

Computation Physics Problem Set 7

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1 Problem 1

1.1 a

First we find the angular velocity of the moon by using the classic equation $F = ma_c = \frac{GMm}{R^2} = m\omega^2 R$. This can be solved to find $\omega^2 = \frac{GM}{R^3}$. To find the point at which the gravitational force from both the moon and the earth match perfectly so the angular acceleration is equal to that of the satellite, we can use a similar equation. $F = ma_c = F_{earth} + F_{moon} = m_{sat}\omega^2 * r = \frac{GMm_{sat}}{r^2} - \frac{G*m_{moon}m_{sat}}{(R-r)^2}$. Canceling m_{sat} and rescaling so $m' = m/M$ and $r' = r/R$ gets the following:

$$\begin{aligned}\frac{GM}{r^2} - \frac{Gm}{(R-r)^2} &= \frac{GM}{R^3}r \\ \frac{GM/m}{(r/R)^2} - \frac{Gm/m}{((R/R) - (r/R))^2} &= \frac{GM}{R^3}r \frac{R^2}{m} \\ \frac{1}{m'r'^2} - \frac{1}{(1-r')^2} &= \frac{r'}{m'} \\ \frac{1}{r'^2} - \frac{m'}{(1-r')^2} - r' &= 0 \\ (1-r')^2 - m'r'^2 - r'^3(1-r')^2 &= 0\end{aligned}$$

1.2 b

We need to find which r will get us the Lagrange point. To do this we will use the secant method, beginning with a bracket of $r' = (0,1)$ since r has to be between 0 and R . Implementing this results in the following:

For the Earth and the moon: (distance from Earth) 326318430.0 meters

For the Earth and the Sun: (distance from Sun) 148108410000.0 meters

For the Sun and a really large Earth: (distance from Sun) 139625450000.0 meters

2 Problem 2

Implementing the brent algorithm on the given function results in 0.2999999999999978. This is very similar to the result from `scipy.optimize.brent` which results in 0.300000000023735.