CHAPTER – 10

PROBABILITY

Probability

This is an important topic for MBA entrance tests. Recent trends show that this importance is increasing. Besides the entrance tests, the topic is an important part of the management courses themselves. Therefore, students aspiring to be future managers need a sound understanding of the basics.

Natural phenomena are of two types - deterministic and probabilistic. For example, the direction in which or the time at which the sun rises everyday is a deterministic phenomenon, while, where or when it may rain is a probabilistic phenomenon. Similarly, **experiments** (the operations of doing or observing something resulting in some final outcomes) are of two types - those in which the outcome is definite and others in which the actual outcome may be any one of the many possible outcomes. For example, hydrogen is allowed to react with oxygen. They react in a certain ratio and produce water. The outcome is definite. Instead if a coin is tossed, it may turn up showing either heads or tails, the outcome may be one of the two possible outcomes. Experiments of the second kind are called **random experiments** (RE). One instance of such an experiment is a **trial**. The set of all possible outcomes for a particular random experiment is its **sample space** S. This corresponds to the concept of the universal set in set theory. Each outcome is said to be a point in S. Thus for the experiment of tossing a coin, the sample space is the set of the two outcomes - heads and tails. For the experiment of rolling a die, it is {1, 2, 3, 4, 5, 6}. For drawing a card from a deck of cards it is the set of all the 52 possible outcomes - corresponding to the 52 cards. Any subset of the sample space is a simple event. Two (or more) events which occur for two (or more) different experiments - or for two (or more) trials of the same experiment are called **compound events**. Thus if a coin is tossed and a die is rolled, the event of getting a head (in the case of the coin) and 5 say (in the case of the die) is a compound event. Similarly, if a die is rolled twice, the event of getting an even number on the first roll and an odd on the second is a compound event.

Equally likely outcomes:

If a normal coin is tossed, it may come up either heads or tails. Both the outcomes are equally likely. We can accept this intuitively, even though at the moment, we do not know how to compute (or measure) the probability of either outcome. For the purpose of this discussion, we start with this assumption - that we can recognise intuitively whether all the possible outcomes are equally likely or not. We know from experience that all coins are not normal. Sometimes, the mass in the coin is so distributed that it shows up one side more than the other. Such coins (or dice) are said to be biased.

We can now consider one definition of probability. This is the only one that we need for the questions that we shall face. For a random experiment with n "equally likely" outcomes, if E is an event which can be considered to have occurred for m of the outcomes, the probability (mathematical probability or a priori probability) of E is $\frac{m}{n}$, i.e. $P(E) = \frac{m}{n}$

The **complement** of an event is the event of the non-occurrence of E. It is denoted by $\overline{E}$ and $P(\overline{E}) = 1 - \frac{m}{n}$.

For example, if the RE is tossing a coin and E is the event of getting heads, $P(E) = \frac{1}{2}$. Also $P(\overline{E}) = 1 - \frac{1}{2} = \frac{1}{2}$.

The complement of getting a head is getting a tail. If the RE is rolling a die and E = {2, 3}, $\overline{E}$ = {1, 4, 5, 6}. In this case, $P(E) = \frac{1}{3}$, $P(\overline{E}) = \frac{2}{3}$

With this definition, we can consider the two extreme cases. An event is any subset of S. If E is the null set P(E) = 0 (an impossible event) and if E = S, P(E) = 1 (a certain event) For example, let the RE be rolling a die and consider the "event" of getting a 0. In our notation, E would be the null set. For no element of S, can it be said that E has occurred. $\overline{E}$ would be the event of not getting a 0 and $P(\overline{E}) = 1$

Instead of E saying that the probability of an event is m/n, we can also say that the odds in favour of the event are m to n – m i.e $P(E)/P(\overline{E})$. Similarly, the odds against the event are n – m to m i.e. $P(\overline{E})/P(E)$.

Mutually Exclusive Events:

If there is a set of events, such that if any one of them occurs, none of the others can occur, the events are said to be mutually exclusive. Consider the RE of rolling a die and the following events.

$E_1 = \{1\}$
$E_2 = \{2, 3\}$
$E_3 = \{4, 5\}$

The three events are mutually exclusive.

Collectively Exhaustive Events:

If there is a set of events such that at least one of them is bound to occur, the events are said to be collectively exhaustive. For the RE considered above, if $E_4 = \{1, 2, 3, 4\}$ and $E_5 = \{3, 4, 5, 6\}$, E_4 and E_5 are collectively exhaustive.

If a set of events are both mutually exclusive and collectively exhaustive, the sum of their probabilities is 1.

Addition Theorem of Probability:

If A and B are two events, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

This result follows from the corresponding result in set theory. If n (X) represents the number of elements in set X, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

Example: If a die is rolled, what is the probability that the number that comes up is either even or prime?

A = The event of getting an even number = {2, 4, 6}
B = The event of getting a prime = {2, 3, 5}
$A \cup B = \{2, 3, 4, 5, 6\}$
$A \cap B = \{2\}$

$P(A) = \frac{3}{6}$, $P(B) = \frac{3}{6}$, $P(A \cup B) = \frac{5}{6}$ and $P(A \cap B) = \frac{1}{6}$.

We can verify that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability and the Multiplication Theorem of Probability

The events that we have considered so far are without reference to other events (or conditions). But, very often, we need to consider events, in relation to other events (or conditions). We can continue with the same RE. Let A be the event of getting a prime and B be the event of getting an even number, i.e. A = {2, 3, 5} and B = {2, 4, 6}, P (A) = $\frac{3}{6}$

But if B is known to have occurred, then P (A) = $\frac{1}{3}$.

We write this as follows:

$P\left(\frac{A}{B}\right) = \frac{1}{3}$, This is read as follows:

The probability of A, given B is $\frac{1}{3}$.

In general,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \text{ or } P(A \cap B) = P(B).P\left(\frac{A}{B}\right) \rightarrow (1)$$

Also, as $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$, it follows that

$$P(B \cap A) = P(A).P\left(\frac{B}{A}\right). \quad \rightarrow \quad (2)$$

This result (1) or (2) is known as the multiplication theorem of probability.

If $P\left(\frac{A}{B}\right)$ = P (A), then A and B are said to be independent.

A is independent of B, because whether B occurs or does not occur, the probability of A does not change.

Example, A number is selected at random from the integers 1 to 50. A is the event of getting a multiple of 5 and B is the event of getting an even number.

Then A = {5, 10, 15....45, 50}, B = {2, 4, ..., 48, 50}

P (A) = $\frac{10}{50}$

$P\left(\frac{A}{B}\right) = \frac{5}{25}$, also $P\left(\frac{A}{\bar{B}}\right) = \frac{5}{25}$

$\therefore$A and B are independent events.

If A and B are independent, $P\left(\frac{A}{B}\right)$ = P(A)

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = P(A) \qquad \text{or } P(A \cap B) = P(A).P(B)$$

If A and B are independent, so are A, $\bar{B}$; $\bar{A}$, B and $\bar{A}$,$\bar{B}$

i.e. if P (A) = $P\left(\frac{A}{B}\right)$ then each of the following is true

$P(A) = P\left(\frac{A}{\bar{B}}\right)$

$P(B) = P\left(\frac{B}{A}\right) = P\left(\frac{B}{\bar{A}}\right)$

We can show the following results

If $P(A) = P\left(\frac{A}{B}\right)$, then $P(A) = \left(\frac{A}{\bar{B}}\right)$.

Also $P\left(\frac{B}{A}\right) = P(B) = P\left(\frac{B}{\bar{A}}\right)$

Pairwise independence and mutual independence:

If A, B, C are three events such that each of the 3 pairs A, B; B, C and C, A are independent. A, B, C are said to be pairwise independent. Let P(A), P(B), P(C) be a, b, c respectively.

