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FORMATION TRACKING CONTROL FOR MULTI-ROBOT SYSTEMS BASED ON REINFORCEMENT LEARNING - ADP APPROACH

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Introduction

1. Introduction

- In this research, a system of multiple wheeled mobile robots is required to track and maintain specific formations while following desired trajectories.
- The objectives of this work are:
  - Introduce the multi-agent consensus control algorithms for single- and double-integrator models, and ensure formation tracking of wheeled mobile robot system in case of no disturbance by traditional control method.
  - Present an ADP-based adaptive optimal controller for the wheeled mobile robot formation tracking, considering wheel slip and external disturbances.

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System modeling

A. Introduction of wheeled mobile robot (WMR)

- Two main driving wheels, one free wheel

Symbol	Meaning
$G(x_G, y_G)$	the center of mass of the platform
$M(x_M, y_M)$	the midpoint of the wheel shaft
F_1, F_2	total longitudinal friction forces at the right and left wheel
F_3	total lateral friction force along the wheel shaft
F_4, μ	external force and moment acting on G, respectively
γ_R, γ_L	longitudinal slip factors of the left and right wheels
η	the lateral slip factor along the wheel shaft
a	the distance between point M and G
b	half of the wheel shaft
θ	the orientation of the WMR
r	the radius of each wheel

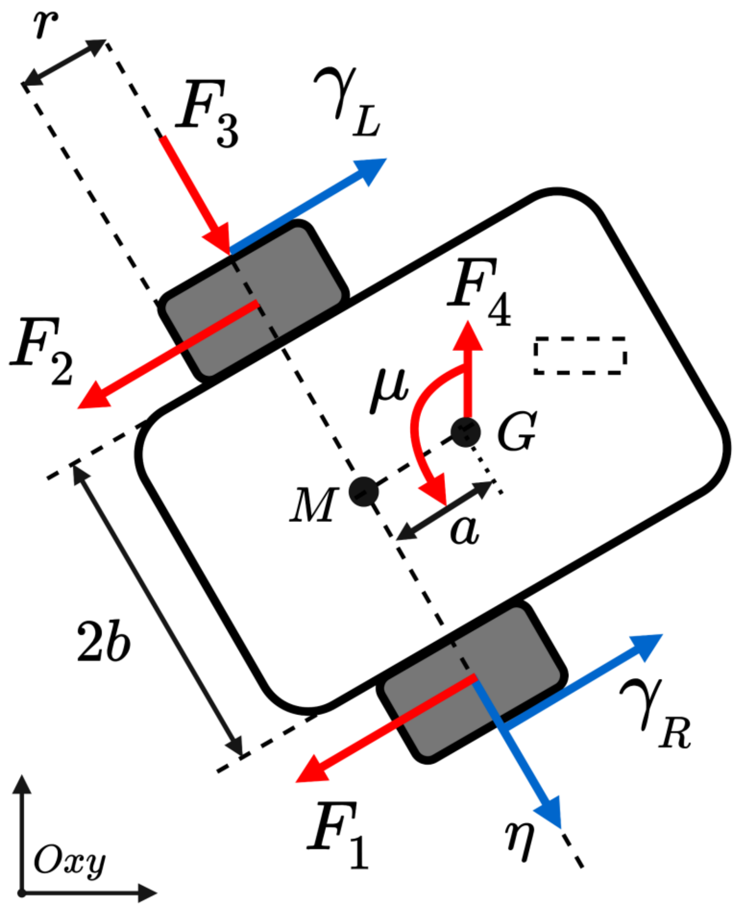


Diagram of the WMR

B. WMR system model:

1. Kinematic model

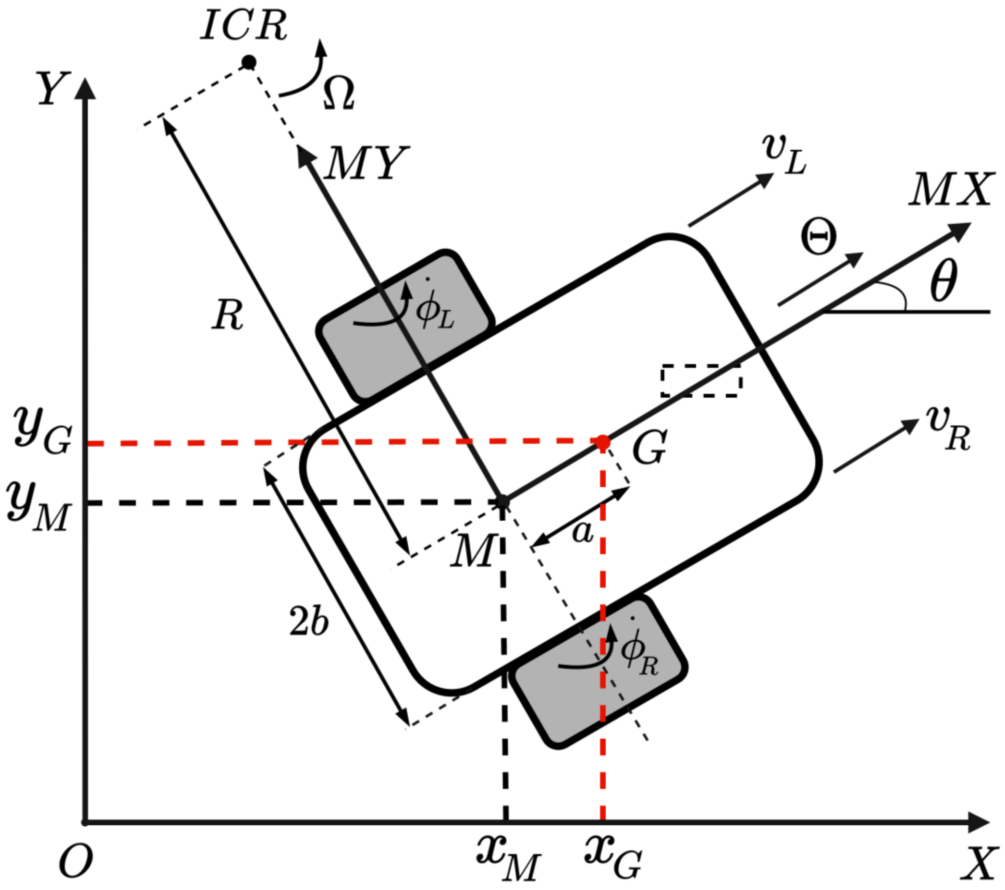
- In case of no disturbance:

The center of mass position is given by:

$$\begin{cases} x_G = x_M + a\cos(\theta) \\ y_G = y_M + a\sin(\theta) \end{cases} \quad (1)$$

The kinematics of the WMR is written as follows:

$$\begin{cases} \dot{x}_G = \Theta\cos(\theta) - \Omega a\sin(\theta) \\ \dot{y}_G = \Theta\sin(\theta) + \Omega a\cos(\theta) \quad (2) \\ \dot{\theta} = \Omega \end{cases}$$



WMR coordinates

Where Θ and Ω are the linear and angular velocities of the WMR:

$\Theta = \frac{r(\dot{\phi}_R+\dot{\phi}_L)}{2}$, $\Omega = \frac{r(\dot{\phi}_R-\dot{\phi}_L)}{2b}$, $\dot{\phi}_R$ and $\dot{\phi}_L$ are the angular velocities of the right and left wheel.

B. WMR system model:

1. Kinematic model

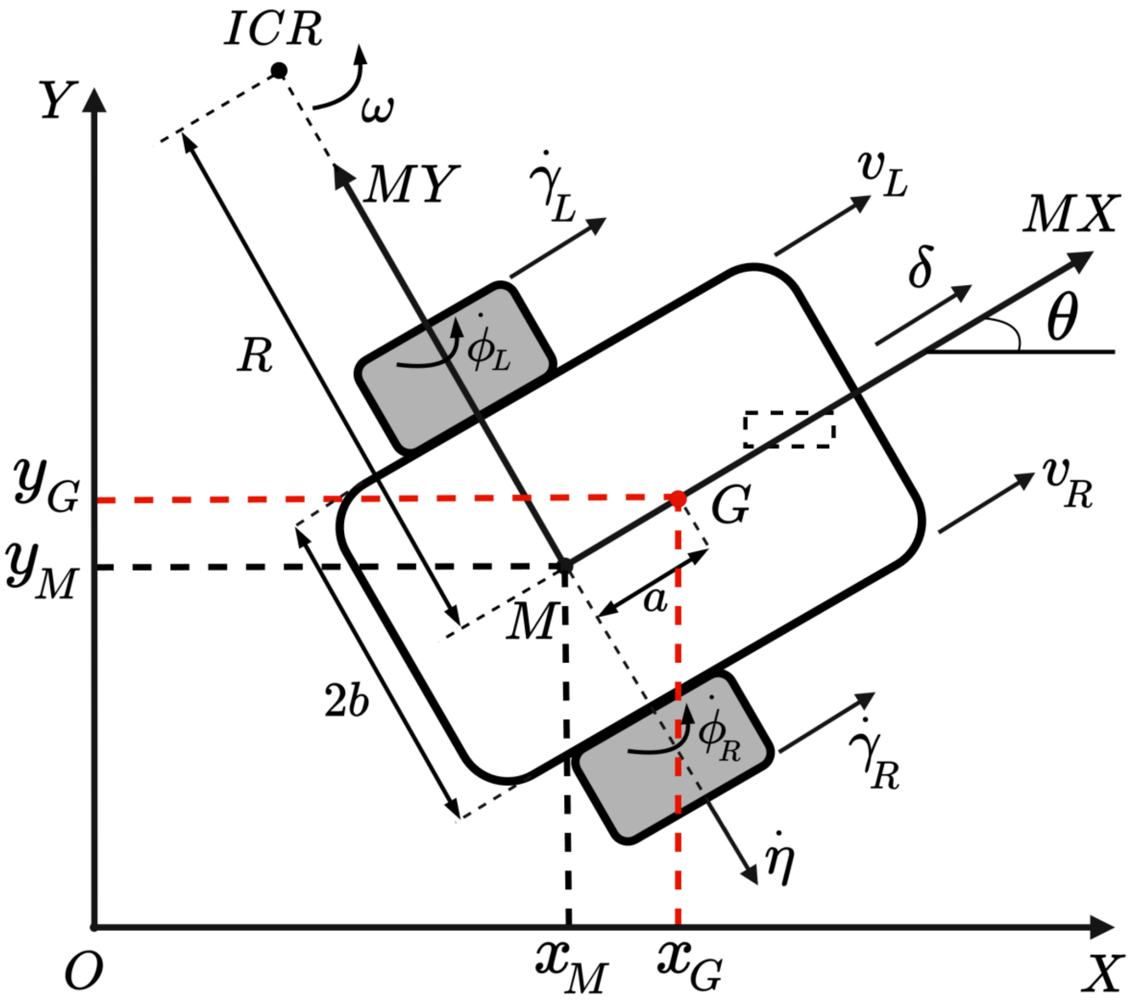
- In case of wheel slip and external disturbances

The actual linear and angular velocities are computed as:

$$\begin{cases} \delta = \dfrac{r(\dot{\phi}_R+\dot{\phi}_L)}{2} + \dfrac{\dot{\gamma}_R+\dot{\gamma}_L}{2} \\ \omega = \dfrac{r(\dot{\phi}_R-\dot{\phi}_L)}{2b} + \dfrac{\dot{\gamma}_R-\dot{\gamma}_L}{2b} \end{cases} \quad (3)$$

The actual kinematics of the WMR is written as follows:

$$\begin{cases} \dot{x}_G = \delta cos(\theta) - \omega a \sin(\theta) - \dot{\eta} \sin(\theta) \\ \dot{y}_G = \delta sin(\theta) + \omega a \cos(\theta) + \dot{\eta} \cos(\theta) \\ \dot{\theta} = \omega \end{cases} \quad (4)$$



WMR coordinates

B. WMR system model:

1. Kinematic model

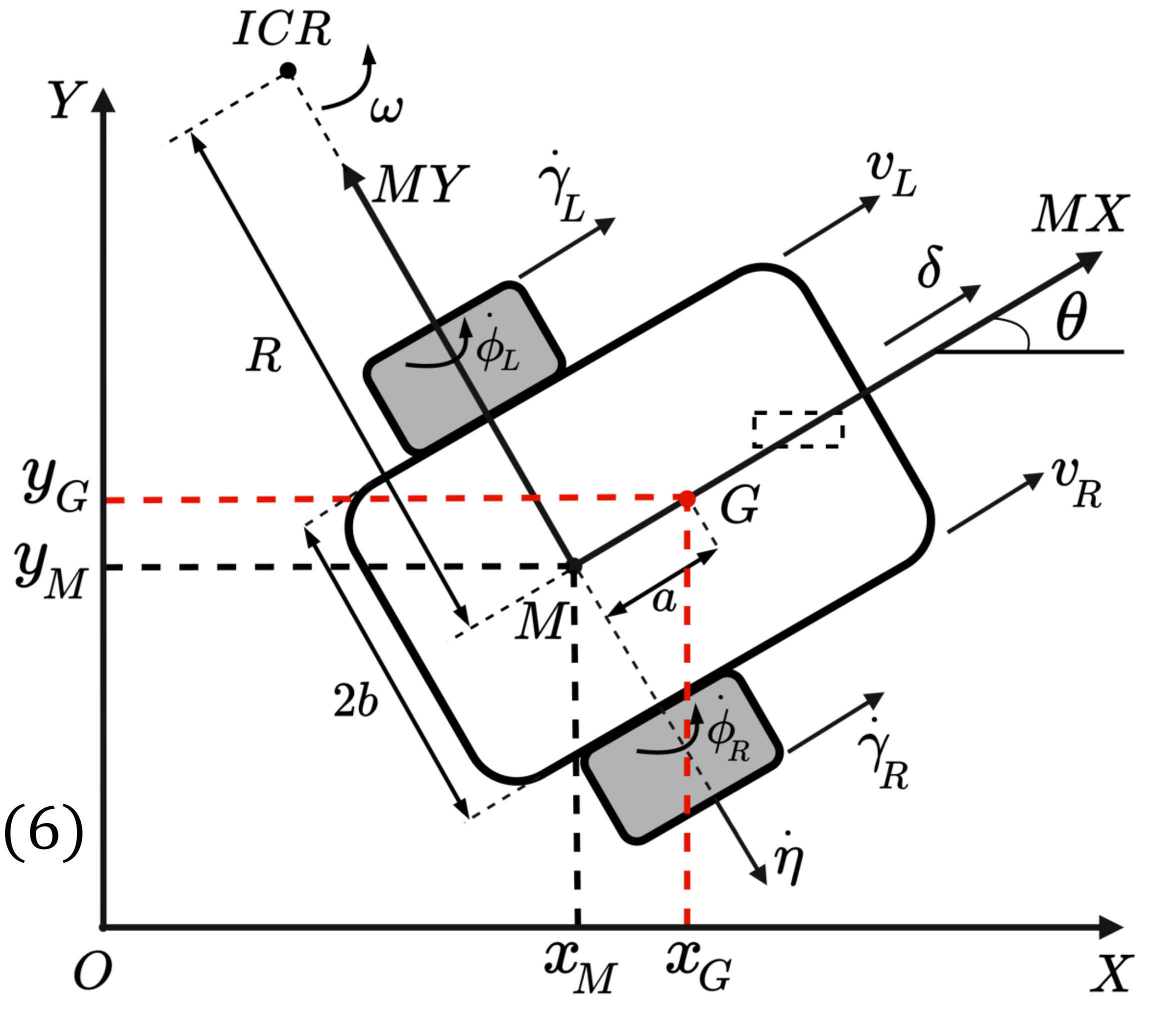
- In case of wheel slip and external disturbances

