

**Spring 2022**

# INTRODUCTION TO COMPUTER VISION

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**Atlas Wang**

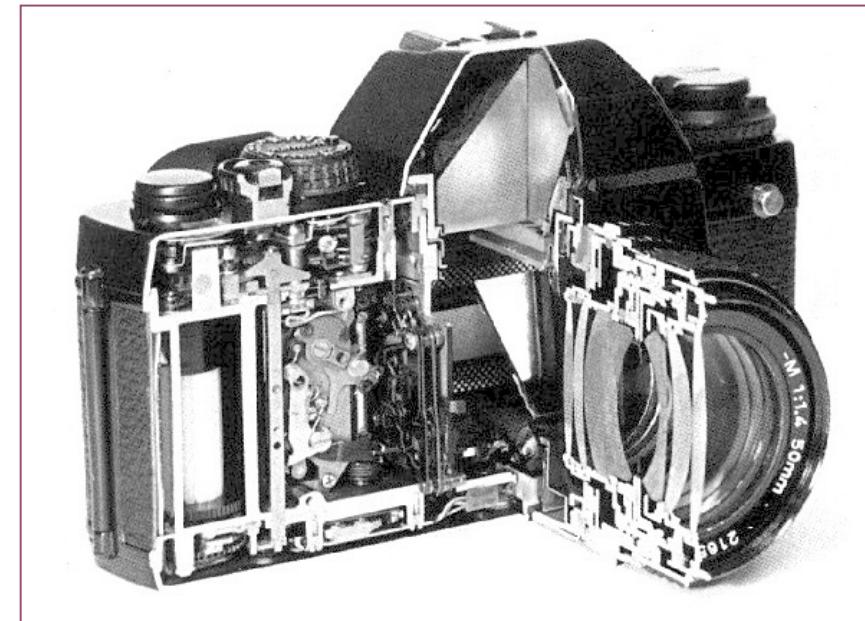
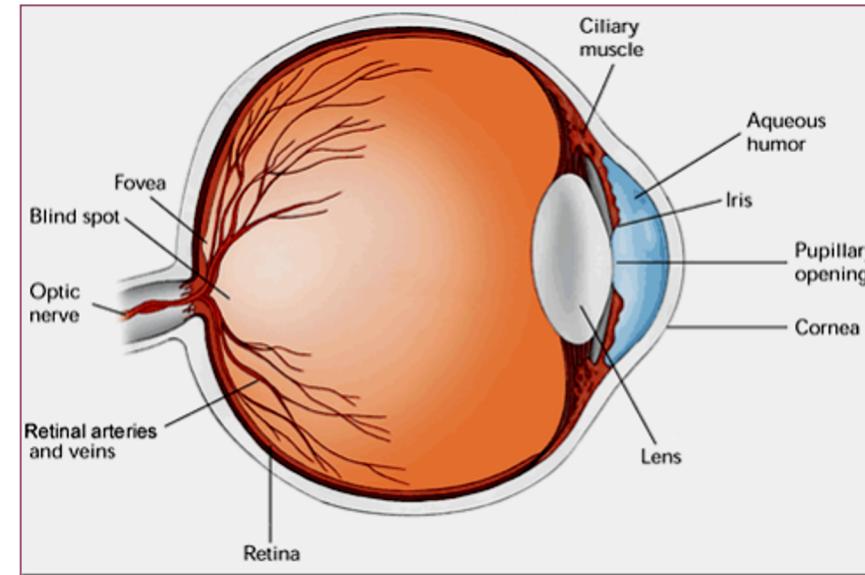
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<https://vita-group.github.io/>

Many slides here were adapted from CMU 16-385 + Brown CSCI 1430

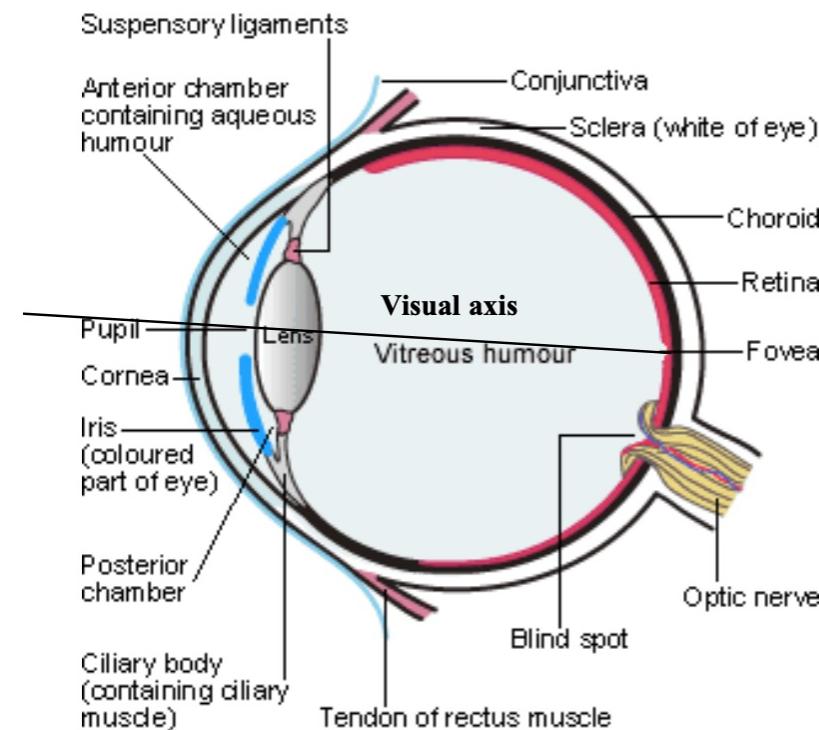
# Image Formulation

- Human: lens forms image on retina, sensors (rods and cones) respond to light
- Computer: lens system forms image, sensors (CCD, CMOS) respond to light



# Overview of Human Vision: “Low-Level”

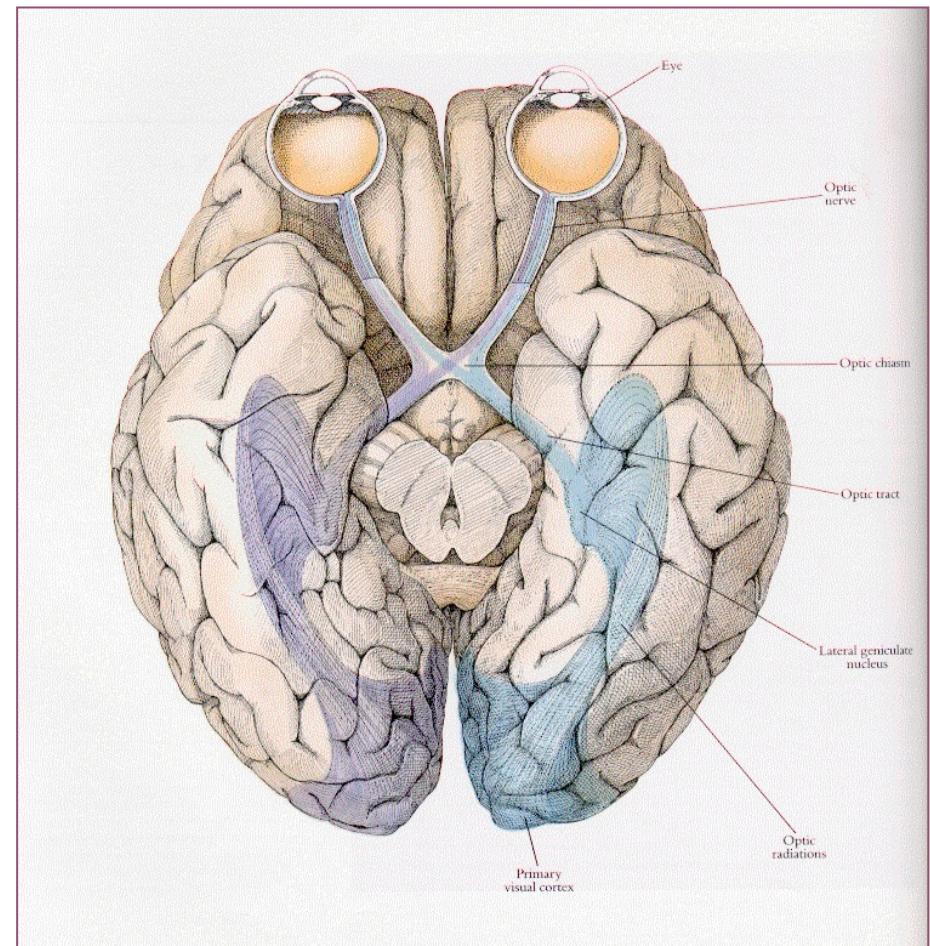
- Human visual perception plays a key role in composing our “computer vision” techniques!
- **Lens and Cornea:** focusing on the objects
- Two receptors in the retina: **Cones and Rods**
  - Cones located in fovea and are sensitive to **color**
  - Rods give a general overall picture of view, are insensitive to color but sensitive to the **level of illumination**

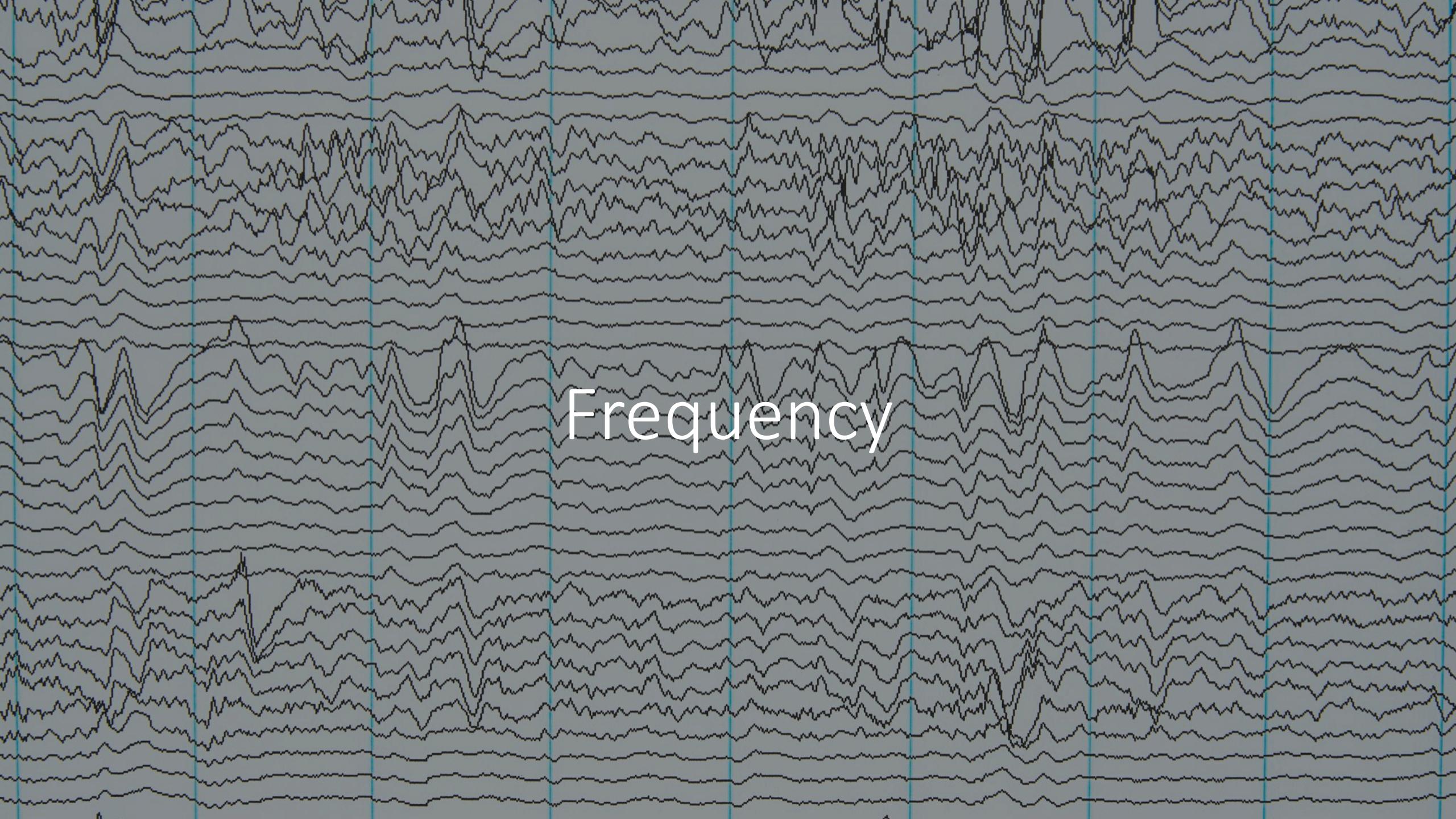


<http://www.mydr.com.au/eye-health/eye-anatomy>

# Overview of Human Vision: “Mid & High-Level”

- Lateral Geniculate Nucleus
  - “Compute” temporal and spatial correlations
- Primary Visual Cortex (V1)
  - Very well-defined mapping of the spatial information
  - “Saliency hypothesis” and gaze shifts
- Further processing starting from V1: “**What-Where Pathway**”
  - Temporal cortex (ventral): what is the object?
  - Parietal cortex (dorsal): where is the object? How do I get it?
- Recognition-by-components (**RBC**) theory

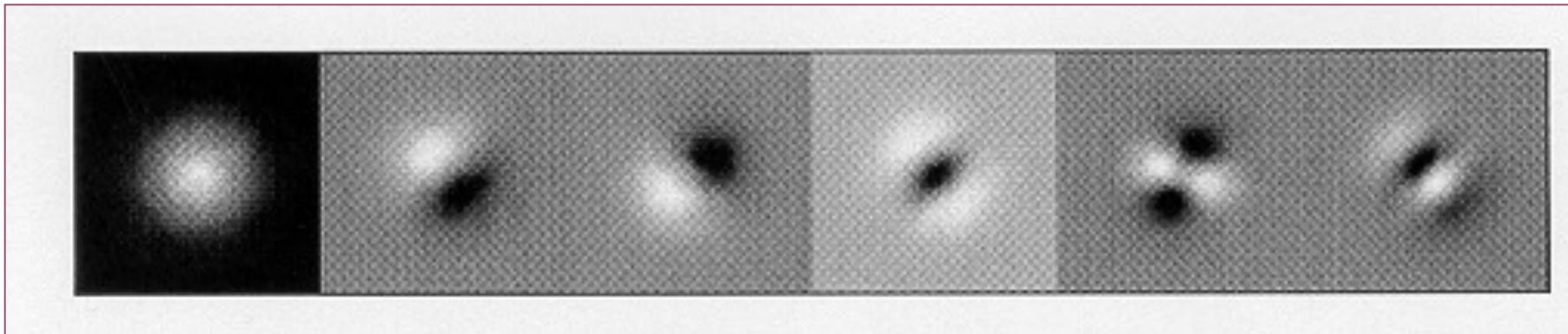




Frequency

# Your Brain Secretly Thinks in the Frequency Domain

- Low-level human vision can be (partially) modeled as a set of *multi-resolution, and multi-orientation* filters

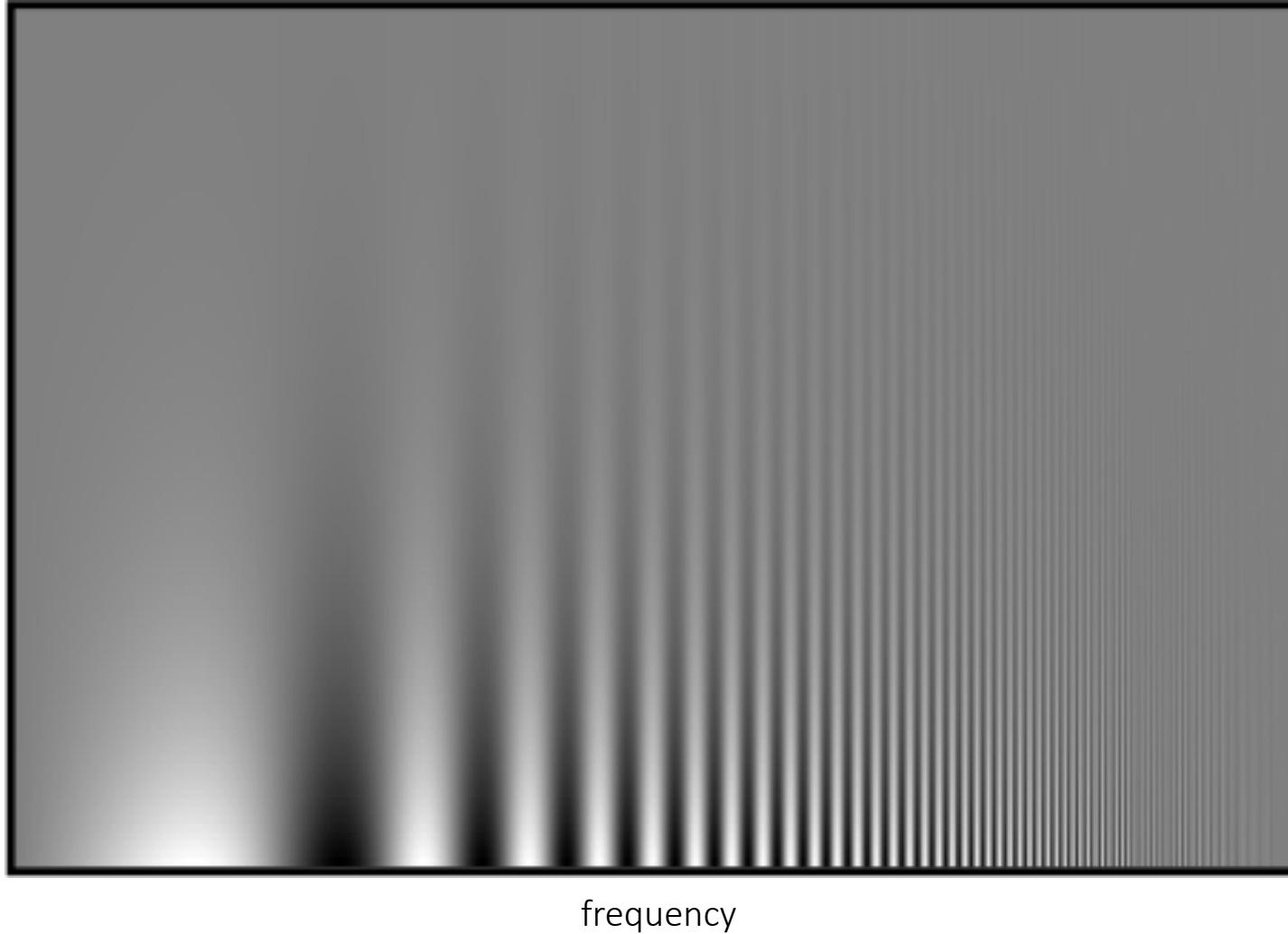


- Human perception cues are dominated by mid- to high-frequency bands
  - The spatial-frequency theory refers to the theory that the visual cortex operates on a code of spatial frequency, not on the code of straight edges and lines *Your brain knows “how to do” Fourier transform, before you know it...*
  - When we see something from a distance, we are effectively subsampling it. *Did this remind you of sampling theorem?*
  - “Depth Cues”: Focus, Vergence, Stereo ...

# Variable frequency sensitivity

Experiment: Where do you see the stripes?

