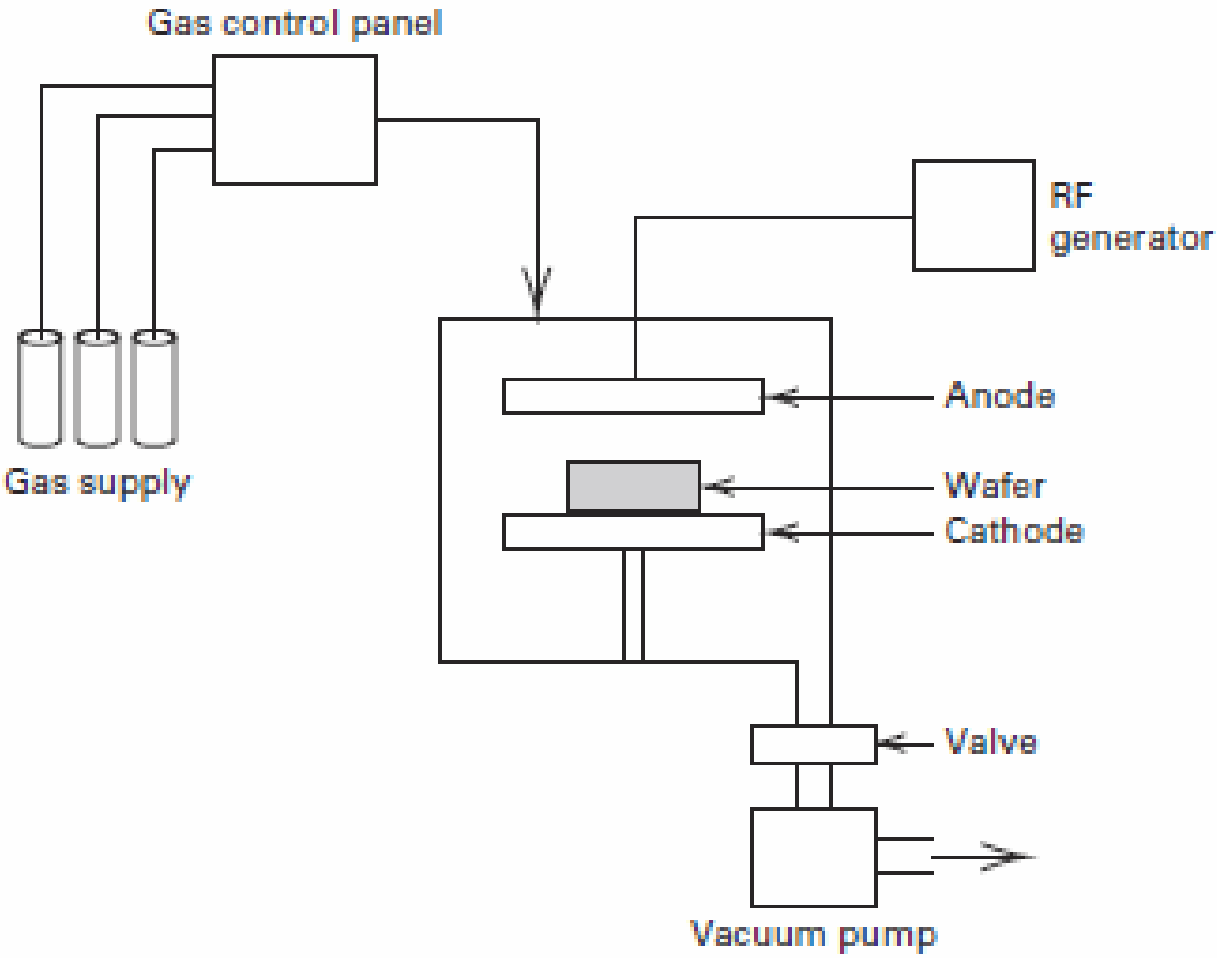
Analysis of Variance

- There are lots of practical situations where there are either more than two levels of interest, or there are several factors of simultaneous interest
- The **analysis of variance** (ANOVA) is the appropriate analysis "engine" for these types of experiments – Chapter 3
- The ANOVA was developed by Fisher in the early 1920s, and initially applied to agricultural experiments
- Used extensively today for industrial experiments

An Example (See pg. 66)

- An engineer is interested in investigating the relationship between the RF power setting and the etch rate for this tool. The objective of an experiment like this is to model the relationship between etch rate and RF power, and to specify the power setting that will give a desired target etch rate.
- The response variable is etch rate.
- She is interested in a particular gas (C_2F_6) and gap (0.80 cm), and wants to test four levels of RF power: 160W, 180W, 200W, and 220W. She decided to test five wafers at each level of RF power.
- The experimenter chooses 4 **levels** of RF power 160W, 180W, 200W, and 220W
- The experiment is **replicated** 5 times – runs made in random order



■ **FIGURE 3.1 A single-wafer plasma etching tool**

An Example (See pg. 66)

Table 3-1 Etch Rate Data (in Å/min) from the Plasma Etching Experiment

Power (W)	Observations 1	2	3	4	5	Totals	Averages
160	575	542	530	539	570	2756	551.2
180	565	593	590	579	610	2937	587.4
200	600	651	610	637	629	3127	625.4
220	725	700	715	685	710	3535	707.0

- Does **changing** the power change the mean etch rate?
- Is there an **optimum** level for power?

