Name: KEY Block: _____

Precalculus | Semester 1 | Review

Perform the operation and write the result in standard form.

1. $(-8 - 3i) + (-1 - 15i)$

$-9-18i$

2. $(10 + \sqrt{-20}) - (4 - \sqrt{-45})$

$10 + 2i\sqrt{5} - 4 + 3i\sqrt{5}$

$6 + 5i\sqrt{5}$

3. $7i(2 - 3i) = 14i - 21i^2$

$= 21 + 14i$

4. $(2 + i)(6 - i) = 12 - 2i + 6i - i^2$

$= 12 - 2i + 6i + 1$

$= 13 + 4i$

5. $(4 + 3i)^2 - (5 + i)^2 = (16 + 24i + 9i^2) - (25 + 10i + i^2)$

$= (7 + 24i) - (24 + 10i)$

$= -17 + 14i$

6. $\frac{8 + 5i}{i} \cdot \frac{(i)}{(i)} = \frac{8i + 5i^2}{i^2} = \frac{8i - 5}{-1} = 5 - 8i$

7. $\frac{5i}{2 - i} \cdot \frac{(2+i)}{(2+i)} = \frac{10i + 5i^2}{4 - i^2} = \frac{-5 + 10i}{5} = -1 + 2i$

8. $(2i - 1) \div (3i + 2)$

$\frac{2i-1}{3i+2} \cdot \frac{(3i-2)}{(3i-2)} = \frac{6i^2 - 4i - 3i + 2}{9i^2 - 4} = \frac{-4 - 7i}{-13} = \frac{4}{13} + \frac{7i}{13}$

Divide using the indicated method.

9. Long Division: $(3x^3 + 4x - 1) \div (x^2 + 1)$

$x^2+0x+1 \;\overline{)\; 3x^3+0x^2+4x-1}$ → $3x + \frac{x-1}{x^2+1}$

$-3x^3+0x^2-3x$

$x-1$

10. Synthetic Division: $(2x^4 - 5x^2 - 3) \div (x - 2)$

2	2	0	−5	0	−3
		4	8	6	12
	2	4	3	6	9

$2x^3 + 4x^2 + 3x + 6 + \frac{9}{x-2}$

Solve the quadratic equation using any applicable method.

11. $x^2 + 54 = 0$

$x^2 = -54$

$x = \pm\sqrt{-54}$

$x = \pm 3i\sqrt{6}$

12. $x^2 - 2x + 8 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(8)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-28}}{2} = \frac{2 \pm 2i\sqrt{7}}{2} = 1 \pm i\sqrt{7}$

Find all zeros for each function. Write the polynomial as a product of linear factors.

13. $g(t) = 2t^4 - 3t^3 + 16t - 24$

−2	2	−3	0	16	−24
		−4	14	−28	24
3/2	2	−7	14	−12	0
		3	−6	12	
	2	−4	8	0	

FREE GCF $2x^2 - 2x + 4$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$

Z: $x = -2, 3/2, 1 \pm i\sqrt{3}$

F: $(x+2)(2x-3)(x-1-i\sqrt{3})(x-1+i\sqrt{3})$

14. $h(x) = 3x^5 + 2x^4 - 3x - 2$

Z: $x = -1, 1, \pm i, -\frac{2}{3}$

F: $(x+1)(x-1)(x-i)(x+i)(3x+2)$

−1	3	2	0	0	−3	−2
		−3	1	−1	1	2
1	3	−1	1	−1	−2	0
		3	2	3	2	
−2/3	3	2	3	2	0	
		−2	0	−2		
	3	0	3	0		

$3x^2 + 3 = 0 \rightarrow x^2 = -1 \rightarrow x = \pm i$

15. $f(x) = 4x^3 - 12x^2 + 9x$

$x(4x^2 - 12x + 9) = 0$

F: $x(2x-3)(2x-3)$

Z: $x = 0, 3/2$

16. $f(x) = x^3 - 7x^2 + 11x + 19$

−1	1	−7	11	19
		−1	8	−19
	1	−8	19	

$x^2 - 8x + 19$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(19)}}{2(1)}$

$x = \frac{8 \pm \sqrt{-12}}{2} = \frac{8 \pm 2i\sqrt{3}}{2} = 4 \pm i\sqrt{3}$

Z: $x = -1, 4 \pm i\sqrt{3}$

F: $(x+1)(x-4-i\sqrt{3})(x-4+i\sqrt{3})$

Use the zero / root feature of a graphing utility to approximate the real zeros of the function and extrema of the functions.

