COL 703

Assignment 5

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a) We start the game. So, we can place A at any square, then B can place and so on

We win the game if we can place 3 A's on the board such that none of them is on the diagonal of a square containing B

~~We define~~

So, outcome can be determined after 5 moves.

P_A, P_B, P_A, P_B, P_A (P_A: player placing A, P_B: player placing B)

We define two variables

A_{i_s,j_s} : A is placed at (i_s, j_s) by P_A in step s

B_{i_s,j_s} : B is placed at (i_s, j_s) by P_B in step s

We have 3-cases

Case-1: ~~Then~~ It is P_A's first move. So, the board is empty.

Then we define ϕ_1 as

$\phi_1 = \exists A_{i_1,j_1} \forall B_{i_2,j_2} \exists A_{i_3,j_3} \forall B_{i_4,j_4} \exists A_{i_5,j_5} \psi$

This is because player is considered existentially, opponent is considered universally.

Let $\psi_1 = \bigwedge \{ \neg a_{ij} \wedge \neg b_{i'j'} \leftrightarrow m_{ij} , \ 1 \le i \le 3, \ 1 \le j \le 3 , \ |i-i'| = |j-j'| \}$

This formula gives conditions when m_{ij} should be true

For m_{ij} to be true, it should not have A, so $\neg a_{ij}$

It should not have B on its diagonal. So, $\neg b_{i'j'}$ for $|i-i'| = |j-j'|$)

Now, we have to encode the condition for winning if we place A at $m_{i,j}$

$$\psi_2 = \Big(m_{ij} \rightarrow \neg\big(\cancel{\ldots} \\ \cancel{\ldots} \\ |i_3 - i_2| = |j_3 - j_2| \vee \\ \cancel{\ldots} \\ |i_5 - i_2| = |j_5 - j_2| \vee \\ |i_5 - i_4| = |j_5 - j_4|\big)\Big) \wedge \{\neg a_{i_k j_k} \wedge \neg b_{i_k j_k} \mid k = 1,2,3,4,5\}$$

This encodes that future moves don't overlap with past

This encodes that if ~~A is~~ m_{ij} is true, then (i_3, j_3) should not be on diagonal to (i_2, j_2), ~~(i_4, j_4)~~ placed at because A is placed at (i_3, j_3) in step 3, B is placed at (i_2, j_2) in step 2. Similarly (i_5, j_5) should not be on diagonal with (i_2, j_2) and (i_4, j_4)

Thus, ψ_2 encodes winning condition:

So, $\psi = \psi_1 \wedge \psi_2 \wedge (i, j_1) = (i, j)$

ψ ~~If ψ is true~~, ~~it implies that~~

If a QBF-SAT solver outputs that a m_{ij} is true it implies that m_{ij} ~~follows the~~ is not on diagonal of any previous B's and if we place A at m_{ij} we eventually win the game.

Hence $\phi_1 = \exists A_{i_1 j_1} \forall B_{i_2 j_2} \exists A_{i_3 j_3} \forall B_{i_4 j_4} \exists A_{i_5 j_5} (\psi_1 \wedge \psi_2 \wedge ((i, j_1) = (i, j)))$

This encodes that we placed at (i,j) where m_{ij} is true

Case-2: This is P_A's 2nd move i.e. 3rd step.

ψ_1 remains same because the conditions where m_{ij} should be true is not changed

$$\psi_2 = (m_{ij} \rightarrow \neg(|i_5 - i_4| = |j_5 - j_4|)) \wedge \{\neg a_{i_k j_k} \wedge \neg b_{i_k j_k} \mid k = 1,2,3,4,5\}$$

Hence $\phi_1 = \exists A_{i_3 j_3} \forall B_{i_4 j_4} \exists A_{i_5 j_5} (\psi_1 \wedge \psi_2 \wedge ((i_3, j_3) = (i, j)))$.

Case-3: This is P_A's 3rd move i.e. 5th step.

ψ_1 is same

$\psi_2 = \wedge \{\neg a_{i_k j_k} \wedge \neg b_{i_k j_k} \mid k = 1,2,3,4,5\}$

Hence, $\phi_1 = \exists A_{i_5 j_5} (\varphi_1 \wedge (i_5, j_5) = (i,j) \wedge \varphi_2)$

So, based on the step number, we feed the ~~required~~ corresponding formula to QBF-SAT solver which outputs the values for m_{ij}

b) In this case, opponent starts the game. It is not possible to give a propositional logic formula ϕ_2 using only the given propositional variables.

The reason is that we can't formulate the opponent universally. But in the previous case we can consider the opponent universally.

Let's assume that we can represent it as formula

$$\phi = \forall B_{i_1 j_1}\ \exists A_{i_2 j_2}\ \forall B_{i_3 j_3}\ \exists A_{i_4 j_4}\ \forall B_{i_5 j_5}\ \psi$$

Now, the move of P_B in step 3 depends on the move of P_A in step 2. ~~So~~, There will be a limited set of moves for B in step 3. ~~But~~ So, we can't formulate it universally. Also, these limited set of moves change based on P_A's previous moves. Hence, we can't represent this case as QBF