Combining (3) and (4), the position of the WMR can be rewritten as:

$$\dot{p} = Sv + d_p \qquad (5)$$

Where,

$$\begin{cases} p = [x_G \quad y_G]^T, \; v = [\dot{\phi}_R \quad \dot{\phi}_L]^T \\ S = \begin{bmatrix} \dfrac{r\cos(\theta)}{2} - \dfrac{ar\sin(\theta)}{2b} & \dfrac{r\cos(\theta)}{2} + \dfrac{ar\sin(\theta)}{2b} \\ \dfrac{r\sin(\theta)}{2} + \dfrac{ar\cos(\theta)}{2b} & \dfrac{r\sin(\theta)}{2} - \dfrac{ar\cos(\theta)}{2b} \end{bmatrix} \\ d_p = \begin{bmatrix} \dfrac{\cos(\theta)(\dot{\gamma}_R+\dot{\gamma}_L)}{2} - \dfrac{a\sin(\theta)(\dot{\gamma}_R-\dot{\gamma}_L)}{2b} - \dot{\eta}\sin(\theta) \\ \dfrac{\sin(\theta)(\dot{\gamma}_R+\dot{\gamma}_L)}{2} + \dfrac{a\cos(\theta)(\dot{\gamma}_R-\dot{\gamma}_L)}{2b} + \dot{\eta}\cos(\theta) \end{bmatrix} \end{cases} \qquad (6)$$



WMR coordinates

B. WMR system model:

2. Dynamic model

- In case of no disturbance

The dynamic model is derived by Lagrange formulation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = F - A^T\lambda \quad (7)$$

Where,

+ L defines the Lagrangian (difference between kinetic and potential energy of the system), given as:

$$L = K - U \quad (8)$$

- $U = 0$: potential energy of the system
- K: kinetic energy of the WMR platform, derived as: $\mathrm{K} = K_G = \frac{1}{2}m_G(\dot{x}_G^2 + \dot{y}_G^2) + \frac{1}{2}I_G\Omega^2 \quad (9)$

m_G: mass of the platform of the WMR without the driving wheels

I_G: moment of inertia of the WMR platform about the vertical axis through G

B. WMR system model:

2. Dynamic model

- In case of no disturbance

The dynamic model is derived by Lagrange formulation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = F - A^T\lambda \quad (7)$$

Where,

+ $\mathrm{q} = [x_G \quad y_G \quad \theta]^T$ is the general coordinate, $\lambda = [\lambda_1 \quad \lambda_2 \quad \lambda_3]^T$ is the Lagrange multiplier.

+ $A(q) = [-\sin(\theta) \quad \cos(\theta) \quad -a]$ is the matrix of kinematic constraint coefficients satisfying:

$$A(q)\dot{q} = 0 \quad (10)$$

+ $\mathrm{F} = [F_x \quad F_y \quad T]$ is the general force acting on the system

- $F_x = \frac{1}{r}(\tau_R + \tau_L)\cos(\theta)$: force in the x-axis direction
- $F_y = \frac{1}{r}(\tau_R + \tau_L)sin(\theta)$: force in the y-axis direction
- $T = \frac{b}{r}(\tau_R - \tau_L)$: the torque on the robot
- τ_R, τ_L: torques applied to the right and left wheels

B. WMR system model:

2. Dynamic model

- In case of no disturbance

According to Lagrange formulation (7), the following differential equations are obtained:

$$\begin{cases} m_G\ddot{x}_G = F_x + \lambda_1 \sin(\theta) \\ m_G\ddot{y}_G = F_y - \lambda_2 \cos(\theta) \\ I_G\ddot{\theta} = T + \lambda_3 a \end{cases} \qquad (11)$$

Or written in matrix form as: $$\bar{M}\ddot{q} = \bar{E}\tau - A^T\lambda \qquad (12)$$

Where,

$$\tau = [\tau_R \quad \tau_L]^T, \quad \bar{M} = \begin{bmatrix} m_G & 0 & 0 \\ 0 & m_G & 0 \\ 0 & 0 & I_G \end{bmatrix}, \quad \bar{E} = \frac{1}{\mathrm{r}}\begin{bmatrix} \cos(\theta) & \cos(\theta) \\ \sin(\theta) & \sin(\theta) \\ b & -b \end{bmatrix}$$

B. WMR system model:

2. Dynamic model

- In case of no disturbance

$$\bar{M}\ddot{q} = \bar{E}\tau - A^T\lambda \qquad (12)$$

The kinematics of the WMR (2) can be written in matrix form as: $\dot{q} = S_0 v_0 \qquad (13)$

$$\text{Where} \quad S_0 = \begin{bmatrix} \cos(\theta) & -a\sin(\theta) \\ \sin(\theta) & a\cos(\theta) \\ 0 & 1 \end{bmatrix}, \quad v_0 = [\Theta \quad \Omega]^T$$

Taking the time derivative of (13): $\ddot{q} = \dot{S}_0 v_0 + S_0 \dot{v}_0 \qquad (14)$

Substitute (14) into (12) : $\bar{M}\dot{S}_0 v_0 + \bar{M} S_0 \dot{v}_0 = \bar{E}\tau - A^T\lambda \qquad (15)$

Multiply (15) by S^T gives: $\boxed{M\dot{v}_0 + \bar{B} v_0 = E\tau} \qquad (16)$

$$\text{Where} \quad \mathrm{M} = \mathrm{S^T \bar{M} S} = \begin{bmatrix} m_G & 0 \\ 0 & m_G a^2 + I_G \end{bmatrix}, \quad \bar{B} = S^T \bar{M} \dot{S} = \begin{bmatrix} 0 & -m_G a\Omega \\ m_G a\Omega & 0 \end{bmatrix}, \quad E = S^T \bar{E} = \begin{bmatrix} \frac{1}{r} & \frac{1}{r} \\ \frac{b}{r} & -\frac{b}{r} \end{bmatrix}$$

B. WMR system model:

2. Dynamic model

- In case of wheel slip and external disturbances

The Lagrangian is defined as: $\mathrm{L} = K - U = K_G + K_R + K_L \quad (17)$

Where
$$\begin{cases} K_G = \frac{1}{2} m_G (\dot{x}_G^2 + \dot{y}_G^2) + \frac{1}{2} I_G \omega^2 & \text{: the kinetic energy of the WMR platform.} \\ K_R = \frac{1}{2} m_W \left[\left(r\dot{\phi}_R + \dot{\gamma}_R \right)^2 + \eta^2 \right] + \frac{1}{2} I_W \dot{\phi}_R^2 + \frac{1}{2} I_D \omega^2 & \text{: the kinetic energy of the right wheel.} \\ K_L = \frac{1}{2} m_W \left[\left(r\dot{\phi}_R + \dot{\gamma}_L \right)^2 + \eta^2 \right] + \frac{1}{2} I_W \dot{\phi}_L^2 + \frac{1}{2} I_D \omega^2 & \text{: the kinetic energy of the left wheel.} \end{cases}$$

- m_W: mass of each wheel
- I_W: the moment of inertia of each driving wheel about its rotational axis
- I_D: the moment of inertia of each driving wheel about its diameter axis

B. WMR system model:

2. Dynamic model

- In case of wheel slip and external disturbances

Based on (4) , the nonholonomic constraint equations can be written as:

$$\begin{cases} \dot{\gamma}_R = -r\dot{\phi}_R + \dot{x}_G\cos(\theta) + \dot{y}_G\sin(\theta) + b\omega \\ \dot{\gamma}_L = -r\dot{\phi}_L + \dot{x}_G\cos(\theta) + \dot{y}_G\sin(\theta) - b\omega \\ \dot{\eta} = -\dot{x}_G\sin(\theta) + \dot{y}_G\cos(\theta) \end{cases} \quad (18)$$

Similarly, the Lagrange formulation can be obtained as: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + \bar{\tau}_d = N\tau - A^T\lambda \quad (19)$

Where $\bar{\tau}_d$ illustrates model uncertainties and unknown disturbances, N is the input transformation matrix.

- $q = [x_G \quad y_G \quad \theta \quad \eta \quad \gamma_R \quad \gamma_L \quad \phi_R \quad \phi_L]^T$ is the general Lagrange coordinate
- $A(q) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & b & 0 & -1 & 0 & -r & 0 \\ \cos(\theta) & \sin(\theta) & -b & 0 & 0 & -1 & 0 & -r \\ -\sin(\theta) & \cos(\theta) & a & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$ which satisfies $A(q)\dot{q} = 0$

B. WMR system model:

2. Dynamic model

- In case of wheel slip and external disturbances

Solve the Lagrange equation gives: $\bar{M}\ddot{q} + \bar{\tau}_d = N\tau - A^T\lambda$ (20) where:

$$M = \begin{bmatrix} m_G & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_G & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_G + 2I_D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2m_W & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_W & 0 & m_W r & 0 \\ 0 & 0 & 0 & 0 & 0 & m_W & 0 & m_W r \\ 0 & 0 & 0 & 0 & m_W r & 0 & m_W r^2 + I_W & 0 \\ 0 & 0 & 0 & 0 & 0 & m_W r & 0 & m_W r^2 + I_W \end{bmatrix}, \mathrm{N} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

B. WMR system model:

2. Dynamic model

- In case of wheel slip and external disturbances

The actual kinematics (6) can be written as: $\dot{q} = S_1(q)v + S_2(q)\dot{\gamma} + S_3(q)\,\dot{\eta}$ (21) where:

- $S_1 = \begin{bmatrix} \frac{r\cos(\theta)}{2} - \frac{ar\sin(\theta)}{2b} & \frac{r\cos(\theta)}{2} + \frac{ar\sin(\theta)}{2b} \\ \frac{r\sin(\theta)}{2} + \frac{ar\cos(\theta)}{2b} & \frac{r\sin(\theta)}{2} - \frac{ar\cos(\theta)}{2b} \\ \frac{r}{2b} & -\frac{r}{2b} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $S_2 = \begin{bmatrix} \frac{cos(\theta)}{2} - \frac{a\sin(\theta)}{2b} & \frac{cos(\theta)}{2} - \frac{a\sin(\theta)}{2b} \\ \frac{\sin(\theta)}{2} + \frac{a\cos(\theta)}{2b} & \frac{\sin(\theta)}{2} - \frac{a\cos(\theta)}{2b} \\ \frac{1}{2b} & -\frac{1}{2b} \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $S_3 = \begin{bmatrix} -\sin(\theta) \\ cos(\theta) \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
- $v = [\dot{\phi}_R \quad \dot{\phi}_L]^T$, $\gamma = [\gamma_R \quad \gamma_L]^T$

B. WMR system model:

2. Dynamic model

- In case of wheel slip and external disturbances

$$\bar{M}\ddot{q} + \bar{\tau}_d = N\tau - A^T\lambda \qquad (20)$$

Taking the time derivative of (21): $\ddot{q} = \dot{S}_1 v + S_1\dot{v} + \dot{S}_2\dot{\gamma} + S_2\ddot{\gamma} + \dot{S}_3\dot{\eta} + S_3\ddot{\eta} \qquad (22)$

Substitute (22) into (20) and multiply both sides by S_1^T gives the actual dynamic model:

$$\boxed{M\dot{v} + Bv + Q\ddot{\gamma} + C\dot{\eta} + G\ddot{\eta} + \tau_d = \tau} \qquad (23)$$

