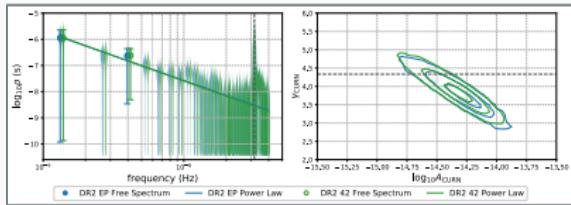
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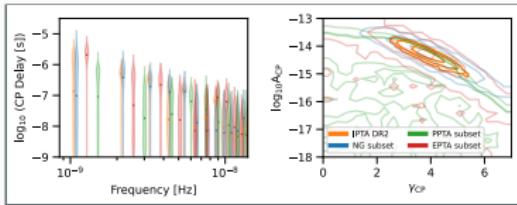
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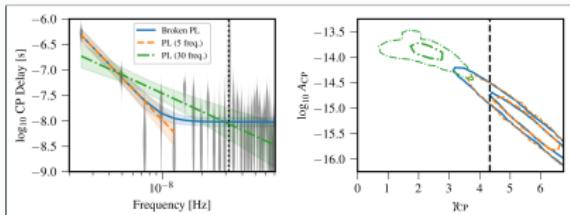
IPTA [2201.03980]



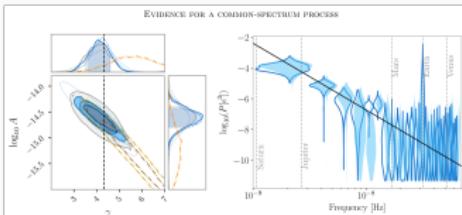
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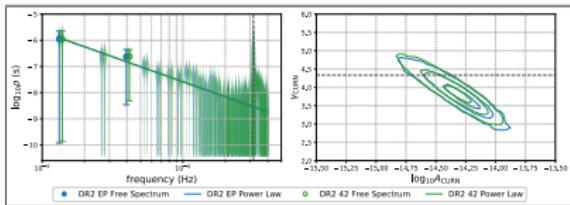
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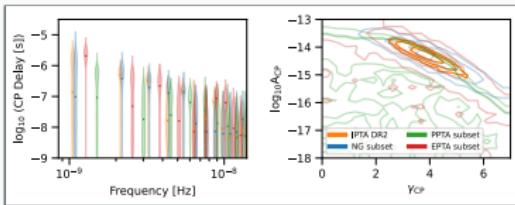
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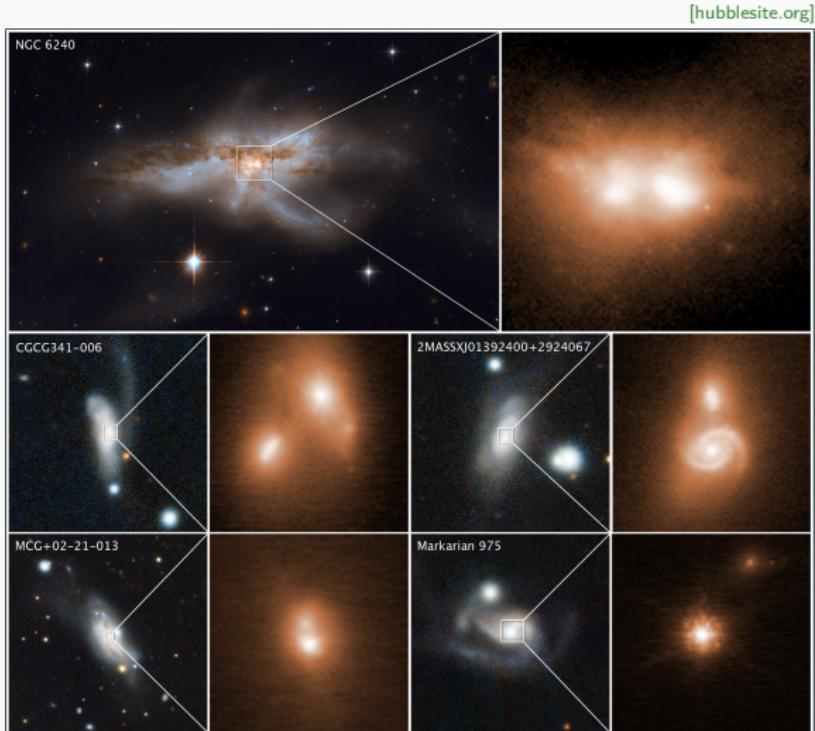


IPTA [2201.03980]



Not yet a detection, but consistent with interpretation in terms of GWs!

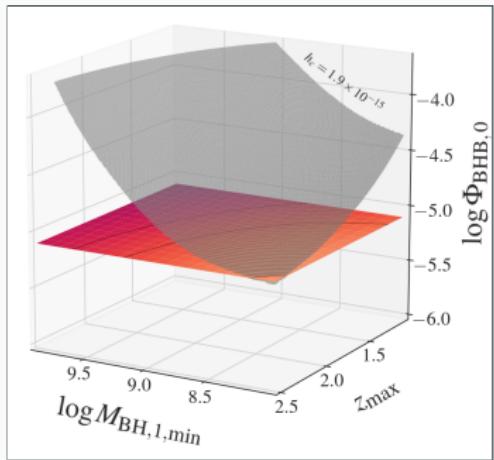
Astrophysical interpretation



Mergers of supermassive black-hole binaries (SMBHBs) after galaxy mergers

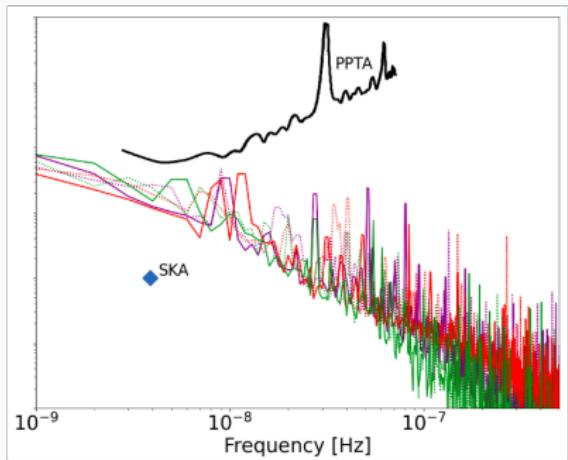
SMBHB models and simulations

SMBHB population model based on empirical quasar population



[Casey-Clyde et al.: 2107.11390]

Simulation based on SHARK galaxy evolution model (delay: 1 Gyr)

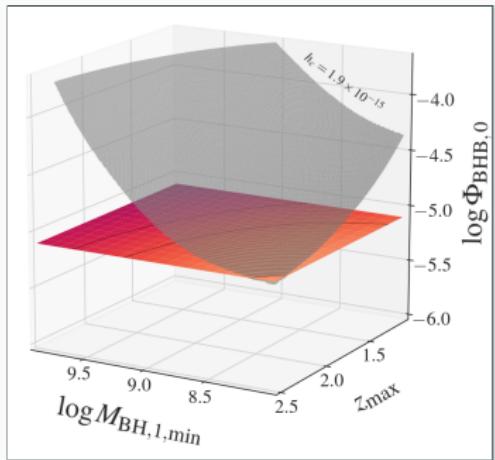


[Curylo, Bulik: 2108.11232]

- **Left:** $\Phi_{\text{BHB},0}$ is $\mathcal{O}(10)$ times larger than previous estimates, e.g., 1708.03491
- **Right:** Time delays of around 100 Myr to 1 Gyr not enough to explain amplitude

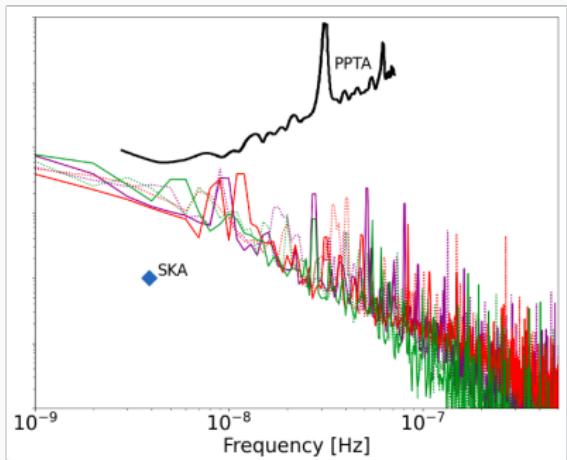
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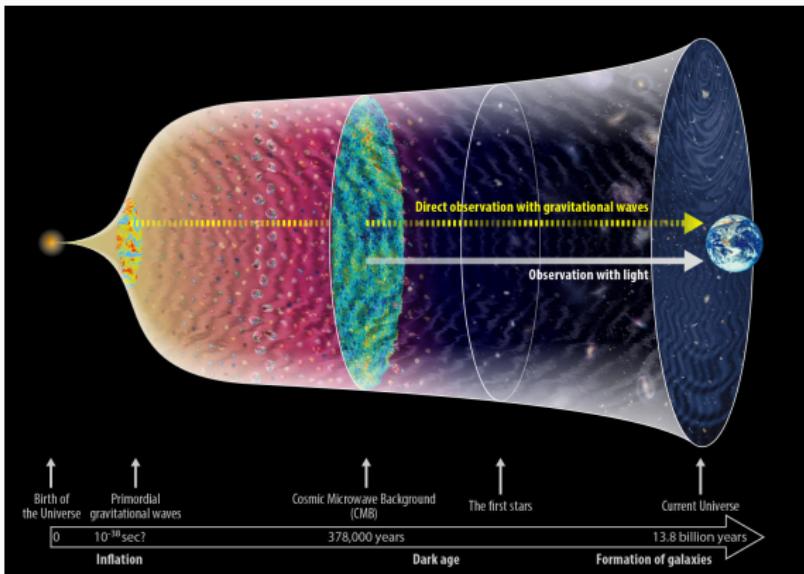
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This might tell us something about our models and simulations ... or about the signal

Cosmological gravitational waves

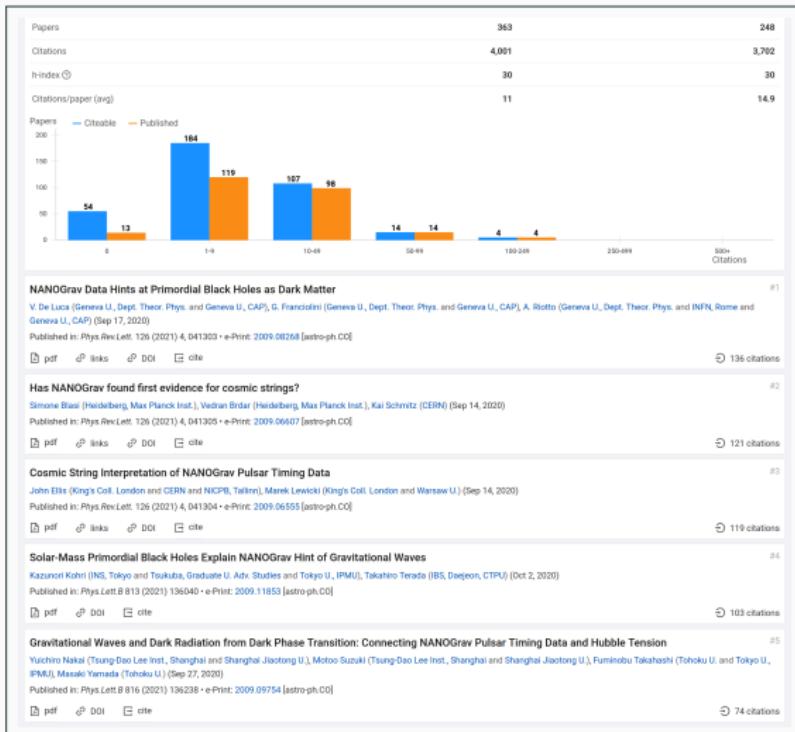
[National Astronomical Observatory of Japan, gwpo.nao.ac.jp]



Viable possibility: Signal receives contributions from SMBHBs + X (or X only?)
→ Probe cosmology of the early Universe and particle physics at high energies

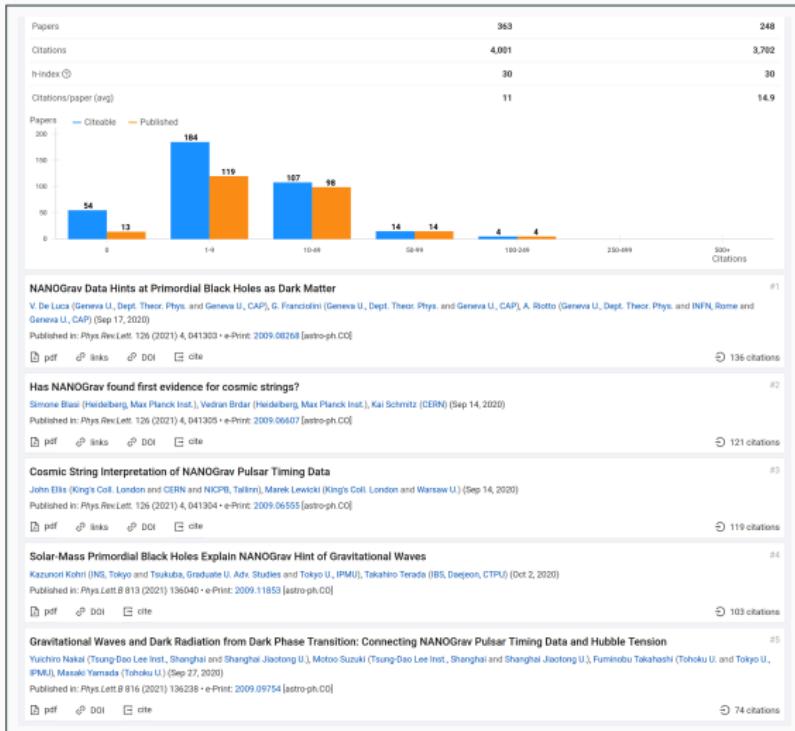
Cosmic strings [2009.06555, 2009.06607, 2009.10649, 2009.13452, 2102.08923] **Primordial black holes** [2009.07832, 2009.08268, 2009.11853, 2010.03976, 2101.11244] **Phase transitions** [2009.09754, 2009.10327, 2009.14174, 2009.14663, 2101.08012] **Audible axions and axion strings** [2009.11875, 2012.06882] **Inflation** [2009.13432, 2010.05071, 2011.03323] **Domain walls** [2009.13893, 2012.14071]

Overwhelming reaction in the particle physics community



NANOGrav 2009.04496: 423 citations and dozens of possible interpretations

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- ① Primordial black holes
- ② Cosmic strings
- ③ Phase transition
- ④ Inflation
- ...

References

Literature

- **Book:** M. Maggiore, *Gravitational Waves. Vol. 1: Theory and Experiments* Oxford University Press (2007), ISBN-13: 978-0198570745.
- **Book:** M. Maggiore, *Gravitational Waves. Vol. 2: Astrophysics and Cosmology* Oxford University Press (2018), ISBN-13: 978-0198570899.
- **Review:** C. Caprini and D. G. Figueroa, *Cosmological Backgrounds of Gravitational Waves*, Class. Quant. Grav. **35** (2018) 163001, [arXiv:1801.04268].

