Computation Physics Problem Set 9

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1 Problem 1

1.1 a

To turn the harmonic oscillator into two coupled first order equations, we define the following:

$$\frac{dx}{dt} = v$$
$$\frac{dv}{dt} = -\omega^2 x$$

The motion of the simple harmonic oscillator with the given conditions and x0 = 1 is shown in figure 1.

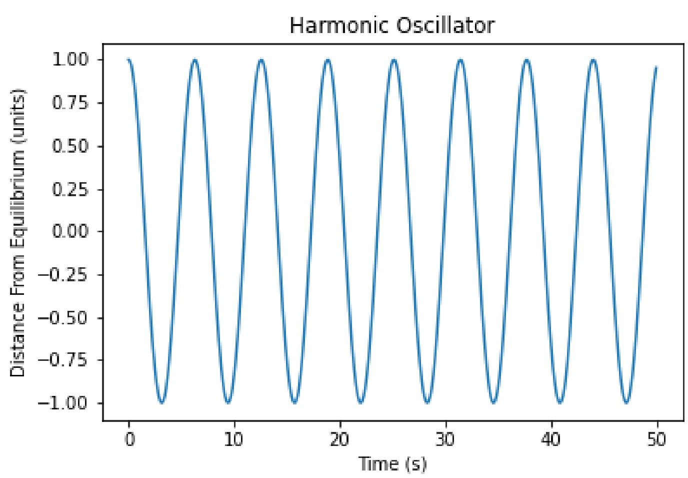

Figure 1: The motion of a simple harmonic oscillator with x0 = 1

1.2 b

The motion of the simple harmonic oscillator with the given conditions and $x0 = 2$ is shown in figure 2. Since both graphs for $x0 = 1$ and $x0 = 2$ have the same x-axis scaling, you can see the periods for both initial conditions are the same ($T = 2\pi$).

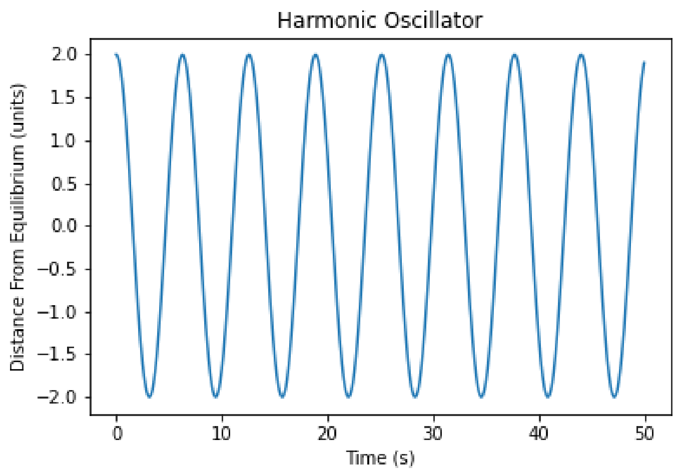

Figure 2: The motion of a simple harmonic oscillator with x0 = 2

1.3 c

The motion of the anharmonic oscillator with $x0 = 1$ and $x0 = 2$ are shown in figures 3&4. The period of oscillation can be observed to ??? as the amplitude doubles.

1.4 d

The ability to create phase space plots was added to the code

1.5 e

The phase space plots for the van der Pol oscillator are included as figures 5 - 7