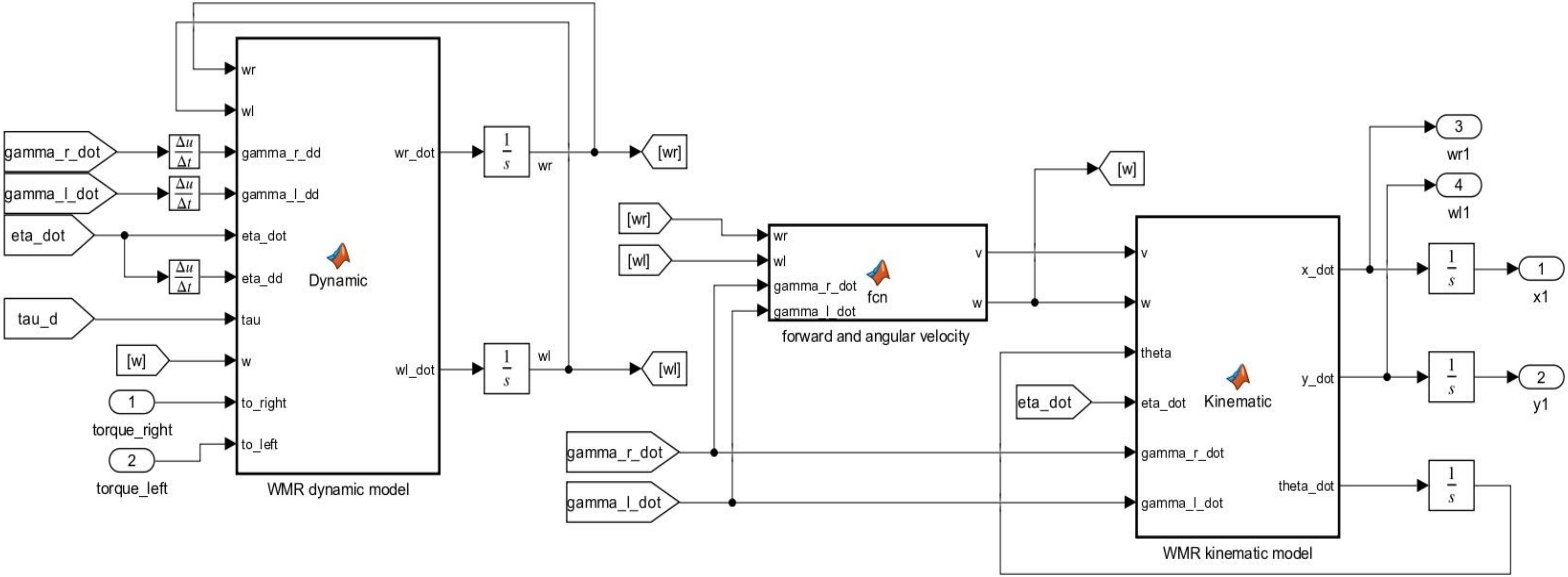
Where $\tau_d = S_1^T\bar{\tau}_d$; $M = S_1^T\bar{M}S_1 = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$; $m_{22} = m_{11}, m_{21} = m_{12}$; $Q = S_1^T\bar{M}S_2 = \begin{bmatrix} Q_1 & Q_2 \\ Q_2 & Q_1 \end{bmatrix}$;

$$\mathrm{G} = \mathrm{S}_1^{\mathrm{T}}\bar{\mathrm{M}}\mathrm{S}_3 = \frac{m_G a r}{2b}\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad B = S_1^T\bar{M}\dot{S}_1 = \frac{m_G a r^2}{2b}\omega\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

$$m_{11} = m_G\left(\frac{r^2}{4} + \frac{a^2r^2}{4b^2}\right) + \frac{r^2}{4b^2}(I_G + 2I_D) + m_W r^2 + I_W \;, m_{12} = m_G\left(\frac{r^2}{4} - \frac{a^2r^2}{4b^2}\right) - \frac{r^2}{4b^2}(I_G + 2I_D)$$

$$Q_{1,2} = m_G\frac{r}{4}\left(1 \pm \frac{a^2}{b^2}\right) \pm \frac{r}{4b}(I_G + 2I_D)$$

C. WMR simulation implementation:



Multi-agent control design using traditional method

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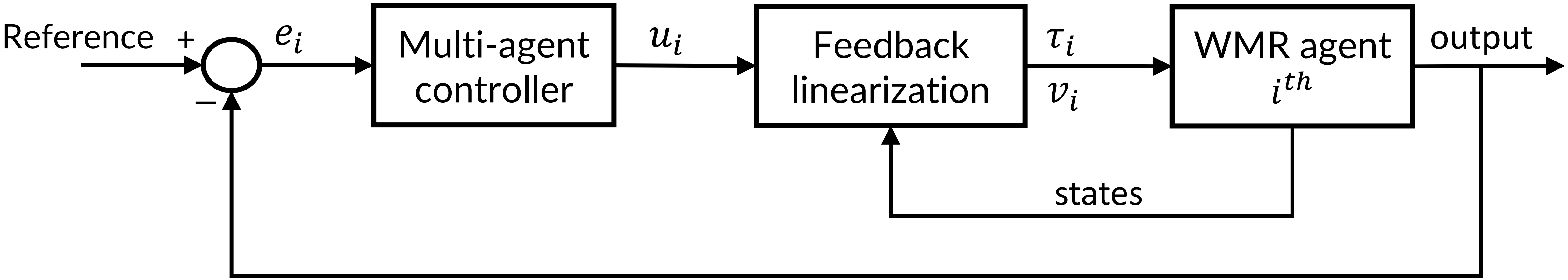
3. Multi-agent control using consensus algorithm

A. Control objective:

- The consensus algorithm is utilized for agents with single-integrator and double-integrator dynamics to achieve common information states.
- Distributed control law is implemented for each agent to ensure formation tracking while accurately following predefined trajectories.

➔ Apply the control algorithm to the WMR system.

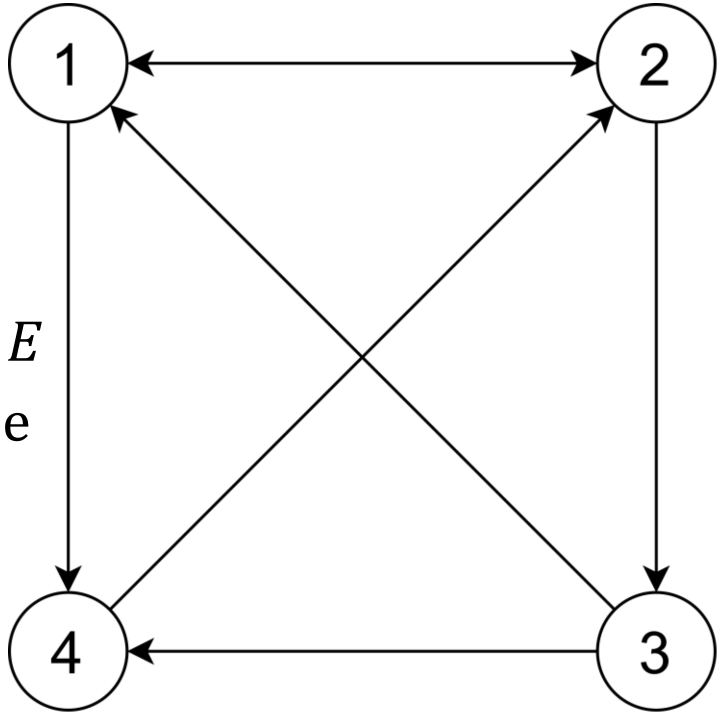


A. Control objective:

- Each agent interacts and exchanges relevant information with other agents through the communication graph.
- Consider a network of n agents with graph $G = \{V, E\}$ where:

- $V = \{1,2, \dots, n\}$ is a set of nodes
- $E \subset V \times V$ is a set of edges
- The adjacency matrix is $A = [a_{ij}]_{n\times n} \in \mathbb{R}^{n\times n}, \begin{cases} a_{ij} = 1 \; if \; (i,j) \in E \\ a_{ij} = 0, \text{otherwise} \end{cases}$
- The degree matrix is $\mathrm{D} = \mathrm{diag}\{\mathrm{d}_1, \dots, \mathrm{d}_\mathrm{n}\} \in \mathbb{R}^{n\times n}, \;\; d_i = \sum_{j=1}^{n} a_{ij}$
- The Laplacian matrix is $\mathcal{L} = D - A = [l_{ij}]_{n\times n} \in \mathbb{R}^{n\times n}$, where:

$$l_{ij} = \begin{cases} -a_{ij} = 1 \,, \; if \; i,j \in \mathrm{V}, \; \mathrm{i} \neq j \\ \sum_{j=1}^{n} a_{ij} \,, \; if \; i = j \end{cases}$$



Interaction graph of 4 agents

B. Consensus algorithm for single-integrator models:

Consider the case where each agent is modeled as single-integrator dynamics, expressed as:

$$\dot{p}_{ri} = u_i \qquad (24)$$

where p_{ri}, and u_i are the position and the control input of agent i^{th}, respectively.

The control law implemented for each agent:

$$u_i = \dot{p}_{ri}^d - k_p\left(p_{ri} - p_{ri}^d\right) + \sum_{j \in N_i} a_{ij}\left(\left(p_{rj} - p_{rj}^d\right) - \left(p_{ri} - p_{ri}^d\right)\right), i = 1, \dots, n \qquad (25)$$

- $\mathrm{N_i} = \{\mathrm{j} \in V | (i,j) \in E\}$: the set of neighborhood of agents i^{th}
- $p_{ri} = [x_{ri} \quad y_{ri}]^T$: actual position
- $p_{ri}^d = \left[x_{ri}^d \quad y_{ri}^d\right]^T$: desired position
- $u_i = [u_{xi} \quad u_{yi}]^T$: control input

Let $p_r = [p_{r1} \quad p_{r2} \quad \cdots \quad p_{rn}]^T$, $p_r^d = [p_{r1}^d \quad p_{r2}^d \quad \dots \quad p_{rn}^d]^T$, the control law is rewritten as:

$$u = -\left(\left(k_p I_n + \mathcal{L}\right) \otimes I_2\right)\left(p_r - p_r^d\right) \qquad (26)$$

Where $u = [u_1 \quad u_2 \quad \cdots \quad u_n]^T$, I_n is the identity matrix of size n

B. Consensus algorithm for single-integrator models:

The positions of each WMR agent under no disturbance (13) can be rewritten as:

$$\dot{p}_i = Sv_i \qquad (27)$$

Where $S = \begin{bmatrix} \cos(\theta) & -a\sin(\theta) \\ \sin(\theta) & a\cos(\theta) \end{bmatrix}, \quad v_i = [\Theta_i \quad \Omega_\mathrm{i}]^T$

Feedback linearization:

$$v_i = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\frac{1}{a}\sin(\theta) & \frac{1}{a}\cos(\theta) \end{bmatrix} u_i \qquad (28), \text{ where } u_i = [\mathrm{u}_{xi} \quad u_{y\mathrm{i}}]^T$$

The model becomes:

$$\dot{p}_i = u_i \qquad (29)$$

➔ Control law (26) can be directly applied.

C. Consensus algorithm for double-integrator models:

In case the dynamics of each agent follows double-integrator dynamics, defined as:

$$\begin{cases} \dot{p}_{ri} = v_{ri} \\ \dot{v}_{ri} = u_i \end{cases} \qquad (30)$$

where p_{ri} , v_{ri} and u_i are the position, velocity and the control input of agent i^{th}, respectively.

The control law implemented for each agent:

$$u_i = -k_p\left(p_{ri} - p_{ri}^d\right) + \sum_{j \in N_i} a_{ij}\left((p_{rj} - p_{rj}^d) - (p_{ri} - p_{ri}^d)\right) - k_v\left(v_{ri} - \dot{p}_{ri}^d\right), i = 1, \dots, n \quad (31)$$

Or rewritten as:

$$u = -\left((k_p I_n + \mathcal{L}) \otimes I_2\right)\left(p_r - p_r^d\right) - k_v\left(v_r - \dot{p}_r^d\right) \qquad (32)$$

C. Consensus algorithm for double-integrator models:

The dynamic model of each WMR agent under no disturbance (16) can be rewritten as:

$$\dot{v}_i = Bv_i + N\tau_i \quad (33)$$

$$\mathrm{B} = -\mathrm{M}^{-1}\overline{\mathrm{B}} = \begin{bmatrix} 0 & a\Omega_\mathrm{i} \\ \frac{-m_G a\Omega}{I_G + m_G a} & 0 \end{bmatrix}, \; N = M^{-1}E = \begin{bmatrix} \frac{1}{m_G r} & \frac{1}{m_G r} \\ \frac{b}{r(I_G + m_G a^2)} & \frac{-b}{r(I_G + m_G a^2)} \end{bmatrix}, \; v_i = \begin{bmatrix} \Theta_i \\ \Omega_i \end{bmatrix}, \; \tau_i = \begin{bmatrix} \tau_{Ri} \\ \tau_{Li} \end{bmatrix}$$

And the kinematic equations (27) obtained as: $\dot{p}_i = Sv_i$. Define: $z_i = Sv_i$ (34)

Derivative of z_i:

$$\dot{z}_i = \dot{S}v_i + S\dot{v}_i \quad (35)$$

Substitute (33) into (35):

$$\dot{z}_i = \left(\dot{S} + SB\right)v_i + SN\tau_i \quad (36)$$

Feedback linearization:

$$\tau_i = N^{-1}S^{-1}\left(u_i - \left(\dot{S} + SB\right)v_i\right) \quad (37)$$

The model becomes:

$$\begin{cases} \dot{p}_i = z_i \\ \dot{z}_i = u_i \end{cases} \quad (38)$$

➔ Control law (32) can be directly applied.

Multi-agent control design using Adaptive Dynamic Programming approach

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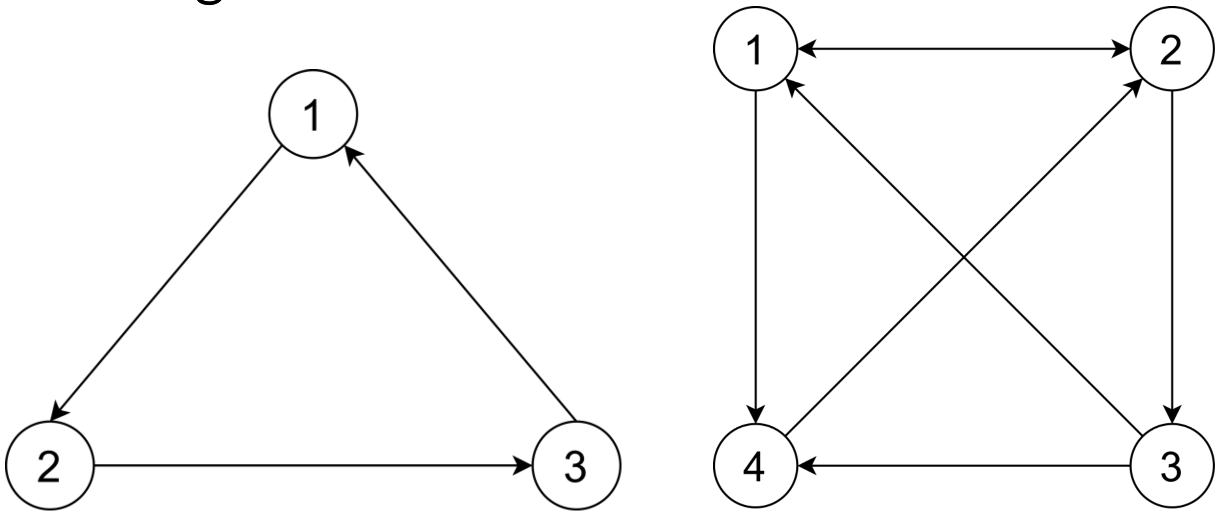
4. Multi-agent control using ADP approach

A. Control objective:

- An adaptive optimal controller in cooperation with a disturbance observer is proposed to ensure formation tracking for WMR agents in case of wheel slip and external disturbances.
- A multi-robot system with n WMRs achieve formation geometry of square and triangle shape, following two types of trajectories: line and circle.
- For comparison with previous approach, the WMR formation is tasked to follow a curved trajectory under square configuration.



