

## CHAPTER 2

# Operational Amplifiers

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## IN THIS CHAPTER YOU WILL LEARN

1. The terminal characteristics of the ideal op amp.
2. How to analyze circuits containing op amps, resistors, and capacitors.
3. How to use op amps to design amplifiers having precise characteristics.
4. How to design more sophisticated op-amp circuits, including summing amplifiers, instrumentation amplifiers, integrators, and differentiators.
5. Important nonideal characteristics of op amps and how these limit the performance of basic op-amp circuits.

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## Introduction

Having learned basic amplifier concepts and terminology, we are now ready to undertake the study of a circuit building block of universal importance: the operational amplifier (op amp). Op amps have been in use for a long time, their initial applications being primarily in the areas of analog computation and sophisticated instrumentation. Early op amps were constructed from discrete components (vacuum tubes and then transistors, and resistors), and their cost was prohibitively high (tens of dollars). In the mid-1960s the first integrated-circuit (IC) op amp was produced. This unit (the  $\mu$ A 709) was made up of a relatively large number of transistors and resistors all on the same silicon chip. Although its characteristics were poor (by today's standards) and its price was still quite high, its appearance signaled a new era in electronic circuit design. Electronics engineers started using op amps in large quantities, which caused their price to drop dramatically. They also demanded better-quality op amps. Semiconductor manufacturers responded quickly, and within the span of a few years, high-quality op amps became available at extremely low prices (tens of cents) from a large number of suppliers.

One of the reasons for the popularity of the op amp is its versatility. As we will shortly see, one can do almost anything with op amps! Equally important is the fact that the IC op amp has characteristics that closely approach the assumed ideal. This implies that it is quite easy to design circuits using the IC op amp. Also, op-amp circuits work at performance levels that are quite close to those predicted theoretically. It is for this reason that we are studying op amps at this early stage. It is expected that by the end of this chapter the reader should be able to successfully design nontrivial circuits using op amps.

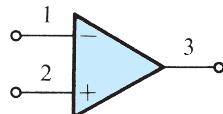
As already implied, an IC op amp is made up of a large number (about 20) of transistors together with resistors, and (usually) one capacitor connected in a rather complex circuit. Since

we have not yet studied transistor circuits, the circuit inside the op amp will not be discussed in this chapter. Rather, we will treat the op amp as a circuit building block and study its terminal characteristics and its applications. This approach is quite satisfactory in many op-amp applications. Nevertheless, for the more difficult and demanding applications it is quite useful to know what is inside the op-amp package. This topic will be studied in Chapter 13. More advanced applications of op amps will appear in later chapters.

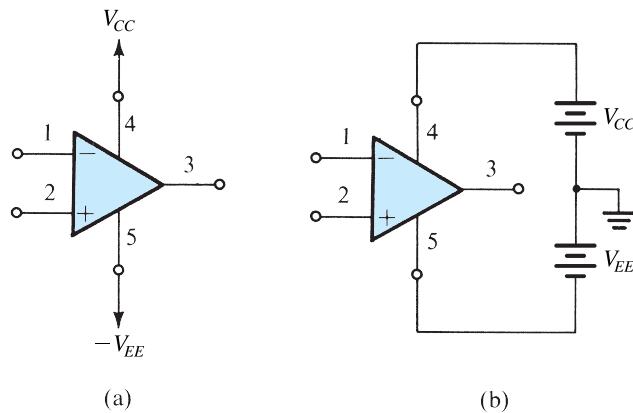
## 2.1 The Ideal Op Amp

### 2.1.1 The Op-Amp Terminals

From a signal point of view the op amp has three terminals: two input terminals and one output terminal. Figure 2.1 shows the symbol we shall use to represent the op amp. Terminals 1 and 2 are input terminals, and terminal 3 is the output terminal. As explained in Section 1.4, amplifiers require dc power to operate. Most IC op amps require two dc power supplies, as shown in Fig. 2.2. Two terminals, 4 and 5, are brought out of the op-amp package and connected to a positive voltage  $V_{CC}$  and a negative voltage  $-V_{EE}$ , respectively. In Fig. 2.2(b) we explicitly show the two dc power supplies as batteries with a common ground. It is interesting to note that the reference grounding point in op-amp circuits is just the common terminal of the two power supplies; that is, no terminal of the op-amp package is physically connected to ground. In what follows we will not, for simplicity, explicitly show the op-amp power supplies.



**Figure 2.1** Circuit symbol for the op amp.



**Figure 2.2** The op amp shown connected to dc power supplies.

In addition to the three signal terminals and the two power-supply terminals, an op amp may have other terminals for specific purposes. These other terminals can include terminals for frequency compensation and terminals for offset nulling; both functions will be explained in later sections.

### EXERCISE

- 2.1** What is the minimum number of terminals required by a single op amp? What is the minimum number of terminals required on an integrated-circuit package containing four op amps (called a quad op amp)?

**Ans.** 5; 14

### 2.1.2 Function and Characteristics of the Ideal Op Amp

We now consider the circuit function of the op amp. The op amp is designed to sense the difference between the voltage signals applied at its two input terminals (i.e., the quantity  $v_2 - v_1$ ), multiply this by a number  $A$ , and cause the resulting voltage  $A(v_2 - v_1)$  to appear at output terminal 3. Thus  $v_3 = A(v_2 - v_1)$ . Here it should be emphasized that when we talk about the voltage at a terminal we mean the voltage between that terminal and ground; thus  $v_1$  means the voltage applied between terminal 1 and ground.

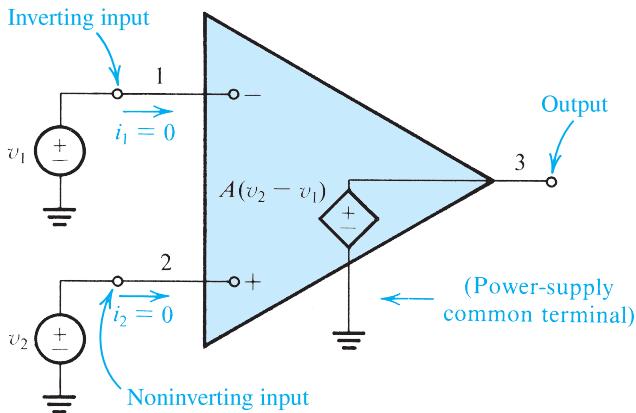
The ideal op amp is not supposed to draw any input current; that is, the signal current into terminal 1 and the signal current into terminal 2 are both zero. In other words, *the input impedance of an ideal op amp is supposed to be infinite*.

How about the output terminal 3? This terminal is supposed to act as the output terminal of an ideal voltage source. That is, the voltage between terminal 3 and ground will always be equal to  $A(v_2 - v_1)$ , independent of the current that may be drawn from terminal 3 into a load impedance. In other words, *the output impedance of an ideal op amp is supposed to be zero*.

Putting together all of the above, we arrive at the equivalent circuit model shown in Fig. 2.3. Note that the output is in phase with (has the same sign as)  $v_2$  and is out of phase with (has the opposite sign of)  $v_1$ . For this reason, input terminal 1 is called the **inverting input terminal** and is distinguished by a “–” sign, while input terminal 2 is called the **noninverting input terminal** and is distinguished by a “+” sign.

As can be seen from the above description, the op amp responds only to the *difference* signal  $v_2 - v_1$  and hence ignores any signal *common* to both inputs. That is, if  $v_1 = v_2 = 1\text{ V}$ , then the output will (ideally) be zero. We call this property **common-mode rejection**, and we conclude that an ideal op amp has zero common-mode gain or, equivalently, infinite common-mode rejection. We will have more to say about this point later. For the time being note that the op amp is a **differential-input, single-ended-output** amplifier, with the latter term referring to the fact that the output appears between terminal 3 and ground.<sup>1</sup>

<sup>1</sup>Some op amps are designed to have differential outputs. This topic will not be discussed in this book. Rather, we confine ourselves here to single-ended-output op amps, which constitute the vast majority of commercially available op amps.



**Figure 2.3** Equivalent circuit of the ideal op amp.

Furthermore, gain  $A$  is called the **differential gain**, for obvious reasons. Perhaps not so obvious is another name that we will attach to  $A$ : the **open-loop gain**. The reason for this name will become obvious later on when we “close the loop” around the op amp and define another gain, the closed-loop gain.

An important characteristic of op amps is that they are **direct-coupled** or **dc amplifiers**, where dc stands for direct-coupled (it could equally well stand for direct current, since a direct-coupled amplifier is one that amplifies signals whose frequency is as low as zero). The fact that op amps are direct-coupled devices will allow us to use them in many important applications. Unfortunately, though, the direct-coupling property can cause some serious practical problems, as will be discussed in a later section.

How about bandwidth? The ideal op amp has a gain  $A$  that remains constant down to zero frequency and up to infinite frequency. That is, ideal op amps will amplify signals of any frequency with equal gain, and are thus said to have *infinite bandwidth*.

