Sec 1

Q.1 [11831809]

A point P(a, b) is on the line 2y = 3x such that both a, b are integers and the point P is the nearest point on this line to the point (7, 5). Find the value of $a^2 + ab + b^2$.

Solution:

Answer key/Solution

Correct Answer : 76

Since P(a, b) lies on the line 2y = 3x.
Therefore, 2b = 3a ⇒ a = 2b/3
Also, a, b both are integers. Therefore, b will be a multiple of 3.
So (a, b) can be … (– 4, – 6), (– 2, – 3), (0, 0), (2, 3), (4, 6), (6, 9), …
Point (4, 6) is the nearest point on this line to the point (7, 5).
So a = 4 and b = 6
Hence, $a^2 + ab + b^2 = 4^2 + 4 \times 6 + 6^2 = 76$.

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Q.2 [11831809]

Sruti and Smriti start simultaneously from the same point on a circular track. If they travel in opposite directions, they meet at 9 distinct points on the track whereas if they travel in the same direction, then they meet at ‘n’ distinct points on it where ‘n’ is a prime number. If Smriti’s speed is S% less than that of Sruti, then which of the following can be a value of S?

1 ◯ 85

2 ◯ 87.5

3 ◯ 50

4 ◯ 70

Solution:

Answer key/Solution

Correct Answer : 2

Let the ratio of the speeds of Sruti and Smriti in the lowest terms be x/y .
Sruti and Smriti will meet at (x + y) distinct points on the track when traveling in opposite directions and (x – y) distinct points on the track when traveling in the same direction.
It is given that x + y = 9
The different possibilities are 8 + 1, 7 + 2, 6 + 3, 5 + 4
For x – y = n, we get the corresponding possibilities as 8 – 1 = 7; 7 – 2 = 5; 6 – 3 = 3; 5 – 4 = 1
As it is given that ‘n’ is a prime number, ‘n’ can be 7, 5 or 3.
Therefore, the ratio of the speeds of Sruti and Smriti can be 8 : 1, 7 : 2 or 6 : 3 (i.e., 2 : 1. This is not possible. In this case the meeting points will be 2 + 1 = 3).
So, Smriti’s speed can be less than Shruti’s speed by:
(8 – 1)/8 × 100 = 87.5% or
(7 – 2)/7 × 100 = 71.4%
Hence, the correct option is 87.5.

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Q.3 [11831809]

It is mandatory for all the students of a class to take part in atleast one sports event and atleast one cultural event. There were 6 sports and N cultural events that were organised. There were 10 participants and 20 participants in each of these events, respectively. Every student participated in 3 sports and 5 cultrual events. What is the value of N?

1 ◯ 5

2 ◯ 6

3 ◯ 8

4 ○ Cannot be determined

Solution:

Correct Answer : 1

Answer key/Solution

Total number of participants in the sports events = 6 × 10 = 60

Total number of students in the class = $\frac{60}{3} = 20$

Total number of participants in cultrual events = 20 × N

Hence, total number of students = $\frac{20N}{5} = 20$

$\Rightarrow N = 5.$

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Q.4 [11831809]

The maximum sum of the digits of a 100 digit number is A. The maximum sum of the digits of a number less than A is B. The maximum sum of the digits of a number less than B is C. What is C?

1 ○ 8

2 ○ 9

3 ○ 10

4 ○ 12

Solution:

Correct Answer : 3

Answer key/Solution

A = 9 × 100 = 900

B = Sum of the digits of 899 = 26

C = Sum of the digits of 19 = 10.

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Q.5 [11831809]

Let A = {1, 3, 9, 27, 81, ..., 3^{100}}. If B is a subset of A such that the geometric mean of no two elements of B is 3^{50}, then what is the maximum number of elements in set B?

1 ○ 50

2 ○ 51

3 ◯ 49

4 ◯ 52

Solution:
Correct Answer : 2

Answer key/Solution

$A = \{1, 3, 9, 27, 81, \ldots, 3^{100}\} = \{1, 3^1, 3^2, 3^3, 3^4, \ldots, 3^{100}\}$
Let two elements in B be of the form 3^a, 3^b.

Then, $\sqrt{3^a 3^b} \neq 3^{50}$

$\Rightarrow 3^{\frac{a+b}{2}} \neq 3^{50}$

$\Rightarrow \frac{a+b}{2} \neq 50$

$\Rightarrow a + b \neq 100$
So (a, b) = (0, 100), (1, 99), (2, 98), …, (49, 51)

We can see that there are 50 such pairs of (a, b).

