This is the third homework assignment. Students should tick in TUWEL problems they have solved and upload their detailed solutions by 20:00 on Monday October 30, 2023.

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1. **Multiple-choice exam**
   Nik takes an exam in Statitstics and Probability Theory that contains 20 multiple-choice questions. Each question has four possible options and only one of the four options is correct. He knows the answer to ten questions, but he has no idea about the other ten questions so he chooses answers randomly. His score X on the exam is the total number of answers he answered correctly.

   (a) Find the probability mass function of X.
   (b) Compute the probability $P(X > 16)$.

2. **Telemarketing phone calls**
   On average, a household receives 6.5 telemarketing phone calls per week. Using the Poisson probability distribution, find the probability that a randomly selected household receives

   (a) exactly two
   (b) not more than five

   telemarketing phone calls during a given week.

3. **Uniform $(0, 1)$-distribution and its related distributions**
   Let X be a Uniform(0,1) random variable.

   (a) Find the cumulative distribution function of $V = e^X$ and its expectation.
   (b) Show that $W = -2 \ln X$ is $\exp(\frac{1}{2})$-distributed.
   (c) Let F be the cumulative distribution function of a continuous random variable. Find the cumulative distribution function of $Y = F^{-1}(X)$.

4. **The Pareto distribution: the 80-20 rule.**
   The Pareto distribution is a power-law probability distribution, i.e. for a fixed baseline m its probability density function depends on the parameter α having the form

$$f(x) = \begin{cases} \frac{\alpha \cdot m^\alpha}{x^{\alpha+1}}, & x \geq m \\ 0, & x < m \end{cases}.$$

   Assume X is a random variable that follows such a distribution.

   (a) Find the cumulative distribution function of X.

(b) The Pareto principle is also known as the 80-20 rule. This principle describes a variety of phenomena. For example, it means that 80% of the wealth is owned by 20% of the people. It may be applied to fundraising (20% of the donors contributing towards 80% of the total), in business management (80% of sales come from 20% of clients), in computer science (80% of a piece of software can be written in 20% of the total allocated time), etc. The 80-20 rule is only exact for the Pareto distribution with $\alpha = \frac{\ln 5}{\ln 4} = 1.16$.
Assuming $\alpha = m = 1$, compute the 0.80-quantile of X.

5. **The Pareto distribution: Generating random numbers**
Problem 3c shows that if $X \sim \mathcal{U}(0,1)$, then $Y = F^{-1}(X)$ generates a random number Y from any continuous distribution with the specified cumulative distribution function F. This gives us the following algorithm to generate random numbers from a random variable Y with the given cumulative distribution function F.

Algorithm: Inversion method

1° compute the inverse F^{-1} of F
2° generate n independent random numbers $x_1, x_2, \ldots, x_n$ from $\mathcal{U}(0,1)$
3° compute $y_1 = F^{-1}(x_1), y_2 = F^{-1}(x_2), \ldots, y_n = F^{-1}(x_n)$.

Then, $y_1, y_2, \ldots, y_n$ are n independent random observations of the random variable Y.

In `R`, generate 15 observations from a random variable X with the Pareto distribution with $\alpha = m = 2$ (see Problem 4) by applying the inversion method.