Computation Physics Problem Set 6

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1 Part a

The plot of the various spectrums for the first five galaxies is shown in figure 1.

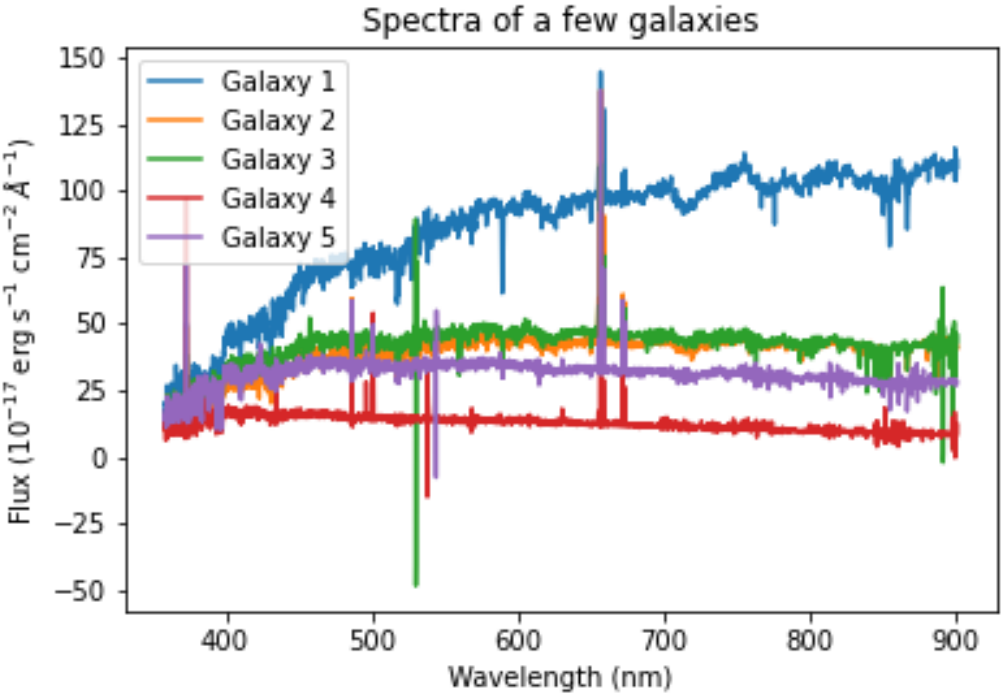

Figure 1: Spectra of the first few galaxies.

From figure 2, we see the wavelengths of photons emitted when electrons from a hydrogen atom drop from various energy levels. We can also see that there are peaks in the galaxy spectra near the 656 nm and the 486 nm wavelengths which would correspond to a hydrogen atom dropping from an n = 3 to 2 energy level and an n = 4 to 3 energy level.

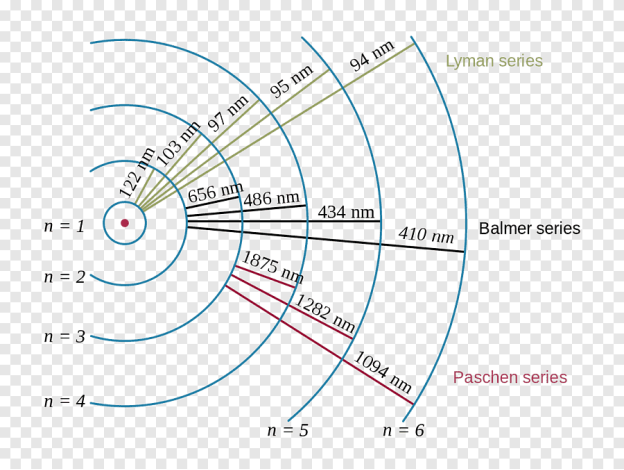

Figure 2: Corresponding photon wavelength for various energy level changes in a hydrogen atom

2 Parts b and c

After normalizing and centering the data, the residuals of the galaxies will look like figure 3.

3 Part d

The first five eigenvectors plotted look like figure 4.

4 Part e

The first five eigenvectors plotted using SVD look like figure 5.