We have discussed all of the properties of the ideal op amp except for one, which in fact is the most important. This has to do with the value of  $A$ . *The ideal op amp should have a gain  $A$  whose value is very large and ideally infinite.* One may justifiably ask: If the gain  $A$  is infinite, how are we going to use the op amp? The answer is very simple: In almost all applications the op amp will *not* be used alone in a so-called open-loop configuration. Rather, we will use other components to apply feedback to close the loop around the op amp, as will be illustrated in detail in Section 2.2.

For future reference, Table 2.1 lists the characteristics of the ideal op amp.

**Table 2.1** Characteristics of the Ideal Op Amp

1. Infinite input impedance
2. Zero output impedance
3. Zero common-mode gain or, equivalently, infinite common-mode rejection
4. Infinite open-loop gain  $A$
5. Infinite bandwidth

### 2.1.3 Differential and Common-Mode Signals

The differential input signal  $v_{Id}$  is simply the difference between the two input signals  $v_1$  and  $v_2$ ; that is,

$$v_{Id} = v_2 - v_1 \quad (2.1)$$

The common-mode input signal  $v_{Icm}$  is the average of the two input signals  $v_1$  and  $v_2$ ; namely,

$$v_{Icm} = \frac{1}{2}(v_1 + v_2) \quad (2.2)$$

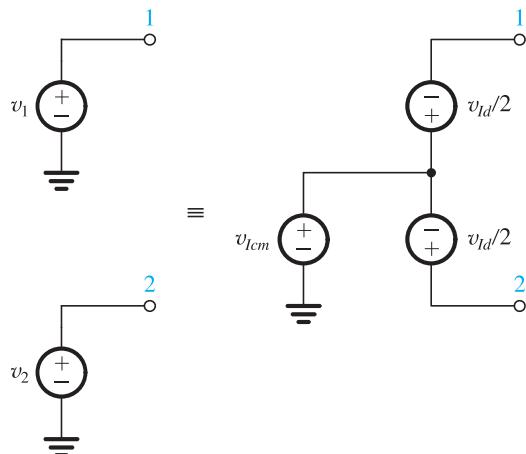
Equations (2.1) and (2.2) can be used to express the input signals  $v_1$  and  $v_2$  in terms of their differential and common-mode components as follows:

$$v_1 = v_{Icm} - v_{Id}/2 \quad (2.3)$$

and

$$v_2 = v_{Icm} + v_{Id}/2 \quad (2.4)$$

These equations can in turn lead to the pictorial representation in Fig. 2.4.



**Figure 2.4** Representation of the signal sources  $v_1$  and  $v_2$  in terms of their differential and common-mode components.

#### EXERCISES

- 2.2** Consider an op amp that is ideal except that its open-loop gain  $A = 10^3$ . The op amp is used in a feedback circuit, and the voltages appearing at two of its three signal terminals are measured. In each of the following cases, use the measured values to find the expected value of the voltage at the third terminal. Also give the differential and common-mode input signals in each case. (a)  $v_2 = 0$  V and  $v_3 = 2$  V; (b)  $v_2 = +5$  V and  $v_3 = -10$  V; (c)  $v_1 = 1.002$  V and  $v_2 = 0.998$  V; (d)  $v_1 = -3.6$  V and  $v_3 = -3.6$  V.
- Ans.** (a)  $v_1 = -0.002$  V,  $v_{Id} = 2$  mV,  $v_{Icm} = -1$  mV; (b)  $v_1 = +5.01$  V,  $v_{Id} = -10$  mV,  $v_{Icm} = 5.005 \approx 5$  V; (c)  $v_3 = -4$  V,  $v_{Id} = -4$  mV,  $v_{Icm} = 1$  V; (d)  $v_2 = -3.6036$  V,  $v_{Id} = -3.6$  mV,  $v_{Icm} \approx -3.6$  V

- 2.3 The internal circuit of a particular op amp can be modeled by the circuit shown in Fig. E2.3. Express  $v_3$  as a function of  $v_1$  and  $v_2$ . For the case  $G_m = 10 \text{ mA/V}$ ,  $R = 10 \text{ k}\Omega$ , and  $\mu = 100$ , find the value of the open-loop gain  $A$ .

**Ans.**  $v_3 = \mu G_m R (v_2 - v_1)$ ;  $A = 10,000 \text{ V/V}$  or  $80 \text{ dB}$

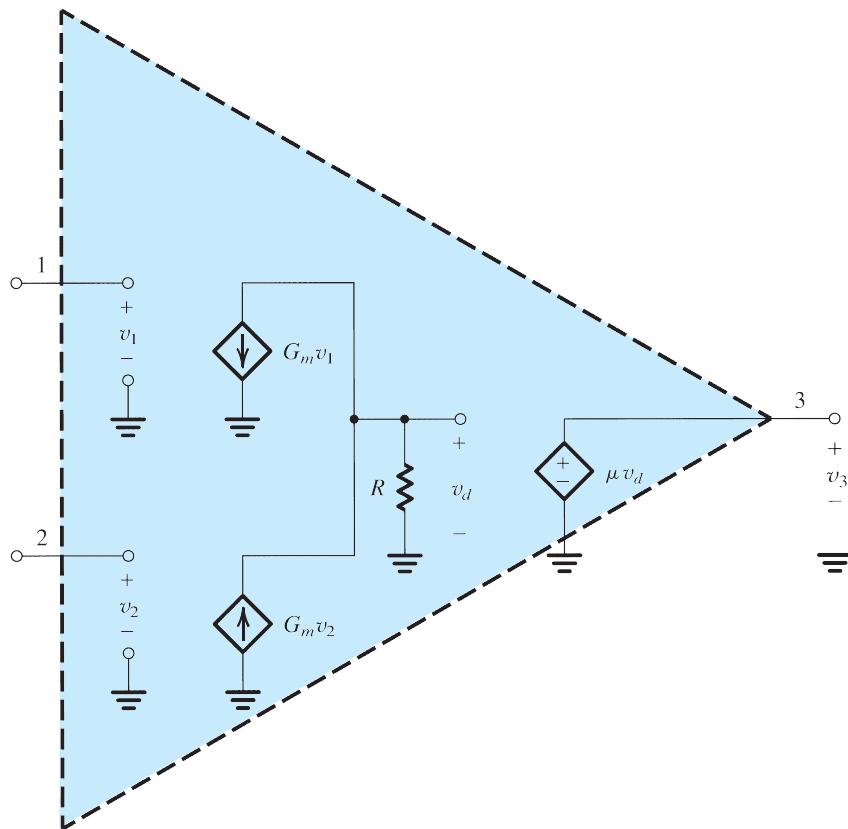
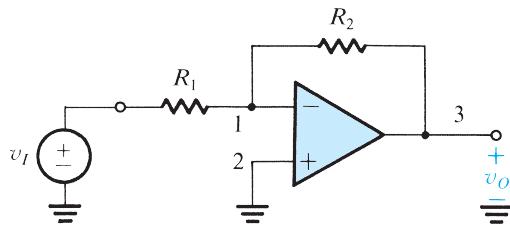


Figure E2.3

## 2.2 The Inverting Configuration

As mentioned above, op amps are not used alone; rather, the op amp is connected to passive components in a feedback circuit. There are two such basic circuit configurations employing an op amp and two resistors: the inverting configuration, which is studied in this section, and the noninverting configuration, which we shall study in the next section.

Figure 2.5 shows the inverting configuration. It consists of one op amp and two resistors  $R_1$  and  $R_2$ . Resistor  $R_2$  is connected from the output terminal of the op amp, terminal 3, *back* to the *inverting* or *negative* input terminal, terminal 1. We speak of  $R_2$  as applying **negative feedback**; if  $R_2$  were connected between terminals 3 and 2 we would have called this **positive feedback**. Note also that  $R_2$  *closes the loop* around the op amp. In addition to adding  $R_2$ , we have grounded terminal 2 and connected a resistor  $R_1$  between terminal 1 and an input signal source



**Figure 2.5** The inverting closed-loop configuration.

with a voltage  $v_I$ . The output of the overall circuit is taken at terminal 3 (i.e., between terminal 3 and ground). Terminal 3 is, of course, a convenient point from which to take the output, since the impedance level there is ideally zero. Thus the voltage  $v_O$  will not depend on the value of the current that might be supplied to a load impedance connected between terminal 3 and ground.

