

## Problem 1

(1) Ans:  $\pi = 4 \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{2n-1}$

The possible errors are roundoff errors, subtractive errors

To minimize the errors

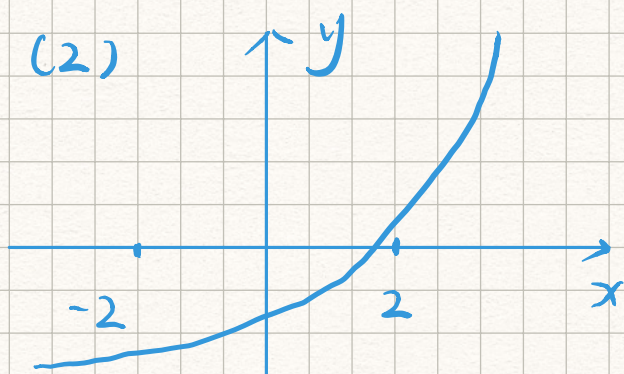
I write  $\pi = \begin{cases} 8 \sum_{n=1,3,5}^N \frac{1}{4n^2-1} & N \text{ is odd} \\ 8 \sum_{n=1,3,5}^{N-1} \frac{1}{4n^2-1} + \frac{4}{2N-1} & N \text{ is even} \end{cases}$

## Problem 2

Ans on ipynb.

## Problem 3.

Ans: (1)  $x_{n+1} = g(x_n) = x_n - \frac{2f(x_n)f'(x_n)}{2(f'(x_n))^2 + f(x_n)f''(x_n)}$



(4) root = 1.7215      count = 18



(4) advantage: <sup>order of</sup> convergence is 3, it is count is smaller than them.

disadvantage: more complex than them  
we should know  $f(x)$ ,  $f'(x)$ ,  $f''(x)$ .

Problem 4.

$$(1) \begin{pmatrix} 7 & -3 \\ -2 & 5 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ -10 \\ 8 \end{pmatrix}$$

(2) and (3) numpy. qr method

$$A x = b \quad A = QR$$

$$QR x = b$$

$$R x = Q^T b$$

$$x = R^{-1} Q^T b$$

code on ipynb

solution  $x_1 = 0.58500436$

$$x_2 = -1.37837838$$

(4) Not satisfied.

Problem 5

Ans: (1) (2) and (3) on ipynb



# Problem 6

Ans: (2)

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{pmatrix}$$