Signal Processing Project Report

Paper: **Quantization and Compensation in Sampled Interleaved Multichannel Systems**
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Overview:
We consider a interleaved, multichannel sampling structure and perform the usual Nyquist-Shannon sampling[1] for a given signal. We show that overall SQNR ratio of the signal can be increased by varying the quantizer step size, changing the relative time-delays between different channels.

Description:
In a multichannel sampling structure we can have various channels sampling the same signal at a time which allows us to sample the signal from each channel at a lower rate (less than Nyquist rate).

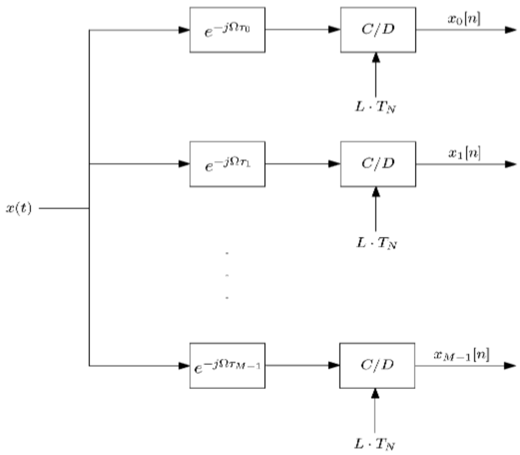

Fig. 1. Multichannel sampling.

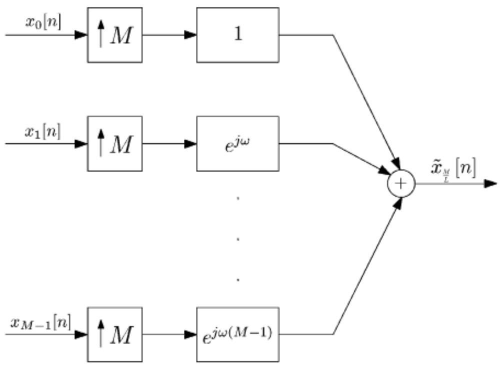

Fig. 2. Interleaving the output samples of the multichannel sampling system of Fig. 1.

Later we interleave the outputs of all these channels to give a uniformly sampled signal, but sometimes we fail to get so due to errors/improper tuning in time delays between the channels. This decreases the overall SQNR ratio of the signal. We can compensate the error due to nonuniform spacing of the channel offsets by:
(i) Appropriately assigning the quantizers of different step sizes to each channel.
(ii) Choosing a set of time delays for all the channel with a common property.

After the signal is passed through the quantizer, the error produced in each channel is represented as additive noise model[2] and is considered to be uniformly distributed between $[\frac{-\Delta_i}{2}, \frac{\Delta_i}{2}]$ where Δ_i is the step size of ith channel quantiser .Now we consider ($\sigma^2{}_e$) the average of ensemble average power of e(t) where e(t) is the total noise component in the interleaved output of the signal. Now we model our simulations to get minimum and it's represented by $\sigma^2{}_{e\ min}$.

1. Section 7.1 , Systems and Signals, ALAN V. OPPENHEIM ,ALAN S. WILLSKY
2. A. B. Sripad and D. L. Snyder, "A necessary and sufficient condition for quantization errors to be uniform and white," IEEE Trans. Acoust., Speech, Signal Process., vol. ASSP-25, no. 5, pp. 442–448, 1977.

Finally we show that we can reduce the average quantisation noise power and compensate the error due to mismatched timings in the channels by appropriately choosing the quantizer step sizes and time-delay between each channel.

Project Idea: Given the relative timing between channels and the available step size quantizers with different bit allocations. We find the best possible arrangement of the different size quantizers available to channels to find the most optimum system to get a better SQNR ratio.

The performance of the system is evaluated by calculating $\sigma^2{}_{e\,min}$ of the output signal. $\sigma^2{}_{e\,min}$ is the average over time of ensemble average power of noise component of the signal. We calculate $\sigma^2{}_{e\,min}$ because it would effectively represents the power of noise in the signal so if the value of $\sigma^2{}_{e\,min}$ is low it would mean that the error in the final interleaved signal is low.

Since the relative timing between the channels is fixed. We will take all possible arrangements of the different bit step size quantizers and find their $\sigma^2{}_{e\,min}$. We then calculate ϒ which represents the reduction in average noise power.

$$\sigma^2{}_{e\,min} = \frac{1}{2\Pi}\int_{-\pi}^{\pi}\sum_{m=0}^{M-1}\frac{\sigma_m^2}{l} * \left|G_m\left(e^{jw}\right)\right|^2 d\omega$$

, $G_m(e^{jw})$ is the reconstruction filter

$$\Upsilon = \frac{\sigma^2}{\sigma_{e\,min}^2}$$

Here σ^2 the average of ensemble average power over t of uniform step size quantizers, $\sigma^2{}_m$ is the variance of error caused due to quantisation in the m^{th} channel.
So, the system with highest ϒ gives the best possible output because higher ϒ represents that $\sigma^2{}_{e\,min}$ is low which implies that the overall power of the noise in the signal is less.

After finding the values of ϒ we plot it against the relative timings of the channels.
Matlab will be used for calculating the values of $\sigma^2{}_{e\,min}$ and ϒ and choosing the best combination of quantiser and time-shift for each channel.

Work Done so far:

We've the code ready for finding the $\sigma^2{}_{e\,min}$ for given quantiser step sizes or for given time-shifts of each channel. We should further organise the code, return the best possible combinations from the obtained $\sigma^2{}_{e\,min}$ values and finally add plots to give a better intuition of why we are choosing a particular set of values.

1. Section 7.1 , Systems and Signals, ALAN V. OPPENHEIM ,ALAN S. WILLSKY
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