Interaction topology considered

4. Multi-agent control using ADP approach

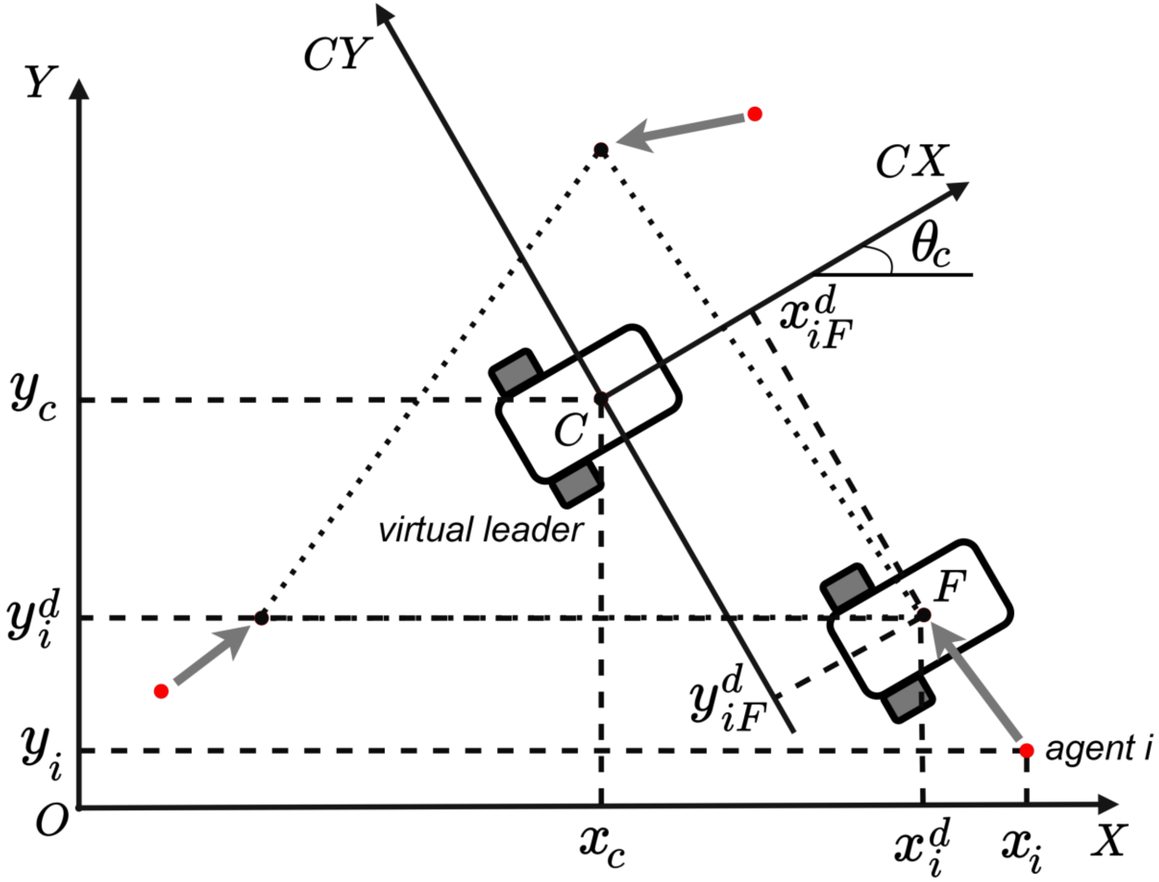
B. Problem formulation:

- Virtual leader/structure approach.
- Decentralized scheme instead of a centralized one.
- Control approach:

+ Define the desired dynamics of the virtual leader or virtual structure.

+ The motion of the virtual leader is translated into desired motions for each robot.

+ Derive distributed tracking control laws for each robot.



Triangle formation with a known virtual leader

4. Multi-agent control using ADP approach

B. Problem formulation:

The formation structure of the WMR system is derived as:

$$p_i^d = p_c + \begin{bmatrix} \cos(\theta_c) & -\sin(\theta_c) \\ \sin(\theta_c) & \cos(\theta_c) \end{bmatrix} p_{iF}^d \qquad (39)$$

- $p_i^d = \begin{bmatrix} x_i^d & y_i^d \end{bmatrix}^T$: desired position
- $p_c = \begin{bmatrix} x_c & y_c \end{bmatrix}^T$: position of virtual leader
- $p_{iF}^d = \begin{bmatrix} x_{iF}^d & y_{iF}^d \end{bmatrix}^T$: desired deviation of the ith robot

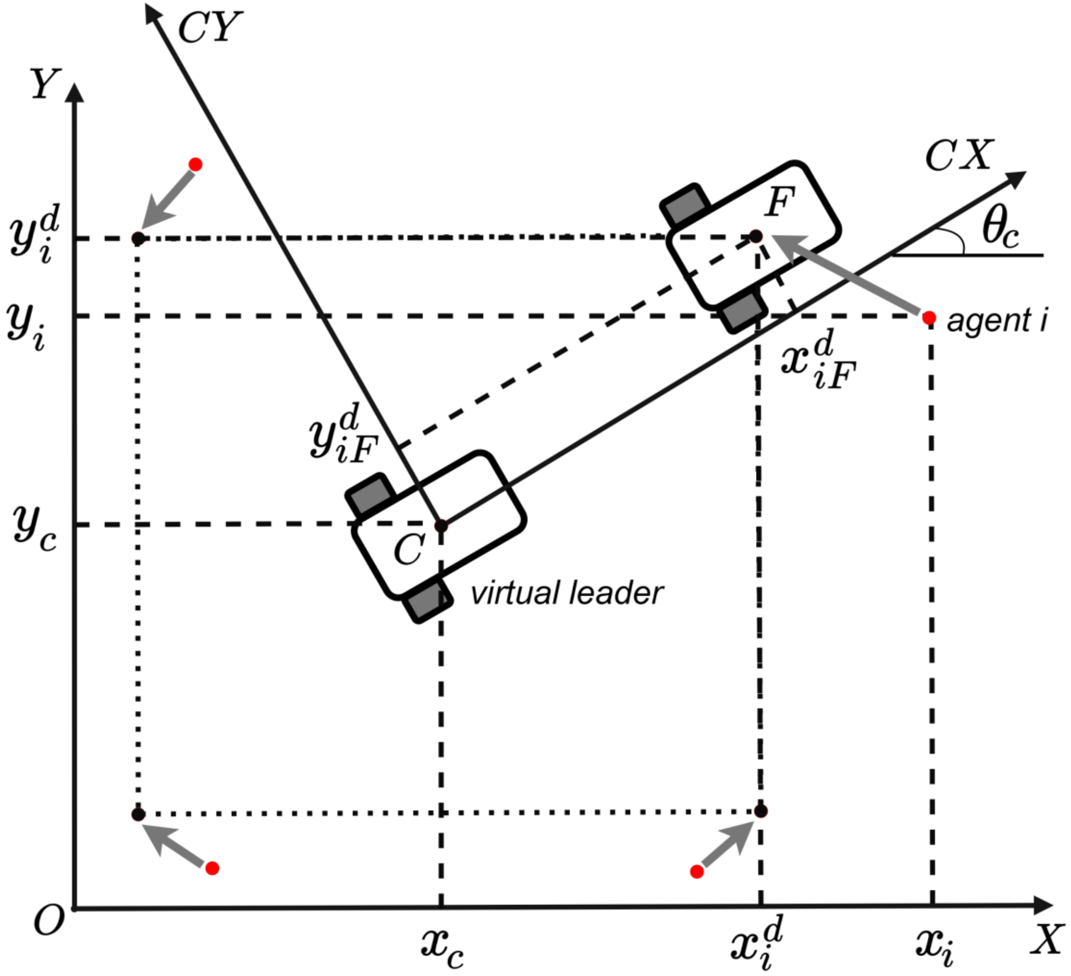
Where $x_{iF}^d = l_i \cos(\phi_i)$, $y_{iF}^d = l_i \sin(\phi_i)$

+ l_i : the distance from the formation center to each agent

+ ϕ_i : is the angular offsets for each agent

$$\phi_i = \frac{\pi}{2}(i-1)\,,\ i = 1, \dots, 4 \text{ for square shape}$$

$$\phi_i = \frac{2\pi}{3}(i-1)\,,\ i = 1, \dots, 3 \text{ for triangle shape}$$



Square formation with a known virtual leader

➔ Control law (32) using consensus algorithm can be applied.

ĐẠI HỌC BÁCH KHOA HÀ NỘI
HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

C. Tracking control design:

1. Control strategy

The consensus-based control law requires each WMR to approximate the double-integrator dynamics (30):

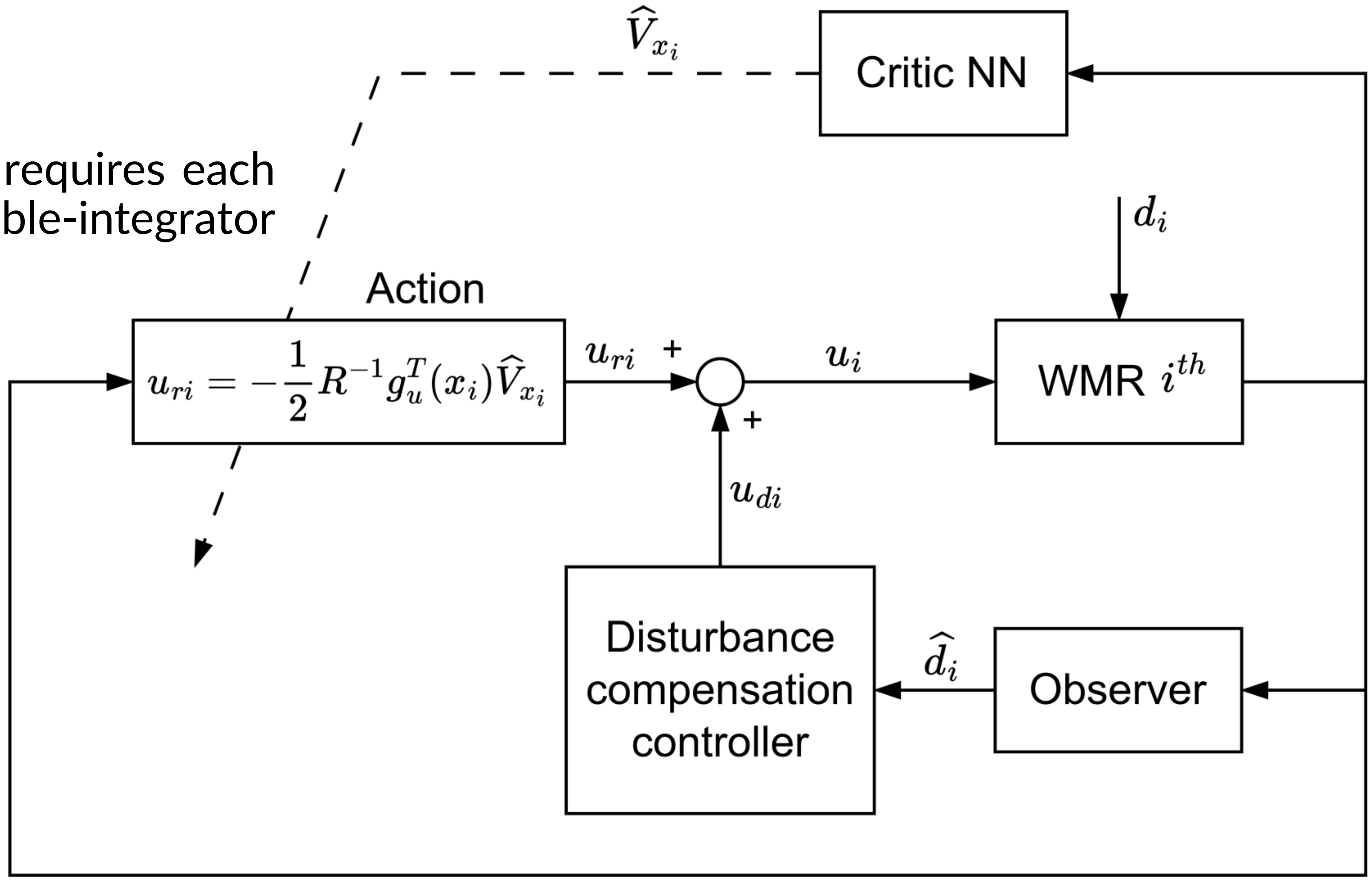
$$\begin{cases} \dot{p}_{ri} = v_{ri} \\ \dot{v}_{ri} = u_i \end{cases}$$

The kinematics of the i^{th} robot under disturbance is obtained from (5):

$$\dot{p}_i = S v_i \quad (40)$$

Define the tracking error:

$$e_i = p_i - p_{ri} \quad (41)$$



ADP control structure combined with disturbance observer

4. Multi-agent control using ADP approach

C. Tracking control design:

1. Control strategy

The error dynamics is derived as:

$$\dot{e}_i = \dot{p}_i - \dot{p}_{ri} = Sv_i + d_p - v_{ri} \qquad (42)$$

The time derivative of (42): $\ddot{e}_i = \dot{S}v_i + S\dot{v}_i + \dot{d}_p - u_i \qquad (43)$

The dynamic model for i^{th} robot (23) can be rewritten as:

$$\dot{v}_i = -M^{-1}Bv_i + M^{-1}\tau_i + d_v \quad (44)\text{ , where } \; d_v = -M^{-1}(Q\ddot{\gamma} + C\dot{\eta} + G\ddot{\eta} + \tau_d)$$

Substitute (44) into (43) : $\ddot{e}_i = -SM^{-1}Bv_i + u_i^{new} + \dot{S}v_i + Sd_v + \dot{d}_p \quad (45)$

Where $u_i^{new} = \mathrm{S}M^{-1}\tau_i - u_i$ is the transformed control input.

