Math3202 Assignment1-Wonder Market

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1 Section A: Report to boss

Sets

I	Set of stores
J	Set of distribution centres
S	Set of Scenarios

Data

C_{ji}	Cost of transporting one truckload from distribution centre $j \in J$ to store $i \in I$
M_j	Maximum capacity at the distribution centres $j \in J$
D_i	Current weekly demand at each stores $i \in I$
$MaxN_j$	Maximum capacity at the distribution centres on the north side of the river $j \in J$
$IsNorth_j$	True if distribution centre j is on the north side of river $j \in J$
	False if distribution centre j is not on the north side of river $j \in J$
DS_{si}	Demand at each store of each scenarios $s \in S$, at each stores $i \in I$

Variables

x_{ji}	Proportion of truckload amount transported from distribution centre $j \in J$ to store in $i \in I$

Objective

$$\min \sum_{j \in J} \sum_{i \in I} C_{ji} x_{ji} D_i$$

Constraints

$$\sum_{j \in J} x_{ji} = 1 \qquad \forall i \in I \tag{1}$$

$$\sum_{i \in I} x_{ji} D_i \leq M_j \qquad \forall j \in J \tag{2}$$

$$\sum_{\substack{j \in J i \in I \\ IsNorth_j}} x_{ji} D_i \leq MaxN_j \qquad \forall i \in I \tag{3}$$

$$\sum_{i \in I} x_{ji} DS_{si} \leq M_j \qquad \forall j \in J, \forall s \in S \tag{4}$$

$$\sum_{\substack{j \in J i \in I \\ IsNorth_j}} x_{ji} DS_{si} \leq MaxN_j \qquad \forall i \in I, \forall s \in S \tag{5}$$

$$x_{ji} \geq 0 \qquad \forall j \in J, \forall i \in I \tag{6}$$

Constraint (1) says that each store should be transported 100 percentages of the truckloads from Distribution Centres

Constraint (2) and (4) say that the amount transported from distribution centre must not exceed its capacity neither current weekly demand nor 5 scenarios

Constraint (3) and (5) say that the amount transported from distribution centre which is on the north side of the river must not exceed its maximum capacity at those distribution centres neither current weekly demand nor 5 scenarios

Constraint (6) enforces the stores cannot be transported negative truckloads.

Comments

Communication 1

Based on the cost of truckload of each distribution centre to each store, and the current weekly demand at each store provided by client, we provide them with the optimal transport cost for meeting their demand needs

The optimal transport cost is: $155758/week

Communication 2

Based on the capacity truckloads of each distribution centre updated by client, we provide them with the optimal transport cost for meeting their demand needs
The optimal transport cost is: \$165603/week

Communication 3

Base on the constraint updated by client that the total capacity truckloads of two distribution centres(DC1 and DC2) which is on the north side of river is restricted to 95 truckloads due to sharing a labour pool, we provide them with the optimal transport cost for meeting their demand needs
The optimal transport cost is: \$174931/week

Communication 4

Table 1 next page shows that the proportion of truckloads amount from each distribution centre to each store could be controlled by client to avoid ”reduce cost”

For example, In the third row, the proportion of truckloads amount is 100% which is greater that 0% and it is a basis variable. The transporting cost (objective value) \$8988 can not be decreasing as the ”reduced cost” \$0. Because DC0 has sent 100% of truckloads to S2.

On the other hand, in the first row, 0% of proportion of truckloads amount transported from DC0 to S0, and it is a non-basis variable. When transporting cost (objective value) \$37788.0 is decreasing as the ”reduced cost” \$10176.0. Because DC0 will start transporting truckloads to S0.

