Analytical Finance I

Delta Hedging of a Call

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MDU
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GitHub: github.com/MMA707
Dashboard: Hedging Dashboard Backtest.

Summary of Delta Hedging Strategy

1. Introduction and Objective

Definition : Delta Hedging is making the price of an asset held to be neutral for some small change in price.

Objective:

- Study of delta hedging for a vanilla call option.
- Include advanced volatility modeling (GARCH, VIX proxies).
- Simulate performance under realistic transaction cost scenarios.

Black-Scholes Framework

Black-Scholes Model: A mathematical model for pricing European options.

Black-Scholes Formula for Call Option:

$$C = S \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2)$$

$$d_1 = \frac{\ln(S/K) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Delta (Δ): The sensitivity of the option price to changes in the underlying asset price.

$$\Delta = N(d_1)$$

Gamma (Γ): The sensitivity of Δ for a change in the underlying asset price.

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{\Delta t}}$$

Methodology

3. Methodology: Summary

1. **Data Acquisition:**
   - Choose Stock Ticker.
   - Source: Yahoo Finance via `yfinance` library.
2. **Volatility Estimation:**
   - GARCH(1,1) model for estimating daily conditional volatility.
   - Annualized volatility: $\sigma_{annual} = \sigma_{daily} \times \sqrt{252}$.
   - Implied Volatility Proxy for SPY: VIX
3. **Delta Hedging Strategy:**
   - At intialization: Buy a call option of parameters:
     - Strike Price $(K) = 1.1 \times$ Initial Stock Price (S)
     - Time to Maturity $(T) = 6$ months
     - Short quantity of Δ underlying
   - During the life of the option:
     - Rebalance the underlying asset: $n_{day,stock}$.
     - Let option expire to maturity
     - **OR** Rolling the option: $n_{day,option}$

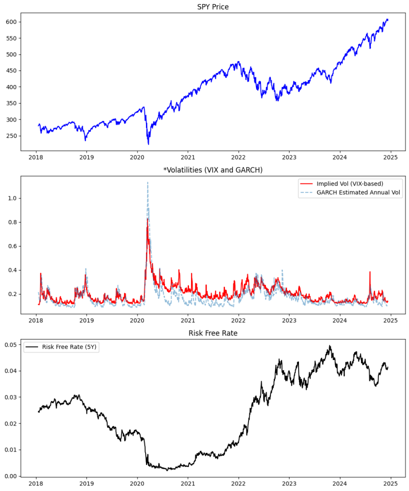
3. Methodology: Data Acquisition, Volatility Estimation

Stock Data:

- SPY daily closing prices.
- Period: Last 6 months.
- Calculated log returns for volatility estimation.

Volatility Estimation:

- Applied GARCH(1,1) model to log returns.
- Smoothed volatility using a rolling window.
- Annualized the conditional volatility estimates.



3. Methodology: Delta Hedging Strategy

Steps Overview

- Initial Setup ($t = 0$): Create hedge.
- Daily ($t \in]0; T[$): Rebalance and/or roll.
- End ($t = T$): Close all.

t=0: Initialization

- Buy 1 call option.
- Compute Δ via Black-Scholes.
- Short $-\Delta$ shares for hedge.

t $\in]0; T[$: Adjustments

- Rebalance hedge: adjust short shares from new computed Δ .
- **OPTIONAL:** Roll option every `n_days_option`, no exercise.

t=T: Final Steps

- Option expires to maturity with payoff $(S_T - K)^+$
- Settle stock and calculate PnL, restart for a new period period T.

Transaction Costs during the life of the Option:

Step	Costs
$t = 0$	Option premium, initial hedging fees
$t \in]0; T[$	Rebalancing fees (shares and/or rolls)
$t = T$	Final settlement fees

3. Methodology: Implementation

Python Implementation Overview

1. **Libraries Used:**
    - `yfinance` for data acquisition.
    - `numpy`, `pandas` for data handling.
    - `scipy.stats` for Normal Cumulative Density Function.
    - `arch` for GARCH modeling.
    - `streamlit` for building the web app tool.
    - `plotly` for plotting.
2. **Key Functions:**
    - `black_scholes`: Computes option price and Δ.
    - `build_initial_positions`: Create the portfolio.
    - `simulate_trades`: Simulates the strategy over time.
3. **Parameters:**
    - Stock ticker, dates, rebalancing frequencies (stock, option), fees, `K_multiplier`, option maturity.

