Ejercicios de deber calculo integral

Grupo 3

Tema: Integrales Múltiples: Calculo de áreas en reales, Reales a funciones, Funciones a Reales y Centro de masas.

Áreas:

1. Real a función:

$$A = \int_{2}^{3}\int_{y^2-3}^{y+3} 3x^2 + y^2 dx\, dy$$

Real a Función

$$A=\int_2^3\int_{y^2-3}^{y+3} 3x^2+y^2\,dx\,dy$$

$$A=\int_{y^2-3}^{y+3} 3x^2+y^2\,dx$$

$$3\int x^2\,dx + y^2\int dx$$

$$\frac{\cancel{3}x^3}{\cancel{3}} + y^2x \Big|_{y^2-3}^{y+3}$$

$$x^3+xy^2\Big|_{y^2-3}^{y+3}$$

$$\left[(y+3)^3+(y+3)(y^2)\right] - \left[(y^2-3)^3+(y^2-3)(y^2)\right]$$

$$= -y^6+8y^4+2y^3-12y^2+27y+54$$

$$\int_2^3 -y^6+8y^4+2y^3-12y^2+27y+54\,dy$$

$$-\int y^6\,dy+8\int y^4\,dy+2\int y^3\,dy-12\int y^2\,dy+27\int y\,dy+54\int dy$$

$$-\frac{y^7}{7}+\frac{8y^5}{5}+\frac{\cancel{2}y^4}{\cancel{4}}-\frac{\cancel{12}y^3}{\cancel{3}}+\frac{27y^2}{2}+54y\Big|_2^3$$

$$-\frac{y^7}{7}+\frac{8y^5}{5}+\frac{y^4}{2}-4y^3+\frac{27y^2}{2}+54y\Big|_2^3$$

$$\left[-\frac{(3)^7}{7}+\frac{8(3)^5}{5}+\frac{(3)^4}{2}-4(3)^3+\frac{27(3)^2}{2}+54(3)\right] - \left[-\frac{(2)^7}{7}+\frac{8(2)^5}{5}+\frac{(2)^4}{2}-4(2)^3+\frac{27(2)^2}{2}+54(2)\right]$$

$$A = 60.01\ u^2$$

2. Real a real:

$$A = \int_0^3 \int_{-1}^1 x - 2y\, dy\, dx$$

Real a Real

$$A = \int_0^3 \int_{-1}^1 x - 2y \quad dy \quad dx$$

$$\int_{-1}^1 x - 2y \quad dy$$

$$x \int dy \quad - 2 \int y \, dy$$

$$xy \quad \frac{-\cancel{2}y^2}{\cancel{2}} \Big|_{-1}^1$$

$$xy - y^2 \Big|_{-1}^1$$

$$\left[x(1) - (1)^2 \right] - \left[x(-1) - (-1)^2 \right]$$

$$x - 1 + x + 1$$

$$= 2x$$

$$\int_0^3 2x \quad dx$$

$$\frac{\cancel{2}x^2}{\cancel{2}} \Big|_0^3$$

$$x^2 \Big|_0^3$$

$$\left[(3)^2 \right] - \left[(0)^2 \right]$$

$$A = 9 \quad u^2$$

3. Función a real:

$$A = \int_{x^2}^{3x+2}\int_0^2 3x - 2y\,dx\,dy$$

de Función a Real

$$A = \int_{x^2}^{3x+2}\int_0^2 3x-2y\,dx\,dy$$

$$\int_0^2\int_{x^2}^{3x+2} 3x-2y\,dy\,dx$$

$$\int_{x^2}^{3x+2} 3x-2y\,dy$$

$$3x\int dy - 2\int y\,dy$$

$$3xy - \frac{2y^2}{2}\Big|_{x^2}^{3x+2}$$

$$\left[3x(3x+2) - \frac{\cancel{2}(3x+2)^2}{\cancel{2}}\right] - \left[3x(x^2) - \frac{\cancel{2}(x^2)^2}{\cancel{2}}\right]$$

