This is the fourth homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by 20:00 on Monday November 6, 2023.

1. **Miraculin**
   Miraculin is a protein naturally produced in a rare tropical fruit. It can convert a sour taste into a sweet taste. Consequently, miraculin has the potential to be an alternative low-calorie sweetener. A group of Japanese environmental scientists investigated the ability of a hybrid tomato plant to produce miraculin. For a particular generation of the tomato plant, the amount X of miraculin produced (measured in micrograms per gram of fresh weight) had a mean 105.3 and a standard deviation of 8.0. Assume that X is normally distributed. Use the table of the cdf of the standard normal distribution to compute

   (a) $P(100 < X < 110)$
   (b) $P(X(X - 120) > 0)$
   (c) the interquartile range of the miraculin production.

   Note: The interquartile range is the difference betwen the upper and the lower quartile of X, i.e. $IQR_X = x_{0.75} - x_{0.25}$.

2. **Cholesterol drug treatement**
   Manufacturing and selling drugs that claim to reduce an individual's cholesterol level is a big business. A company would like to market their drug to women if their cholesterol is in the top 15%. Assume the cholesterol levels of adult women can be desribed by a normal model with an expectation of 188 mg/dL and a standard deviation of 24. Use R to answer the following questions (the commands `pnorm()`, `qnorm()`, and `dnorm()` might be useful)

   (a) Draw and label the normal model.
   (b) What percent of adult women is expected to have cholesterol levels between 150 mg/dL and 170 mg/dL?
   (c) Compute the interquartile range of the cholesterol levels.

3. **Acrylic nails**
   One of the most popular types of artificial nails are acrylic nails. These full nail extensions are made from a combination of liquid monomer and powder polymer. Given that 10% of the acrylic nails made using a certain manufacturing process have a length less than 5.8 centimeters, while 5% have a length greater than 6.7 centimeters, what are the mean and the standard deviation of the lengths of the nails? Assume that the lengths have a normal distribution.

4. **The winning result**
   Assume that the times of fifteen 100-metres sprinters are independent random variables with common $\mathcal{N}(10, 0.125^2)$-distribution. Compute the probability that the winning results is below 9.75, i.e. compute the probability that the minimum of their running times is less than 9.75.

5. **Coffee and doughnuts**

   At a certain coffee shop, all the customers can buy a cup of coffee and also a doughnut. The shop owner believes that the number of cups she sells each day is normally distributed with an expectation of 320 cups and a standard deviation of 20 cups. She also believes that the number of doughnuts she sells each day is independent of the coffee sales and is normally distributed with an expectation of 150 doughnuts and a standard deviation of 12.

   (a) What is the probability that on any given day she will sell at least 500 of her products, be it cups of coffee or doughnuts?

   (b) The shop is open every day including Sunday. Assuming day-to-day sales are independent, what is the probability she will sell more than 1000 doughnuts in a week?