Optimising Portfolio Allocation

Integrating Value Investing Principles with Modern Portfolio Theory

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Abstract

This study explores the integration of value investing principles with Modern Portfolio Theory (MPT) to optimise portfolio allocation. The research aims to construct a portfolio that achieves superior risk-adjusted returns by leveraging advanced valuation models, quantitative financial analysis, and risk management techniques. Key methodologies include discounted cash flow analysis, efficient allocation using machine learning in Python, and back-testing against traditional benchmarks. Results demonstrate that the optimised portfolio significantly outperformed the S&P 500 over a 1-year period, highlighting the potential of combining fundamental analysis with robust diversification strategies. Despite limitations like a limited sample size and elevated volatility, the findings underscore the viability of this approach in addressing dynamic market conditions.

1. Introduction

Portfolio allocation is a cornerstone of modern investment strategies, aiming to balance risk and return through diversification—a challenge made increasingly complex by heightened market volatility, rapid technological advancements, and shifting economic landscapes. The advent of Modern Portfolio Theory (MPT) by Markowitz introduced mathematical rigour to portfolio construction, emphasising efficient allocation across assets to optimise returns relative to risk. However, integrating fundamental investment principles, such as value investing, with MPT presents an opportunity to refine the process further. Value investing, popularised by Graham and later advanced by practitioners like Zweig, focuses on identifying undervalued securities with robust growth potential. This thesis seeks to explore the integration of value investing principles with MPT to develop an optimised portfolio that outperforms traditional benchmarks.

The primary objective of this research is to construct a portfolio that balances the principles of value investing with diversification and risk management techniques rooted in MPT. By incorporating quantitative methodologies, such as discounted cash flow models for precise valuation and machine learning in Python, this approach leverages innovative tools to refine company selection. These methodologies not only enhance the identification of high-growth, undervalued companies but also strengthen the portfolio's ability to withstand market fluctuations. This study also evaluates the practical implications of these methodologies through back-testing.

2. Literature

Modern Portfolio Theory (MPT) has been a foundational framework for portfolio construction, focusing on diversification to minimise risk without sacrificing returns (Stuart & Markowitz, 1959). However, MPT has faced critiques for its reliance on assumptions such as normal distribution of returns and static correlations, which may not hold in dynamic, real-world markets. Recent enhancements to MPT, including the integration of behavioural finance insights and nonlinear risk models, have sought to address these limitations, offering more robust tools for portfolio optimisation. By considering asset correlations and volatility, MPT introduced the concept of the efficient frontier, which identifies the set of optimal portfolios for a given risk level. The Sharpe Ratio further enhanced this framework by quantifying risk-adjusted returns (Sharpe, 1966).

Despite its widespread adoption, MPT faces limitations in addressing real-world market inefficiencies, such as its reliance on static correlations and assumptions of normal return distributions, which may oversimplify the complexities of financial markets. For example, the 2008 financial crisis highlighted how extreme events and systemic risks can deviate significantly from MPT's foundational assumptions. Value investing, as championed by Graham and Zweig (2024), offers a complementary perspective. This approach emphasises intrinsic value derived from fundamental analysis, seeking securities that trade below their fair value. Metrics such as the price-to-earnings (P/E) ratio and earnings per share (EPS) growth, and discounted cash flow analysis are central to this methodology.

Recent studies, such as Damodaran (2012, 2024), have advanced valuation techniques, integrating macroeconomic variables and sector-specific growth rates. The use of risk metrics like Value at Risk (VaR) and Conditional VaR has also gained prominence, offering more nuanced assessments of downside risk (Jorion, 2011). Bodie et al. (2024) highlighted the significance of skewness and kurtosis adjustments in risk modelling, enhancing the accuracy of traditional Gaussian VaR calculations.

The marginal benefits of diversification have also been scrutinised. Statman (1987) and Eling (2009) observed diminishing returns as portfolio size increases beyond 20-30 assets. This phenomenon underscores the importance of balancing diversification with transaction costs and portfolio complexity. Luijendijk (2024) examined over-diversification in exchange-traded funds (ETFs), revealing potential inefficiencies in excessively large portfolios.

This literature underscores the potential for integrating value investing principles with MPT to address its limitations. By leveraging advanced valuation models, risk metrics, and diversification strategies, this thesis aims to construct a portfolio that achieves superior risk-adjusted returns.

3. Methodology

The first step in our process is to identify the type of businesses we want to analyse before conducting a deeper quantitative evaluation. To capture a broad representation of the market, we will include companies from all industries and sectors in our screener. However, to align with our benchmark, we will focus exclusively on US-listed companies.

The initial screening will apply two criteria: companies must qualify as both "undervalued" (Investopedia, 2020) and demonstrate potential for growth. While these criteria may appear contradictory, they serve as essential filters for our screener:

- P/E ratio < 38.55,
- 1-Year EPS Growth ≥ 15%.

The threshold for the P/E ratio is based on the S&P 500's Shiller P/E ratio, which currently stands at 38.55 (Multpl, 2024a). We use the Shiller P/E ratio because it accounts for earnings over the previous ten years and adjusts for inflation, making it less susceptible to short-term fluctuations (Multpl, 2024b). To calculate this ratio, we take the yearly earnings of the S&P 500 for each of the last ten years, adjust them for inflation using the Consumer Price Index (CPI), and calculate the average (e10). Dividing the current price of the S&P 500 by e10 yields the Shiller P/E ratio (Multpl, 2024b).

For the growth criterion, the EPS growth threshold of 15% represents a high-growth benchmark. EPS was chosen over other metrics for two reasons. First, EPS is a critical metric in this paper's valuation methods, meaning companies with high EPS growth are likely to perform well in the subsequent financial analysis. Second, this approach allows us to capture aggressive growth without relying on high P/E/G ratios, which often include inflated prices based on future growth expectations. We then select the top 100 firms based on their 5-year revenue CAGR, as total revenue has a direct impact on EBIT and overall profitability.

Once the top 100 stocks are selected, each will undergo a detailed quantitative financial analysis to assess their suitability as sound investment opportunities. We will be calculating this intrinsic value using a revised version of Graham's Formula (Graham & Zweig, 2024), where P/E for zero-growth firms is 8.5x, growth multiplier is 1x, and average and current yield AAA corp. bonds are 4.07% and 4.60%, respectively (YCharts, 2024):

$$\frac{\text{EPS} \times \left(8.5 + (\text{Projected Growth Rate} \times 1g)\right) \times \text{Avg. yield of AAA corp. bonds}}{\text{Current yield of AAA corp. bonds}}$$

The current stock price falling below the intrinsic value is not sufficient on its own. To ensure robust risk management, we must also define a margin of safety to determine an acceptable buy price. This margin of safety will be calculated using the Gaussian VaR at a 95% confidence level, based on the stock's monthly price changes over the past five years. If the margin of safety equals the Gaussian VaR, the acceptable buy price is calculated as follows:

$$\text{Acceptable Buy Price} = (1 - \text{Gaussian VaR}) \times \text{Intrinsic Value}$$

The Gaussian VaR itself is derived using the Cornish-Fisher expansion, which adjusts for skewness and kurtosis in the normal distribution curve (Fisher, 1950).