Lecture notes (slides)

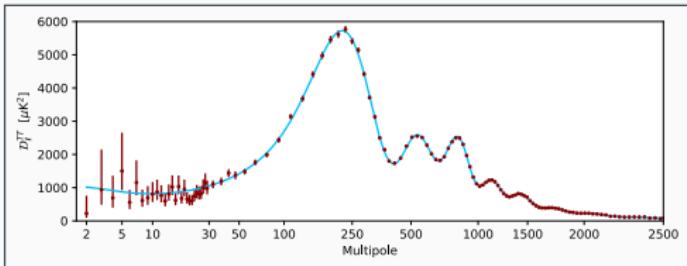
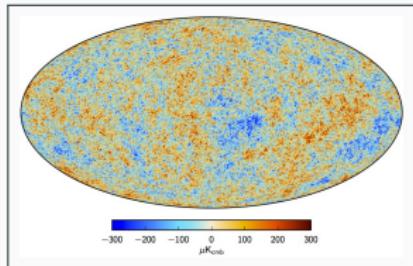
- K. Schmitz, Lectures at Heidelberg Graduate Days 2021
<http://doi.org/10.5281/zenodo.4678780> (881-page PDF file)

YouTube videos

- K. Schmitz, Lectures at Chung-Ang University, <https://youtu.be/0-tMnRArrZA>, https://youtu.be/nJ9of9dt_1c, <https://youtu.be/xFI8PBsAoqY> (10 hours)

Primordial GWs from cosmic inflation

Cosmic inflation

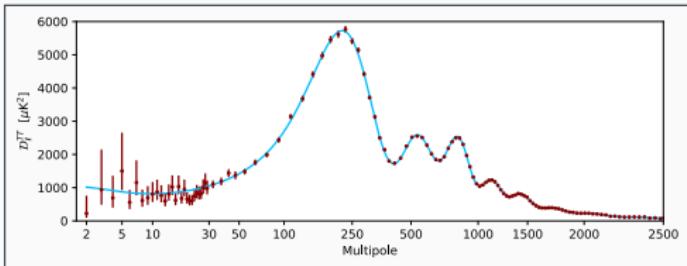
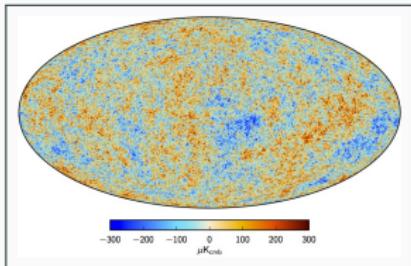


[PLANCK Collaboration 2018]

Inflation: Stage of accelerated expansion in the early universe

- Explains size, homogeneity, and isotropy of our Universe on cosmological scales
- Quantum fluctuations during inflation seed structure formation on galactic scales

Cosmic inflation



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Microscopic dynamics

- Scalar “inflaton” field ϕ slowly rolls down scalar potential (“slow-roll inflation”)
- $V(\phi) \approx \text{const} \approx$ slowly decaying cosmological constant \rightarrow exponential expansion
- Vacuum continuously sources perturbations in the inflaton field and in the metric

[Guth 1981] [Linde 1982] [Albrecht & Steinhardt 1982]

Primordial tensor perturbations

Action for transverse–traceless tensor perturbations from Einstein–Hilbert action

$$S = \frac{m_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} R \quad \supset \quad S_h^{(2)} = -\frac{m_{\text{Pl}}^2}{8} \int d\eta d^3x a^2(\eta) \eta^{\mu\nu} \partial_\mu \mathbf{h}_{ij} \partial_\nu \mathbf{h}_{ij}$$

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Go to Fourier space; introduce new field variables $v_p(\eta, \mathbf{k}) = m_{\text{Pl}}/\sqrt{2} a(\eta) h_p(\eta, \mathbf{k})$:

$$S_h^{(2)} = \frac{1}{2} \sum_{p=+, \times} \int d\eta \frac{d^3k}{(2\pi)^3} \left[|v'_p(\eta, \mathbf{k})|^2 - \left(k^2 - \frac{a''}{a} \right) |v_p(\eta, \mathbf{k})|^2 \right]$$

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Solve equation of motion in inflationary background, $a(t) \approx a_0 e^{Ht}$, $a(\eta) \approx -1/(H\eta)$

$$v_k'' + \left(k^2 - \frac{a''}{a} \right) v_k = 0$$

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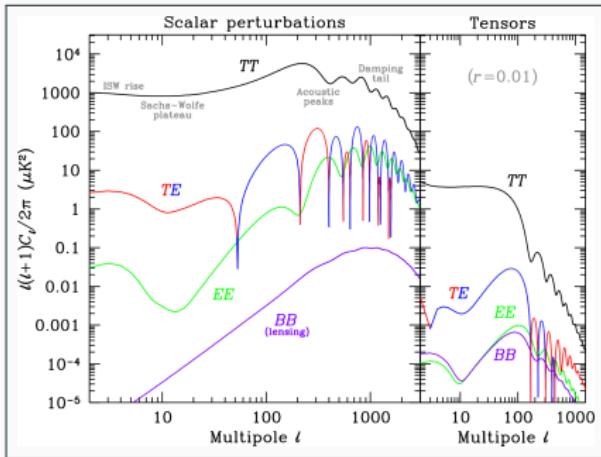
Primordial tensor spectrum

$$\langle 0 | h_{ij}(\eta, \mathbf{k}) h_{ij}^*(\eta, \ell) | 0 \rangle = \frac{2\pi^2}{k^3} \delta^{(3)}(\mathbf{k} - \ell) \mathcal{P}_h(k),$$

$$\boxed{\mathcal{P}_h(k) \approx 2 \left(\frac{H_k}{\pi m_{\text{Pl}}} \right)^2}$$

CMB observables

[Review of Particle Physics (2020), pdg.lbl.gov]



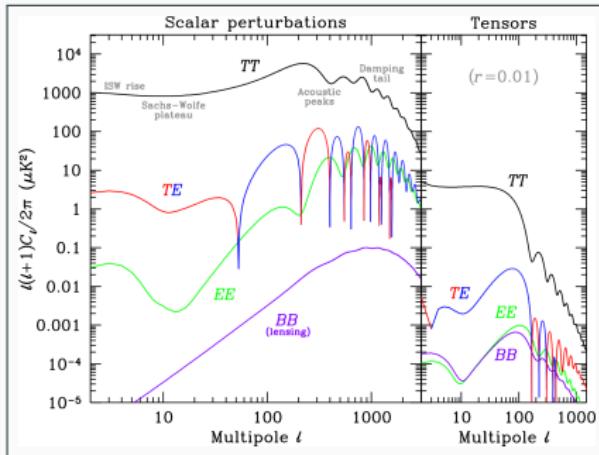
Primordial scalar + tensor perturbations source temperature + polarization anisotropies

$$\mathcal{P}_R(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1} \approx \frac{1}{8\epsilon} \left(\frac{H_k}{\pi m_{Pl}} \right)^2, \quad \mathcal{P}_h(k) = r A_s \left(\frac{k}{k_*} \right)^{n_t} \approx 2 \left(\frac{H_k}{\pi m_{Pl}} \right)^2$$

with the slow-roll parameter $\epsilon = -\dot{H}/H^2$ measuring the deviation from dS expansion

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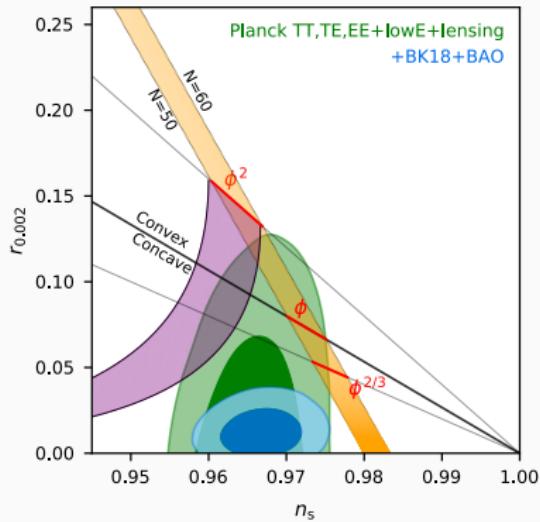
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Consistency relation in single-field slow-roll inflation:

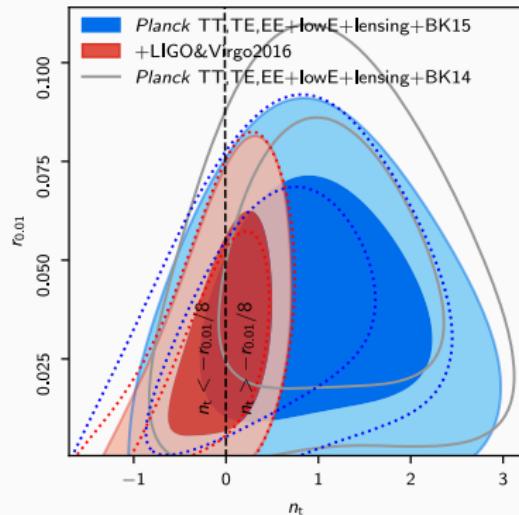
$$n_t = \frac{d \ln \mathcal{P}_h}{d \ln k} \approx -2\epsilon = -\frac{r}{8}$$

Observational constraints

Scalar spectral index versus $r_{0.002}$

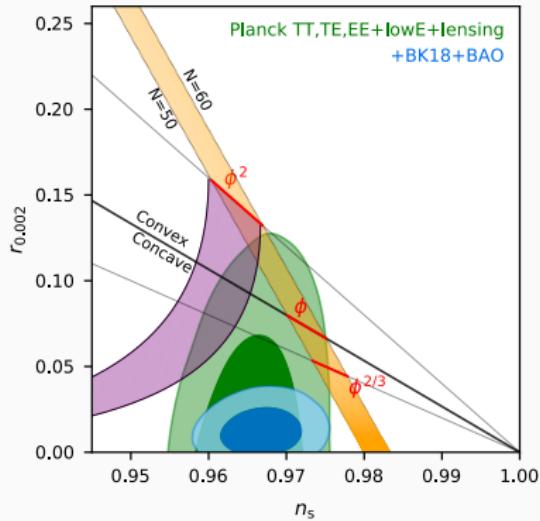


Tensor spectral index versus $r_{0.01}$

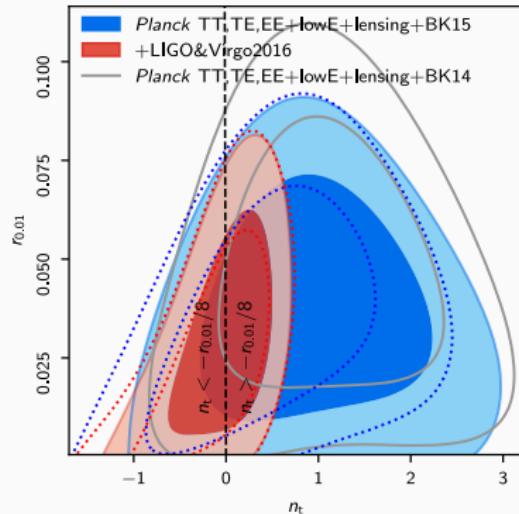


Observational constraints

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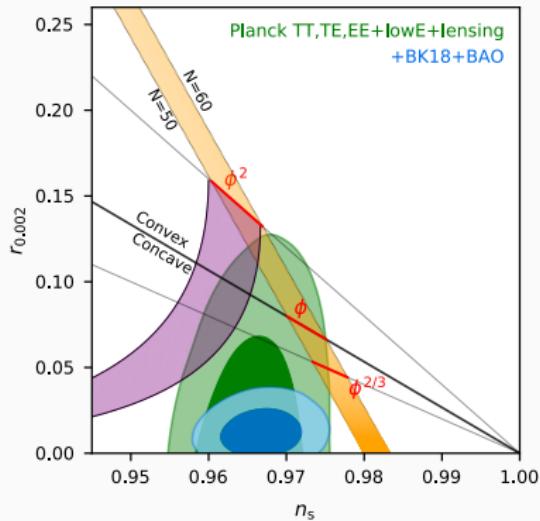
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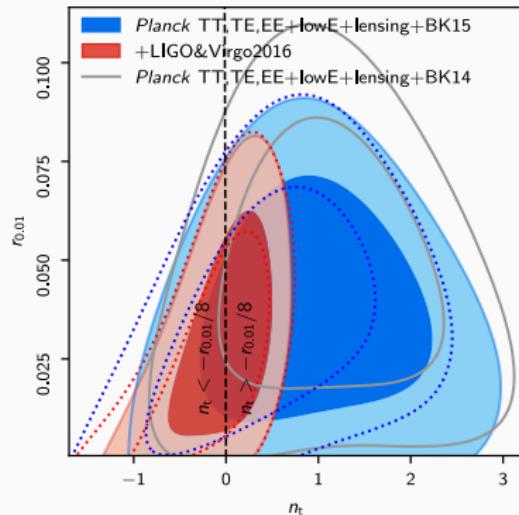
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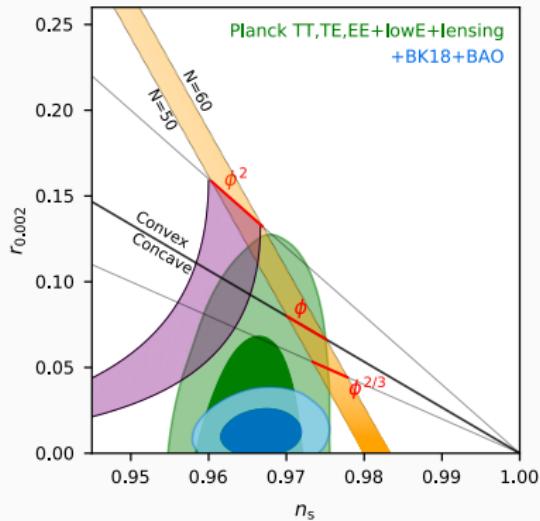
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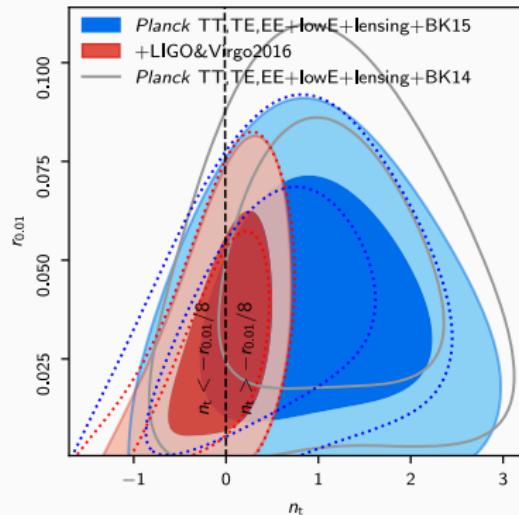
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- Future CMB polarization experiments (e.g., LiteBIRD) will probe $r \gtrsim 10^{-4\cdots 3}$