### 2.2.1 The Closed-Loop Gain

We now wish to analyze the circuit in Fig. 2.5 to determine the **closed-loop gain**  $G$ , defined as

$$G \equiv \frac{v_O}{v_I}$$

We will do so assuming the op amp to be ideal. Figure 2.6(a) shows the equivalent circuit, and the analysis proceeds as follows: The gain  $A$  is very large (ideally infinite). If we assume that the circuit is “working” and producing a finite output voltage at terminal 3, then the voltage between the op-amp input terminals should be negligibly small and ideally zero. Specifically, if we call the output voltage  $v_O$ , then, by definition,

$$v_2 - v_1 = \frac{v_O}{A} = 0$$

It follows that the voltage at the inverting input terminal ( $v_1$ ) is given by  $v_1 = v_2$ . That is, because the gain  $A$  approaches infinity, the voltage  $v_1$  approaches and ideally equals  $v_2$ . We speak of this as the two input terminals “tracking each other in potential.” We also speak of a “virtual short circuit” that exists between the two input terminals. Here the word *virtual* should be emphasized, and one should *not* make the mistake of physically shorting terminals 1 and 2 together while analyzing a circuit. A **virtual short circuit** means that whatever voltage is at 2 will automatically appear at 1 because of the infinite gain  $A$ . But terminal 2 happens to be connected to ground; thus  $v_2 = 0$  and  $v_1 = 0$ . We speak of terminal 1 as being a **virtual ground**—that is, having zero voltage but not physically connected to ground.

Now that we have determined  $v_1$  we are in a position to apply Ohm’s law and find the current  $i_1$  through  $R_1$  (see Fig. 2.6) as follows:

$$i_1 = \frac{v_I - v_1}{R_1} = \frac{v_I - 0}{R_1} = \frac{v_I}{R_1}$$

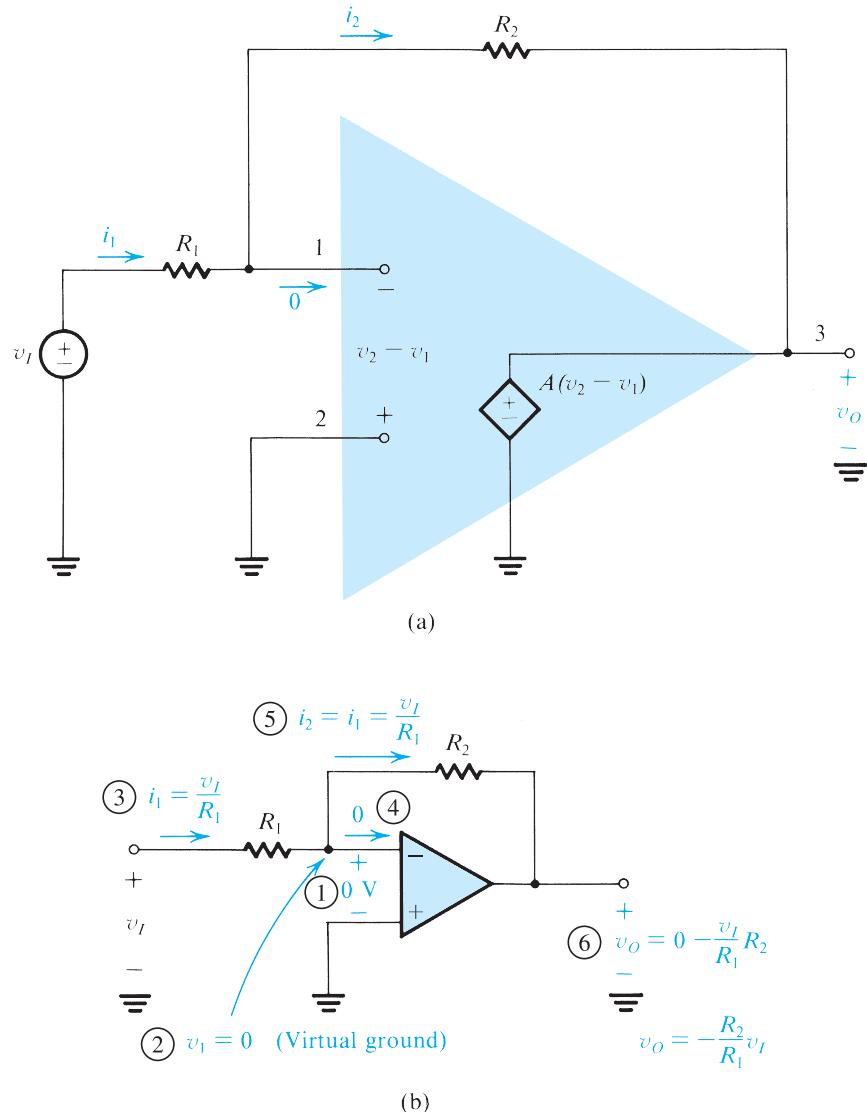
Where will this current go? It cannot go into the op amp, since the ideal op amp has an infinite input impedance and hence draws zero current. It follows that  $i_1$  will have to flow through  $R_2$  to the low-impedance terminal 3. We can then apply Ohm’s law to  $R_2$  and determine  $v_O$ ; that is,

$$\begin{aligned} v_O &= v_1 - i_1 R_2 \\ &= 0 - \frac{v_I}{R_1} R_2 \end{aligned}$$

Thus,

$$\frac{v_O}{v_I} = -\frac{R_2}{R_1}$$





**Figure 2.6** Analysis of the inverting configuration. The circled numbers indicate the order of the analysis steps.

which is the required closed-loop gain. Figure 2.6(b) illustrates these steps and indicates by the circled numbers the order in which the analysis is performed.

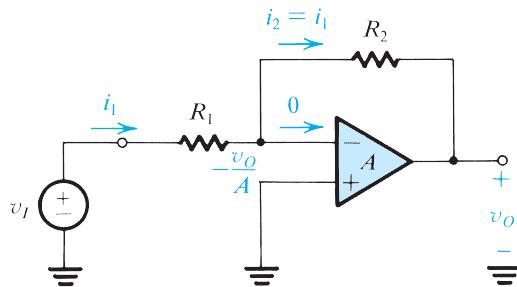
We thus see that the closed-loop gain is simply the ratio of the two resistances  $R_2$  and  $R_1$ . The minus sign means that the closed-loop amplifier provides signal inversion. Thus if  $R_2/R_1 = 10$  and we apply at the input ( $v_I$ ) a sine-wave signal of 1 V peak-to-peak, then the output  $v_O$  will be a sine wave of 10 V peak-to-peak and phase-shifted 180° with respect to the input sine wave. Because of the minus sign associated with the closed-loop gain, this configuration is called the **inverting configuration**.

The fact that the closed-loop gain depends entirely on external passive components (resistors  $R_1$  and  $R_2$ ) is very significant. It means that we can make the closed-loop gain as accurate as we want by selecting passive components of appropriate accuracy. It also means that the closed-loop gain is (ideally) independent of the op-amp gain. This is a dramatic illustration of negative feedback: We started out with an amplifier having very large gain  $A$ , and through applying negative feedback we have obtained a closed-loop gain  $R_2/R_1$  that is much smaller than  $A$  but is stable and predictable. That is, we are trading gain for accuracy.

## 2.2.2 Effect of Finite Open-Loop Gain

The points just made are more clearly illustrated by deriving an expression for the closed-loop gain under the assumption that the op-amp open-loop gain  $A$  is finite. Figure 2.7 shows the analysis. If we denote the output voltage  $v_o$ , then the voltage between the two input terminals of the op amp will be  $v_o/A$ . Since the positive input terminal is grounded, the voltage at the negative input terminal must be  $-v_o/A$ . The current  $i_1$  through  $R_1$  can now be found from

$$i_1 = \frac{v_I - (-v_o/A)}{R_1} = \frac{v_I + v_o/A}{R_1}$$



**Figure 2.7** Analysis of the inverting configuration taking into account the finite open-loop gain of the op amp.

The infinite input impedance of the op amp forces the current  $i_1$  to flow entirely through  $R_2$ . The output voltage  $v_o$  can thus be determined from

$$\begin{aligned} v_o &= -\frac{v_o}{A} - i_1 R_2 \\ &= -\frac{v_o}{A} - \left( \frac{v_I + v_o/A}{R_1} \right) R_2 \end{aligned}$$

Collecting terms, the closed-loop gain  $G$  is found as

$$G \equiv \frac{v_o}{v_I} = \frac{-R_2/R_1}{1 + (1 + R_2/R_1)/A} \quad (2.5)$$

We note that as  $A$  approaches  $\infty$ ,  $G$  approaches the ideal value of  $-R_2/R_1$ . Also, from Fig. 2.7 we see that as  $A$  approaches  $\infty$ , the voltage at the inverting input terminal approaches zero. This is the virtual-ground assumption we used in our earlier analysis when the op amp was

assumed to be ideal. Finally, note that Eq. (2.5) in fact indicates that to minimize the dependence of the closed-loop gain  $G$  on the value of the open-loop gain  $A$ , we should make

$$1 + \frac{R_2}{R_1} \ll A$$

### Example 2.1

Consider the inverting configuration with  $R_1 = 1 \text{ k}\Omega$  and  $R_2 = 100 \text{ k}\Omega$ , that is, having an ideal closed-loop gain of  $-100$ .