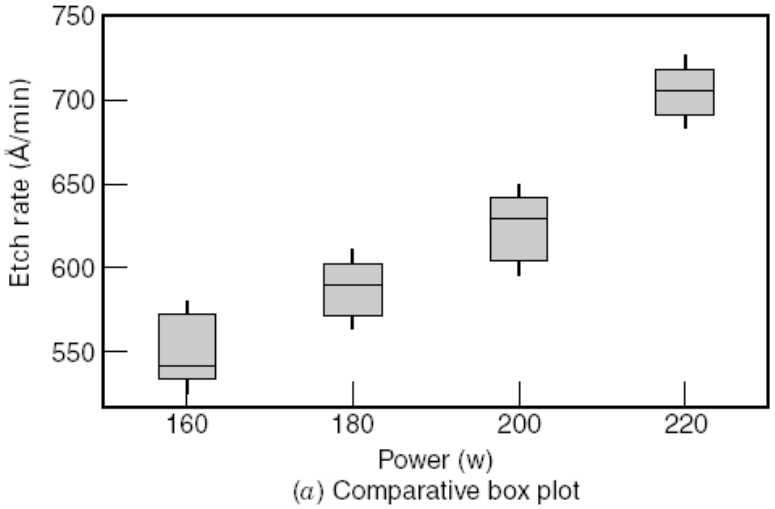


Figure 3-2 Box plots and scatter diagram of the etch rate data.

The Analysis of Variance (Sec. 3-2, pg. 68)

Table 3-2 Typical Data for a Single-Factor Experiment

Treatment (level)	Observations				Totals	Averages
1	y_{11}	y_{12}	$\cdots$	y_{1n}	$y_{1.}$	$\bar{y}_{1.}$
2	y_{21}	y_{22}	$\cdots$	y_{2n}	$y_{2.}$	$\bar{y}_{2.}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
a	y_{a1}	y_{a2}	$\cdots$	y_{an}	$y_{a.}$	$\bar{y}_{a.}$
					$y_{..}$	$\bar{y}_{..}$

- In general, there will be *a* **levels** of the factor, or *a* **treatments, and** *n* **replicates** of the experiment, run in **random order…**a completely randomized design **(CRD)**
- $N = an$ total runs
- We consider the **fixed effects** case…the **random effects** case will be discussed later
- Objective is to test hypotheses about the equality of the *a* treatment means

The Analysis of Variance

- The name "analysis of variance" stems from a **partitioning** of the total variability in the response variable into components that are consistent with a **model** for the experiment

- The basic single-factor ANOVA model is

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}, \begin{cases} i = 1, 2, ..., a \\ j = 1, 2, ..., n \end{cases}$$

Number of Treatments

Number of Replicates

$\mu =$ an overall mean, $\tau_i = ith$ treatment effect,

$\varepsilon_{ij} =$ experimental error, $NID(0, \sigma^2)$

Models for the Data

There are several ways to write a model for the data:

$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$ is called the effects model

Let $\mu_i = \mu + \tau_i$, then

$y_{ij} = \mu_i + \varepsilon_{ij}$ is called the means model

Regression models can also be employed

The Analysis of Variance

- Using the models discussed, we would like to be able to differentiate between:
  - Random noise
  - Random noise + treatment effect

- Ratio of Effects: $\frac{\text{Random + Treatment}}{\text{Random}}$

- If this ratio is near 1, then treatment effect is near 0.

The Analysis of Variance

- **Total variability** is measured by the total sum of squares:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2$$

- The basic ANOVA partitioning is:

$$\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{a}\sum_{j=1}^{n}[(\bar{y}_{i.} - \bar{y}_{..}) + (y_{ij} - \bar{y}_{i.})]^2$$

$$= n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2 + \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i.})^2$$

$$SS_T = SS_{Treatments} + SS_E$$

Partitioning the Variance

Table 3-2 Typical Data for a Single-Factor Experiment

Treatment (level)	Observations				Totals	Averages
1	y_{11}	y_{12}	$\cdots$	y_{1n}	$y_{1.}$	$\bar{y}_{1.}$
2	y_{21}	y_{22}	$\cdots$	y_{2n}	$y_{2.}$	$\bar{y}_{2.}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
a	y_{a1}	y_{a2}	$\cdots$	y_{an}	$y_{a.}$	$\bar{y}_{a.}$
					$y_{..}$	$\bar{y}_{..}$