17. $f(x) = x^4 - x^3 - 1$

Zeros: $x = -0.819, 1.380$

Min: $(0.750, -1.150)$

Max: no max

18. $f(x) = 3x^5 + 2x^4 - 12x - 8$

Zeros: $x = -1.414, -0.667, 1.414$

Min: $(0.836, -15.830)$

Max: $(-1.113, 3.301)$

19. Identify the vertex and intercepts of the graph of $y = x^2 + 5x + 6$.

Vertex: $\left(-\frac{5}{2}, -\frac{1}{4}\right)$

$x = \frac{-b}{2a} = \frac{-5}{2(1)} = -\frac{5}{2}$

$y = \left(-\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) + 6 = -\frac{1}{4}$

Intercepts: $(-3, 0)$ $(-2, 0)$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3, -2$

20. Sketch the graph of $f(x) = -x^3 + 7x + 6$.

$-1 \mid -1 \;\; 0 \;\; 7 \;\; 6$

$\quad\quad\;\; 1 \;\; -1 \;\; -6$

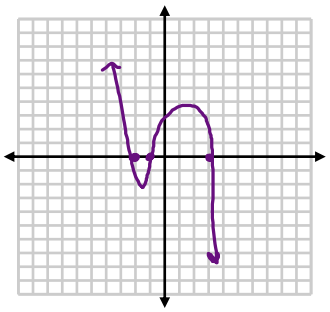
$\quad -1 \;\; 1 \;\; 6 \;\; 0$

$(x+1)(-x^2 + x + 6) = 0$

$-(x+1)(x^2 - x - 6) = 0$

$-(x+1)(x-3)(x+2) = 0$

$x = -1, 3, -2$



Solve the inequality and graph the solution on the number line. State your solution in interval notation.

21. $x^3 + x^2 < 6x$

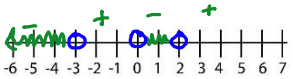
$x^3 + x^2 - 6x < 0$ (−)

$x(x^2 + x - 6) = 0$

$x(x+3)(x-2) = 0$

$x = 0, -3, 2$

$(-\infty, -3) \cup (0, 2)$



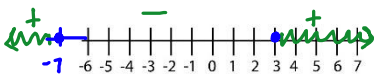
22. $x^2 + 4x \geq 21$

$x^2 + 4x - 21 \geq 0$ (+)

$(x+7)(x-3) \geq 0$

$x = -7, 3$

$(-\infty, -7] \cup [3, \infty)$



23. $x^3 + 18 > 4x^2 + 3x$

$x^3 - 4x^2 - 3x + 18 > 0$ (+)

$-2 \mid 1 \;\; -4 \;\; -3 \;\; 18$

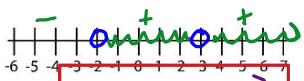
$\quad\quad -2 \;\; 12 \;\; -18$

$\quad\quad 1 \;\; -6 \;\; 9 \;\; 0$

$(x+2)(x^2 - 6x + 9) = 0$

$(x+2)(x-3)(x-3)$

$x = -2, 3$



$(-2, 3) \cup [3, \infty)$

Solve each inequality. Graph each solution set on the provided grid.

24. $f(x) \leq x^3 - 4x^2 - 7x + 10$

$-2 \mid 1 \;\; -4 \;\; -7 \;\; 10$

$\quad\quad -2 \;\; 12 \;\; -10$

$\quad\quad 1 \;\; -6 \;\; 5 \;\; 0$

$(x+2)(x^2 - 6x + 5) = 0$

$(x+2)(x-5)(x-1) = 0$

$x = -2, 5, 1$

$x = -\frac{1}{2}$

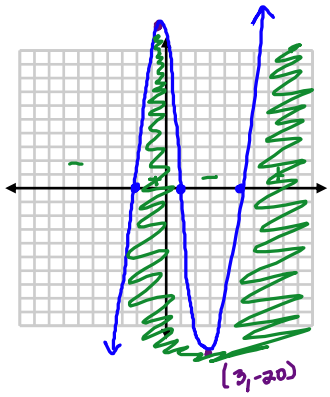
$y = (-\frac{1}{2})^3 - 4(-\frac{1}{2})^2 - 7(-\frac{1}{2}) + 10$