Both figures show the same eigenvectors for both types of computation. The only difference is the purple (eigenvector #5), which seems to differ between both computations.

The computational cost can be estimated using the time required for the computation. The time required when using the covariance matrix was 101.0785096900072 seconds while the time required when using SVD was 1.909883660991909 seconds.

5 Part f

The SVD was significantly faster and less computationally costly than using the covariance matrix which would make it the preferred method to compute the eigenvectors. Also, the condition number of the covariance matrix is 36005756 while the covariance matrix of R is 453044.3. A higher condition value indicates the matrix is closer to being nearly singular, meaning it is less numerically stable and would be more prone to overflow/underflow problems during computations. To avoid these problems, doing an SVD decomposition of the matrix would be preferred.

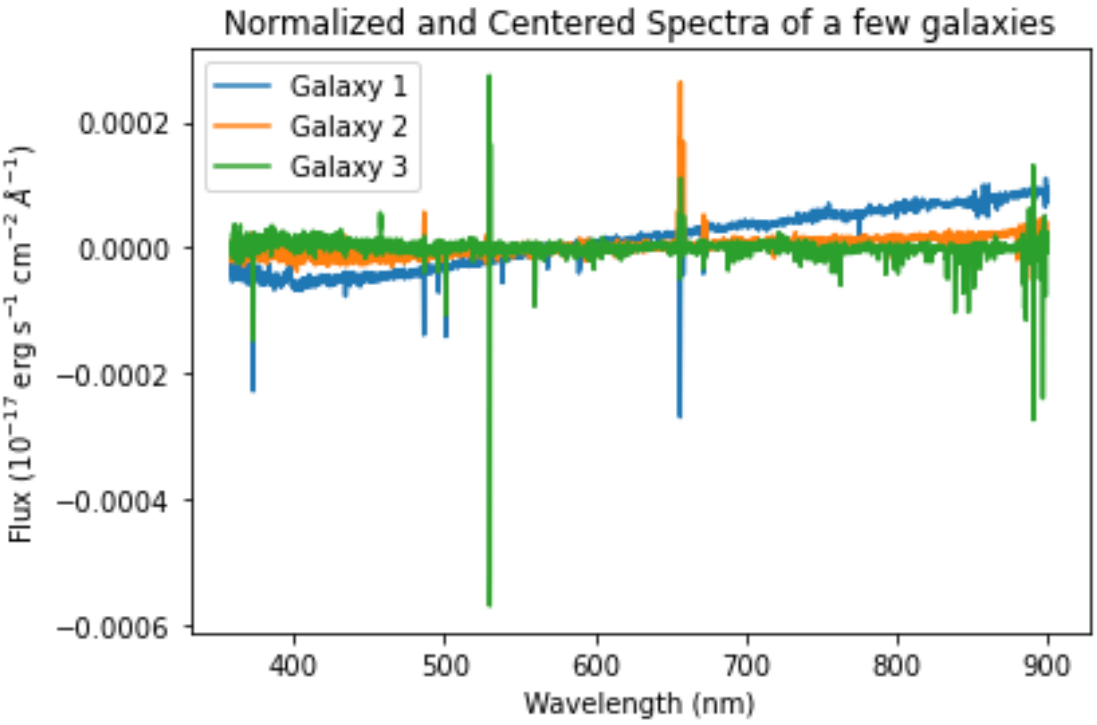

Figure 3: The normalized and centered spectra of the first five galaxies

6 Part g

The reconstructed galaxy spectra for the first five galaxies using only the first five coefficients are shown in figures 6 - 10.

7 Part h

: The plots of c_0 vs c_1 and c_0 vs c_2 are shown in figures 11 & 12.

8 Part i

Plotting the root mean squared residuals when using various numbers of coefficients results in figure 13

The final root-mean-squared residual for $Nc = 20 = 1.1929113547875935e^{-5}$.