Total Variability

Random Variability

Treatment Variability

The Analysis of Variance

$$SS_T = SS_{Treatments} + SS_E$$

- A large value of $SS_{Treatments}$ reflects large differences in treatment means
- A small value of $SS_{Treatments}$ likely indicates no differences in treatment means
- Formal statistical hypotheses are:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a$$

$$H_1 : \text{ At least one mean is different}$$

The Analysis of Variance

- While sums of squares cannot be directly compared to test the hypothesis of equal means, **mean squares** can be compared.
- A mean square is a sum of squares divided by its degrees of freedom:

$$df_{Total} = df_{Treatments} + df_{Error}$$

$$an - 1 = a - 1 + a(n-1)$$

$$MS_{Treatments} = \frac{SS_{Treatments}}{a-1}, MS_E = \frac{SS_E}{a(n-1)}$$

- **If the treatment means are equal, the treatment and error mean squares will be (theoretically) equal.**
- **If treatment means differ, the treatment mean square will be larger than the error mean square.**

The Analysis of Variance

- Let's take a look at the expected values of the mean squares for the fixed effect model

$$E\left(MS_{Treatments}\right) = \sigma^2 + \frac{n\sum_{i=1}^{a}\tau_i^2}{a-1}$$

$$E\left(MS_{Error}\right) = \sigma^2$$

The Analysis of Variance

- Remember the hypotheses

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a$$

H_1 : At least one mean is different

- Another interpretation of the null hypothesis

$$H_0 : \tau_1 = \tau_2 = \ldots = \tau_a = 0$$

- Hence **UNDER THE NULL HYPOTHESIS**

$$\sum_{i=1}^{a} \tau_i^2 = 0$$

The Analysis of Variance

- Once again, **UNDER THE NULL HYPOTHESIS**

$$E\left(MS_{Treatments}\right)=\sigma^2$$

- Therefore **if the null hypothesis is correct (under the null hypothesis) which means no significant treatment effect** ,we would have

$$\frac{E\left(MS_{Treatments}\right)}{E\left(MS_{Error}\right)}=1$$

The Analysis of Variance

- In a given data set, we do not know the expected values of the mean squares but their single realization for that data set
- Hence all we can do is to look at the following ratio

$$\frac{MS_{Treatment}}{MS_{Error}}$$

- Even if there is no significant treatment effect (the null hypothesis is correct), this ratio will not necessarily (and almost certainly) be equal to 1 exactly
- But if it is small enough, we would say that we do not have enough evidence to reject the null hypothesis that there is no significant treatment effect

The Analysis of Variance is Summarized in a Table

Table 3-3 The Analysis of Variance Table for the Single-Factor, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between treatments	$SS_{\text{Treatments}} = n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2$	$a - 1$	$MS_{\text{Treatments}}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Error (within treatments)	$SS_E = SS_T - SS_{\text{Treatments}}$	$N - a$	MS_E	
Total	$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2$	$N - 1$		

- Computing…see text, page 69
- The **reference distribution** for F_0 is the $F_{a\text{-}1,\, a(n\text{-}1)}$ distribution
- **Reject** the null hypothesis (equal treatment means) if

$$F_0 > F_{\alpha,a-1,a(n-1)}$$

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{n} y_{ij}^2 - \frac{y_{..}^2}{N} \qquad \textbf{(3.8)}$$

$$SS_{\text{Treatments}} = \frac{1}{n} \sum_{i=1}^{a} y_{i.}^2 - \frac{y_{..}^2}{N} \qquad \textbf{(3.9)}$$

$$SS_E = SS_T - SS_{\text{Treatments}} \qquad \textbf{(3.10)}$$

ANOVA Table
Example 3-1

$$SS_T = \sum_{i=1}^{4}\sum_{j=1}^{5} y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$= (575)^2 + (542)^2 + \cdots + (710)^2 - \frac{(12{,}355)^2}{20}$$

$$= 72{,}209.75$$

$$SS_{\text{Treatments}} = \frac{1}{n}\sum_{i=1}^{4} y_{i.}^2 - \frac{y_{..}^2}{N}$$

$$= \frac{1}{5}[(2756)^2 + \cdots + (3535)^2] - \frac{(12{,}355)^2}{20}$$

$$= 66{,}870.55$$

$$SS_E = SS_T - SS_{\text{Treatments}}$$
$$= 72{,}209.75 - 66{,}870.55 = 5339.20$$