$$9x^2+6x-9x^2-12x-4-3x^3+x^4$$

$$= x^4-3x^3-6x-4$$

$$\int_0^2 x^4-3x^3-6x-4\,dx$$

$$\int x^4\,dx - 3\int x^3\,dx - 6\int x\,dx - 4\int dx$$

$$\frac{x^5}{5}-\frac{3x^4}{4}-\frac{\cancel{6}x^2}{\cancel{2}}-4x\Big|_0^2$$

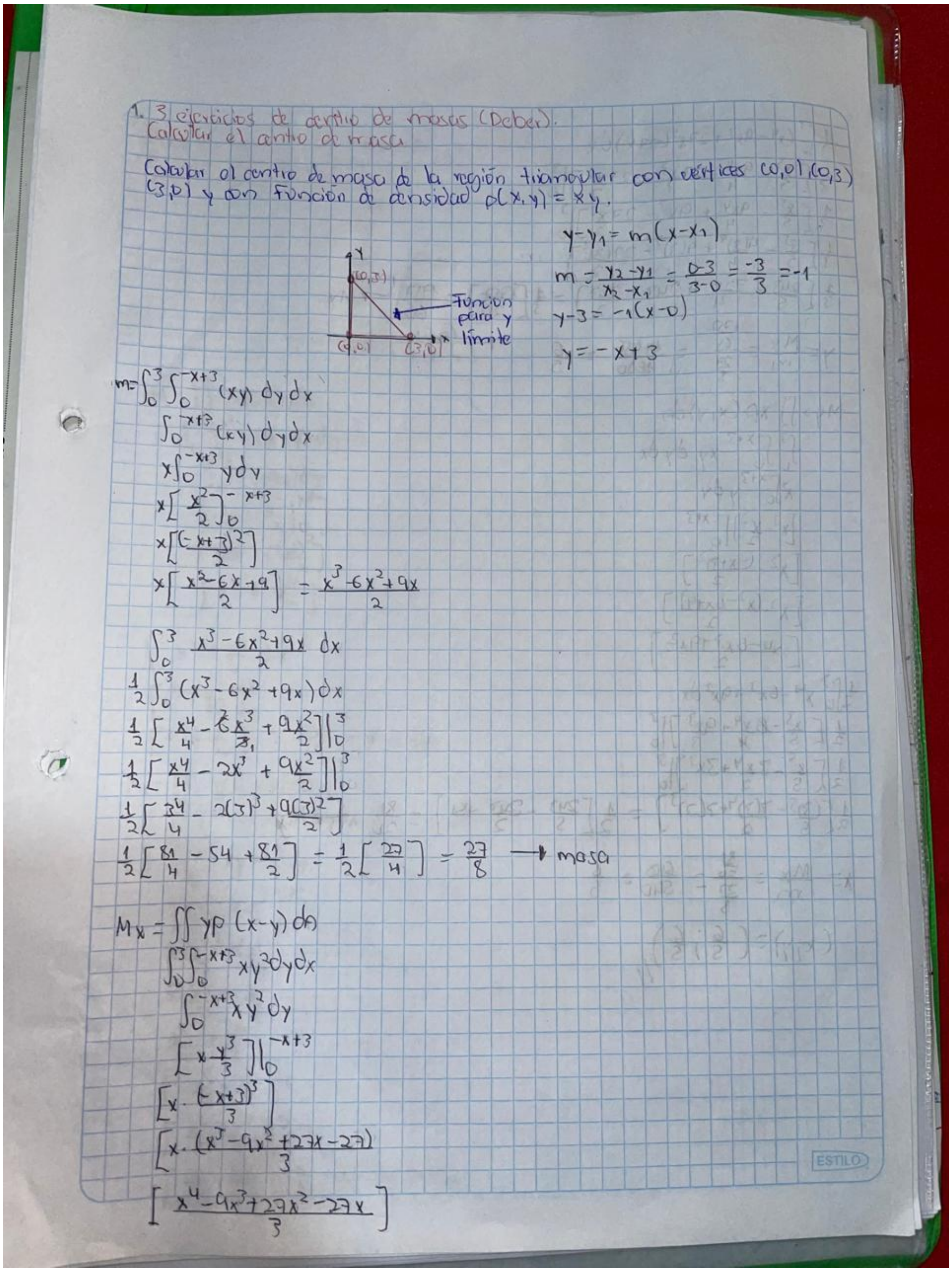
$$\frac{x^5}{5}-\frac{3x^4}{4}-3x^2-4x\Big|_0^2$$

$$\frac{(2)^5}{5}-\frac{3(2)^4}{4}-3(2)^2-4(2)$$

$$A = 25.6\,u^2$$

Ejercicios de Deber sobre centro de masas (3):