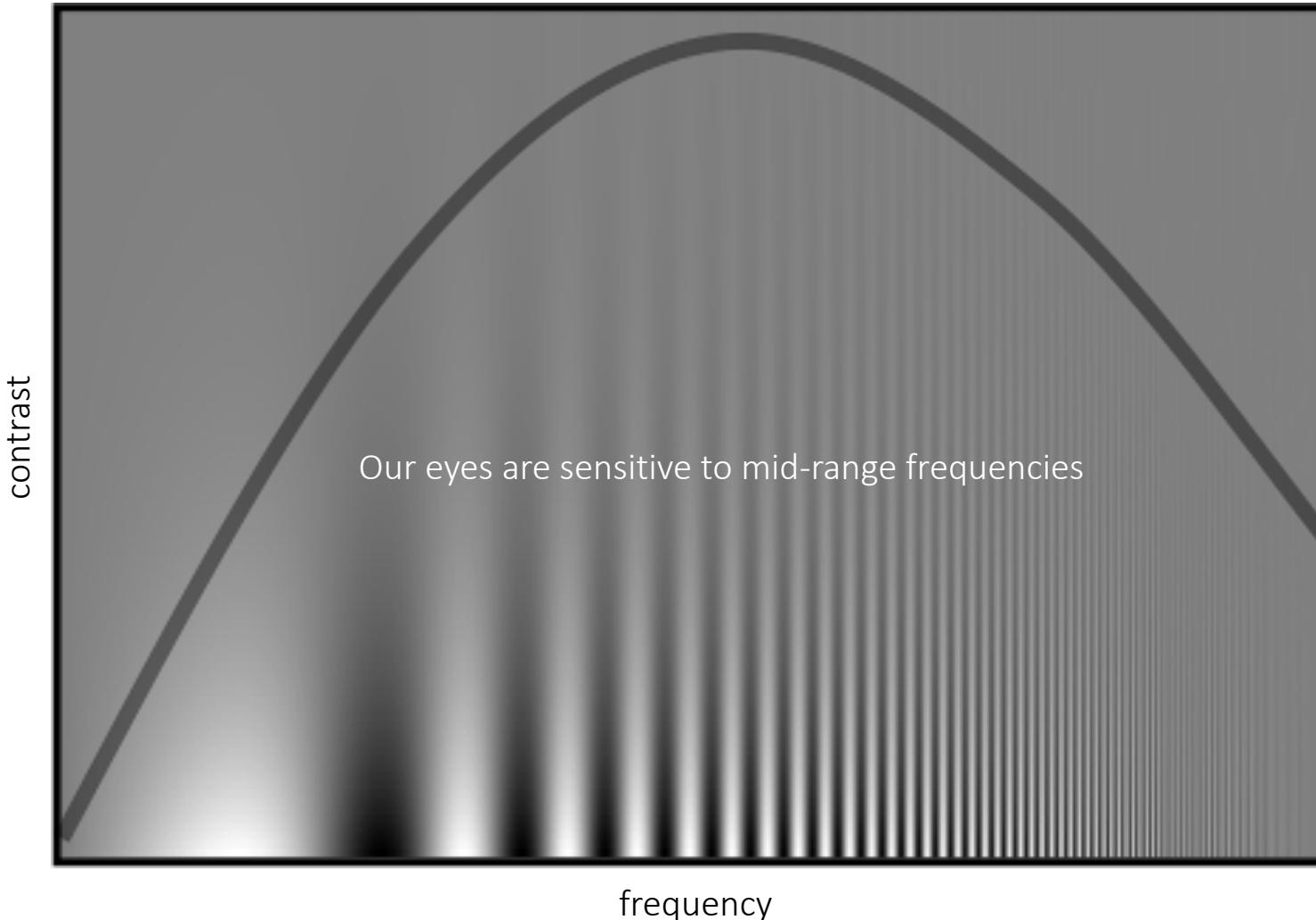
contrast



frequency

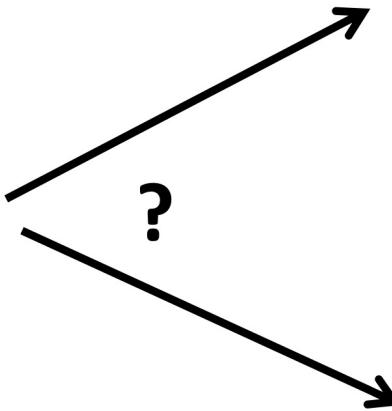
# Variable frequency sensitivity

Campbell-Robson contrast sensitivity curve



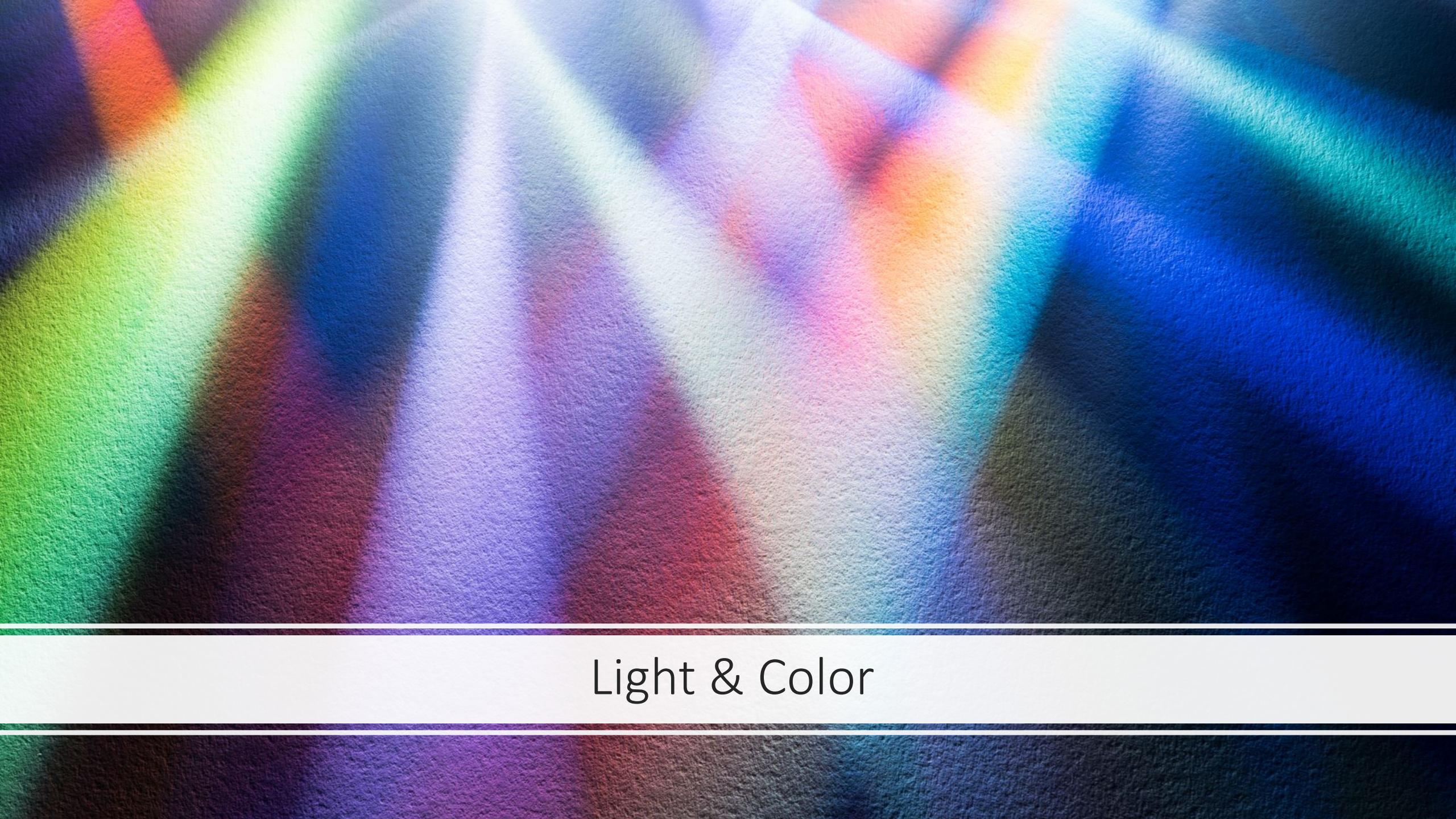
- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid frequencies dominate perception

# Example: Hybrid Images (distance -> sampling)



Distance-dependent  
perception of hybrid  
images by human

*Are you still complaining  
deep networks are easily  
fooled? ☺*



# Light & Color

# Our perceived brightness is often “relative”

- The apparent brightness depends on the surrounding region
  - **Brightness contrast:** a constant-colored region seem lighter or darker depending on the surround:

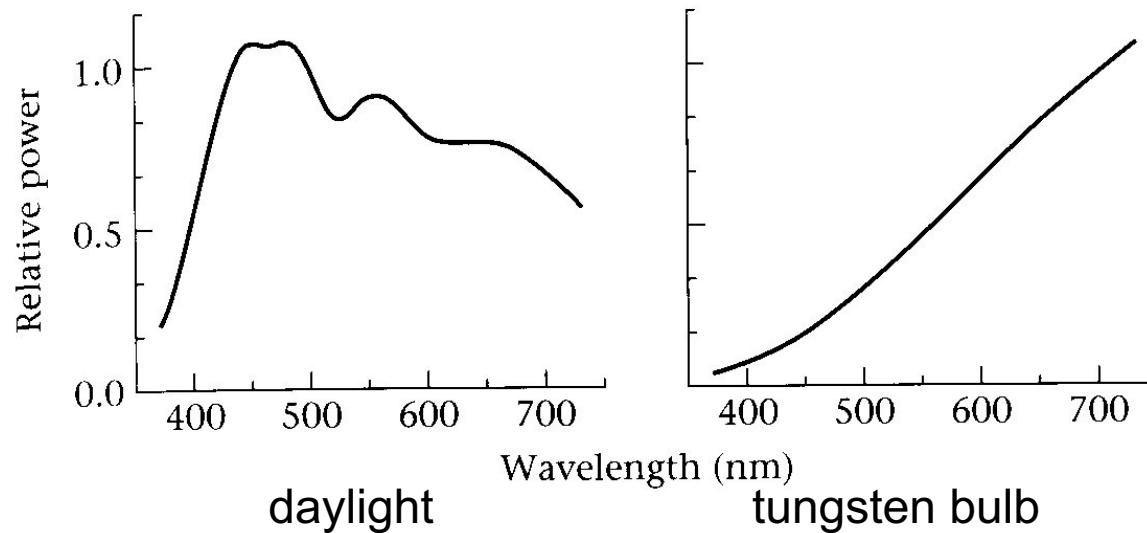


- [http://www.sandlotsscience.com/Contrast/Checker\\_Board\\_2.htm](http://www.sandlotsscience.com/Contrast/Checker_Board_2.htm)

- **Brightness constancy:** a surface looks the same under widely varying lighting conditions
  - For example, something white will appear to be the same shade of white no matter how much light it is being exposed to - noontime sunlight or a soft lamplight at night.
  - A type of psychological “perceptual constancy” (other constancy forms: color, shape ...)

# Light spectrum

- The appearance of light depends on its power **spectrum**
  - How much power (or energy) at each wavelength



Our visual system converts a light spectrum into “color”

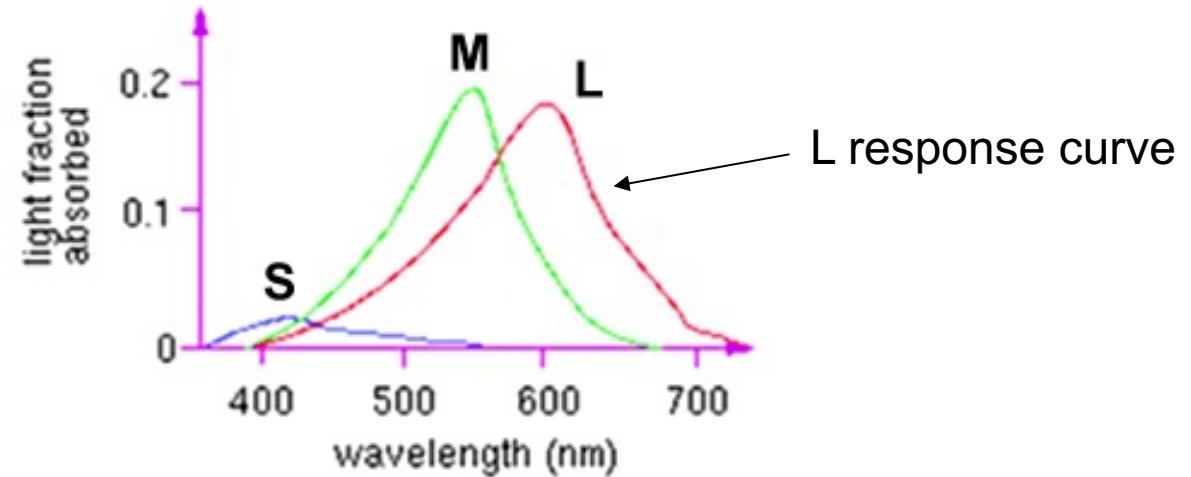
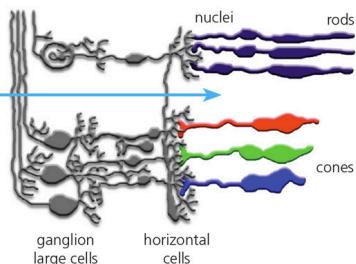
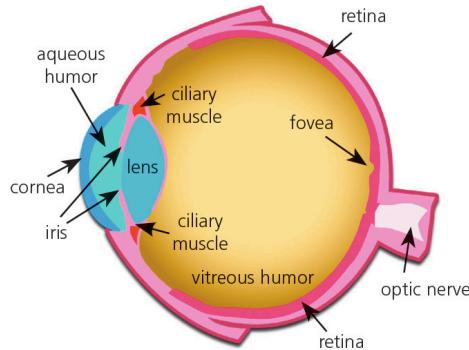
- This is a rather complex transformation
- **Color is an extended concept of “brightness”**

# Colors are almost always “mixtures”

- We almost never see a “pure” wavelength of light; rather a mixture of wavelengths, each with a different “power”
- Only some colors occur as pure wavelengths; most are mixtures of pure colors (e.g. white)

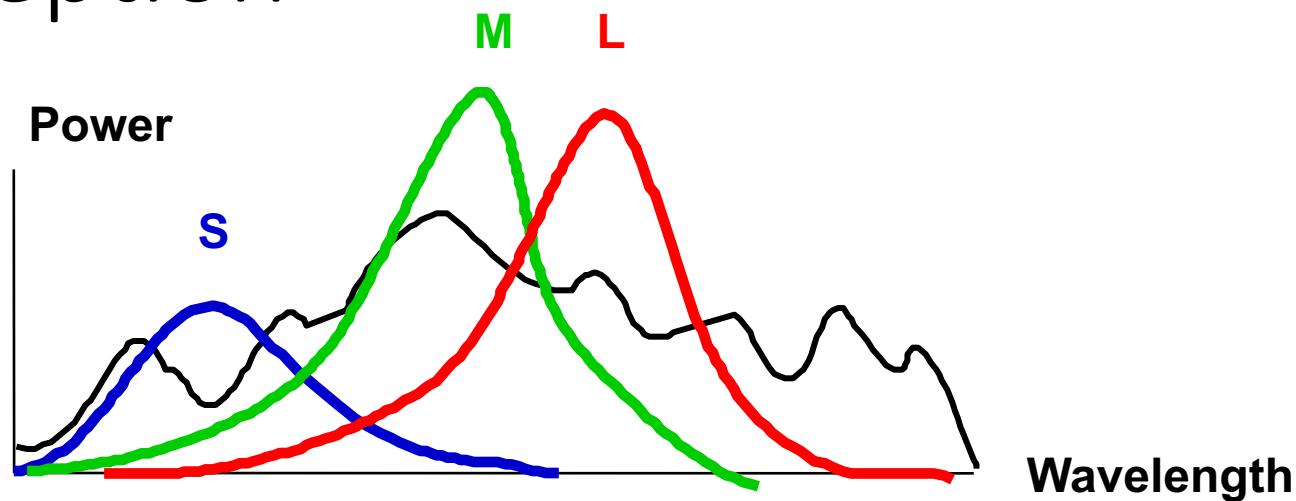


# Color perception



- Three types of cones
  - Each is sensitive in a different region of the spectrum, but regions overlap
    - Short (S) corresponds to blue
    - Medium (M) corresponds to green
    - Long (L) corresponds to red
  - Different sensitivities: we are more sensitive to green than red
    - varies from person to person (and with age)
  - Colorblindness—deficiency in at least one type of cone

# Color perception



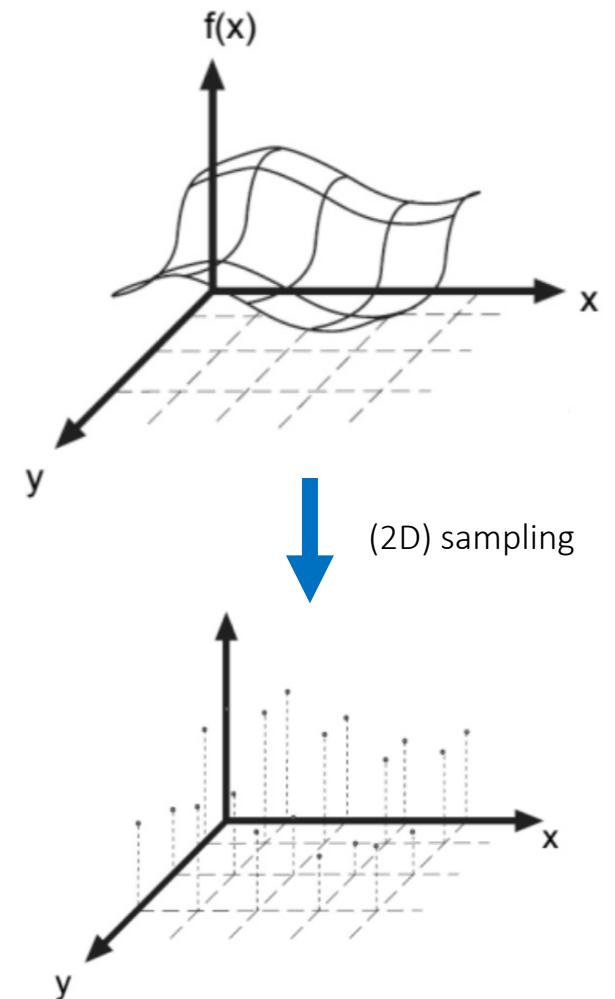
- Rods and cones act as filters on the spectrum
  - To get the output of a filter, multiply its response curve by the spectrum, integrate over all wavelengths
    - Each cone yields one number
  - Q: How can we represent an entire spectrum with 3 numbers?
  - A: We can't! Most of the information is lost.
    - As a result, two different spectra may appear indistinguishable by human eyes
    - Just like spatial “resolution”, human eyes also have limited “color resolution”

The background is a dynamic, colorful digital landscape. It features a grid of binary digits (0s and 1s) in various colors like red, green, blue, and yellow, some of which are highlighted with a bright glow. These digits are set against a dark, almost black, background. Interspersed among the digits are several large, semi-transparent, glowing numbers in red, orange, and yellow. These numbers appear to be floating or moving through the space. In the foreground, there are numerous thin, glowing lines in shades of red, orange, and yellow, creating a sense of depth and motion, reminiscent of light trails or data streams.

Now, Digital Images!

# Digital Image: Sampling of Continuous Visual World

- **Signal:** function depending on some variable with physical meaning
  - Our real visual world is always a continuous signal, do you agree?
- **Digital Image:** sampling of that function, dependent on **variables** of:
  - Two-axis: x-y coordinates
  - Three-axis: x-y-time (**video**)
- “Brightness/Color” is the **value** of the function for visible light, a.k.a. **pixel**
- Other possible function values in various “images”: depth, heat...



