This is the eleventh homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by 20:00 on Monday January 8, 2024.

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1. **Rolling die, part 1**
   A d-sided die with colored sides was rolled n times. The outcomes are stored in the file `die.Rdata`. (Each side appeared at least once.)

   (a) What are the sample size n and the number of sides d of the die rolled?

   (b) Visualize the relative frequencies in a colored barplot and add the standard error of each frequency.

   (c) Given your graphic, what is your opinion on the assertion: 'the die is fair'?

2. **Rolling die, part 2**
   Test the null hypothesis that the die is fair with a χ^2-test on the 5%-significance level, without using R-command `chisq.test()`.

   (a) What are the observed (absolute) frequencies?

   (b) What are the expected frequencies under the null hypothesis?

   (c) What is the value x^2 of the χ^2-statistic?

   (d) In the context of the associated model, how is the χ^2-statistic X^2 distributed under the null hypothesis?

   (e) What is the rejection area R? Compute also the p-value. Do you reject the null hypothesis?

   (f) Interpret your result.

3. **An experiment producing numerical data**
   Suppose we have an experiment that produces numerical data with the possible outcomes -2, -1, 0, 1, 2, 3 or more. We run 55 trials and count the frequency of each outcome, getting the following data:

Outcomes	-2	-1	0	1	2	3or more
Observed frequences	3	11	16	14	8	3

   Use the χ^2-test for goodness of fit to test the null hypothesis H_0 that the data is drawn from 55 trials of a binomial distribution $B(8, 0.5)$ against the alternative hypothesis H_A that the data is drawn from some other distribution with the level of significance $\alpha = 1\%$. Compute the p-value and give your decision on the test.

4. **χ^2-test for independence (without R)**
   120 students from major computer science of three Viennese Universities were randomly chosen and asked which lecture of mathematics (a: calculus, b: algebra, c: probability) they enjoyed most. The frequencies are given in the following table:

	Uni A	Uni B	Uni C
calculus	15	10	5
algebra	10	20	15
probability	20	10	m

   Obtain the missing observed frequency m and perform a χ^2- test to test whether the preference for a lecture is independent from the university, on a 5% significance level. Only use the following table which gives the 95%−quantile q of the χ^2-distribution with df degrees of freedom.

df	1	2	3	4	5	6	7	8	9
q	3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92

5. **χ^2-test for independence (with R)**
   (a) Solve the previous exercise using R.
   (b) Can you also reject for a significance level of $\alpha = 0.1\%$?
   (c) Double observed frequencies in each cell and perform the test on the 0.1% level. Did the decision change comparing to the one from (b)?