Computation Physics Problem Set 5

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1 Problem 1

Analytically calculate the derivative of the function $f(x) = 1 + \frac{1}{2}tanh(2x)$.

$$f'(x) = 0 + \frac{d}{dx}\frac{1}{2}tanh(2x) = \frac{1}{2}sech^2(2x) * 2 = sech^2(2x) \quad (1)$$

Figure 1 shows that using the central limit and jax results in the same thing as the analytical derivative of $f(x) = 1 + \frac{1}{2}tanh(2x)$.

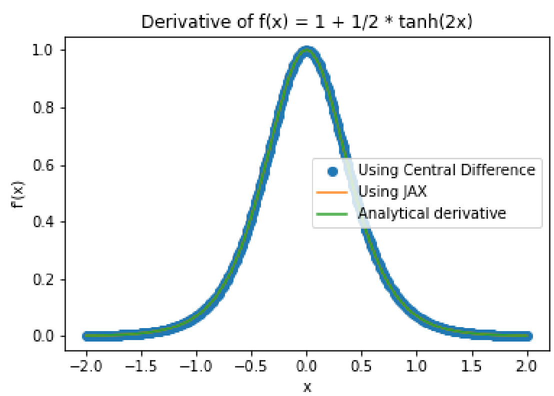

Figure 1: Derivative of function f(x) calculated analytically and numerically using Central Difference and jax

2 Problem 2

2.1 a

Figure 2 shows the integrand of the gamma function for different values of a.

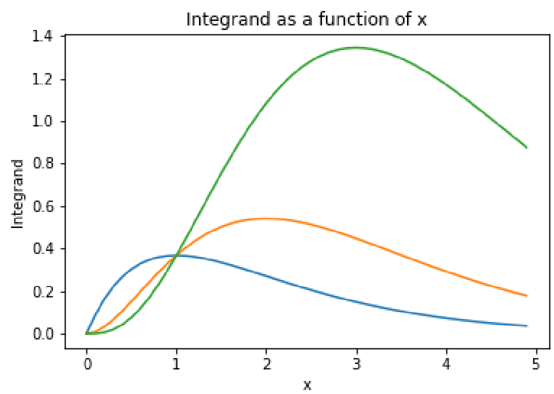

Figure 2: Integrand of the gamma function

2.2 b

To find the maximum of the function, we need to take the derivative and find when it equals 0. This will find the critical point, which we can see via the previous graphs, will be the maximum.

$$\begin{aligned}
\frac{d}{dx}(x^{a-1} * e^{-x}) &= (a-1)x^{a-2} * e^{-x} + x^{a-1} * (-1)e^{-x} \\
&= e^{-x} * x^{a-2}((a-1) - x) \\
&= e^{-x} * x^{a-2}(a-1-x)
\end{aligned}$$

Now setting this expression equal to zero gets us that either $e^{-x} = 0$, $x^{a-2} = 0$, or $(a-1-x) = 0$. This results in finding that the critical points must occur at $x = \infty, 0, (a-1)$. Since graphically, we know that at 0, the function is at a min and that at infinity the function is decaying, we know that the maximum point must be $x = a - 1$.

2.3 c

If we want the peak of the integrand to lie at $z = 1/2$, then we need the function to return $z(x = a-1) = 1/2$. Therefore we want $z = \frac{x}{c+x} = \frac{a-1}{c+a-1} = 1/2$. This can be simplified as $2(a-1) = 2a-2 = (c+a-1)$, which you can solve to get $c = a-1$. This results in the change of variables being $z(x) = \frac{x}{a-1+x}$.

2.4 d

if we instead write $x^{a-1} = e^{(a-1)ln(x)}$, we can write $x^{a-1} * e^{-x} = e^{(a-1)ln(x)}e^{-x} = e^{(a-1)ln(x)-x}$ This is better because you avoid multiplying a really big number (x^{a-1}) with a really small number e^{-x} for large x. In this new form, the logarithm will keep the exponent in a more manageable range and avoid possible overflow or underflow errors.

2.5 e

Using these changes to calculate the integral results in a result of $\Gamma(3/2) = 0.8862269613087249$. (which is close to what we expect)

2.6 f

For a = 3,6,10, we get $\Gamma(3) = 2.0000000000000084, \Gamma(6) = 120.00000000000043, \Gamma(10) = 362880.00000000163$, which is close to what we expect.

3 Problem 3

3.1 a

Figure 3 shows a plot of the signal data from the provided link.