Observational constraints

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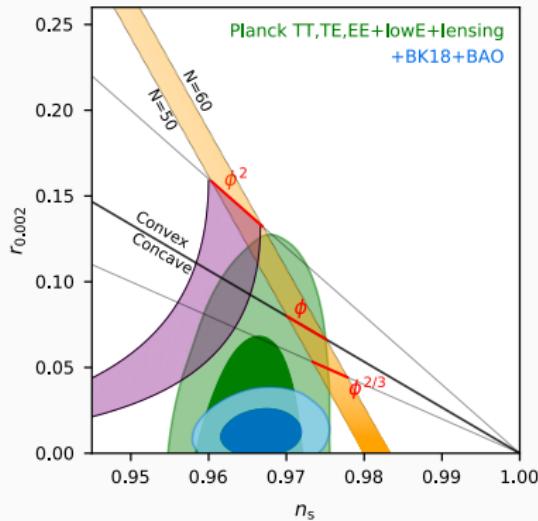
Tensor spectral index versus $r_{0.01}$



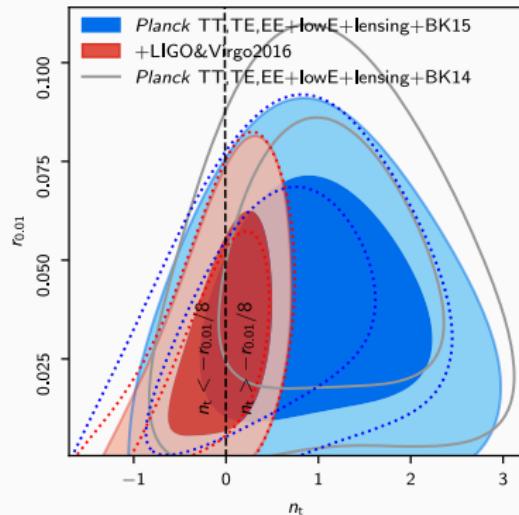
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- In absence of a GW signal, tensor index only poorly constrained by CMB

Observational constraints

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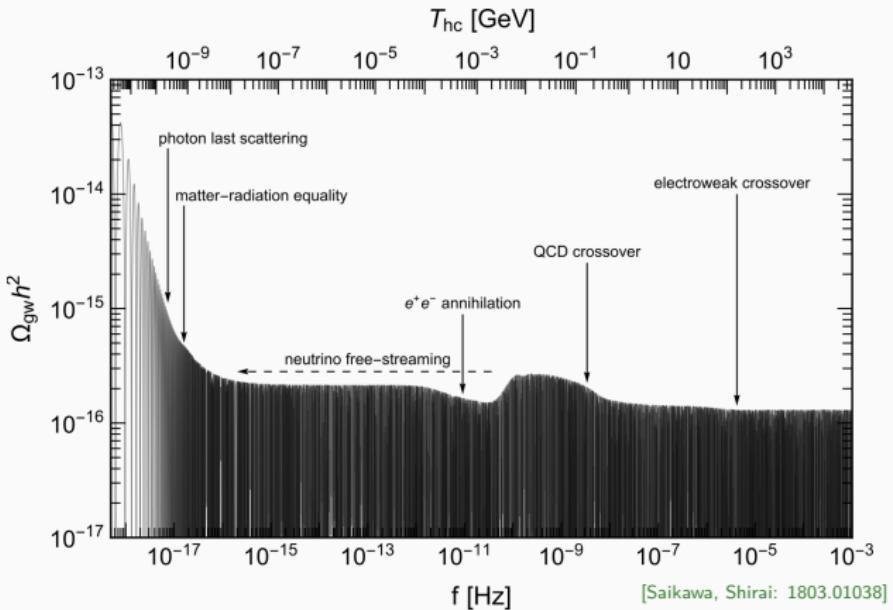


Tensor spectral index versus $r_{0.01}$



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- Future CMB polarization experiments (e.g., LiteBIRD) will probe $r \gtrsim 10^{-4\cdots 3}$
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- Tighter constraints if power law naively extrapolated to higher frequencies

Logbook of the expansion history

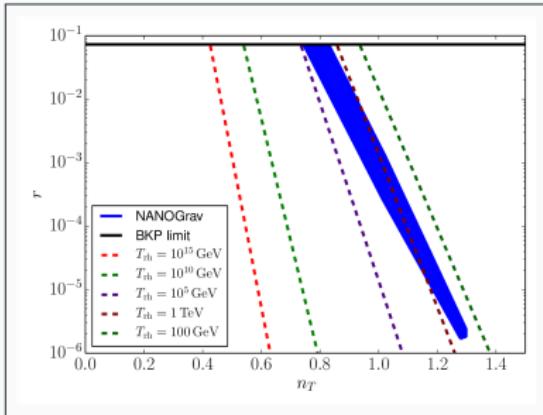


Scale-invariant GW spectrum from inflation, redshifted to the present epoch

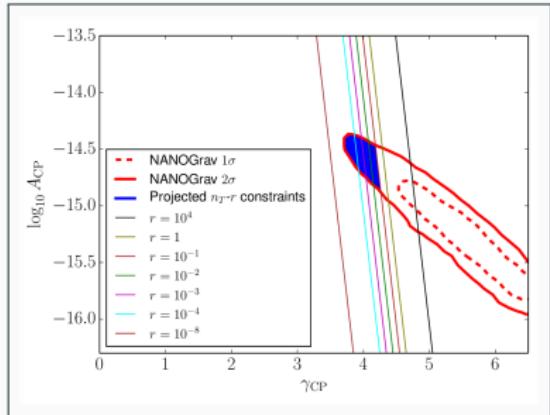
- Cosmic logbook encoding the entire expansion history of the early Universe
- Major events in the early Universe leave their imprint in the SGWB signal
- Approximately flat plateau at $\Omega_{\text{gw}} \sim \Omega_r / 24 r A_s \sim 2 \times 10^{-16}$ ($r/0.044$)

Inflationary GWs in the PTA band

Index and amplitude at low f (CMB)



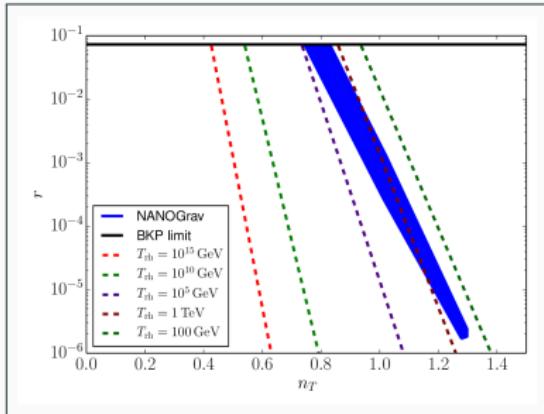
Index and amplitude at high f (PTA)



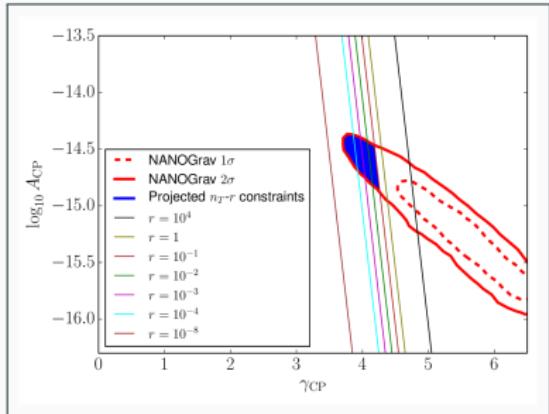
[Vagnozzi: 2009.13432]

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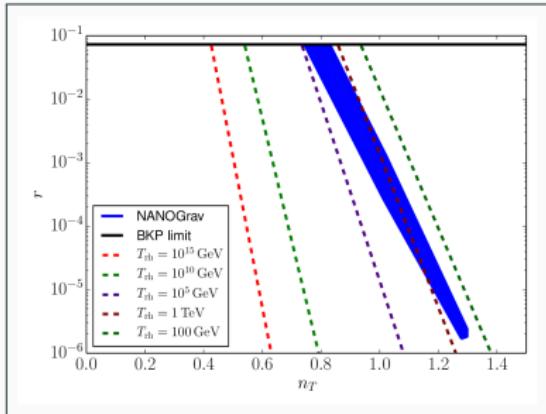


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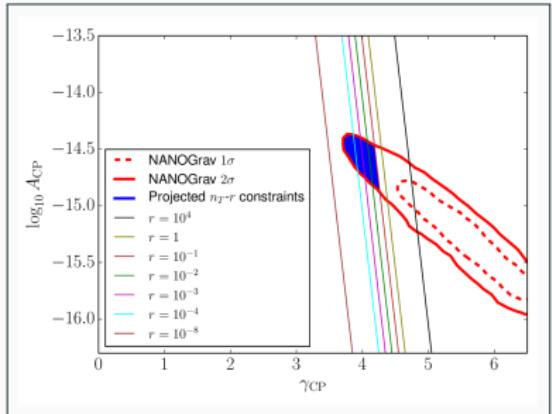
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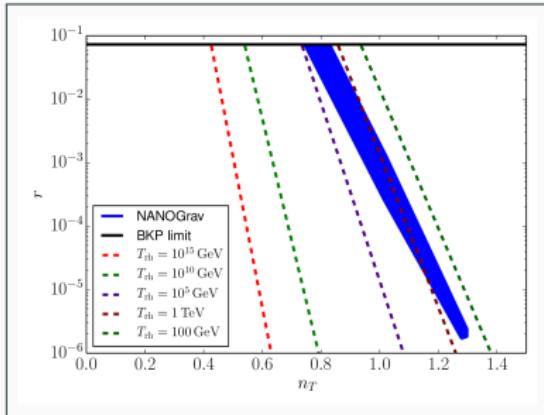


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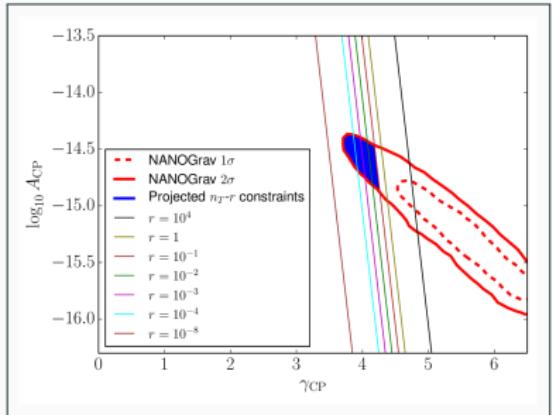
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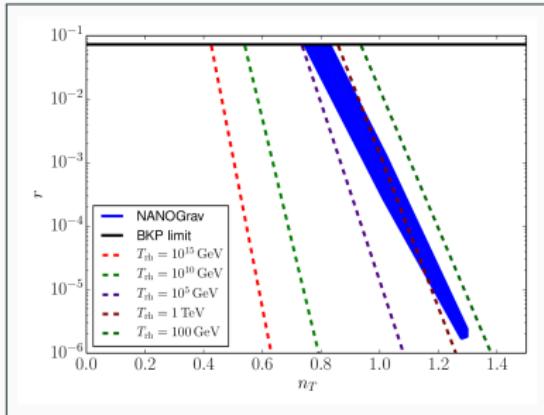


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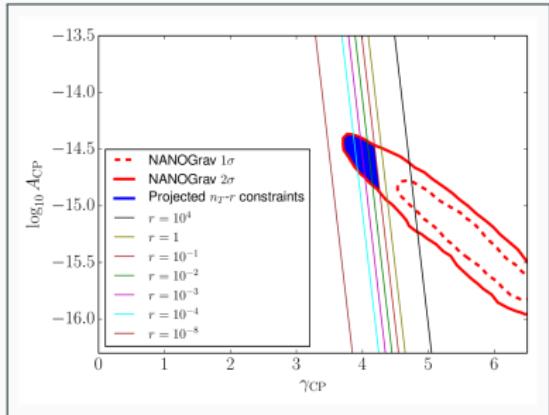
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- Violates consistency relation; consider nonminimal sources of GW production
- If naively extrapolated too far, too much GW radiation (N_{eff}); bound on T_{rh}

Inflationary GWs in the PTA band

Index and amplitude at low f (CMB)



Index and amplitude at high f (PTA)

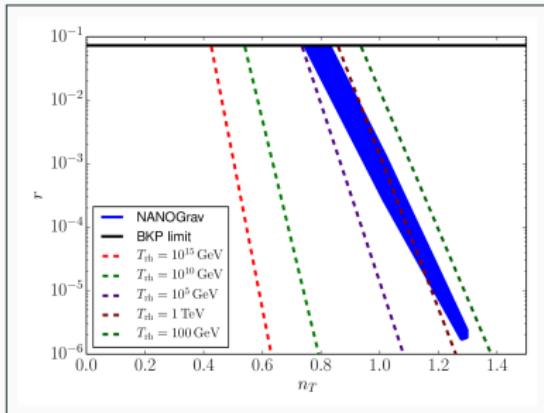


[Vagnozzi: 2009.13432]

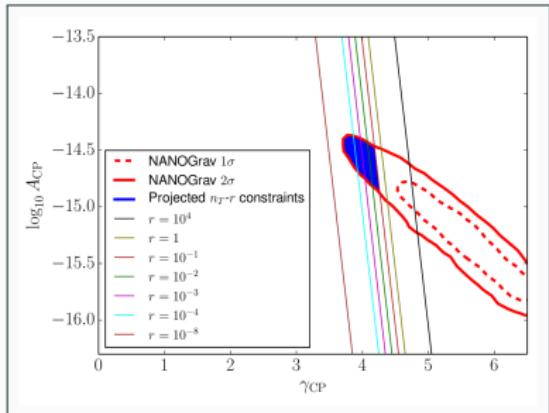
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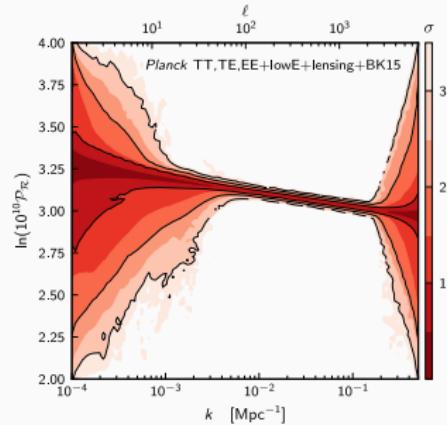


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Primordial black holes (PBHs)

Enhanced curvature perturbations

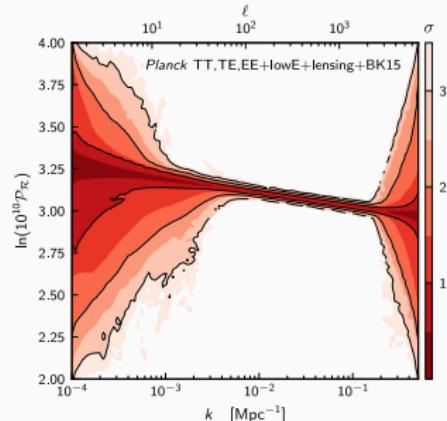


[PLANCK Collaboration: 1807.06211]

Primordial scalar power spectrum only well known at CMB scales

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{24\pi^2} \frac{1}{\epsilon} \frac{V}{m_{\text{Pl}}^4} \simeq (2.10 \pm 0.06) \times 10^{-9} \quad \text{at} \quad k_{\text{CMB}} = 0.05 \text{ Mpc}^{-1}$$

