Module 2
Scale Space for Biological Image Segmentation

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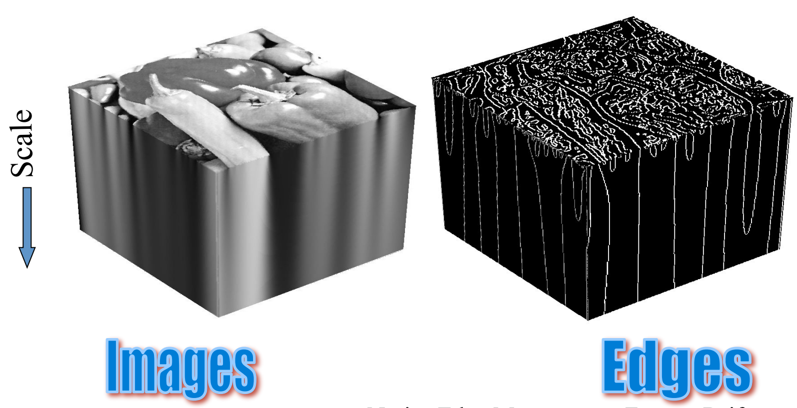
First Approach

- Create a *scale space* via *area morphology* (connected filters)
- Cluster vectors in the scale space to extract segments

Objective

- The goal of the work is to investigate segmentation procedures that can be used for biological imagery
- Ability to specify the minimum-sized segments
- Provide segments that minimize classification error
- Provide segments that minimize intra-object classification error

Scale-space Generation from Linear Filtering

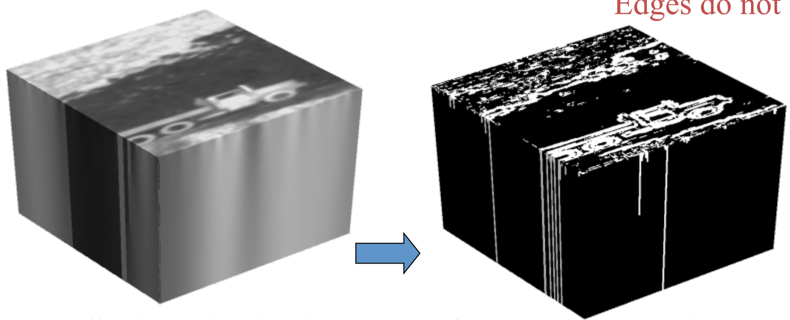


- Notice Edge Movement -- Feature Drift as scale increases
- Difficult to use in multi-scale image analysis tasks

Scale-space from Anisotropic Diffusion

• Diffusion/smoothing is inhibited where the local gradient magnitude is large...

Edges do not drift



• Causality is maintained -- no new features are created with increased scale

• Can be used in multi-scale tasks such as coarse-to-fine searches

Area Morphology

- Area morphology is based on the manipulation of connected components within the image level sets according to their area
- Let $\mathbf{I}$ represent an image with level sets $\mathbf{B}(\mathbf{I}, t)$:
- Within a particular level set $\mathbf{B}$, we have a connected component of 1's at (x, y) represented by $\mathbf{C}_{\mathbf{B}}(x, y)$

$$(x, y) \in \mathbf{B}(\mathbf{I}, t) \text{ if } I(x, y) \geq t$$

Area Open and Close Operators

- For a particular level set $\mathbf{B}$, the area open and close operators are defined by

$$(x, y) \in \overset{a}{\circ}(\mathbf{B}) \text{ if } |\mathbf{C}_{\mathbf{B}}(x, y)| \geq a$$

$$(x, y) \in \overset{a}{\bullet}(\mathbf{B}) \text{ if } |\mathbf{C}_{\mathbf{B}}^{c}(x, y)| \geq a$$

