Computation Physics Problem Set 1

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1 Problem 1

Here are the results from using both methods of multiplication for matrices of different size (NxN). Both plots have been plotted on logarithmic scales. The slope of the plots will correspond to the power of N the efficiency corresponds to. The lines of best fit, while not perfect, are $log(time) = 3*log(N) - 6.07$ for the explicit multiplication function and $log(time) = 2.1*log(N) - 7.6$ when using the dot function. This means the the explicit multiplication function will be $O(N^3)$ as expected but the dot function will be closer to $O(N^{2.1})$

The plots for the two functions with Matrix size versus computation time have been included in figure 1.

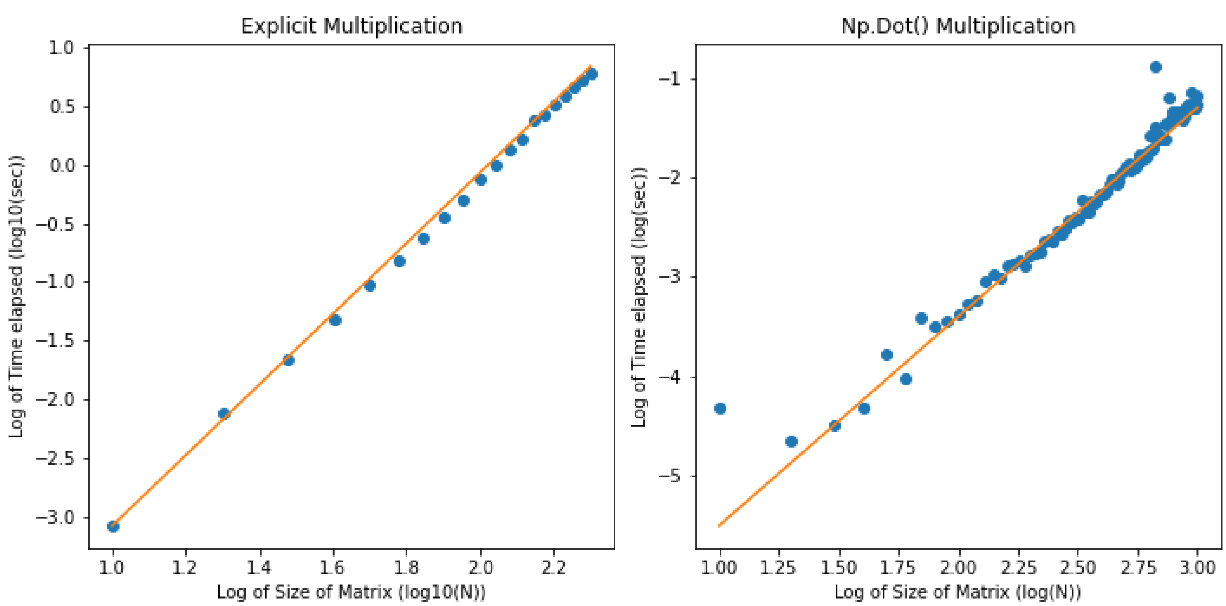

Figure 1: Plots for time required for both explicit multiplication and the dot function for matrix multiplication

2 Problem 2

Exercise 10.2

2.1 a

This was coded as function PbDecay(Pb, Bi209, dt).

2.2 b

This was coded as function TiDecay(Pb, Bi209, dt).

2.3 c

This was coded as function Bi213Decay(Pb, Bi209, dt).

2.4 d (and final results)

The results for the number of each atom at each second for 20000 seconds are shown in figure 2.