$y = 12.375$

$x = 3$

$y = 3^3 - 4(3)^2 - 7(3) + 10$

$y = -20$



25. $g(x) > x^3 + 5x^2$

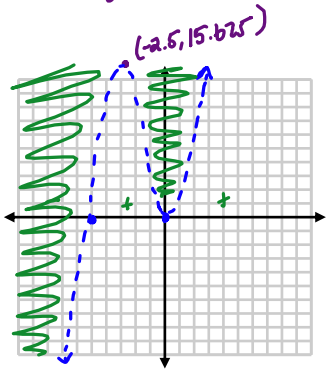
$x^2(x+5)$

$x = 0 \quad x = -5$

$x = -2.5$

$y = (-2.5)^3 + 5(-2.5)^2$

$y = 15.625$



26. $h(x) \leq x^3 - 12x - 16$

$-2 \mid 1 \;\; 0 \;\; -12 \;\; -16$

$\quad\quad -2 \;\; 4 \;\; 16$

$\quad\quad 1 \;\; -2 \;\; -8 \;\; 0$

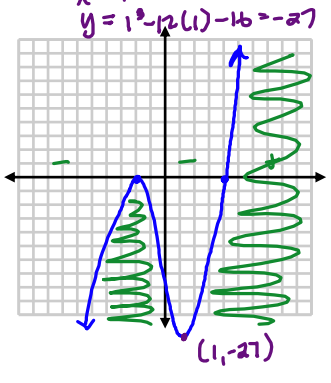
$(x+2)(x^2 - 2x - 8)$

$(x+2)(x-4)(x+2)$

$x = -2, 4, -2$

$x = 1$

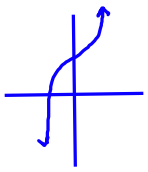
$y = 1^3 - 12(1) - 16 = -27$



Unit 2: Rational Functions

Using a graphing utility, verify that the inverse of the function exists. If it does exist, algebraically find the inverse. If it does not exist, place the proper restrictions on f so that the inverse can exist.

27. $f(x) = x^3 + 8$



passes HLT ✓

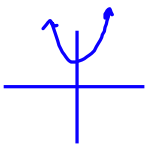
$x = y^3 + 8$

$y^3 = x - 8$

$y = \sqrt[3]{x-8}$

$f^{-1}(x) = \sqrt[3]{x-8}$

28. $f(x) = x^2 + 6$

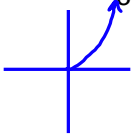


Fails HLT!

Restrictions:

$x \geq 0$ or $x \leq 0$

29. $f(x) = \frac{3x^{\frac{3}{2}}}{8}$



passes HLT ✓

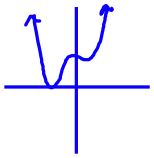
$x = \frac{3}{8} y^{\frac{3}{2}}$

$\frac{8x}{3} = y^{\frac{3}{2}}$

$y^3 = \left(\frac{8x}{3}\right)^2$

$f^{-1}(x) = \sqrt[3]{\left(\frac{8x}{3}\right)^2}$

30. $f(x) = x^4 - 3x^2 + 2x + 5$



Fails HLT!

Restrictions:

$x \leq -1.366$

$-1.366 \leq x \leq 0.366$

$0.366 \leq x \leq 1.00$

$x \geq 1$

any of these will work

Algebraically determine if the two functions are inverses of each other.

31. $f(x) = \frac{5}{2}x + 3$, $g(x) = \frac{2(x-3)}{5}$

$f(g(x)) = \frac{5}{2}\left(\frac{2(x-3)}{5}\right) + 3 = x - 3 + 3 = x$ ✓

$g(f(x)) = \frac{2\left(\frac{5}{2}x + 3 - 3\right)}{5} = \frac{2\left(\frac{5}{2}x\right)}{5} = \frac{5x}{5} = x$ ✓

Yes, Inverses!

32. $f(x) = 4x - 5$, $g(x) = \frac{x}{4} + 5$

$f(g(x)) = 4\left(\frac{x}{4} + 5\right) - 5 = x + 20 - 5 = x + 15 \neq x$

$g(f(x)) = \frac{4x-5}{4} + 5 = x - \frac{5}{4} + 5 \neq x$

No, not Inverses

Sketch the graph of the rational function. List all asymptotes, holes, intercepts, and at least one point per domain section.