As A, B are independent, P (A $\cap$ B) = ab,
As B, C are independent, P (B $\cap$ C) = bc,
As A, C are independent, P (C $\cap$ A) = ca

Even if these three conditions are true, P (A $\cap$ B $\cap$ C) is not necessarily equal to abc. In case it is, the events are said to be mutually independent. We note that mutual independence is a stronger condition. It implies pairwise independence (while pairwise independence does not necessarily mean mutual independence)

For more events, we can generalise the concept, we can talk of pairwise (or 2-wise), tripletwise (or 3-wise), quadrapletwise (or 4-wise) independence and so on. Mutual independence of n events would mean i-wise independence for i = 2, 3, 4....n

Example:

A positive integer from 1 to 60 is selected at random.
A is the event of selecting a multiple of 3.
B is the event of selecting a multiple of 4.
C is the event of selecting a multiple of 5.
The following results are true.

$a = P(A) = \frac{20}{60} = \frac{1}{3}$	$P(A \cap B) = \frac{5}{60}$	$P(A \cap B \cap C) = \frac{1}{60}$
$b = P(B) = \frac{15}{60} = \frac{1}{4}$	$P(A \cap C) = \frac{4}{60}$	
$c = P(C) = \frac{12}{60} = \frac{1}{5}$	$P(B \cap C) = \frac{3}{60}$	

Thus, P (A $\cap$ B) = ab, P (A $\cap$ C) = ac, P (B $\cap$ C) = bc and P (A $\cap$ B $\cap$ C) = abc,

$\therefore$ A, B, C are not merely pairwise independent but also mutually independent

Bayes' Theorem:

Let A_1, A_2 .... A_n be mutually exclusive and collectively exhaustive events with respective probabilities of p_1, p_2...... p_n. Let B be an event such that P (B) $\neq$ 0 and $P\left(\frac{B}{A_i}\right)$ for i = 1 to n be q_1, q_2.....q_n. Then the conditional probability of A_i given B is $\frac{p_i q_i}{p_1 q_1 + p_2 q_2 + ... + p_n q_n}$

Example:

Box 1 contains 3 white and 2 black balls. Box 2 contains 1 white and 4 black balls. A ball is picked from one of the two boxes. It turns out to be black. Find the probability that it was drawn from box 1.

The data is tabulated below

	Box 1	Box 2
White	3	1
Black	2	4

The event that box 1 is selected is say A_1.
The event that box 2 is selected is A_2

$\therefore$ $p_1 = P(A_1) = \frac{1}{2}$ and $p_2 = P(A_2) = \frac{1}{2}$

Let B be the event that a black ball is selected

$\therefore P\left(\frac{B}{A_1}\right) = \frac{2}{5}$ and $P\left(\frac{B}{A_2}\right) = \frac{4}{5}$

Now it is given that a black ball has been drawn and we need to find P (A_1) i.e. we need $P\left(\frac{A_1}{B}\right)$. We use the result above

$$P\left(\frac{A_1}{B}\right) = \frac{P(A_1)P\left(\frac{B}{A_1}\right)}{P(A_1)P\left(\frac{B}{A_1}\right) + P(A_2)P\left(\frac{B}{A_2}\right)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{2}{5}\right)}{\left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)} = \frac{2}{2+4} = \frac{1}{3}$$

Expected value

The concept of "expected value" is very important in the Theory of Probability. This concept is very useful in managerial decision-making.

The theory of probability has its origin in gambling. When people went to gambling houses or casinos where they used to get certain money if they achieved a certain result in the game. Mathematicians wanted to find out as to how much a person will earn if the game is played a large number of times.

Let us say that a man is playing a game of "throwing a die". He is given ₹6 if he throws a "four" and ₹9 if he throws a "six" on the die and not paid anything if he throws any other number (of course, he will have to pay some amount to the gambling house owner each time he wants to throw the die and this aspect will be considered later). Suppose he throws the die a large number of times - say 6,00,000 times. As the number of times the experiment is repeated becomes very large, we know that the number of times each event will occur is given by probability.

A "four" will appear with a probability of 1/6, i.e., it is expected to appear 1,00,000 times out of a total of 6,00,000 times the die is thrown. Similarly, a "six" will appear 1,00,000 times (because the probability is 1/6). Hence, the amount he will get in the long run will be $1{,}00{,}000 \times 6 + 1{,}00{,}000 \times 9 = 15{,}00{,}000$. The amount he gets per throw will be 1500000/600000 = ₹2.5.

We say that the person's **expected value** in this game per throw in the long run is ₹2.5.

This can be calculated without the number of throws coming into the picture. Once the events are defined, we should have the probabilities of all the events and the monetary value associated with each event (i.e., how much money is earned or given away if that particular event occurs). Then,

$$\boxed{\text{Expected Value} = \sum_i [\text{Probability } (E_i)] \times [\text{Monetary value associated with event } E_i]}$$

Example:

A person tosses a coin. If it comes up heads, he gets ₹10. If it comes up tails, he has to pay ₹5. What is his expected value?

Event	Heads	Tails
Probability	$\frac{1}{2}$	$\frac{1}{2}$
Expectation	10	–5

$$\therefore E = \frac{\left(\frac{1}{2}\right)10 + \left(\frac{1}{2}\right)(-5)}{\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)} = 2.5$$

Examples:

10.01. Two fair dice are thrown simultaneously. What is the probability that one die shows up a number greater than '4' and the other shows up a number less than '3'?

Sol. When two dice are thrown simultaneously, the total number of outcomes is $6 \times 6 = 36$.
Out of these, favourable cases are 8 i.e. (5, 1), (5, 2), (6, 1), (6, 2), (1, 5), (2, 5), (1, 6) and (2, 6).

Hence, the required probability = $\frac{8}{36} = \frac{2}{9}$.

10.02. When two fair dice are thrown simultaneously, what is the probability that the sum obtained is not equal to 7?

Sol. We will have more cases to deal for the sum not being 7.
Rather than looking at finding the probability of this event directly, we will find the probability of non-occurrence of this event and subtract it from 1 to get the required probability.
So, we deal with the case where the sum is equal to 7.
The sum being 7 : (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).
There are 6 favourable occurrences.

$\therefore$P (sum is 7) = $\frac{6}{36} = \frac{1}{6}$.

Hence, the required probability = $1 - \frac{1}{6} = \frac{5}{6}$.

10.03. When three fair dice are thrown simultaneously, what is the probability that the first die shows up an even number, second die shows up an even prime number and third die shows up a composite number?

Sol. Let A: the event of the first die showing up an even number i.e. 2, 4, or 6.
B: the event of the second die showing up an even prime number i.e. 2.
C: the event of the third die showing up a composite number i.e. 4 or 6.
Their respective probabilities being,

$P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{1}{6}$ and $P(C) = \frac{2}{6} = \frac{1}{3}$.

The required probability is obtained by compounding these events i.e. by multiplying individual probabilities. Hence, the required probability is $\frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} = \frac{1}{36}$.

10.04. If four fair dice are thrown simultaneously, what is the probability that the sum of the numbers is more than 21?

Sol. Total number of cases = 6^4 = 1296.
The various combinations for the sum being greater than 21 and the corresponding number of arrangements in each case are as follows:
Sum being 22:

(6, 6, 6, 4) $\rightarrow$ $\frac{4!}{3!} = 4$

(6, 6, 5, 5) $\rightarrow$ $\frac{4!}{2!\,2!} = 6$

Sum being 23: (6, 6, 6, 5) $\rightarrow$ $\frac{4!}{3!} = 4$

Sum being 24: (6, 6, 6, 6) $\rightarrow$ = 1
The number of favourable cases is 15.

Hence, the required probability $= \frac{15}{1296} = \frac{5}{432}$.

10.05. If 4 fair coins are tossed together, what is the probability of getting exactly 3 heads?

Sol. The event of getting exactly 3 heads will be the combination of 3 heads and 1 tail. The number of arrangements with this combination is $\frac{4!}{3!\,1!} = 4$.

Hence, the probability is $\frac{4}{2^4} = \frac{1}{4}$.

10.06. If five fair coins are tossed together, what is the probability of getting at the most 3 tails?

Sol. P (getting at most 3 tails)
= 1 – P (getting at least 4 tails)
= 1 – P (getting 4 tails or 5 tails)
The event of getting exactly 4 tails is same as obtaining 4 tails and 1 head in any order and the total occurrences in which case are $\frac{5!}{4!} = 5$.