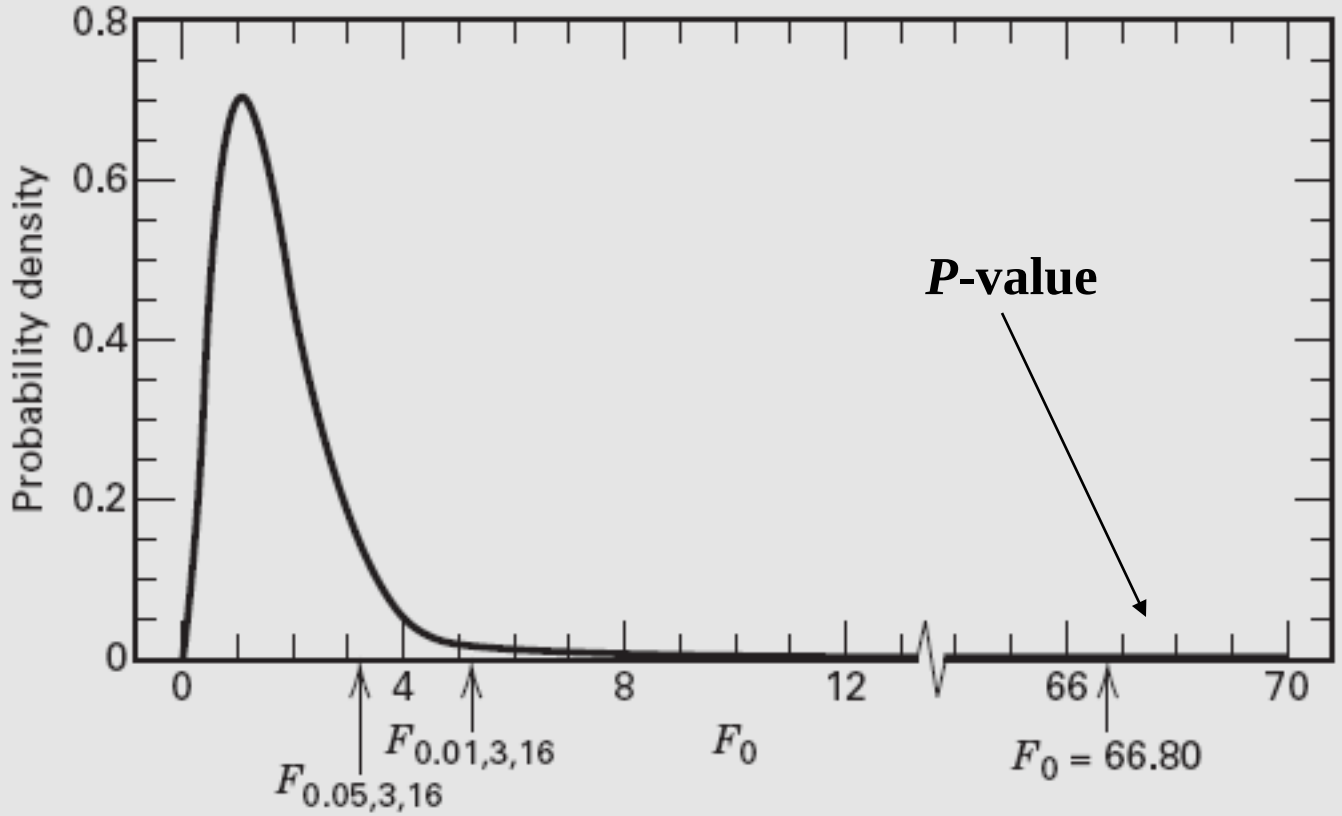
Usually, these calculations would be performed on a computer, using a software package with the capability to analyze data from designed experiments.

The ANOVA is summarized in Table 3.4. Note that the RF power or between-treatment mean square (22,290.18) is many times larger than the within-treatment or error mean square (333.70). This indicates that it is unlikely that the treatment means are equal. More formally, we can compute

■ **TABLE 3.4**
ANOVA for the Plasma Etching Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	*P*-Value
RF Power	66,870.55	3	22,290.18	$F_0 = 66.80$	<0.01
Error	5339.20	16	333.70		
Total	72,209.75	19			

The Reference Distribution:



■ FIGURE 3.3 The reference distribution ($F_{3,16}$) for the test statistic F_0 in Example 3.1

ANOVA Table
Example 3-1

Table 3-4 ANOVA for the Plasma Etching Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
RF Power	66,870.55	3	22,290.18	$F_0 = 66.80$	<0.01
Error	5339.20	16	333.70		
Total	72,209.75	19			

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_a$$

H_1 : At least one mean is different

The probability of wrongly rejecting the null hypothesis is less than 1%

The Reference Distribution:

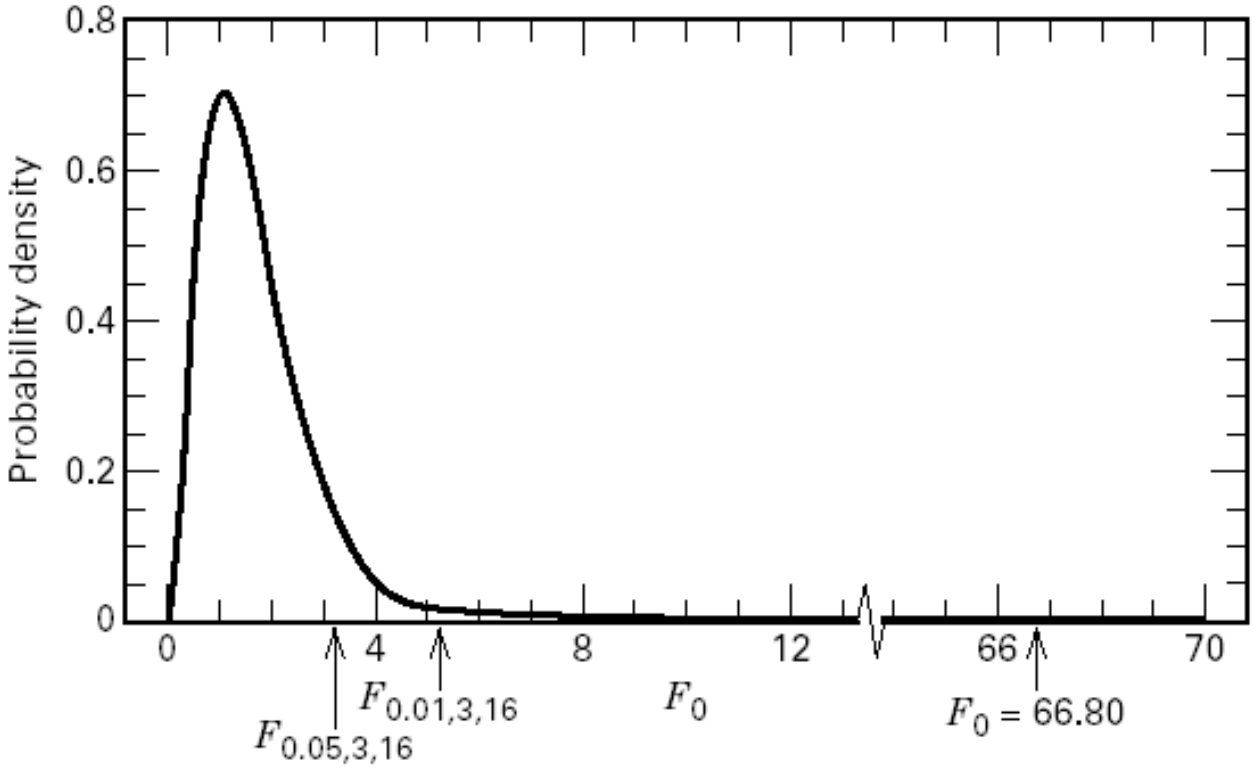


Figure 3-3 The reference distribution ($F_{3,16}$) for the test statistic F_0 in Example 3-1.

Shaking out rust – Reading an F table

IV. Percentage Points of the F Distribution (*continued*)

$F_{0.05,\nu_1,\nu_2}$

ν_2 \ ν_1	Degrees of Freedom for the Numerator (ν_1)																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Degrees of Freedom for the Denominator (ν_2)

ANOVA calculations are usually done via computer

- Text exhibits sample calculations from two software packages, Design-Expert and Minitab
- See page 103 for Design-Expert, page 104 for Minitab
- Text discusses some of the summary statistics provided by these packages

Model Adequacy Checking in the ANOVA
Text reference, Section 3-4, pg. 80

- **Checking assumptions** is important
- Normality
- Constant variance
- Independence
- Have we fit the right model?
- Later we will talk about what to do if some of these assumptions are **violated**

Model Adequacy Checking in the ANOVA

- Examination of **residuals** (see text, Sec. 3-4)

$$e_{ij} = y_{ij} - \hat{y}_{ij}$$
$$= y_{ij} - \bar{y}_{i.}$$

- **Residual plots** are very useful
- An adequate model produces residual plots that are **structureless**