33. $h(x) = \frac{4}{x^2} - 1 = \frac{4 - x^2}{x^2} = \frac{-x^2 + 4}{x^2}$

VA: $x = 0$

Hole: none

HA: $y = -1$

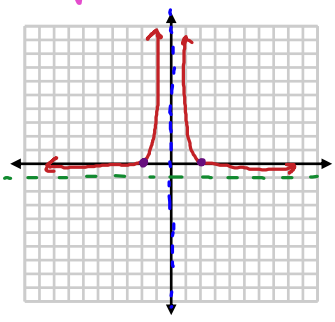
x-int: $(2,0)$ $(-2,0)$

y-int none

$-x^2 = -4$

$x^2 = 4$

$x = \pm 2$



34. $g(x) = \frac{x^2 - 2}{x - 1}$

VA: $x = 1$

HA: none

SA: $y = x + 1$

x-int: none

y-int: $(0, -2)$

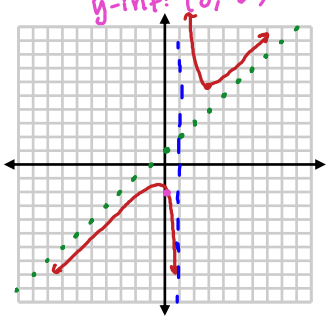
1 | 1 0 2

1 1 1

$x^2 = -2$

$x = \pm\sqrt{-2}$

x	y
3	$\frac{11}{2}$



35. $f(x) = \frac{2x^2 + 9}{5x^2 - 2}$

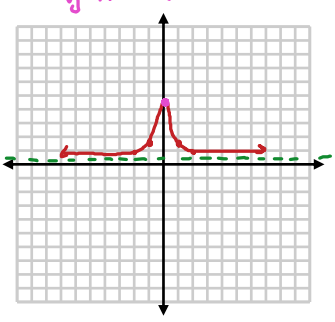
VA: none

HA: $y = \frac{2}{5}$

x-int: none

y-int: $(0, -9/2)$

x	y
-2	$\frac{17}{22}$
-1	$\frac{11}{7}$
1	$\frac{11}{7}$
2	$\frac{17}{22}$



Solve the inequality and graph the solution on the number line. State your solution in interval notation.

36. $\frac{1}{x} - x > 0$

$\frac{1-x^2}{x} > 0$ (+)

$\frac{-x^2+1}{x} > 0$

VA: $x=0$

Hole: none

x-int: $(1,0)$ $(-1,0)$

$-x^2+1=0$

$-x^2=-1$

$x^2=1$

$x=\pm 1$

$(-\infty,-1) \cup (0,1)$

37. $\frac{5x}{x^2-4} \leq 0$ (−)

VA: $x=-2, 2$

$x^2=4$

$x=\pm 2$

Hole: none

x-int: $(0,0)$

$(-\infty,-2) \cup [0,2)$

Solve each inequality. Graph each solution set on the provided grid.

38. $y \geq \frac{x+6}{x+1} - 2$

$y \geq \frac{x+6-2(x+1)}{x+1}$

$y \geq \frac{-x+4}{x+1}$ (−)

x	y
-3	-1/2
-1	2

VA: $x=-1$

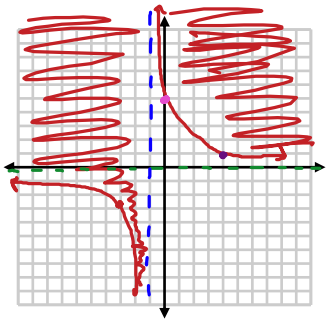
Hole: none

HA: $y=-1$

SA: none

x-int: $(4,0)$

y-int: $(0,4)$



39. $y < \frac{-2}{x^2+5x+4}$

$y < \frac{-2}{(x+4)(x+1)}$ (+)