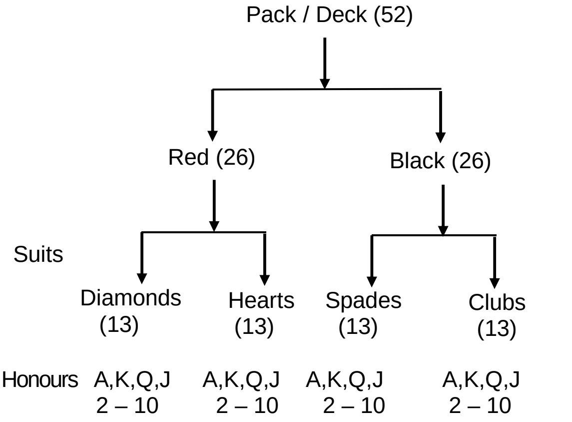
Hence, P(getting exactly 4 tails) = $\frac{5}{32}$.

P (getting exactly 5 tails) = $\frac{1}{32}$.

$\therefore$P(getting at most 3 tails) = $1 - \left[\frac{5}{32} + \frac{1}{32}\right] = \frac{13}{16}$.

We will now take up a few examples based on cards. Before we take up the examples, we will look at a few basics pertaining to cards. A standard pack has 52 cards. In a pack of 52 cards, there are 4 different suits - clubs, hearts, diamonds and spades. Clubs and spades are black in colour and hearts and diamonds are red in colour. Each suit has 13 cards - 2, 3, 4, …., 10, Jack, Queen, King and Ace. There are 26 red and 26 black cards in a pack of 52 cards. There are four Aces, four Kings, four Queens, ..., four 3's and four 2's in a pack of cards. In each suit the four cards Ace, King, Queen and Jack are called "honours". So in a pack of cards there are 16 honours, out of which 8 are red and 8 are black. Jack is also called Knave.

Tree diagram showing the classification in a pack of cards:



10.07. A card is drawn from a well shuffled pack of cards. Find the probability that it is
(i) an ace.
(ii) a red numbered card.
(iii) a spade.
(iv) a black king.

Sol. The number of ways of selecting one card out of 52 cards is ${}^{52}C_1 = 52$.

(i) One card can be drawn from 4 aces in ${}^{4}C_1 = 4$ ways.
Hence, the required probability is
$\frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$

(ii) A pack of cards has 18 red numbered cards. A red numbered card can be drawn in ${}^{18}C_1 = 18$ ways.
Hence the probability of drawing a red numbered card is $\frac{18}{52} = \frac{9}{26}$.

(iii) A pack of cards has 13 spades. Hence the probability of drawing a spade is $\frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4}$.

(iv) A pack of cards has 2 black kings. Hence the probability of drawing a black king is $\frac{{}^{2}C_1}{{}^{52}C_1} = \frac{2}{52} = \frac{1}{26}$.

10.08. If two cards are drawn simultaneously from a well shuffled pack of cards, then find the probability of both being
(i) queens.
(ii) honours.
(iii) red honours.

Sol. Two cards can be dawn from a pack of cards in ${}^{52}C_2$ ways.

(i) There are 4 queens in a pack of cards. Two queens can be drawn in ${}^{4}C_2$ ways.
Probability of both being queens is $\frac{{}^{4}C_2}{{}^{52}C_2}$.

(ii) There are 16 honours in a pack of cards. Probability of both being honours is $\frac{^{16}C_2}{^{52}C_2}$.

(iii) There are 8 red honours in a pack of cards. Probability of both the cards drawn being red honours is $\frac{^{8}C_2}{^{52}C_2}$.

10.09 If two cards are drawn at random from a pack of cards, what is the probability that one of them is a numbered card and the other is a king?

Sol. Two cards can be drawn from a pack of cards in $^{52}C_2$ ways.
A numbered card can be drawn in $^{36}C_1$ ways and a king can be drawn in $^{4}C_1$ ways.
Hence, the required probability is $\frac{^{36}C_1 \times {}^{4}C_1}{^{52}C_2}$.

10.10. If two cards are drawn simultaneously from a pack of cards, what is the probability that both are spades or both are clubs?

Sol. Here both the cards should be spades or both should be clubs. These are two mutually exclusive events. Let A denote the event of getting both spades and B denote the event of getting both clubs.
n(A) = $^{13}C_2$, n(B) = $^{13}C_2$ and n(A $\cap$ B) = 0.
We need to compute P (A $\cup$ B).
P(A $\cup$ B) = P(A) + P(B) = $\frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2}$.

10.11. When two cards are drawn simultaneously from a pack of cards, what is the probability that both are red or both are kings?

Sol. Here, we have two events which are not mutually exclusive. While the two cards are red they can simultaneously be kings also. Let A and B be the events of selecting both red or both kings respectively.
Then we need to find out P(A $\cup$ B)
P(A $\cup$ B) = P(A) + P(B) – P(A $\cap$ B)
= $\frac{^{26}C_2}{^{52}C_2} + \frac{^{4}C_2}{^{52}C_2} - \frac{^{2}C_2}{^{52}C_2}$

10.12. When three cards are drawn in succession from a pack of cards with replacement, what is the probability that the first drawn card is a heart, the second is a red card and the third an honour?

Sol. Let E_1, E_2 and E_3 denote the events of drawing a heart, a red card and a honour card in the first, second and third draws, with replacement in that order. As the cards drawn are being replaced, E_1, E_2, E_3 are independent.
Hence the required probability
= P($E_1 \cap E_2 \cap E_3$)= P(E_1) . P(E_2) . P(E_3)
= $\frac{^{13}C_1}{^{52}C_1} \times \frac{^{26}C_1}{^{52}C_1} \times \frac{^{16}C_1}{^{52}C_1}$ = 1/4 $\times$ 1/2 $\times$ 4/13
= 1/26.

10.13. When three cards are drawn simultaneously from a pack of cards, what is the probability that one of them is a black card, another a heart, and the third a diamond?

Sol. There are 26 black cards, 13 hearts and 13 diamonds in a pack of cards. The required probability is $\frac{^{26}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{^{52}C_3}$.

10.14. When two balls are drawn in succession without replacement from a box containing 4 red balls and 7 green balls, find the probability that
(i) the first one is red and the second one is green.
(ii) both are red.

Sol. The first ball can be drawn in $^{11}C_1$ = 11 ways and as the ball is not being replaced, the second ball can be drawn in $^{10}C_1$ = 10 ways.
(i) A red ball can be drawn in $^{4}C_1$ = 4 ways in the first draw and a green ball can be drawn in the second draw in $^{7}C_1$ = 7 ways.
$\therefore$The required probability = $\frac{4}{11} \times \frac{7}{10} = \frac{14}{55}$
(ii) A red ball can be drawn in $^{4}C_1$ = 4 ways in the first draw. Since this ball is not replaced, a red ball in the second draw can be drawn in $^{3}C_1$ = 3 ways.
$\therefore$The required probability = $\frac{4}{11} \times \frac{3}{10} = \frac{6}{55}$.

10.15. A bag contains 5 black balls, 6 red balls and 3 white balls. If a ball is drawn at random, what is the probability that it is
(i) not a white ball?
(ii) a red ball?

Sol. One ball can be drawn out of the 14 balls present in the bag in $^{14}C_1$ ways.
(i) A ball other than a white ball (5 + 6 = 11) can be drawn in $^{11}C_1$ ways. Hence, the probability of not drawing a white ball is $\frac{^{11}C_1}{^{14}C_1} = \frac{11}{14}$.
(ii) One red ball out of the six red balls present in the bag can be drawn in $^{6}C_1$ ways.
Hence, the probability of drawing a red ball is $\frac{^{6}C_1}{^{14}C_1} = \frac{3}{7}$.

10.16. A bag contains 7 blue, 5 red and 4 green balls. If three balls are drawn simultaneously at random, what is the probability that
(i) one is blue, one is red and the other is green?
(ii) two are red and one is green?
(iii) all three are of the same colour?

Sol. Three balls can be drawn from 16 balls in $^{16}C_3$ ways.