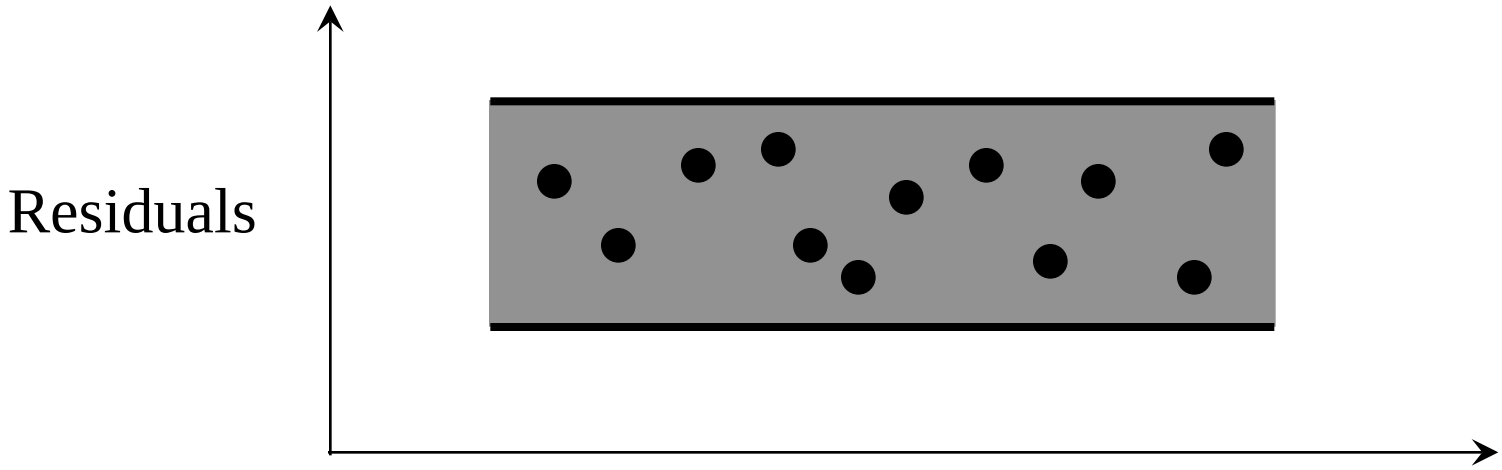
Normality

Indepedence

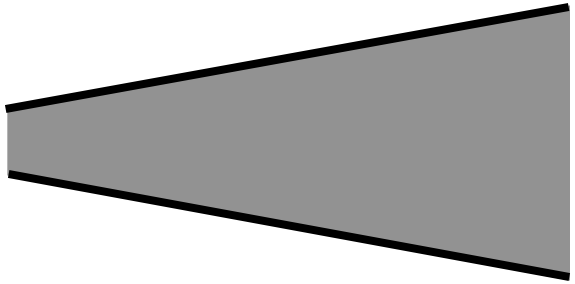
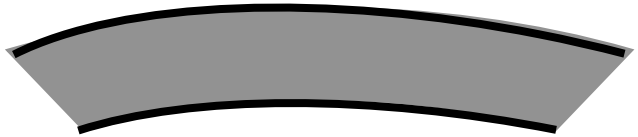
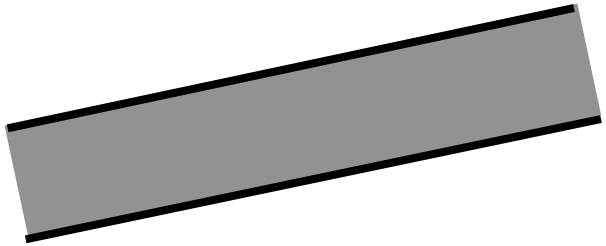
Constant Variance

Model Adequacy Checking

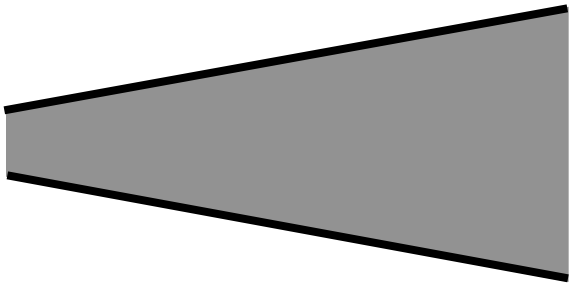
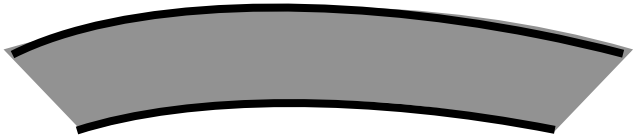
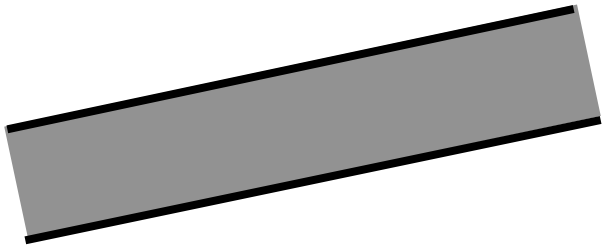
Plot of Residuals vs. Anything Should be a Band of Random "Noise"



Residuals vs. Fitted Values, $\hat{y}$

Pattern	Interpretation
	**Error variance depends on the mean (Transformation)**
	**Possible need for second order term or transformation**
	**Error in analysis (Possible omission of intercept)**

Residuals vs. Factor Levels, *x*

Pattern	Interpretation
	**Error variance depends on the factor (Variance Effect)**
	**Possible need for that factor's quadratic term or transformation**
	**Error in analysis (Possible need to include the main effect of the factor)**

Data Transformation

- To stabilize the variance
- Normalization of errors
- Model simplicity; e.g. non-linear to linear

$$y = b_0 e^{-b_1 x} \Rightarrow \ln y = \ln b_0 - b_1 x$$

Variance Stabilizing Transformations

- If the variance of *y* changes with the mean of *y*, that is

$$\sigma_y \propto \mu^{\alpha}$$

Then a power transformation of the original data may stabilize the variance

$$y^* = y^{\lambda}$$

Variance Stabilizing Transformations

Relationship between σ_y and μ	λ	Transformation
$\sigma_y \propto$ constant	1	No Transformation
$\sigma_y \propto \mu^{1/2}$	1/2	Square Root
$\sigma_y \propto \mu$	0	Log
$\sigma_y \propto \mu^{3/2}$	-1/2	Reciprocal Square Root
$\sigma_y \propto \mu^{2}$	-1	Reciprocal

See Box and Cox (1964) for more detailed discussion

Model Adequacy Checking -- Normality

- **Normal probability plot** of residuals checks assumption that errors are NID
- If normal will resemble straight line
  - Moderate departures little concern for fixed effect
  - Distribution that has thicker or thinner tails is of more concern
- Departures will cause true significance level and power to differ from designed values
- Presence of outliers can seriously distort ANOVA