VA: $x=-4, -1$

Hole: none

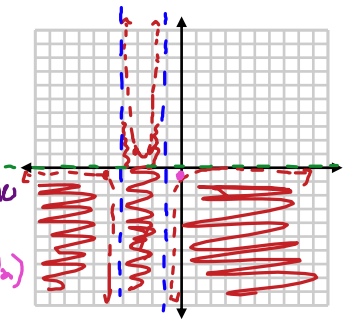
HA: $y=0$

SA: none

x-int: none

y-int: $(0,-1/2)$

x	y
-5	-1/2
-2	1
-3	1

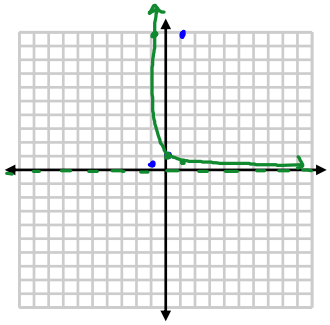


Unit 3: Exponential & Logarithmic Functions

Construct a table of values for the parent function. Then use transformations to graph the function. Identify the asymptote and turning point.

40. $f(x) = 10^{-x}$ flip y

x	y
-2	1/100
-1	1/10
0	1
1	10
2	100

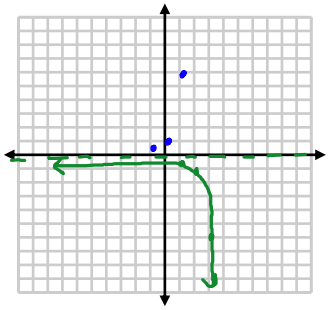


TP: $(0,1)$

Asy: $y=0$

41. $f(x) = -6^{x-2}$ flip x, right 2

x	y
-2	1/36
-1	1/6
0	1
1	6
2	36

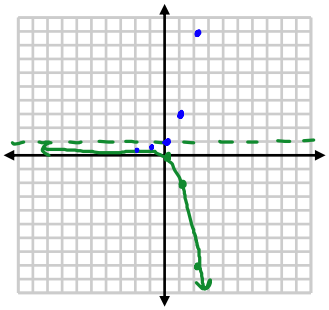


TP: $(2,-1)$

Asy: $y=0$

42. $f(x) = 1 - 3^x$ up 1, flip x

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9



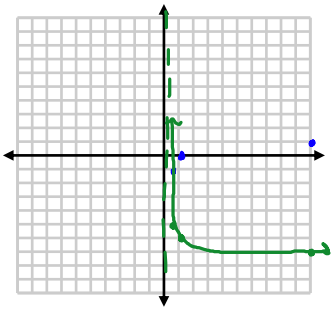
TP: $(0,0)$

Asy: $y=1$

43. $f(x) = -\log_{10} x - 6$ flip x, down 6

$x = 10^y$

x	y
1/100	-2
1/10	-1
1	0
10	1
100	2



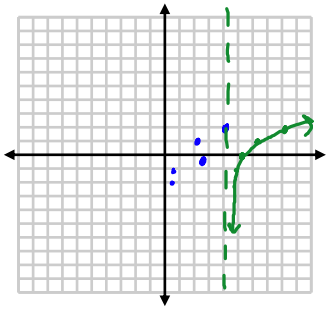
TP: $(1,-6)$

Asy: $x=0$

44. $f(x) = \log_2 (x-4)$ right 4

$x = 2^y$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2



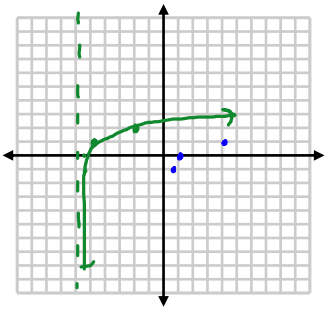
TP: $(5,0)$

Asy: $x=4$

45. $f(x) = 1 + \log_4 (x+6)$ up 1, left 6

$x = 4^y$

x	y
1/16	-2
1/4	-1
1	0
4	1
16	2



TP: $(-5,1)$

Asy: $x=-6$

Evaluate each expression. Round results to the nearest thousandth.

46. $\log_7 7^{-0.89}$ $= -0.89$

47. $4.6 \ln e^2$ $= 4.6(2) = 9.2$

48. $-\log_{10} 1000$ $-\log_{10} 10^3 = -3$

Evaluate each logarithm using the change of base formula. Round results to the nearest thousandth.

49. $\log_7 44 = \frac{\log 44}{\log 7} = 1.945$

50. $\log_{2/5} 1.3 = \frac{\log 1.3}{\log 2/5} = -0.286$

51. $\log_{12} 64 = \frac{\log 64}{\log 12} = 1.674$

Use the properties of logarithms to expand the expression as a sum, difference or multiple of logarithms.