Area open-close (AOC) is simply the concatenation of area open and close

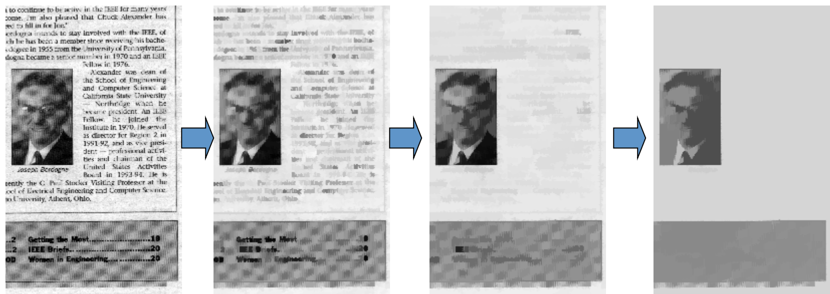
Area Open-Close Scale Space

- For an image, the AOC is computed by stacking the processed level sets

$$I(x, y) = \sum_{i=0}^{K-1} 1_{(x,y) \in L(\mathbf{I},i)}$$

- The scale space is created recursively
- The AOC scale space maintains the fidelity, causality, strong causality and Euclidean invariance properties
- Fast algorithms exist to compute the AOC (queue, tree and pyramid-based)

Non-bio Area Open-Close Scale Space Example



8-bit, 292x176 scan of '*The Institute*'

Clustering

- Let the 1-D signal $\mathbf{I}(x, y)$ represent the scale space evolution of $I(x, y)$
- $\mathbf{I}(x, y)$ is a scale space vector -- we cluster the scale space vectors to segment the image
- The distance between two scale space vectors is defined by

$$d(\mathbf{I}(x_1, y_1), \mathbf{I}(x_2, y_2)) = \left[\sum_{\Omega_s} \left| I_s(x_1, y_1) - I_s(x_2, y_2) \right|^p \right]^{1/p}$$

Fuzzy c-Means Clustering

Minimize:

$$J_m(U,\boldsymbol{\mu}) = \sum_{\Omega} \sum_{i=1}^{C} (u_i(x,y))^m \, \| d_i(x,y) \|^2$$

Sum over Scales

Sum over Classes

Class Membership

Distance of scale space vector to cluster center

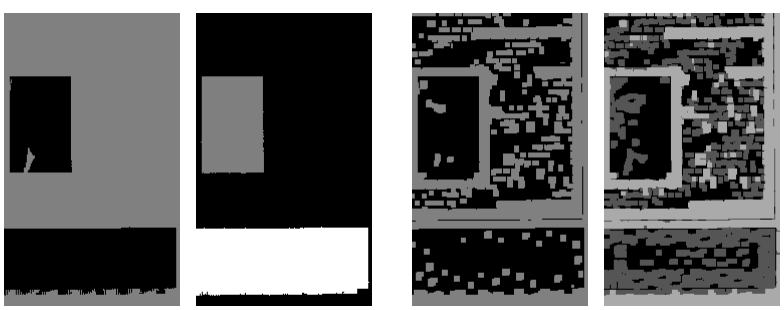
Update Memberships:

$$u_i(x,y) = 1/[\sum_{j=1}^{C} \left(\frac{d_i(x,y)}{d_j(x,y)} \right)^{2/(m-1)}]$$

Update Cluster Centers:

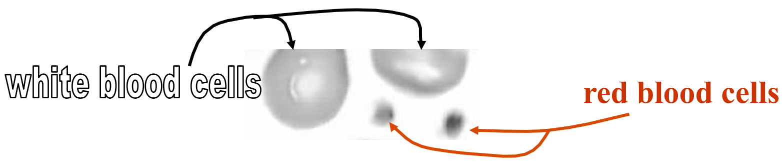
$$\boldsymbol{\mu}_i = \frac{\sum_{\Omega} (u_i(x,y))^m \, \mathbf{I}(x,y)}{\sum_{\Omega} (u_i(x,y))^m}$$

Clustering Results



Using AOC scale space	Using AOC scale space	Using open scale space	Using open scale space
2 Classes	3 Classes	2 Classes	3 Classes

Cellular Example of Intra-object Error



Original blood image

Standard Classification without AOC Scale-space

AOC Scale-space Classification

Second Approach

- Use a partial differential equation (PDE)-driven diffusion to encourage intra-region smoothing and
  - Discourage inter-region smoothing
- This diffusion can be used to generate space space

Diffusion

- Diffusion is a mathematical model for several physical processes: the migration of bacteria, heat transfer, etc.
- The same partial differential equations (PDEs) may also be used for signal / image smoothing:

