— (x) → set of input / observed variables

(z) → be latent variables.

$$\boxed{P(z, x)}$$ joint distribution

(Inference Problem
✓ to compute conditional distribution
of latent variables
$$\boxed{P\left(\dfrac{z}{x}\right)}$$

Observation
(x)

$$\boxed{P\left(\dfrac{z}{x}\right) = \dfrac{P\left(\dfrac{x}{z}\right) \cdot P(z)}{P(x)}} \longrightarrow (i)$$

(i) → $P(x)$ can't be solved

intractable problem

$$\nearrow \boxed{P(x) = \int_{z} \theta\left(\dfrac{x}{z}\right) p(z) \, dz}$$

How many?

+ Not a closed form!
+ Intracable

④ Approximate $P\left(\dfrac{z}{n}\right)$

by another distribut-
$q\left(\dfrac{z}{n}\right)$ such th

THE solution become

tracable

Variational Inference

$$P\left(\frac{z}{x}\right) \qquad Q\left(\frac{z}{x}\right)$$

$$D_{KL}\left(Q_\phi\left(\frac{z}{x}\right) \;\middle\|\; P_\theta\left(\frac{z}{x}\right)\right)$$

$$= \int Q_\phi\left(\frac{z}{x}\right) \log\left(\frac{Q_\phi(z_x)}{P_\theta(z_x)}\right) dz$$

$$= \sum Q_\phi\left(\frac{z}{x}\right) \log \frac{Q_\phi\left(\frac{z}{x}\right)}{P_\theta\left(\frac{z}{x}\right)}$$

MLE

$$\left[\mathbb{E}_{z \sim Q\left(\frac{z}{u}\right)} \log \frac{Q_\phi\left(\frac{z}{u}\right)}{P_\theta\left(\frac{z}{u}\right)} \right]$$

$$\left[\mathbb{E}_{z \sim Q\left(\frac{z}{u}\right)} \left(\log Q_\phi\left(\frac{z}{u}\right) - \log P_\theta\left(\frac{z}{u}\right) \right) \right]$$

(ii)

$$\mathbb{E}_{z \sim Q\left(\frac{z}{u}\right)} \left(\log Q_\phi\left(\frac{z}{u}\right) - \log \left(\frac{P\left(\frac{x}{z}\right)}{P(x)} \right) \right)$$