The control torque for the i^{th} WMR is calculated as:

$$\boxed{\tau_i = MS^{-1}(u_i^{new} + u_i) \quad (46)}$$

4. Multi-agent control using ADP approach

C. Tracking control design:

1. Control strategy

Define $x_{1i} = e_i$, $x_{2i} = \dot{x}_{1i} + \lambda x_{1i}$, the time derivative of x_{1i} and x_{2i} is obtained as:

$$\begin{cases} \dot{x}_{1i} = \dot{e}_i = Sv_i + d_p - v_{ri} \\ \dot{x}_{2i} = Ex_{2i} - E\lambda x_{1i} + u_i^{new} + d_i \end{cases} \quad (47)$$

Where: $E = E_1 S^{-1}$, $E_1 = -SM^{-1}B$, $d_i = \xi_{2i} - E\xi_{1i}$,

$$\xi_{1i} = d_p - v_{ri} \;,\; \xi_{2i} = Sd_v + \dot{S}v_i + \dot{d}_p + \lambda Sv_i + \lambda\xi_{1i}$$

Rewrite (47) in state space form: $\dot{x}_i = f(x_i) + g_u u_i^{new} + g_d d_i \quad (48)$

Where $x_i = \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix}$, $f(x_i) = \begin{bmatrix} x_{2i} - \lambda x_{1i} \\ Ex_{2i} - \lambda Ex_{1i} \end{bmatrix}$, $g_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $g_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

The control input is derived as follows to achieve optimal performance :

$$\boxed{u_i^{new} = u_{ri}(x_i) + u_{di}(x_i) \quad (49)}$$

C. Tracking control design:

2. Observer-based controller design

A disturbance observer is used to estimate disturbance d_i as follows:

$$\begin{cases} \hat{d}_i = z_i + \rho(x_i) \\ \dot{z}_i = -h(x_i)\{g_d(x_i)[z_i + \rho(x_i)] + f(x_i) + g_u(x_i)u_i^{new}\} \end{cases} \qquad (50)$$

Where:

- $\hat{d}_i$: the estimation of d_i
- z_i : the internal state of the observer
- $\rho(x_i)$: the nonlinear function vector to be designed
- $h(x_i)$: the gain of the observer, defined by $h(x_i) = \frac{\partial \rho(x_i)}{\partial x_i}$

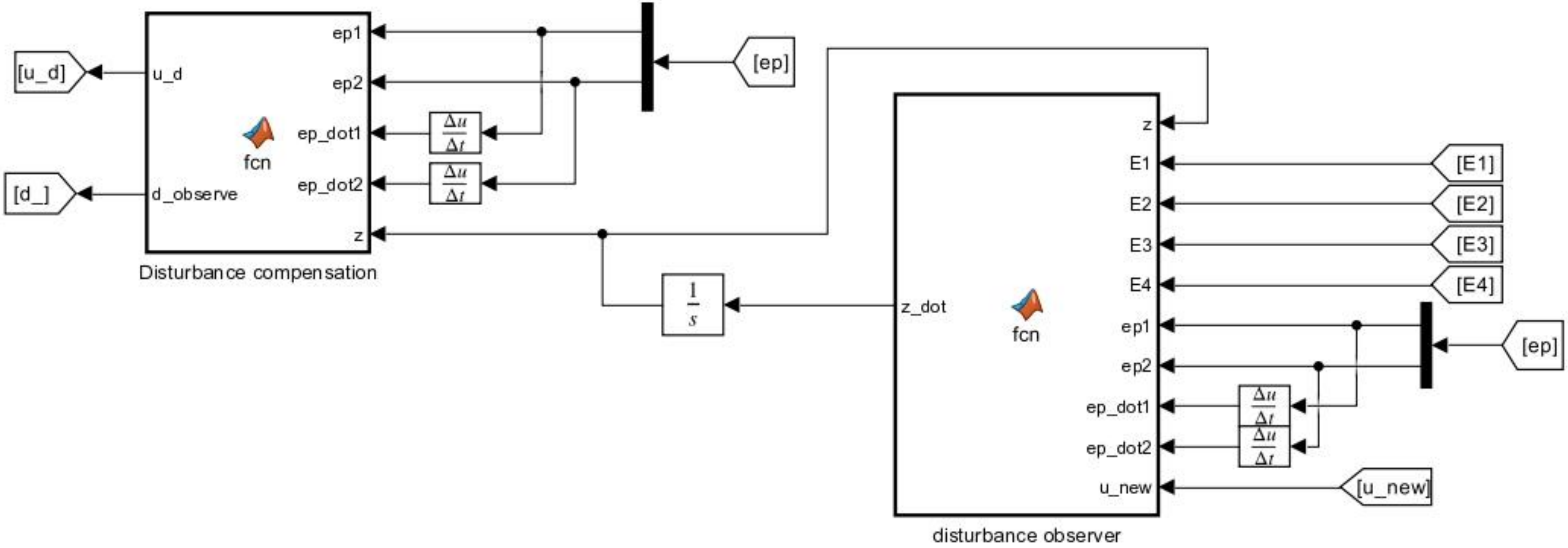
The compensation control component is derived as:

$$\boxed{u_{di}(x_i) = -\hat{d}_i \qquad (51)}$$

C. Tracking control design:

2. Observer-based controller design

The observer-based controller simulation implementation:



C. Tracking control design:

3. Adaptive optimal controller design

In the case of $d_i = 0$ the error system (48) is rewritten as:

$$\dot{x}_i = f(x_i) + g_u u_i^{new} \qquad (52)$$

Define the following cost function:

$$J(x_i) = \int_t^{\infty} r\left(x_i(\tau_i), u_{ri}\big(x_i(\tau_i)\big)\right) d\tau_i \qquad (53)$$

Where the utility function is defined as: $r = x_i^T(\tau_i) Q x_i(\tau_i) + u_{ri}^T(x_i) R u_{ri}(x_i)$

Define the Hamiltonian function of the problem as:

$$H = \left(\frac{\partial J}{\partial x_i}\right)^T + r(x_i, u_{ri}) = \left(\frac{\partial J}{\partial x_i}\right)^T + x_i^T Q x_i + u_{ri}^T R u_{ri} \qquad (54)$$

C. Tracking control design:

3. Adaptive optimal controller design

For the system (52) to have the optimal solution, there must exist an optimal cost function $V(x_i, u_{ri})$ defined by:

$$V(x_i, u_{ri}) = \min_{u_{ri}}\big(J(x_i)\big) \qquad (55)$$

Which satisfies the HJB equation:

$$0 = \min_{u_{ri}}\big(H(x_i, u_{ri}, V)\big) = \left(\frac{\partial V}{\partial x_i}\right)^T \dot{x}_i + x_i^T Q x_i + u_{ri}^T R u_{ri} \qquad (56)$$

The optimal control component u_{ri} can be derived as:

$$u_{ri} = \underset{u_{ri}}{\operatorname{argmin}}\big(H(x_i, u_{ri}, V)\big) \qquad (57)$$

Solve (57) using (56) obtains:

$$\boxed{u_{ri} = -\frac{1}{2} R^{-1} g_u^T \frac{\partial V}{\partial x_i} \qquad (58)}$$

C. Tracking control design:

3. Adaptive optimal controller design

The value function $V(x_i, u_{ri})$ is approximated by a NN as:

$$V(x_i, u_{ri}) = W_i^T \phi(x_i) + \epsilon(x_i) \quad (59)$$

The estimated cost function:

$$\hat{V}\,(x_i, u_{ri}) = \widehat{W}_i^T \phi(x_i) \qquad (60)$$

The approximated Hamiltonian function:

$$H\big(x_i, u_{ri}, \widehat{W}_i\big) = \widehat{W}_i^T \left(\frac{\partial \phi}{\partial x_i}\right) \dot{x}_i + x_i^T Q x_i + u_{ri}^T R u_{ri} = e_i \quad (61)$$

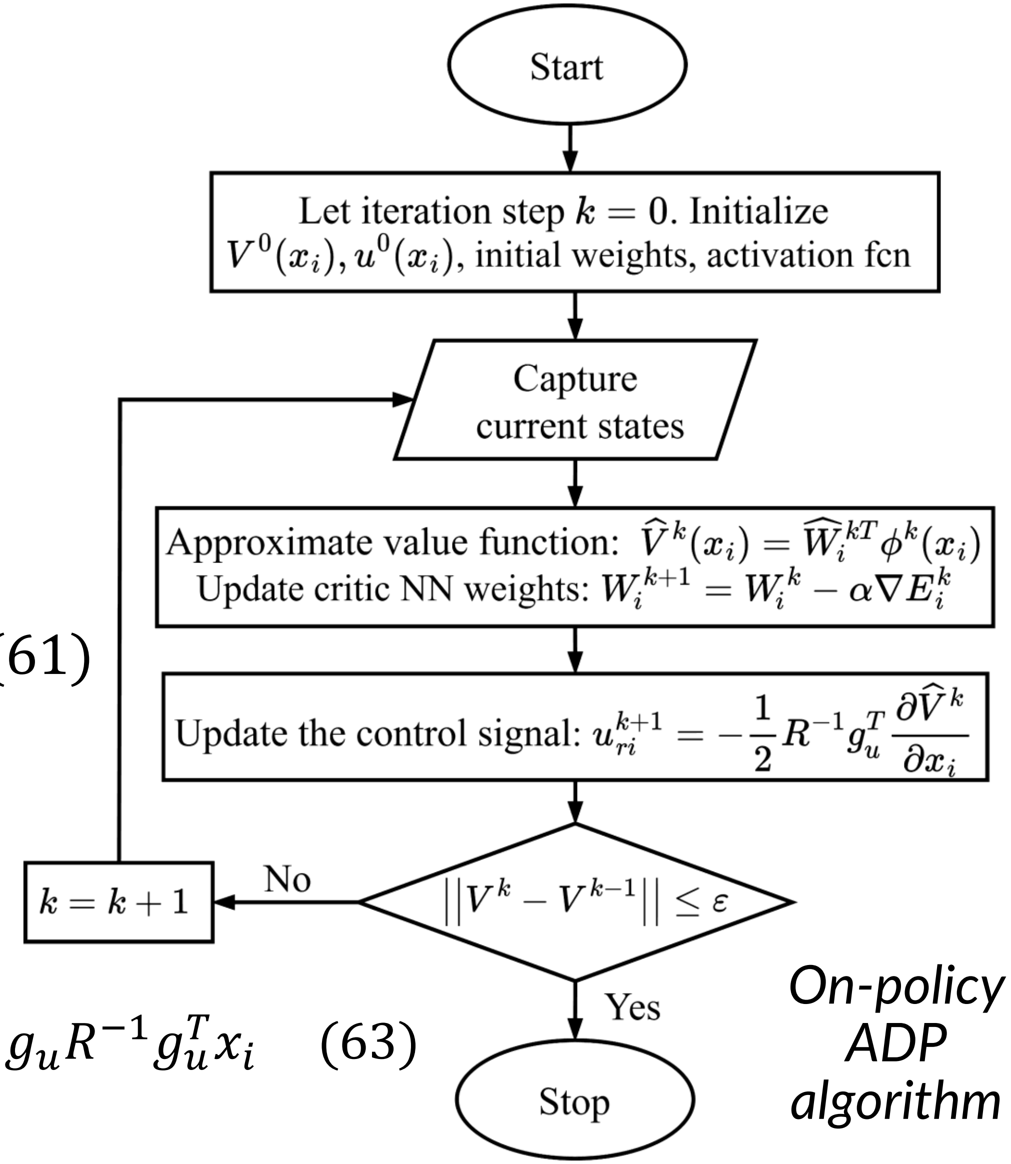
Update $\widehat{W}_i$ to minimize the squared residual error:

$$E_i = \frac{1}{2} e_i^T e_i \quad (62)$$

The weight update law is given by:

$$\dot{\widehat{W}}_i = -\alpha_1 \frac{\hat{\sigma}_i}{\left(\hat{\sigma}_i^T \hat{\sigma}_i + 1\right)^2} \left(\hat{\sigma}_i^T \widehat{W}_i + Q(x_i) + u_{ri}^T R u_{ri}\right) + \frac{1}{2} \alpha_2 \frac{\partial \phi}{\partial x_i} g_u R^{-1} g_u^T x_i \quad (63)$$

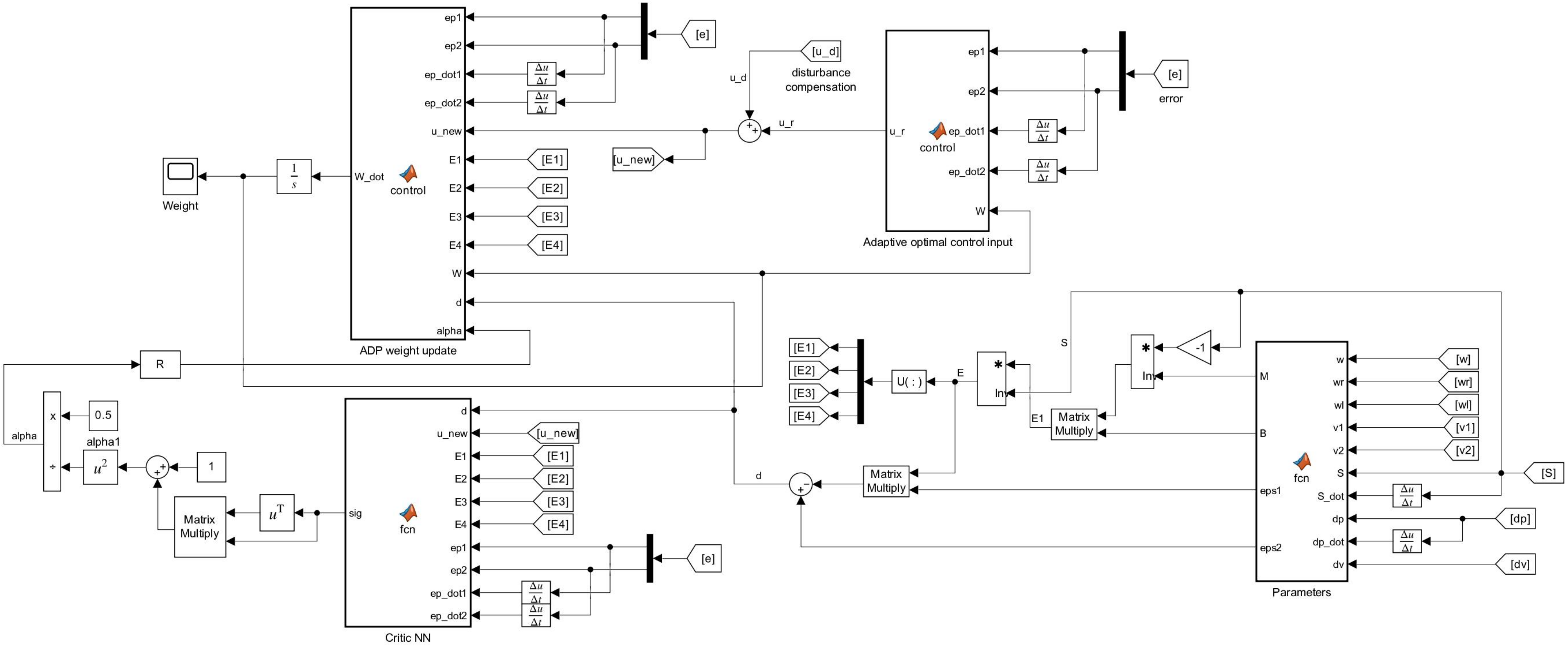
Where $\hat{\sigma}_i = (\partial \phi / \partial x_i) \dot{x}_i$



