Solutions for SM1002110

Chapter – 1
(Special Equations)

Concept Review Questions

Solutions for questions 1 to 15:

1. Given Rem $\left(\frac{4S}{7}\right) = 3$

By trial and error method when S = 6 then Rem $\left(\frac{4S}{7}\right) = 3$.

$\therefore$ The possible values of S are 6, 13, 20, etc.
Choice (B)

2. Given 80a + 3b + 2c = 101.
Clearly $a \le 1 \Rightarrow a = 1$ ($\because$ a, b, c are natural numbers)
Amount left for purchasing the items b and c is 101 – 80 = 21.
Possible values are b = 3, c = 6.
$\therefore$ Total items purchased = 1 + 3 + 6 = 10. Ans : (10)

3. Clearly choice D alone satisfies the given equation.
Choice (D)

4. Given 3x + 7y = 37
When y = 1, x = 10
again when y = 4, x = 3 ($\because x, y \in z^+$)
only two combinations possible. Choice (B)

5. Given 5x + 16y = 100
Clearly x and 100 are multiples of 5
$\therefore$ y must also be multiple of 5. Choice (C)

6. Given Rem $\left(\frac{4Q}{5}\right) = 4$

4 Rem $\left(\frac{Q}{5}\right) = 4$

$\Rightarrow$ Rem $\left(\frac{Q}{5}\right) = 1 \Rightarrow Q = 5k + 1$ Choice (C)

7. The required solutions are: (1, 4) and (4, 6)
$\therefore$ The number of positive integral solution = 2.
Ans : (2)

8. The required solutions are: (0, 4) and (5, 1)
$\therefore$ The number of solutions = 2. Ans : (2)

9. By trial and error method, we can find that only when A = 14, B is a positive integer. Choice (C)

10. Given, 5X + 7Y = 135
As 5X and 135 are divisible by 5, 7Y should also be divisible by 5
$\Rightarrow$ Y = 15 or 20
But when Y = 20, 7Y = 140 > 135.
$\therefore$ Y = 15 Choice (C)

11. Let the number of pencils bought be x and the number of erasers bought be y.
Then 4x + 3y = 15 3y and 15 are divisible by 3
$\Rightarrow$ 4x should also be divisible by 3
$\Rightarrow$ x = 3, 6, 9, ….But for $x \ge 6$, 4 x > 15
$\therefore$x = 3
Hence, the number of pencils bought by the Ramesh is 3
Choice (A)

12. Let the number of parrots be x and the number of rabbits be y.
Then, 4x + 7y = 29
Dividing though out the equation by 4, we get
$\text{rem}\left(\frac{4x+7y}{4}\right) = \text{rem}\left(\frac{29}{4}\right), \Rightarrow \text{rem}\left(\frac{3y}{4}\right) = 1$
By trial y = 3 satisfies the above condition and the other values of y can be obtained at an interval of 4 thereof.
$\therefore$y = 3, 7, 11, … But for $y \ge 7$, 7y > 29
$\therefore$y = 3 $\Rightarrow$ x = 2
Hence, the number of parrots with Shakuntala is 2.
Ans : (2)

13. Let x, y be the number of ₹2 and ₹5 coins with Ritish. Then 2x + 5y = 13
Dividing throughout the equation by 2, we get
$\text{Rem}\left(\frac{2x+5y}{2}\right) = \text{Rem}\left(\frac{13}{2}\right), \Rightarrow \text{Rem}\left(\frac{y}{2}\right) = 1$
So, y = 3 satisfies the above condition and the other values of y can be obtained at an interval of 2 thereof.
y = 1, 3, 5
but when $y \ge 3$ $5y \ge 13$
$\therefore$y = 1 $\Rightarrow$ x = 4
Hence, the number of coins with Ritish is 1 + 4 = 5
Ans : (5)

14. Let the number of scales and charts purchased be x and y respectively, 7x + 5y = 42.
As 7x and 42 are divisible by 7, 5y should also be divisible by 7.
$\Rightarrow$ y = 7, 14, 21, …
But for $y \ge 14$, 5y > 42
$\therefore$y = 7 $\Rightarrow$ x = 1
Hence, the number of items purchased is 1 + 7 = 8
Choice (A)

15. Let x, y be the number of cricket balls and shuttle cocks purchased respectively, 10x + 11y = 130.
As 10x and 130 are divisible by 10, 11y should also be divisible by 10
$\Rightarrow$ y = 10, 20, 30, … But for $y \ge 20$, 11y > 130
$\therefore$ y = 10 $\Rightarrow$ x = 2
Hence, the number of shuttle cocks purchased is 10.
Ans : (10)

Exercise – 1(a)

Solutions for questions 1 to 30:

1. Let D be the date of birth and M be the month of birth of Rajesh.
Then, 25D + 9M = 563 ------- (1)
Dividing the equation by 9, we get
Rem $\left(\frac{25D+9M}{9}\right)$ = Rem $\left(\frac{563}{9}\right)$,

(where Rem $\left(\frac{x}{y}\right)$ is the remainder when x is divided by y)

$\Rightarrow$ Rem $\left(\frac{7D}{9}\right) = 5$

By trial, D = 2 satisfies the above condition and the other values of D can be obtained by adding 9 successively.
$\therefore$ D = 2, 11, 20
The corresponding values of M are 57, 32, 7 –18, ……
As $0 < M \le 12$, M = 7. Choice (B)

2. Let x, y be the number of shuttlecocks and cricket balls purchased. Then, 8x + 15y = 769
Dividing the equation by 8, we get
Rem $\left(\frac{8x+15y}{8}\right)$ = Rem $\left(\frac{769}{8}\right) \Rightarrow$ Rem $\left(\frac{7y}{8}\right) = 1$
By trial, y = 7 satisfies the above condition, and the other values of y can be obtained by adding 8 successively.
$\therefore$ y = 7, 15, 23, 31, 39, 47, 55, …….
But for $y \ge 55$,15y > 769. So, y < 55.
The corresponding values of x are 83, 68, 53, 38, 23, 8.
Hence, the items can be purchased in 6 different combinations.
Ans : (6)

3. Let the number of days on which Kushal met the target be x and the number of days on which he did not meet the targets be y.
Then, 105x + 87y = 2988
$\Rightarrow$ 35x + 29y = 996
$\Rightarrow$ Rem $\left(\frac{6x}{29}\right)$ = Rem $\left(\frac{996}{29}\right)$ = 10 $\Rightarrow$ $6x = 29x_1 + 10$
$\Rightarrow$ Rem $\left(\frac{5x_1}{6}\right)$ = 2 $\Rightarrow$ $5x_1 = 6x_2 + 2$
$\Rightarrow$ Rem $\left(\frac{x_2}{5}\right)$ = 3 $\therefore$ x_2 = 3, 8, 13 etc.
$x_2 = 3 \Rightarrow x_1 = 4 \Rightarrow x = 21 \Rightarrow y = \frac{996 - (35)(21)}{29} = 9$
$\therefore$ Other values of x are obtained by adding 29 successively while those of y are obtained by adding 35 successively. But as x ≤ 31, y ≤ 31, (x, y) = (21, 9)
i.e. Kushal met the target on 21 days. Choice (A)

4. Let the number of coins with John and Rosy be x and y respectively. Then, x + k = 4(y − k)
$\Rightarrow$ x − 4y = − 5k ----- (1)
and x − k = 3(y + k)
$\Rightarrow$ x − 3y = 4k ----- (2)
Eliminating k from (1) and (2), we get 9x − 31y = 0
$\Rightarrow$ x = $\frac{31y}{9}$ $\Rightarrow$ y = 9, 18, 27, 36, ………..and the corresponding values of x are 31, 62, 93, 124, …….
But x + y < 90 (given).
$\therefore$ The possible number of coins with John and Rosy together is 9 + 31, 18 + 62. i.e. 40 or 80. Choice (A)

5. Let the weight of the man who left the group be k kg. Then, 72n + (140 − k) = 75(n + 1) $\Rightarrow$ k = 65 − 3n
when n = 2, k = 59, and when n = 3, k = 56.
But k > 58. $\therefore$ k = 59. Ans : (59)

6. Let m, n be the two parts such that m = 9p and n = 17q, where p, q $\in$ N. Then, 9p + 17q = 284.
Dividing the equation throughout by 9, we get
Rem$\left(\frac{9p+17q}{9}\right)$ = Rem$\left(\frac{284}{9}\right)$ $\Rightarrow$ Rem$\left(\frac{8q}{9}\right)$ = 5
By trial, q = 4 satisfies the above equation, and the other values of q can be obtained by adding 9 successively.
$\therefore$ q = 4, 13, 22, ... But for q ≥ 22, 17q ≥ 284.
So, q = 4, 13 and the corresponding values of p are 24, 7. Hence, the required number of ways of dividing the number 284 is 2. Ans : (2)

7. Let x, y and z be the number of apples, oranges and jack fruits respectively bought by Jasmine.
Then, 8x + 3y + 13z = 112
Put x = X + 4, y = Y + 4 and z = Z + 4
We get, 8(X + 4) + 3 (Y+ 4) + 13(Z+ 4) = 112.
$\Rightarrow$ 8X + 3Y + 13Z = 16.
By trial and error, we can find that the solutions of the above equation are X = 0, Y = 1, Z = 1; X = 2, Y = 0, Z = 0. Hence, Jasmine can buy the fruits in two different combinations. Choice (B)

8. Let x, y and z be the number of 10 marks, 5 marks and 2 marks questions respectively.
Then, x + y + z = 35 ----- (1)
and 10x + 5y + 27z = 100 ----- (2)
(2) − 2 · (1) gives, 8x + 3y = 30.
Since, 3y and 30 are multiples of 3, 8x must be a multiple of 3.
$\Rightarrow$ x = 3, 6, 9, 12, ... But for x ≥ 6, 8x ≥ 30.
$\therefore$ x = 3
$\Rightarrow$ y = 2 and z = 30. Hence, the number of 5 marks questions in the paper is 2. Choice (B)

9. Let x, y and z be the number of woollen jackets, sweaters and gloves respectively sold by the dealer. Then, 300x + 175y + 100z = 1175
$\Rightarrow$ 12x + 7y + 4z = 47.
Dividing the equation by 4, we get
Rem $\left(\frac{12x+7y+4z}{4}\right)$ = Rem $\left(\frac{47}{4}\right)$ $\Rightarrow$ Rem $\left(\frac{3y}{4}\right)$ = 3.
By trial, y = 1 satisfies the above equation, and the other values of y can be obtained at by adding 4 successively.
$\therefore$ y = 1, 5, 9, 13, ... but for y ≥ 9, 7y > 47.
$\therefore$ y = 1, 5. When y = 5, 12x + 3y = 12
This equation does not have positive integral solutions.
$\therefore$ y = 1. Hence, only one sweater is sold by the dealer.
Choice (A)

10. Let x, y and z be the number of employees in the sections A, B and C respectively.
Then, x + y + z = 14. ----- (1)
and 11000x + 6000y + 2000z = 84000
$\Rightarrow$ 11x + 6y + 2z = 84 ----- (2)
(2) − 2(1) gives 9x + 4y = 56.
Clearly x = 4 and y = 5 satisfy the above equation and there is no other solution for the equation. Hence, the number of employees in the section A is 4. Ans : (4)

11. Let A, B and C be the angles of the triangle such that 13A = 17B $\Rightarrow$ A = $\frac{17B}{13}$
As A, B and C are integers, B must be a multiple of 13.
So, the possible values of A, B and C are listed below:

B	13	26	39	52	65	78
A	17	34	51	68	85	X
C	X	X	X	60	30	X

$\therefore$ The least possible angle in the triangle is 30°.
Ans : (30)

Solutions for questions 12 and 13:

Let x, y be the number of black boards and white chalk boxes bought.
Then, 25x+ 5y = 1000 $\Rightarrow$ 5x +y = 200 ----- (1)
and 25y + 5x < 500 $\Rightarrow$ 5y + x < 100
$\Rightarrow$ 5(200 − 5x) + x < 100 (using (1))
$\Rightarrow$ 900 < 24x (or) x > $\frac{225}{6}$ $\Rightarrow$ x = 38, 39 40, 41, ...
But for x ≥ 40, 25x ≥ 1000.
$\therefore$ x = 38, 39 and the corresponding values of y are 10, 5.

12. Ajay can buy the items in two different combinations.
Choice (B)

13. If he bought at least 10 of each, he bought 10 chalk boxes and 38 black boards. Choice (A)

Solutions for questions 14 and 15:

Let x, y and z be the number of CDs, DVDs and cassettes purchased. Then, x + y + z = 38 ----- (1)
and 80x + 150y + 30z = 354 ----- (2)
(2) − 3(1) gives, 5x + 12y = 240.
As 12y, 240 are multiples of 12, 5x must be a multiple of 12.
so, x = 12, 24, 36, 48, 60, ...
But x + y + z = 38. so, x = 12, 24, 36.
The corresponding values of y are 15, 10, 5.
But x + y + z = 38.
$\therefore$ x = 36 and y = 5 is not possible.
So, x = 12, 24 and y = 15, 10, and the corresponding values of z are 11, 4.

14. When x is maximum, y + z = 10 + 4 = 14. Choice (C)

15. When y is minimum, z =4. Choice (A)

16. Let N be the required number. Then, N = 17p + 5 and N = 11q + 6. So, 17p + 5 = 11q + 6
$\Rightarrow$ 17p − 11q = 1.
Dividing the equation by 11, we get
Rem $\left(\frac{17p-11q}{11}\right)$ = Rem $\left(\frac{1}{11}\right)$ $\Rightarrow$ Rem $\left(\frac{6p}{11}\right)$ = 1
By trial, p = 2 satisfies the above condition, and the corresponding values of p can be obtained by adding 11 successively.
$\therefore$ p = 2, 13, 24, 35, 46, 57, ...
But for p $\geq$ 68, N > 1000. So, p = 2, 13, 24, 35, 46, 57. Hence, the required number of 3 digit numbers is 6. Ans : (6)

17. Let x, y be the number of packs of ice – cream of vanilla and strawberry flavors respectively purchased by Meghana. Then, 55x + 80y = 710. $\Rightarrow$ 11x + 16y = 142
Dividing the equation throughout by 11, we get
Rem $\left(\frac{11x+16y}{11}\right)$ = Rem $\left(\frac{142}{11}\right)$ $\Rightarrow$ Rem $\left(\frac{5y}{11}\right)$ = 10
By trial, y = 2 satisfies the above condition, and the other values of y can be obtained by adding 11 successively.
$\therefore$ y = 2, 11, 13, 24, ...
But for y $\geq$ 13, 11y $\geq$ 142
$\therefore$ y = 2 which in turn gives x = 10. The total number of packs purchased is 2 + 10 or 12. Ans : (12)

Solutions for questions 18 and 19:

Let x be the number of sixers hit, y be the number of two's taken and z be the number of singles taken.
Then, 6x + 2y +z = 130 ----- (1)
and x+ y +z = 42 ----- (2)
Also, x − y $\geq$ 8 ----- (1)
and z− y $\leq$ 10 ----- (2)
(1) − (2) gives, 5x + y = 88 ----- (3)
from (1) and (3), we have x − (88 − 5x) $\geq$ 8
$\Rightarrow$ 6x $\geq$ 96 $\Rightarrow$ x $\geq$ 16----- (4)
from (2) and (1), we have (42 − x − y) − y $\leq$ 10
$\Rightarrow$ 42 − x − 2(88 − 5x) $\leq$ 10 $\Rightarrow$ 9x $\leq$ 144 $\Rightarrow$ x $\leq$ 16 ------ (5)
$\therefore$ from (4) and (5), we get x = 16
$\Rightarrow$ y = 88 − 5(16) = 8 and z = 18.

