This is the first homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by 20:00 on Monday October 16, 2023.

1. **Random walk of a robot**
   A robot is placed at the origin (the point $(0,0)$) on a two-dimension integer grid (see the figure below). Denote the position of the robot by (x, y). The robot can either move right to $(x+1, y)$ or move up to $(x, y+1)$.
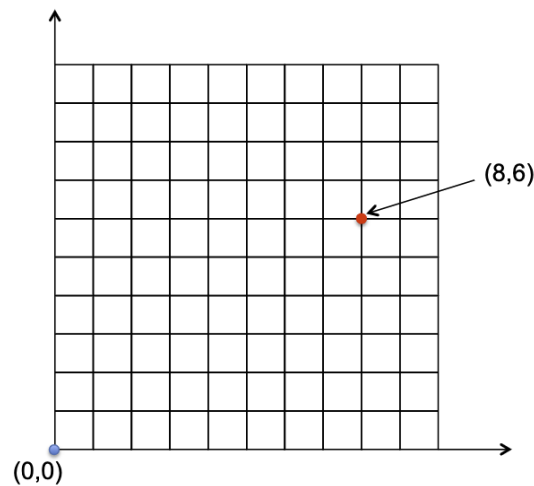
   Suppose each time the robot randomly moves right or up with equal chance. What is the probability that the robot will ever reach the point $(8, 6)$?

2. **Passwords**
   Anna needs to choose a password for her computer account, that is a sequence of eight characters consisting of uppercase letters, lowercase letters and digits. The rule is that the password must consist of two lowercase letters (a to z) followed by one uppercase letter (A to Z) followed by five digits $(0, 1, \ldots, 9)$. For example, `mhT26039` is a valid password. A hacker has been able to write a program that randomly and independently generates 10^7 passwords according to the above rule. Note that the same password could be generated more than once. If one of the randomly chosen passwords matches Anna's password, then the hacker can access her account information. What is the probability that the hacker is successful in accessing her account information?

3. **Independent events/disjoint events**
   For three events A, B and C we know that A and C are independent, B and C are independent, and A and B are disjoint. The probabilities $P(A \cup C) = \frac{2}{3}$, $P(B \cup C) = \frac{3}{4}$, $P(A \cup B \cup C) = \frac{11}{12}$ are also given. Find $P(A)$, $P(B)$ and $P(C)$.

4. **Spam filter**
   One way to design a spam filter is to look at the phrases in an email. In particular, some phrases are more frequent in spam emails. Suppose that we have the following information: 30% of emails are spam, 1% of spam emails contain the phrase "filled with joy"; 0.2% of non-spam emails contain the phrase "filled with joy". Suppose that an email is checked and found to contain the phrase "filled with joy". What is the probability that the email is spam?

5. **Coin toss game**
   Anna and Nick play the following game: Anna repeatedly tosses an unfair coin with the probability p, $0 < p \neq \frac{1}{2}$ of obtaining a head (H). The game ends the first time that two consecutive heads (HH) or two consecutive tails (TT) are observed. Anna wins if (HH) is observed and Nik wins if (TT) is observed. For example, if the outcome is $HTHTT$, Nik wins while if the outcome is $HTHTHH$, Anna wins. Given that Anna won the game, what is the probability that the first coin toss resulted in head?