The M-dimensional signal

the diffusion coefficient

$$\frac{\partial I(\mathbf{x})}{\partial t} = \operatorname{div}\left[c(\mathbf{x})\nabla I(\mathbf{x})\right]$$

time = scale

local gradient

Motivation

- Benefits of the Diffusion Technique
  - Diffusion can be **efficiently implemented** using locally connected units on a parallel processor
  - Intra-region smoothing vs. inter-region smoothing (edge and **feature preservation**)
  - Control of feature **scale** to create a *scale-space* -- a family of signals that vary from coarse to fine

Diffusion on Digital Imagery

- The diffusion PDE must be discretized for implementation on digital images

New Image Intensity at iteration t+1

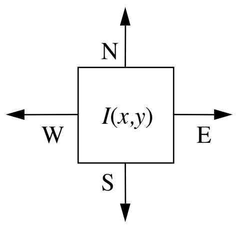
Diff. Coef. in p^{th} direction

$$[I(\mathbf{x})]_{t+1} \leftarrow \left[I(\mathbf{x}) + \Delta T \sum_{p=1}^{\Omega} c_p(\mathbf{x}) \nabla I_p(\mathbf{x}) \right]_t$$

Time step

Diffusion direction

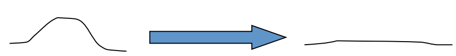
Simple difference in p^{th} direction



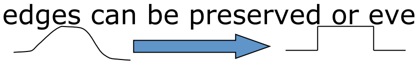
For 2-D images, there are Ω=4 directions of diffusion (N, S, E, W)

The Diffusion Coefficient

- If the diffusion coefficient is constant at all image locations, isotropic diffusion (Gaussian smoothing) is enacted
  - Problem: isotropic diffusion does not preserve edges



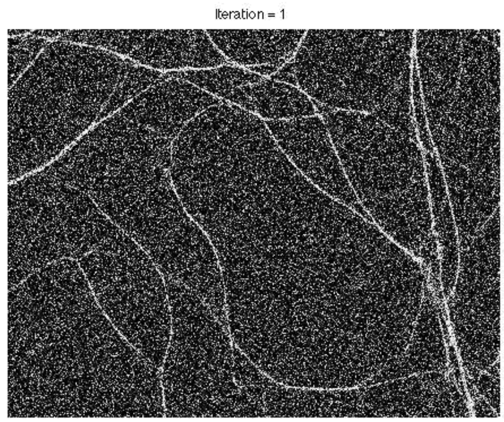
- If the diffusion coefficient is allowed to vary with the local image gradient magnitude, we have **anisotropic diffusion**
  - Benefit: edges can be preserved or even enhanced



Form: typically a non-increasing function of image gradient magnitude…

$$c_p(\mathbf{x}) = f\left(\left|\nabla I_p(\mathbf{x})\right|\right)$$

An example on a noisy image of axons



Original

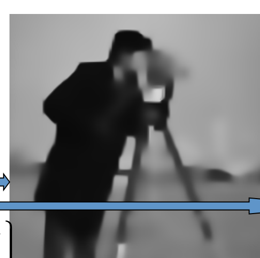
$$c_p(\mathbf{x}) = \exp\left\{-\left[\frac{\nabla I_p(\mathbf{x})}{k}\right]^2\right\}$$

Traditional Anisotropic Diffusion

Preserves Noise, Small Details

Anisotropic Diffusion with Gradient Prefilter

$$c_p(\mathbf{x}) = \exp\left\{-\left[\frac{\nabla S_p(\mathbf{x})}{k}\right]^2\right\}$$



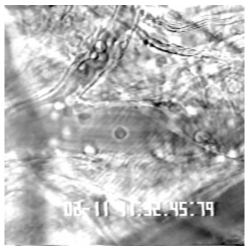
$$\mathbf{S} = \mathbf{I} * \mathbf{G}(\sigma)$$

$$\mathbf{S} = (\mathbf{I} \circ \mathbf{B}) \bullet \mathbf{B}$$

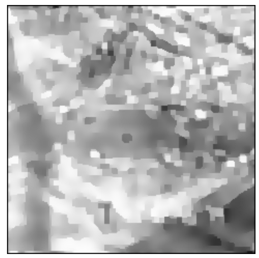
Morphological

Image Enhancement

- We know the scale and shape of the leukocytes (roughly)
- We can exploit that knowledge in removing noise and clutter – using morphological anisotropic diffusion



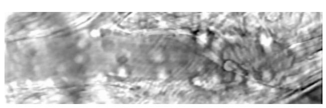
Input



After Diffusion w/ Morph. Diffusion coef.