Results

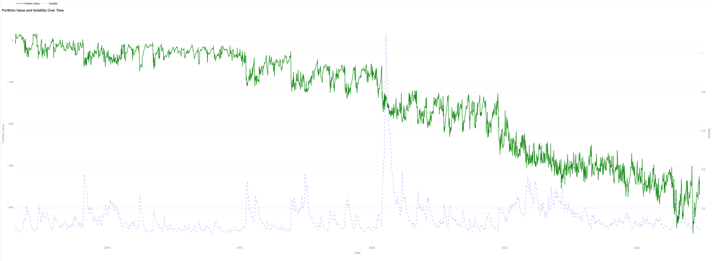
4. Results: Parameters Used

Parameters used for the simulation:

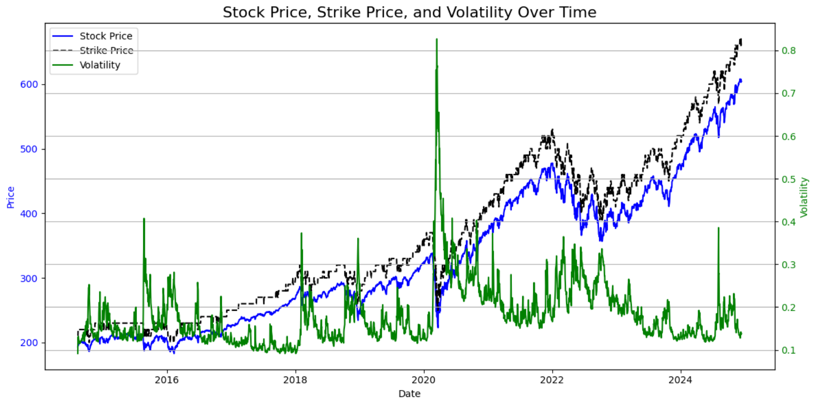
- Backtest Period: 15 Years
- Option Maturity: 6 months
- Volatility Proxy: `GARCH(1,1)`
- Risk free proxy: `FVX`
- Rebalancing Underlying: 1 day
- Rolling Option Position: `Never`
- Fee Rate: 1%

Portfolio Performance:

- Initial Portfolio Value: + $\Delta S - N_c C = 31\$$
- Final Portfolio Value: −$543
- Total Fees Paid: $193
- Cumulative Delta Drift: $0

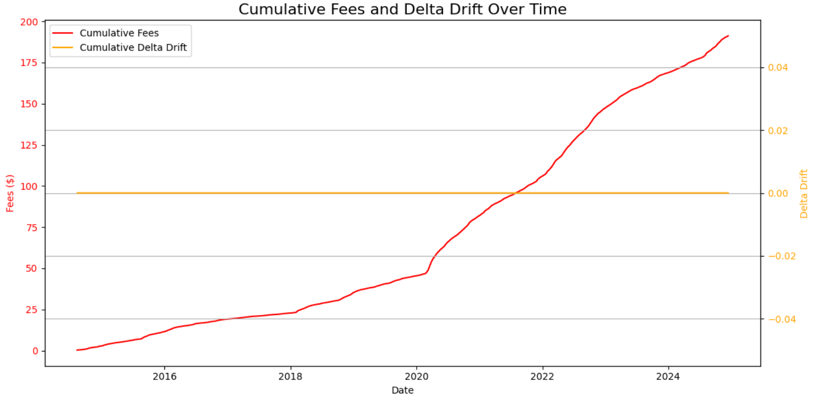


Stock and Strike Price Over Time



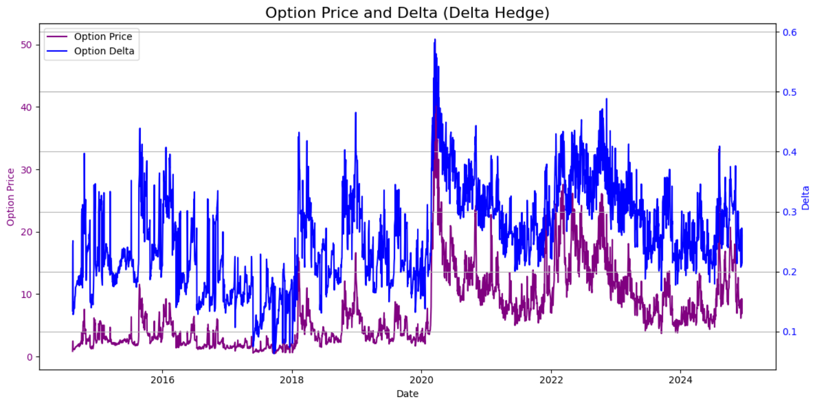
4. Results: Total Fees

Cumulative Fees and Delta Drift



4. Results: Option Price, Option Delta

Delta Over Time of Option



Live Demonstration of the tool

Conclusion

5. Conclusion

Analysis:

- Gradual decline in portfolio value over time.
- Losses were primarily due to transaction costs, time decay (theta), and gamma risk.
- Results fits the theoretical expectations of buying a Call Option under non-constant volatility.

