The search of the Third Integral of Motion: Some Numerical Investigations

Victor Velev
velev@wisc.edu

Introduction

In the early 60's astronomers were up against a wall. Careful measurements of the motion of nearby starts in the galaxy allowed for computing averages of the observed motions. Unfortunately, the averages where way of from the theoretical predictions. Several assumptions where made such as the axisymmetricity of the galaxy, which implied an axisymmetric central potential. Perhaps the galaxy has non-axissymmetric components of the potential which were unfairly neglected? It turned out that the problem was much deeper. The understanding of motion was wrong.

Consider the phase-space distribution function $f(\vec{x}, \vec{p})$, which gives the probability density of finding a star at a position $\vec{x}$ with momentum $\vec{p}$. In terms of f, the statistical average of any dynamical quantity w over some volume of space V is just

$$\langle w \rangle_V = \int_V f w$$

Theoretical considerations concluded that the velocity dispersion in the radial direction should be about the same as the velocity dispersion in the direction orthogonal to the galactic plane. In cylindrical polar coordinates (r, θ, z):

$$\sigma_r = \langle (\dot{r} - \langle \dot{r} \rangle)^2 \rangle^{\frac{1}{2}}$$
$$\sigma_z = \langle (\dot{z} - \langle \dot{z} \rangle)^2 \rangle^{\frac{1}{2}}$$

And the conclusion being $\sigma_r \approx \sigma_z$. The measurements showed $\sigma_r \approx 2\sigma_z$.

In physics, questions about motions are solved with the help of *integrals of motion* - constants which are conserved throughout the trajectory of the moving body. Because of the vastness of the galaxy, it is not unreasonable to assume that the distribution of stars in the galaxy does not change much with time, or changes only very slowly. That is to say, close encounters with other stars are very rare. Therefore, we can consider each individual star as moving in a time-independent, axisymmetric central potential. In a system of cylindrical polar coordinates r, θ, z the Hamiltonian is

a_{theta}

$$T + V = \frac{1}{2m}\left[{p_r}^2 + \frac{(p_{\text{theta}})^2}{r^2} + p_z\right] + V(r, z)$$

Related Work

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aeque doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e patre audiebam facete et urbane Stoicos irridente, statua est in quo a nobis philosophia defensa et collaudata est, cum id, quod maxime placeat, facere possimus, omnis voluptas assumenda est, omnis dolor repellendus. Temporibus autem quibusdam et aut officiis debitis aut rerum necessitatibus saepe eveniet, ut et voluptates repudiandae sint et molestiae non recusandae. Itaque earum rerum defuturum, quas natura non depravata desiderat. Et quem ad me accedis, saluto: 'chaere,' inquam, 'Tite!' lictores, turma omnis chorusque: 'chaere, Tite!' hinc hostis mi Albucius, hinc inimicus. Sed iure Mucius. Ego autem mirari satis non queo unde hoc sit tam insolens domesticarum rerum fastidium. Non est omnino hic docendi locus; sed ita prorsus existimo, neque eum Torquatum, qui hoc primus cognomen invenerit, aut torquem illum hosti detraxisse, ut aliquam ex eo est consecutus? – Laudem et caritatem, quae sunt vitae.