Background Subtraction

- Subtract background image from each registered frame to reveal foreground objects (leukocytes)



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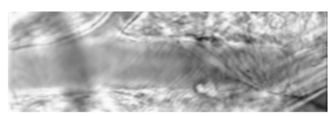

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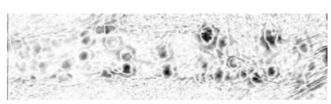

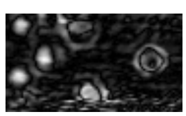

Original Subimage After Background Sub. After Diffusion w/ Morph diffusion coef.

Anisotropic diffusion with signal-dependent noise

- Previous AD assumes additive noise that is independent of signal
- What to do when noise is signal-dependent?
- Base diffusion coefficient on Standard Deviation / Mean instead of Gradient magnitude...
- Let $q(x,y) = \sigma/\mu$ – the coef. of variation!
- Called SRAD: speckle reducing anisotropic diffusion

Diffusion Coefficient

$$c(q) = \frac{1}{1 + \left[q^2(x,y;t) - q_0^2(t)\right] / \left[q_0^2(t)\left(1 + q_0^2(t)\right)\right]}$$

ICOV of fully developed speckle

Instantaneous Coef. Of Variation (ICOV)

Gradient

$$q(x,y) = \sqrt{\frac{\left|(1/2)\left|\nabla I(x,y)\right|^2 - (1/16)\left(\nabla^2 I(x,y)\right)^2\right|}{\left[I(x,y) + (1/4)\nabla^2 I(x,y)\right]^2}}$$

Derived from coefficient of variation

Laplacian

Enhancement via SRAD can improve segmentation results

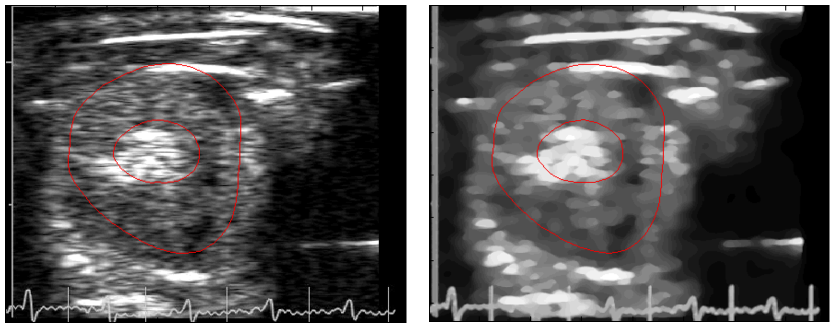

Image segmentation fails due to weak bottom left edge

SRAD improves contrast, intraregion homogeneity

END

- Onward to graphs!

Intra-object Classification Error

$$e_s(x_1,y_1) = \left|\left\{(x_2,y_2):(x_2,y_2)\in O_s(x_1,y_1),(x_2,y_2)\in\gamma_{c_2},(x_1,y_1)\in\gamma_{c_1},c_1\neq c_2\right\}\right|$$

Members of same object (member of the same connected component within an image level set)

Members of different classes

$$e = \frac{\sum_{\Omega_s}\sum_{\Omega} e_s(x,y)}{|\Omega_s||\Omega|}$$

Errors summed over all scales and positions (and normalized)

Example	Scale Space Fuzzy c-means	Scale Space k-means	Fuzzy c-means	k-means	Bayesian Classifier
1	133.58	139.64	171.62	167.77	168.10
2	125.5	115.95	174.74	174.74	173.84
3	365.55	399.84	416.30	419.95	463.93
4	122.1	120.55	159.98	158.99	143.23
5	169.98	168.78	211.7	209.53	188.89