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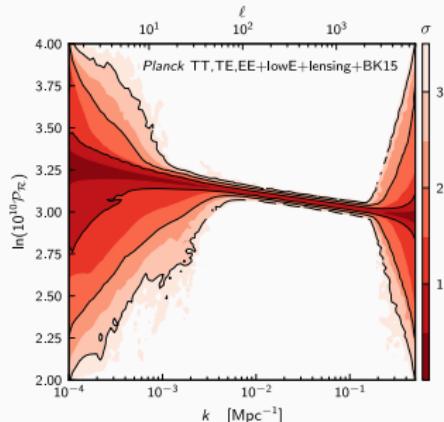
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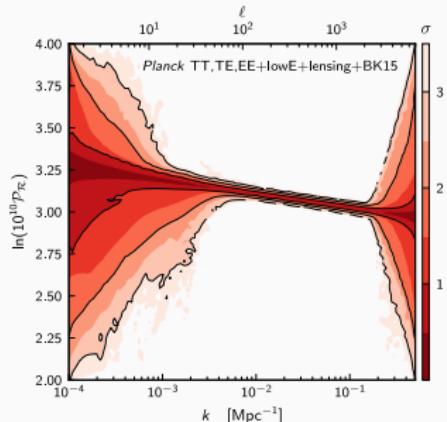
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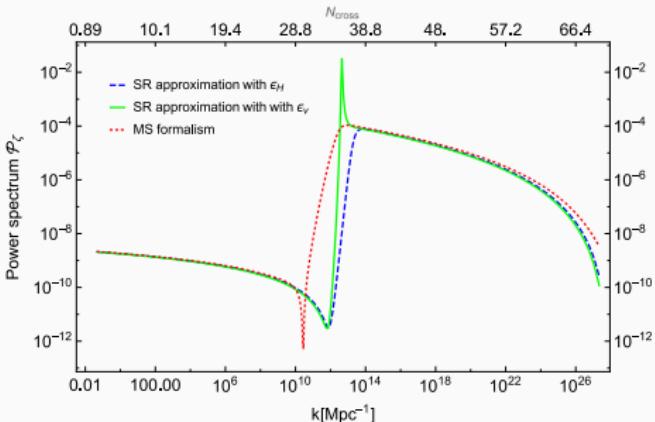
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[Drees, Xu: 1905.13581]

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- E.g.: Critical Higgs inflation, saddle point in scalar potential at low field values

Scalar-induced GWs (SIGWs) at second order in perturbation theory

Cosmological perturbation theory

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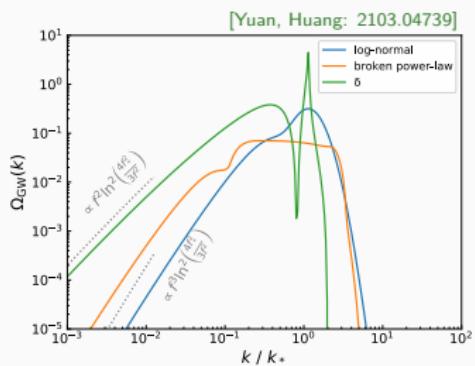
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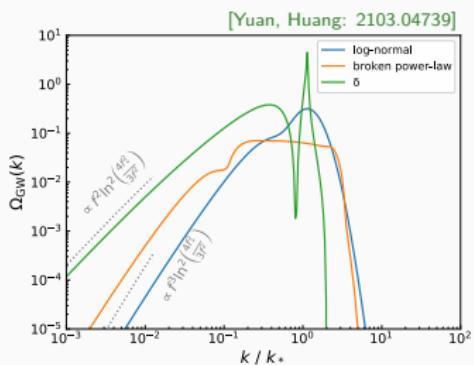
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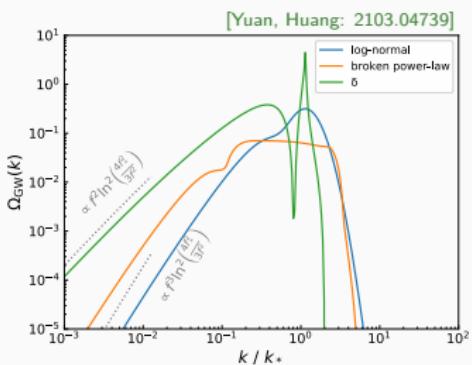
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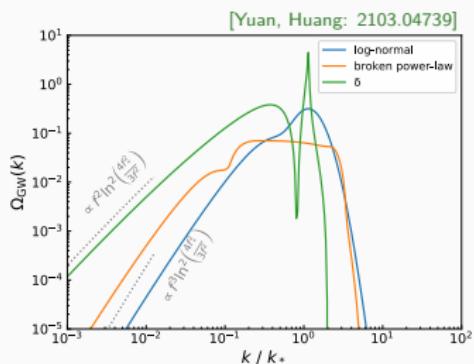
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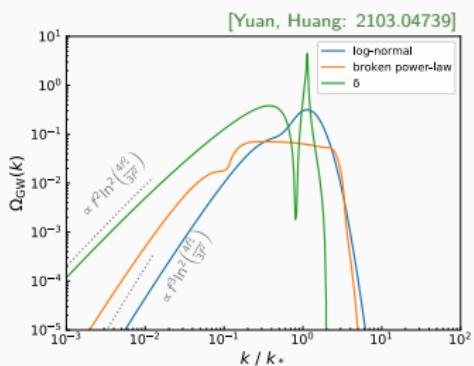
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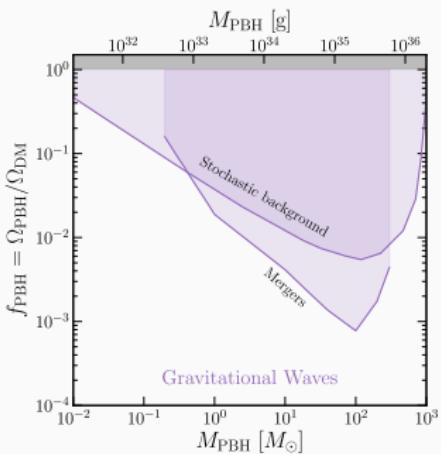
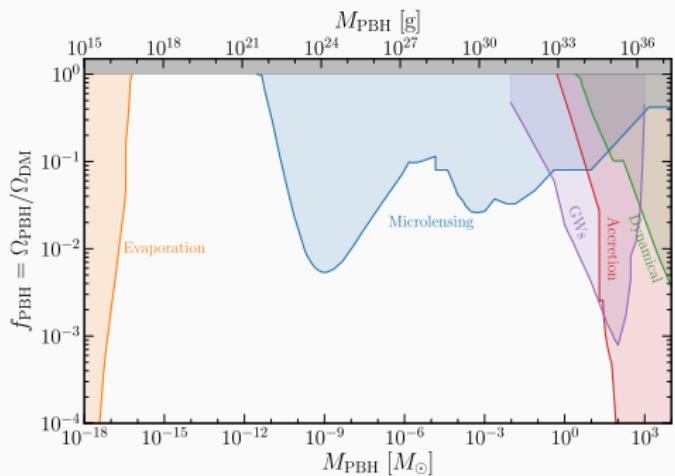
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Depending on \mathcal{P}_ζ , second-order scalar-induced GWs can dominate the SGWB signal!

Associated PBH formation

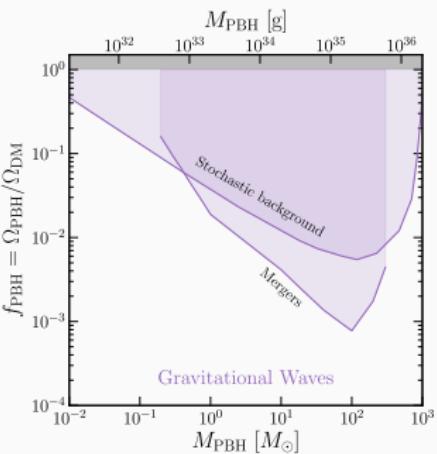
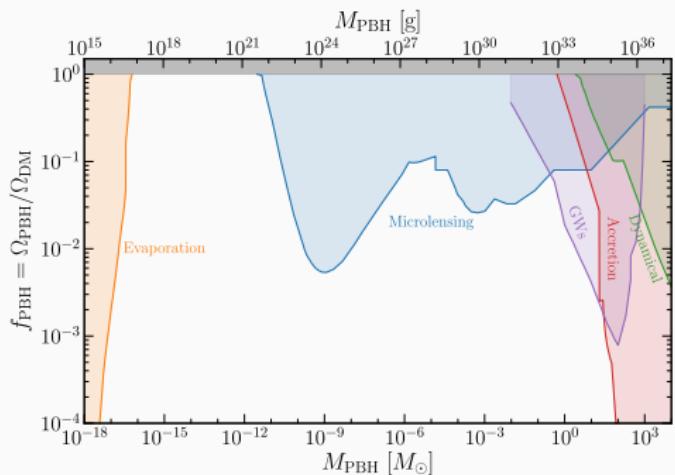


[Green, Kavanagh: 2007.10722]

Collapse of horizon mass when overdense regions re-enter the causal horizon

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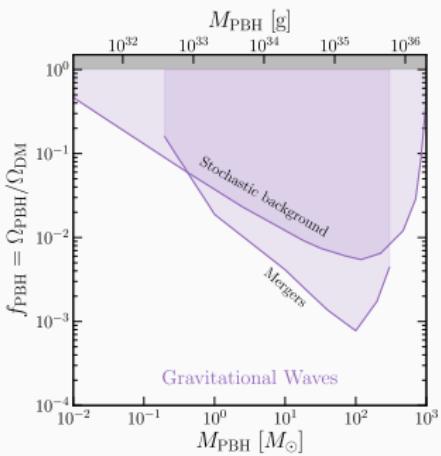
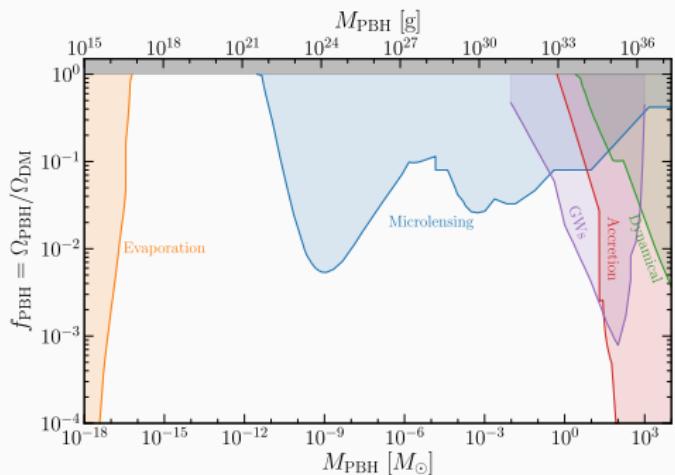


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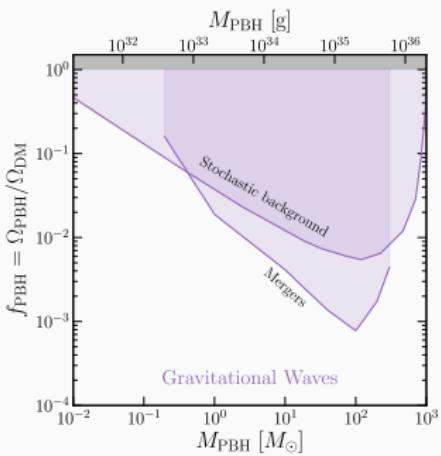
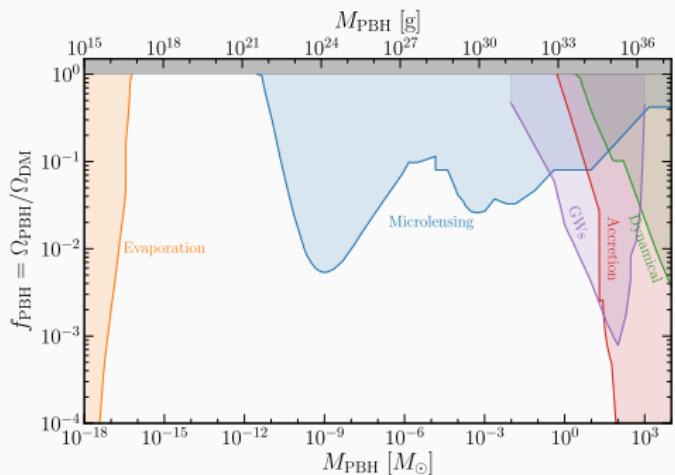


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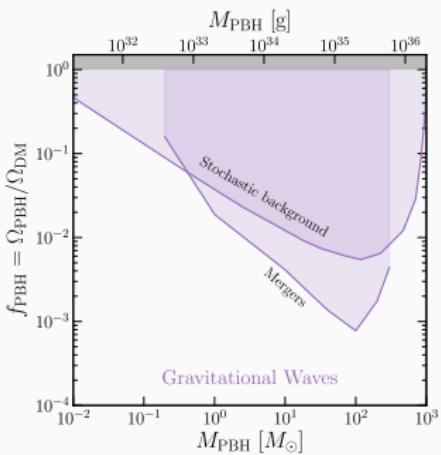
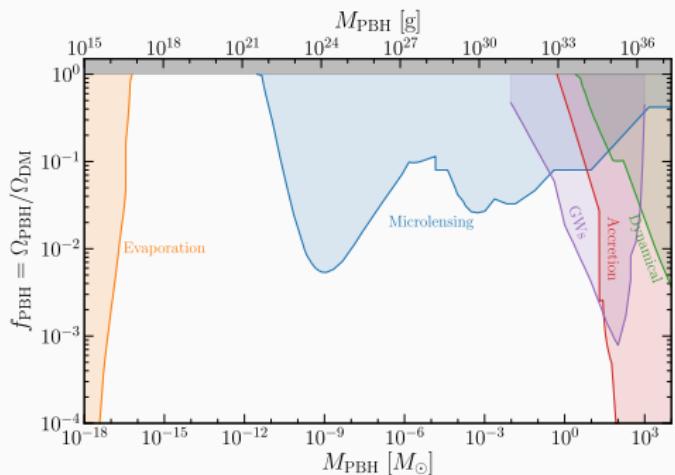


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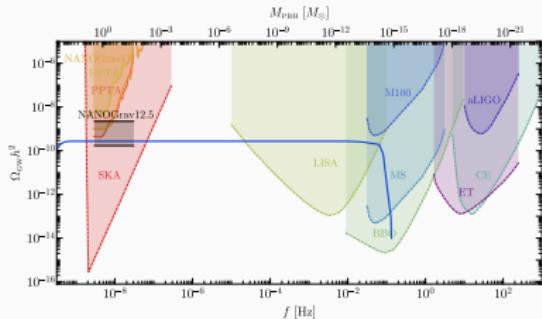
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- ... as well as for the GW signals / stochastic background from PBH mergers

PBH interpretation of the PTA signal

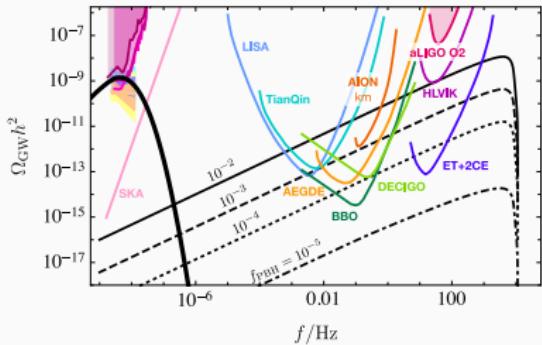
[De Luca, Franciolini, Riotto: 2009.08268]