18. Number of sixers hit is 16. Choice (A)

19. All the given options are true. Choice (D)

20. Let y pots be arranged per row and let x be the numbers of rows.
Then, xy < 300 and (x − 10) (y + 6) = xy
$\Rightarrow$xy + 6x − 10y − 60 = xy
$\Rightarrow$ 6x − 10y = 60 $\Rightarrow$ 3x – 5y = 30
We see that y is a multiple of 3 and x of 5.
By trial, x = 10, y = 0 is a solution. The other solutions can be obtained by subtracting the coefficient of y (i.e. –5) successively from the value of x and adding the coefficient of x (i.e. 3) successively to the value of y.
i.e. (x, y) = (10, 0), (15, 3) (20, 6) (25, 9) (30, 12) etc.
But as x > 0, y > 0 and xy < 300, only (15, 3), (20, 6) and (25, 9) are possible values of (x, y)
$\therefore$ The required number of pots = 25 $\times$ 9 = 225.
Ans : (225)

21. Let x, 4y, y be the number of ₹5, ₹2 and ₹1 coins respectively with Pallavi. Then, 5x + 8y + y = 135
$\Rightarrow$ 5x + 9y = 135. As 9y and 189 are multiples of 9, 5x must be a multiple of9.
$\Rightarrow$ x = 9, 18, 27, ... But x $\geq$ 10 and for x $\geq$ 27, 5x $\geq$ 135. $\therefore$ x = 18 which in turn gives y = 5. Hence, the number of coins with Pallavi is x + 5y = 18 + 25 = 43.
Ans : (43)

Solutions for questions 22 and 23:

Let x, y and z be the number of diamonds, rubies and sapphires purchased respectively. Then, x + y + z = 42 ----- (1)
and 1800x + 2700y + 1200z = 75000
$\Rightarrow$ 18x + 27y + 12z = 750
$\Rightarrow$ 6x + 9y + 4z = 250 ----- (2)
(2) − 4(1) gives,
2x + 5y = 82
$\Rightarrow$ y = 2, 4, 6, 8, 10, 12, 14, 16, ...
But y $\geq$ 5. So, y = 6, 8, 10, 12, 14, 16, 18, ...
The corresponding values of x are 26, 21, 16, 11, 6, 1, . .
But x $\geq$ 5. $\therefore$ The possible values of x, y and z are tabulated below:

X	26	21	16	11	6
Y	6	8	10	12	14
Z	10	13	16	19	22

22. The number of rubies purchased is 10. Choice (B)

23. The number of diamonds purchased in this case is 6.
Choice (A)

24. Let the number of insects of the first and second kind be a and b respectively. After 4 days, the total number is 81a + 1296b $\therefore$ 81a + 1296b = 6804
$\Rightarrow$ a + 16b = 84 As Rem $\left(\frac{84}{16}\right)$ = 4, Rem $\left(\frac{a}{16}\right)$ = 4
$\therefore$ (a, b) = (4, 5), (20, 4), (36, 3), (52, 2) and (68, 1)
As a > 25 and b > 2, (a, b) = (36, 3). a + b = 39.
Choice (C)

25. Let x, y and 2x be the number of pens, rulers and refills purchased. Then, 10x + 15y + 10x = 350
$\Rightarrow$ 4x + 3y = 70. Dividing the equation by 3, we get Rem $\left(\frac{4x+3y}{3}\right)$ = Rem $\left(\frac{70}{3}\right)$ $\Rightarrow$ Rem $\left(\frac{x}{3}\right)$ = 1.
By trial, x = 1 satisfies the above equation, and the other values of x can be obtained by adding 3 successively
$\therefore$ x = 1, 4, 7, 10, 13, 16, 19, ...
But for x $\geq$ 19, 4x > 70. So, x = 1, 4, 7, 10, 13, 16 and the corresponding values of y are 22, 18, 14, 10, 6, 2.
But the number of pens purchased is at least 10 more than the number of pencils. So, x = 16 and y = 2. Hence, the total number of items purchased by Arpitha is 3x + y = 48 + 2 = 50. Ans : (50)

26. $\frac{1}{x}+\frac{1}{y}=\frac{1}{12}$ $\Rightarrow$ xy – 12x – 12y + 144 = 144
$\Rightarrow$ (x – 12) (y – 12) = $12^2 = 2^4 3^2$
As x > 0, y > 0, x – 12 > – 12 and y – 12> – 12
If we assign a negative value to either bracket, we have to assign negative values to both the brackets. If one of the values is numerically less than 12, the other would be numerically greater. It is not possible that both factors can be numerically less.
$\therefore$ We can assign only positive values to either bracket, viz any of the 15 values. Correspondingly, we get 15 values for (x, y) Ans : (15)

27. $\frac{5}{x}+\frac{1}{y}=\frac{1}{31}$ $\Rightarrow$ xy – 31x – 5(31) y + 5(31)^2 = 5(31)^2
$\Rightarrow$[x – 5(31)] (y – 31) = 5(31)^2
The number on the RHS has 6 positive integral factors, and hence 12 integral factors (positive or negative) As y > 0, y – 31 > –31
The factors numerically less than 31 are 1 and 5.
$\therefore$ y – 31 can be –5, –1, 1, 5, 31, 5(31), 31^2, 5(31^2).There are 8 possible values and in each case x would have a corresponding integral value (positive or negative)
Choice (D)

28. $\frac{11}{x}-\frac{2}{y}=\frac{1}{9}$ $\Rightarrow$ xy + 18x – 99y – 18(99) = – 18(99)
$\Rightarrow$ (x – 99) (y + 18) = – 18(99) = – 2(3^4)(11) = – N (say)
N has 20 positive integral factors. And hence 40 integral factors (positive or negative). Each bracket can be assigned

any of these 40 values. The other bracket would have to be assigned the corresponding factor .Each such assignment produces an integral value of (x, y). Then, there are 0 values. Choice (D)

29. $x^2 - y^2 = (x - y)(x + y) = 3(5)(7)(11)$
The number on the RHS has 16 factors. As $x > 0$, $y > 0$, it follows that $x + y > 0$ and hence $x - y > 0$, i.e. $x > y$. Thus $x - y$ can be assigned any of the 8 smaller values. $x + y$ would have to take the corresponding value. Each such assignment produces a unique solution. Thus there are 8 positive integral values of (x, y). Choice (C)

30. $a^2 - b^2 = (a - b)(a + b) = 2^2(3)(11)$
The number on the RHS has 12 positive factors ($\therefore$ 24 integral factors (+ve or –ve)). Unlike the earlier problems, in which values could be assigned to the two brackets independently, here we cannot do so. The sum and the difference of the two brackets are both even. $\therefore$ the two 2's on the RHS have to be assigned to the two brackets, one to each bracket. The other factors can be assigned to either bracket. The possible assignments and the corresponding values of a, b are tabulated below.

a – b	a + b	a	b
2	2(3)(11)	34	32
2(3)	2(11)	14	8
2(11)	2(3)	14	–8
2(3)(11)	2	34	–32

By changing the sign of both a – b and a + b ($\therefore$ of both a and b), we get 4 more pairs. Thus there are 8 integral values that (a, b) can take. Ans : (8)

Exercise – 1(b)

Solutions for questions 1 to 30:

1. Let the date on which Ramu was born be d and the month be m.
We have, $8d + 15m = 240$
Dividing by the least coefficient, we get
$$d + m + \left(\frac{7m}{8}\right) = 30$$
In the equation above, $\frac{7m}{8}$ must be an integer.
$\therefore$ m must be divisible by 8.
$\therefore$ m can only be 8.
Therefore the month in which Ramu was born was August.
Choice (D)

2. Let the number of mangoes sold be m and the number of apples sold be a. We have $5m + 6a = 100$
Dividing by the least coefficient, we have
$m + a + (a/5) = 20$
a/5 must be an integer. $\therefore$ a must be divisible by 5
$\therefore$ a = 5, 10, 15, 20, ……….
But when $a \geq 20$, $6a \geq 120$ which is not possible. Therefore only three values of a are possible, viz., a = 5, 10 and 15 and the corresponding values of m are 14, 8 and 2. Therefore, he can sell the fruits in three combinations.
Ans : (3)

3. Let the number of days on which he completed the task be x and number of days on which he did not complete the task be y.
We have, $x + y \leq 30 \to$ (i),
and $50x + 30y = 1430$
$\Rightarrow 5x + 3y = 143$
Dividing by the least coefficient, we have
Rem $(2x/3) = 2 \Rightarrow x = 1, 4, 7, 10, \ldots28$ and the corresponding values of y are 46, 41, 36, 31, ….., 1
But considering the inequality (i) only one combination is possible which is x = 28 and y = 1. Choice (B)

4. The craftsman reported for work for 28 + 1 = 29 days.
Choice (A)

5. Let p and q be the two parts such that p = 5x and q = 8y.
Then, $5x + 8y = 149$ Rem $(3y/5) = 4$
The possible values of y are 3, 8, 13, 18 and the corresponding values of x are 25, 17, 9 and 1.
Hence, we have 4 possible ways. Ans : (4)

6. Let the number be n
Then $n = 17x + 1$ and $n = 3y + 2$
$\therefore 17x + 1 = 3y + 2$
$\Rightarrow 17x - 3y = 1$
Dividing by the least coefficient, we have
Rem $\left(\frac{2x}{3}\right) = 1$
$\Rightarrow$ x = 2, 5 (only) (because when x = 8, (17x + 1) is not a two-digit number)
$\therefore$ There are two numbers of the required kind.
Ans : (2)

7. Let the number of chocolates with A be a, and with B be b.
Then, $a + b < 60$
Let the number of chocolates which A gives to B be equal to x.
Then, $3(a - x) = b + x$ and $a + x = b - x$
$\Rightarrow 2x = b - a$ and $4x = 3a - b$
$\Rightarrow 3a - b = 2b - 2a,$
$\Rightarrow 5a = 3b$
$b = \frac{5a}{3} \Rightarrow$ a has to be a multiple of 3.
So, choice (A) is the right option. Choice (A)

8. Let the number of students in the group be 'n' and let the percentile of the person who left the group be x.
Then,
$$\left(\frac{75n + 75 + 85 + 99 - x}{n + 2f}\right) = 77$$
$75n + 259 - x = 77n + 154$
$\Rightarrow 105 = 2n + x$
$\Rightarrow x = 105 - 2n$ But, $94 < x < 100$
$94 < 105 - 2n < 100$
$\Rightarrow -11 < -2n < -5$
$\Rightarrow 5.5 > n > 2.5$
$\Rightarrow 2.5 < n < 5.5$
Since, n is given to be a multiple of 5, n = 5
Therefore, the number of students in the group now is
= (5 + 2) = 7. Ans : (7)

9. Suppose Ravi buys a pens, b pencils, and c notebooks
Then, $6a + 5b + 3c = 50$.
Since he has to buy at least 3 items of each kind, ₹42 has been spent and with the remaining ₹8, it is only possible to buy 1 pencil and 1 note book. Thus, there is a unique combination in which the items can be bought
Choice (B)

10. Let the number of art books sold be a, science books sold be s, and magazines sold be m.
Then,
$120s + 100a + 25m = 685 \Rightarrow 24s + 20a + 5m = 137$
Dividing by the least coefficient, we have
Rem $\left(\frac{4s}{5}\right) = 2 \Rightarrow s = 3, 8, 13 \ldots\ldots$
But 's' cannot be greater than 3 in the possible set
$\therefore s = 3$
$\Rightarrow 100a + 25m = 685 - 360 \Rightarrow 100a + 25m = 325$
$4a + m = 13$
The possible values of 'a' and 'm', such that a > 0 and m > 0 are
a = 1, m = 9 a = 2, m = 5 a = 3, m = 1
$\therefore$ The maximum possible number of books that could have been sold is 3 + 1 + 9 = 13 Ans : (13)

Solutions for questions 11 and 12:

11. Suppose the owner purchased x gift articles of the kind A and y gift articles of the kind B. Then, 100x + 20y = 4000
5x + y = 200 ------- (1)
Also, 100y + 20x < 2000
5y + x < 100 ⇒ 5 (200 – 5x) + x < 100
$x > \frac{900}{24} = 37 \cdot 5$
⇒ x ≥ 38
But for x ≥ 40, 5x ≥ 200.
So, x = 38, or x = 39 which implies that he can buy the items in two possible combinations. Ans : (2)

12. When x = 38, y = 10 and when x = 39, y = 5
Now since a minimum of 10 pieces of each variety are purchased, the only combination possible is x = 38 and y = 10. So, the total number of gifts the shop owner must have bought is 48. Ans : (48)

13. Let x°, y° and z° be the three angles of the trian gle.
Given 19x = 21y (say)
$\Rightarrow x = \frac{21y}{19}$.
As x, y and z are positive integers, y should be a multiple of 19. The possible combinations of the values of x, y and z are listed below: ('–' indicates that we don't need to consider since z would be obtuse)

x	y	z
21	19	–
42	38	–
63	57	60
84	76	20

∴ The least possible angle in the triangle is 20°.
Choice (B)

14. Given, a + b + c = 40 → (i)
24a + 50b + 30c = 1420 → (ii)
Eliminating a from the above equations, we get
13b + 3c = 230 → (iii)
$\Rightarrow \text{Rem}\left(\frac{13\,b}{3}\right) = 2$
⇒ b = 2, 5, 8, 11, 14 and 17
∴ The maximum possible value of b is 17
∴ A and C together would buy (40 – 17) = 23 pens
Choice (A)

15. Let the number of art books purchased be x and the number of science books purchased be y.
Then, 72x + 110y = 1020 i.e., 36x + 55y = 510
We note that 55y and 510 are multiples of 5. So, 36x must also be a multiple of 5. This means x should be a multiple of 5.
The possible values of x and the corresponding values of y are listed below
36 x + 55y = 510 i.e., 36(5) + 55(6) = 510
36(60) + 55(–30) = 510
After getting one value for (x, y), the other values are obtained by increasing x by 55, (the coefficient of y) and decreasing y by 36, (the coefficient of x). We see that only (5, 6) is a feasible value for (x, y). Therefore, the total number of books bought by Ram is 5 + 6 = 11. Choice (C)

16. Let the number of éclairs, caramels and mints purchased be x, y and z respectively.
Then, 3x + 2y + z= 45 and z = 2y
∴3x + 4y = 45
$\Rightarrow \text{Rem}\left(\frac{y}{3}\right) = 0$ (Dividing by the least coefficient)
⇒ y = 3, 6, 9 and corresponding x = 11, 7, 3.
Therefore, a possible value of the number of eclairs bought is 7. Choice (B)

Solutions for questions 17 and 18:

Let the number of precious stones purchased of the varieties A, B and C be a, b and c respectively
Then, a + b + c = 27 ------ (i)
750a + 1000b + 1250c = 30000
3a + 4b + 5c = 120 ------ (ii)
Eliminating 'a' from (i) and (ii), we get
b + 2c = 39
⇒ b = 39 – 2c ⇒ c ≤ 17 (as b > 3)
Substituting for b in (i), we get
a + 39 – 2c + c = 27 ⇒ c = 12 + a a ≥ 4
When c = 17, we get a = b = 5, which contradicts the condition that a ≠ b). When c = 16, a = 4 and b = 7.
When c ≤ 15, a ≤ 3 which is not possible.
∴ c = 16, a = 4 and b = 7 is the only feasible solution.