To apply Graham's revised formula, we need to determine a projected growth rate. Given that we are analysing 100 different firms, we will use a method that balances efficiency with accuracy, ensuring reliable comparisons. The projected growth rate will be derived from a combination of the firm's historical and projected performance (SeekingAlpha, 2024), as well as the historical and projected 5-year CAGRs of the broader sector (Damodaran, 2024):

$$\text{Projected Growth Rate} = \frac{\text{Firm 5YR Rev. CAGR} + \text{Firm 5YR Exp. Rev. CAGR} + \text{Sector 5YR Rev. CAGR} + \text{Sector 5YR Exp. Rev. CAGR}}{4}$$

Using 2-year moving averages starting with FY 2023, projected operating and net margins provide a basis for calculating nominal EBIT and net profit. This modelling approach emphasises terminal value rather than exact year-by-year fluctuations for comparative purposes. CapEx is calculated similarly, but looking directly at nominal CapEx change instead.

We now have all the required values to discount future cash flows using a WACC calculator for the DCF model. I will be using this formula to calculate expected FCF (Damodaran, 2012):

$$\text{FCF} = \text{EBIT} \cdot (1 - \text{TR}) + D - \Delta\text{NetWC} - \text{CapEx}$$

Where:

- EBIT: Earnings Before Interest and Taxes,
- TR: Tax Rate,
- *D*: Depreciation,
- ΔNetWC: Change in Net Working Capital,
- CapEx: Capital Expenditures.

The formula for WACC is (Damodaran, 2011):

$$\text{WACC} = \left(\frac{D}{V}\right) \cdot \left(r_d \cdot (1 - T)\right) + \left(\frac{P}{V}\right) \cdot r_{ps} + \left(\frac{S}{V}\right) \cdot r_e$$

Where:

- *D*, *P*, *S*: Market values of debt, preferred equity, and common equity, respectively,
- *V*: Total capital ($V = D + P + S$),
- r_d, r_{ps}, r_e: Required rates of return for debt, preferred stock, and equity,
- *T*: Corporate tax rate

Using a perpetuity growth model, I estimated the terminal value with this formula (Damodaran, 2012):

$$TV = \frac{\text{FCF}_{\text{Final Year}} \cdot (1 + \text{Terminal Growth Rate})}{\text{WACC} - \text{Terminal Growth Rate}}$$

To calculate the present value of future cash flows, I discounted both the forecasted FCFs and the terminal value using the WACC (Damodaran, 2012):

$$PV = \sum \left(\frac{\text{FCF}_t}{(1 + \text{WACC})^t} + \frac{TV}{(1 + \text{WACC})^t}\right)$$

Where:

- *t*: Final forecast year
- *TV*: Terminal Value

With the EV now calculated, I will determine the stock's price target for a given year by calculating the annualised growth multiple (GR), which will be derived by averaging several growth factors based on their respective 5-year CAGRs:

- Total revenues,
- Operating income,
- Net income,
- Free cash flow,
- Intrinsic value,
- Enterprise value.

The resulting GR factor will be multiplied by the current stock price, raised to the power of the forecasted period, *t,* in the DCF model. The final value will determine the target exit price:

$$\text{Target Exit Price} = (GR \cdot \text{Curr. Stock Price})^t$$

For a firm to pass this financial analysis:

- The projected intrinsic value must be above the current price, adjusted for the margin of safety such that the current price is under the acceptable buy price,
- The projected enterprise value is above the current enterprise value,
- The GR factor is positive.

Once the eligible firms have been selected, a comprehensive risk analysis will be performed to identify the optimal allocation for the index comprising these companies. This analysis will leverage the monthly percentage changes in stock prices over the past five years. Upside potential will be measured through the annualised returns for the period, while downside risk will be assessed using the Gaussian Value at Risk (VaR). To ensure greater accuracy, the Cornish-Fisher expansion will be applied to account for skewness and kurtosis in the return distributions.

To assess the downside risk of an asset, we calculate drawdowns, which represent the decline from the asset's peak value over time. This process is performed using three key metrics (Bodie et al., 2024):

- Wealth Index (WI_t) tracks the growth of an investment over time, assuming reinvestment of returns:

$$WI_t = WI_{t-1} \cdot (1 + R_t)$$

Where R_t is the return at time t.

- Previous Peaks (PP_t) identifies the maximum value of the Wealth Index up to a given time:

$$PP_t = \max(WI_t \;\; t \leq T)$$

- Drawdown (DD_t) measures the percentage decline from the peak value:

$$DD_t = \frac{WI_t - PP_t}{PP_t}$$

Semi-deviation focuses on negative returns to assess downside volatility. It is computed as the standard deviation of all negative returns (Sortino & van der Meer, 1991):

$$\text{Semi-Deviation} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(R_t - \bar{R})^2 \quad \text{where } R_t < 0}$$

Skewness measures the asymmetry of return distributions, and kurtosis indicates the ‘peakedness’ of these distributions (Hull, 2022):

$$\text{Skewness} = \frac{\mathbb{E}[(R-\mu)^3]}{\sigma^3}$$

$$\text{Kurtosis} = \frac{\mathbb{E}[(R-\mu)^4]}{\sigma^4}$$

VaR quantifies the potential loss in portfolio value with a specified confidence level. Gaussian VaR assumes normal distribution of returns (Jorion, 2011):

$$\text{VaR}_{\text{gauss}} = -(\mu + Z \cdot \sigma)$$

Here, Z represents the standard normal quantile. Adjustments for skewness and kurtosis are made using the Cornish-Fisher expansion (Fisher, 1950).

Conditional VaR, also known as Expected Shortfall, calculates the average loss beyond the VaR threshold (Jorion, 2011):

$$\text{CVaR} = \mathbb{E}[R \quad R \leq \text{VaR}]$$

The efficient frontier represents the set of portfolios offering the highest return for a given level of risk. This is achieved by minimising portfolio volatility for a range of target returns (Stuart & Markowitz, 1959):

$$W* = \arg\min_{W} \sqrt{W^T \cdot \Sigma \cdot W} \quad \text{subject to} \quad W^T \cdot R = R_p$$

Where:

- R_p is the weighted sum of individual asset returns,
- W is the weight vector and R is the return vector,
- Σ is the covariance matrix of returns.

The Sharpe Ratio measures the excess return per unit of risk. The portfolio with the maximum Sharpe Ratio is considered the most efficient (Sharpe, 1966):

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \quad \text{subject to} \quad \sigma_p = \sqrt{W^T \cdot \Sigma \cdot W}$$

Where:

- σ_p is the standard deviation of portfolio returns, accounting for asset correlations,
- R_f is the risk-free rate.