$10^{-12} M_{\odot}$ PBHs + PBH dark matter



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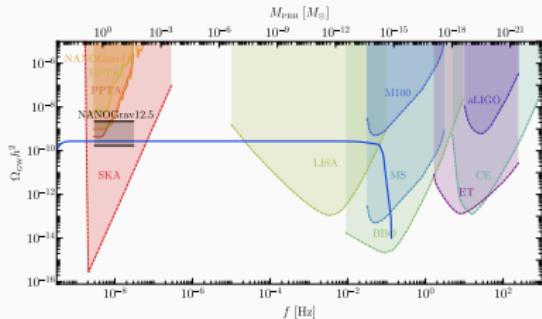


Two representative benchmark models

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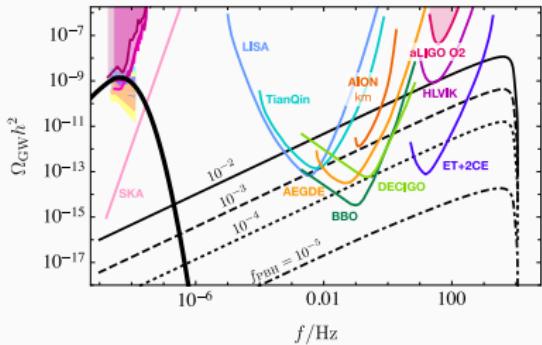
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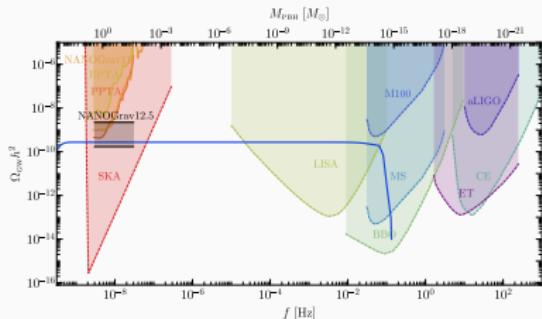
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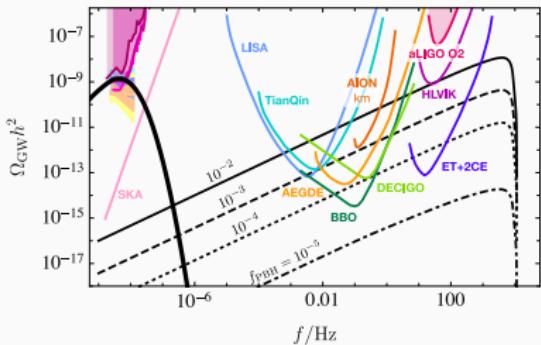
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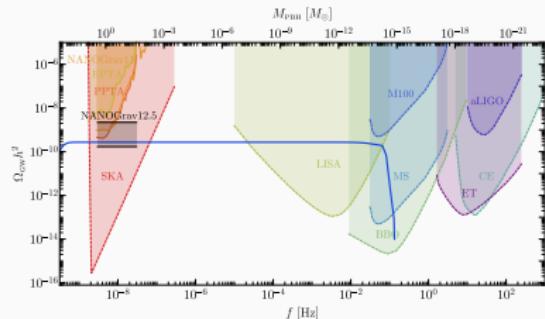
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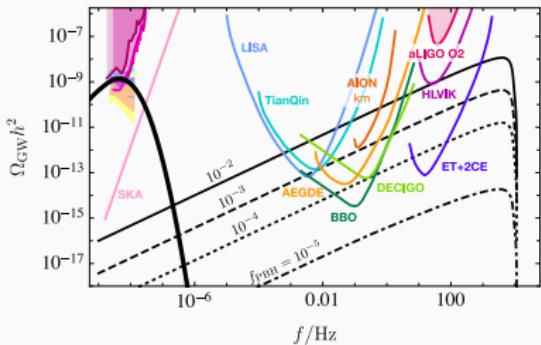
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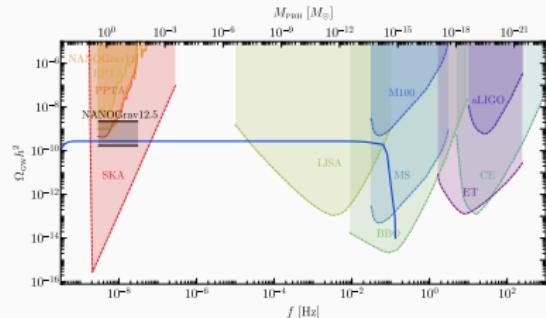
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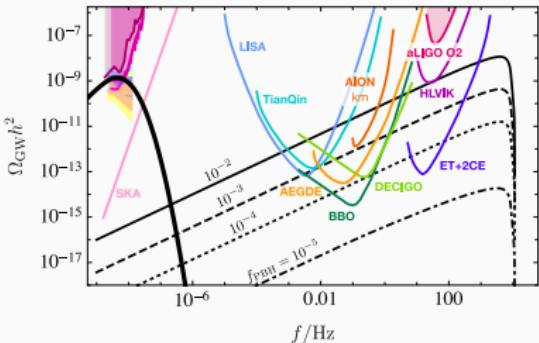
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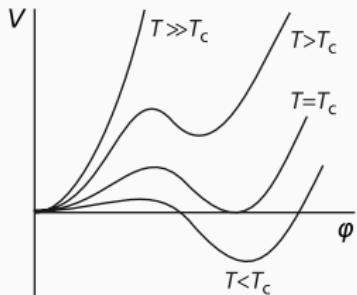
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- **Project idea ③:** Survey range of viable PBH scenarios and their nHz GW signal

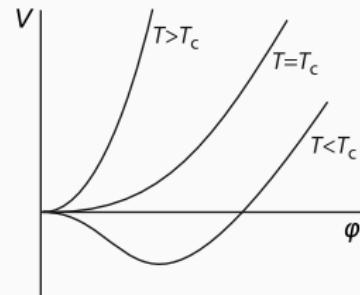
Cosmological phase transitions

First-order phase transitions

First-order phase transition



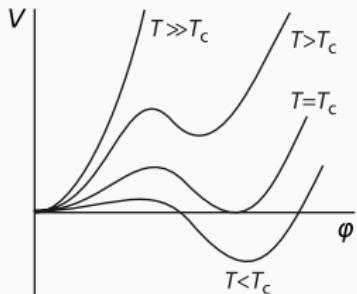
Second-order phase transition



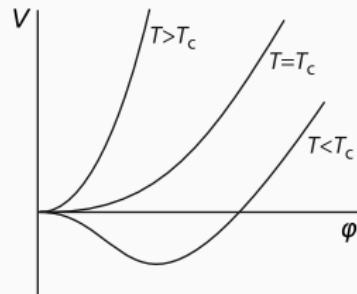
[Kinnunen, Baarsma, Martikainen, Törmä: 1706.07076]

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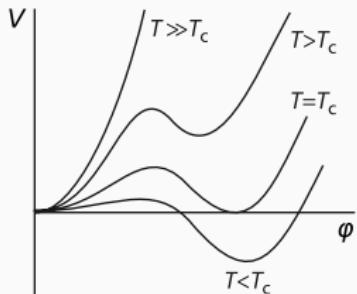


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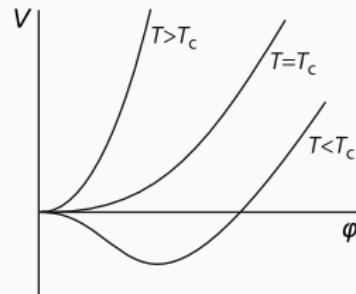
GWs from strong first-order phase transitions (SFOPTs)

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Second-order phase transition



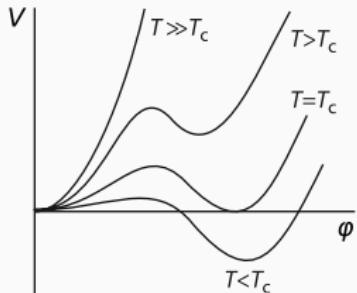
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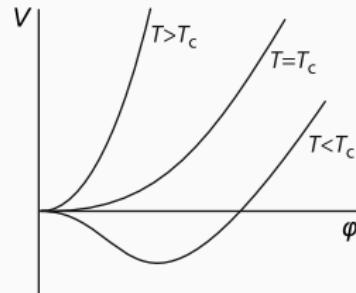
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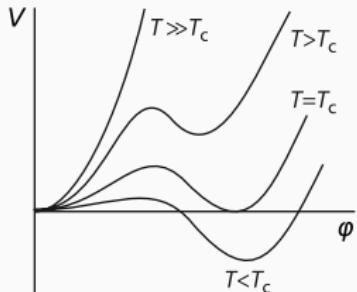
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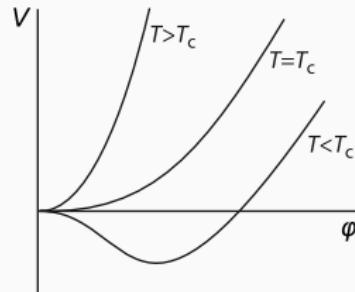
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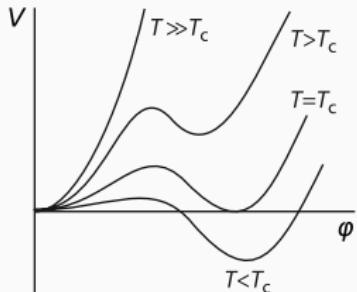
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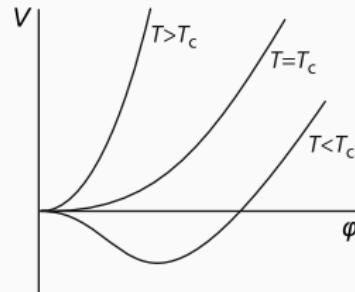
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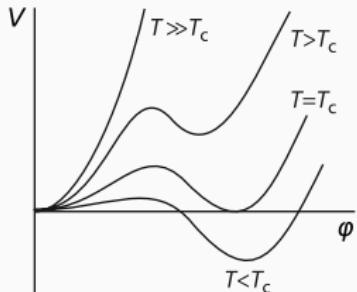
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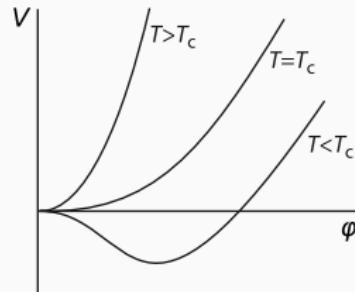
Three sources of GWs

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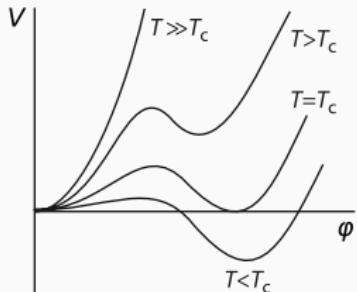
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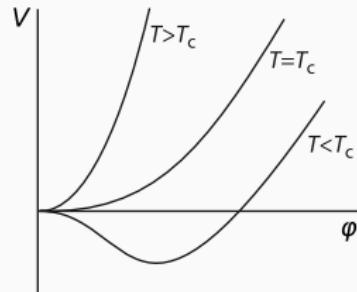
- Ω_b : Bubble collisions, gradient energy in the scalar field

First-order phase transitions

First-order phase transition



Second-order phase transition



[Kinnunen, Baarsma, Martikainen, Törmä: 1706.07076]

GWs from strong first-order phase transitions (SFOPTs)

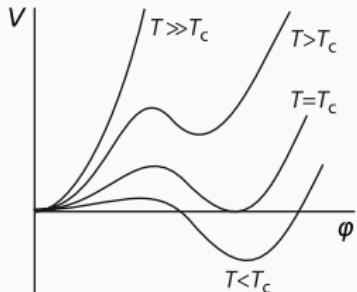
- Barrier in the effective potential of the order parameter (scalar field)
- Thermal jump or quantum tunneling \rightarrow bubble nucleation in position space
- Bubbles expand, accelerate, transfer energy to the surrounding plasma, and collide

Three sources of GWs

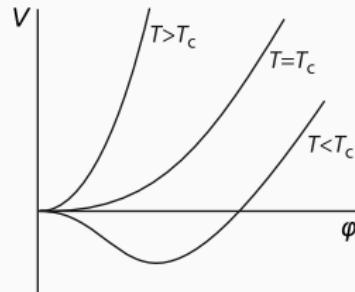
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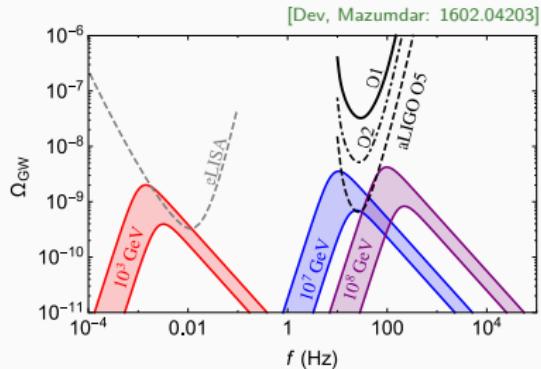
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- Ω_b : Bubble collisions, gradient energy in the scalar field
- Ω_s : Sound waves, compression and rarefaction waves in the bulk plasma
- Ω_t : Magnetohydrodynamic turbulence, vortical motion in the bulk plasma

Example: GWs from sound waves



Relevant parameters

- Strength parameter $\alpha = \Delta\theta/\rho_{\text{rad}}$
- Inverse duration $\beta = -d/dt S_E$
- Temperature scale T_*
- Bubble wall velocity v_w

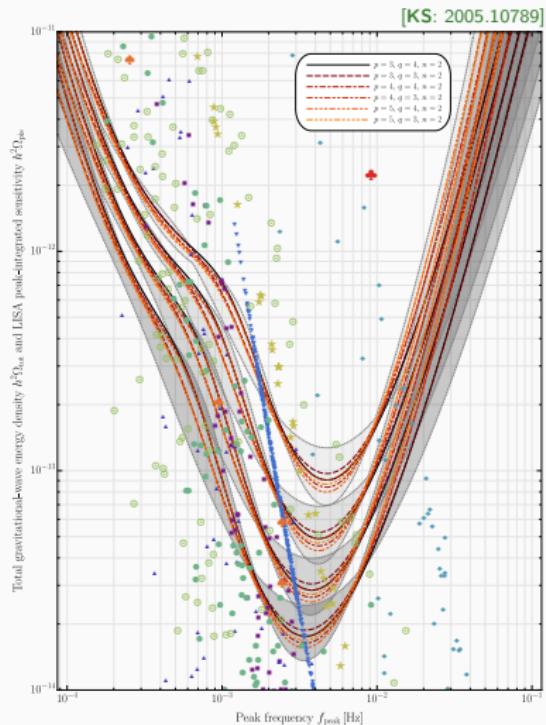
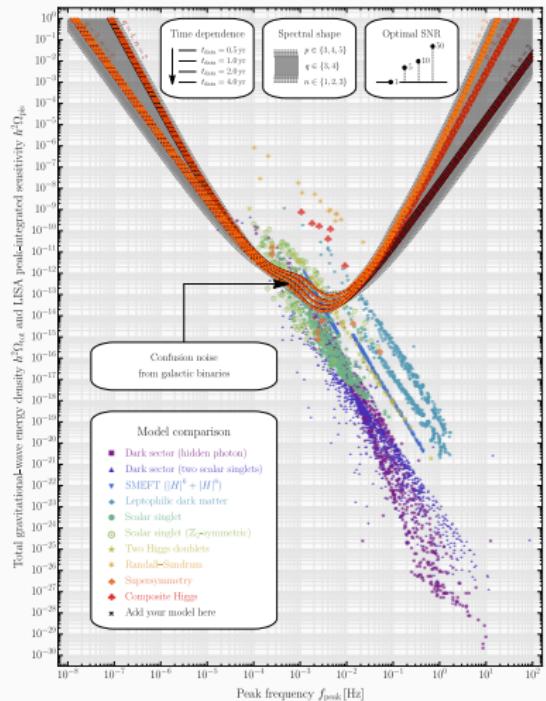
Frequency: $f_{\text{sw}}^{\text{peak}} \sim 1 \text{ mHz} \left(\frac{1}{v_w} \right) \left(\frac{\beta/H_*}{100} \right) \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_*}{100 \text{ GeV}} \right)$