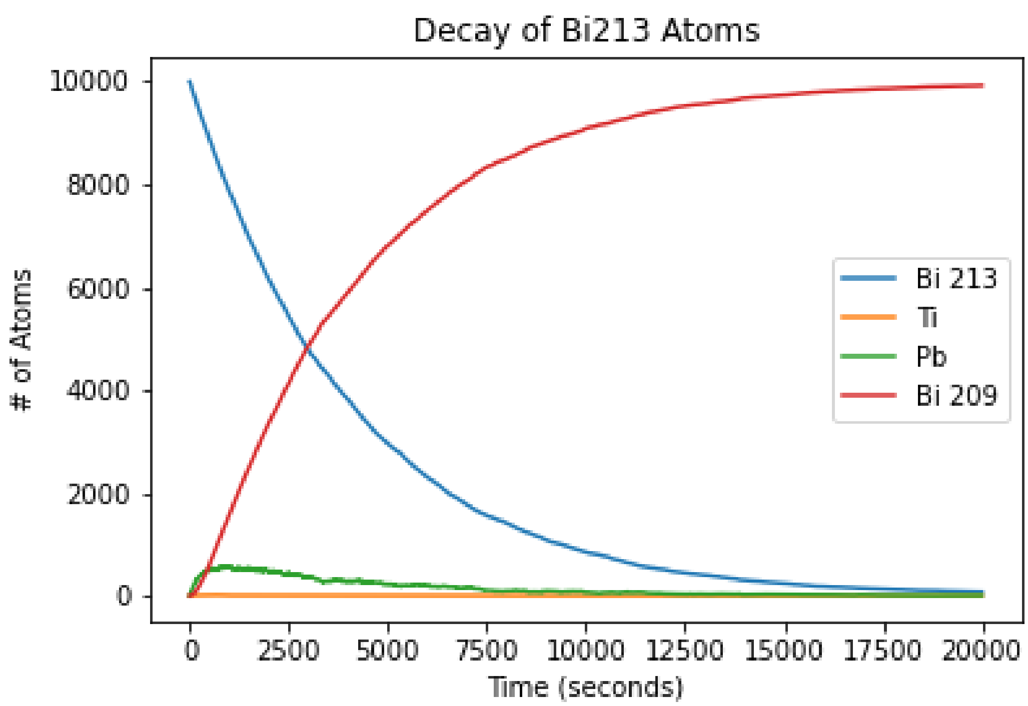

Figure 2: Number of atoms as Bi 213 decays into Bi 209.

3 Problem3

Exercise 10.3
Figure 3 shows the results of the decay of Thallium 208.

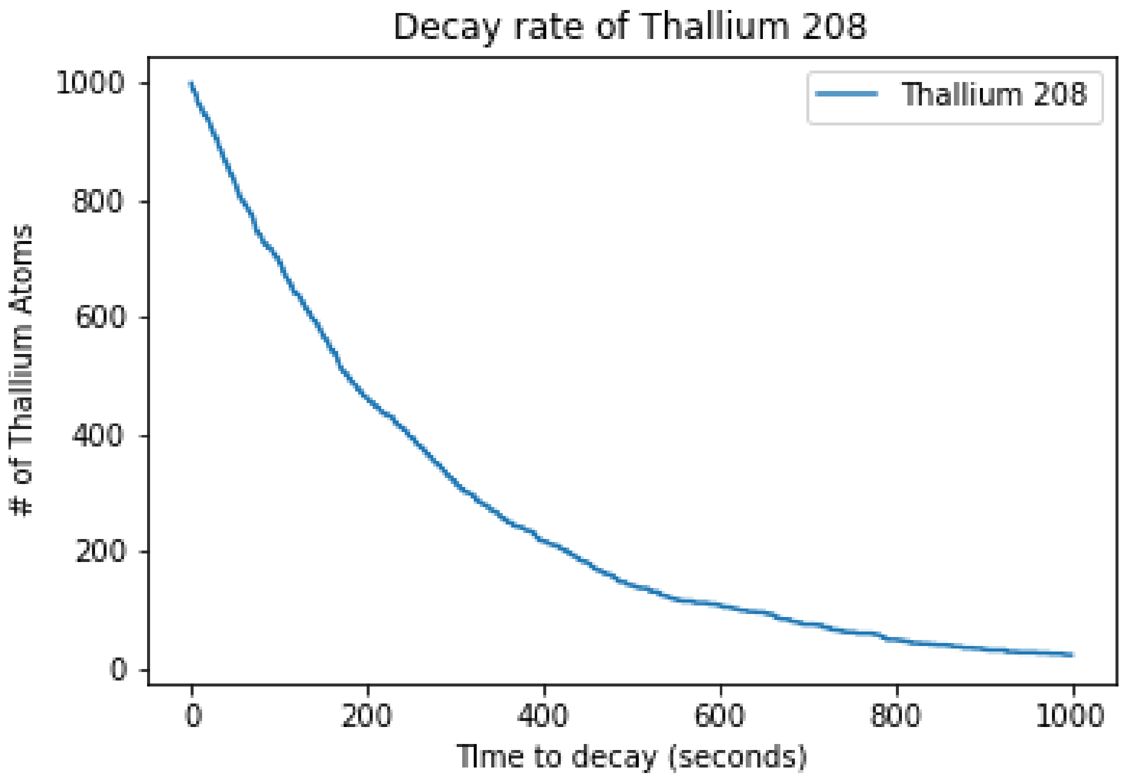

Figure 3: A plot showing the number of thallium atoms remaining corresponding to time.

4 Problem4

The central limit theorem states that the distribution of sample means approximates a normal distribution as the sample size gets larger, regardless of the population's distribution.

Specifically, we are checking the function $y = \frac{1}{N}\sum_{i=0}^{N} x_i$ where x is a random variate distributed as e^{-x}.