The Capital Market Line (CML) illustrates the relationship between expected returns and risk for portfolios including a risk-free asset (Sharpe, 1964). It is defined as:

$$CML : R = R_f + \frac{\sigma}{\sigma_m}(R_m - R_f)$$

Where R_m and σ_m represent the return and volatility of the market portfolio.

4. Marginal Returns of Diversification

The marginal return of adding an asset to a portfolio can be analysed using the concept of the efficient frontier. As additional assets are included in the portfolio, the efficient frontier becomes smoother, and the incremental benefit of adding further assets diminishes (Stuart & Markowitz, 1959).

Mathematically, the marginal return of diversification can be expressed as the derivative of the portfolio return with respect to the addition of a new asset. If the new asset has a return R_i and the existing portfolio has a return R_p , the marginal return is:

$$MR = \frac{\partial R_p}{\partial w_i} = R_i$$

Where:

- w_i is the weight of the new asset in the portfolio,
- R_i is its expected return.

Empirical research supports the idea that the marginal returns from diversification decrease as the number of assets in the portfolio increases. Statman (1987) found that adding a small number of assets—say 10—significantly reduces portfolio risk. However, once the portfolio exceeds 20-30 assets, the reduction in risk becomes marginal, indicating that the portfolio has reached a sufficient level of diversification.

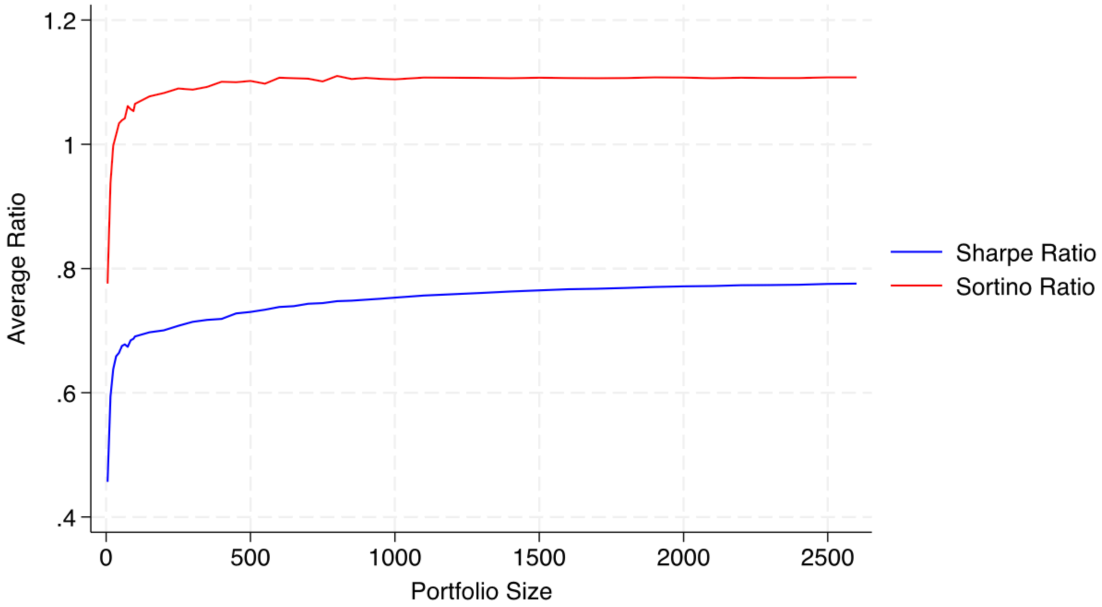

Fig. 1. Risk-adjusted Marginal Returns of Diversification (Luijendijk, 2024).

In a study by Eling in 2009, it was shown that as the number of stocks in a portfolio increases beyond 20, the benefits of adding more stocks to reduce portfolio risk diminished, especially in markets where asset correlations were high.

Therefore, while the goal is to achieve a well-diversified portfolio with reduced risk, there is a point beyond which further diversification offers diminishing returns. The key challenge is balancing the costs associated with managing a large number of assets—such as transaction costs and portfolio rebalancing—against the marginal benefits of adding additional assets. The aim for the portfolio is to have 10-20 stocks across all industries.

5. Implementing risk management tools using machine learning in Python

This section details the implementation of risk management tools using machine learning in Python, incorporating the methodologies outlined earlier in this thesis. These tools integrate advanced risk-adjusted metrics, such as the Sharpe Ratio and the Cornish-Fisher expansion, with MPT principles to enhance portfolio optimisation. For demonstration purposes, the latest available data is utilised to illustrate the implementation process, while back-testing employs only historical data available at the time of allocation or rebalancing. A complete list of the functions used in the analysis is provided in the Appendix (Pg. 23). These functions are referenced in the code under the alias *rkit* and are called upon as needed throughout the analysis.

The first step involves importing the index data (ind) and calculating the expected returns (er) and covariance matrix (cov). A five-year historical sample is annualised to establish these metrics:

```
<py>
ind = rkit.get_ind_returns()
er = rkit.annualise_rets(ind["2019-12":"2024-12"], 12)
cov = ind[“2019-12":"2024-12"].cov()
</py>
```

To address the limitations of standard MPT models, the Cornish-Fisher expansion is applied when calculating the Gaussian VaR. This approach adjusts for skewness and kurtosis in return distributions, offering a more accurate representation of tail risks. The results are visualised as follows:

```
<py>
rkit.gaussian_var(ind, modified = True).sort_values().plot.bar(figsize = (20, 12), title = "Value at Risk", color = “Purple")
</py>
```

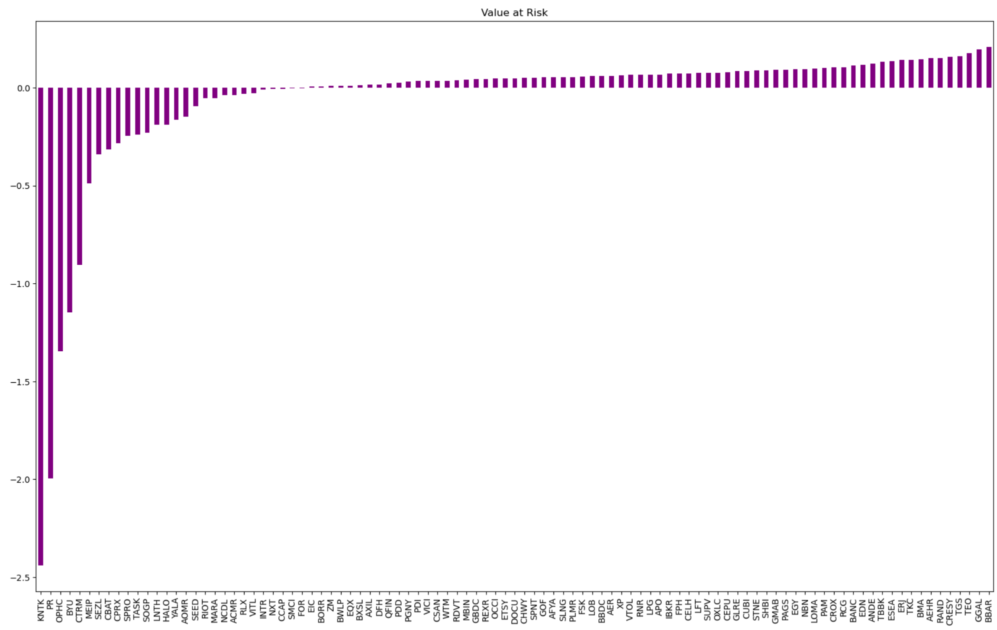

Fig. 2. Value at Risk Chart.