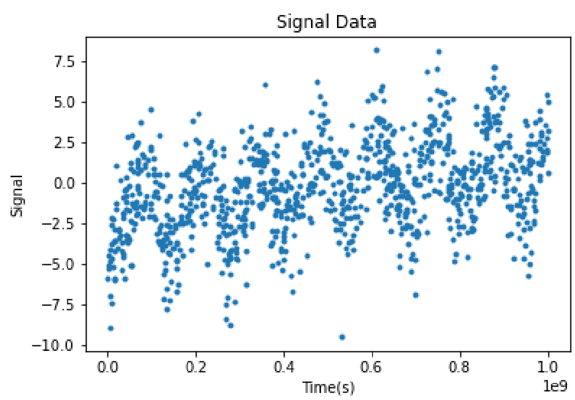

Figure 3: Signal data plotted against time

3.2 b

Figure 4 shows a 3rd degree polynomial fitted to the data.

3.3 c

Figure 5 shows the residuals from fitting a 3rd degree polynomial to the data. As you can see, a periodic correlation is still visible. Any measurement should be seen as random variation around the 0.0 signal line, but the still visible pattern makes it certain that there is still more information to be found from the data that can be seen with just a 3rd degree polynomial.

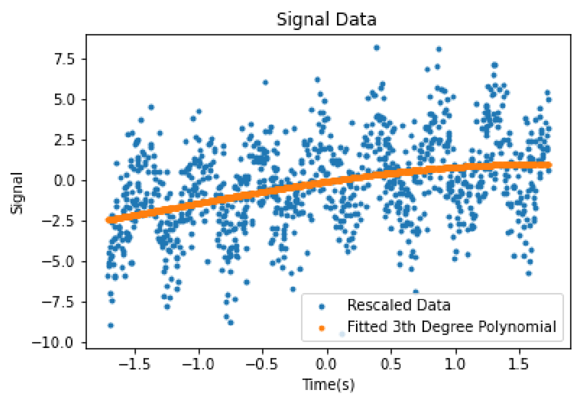

Figure 4: 3rd degree polynomial fit to rescaled signal data

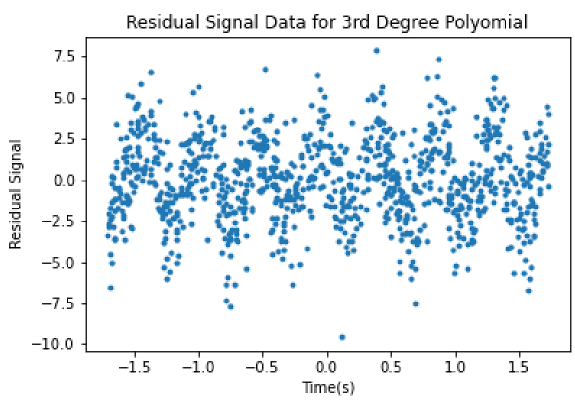

Figure 5: Residuals from the 3rd degree fitted polynomial

3.4 d

The best fit for the signal data I could get was with a 25th degree polynomial. This, however, had a condition number of 38107280470.58172. Any lower or higher degree polynomial had a worse fit with just as bad of a condition number. Basically, there is no reasonable polynomial that you could use to get a good fit of the data. This fit is shown in Figure 6

3.5 e

With a sine and cosine function and a fourier series going to the 8th term, I was able to get a condition number of 1.6200220571412896 with a visibly much better fit. This is shown in figure 7. This fit does a better job describing the visibly periodic nature of the data.

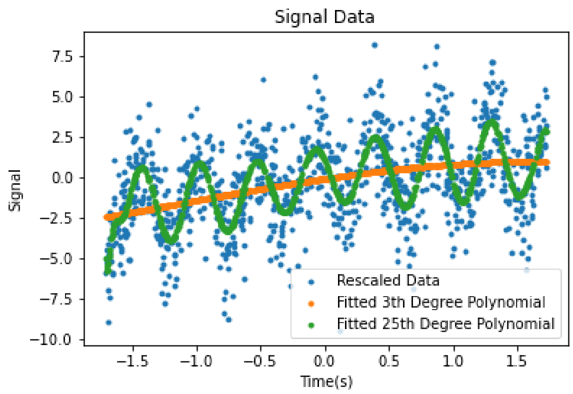

Figure 6: 25th degree polynomail fit to rescaled signal data

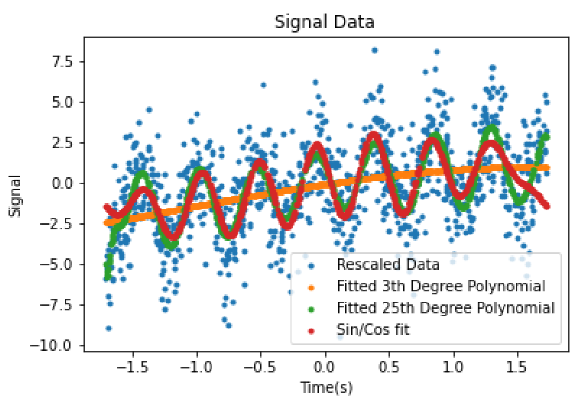

Figure 7: Sin and Cosine fit to rescaled signal data