17. The total number of pieces of A and B purchased together is 11. Choice (A)

18. C is bought in the greatest possible number.
Choice (C)

Solutions for questions 19 and 20:

Let a, b and c be the number of ₹50, ₹10 and ₹5 notes respectively.
Given that, a + b + c = 27 …..(1)
And 50a + 10b + 5c = 700.
⇒ 10a + 2b + c = 140 …..(2)
Given, ₹5 notes are less than ₹50 notes by at most 2.
⇒ Number of ₹5 notes can be two less than or one less than that of ₹50 notes.
C = a – 2 or a – 1

Case (i): c = a – 2
⇒ a = c + 2
Substituting in (1) and (2)
2c + b = 25…..(3)
11c + 2b = 120 …..(4)
Solving (3) and (4)
C = 10, b = 5 and a = 12

Case (ii): a = c + 1
Substituting in (1) and (2)
2c + b = 26
11c + 2b = 130
When we solve the above two equations, the values of a, b and c won't be integers. Hence, c = 10, b = 5 and a = 12.

19. The number of ₹10 notes is 5. Ans : (5)

20. If the cashier loses two ₹50 notes, the number of ₹50 notes remaining with him would be 12 – 2, i.e., 10.
Ans : (10)

21. Let x be the number of boxes packed and y be the number of fruits packed per box.
Then, (x) (y) < (21) (12) i.e., (x) (y) < 252 --------- (i)
Also, x ≥ 16 --- (ii)
and xy = (x + 10) (y – 5)
⇒ x = 2y – 10
The possible values of x and y (for x > 15) and the corresponding value of (x) (y) are tabulated below

x	y	(x) (y)
16	13	208
18	14	252
20	15	300

∴ Only when (x, y) = (16, 13), (x) (y) is less than 252 and in this case (x) (y) = 208 Choice (A)

22. Let r, t and f respectively denote the number of one-rupee, three-rupee and five-rupee stamps
Then, r + 3t + 5f = 75 ---------- (1)
r ≥ 20 and 5t = r ---------- (2)
∴ From (1) and (2), we have 8t + 5f = 75
Dividing by the least coefficient, we have
Rem (3t/5) = 0
⇒ t = 5, 10, 15…….75
But 8t ≤ 75, ⇒ the only possible value of t is 5
For t = 5, f = $\frac{(75-40)}{5} = \frac{35}{5} = 7$ and r = 25 (∵ from (1))
∴ The total number of stamps with Madhavi is
5 + 7 + 25 = 37. Choice (B)

23. Let the initial number of red flowers be x, and that of yellow flowers be y.
Due to Ram's magic, the number of red flowers at the end of one minute is 729x.
Due to Ravi's magic, the number of yellow flowers at the end of one minute is 64y.
Given, 729x + 64y = 1049 where x and y are integers.
For x > 2, 729x > 1049,
∴ x = 1.The corresponding value of y is $(1049 - 729)/64$
= $320/64 = 5$
Therefore, the total number of flowers initially is x + y i.e., 6.
Ans : (6)

24. $\frac{1}{x}+\frac{1}{y}=\frac{1}{7}$ ⇒ xy – 7x – 7y + 49 = 49
⇒ (x – 7) (y – 7) = 49
As x > 0, y > 0, it follows that x – 7 and y – 7 > –7
∴ (x – 7, y – 7) = (1, 49), (7, 7) or (49, 1)
⇒ (x, y) = (8, 56) (14, 14) or (56, 8)
(x, y) has 3 possible values. Choice (C)

25. $a^2 - b^2$ = (a – b) (a + b) = 7 (11) (13)
As a > 0, b> 0 it follows that a + b > 0. ∴ a – b > 0 as a > b.
Therefore there are 8 factors of the number on the RHS.
∴ (a + b) can have the 4 higher values and (a, b) would have 4 values thus (a – b) (a + b)
= 1(1001)
= 7(143)
= 11(91) = 13(77)
Corresponding (a, b) = (501, 500), (75, 68), (51, 40), (45, 32)
Choice (A)

26. Let the number of employees in each of the departments A, B and C be x, y and z respectively.
Then, x + y + z= 36 --- (1)
and 5000x + 3000y + 2000z = 1,22,000
5x + 3y + 2z = 122 --- (2)
Eliminating z from (1) and (2), we have
3x + y = 50
Considering possible values,
x = 10, y = 20, z = 6 (not possible ∵ each department has at least 10 employees)
x = 11, y = 17, z = 8 (not possible)
x = 12, y = 14, z = 10 (possible)
x = 13, y = 11, z = 12 (possible)
x = 14, y = 8, z = 14 (not possible)
Therefore, we have two possible combinations.
The maximum possible number of people in the department C is 12. Ans : (12)

27. Let the number of scales bought be s, number of pencils be p and number of crayons be c.
Then, c = 2p and p ≥ s + 1
Cost of each scale is ₹2
Cost of each pencil is ₹6
Cost of each crayon is ₹4
6p + 2s + 4c = 94
⇒ 3p + s + 2c = 47
s + 4p + 3p = 47
⇒ s = 47 – 7p
⇒ p ≤ 6 ----- (i)
Also p ≥ s + 1 ⇒ p ≥ 47 – 7p + 1
⇒ 8p ≥ 48
⇒ s > 0 ⇒ p ≥ 6 ----- (ii)
∴From (i) and (ii), p = 6 ⇒ c = 12 and s = 5
So, s + p + c = 5 + 6 + 12 = 23. Choice (A)

28. $\frac{3}{x}+\frac{4}{y}=\frac{1}{5}$ ⇒ xy – 20x – 15y +300 = 300 = $2^2(3)(5^2)$
∴ (x – 15) (y – 20) = 300
As x > 0, it follows that x – 15 > – 15
The number of positive factors of 300 is 3(2)(3) or 18.
The negative factors greater than –15 are –1, –2, –3, –4, –5, –6, – 10 and –12.
∴ x – 15 can have 18 + 8 or 26 possible values, for which x > 0. Correspondingly, y would have some value, positive or negative. Ans : (26)

29. $\frac{37}{x}-\frac{4}{y}=\frac{1}{13}$ ⇒13(37y – 4x) = xy
⇒ xy + 13(4)x – 13(37)y – 13^2(37)(4) = – 13^2(37)(4)
⇒ [x – 13(37)][y + 13(4)] = –13^2(37)(4)
⇒[13(37) – x] [y + 52] = $2^2 13^2 37$
The number on the RHS has 18 factors, to the first bracket we have to assign a factor that is less than 13(37) to the second bracket, we have to assign a factor that is greater than 52. If the first bracket is assigned the value 13(37), the second would be assigned 13(4). ∴ If the first bracket is assigned a value less than 13(37), the second would automatically be assigned a value greater than 13(4).
Therefore 13(37) – x = 1, 2, 4, 13, 26, 37, 52, 74, 148, 169 or 338 i.e., x has 11 possible values and corresponding y has 11 possible values, i.e. (x, y) has 11 values for which x > 0, y > 0. Choice (B)

30. $4x^2 - 9y^2$ = 2100 ⇒ (2x – 3y) (2x + 3y) = $2^2 3^1 5^2 7^1$
Unlike the earlier problems, in which values could be assigned to the two brackets independently, here we cannot do so. The sum of the two factors on the LHS is 4x. Therefore, the two 2's cannot be assigned to the same bracket. (one bracket would be a multiple of 4, the other would be odd. The sum would be odd.) The difference of the two factors on the LHS is 6y i.e., a multiple of 3. There is only one 3 on the RHS. No matter to which bracket it is assigned the difference of the brackets would not be a multiple of 3.
∴ There are no integral solutions. Choice (D)

Chapter – 2
(Quadratic Equations)

Concept Review Questions

Solutions for questions 1 to 25:

1. $x^2 - 9x + \frac{41}{4} = 0$ x = $\frac{9 \pm \sqrt{(-9)^2 - 4(1)(\frac{41}{4})}}{2}$
= $\frac{9 \pm 2\sqrt{10}}{2}$ Choice (D)

2. Any quadratic equation in x,
can be expressed as x^2 – (sum of the roots) x + (product of the roots) = 0.
In the given problem, as the roots are 5 and 6, the quadratic equation in x is $x^2 - 11x + 30 = 0$ Choice (C)

3. As the product of the roots is unknown, the quadratic equation cannot be uniquely determined. Choice (D)

4. $9x^2 - 144x + 92 = 0$ will have the sum of its roots equal to $-\left(\frac{-144}{9}\right) = 16$ Ans : (16)

5. The equation is $x^2 - 20x + 36 = 0$ Choice (B)

6. Going by the choices, only choice (D) satisfies the given conditions. Choice (D)

7. Discriminant = $8^2 - (4)(4)(-11) = 240$ which is positive but not a perfect square
$\therefore$ The roots of $4x^2 + 8x - 11 = 0$ will be conjugate surds. Choice (C)

8. As the equation $x^2 + px + 81 = 0$ has equal roots,
$p^2 - 4(1)(81) = 0$
$\Rightarrow p^2 = 324$
$\Rightarrow p = \sqrt{324} = \pm 18$ Choice (D)

9. $(x^n - b)^2 = 0$ can also be written as $x^{2n} - 2x^n + b^2 = 0$ as the degree of this equation is 2n, it has 2n roots. Choice (C)

10. As the sum of the roots as well as the product of the roots is positive, both roots are positive. Choice (A)

11. Let the roots of $x^2 - 4x + 10 = 0$
be α and β. $\alpha + \beta = 4$ and $\alpha\beta = 10$
The required equation will have roots of $\alpha + 3$ and $\beta + 3$
$\alpha + 3 + \beta + 3 = \alpha + \beta + 6 = 10$
$(\alpha + 3)(\beta + 3) = \alpha\beta + 3(\alpha + \beta) + 9 = 31$
$\therefore$ Required equation is $x^2 - 10x + 31 = 0$. Choice (D)

12. The quadratic equation whose roots are reciprocal of $2x^2 + 5x + 3 = 0$ can be obtained by replacing x by $\frac{1}{x}$ Hence the required equation is $2\left(\frac{1}{x}\right)^2 + 5\left(\frac{1}{x}\right) + 3 = 0$
$\Rightarrow 3x^2 + 5x + 2 = 0$ Choice (B)

13. Product of the roots is $\frac{a}{a} = 1$.
As the product of the roots is 1, the roots are reciprocals of each other.
Hence choice (B) follows. Choice (B)

14. The equation whose roots are the reciprocals of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$
$\therefore$ The required equation is $2x^2 + 4x + 3 = 0$ Choice (A)

15. The required equation will have its sum of the roots equal to that of half that of the given equation. The product of the roots will be one-fourth of that of the given equation.
$\therefore$ The required equation is $x^2 + \frac{7}{2}x + \frac{11}{4} = 0$
i.e. $4x^2 + 14x + 11 = 0$ Choice (B)

16. Maximum / minimum value of the quadratic expression $px^2 + qx + r$ occurs at $x = \frac{-q}{2p}$ Choice (A)

17. For the quadratic expression $px^2 + qx + r$, $\frac{4pr - q^2}{4p}$ is the maximum value when $p < 0$ and the minimum value when $p > 0$. Choice (C)

18. A quadratic expression of the form of $ax^2 + bx + c = 0$, where b and c are real numbers always has a minimum value when a is positive and has a maximum value when a is negative. Its maximum/minimum value is given by
$= \frac{4ac - b^2}{4a}$
For the given expression a = 1, b = 1 and c = 5. Hence it has a minimum value, The minimum value is given by
$\frac{4(1)(5) - 1^2}{4(1)} = \frac{19}{4}$ Choice (A)

19. The product of the reciprocal roots = $\frac{c}{a} = 1$
c = 3, a = 3. Choice (B)

20. If the root of a quadratic equation with rational coefficient is $3 + 2\sqrt{2}$, the other root is $3 - 2\sqrt{2}$.
Therefore, their sum is 6. Choice (C)

21. Let $f(x) = x^3 + 7x^2 + 4x + 5 = x(x^2 + 7x + 4) + 5$
If any polynomial E(x) is divided by another polynomial F(x) of lower degree, then there is a resulting quotient Q(x) and the resulting remainder R(x).
E(x) = F(x) Q(x) + R(x)
In the given problem, E(x) = f(x), F(x) = x, $Q(x) = x^2 + 7x + 4$ and R(x) = 5. Ans : (5)

22. $3x^2 - 5x - 2 = (3x + 1)(x - 2)$
$3x^2 - 5x - 2$ is divisible by $x - 2$
$\therefore$ Remainder is 0. Ans : (0)

23. $5x^3 - 2x^2 - 3x - 2 = x(5x^2 - 2x - 3) - 2)$
$= x(x - 1)(5x + 3) - 2$
If 2 is added to $5x^3 - 2x^2 - 3x - 2$, then the resulting expression is exactly divisible by $x - 1$. Choice (D)

24. Let $g(x) = ax^2 - bx - c$
$x + 1$ i.e. $x - (-1)$ is a factor of g(x) - (1)
Factor Theorem: x – a is a factor of f(x) if and only if f(a) = 0.
$g(-1) = 0$ ($\because$ From (1))
$g(-1) = a + b - c$. $\therefore a + b - c = 0$
$a + b = c$ Choice (C)