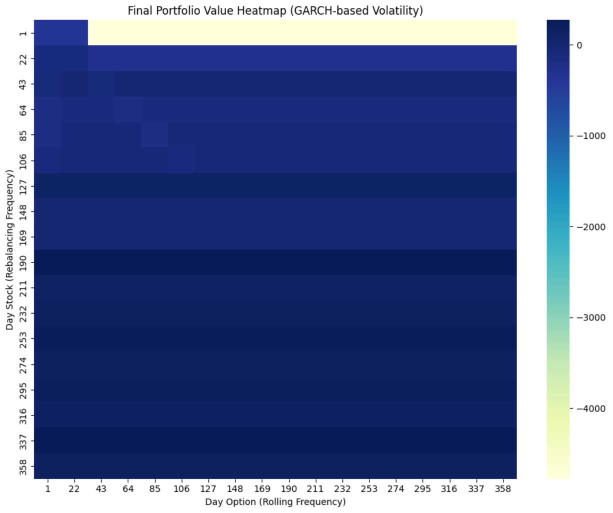
Limitations:

- Simplified transaction cost model.
- Supposed each option existed (of $K = 1.1 \times S$).
- Use of Black-Scholes model (Heston, SABR are better for local volatility).

Appendix

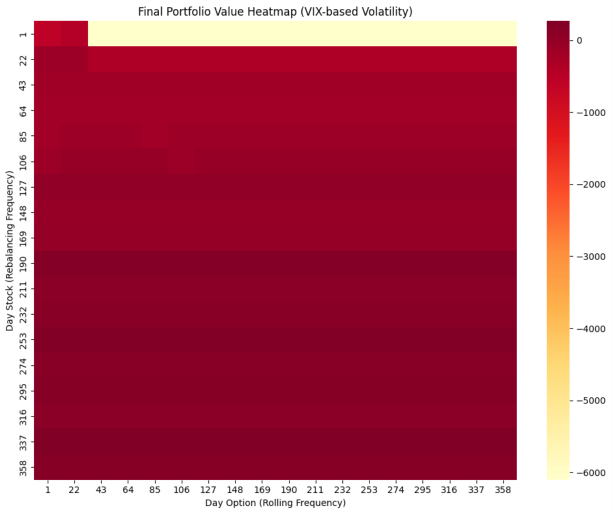
Parameters Sweep: GARCH

Portfolio values based on different rebalancing days



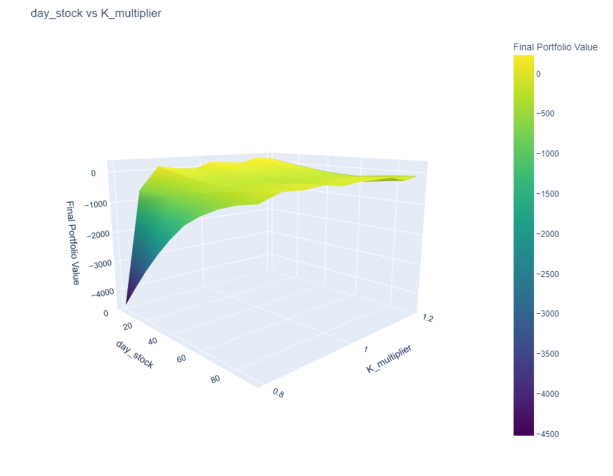
Parameters Sweep: VIX

Portfolio values based on different rebalancing days



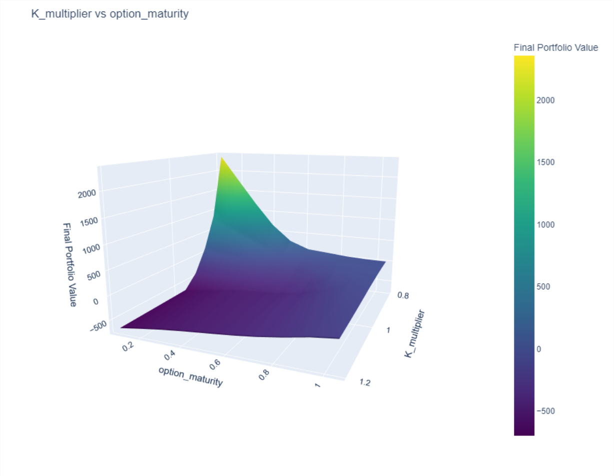
Portfolio Sensibility: Rebalancing Day for Stock — K