Sensitivity Analysis for Primal Variables						
S / D	**Proportion**	**Reduce Cost**	**Cost**	**Objective**	**Lower Bound**	**Upper Bound**
[0, 0]	0.0	10176.0	3149	37788.0	27612.0	1e+100
[0, 1]	0.0	16982.0	3761	52654.0	35672.0	1e+100
[0, 2]	1.0	0.0	1498	8988.0	-1e+100	10110.0
[0, 3]	0.0	22620.0	3592	71840.0	49220.0	1e+100
[0, 4]	0.0	17850.0	3950	55300.0	37450.0	1e+100
[0, 5]	0.0	20880.0	2385	35775.0	14895.0	1e+100
[0, 6]	0.0	12369.0	2522	17654.0	5285.0	1e+100
[0, 7]	0.0	19248.0	2976	47616.0	28368.0	1e+100
[0, 8]	0.0	10392.0	3691	29528.0	19136.0	1e+100
[0, 9]	1.0	0.0	616	4312.0	-1e+100	17178.0
[1, 0]	0.0	7128.0	2895	34740.0	27612.0	1e+100
[1, 1]	0.79	0.0	2548	35672.0	23142.0	35952.0
[1, 2]	0.0	1122.0	1685	10110.0	8988.0	1e+100
[1, 3]	0.0	11880.0	3055	61100.0	49220.0	1e+100
[1, 4]	0.0	1498.0	2782	38948.0	37450.0	1e+100
[1, 5]	1.0	0.0	993	14895.0	-1e+100	35775.0
[1, 6]	1.0	0.0	755	5285.0	-1e+100	17654.0
[1, 7]	0.0	8800.0	2323	37168.0	28368.0	1e+100
[1, 8]	0.0	160.0	2412	19296.0	19136.0	1e+100
[1, 9]	0.0	12866.0	2454	17178.0	4312.0	1e+100
[2, 0]	1.0	0.0	1406	16872.0	-1e+100	24000.0
[2, 1]	0.21	0.0	1653	23142.0	22862.0	35672.0
[2, 2]	0.0	6984.0	1767	10602.0	3618.0	1e+100
[2, 3]	1.0	0.0	1566	31320.0	-1e+100	43200.0
[2, 4]	1.0	0.0	1780	24920.0	-1e+100	26418.0
[2, 5]	0.0	20925.0	1493	22395.0	1470.0	1e+100
[2, 6]	0.0	12628.0	1664	11648.0	-980.0	1e+100
[2, 7]	1.0	0.0	878	14048.0	-1e+100	22848.0
[2, 8]	1.0	0.0	1497	11976.0	-1e+100	12136.0
[2, 9]	0.0	20209.0	2608	18256.0	-1953.0	1e+100

Table 1: The primal variables by sensitivity analysis

Table 2 below shows the optimal capacity of each distribution centres for current weekly demand and five scenarios

For Scenario 2 and Scenario 3, the slack value of DC0 of them are 0, which means that increasing one unit of capacity will increase the cost of transporting by 696.24 and 4188.1, respectively.

For other scenarios, the slack value of each distribution is not 0, which means that increasing one unit of capacity of each distribution will not influence the cost of transporting, because the dual values are 0, the basis we get would remain optimal

Sensitivity Analysis for Dual Variables						
	Distribution Centre	**Capacity**	**Slack**	**Dual Variable**	**Lower Bound**	**Upper Bound**
Current	DC0	50.0	23.49	0.0	26.51	1e+100
	DC1	50.0	27.49	0.0	22.51	1e+100
	DC2	73.0	3.03	0.0	69.97	1e+100
Scenario 0	DC0	50.0	2.0	0.0	48.0	1e+100
	DC1	50.0	26.22	0.0	23.78	1e+100
	DC2	73.0	1.78	0.0	71.22	1e+100
Scenario 1	DC0	50.0	2.82	0.0	47.18	1e+100
	DC1	50.0	26.16	0.0	23.84	1e+100
	DC2	73.0	3.03	0.0	69.97	1e+100
Scenario 2	DC0	50.0	0.0	-696.24	48.0	50.58
	DC1	50.0	26.97	0.0	23.03	1e+100
	DC2	73.0	3.03	0.0	69.97	1e+100
Scenario 3	DC0	50.0	0.0	-4188.1	49.0	50.42
	DC1	50.0	25.97	0.0	24.03	1e+100
	DC2	73.0	3.03	0.0	69.97	1e+100
Scenario 4	DC0	50.0	6.0	0.0	44.0	1e+100
	DC1	50.0	27.49	0.0	22.51	1e+100
	DC2	73.0	0.51	0.0	72.49	1e+100

Table 2: The dual variables by sensitivity analysis

2 Section B: Report to client

Communication 1

Dear Customer,

Table 3 shows the optimal transport of goods as follow:

Table 4 shows that the proportion from each distribution centre as follow:

the estimated optimal transport cost is $155758/week.