(i) We can draw one blue ball out of 7 blue balls in $^{7}C_1$ ways, one red ball out of 5 red balls in $^{5}C_1$ ways and 1 green ball out of 4 green balls in $^{4}C_1$ ways.

Required Probability = $\frac{{}^7C_1 \times {}^5C_1 \times {}^4C_1}{{}^{16}C_3}$.

(ii) We can select two red out of 5 red balls in ${}^5C_2$ ways and one green out of 4 green balls in ${}^4C_1$ ways.

The required probability = $\frac{{}^5C_2 \times {}^4C_1}{{}^{16}C_3}$.

(iii) To select three balls of the same colour, we can choose either 3 blue balls from 7 blue balls or 3 red balls from 5 red balls or 3 green balls from 4 green balls in ${}^7C_3$, ${}^5C_3$ and ${}^4C_3$ ways, respectively.

The required probability = $\frac{{}^7C_3 + {}^5C_3 + {}^4C_3}{{}^{16}C_3}$.

10.17. Nakul Kumar has 3 fifty rupee notes, 4 hundred rupee notes and 6 five hundred rupee notes in his pocket. If 2 notes are taken at random, what are the odds in favour of both notes being of hundred rupee denominations?

Sol. Total number of ways in which 2 notes can be taken from the pocket containing 13 notes is ${}^{13}C_2$ and the number of ways in which 2 hundred rupee notes can be taken is ${}^4C_2$.

$\therefore$The probability of choosing 2 hundred rupee notes is $\frac{{}^4C_2}{{}^{13}C_2} = \frac{4\times3}{13\times12} = \frac{1}{13}$.

Odds in favour of the event
= favourable ways : unfavourable ways
= 1 : 12

10.18. Shivcharan, who is interested in Philately saw 4 Indonesian, 5 Mexican and 6 Egyptian stamps in a box. He drew 2 stamps from the box one after the other. What is the probability that the stamp drawn second is Mexican when the stamp drawn first is
(i) replaced?
(ii) not replaced?

Sol. (i) If the stamp drawn first is replaced, the box will contain all the stamps it originally had. The probability of drawing a Mexican stamp is 5/15 = 1/3

(ii) If the stamp is not replaced, there are 2 cases to deal with.
(a) The first draw yielding a Mexican stamp.
(Or)
(b) The first draw not yielding a Mexican stamp. The corresponding probabilities being

Case (a) $5/15 \times 4/14 = 2/21$
Case (b) $10/15 \times 5/14 = 5/21$

10. 19. The probability of a snake being venomous is 0.5, the probability of a snake being not venomous or not an instant killer is 0.8. If it is known that a snake is venomous, find the probability that it is an instant killer.

Sol. Let A be the event that the snake is venomous, and B be the event that the snake is an instant killer.
It is given that
P(A) = 0.5

$P(\overline{A} \cup \overline{B}) = 0.8$

$\Rightarrow P(\overline{A \cap B}) = 0.8$

$\Rightarrow P(A \cap B) = 1 - 0.8 = 0.2$

$\therefore P(B/A) = \frac{P(A \cap B)}{P(A)}$

$= \frac{0.2}{0.5} = \frac{2}{5}$

10.20. One number is selected at random from the set.
S = {2222, 4422, 6622, 2244, 4444, 6644, 2266, 4466, 6666}
A is the event that the number start with 22.
B is the event that the number end with 44.
C is the event that the unit's digit and thousands digit are equal.
Which of the following is true?
(A) A and B are dependent.
(B) B and C are dependent.
(C) A, B, C are mutually independent.
(D) A, B, C are pair wise independent.

Sol. $P(A) = \frac{3}{9} = \frac{1}{3}$ $P(A \cap B) = \frac{1}{9}$

$P(B) = \frac{3}{9} = \frac{1}{3}$ $P(A \cap C) = \frac{1}{9}$

$P(C) = \frac{3}{9} = \frac{1}{3}$ $P(B \cap C) = \frac{1}{9}$

$P(A \cap B) = P(A)\,P(B)$,
$P(B \cap C) = P(B)\,P(C)$
$(C \cap A) = P(C)\,P(A)$
but $P(A)P(B)P(C) \neq P(ABC)$
They are pairwise independent but not mutually independent.

10.21. During the next one year, the probability that Apple releases a product is 0.6. The probability that a product is a success, given that it is released by Apple is 0.7. The probability that a product is a success and released by Google is 0.4. A product released by either Apple or Google during the next one year is a success. Find the probability that it is released by Apple.

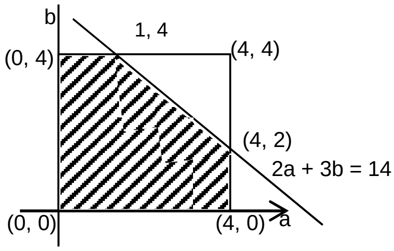
Sol. Let A-be the event of a product being released by Apple.
Let B-be the event of a product being released by Google.
Let E be the event of a product being a success.
P(A) = 0.6
$P(E/A) = 0.7$
$P(E \cap B) = 0.4$

$$P(A/E) = \frac{P(A).P(E/A)}{P(A).P(E/A) + P(B).P(E/B)}$$

$$= \frac{0.6(0.7)}{0.6(0.7) + 0.4} = \frac{0.42}{0.82} = \frac{21}{41}$$

10.22. a, b are two positive numbers such that a<4, b<4, Find the probability that 2a + 3b<14.

Sol.



If we add a-on x-axis, and b-on y-axis,
2a + 3b = 14 is a line as shown above,
In the shaded region 2a + 3b < 14
Required probability

$$= \frac{4(4) - \frac{1}{2}(3)(2)}{4(4)} = \frac{13}{16}$$

10.23. Subash participates in a game that involves throwing an unbiased die, where a participant is given twice as many rupees as the number that turns up on the die if it is prime and thrice as many rupees as the number that turns up on the die, if it is composite. What will be the expected value per throw in the long run, if the participant has to pay ₹42, if one turns upon the die?

Sol. When an unbiased die is rolled, the probability of getting a 1 is 1/6. Similar is a case for the outcomes 2, 3, 4, 5 and 6 i.e. P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6.
Monetary value when a prime number (i.e. 2, 3 or 5) turns up = 2 times the number on die.
Monetary value when a composite number (i.e. 4 or 6) turns up = 3 times the number on the die.
Monetary value when 1 turns up = –42.
(As the participant has to pay in this case).

$\therefore$ Expected value

$$= \frac{1}{6}\times 2\times(2) + \frac{1}{6}\times 2\times(3) + \frac{1}{6}\times 3\times(4)$$

$$+ \frac{1}{6}\times 2\times(5) + \frac{1}{6}\times 3\times(6) - \frac{1}{6}\times 42$$

$$= \frac{8}{6} = ₹1.33 \text{ (approx)}$$

10.24. A game involving a biased die is such that ₹5 is paid each time the die shows up a score of 3, while ₹8 is paid for every other score on the die. The die is such that the score of 3 occurs 4 times as frequently as any other score. How much would a person be willing to pay as entry fee each time, if in the long run, there has to be neither a profit nor a loss for taking part in this game?

Sol. As the game should result neither in profit nor in loss, the entry fee should be equal to the expected value of the game.
To calculate the expected value, we need the probability of the events involved.
Let us assume that the probability of getting any number other than three as p. The probability of getting a three is 4p.
Since all the events are mutually exclusive and collectively exhaustive, the sum of their probabilities should be equal to 1.
Hence p + p + 4p + p + p + p = 1 $\Rightarrow$ 9p = 1
or p = 1/9
$\therefore$ The probability that the number three appears is 4/9 and the probability that any other number appears is 1/9.
Hence the expected value is

$$= \frac{4}{9}(5) + \frac{1}{9}(8 + 8 + 8 + 8 + 8)$$

$$= \frac{20}{9} + \frac{40}{9} = \frac{20}{3} = 6.66$$

Hence, the person should be willing to pay ₹6.66, as entry fee.