On-policy ADP algorithm

4. Multi-agent control using ADP approach

The adaptive optimal control simulation implementation:

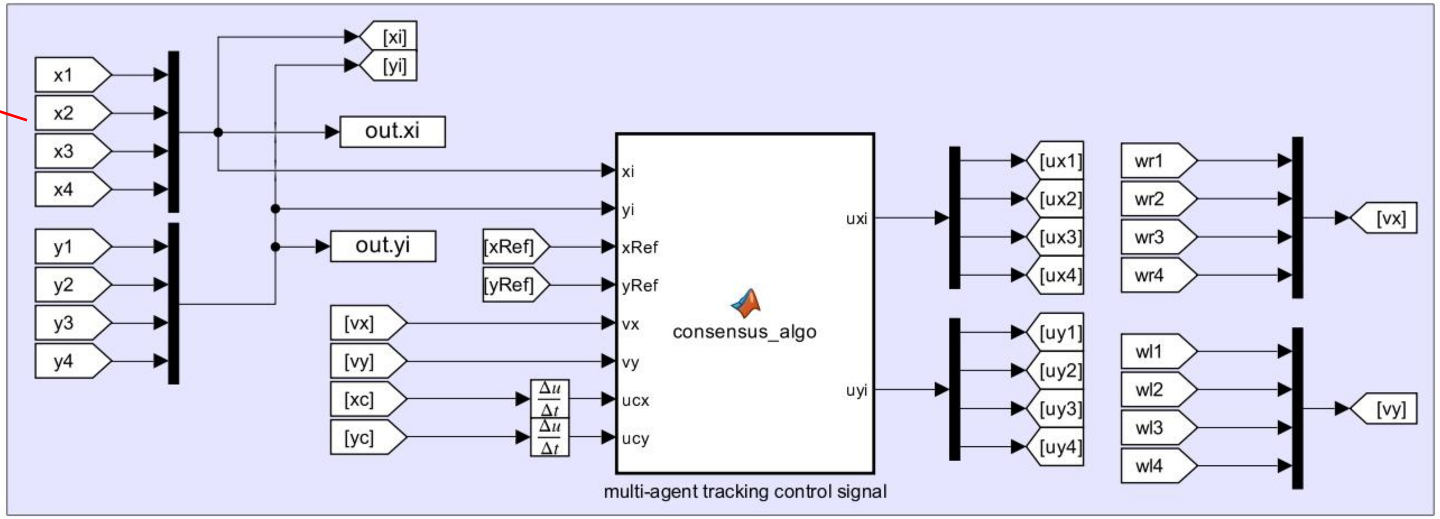


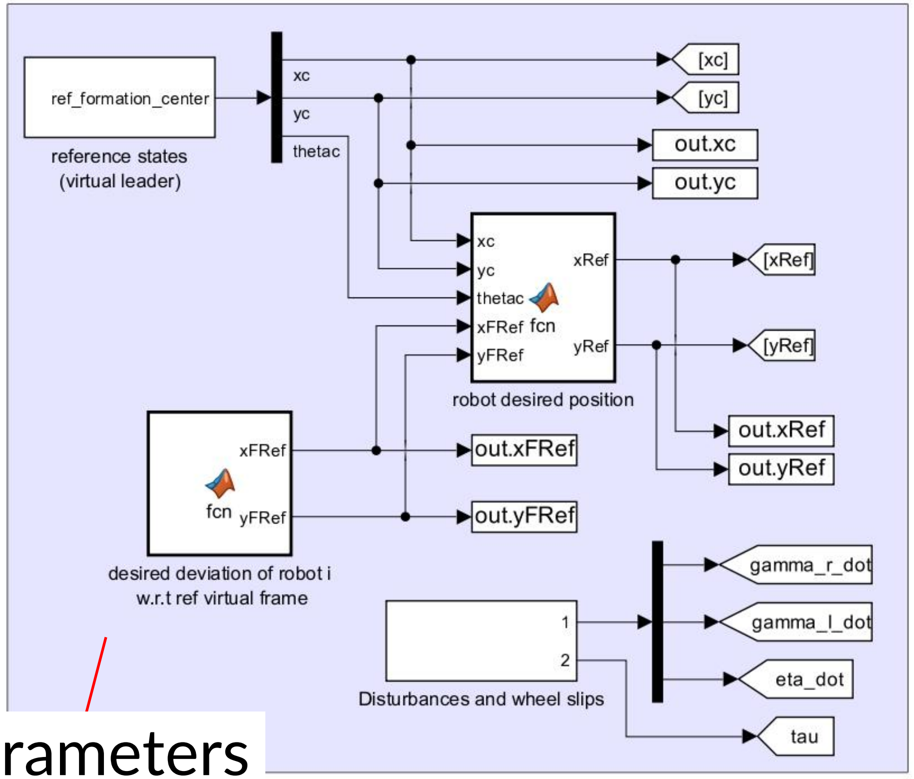
Simulation and Results

5. Simulation implementation

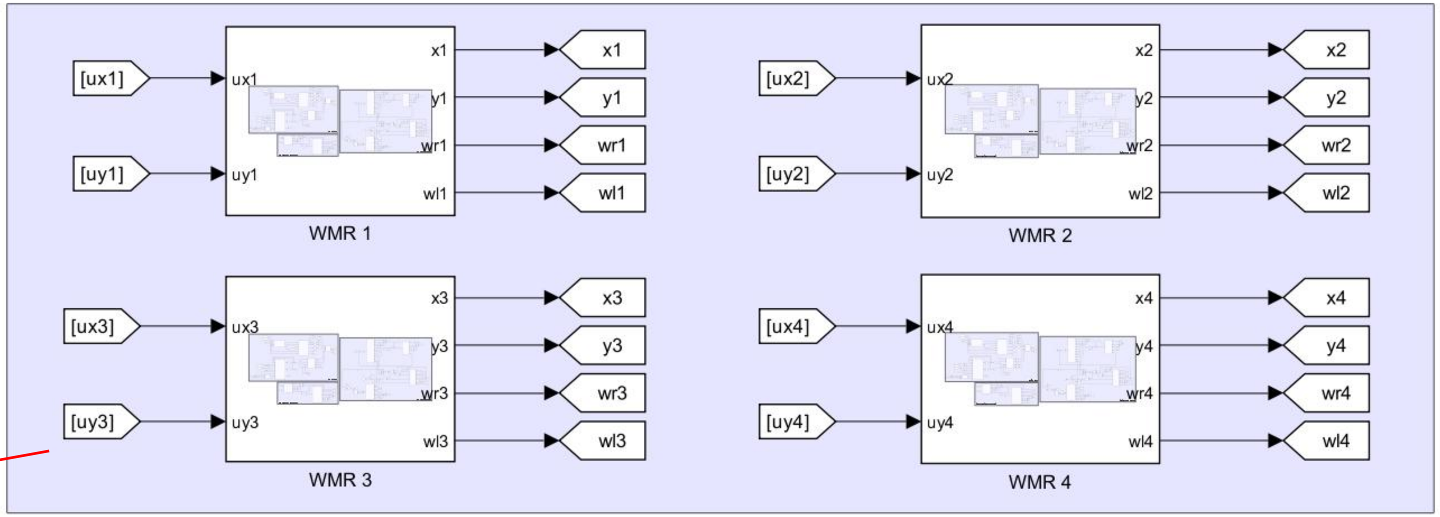
Overall system simulation :

Multi-agent control





Parameters generation



WMR system

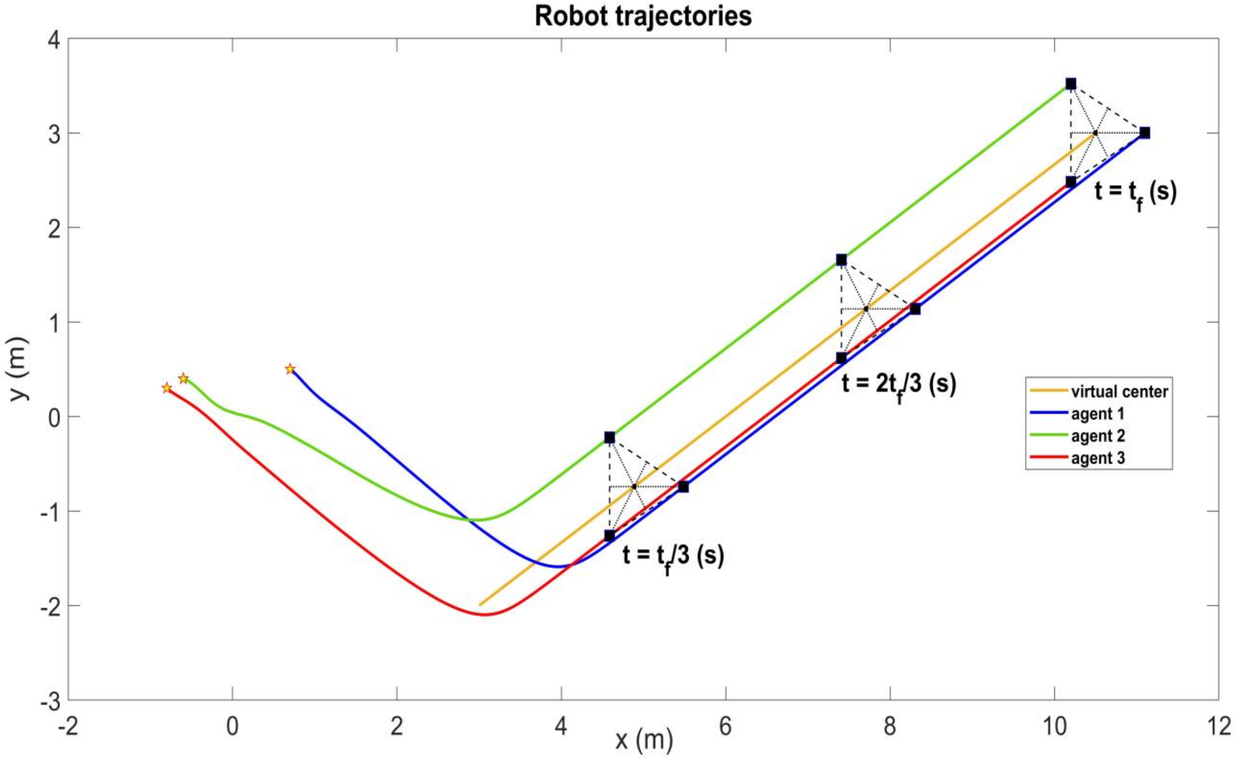
5. Simulation results

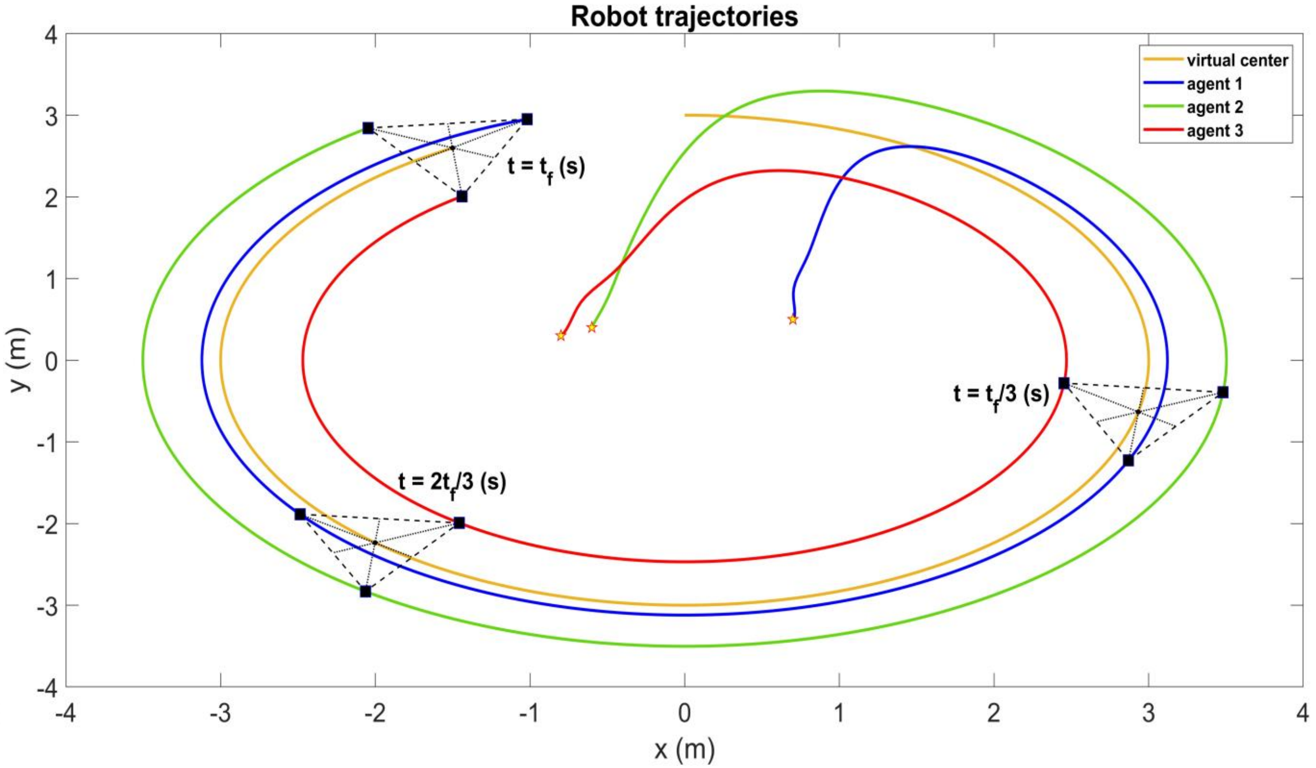
A. Formation tracking results of multiple WMRs