25. Remainder theorem: If f(x) is divided by x − a, then the remainder of the division is f(a).
Let $f(x) = 9x^2 - 13x + c$
The remainder of the division of f(x) by x + 1 i.e., $x - (-1)$ is $f(-1)$.
This is also equal to 12.
$9(-1)^2 - 13(-1) + c = 12 \Rightarrow c = -10$. Ans : (–10)

Exercise – 2(a)

Solutions for questions 1 to 40:

1. (i) Let, $x^2 = y$
$y^2 - 41y + 400 = 0$
$\Rightarrow (y - 25)(y - 16) = 0$
Either, y = 25 or 16 When, y = 16, $x^2 = 16$
$\Rightarrow x = \pm 4$
When y = 25, $x^2 = 25 \Rightarrow x = \pm 5$ Choice (B)

(ii) $3.2^{2x+1} - 5.2^{x+2} + 16 = 0$
$\Rightarrow 6.2^{2x} - 20.2^x + 16 = 0$
Let, $2^x = y$
$\therefore 6y^2 - 20y + 16 = 0$
$\Rightarrow (3y - 4)(2y - 4) = 0$
Either y = 2 or 4/3. When, y = 2, $2^x = 2$
$\Rightarrow x = 1$. Choice (A)

(iii) $\sqrt{10x - 4} = 7 - \sqrt{2x + 5}$
Squaring both sides,
$10x - 4 = 49 + 2x + 5 - 14\sqrt{2x + 5}$
$\Rightarrow 8x - 58 = \sqrt{2x + 5}\, - 14$
$\Rightarrow 7\sqrt{2x + 5} = 29 - 4x$
Squaring both sides again,
$98x + 245 = 841 + 16x^2 - 232x$
$\Rightarrow 16x^2 - 330x + 596 = 0$
$\Rightarrow 8x^2 - 165x + 298 = 0$
$\Rightarrow x = 2, 149/8$
But, x = 149/8 is an extraneous root.
$\therefore$ x = 2. Choice (A)

(iv) $\sqrt{3x+4} - \sqrt{2x+1} = \sqrt{x-3}$

Squaring both sides,

$3x + 4 + 2x + 1 - 2\sqrt{6x^2+11x+4} = x - 3$

$\Rightarrow 2\sqrt{6x^2+11x+4} = 4x + 8$

$\Rightarrow \sqrt{6x^2+11x+4} = 2x + 4$

Squaring both sides again,

$6x^2 + 11x + 4 = 4x^2 + 16x + 16$

$\Rightarrow 2x^2 - 5x - 12 = 0$

$\therefore x = 4$ or $-3/2$

But, $x = -3/2$ is not possible.

$\therefore x = 4$ Choice (C)

(v) $\sqrt{x^2-x+2} + \sqrt{6x-8+x^2} = \sqrt{2x^2-x+6}$

$\Rightarrow \sqrt{(x-2)(x+1)} + \sqrt{(x-2)(4-x)} = \sqrt{(x-2)(2x+3)}$

$\Rightarrow \sqrt{x-2}\left(\sqrt{x+1}+\sqrt{4-x}-\sqrt{2x+3}\right) = 0$

$\sqrt{x-2} = 0 \Rightarrow x = 2$

From $\sqrt{x+1} + \sqrt{4-x}$ $\sqrt{4-x} = \sqrt{2x+3}$

Squaring both sides,

$x + 1 + 4 - x + 2\sqrt{4+3x} - x^2 = 2x + 3$

$\Rightarrow 2\sqrt{4+3x-x^2} = 2x - 2$

$\Rightarrow \sqrt{4+3x-x^2} = x - 1$

Squaring both sides again, $4 + 3x - x^2 = x^2 - 2x + 1$

$\Rightarrow 2x^2 - 5x - 3 = 0 \Rightarrow x = 3, -1/2$

$x = -1/2$ will not satisfy the condition given their kind of roots are called extraneous roots. Choice (B)

2. Let the roots be α and 2α

$\alpha + 2\alpha = 3\alpha = p \Rightarrow \alpha = p/3$ ----- (1)

and $\alpha x\ 2\alpha = 2\alpha^2 = 10p \Rightarrow \alpha^2 = 5p$ ---- (2)

From (1) and (2) $\frac{p^2}{q} = 5p \Rightarrow p = 0$ or 45

But we are given $p \neq 0$.

$\therefore p = 45$ Ans : (45)

3. Let the roots be α and β

$\alpha + \beta = m/2$ ---- (1)

$\alpha\beta = 15/2$ ----- (2)

$\alpha - \beta = 1/2$ ----- (3)

Solving (2) and (3) we get $\alpha = 3$, $\beta = 5/2$ or $\alpha = -5/2$, $p = -3$

$\therefore \alpha + \beta = 11/2$ or $-11/2$ Now $m/2 = 11/2$ or $-11/2$

$\therefore m = \pm 11$ Choice (B)

4. Let the roots be 2α and 3α $2\alpha + 3\alpha = -b/a$

$\Rightarrow \alpha = -b/5a$

Also, 2α x $3\alpha = c/a \Rightarrow \alpha^2 = c/6a$

$\therefore b^2/25a^2 = c/6a \Rightarrow 6b^2 = 25ac$ Choice (B)

5. Let the roots be α and α^2

$\alpha + \alpha^2 = -p$

$\alpha^3 = -q$ ------ (2)

$(\alpha + \alpha^2)^3 = -p^3$

$\Rightarrow \alpha^3 + (\alpha^3)^2 + 3\alpha^3(\alpha + \alpha^2) = -p^3$

$\Rightarrow -q + q^2 + 3pq = -p^3$

$\Rightarrow p^3 + q^2 = q(1 - 3p)$ Choice (B)

6. $(x + 1)(x + 3)(x + 5)(x + 7) = 5760$

$\Rightarrow \{(x + 1)(x + 7)\}\{(x + 3)(x + 5)\} = 5760$

$\Rightarrow (x^2 + 8x + 7)(x^2 + 8x + 15) = 5760$

Let $x^2 + 8x + 7 = y$

$y(y + 8) = 5760 \Rightarrow y = 72, -80$ When, $y = 72$

$x^2 + 8x + 7 = 72 \Rightarrow x^2 + 8x - 65 = 0$

$\Rightarrow x = 5, -13$

When $y = -80$ $x^2 + 8x + 7 = -80$

$\Rightarrow x^2 + 8x + 87 = 0$

$D = \sqrt{64} - 348$ and hence x won't have real roots

$\therefore x = 5$ or -13 Choice (A)

7. $4(x^2 + 1/x^2) + 16(x + 1/x) - 57 = 0$

Let $x + 1/x = y$ $x^2 + 1/x^2 = y^2 - 2$

$\therefore 4(y^2 - 2) + 16y - 57 = 0$ $4y^2 + 16y - 65 = 0$

$\Rightarrow y = \frac{-16 \pm \sqrt{256+4\times4\times65}}{2\times4} = \frac{-4\pm\sqrt{81}}{2}$

$= -13/2$ or $5/2$.

when, $y = 5/2$

$\Rightarrow x + 1/x = 5/2 \Rightarrow x = 2$ or $1/2$

when, $y = -13/2$

$\Rightarrow x + 1/x = -13/2$ $2x^2 + 13x + 2 = 0$

$\Rightarrow x = \frac{-13\pm\sqrt{169-16}}{4} = \frac{-13\pm\sqrt{153}}{4}$

Since x is rational, $x = 2$ or $1/2$. Choice (A)

8. $\frac{p}{x-p} + \frac{q}{x-q} = r$

$\Rightarrow (p + q)x - 2pq = r[x^2 - (p + q)x + pq]$

$\Rightarrow rx^2 + (r + 1)(p + q)x - (r + 2)pq = 0$

Sum of the roots = 0

$\Rightarrow (r + 1)(p + q) = 0$

$r + 1 = 0$ or $p + q = 0$ Choice (D)

9. $\alpha + \beta = 8/2 = 4$ $\alpha\beta = 5/2$ $(\alpha/\beta + \beta/\alpha) + (1/\alpha + 1/\beta) - 2\alpha\beta$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + \frac{\alpha+\beta}{\alpha\beta} - 2\alpha\beta = \frac{16-5}{5/2} + \frac{4}{5/2} - 5$

$= 22/5 + 8/5 - 5 = 6 - 5 = 1$ Choice (D)

10. $ax^2 + bx + c = 0$ and $a = c$ $\therefore \alpha\beta = \frac{c}{a} = 1$

$\alpha + \beta = \frac{-b}{a}$ As $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\left(\frac{-b}{a}\right)^2 = \alpha^2 + \beta^2 + 2(1)$ $\left(\frac{-b}{a}\right)^2 = \alpha^2 + \beta^2 + 2.$

The roots α and β may be real or complex, but $(-b/a)^2$ is always non-negative Choice (D)

11. α and β are roots of $ax^2 + bx + c = 0$

$\therefore \alpha + \beta = \frac{-b}{a}$, $\alpha . \beta = \frac{c}{a}$

Sum of the roots of the new equation

$= \left(\frac{b}{c}\right)^2 - 2\left(\frac{a}{c}\right) = \frac{b^2 - 2ac}{c^2}$

$= \frac{\left(\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$

As $\frac{c}{a} = \alpha\beta$ and $\frac{-b}{a} = \alpha + \beta$

$\therefore$ the sum of the roots of the second equation

$= \frac{(\alpha+\beta)^2 - 2\ \alpha\beta}{(\alpha\beta)^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \frac{1}{\beta^2} + \frac{1}{\alpha^2}$ Choice (D)

12. Let α be the root of $x^2 + mx + n = 0$ and $x^2 + nx + m = 0$

$\alpha^2 + m\alpha + n = \alpha^2 + n\alpha + m$

$\Rightarrow \alpha(m - n) = m - n \Rightarrow \alpha = 1$ ($\because m \neq n$)

$\Rightarrow (1)^2 + m(1) + n = 0 \Rightarrow m + n = -1$ Ans : (–1)

13. Let the original number be x

The numbers obtained are

$x - 4$, $x - 3$, $x - 2$ and $x + 1$

Given $(x - 4)(x + 1) = (x - 3)(x - 2)$

$\Rightarrow x^2 - 4x + x - 4 = x^2 - 3x - 2x + 6$

$\Rightarrow x^2 - 3x - 4 = x^2 - 5x + 6$

$\Rightarrow 2x = 10 \Rightarrow x = 5$ Choice (C)

14. $10x + y = 4(x + y) \Rightarrow 3y = 6x$ or $y = 2x$
Also $10x + y = 2xy \Rightarrow 12x = 4x^2 \Rightarrow x = 3$
$\therefore$ y = 6 $\therefore$ The required number is 36 Ans : (36)

15.

	A	B
Number of items	x	y
Price per item	a	b

Now ax = by a/b = y/x Also, ay = 576
And bx = 676 $\Rightarrow$ ay/bx = 576/676
$\Rightarrow$ a/b x a/b = 144/169 (since a/b = y/x)
$\Rightarrow (a/b)^2 = (12/13)^2 \therefore$ a : b = 12 : 13
$\therefore$ A had 12 items Choice (A)

16. For real roots, $B^2 - 4(A)(2) \geq 0$ i.e., $B^2 \geq 8A$ must hold.
$A \geq 2$.
$\therefore 8A \geq 16$. $B^2 \geq 8A \geq 16$.
$B^2 \geq 16$ ----------(1)
$B \leq 6$. Also B is an integer ----------(2)
From (1) and (2), $16 \leq B^2 \leq 36$
$B^2 = 16, 25$ or 36
If $B^2 = 16$, $8A \leq 16$. $A \leq 2$. As $A \geq 2$, A = 2.
If $B^2 = 25$, $8A \leq 25$. $A \leq \frac{25}{8} = 3\frac{1}{8}$. As $A \geq 2$, A = 2 or 3.
If $B^2 = 36$, $8A \leq 36$. $A \leq \frac{36}{8} = 4\frac{1}{2}$. As $A \geq 2$, A = 2, 3 or 4.
(A, B) = (2, ± 4), (2, ± 5), (3, ± 5), (2, ± 6), (3, ± 6), (4, ± 6).
Number of values of (A, B) = 12. Choice (A)

17. Let $4\sqrt{12-4\sqrt{12-4\sqrt{12\,.....}}} = x \Rightarrow 4\sqrt{12-x} = x$
Squaring both sides, we have $x^2 = 192 - 16x$
$\Rightarrow x^2 + 16x - 192 = 0 \Rightarrow (x + 24)(x - 8) = 0 \Rightarrow x = 8$
($\because \sqrt{y}$ denotes a positive quantity)
Required value = $\sqrt{12x} = \sqrt{12(8)} = 4\sqrt{6}$
Choice (C)

18. Let, $\frac{x^2-2x+4}{x^2+2x+4} = k$
$\Rightarrow (k - k)x^2 + 2(k + 1)x + 4(k - 1) = 0$
for x to be real; $D \geq 0$
$\Rightarrow 4(k + 1)^2 - 16(k - 1)^2 \geq 0 \Rightarrow 12k^2 - 40k + 12 \leq 0$
$\Rightarrow 3k^2 - 10k + 3 \leq 0 \Rightarrow (3k - 1)(k - 3) \leq 0$
$\Rightarrow k \in [1/3, 3]$ Choice (A)

19. Let the equation be $ax^2 + bx + c = 0$
The coefficients that Mohan, wrote correctly are a and b.
$\therefore -b/a = 28$
The correct constant term was noted down by Sohan.
$\therefore c/a = 8(24) = 192$
$\therefore$ The correct equation is $(x^2 - 28x + 192) = 0$
$\therefore$ The correct roots are 16 and 12. Choice (B)

20. The discriminant of the quadratic equation is $q^2 - 4p$
This must be non-negative since the roots are real.
$\therefore$ If p = 1, q has 4 possibilities.
If p = 2, q has 3 possibilities
If p = 3, q has 2 possibilities
If p = 4, q has 2 possibilities
If p = 5, q has 1 possibility
$\therefore$ Total number of equations = 12. Ans : (12)

21. If P = 0, then $y = Q^2$. There is only 1 root.
If Q = 0, then $y = P^2 - 1$. There is only 1 root.
If neither P nor Q is 0, there are 2 roots. Choice (D)

22. $x^2 - 19x + 60 = 0$
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = \frac{19 \pm \sqrt{361-240}}{2(1)} = \frac{19 \pm \sqrt{121}}{2}$
$\therefore p = \frac{19+\sqrt{121}}{2}, q = \frac{19-\sqrt{121}}{2}$
$\sqrt{\frac{p}{q}} = \sqrt{\frac{\frac{(19)+11}{2}}{\frac{(19-11)}{2}}} = \sqrt{\frac{15}{4}}$
$\therefore \sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}} = \frac{\sqrt{4}}{\sqrt{15}} = \frac{2}{\sqrt{15}}$
$\sqrt{\frac{q}{p}} = \sqrt{\frac{15}{4}} = \frac{\sqrt{15}}{2}$
$\therefore \frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} = \frac{2}{\sqrt{15}} + \frac{\sqrt{15}}{2} = \frac{4+15}{2\sqrt{15}} = \frac{19}{2\sqrt{15}}$