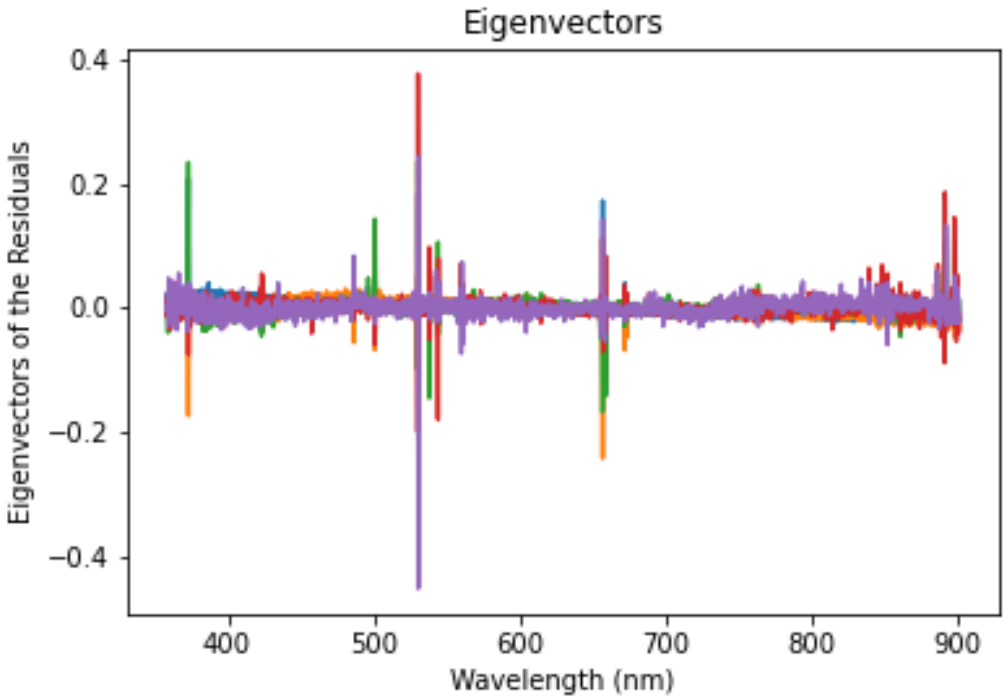

Figure 4: The first five eigenvectors using the Covariance Matrix

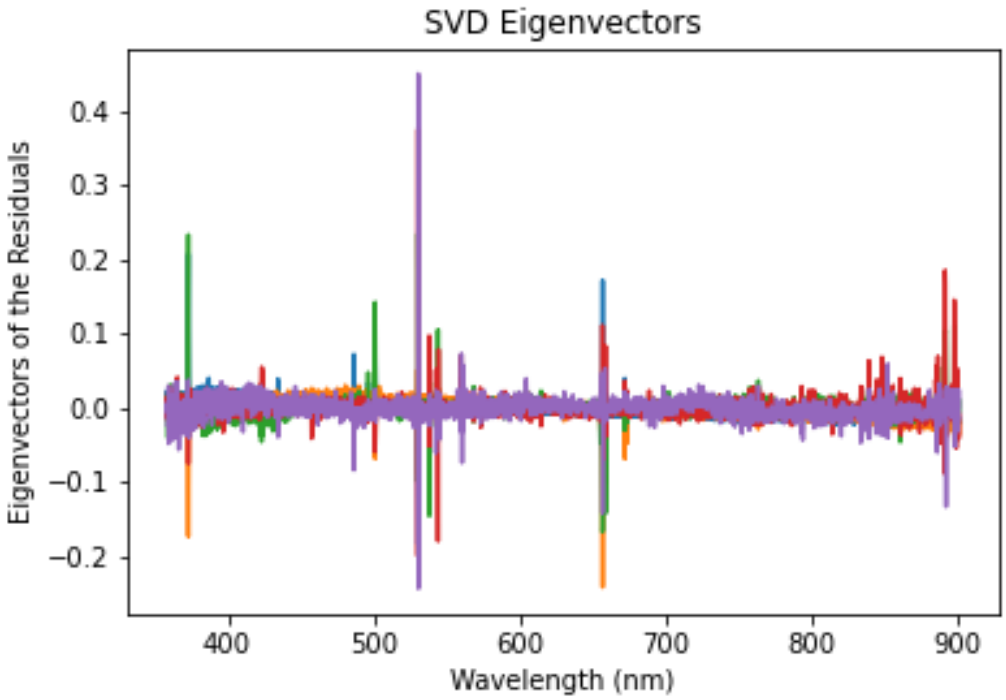

Figure 5: The first five eigenvectors using SVD decomposition of the R matrix

Figure 6: The reconstructed spectrum plotted atop the original spectrum for galaxy #0