- (a) Find the closed-loop gain for the cases  $A = 10^3, 10^4$ , and  $10^5$ . In each case determine the percentage error in the magnitude of  $G$  relative to the ideal value of  $R_2/R_1$  (obtained with  $A = \infty$ ). Also determine the voltage  $v_i$  that appears at the inverting input terminal when  $v_t = 0.1 \text{ V}$ .
- (b) If the open-loop gain  $A$  changes from 100,000 to 50,000 (i.e., drops by 50%), what is the corresponding percentage change in the magnitude of the closed-loop gain  $G$ ?

#### Solution

- (a) Substituting the given values in Eq. (2.5), we obtain the values given in the following table, where the percentage error  $\epsilon$  is defined as

$$\epsilon \equiv \frac{|G| - (R_2/R_1)}{(R_2/R_1)} \times 100$$

The values of  $v_i$  are obtained from  $v_i = -v_o/A = Gv_t/A$  with  $v_t = -0.1 \text{ V}$ .

| $A$    | $ G $ | $\epsilon$ | $v_i$    |
|--------|-------|------------|----------|
| $10^3$ | 90.83 | -9.17%     | -9.08 mV |
| $10^4$ | 99.00 | -1.00%     | -0.99 mV |
| $10^5$ | 99.90 | -0.10%     | -0.10 mV |

- (b) Using Eq. (2.5), we find that for  $A = 50,000$ ,  $|G| = 99.80$ . Thus a -50% change in the open-loop gain results in a change in  $|G|$  from 99.90 to 99.80, which is only -0.1%!

### 2.2.3 Input and Output Resistances

Assuming an ideal op amp with infinite open-loop gain, the input resistance of the closed-loop inverting amplifier of Fig. 2.5 is simply equal to  $R_1$ . This can be seen from Fig. 2.6(b), where

$$R_i \equiv \frac{v_i}{i_i} = \frac{v_i}{v_t/R_1} = R_1$$

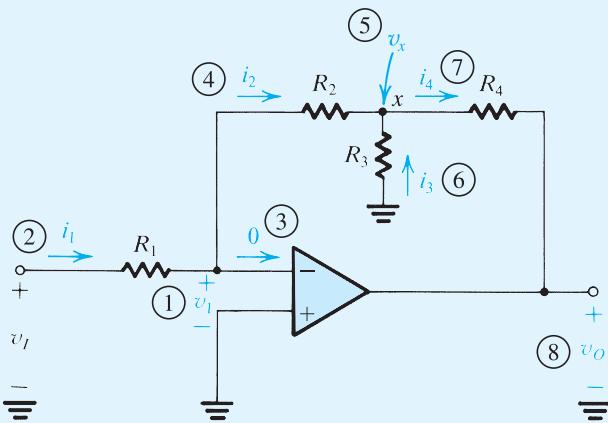
Now recall that in Section 1.5 we learned that the amplifier input resistance forms a voltage divider with the resistance of the source that feeds the amplifier. Thus, to avoid the loss of signal strength, voltage amplifiers are required to have high input resistance. In the case of the inverting op-amp configuration we are studying, to make  $R_i$  high we should select a high value for  $R_1$ . However, if the required gain  $R_2/R_1$  is also high, then  $R_2$  could become impractically large

(e.g., greater than a few megohms). We may conclude that the inverting configuration suffers from a low input resistance. A solution to this problem is discussed in Example 2.2 below.

Since the output of the inverting configuration is taken at the terminals of the ideal voltage source  $A(v_2 - v_1)$  (see Fig. 2.6a), it follows that the output resistance of the closed-loop amplifier is zero.

### Example 2.2

Assuming the op amp to be ideal, derive an expression for the closed-loop gain  $v_o/v_I$  of the circuit shown in Fig. 2.8. Use this circuit to design an inverting amplifier with a gain of 100 and an input resistance of  $1 \text{ M}\Omega$ . Assume that for practical reasons it is required not to use resistors greater than  $1 \text{ M}\Omega$ . Compare your design with that based on the inverting configuration of Fig. 2.5.



**Figure 2.8** Circuit for Example 2.2. The circled numbers indicate the sequence of the steps in the analysis.

### Solution

The analysis begins at the inverting input terminal of the op amp, where the voltage is

$$v_1 = \frac{-v_o}{A} = \frac{-v_o}{\infty} = 0$$

Here we have assumed that the circuit is “working” and producing a finite output voltage  $v_o$ . Knowing  $v_1$ , we can determine the current  $i_1$  as follows:

$$i_1 = \frac{v_I - v_1}{R_1} = \frac{v_I - 0}{R_1} = \frac{v_I}{R_1}$$

Since zero current flows into the inverting input terminal, all of  $i_1$  will flow through  $R_2$ , and thus

$$i_2 = i_1 = \frac{v_I}{R_1}$$

Now we can determine the voltage at node  $x$ :

$$v_x = v_1 - i_2 R_2 = 0 - \frac{v_I}{R_1} R_2 = -\frac{R_2}{R_1} v_I$$

**Example 2.2** continued

This in turn enables us to find the current  $i_3$ :

$$i_3 = \frac{0 - v_x}{R_3} = \frac{R_2}{R_1 R_3} v_I$$

Next, a node equation at  $x$  yields  $i_4$ :

$$i_4 = i_2 + i_3 = \frac{v_I}{R_1} + \frac{R_2}{R_1 R_3} v_I$$

Finally, we can determine  $v_O$  from

$$\begin{aligned} v_O &= v_x - i_4 R_4 \\ &= -\frac{R_2}{R_1} v_I - \left( \frac{v_I}{R_1} + \frac{R_2}{R_1 R_3} v_I \right) R_4 \end{aligned}$$

Thus the voltage gain is given by

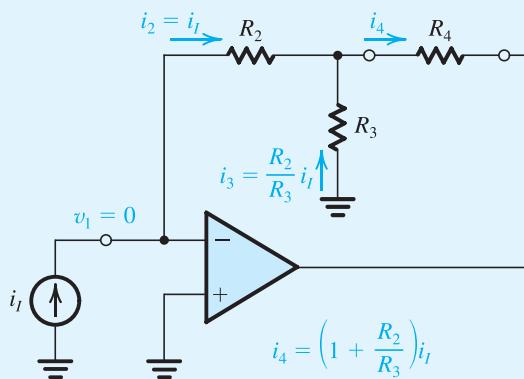
$$\frac{v_O}{v_I} = - \left[ \frac{R_2}{R_1} + \frac{R_4}{R_1} \left( 1 + \frac{R_2}{R_3} \right) \right]$$

which can be written in the form

$$\frac{v_O}{v_I} = - \frac{R_2}{R_1} \left( 1 + \frac{R_4}{R_2} + \frac{R_4}{R_3} \right)$$

Now, since an input resistance of  $1 \text{ M}\Omega$  is required, we select  $R_1 = 1 \text{ M}\Omega$ . Then, with the limitation of using resistors no greater than  $1 \text{ M}\Omega$ , the maximum value possible for the first factor in the gain expression is 1 and is obtained by selecting  $R_2 = 1 \text{ M}\Omega$ . To obtain a gain of  $-100$ ,  $R_3$  and  $R_4$  must be selected so that the second factor in the gain expression is 100. If we select the maximum allowed (in this example) value of  $1 \text{ M}\Omega$  for  $R_4$ , then the required value of  $R_3$  can be calculated to be  $10.2 \text{ k}\Omega$ . Thus this circuit utilizes three  $1\text{-M}\Omega$  resistors and a  $10.2\text{-k}\Omega$  resistor. In comparison, if the inverting configuration were used with  $R_1 = 1 \text{ M}\Omega$  we would have required a feedback resistor of  $100 \text{ M}\Omega$ , an impractically large value!

Before leaving this example it is insightful to inquire into the mechanism by which the circuit is able to realize a large voltage gain without using large resistances in the feedback path. Toward that end, observe that because of the virtual ground at the inverting input terminal of the op amp,  $R_2$  and  $R_3$  are in effect in parallel. Thus, by making  $R_3$  lower than  $R_2$  by, say, a factor  $k$  (i.e., where  $k > 1$ ),  $R_3$  is forced to carry a current  $k$ -times that in  $R_2$ . Thus, while  $i_2 = i_I$ ,  $i_3 = ki_I$  and  $i_4 = (k+1)i_I$ . It is the current multiplication by a factor of  $(k+1)$  that enables a large voltage drop to develop across  $R_4$  and hence a large  $v_O$  without using a large value for  $R_4$ . Notice also that the current through  $R_4$  is independent of the value of  $R_4$ . It follows that the circuit can be used as a current amplifier as shown in Fig. 2.9.



**Figure 2.9** A current amplifier based on the circuit of Fig. 2.8. The amplifier delivers its output current to  $R_4$ . It has a current gain of  $(1 + R_2/R_3)$ , a zero input resistance, and an infinite output resistance. The load ( $R_4$ ), however, must be floating (i.e., neither of its two terminals can be connected to ground).

