$C\ =< x^4 + x^2 + x + 1 >$, цикличен код с дължина 7, над F_2

$g(x)\ =\ x^4 + x^2 + x + 1, deg(g(x)) = 4 = n - k, n\ =\ 7, k\ =\ n - 4\ =\ 7 - 4\ =\ 3,$

C е [7, 3] цикличен код над F_2

$$h(x) = \frac{x^7 - 1}{g(x)} = \frac{x^7 - 1}{x^4 + x^2 + x + 1} = x^3 - x - 1 + \frac{2x^2 + 2x}{x^4 + x^2 + x + 1},$$

$$\text{тъй като } 2x^2 + 2x \equiv 0\ (mod\ 2), \text{то следва, че } g(x) \mid x^7 - 1 \text{ без остатък}$$

$$\text{в } F_2, x^3 - x - 1 \equiv x^3 + x + 1\ (mod\ 2), \text{което е пораждащ полином на дулания код на } C$$

$$H = \begin{matrix} 1110100 \\ 0111110 \\ 0010001 \end{matrix}$$

a)

$$H\ e\ 3x7 \text{ матрица, тогава } n -\ k\ =\ 3, n\ =\ 7, \text{от тук, } k\ =\ 4. H_3[7,4,3]_2$$

b)

$$v \text{ е код, тогава и само тогава, когато } vH^T\ = 0.$$

Това са векторите

$\{(0,0,0,0,0,0,0), (1,1,0,1,0,0,0), (0,1,0,0,1,0,0), (1,0,0,1,1,0,0),$

$(1,1,0,0,0,1,0), (0,0,0,1,0,1,0), (1,0,0,0,1,1,0), (0,1,0,1,1,1,0),$

$(0,1,1,0,0,0,1), (1,0,1,1,0,0,1), (0,0,1,0,1,0,1), (1,1,1,1,1,0,1),$

$(1,0,1,0,0,1,1), (0,1,1,1,0,1,1), (1,1,1,0,1,1,1), (0,0,1,1,1,1,1)\},$

16 на брой.

c)

$$G = \begin{matrix} 1000111 \\ 0100101 \\ 0010110 \\ 0001011 \end{matrix}$$

$$\text{Намираме } (0,1,0,1)G\ =\ (0,1,1,0,1,1,0), (1,0,1,0)G\ =\ (1,1,0,1,0,1,1)$$

d) $n -\ k\ =\ 3, n\ =\ 7,$ тогава $H_3[7,3,3]_2$ двоичен код.

e) Умножаваме всички бинарни вектори с дължина 3 с матрицата H и получаваме

$\{(0,0,0,0,0,0,0), (1,1,0,1,0,0,1), (1,0,1,1,0,1,0), (0,1,1,0,0,1,1),$

$(1,1,1,0,1,0,0), (0,0,0,1,1,1,1), (0,1,1,0,1,1,0), (1,0,1,1,1,1,1)\},$

8 на брой.