To find the mean of y, we will first find the expected value of an exponential distribution, which is E[x] = 1. Using this information, the mean is $y_{\text{mean}} = \frac{1}{N}\sum_{i=0}^{N}\frac{1}{N}\sum_{i=0}^{N} x_i = \frac{1}{N}\sum_{i=0}^{N} E[x] = \frac{1}{N}\sum_{i=0}^{N} 1 = \frac{1}{N} * N = 1$. This means the mean of y would not depend on N.

To find the variance of y, we can do the following: $Var(y) = E(\sigma^2) = Var(\frac{1}{N}\sum_{i=0}^{N} x_i) =$

$\frac{1}{N^2} * Var(\sum_{i=0}^{N} x_i) = \frac{1}{N^2} * N * Var(x_i)$. Since the variance of the exponential distribution is 1, the Variance is $1/N^2 * N = 1/N$.

The figures below show how y approaches a Gaussian as N gets larger. Figure 4, with N set to 7, shows the y distribution as very skewed to the right, while Figure 5, with N set to 1000, shows the y distribution to be significantly closer to a Gaussian.

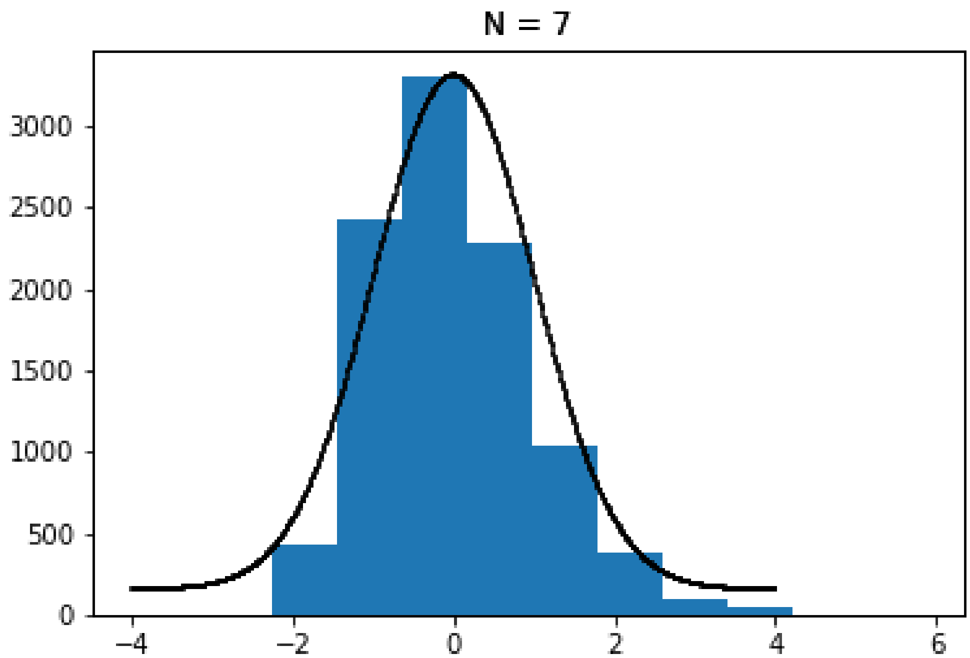

Figure 4: The distribution of y with N = 7

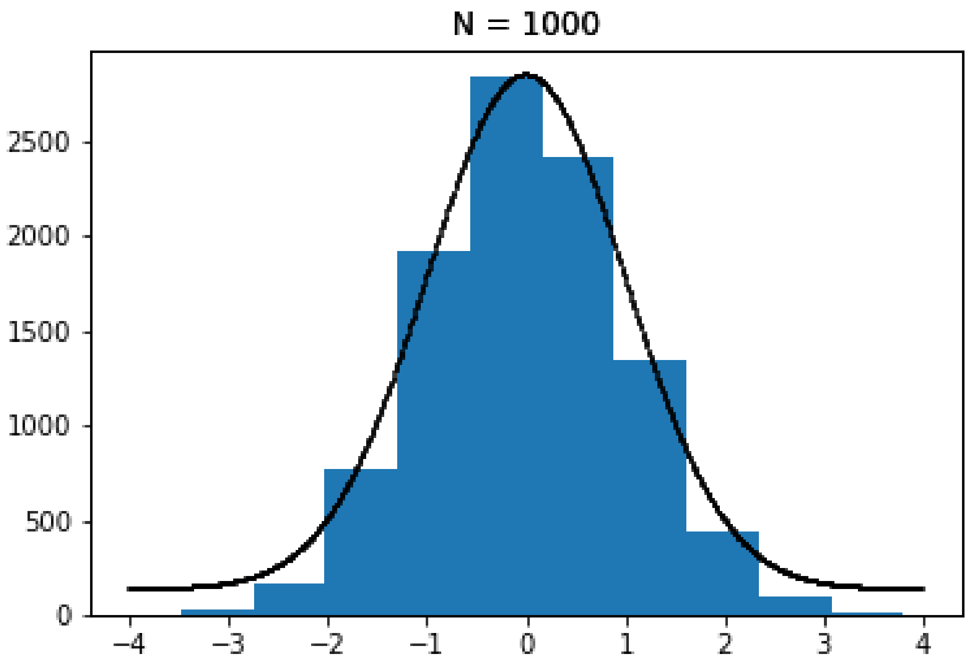

Figure 5: The distribution of y with N = 1000

Figure 6 below shows how the mean, variance, skewness, and kurtosis of the distribution change with respect to N. Both the mean and variance follow the pattern determined above.