# Digital Image Representation

Binary



Gray scale



Color



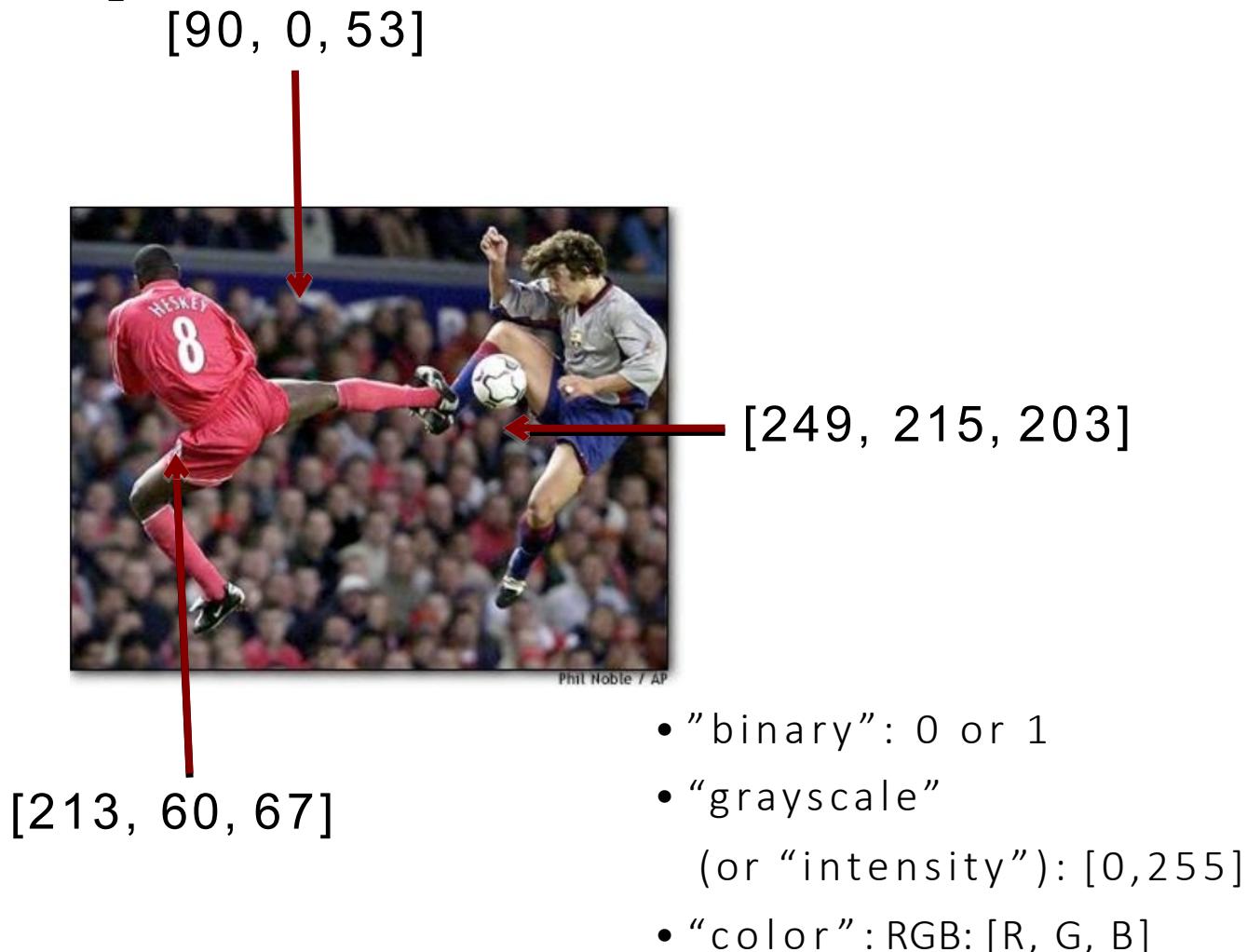
Phil Noble / AP

# Digital Images are Sampled and Quantized

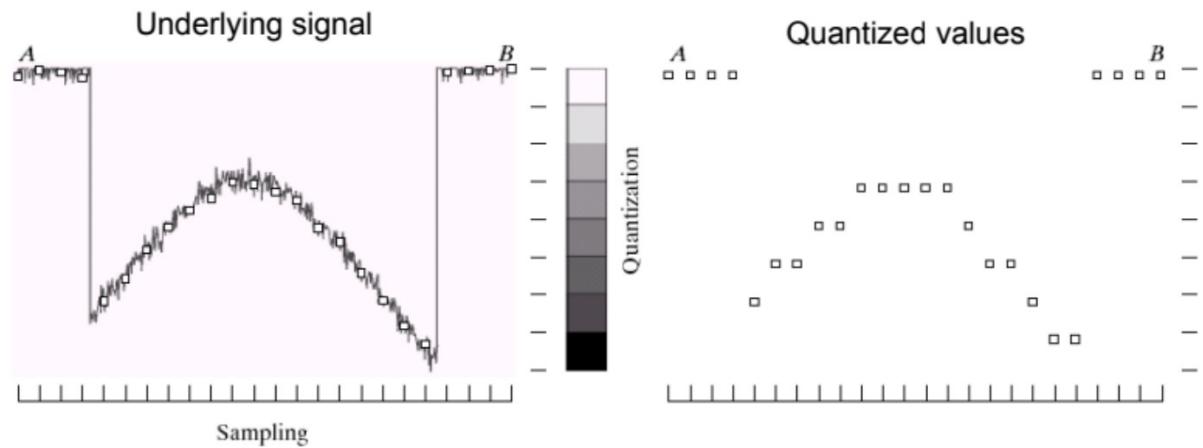
- An image contains **discrete number of pixels**, and each pixel has **discrete number of values**
- **Remember:** you discretize both the spatial (2D or 3D) and spectral(pixel value) dimensions, either at certain “resolution”

Samples = pixels

Quantization = number of bits per pixel



# Digital Values can be Quantized Further

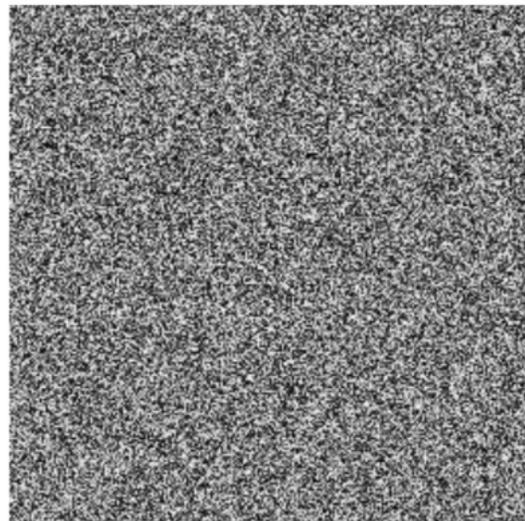


- We often call this *bit depth*
- For photography, this is also related to *dynamic range*

# Is An Image Just A Matrix?

```
>>> from matplotlib import pyplot as p  
>>> I = r.rand(256,256);  
>>> p.imshow(I);  
>>> p.show();
```

Is it an image?



**Image is a high-structured 2D signal!!**

- *(piece-wise) smoothness, self-similar patterns (fractal), “reducible” to the composition of basic units (subspace)...*
  - *A wealth of “image priors”, although not always explicit*
- It takes great luck for a 2D matrix to be an image!

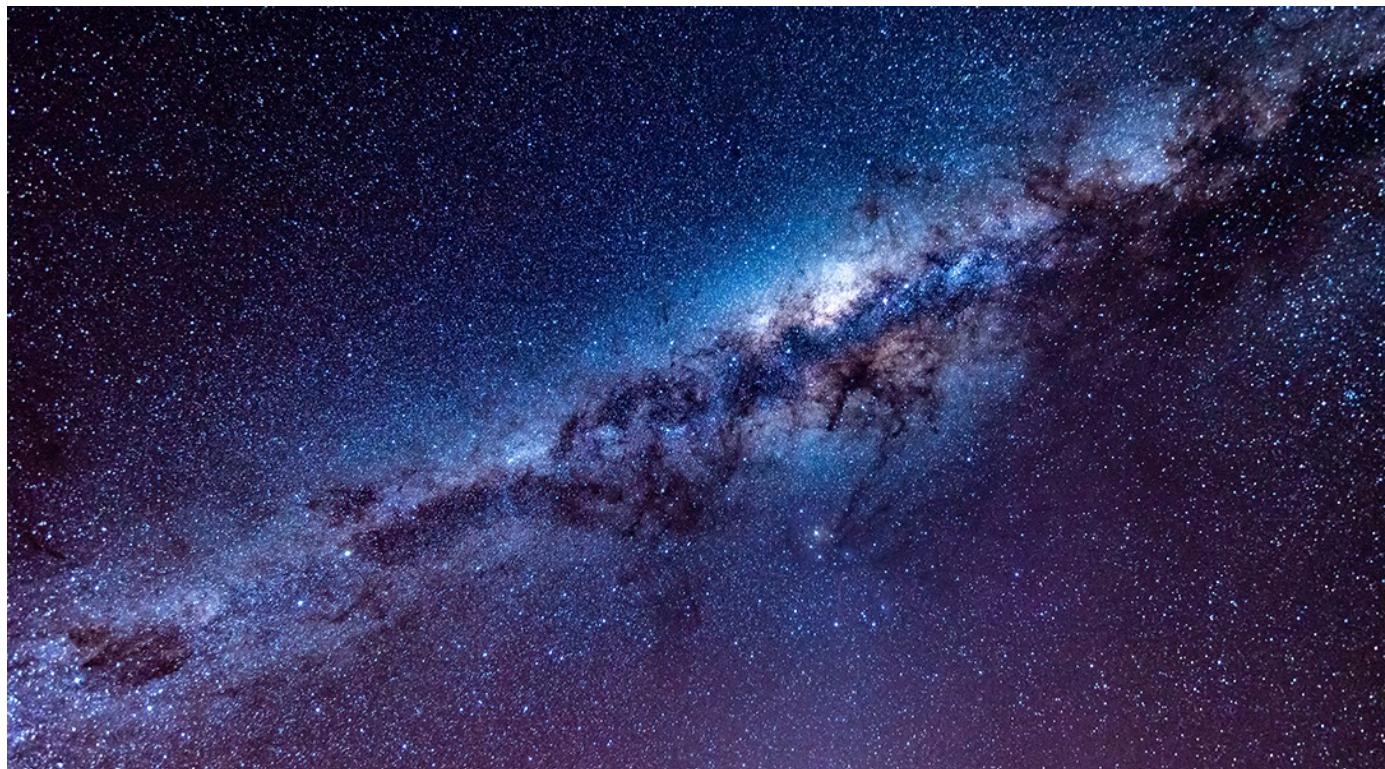
*8bit = 256 values ^ 65,536*

Computer vision makes sense of an extremely high-dimensional space

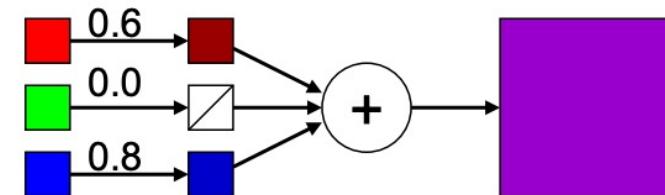
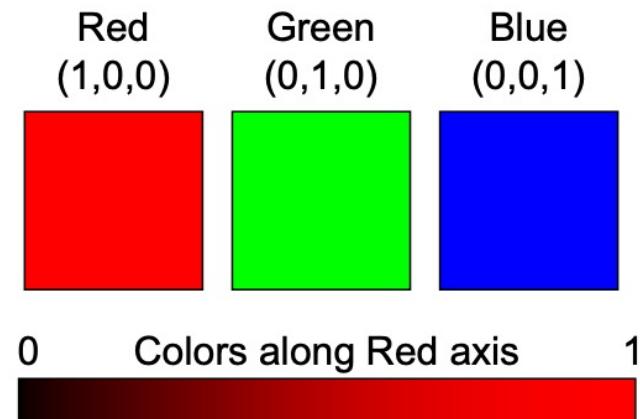
- *Using low-dimensional, explainable models*

# “Natural Image Manifold”

- The distribution of natural images (or patches) is similar to the mass distribution in the universe, where there are high-density and low-density areas
- This “manifold” has to be highly nonlinear, inherently **low-dimensional**, and **locally smooth** ... (do you understand why?)



# Color Image: Three-Channel RGB Model

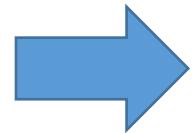
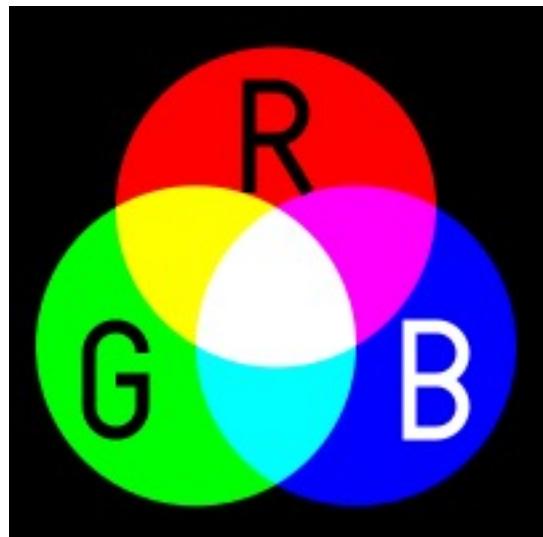


*Universal, yet non-perceptual...*

- The three channels are strongly correlated!*



# Color Space Representations



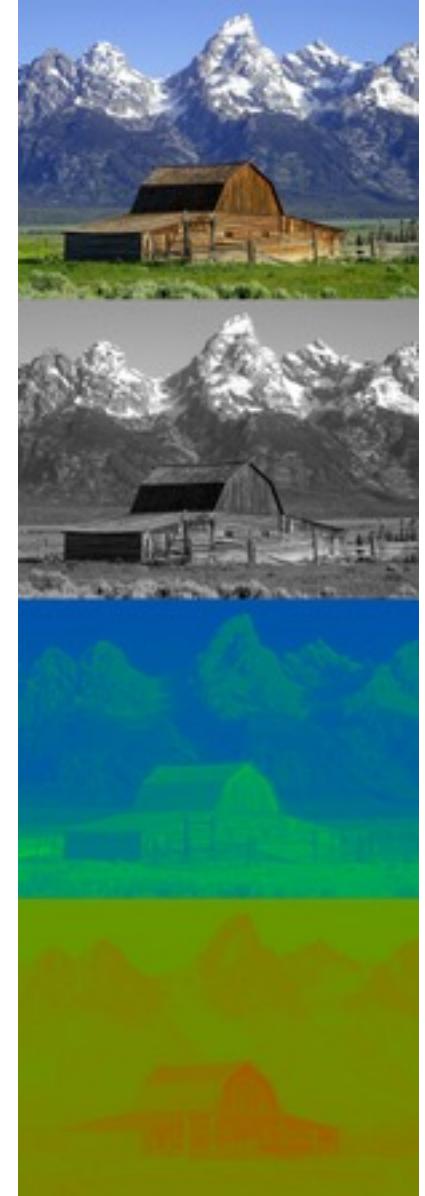
**RGB system (most common):**  
linear additive color mixing

$$\begin{bmatrix} Y' \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.14713 & -0.28886 & 0.436 \\ 0.615 & -0.51499 & -0.10001 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix},$$

$$\begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1.13983 \\ 1 & -0.39465 & -0.58060 \\ 1 & 2.03211 & 0 \end{bmatrix} \begin{bmatrix} Y' \\ U \\ V \end{bmatrix}.$$

**YUV system (popular in color TV):**

- Y stands for the luma component ( brightness)
- U and V are the chrominance (color) components



# Bit Color Depth



1 bit

2 bits

4 bits

8 bits

24 bits

1= ON 0 =OFF

00 01 10 11  
Different shades of gray

24-bit **TrueColor** can represent more than 16.7 million unique colors

- More colors than the human eye can distinguish!

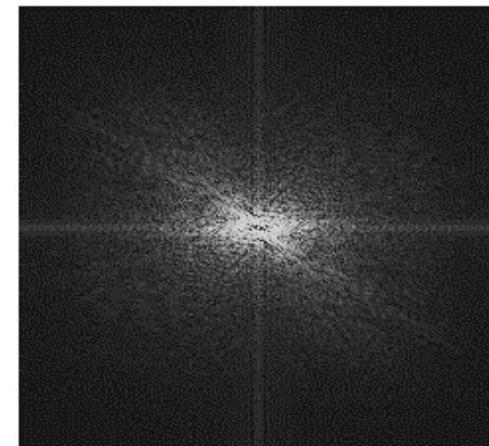
# Video: Frame-by-Frame Image Sequence

30 frames/second



# Spatial and Frequency Domains

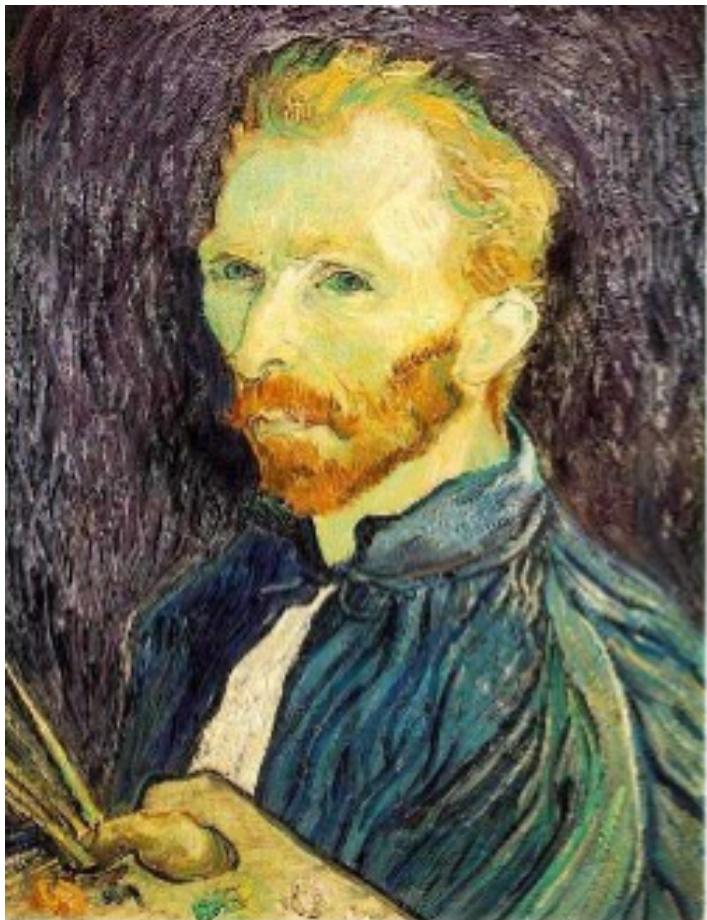
- Spatial domain
  - refers to planar region of **intensity values at time  $t$**
- **Frequency domain**
  - think of each color plane as a **sinusoidal function of changing intensity values**
  - refers to organizing pixels according to their changing intensity (frequency)





This image is too big to fit on the screen.  
How would you reduce it to half its size?

# Naïve image downsampling



1/2

Throw away half the rows and columns

delete even rows  
delete even columns



1/4

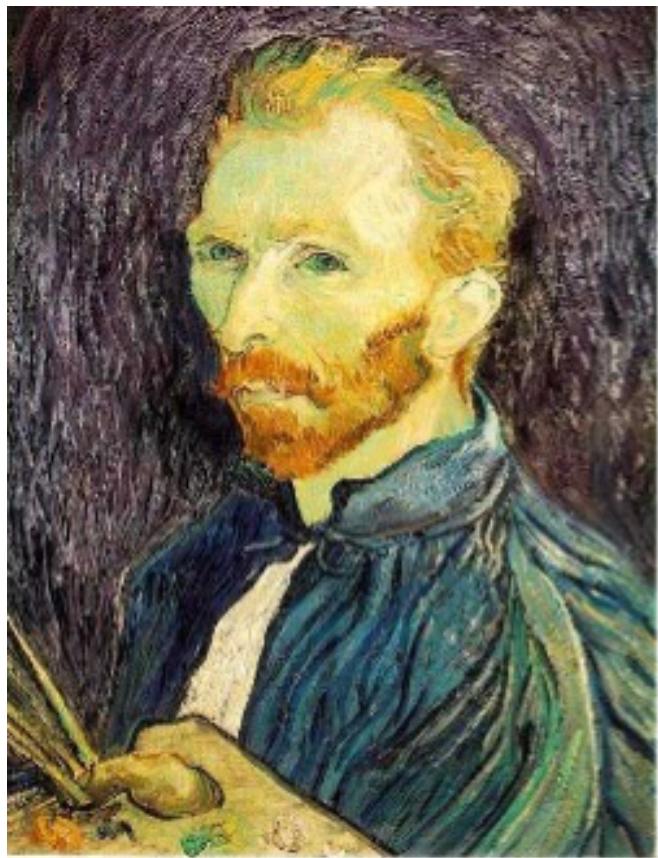
delete even rows  
delete even columns



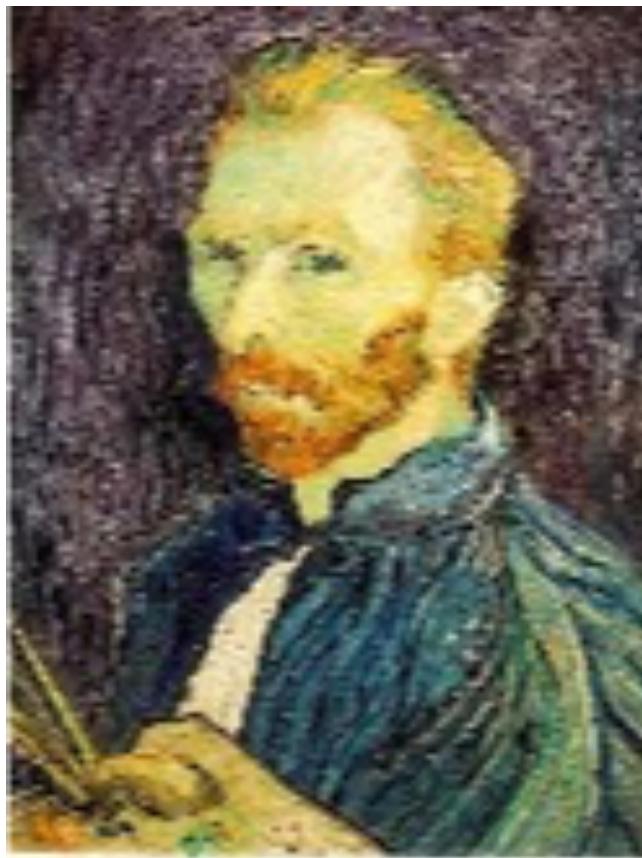
1/8

What is the problem with this approach?

# Naïve image downsampling



1/2



1/4 (2x zoom)

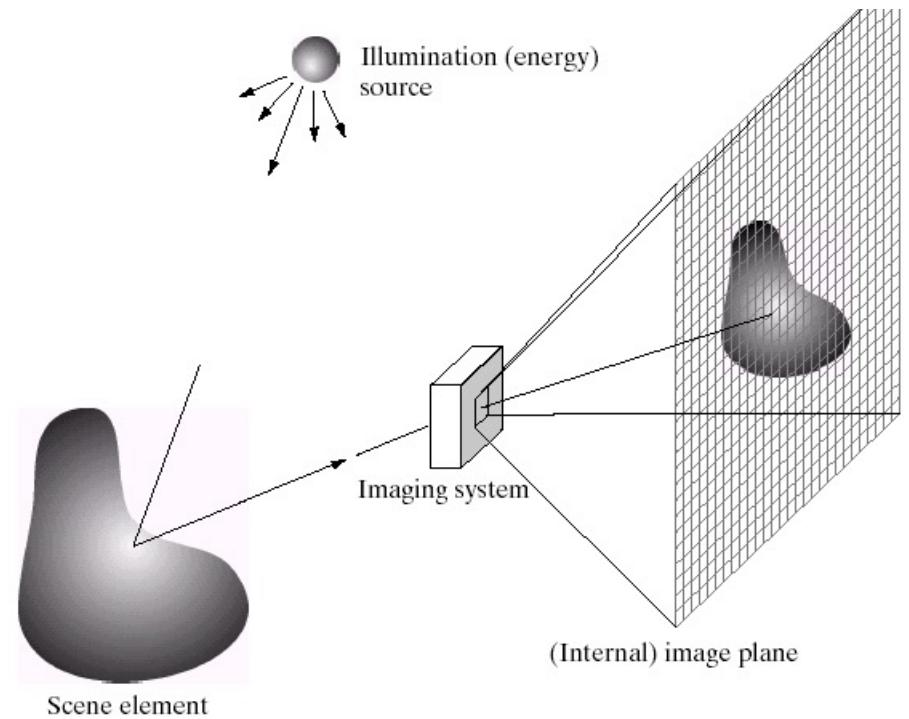


1/8 (4x zoom)

What is the 1/8 image so pixelated (and do you know what this effect is called)?