Amplitude: $\Omega_{\text{sw}}^{\text{peak}} \sim 10^{-8} \min \{1, H\tau_{\text{sh}}\} \left(\frac{v_w}{1} \right) \left(\frac{100}{\beta/H_*} \right) \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\kappa \alpha}{1+\alpha} \right)^2$

Shape: $S_{\text{sw}}(f) = \frac{(f/f_{\text{peak}})^p}{[q/(p+q)+p/(p+q)(f/f_{\text{peak}})]^{n(p+q)/n}}, \quad \{p, q, n\} = \{3, 4, 2\}$

[Hindmarsh: 1608.04735] [Hindmarsh, Huber, Rummukainen, Weir: 1704.05871] [Ellis, Lewicki, No: 1809.08242, 2003.07360]
 [Ellis, Lewicki, No, Vaskonen: 1903.09642] [Hindmarsh, Hijazi: 1909.10040] [Giese, Konstandin, van de Vis: 2004.06995]

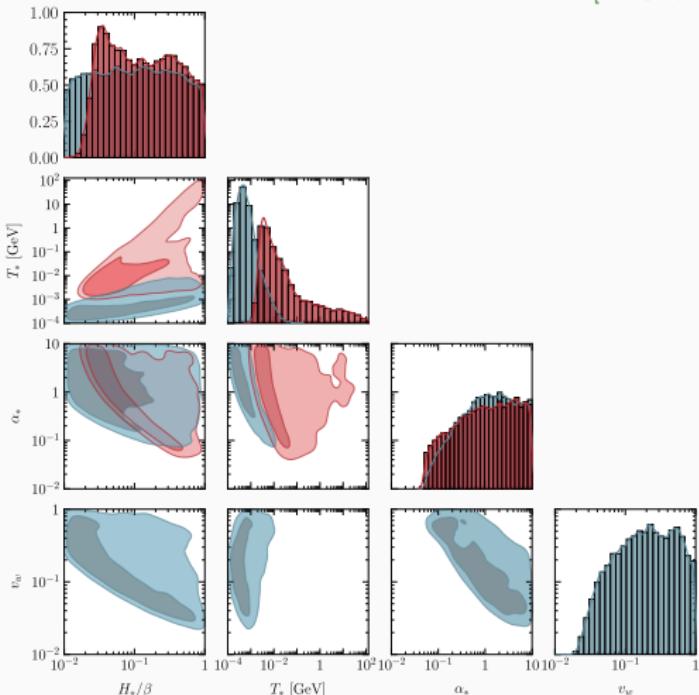
LISA sensitivity to GWs from sound waves



- Peak-integrated sensitivity: LISA sensitivity projected onto $f_{\text{peak}} - h^2 \Omega_{\text{tot}}$ plane
- Project idea ④:** Repeat analysis for PTAs, compile “atlas” of relevant models

Search for GWs from a SFOPT with the NANOGrav 12.5-year data

[NANOGrav collaboration: 2104.13930]

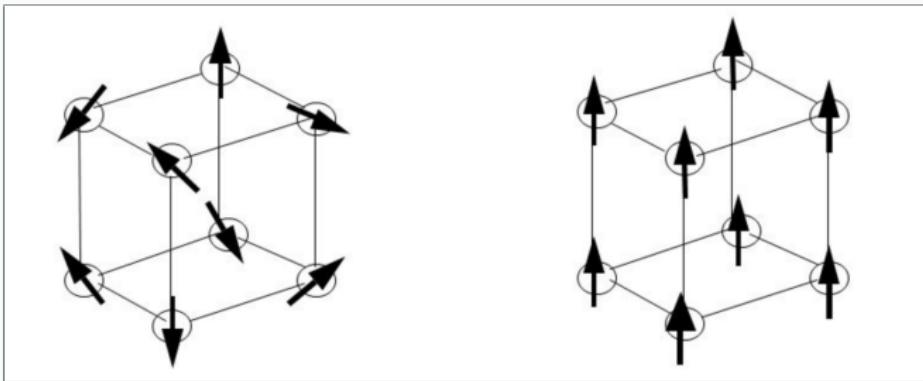


- Blue: GWs from sound waves only; red: GWs from bubble collisions only
- Preference for low temperature scale T_* ; phase transition in a hidden sector

Cosmic defects

Magnetic domains in a ferromagnet

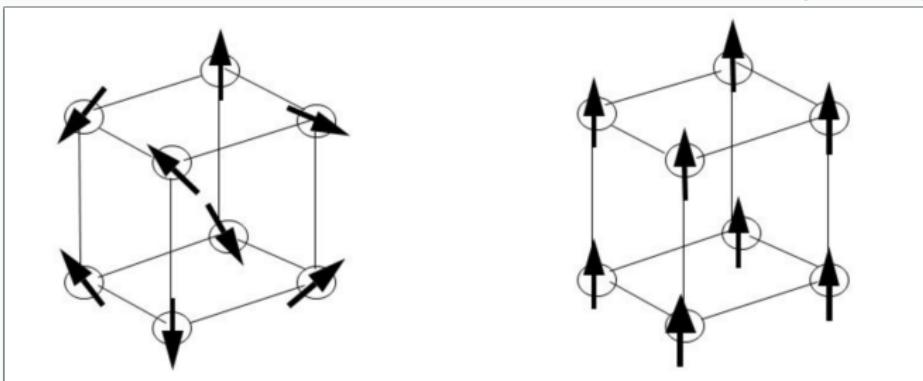
[wikimedia.org]



Magnetization in a ferromagnet

Magnetic domains in a ferromagnet

[wikimedia.org]

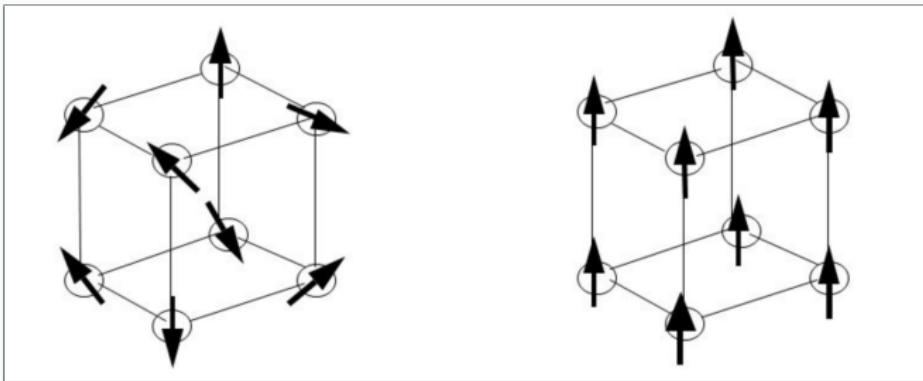


Magnetization in a ferromagnet

- Phase transition at the Curie temperature: paramagnet \rightarrow ferromagnet

Magnetic domains in a ferromagnet

[wikimedia.org]

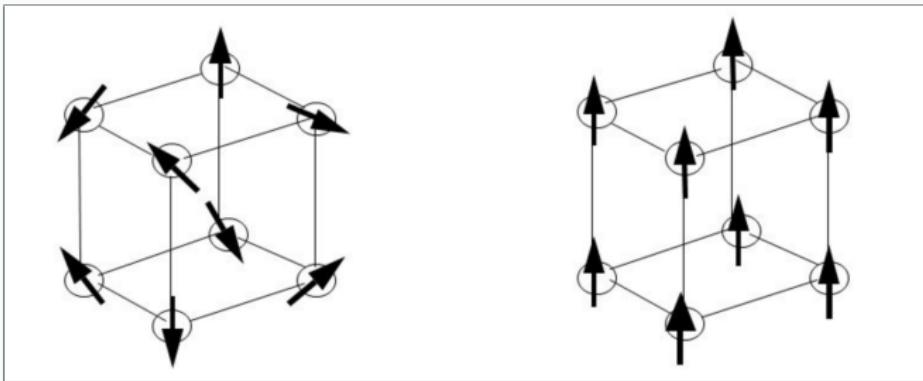


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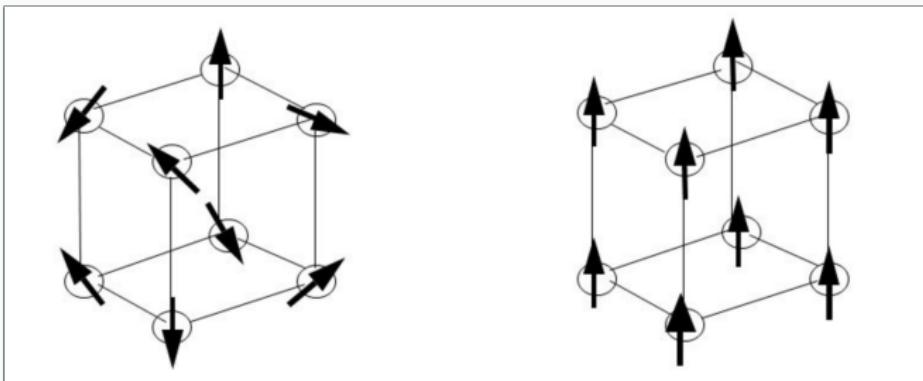


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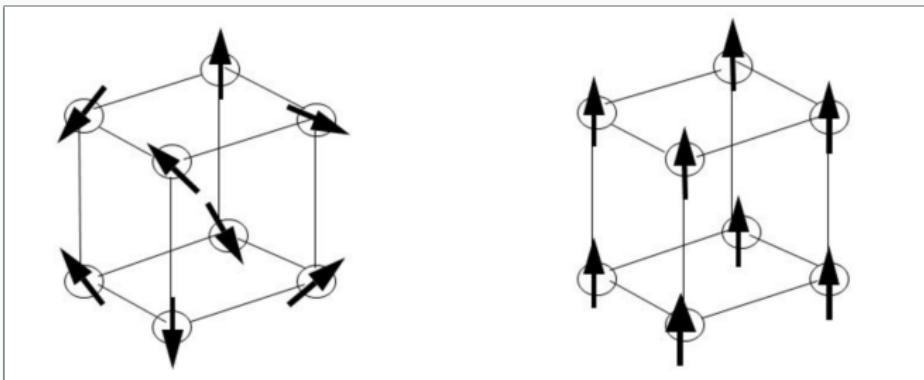


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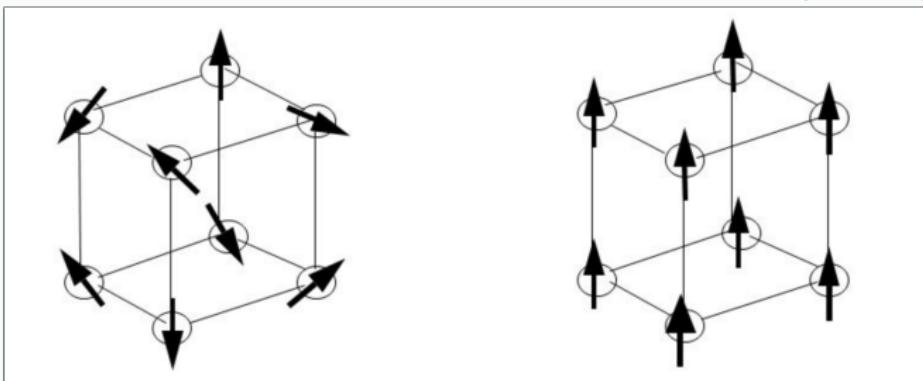


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Magnetic domains in a ferromagnet

[wikimedia.org]



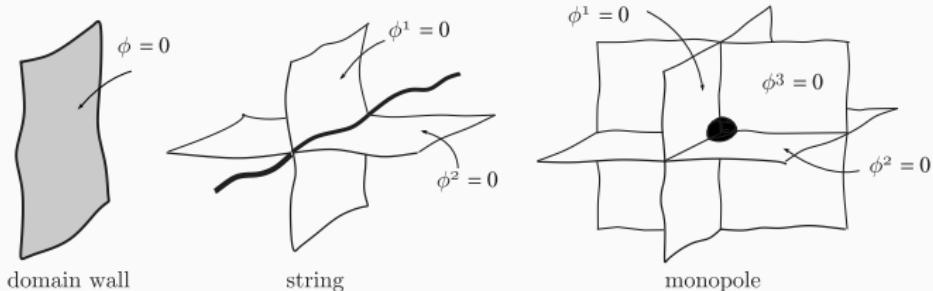
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Similar phenomenology after phase transitions in the early Universe!