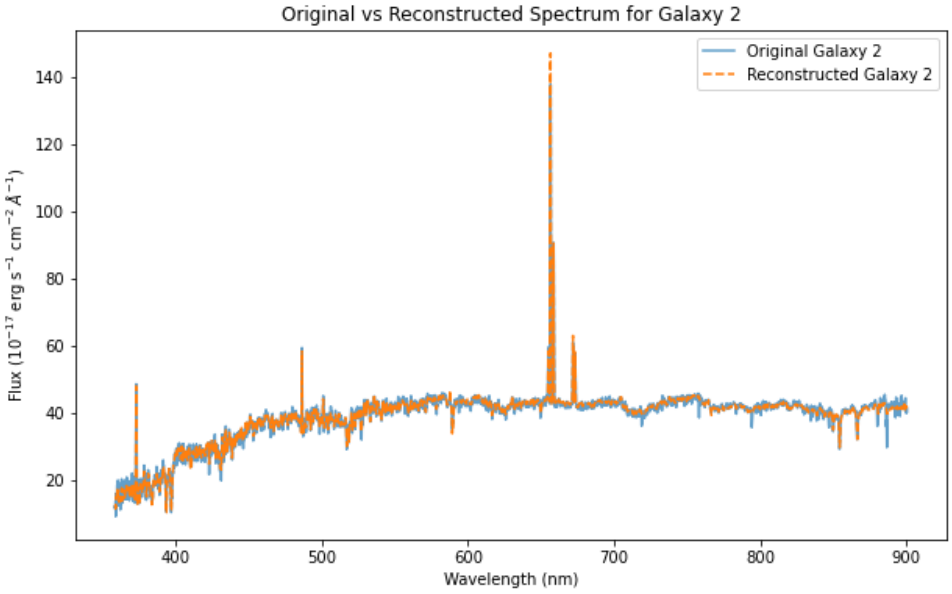

Figure 7: The reconstructed spectrum plotted atop the original spectrum for galaxy #1

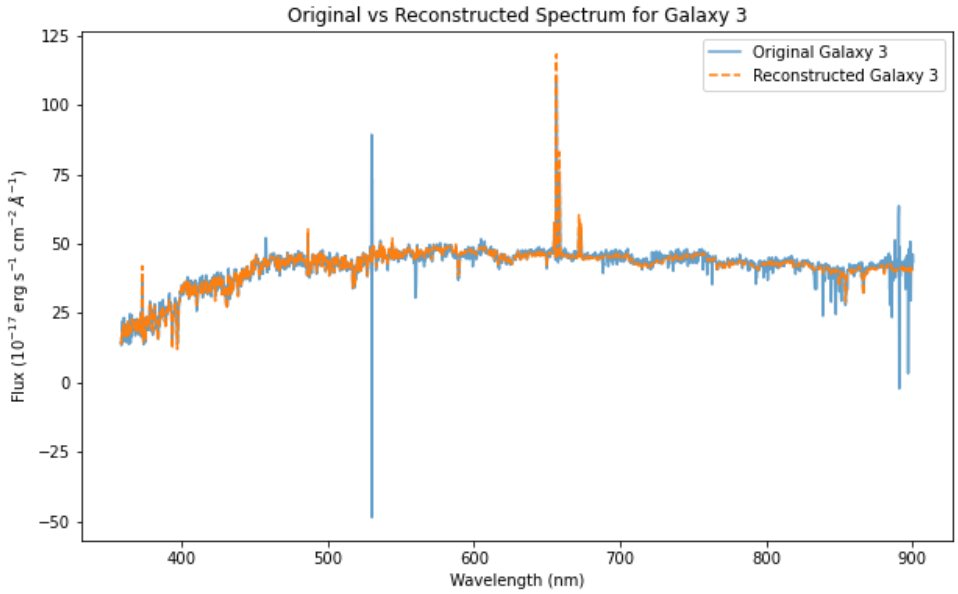

Figure 8: The reconstructed spectrum plotted atop the original spectrum for galaxy #2