The analysis also visualises annualised returns, complementing the VaR assessment by providing insight into the portfolio's growth potential:

```
<py>
rkit.annualise_rets(ind, modified = True).sort_values().plot.bar(figsize = (20, 12), title = “Annualised Returns", color = “Green")
</py>
```

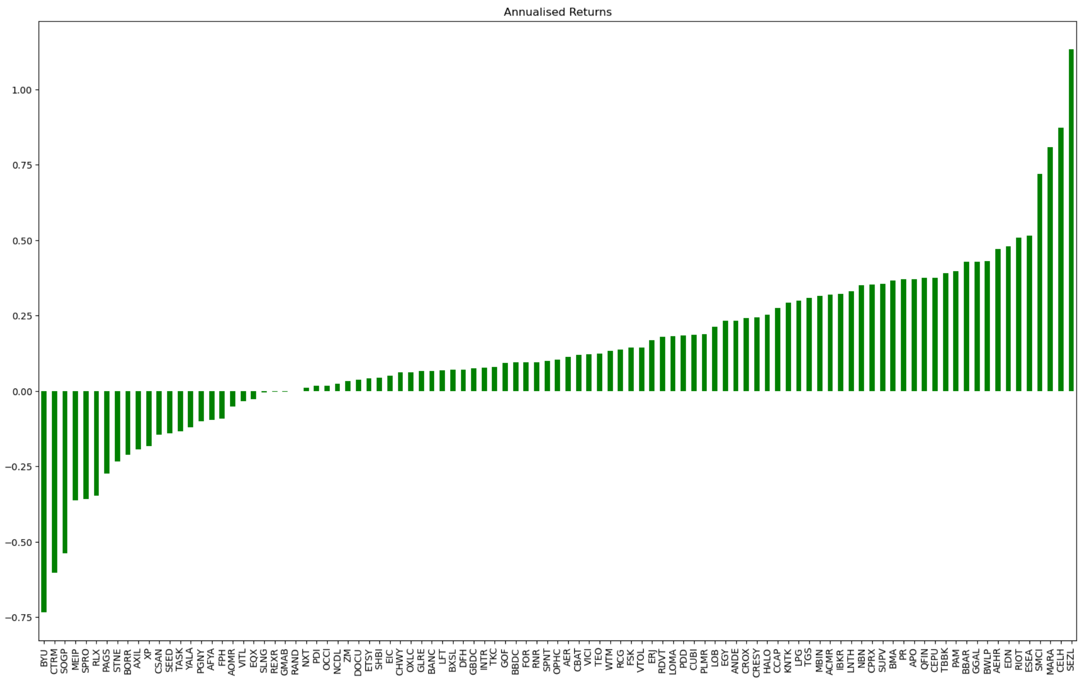

Fig. 3. Annualised Returns Chart.

To evaluate the portfolio's risk-adjusted performance, the Sharpe Ratio is calculated using a risk-free rate of 4.25%, corresponding to the long-term average yield of a 10-year U.S. Treasury bond:

```
<py>
rkit.sharpe_ratio(ind, 0.0425, 12).sort_values().plot.bar(figsize = (20, 12), title = "Sharpe Ratio”)
</py>
```

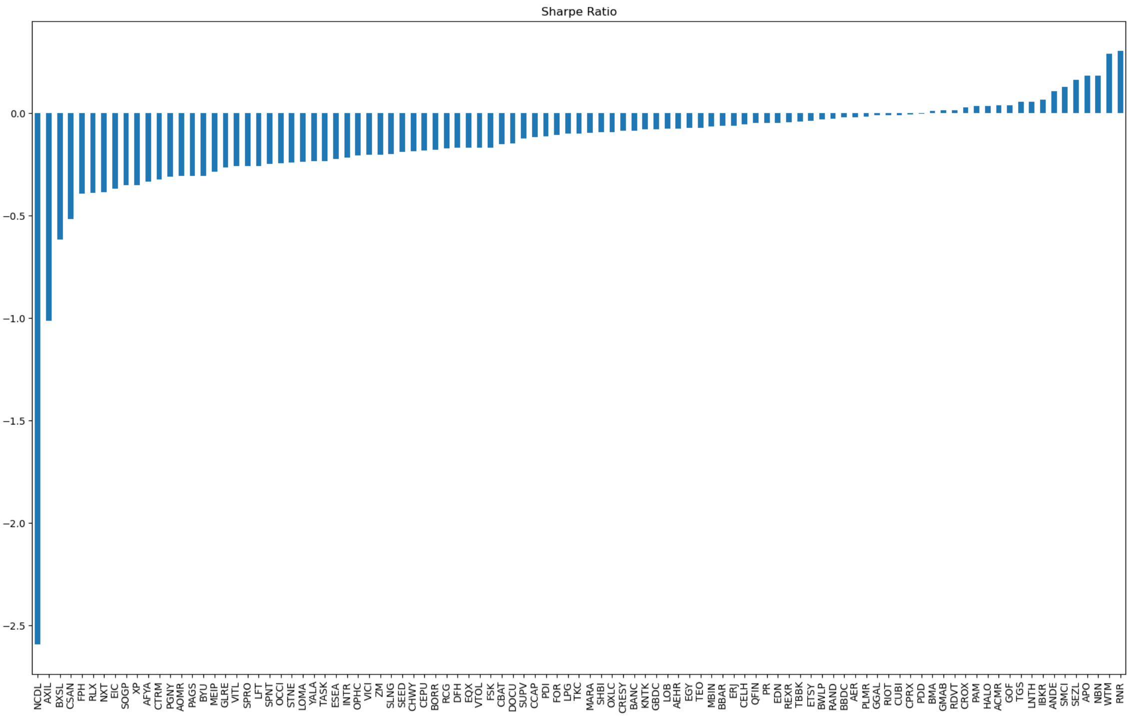

Fig. 4. Annualised Share Ratio Chart.