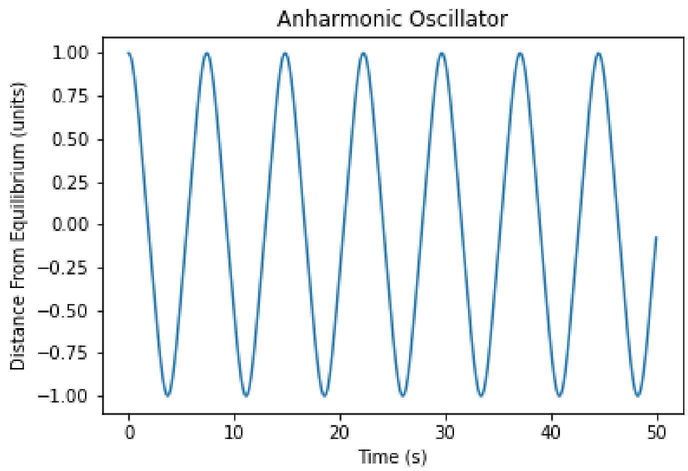

Figure 3: The motion of an anharmonic oscillator with x0 = 1

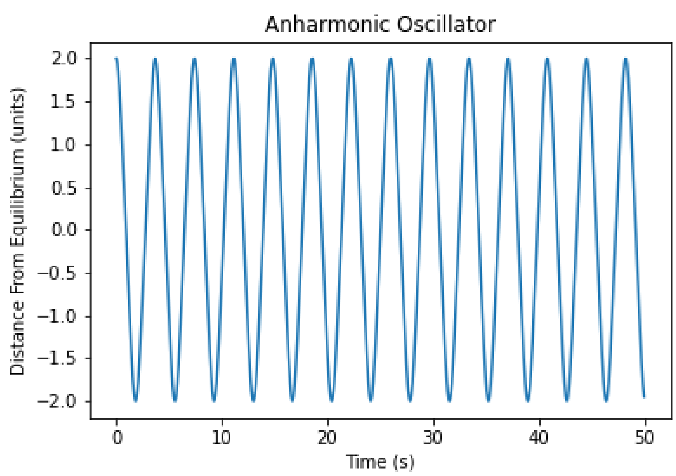

Figure 4: The motion of an anharmonic oscillator with x0 = 2

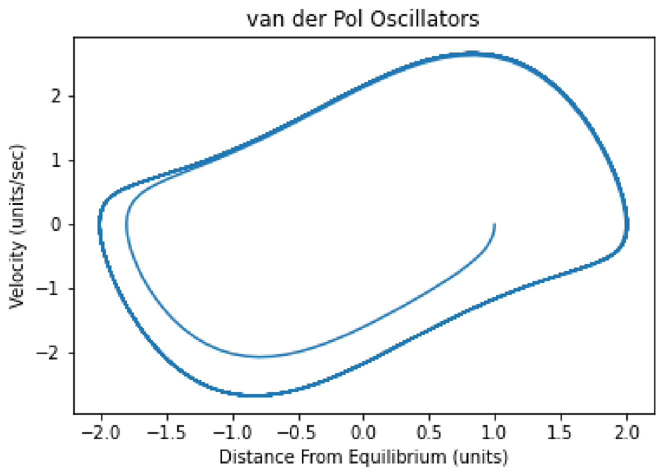

Figure 5: The phase space diagram of a van der Pol oscillator with $\mu = 1$

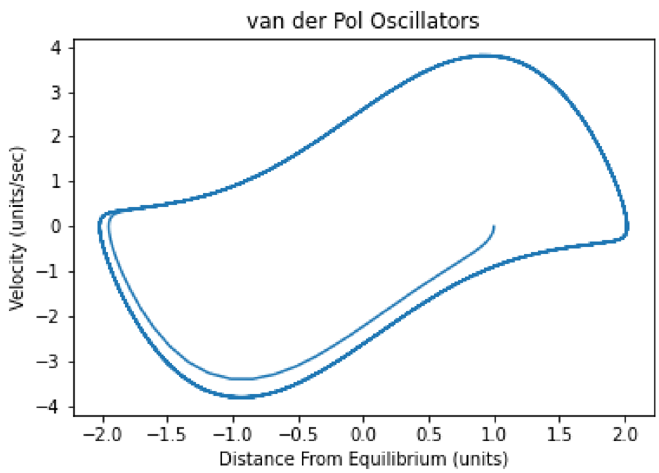

Figure 6: The phase space diagram of a van der Pol oscillator with $\mu = 2$

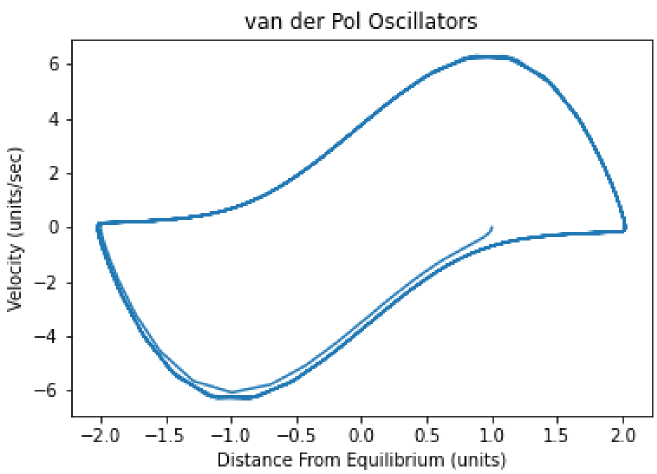

Figure 7: The phase space diagram of a van der Pol oscillator with $\mu = 4$

2 Problem 2

2.1 a

The forces on the cannonball obey the following equation:

$$\vec{F} = m\ddot{\vec{r}} = \frac{-1}{2}\pi R^2 \rho C \dot{r}^2 \hat{r} - mg\hat{y} \tag{1}$$

Which can be rewritten as:

$$\ddot{x}\hat{x} + \ddot{y}\hat{y} = \frac{-1}{2m}\pi R^2 \rho C(\dot{x}^2 + \dot{y}^2)\frac{\dot{x}\hat{x} + \dot{y}\hat{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} - g\hat{y} \tag{2}$$

Splitting this into x and y components we get:

$$\hat{x} : \ddot{x} = \frac{-1}{2m}\pi R^2 \rho C \dot{x}\sqrt{\dot{x}^2 + \dot{y}^2} \tag{3}$$