52. $\log_2 3a^4$

$\log_2 3 + 4\log_2 a$

53. $\ln \frac{5\sqrt{x}}{6}$

$\ln 5 + \frac{1}{2}\ln x - \ln 6$

54. $\ln \frac{x\sqrt{x+1}}{2e^4}$

$\ln x + \frac{1}{2}\ln(x+1) - \ln 2 - 4$

Use the properties of logarithms to condense the expression to the logarithm of a single quantity.

55. $\log_3 13 + \log_3 y$

$\log_3(13y)$

56. $4 \ln x - 4 \ln y$

$\ln \frac{x^4}{y^4}$

57. $\ln x - \ln(x+2) + \ln(2x-3)$

$\ln \left[\frac{x(2x-3)}{x+2}\right]$

Solve each equation. Round results to the nearest thousandth.

58. $3^x = 81$ $3^x = 3^4$ $x = 4$

59. $5^{2x} = 2500$ $2x = \log_5 2500$ $2x = 4.861$ $x = 2.431$

60. $\log_7 x = 6$ $x = 7^6$ $x = 117,649$

61. $\log_{10}(x-4) = 5$ $x - 4 = 10^5$ $x - 4 = 100,000$ $x = 100,004$

62. $\frac{1025}{8 + e^{4x}} = 5$ $1025 = 5(8 + e^{4x})$ $8 + e^{4x} = 205$ $e^{4x} = 197$ $4x = \ln 197$ $x = \frac{\ln 197}{4} = 1.321$

63. $-xe^{-x} + e^{-x} = 0$ $e^{-x}(-x+1) = 0$ $e^{-x} = 0$ $x + 1 = 0$ $-x = \ln 0$ $x = 1$

64. $\log x - \log(8 - 5x) = 2$ $\log_{10} \frac{x}{8-5x} = 2$ $\frac{x}{8-5x} = 10^2$ $\frac{x}{8-5x} = 100$ $x = 100(8-5x)$ $x = 800 - 500x$ $501x = 800$ $x = 1.597$

65. $2x \ln x - x = 0$ $x(2\ln x - 1) = 0$ $x \neq 0$ (extraneous) $2\ln x - 1 = 0$ $\ln x = \frac{1}{2}$ $x = e^{1/2} = 1.649$

66. The half-life of radioactive actinium(^{227}Ac) is 22 years. The equation for this situation would be $y = \left(\frac{1}{2}\right)^{t/22}$ What percent of the present amount of radioactive actinium will remain after 15 years?

$t = 15$ $y = \left(\frac{1}{2}\right)^{15/22} = 0.623 = 62.3\%$

Unit 4: Systems & Matrices

Solve each system with a method of your choice.

67. $x - y = 6$ (5)
$3x + 5y = 2$

$5x - 5y = 30$
$+\ 3x + 5y = 2$
$8x = 32$
$x = 4$

$4 - y = 6$
$-y = 2$
$y = -2$

$(4, -2)$

68. $y = x - 1$
$y = (x-1)^3$

$y = (y)^3$
$y^3 - y = 0$
$y(y^2 - 1) = 0$
$y(y-1)(y+1) = 0$
$y = 0, 1, -1$

$y = 0$: $0 = x - 1$, $x = 1$ → $(1, 0)$
$y = 1$: $1 = x - 1$, $x = 2$ → $(2, 1)$
$y = -1$: $-1 = x - 1$, $x = 0$ → $(0, -1)$

69. $4x - y^2 = 7$
$x - y = 3$

$x = y + 3$
$4(y+3) - y^2 = 7$
$4y + 12 - y^2 = 7$
$y^2 - 4y - 5 = 0$
$(y-5)(y+1) = 0$
$y = 5 \quad y = -1$

$y = 5$: $x = 5 + 3 = 8$ → $(8, 5)$
$y = -1$: $x = -1 + 3 = 2$ → $(2, -1)$

70. $2x + 5y = -11$
$5x - y = 19$ (5)

$2x + 5y = -11$
$+\ 25x - 5y = 95$
$27x = 84$
$x = \frac{28}{9}$

$5\left(\frac{28}{9}\right) - y = 19$
$\frac{140}{9} - y = 19$
$y = -\frac{31}{9}$