Building on these calculations, the efficient frontier curve is plotted, incorporating the Capital Market Line (CML). The CML is derived from the specified risk-free rate of 4.25%. To simulate portfolio allocations, 1,000 individual portfolios are generated using the covariance matrix and expected returns:

```
<py>
rkit.plot_efn(1000, er, cov, show_cml = True, riskfree_rate = 0.0425)
</py>
```

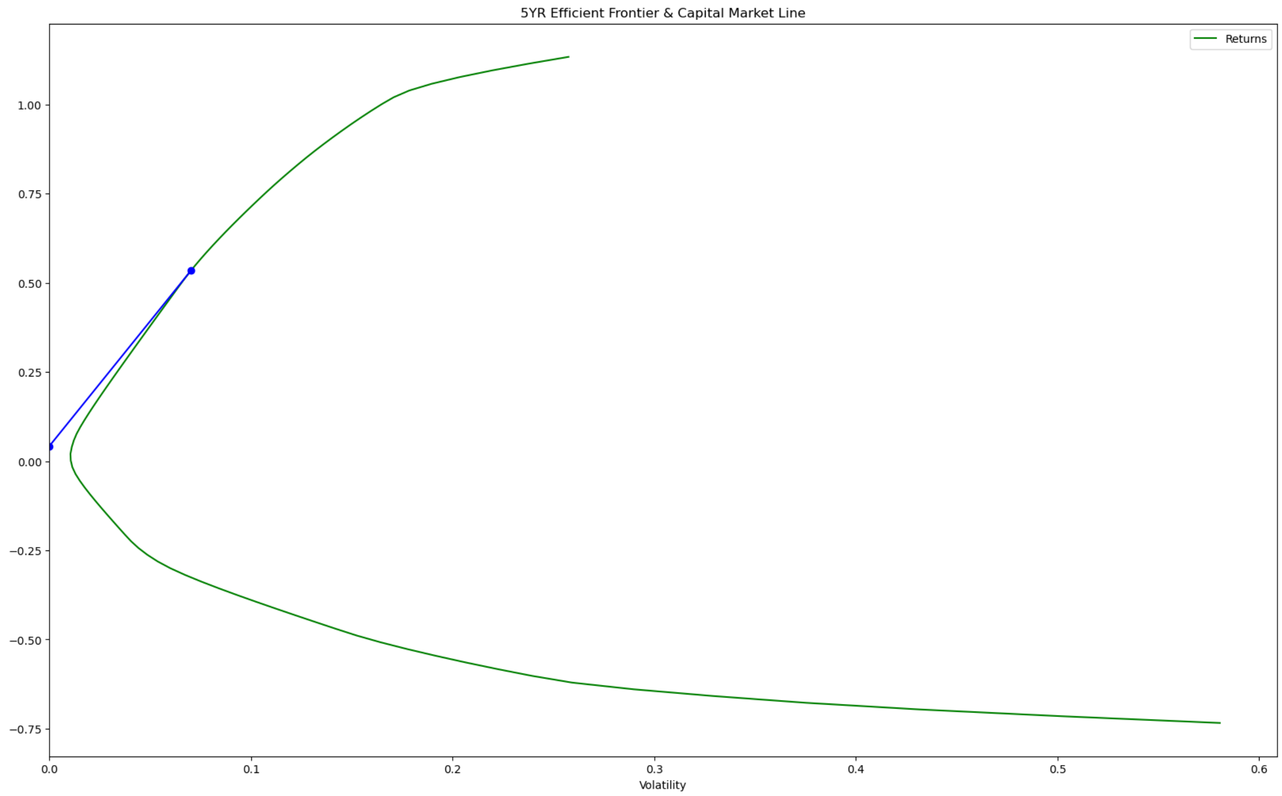

Fig. 5. 5-Year Efficient Frontier Allocation.

The portfolio with the highest Sharpe Ratio is identified as the most efficient. This portfolio aligns with the CML, optimising the trade-off between risk and return. The implementation demonstrates the thesis's methodology of integrating value investing principles with quantitative techniques to construct portfolios that achieve superior risk-adjusted returns. By combining theoretical rigour with practical application, this analysis underscores the potential of advanced risk management tools to enhance portfolio performance in complex financial markets.

Index

	Dec 23	Jan 24	Feb 24	Mar 24	Apr 24	May 24	Jun 24	Jul 24	Aug 24	Sep 24	Oct 24	Nov 24
AEHR	0.00%	0.00%	0.19%	0.00%	0.00%	0.00%	0.00%	1.09%	2.59%	1.59%	1.59%	1.28%
ANDE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.83%
BWLP	0.00%	0.00%	0.00%	0.00%	0.00%	2.18%	0.73%	0.24%	0.23%	0.00%	1.70%	2.86%
CCAP	22.70%	17.04%	18.46%	16.86%	24.89%	24.44%	29.22%	30.47%	27.09%	30.12%	31.16%	27.16%
CELH	18.28%	15.79%	15.76%	18.45%	15.84%	17.15%	9.78%	6.18%	4.80%	6.47%	8.97%	8.68%
CPRX	3.84%	6.29%	3.52%	1.71%	0.00%	0.00%	6.05%	8.47%	7.15%	1.57%	5.97%	6.49%
ESEA	0.00%	2.21%	1.47%	2.54%	1.44%	1.04%	3.32%	5.13%	8.60%	5.66%	3.76%	3.42%
LNTH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.32%	2.80%	6.63%
LPG	31.70%	29.27%	32.80%	34.21%	37.58%	34.72%	29.31%	24.74%	25.01%	16.92%	13.70%	2.33%
NBN	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.47%
PAM	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.32%	5.88%	10.77%
PLMR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	3.42%	3.14%	0.00%	0.00%	0.00%
QFIN	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.37%	1.78%	1.81%
RNR	3.39%	6.48%	4.00%	1.66%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
SEZL	7.22%	7.81%	8.29%	11.18%	6.24%	8.11%	6.81%	6.73%	11.78%	13.95%	17.03%	16.49%
SMCI	12.87%	15.10%	15.52%	13.38%	14.01%	12.36%	14.78%	13.52%	9.60%	8.71%	5.66%	5.78%

Tab. 1. Historical Efficient Portfolio Allocations over the last year.

6. Empirical Results

The back-test results, based on the period from December 1, 2023, to December 1, 2024, are summarised as follows:

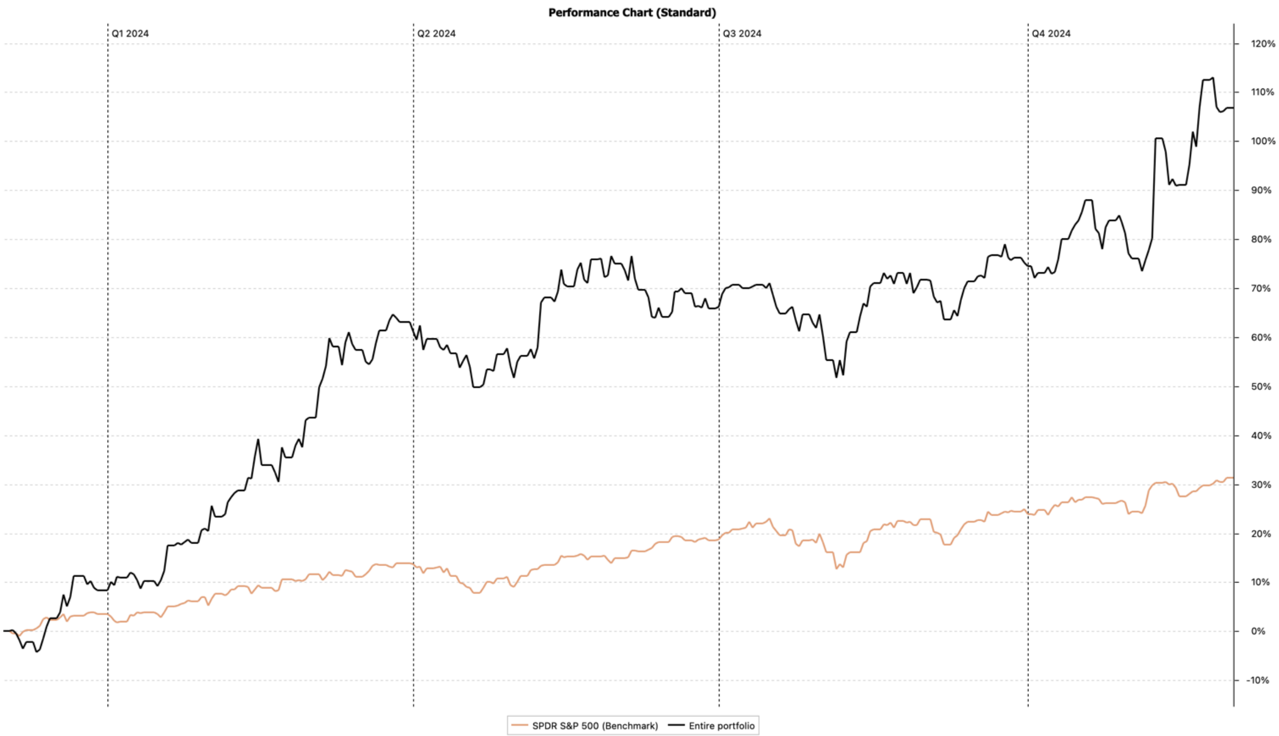

Fig. 6. 1-Year Cumulative Returns.

The optimised portfolio constructed using the methodology outlined in this thesis has significantly outperformed the S&P 500 over the 1-year period. The cumulative return for the optimised portfolio during this time was 106.78%, compared to the S&P 500's 31.25%.

Monthly Growth Rates

Date	Dec 23	Jan 24	Feb 24	Mar 24	Apr 24	May 24	Jun 24	Jul 24	Aug 24	Sep 24	Oct 24	Nov 24
S&P 500	3.53%	1.65%	5.49%	3.27%	(4.59%)	5.53%	3.67%	1.44%	2.80%	2.20%	(1.12%)	7.39%
Optimised Portfolio	8.38%	12.10%	22.63%	19.94%	(9.00%)	21.04%	(9.30%)	(1.08%)	6.94%	3.61%	1.75%	29.77%

Tab. 2. Monthly Growth Rates.

It is worth noting that the maximum drawdown for the optimised portfolio was higher than that of the benchmark, at 14.09% compared to the S&P 500's 8.41%. However, despite the slightly higher risk, the optimised portfolio delivered substantially better risk-adjusted returns, as evidenced by its Sharpe ratio of 3.73, compared to the S&P 500's Sharpe ratio of 2.55.

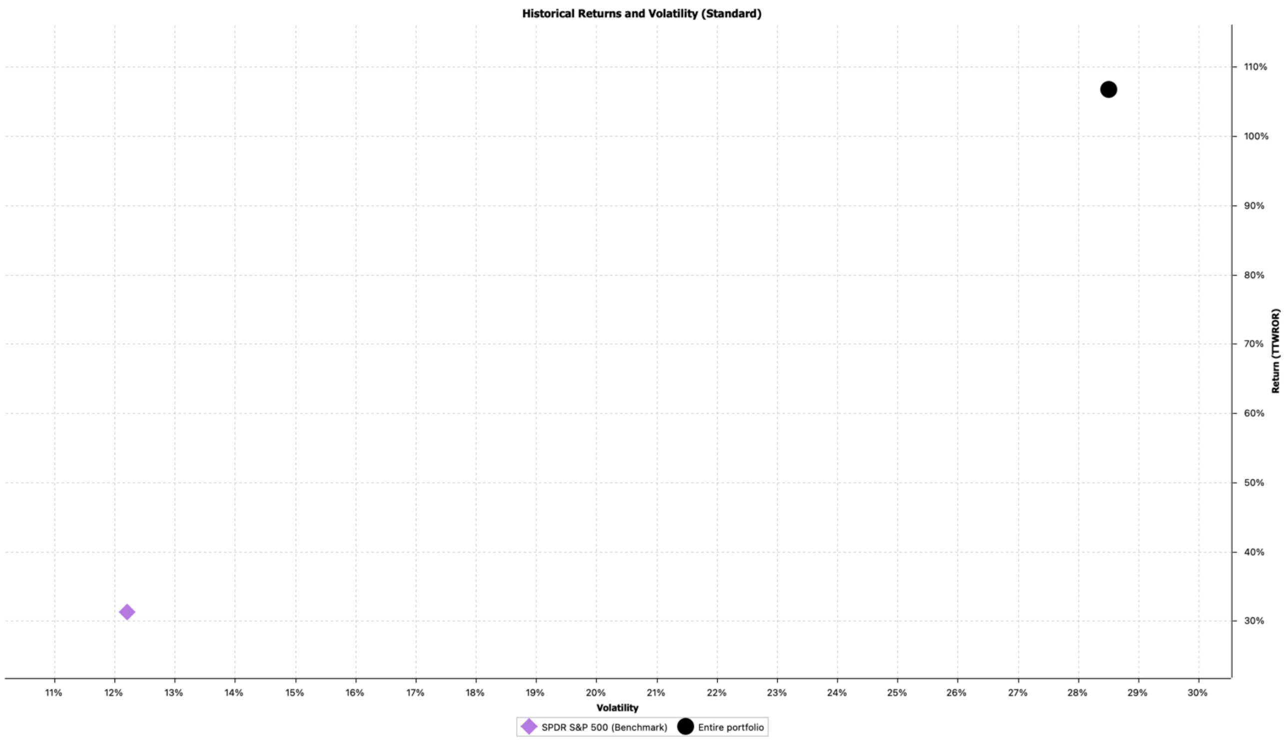

Fig. 7. Historical Return vs. Risk.

While the results are highly promising, it is important to recognise the limitations of the analysis. The findings are based on a single 1-year period, which represents a relatively small sample size. Furthermore, during a period of prolonged drawdown exceeding three months, the portfolio may become illiquid, or potentially incur losses during rebalancing.

That said, the increased volatility associated with the optimised portfolio is more than compensated for by its outstanding returns over the test period. This demonstrates the potential for a well-constructed portfolio to outperform traditional benchmarks, even with slightly elevated risk levels.

7. Conclusion

This research demonstrates the viability of integrating value investing principles with Modern Portfolio Theory to optimise portfolio allocation. By combining fundamental analysis with advanced risk management techniques, the methodology outlined in this thesis identifies undervalued companies with high growth potential while maintaining robust diversification. The back-test results highlight the effectiveness of this approach, with the optimised portfolio significantly outperforming the S&P 500 benchmark over a 1-year period.

