# Computation Physics Problem Set 4

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October 1, 2024

## 1 Problem 1

### 1.1 a

This problem used integration to find the heat capacity of a solid, specifically the heat capacity of a block of aluminum. Using Gaussian quadrature and N = 50 sample points, the heat capacity of the solid at various temperatures was calculated in the function cv(T).

### 1.2 b

The heat capacity of the solid was then plotted against temperature in figure 1

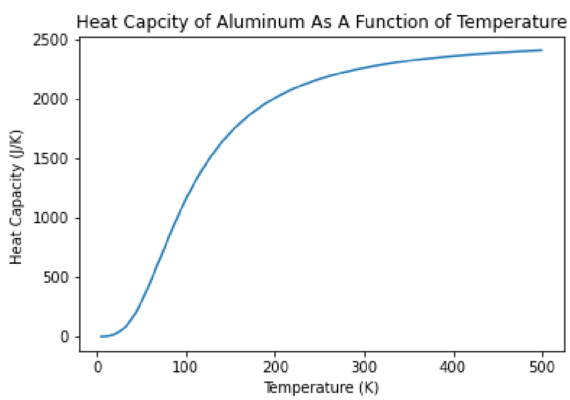


Figure 1: The heat capacity of Aluminum at various temperatures

### 1.3 c

Convergence was tested with N = 10, 20, 30, 40, 50, 60, 70. The specific heat at T = 5 was plotted with its corresponding N value. You can see that at any N higher than 30, the Gaussian quadrature shows extreme accuracy in calculating the specific heat. For N = 10, the heat capacity is highly inaccurate and for N = 20, while the heat capacity is significantly more accurate it is still worse than the N = 30 and higher calculations. For higher temperatures, this convergence occurs even faster with lower values of N.

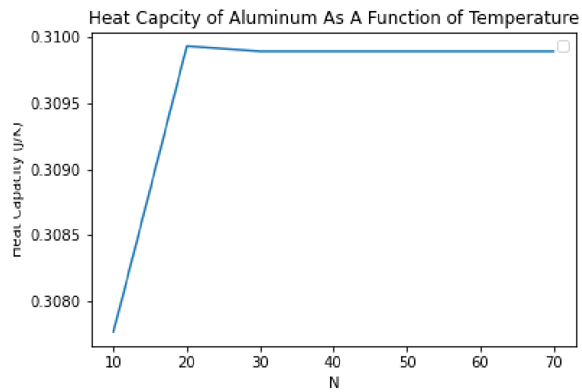


Figure 2: The heat capacity of aluminum with respect to temperature for various number of sample points (N)

# 2 Problem 2

## 2.1 a

First the derivation of the integral. We start with the equation for Energy $E = \frac{1}{m}(\frac{dx}{dt})^2 + V(x)$. We also know that at t=0, x=a, E = V(a), and since energy is conserved, E = V(a) is a constant. We also are integrating over 1/4 of the full oscillation, from t = 0, x=a to t=T/4,x=0 where T = the total period. With some rearranging, we get the following:

$$
\begin{aligned}
E &= \frac{1}{m}(\frac{dx}{dt})^2 + V(x) \\
\frac{2(E - V(x))}{m}) &= (\frac{dx}{dt})^2 \\
\sqrt{\frac{2(V(a) - V(x))}{m}} &= \frac{dx}{dt} \\
\frac{dx}{\sqrt{\frac{2(V(a)-V(x))}{m}}} &= dt \\
\int_{x=a}^{x=0} \frac{dx}{\sqrt{\frac{2(V(a)-V(x))}{m}}} &= \int_{i=0}^{i=T/4} dt \\
T/4 &= \int_a^0 \frac{dx}{\sqrt{\frac{2(V(a)-V(x))}{m}}} \\
T/4 &= \sqrt{m/2} \int_a^0 \frac{dx}{\sqrt{V(a) - V(x)}} \\
T &= \sqrt{8m} \int_a^0 \frac{dx}{\sqrt{V(a) - V(x)}} \\
T &= \sqrt{8m} \int_0^a \frac{dx}{\sqrt{V(a) - V(x)}}
\end{aligned}
$$

Please note that switching the bounds in the last step was possible because of the symmetry of the potential function.