Hence, there can be a maximum of (101 - 50 =) 51 elements in set B

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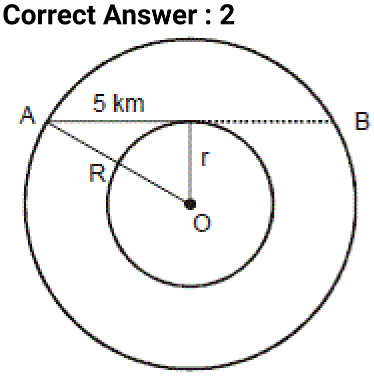
Q.6 [11831809]

There are two concentric circles. A person starts running from a point on the outer circle in a straight line which touches the inner circle. After running for 50 minutes at a speed of 12 km/h, he reaches only a point in the outer circle. If the radii of both the circles are integers, then what is the difference (in km) between the diameters of the two circles?

Solution:
Correct Answer : 2

Answer key/Solution



Length of straight line = 12 × 50/60 = 10 km
Let R and r be the radii of the outer and inner circles respectively.
Then, $R^2 = r^2 + (10/2)^2$ … (i)
Since R and r both are integers.
Therefore, R = 13 km and r = 12 km only satisfy (i).
Hence, the required difference between the diameters = 26 – 24 = 2 km.

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Q.7 [11831809]
If x + y = 1, the maximum value of $xy^4 + x^4y$ is

1 ◯ 1/32

2 ◯ 1/18

3 ◯ 1/12

4 ◯ 1/4

Solution:
Correct Answer : 3

Answer key/Solution

$xy^4 + x^4y = xy(x^3 + y^3)$
$= xy[(x + y)^3 - 3xy(x + y)]$
$= xy(1 - 3xy)$
Think of xy = k
We want to maximise k(1 – 3k).

Therefore, $k(1 - 3k) = \frac{1}{3}[(3k)(1-3k]$

$\Rightarrow$ Maximum when 3k = 1 – 3k or k = 1/6
Therefore, k(1 – 3k) is maximised at k = 1/6.

Hence, $\frac{1}{6}\left(1-\frac{1}{2}\right)=\frac{1}{12}$.

(**Note:** k can take a value $\frac{1}{6}$ since maximum value of xy (i.e., k) is $\frac{1}{4}$.)

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Q.8 [11831809]
In a tuition, 70% of the students are male and x% of the male students and y% of the female students are IIT aspirants. The total number of IIT aspirants who are male and non-IIT aspirants who are female is equal to the total number of non-IIT aspirants who are male and IIT aspirants who are female. Which of the following could be the possible value of (x, y)?

1 ◯ (50, 30)

2 ◯ (40, 30)

3 ◯ (45, 30)

4 ◯ (35, 15)

Solution:
Correct Answer : 4

Answer key/Solution

Let there be 100a students in the tuition. Then, there will be 70a male students and 30a female students.
0.7ax male and 0.3ay female students are IIT aspirants. And, 0.7a (100 – x) males and 0.3a (100 – y) females are non-IIT aspirants.
According to the question,
$0.7ax + 0.3a(100 - y) = 0.7a(100 - x) + 0.3ay$
$\Rightarrow 0.7ax + 30a - 0.3ay = 70a - 0.7ax + 0.3ay$
$\Rightarrow 1.4ax = 0.6ay + 40a$
$\Rightarrow 7x - 3y = 200 \quad \ldots \text{(i)}$
Hence, among the given options, x = 35 and y = 15 satisfy the equation (i).

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Q.9 [11831809]
Jar A has 4 liters of a solution that is 35% acid. Jar B has 6 liters of a solution that is 60% acid. Jar C has 5 liters of a solution that is x% acid. If the entire contents from Jars A, B and C are mixed together such that the new mixture has 55% acid, then find the value of x?