However, the findings also underscore certain limitations, such as the reliance on a relatively small sample size and the challenges posed by illiquidity during a prolonged drawdown. Additionally, while the Sharpe Ratio and other metrics indicate superior risk-adjusted returns, the increased volatility associated with the optimised portfolio warrants further investigation into managing downside risk.

Future research could explore longer back-test periods to examine performance across various economic cycles, focus on emerging and frontier markets to capture untapped growth opportunities, and refine risk-adjusted metrics to enhance the robustness of the portfolio construction process. By addressing these areas, the integration of value investing principles with MPT has the potential to further enhance portfolio performance and provide a valuable framework for investors seeking superior returns in an increasingly complex financial landscape.

8. Acknowledgements

I extend my gratitude to Dr. Damodaran for his invaluable assistance in providing access to publicly available resources and for his guidance on the principles of valuation, particularly in the application of discounted cash flow analysis. I also wish to thank the faculty at EDHEC Business School for their contributions through published courses, which enabled me to implement machine learning techniques in Python. These tools greatly facilitated the process of risk analysis and the computation of an efficient frontier comprising up to a thousand individual portfolios, based on an index of 100 firms. This approach significantly reduced the time required for backtesting and enhanced the overall efficiency of the research.

9. Declaration of Interest

The author declares no conflict of interest in the preparation of this research. The study was conducted independently, and no external funding, sponsorship, or affiliations influenced the methodologies, analyses, or conclusions presented in this work. All data sources and tools used are publicly available or explicitly cited.

10. Appendix

```
<py>
import pandas as pd
import scipy.stats
import numpy as np

def drawdown(return_series: pd.Series):
    """
    Takes a time series of asset returns.

    Returns a DataFrame with columns for the:
    - Wealth Index
    - Previous Peaks
    - Percentage Drawdown
    """

    # Convert DataFrame to Series in case of 'ValueError'
    if isinstance(return_series, pd.DataFrame):
        return_series = return_series.squeeze()

    wealth_index = 1000*(1+return_series).cumprod()
    previous_peaks = wealth_index.cummax()
    drawdowns = (wealth_index - previous_peaks)/previous_peaks

    return pd.DataFrame({"Wealth Index": wealth_index,
                         "Previous Peaks": previous_peaks,
                         "Drawdowns": drawdowns},
                        index = return_series.index)

def get_ind_returns():
    """
    - Load and format given CSV file for stock returns.
    - Has to be given the price change % month to month.
    """

    ind = pd.read_csv(“files.csv",
                      header = 0, index_col = 0, parse_dates = True
                     )

    ind.index = pd.to_datetime(ind.index, format = "%Y%m").to_period("M")
    ind.columns = ind.columns.str.strip()

    return ind

def semideviation(r):
    """
    - Returns the semi-deviation (or negative std.) of r
    - r must be a Series or DataFrame
    """

    is_negative = r < 0

    return r[is_negative].std(ddof = 0)

def skewness(r):
    """
    - Computes the skewness of the supplied Series or DataFrame
    - Returns a Float or a Series
```

```
    """

    demeaned_r = r - r.mean()

    # Use of the population std. therefore dof = 0
    sigma_r = r.std(ddof = 0)
    exp = (demeaned_r**3).mean()

    return exp/sigma_r**3

def kurtosis(r):
    """
    - Computes the kurtosis of the supplied Series or DataFrame
    - Returns a Float or a Series
    """

    demeaned_r = r - r.mean()

    # Use of the population std. therefore dof = 0
    sigma_r = r.std(ddof = 0)
    exp = (demeaned_r**4).mean()

    return exp/sigma_r**4

def is_normal(r, level = 0.01):
    """
    - Applies the Jarque-Bera test to determine if a Series is normal or not
    - Test is applied at the 1% level by default
    - Returns True if the normality hypothesis is accepted, False otherwise
    """

    statistic, p_value = scipy.stats.jarque_bera(r)

    return p_value > level

def historic_var(r, level = 5):
    """
    Returns the historic Value at Risk at a specified level of a Series or DataFrame.
    i.e. returns the number such that "level" percent of the returns fall below that number and
the (100-level) percent are above.
    """

    if isinstance(r, pd.DataFrame):
        return r.aggregate(historic_var, level = level)

    elif isinstance(r, pd.Series):
        return -np.percentile(r, level)

    else:
        raise TypeError("r needs to be a Series or DataFrame")

from scipy.stats import norm
def gaussian_var(r, level = 5, modified = False):
    """
    - Returns the Gaussian Parametric VaR of a Series or DataFrame.
    - If "modified" is True, the modified VaR is returned using the Cornish-Fisher Method.
    """

    # Compute the Z-Value assuming it was Gaussian
    z = norm.ppf(level / 100)
```

```
    if modified:
        # Modify the Z-Value based on the observed skewness and kurtosis
        s = skewness(r)
        k = kurtosis(r)
        z = (z +
             (z**2 - 1)*s/6 +
             (z**3 - 3*z)*(k - 3)/24 -
             (2*z**3 - 5*z)*(s**2)/36
            )

    return -(r.mean() + z*r.std(ddof = 0))

def historic_cvar(r, level = 5):
    """
    Computes the Conditional VaR of a Series or DataFrame.
    """

    if isinstance(r, pd.Series):
        is_beyond = r <= -historic_var(r, level = level)

        return -r[is_beyond].mean()

    elif isinstance(r, pd.DataFrame):
        return r.aggregate(historic_cvar, level = level)

    else:
        raise TypeError("r needs to be a Series or DataFrame")

def annualise_rets(r, periods_per_year):
    """
    Annualises a set of returns.
    """
    # Converts a single return value to a Series in case of AttributeError
    if isinstance(r, (float, int)):
        r = pd.Series([r])

    compounded_growth = (1 + r).prod()
    n_periods = r.shape[0]

    return compounded_growth**(periods_per_year/n_periods) - 1

def annualise_vol(r, periods_per_year):
    """
    Annualises the volatility of a set of returns.
    """

    return r.std()*(periods_per_year**0.5)

def sharpe_ratio(r, riskfree_rate, periods_per_year):
    """
    Computes the annualised Sharpe Ratio of a set of returns.
    """

    rf_per_period = (1 + riskfree_rate)**(1/periods_per_year) - 1
    excess_ret = r - rf_per_period
    ann_ex_ret = annualise_rets(excess_ret, periods_per_year)
    ann_vol = annualise_vol(r, periods_per_year)

    return ann_ex_ret/ann_vol
```

```
def portfolio_returns(weights, returns):
    """
    Weights to returns.
    """

    return weights.T @ returns

def portfolio_vol(weights, covmat):
    """
    Weights to volatility.
    """

    return (weights.T @ covmat @ weights)**0.5

def plot_ef2(n_points, er, cov):
    """
    Plots a 2-Asset Efficient Frontier.
    """

    if er.shape[0] != 2:
        raise ValueError("plot_ef2 can only plot 2-Asset frontiers")

    weights = [np.array([w, 1-w]) for w in np.linspace(0, 1, n_points)]
    rets = [portfolio_returns(w, er) for w in weights]
    vol = [portfolio_vol(w, cov) for w in weights]
    ef = pd.DataFrame({
        "Returns": rets,
        "Volatility": vol,
    })

    return ef.plot.line(x = "Volatility", y = "Returns", figsize = (10, 6), title = "Efficient Frontier",
color = "Green", style = ".-")

from scipy.optimize import minimize
def minimize_vol(target_return, er, cov):
    """
    Target returns to weights vector.
    """

    n = er.shape[0]
    init_guess = np.repeat(1/n, n)
    bounds = ((0.0, 1.0),)*n
    return_is_target = {
        "type": "eq",
        "args": (er,),
        "fun": lambda weights, er: target_return - portfolio_returns(weights, er)
    }
    weights_sum_to_1 = {
        "type": "eq",
        "fun": lambda weights: np.sum(weights) - 1
    }

    results = minimize(portfolio_vol, init_guess,
                       args = (cov,),
                       method = "SLSQP", # Optimizer Method
                       constraints = (return_is_target, weights_sum_to_1),
                       bounds = bounds,
                       options = {"disp": False}
                      )
```

```
    return results.x

def optimal_weights(n_points, er, cov):
    """
    Generates a list of weights for N assets to run the optimiser on in order to minimise volatility.
    """

    target_rs = np.linspace(er.min(), er.max(), n_points)
    weights = [minimize_vol(target_return, er, cov) for target_return in target_rs]

    return weights

from scipy.optimize import minimize
def msr(riskfree_rate, er, cov):
    """
    Risk-Free Rate to weights vector.
    """

    n = er.shape[0]
    init_guess = np.repeat(1/n, n)
    bounds = ((0.0, 1.0),)*n
    weights_sum_to_1 = {
        "type": "eq",
        "fun": lambda weights: np.sum(weights) - 1
    }

    def neg_sharpe_ratio(weights, riskfree_rate, er, cov):
        """
        Returns the negative of the Sharpe Ratio, given weights.
        """

        r = portfolio_returns(weights, er)
        vol = portfolio_vol(weights, cov)

        return -(r - riskfree_rate)/vol

    results = minimize(neg_sharpe_ratio, init_guess,
                   args = (riskfree_rate, er, cov,),
                   method = "SLSQP", # Optimizer Method
                   constraints = (weights_sum_to_1),
                   bounds = bounds,
                   options = {"disp": False}
                  )

    return results.x

def plot_efn(n_points, er, cov, show_cml = False, riskfree_rate = 0):
    """
    Plots a Multi-Asset Efficient Frontier and Capital Market Line.
    """

    weights = optimal_weights(n_points, er, cov)
    rets = [portfolio_returns(w, er) for w in weights]
    vol = [portfolio_vol(w, cov) for w in weights]
    ef = pd.DataFrame({
        "Returns": rets,
        "Volatility": vol,
    })
```

```
        ax = ef.plot.line(x = "Volatility", y = "Returns", figsize = (20, 12), title = "5YR Efficient Frontier
& Capital Market Line", color = "Green", style = "-")

        if show_cml:
            ax.set_xlim(left = 0)

            rf = 0.1
            w_msr = msr(riskfree_rate, er, cov)
            r_msr = portfolio_returns(w_msr, er)
            vol_msr = portfolio_vol(w_msr, cov)

            cml_x = [0, vol_msr]
            cml_y = [riskfree_rate, r_msr]

            ax.plot(cml_x, cml_y, color = "Blue", marker = "o")

        return ax
    </py>
```