Alternate method
$\frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} = \frac{p+q}{\sqrt{pq}} = \frac{19}{\sqrt{60}} = \frac{19}{2\sqrt{15}}$ Choice (C)

23. If k = 2, the equation
$x^2 - (2k^2 - 3)x + 2k^3 - 3k^2 + k - 5 = 0$ reduces to
$x^2 - 5x + 1 = 0$
Then $\alpha_1 + \alpha_2 = 5$ and $\alpha_1 \alpha_2 = 1$
$\alpha_1^3\alpha_2 + \alpha_1\alpha_2^3 = \alpha_1\alpha_2(\alpha_1^2 + \alpha_2^2)$
$= \alpha_1\alpha_2\{(\alpha_1 + \alpha_2)^2 - 2\alpha_1\alpha_2)\} = 1\{5^2 - 2\} = 23$
Choice (A)

24. Let $f(x) = \frac{x-1}{x^2-x+4} = y$. $x^2y - xy + 4y - x + 1 = 0$
$\Rightarrow x^2y - x(y + 1) + 4y + 1 = 0$
f(x) can have any value y, provided the roots of the above equation in x are real
$\therefore b^2 - 4ac \geq 0$
$\Rightarrow (y+1)^2 - 4y(4y+1) \geq 0 \Rightarrow y^2 + 2y + 1 - 16y^2 - 4y \geq 0$
$\Rightarrow 15y^2 + 2y - 1 \leq 0$
i.e., $(3y + 1)(5y - 1) \leq 0$
$\Rightarrow y \in \left[-\frac{1}{3}, \frac{1}{5}\right]$
$\therefore$ The maximum value of $\frac{x-1}{x^2-x+4}$ is $\frac{1}{5}$
Choice (C)

25. Let α, β be the roots of the equation $x^2 + kx + k = 0$
$\alpha + \beta = -k$, $\alpha\beta = k$
Given $|\alpha - \beta| < 6$ or $(\alpha - \beta)^2 < 36$
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta < 36$
$k^2 - 4k - 36 < 0$
$\Rightarrow k = \frac{4 \pm \sqrt{16+144}}{2} = \frac{4 \pm 4\sqrt{10}}{2}$
$k = 2 \pm 2\sqrt{10}$
$\therefore$ Range of k for $k^2 - 4k - 36 < 0$ is
$\Rightarrow k \in \left(2 - 2\sqrt{10}, 2 + 2\sqrt{10}\right)$ Choice (C)

26. Given $y = 2x^2 - 3x + 5$ $y - 2x = 2x^2 - 5x + 5$
The minimum value of $ax^2 + bx + c$ is $\frac{4ac-b^2}{4a}$
$\therefore$ The required minimum value is
i.e., $\frac{4(2)(5)-25}{4(2)} = \frac{15}{8}$ Choice (A)

27. The product of the roots is 18 also. As the roots are not coprime, 3 and 6 are the roots of the equation
$\therefore a = \alpha + \beta = 3 + 6 = 9 \Rightarrow a^2 = 81$ Choice (A)

28. Consider the equation f(x) = 0(I), where f(x) is a polynomial. Let the number of sign changes in f(x) be s and the number of sign changes in f(–x) be r. Let the number of positive roots of (I) be p and the number of negative roots of (I) be n.
If the number of sign changes is given, we can say something about the roots
(1) If s is given, p could be s, s – 2, s – 4, …….
(2) If r is given, n could be r, r – 2, r – 4, …….
If the number of roots is given, we can say something about the sign changes.
(3) If p is given, s could be p, p + 2, p + 4, …….
(4) If n is given, r could be n, n + 2, n + 4, …….
Given $f(x) = x^3 + 3x + a$. ∴ $f(-x) = -x^3 - 3x + a$. We are given that (I) has 0 negative roots (n = 0)
∴ r = 0 or 2. There has to be 0 or 2 sign changes in f(–x)
But we can see that there is no scope for 2 sign changes in f(–x). There can only be 0 sign changes
∴ a must be negative. Choice (C)

29. $f(x) = x^6 + 5x^5 + 11x^4 + 25x^3 + 34x^2 + 20x + 24 = 0$.
There are no changes of sign in f(x), f(x) = 0 has no positive roots given f(x) = 0 has four complex roots f (x) = 0 has two negative roots. The number of sign changes in f(–x) has to be more than 2 by an even number. In fact there are four sign changes in f(–x). Ans : (2)

30. Let $f(x) \equiv x^5 + 5x^4 - 103x^3 - 329x^2 + 2802x + 3024 = 0$
f(x) has two sign changes
∴ f(x) = 0 has 2 or 0 positive roots
But it is given that it has one positive root. With this we conclude that f(x) = 0 has two positive roots.
$f(-x) = -x^5 + 5x^4 + 103x^3 - 329x^2 - 2802x + 3024 = 0$
f(–x) has 3 sign changes
∴ f(–x) = 0 has 3 or 1 negative roots.
But it is given that f(x) = 0 has two negative roots. With this we conclude that f(x) = 0 has 3 negative roots
∴All the five roots are accounted for.
∴ f(x) = 0 has zero non-real roots. Ans : (0)

31. The given equation is $x^3 + 5x^2 - 12x - 36 = 0$ ------(1)
Let the roots be α, 3α and β.
∴ $4\alpha + \beta = -5$ ------(2)
$3\alpha^2 + 4\alpha\beta = -12$ ------(3)
and $3\alpha^2\beta = 36$ ------(4)
Combining (2),(4) would produce a cubic equation, while combining (2), (3) would produce a quadratic.
(2), (3) ⇒ $3\alpha^2 + 4\alpha(-5 - 4\alpha) = -12$
⇒ $13\alpha^2 + 20\alpha - 12 = 0 \Rightarrow (\alpha + 2)(13\alpha - 6) = 0$.
∴ α = –2 or 6/13
∴ Let us find β in each case from (2) When α = –2, β = 3 when α = 6/13, β = –89/13. Only in the first case, (4) is satisfied.
∴ α = 2 and β = 3 i.e. the third root is 3.
∴ The third root is 3 Choice (B)

32. Let one root of the first equation be α, then one of the roots of the second equation will be 2α
$\alpha^2 - 3\alpha + 2k = 0$ ------------- (1)
and $4\alpha^2 - 20\alpha + 24k = 0$ ------------- (2)
from (1), we have $2k = 3\alpha - \alpha^2$
⇒ $4\alpha^2 - 20\alpha + 12(2k) = 0$
⇒ $4\alpha^2 - 20\alpha + 12(3\alpha - \alpha^2) = 0$
⇒ $4\alpha^2 - 20\alpha + 36\alpha - 12\alpha^2 = 0$
⇒ $16\alpha - 8\alpha^2 = 0 \Rightarrow \alpha = 0_{12}$ But α ≠ 0; ∴ α = 2
If α = 2, 4 – 6 + 2k = 0 ⇒ 2k = 2
k = 1 (or) 4(4) – 20(2) + 24k = 0 24k = 24 ⇒ k = 1
Choice (A)

33. Given 2 is a root of $x^3 + \ell x^2 + mx + n = 0$
⇒ $2^3 + \ell(2)^2 + m(2) + n = 0$
⇒ $8 + 4\ell + 2m + n = 0$
Also $\ell - 3 = m$ and $m + 8 = n$
⇒ $8 + 4\ell + 2(\ell - 3) + (m + 8) = 0$
or
$8 + 4\ell + 2\ell - 6 + \ell - 3 + 8 = 0$
⇒ $7\ell = -7 \Rightarrow \ell = -1$
$m = \ell - 3 = -4$ and $n = m + 8 = 4$
∴ The given equation is $x^3 - x^2 - 4x + 4 = 0$
x = 2 is a root.
When x = –2 and x = 1, the given equation is satisfied.
∴ –2, 1 and 2 are the roots of the given equation.
As α < β < γ, α = –2, β = 1 and γ = 2.
∴ $\frac{\beta}{\gamma} = \frac{1}{2}$ Choice (A)

34. α, β, γ are the roots of the equation $x^3 - 7x - 6 = 0$
⇒ α + β + γ = 0, ------ (1)
αβγ = 6 ------ (2)
Also, 2, α, α – γ are in arithmetic progression
⇒ α – 2 = α – γ – α ⇒ α + γ = 2
From (1) α + γ + β = 0
⇒ β = – 2
α + γ = 2 , αβγ = 6 ⇒ αγ = –3
∴ $(\alpha - \gamma) = \sqrt{(\alpha + \gamma)^2 - 4\alpha\gamma} = \sqrt{4 + 4.3} = \pm 4$
α + γ = 2, α – γ = 4,
⇒ α = 3, and γ = –1 and if α + γ = 2 and α – γ = –4, then α = –1 and γ = 3 (∴ α + γ = 2 and α γ = – 3)
∴ The equation with roots α and γ is $x^2 - (\alpha + \gamma)x + \alpha\gamma = 0$
i.e. $x^2 - 2x - 3 = 0$ Choice (C)

35. Let $f(x) = x^3 - 6x^2 - 2x + 5$
f(−1) = 0
x + 1 is a factor of f(x)
$f(x) \equiv x^3 - 6x^2 - 2x + 5 = (x + 1)(x^2 + ax + b)$ (say)
Coefficients of x^3, x^2 and x on the two sides must be equal.
The constant terms on both sides must also be equal.
∴ −6 = 1 + a ⇒ a = - 7and b = 5.
a = −7
$(x + 1)(x^2 - 7x + 5) = 0$
x = −1 or $\frac{7 + \sqrt{29}}{2}$
The only rational root is −1 Ans : (1)

36. Let α, β, γ, δ be the roots of the given equation.
α + β + γ + δ = 2
αβ + βγ + αγ + αδ + βδ + γδ = –13
αβγ + αγδ + αβδ + βγδ = –14
αβγδ = 24
From the options, we notice that only the values in the option (D) satisfy the above equations. Choice (D)

37. Let α, β, γ, δ be the roots of the given equation
α + β + γ + δ = 12
and αβγδ = 81
AM of roots = $\frac{12}{4}$= 3 and GM = $\sqrt[4]{81}$= 3
AM = GM ⇒ α = β = γ = δ = 3
∴ a = αβ + αγ + αδ + βγ + βδ + γδ
= 6(9) = 54
– b = αβγ + αγδ + βγδ + αβδ
= 4 (27) = 108
∴ (a, b) = (54,–108) Choice (C)

38. Let $f(x) = x^{72} + x^{60} + x^{48} + x^{36} + x^{24}$
If A and B are functions of x, then the remainder of Ax divided by Bx is x times the remainder of A divided by B.
The remainder of the given expression E(x) (say) divided by $x^3 - x$ is x times the remainder of $\frac{E(x)}{x}$ g=F(x)(say) divided by $x^2 - 1$.
$F(x) = x^{71} + x^{59} + x^{47} + x^{35} + x^{23}$………(1)
As $x^2 - 1$ is a quadratic expression, the remainder of F(x) divided by $x^2 - 1$ is a linear expression.
Let this remainder be ax + b.
F(x) = (x + 1) (x – 1)Q(x) + ax + b…….(2)
From (1), F(–1) = - 5 and F(1) =5
From (2), F(–1) = a(–1) + b and F(1) = a(1) + b
∴a(–1) + b = –5 and a(1) + b = 5

a = 5 and b = 0
ax + b = 5x

Required remainder = Rem $\left(\frac{E(x)}{x^3-x}\right)$ = x Rem $\left(\frac{F(x)}{x^2-1}\right)$

= x(5x) = $5x^2$ Choice (B)

39. $x+\frac{6}{x}=2\sqrt{3}$

Squaring both sides, $x^2+\frac{36}{x^2}+12=12$

$x^2+\frac{36}{x^2}=0$

$x^2=\frac{-36}{x^2}$

$x^4=-36$

$x^8+x^{12}=x^8(1+x^4)=(x^4)^2(1+x^4)=(-36)^2(1+(-36))$

$=36^2(-35)=-36^2(35)$ Choice (D)

40. Let q(x) and r(x) be the quotient and the remainder respectively when F(x) is divided by (x − 4)(x − 6)

F(x) = (x – 5)(x – 6)q(x) + r(x)

As the divisor (x – 5)(x – 6) is a quadratic expression, the remainder r(x) must be a linear expression.

Let r(x) = ax + b, where a and b are constants.

F(x) = (x – 5)(x – 6)q(x) + ax + b

Remainder theorem: If f(x) is divided by x − a, the remainder of the division is f(a)

The remainders of the divisions of F(x) by x – 5 and x – 6 are F(5) and F(6) respectively.

