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II

Total No. of Questions : **24**
Total No. of Printed Pages : **3**

Regd. No.

Part - III
MATHEMATICS - PAPER - I (A)
(English Version)

Time : 3 Hours **Max. Marks : 75**

Note : *This question paper consists of Three Sections - A, B and C.*

SECTION - A

I. Very Short Answer Type Questions. **10x2=20**

(i) Answer *all* the questions.

(ii) Each question carries *two* marks.

1. Find the domain of the real valued function $f(x) = \frac{1}{\sqrt{1-x^2}}$.

2. If A={1, 2, 3, 4} and $f : A \to R$ is a function defined by $f(x) = \frac{x^2 - x + 1}{x + 1}$, then find the range of f.

3. Find the co-factors of the elements 2, −5 in the matrix $\begin{bmatrix} -1 & 0 & 5 \\ 1 & 2 & -2 \\ -4 & -5 & 3 \end{bmatrix}$.

4. If $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$ then find the values of x, y, z and a.

5. Find a vector in the direction of vector $a = i - 2j$ that has magnitude 7 units.

6. Find the vector equation of the line joining the points $2i+j+3k$ and $-4i+3j-k$.

7. Find the angle between the planes $r.(2i-j+2k)=3$ and $r.(3i+6j+k)=4$.

8. Find the maximum and minimum values of $3\sin x - 4\cos x$.

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9. If $\tan 20° = \lambda$, then show that $\dfrac{\tan 160° - \tan 110°}{1 + \tan 160° \cdot \tan 110°} = \dfrac{1 - \lambda^2}{2\lambda}$.

10. Show that $\tan h^{-1}\left(\dfrac{1}{2}\right) = \dfrac{1}{2} \log_e 3$.

SECTION - B

II. Short Answer Type Questions. **5x4=20**

(i) Answer *any five* questions.

(ii) Each question carries *four* marks.

11. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$ then find $A + A'$ and AA'.

12. If the points whose position vectors are $3i - 2j - k$, $2i + 3j - 4k$, $-i + j + 2k$ and $4i + 5j + \lambda k$ are coplanar, then show that $\lambda = -\dfrac{146}{17}$.

13. Find the volume of the tetrahedron whose vertices are (1, 2, 1), (3, 2, 5), (2, −1, 0) and (−1, 0, 1).

14. Prove that $\sin^2\theta + \sin^2\left(\theta + \dfrac{\pi}{3}\right) + \sin^2\left(\theta - \dfrac{\pi}{3}\right) = \dfrac{3}{2}$.

15. Solve $\tan\theta + 3\cot\theta = 5\sec\theta$.

16. Prove that $\cos\left(2\tan^{-1}\dfrac{1}{7}\right) = \sin\left(2\tan^{-1}\dfrac{3}{4}\right)$.

17. Prove that $\dfrac{a}{bc} + \dfrac{\cos A}{a} = \dfrac{b}{ca} + \dfrac{\cos B}{b} = \dfrac{c}{ab} + \dfrac{\cos C}{c}$.

SECTION - C

III. Long Answer Type Questions. **5x7=35**

(i) Answer *any five* questions.

(ii) Each question carries *seven* marks.

18. Let $f : A \to B$ be a bijection. Then prove that $fof^{-1} = I_B$ and $f^{-1}of = I_A$.

19. Using mathematical induction, prove that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots\cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}, \text{ for all } n \in \mathbf{N}.$$

20. Solve the following system of equations by using matrix inversion method.

$x+y+z=1$,

$2x+2y+3z=6$,

$x+4y+9z=3$.

21. Show that $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$.

22. Find the shortest distance between the skew lines $r=(6i+2j+2k)+t(i-2j+2k)$ and $r=(-4i-k)+s(3i-2j-2k)$.

23. If $A+B+C=\pi$, then prove that

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right).$$

24. Show that $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$.

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