11. Tables & Figures

12. References

1. Bodie, Z., Kane, A. and Marcus, A.J. (2024) Investments. New York, NY: McGraw Hill.
2. Damodaran, A. (2011) Applied Corporate Finance. S.l.: s.n.
3. Damodaran, A. (2012) Investment valuation: Tools and techniques for determining the value of any asset. Hoboken: Wiley.
4. Damodaran, A. (2024) Historical (compounded annual) growth rates by sector, Historical Growth Rates. Available at: https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histgr.html (Accessed: 13 December 2024).
5. Fisher, R.A. (1950) Contributions to mathematical statistics. New York: Wiley.
6. Graham, B. and Zweig, J. (2024) The intelligent investor: The definitive book on value investing. New York, NY: Harper Business, an Imprint of HarperCollins Publishers.
7. Hull, J. (2022) Options, futures, and other derivatives. Harlow, England: Pearson.
8. Investopedia (2020) What does undervalued mean? definition in value investing, Investopedia. Available at: https://www.investopedia.com/terms/u/undervalued.asp (Accessed: 13 December 2024).
9. Jorion, P. (2011) Value at risk: The new benchmark for managing financial risk. New York: McGraw Hill Professional.
10. Multpl (2024a) Shiller Pe ratio, Multpl. Available at: https://www.multpl.com/shiller-pe (Accessed: 13 December 2024).

11. Multpl (2024b) Shiller Pe Ratio FAQ, Multpl. Available at: https://www.multpl.com/shiller-pe/faq (Accessed: 13 December 2024).

12. SeekingAlpha (2024) Stock market analysis & tools for investors, Seeking Alpha. Available at: https://seekingalpha.com/ (Accessed: 13 December 2024).

13. Sortino, F.A. and van der Meer, R. (1991) 'Downside risk', The Journal of Portfolio Management, 17(4), pp. 27–31. doi:10.3905/jpm.1991.409343.

14. YCharts (2024) US corporate AAA effective yield (I:USCAAAEY), YCharts. Available at: https://ycharts.com/indicators/us_coporate_aaa_effective_yield (Accessed: 13 December 2024).

15. Eling, M. (2009) 'Does hedge fund performance persist? overview and new empirical evidence', European Financial Management, 15(2), pp. 362–401. doi:10.1111/j.1468-036x.2008.00471.x.

16. Luijendijk, J.S. (2024) Exploring Over-Diversification in ETF Portfolios, Erasmus Rotterdam University. Available at: https://thesis.eur.nl/pub/73609/Thesis_final_version_JS_Luijendijk.pdf (Accessed: 13 December 2024).

17. Sharpe, W.F. (1964) 'Capital asset prices: A theory of market equilibrium under conditions of risk', The Journal of Finance, 19(3), p. 425. doi:10.2307/2977928.

18. Sharpe, W.F. (1966) 'Mutual Fund Performance', The Journal of Business, 39(S1), p. 119. doi:10.1086/294846.

19. Statman, M. (1987) 'How many stocks make a diversified portfolio?', The Journal of Financial and Quantitative Analysis, 22(3), p. 353. doi:10.2307/2330969.

20. Stuart, A. and Markowitz, H.M. (1959) 'Portfolio selection: Efficient Diversification of Investments', OR, 10(4), p. 253. doi:10.2307/3006625.