1 ◯ 75

2 ◯ 60

3 ◯ 70

4 ◯ 65

Solution:
Correct Answer : 4

Answer key/Solution

Jar A has 35% acid and Jar B has 60% acid.
The total quantity of new mixture after mixing solutions from Jars A, B and C = 4 + 6 + 5 = 15 liters
Let the quantity of acid in Jar C be y liters.
The quantity of acid in the new mixture = (0.35 × 4) + (0.6 × 6) + y
Given that the new mixture has 55% acid.
$\Rightarrow$ **5 + y = 0.55 × 15** $\Rightarrow$ **y = 3.25**
Therefore, percentage of acid in Jar C = 3.25/5 × 100 = 65%
Hence, x = 65.

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Q.10 [11831809]
The base of a regular pyramid is a square and each of the other four sides is an equilateral triangle. If the length of each side is 10 cm, then find the vertical height (in cm) of the pyramid.

1 ◯ 5√3

2 ◯ 5√2

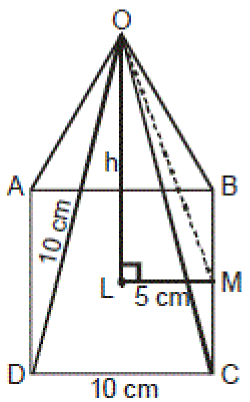
3 ◯ 4√3

4 ◯ 10√2

Solution:

Correct Answer : 2

Answer key/Solution



Length of base = 10 cm

Length of hypotenuse = Altitude of the equilateral triangle $= \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3}$ cm

Hence, vertical height, h = $\sqrt{(5\sqrt{3})^2 - 5^2} = 5\sqrt{2}$ cm.

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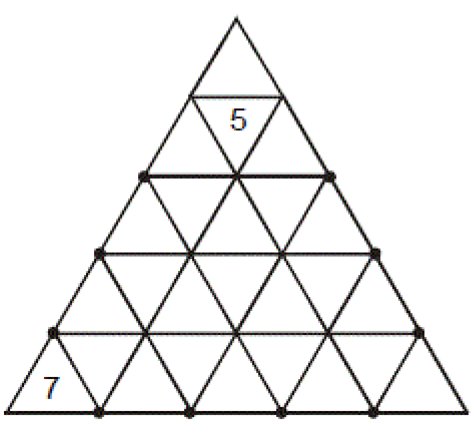
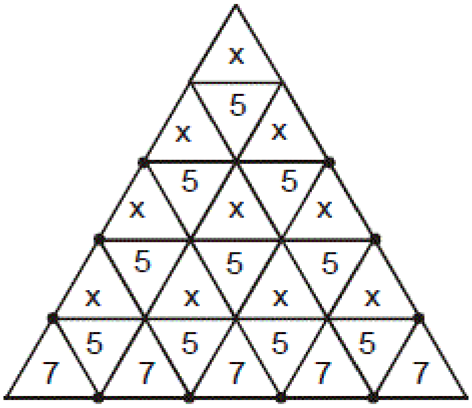
Q.11 [11831809]



In the figure above, all triangular cells are filled with numbers such that the sum of numbers in any two pairs of cells that have a common cell is equal. Sum of all entries in the cells is ____________.

Solution:
Correct Answer : 155



$5 + x = 5 + 7 \Rightarrow x = 7$

Hence, sum of all entries in the cells = $(15 \times 7) + (10 \times 5) = 155$.

Answer key/Solution

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Q.12 [11831809]

Anna, Ben and Clark have an average amount of Rs. 93. If Anna gives Rs. 12 to Ben, then average amount with Ben and Clark would be Rs. 108 whereas if Clark gives Rs. 25 to Anna, then average amount with Anna and Ben would be Rs. 98. What is the amount (in Rs.) with Ben?

Solution:
Correct Answer : 96
The total amount with Anna, Ben and Clark is Rs. 93 × 3 = Rs. 279
If Anna gives Rs. 12 to Ben, then total amount with Ben and Clark will be = Rs. 216
So, Amount with Anna is = 279 – 216 + 12 = Rs. 75
Similarly, if Clark gives Rs. 25 to Anna, then total amount with Anna and Ben will be = Rs. 196
So, amount with Clark will be = 279 – 196 + 25 = Rs. 108
Hence, amount with Ben = Rs. 96.

Answer key/Solution

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Q.13 [11831809]

A project can be completed by ten workers in a certain number of days. If there were two workers less, it would have taken three more days to complete the project. If one worker starts the project and every day a new worker joins, then in how many days will the project be completed?