## EXERCISES

- D2.4** Use the circuit of Fig. 2.5 to design an inverting amplifier having a gain of  $-10$  and an input resistance of  $100\text{ k}\Omega$ . Give the values of  $R_1$  and  $R_2$ .

**Ans.**  $R_1 = 100\text{ k}\Omega$ ;  $R_2 = 1\text{ M}\Omega$

- 2.5** The circuit shown in Fig. E2.5(a) can be used to implement a transresistance amplifier (see Table 1.1 in Section 1.5). Find the value of the input resistance  $R_i$ , the transresistance  $R_m$ , and the output resistance  $R_o$  of the transresistance amplifier. If the signal source shown in Fig. E2.5(b) is connected to the input of the transresistance amplifier, find the amplifier output voltage.

**Ans.**  $R_i = 0$ ;  $R_m = -10\text{ k}\Omega$ ;  $R_o = 0$ ;  $v_o = -5\text{ V}$

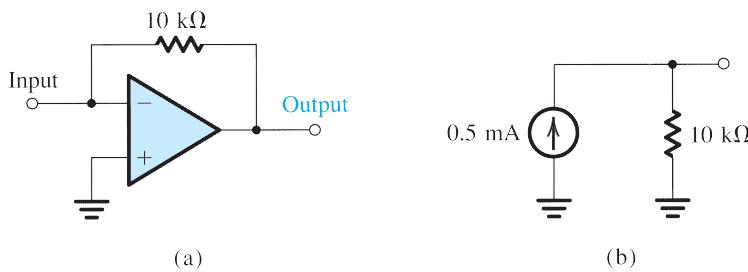


Figure E2.5

- 2.6** For the circuit in Fig. E2.6 determine the values of  $v_1, i_1, i_2, v_o, i_L$ , and  $i_o$ . Also determine the voltage gain  $v_o/v_1$ , current gain  $i_L/i_1$ , and power gain  $P_o/P_i$ .

**Ans.** 0 V; 1 mA; 1 mA;  $-10\text{ V}$ ;  $-10\text{ mA}$ ;  $-11\text{ mA}$ ;  $-10\text{ V/V}$  (20 dB),  $-10\text{ A/A}$  (20 dB); 100 W/W (20 dB)

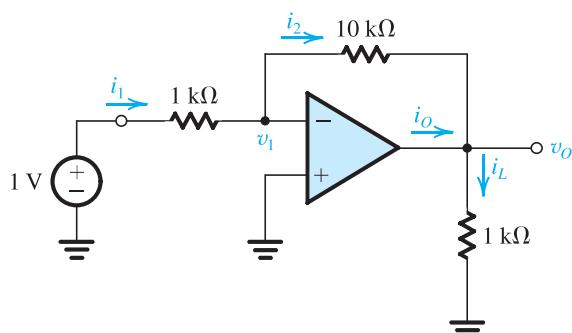


Figure E2.6

## 2.2.4 An Important Application—The Weighted Summer

A very important application of the inverting configuration is the weighted-summer circuit shown in Fig. 2.10. Here we have a resistance  $R_f$  in the negative-feedback path (as before), but we have a number of input signals  $v_1, v_2, \dots, v_n$  each applied to a corresponding resistor

$R_1, R_2, \dots, R_n$ , which are connected to the inverting terminal of the op amp. From our previous discussion, the ideal op amp will have a virtual ground appearing at its negative input terminal. Ohm's law then tells us that the currents  $i_1, i_2, \dots, i_n$  are given by

$$i_1 = \frac{v_1}{R_1}, \quad i_2 = \frac{v_2}{R_2}, \quad \dots, \quad i_n = \frac{v_n}{R_n}$$

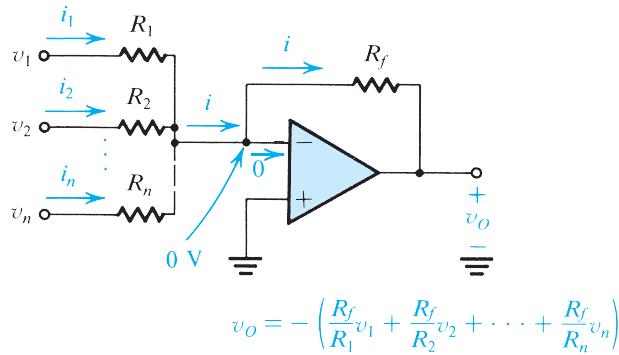


Figure 2.10 A weighted summer.

All these currents sum together to produce the current  $i$ ,

$$i = i_1 + i_2 + \dots + i_n \quad (2.6)$$

which will be forced to flow through  $R_f$  (since no current flows into the input terminals of an ideal op amp). The output voltage  $v_o$  may now be determined by another application of Ohm's law,

$$v_o = 0 - iR_f = -iR_f$$

Thus,

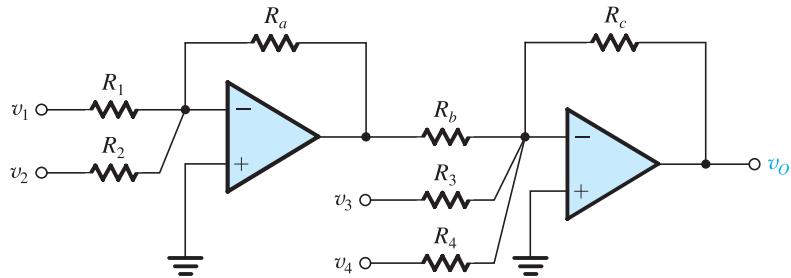
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$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \dots + \frac{R_f}{R_n}v_n\right) \quad (2.7)$$

That is, the output voltage is a weighted sum of the input signals  $v_1, v_2, \dots, v_n$ . This circuit is therefore called a **weighted summer**. Note that each summing coefficient may be independently adjusted by adjusting the corresponding "feed-in" resistor ( $R_1$  to  $R_n$ ). This nice property, which greatly simplifies circuit adjustment, is a direct consequence of the virtual ground that exists at the inverting op-amp terminal. As the reader will soon come to appreciate, virtual grounds are extremely "handy." In the weighted summer of Fig. 2.10 all the summing coefficients must be of the same sign. The need occasionally arises for summing signals with opposite signs. Such a function can be implemented, however, using two op amps as shown in Fig. 2.11. Assuming ideal op amps, it can be easily shown that the output voltage is given by

$$v_o = v_1\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) + v_2\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) - v_3\left(\frac{R_c}{R_3}\right) - v_4\left(\frac{R_c}{R_4}\right) \quad (2.8)$$

Weighted summers are utilized in a variety of applications including in the design of audio systems, where they can be used in mixing signals originating from different musical instruments.



**Figure 2.11** A weighted summer capable of implementing summing coefficients of both signs.

## EXERCISES

**D2.7** Design an inverting op-amp circuit to form the weighted sum  $v_o$  of two inputs  $v_1$  and  $v_2$ . It is required that  $v_o = -(v_1 + 5v_2)$ . Choose values for  $R_1$ ,  $R_2$ , and  $R_f$  so that for a maximum output voltage of 10 V the current in the feedback resistor will not exceed 1 mA.

**Ans.** A possible choice:  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 2 \text{ k}\Omega$ , and  $R_f = 10 \text{ k}\Omega$

**D2.8** Use the idea presented in Fig. 2.11 to design a weighted summer that provides

$$v_o = 2v_1 + v_2 - 4v_3$$

**Ans.** A possible choice:  $R_1 = 5 \text{ k}\Omega$ ,  $R_2 = 10 \text{ k}\Omega$ ,  $R_a = 10 \text{ k}\Omega$ ,  $R_b = 10 \text{ k}\Omega$ ,  $R_3 = 2.5 \text{ k}\Omega$ ,  $R_c = 10 \text{ k}\Omega$

## 2.3 The Noninverting Configuration

The second closed-loop configuration we shall study is shown in Fig. 2.12. Here the input signal  $v_I$  is applied directly to the positive input terminal of the op amp while one terminal of  $R_1$  is connected to ground.

### 2.3.1 The Closed-Loop Gain

Analysis of the noninverting circuit to determine its closed-loop gain ( $v_o/v_I$ ) is illustrated in Fig. 2.13. Again the order of the steps in the analysis is indicated by circled numbers. Assuming that the op amp is ideal with infinite gain, a virtual short circuit exists between its two input terminals. Hence the difference input signal is

$$v_{Id} = \frac{v_o}{A} = 0 \quad \text{for } A = \infty$$

Thus the voltage at the inverting input terminal will be equal to that at the noninverting input terminal, which is the applied voltage  $v_I$ . The current through  $R_1$  can then be determined as  $v_I/R_1$ . Because of the infinite input impedance of the op amp, this current will flow through  $R_2$ , as shown in Fig. 2.13. Now the output voltage can be determined from

$$v_o = v_I + \left( \frac{v_I}{R_1} \right) R_2$$

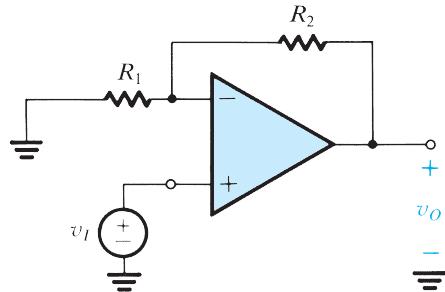


Figure 2.12 The noninverting configuration.