### 2.2 b

The integral to find the period (T) was solved using the python function T(a). This was plotted as a function of the amplitude in figure 3.

### 2.3 c

These results make sense for both a large amplitude and a small amplitude. For small amplitudes, the change in potential is small so (V(a) - V(x)) would also be small. For a small amplitude, the denominator is close to zero so the integrand would get closer to infinity. A smaller amplitude would also mean a smaller restoring force, meaning the period would get infinitely larger.

For a larger amplitude, the distance to travel to reach the restoring force would get very large and the integrand would get very small (since V(a) - V(x) would get infinitely large), so the period would get infinitely smaller (closer to an asymptote of 0).

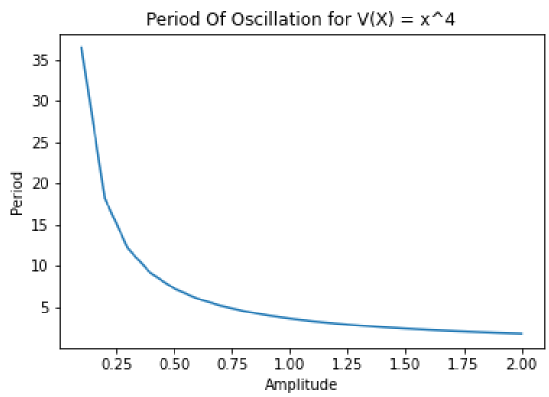


Figure 3: Period of oscillation with respect to the amplitude

# 3 Problem 3

## 3.1 a

There is a python function H(n,x). Using those values, the wavefunction for various n values have been plotted in figure 4

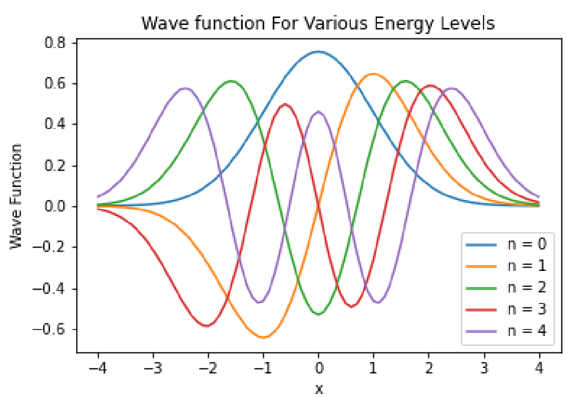


Figure 4: Wave function for various n values

## 3.2 b

The wavefunction for n=30 has been plotted in figure 5

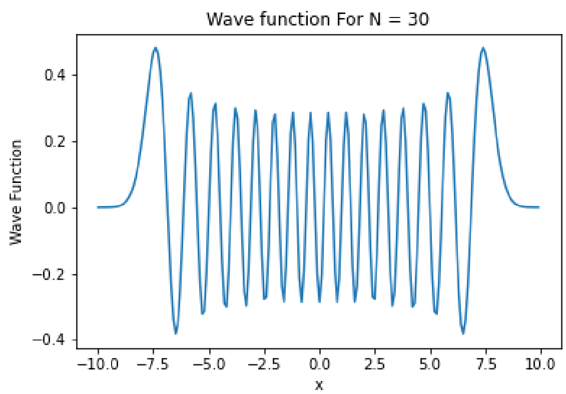


Figure 5: Wave function for n = 30

### 3.3 c

Evaluation of the integral using the Guassian quadrature resulted in an uncertainty of 6.12589534654262e-09.

### 3.4 d

Evaluation of the integral using Gauss-Hermite quadrature resulted in an uncertainty of 0.