F(5) = 17 and F(6) = 19

F(5) = a(5) + b and F(6) = a(6) + b

a(5) + b = 17 and a(6) + b = 19

5a + b = 17 and 6a + b = 19

Solving these, a = 2 and b = 7

The remainder when F(x) is divided by (x – 5) (x – 6) is 2x + 7.
Choice (D)

Exercise – 2(b)

Solutions for questions 1 to 45:

1. $2x^6+5x^3-7=0$ Let $x^3=y$

The given equation is $2(x^3)^2+5(x^3)-7=0$

or $2y^2+5y-7=0 \Rightarrow 2y^2-2y+7y-7=0$

$\Rightarrow 2y(y-1)+7(y-1)=0$

$\Rightarrow y=1$ (or) $y=\frac{7}{2}$

$\Rightarrow x^3=1$ (or) $x^3=\frac{-7}{2}$ $\therefore$ x = 1 (or) $x=\sqrt[3]{-\frac{7}{2}}$

As x > 0, x =1 Choice (A)

2. $\sqrt{x^2+6}+\sqrt{x^2+3}=5$

squaring both sides,

$(x^2+6)+(x^2+3)+2\sqrt{(x^2+6)(x^2+3)}=25$

$\Rightarrow 2x^2+9+2\sqrt{(x^2+6)(x^2+3)}=25$

$\Rightarrow 2\sqrt{(x^2+6)(x^2+3)}=16-2x^2$

Again squaring both sides

$4(x^2+6)(x^2+3)=(16-2x^2)^2$

$\Rightarrow 4[x^4+9x^2+18]=256+4x^4-64x^2$

$\Rightarrow 4x^4+36x^2+72=256+4x^4-64x^2$

$\Rightarrow 100x^2=184$

$\Rightarrow x^2=\frac{184}{100}\Rightarrow x=\pm\frac{\sqrt{46}}{5}$ Choice (A)

3. As the roots of $x^2-px+q=0$ are equal, discriminant is zero.

$\Rightarrow p^2-4q=0$

$\Rightarrow \frac{p}{2}=\pm\sqrt{q}$ Choice (A)

4. For equal roots, discriminant = 0

i.e. (–2k) – 4(1) (9) = 0

$\Rightarrow 4k^2=36$

$\Rightarrow k=\pm 3$ Choice (B)

5. Let x be the positive number. From the given condition we get $(x-2)^2=x-2$

$\Rightarrow x^2-4x+4=x-2$

$\Rightarrow x^2-5x+6=0$

$\Rightarrow x^2-3x-2x+6=0$

$\Rightarrow x(x-3)-2(x-3)=0$

$\Rightarrow (x-3)(x-2)=0 \Rightarrow x=3$ or 2 Choice (C)

6. Let the number be x. Given that

$\Rightarrow x+4=\frac{2}{x}+5 \Rightarrow x+4=\frac{2+5x}{x}$

$\Rightarrow x^2+4x-5x-2=0 \Rightarrow x^2-x-2=0$

$\Rightarrow x^2-2x+x-2=0 \Rightarrow x(x-2)+1(x-2)=0$

$\Rightarrow (x-2)(x+1)=0 \Rightarrow x=2$ (or) –1 Choice (D)

7. Let my usual speed be x km/hr

(12/x) – (12/x + 2) = 1 $\Rightarrow$ x = 4 Choice (D)

8. Let the two numbers be x and y (x > y)

$x^2+y^2=185$ ----- (1)

x + 3y = 35

$\Rightarrow$ x = 35 – 3y ----- (2)

Substituting in (1) we get;

$(35-3y)^2+y^2=185$

$\Rightarrow 10y^2-210y+1040=0 \Rightarrow y^2-21y+104=0$

y = 8 or 13 When y = 8, x = 11

When y = 13, x = –4 $\therefore$ x = 11 Choice (A)

9. Let the positive number be x.

$x-\frac{1}{x}=\frac{168}{13}$

$13x^2-13=168x$

$\Rightarrow 13x^2-168x-13=0$

$13x^2-169x+x-13=0$

$13x(x-13)+1(x-13)=0$

$\Rightarrow (x-13)(13x+1)=0$

x – 13 = 0 or 13x + 1 = 0 $\Rightarrow$ x = 13 Choice (B)

10. Let the roots be α and $\alpha+3$

$2\alpha+3=7 \Rightarrow \alpha=2$

$\therefore$ The roots are 2 and 5

Now, product of the roots = 2p = 10 $\Rightarrow$ p = 5
Choice (B)

11. Let the roots be $(\alpha-1)$ and $(\alpha+1)$

Product of the roots = $\alpha^2-1=3$

$\Rightarrow \alpha=\pm 2$

But α = +2, as the roots are positive.

$\therefore$ The roots are 1 and 3.

Sum of the roots = –p = 1 + 3.

$\Rightarrow$ p = –4 Choice (B)

12. The equation formed on swapping a and b is

$bx^2+ax+c=0$

$\therefore$ sum of its roots = $\frac{-a}{b}=\alpha+\beta$

Product of its roots = $\frac{c}{b}=\alpha.\beta$

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2.\beta^2}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(\frac{-a}{b}\right)^2-2.\frac{c}{b}=\frac{a^2-2cb}{b^2}$ $(\alpha.\beta)^2=\left(\frac{c}{b}\right)^2$

$\therefore \frac{\alpha^2+\beta^2}{\alpha.\beta^2}=\frac{a^2-2cb}{b^2}\times\frac{b^2}{c^2}=\left(\frac{a}{c}\right)^2-2\left(\frac{b}{c}\right)$

Choice (A)

13. Product of the roots = $4y^{\log_y k^2} = 4k^2$

$\therefore 4k^2 = 256 \Rightarrow k^2 = 64$

$\Rightarrow k = \pm 8$

But k cannot be negative $\therefore$ k = 8 Choice (A)

14. Let the speed of the ordinary train be x km/hr

$\frac{1600}{x} - \frac{1600}{x+15} = 24$

Solving we get; x = 25 Ans: (25)

15. Let the smaller number be x, then the other number will be (24 – x)

$2x^2 - (24 - x)^2 = 4$

$\Rightarrow x^2 + 48x - 580 = 0$ x = 10, –58

$\therefore$ The larger of the two numbers is 24 – 10 i.e. 14

Ans : (14)

16. $2x^2 + 5x + 2 = 0$

$\Rightarrow 2x^2 + 4x + x + 2 = 0$

$2x(x + 2) + 1(x + 2) = 0$

$\therefore x = -2$ (or) $x = \frac{-1}{2}$

The roots of $2x^2 + 5x + 2 = 0$ are k less than the roots of $px^2 + qx + r = 0$ $\therefore$ The roots of $px^2 + qx + r = 0$ are –2 + k and – ½ + k $\therefore$ –2 + k satisfies

$p(-2 + k)^2 + q(-2 + k) + r = 0$

$\therefore p(k - 2)^2 + q(k - 2) + 2 = 0$

$p(k^2 + 4 - 4k) + q(k - 2) + r = 0$

$pk^2 + 4p - 4pk + qk - 2q + r = 0$

$k(pk - 4p + q) + 4p - 2q + r = 0$

$\therefore$ Choice (B) could be true Choice (B)

17. Let the amount spent by each boy be ₹x

Total amount spent by the boys = ₹2x

Amount spent by each girl = ₹(x – 50)

Total amount spent by girls = 2(x – 50)

$\therefore$ Product = (2x) 2 (x – 50) = 416

$\Rightarrow x^2 - 50x - 104 = 0$

$\Rightarrow x^2 - 52x + 2x - 104 = 0$

$\Rightarrow x(x - 52) + 2(x - 52) = 0$

$\Rightarrow$ x = 52 (or) x = –2

As x > 0, x = 52

$\therefore$ Each boy spent ₹52

$\therefore$ Amount spent by each girl = 52 – 50 = ₹2. Ans : (2)

18. Given:

$4x(4x - 4) = 3x(3x - 4) + 2x(2x - 4) + 832$

$\Rightarrow 16x^2 - 16x = 9x^2 - 12x + 4x^2 - 8x + 832$

$\Rightarrow 3x^2 + 4x - 832 = 0$

$\Rightarrow 3x^2 - 48x + 52x - 832 = 0$

$\Rightarrow 3x(x - 16) + 52(x - 16) = 0$

$\Rightarrow$ x = 16 (or) x = –52/3

As x > 0, x = 16 Ans : (16)

19. As c/a is rational, the product of the roots is rational.

As one root is $2/\sqrt{3}+\sqrt{5}$ i.e. $\sqrt{5}-\sqrt{3}$, the other root is $q(\sqrt{5}+\sqrt{3})$, where q is some rational number.

The sum of the roots = $(q + 1)\sqrt{5} + (q - 1)\sqrt{3} = -b/a$ (1)

The product of the roots = 2q = c/a ---- (2)

Substituting for q in (1), we get

$(c/2a + 1)\sqrt{5} + (c/2a - 1)\sqrt{3} = -b/a$

Multiplying throughout by 2a and separating the terms in a, b, c we get $(c + 2a)\sqrt{5} + (c - 2a)\sqrt{3} = -2b$ and $c(\sqrt{5}+\sqrt{3}) + 2a(\sqrt{5}-\sqrt{3}) = -2b$

Choice (C)

20. $cx^2 + bx + a = 0$ $\alpha + \beta = \frac{-b}{c}$ and $\alpha\beta = \frac{a}{c}$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \left(\frac{-b}{c}\right)^3 - 3\frac{a}{c}\left(\frac{-b}{c}\right)$

$= \frac{-b^3}{c^3} + \frac{3ab}{c^2} = \frac{3abc - b^3}{c^3}$ $\alpha^3\beta^3 = (\alpha\beta)^3 = \left(\frac{a}{c}\right)^3 = \frac{a^3}{c^3}$

The quadratic equation is $x^2 - (\alpha^3 + \beta^3)x + \alpha^3\beta^3 = 0$

or $x^2 - \left(\frac{3abc - b^3}{c^3}\right)x + \frac{a^3}{c^3} = 0$

or $c^3x^2 - (3abc - b^3)x + a^3 = 0$ Choice (C)

21. x = 5 – 2y, $x^2+y^2 = (5 - 2y)^2+y^2 = 5y^2 - 20y + 25$ which is a quadratic expression in y^2.

It has a minimum value since the coefficient of y^2 is positive.

Its minimum value = $\frac{(4)(5)(25)-20^2}{4(5)} = 5$.

Choice (C)

22. As the roots of $x^2 + Kx + 4 = 0$ are real and unequal, $(K)^2 - 4(1)(4) > 0$. Also $K^2 \le 36$

$K^2 > 16$ and $K^2 \le 36$ As K > 0, K = 5 or 6

$\therefore$ There are 2 equations Ans : (2)

23. $x^2 - 5x + 6 = 0$

The second equation say (B) is $x^2 - 3x - 2x + 6 = 0$

x(x – 3) – 2(x – 3) = 0 x = 3 (or) x = 2

Twice of one root of x^2-5x+6 = 0 is equal to one root of the first equation, say A, viz

$x^2 - 2Rx + 6 = 0$ If 2(3) = 6 is one root, of A

$(6)^2 - 2R(6) + 6 = 0 \Rightarrow R = 7/2$

If 2(2) = 4 is one root of A then $(4)^2 - 2R(4) + 6 = 0 \Rightarrow R$

$= 11/4$ $\therefore R = \frac{7}{2}$ or $\frac{11}{4}$ Choice (C)

24. $b^2 - 4ac = (-2p)^2 - 4(1)(2p) = 4p^2 - 8p = 4p(p-2)$

If p > 2, b2-4ac is positive.

In this case the roots will be real and unequal.

If p = 2, b-4ac = 0.

In this case the roots will be real and equal.

If 0 < p < 2, $b^2 - 4ac < 0$.

In this case the roots will be complex conjugates

$\therefore$ the nature of the roots cannot be determined.

Choice (D)

25. Let the number of chocolates with A and B be x and y Given x + y = 50 and $x^2 + y^2 = 1300$.

By trial x = 20, y = 30 or x = 30, y = 20

The possible number of chocolates with A and B, the rates at which they sell their chocolates and the possible amounts they get are tabulated below.

	A	B	A	B
Number	20	30	30	20
Rate	5	6	5	6
Amount	100	180	150	120
Total Amount for A, B	280		270	

$\therefore$ The total amount with A and B could be ₹280. If each had as many chocolates as the other, they would get ₹10 less. Alternately the total amount they get could be ₹270. If each had as many as the other had, they would get ₹10 more.

Choice (D)

26. If α and β are the roots of $x^2+bx+c = 0$,

$\alpha_1 + \beta_1 = \frac{-b}{a}$ $\alpha_1\beta_1 = \frac{c}{a}$

$\therefore \alpha_1^2 + \beta_1^2 = (\alpha_1 + \beta_1)^2 - 2\alpha_1\beta_1$

$= \frac{b^2}{a^2} - 2.\frac{c}{a} = \frac{b^2 - 2ac}{a^2}$ If α_2 and β_2 are the roots of $cx^2 + bx + a = 0$, $\alpha_2 + \beta_2 = \frac{-b}{c}$ $\alpha_2 \beta_2 = \frac{a}{c}$

$\therefore \alpha_2^2 + \beta_2^2 = \left(\frac{b}{c}\right)^2 - 2.\frac{a}{c} = \frac{b^2 - 2ac}{c^2}$

As we are given that the sum of the squares of both the equations are the same,

$\frac{b^2 - 2ac}{a^2} = \frac{b^2 - 2ac}{c^2} \Rightarrow (b^2 - 2ac)(c^2 - a^2) = 0$

As $b^2 - 2ac \neq 0$, $\Rightarrow c = a$ (or) $c = -a$ Choice (A)

27. Let, $\left(\frac{x+3}{x-1}\right)^2 = y$

$\therefore y^2 + 2y - 8 = 0 \Rightarrow (y + 4)(y - 2) = 0$

$y = -4$ or 2 but, $\left(\frac{x+3}{x-1}\right)^2 \neq -4$ $\therefore \left(\frac{x+3}{x-1}\right)^2 = 2$

$\Rightarrow x^2 + 6x + 9 = 2x^2 - 4x + 2$

$\Rightarrow x^2 - 10x - 7 = 0$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4x(1)(-7)}}{2(1)} = \frac{10 \pm \sqrt{128}}{2}$

$= 5 \pm 4\sqrt{2}$ Choice (D)

28. The roots of the equation $x^2 - 8x + 15 = 0$ are 5, 3.

Let 5 be the common root with $x^2 - 10x + k = 0$ and let β be the other root. Then $(x - 5)(x - \beta) = x^2 - 10x + k = 0$

$5 + \beta = 10$ and $5\beta = k \Rightarrow \beta = 5 \Rightarrow k = 25$

Similarly, let 3 be the common root with $x^2 - 10x + k = 0$ and let β be the other root. Then $(x - 3)(x - \beta)$

$= x^2 - 10x + k = 0,$

$3 + \beta = 10$

and $3\beta = k \Rightarrow \beta = 7$ and $k = 21$ So, $k = 21$ or 25

Choice (C)

29. The student who copied the co–efficient of x in correctly must have copied the constant term correctly. For the equation, $x^2 + bx + c = 0$, the constant term = c = Product of the roots = 54. The student who copied the constant term incorrectly must have copied the coefficient of x correctly. The coefficient of x = b.

As sum of the roots = – (coefficient of x) = 15, b = –15

Hence the correct equation is $x^2 - 15x + 54 = 0$.

Choice (B)

30. Let the price of each chocolate be ₹p and the number of chocolates bought earlier be c.

Original revenue = pc = 1800

After the increase in the price of the chocolates, revenue realised becomes

$(p + 5)(c - 18) = 1800$

$\Rightarrow pc + 5c - 18p - 90 = pc$

$\Rightarrow 5c - 18\left(\frac{1800}{c}\right) - 90 = 0$

$\Rightarrow 5c^2 - 90c - 32400 = 0$

(Multiplying both sides by c)

$\Rightarrow 5c^2 - 450c + 360c - 32400 = 0$

$\Rightarrow 5c(c - 90) + 360(c - 90) = 0$

$5(c + 72)(c - 90) = 0$

$c + 72 = 0$ or $c - 90 = 0$

As $c > 0$, $c = 90$. Choice (B)

31. Let the numerator and denominator be n and d respectively.

$d = n^2 + 1$ —— (1)

$\frac{n+2}{d+9} = \frac{1}{5}$

$\Rightarrow 5n + 10 = d + 9 = n^2 + 10$

$\Rightarrow n^2 - 5n = 0$

$\Rightarrow n(n - 5) = 0$

$\Rightarrow n = 0$ or $n - 5 = 0$

$\Rightarrow n = 0$ or 5

As the fraction has a positive numerator, n is not 0. hence n is 5.