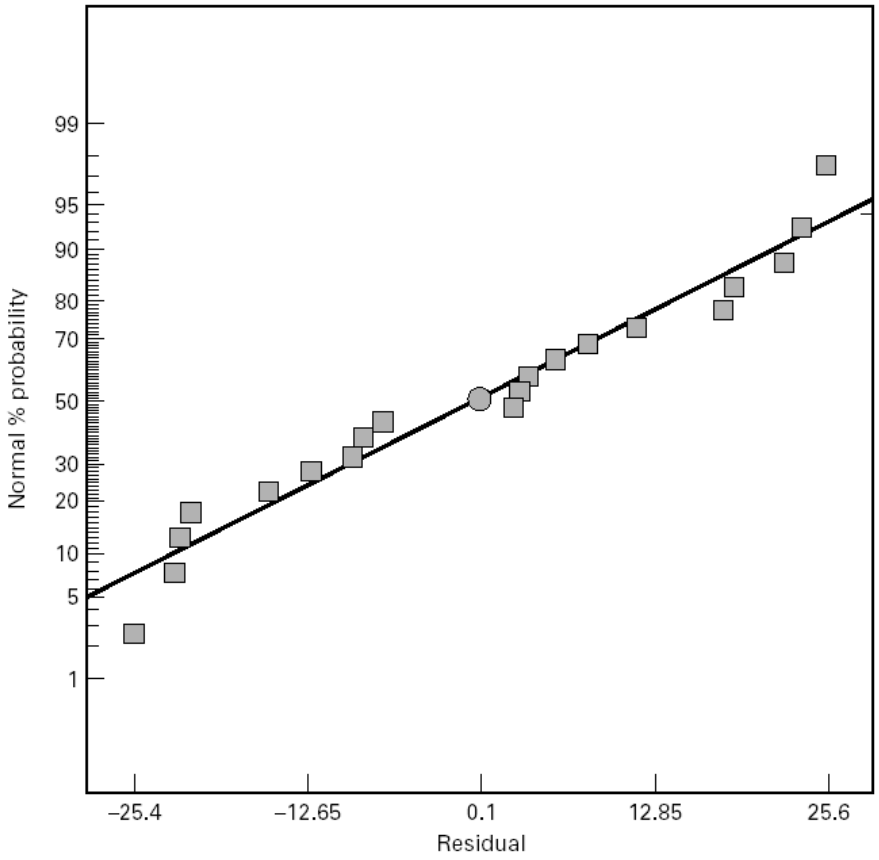


Figure 3-4 Normal probability plot of residuals for Example 3-1.

Model Adequacy Checking -- Independence

- **Time sequence plots** are used to verify that errors are independently distributed.
- Ideal plot shows no structure
- Problematic chart will show funnel effect
- Can occur with:
  - Equipment drift
  - Operator “skill”

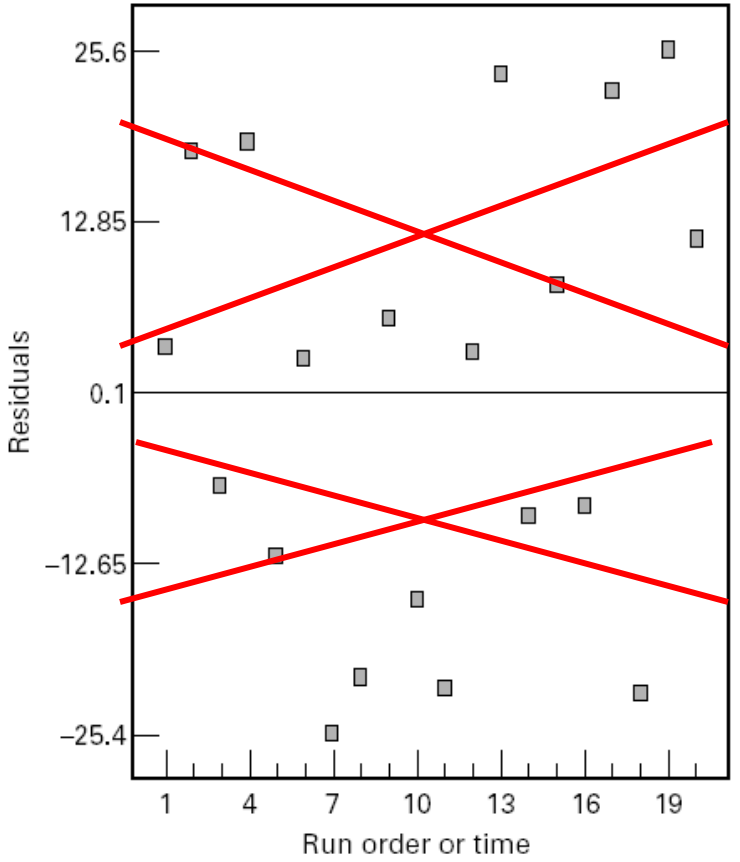


Figure 3-5 Plot of residuals versus run order or time.

