Matrix

Lecture_1

⇒ Why Matrix is required

B. Tech
M. Tech
PhD

$2x + 3y = 5$ — (I)
$3x + 4y = 10$ — (II)

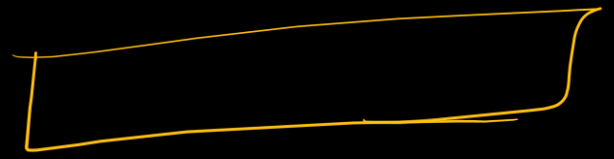
$5x = 15$ — (1)
$x = \frac{15}{5} = 3$

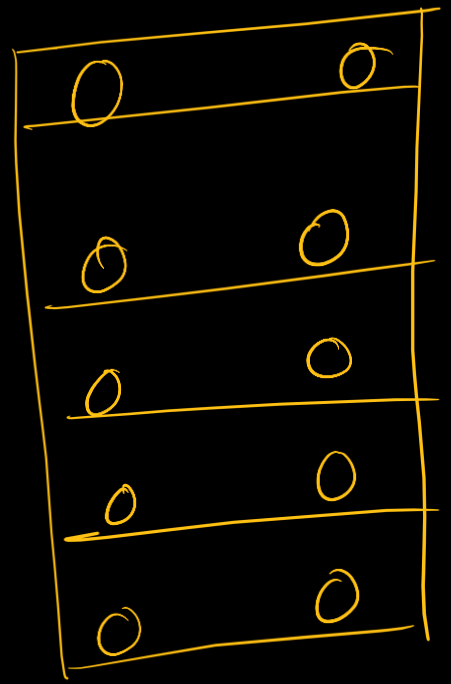
equations

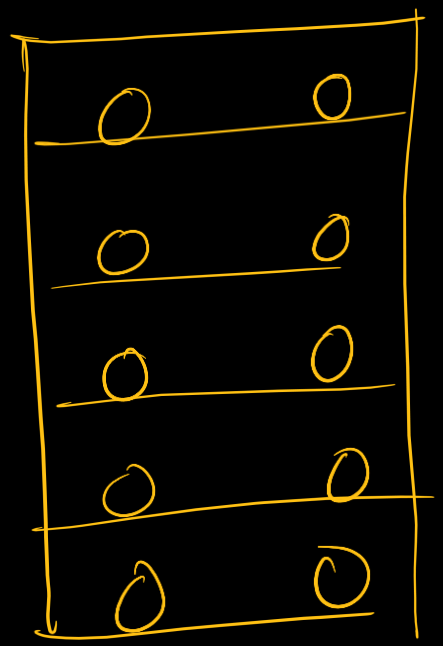
Matrix

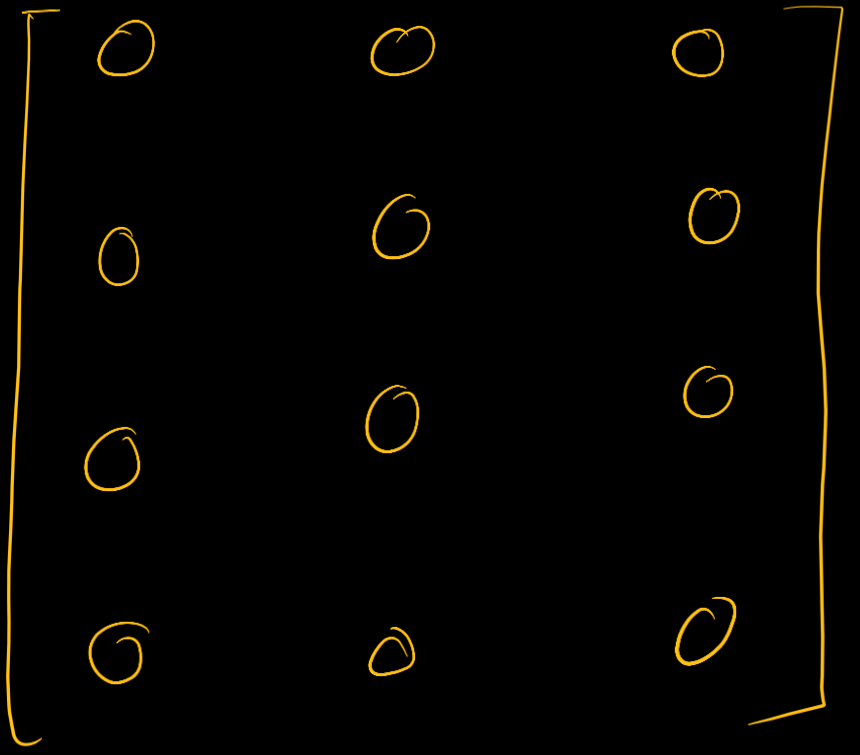
→ What is Matrix

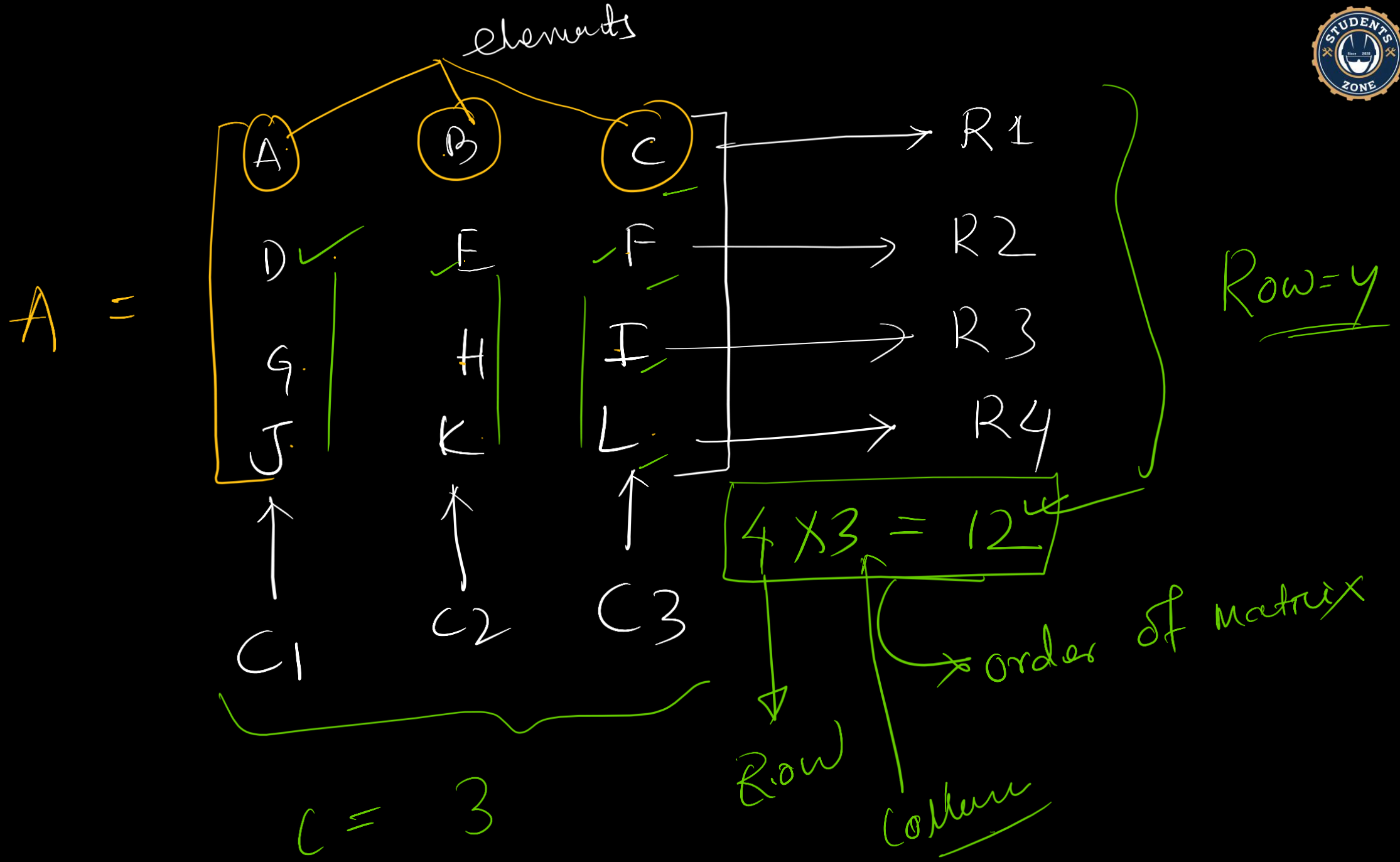
↳ Arrangement










elements
A
B
C
D
E
F
G
H
I
J
K
L
R1
R2
R3
R4
A =
Row = 4
4 X 3 = 12
order of matrix
C1
C2
C3
C = 3
Row
Column

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \\ 0 & 2 & 5 \end{bmatrix}_{3\times 3}$$

R1, R2, R3

$$D = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix}_{3\times 2}$$

R1, R2, R3

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$$

R1, R2

Ex Let A is 4 × 9 matrix, then how many columns are there

(a) 4

(b) 9

(c) 12

(d) NOT

4×9 → order of matrix

R C B

Row column

Q If B is a matrix of 3×10 order then total no. of elements in Matrix B will be

(a) 3

(b) 10

(c) 30

(d) NOT

$3 \times 10 = 30$

Q Can you form a $m \times n$ matrix, given that m and n are known values.