$$\hat{y} : \ddot{y} = \frac{-1}{2m}\pi R^2 \rho C \dot{y}\sqrt{\dot{x}^2 + \dot{y}^2} - g \tag{4}$$

2.2 Rescaling

To rescale the ODE, we first define the following unitless variables:

$$\begin{aligned}
t' &= t/T \to t = Tt' \\
x' &= x/L \to x = Lx' \\
y' &= y/L \to y = Ly' \\
\dot{x}' &= \frac{T}{L}\dot{x} \to \dot{x} = (L/T)\dot{x}' \\
\ddot{x}' &= \frac{T^2}{L}\ddot{x} \to \ddot{x} = (L/T^2)\ddot{x}' \\
\dot{y}' &= \frac{T}{L}\dot{y} \to \dot{y} = (L/T)\dot{y}' \\
\ddot{y}' &= \frac{T^2}{L}\ddot{y} \to \ddot{y} = (L/T^2)\ddot{y}' \\
m' &= m/M \to m = Mm' \\
R' &= R/L \to R = LR' \\
\rho' &= (L^3/M)\rho \to \rho = (M/L^3)\rho' \\
g' &= (T^2/L)g \to g = (L/T^2)g'
\end{aligned}$$

Plugging each of these values in equations 3 & 4, results in the following:

$$\hat{x} : \frac{L}{T^2}\ddot{x}' = \frac{-1}{2Mm'}\pi(LR')^2\frac{M}{L^3}\rho' C\frac{L}{T}\dot{x}'\frac{L}{T}\sqrt{\dot{x}'^2 + \dot{y}'^2} \tag{5}$$

$$\hat{y} : \frac{L}{T^2}\ddot{y}' = \frac{-1}{2Mm'}\pi(LR')^2\frac{M}{L^3}\rho' C\frac{L}{T}\dot{y}'\frac{L}{T}\sqrt{\dot{x}'^2 + \dot{y}'^2} - \frac{L}{T^2}g' \tag{6}$$

which simplify to the following:

$$\hat{x} : \ddot{x}' = \frac{-1}{2m'}\pi(R')^2\rho' C\dot{x}'\sqrt{\dot{x}'^2 + \dot{y}'^2} = \frac{-\pi}{2}\frac{R'^2\rho' C}{m'}\dot{x}'\sqrt{\dot{x}'^2 + \dot{y}'^2} \tag{7}$$

$$\hat{y} : \ddot{y}' = \frac{-1}{2m'}\pi(R')^2\rho' C\dot{y}'\sqrt{\dot{x}'^2 + \dot{y}'^2} - g' = \frac{-\pi}{2}\frac{R'^2\rho' C}{m'}\dot{y}'\sqrt{\dot{x}'^2 + \dot{y}'^2} - g' \tag{8}$$

Defining $\beta = \frac{R'^2\rho' C}{m'}$ as a unitless constant that is determined by R, ρ, C, and m means that those equations can be rewritten as the following:

$$\hat{x} : \ddot{x}' = \frac{-\pi}{2}\beta\dot{x}'\sqrt{\dot{x}'^2 + \dot{y}'^2} \tag{9}$$

$$\hat{y} : \ddot{y}' = \beta\dot{y}'\sqrt{\dot{x}'^2 + \dot{y}'^2} - g' \tag{10}$$

Since g is a known constant, this means that the equations only depend on one unitless parameter β and that the possible values of R, ρ, C, m, and g map to a one-parameter family of solutions.

2.3 b

The plot of the trajectory of the canonball is shown in figure 8.

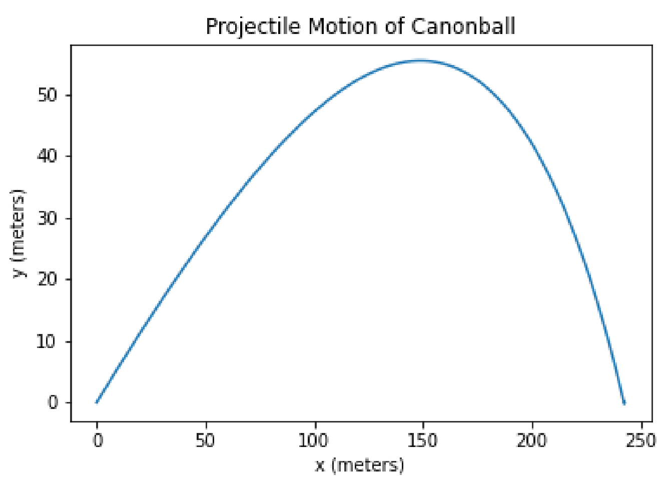

Figure 8: Projectile Motion of the canon ball with the desired values and Mass = 1kg

2.4 c

The plot of the trajectories of cannonballs of various masses is shown in figure 9. As can be seen in the graph, cannonballs of larger masses are able to travel further distances.

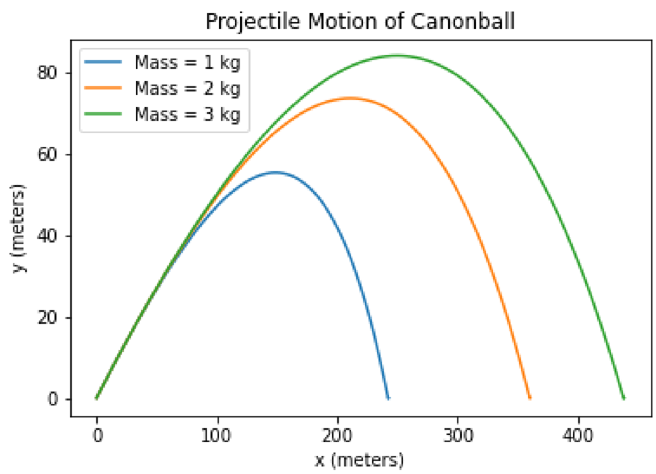

Figure 9: Projectile Motion of the canon ball with the desired values and various masses