Topological defects in the early Universe

[Viatcheslav Mukhanov: Physical Foundations of Cosmology, Cambridge University Press (2005)]

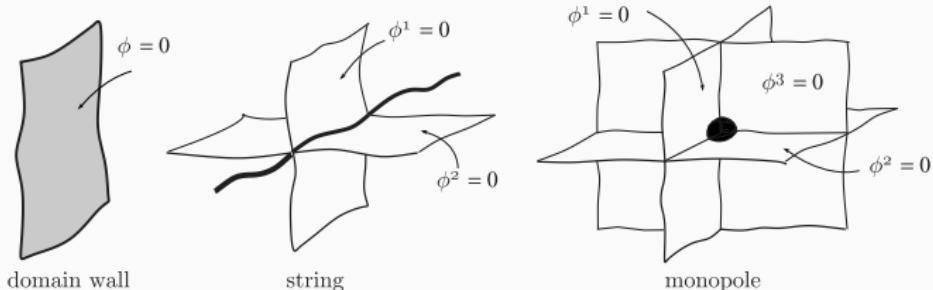


Consider spontaneous symmetry breaking in an N -dimensional scalar field space:

$$V(\Phi) = \frac{\lambda}{4} (\Phi^2 - v^2)^2, \quad \Phi = \frac{1}{\sqrt{N}} (\phi_1, \phi_2, \dots, \phi_N)^T$$

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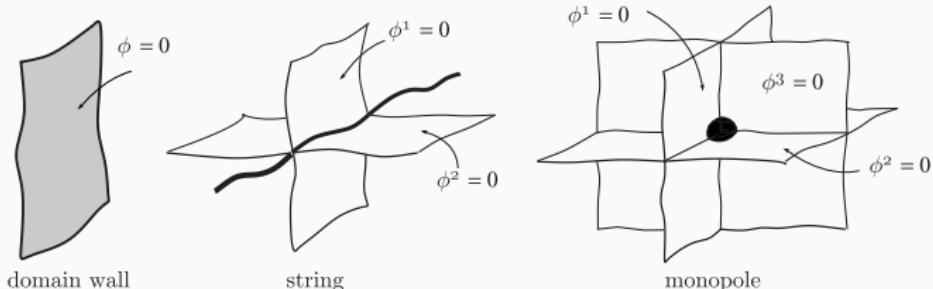
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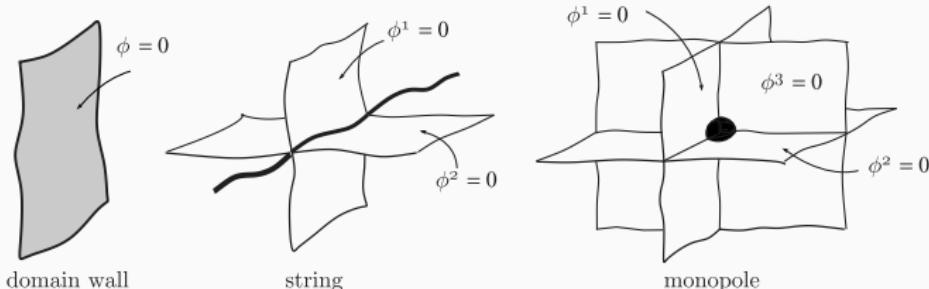
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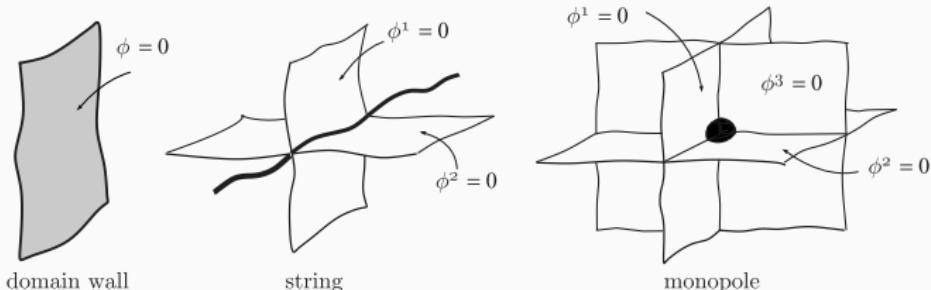
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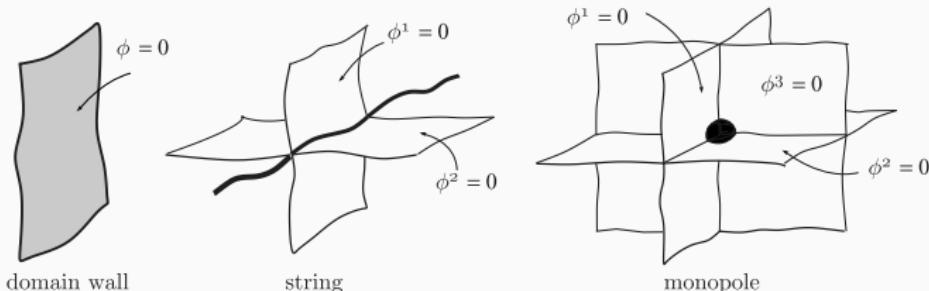
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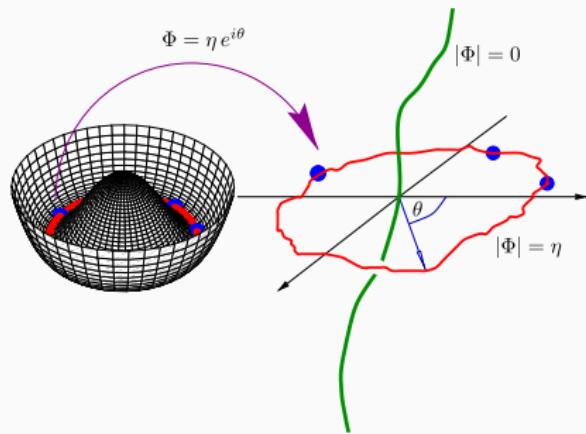
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- Formal description in terms of topology of vacuum manifold $\mathcal{M}(\Phi) \rightarrow$ stability
- In addition, whole zoo of composite defects, non-topological defects, etc.

Cosmic strings

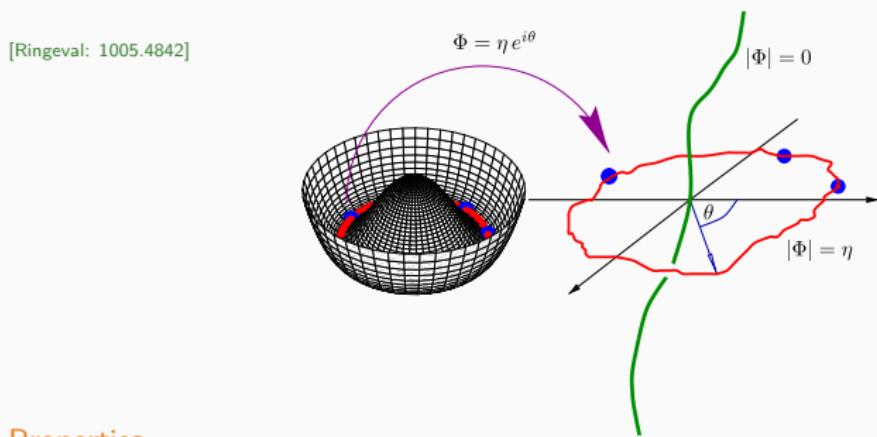
[Ringeval: 1005.4842]



Properties

- Topological defects after spontaneous $U(1)$ breaking in the early Universe

Cosmic strings

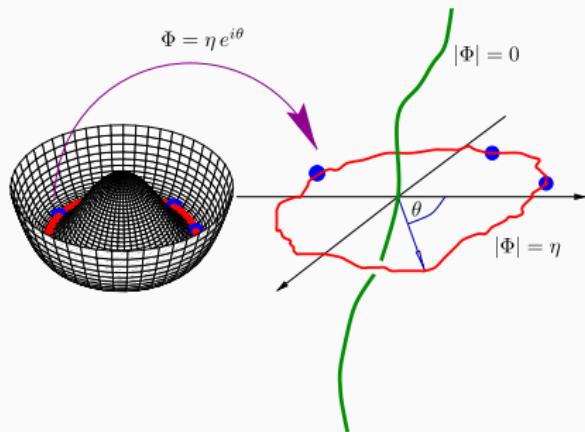


Properties

- Topological defects after spontaneous $U(1)$ breaking in the early Universe
- Global / local $U(1)$ symmetry restored (never broken) at the core of strings

Cosmic strings

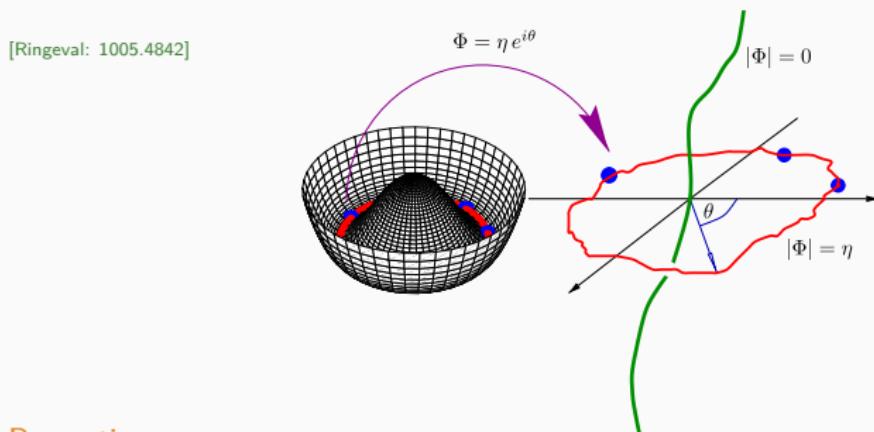
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Cosmic strings



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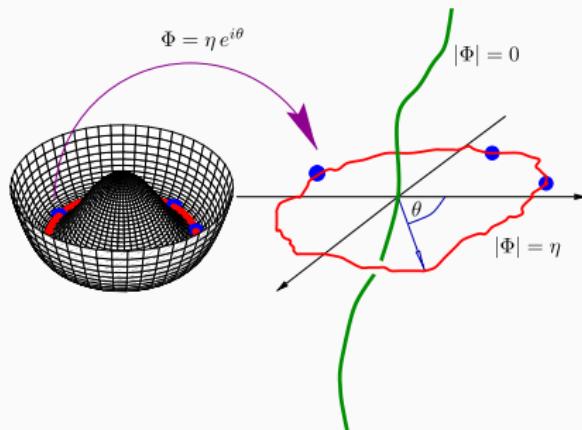
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- $G\mu$: String tension = energy per unit length, in units of $G = 1/M_P^2$

Cosmic strings

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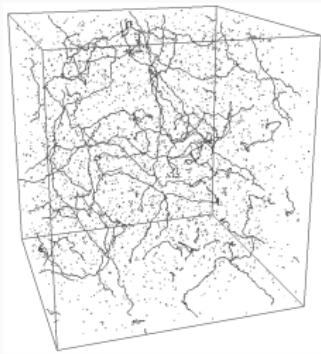
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- α : Size of string loops at the time of formation, in units of H^{-1}

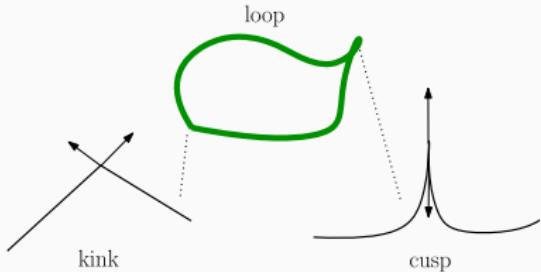
Gravitational waves from cosmic strings

[Allen, Martins, Shellard: ctc.cam.ac.uk/outreach]



Infinite strings and string loops;
scaling regime: $\rho_{\text{cs}} \propto \rho_{\text{crit}} \propto H^2$

[Gouttenoire, Servant, Simakachorn: 1912.02569]

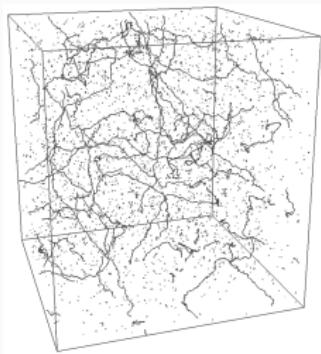


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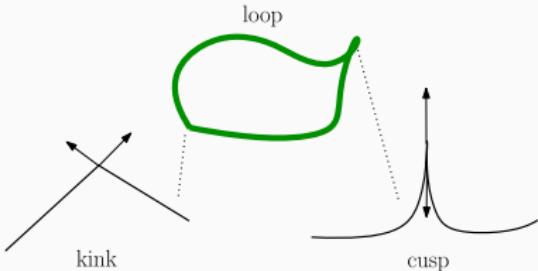
- Cusps
- Kinks
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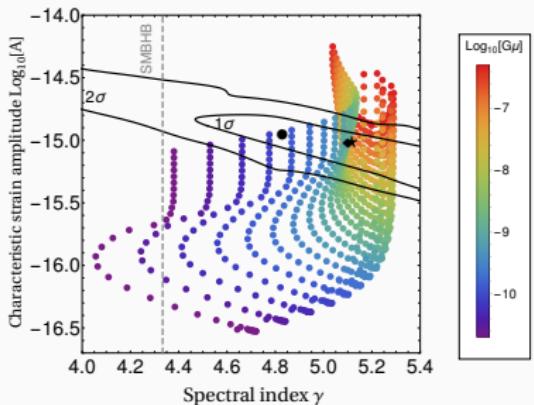
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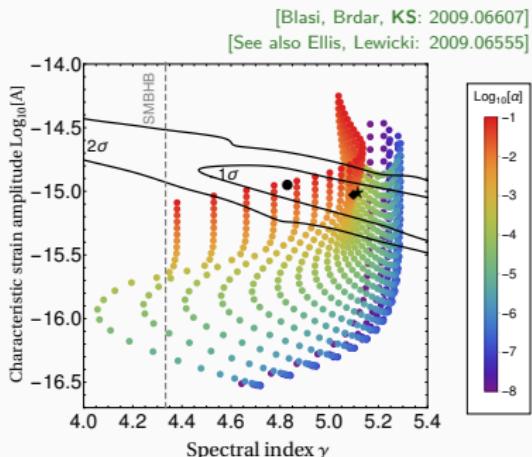
-
- **Nambu–Goto action:** infinitely thin strings, no particle emission
 - **Abelian-Higgs model:** short-lived loops, decay into massive particles
 - **Nonminimal models:** Metastable strings, current-carrying cosmic strings, ...

[Vachaspati, Vilenkin: PRD 31 (1985) 3052] [LISA Cosmology Working Group, Auclair et al.: 1909.00819]

Cosmic-string interpretation of the PTA signal



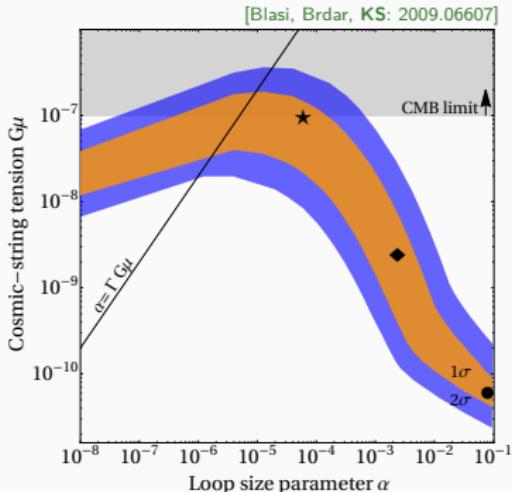
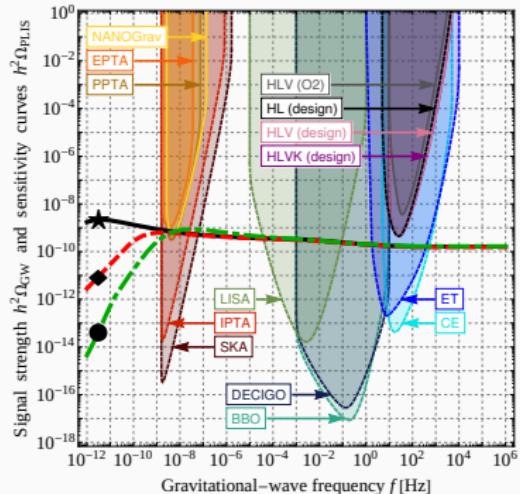
Color code: String tension $G\mu$



Color code: Loop size α

- Fit GW spectrum in the NANOGrav frequency range by a power law
- Straightforward to populate NANOGrav 1σ and 2σ posteriors on A and γ
- IPTA DR2 seems to prefer smaller γ and larger $A \rightarrow$ metastable strings?