1. Calcular el centro de masa de la region triangular con vertices (0,0), (0,3), (3,0) y con funcion de densidad $\rho(x, y) = xy$



1. 3 ejercicios de centro de masas (Deber).
Calcular el centro de masa

Calcular el centro de masa de la región triangular con vértices (0,0), (0,3) (3,0) y con función de densidad $\rho(x,y) = xy$.

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{3-0} = \frac{-3}{3} = -1$$

$$y - 3 = -1(x - 0)$$

$$y = -x + 3$$

$$m = \int_0^3 \int_0^{-x+3} (xy)\, dy\, dx$$

$$\int_0^{-x+3} (xy)\, dy\, dx$$

$$x \int_0^{-x+3} y\, dy$$

$$x\left[\frac{x^2}{2}\right]_0^{-x+3}$$

$$x\left[\frac{(-x+3)^2}{2}\right]$$

$$x\left[\frac{x^2 - 6x + 9}{2}\right] = \frac{x^3 - 6x^2 + 9x}{2}$$

$$\int_0^3 \frac{x^3 - 6x^2 + 9x}{2}\, dx$$

$$\frac{1}{2}\int_0^3 (x^3 - 6x^2 + 9x)\, dx$$

$$\frac{1}{2}\left[\frac{x^4}{4} - \frac{\cancel{6}^{\,2} x^3}{\cancel{3}_{\,1}} + \frac{9x^2}{2}\right]\Bigg|_0^3$$

$$\frac{1}{2}\left[\frac{x^4}{4} - 2x^3 + \frac{9x^2}{2}\right]\Bigg|_0^3$$

$$\frac{1}{2}\left[\frac{3^4}{4} - 2(3)^3 + \frac{9(3)^2}{2}\right]$$

$$\frac{1}{2}\left[\frac{81}{4} - 54 + \frac{81}{2}\right] = \frac{1}{2}\left[\frac{27}{4}\right] = \frac{27}{8} \longrightarrow \text{masa}$$

$$M_x = \iint y\rho(x-y)\, dA$$

$$\int_0^3 \int_0^{-x+3} xy^2\, dy\, dx$$

$$\int_0^{-x+3} xy^2\, dy$$

$$\left[x \cdot \frac{y^3}{3}\right]\Bigg|_0^{-x+3}$$

$$\left[x \cdot \frac{(-x+3)^3}{3}\right]$$

$$\left[x \cdot \frac{(x^3 - 9x^2 + 27x - 27)}{3}\right]$$

$$\left[\frac{x^4 - 9x^3 + 27x^2 - 27x}{3}\right]$$

$$\frac{1}{3}\int_0^3 (x^4 - 9x^3 + 27x^2 (-27x)\, dx$$

$$\frac{1}{3}\left[\frac{x^5}{5} - \frac{9x^4}{4} + 27\frac{x^3}{3} - 27\frac{x^2}{2}\right]\Big|_0^3$$

$$\frac{1}{3}\left[\frac{x^5}{5} - \frac{9x^4}{4} + 9x^3 - 27\frac{x^2}{2}\right]\Big|_0^3$$

$$\frac{1}{3}\left[\frac{3^5}{5} - \frac{9(3)^4}{4} + 9(3)^3 - 27\frac{(3)^2}{2}\right]$$

$$\frac{1}{3}\left[\frac{243}{5} - \frac{720}{4} + 243 - \frac{243}{2}\right] = \frac{1}{3}\left[\frac{1757}{20}\right] = \frac{1757}{60} \rightarrow M_x$$

$$y = \frac{M_x}{m} = \frac{\frac{1757}{60}}{\frac{27}{8}} = \frac{14056}{1620} = \frac{6}{5}$$