$\left(\frac{28}{9}, -\frac{31}{9}\right)$

71. $2x - 3y + z = 3$
$3x - y - 4z = 8$
$-2x + 2y + 3z = -4$

$$\begin{bmatrix} 2 & -3 & 1 \\ 3 & -1 & -4 \\ -2 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5/17 & 11/17 & 13/17 \\ -1/17 & 5/17 & 4/17 \\ 4/17 & 2/17 & 7/17 \end{bmatrix}\begin{bmatrix} 3 \\ 8 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

$(3, 1, 0)$

72. $x - 2y + 3z = -5$
$2x - z = -4$
$3y + z = 17$

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -1 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3/25 & 11/25 & 2/25 \\ -2/25 & 1/25 & 7/25 \\ 6/25 & -3/25 & 4/25 \end{bmatrix}\begin{bmatrix} -5 \\ -4 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$$

$(-1, 5, 2)$

Given $A = \begin{bmatrix} 5 & 4 & 4 \\ -4 & -4 & 0 \\ 1 & 2 & 0 \end{bmatrix}$ **and** $B = \begin{bmatrix} 4 & 4 & 0 \\ 3 & 2 & 1 \\ 1 & -2 & 0 \end{bmatrix}$**, evaluate the expressions.**

73. $A - B$

$$\begin{bmatrix} 1 & 0 & 4 \\ -7 & -6 & -1 \\ 0 & 4 & 0 \end{bmatrix}$$

74. $3A$

$$\begin{bmatrix} 15 & 12 & 12 \\ -12 & -12 & 0 \\ 3 & 6 & 0 \end{bmatrix}$$

75. $3A - 2B$

$$\begin{bmatrix} 15 & 12 & 12 \\ -12 & -12 & 0 \\ 3 & 6 & 0 \end{bmatrix} - \begin{bmatrix} -8 & -8 & 0 \\ -6 & -4 & -2 \\ -2 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 & 12 \\ -18 & -16 & -2 \\ 1 & 10 & 0 \end{bmatrix}$$

76. AB

$$\begin{bmatrix} 20+12+4 & 20+8-8 & 0+4+0 \\ -16-12+0 & -16-8+0 & 0-4+0 \\ 4+6+0 & 4+4+0 & 0+2+0 \end{bmatrix}$$

$$\begin{bmatrix} 36 & 20 & 4 \\ -28 & -24 & -4 \\ 10 & 8 & 2 \end{bmatrix}$$

77. Given $A = \begin{bmatrix} -6 & 4 \\ 10 & -5 \end{bmatrix}$

a. Find A^{-1}

$\text{Det} = \begin{vmatrix} -6 & 4 \\ 10 & -5 \end{vmatrix} = 30 - 40 = -10$ $\quad A^{-1} = \frac{1}{-10}\begin{bmatrix} -5 & -4 \\ -10 & -6 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{1}{2} & \frac{2}{5} \\ 1 & \frac{3}{5} \end{bmatrix}}$

b. Use A^{-1} to solve $\begin{matrix} -6x + 4y = 10 \\ 10x - 5y = 20 \end{matrix}$

$\begin{bmatrix} -6 & 4 \\ 10 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$ $\quad \boxed{(13, 22)}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{2}{5} \\ 1 & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 10 \\ 20 \end{bmatrix}$

Find the determinant of each matrix.

78. $\begin{bmatrix} -25 & 18 \\ 6 & -7 \end{bmatrix}$

$\text{Det} = \begin{vmatrix} -25 & 18 \\ 6 & -7 \end{vmatrix} = 175 - 108 = \boxed{67}$

79. $\begin{bmatrix} 4 & 0 & 3 \\ 1 & -8 & 2 \\ 3 & 2 & 2 \end{bmatrix}$

$-72 + 16 + 0 = -56$

$\text{Det} = \begin{vmatrix} 4 & 0 & 3 \\ 1 & -8 & 2 \\ 3 & 2 & 2 \end{vmatrix} = -58 - (-56) = \boxed{-2}$

$-64 + 0 + 6 = -58$

Determine if the points are collinear. If they are not collinear, find the area of the triangle formed by the points.