# The Devil of Digital Sampling: Aliasing

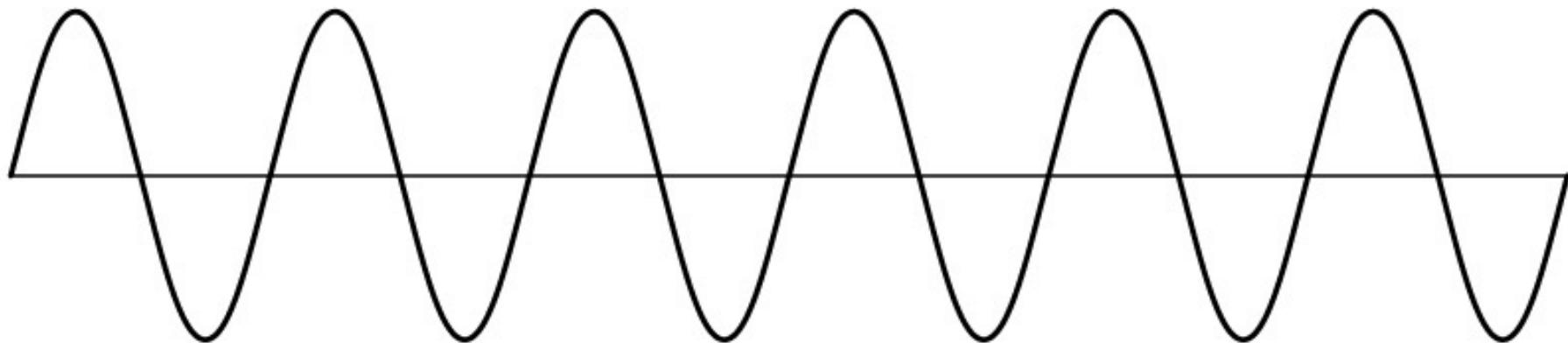
# Reminder



Images are a *discrete*, or *sampled*, representation of a *continuous* world

# Sampling

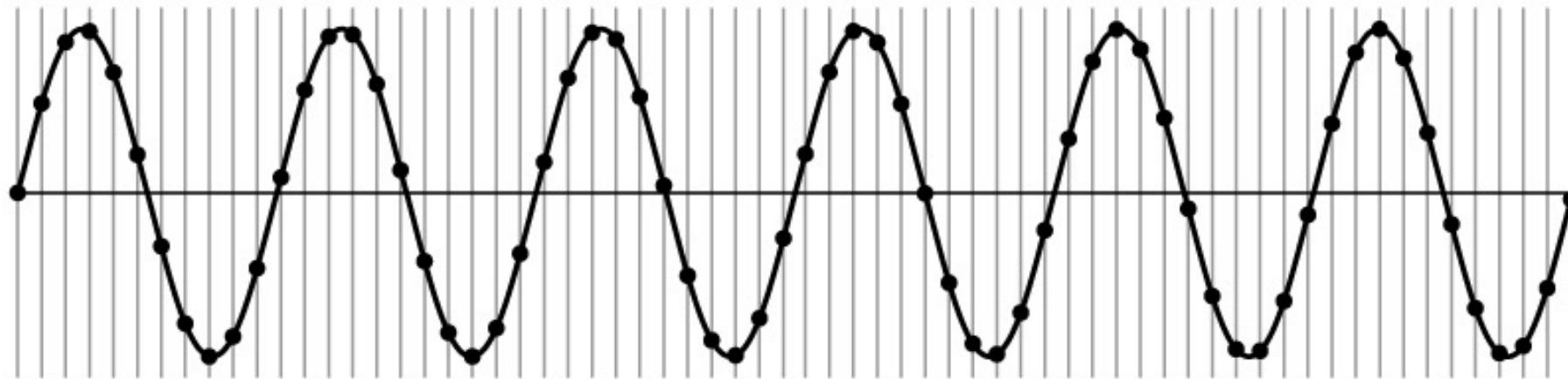
Very simple example: a sine wave



How would you discretize this signal?

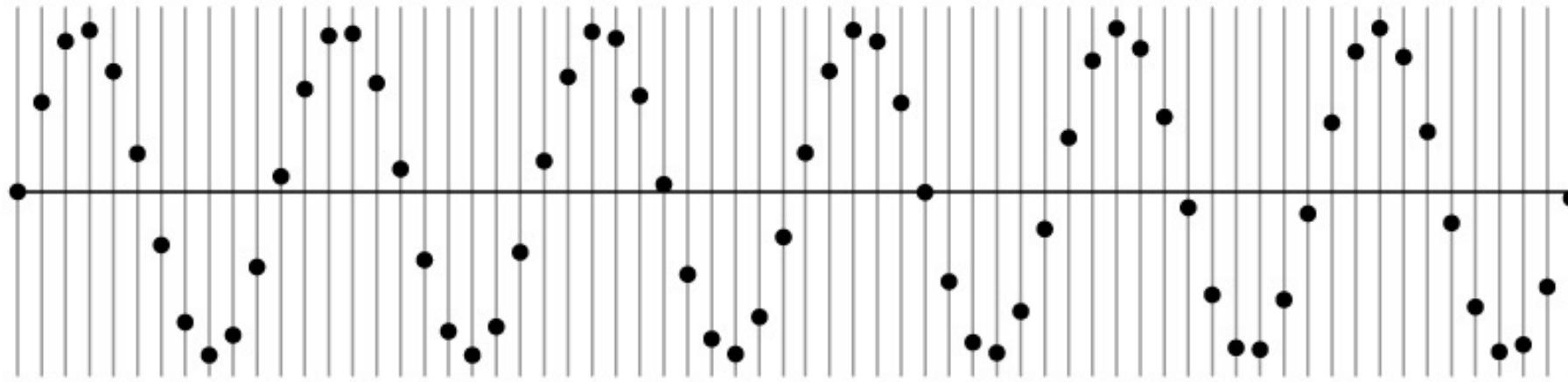
# Sampling

Very simple example: a sine wave



# Sampling

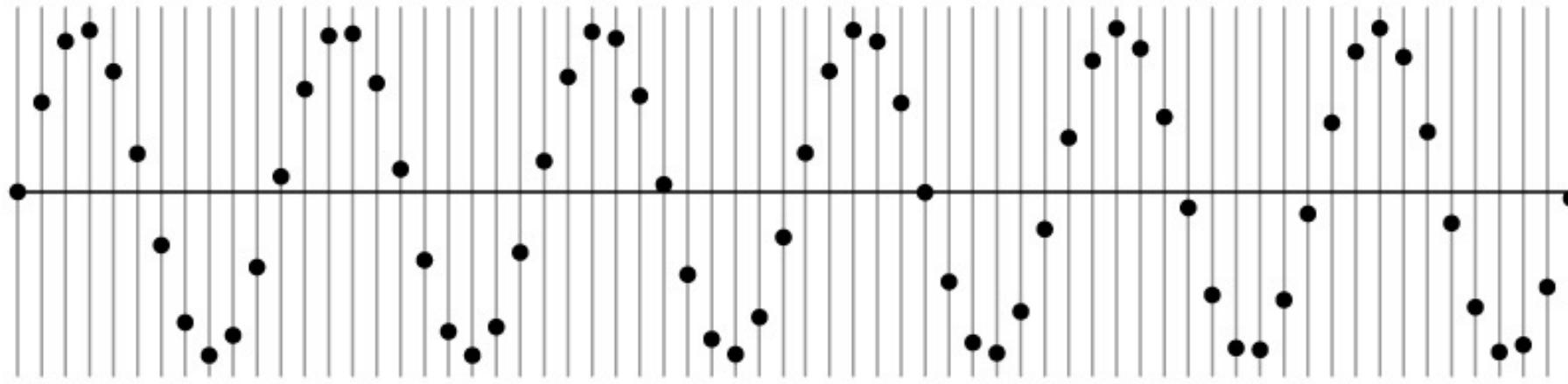
Very simple example: a sine wave



How many samples should I take?  
Can I take as *many* samples as I want?

# Sampling

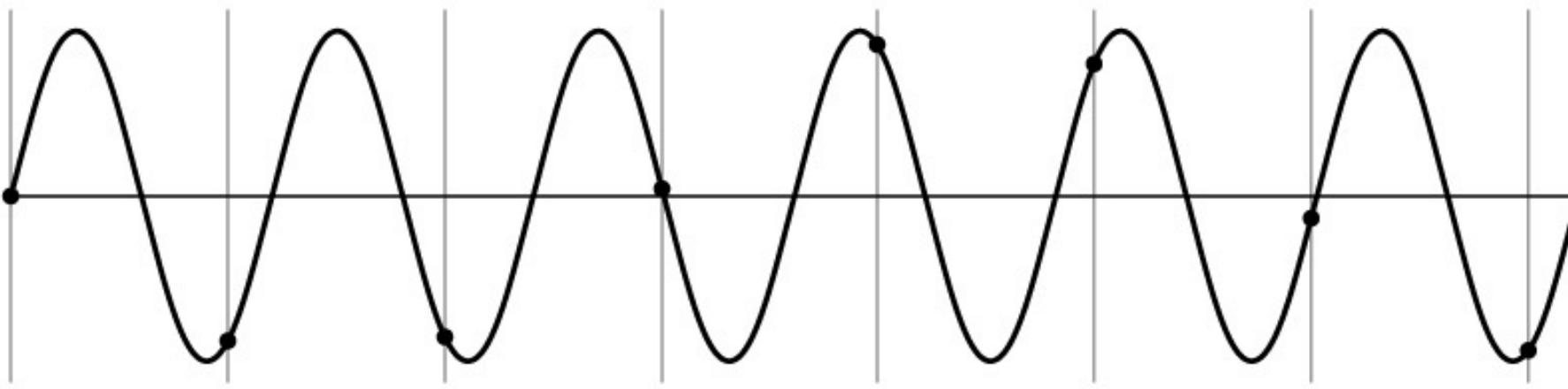
Very simple example: a sine wave



How many samples should I take?  
Can I take as *few* samples as I want?

# Undersampling

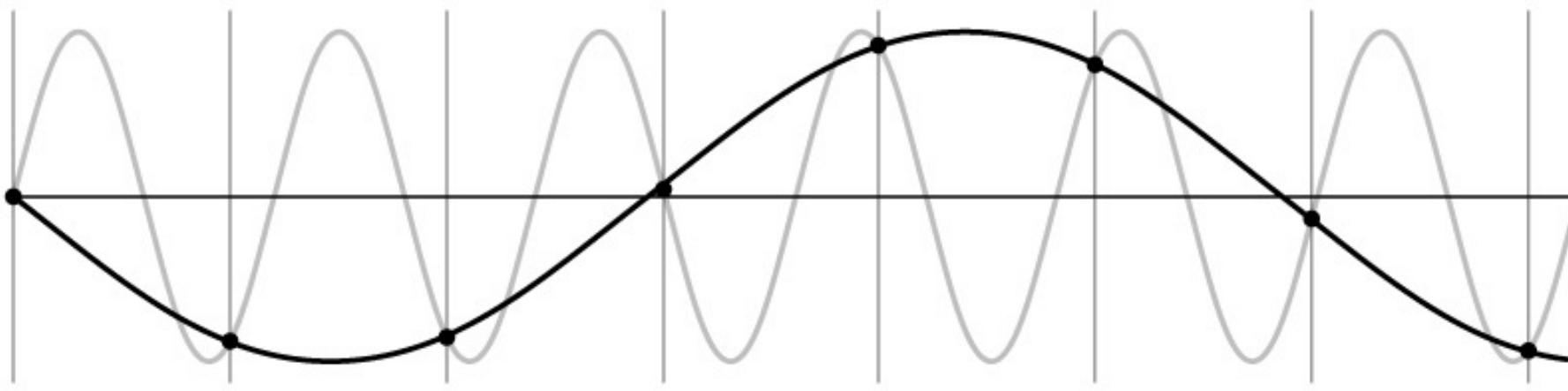
Very simple example: a sine wave



Unsurprising effect: information is lost.

# Undersampling

Very simple example: a sine wave

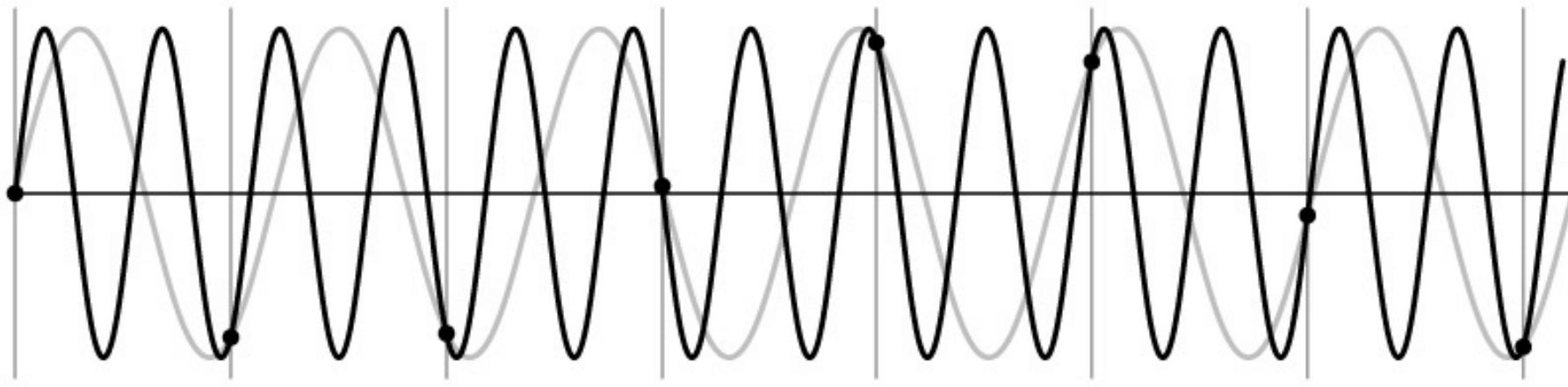


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

# Undersampling

Very simple example: a sine wave



Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

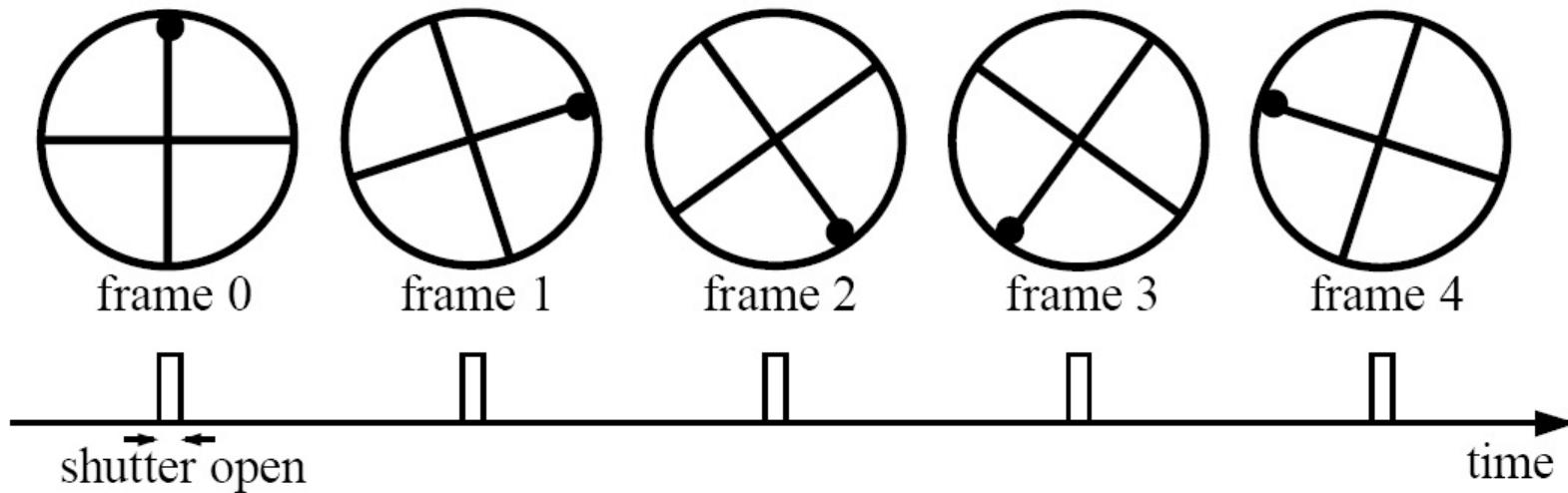
Note: we could always confuse the signal with one of *higher* frequency.

# Temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time =  $1/30$  sec. for video,  $1/24$  sec. for film):



Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)



# Anti-aliasing

How would you deal with aliasing?

# Anti-aliasing

How would you deal with aliasing?

Approach 1: Oversample the signal

# Anti-aliasing

How would you deal with aliasing?

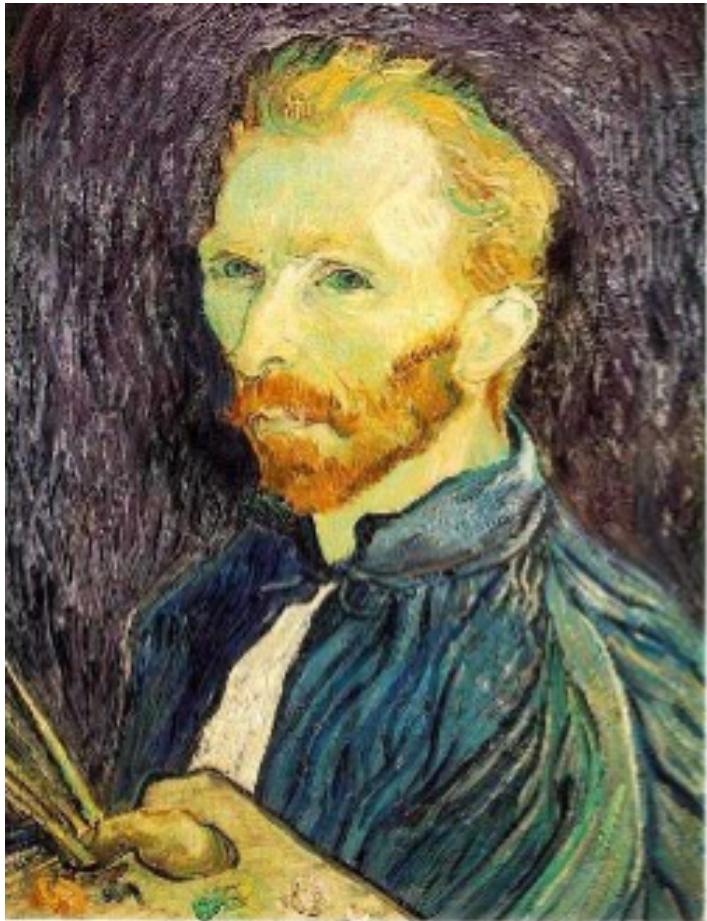
Approach 1: Oversample the signal

Approach 2: Smooth the signal

- Remove some of the detail effects that cause aliasing.
- Lose information, but better than aliasing artifacts.

How would you smooth a signal?