$$M_y = \iint x\rho(x,y)\, dA$$

$$\int_0^3 \int_0^{-x+3} xy\, dy\, dx$$

$$x^2 \int_0^{-x+3} y\, dy$$

$$\left[x^2 \frac{y^2}{2}\right]\Big|_0^{-x+3}$$

$$\left[x^2 \frac{(-x+3)^2}{2}\right]$$

$$\left[x^2 \frac{(x^2-6x+9)}{2}\right]$$

$$\left[\frac{x^4 - 6x^3 + 9x^2}{2}\right]$$

$$\frac{1}{2}\int_0^3 x^4 - 6x^3 + 9x^2\, dx$$

$$\frac{1}{2}\left[\frac{x^5}{5} - \cancel{6}\frac{x^4}{\cancel{4}} + \frac{9x^3}{3}\right]\Big|_0^3$$

$$\frac{1}{2}\left[\frac{x^5}{5} - 3\frac{x^4}{2} + 3x^3\right]\Big|_0^3$$

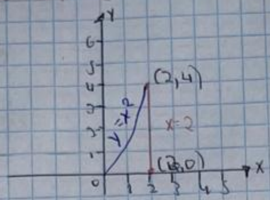
$$\frac{1}{2}\left[\frac{(3)^5}{5} - \frac{3(3)^4}{2} + 3(3)^3\right] = \frac{1}{2}\left[\frac{243}{5} - \frac{243}{2} + 81\right] = \frac{81}{20} \rightarrow M_y$$

$$x = \frac{M_x}{m} = \frac{\frac{81}{20}}{\frac{27}{8}} = \frac{648}{540} = \frac{6}{5}$$

$$(x;y) = \left(\frac{6}{5} ; \frac{6}{5}\right) //$$

2. Halle las masa, los momentos y el centro de masa de la lamina de densidad $\rho(x, y) = x + y$ que ocupa la region R bajo la curva y = x^2 en el interval $0 \leq x \leq 2$.

2. Halle la masa, los momentos y el centro de masa de la lámina de densidad $\rho(x,y) = x+y$ que ocupa la región R bajo la curva $y = x^2$ en el intervalo $0 \leq x = 2$



$m = \iint \rho(x,y)\,dA$

$\int_0^2 \int_0^{x^2} (x+y)\,dy\,dx$

$\int_0^{x^2} (x+y)\,dy$

$\left[xy + \frac{y^2}{2}\Big|_0^{x^2}\right]$

$\left[x(x^2) + \frac{(x^2)^2}{2}\right]$

$\left[x^3 + \frac{x^4}{2}\right]$

$\int_0^2 x^3 + \frac{x^4}{2}\,dx$

$\left[x^4 + \frac{x^5}{10}\right]_0^2$

$\left[2^4 + \frac{2^5}{10}\right] = \frac{76}{5} \rightarrow$ masa

$M_x = \iint_R y\rho(x+y)\,dA$

$\int_0^2 \int_0^{x^2} y(x+y)\,dy\,dx$

$\int_0^{x^2} xy + y^2\,dy$

$\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_0^{x^2}$

$x\frac{(x^2)^2}{2} + \frac{(x^2)^3}{3}$

$x \cdot \frac{x^4}{2} + \frac{x^6}{3}$

$\frac{x^5}{2} + \frac{x^6}{3}$

$\int_0^3 \frac{x^5}{2} + \frac{x^6}{3}\,dx$

$\left[\frac{x^6}{12} + \frac{x^7}{21}\right]\Big|_0^2$

$\left[\frac{64}{12} + \frac{128}{21}\right] = \frac{80}{7} \rightarrow M_x$

$$M_y = \iint_R x\rho(x,y)\,dA$$

$$\int_0^2 \int_0^{x^2} x(x+y)\,dy\,dx$$

$$\int_0^{x^2} x^2 + xy\,dy$$

$$\left[x^2 y + x\frac{y^2}{2}\right]_0^{x^2}$$

$$\left[x^2 \cdot x^2 + x\cdot\frac{(x^2)^2}{2}\right]$$

$$\left[x^4 + \frac{x^5}{2}\right]$$

$$\int_0^2 x^4 + \frac{x^5}{2}\,dx$$

$$\left[\frac{x^5}{5} + \frac{x^6}{12}\right]\Big|_0^2$$

$$\left[\frac{2^5}{5} + \frac{2^6}{12}\right] = \frac{32}{5} + \frac{64}{12} = \frac{176}{15} \rightarrow M_y$$