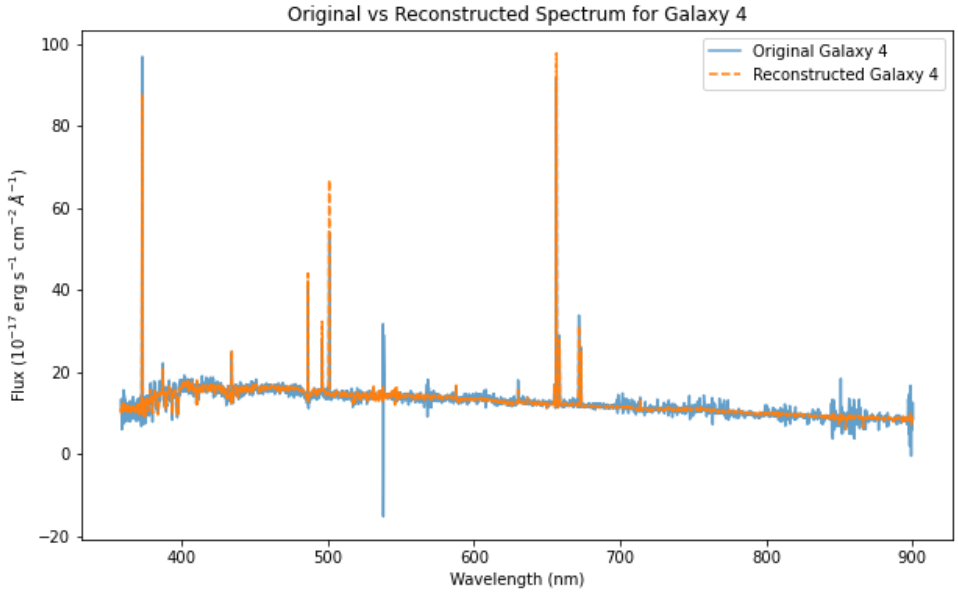

Figure 9: The reconstructed spectrum plotted atop the original spectrum for galaxy #3

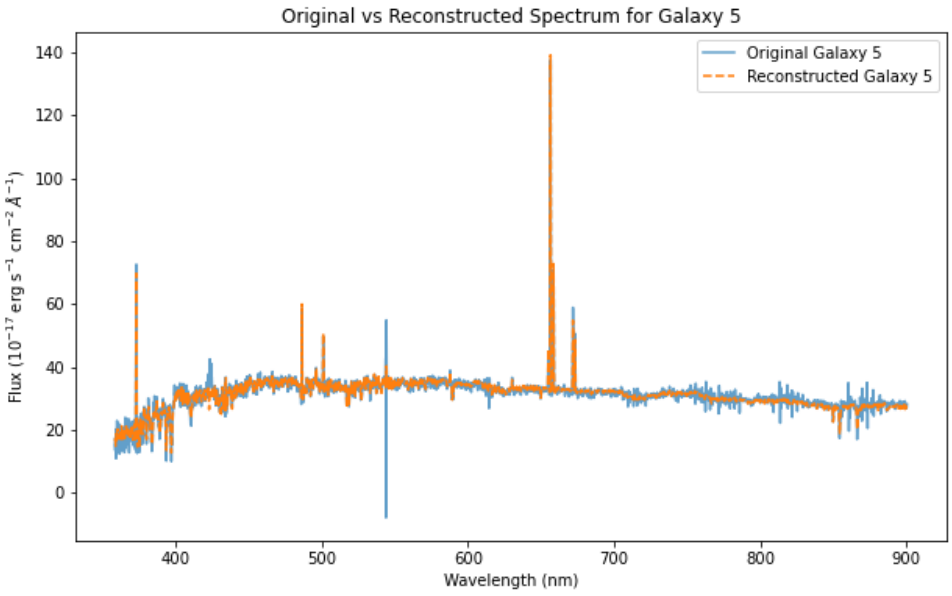

Figure 10: The reconstructed spectrum plotted atop the original spectrum for galaxy #4