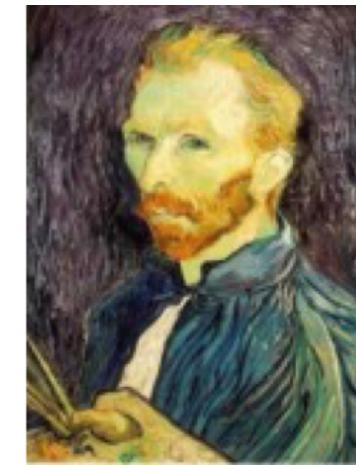
# Better image downsampling



1/2

Apply a smoothing filter first, then throw away half the rows and columns

Gaussian filter  
delete even rows  
delete even columns



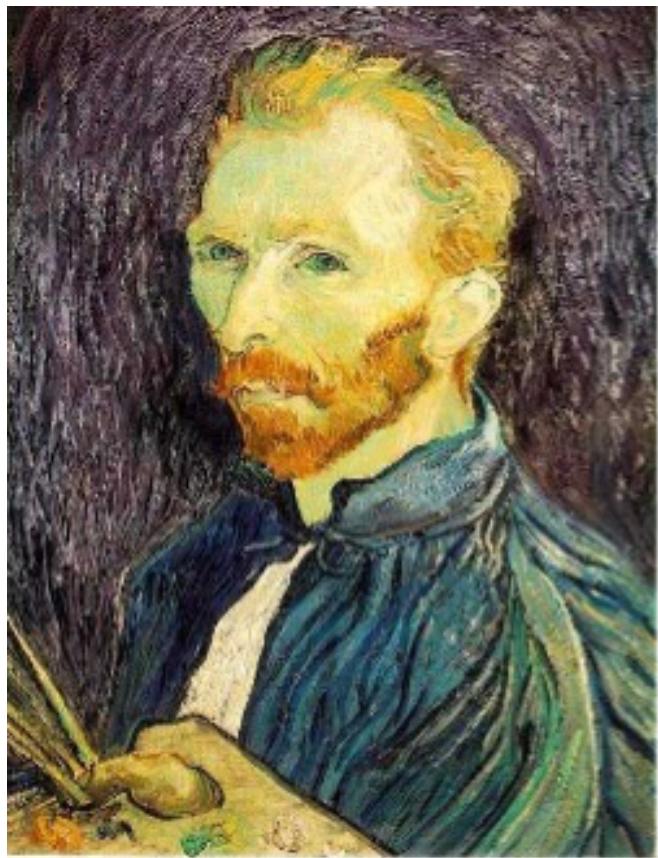
1/4

Gaussian filter  
delete even rows  
delete even columns

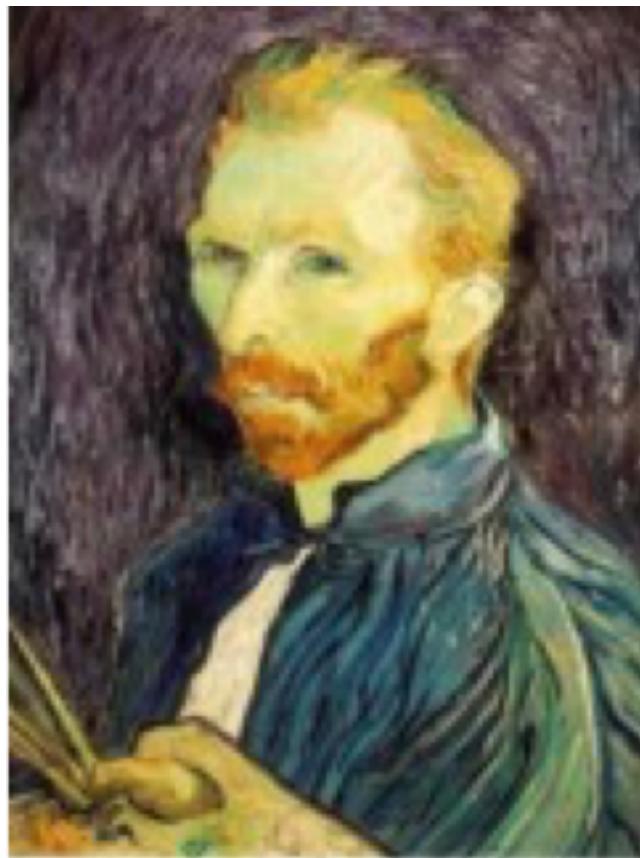


1/8

# Better image downsampling



1/2

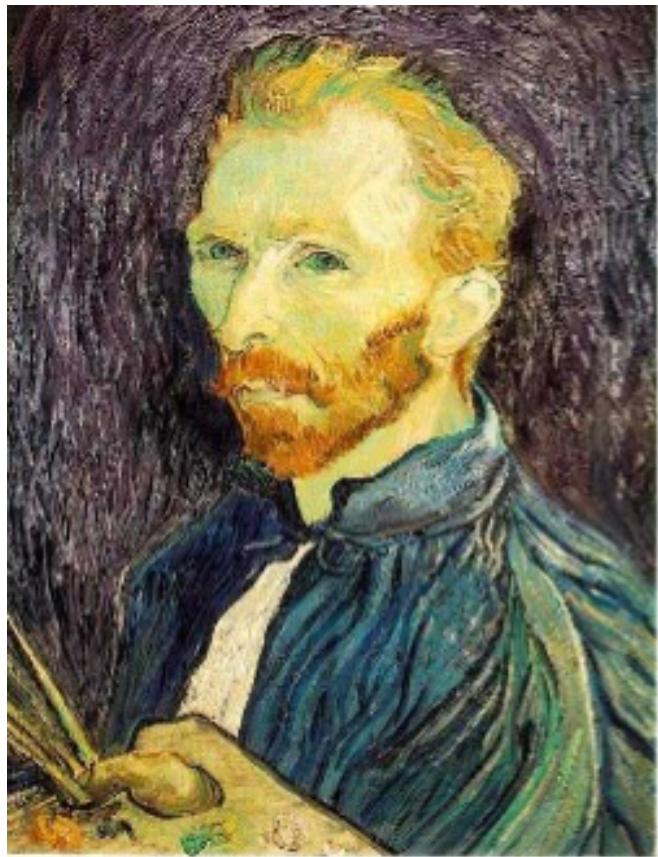


1/4 (2x zoom)

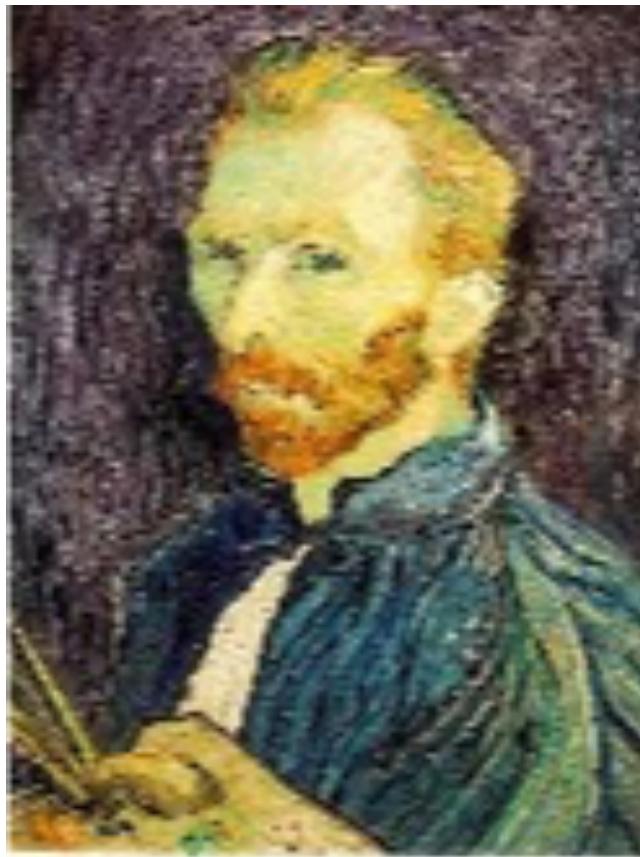


1/8 (4x zoom)

# Naïve image downsampling



1/2



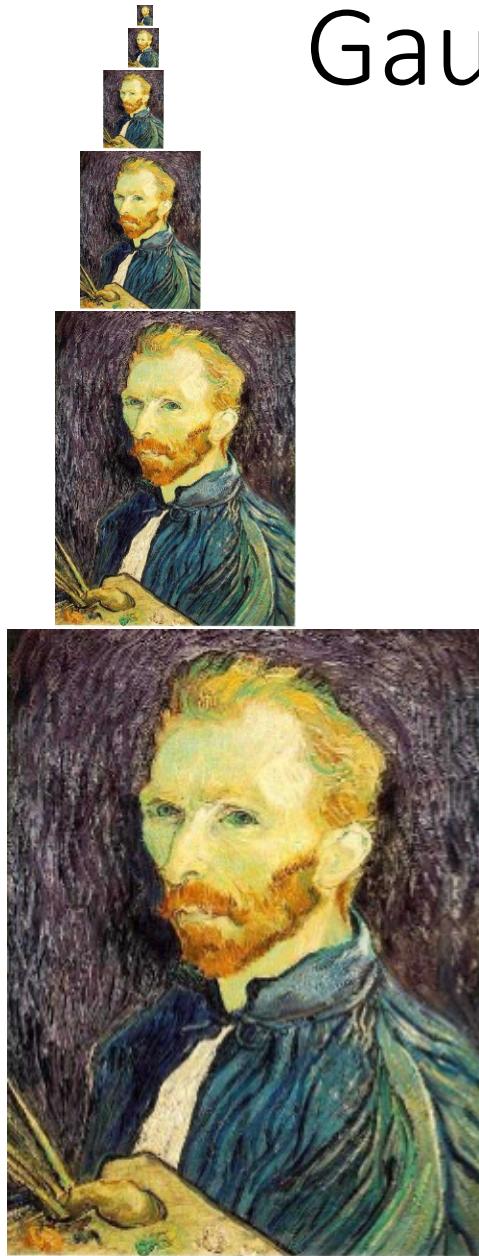
1/4 (2x zoom)



1/8 (4x zoom)

A photograph of a large pyramid, likely the Great Pyramid of Giza, viewed from a low angle. The pyramid's surface is composed of many small, rectangular stone blocks. The sky above is a uniform, clear blue.

# Image pyramid: Gaussian and Laplacian



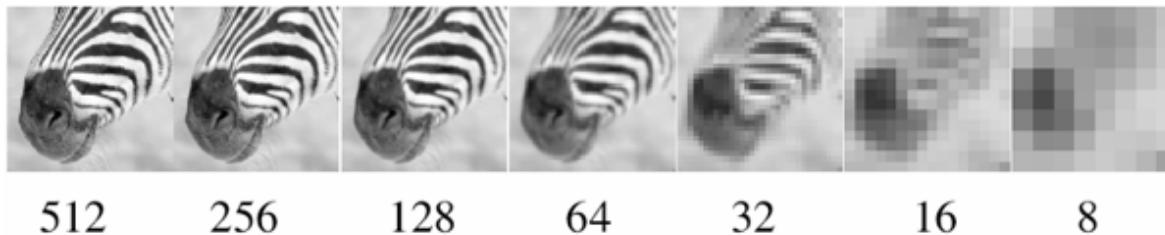
# Gaussian image pyramid

The name of this sequence of subsampled images

Algorithm

```
repeat:  
    filter  
    subsample  
until min resolution reached
```

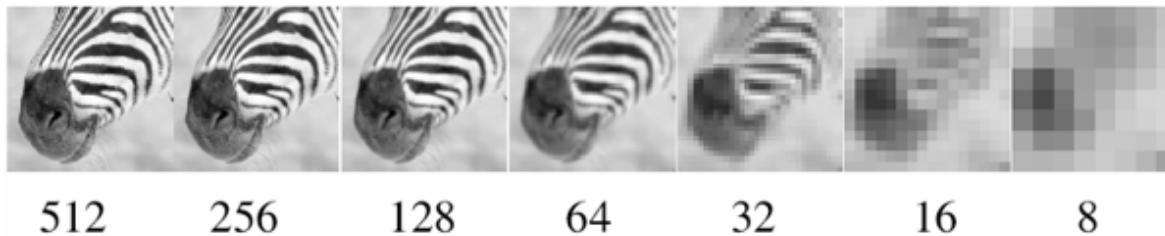
# Some properties of the Gaussian pyramid



What happens to the details of the image?



# Some properties of the Gaussian pyramid



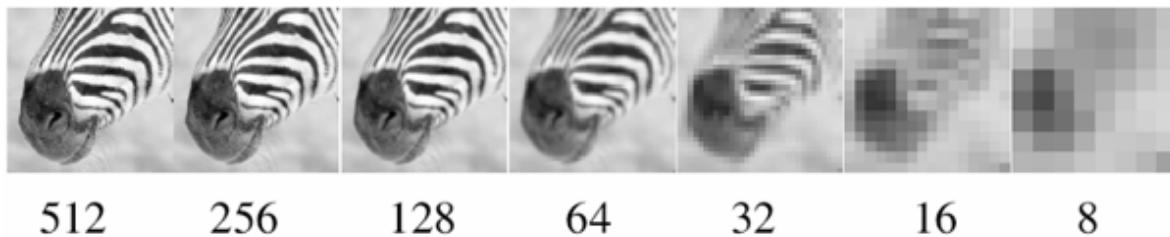
What happens to the details of the image?

- They get smoothed out as we move to higher levels.



What is preserved at the higher levels?

# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

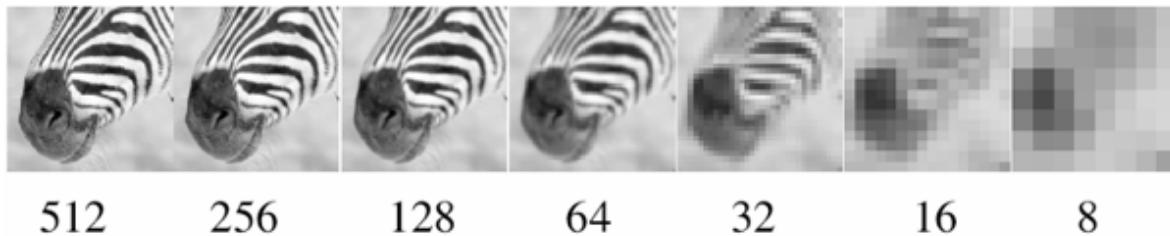


What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.



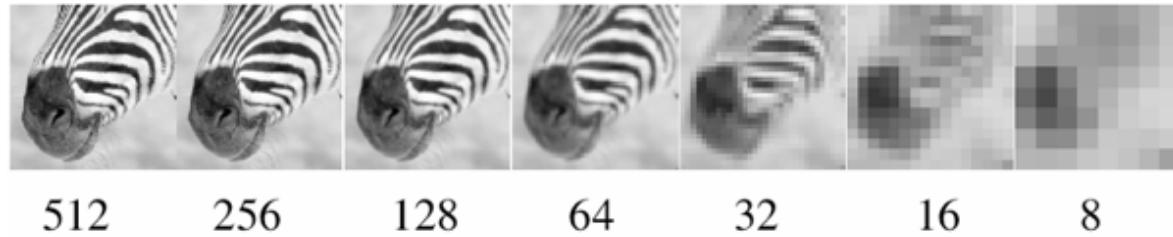
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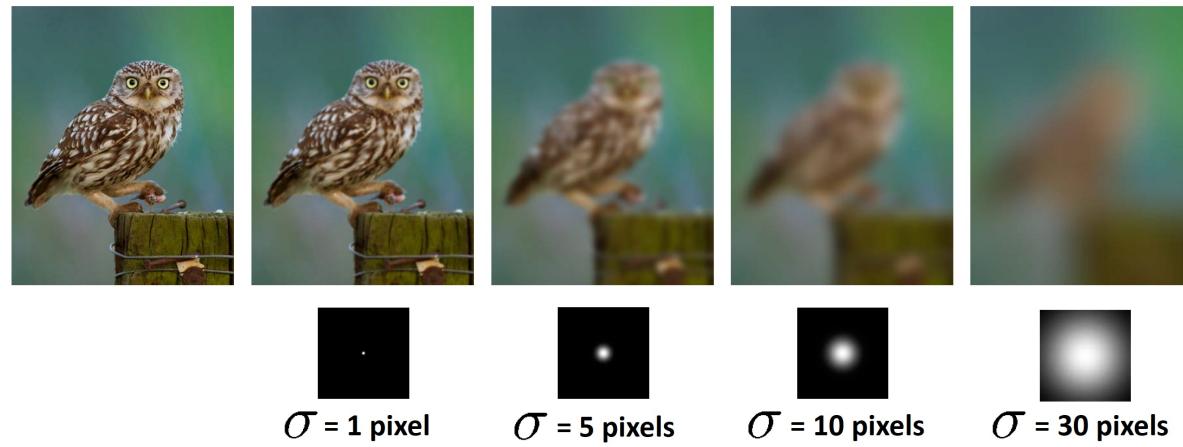
- That's not possible.

# Relating Nyquist-Shannon theorem to Gaussian pyramid



- Gaussian blurring is low-pass filtering.
- By blurring we try to sufficiently decrease the Nyquist frequency to avoid aliasing.

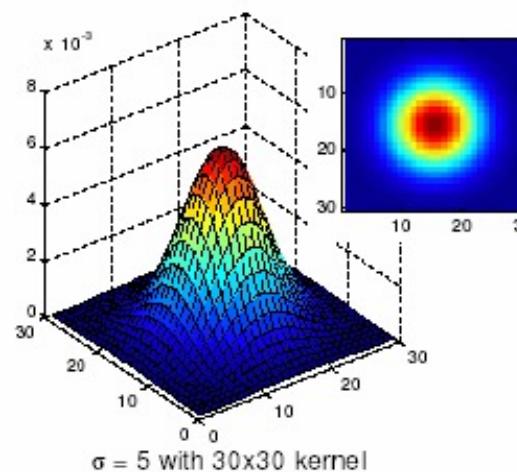
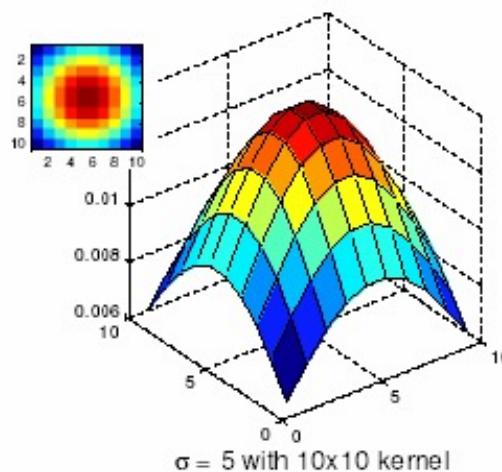
*How large should the Gauss blur we use be?*



# Choosing blur level & kernel width

Practically, you have two parameters to choose: blur level  $\sigma$ , and the (discrete) filter size

- Q1: How to choose appropriate  $\sigma$ , knowing the down-sampling rate  $s$ ?
- One plausible empirical rule:  $\sigma = \sqrt{s/2}$



- Q2: The Gaussian function has infinite support, but discrete filters use finite kernels!
- Values at edges should be near zero. Practically, we set filter half-width to about  $3\sigma$

# Blurring is lossy



level 0



level 1 (before downsampling)



residual

What does the residual look like?

# Blurring is lossy



level 0



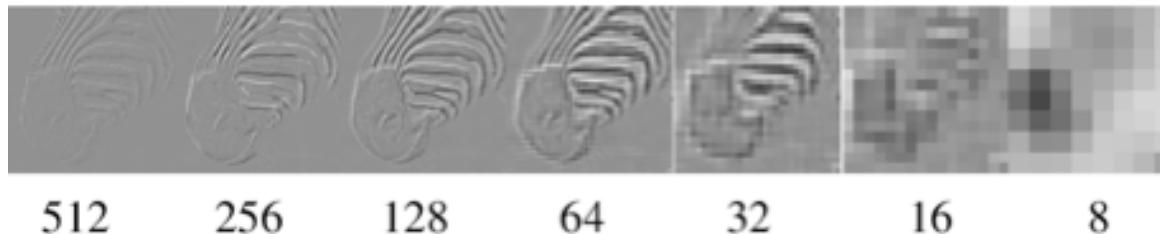
level 1 (before downsampling)



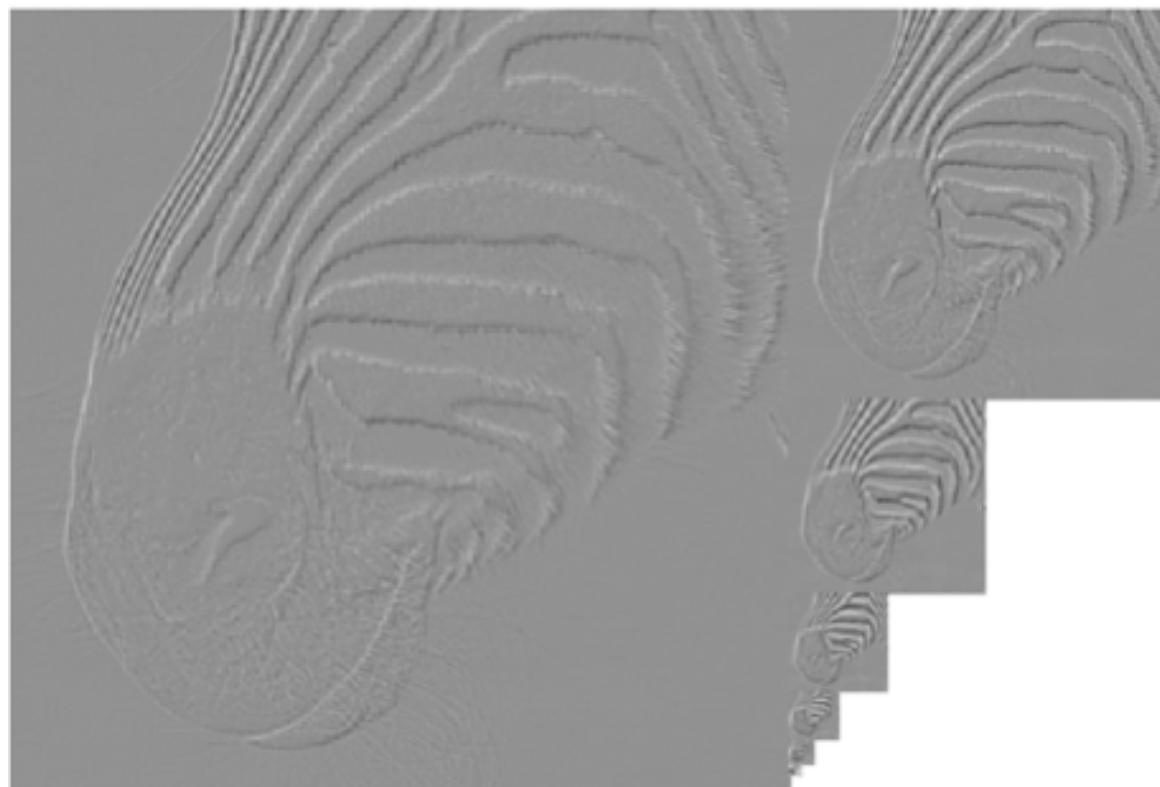
residual

Can we make a pyramid that is lossless?