Varying K multiplier and Rebalancing Days for Stock



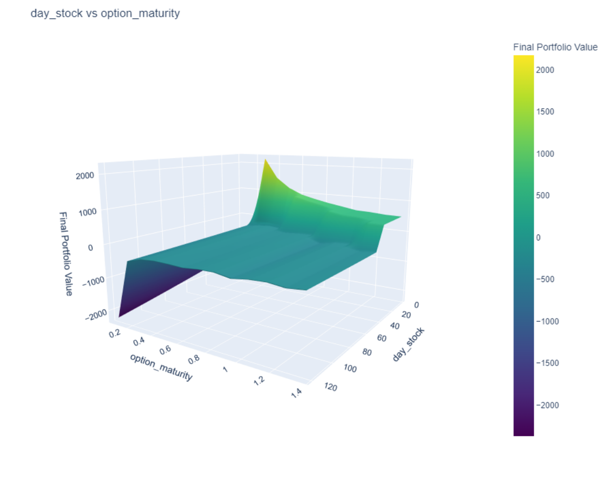
Portfolio Sensibility: Option Maturity — K

Varying K multiplier and Option Maturity



Portfolio Sensibility: Option Maturity — Rebalancing Day for Stock

Varying Rebalancing Days for stock and Option Maturity

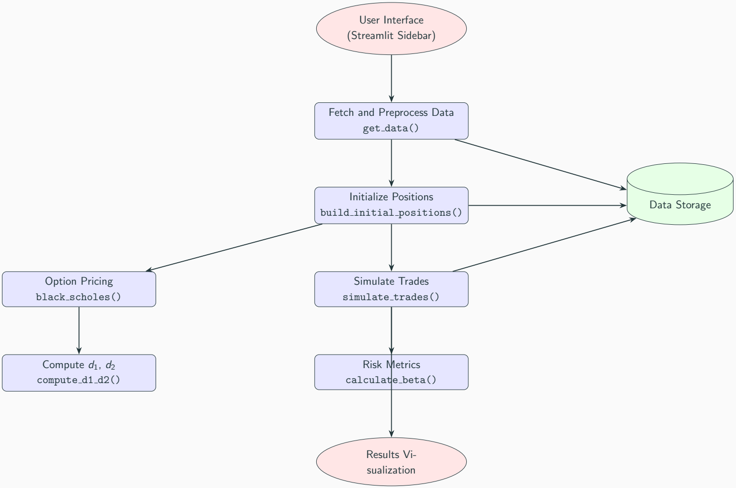


Appendix: Technical Implementation

User Interface & Interaction

- **Sidebar Navigation**
  - Pages: Simulation, Volatility Visualization, Hedging Computations, Positions Overview
  - Parameter inputs: Stock ticker, history period, rebalancing frequency, fees, etc.
- **Visualization**
  - Interactive plots for portfolio value, volatility, option metrics
  - Detailed tables for simulation results and portfolio positions
- **Performance Optimization**
  - Caching of data and computations to enhance responsiveness
  - Efficient handling of large datasets and complex calculations

Appendix: Function Interaction Diagram



Appendix: Technical Implementation

Technologies & Libraries

- **Programming Language:** Python
- **Framework:** Streamlit for interactive web app
- **Data Handling:** Pandas, NumPy
- **Data Sources:** Yahoo Finance (`yfinance`)
- **Statistical Modeling:** `scipy.stats`, `arch` for GARCH models
- **Visualization:** Plotly (`plotly.graph_objects`, `plotly.subplots`)
- **Caching:** Streamlit's `@st.cache_data` for optimized performance

Appendix: Technical Implementation

Core Functionalities

- **Data Fetching & Preprocessing**
  - Retrieves stock, risk-free rate, and volatility proxy data
  - Calculates log returns and handles missing data
- **Volatility Estimation**
  - Supports multiple proxies: GARCH, VIX, VXN, Realized Volatility, EWMA, ATR
  - Implements rolling and exponential weighted calculations
- **Option Pricing & Hedging**
  - Black-Scholes model for option pricing and delta calculation
  - Delta hedging strategy with dynamic rebalancing
- **Trade Simulation**
  - Simulates portfolio over time with configurable parameters
  - Accounts for transaction fees and option maturities