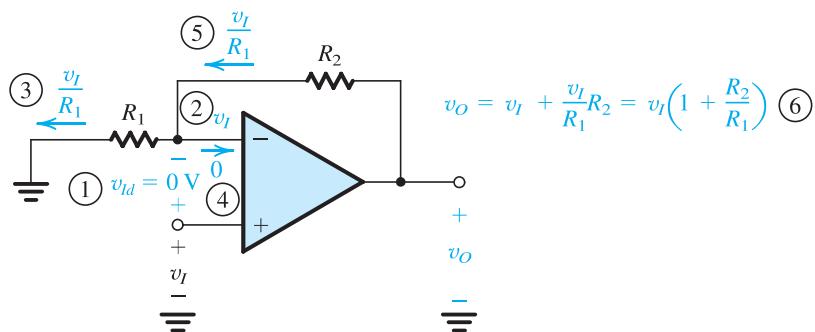


Figure 2.13 Analysis of the noninverting circuit. The sequence of the steps in the analysis is indicated by the circled numbers.

which yields

$$\frac{v_O}{v_I} = 1 + \frac{R_2}{R_1} \quad (2.9)$$

Further insight into the operation of the noninverting configuration can be obtained by considering the following: Since the current into the op-amp inverting input is zero, the circuit composed of  $R_1$  and  $R_2$  acts in effect as a voltage divider feeding a fraction of the output voltage back to the inverting input terminal of the op amp; that is,

$$v_I = v_O \left( \frac{R_1}{R_1 + R_2} \right) \quad (2.10)$$

Then the infinite op-amp gain and the resulting virtual short circuit between the two input terminals of the op amp forces this voltage to be equal to that applied at the positive input terminal; thus,

$$v_O \left( \frac{R_1}{R_1 + R_2} \right) = v_I$$

which yields the gain expression given in Eq. (2.9).

This is an appropriate point to reflect further on the action of the negative feedback present in the noninverting circuit of Fig. 2.12. Let  $v_I$  increase. Such a change in  $v_I$  will cause  $v_{Id}$  to increase, and  $v_O$  will correspondingly increase as a result of the high (ideally infinite) gain of the op amp. However, a fraction of the increase in  $v_O$  will be fed back to the inverting input terminal of the op amp through the  $(R_1, R_2)$  voltage divider. The result of this feedback will be to counteract the increase in  $v_{Id}$ , driving  $v_{Id}$  back to zero, albeit at a higher value of  $v_O$  that corresponds to the increased value of  $v_I$ . This **degenerative** action of negative feedback gives it the alternative name **degenerative feedback**. Finally, note that the argument above applies equally well if  $v_I$  decreases. A formal and detailed study of feedback is presented in Chapter 11.

### 2.3.2 Effect of Finite Open-Loop Gain

As we have done for the inverting configuration, we now consider the effect of the finite op-amp open-loop gain  $A$  on the gain of the noninverting configuration. Assuming the op amp to be ideal except for having a finite open-loop gain  $A$ , it can be shown that the closed-loop gain of the noninverting amplifier circuit of Fig. 2.12 is given by

$$G \equiv \frac{v_o}{v_i} = \frac{1 + (R_2/R_1)}{1 + \frac{1 + (R_2/R_1)}{A}} \quad (2.11)$$

Observe that the denominator is identical to that for the case of the inverting configuration (Eq. 2.5). This is no coincidence; it is a result of the fact that both the inverting and the noninverting configurations have the same feedback loop, which can be readily seen if the input signal source is eliminated (i.e., short-circuited). The numerators, however, are different, for the numerator gives the ideal or nominal closed-loop gain ( $-R_2/R_1$  for the inverting configuration, and  $1 + R_2/R_1$  for the noninverting configuration). Finally, we note (with reassurance) that the gain expression in Eq. (2.11) reduces to the ideal value for  $A = \infty$ . In fact, it approximates the ideal value for

$$A \gg 1 + \frac{R_2}{R_1}$$

This is the same condition as in the inverting configuration, except that here the quantity on the right-hand side is the nominal closed-loop gain. The expressions for the actual and ideal values of the closed-loop gain  $G$  in Eqs. (2.11) and (2.9), respectively, can be used to determine the percentage error in  $G$  resulting from the finite op-amp gain  $A$  as

$$\text{Percent gain error} = -\frac{1 + (R_2/R_1)}{A + 1 + (R_2/R_1)} \times 100 \quad (2.12)$$

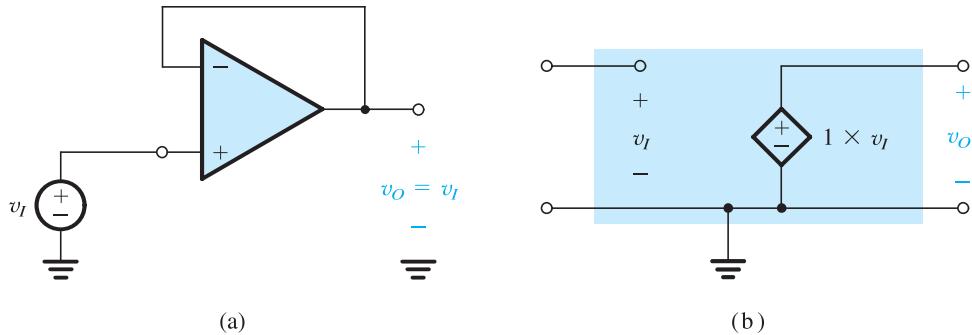
Thus, as an example, if an op amp with an open-loop gain of 1000 is used to design a noninverting amplifier with a nominal closed-loop gain of 10, we would expect the closed-loop gain to be about 1% below the nominal value.

### 2.3.3 Input and Output Resistance

The gain of the noninverting configuration is positive—hence the name *noninverting*. The input impedance of this closed-loop amplifier is ideally infinite, since no current flows into the positive input terminal of the op amp. The output of the noninverting amplifier is taken at the terminals of the ideal voltage source  $A(v_2 - v_1)$  (see the op-amp equivalent circuit in Fig. 2.3), and thus the output resistance of the noninverting configuration is zero.

### 2.3.4 The Voltage Follower

The property of high input impedance is a very desirable feature of the noninverting configuration. It enables using this circuit as a buffer amplifier to connect a source with a high impedance to a low-impedance load. We discussed the need for buffer amplifiers in Section 1.5. In many applications the buffer amplifier is not required to provide any voltage gain; rather, it is used mainly as an impedance transformer or a power amplifier. In such cases we may make  $R_2 = 0$  and  $R_1 = \infty$  to obtain the **unity-gain amplifier** shown in Fig. 2.14(a). This circuit is commonly referred to as a **voltage follower**, since the output “follows” the input. In the ideal case,  $v_o = v_i, R_{in} = \infty, R_{out} = 0$ , and the follower has the equivalent circuit shown in Fig. 2.14(b).



**Figure 2.14** (a) The unity-gain buffer or follower amplifier. (b) Its equivalent circuit model.

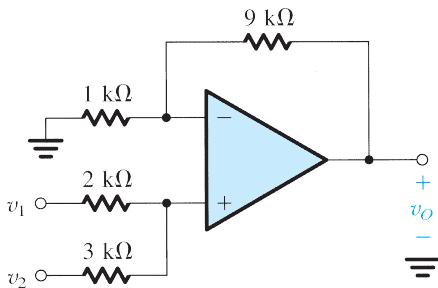
Since in the voltage-follower circuit the entire output is fed back to the inverting input, the circuit is said to have 100% negative feedback. The infinite gain of the op amp then acts to make  $v_{Id} = 0$  and hence  $v_o = v_i$ . Observe that the circuit is elegant in its simplicity!

Since the noninverting configuration has a gain greater than or equal to unity, depending on the choice of  $R_2/R_1$ , some prefer to call it "a follower with gain."

# EXERCISES

- 2.9** Use the superposition principle to find the output voltage of the circuit shown in Fig. E2.9.

**Ans.**



## Figure E2.9

- 2.10** If in the circuit of Fig. E2.9 the  $1-k\Omega$  resistor is disconnected from ground and connected to a third signal source  $v_3$ , use superposition to determine  $v_o$  in terms of  $v_1$ ,  $v_2$ , and  $v_3$ .

**Ans.**  $v_o = 6v_1 + 4v_2 - 9v_3$

- D2.11** Design a noninverting amplifier with a gain of 2. At the maximum output voltage of 10 V the current in the voltage divider is to be 10  $\mu$ A.