From (1) d = 26

Hence $\frac{n}{d} = \frac{5}{26}$ Choice (D)

32. $f(x) = 3x^4 - 13x^3 + 7x^2 + 17x + a - 10 = 0$ has 3 positive roots. The number of sign changes in f(x) have to be 3, 5, ….. as f(x) is a 4th degree polynomial there have to be exactly 3 sign changes a – 10 must be negative i.e a < 10. a can be 4.

Choice (B)

33. Let $f(x) = x^{15} + 1$

f(x) = 0 has no sign changes

$\therefore$ The number of positive roots is zero and also x = 0 is not a root.

$f(-x) = -x^{15} + 1$

f(– x) = 0 has one change of sign

$\therefore$f(x) has only one negative root

$\therefore$ The number of non'–'real roots of f(x) = 0 is 14.

Ans : (14)

34. $p^2 - q^2 = 0$

$\Rightarrow p^2 = q^2 (p + r)^2 + q^2 = 1$

$\Rightarrow 2p^2 + 2pr + r^2 - 1 = 0$

$p = \frac{-2r \pm \sqrt{4(r^2 - 2(r^2 - 1))}}{2}$

$= \frac{-2r \pm 2\sqrt{2 - r^2}}{2} = -r \pm \sqrt{2 - r^2}$

As the solution for p must be unique, $2 - r^2 = 0$.

As it must be positive, r must be negative.

$\therefore r = -\sqrt{2}$ Choice (C)

35. Given $x^2 - 2x - 15 = 0$ i.e, $(x - 5)(x + 3) = 0$ is a factor of $x^4 + px^2 + q$

–3, 5 are the roots of $x^4 + px^2 + q = 0$

$\Rightarrow (-3)^4 + p(-3)^2 + q = 0$ and $(5)^4 + p(5)^2 + q = 0$

$9p + q = -81$ and $25p + q = -625$

Solving the above two equations, we get p = – 34 and q = 225

$\therefore (p, q) = (-34, 225)$ Choice (A)

36. The given equation is $x^4 + \ell x^3 + mx^2 + nx + 24 = 0$. Let the roots 3, 1, –2 and the fourth root be denoted by β, γ, δ and α respectively.

The product of the roots = 24

If the fourth root is α,

$3(1)(-2)\alpha = 24$

$\Rightarrow \alpha = -4$

$\therefore$ The roots of the equation are $\alpha = -4$, $\beta = 3$, $\gamma = 1$ and $\delta = -2$

$-\ell$ = (Sum of roots) = $(-4 + 3 + 1 - 2) = -2$ $\therefore$ $\ell = 2$.

$m = (\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta) = -13$

$-n = (\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta) = 14$

$\therefore n = -14$

$\ell + m - n = 2 - 13 + 14 = 3$ Choice (D)

37. Given α, β, γ are the roots of the equation

$x^3 + ax^2 + bx + c = 0$

$\gamma = -1$ From the roots and the coefficients relations

$\alpha + \beta - 1 = -a$ i.e. $-\alpha - \beta + 1 = a$ ------ (1)

$\alpha\beta - \beta - \alpha = b$ ------ (2)

$\alpha\beta(-1) = -1$ i.e. $\alpha\beta = c$ ------ (3)

Also given a, b, c are in arithmetic progression

$\Rightarrow 2b = a + c$ ---------- (4)

From (1) and (3) $a + c = -\alpha - \beta + 1 + \alpha\beta$

From (4), $2(\alpha\beta - \beta - \alpha) = \alpha\beta - \alpha - \beta + 1$

$\Rightarrow \alpha\beta - \beta - \alpha = 1$ Choice (D)

38. $12x^2 + 25x + k = (3x + 4)(ax + b)$ (say)
$= 3ax^2 + 4ax + 3bx + 4b$
Comparing the two sides,
- The coefficients of x^2 have to be equal i.e.,
$3a = 12$. $\therefore a = 4$
- The coefficients of x have to be equal i.e.,
$4a + 3b = 25$
As a = 4, b has to be 3.
$ax + b = 4x + 3$. $\therefore$ $4x + 3$ is a factor of $12x^2 + 25x + k$
Choice (B)

39. $2x + \frac{1}{x} = -\sqrt{6}$

Cubing both sides, we get $8x^3 + \frac{1}{x^3} + 3(2)(2x + \frac{1}{x})$

$= -6\sqrt{6}$

$8x^3 + \frac{1}{x^3} + 3(2)(-\sqrt{6}) = -6\sqrt{6}$

$8x^3 + \frac{-1}{x^3} = 0$

$8x^6 = -1$

$x^6 = -\frac{1}{8}$

$x^{12} - x^6 = \frac{-1}{8}\left(\frac{-1}{8} - 1\right) = \frac{9}{64}$ Choice (A)

40. Let $g(x) = x^{1021} + x^2 + x + 5$.
Remainder theorem: If f(x) is divided by x – a, the remainder of the division is f(a).
The remainder of g(x) divided by x + 1 i.e., x – (−1) is g(−1).
$g(-1) = (-1)^{1021} + (-1)^2 + (-1) + 5$
$= (-1) + 1 + (-1) + 5 = 4$ Ans : (4)

41. The first equation (A) is $x^3 - 3x^2 + 4 = 0$.
We note that $(-1)^3 - 3(-1)^2 + 4 = 0$.
$\therefore$ By the factor theorem, x+1 is a factor of $x^3 - 3x^2 + 4$. Dividing $x^3 - 3x^2 + 4$ by x+1, we get $x^2 - 4x + 4$ in the quotient i.e. $x^3 - 3x^2 + 4 = (x + 1)(x - 2)^2$
The second equation (B) is $x^2 + x - 6 = 0$
$\Rightarrow (x + 3)(x - 2) = 0$.The roots of A are –1, 2, 2
The roots of B are –3 and 2.
We see that 2 roots of A are also roots of B, while only one root of B is also a root of A. Ans : (2)

42. The given equation is $x^3 + 3x^2 - 10x - 24 = 0$ ----(1)
Let the roots of the equation be $\alpha, 2\alpha, \beta$
$\therefore 3\alpha + \beta = -3$ ----(2)
$\alpha(2\alpha) + 2\alpha\beta + \alpha\beta = -10$ i.e. $2\alpha^2 + 3\alpha\beta = -10$ ----(3)
and $2\alpha^2\beta = 24$ ----(4)
(2),(3) $\Rightarrow$
$2\alpha^2 + 3\alpha(-3 - 3\alpha) + 10 = 0$
$\Rightarrow 2\alpha^2 - 9\alpha - 9\alpha^2 + 10 = 0$
$\Rightarrow 7\alpha^2 + 9\alpha - 10 = 0$
$(\alpha + 2)(7\alpha - 5) = 0$
$\Rightarrow \alpha = -2$ or $\frac{5}{7}$
Let us find β in each case from (2)
When $\alpha = -2$, $\beta = 3$. When $\alpha = 5/7$, $\beta = 36/7$. Only in the first case, (4) is satisfied.
$\therefore$ $\alpha = -2$ and $\beta = 3$ i.e. the third root is 3. Ans : (3)

43. The given equation is $x^3 - 7x^2 + 36 = 0$ --- (1)
Let α, -3α, β be the roots of the equation
$\alpha - 3\alpha + \beta = 7 \Rightarrow \beta - 2\alpha = 7$ --- (2)
$\alpha(3\alpha) + (-3\alpha)\beta + \beta(\alpha) = 0$
$\Rightarrow -3\alpha^2 - 2\alpha\beta = 0$ --- (3)
(2), (3) $\Rightarrow -3\alpha^2 - 2\alpha(7 + 2\alpha) = 0 \Rightarrow -3\alpha^2 - 14\alpha - 4\alpha^2 = 0$
$\Rightarrow 7\alpha^2 + 14\alpha = 0 \Rightarrow 7\alpha(\alpha + 2) = 0$
$\alpha = 0$, or $\alpha = -2$
$\alpha = -2$ satisfies equation (1) while $\alpha = 0$ does not
$\beta = 7 + 2\alpha = 7 - 4 = 3$
$\therefore$ The roots of the equation are –2, 6, 3
The difference of the greatest two roots = 6 – 3 = 3
Ans : (3)

44. $x^2 - 12x + k = 0$

$x = \frac{12 \pm \sqrt{144 - 4k}}{2}$

$= 6 \pm \sqrt{36 - k}$

One of the roots of $x^2 - 12x + k = 0$ lies between 0 and 1 $6 + \sqrt{36 - k}$ is at least 6 and $6 - \sqrt{36 - k}$ is at most 6.

$\therefore$ The root lying between 0 and 1 has to be $6 - \sqrt{36 - k}$

$0 < 6 - \sqrt{36 - k} < 1$

$-6 < -\sqrt{36 - k} < -5$

$5 < \sqrt{36 - k} < 36$

$0 < k < 11$

Alternative solution
One of the roots of $f(x) \equiv x^2 - 12x + k = 0$ lies between 0 and 1.
F(x) = k and f(1) = 1-12 +k =k-11
There two expressions have to opposite signs
As k > k – 11, k has to be positive and k -11 has to be negative, i.e.$0 < k < 11$. Choice (D)

45. The dividend is a polynomial of degree 20 and it has 11 terms. The divisor is $x^3 + x$, a binomial of degree 3. The degree of the remainder of degree 0, 1 or 2. The dividend can be split into the 11 terms and the remainder of each term can be obtained. The required remainder is the sum of these 11 remainders. The various terms, including the monomials of lower degree and the corresponding remainders are shown below. The divisor is $x^3 + x$.

Monomial	Remainder
1 x x^2	1 x x^2
x^3 x^4	-x $-x^2$
x^5 x^6	$-x^3 \equiv x$ x^2

We see that the remainder can only be –x, $-6-x^2$, x or x^2. These 4 possible remainders, and the corresponding terms (from among the given 11 terms) are tabulated below.

Monomial	Remainder
-x	x^{11}, x^{15}, x^{19}
$-x^2$	x^{12}, x^{16}, x^{20}
x	x^{13}, x^{17}
x^2	x^{10}, x^{14}, x^{18}

$\therefore$The net remainder is –x. Choice (B)

Solutions for questions 46 to 55:

46. From statement I, $k^2 - 4a(72) = 0 \Rightarrow k^2 = 288a$
But as we do not know the value of a, we cannot find k
$\therefore$Statement I alone is not sufficient
From statement II, a = 8
But no further information is given.
$\therefore$ Statement II alone is not sufficient.
By combining I and II,
$k^2 = 288(8) \Rightarrow k = \pm 48$
But given k is positive $\Rightarrow$ k = 48 Choice (C)

47. $x^2 + y^2 + 4x - 6y + b = 0$
From statement (1) we get
$x^2 + y^2 + 4x - 6y + 13 = 0$
$(x + 2)^2 + (y - 3)^2 = 0$
Since it is expressed as a sum of two squares, both terms must be 0.
$\therefore$ x = – 2, y = 3,i.e., there is a unique solution for (x, y)
Hence statement I alone is sufficient.
From statement II we get
$x^2 + y^2 + 4x + 6y + 0 = 0$
$\Rightarrow (x + 2)^2 + (y - 3)^2 = 13$ i.e., (x, y) can have many values, i.e., the equation does not have a unique solution.
$\therefore$ Statement II alone is also sufficient Choice (B)

48. The minimum value of the expression $3x^2 + bx + c$ is

$$\frac{4(3)(c)-b^2}{4(3)} = c - \frac{b^2}{12}$$

From statement I , the product of the roots = c/3 = 1
$\Rightarrow$ c = 3. But we do not know the value of b
$\therefore$Statement I is not sufficient.
From statement II, –b/3= 2 $\Rightarrow$ b = – 6
But we don't know the value of c
$\therefore$Statement II is not sufficient.
By combining I and (II),
the minimum value = 3 – 36/12 = 0 Choice (C)

49. From statement I, as one of the roots is complex the other root is also complex.
$\therefore$Statement I alone is sufficient.
From statement II,one of the roots is real. This means that the other root is also real. Hence statement (II) is also sufficient. Choice (B)

50. From statement I,
Let the roots be α and $1/\alpha$

$$\therefore \alpha + \frac{1}{\alpha} = \frac{-b}{5} \Rightarrow 2 \le \frac{-b}{5} \Rightarrow -b \ge 10 \Rightarrow b \le -10$$

$\therefore$ b is not greater than 0
$\therefore$ statement I alone is sufficient
From statement II, $b^2 = 20c$
$\therefore$b can be positive, negative or zero.
$\therefore$ statement II is not sufficient. Choice (A)

51. The roots of $x^2 - 3x - 4 = 0$ i.e.,(x – 4) (x + 1) = 0 are
x = 4 or x = – 1
if x = 4 is the common root and then the other root of $2x^2 + kx - 5 = 0$ is P_2, then $4P_2 = -\frac{5}{2}$ or $P_2 = -\frac{5}{8}$

$$\therefore -\frac{k}{2} = 4 + -\left(\frac{5}{8}\right) = \frac{27}{8} \quad \therefore k = -\frac{27}{4}$$

if x = – 1 is the common root then $-P_2 = -\frac{5}{2}$

$$P_2 = \frac{5}{2} \text{ i.e. } -\frac{k}{2} = -1 + \frac{5}{2} = \frac{3}{2}$$

k = – 3
From statement I, k is an integer. Hence k = – 3.
Hence Statement I alone is sufficient.
From statement II, k + 5 is a natural number.
This is possible only when k = –3.
Hence statement II is also sufficient. Choice (B)

52. The roots of the equation $ax^2 - (a + 1)x + 1 = 0$ are

$$\frac{a+1 \pm \sqrt{(a+1)^2 - 4a}}{2a}$$

$$= \frac{(a+1) \pm (a-1)}{2a} = \frac{2a}{2a} \text{ or } \frac{2}{2a} = 1 \text{ or } \frac{1}{a}$$

if a = 1 or -1 then both roots of the equation are integers Otherwise there is only one integral root. From statement I there is only one integral solution.
Statement II allows the possibility x = –1 which results in an integer and for other values result is a non-integer.
So II alone is not sufficient. Choice (A)

53. $x^2 - 5x + a = 0$
p and q are the roots.
p + q = 5 pq = a

$$\frac{(p^3+q^3)}{pq} = \frac{(p+q)^3 - 3pq(p+q)}{pq}$$

$$= \frac{5^3 - 3a(5)}{a} = \frac{125 - 15a}{a} = \frac{125}{a} - 15$$

From statement I, a = 2, 3 or 4 At any of these values,
$\frac{125}{a} - 15$ is not an integer
$\therefore$ Statement I alone is sufficient
From statement II, when a = 1 or 5, 125/a – 15 is an integer and in the other cases it is not.
$\therefore$Statement II is not sufficient. Choice (A)

54. For the equation $ax^2 + 3x + 2 = 0$, the roots are $\propto$ and β

$$\therefore \alpha+\beta = \frac{-3}{a}, \ \alpha\beta = \frac{2}{a}$$

$$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3 - 3(\alpha+\beta)\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-3}{a}\right)^3 - 3\left(\frac{-3}{a}\right)\left(\frac{2}{a}\right)}{\frac{2}{a}}$$

$$= \frac{\left(\frac{-27}{a^3} + \frac{18}{a^2}\right)}{\frac{2}{a}} = \frac{18a - 27}{2a^2} = \frac{9}{2a^2}(2a - 3), \text{ which is}$$

negative if a < 3/2, zero if a = 3/2 and positive if a > 3/2.

