Assignment 1 (*Marks: 7, Due date: Sept. 12*) **COL703, Aug-Nov 2022**

You are required to submit answers of only those questions that carry marks.

Please note that there will be zero tolerance for dishonest means like copying solutions from others, and even letting others copy your solution, in any assignment/quiz/exam. If you are found indulging in such an activity, your answer-paper/code will not be evaluated and your participation/submission will not be counted. Second-time offenders will be summarily awarded an F grade. The onus will be on the supposed offender to prove his or her innocence.

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1. For the sequents below, show which ones are valid and which ones aren't:
   (a) $\neg p \to \neg q \vdash q \to p$
   (b) **[0.5 marks]** $\neg p, p \lor q \vdash q$
   (c) $p \lor q, \neg q \lor r \vdash p \lor r$
   (d) **[0.5 marks]** $p \land \neg p \vdash \neg(r \to q) \land (r \to q)$
2. Show that a formula ϕ is valid iff $\top \equiv \phi$, where $\top$ is an abbreviation for an instance $p \lor \neg p$ of LEM.
3. **[1 marks]** Let us introduce a new connective $\phi \leftrightarrow \psi$ which should abbreviate $(\phi \to \psi) \land (\psi \to \phi)$. Design introduction and elimination rules for $\leftrightarrow$ and show that they are derived rules if $\phi \leftrightarrow \psi$ is interpreted as $(\phi \to \psi) \land (\psi \to \phi)$.
4. Prove the validity of the following sequents:
   (a) $\vdash \neg p \to (p \to (p \to q))$
   (b) $p \land q \vdash \neg(\neg p \lor \neg q)$
   (c) **[0.5 marks]** $(p \to r) \land (q \to r) \vdash p \land q \to r$
   (d) **[0.5 marks]** $p \to q \land r \vdash (p \to q) \land (p \to r)$
5. Write a python program that takes two inputs: i) a *formula* file, containing a CNF formula in DIMACS format, and ii) a *proof* file that contains a resolution proof, and prints "correct" if the resolution proof is correct, and "incorrect" otherwise. The *proof* file contains a line for each application of the resolution rule, in the following format:

   `ip jf l`$_{i_1}$ `l`$_{i_2}$ `... l`$_{i_k}$ `0`

   which means that the clause in line i of the *proof* file (denoted by the letter 'p' after the number i) and j of the *formula* file (denoted by the letter 'f' after the number j) have been resolved to get a clause with the literals $l_{i_1}, l_{i_2}, \ldots, l_{i_k}$ (for example, `3p 7f 2 -4 5 0` denotes that the clause in line 3 of the *proof* file and line 7 of the *formula* file have been resolved, and the resulting clause is $(p_2 \lor \neg p_4 \lor p_5)$ assuming p_i is the name of the i^{th} variable).

   You are required to submit the following:
   (a) **[1.5 marks]** Your code, along with a `README` file containing instructions run the code.
   (b) **[0.5 marks]** A *formula* file that has at least 5 different clauses, and three proof files – two correct, and one incorrect – such that each proof file has at least four lines, and no two lines in any single file resolve the same two clauses.

Here's a sample *formula*:

```
c CNF formula (p1 ∨ !p2) ∧ (p2 ∨ p3) ∧ (!p1 ∨ !p2 ∨ p3) ∧ (!p3)
p cnf 3 4
1 -2 0
2 3 0
-1 -2 3 0
-3 0
```

And, here is a sample *proof*:

```
3f 4f 1 3 0
4f 5f -1 3 0
1p 2p 3 0
6f 3p 0
```

Note that this proof is correct (we have seen this example in the class), and your code should print "correct" for the above inputs.

6. Find a propositional logic formula ϕ which contains only the atoms p, q, and r, and which is true only when p and q are false, or when $\neg q \wedge (p \vee r)$ is true.

7. **[1 marks]** An adequate set of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. For example, $\{\neg, \vee\}$ is adequate. Is $\{\leftrightarrow, \neg\}$ adequate. Justify your answer. Recall that $\phi \leftrightarrow \psi$ is interpreted as $(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$.

8. Show that the following sequents are not valid by finding a valuation in which the truth values of the formulas to the left of $\vdash$ are `T` and the truth value of the formula to the right of $\vdash$ is `F`.

   (a) $\neg p \vee (q \rightarrow p) \vdash \neg p \wedge q$

   (b) **[0.5 marks]** $\neg r \rightarrow (p \vee q), r \wedge \neg q \vdash r \rightarrow q$

   (c) $p \rightarrow (\neg q \vee r), \neg r \vdash \neg q \rightarrow \neg p$

   (d) **[0.5 marks]** $p \rightarrow (q \rightarrow r) \vdash p \rightarrow (r \rightarrow q)$

9. Give a natural deduction proof of PBC using only basic rules.