Solution:
Correct Answer : 15

Answer key/Solution

Let, the number of days taken by 10 workers to complete the project be x.
$10x = 8(x + 3) \Rightarrow x = 12$
Let the total work be 120 units. One worker does 1 unit in a day.
With one worker joining everyday, the units of work completed everyday will be 1, 2, 3, 4, ...
Let the project get completed in n days.
$\Rightarrow n(n + 1)/2 = 120 \Rightarrow n = 15$ days.

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Q.14 [11831809]
For all integers x, $f(f(x)) = f(x + 2) - 3$. If $f(1) = 4$; $f(4) = 3$; $f(5) =$ ______.

Solution:
Correct Answer : 12

Answer key/Solution

$f(1) = 4$; So $f(f(1)) = f(4) = 3$
$f(f(1)) = f(3) - 3$;
So $f(3) = 6$ ... (i)
$f(f(3)) = f(6) = f(5) - 3$
$\Longrightarrow f(5) = f(6) + 3$ ... (ii)
From (i) and (ii), $f(6) = f(f(4)) + 3 = f(3) + 3 = 9$
Hence, from (ii), $f(5) = 9 + 3 = 12$.

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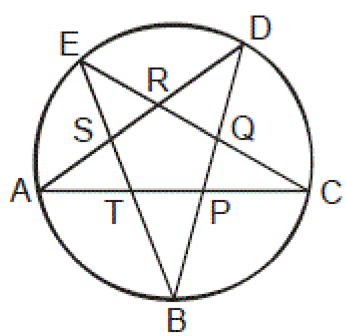
Q.15 [11831809]

In the figure given below, PQRST is a regular pentagon drawn in a circle. Which of the following options represents the ratio of the measures of $\angle SAT : \angle BPT : \angle ERD$?



1 ◯ 2 : 4 : 5

2 ◯ 1 : 2 : 5

3 ◯ 1 : 2 : 3

4 ◯ 3 : 2 : 5

Solution:

Correct Answer : 3

Answer key/Solution

Given that PQRST is a regular pentagon.
So each of the exterior angles is equal to 360°/5 = 72°
⇒ ∠BPT = 72°
∠SRQ is an Interior angle = 180° – 72° = 108°
⇒ ∠ERD = ∠SRQ = 108° (vertically opposite angles)
Also, ∠AST and ∠ATS are exterior angles and are equal to 72°.
Therefore, ∠SAT = 180° – (72° + 72°) = 36° (Sum of the angles of a triangle is 180°.)
Hence, required ratio = 36 : 72 : 108 = 1 : 2 : 3.

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Q.16 [11831809]

Find the number of integral solutions for the inequality (|x – 1| –5)(|x + 2| –4) < 0.

Solution:

Correct Answer : 4

Answer key/Solution

Case 1: |x – 1| – 5 < 0 and |x + 2| – 4 > 0
Or, |x – 1| < 5 and |x + 2| > 4
Or, –4 < x < 6 and x > 2 and x < –6
Common values are: 3, 4 and 5
Case 2: |x – 1| – 5 > 0 and |x + 2| – 4 < 0
Or, |x – 1| > 5 and |x + 2| < 4
Or, –4 > x and x < 6 and –6 < x < 2
Common values are: –5
Hence, total integral values satisfying the inequality = 4.

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Q.17 [11831809]

How many sets of 9 consecutive positive integers are there such that their sum does not exceed 500?

1 ◯ 49

2 ◯ 50

3 ◯ 51

4 ◯ 52

Solution:

Correct Answer : 3

We know

$1 + 2 + 3 + \ldots + 9 = 45$

Let the numbers be $(k + 1), (k + 2), \ldots, (k + 9)$

So $9k + 45 \leq 500$

Or, $9k \leq 455$ or, $k \leq 50.55$

Since $k = 0, 1, 2, 3, \ldots, 50$

Hence, required number of sets = 51.

Answer key/Solution

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Q.18 [11831809]

Let $Ax^2 + Bx + C = 0$ be a quadratic equation such that A, B, C are rational numbers and $A \neq 0$. If the sum of the roots of the equation is 3 and sum of their squares is 1, then the value of C/B is

1 $-2/3$

2 $-4/3$

3 $5/4$

4 $-5/3$

Solution:

Correct Answer : 2

Let p and q be the roots of the given quadratic equation $Ax^2 + Bx + C = 0$.