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
DC0			**6.0**							**7.0**
DC1						**15.0**	**7.0**			
DC2	**12.0**	**14.0**		**20.0**	**14.0**			**16.0**	**8.0**	
Total Cost	16872	23142	8988	31320	24290	14895	5285	14048	11976	4312
Optimal Transport cost: $155758										

Table 3: The optimal transport of goods in without capacity, in truckloads

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
DC0			**100%**							**100%**
DC1						**100%**	**100%**			
DC2	**100%**	**100%**		**100%**	**100%**			**100%**	**100%**	

Table 4: The optimal transport of truckloads, in proportion

The cost of transporting from DC2 to S0 S1 S3 S3 S7 and S8 is the cheapest;

The cost of transporting from DC0 to S2 and S9 is the cheapest;

The cost of transporting from DC1 to S5 and S6 is the cheapest.

So, we recommend the optimal truckloads transporting from that distribution centre to avoid a higher cost of transporting from other distribution centres.

Sincerely,

JiaHao Wu, Ling Xu

Communication 2

Dear Customer,

Table 5 shows each distribution center has limited capacity, the new optimal transport of goods are as follows:

Table 6 shows that the proportion from each distribution centre as follow:

the estimated optimal transport cost is $165603/week.

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
DC0			**6.0**							**7.0**
DC1		**11.0**				**15.0**	**7.0**			
DC2	**12.0**	**3.0**		**20.0**	**14.0**			**16.0**	**8.0**	
Total Cost	16872	32987	8988	31320	24920	14895	5285	14048	11976	4312
optimal transport cost: $165603.0										

Table 5: The optimal transport of goods in limited capacity, in truckloads

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
DC0			**100%**							**100%**
DC1		**79%**				**100%**	**100%**			
DC2	**100%**	**21%**		**100%**	**100%**			**100%**	**100%**	

Table 6: The optimal transport of truckloads, in proportion

There is some change of the solution because for DC2, if we insist on the original plan, then the total truckloads would be 84, which is higher than the maximum capacity of 73. So we have to allocate some transportation job from DC2 to DC1 or DC0.

In the currently transported by DC2, we have S0,S1,S3,S4,S7,S8, totally six choices. We select to allocate S1 from DC2 to DC1 by 11 truckloads. Because the dual cost of each truckloads from DC2 to DC1 is just $895, which is the lowest cost per truck load among these six choices and adding this 11 truckloads to DC1 will not make the DC1 exceeds its maximum capacity.

Due to the limited capacity of each distribution center, we recommend transporting as much as possible of goods from DC1 to S1 to avoid a high cost of transporting.

Sincerely,

JiaHao Wu, Ling Xu

Communication 3

Dear Customer,

Table 7 shows the limited capacity for distribution centres at the north side of river, the new optimal transport of goods are as follows:

Table 8 shows that the proportion from each distribution centre as follow:

the estimated optimal transport cost is $174931/week.

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
DC0	**11.0**		**6.0**							**7.0**
DC1						**15.0**	**7.0**			
DC2	**1.0**	**14.0**		**20.0**	**14.0**			**16.0**	**8.0**	
Total Cost	36045	32987	8988	31320	24920	14895	5285	14048	11976	4312
optimal transport cost: $174931.0										

Table 7: The optimal transport of truckloads in limited capacity at the north side of river

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
DC0	**92%**		**100%**							**100%**
DC1						**100%**	**100%**			
DC2	**8%**	**100%**		**100%**	**100%**			**100%**	**100%**	

Table 8: The % of optimal transport of goods in limited capacity at the north side of river

We made this adjustment because the truckloads from DC1 and DC2 last communication was total 106 which is greater than their total capacity of 95. So firstly we have to reduce 11 truckload from DC1 or DC2 to DC0. There are S0,S1,S3,S4,S5,S6,S7,S8 totally 8 choices to shift to DC0. We choose to shift 11 truckload from distributed by DC2 to distributed by DC0 of S0. This is because shift from DC2 to DC0 of the S0 just raise the cost per truckload to $1743 which is the lowest extra cost in these 8 choices.