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. When three dice are rolled, the number of possible cases that the total score is greater than 16 is

$\boxed{\quad}$

2. When 'n' dice are rolled, the probability that all the dice show '1' is

(A) $\frac{1}{6^{n-1}}$ (B) $\frac{2}{6^n}$

(C) $\frac{1}{6^n}$ (D) $\frac{10}{6^n}$

3. When five coins are tossed, the number of possible outcomes are

$\boxed{\quad}$

4. If A and B are independent events such that P(A) = 1/3 and P(B) = 2/7, then the probability that both the events occur is _______.

(A) $\frac{13}{21}$ (B) $\frac{5}{24}$ (C) $\frac{1}{7}$ (D) $\frac{2}{21}$

5. When four dice are rolled, the probability that the total score on the four dice is maximum is _______.

(A) $\frac{1}{1296}$ (B) $\frac{1}{216}$ (C) $\frac{1}{432}$ (D) $\frac{5}{1296}$

6. If A and B are mutually exclusive and collectively exhaustive events and the probability that the non-occurrence of A is 3/4, then the probability of occurrence of B is $\boxed{\quad}$.

7. A bag contains 5 red balls, 3 green balls and 2 white balls. If one ball is drawn from the bag, then the probability that it is a green ball is $\boxed{}$.

8. When a dice is thrown, the probability that the number on the dice is even or odd is $\boxed{}$.

9. If P(A) = $\frac{3}{4}$, P(B) = $\frac{7}{10}$ and P(A ∩ B) = $\frac{3}{5}$, then P(A ∪ B) is _______.
(A) $\frac{17}{20}$ (B) $\frac{13}{20}$ (C) $\frac{3}{7}$ (D) $\frac{7}{10}$

10. A card is drawn from a pack of cards. The probability that it is a red card is $\boxed{}$.

11. When 7 coins are tossed, the probability that all of them show up either heads or tails is _______.
(A) $\frac{1}{2^7}$ (B) $\frac{5}{2^7}$ (C) $\frac{3}{2^7}$ (D) $\frac{1}{2^6}$

12. If the probability of occurrence of an event A is $\frac{3}{7}$, then the probability of non-occurrence of A is _______.
(A) $\frac{4}{7}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) $\frac{3}{7}$

13. If A_1, A_2,……..A_n are mutually exclusive and collectively exhaustive events of an experiment, then the sum of the probabilities of all the events is $\boxed{}$.

14. In the month of February of a non-leap year, the probability that it will have 5 Saturdays is $\boxed{}$.

15. Four letters and four addressed envelopes are given to a dispatching clerk to dispatch. If he inserts the letters into the envelopes randomly, find the probability that all the letters are inserted into corresponding addressed envelopes.
(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4!}$ (D) 0

16. In the above problem the probability that exactly one letter goes to a non-corresponding addressed envelope is $\boxed{}$.

17. When two fair coins are tossed together, what is the probability that they both show the same face?
$\boxed{}$

18. When a fair dice is rolled once, the probability that a composite number turns up is _______.
(A) 1/4 (B) 1/3 (C) 2/3 (D) 1/2

19. When a letter is selected at random from the English alphabet, the probability that it is a consonant is _______.
(A) 5/26 (B) 10/13 (C) 21/26 (D) 7/13

20. A card is selected at random from a pack of cards. What is the probability that it is a black king?
(A) 2/13 (B) 1/13 (C) 12/13 (D) 1/26

21. When two fair dice are rolled together, what is the probability that they both show even numbers?
$\boxed{}$

22. If four cards are drawn at random from a well shuffled pack of cards, what is the probability that each card is an ace?
(A) $\frac{6}{^{52}C_4}$ (B) $\frac{4}{^{52}C_4}$
(C) $\frac{1}{^{52}C_4}$ (D) None of these

23. If a card is drawn at random from a pack, the probability that it is a black jack is ______.
(A) 1/52 (B) 1/26 (C) 3/26 (D) 1/13

24. If a dice is rolled two times, what is the probability of the product of the numbers obtained being 12?
(A) 1/9 (B) 8/9 (C) 5/36 (D) 1/6

25. A number is chosen at random from {1, 2, 3, 4, 5, 6, 7, 8, 9}. The probability that it will be a multiple of 3 is _______.
(A) 1/3 (B) 2/3 (C) 3/4 (D) 5/9

26. The probability of rolling a total of 3 when two dice are rolled is _______.
(A) 1/36 (B) 1/18 (C) 5/36 (D) 35/36

27. What is the probability that a non-leap year selected at random has 53 Mondays?
(A) $\frac{6}{7}$ (B) 1 (C) $\frac{2}{7}$ (D) $\frac{1}{7}$

28. A and B are events of a certain random experiment. If P(A) = $\frac{1}{2}$, P(B) = $\frac{5}{12}$ and
P (A∪B) = $\frac{7}{12}$, then P(A∩B) = _______.
(A) $\frac{2}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

29. A, B and C are three mutually exclusive and collectively exhaustive events of a certain random experiment. If P(A) = 0.2 and P(B) = 0.6, then find P(C).
$\boxed{}$

30. A and B appear for an interview. The probability that A is shortlisted in the interview is 3/7 and that of B is 1/7. The probability that both are shortlisted for the interview is _______.
(A) 24/49 (B) 4/7 (C) 1/49 (D) 3/49

Exercise – 10(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If three consecutive letters are selected at random from the English alphabet, then the probability that all the three are consonants is _______.

(A) $\frac{1}{2}$ (B) $\frac{11}{24}$ (C) $\frac{5}{12}$ (D) $\frac{7}{12}$

2. Three letters are selected at random from the English alphabet. What is the probability that

(i) all are consonants?

(A) $\frac{25}{132}$ (B) $\frac{163}{260}$

(C) $\frac{33}{65}$ (D) $\frac{133}{260}$

(ii) one is a consonant and the other two are vowels?

(A) $\frac{7}{180}$ (B) $\frac{33}{65}$

(C) $\frac{21}{260}$ (D) $\frac{133}{260}$

3. If six unbiased coins are tossed together, then the probability of getting at least two tails is _______.

(A) $\frac{61}{64}$ (B) $\frac{7}{8}$ (C) $\frac{63}{64}$ (D) $\frac{57}{64}$

4. If eight unbiased coins are tossed together, then the probability that the number of heads exceeds the number of tails is _______.

(A) $\frac{31}{128}$ (B) $\frac{1}{2}$ (C) $\frac{93}{256}$ (D) $\frac{57}{256}$

5. If an unbiased coin is tossed four times, then the probability that the same face does not show up in any two consecutive tosses is

$\boxed{\quad}$

6. A fair dice is rolled twice. What is the probability that the number obtained in the first roll is a factor of the number obtained in the second roll?

(A) $\frac{2}{3}$ (B) $\frac{5}{12}$ (C) $\frac{7}{18}$ (D) $\frac{11}{36}$

7. If four fair dice are rolled together, then the probability that the total score on the four dice is less than 22 is _______.

(A) $\frac{26}{27}$ (B) $\frac{3}{432}$ (C) $\frac{427}{432}$ (D) $\frac{83}{108}$

8. If a number is selected randomly from the natural numbers 1 to 30, the probability that the number is divisible by either 4 or 7 is _______.

(A) $\frac{2}{5}$ (B) $\frac{7}{15}$ (C) $\frac{11}{30}$ (D) $\frac{1}{3}$

9. The probability that a square selected at random from a 8×8 chessboard is of size 3×3 is _______.

(A) $\frac{8}{51}$ (B) $\frac{14}{17}$ (C) $\frac{3}{17}$ (D) $\frac{25}{204}$

10. Eight letters are to be placed in eight addressed envelopes. If the letters are placed at random into the envelopes, the probability that

(i) exactly one letter is placed in a wrongly addressed envelope is $\boxed{\quad}$.

(ii) at least seven letters are placed into right envelopes is _______.

(A) $\frac{1}{8!}$ (B) $\frac{9}{8!}$ (C) 1 (D) 0

(iii) at least two letters are placed into wrongly addressed envelope is _______.

(A) $\frac{7!-1}{8!}$ (B) $\frac{1}{8!}$

(C) $1-\frac{1}{8!}$ (D) $1-\frac{15}{8!}$

(iv) none of the eight letters is placed into its corresponding envelope is _______.

