Lab Session

16-04-2020

1. Find the smallest positive zero of

$$f(x) = x^4 - 6.4x^3 + 6.45x^2 + 20.538x - 31.752$$

2. Create the second difference matrix with size N, for example $N = 5$:

$$A = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

3. A is a square matrix? A is a diagonal matrix?
4. Use the results of LU decomposition

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 11/13 & 1 \end{pmatrix} \begin{pmatrix} 2 & -3 & -1 \\ 0 & 13/2 & -7/2 \\ 0 & 0 & 32/13 \end{pmatrix}$$

   to solve $Ax = b$, where $b^T = (1\ -1\ 2)$.
5. Read the scripyt `lab_20200416_jaan1.py` . What is the purpose of the function? Can you find the errors in the script and correct them?
6. Solve the following n simultaneous equations by the SOR method, A is the second difference matrix,

$$Ax = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

   In this case the iterative formulas are

$$\begin{aligned} x_1 &= \omega(x_2 - x_n)/2 + (1-\omega)x_1 \\ x_i &= \omega(x_{i-1} + x_{i+1})/2 + (1-\omega)x_i,\ i = 2, 3, \cdots, n-1 \\ x_n &= \omega(1 - x_1 + x_{n-1})/2 + (1-\omega)x_n \end{aligned}$$

(1) Read the script `gaussSeidel.py`. Can you find out the formula for the relaxation factor ω?

(2) Can you code `iterEqs(x, omega)` and integrate it into the main program `lab_20200416_jaan2.py`?

(3) Run the program with $n = 20$, $\epsilon = 1e-9$. What is the value of ω? The exact solution can be shown to be $x_i = -n/4 + i/2, i = 1, 2, \cdots, n$.

(4) Is the convergence slow or fast? Why?

(5) If we were to change each diagonal term of the coefficient from 2 to 4, how many iterations will be needed?