- Scenario 1: Triangle formation

Line trajectory: $\begin{cases} x_c = 3 + 0.75t \\ y_c = -2 + 0.5t \end{cases}$

Circle trajectory: $\begin{cases} x_c = 3\sin\left(\frac{\pi}{6}t\right) \\ y_c = 3\cos\left(\frac{\pi}{6}t\right) \end{cases}$

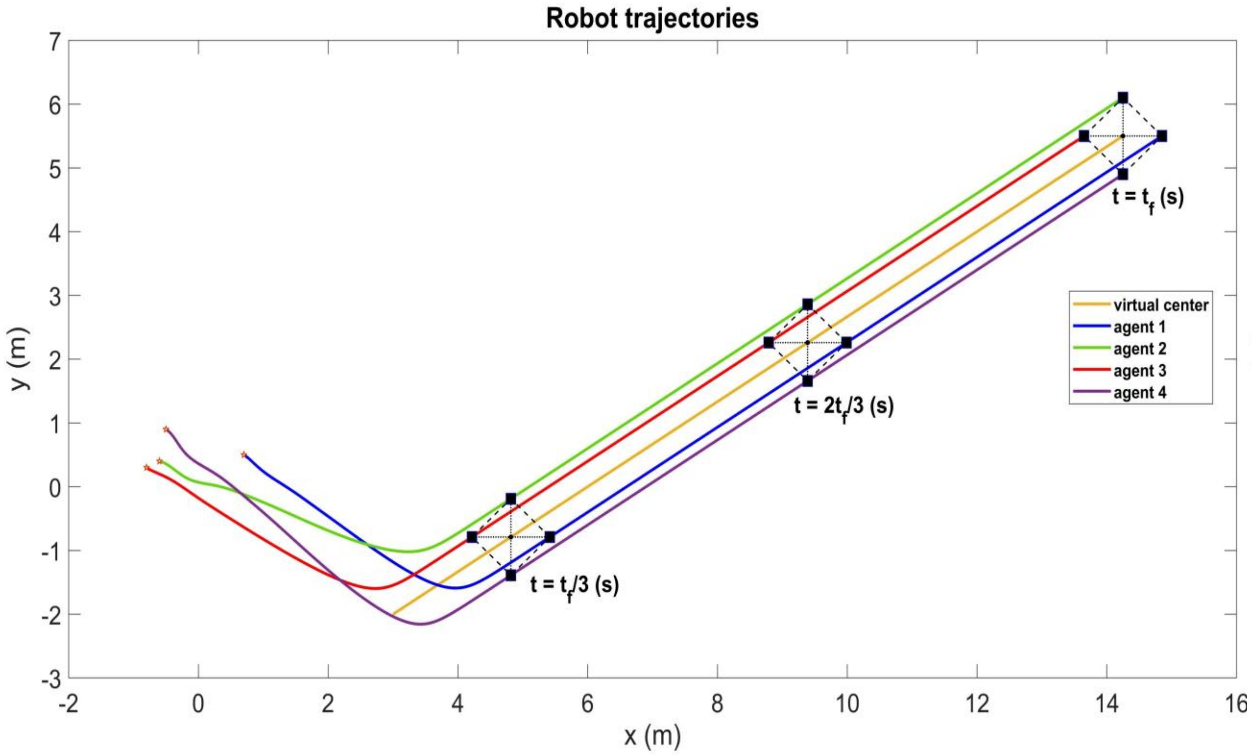




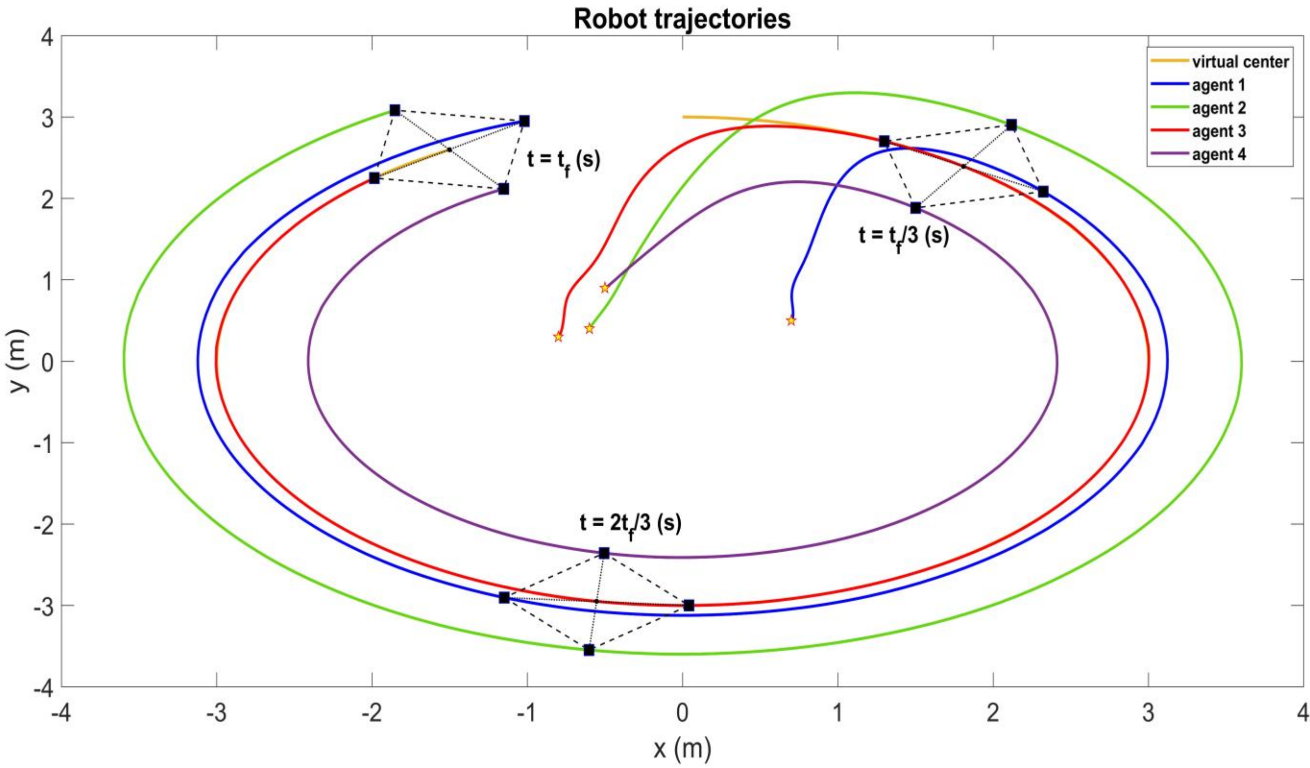
A. Formation tracking results of multiple WMRs