**Ans.**  $R_1 = R_2 = 0.5 \text{ M}\Omega$

- 2.12** (a) Show that if the op amp in the circuit of Fig. 2.12 has a finite open-loop gain  $A$ , then the closed-loop gain is given by Eq. (2.11). (b) For  $R_1 = 1 \text{ k}\Omega$  and  $R_2 = 9 \text{ k}\Omega$  find the percentage

deviation  $\epsilon$  of the closed

- deviation  $\epsilon$  of the closed-loop gain from the ideal value of  $(1 + R_2/R_1)$  for the cases  $A = 10^3, 10^4$ , and  $10^5$ . For  $v_i = 1$  V, find in each case the voltage between the two input terminals of the op amp.

**Ans.**  $\epsilon = -1\%, -0.1\%, -0.01\%$ ;  $v_2 - v_1 = 9.9$  mV, 1 mV, 0.1 mV

- 2.13** For the circuit in Fig. E2.13 find the values of  $i_L$ ,  $v_1$ ,  $i_1$ ,  $i_2$ ,  $v_o$ ,  $i_L$ , and  $i_o$ . Also find the voltage gain  $v_o/v_I$ , the current gain  $i_L/i_I$ , and the power gain  $P_o/P_I$ .

**Ans.** 0; 1 V; 1 mA; 1 mA; 10 V; 10 mA; 11 mA; 10 V/V (20 dB);  $\infty$ ;  $\infty$

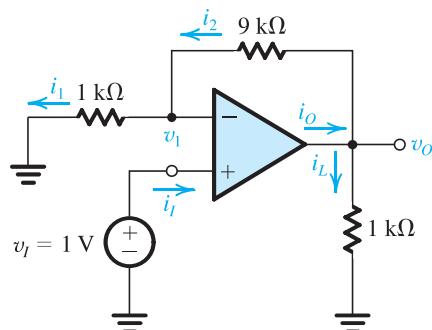


Figure E2.13

- 2.14** It is required to connect a transducer having an open-circuit voltage of 1 V and a source resistance of  $1 \text{ M}\Omega$  to a load of  $1\text{-k}\Omega$  resistance. Find the load voltage if the connection is done (a) directly, and (b) through a unity-gain voltage follower.

**Ans.** (a) 1 mV; (b) 1 V

## 2.4 Difference Amplifiers

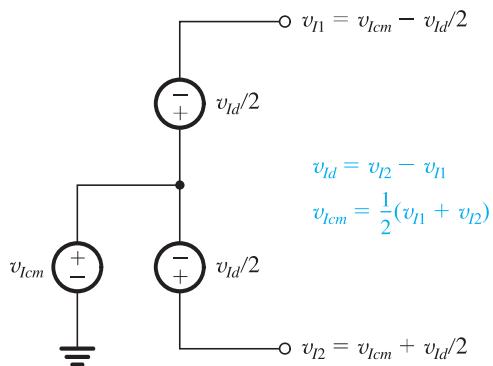
Having studied the two basic configurations of op-amp circuits together with some of their direct applications, we are now ready to consider a somewhat more involved but very important application. Specifically, we shall study the use of op amps to design difference or differential amplifiers.<sup>2</sup> A difference amplifier is one that responds to the difference between the two signals applied at its input and ideally rejects signals that are common to the two inputs. The representation of signals in terms of their differential and common-mode components was given in Fig. 2.4. It is repeated here in Fig. 2.15 with slightly different symbols to serve as the input signals for the difference amplifiers we are about to design. Although ideally the difference amplifier will amplify only the differential input signal  $v_{Id}$  and reject completely the common-mode input signal  $v_{Icm}$ , practical circuits will have an output voltage  $v_O$  given by

$$v_O = A_d v_{Id} + A_{cm} v_{Icm} \quad (2.13)$$

where  $A_d$  denotes the amplifier differential gain and  $A_{cm}$  denotes its common-mode gain (ideally zero). The efficacy of a differential amplifier is measured by the degree of its rejection of common-mode signals in preference to differential signals. This is usually quantified by a measure known as the **common-mode rejection ratio (CMRR)**, defined as

$$\text{CMRR} = 20 \log \frac{|A_d|}{|A_{cm}|} \quad (2.14)$$

<sup>2</sup>The terms *difference* and *differential* are usually used to describe somewhat different amplifier types. For our purposes at this point, the distinction is not sufficiently significant. We will be more precise near the end of this section.



**Figure 2.15** Representing the input signals to a differential amplifier in terms of their differential and common-mode components.

The need for difference amplifiers arises frequently in the design of electronic systems, especially those employed in instrumentation. As a common example, consider a transducer providing a small (e.g., 1 mV) signal between its two output terminals while each of the two wires leading from the transducer terminals to the measuring instrument may have a large interference signal (e.g., 1 V) relative to the circuit ground. The instrument front end obviously needs a difference amplifier.

Before we proceed any further we should address a question that the reader might have: The op amp is itself a difference amplifier; why not just use an op amp? The answer is that the very high (ideally infinite) gain of the op amp makes it impossible to use by itself. Rather, as we did before, we have to devise an appropriate feedback network to connect to the op amp to create a circuit whose closed-loop gain is finite, predictable, and stable.

### 2.4.1 A Single-Op-Amp Difference Amplifier

Our first attempt at designing a difference amplifier is motivated by the observation that the gain of the noninverting amplifier configuration is positive,  $(1 + R_2/R_1)$ , while that of the inverting configuration is negative,  $(-R_2/R_1)$ . Combining the two configurations together is then a step in the right direction—namely, getting the difference between two input signals. Of course, we have to make the two gain magnitudes equal in order to reject common-mode signals. This, however, can be easily achieved by attenuating the positive input signal to reduce the gain of the positive path from  $(1 + R_2/R_1)$  to  $(R_2/R_1)$ . The resulting circuit would then look like that shown in Fig. 2.16, where the attenuation in the positive input path is achieved by the voltage divider  $(R_3, R_4)$ . The proper ratio of this voltage divider can be determined from

$$\frac{R_4}{R_4 + R_3} \left( 1 + \frac{R_2}{R_1} \right) = \frac{R_2}{R_1}$$

which can be put in the form

$$\frac{R_4}{R_4 + R_3} = \frac{R_2}{R_2 + R_1}$$

This condition is satisfied by selecting

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} \quad (2.15)$$

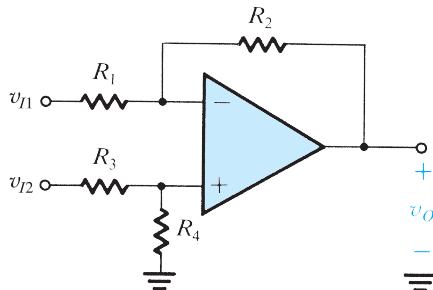


Figure 2.16 A difference amplifier.

This completes our work. However, we have perhaps proceeded a little too fast! Let's step back and verify that the circuit in Fig. 2.16 with  $R_3$  and  $R_4$  selected according to Eq. (2.15) does in fact function as a difference amplifier. Specifically, we wish to determine the output voltage  $v_o$  in terms of  $v_{I1}$  and  $v_{I2}$ . Toward that end, we observe that the circuit is linear, and thus we can use superposition.

To apply superposition, we first reduce  $v_{I2}$  to zero—that is, ground the terminal to which  $v_{I2}$  is applied—and then find the corresponding output voltage, which will be due entirely to  $v_{I1}$ . We denote this output voltage  $v_{O1}$ . Its value may be found from the circuit in Fig. 2.17(a), which we recognize as that of the inverting configuration. The existence of  $R_3$  and  $R_4$  does not affect the gain expression, since no current flows through either of them. Thus,

$$v_{O1} = -\frac{R_2}{R_1} v_{I1}$$

Next, we reduce  $v_{I1}$  to zero and evaluate the corresponding output voltage  $v_{O2}$ . The circuit will now take the form shown in Fig. 2.17(b), which we recognize as the noninverting configuration with an additional voltage divider, made up of  $R_3$  and  $R_4$ , connected to the input  $v_{I2}$ . The output voltage  $v_{O2}$  is therefore given by

$$v_{O2} = v_{I2} \frac{R_4}{R_3 + R_4} \left( 1 + \frac{R_2}{R_1} \right) = \frac{R_2}{R_1} v_{I2}$$

where we have utilized Eq. (2.15).