(A) $\frac{1}{8!}$ (B) $1-\frac{1}{8!}$

(C) $1-\frac{1}{7!}$ (D) None of these

11. Two cards are drawn at random from a well shuffled pack of cards. What is the probability that

(i) both are spades or both are diamonds?

(A) $\frac{4}{17}$ (B) $\frac{3}{17}$

(C) $\frac{1}{17}$ (D) $\frac{2}{17}$

(ii) both are queens or both are red coloured?

(A) $\frac{331}{1326}$ (B) $\frac{55}{221}$

(C) $\frac{10}{221}$ (D) $\frac{200}{221}$

(iii) both are diamonds or neither is a king?

(A) $\frac{331}{1326}$ (B) $\frac{55}{221}$

(C) $\frac{10}{221}$ (D) $\frac{190}{221}$

12. If four cards are drawn at random from a well shuffled pack of cards, then what is the probability that

(i) all of them are honours?

(A) $\frac{^{12}C_4}{^{52}C_4}$ (B) $\frac{^{16}C_4}{^{52}C_4}$

(C) $\frac{^{12}C_4}{^{38}C_4}$ (D) $\frac{^{36}C_4}{^{52}C_4}$

(ii) the cards are honours of four different suits?

(A) $\frac{^{16}C_4}{^{52}C_4}$ (B) $\frac{16}{^{52}C_4}$

(C) $\frac{256}{^{52}C_4}$ (D) $\frac{64}{^{52}C_4}$

(iii) three of them are number cards of the same colour and the 4^{th} card is a numbered card of different colour?

(A) $\frac{^{18}C_1{}^{18}C_3}{^{52}C_4}$ (B) $\frac{\left(^{18}C_1{}^{18}C_3\right)^2}{^{52}C_4}$

(C) $\frac{\left(^{16}C_1{}^{16}C_1\right)^2}{^{52}C_4}$ (D) $\frac{2.{}^{18}C_1{}^{18}C_3}{^{52}C_4}$

13. A five digit number is formed using the digits 0, 1, 2, 3, 4 and 5 at random but without repetition. The probability that the number so formed is divisible by 5 is $\boxed{}$.

14. The odds against an event are 4 to 5 and the odds in favour of another independent event are 3 to 7. The probability that

(i) exactly one of them will occur is _______.

(A) $\frac{47}{90}$ (B) $\frac{7}{18}$

(C) $\frac{2}{15}$ (D) $\frac{7}{135}$

(ii) neither of them will occur is ______.

(A) $\frac{7}{45}$ (B) $\frac{31}{45}$

(C) $\frac{14}{45}$ (D) $\frac{7}{15}$

15. A bag contains 4 five rupee coins, 7 two rupee coins and 9 one-rupee coins. If three coins are drawn at random from the bag, then find the odds against drawing the minimum possible amount.

(A) 95 : 88 (B) 7 : 88 (C) 88 : 95 (D) 88 : 7

16. From a box containing 18 bulbs, of which exactly $1/3^{rd}$ are defective, five bulbs are chosen at random to fit into the five bulb holders in a room. The probability that the room gets lighted is _______.

(A) $1-\frac{^{6}C_5}{^{18}C_5}$ (B) $\frac{^{6}C_5}{^{18}C_5}$

(C) $\frac{^{12}C_5}{^{18}C_5}$ (D) $1-\frac{^{12}C_5}{^{18}C_5}$

17. If A and B are two possible events of an experiment such that $P(A \cup B) = 0.6$ and $P(A) = 0.3$ then find P(B) given that

(i) A and B are mutually exclusive events.

$\boxed{}$

(ii) A and B are independent events.

(A) $\frac{4}{7}$ (B) $\frac{9}{10}$ (C) $\frac{3}{10}$ (D) $\frac{3}{7}$

18. Three persons Shiva, Jagan and Rohit aim at a target. Their respective probabilities of hitting the target are 2/3, 5/7 and 3/8. What is probability that

(i) none of them hits the target?

(A) $\frac{7}{92}$ (B) $\frac{5}{28}$

(C) $\frac{5}{84}$ (D) $\frac{10}{198}$

(ii) at least two of them hit the target?

(A) $\frac{9}{14}$ (B) $\frac{107}{168}$

(C) $\frac{77}{168}$ (D) $\frac{79}{84}$

(iii) exactly one of them hits the target?

(A) $\frac{17}{56}$ (B) $\frac{107}{168}$

(C) $\frac{5}{28}$ (D) $\frac{23}{28}$

19. A and B pick a card at random from a well shuffled pack of cards, one after the other replacing it every time till one of them gets a spade. The person who picks a spade is declared the winner. If A begins the game, then the probability that B wins the game is______.

(A) $\frac{5}{9}$ (B) $\frac{4}{9}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$

20. Two balls are drawn, one after the other, from a bag containing 8 pink and 6 orange balls. The probability of drawing pink and orange balls in succession in that order, when the ball that is drawn first is

(i) not replaced is ______.

(A) $\frac{24}{91}$ (B) $\frac{12}{49}$

(C) $\frac{7}{13}$ (D) $\frac{19}{91}$

(ii) replaced is _______.

(A) $\frac{6}{49}$ (B) $\frac{20}{91}$

(C) $\frac{12}{49}$ (D) $\frac{13}{49}$

21. Three balls are drawn at random from a bag containing 6 white, 5 green and 4 red balls. What is the probability that

(i) the three balls are of different colours?

(A) $\frac{67}{91}$ (B) $\frac{5}{91}$

(C) $\frac{1}{455}$ (D) $\frac{24}{91}$

(ii) the three balls are of same colour?

(A) $\frac{68}{455}$ (B) $\frac{80}{455}$ (C) $\frac{34}{455}$ (D) $\frac{30}{65}$

(iii) two of them are of same colour and the third is of a different colour?

(A) $\frac{301}{455}$ (B) $\frac{34}{455}$

(C) $\frac{68}{455}$ (D) $\frac{202}{455}$

22. A bag contains 9 white and 5 yellow balls, and another bag contains 6 white and 8 yellow balls. If one of the bags is selected at random and two balls are drawn at random from the bag, then the probability that both the balls are white is _______.

(A) $\frac{15}{26}$ (B) $\frac{51}{182}$ (C) $\frac{40}{91}$ (D) $\frac{131}{182}$

23. If $P(A \cap B) = 0.2$, $P(A) = 0.5$ and $P(A \cup B) = 0.7$, then $P(\overline{B}) =$ ☐.

24. If $P(A) = 1/4$, $P(B) = 2/5$ and $P(A \cup B) = 1/2$, then $P(A \cap B)$ is ☐.

25. On a biased dice, any even number appears four times as frequently as any odd number. If the dice is rolled thrice what is the probability that the sum of the scores on them is more than 16?

(A) $\frac{26}{375}$ (B) $\frac{112}{375}$

(C) $\frac{26}{3375}$ (D) $\frac{112}{3375}$

26. Two independent witnesses A and B whose chances of speaking truth are 3 out of 4 and 7 out of 10 respectively agree in making a certain statement, then the probability that the statement is true is ☐.

27. A basket contains 8 green balls, 11 white balls and 12 black balls. If 20 balls are picked at random, then the probability that they do not contain a green ball is ____.

(A) $\frac{{}^{5}C_{4}}{{}^{15}C_{4}}$ (B) $\frac{{}^{23}C_{11}}{{}^{31}C_{20}}$

(C) $\frac{{}^{11}C_{8}}{{}^{31}C_{8}}$ (D) $\frac{{}^{11}P_{8}}{{}^{31}P_{8}}$

28. The probability that two particular persons do not have their birthdays on the same day (where they were born in a non-leap year) is ____.

(A) $\frac{2}{365}$ (B) $\frac{363}{365}$

(C) $\frac{1}{365}$ (D) $\frac{364}{365}$

29. If a line segment joining two vertices of a regular hexagon is chosen at random, what is the probability of the line segment to be a diagonal?

☐

30. An unbiased coin is tossed a fixed number of times. If the probability of getting 4 heads equals the probability of getting 7 heads, then what is the probability of getting 2 heads?