# Laplacian image pyramid

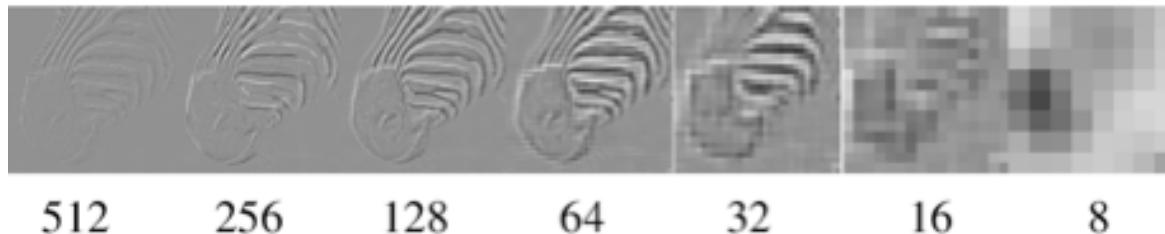


At each level, retain the residuals instead of the blurred images themselves.

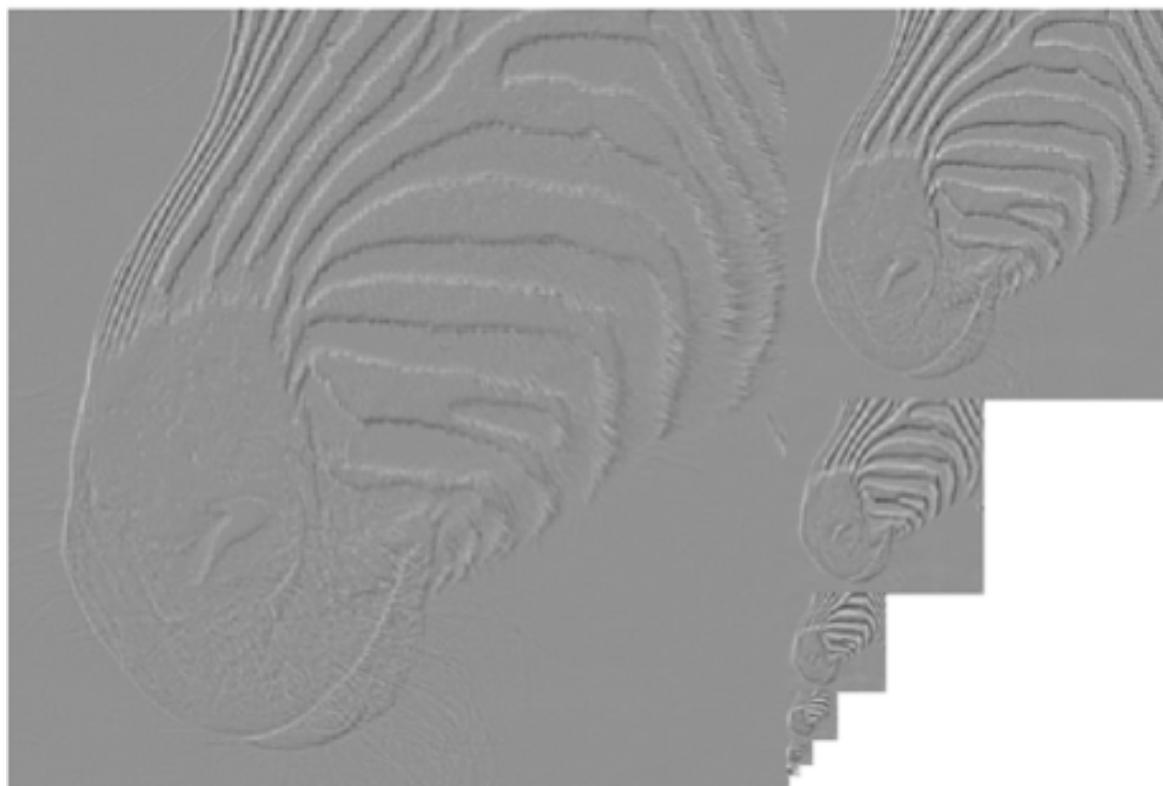


Can we reconstruct the original image using the pyramid?

# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.



Can we reconstruct the original image using the pyramid?

- Yes we can!

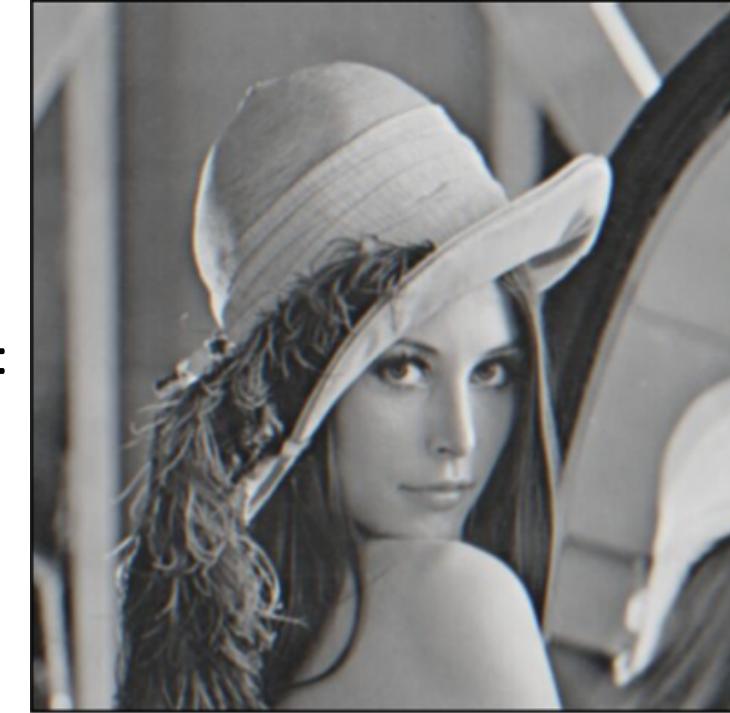


What do we need to store to be able to reconstruct the original image?

# Let's start by looking at just one level



level 0



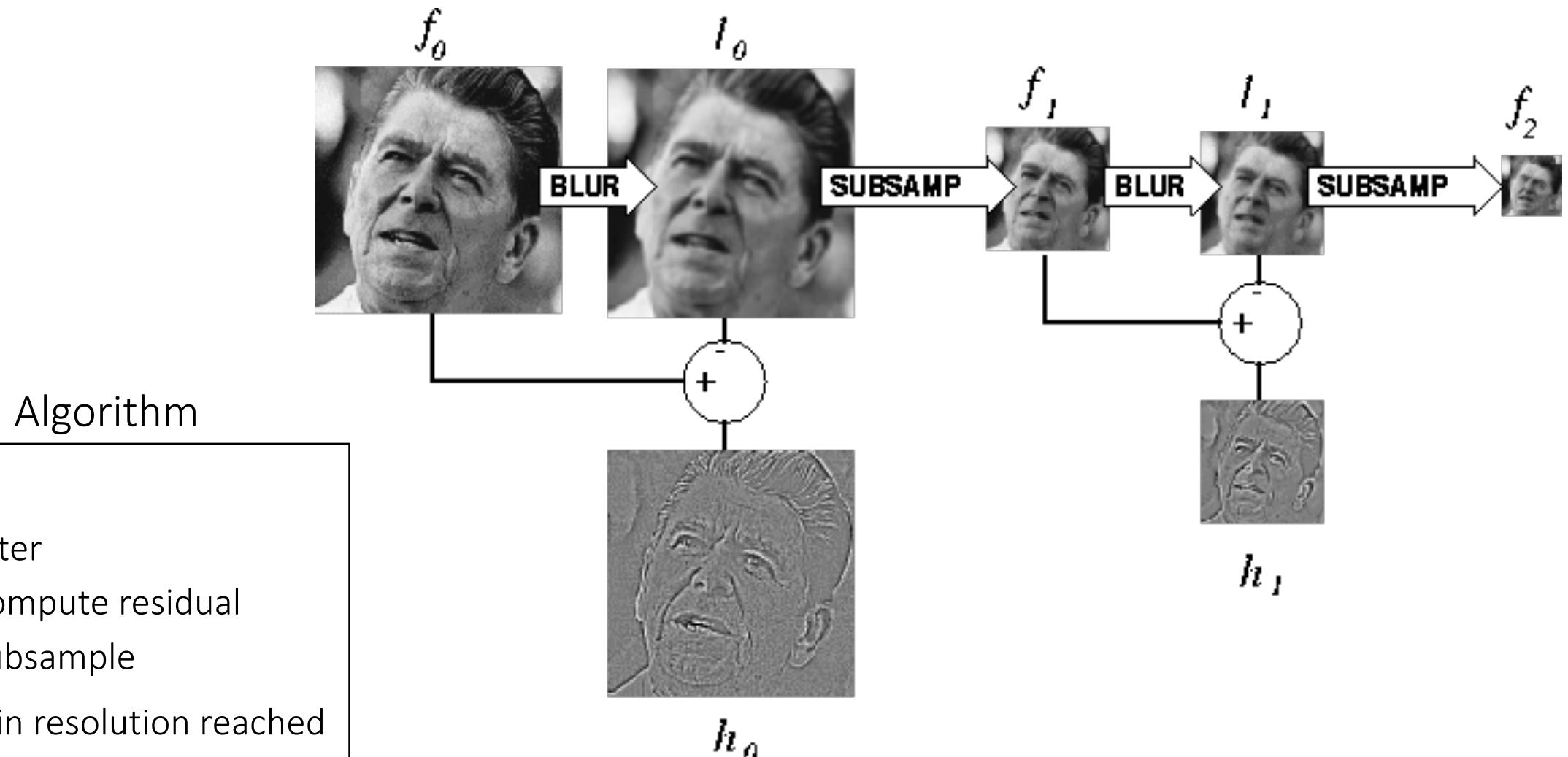
level 1 (upsampled)



residual

Does this mean we need to store both residuals and the blurred copies of the original?

# Constructing a Laplacian pyramid



# Constructing a Laplacian pyramid

What is this part?

Algorithm

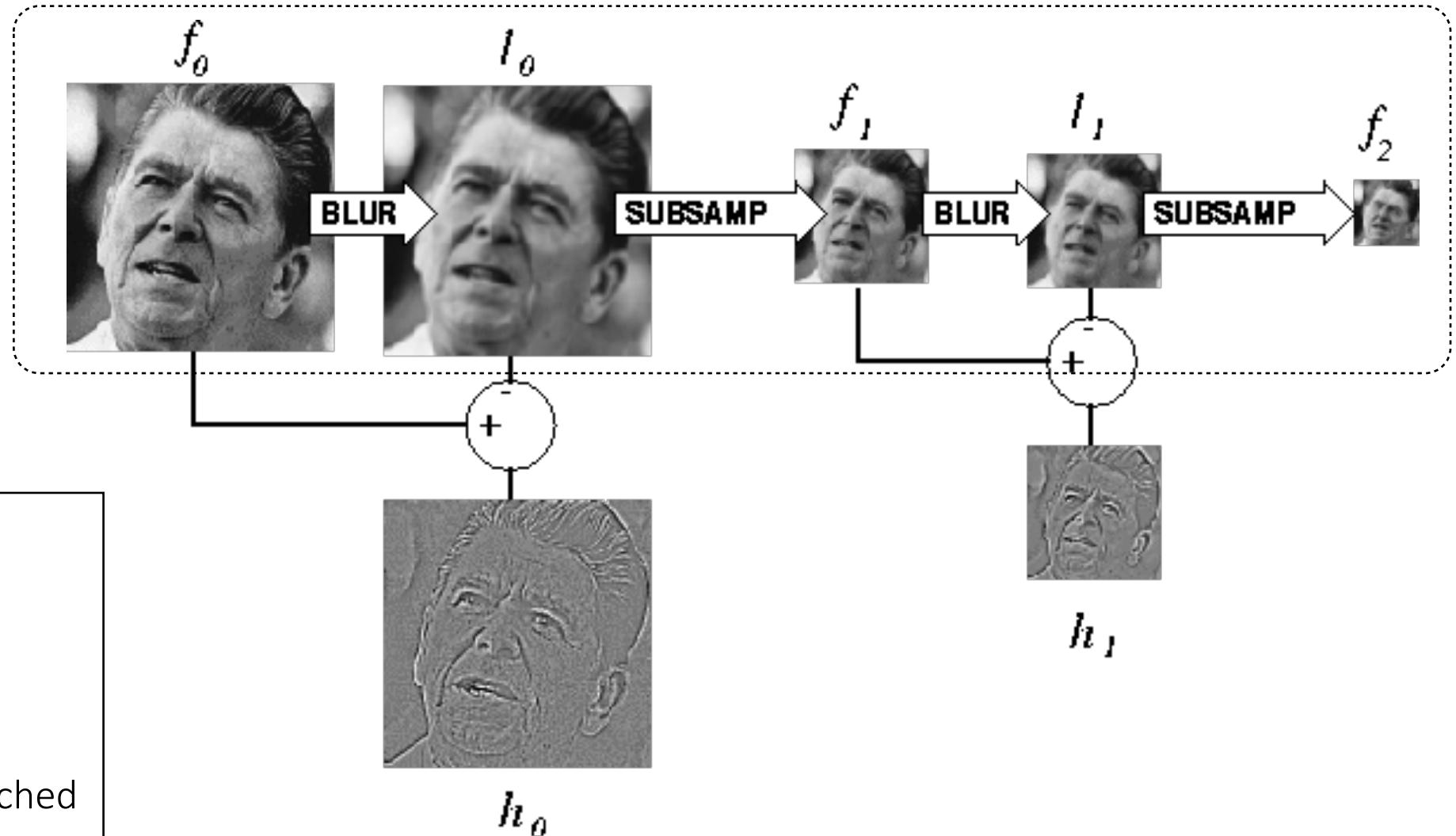
repeat:

filter

compute residual

subsample

until min resolution reached



# Constructing a Laplacian pyramid

It's a Gaussian pyramid.

Algorithm

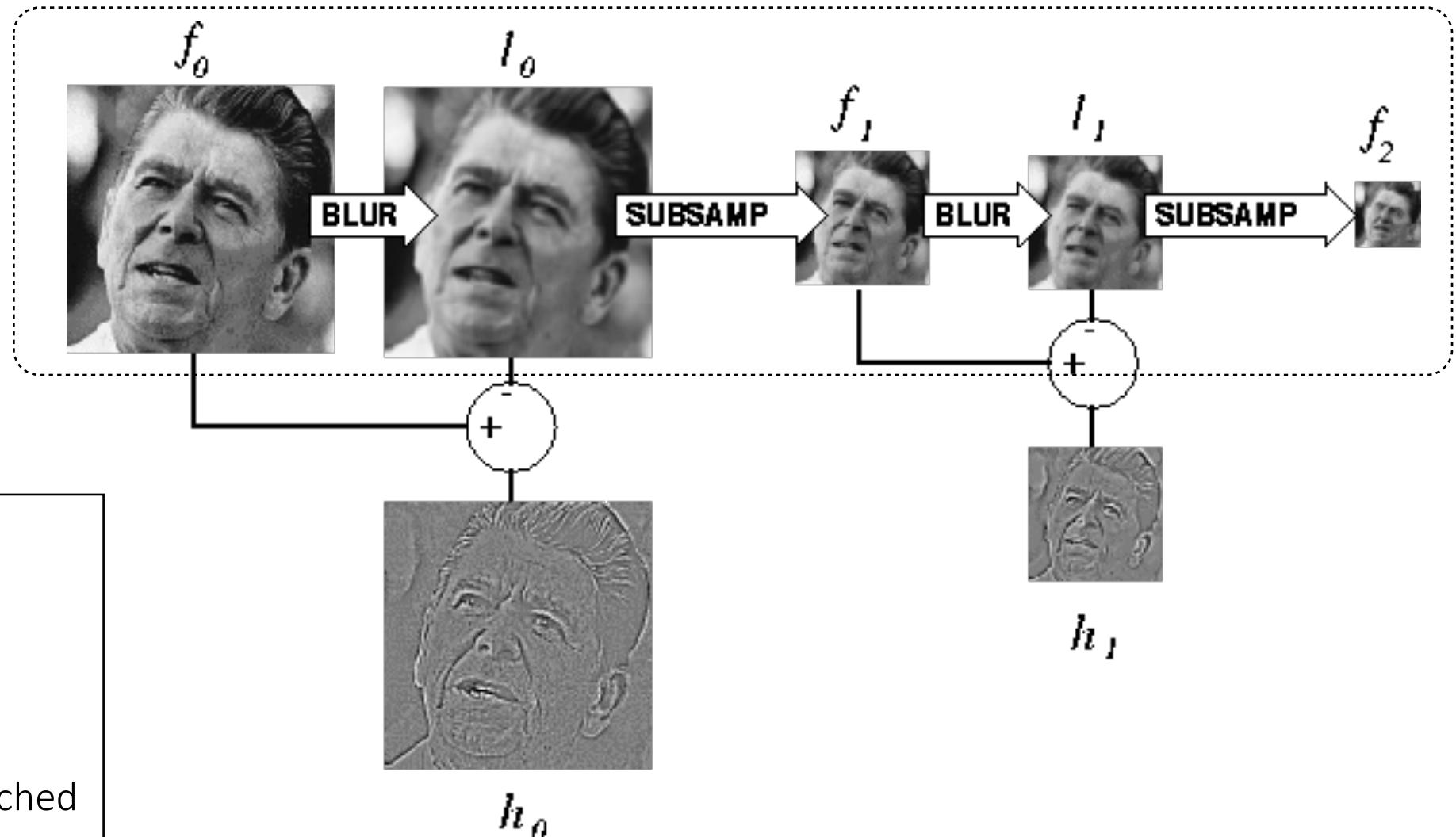
repeat:

filter

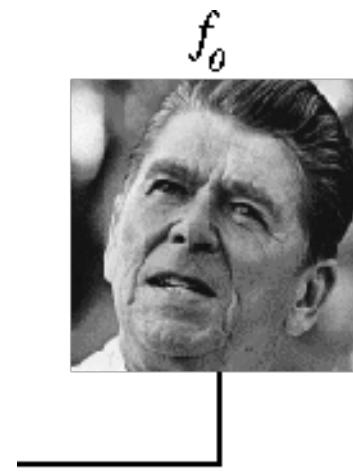
compute residual

subsample

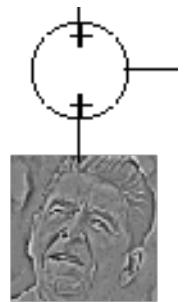
until min resolution reached



What do we need to construct the original image?