The superposition principle tells us that the output voltage  $v_o$  is equal to the sum of  $v_{O1}$  and  $v_{O2}$ . Thus we have

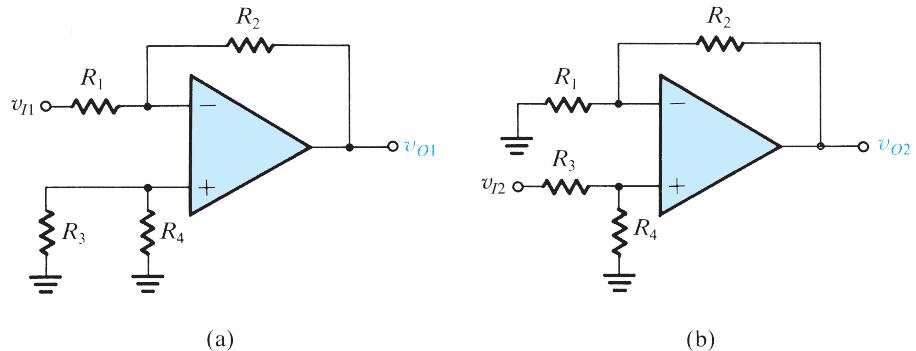
$$v_o = \frac{R_2}{R_1} (v_{I2} - v_{I1}) = \frac{R_2}{R_1} v_{Id} \quad (2.16)$$

Thus, as expected, the circuit acts as a difference amplifier with a differential gain  $A_d$  of

$$A_d = \frac{R_2}{R_1} \quad (2.17)$$

Of course this is predicated on the op amp being ideal and furthermore on the selection of  $R_3$  and  $R_4$  so that their ratio matches that of  $R_1$  and  $R_2$  (Eq. 2.15). To make this matching requirement a little easier to satisfy, we usually select

$$R_3 = R_1 \quad \text{and} \quad R_4 = R_2$$



**Figure 2.17** Application of superposition to the analysis of the circuit of Fig. 2.16.

Let's next consider the circuit with only a common-mode signal applied at the input, as shown in Fig. 2.18. The figure also shows some of the analysis steps. Thus,

$$i_1 = \frac{1}{R_1} \left[ v_{Icm} - \frac{R_4}{R_4 + R_3} v_{Icm} \right] \\ = v_{Icm} \frac{R_3}{R_4 + R_3} \frac{1}{R_1} \quad (2.18)$$

The output voltage can now be found from

$$v_o = \frac{R_4}{R_4 + R_3} v_{lcm} - i_2 R_2$$

Substituting  $i_2 = i_1$  and for  $i_1$  from Eq. (2.18),

$$v_O = \frac{R_4}{R_4 + R_3} v_{Icm} - \frac{R_2}{R_1} \frac{R_3}{R_4 + R_3} v_{Icm}$$

$$= \frac{R_4}{R_4 + R_3} \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right) v_{Icm}$$

Thus,

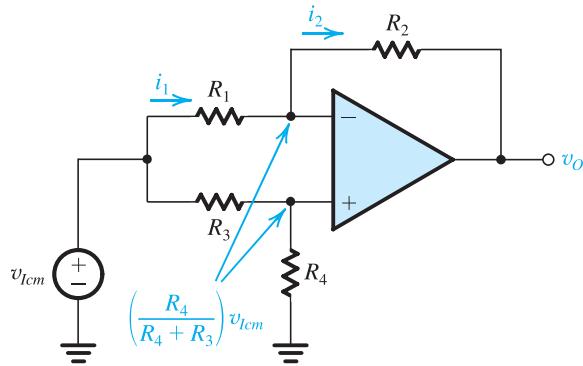
$$A_{cm} \equiv \frac{v_O}{v_{I_{cm}}} = \left( \frac{R_4}{R_4 + R_3} \right) \left( 1 - \frac{R_2}{R_1} \frac{R_3}{R_4} \right) \quad (2.19)$$

For the design with the resistor ratios selected according to Eq. (2.15), we obtain

$$A_{cm} = 0$$

as expected. Note, however, that any mismatch in the resistance ratios can make  $A_{cm}$  nonzero, and hence CMRR finite.

In addition to rejecting common-mode signals, a difference amplifier is usually required to have a high input resistance. To find the input resistance between the two input terminals



**Figure 2.18** Analysis of the difference amplifier to determine its common-mode gain  $A_{cm} \equiv v_o/v_{icm}$ .

(i.e., the resistance seen by  $v_{ld}$ ), called the **differential input resistance**  $R_{id}$ , consider Fig. 2.19. Here we have assumed that the resistors are selected so that

$$R_3 = R_1 \quad \text{and} \quad R_4 = R_2$$

Now

$$R_{id} \equiv \frac{v_{ld}}{i_l}$$

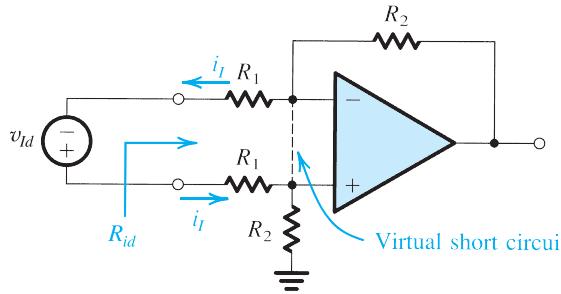
Since the two input terminals of the op amp track each other in potential, we may write a loop equation and obtain

$$v_{ld} = R_1 i_l + 0 + R_1 i_l$$

Thus,

$$R_{id} = 2R_1 \quad (2.20)$$

Note that if the amplifier is required to have a large differential gain ( $R_2/R_1$ ), then  $R_1$  of necessity will be relatively small and the input resistance will be correspondingly low, a drawback of this circuit. Another drawback of the circuit is that it is not easy to vary the differential gain of the amplifier. Both of these drawbacks are overcome in the instrumentation amplifier discussed next.



**Figure 2.19** Finding the input resistance of the difference amplifier for the case  $R_3 = R_1$  and  $R_4 = R_2$ .

## EXERCISES

- 2.15** Consider the difference-amplifier circuit of Fig. 2.16 for the case  $R_1 = R_3 = 2 \text{ k}\Omega$  and  $R_2 = R_4 = 200 \text{ k}\Omega$ .  
 (a) Find the value of the differential gain  $A_d$ . (b) Find the value of the differential input resistance  $R_{id}$  and the output resistance  $R_o$ . (c) If the resistors have 1% tolerance (i.e., each can be within  $\pm 1\%$  of its nominal value), use Eq. (2.19) to find the worst-case common-mode gain  $A_{cm}$  and hence the corresponding value of CMRR.

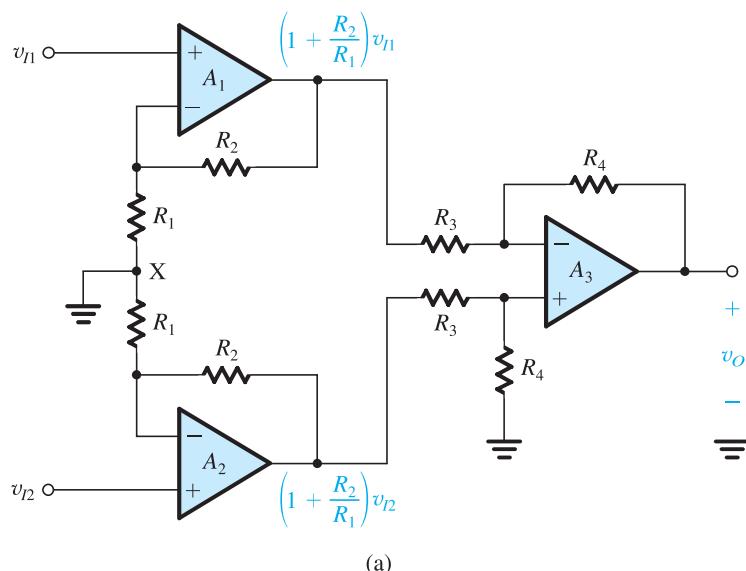
**Ans.** (a) 100 V/V (40 dB); (b) 4 k $\Omega$ , 0  $\Omega$ ; (c) 0.04 V/V, 68 dB

- D2.16** Find values for the resistances in the circuit of Fig. 2.16 so that the circuit behaves as a difference amplifier with an input resistance of 20 k $\Omega$  and a gain of 10.

**Ans.**  $R_1 = R_3 = 10 \text{ k}\Omega$ ;  $R_2 = R_4 = 100 \text{ k}\Omega$

### 2.4.2 A Superior Circuit—The Instrumentation Amplifier

The low-input-resistance problem of the difference amplifier of Fig. 2.16 can be solved by using voltage followers to buffer the two input terminals; that is, a voltage follower of the type in Fig. 2.14 is connected between each input terminal and the corresponding input terminal of the difference amplifier. However, if we are going to use two additional op amps, we should ask the question: Can we get more from them than just impedance buffering? An obvious answer would be that we should try to get some voltage gain. It is especially interesting that



**Figure 2.20** A popular circuit for an instrumentation amplifier. (a) Initial approach to the circuit. (b) The circuit in (a) with the connection between node X and ground removed and the two resistors  $R_1$  and  $R_1$  lumped together. This simple wiring change dramatically improves performance. (c) Analysis of the circuit in (b) assuming ideal op amps.