(A) $\frac{45}{1024}$ (B) $\frac{55}{1024}$

(C) $\frac{55}{2048}$ (D) $\frac{27}{1024}$

31. At the Wimbledon, the probability that Federer qualifies for the final is 0.7, and the probability that Nadal qualifies for the semifinal is 0.5. The probability that Federer qualifies for the final or Nadal qualifies for the semifinal is 0.8. Given that Nadal qualifies for the semifinal, find the probability that Federer qualifies for the final.

☐

32. A frog fell into a box, which had four holes. Through one hole it can come out of the box in 4 minutes, through another hole it takes 3 minutes, through a third hole, it takes 2 minutes. But through the fourth hole, after travelling for 3 minutes, the frog falls back into the box. Every time it falls back into the box, the properties of the 4 holes get jumbled and the frog is equally likely to try out any one of the 4 holes. Find the expected value for the time the frog takes to come out of the box (in minutes).

☐

33. Raju throws a fair dice. He is promised an amount (in ₹), which is twice the number showing up if that number is odd and an amount thrice the number showing up, if it is even. What is the maximum amount Raju would be willing to pay each time to throw the dice, if in the long run he wants to make an average profit of ₹7 per throw?

☐

34. Ganesh picks a card at random from the set of cards numbered from 201 to 350. If the number on the card that he picks up is a multiple of 7, he wins ₹25. If it is a multiple of 13, he wins ₹60 and if it is multiple of both 7 and 13 he wins ₹100. In the long run, what is the approximate amount Ganesh can win on an average, per draw, if he has to pay ₹4 as a participation fee for each draw?

☐

35. Anil throws a biased coin on which heads appears in 70% of the trials. In a game involving this coin, if Anil receives ₹20 for a head and if he has to pay ₹25 for a tail, then in the long run per game, Anil makes an average profit / loss of ₹ ☐.

Exercise – 10(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. 7 letters are to be placed in seven addressed envelopes. If the letters are placed at random into the envelopes, the probability that

(i) all of them are placed in the corresponding envelopes is _______.

(A) 1 (B) $\frac{1}{6!}$ (C) $\frac{1}{7!}$ (D) $\frac{1}{7^7}$

(ii) Exactly six letters are placed in their corresponding envelopes is $\boxed{}$

2. If three cards are drawn at random, from a well shuffled pack of cards, then what is the probability that

(i) all of them are from the same suit?

(A) $\frac{4\left({}^{13}C_3\right)}{{}^{52}C_3}$ (B) $\frac{\left({}^{13}C_3\right)^3}{{}^{52}C_3}$

(C) $\frac{4.\left({}^{13}C_3\right)^3}{{}^{52}C_3}$ (D) $\frac{{}^{13}C_3}{{}^{52}C_3}$

(ii) all of them are from different suits?

(A) $\frac{13}{{}^{52}C_3}$ (B) $\frac{4(13)^3}{{}^{52}C_3}$

(C) $\frac{{}^{39}C_3}{{}^{52}C_3}$ (D) $\frac{(13)^4}{{}^{52}C_3}$

(iii) Two of them are numbered cards from the same suit and the 3rd number card is from a different suit?

(A) $\frac{{}^9C_2\,{}^9C_1}{{}^{52}C_3}$ (B) $\frac{{}^9C_2\,{}^{27}C_1}{{}^{52}C_3}$

(C) $\frac{4\left({}^9C_2\,{}^{27}C_1\right)}{{}^{52}C_3}$ (D) $\frac{4\left({}^9C_2\,{}^9C_1\right)}{{}^{52}C_3}$

3. The probability of drawing a blue and an orange ball in succession in that order from a bag containing 6 blue and 4 orange balls, when the ball that is drawn first is

(i) not replaced is _______.

(A) $\frac{11}{15}$ (B) $\frac{4}{15}$ (C) $\frac{4}{9}$ (D) $\frac{3}{5}$

(ii) replaced is $\boxed{}$

4. Two cards are drawn at random from a well shuffled pack of cards. Given that both are black, what is the probability that the cards have

(i) the same honour on them?

(A) $\frac{8}{325}$ (B) $\frac{4}{325}$ (C) $\frac{1}{65}$ (D) $\frac{2}{65}$

(ii) different honours and belong to different suits?

(A) $\frac{9}{325}$ (B) $\frac{18}{325}$

(C) $\frac{36}{325}$ (D) $\frac{12}{325}$

5. A number is randomly chosen from the numbers 10 to 99. It is observed that the sum of the digits of the number is ten. Find the probability that it is divisible by five.

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{9}$

Moderate

6. If three consecutive letters are selected at random from the English alphabet, then what is the probability that at least one letter is a vowel?

(A) $\frac{1}{2}$ (B) $\frac{13}{24}$ (C) $\frac{7}{12}$ (D) $\frac{11}{24}$

7. Two numbers are selected at random from the set of first fifty natural numbers. What is the probability that

(i) both are even numbers?

(A) $\frac{12}{49}$ (B) $\frac{24}{49}$ (C) $\frac{1}{2}$ (D) $\frac{37}{49}$

(ii) one is even and the other is odd?

(A) $\frac{25}{98}$ (B) $\frac{24}{49}$ (C) $\frac{25}{49}$ (D) $\frac{24}{98}$

8. Eight unbiased coins are tossed together. The probability that the number of heads is equal to the number of tails is _______.

(A) $\frac{1}{64}$ (B) $\frac{1}{128}$ (C) $\frac{35}{64}$ (D) $\frac{35}{128}$

9. Two fair dice are thrown one after the other. What is the probability that the number on the first die is a factor of the number on the second die?

(A) $\frac{2}{9}$ (B) $\frac{7}{9}$ (C) $\frac{5}{18}$ (D) $\frac{7}{18}$

10. If four dice are thrown together, then the probability that the product of the numbers showing up on them is a prime number is _______.

(A) $\frac{1}{54}$ (B) $\frac{1}{108}$ (C) $\frac{1}{432}$ (D) 0

11. Two cards are drawn at random from a well shuffled pack of cards. What is the probability that

(i) both are spades or both are hearts?

(A) $\frac{3}{17}$ (B) $\frac{2}{17}$ (C) $\frac{1}{17}$ (D) $\frac{15}{17}$

(ii) both are numbered cards or both are red cards?

(A) $\frac{101}{1326}$ (B) $\frac{955}{1526}$ (C) $\frac{401}{663}$ (D) $\frac{262}{663}$

(iii) both are kings or both are diamonds?

(A) $\frac{7}{221}$ (B) $\frac{41}{663}$ (C) $\frac{1}{13}$ (D) $\frac{14}{221}$

12. Five digit numbers are formed using the digits 0 to 5 without repetition. The probability that the number so formed is divisible by 5 is $\boxed{}$.

13. The odds against an event are 4 to 5 and the odds in favour of another independent event are 3 : 7. The probability that

(i) exactly one of them occurs is _______.

(A) $\frac{7}{18}$ (B) $\frac{47}{90}$ (C) $\frac{2}{15}$ (D) $\frac{43}{90}$

(ii) none of them occur is _______.

(A) $\frac{14}{45}$ (B) $\frac{31}{45}$ (C) $\frac{7}{15}$ (D) $\frac{8}{15}$

(iii) at least one of them occurs is ______.

(A) $\frac{11}{32}$ (B) $\frac{13}{61}$ (C) $\frac{31}{45}$ (D) $\frac{21}{45}$

14. A and B pick up a card at random from a well shuffled pack of cards one after the other, replacing it every time till one of them gets a queen. If A starts the game, then the probability that B wins the game is

$\boxed{}$

15. Three balls are drawn at random from an urn containing 7 green, 6 black and 2 pink balls. What is the probability that

(i) the three balls are of the same colour?

(A) $\frac{80}{91}$ (B) $\frac{33}{56}$ (C) $\frac{56}{455}$ (D) $\frac{11}{91}$

(ii) two of them are of same colour and the third is of a different colour?

(A) $\frac{414}{455}$ (B) $\frac{403}{455}$ (C) $\frac{316}{455}$ (D) $\frac{217}{455}$

(iii) the three balls are of different colours?