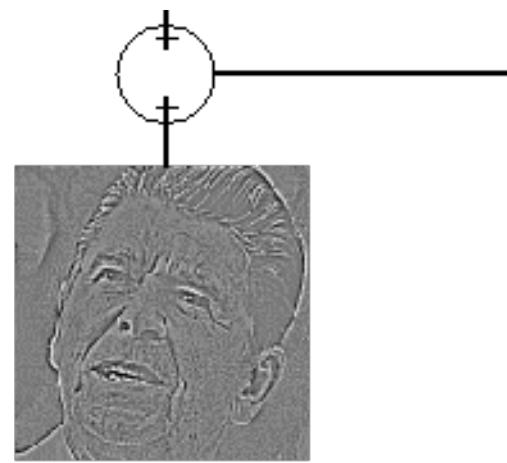


# What do we need to construct the original image?



$h_1$

(1) residuals



$h_0$

# What do we need to construct the original image?

(2) smallest  
image



$h_1$

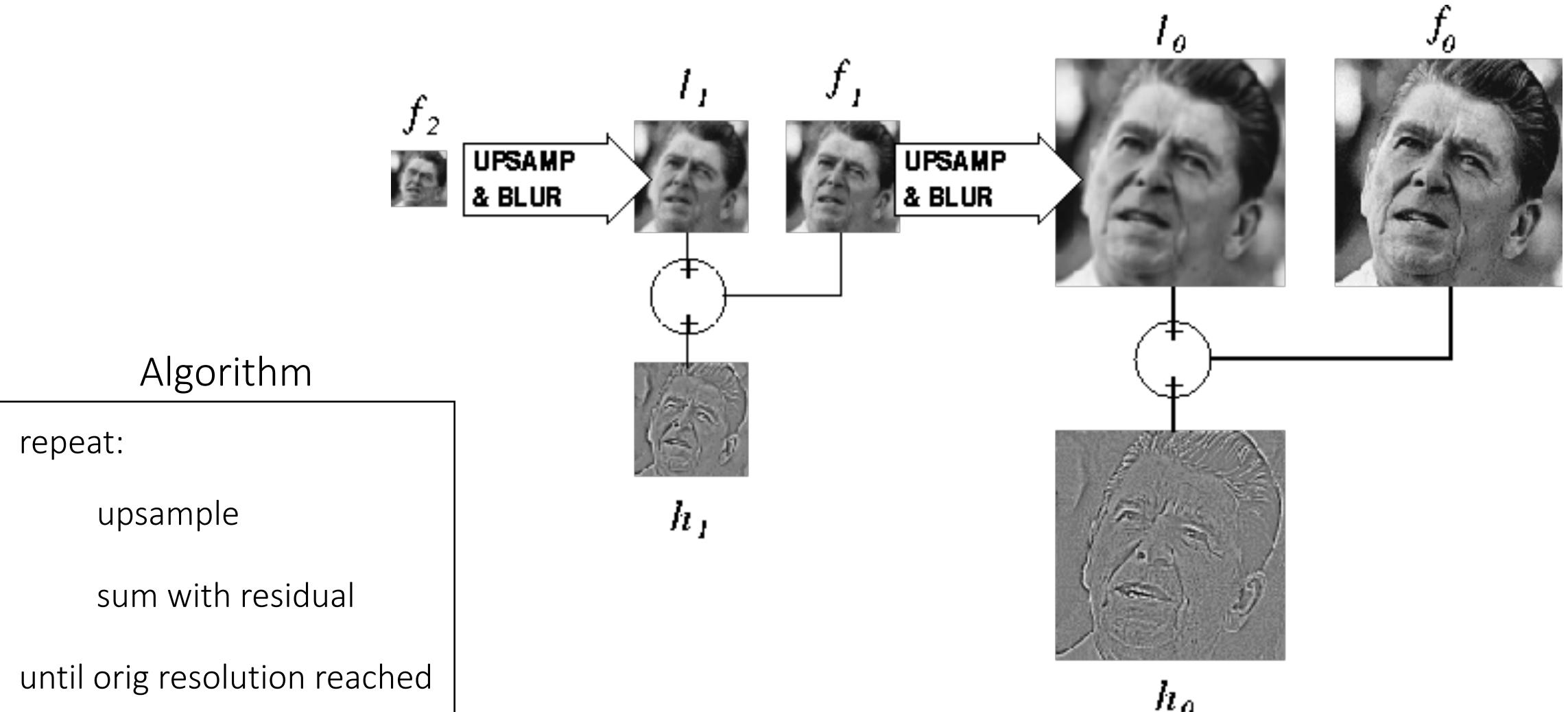
(1) residuals



$h_0$



# Reconstructing the original image

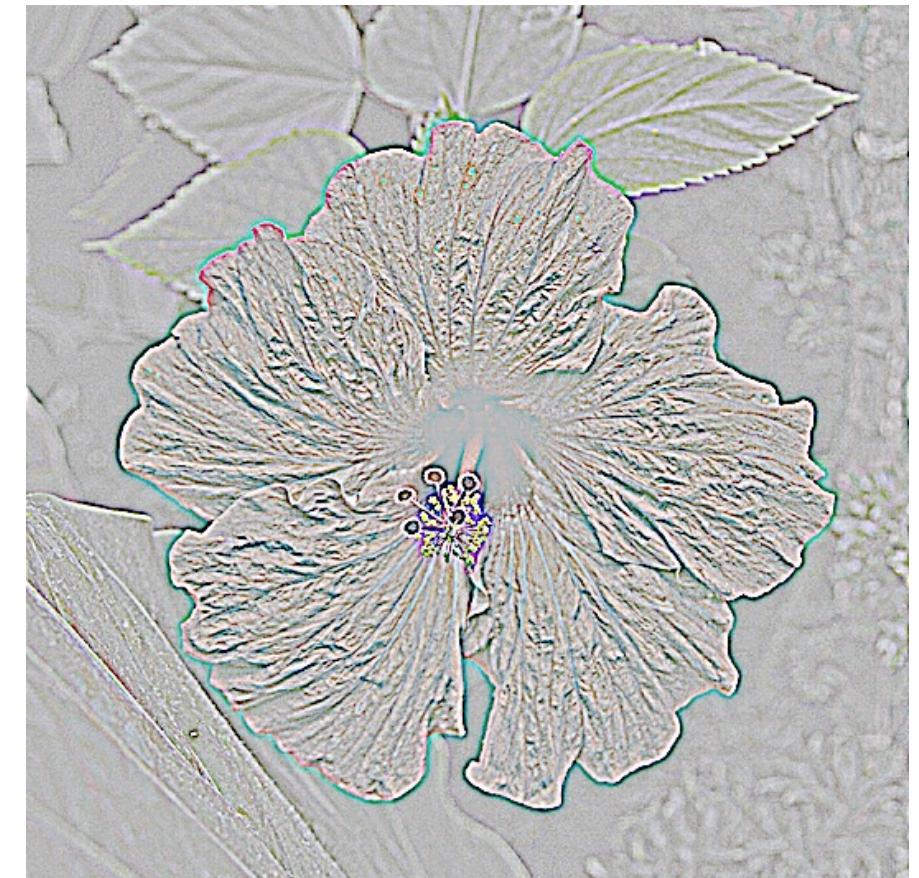
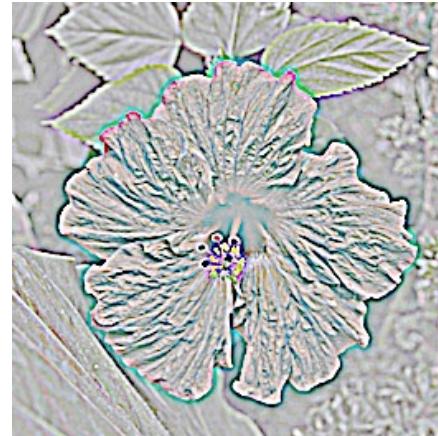
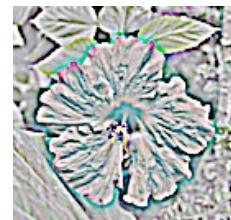


# Gaussian vs Laplacian Pyramid



Shown in opposite  
order for space.

Which one takes  
more space to store?



# Still used extensively



input image



foreground details enhanced, background details reduced

user-provided mask

# Other types of pyramids

Steerable pyramid: At each level keep multiple versions, one for each direction.



Wavelets: Huge area in image processing



# What are image pyramids used for?

image compression



multi-scale  
texture mapping

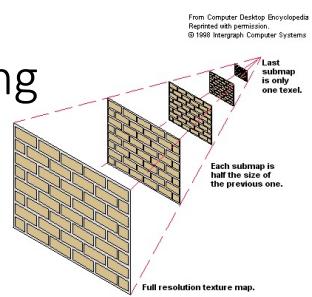
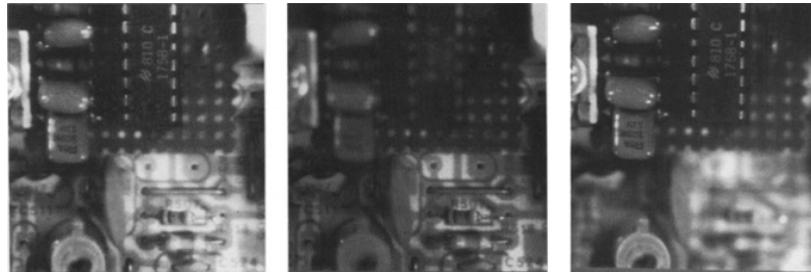


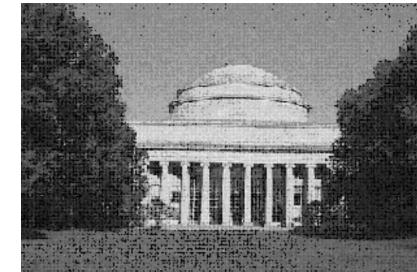
image blending



focal stack compositing



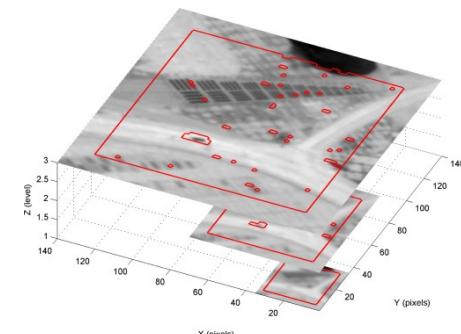
denoising



multi-scale detection



multi-scale registration



# Fourier transform

---

# Fourier transform

continuous

$$F(k) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi kx}dx$$

Fourier transform

inverse Fourier transform

$$f(x) = \int_{-\infty}^{\infty} F(k)e^{j2\pi kx}dk$$

discrete

$$F(k) = \frac{1}{N} \sum_{x=0}^{N-1} f(x)e^{-j2\pi kx/N}$$

$k = 0, 1, 2, \dots, N-1$

$$f(x) = \sum_{k=0}^{N-1} F(k)e^{j2\pi kx/N}$$

$x = 0, 1, 2, \dots, N-1$

'summation of sine waves'

# Computing the discrete Fourier transform (DFT)

$F(k) = \frac{1}{N} \sum_{x=0}^{N-1} f(x)e^{-j2\pi kx/N}$  is just a matrix multiplication:

$$\mathbf{F} = \mathbf{W}\mathbf{f}$$

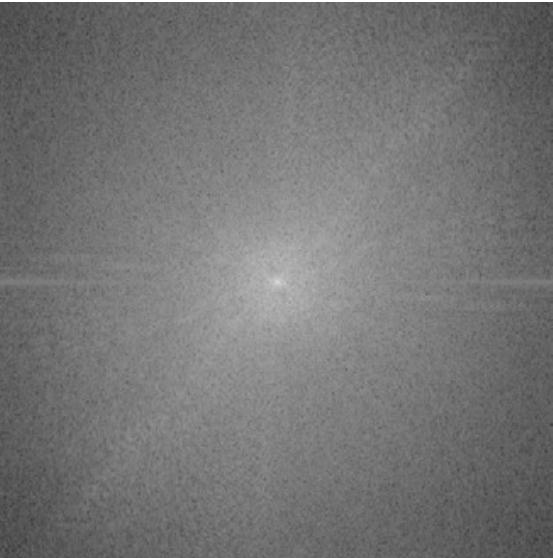
$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \\ \vdots \\ F(N-1) \end{bmatrix} = \begin{bmatrix} W^0 & W^0 & W^0 & W^0 & \dots & W^0 \\ W^0 & W^1 & W^2 & W^3 & \dots & W^{N-1} \\ W^0 & W^2 & W^4 & W^6 & \dots & W^{N-2} \\ W^0 & W^3 & W^6 & W^9 & \dots & W^{N-3} \\ \vdots & \vdots & & & \ddots & \vdots \\ W^0 & W^{N-1} & W^{N-2} & W^{N-3} & \dots & W^1 \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \\ \vdots \\ f(N-1) \end{bmatrix} \quad W = e^{-j2\pi/N}$$

In practice this is implemented using the *fast Fourier transform* (FFT) algorithm.

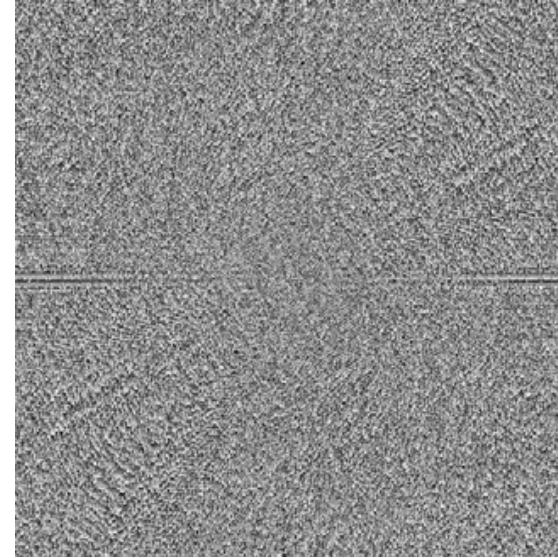
# Fourier transforms of natural images



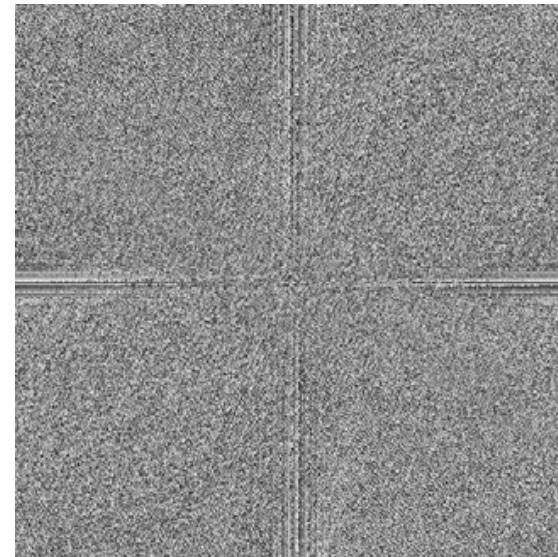
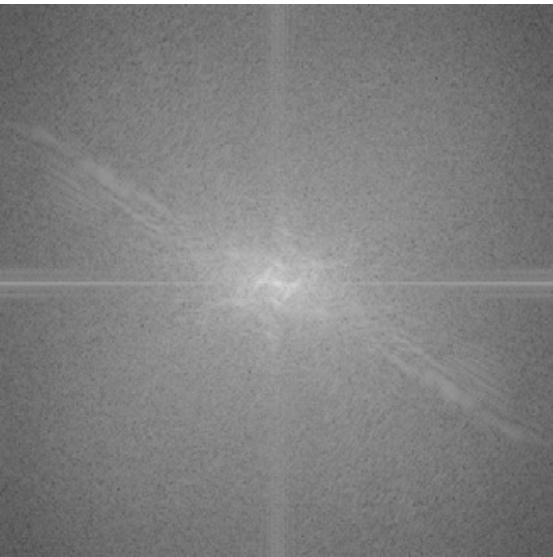
original



amplitude

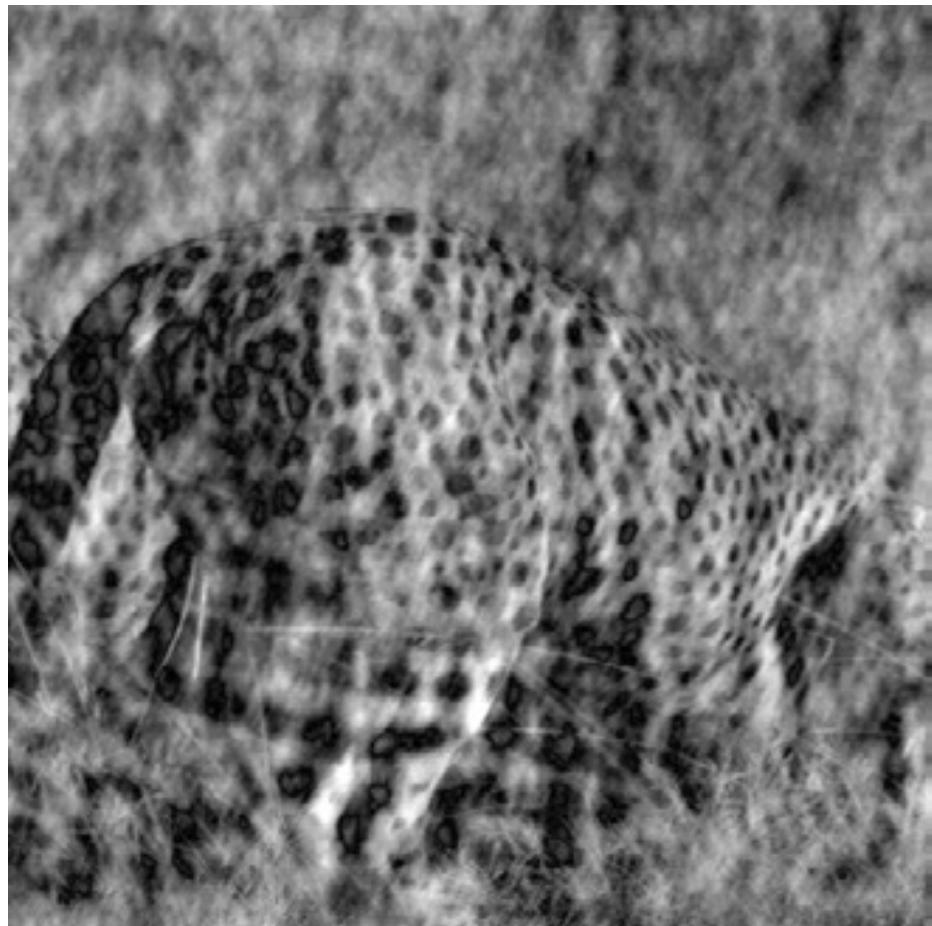


phase

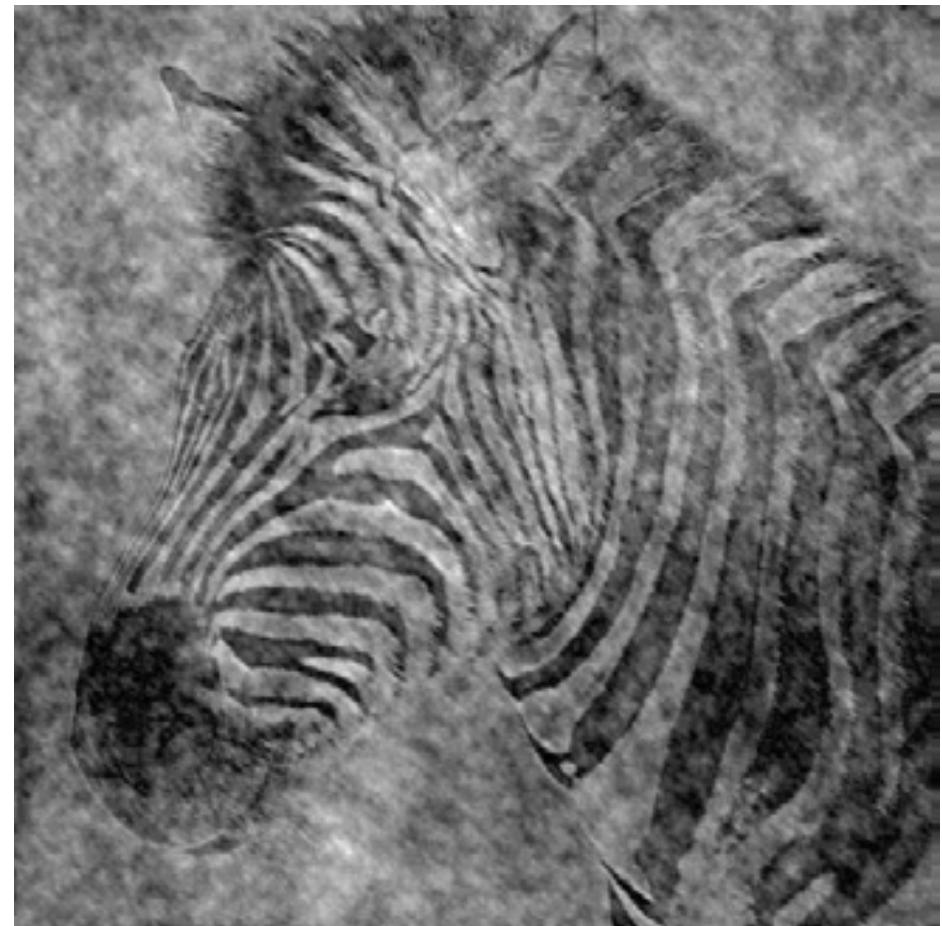


# Fourier transforms of natural images

Image phase matters!



cheetah phase with zebra amplitude



zebra phase with cheetah amplitude

# Frequency-Domain View of Filtering

# The convolution theorem

The Fourier transform of the convolution of two functions is the product of their Fourier transforms:

$$\mathcal{F}\{g * h\} = \mathcal{F}\{g\}\mathcal{F}\{h\}$$

The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms:

$$\mathcal{F}^{-1}\{gh\} = \mathcal{F}^{-1}\{g\} * \mathcal{F}^{-1}\{h\}$$

Convolution in spatial domain is equivalent to multiplication in frequency domain!

