MATH3202/7232 Operations Research & Mathematical Planning 2017

Assignment 3 – Optimal Fertiliser Application

This assignment is due by 6pm on Friday, May 26th and is worth 10% of your final grade. You can do each assignment in pairs, with a single submission.

Impressed by your previous work, **Relit Fires** has now engaged you as a consultant to help their customers improve the application of nitrogen fertiliser products over time.

One customer grows forage crops, such as hay, and it is known that plant-available nitrogen and weather conditions are important factors influencing forage yields. A previous study has used a regression model to estimate that the hay yield (pounds per acre) in a particular year t is given by

$$\begin{aligned} Y_t \;=\; & -476.25 - 5.5042N_t + 245.049W_t + 0.00433N_t^2 - 9.9584W_t^2 \\ & +2.4524N_tW_t - 0.00336N_t^2W_t - 0.03314N_tW_t^2 \end{aligned}$$

where N_t is the nitrogen (pounds per acre) available to the plants in year t and W_t is the growing season precipitation (inches) in year t. The value of the yield is \$0.01 per pound.

The available nitrogen, N_t, is the sum of the nitrogen applied using fertiliser in year t, denoted by A_t, and the nitrogen already in the soil at the start of year t, denoted by R_t. It costs the grower \$0.10 per pound of nitrogen plus an application cost of \$1.00 per acre if any nitrogen is used in a year. The nitrogen retained in the soil from one growing season to the next is estimated by

$$R_t = 3.9209(A_{t-1} + R_{t-1})^{1.0405}W_{t-1}^{-1.1566}$$

The grower would like your recommendations for the following questions:

1. You are given historical precipitation data for a 10-year period. Supposing the grower had known these values in advance, what would have been the optimal fertiliser application to maximise profit? The grower can then compare this 'oracle' solution with her existing practice to see how dynamic programming might improve her production.

2. Given forecast probabilities for future precipitation levels, what is the optimal fertiliser application strategy to pursue?

It is reasonable for your model to consider integer values of A_t and R_t only, rounding the value of R_t as needed.

Precipitation data and forecasts will be provided by Relit Fires.

Submission

You will need to submit the following through Blackboard:

- A general formulation that describes the states and return function used in your two models. *6 marks*
- A single Python file with your implementations. This should be easy to relate back to the formulation. We will attempt to execute this. *8 marks*
- A brief report addressing the two questions above. *6 marks*