Observational prospects



Complementarity

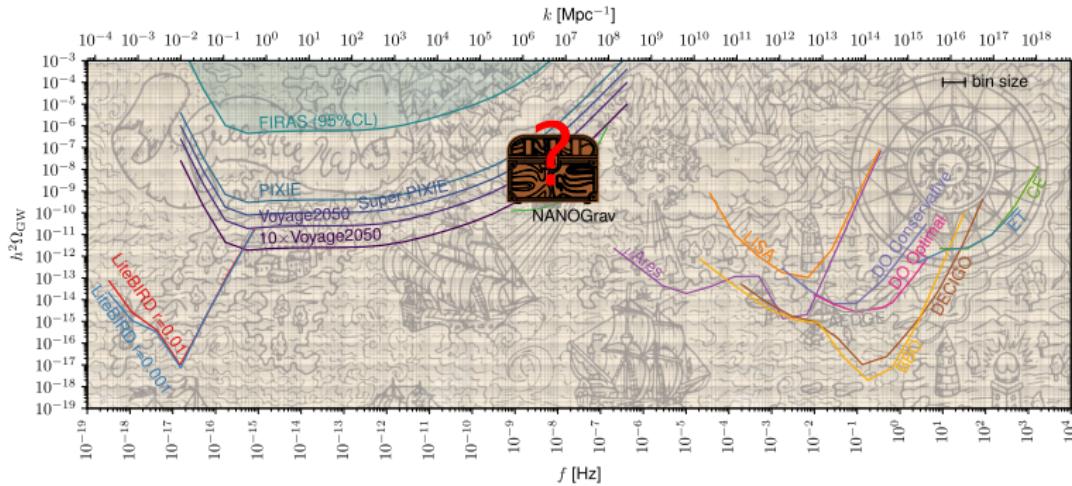
- Entire viable parameter space will be probed in future GW experiments

Relation to grand unification

- $G_\mu \sim 10^{-(10 \dots 7)}$ points to $U(1)$ breaking scale $v \sim 10^{14 \dots 16}$ GeV

Conclusions

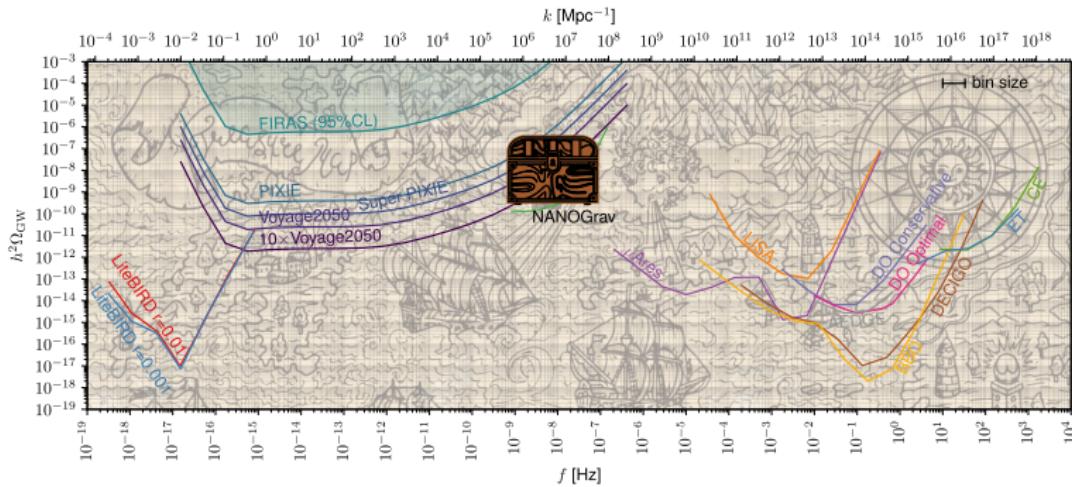
Summary



Cosmological GWs: Primary probe of fundamental physics in the 21st century

- Window onto the early Universe and particle physics at highest energies
 - Insights into cosmic inflation, phase transitions, grand unification, etc.
 - PTAs complementarity to other GW searches and lab searches for new physics

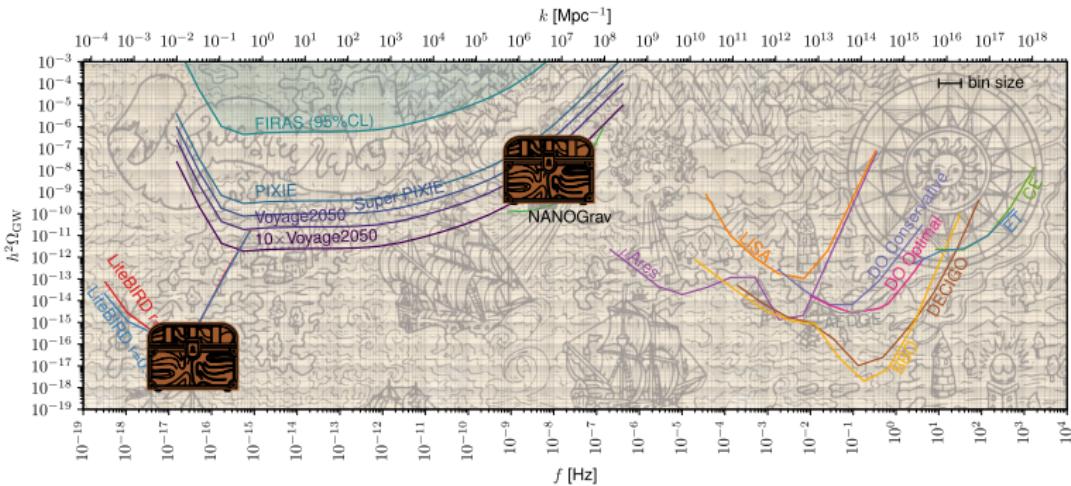
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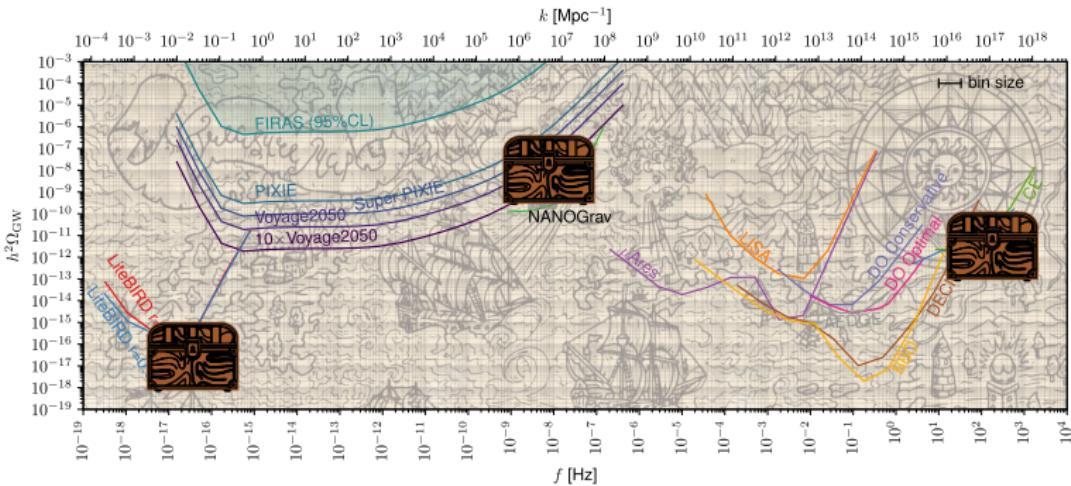
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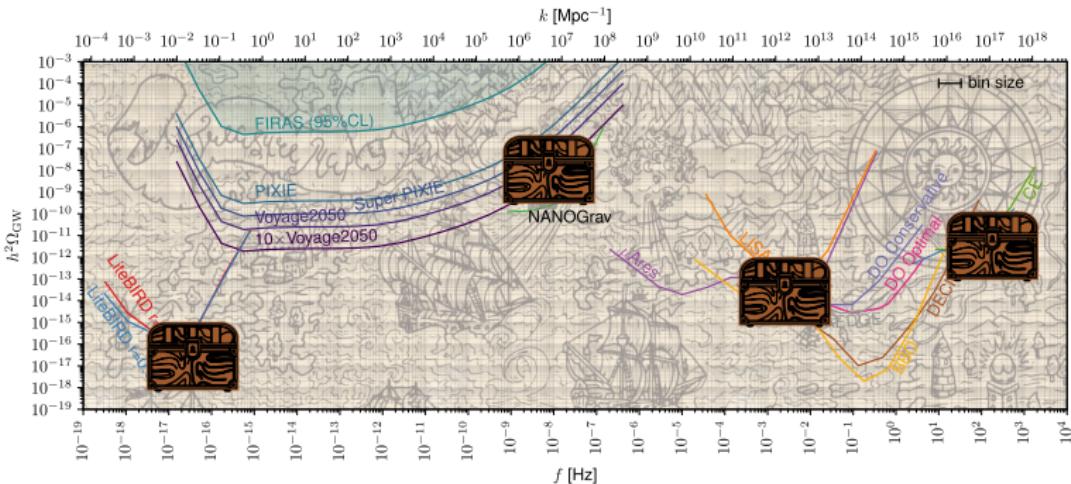
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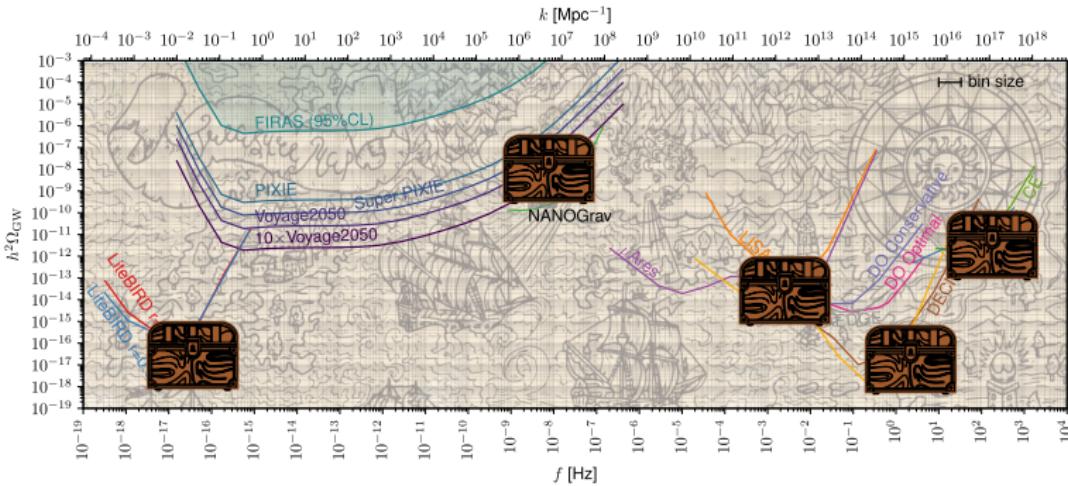
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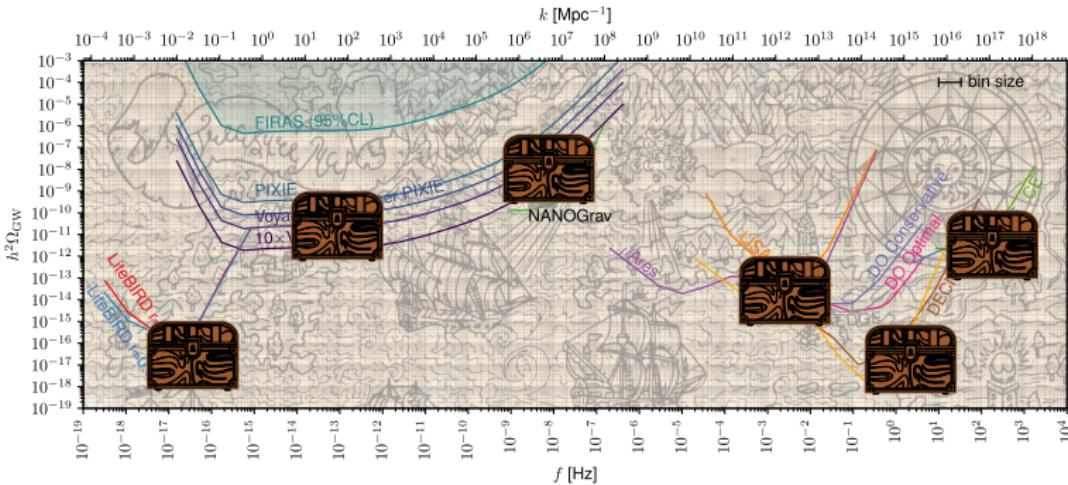
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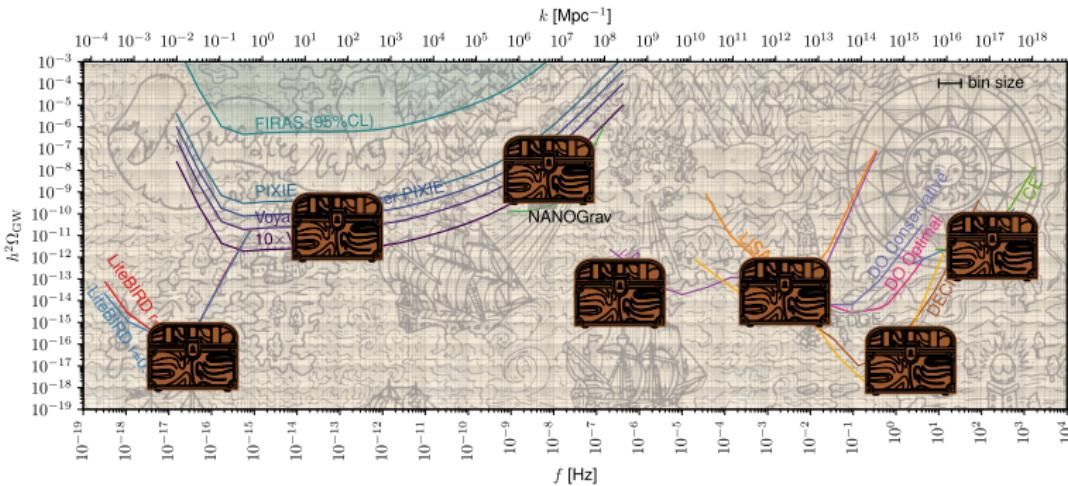
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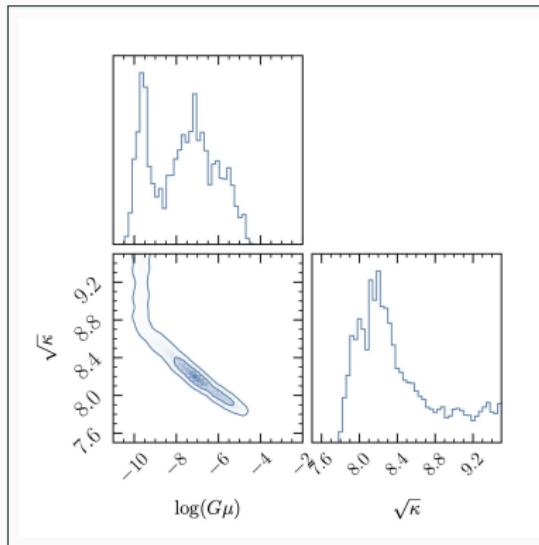


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Outlook

Preliminary results!!! IPTA DR2 search for metastable strings



On-going analyses

- **IPTA DR2:** Search for GWs from cosmic strings (stable, metastable, ...)
- **IPTA DR2:** Search for GWs from a cosmological phase transition
- **NANOGrav 15-year data:** Search for a whole range of exotic GW sources

Stay tuned!

And thanks a lot for your attention