(A) $\frac{53}{65}$ (B) $\frac{43}{65}$ (C) $\frac{1}{5}$ (D) $\frac{12}{65}$

16. A bag contains 6 five rupee coins, 5 two rupee coins and 4 one rupee coins. If 5 coins are selected at random from the bag, then find the odds in favour of the draw yielding the minimum possible amount.

(A) 1 : 3002 (B) 5 : 2998
(C) 2 : 3003 (D) 5 : 3003

17 A bag contains 7 red and 3 blue balls and another bag contains 6 blue and 4 red balls. If one of the bags is selected at random and two balls are drawn at random from the bag thus selected, the probability that the two balls are of same colour is

$\boxed{}$

18. If A and B are two events of an experiment such that $P(A \cup B) = \frac{3}{4}$, $P(A) = \frac{7}{20}$, then find P(B) given that

(i) A and B are mutually exclusive.

$\boxed{}$

(ii) A and B are equally likely.

$\boxed{}$

(iii) A and B are independent events.

(A) $\frac{7}{13}$ (B) $\frac{8}{13}$ (C) $\frac{6}{13}$ (D) $\frac{2}{5}$

19. In a biased coin, head occurs three times as frequently as tail occurs. If the coin is tossed 3 times, what is the probability of getting two heads?

(A) $\frac{3}{32}$ (B) $\frac{3}{64}$ (C) $\frac{9}{64}$ (D) $\frac{27}{64}$

20. If a square is selected at random from a 8 x 8 chess board, what is the probability that the square is of dimension 1 x 1?

(A) 1 (B) 0 (C) $\frac{16}{51}$ (D) $\frac{17}{108}$

21. Five unbiased coins are tossed together. The probability that the number of heads exceeds the number of tails is $\boxed{}$.

22. Find the probability that a number between 1 to 25 selected at random is divisible by either 2 or 7.

(A) $\frac{15}{23}$ (B) $\frac{14}{25}$ (C) $\frac{14}{23}$ (D) $\frac{3}{5}$

23. From a box containing 24 bulbs of which exactly $\frac{3}{4}^{th}$ of them are good three bulbs are chosen at random to fit into the three bulb holders in a room. The probability that the room is lighted is _______.

(A) $\frac{248}{253}$ (B) $\frac{4}{507}$ (C) $\frac{5}{506}$ (D) $\frac{501}{506}$

24. Three mountaineers Akil, Dikil, and Sunil are climbing up a mountain with their respective probability of reaching the summit being $\frac{2}{3}$, $\frac{5}{8}$ and $\frac{4}{7}$ respectively. What is the probability that

(i) none of them reach the summit?

(A) $\frac{1}{14}$ (B) $\frac{3}{56}$
(C) $\frac{5}{56}$ (D) $\frac{3}{14}$

(ii) exactly two of them reaches the summit?

(A) $\frac{37}{84}$ (B) $\frac{5}{12}$ (C) $\frac{19}{28}$ (D) $\frac{6}{17}$

(iii) at least two of them reaches the summit?

(A) $\frac{5}{21}$ (B) $\frac{3}{56}$ (C) $\frac{37}{84}$ (D) $\frac{19}{28}$

25. I had to type a 6 character password. The probability that I make a mistake in typing a character is 0.3. The password that I typed turned out to be wrong. Find the probability that only the last character that I entered is wrong.

(A) $\frac{(0.7)^5 \times (0.3)}{1-(0.7)^6}$ (B) $\frac{(0.7)^6}{1-(0.7)^6}$

(C) $\frac{(0.7)^5}{1-(0.7)^6}$ (D) $\frac{(0.7)(0.3)^5}{1-(0.7)^6}$

26. Sridhar throws a dice. He is promised an amount twice the value of the number showing up if the number showing up is odd and an amount thrice the value of the number showing up if the number showing up is even. What is the maximum amount that Sridhar is willing to pay each time to throw the dice if in the long run he wants to make an average profit of ₹6 per throw?

$\boxed{}$

27. Sanjay throws a biased coin on which the head appears in 70% of the situations. In a game involving this coin, if Sanjay is paid ₹20 per head and he has to pay ₹25 for a tail, then in the long run, per game Sanjay makes an average profit/loss of $\boxed{}$ ₹ per game.

28. Ten people are to be seated in a row. The probability that two particular persons never sit together is $\boxed{}$

29. If P (A) = $\frac{1}{4}$, P (B) = $\frac{2}{3}$ and P (A $\cup$ B) = $\frac{3}{4}$, then A and B are

(A) mutually exclusive and equally likely events.
(B) independent events but not equally likely.
(C) equally likely and Independent events.
(D) None of these.

30. The odds against an event A are 3 to 2 while the odds in favor of another independent event B are 2 to 5. The probability that at least one of them would happen is _______.

(A) 5/7 (B) 2/7 (C) 4/7 (D) 3/7

31. Arun speaks truth in 70% of cases while Bhargav speaks truth in 65% of cases. The probability that they will contradict each other while stating the same fact is $\boxed{}$

Difficult / Very Difficult

32. If an unbiased coin is flipped 7 times, the probability that the same face shows up in exactly any four consecutive flips is _______.

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{35}{128}$

33. Rahul picks up a card at random from a set of cards numbered from 201 to 350. If the number on the card that he picks up is a multiple of 8, he wins ₹15. If it is a multiple of 13, he wins ₹40, and if it is a multiple of both 8 and 13, he wins ₹80. In the long run, what is the approximate amount in ₹ that Rahul will gain on an average if he has to pay ₹2 as a participation fee for each draw?

$\boxed{}$

34. Let X and Y be two events such that P(X $\cup$ Y) = $\frac{3}{5}$; P ($\overline{X}$) = $\frac{2}{3}$; P ($\overline{Y}$) = $\frac{3}{5}$; then events x and y are _______.

(A) independent and equally likely
(B) independent but not equally likely
(C) mutually exclusive and independent
(D) None of these

35. A letter is selected at random from each of the words "PLASMA" and "MANASA". The probability that the two letters are not the same, is _______.

(A) $\frac{2}{3}$ (B) $\frac{1}{7}$ (C) $\frac{3}{5}$ (D) $\frac{7}{9}$

Key

Concept Review Questions

1. 4
2. C
3. 32
4. D
5. A
6. 0.75
7. 0.3
8. 1
9. A
10. 0.5
11. D
12. A
13. 1
14. 0
15. C
16. 0
17. 0.5
18. B
19. C
20. D
21. 0.25
22. C
23. B
24. A
25. A
26. B
27. D
28. D
29. 0.2
30. D

Exercise – 10(a)

1. B
2. (i) D
   (ii) C
3. D
4. C
5. 0.125
6. C
7. C
8. D
9. C
10. (i) 0
    (ii) A
    (iii) C
    (iv) D
11. (i) D
    (ii) B
    (iii) D
12. (i) B
    (ii) C
    (iii) D
13. 0.36
14. (i) A
    (ii) C
15. D
16. A
17. (i) 0.3
    (ii) D
18. (i) C
    (ii) B
    (iii) A
19. C
20. (i) A
    (ii) C
21. (i) D
    (ii) C
    (iii) A
22. B
23. 0.6
24. 0.1
25. D
26. 0.875
27. D
28. D
29. 0.6
30. C
31. 0.8
32. 4
33. 2
34. 4.16
35. 6.50

Exercise – 10(b)

1. (i) C
   (ii) 0
2. (i) A
   (ii) B
   (iii) C
3. (i) B
   (ii) 0.24
4. (i) B
   (ii) D
5. A
6. B
7. (i) A
   (ii) C
8. D
9. D
10. B
11. (i) B
    (ii) C
    (iii) D
12. 0.36
13. (i) B
    (ii) A
    (iii) C
14. 0.48
15. (i) D
    (ii) C
    (iii) D
16. B
17. 0.5
18. (i) 0.4
    (ii) 0.35
    (iii) B
19. D
20. C
21. 0.5
22. C
23. D
24. (i) B
    (ii) A
    (ii) D
25. A
26. 3
27. 6.50
28 0.8
29. B
30. C
31. 0.44
32. C
33. 3.06
34. B
35. D