$$\left.\begin{aligned} x &= \frac{M_y}{m} = \frac{\frac{176}{15}}{\frac{36}{5}} = \frac{880}{540} = \frac{44}{27} \\ y &= \frac{M_x}{m} = \frac{\frac{80}{7}}{\frac{36}{5}} = \frac{400}{252} = \frac{100}{63} \end{aligned}\right\} \text{Centros de Masa } (x, y) = \left(\frac{44}{27}, \frac{100}{63}\right) //$$

3. Calcule la masa, los momentos y el centro de masa de la region entre las curvas y = x como $y = x^2$ con la funcion de densidad $\rho(x, y) = x$ en el intervalo $0 \leq x \leq 1$.

3. Calcule la masa, los momentos y el centro de masa de la región entre las curvas $y = x$ como $y = x^2$ con la función de densidad $\rho(x,y) = x$ en el intervalo $0 \leq x \leq 1$.

$$m = \iint_R \rho(x,y)\, dA$$

$$\int_0^1 \int_{x^2}^{x} x \, dy \, dx$$

$$\int_{x^2}^{x} x \, dy$$

$$\left[x y \right]\Big|_{x^2}^{x}$$

$$\left[x \cdot x \right] - \left[x \cdot x^2 \right]$$

$$\left[x^2 - x^3 \right]$$

$$\int_0^1 x^2 - x^3 \, dx$$

$$\left[\frac{x^3}{3} - \frac{x^4}{4} \right]\Big|_0^1$$

$$\left[\frac{1^3}{3} - \frac{1^4}{4} \right] = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \rightarrow \text{masa}$$

$$M_x = \iint_R y \rho(x,y)\, dA$$

$$\int_0^1 \int_{x^2}^{x} x y \, dy \, dx$$

$$\int_{x^2}^{x} x y \, dy$$

$$\left[\frac{y^2}{2} \right]\Big|_{x^2}^{x}$$

$$\left[\frac{x^2}{2} - \frac{(x^2)^2}{2} \right]$$

$$\left[\frac{x^2}{2} - \frac{x^4}{2} \right]$$

$$\int_0^1 x \left(\frac{x^2}{2} - \frac{x^4}{2} \right) dx$$

$$\int_0^1 \frac{x^3}{2} - \frac{x^5}{2} \, dy$$

$$\left[\frac{x^4}{8} - \frac{x^6}{12} \right]\Big|_0^1 = \frac{1^4}{8} - \frac{1^6}{12} = \frac{1}{8} - \frac{1}{12} = \frac{1}{24} \rightarrow M_x$$

$$M_y = \int_R\int x\rho(x,y)\,dA$$

$$\int_R\int x(x)\,d_y dx$$

$$\int_0^1\int_{x^2}^{x} x^2\,dy\,dx$$

$$\int_{x^2}^{x} x^2\,d_y$$

$$x^2 y\Big|_{x^2}^{x}$$

$$x^2 x - x^2 x^2$$

$$x^3 - x^4$$

$$\int_0^1 (x^3 - x^4)\,dx$$

$$\left[\frac{x^4}{4} - \frac{x^5}{5}\right]\Big|_0^1$$

$$\left[\frac{1^4}{4} - \frac{1^5}{5}\right] = \frac{1}{4} - \frac{1}{5} = \frac{1}{20} \longrightarrow M_y$$

$$\left.\begin{aligned} x &= \frac{M_y}{m} = \frac{\frac{1}{20}}{\frac{1}{12}} = \frac{12}{20} = \frac{3}{5} \\ y &= \frac{M_x}{m} = \frac{\frac{1}{24}}{\frac{1}{12}} = \frac{12}{24} = \frac{1}{2} \end{aligned}\right\} (x;y) = \left(\frac{3}{5};\frac{1}{2}\right)_{//}$$

Nombre