

This is the ninth homework assignment. Students should tick in [TUWEL](#) problems they have solved and upload their detailed solutions by **20:00 on Monday December 11, 2023**.

1. Two-sample t -test

Create two independent samples from the normal distribution. The first sample of size 10 shall be taken from the standard normal distribution. The second sample of size 20 shall be taken from the $\mathcal{N}(1, 1)$ -distribution. Test the null hypothesis that the populations means are equal with a (two-sided) two-sample t -test at the 5%-significance level:

- (a) Calculate the t -statistic (without `t.test()`).
- (b) Compare it to the output of `t.test()`.
- (c) Interpret the result of the test.

2. Two-sample t -test using normal approximation

Messages are frequently sent from a sender to either receiver 1 or receiver 2. For both receivers, several times for the transfer were measured (in seconds) and stored in the file `waitingtimes2.Rdata`.

- (a) Visualize both data sets. Are the distributions approximately bell-shaped?
- (b) Test the null-hypothesis of equal mean transfer times for both receivers on the 1%-level with a two sample t -test (using the normal approximation).
- (c) Compare your result to the output of `t.test()`.

3. Corneal thickness

To determine whether glaucoma affects the corneal thickness, measurements were made in nine people affected by glaucoma in one eye but not in the other. The corneal thicknesses (in microns) were as follow

Person	1	2	3	4	5	6	7	8	9
Eye affected by glaucoma	488	478	480	426	440	410	458	460	468
Eye not affected by glaucoma	484	478	492	444	436	398	464	476	420

Assume the corneal thicknesses are normally distributed with expectation μ_1 and variance σ_1^2 for eyes affected by glaucoma, and expectation μ_2 and variance σ_2^2 for eyes not affected by glaucoma. Test $H_0 : \mu_1 = \mu_2$ against $H_A : \mu_1 < \mu_2$ using $\alpha = 10\%$. What kind of test will you perform? Base your conclusion on a 90% confidence interval.

4. Which statement is correct?

In the situation of a (two-sided) two-sample t -test at 5%-significance level, assume that the null hypothesis $H_0 : d = d_0$ was rejected. Comment on the following statements.

- (a) The null hypothesis would also be rejected at the 7%-level.
- (b) The equivalent 99% confidence interval does not contain d_0 .
- (c) If both sample sizes are increased by a factor 4, then the value of the t -statistic is halved if all other estimates remain the same.
- (d) If one of the sample sizes is increased, then the width of the 95%-confidence interval is increased (if all other estimates remain the same)
- (e) There is a 5% chance that the null hypothesis is true.
- (f) If the null hypothesis is true, there is a 5% chance that we came to the wrong conclusion.

5. Simulation of test-power

Simulate the distribution of the p -value in the two-sided two-sample t -test: Let $X_1, \dots, X_{20}, Y_1, \dots, Y_{20}$ be independent random variables with $X_i \sim N(0, 1)$ and $Y_i \sim N(d, 1)$ for all $i = 1, 2, \dots, 20$. For each $d \in \{0, 0.25, 0.5\}$, derive p -values in 10000 simulations ($H_0 : d = 0$) and plot them in a histogram of unit area. Comment on your three histograms.