$C \ =< x^4 + x^2 + x + 1 >$, цикличен код с дължина 7, над F_2

$g(x) \ = \ x^4 + x^2 + x + 1, deg(g(x)) = 4 = n - k, n \ = \ 7, k \ = \ n - 4 \ = \ 7 - 4 \ = \ 3,$

С е [7, 3] цикличен код над F_2

$$h(x) = \frac{x^7 - 1}{g(x)} = \frac{x^7 - 1}{x^4 + x^2 + x + 1} = x^3 - x - 1 + \frac{2x^2 + 2x}{x^4 + x^2 + x + 1},$$

тъй като $2x^2 + 2x \equiv 0 \ (mod \ 2)$, то следва, че $g(x) \mid x^7 - 1$ без остатък

в $F_2, x^3 - x - 1 \equiv x^3 + x + 1 \ (mod \ 2)$, което е пораждащ полином на дулания код на C

$$H = \begin{matrix} 1110100 \\ 0111110 \\ 0010001 \end{matrix}$$

a)

H e $3x7$ матрица, тогава $n - \ k \ = \ 3, n \ = \ 7$, от тук, $k \ = \ 4. H_3[7, 4, 3]_2$

b)

v е код, тогава и само тогава, когато $vH^T \ = 0.$

Това са векторите

{(0,0,0,0,0,0,0), (1,1,0,1,0,0,0), (0,1,0,0,1,0,0), (1,0,0,1,1,0,0),

(1,1,0,0,0,1,0), (0,0,0,1,0,1,0), (1,0,0,0,1,1,0), (0,1,0,1,1,1,0),

(0,1,1,0,0,0,1), (1,0,1,1,0,0,1), (0,0,1,0,1,0,1), (1,1,1,1,1,0,1),

(1,0,1,0,0,1,1), (0,1,1,1,0,1,1), (1,1,1,0,1,1,1), (0,0,1,1,1,1,1)},

16 на брой.

c)

$$G = \begin{matrix} 1000111 \\ 0100101 \\ 0010110 \\ 0001011 \end{matrix}$$

Намираме $(0, 1, 0, 1)G \ = \ (0, 1, 1, 0, 1, 1, 0), (1, 0, 1, 0)G \ = \ (1, 1, 0, 1, 0, 1, 1)$

d) $n - \ k \ = \ 3, n \ = \ 7$, тогава $[7, 3, 4]$ линеен код.

e) Умножаваме всички бинарни вектори с дължина 3 с матрицата H и получаваме

{(0,0,0,0,0,0,0), (1,1,0,1,0,0,1), (1,0,1,1,0,1,0), (0,1,1,0,0,1,1),

(1,1,1,0,1,0,0), (0,0,0,1,1,1,1), (0,1,1,0,1,1,0), (1,0,1,1,1,1,1)},

8 на брой.