COL 351 : Analysis and Design of Algorithms
Semester I, 2021-22, CSE, IIT Delhi

Assignment - 4 (due on 13th November, 11:00 PM)

Important Guidelines:

- Each assignment must be done in a group of size at most two.
- Handwritten submissions will not be accepted. Solutions must be typed-up (in Latex, Microsoft Word, etc.), and submitted in pdf format. Each solution must start on a new page.
- **Your answer to each question must be formal and have a proper correctness proof.** No marks will be granted for vague answers with intuition or for algorithms without proof. You must be very rigorous in providing mathematical detail in support of your arguments.
- Cheating of any form will lead to strict penalty.

1 Flows and Min-Cuts

Let $G = (V, E)$ be a directed graph with source s and $T = \{t_1, \ldots, t_k\} \subseteq V$ be a set of terminals. For any $X \subseteq E$, let $r(X)$ denote the number of vertices $v \in T$ that remains reachable from s in $G - X$.[1]

Give an $O(|T| \cdot |E|)$ time algorithm to find a set X of edges that minimizes the quantity $r(X) + |X|$. (Note that setting X equal to the empty-set is allowed). [20 marks]

Hint: Look at (s, t)-max-flow (and corresponding mi-cut) in an appropriate auxiliary graph H computed from G.

2 Hitting Set

Consider a set $U = \{u_1, \ldots, u_n\}$ of n elements and a collection $A_1, A_2, \ldots, A_m$ of subsets of U. That is, $A_i \subseteq U$, for $i \in [1, m]$. We say that a set $S \subseteq U$ is a hitting-set for the collection $A_1, A_2, \ldots, A_m$ if $S \cap A_i$ is non-empty for each i.

The *Hitting-Set Problem* (HS) for the input $(U, A_1, \ldots, A_m)$ is to decide if there exists a hitting-set $S \subseteq U$ of size at most k.

1. Prove that Hitting-Set problem is in NP class. [5 marks]
2. Prove that Hitting Set is NP-complete by reducing Vertex-cover to Hitting Set. [15 marks]

3 Feedback Set

Given an undirected graph $G = (V, E)$, a feedback-set is a set $X \subseteq V$ satisfying that $G - X$ has no cycle. The *Undirected Feedback Set Problem* (UFS) asks: Given G and k, does there exist a feedback set of size at most k.

1. Prove that Undirected Feedback Set Problem is in NP class. [5 marks]
2. Prove that Undirected Feedback Set Problem is NP-complete by reducing Vertex-cover to Undirected Feedback Set Problem. [15 marks]

Hint: Consider the polynomial time reduction of vertex-cover to dominating-set covered in Lecture.

[1] $G - X$ is defined as $(V, E \setminus X)$