Secondly, since we had shifted 11 truckload from DC2 to DC0, now the total truckload of DC2 is 73. This is now under the maximum capacity of DC2. Since in communication one, we shift some capacity from DC2 to DC1 due to DC2 exceeded its capacity and DC1's cost is generally higher than DC2, now we can move back some capacity from DC1 to DC2 or from DC1 to DC0. We choose to move 11 truckload from DC1 to DC2 for S1. Because among S1,S5,S6 these three choices that was distributed by DC1, if we choose to move truckload from DC1 to DC2 for S1. That will reduce our cost per truckload by $895, which is the maximum cost reduction that we can reduce per truckload from DC1 to other distribution centre.

Due to the limited capacity of these two distribution centers on the north side of the river, we recommend transporting as much as possible of truckloads from DC0 to S0 and then move the truckload from DC1 to DC2 for S1 back to avoid a high cost of transporting.

Sincerely,
JiaHao Wu, Ling Xu

Communication 4

Dear Customer,

Table 9 shows that for the situation we need to take surges in demand, the proportion of transport demand from each distribution centre to each store should be the same as it in each scenarios

Table 10 shows that the optimal transport in current demand and each Scenario for comparing and find the surge in demand

The estimated optimal transport cost is $ 183703.32.

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
DC0	59%		94%						87%	98%
DC1			6%			100%	100%			2%
DC2	41%	100%		100%	100%			100%	13%	

Table 9: The proportion of transporting from each distribution centre to each store

	S / DC	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
Current Weekly Demand	DC0	7.03	0.0	5.64	0.0	0.0	0.0	0.0	0.0	6.99	6.85
	DC1	0.0	0.0	0.36	0.0	0.0	15.0	7.0	0.0	0.0	0.15
	DC2	4.97	14.0	0.0	20.0	14.0	0.0	0.0	16.0	1.01	0.0
Scenario0	DC0	8.79	0.0	25.37	0.0	0.0	0.0	0.0	0.0	6.99	6.85
	DC1	0.0	0.0	1.63	0.0	0.0	15.0	7.0	0.0	0.0	0.15
	DC2	6.21	14.0	0.0	20.0	14.0	0.0	0.0	16.0	1.01	0.0
Scenario1	DC0	7.03	0.0	26.31	0.0	0.0	0.0	0.0	0.0	6.99	6.85
	DC1	0.0	0.0	1.69	0.0	0.0	15.0	7.0	0.0	0.0	0.15
	DC2	4.97	14.0	0.0	20.0	14.0	0.0	0.0	16.0	1.01	0.0
Scenario2	DC0	7.03	0.0	5.64	0.0	0.0	0.0	0.0	0.0	6.99	30.34
	DC1	0.0	0.0	0.36	0.0	0.0	15.0	7.0	0.0	0.0	0.66
	DC2	4.97	14.0	0.0	20.0	14.0	0.0	0.0	16.0	1.01	0.0
Scenario3	DC0	7.03	0.0	29.12	0.0	0.0	0.0	0.0	0.0	6.99	6.85
	DC1	0.0	0.0	1.88	0.0	0.0	15.0	7.0	0.0	0.0	0.15
	DC2	4.97	14.0	0.0	20.0	14.0	0.0	0.0	16.0	1.01	0.0
Scenario4	DC0	7.03	0.0	5.64	0.0	0.0	0.0	0.0	0.0	24.48	6.85
	DC1	0.0	0.0	0.36	0.0	0.0	15.0	7.0	0.0	0.0	0.15
	DC2	4.97	14.0	0.0	20.0	14.0	0.0	0.0	16.0	3.52	0.0

Table 10: The optimal transport in current demand and each Scenario, in truckloads

Due to the surge in demand(S0, S2, S8 and S9), different distribution centre does not have enough capacity which will result in an increase in truckloads transporting by the other distribution centres, as the cost of transporting increasing.

We recommend transporting as much as possible of goods from the other distribution to avoid high cost of transporting. We hope what we estimate could help you make decisions.

Sincerely,

JiaHao Wu, Ling Xu