80. (-5, 0), (4, 4), and (3, 2)

$\det = \begin{vmatrix} -5 & 0 & 1 \\ 4 & 4 & 1 \\ 3 & 2 & 1 \end{vmatrix} = -14$

Not Collinear

$A = -\frac{1}{2}(-14) = 7u^2$

81. (-7, 4), (0, 6), and (14, 10)

$\det = \begin{vmatrix} -7 & 4 & 1 \\ 0 & 6 & 1 \\ 14 & 10 & 1 \end{vmatrix} = 0$

Collinear

Use Cramer's Rule to solve the systems.

82. $\begin{matrix} 20x + 8y = 11 \\ 12x - 24y = 21 \end{matrix}$

$D = \begin{vmatrix} 20 & 8 \\ 12 & -24 \end{vmatrix} = -480 - 96 = -576$

$D_x = \begin{vmatrix} 11 & 8 \\ 21 & -24 \end{vmatrix} = -264 - 168 = -432$

$D_y = \begin{vmatrix} 20 & 11 \\ 12 & 21 \end{vmatrix} = 420 - 132 = 288$

$x = \frac{-432}{-576} = \frac{3}{4}$ $\quad y = \frac{288}{-576} = -\frac{1}{2}$

$\boxed{\left(\frac{3}{4}, -\frac{1}{2}\right)}$

83. $\begin{matrix} 6x - 5y - 2z = 10 \\ -2x + 2y - 3z = -13 \\ -2x + 5y - 4z = -4 \end{matrix}$

$D = \begin{vmatrix} 6 & -5 & -2 \\ -2 & 2 & -3 \\ -2 & 5 & -4 \end{vmatrix} = 64$ $\quad D_y = \begin{vmatrix} 6 & 10 & -2 \\ -2 & -13 & -3 \\ -2 & -4 & -4 \end{vmatrix} = 256$

$D_x = \begin{vmatrix} 10 & -5 & -2 \\ -13 & 2 & -3 \\ -4 & 5 & -4 \end{vmatrix} = 384$ $\quad D_z = \begin{vmatrix} 6 & -5 & 10 \\ -2 & 2 & -13 \\ -2 & 5 & -4 \end{vmatrix} = 192$

$x = \frac{384}{64} = 6$ $\quad y = \frac{256}{64} = 4$ $\quad z = \frac{192}{64} = 3$

$\boxed{(6, 4, 3)}$

Unit 5: Counting & Probability

Simplify each factorial expression.

84. $\frac{11! \cdot 4!}{4! \cdot 7!}$ $= \frac{11!}{7!} = 11 \cdot 10 \cdot 9 \cdot 8 = \boxed{7920}$

85. $\frac{n!}{(n+1)!}$ $= \frac{n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1}{(n+1)(n)(n-1)\cdots 3 \cdot 2 \cdot 1}$ $= \boxed{\frac{1}{n+1}}$

86. $\frac{2 \cdot n!}{(n-1)!}$ $= \frac{2n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1}{(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1}$ $= \boxed{2n}$

Evaluate each expression.

87. ${}_9C_3 = \boxed{84}$

88. ${}_{20}C_3 = \boxed{1140}$

89. ${}_9P_2 = \boxed{72}$

90. ${}_{70}P_3 = \boxed{328,440}$

91. How many distinct license plates can be issued consisting of one letter followed by a three digit number?

$24 \cdot 10 \cdot 10 \cdot 10 = \boxed{24,000 \text{ plates}}$

92. Four students are randomly selected from a class of 25 to answer questions from a math assignment. In how many ways can the four be selected?

${}_{25}C_4 = \boxed{12,650 \text{ ways}}$

93. A card is drawn from a standard deck of 52 playing cards. Find the probability that it is a red face card.

$P(RF) = \frac{6}{52} = \boxed{\frac{3}{26}}$

94. Two spark plugs require replacement in a four cylinder engine. The mechanic randomly removes two plugs. Find the probability that they are the two defective plugs.

$P(2D) = \frac{{}_2C_2}{{}_4C_2} = \boxed{\frac{1}{6}}$

95. Two integers from 1 to 60 are chosen by a random number generator. What is the probability that:

a. Both numbers are odd

$P(\text{odd and odd}) = \frac{30}{60} \cdot \frac{30}{60} = \boxed{\frac{1}{4}}$

b. Both numbers are less than 12

$P(\text{both less than 12}) = \frac{11}{60} \cdot \frac{11}{60} = \boxed{\frac{121}{3600}}$

c. The same number is chosen twice

$P(\text{same number}) = \boxed{\frac{1}{60}}$