# Convolution for 1D continuous signals

Definition of linear shift-invariant filtering as convolution:

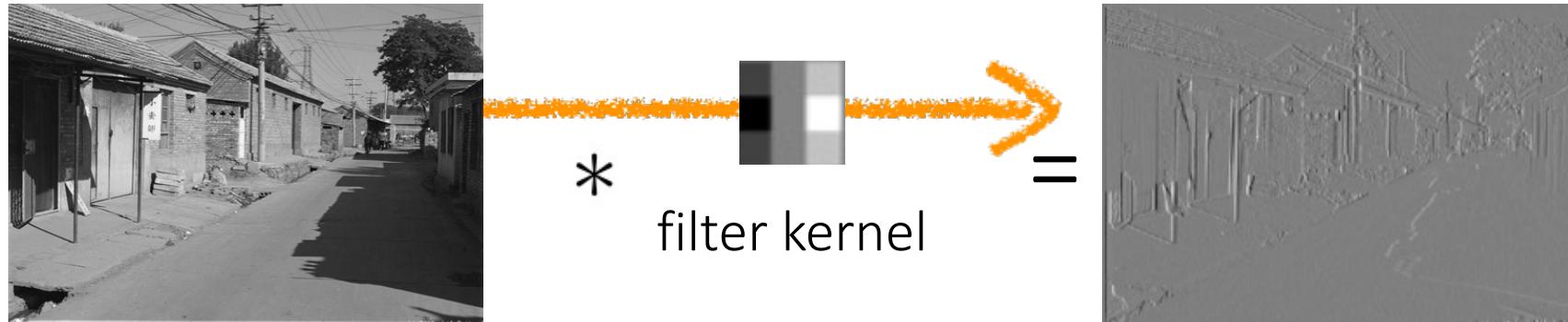
$$(f * g)(x) = \int_{-\infty}^{\infty} f(y)g(x - y)dy$$

filtered signal      ← filter      ← input signal

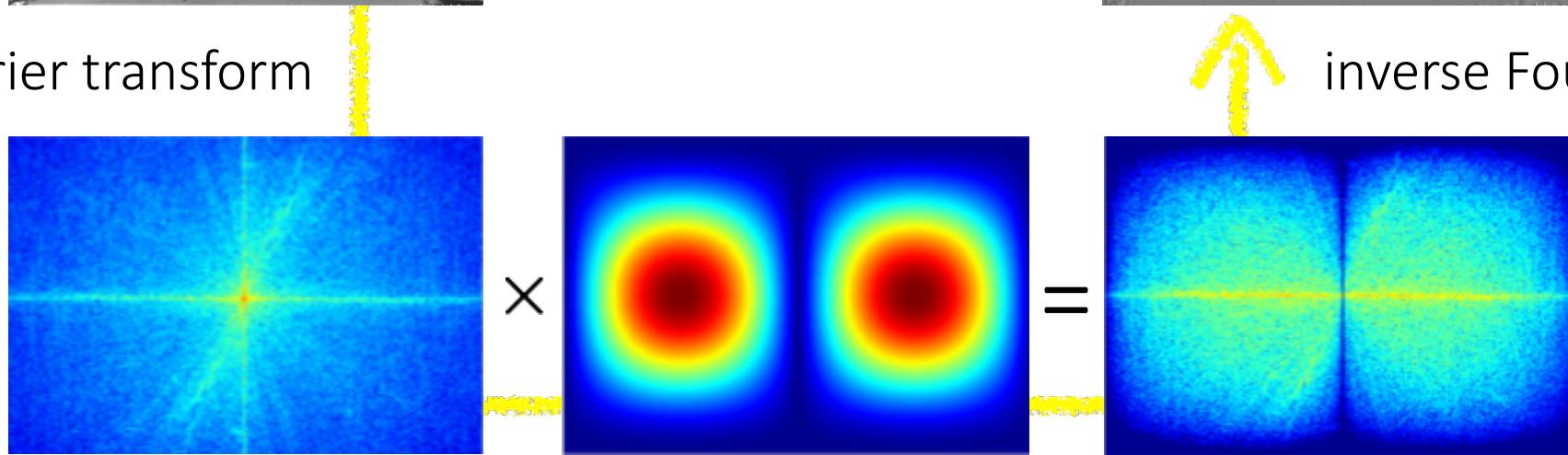
Using the convolution theorem, we can interpret and implement all types of linear shift-invariant filtering as multiplication in frequency domain.

Why implement convolution in frequency domain?

# Spatial domain filtering

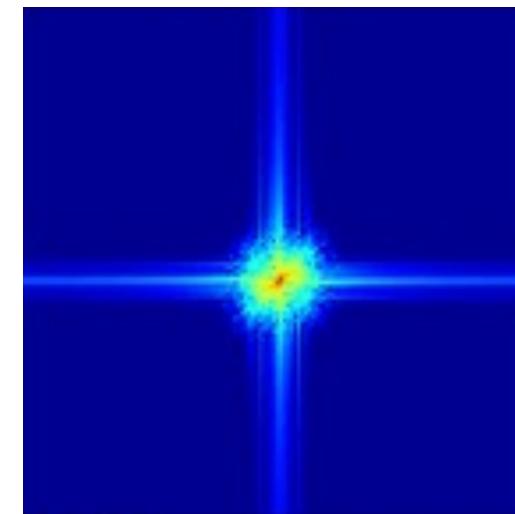
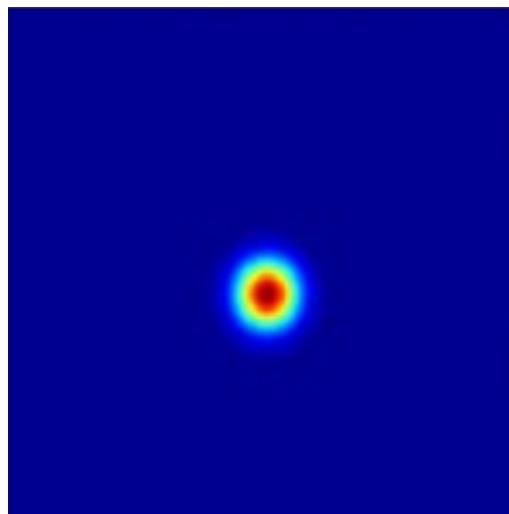
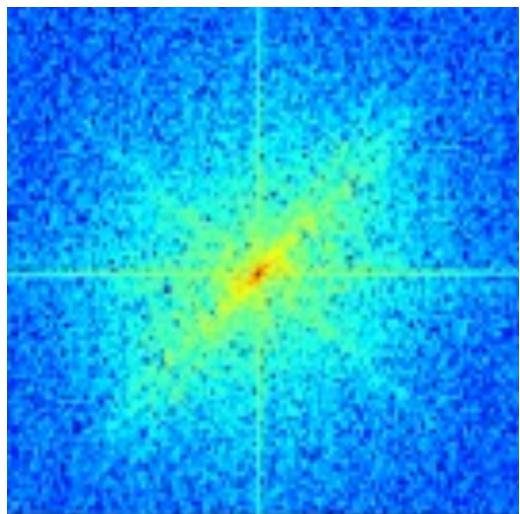
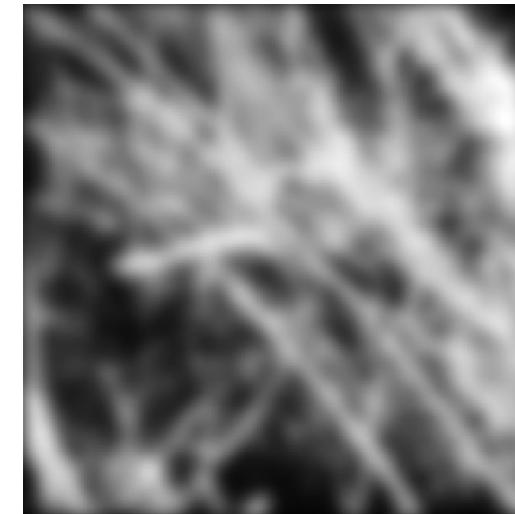
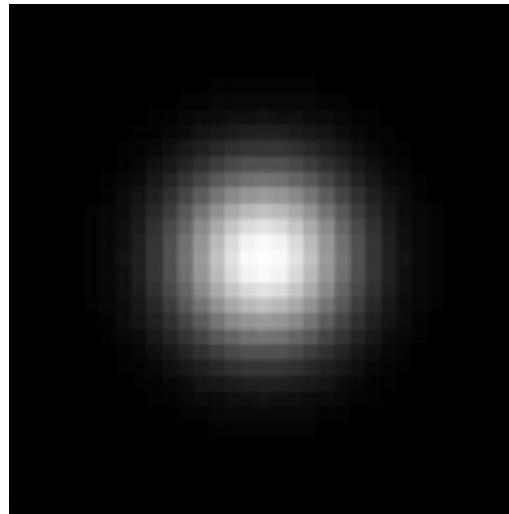


Fourier transform

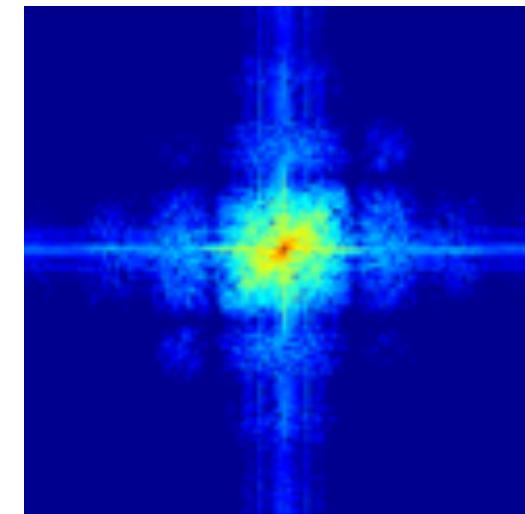
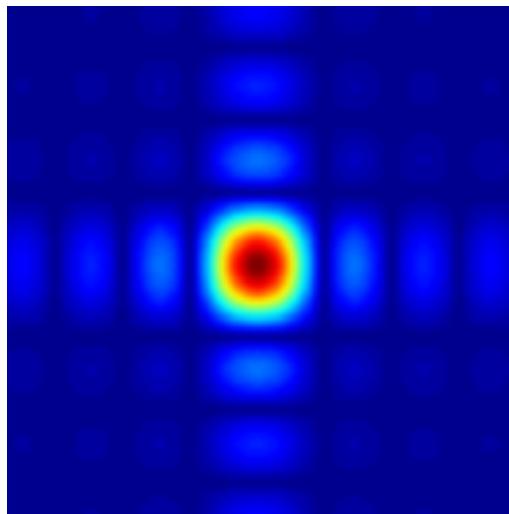
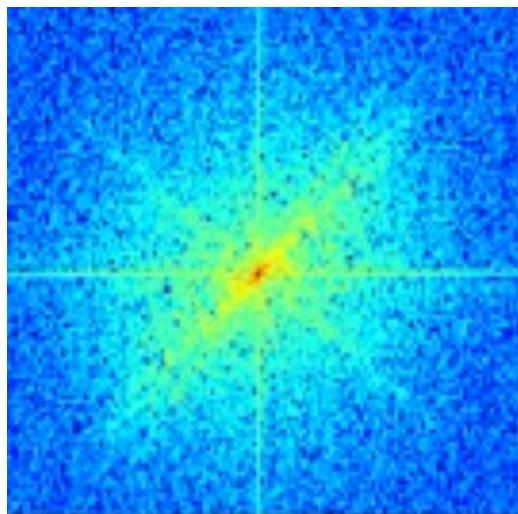
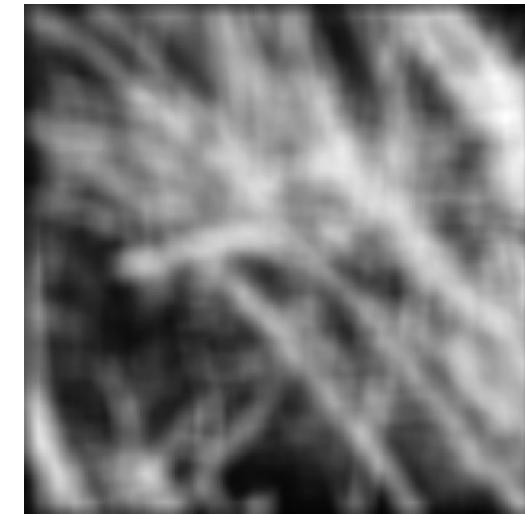
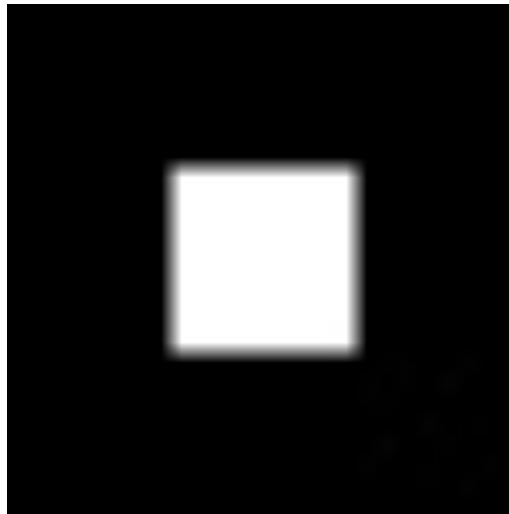


# Frequency domain filtering

# Gaussian blur

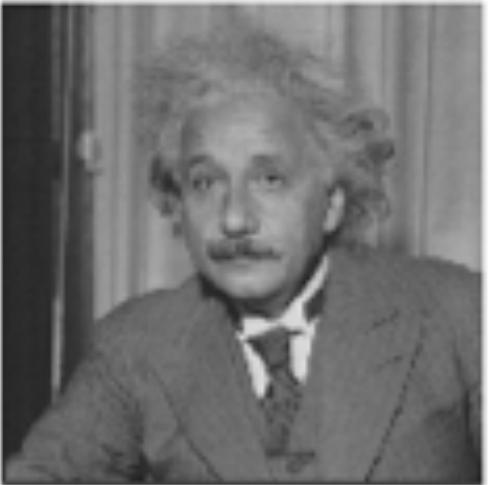


# Box blur

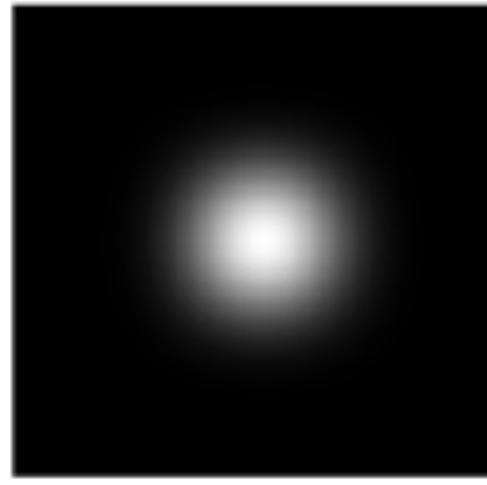


# More filtering examples

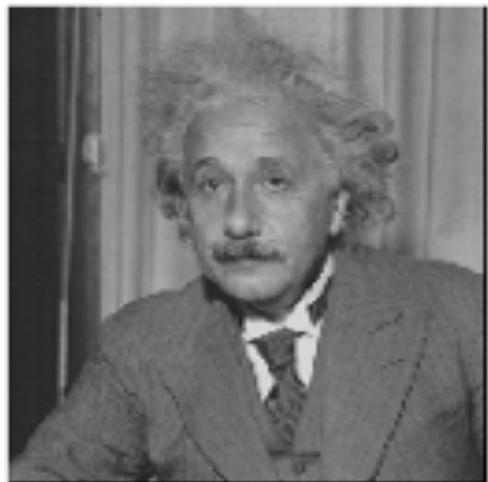
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?



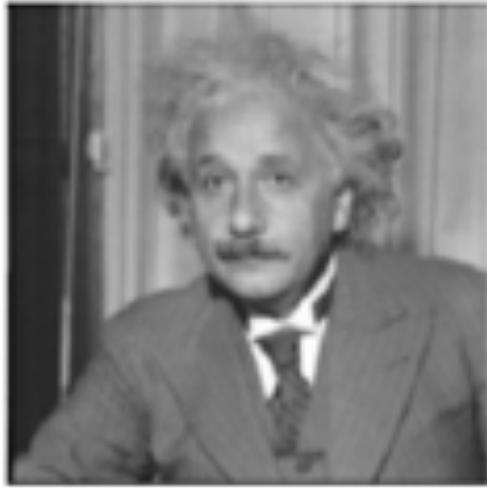
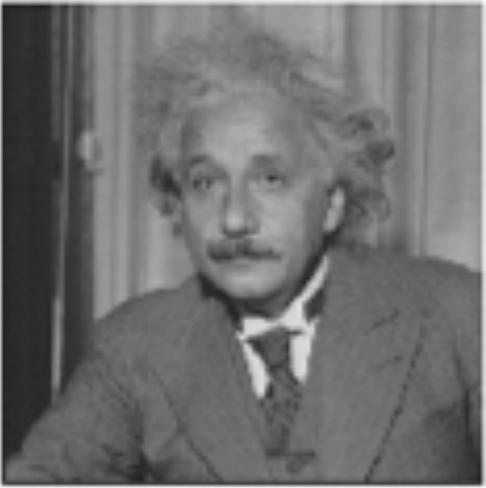
filters shown  
in frequency-  
domain



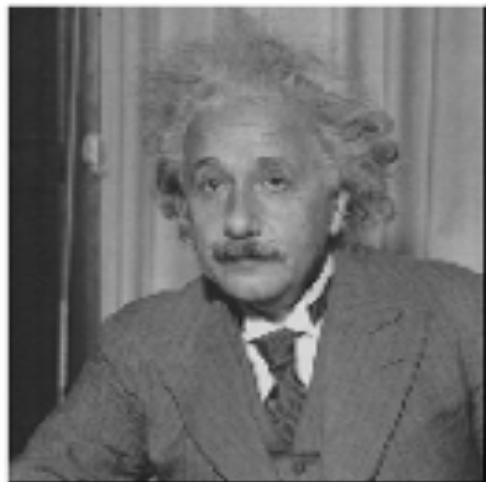
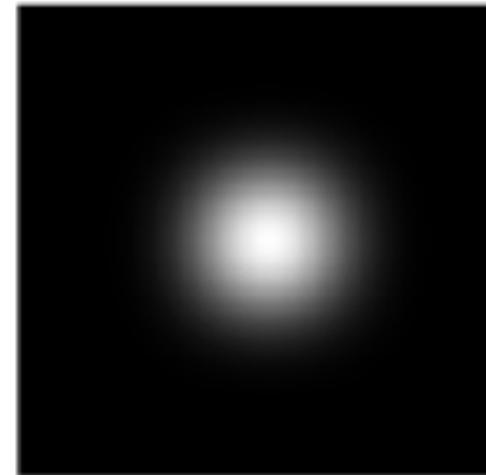
?



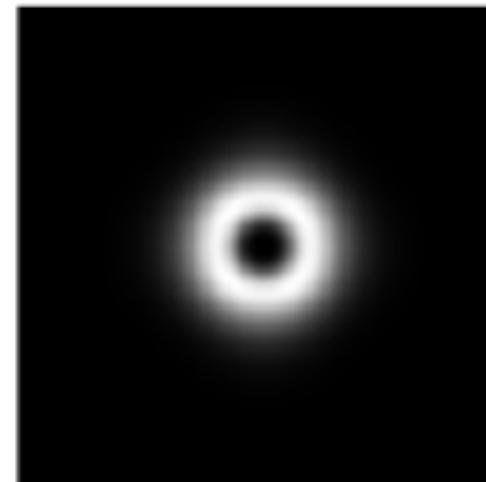
# More filtering examples



low-pass



band-pass

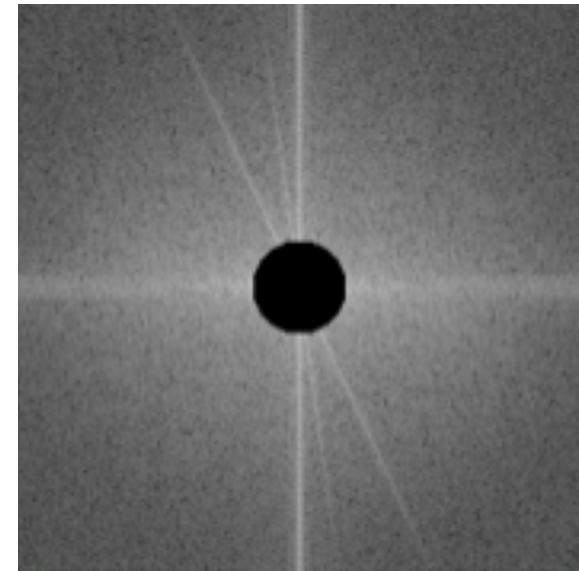


filters shown  
in frequency-  
domain

# More filtering examples

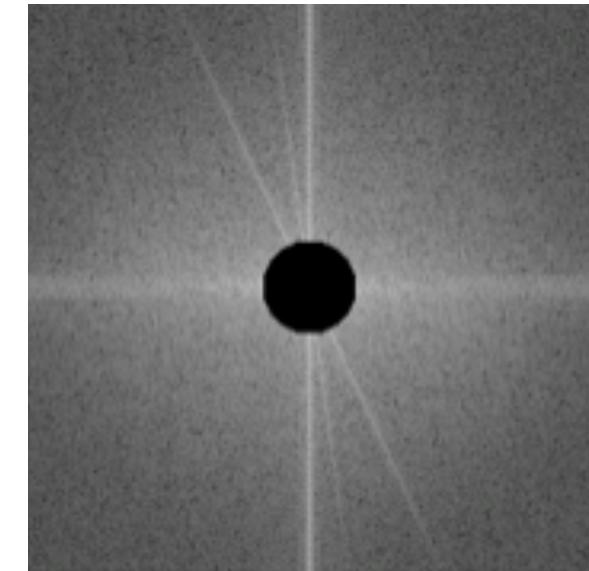


?



high-pass

# More filtering examples



high-pass



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