Math 3202

Operations Research

Assignment 1

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Seciton A

Sets

- ***Ports*** = [Brisbane, Sydney, Melbourne]
- ***P*** = [1, 2, 3]
- ***Quarter*** = [Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8]
- ***Q*** = [1, 2, 3, 4, 5, 6, 7, 8]

A port set includes three ports that raw material is imported in. P set is the number representation of those three ports. We can use those numbers to build matrix index.

A quarter set includes eight quarters. Q set is the number representation of those eight quarters. Similarly we can use those numbers as the matrix index later.

Data

1460 1080 950

830 990   1170

940   960   1430

1240 1120 1180

D *=830   1060 1410*

1590 1030 1451

1450 960 1590

1110 1410 890

- ***C*** = [65 99 96 66 70 61 66 89]
- ***ShipCapcity*** = 5000
- ***InitialAmount*** = [1500 2100 1800]
- ***StoreCapacity*** = [1500 3400 2300]

D is actually an 8 X 3 matrix containing the demanding amount in each of three ports and for each of those eight quarters.

C is the cost of import for each quarter.

ShipCapacity is the ship's importing capacity which is 5000 tonnes.

InitialAmount is how much material is left at the beginning for each port

StoreCapacity is the maximum amount that material can be stored in each port for every quarter.

Variable

- I_{qp} is the imported amount for p port in q quarter. (all p in set P and all q in set Q)
- S_{qp} is the stored material for p port in q quarter. (all p in set P and all q in set Q)

Constraints

1) Constraint for stored amount after the first quarter

$$S_{01} = InitialAmount_0 + I_{01} - D_{01}$$

$$S_{02} = InitialAmount_1 + I_{02} - D_{02}$$

$$S_{03} = InitialAmount_2 + I_{03} - D_{03}$$

- S_{0p} is the stored amount after the first quarter for p port. (all p in set P)

- $InitialAmount_p$ is the initial amount stored in port p at the beginning. (all p in set P)
- I_{0p} is the imported amount for each p on the first quarter. (all p in set P)
- D_{0p} is the demand for port p in the first quarter. (all p in set P)

Hence this constraint ensures that S_{0p} must equal to the $InitialAmount_p$ plus the I_{0p} and minus D_{0p} for all p in set P.

2) Constraint for stored amount after the first quarter

$$S_{qp} = S_{(q-1)p} + I_{qp} - D_{qp} \text{ for } q \geq 1 \text{ and } q \in Q \text{ and } p \in P$$

This constraint ensures that after the first quarter, the stored amount for each following quarter must equal to the sum of stored amount from last quarter for each port ($S_{(q-1)p}$) and imported amount for each quarter in each port (I_{qp}) minus the demanded amount for each quarter in each port (D_{pq}).

3) Constraint for demanded amount after the first quarter

$$I_{qp} + S_{(q-1)p} \geq D_{qp} \text{ for } q \in Q \text{ and } p \in P$$

This constraint ensures that the sum of imported amount for each quarter in each port (I_{qp}) must be greater or equal to demanded amount for each quarter in each port (D_{pq}).

4) Constraint for ship capacity

$$\sum_{q=1}^{8} \sum_{p=1}^{3} I_{qp} \leq ShipCapacity \text{ for } q \in Q \text{ and } p \in P$$

This constraint ensures that the total imported amount for each quarter and every port must be smaller or equal to the ship capacity.

Objective

$$Minimise \sum_{q=1}^{8}\sum_{p=1}^{3}\left(I_{qp}C_q + 1.5S_{qp}\right)$$

The objective is to minimise this cost function because this is the total amount of money paid for all quarters.

Slack Variables

Slack variable is used to form the matrix operation by adding the slack variable to those inequality equations. The slack variable also determines if one of the constraints has reached its limit so that there is no more improvement in this constraint.

For example, in constraint 3),

$$I_{qp} + S_{(q-1)p} \geq D_{qp}\ for\ q\ \in Q\ and\ p\ \in P,$$

a slack variable $Slack_{qp}$ is added into the constraint to make

$$I_{qp} + S_{(q-1)p} + \boldsymbol{Slack_{qp}} = D_{qp}\ for\ q\ \in Q\ and\ p\ \in P.$$

In this problem, we are trying to minimise both $\mathrm{I_{qp}}$ and $\mathrm{S_{(q-1)p}}$ which means the slack variables will be kept as large as possible. Then, after the optimisation is done, slack variable tells us how much more amount could we still import or store if we do not consider to minimise both $\mathrm{I_{qp}}$ and $\mathrm{S_{(q-1)p}}$.