- Scenario 2: Square formation

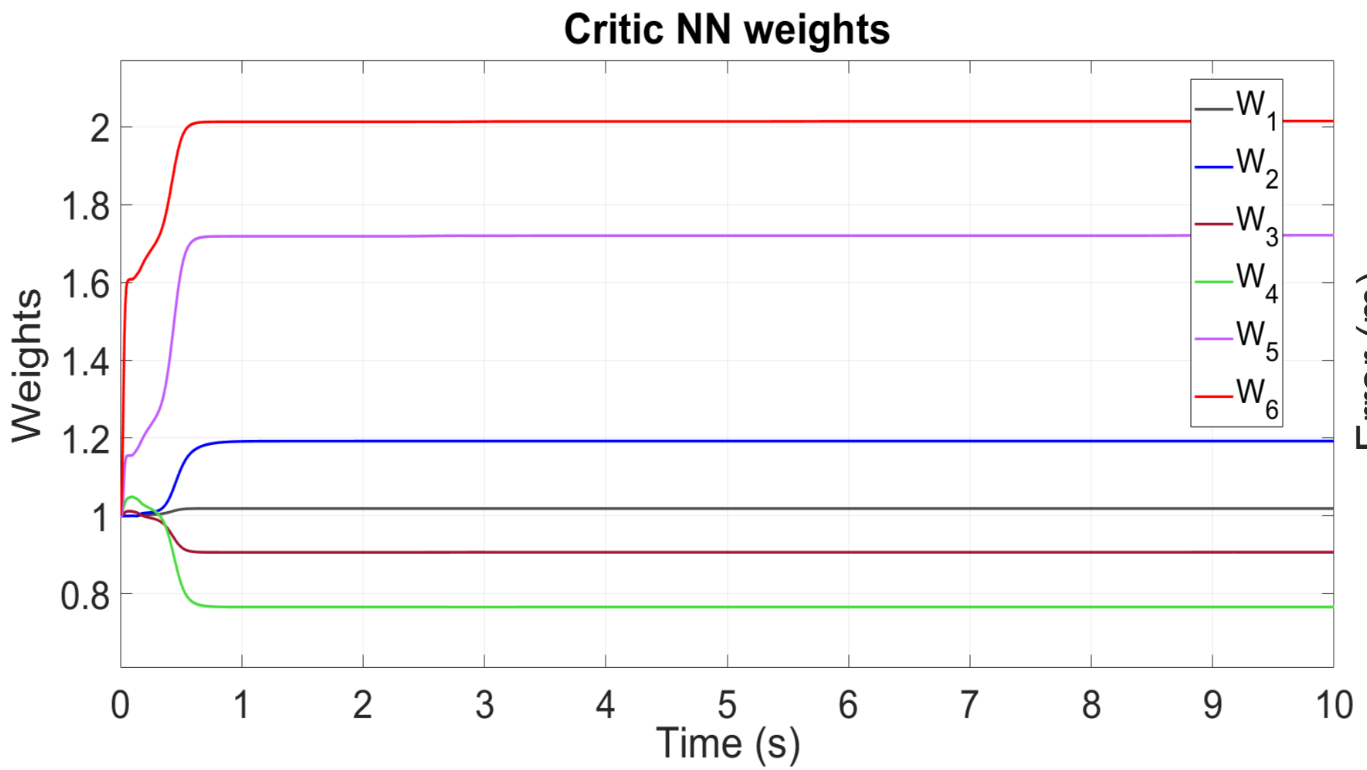
Line trajectory

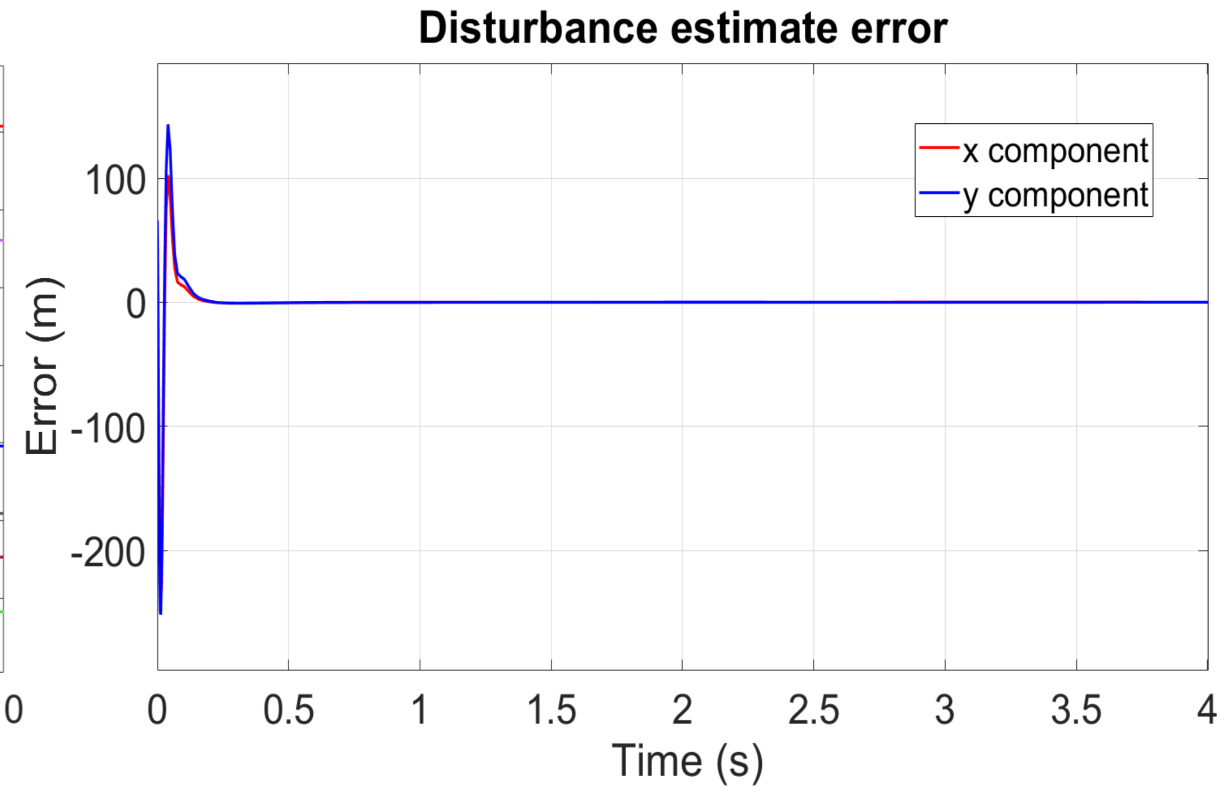


Circle trajectory



A. Formation tracking results of multiple WMRs



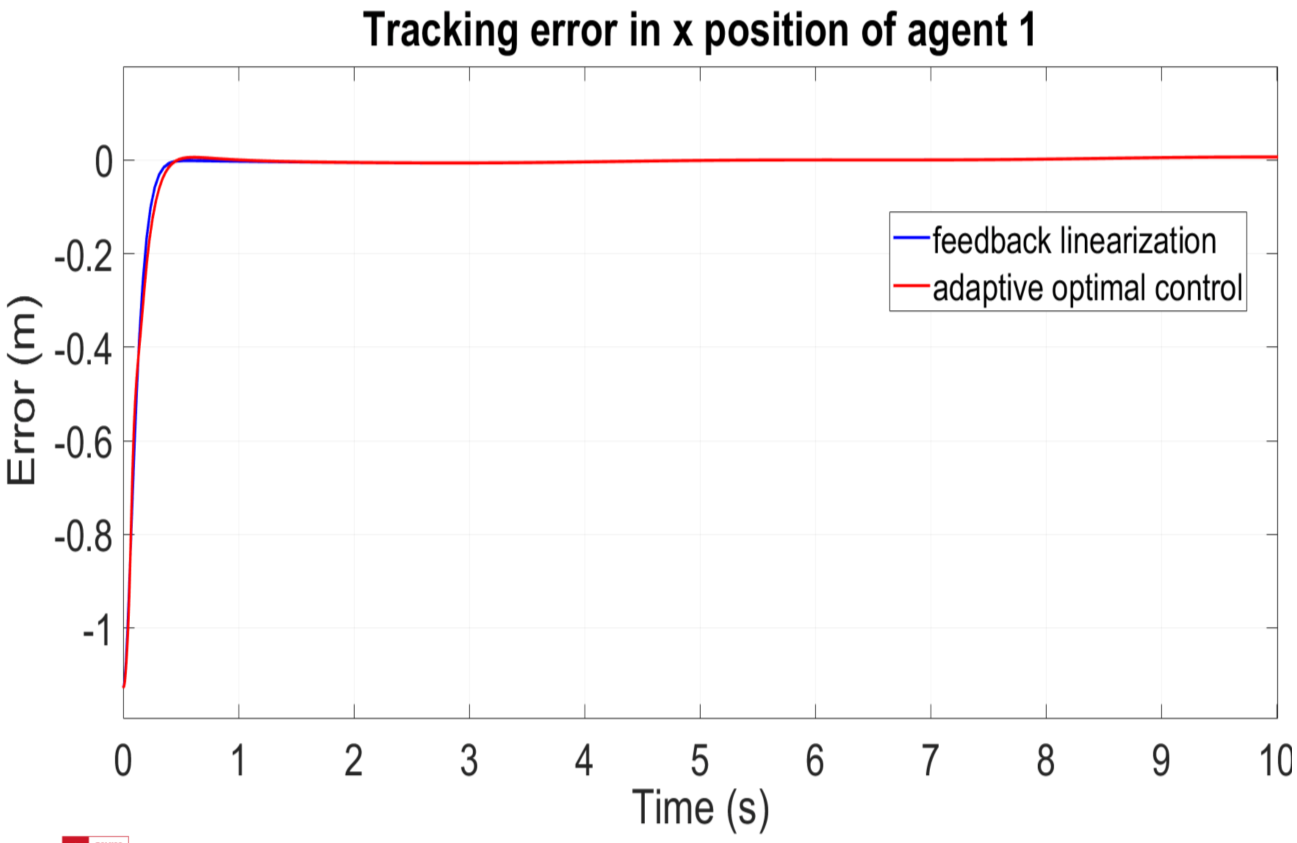


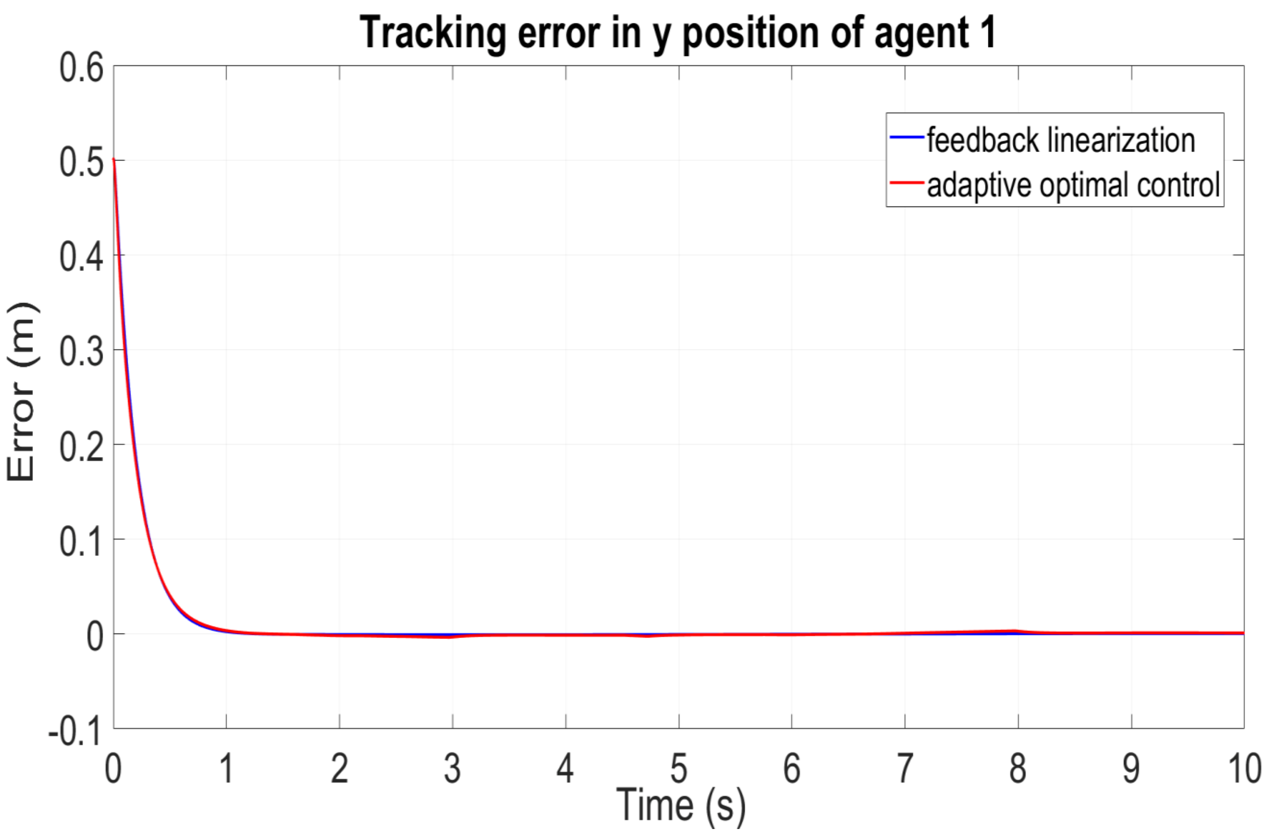
Results of ADP-based optimal control for agent 1 tracking triangle formation while following line trajectory

B. Comparison between ADP-based optimal control method and conventional feedback linearization control method:

In case of no disturbance:

Curved trajectory: $\begin{cases} x_c = -t \\ y_c = \sin(0.5t) + 0.5t + 1 \end{cases}$

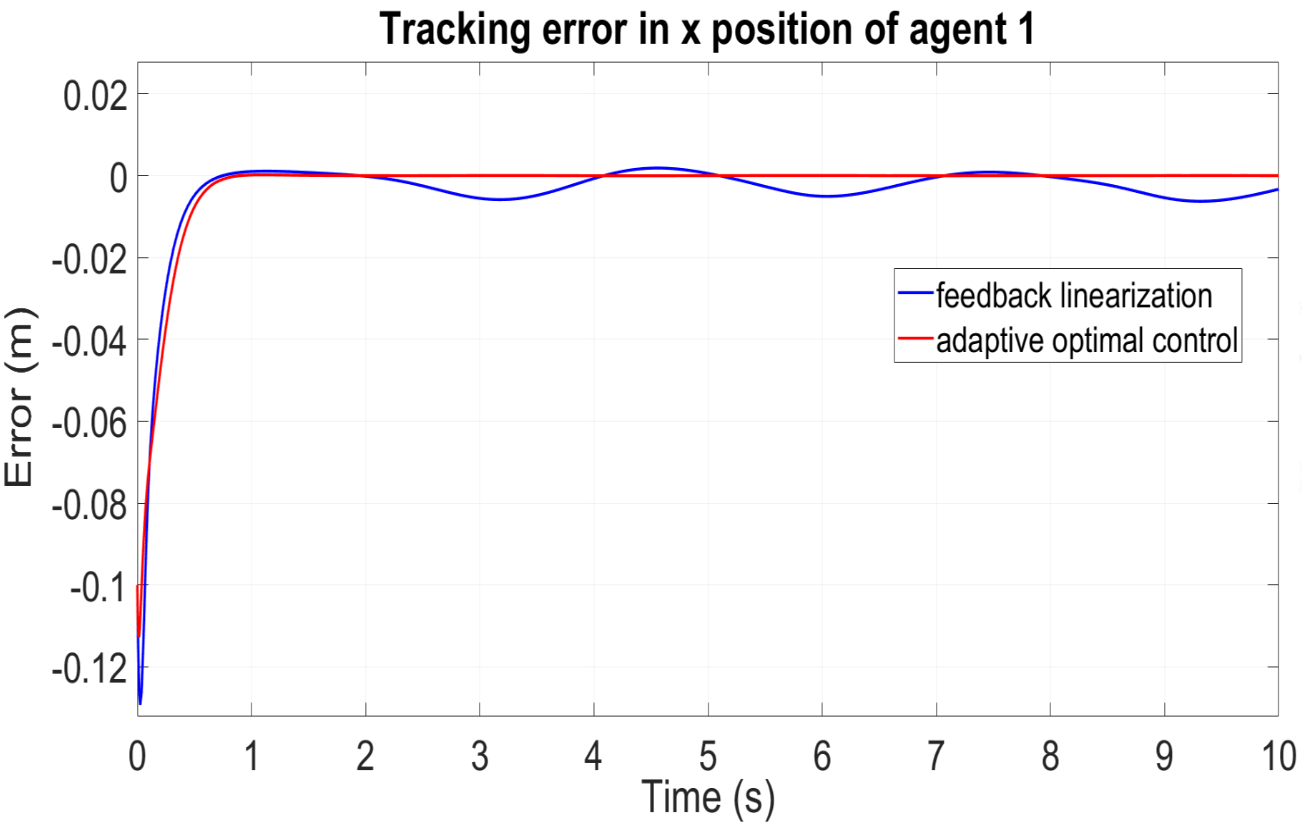


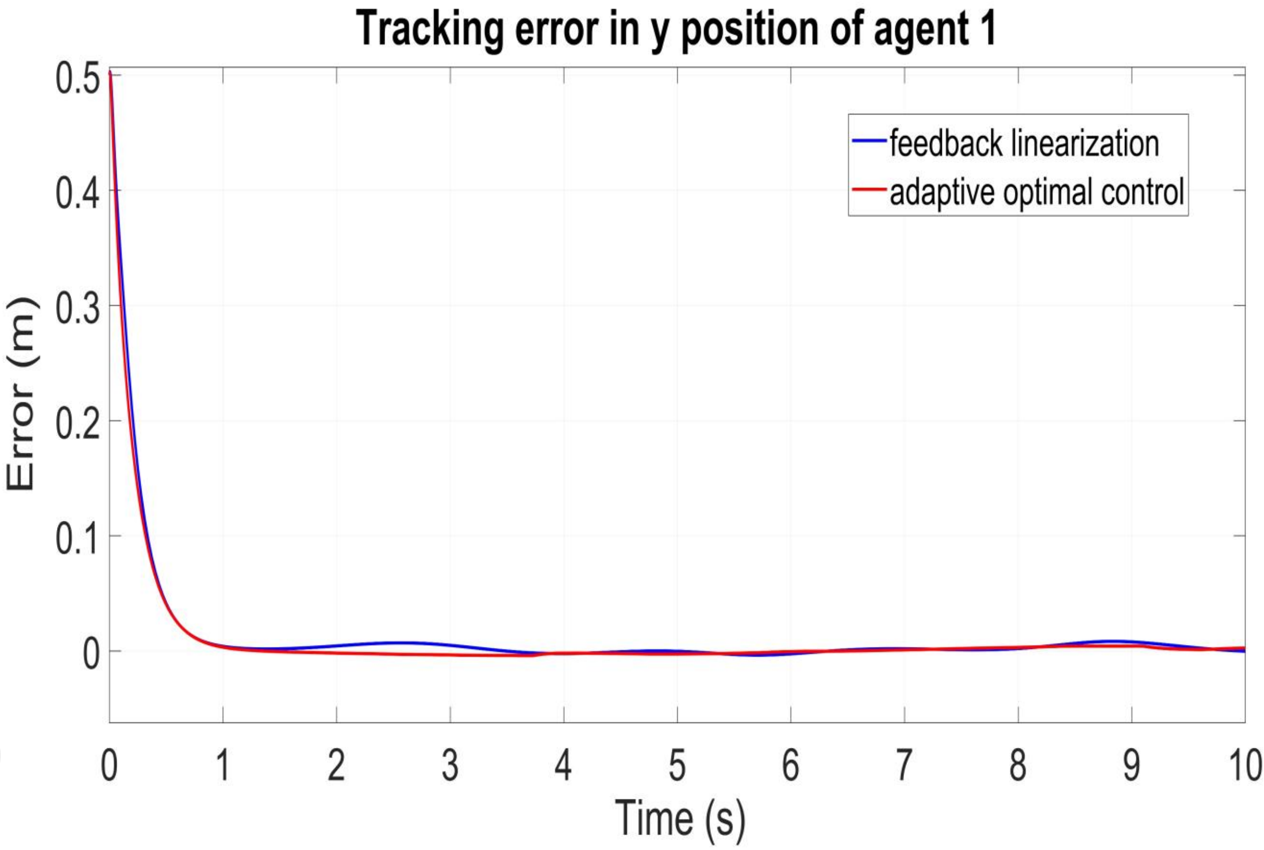


5. Simulation results

B. Comparison between ADP-based optimal control method and conventional feedback linearization control method:

In case of wheel slip and external disturbance:





6. Conclusion

- In general, the WMR system achieves the desired formations while following the predefined trajectories.
- The approach assumes each agent has consistent understanding of the formation center. Future study can focus on designing consensus algorithm to drive each agent’s instantiation of the virtual center to a common value.
- The proposed method requires knowledge of the system’s internal dynamics. Future improvement may focus on developing model-free control algorithms that rely solely on system state data.
- Possibly develop more effective multi-agent control algorithms. These advancements may include distance-based control, direction vector-based control, and graph rigidity theory-based methods.

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