This is the second homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by 20:00 on Monday October 23, 2023.

1. **Winnings**
   A box contains 5 red and 9 blue marbles. Two marbles are withdrawn randomly. If the two marbles are of the same color, then you win 2 Euro and if they are of different colors then you lose 1.2 Euro. Compute the expected value and the variance of the amount you win.

2. **Continuous random variable**
   Let X a continuous random variable with the probability density function (pdf)

   $$f_X(x) = \begin{cases} \alpha e^{-x}, & x \in (0, \ln 2) \\ 0, & \text{otherwise} \end{cases},$$

   where α is a positive real constant and ln denotes the natural logarithm.

   (a) Determine the value of α.
   (b) Find the cumulative distribution function (cdf) of X.
   (c) Compute the first two moments of X.

3. **The Cauchy cumulative distribution function**
   Let
   $$F(y) = \frac{1}{2} + \frac{1}{\pi} \arctan y, \quad y \in \mathbb{R}.$$

   (a) Show that F is a cumulatiove distribution function of a certain random variable Y.
   (b) Find the density function and use `R` to sketch its graph.
   (c) Find $a \in \mathbb{R}$ such that $P(Y > a) = 0.1$.

4. **Transformations**
   Let
   $$f_X(x) = \begin{cases} \frac{1}{2} \cdot (x + 1), & x \in (-1, 1) \\ 0, & \text{otherwise} \end{cases}$$

   be the probability density function of a random variable X.
   Find the cumulative distribution function of
   $$Y = \frac{9}{(X - 2)^2}.$$

5. **R -functions**

   `trees` is the `R` Dataset Package containing Diameter, Height and Volume for 31 Black Cherry trees. Using `R` define a vector h that contains the values of the column `Height` and define a vector v that contains the values of the column `Volume` from the dataset `trees`. Use the command `table(h)` to obtain the frequency table of the vector h. What are the outputs of the commands `summary(h)` and `factor(h)`? Compute the sample means `mean()` and the sample variances `var()` for both vectors. Use the command `plot()` to plot the points (h, v).