Then, $p + q = -B/A = 3 \Rightarrow A = -B/3$ and $pq = C/A$

So $p^2 + q^2 = (p + q)^2 - 2pq$

$\Rightarrow 1 = 3^2 - 2C/A$

$\Rightarrow C/A = 4$

$\Rightarrow C = 4A = -4B/3$

$\Rightarrow C/B = -4/3$.

Answer key/Solution

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Q.19 [11831809]

The lotuses in the Jubilee lake started blooming from the month of January. In every alternate month starting from January, 80 more flowers appeared than there were at the end of the previous month. In every alternate month starting from February, the number of flowers became half the number there were at the end of previous month. If 250 lotuses bloomed in the month of July, then how many lotuses bloomed in the month of January?

Solution:

Correct Answer : 880

Answer key/Solution

Let the number of lotuses blooming in the month of January be x.
The number for successive months:

Month	Number of Lotuses blooming
January	x
February	$x/2$
March	$x/2 + 80$
April	$x/4 + 40$
May	$x/4 + 120$
June	$x/8 + 60$
July	$x/8 + 140$

$\therefore$ Number of lotuses blooming in July is $x/8 + 140 = 250 \Rightarrow x = 880$
Hence, 880 lotuses bloomed in January.

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Q.20 [11831809]

If $4|x| + 3y = 18$ and $3x + 2|y| + 5 = 0$, then $6x - 5y$ is

1 ◯ −676

2 ◯ −28

3 ◯ −34

4 ◯ 27

Solution:

Correct Answer : 2

Answer key/Solution

The given equations are

$4|x| + 3y = 18$ ... (i)

and $3x + 2|y| + 5 = 0$ ... (ii)

Case 1: When x and y both are positive.

$4x + 3y = 18$ and $3x + 2y = –5$

Solving (i) and (ii), we get

$x = – 51$ and $y = 74$ which is not possible.

Case 2: When x and y both are negative.

$–4x + 3y = 18$ and $3x – 2y = –5$

Solving (i) and (ii), we get

$x = 21$ and $y = 34$ which is not possible.

Case 3: When x is positive and y is negative.

$4x + 3y = 18$ and $3x – 2y = –5$

Solving (i) and (ii), we get

$x = 21/17$ and $y = 74/17$ which is not possible.

Case 4: When x is negative and y is positive.

$–4x + 3y = 18$ and $3x + 2y = –5$

Solving (i) and (ii), we get

$x = –3$ and $y = 2$ which is possible.

Hence, $6x – 5y = 6 \times (–3) – 5 \times 2 = –28$.

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Q.21 [11831809]

Two teams A and B participated in a game with three rounds. The ratio of their scores in each round were 3 : 4, 7 : 5 and 4 : 7 respectively. The scores of team A were in increasing arithmetic progression and scores of team B in first two rounds were in the ratio 8 : 5. Which of the following cannot be the total scores of team A and B in the third round?

1 ◯ 154

2 ◯ 110

3 ◯ 132

4 ◯ 143

Solution:
Correct Answer : 4

Answer key/Solution

	A	B
Round 1	3x	4x
Round 2	7y	5y
Round 3	4z	7z

Since the scores of team A were in increasing arithmetic progression, therefore,

$$\frac{3x+4z}{2}=7y \quad \text{... (i)}$$

And, $\frac{4x}{5y}=\frac{8}{5} \Rightarrow x=2y$ ... (ii)

$$\therefore \frac{3x+4z}{2}=7y \Rightarrow z=2y \quad \text{... (iii)}$$

From (i), (ii) and (iii), x = z = 2y
Hence, the total score in third round = 4z + 7z = 11z = 22y, only 143 doesn't satisfy the criterion.

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Q.22 [11831809]
Two of the sides of a triangle are 25, 19. The length of the third side is also a natural number.
If a > Any possible value of the perimeter of the triangle
and b < Any possible value of the perimeter of the triangle.
What is the minimum value of a – b? (a and b are both integers.)

1 ◯ 19

2 ◯ 21

3 ◯ 38

4 ◯ 40

Solution:
Correct Answer : 3

Answer key/Solution

Let S be the third side.
Then, 6 < S < 44
So largest value of S = 43; the smallest value of S = 7
So 51 ≤ Perimeter of the triangle ≤ 87
Therefore, least value of a = 88; Maximum value of b = 50
Hence, the minimum value of a – b = 88 – 50 = 38.

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