(a) yes

(b) No

Ex

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & & a_{mn} \end{bmatrix}$$

C_1 C_2 C_3 C_n

$R1$ $R2$ $R3$

RC

Row of matrix

column of matrix

$m \times n$

R C

a_{11} = R C

a_{32} = $R = 3$, $C = 2$

a_{23} : $R = 2$, $C = 3$

$A = [a_{ij}]_{m \times n}$

* In matrix if you see this can be read as

m × n
m cross n
m by n

or

a_{ij} → Element

i → Row of element

j → column of element

Q Which of the following is correct ?

(a) elements involved in matrix

(b) Matrix involved in ement.

Q If India is a matrix then Odisha will be _____

(a) element

(b) component

(c) Matrix

(d) NOT

Ex If $B = [b_{ij}]_{3\times y}$ then total elements in this Matrix will be

(a) 3

(b) y

(c) 12

(d) NOT

Or in matrix form $C = [c_{ij}]_{2\times 2}$

Sol

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}_{2\times 2}$$

Or

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

ex

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

C1 C2 C3

R1 R2 R3

$a_{23}^{RC} = ?$

(a) 3

(b) 5

(c) 6

(d) 9

$a_{32} = ?$

(a) 7

(b) 8 ✓

(c) 9

(d) NOT

$a_{13} =$

(a) 1

(b) 2

(c) 4

(d) 3 ✓

Ex

$$B = \begin{bmatrix} \sin x & \cos x \\ \tan x & -\cot x \end{bmatrix}_{2\times 2}$$

$b_{22} = ?$

Sol $\rightarrow -\cot x$

$b_{23} = ?$

(a) $\sin x$

(b) $\cos x$

(c) $\tan x$

(d) Not

$b_{21} = ?$

Sol $\tan x$

Ex construct a 2×2 matrix $A=[a_{ij}]$ whose elements are given by $a_{ij}=i+j$

Ans $A=\begin{bmatrix}2&3\\3&4\end{bmatrix}$

(a) $\begin{bmatrix}1&2\\3&4\end{bmatrix}$

(b) $\begin{bmatrix}2&4\\3&1\end{bmatrix}$

(c) $\begin{bmatrix}2&4\\3&4\end{bmatrix}$

(d) $\begin{bmatrix}2&3\\3&4\end{bmatrix}$ ✓

Sol $A=[a_{ij}]_{2\times2}=\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}$

$a_{11}=i+j=1+1=2$

$a_{12}=1+2=3$ ✓

$a_{21}=2+1=3$

$a_{22}=2+2=4$ ✓

ex construct a 2×2 matrix $A=[a_{ij}]$ whose elements are given by $a_{ij}=i-j$

(a) $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$

(b) $\begin{bmatrix}2 & 3\\ 3 & 4\end{bmatrix}$

(c) $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

(d) NOT

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 1-2\\ 2-1 & 2-2\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

Q Form a matrix by taking the example of a student who have 6 shirts. How many no. of ways are there to form this matrix.

(a) 1

(b) 2

(c) 3

(d) more then 3 ways

Sol $a = \begin{bmatrix} A & B & C & D & E & F \end{bmatrix}_{1\times 6}$

$b = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix}_{6\times 1}$

$c = \begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}_{2\times 3}$

$d = \begin{bmatrix} A & B \\ C & D \\ E & F \end{bmatrix}_{3\times 2}$

Types of Matrix :-

Row Matrix :- → A matrix having only one row is called row matrix.

→ While writing the order of matrix the first letter must be 1.

eg $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1 \times 4}$ (C_1 C_2 C_3 C_4)

eg $B = [1, 2, a, b, \ldots\ldots n]_{1 \times n}$

Q If a row matrix of order $m \times n$, then $m =$ ____ ?

(a) 1 ✓

(b) 2

(c) 3

(d) ∞

Q. If a row matrix of order $n \times n$, then $n =$ ____ ?

(a) 1

(b) 4

(c) 0

(d) Any Natural number

$n = -3$

$n = \frac{2}{3}$

$n = \pi$

Numbers

1. Natural Number :- $1, 2, 3, 4, 5, 6 \ldots\ldots \infty$

2. Whole Number :- $0, 1, 2, 3, 4, \ldots\ldots\ldots \infty$

3. Integers :- $-\infty, \ldots\ldots -4, -3, -2, -1, 0, 1, 2, \ldots\ldots \infty$

4. [Real Number :- $-\infty, -4, \frac{2}{3}, \sqrt{2}, \pi, 2, 3 \ldots$

5. Imaginary Number :- $i, 2i$]

6. Rational Number :- $\frac{2}{3}, 2, \frac{1}{2}, \frac{p}{q} \ (q \neq 0)$

7. Irrational Number :- $\sqrt{2}, \pi, \sqrt{3}$

2. Column Matrix :- A matrix having only one column.

or

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4\times 1} \quad \begin{matrix} \rightarrow R1 \\ \rightarrow R2 \\ \rightarrow R3 \\ \rightarrow R4 \end{matrix}$$