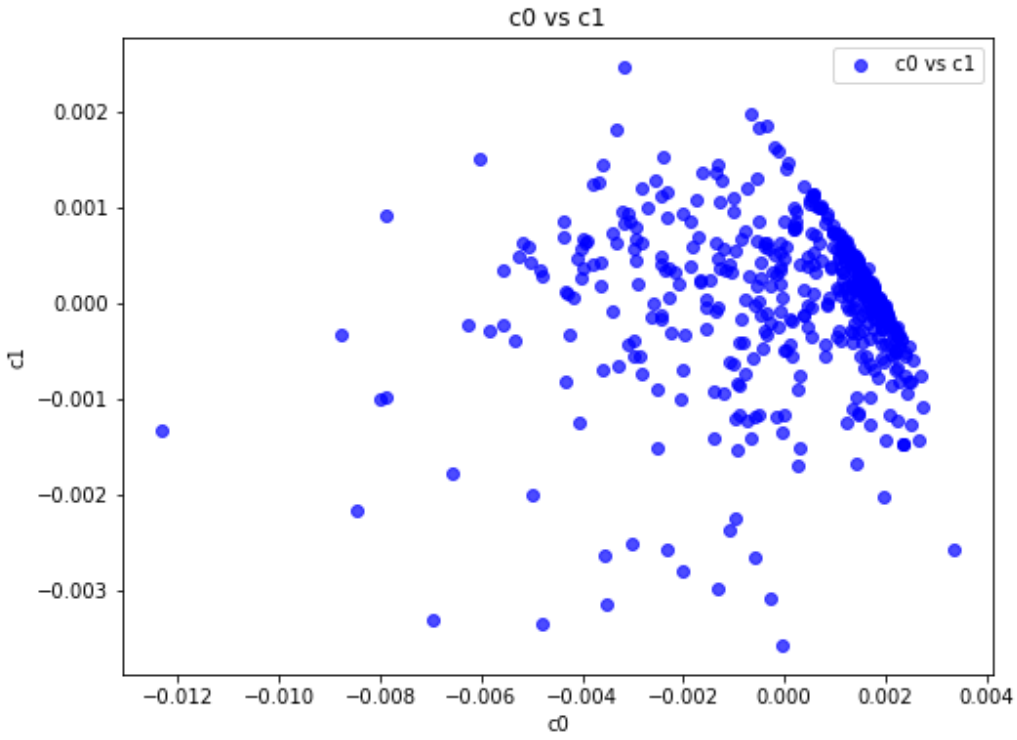

Figure 11: The first coefficient vs the second

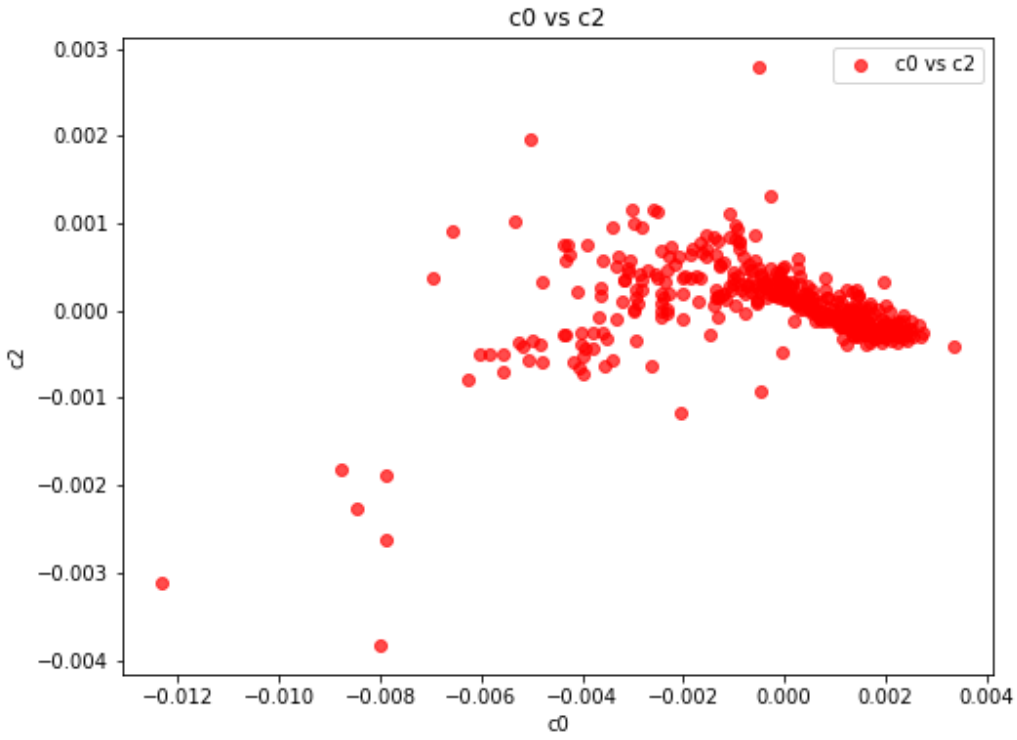