Finally, we will estimate when the skewness and kurtosis have reached 1% of their value for N=1. Since the skewness and kurtosis have been approximately 2.00 and 6.00 for N = 1, lines were drawn at approximately 0.2 and 0.6. These correspond to N = 100 for skewness to be approximately 1% of N = 1 and N = 1000 for kurtosis to be approximately 1% of N = 1.

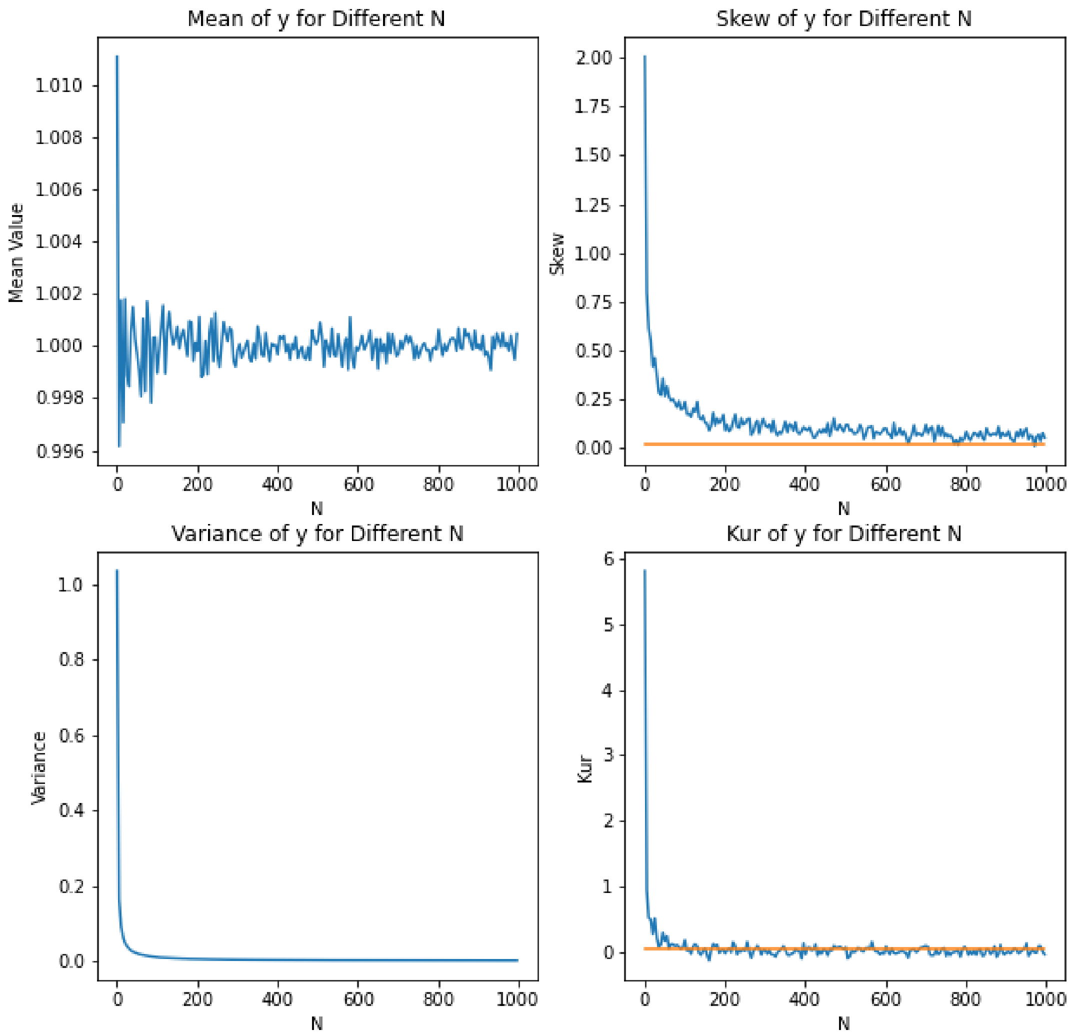

Figure 6: The mean, variance, skew, and kurtosis of y for different N values