Post-ANOVA Comparison of Means

- The analysis of variance tests the hypothesis of equal treatment means
- Assume that residual analysis is satisfactory
- If that hypothesis is rejected, we don't know **which specific means** are different
- Determining which specific means differ following an ANOVA is called the **multiple comparisons problem**
- There are **lots** of ways to do this…see text, Section 3-5, pg. 87
- We will use pairwise *t*-tests on means…sometimes called Fisher's Least Significant Difference (or Fisher's **LSD**) Method

Graphical Comparison of Means
Text, pg. 91

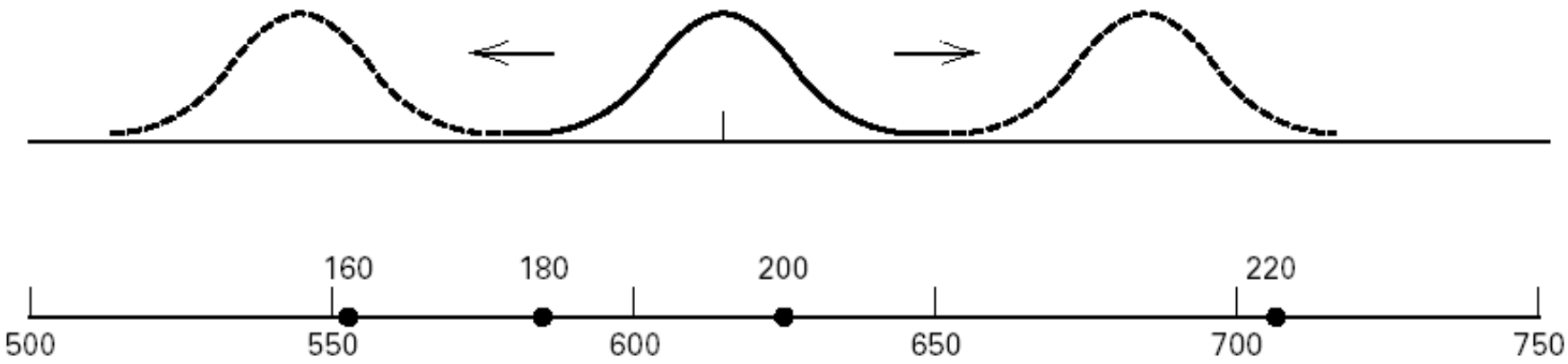


Figure 3-11 Etch rate averages from Example 3-1 in relation to a t distribution with scale factor $\sqrt{MS_E/n} = \sqrt{330.70/5} = 8.13$.

Fisher's LSD Test

$$\sqrt{\frac{MS_E}{n}}$$

$$\sqrt{MS_E\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$$

$$\frac{\text{Difference}}{\text{Std. Error}}$$

Treatment Means (Adjusted, If Necessary)

	Estimated Mean	Standard Error
1–160	551.20	8.17
2-180	587.40	8.17
3-200	625.40	8.17
4-220	707.00	8.17

Treatment	Mean Difference	DF	Standard Error	t for H_0 Coeff=0	Prob > |t|
1 vs 2	–36.20	1	11.55	–3.13	0.0064
1 vs 3	–74.20	1	11.55	–6.42	<0.0001
1 vs 4	–155.80	1	11.55	–13.49	<0.0001
2 vs 3	–38.00	1	11.55	–3.29	0.0046
2 vs 4	–119.60	1	11.55	–10.35	<0.0001
3 vs 4	–81.60	1	11.55	–7.06	<0.0001

Values of "Prob > |t|" less than 0.0500 indicate the difference in the treatment means is significant.
Values of "Prob > |t|" greater than 0.1000 indicate the difference in the two treatment means is not significant.

Sample Size Determination
Text, Section 3-7, pg. 105

- **FAQ** in designed experiments
- Answer depends on lots of things; including what type of experiment is being contemplated, how it will be conducted, resources, and desired **sensitivity**
- Sensitivity refers to the **difference in means** that the experimenter wishes to detect
- Generally, **increasing** the number of **replications increases** the **sensitivity** or it makes it easier to detect small differences in means

Sample Size Determination
Fixed Effects Case

- Can choose the sample size to detect a specific difference in means and achieve desired values of **type I and type II errors**
- Type I error – reject H_0 when it is true (α)
- Type II error – fail to reject H_0 when it is false (β)
- **Power** = 1 - β
- **Operating characteristic curves** plot β against a parameter Φ where

$$\Phi^2 = \frac{n\sum_{i=1}^{a}\tau_i^2}{a\sigma^2}$$

Sample Size Determination
Fixed Effects Case---use of OC Curves

- The **OC curves** for the fixed effects model are in the Appendix, Table V, pg. 693
- A very common way to use these charts is to define a difference in two means D of interest, then the minimum value of Φ^2 is

$$\Phi^2 = \frac{nD^2}{2a\sigma^2}$$

- Typically work in terms of the ratio of D/σ and try values of n until the **desired power** is achieved
- Most software packages will perform power and sample size calculations

Example

- For our previous experiment, suppose we would like to reject the null hypothesis with a probability of at least 0.90 if any two treatment means differed by as much as 75 A/minute and $\alpha = 0.01$.
- We are assuming that $\sigma = 25$ psi and the minimum value of Φ^2 is

$$\Phi^2 = \frac{n(75^2)}{2(4)(25^2)} = 1.125n$$

n	Φ2	Φ	a(n-1)	β	Power (1-β)
4	4.50	2.12	12	0.35	0.65

V. Operating Characteristic Curves for the Fixed Effects Model Analysis of Variance (*continued*)