Figure 12: The first coefficient vs the third

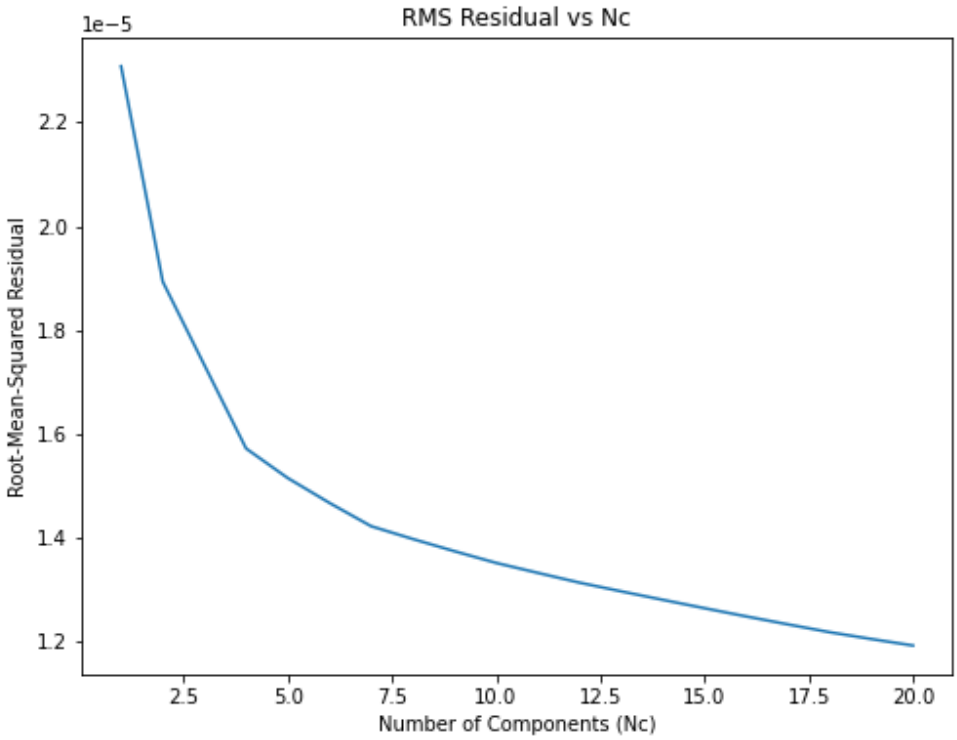

Figure 13: The root mean squared residuals when using various numbers of coefficients