Seciton B

Additional Constraint

1) Constraint on stored amount after last quarter

Currently, we do not have constraints on the stored amount after the eighth quarter. That will cause the stored amount for each port to be as small as possible after the eighth quarter.

As client requests the stored amount after eighth quarter for each port has to be the same as the initial stored amount at the beginning of the first quarter. This will definitely affect optimised result since we might have to import more materials at the eighth quarter than without this constraint.

2) Constraint on stored amount for each quarter

Also, we do not have constraints on the storing capacity for each port in each quarter. That means we can store as much as we need to optimise the result.

However, As the client responses *"We have a maximum storage capacity of 1500 tonnes in Brisbane, 3400 tonnes in Sydney and 2300 tonnes in Melbourne",* those constraints will also lower the stored amount in each port for each quarter comparing to the stored amount without those constraints and hence change the optimised result.

Solution for Additional Constraints

1) Solution for additional constraint 1)

$$S_{7p} = InitialAmount_p \ for\ p \ \in P$$

In order to fulfil the client request, one more constraint is added to ensure that at the end of eighth quarter, the left over amount is equal to the initial amount at the beginning of the first quarter. Before this constraint is added in, objective value is $1576270 and imported values and stored values are presented in the following tables

- **Imported amount**

Quarters/Ports	Brisbane	Sydney	Melbourne
Quarter 1	1730	930	1750
Quarter 2	0	0	0
Quarter 3	0	0	0
Quarter 4	1240	1950	1810
Quarter 5	2460	230	1560
Quarter 6	0	3400	1600
Quarter 7	2520	0	2480
Quarter 8	0	0	0

- **Stored amount**

Quarters/Ports	Brisbane	Sydney	Melbourne
Quarter 1	1770	1950	2600
Quarter 2	940	960	1430
Quarter 3	0	0	0
Quarter 4	0	830	0
Quarter 5	1630	0	150
Quarter 6	40	2370	0
Quarter 7	1110	1410	890
Quarter 8	0	0	0

But after the constraint is added in, the minimised value becomes $2046130 which means we have to import or store more materials to satisfy this constraint. The imported and stored amounts are presented below.

- **Imported amount**

Quarters/Ports	Brisbane	Sydney	Melbourne
Quarter 1	1730	***1520***	1750
Quarter 2	0	0	0
Quarter 3	0	0	0
Quarter 4	1240	***1590***	***2170***
Quarter 5	***3950***	***0***	***1050***
Quarter 6	0	***1030***	***3970***
Quarter 7	***0***	***4470***	***530***
Quarter 8	***2530***	***0***	***1530***

- **Stored amount**

Quarters/Ports	Brisbane	Sydney	Melbourne
Quarter 1	1770	***2540***	2600
Quarter 2	940	***1550***	1430
Quarter 3	0	***590***	0
Quarter 4	0	***1060***	***360***
Quarter 5	***3120***	0	***0***
Quarter 6	***1530***	***0***	***2220***
Quarter 7	***80***	***3510***	***1160***
Quarter 8	***1500***	***2100***	***1800***

The value which is different from the previous one has been highlighted.

2) Solution for additional constraint 2)

$$S_{qp} \leq StoreCapacity_p \text{ for } q \geq 1 \text{ and } q \in Q \text{ and } p \in P$$

This ensures S_{qp} must be smaller or equal to the store capacity for each port in every quarter. As this constraint is added, the object value becomes \$2054395 which is more than what we have before this constraint added which is \$2046130.

- **Imported amount**

Quarters/Ports	Brisbane	Sydney	Melbourne
Quarter 1	***1460***	***2090***	***1450***
Quarter 2	0	0	0
Quarter 3	***270***	0	***300***
Quarter 4	***2740***	***450***	***1810***
Quarter 5	***0***	***1290***	***3710***
Quarter 6	***1290***	***3710***	***0***
Quarter 7	***1080***	***960***	***2960***
Quarter 8	***2610***	***110***	***770***

- **Stored amount**

Quarters/Ports	Brisbane	Sydney	Melbourne
Quarter 1	***1500***	***3110***	***2300***
Quarter 2	***670***	***2120***	***1130***
Quarter 3	0	***1160***	0
Quarter 4	***1500***	***490***	***0***
Quarter 5	***670***	***720***	***2300***
Quarter 6	***370***	***3400***	***550***
Quarter 7	***0***	***3400***	***1920***
Quarter 8	1500	2100	1800

The value which is different from the previous one has been highlighted.