$m \times n$

$$B = \begin{bmatrix} a \\ b \end{bmatrix}_{2\times 1}$$

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ m \end{bmatrix}_{m\times 1}$$

Q If a column matrix written in $m \times n$ order then $n =$ ___ ?

(a) 1

(b) 2

(c) 0

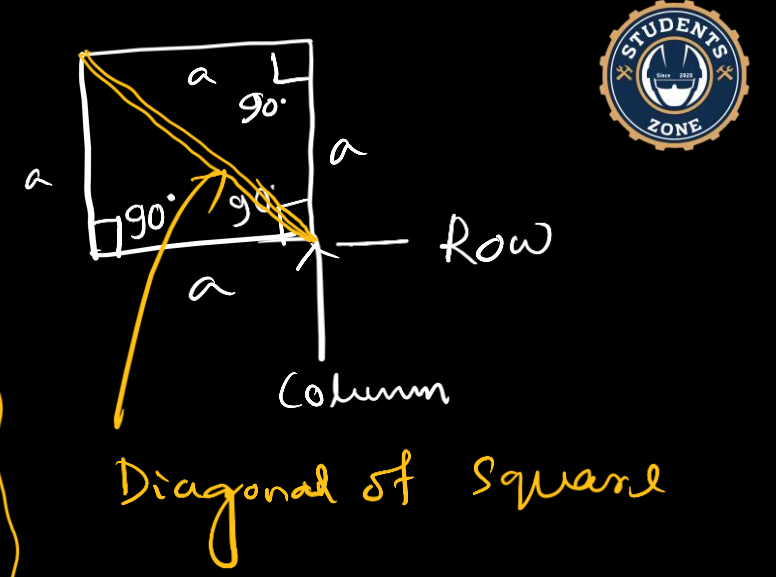
(d) Any Natural Number.

Square Matrix :-

A matrix in which number of rows is equal to number of column is called a square matrix.

→ $m = n$

→ $A = [a_{ij}]_{n \times n}$ $\quad n$ = order of Matrix



Or

$A =$ $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$ — off diagonal elements ✗

leading diagonal or principal diagonal.

$A =$ $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$

a_{11}, a_{22}, a_{33} → Diagonal elements

Ex

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

Here diagonal elements are 1 & 4

Ex

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Here the diagonal elements are

(a) 1, 2, 3

(d) 1, 5, 9 ✓

(b) 3, 5, 7

(c) 1, 5, 8

Q1

$$D = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}_{2\times 3}$$

Here diagonal elements are

(a) 2, 7

(b) 2, 6

(c) 4, 5

(d) None of the above

Sol This is not a square matrix, hence diagonal elements can't be determined.

Square Matrix

The elements a_{ij} of a square matrix $A = [a_{ij}]_{n \times n}$ for which $i = j$ i.e. the elements $a_{11}, a_{22}, a_{33} \ldots a_{nn}$ are called diagonal elements.

* A matrix $A = [a_{ij}]_{m \times n}$, where $m = n$, that is called a square matrix.

Diagonal Matrix :-

A Square matrix $A = [a_{ij}]_{n\times n}$ is called a diagonal matrix if all the elements, except those in leading diagonal are zero i.e.

$$a_{ij} = 0 \quad \text{for all} \quad i \neq j$$

ex

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}_{2\times 2}$$

$$A = \text{diag}[a_{11}, a_{12}]$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}$$

$$B = \text{diag}[b_{11}, b_{22}, b_{33}]$$

* A Square matrix in which all the elements are Zero except diagonal elements is called diagonal matrix.

ex

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$C = \text{diag}[1, 2, 3]$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$D = \text{diag}[1, 5, 6]$$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = \text{diag}[0, 0, 1]$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$$

$$F = \text{diag}[0, 2]$$

Q Whether all Square matrix is a diagonal matrix ?

(a) yes

(b) No

Q Whether all diagonal matrix is a Square matrix ?

(a) yes

(b) No

Q $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ 0

this is a ____ matrix

(a) diagonal

(b) Square

(c) Scaler

(d) Both a & b

Sol As a_{23} element is non zero, hence this is not a diagonal matrix.

∴ This is a Square matrix.