From statement I, if a is –ve, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} < 0$.

Hence I alone is sufficient.

If a > 0 and lesser than $\frac{3}{2}$ then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} < 0$

But if $a > \frac{3}{2}$, then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} > 0$

Hence statement II is not sufficient Choice (A)

55. Sum of the roots of $ax^2 + bx + c = 0$ is $\frac{-b}{a}$

From statement I, b/c = 3
This statement alone is not sufficient.
From statement II, sum of the roots of $cx^2 + ax + b = 0$ is 4, i.e., – a/c = 4. This statement alone is not sufficient.

But if we use both statements, we have $\frac{b}{c} = 3, \ \frac{-a}{c} = 4$

$$\text{Hence } \frac{-b}{a} = \frac{\frac{b}{c}}{\frac{-a}{c}} = \frac{3}{4}$$

$\therefore$I and II together are sufficient. Choice (C)

Chapter – 3
(Inequalities and Modulus)

Concept Review Questions

Solutions for questions 1 to 25:

1. (2, 5) (By definition) Choice (D)

2. (– 5, 2] (By definition) Choice (C)

3. When a = 2 and b = 3, a – b = 2 – 3 = – 1
$\therefore$ a – b need not always be positive. Choice (B)

4. Since 'a' and 'b' are real numbers, ab, a + b and a – b need not always be positive. Choice (D)

5. Choice (A) is a standard result
Consider choice (B),
4 > 2 and –3 > –4
But 4(–3) < 2(–4)
Consider choice (C),
4 > 2 and –2 > –5
But 4 + 2 < 2 + 5 Choice (A)

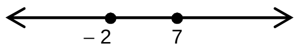
6. We know that, when a > 0, b > 0 and a > b, then $\frac{a}{b} > 1$.
$\therefore$ Choice (C) is always true.
When a = – 4 and b = – 2, $\frac{a}{b} = \frac{-4}{-2}$
= 2 > 1. But – 4 < – 2
$\therefore$ Choice (A) is not always true.
When a = – 2 and b = – 4,
$\frac{a}{b} = \frac{-2}{-4} = \frac{1}{2} < 1$
But – 2 < – 4
$\therefore$ Choice (B) is not always true. Choice (C)

7. Given, x + 2 ≥ 5
$\Rightarrow$ x ≥ 5 – 2
$\Rightarrow$ x ≥ 3
$\therefore$ x $\in$ [3, ∞) Choice (A)

8. Given, 3 – x ≤ 4
$\Rightarrow$ – x ≤ 4 – 3 $\Rightarrow$ – x ≤ 1
$\Rightarrow$ x ≥ – 1 Choice (C)

9. Given, 2x + 3 > 4
$\Rightarrow$ 2x > 4 – 3
$\Rightarrow$ 2x > 1 $\Rightarrow$ x > $\frac{1}{2}$
$\therefore$ The required solution set is $\left(\frac{1}{2}, \infty\right)$ Choice (D)

10. Given, $x^2 – 5x – 14 \le 0$
$\Rightarrow x^2 – 7x + 2x – 14 \le 0$
$\Rightarrow$ x (x – 7) + 2 (x – 7) ≤ 0
$\Rightarrow$ (x – 7) (x + 2) ≤ 0
The critical values are – 2, 7



Clearly '0' satisfies the given inequation.
So, the solution set is [– 2, 7]
The integers in the solution
Set are – 2, – 1, 0, 1, 2, 3, 4, 5, 6 and 7. Choice (A)

11. Since 'a' is a positive real number,
A.M. $\left(a, \frac{1}{a}\right) \ge$ G.M. $\left(a, \frac{1}{a}\right)$
$\Rightarrow \frac{\left(a + \frac{1}{a}\right)}{2} \ge \sqrt{a \cdot \frac{1}{a}} \Rightarrow a + \frac{1}{a} \ge 2$
$\therefore$ The minimum value of $a + \frac{1}{a}$ is 2. Ans : (2)

12. We have, $x^2 – 3x + 2 = (x – 1)(x – 2)$
Which is not always positive
$x^2 + 2x – 35 = (x + 7)(x – 5)$ which is not always positive
$x^2 + 4x + 5 = (x + 2)^2 + 1$ which is always positive
$x^2 – x – 6 = (x – 3)(x + 2)$ which is not always positive.
Choice (C)

13. We know that, a > b $\Rightarrow a^2 > b^2$
if a and b are positive real numbers. Choice (C)

14. – 5 ≤ x < 7 means x can be any real number lying between –5 and 7 including –5 but not 7. One way to represent the solution set is [-5,7]. Choice (C)

15. α, β (α < β) are the roots of the equation (x –α) (x – β) = 0 and the solution sets for the inequality (x – α) (x – β) ≥ 0
(-α, α] U [β, α).
$\Rightarrow$ Here α =–3 and β = 5, α
$\therefore$ The required equation is (x+3) (x–5) ≥ 0.
Choice (A)

16. 2x –5 ≥ 7x +10
$\Rightarrow$ 2x –7x ≥ 10 + 5
$\Rightarrow$ –5x ≥ 15
x ≤ –3 The solution set is (–∞,–3] Choice (C)

17. $\Rightarrow$ 3x +4 ≤ –5x+12 $\Rightarrow$ 3x+5x ≤ 12–4
$\Rightarrow$ 8x ≤8 $\Rightarrow$ x ≤ 1 Choice (B)

18. If (x – α) (x – β) ≥ 0 then
x ≤ α, or x ≥ β (α < β)
Here, α = –13 and β = 15
x ≤ –13 or x ≥ 15. Choice (C)

19. The minimum value of |x| is 0.
$\therefore$ The minimum value of 10 + |x| is 10 + 0 = 10
Ans : (10)

20. The expression 5 – |2 – x| will have a maximum value when |2 – x| = 0
i.e., when x = 2. Ans : (2)

21. Given, |x + 2| = 2x – 1
Case (i) : Let x < – 2
Then |x + 2| = – (x + 2)
$\therefore$ |x + 2| = 2x – 1
$\Rightarrow$ – (x + 2) = 2x – 1
$\Rightarrow$ – x – 2 = 2x – 1
$\Rightarrow$ 3x = – 1 $\Rightarrow$ $x = \frac{-1}{3}$
But $x = \frac{-1}{3}$ does not satisfy the given equation
Case (ii):
Let x ≥ –2
Then, |x + 2| = x + 2
$\therefore$ |x + 2| = 2x – 1 $\Rightarrow$ x + 2 = 2x – 1
$\Rightarrow$ x = 3
Clearly, x = 3 satisfies the given equation
Hence, the required number of solutions is 1.

Alternate method:

If x ≤ – 2, $|x + 2| \ge 0$, while 2x – 1 < 0. There are no solutions
If x > – 2, $|x + 2|$ = x + 2
$\therefore$ x + 2 = 2x – 1 $\Rightarrow$ x = 3 Ans : (1)

22. For every real value of x, |x – 1| ≥ 0 > –2
$\therefore$ The required solution set is (–∞, ∞). Choice (C)

23. For every real value of x, |x + 2| ≥ 0
So, no real value of x satisfies the inequation |x + 2| < –3
$\therefore$ There is no solution for the given inequation.
Choice (A)

24. Given, |x + 3| ≤ 5
$\Rightarrow$ – 5 ≤ x + 3 ≤ 5 $\Rightarrow$ – 5 – 3 ≤ x ≤ 5 – 3
$\Rightarrow$ – 8 ≤ x ≤ 2
$\therefore$ The integers which satisfy the given inequation are –8, – 7, – 6, – 5, – 4, – 3, – 2, – 1, 0, 1 and 2.
Hence, the required number of integers is 11. Ans : (11)

25. Given, $|x - 2| \le 3$
$\Leftrightarrow -3 \le x - 2 \le 3$
$\Leftrightarrow -3 + 2 \le x \le 3 + 2$
$\Leftrightarrow -1 \le x \le 5$
$\therefore$ The required solution set is $[-1, 5]$ Choice (C)

Exercise – 3(a)

Solutions for questions 1 to 30:

1. (i) Given, $4x + 3 > 6x + 7$
$\Rightarrow 4x - 6x > 7 - 3$
$\Rightarrow -2x > 4$
$\Rightarrow x < -2$ Choice (C)

(ii) Given, $7x - 5 > 4x + 13$
$\Rightarrow 7x - 4x > 13 + 5$
$\Rightarrow 3x > 18$
$\Rightarrow x > 6$
$\therefore x \in (6, \infty)$ Choice (B)

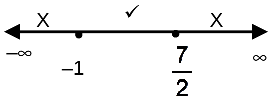
(iii) Given, $6x - 9 \ge 3x + 5$
$6x - 3x \ge 5 + 9$
$3x \ge 14$
$x \ge \dfrac{14}{3}$ Choice (A)

(iv) Given, $7x + 5 \le 3x - 11$
$\Rightarrow 7x - 3x \le -11 - 5$
$\Rightarrow 4x \le -16$
$\Rightarrow x \le -4$
$\therefore x \in (-\infty, -4]$ Choice (D)

(v) Given, $10x - 13 > 7x + 9$
$\Rightarrow 10x - 7x > 9 + 13$
$\Rightarrow 3x > 22$
$\Rightarrow x > \dfrac{22}{3}$
$\therefore x \in \left(\dfrac{22}{3}, \infty\right)$ Choice (B)

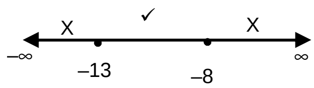
2. Given,
$6x + 9 < 3x + 5$; $4x + 7 > 2x - 5$
$6x - 3x < 5 - 9$; $4x - 2x > -5 - 7$
$3x < -4$; $2x > -12$
$\Rightarrow x < -\dfrac{4}{3}$ …..(A) ; $x > -6$ ……(B)
$\therefore$ From (A) and (B)
we have $x \in \left(-6, -\dfrac{4}{3}\right)$ Choice (C)

3. Given, $5x + 7 - 2x^2 > 0$
$\Rightarrow 2x^2 - 5x - 7 < 0$
$\Rightarrow 2x^2 - 7x + 2x - 7 < 0$
$\Rightarrow x(2x - 7) + 1(2x - 7) < 0$
$\Rightarrow (x + 1)(2x - 7) < 0$
So, the critical points are -1, $\dfrac{7}{2}$



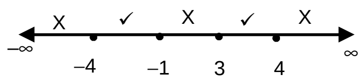
Now, when x = 0 the inequality is true.
$\therefore$ When $x \in \left(-1, \dfrac{7}{2}\right)$ the inequality is true.
Hence, the required solution set is $\left(-1, \dfrac{7}{2}\right)$ Choice (A)

4. Given, $\dfrac{x-7}{x+8} > 4$
$\Rightarrow \dfrac{x-7}{x+8} - 4 > 0 \quad \Rightarrow \dfrac{x-7-4(x+8)}{x+8} > 0$
$\Rightarrow \dfrac{x-7-4x-32}{x+8} > 0 \quad \Rightarrow \dfrac{-3x-39}{x+8} > 0$
$\Rightarrow \dfrac{-3(x+13)}{x+8} > 0$
$\Rightarrow \dfrac{x+13}{x+8} < 0$
$\Rightarrow \dfrac{(x+13)(x+8)}{(x+8)^2} < 0$
$\Rightarrow (x + 13)(x + 8) < 0 \quad (\because (x + 8)^2 > 0$ for $x \ne 8)$
The critical points are –13, –8



When x = 0, the inequality is not true.
$\therefore$ The solution set is $(-13, -8)$ Choice (C)

5. Given, $\dfrac{x^2+5x+4}{x^2-7x+12} \le 0$
$\Rightarrow \dfrac{(x+4)(x+1)}{(x-4)(x-3)} \le 0$
$\Rightarrow \dfrac{(x+4)(x+1)(x-4)(x-3)}{(x-4)^2(x-3)^2} \le 0$
$\Rightarrow (x + 4)(x + 1)(x - 4)(x - 3) \le 0$
The critical points are –4, –1, 3 and 4



When x = 0, the inequality is not true.
Hence, when $x \in [-4, -1]$ or $(3, 4)$ the inequality is true.
$\therefore$The required solution set: $[-4, -1] \cup (3, 4)$. Choice (A)

6. min $(x - 1, x + 2) = x - 1$
max$(x + 3, x + 5) = x + 5$
max $(x - 1, x + 5) = x + 5$ Choice (B)

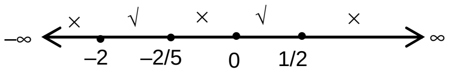
7. Let $x^3 + 1 > x^2 + x$
$\Rightarrow x^3 - x^2 - x + 1 > 0$
$\Rightarrow x^2(x - 1) - 1(x - 1) > 0$
$\Rightarrow (x - 1)(x^2 - 1) > 0$
$\Rightarrow (x - 1)^2(x + 1) > 0$
$\Rightarrow x + 1 > 0$ and $x \ne 1$
$[\because (x - 1)^2 \ge 0 \; x \in R]$
$\Rightarrow x > 1$ and $x \ne 1$
$\Rightarrow x \in (-1. 1) \cup (1, \infty)$
$\therefore$ The solution set is $(-1,1) \cup (1, \infty)$ Choice (A)

8. Given $2 - \dfrac{1}{n} < x \le 4 + \dfrac{1}{n}$
$n = 1 \Rightarrow 2 - 1 < x \le 4 + 1$
$1 < x \le 5$
$x = 2 \Rightarrow \dfrac{3}{2} < x < \dfrac{9}{2}$ and so on
The range of x is (1, 5] Choice (A)

9. Given $\dfrac{5x}{x+2} - \dfrac{1}{2x} < 0 \quad \dfrac{10x^2-x-2}{2x(x+2)} < 0$
$\dfrac{10x^2-5x+4x-2}{x(x+2)} < 0 \Rightarrow \dfrac{5x(2x-1)+2(2x-1)}{x(x+2)} < 0$
$\Rightarrow x(x + 2)(5x + 2)(2x - 1) < 0$

The critical points are x = 0, x = –2, x = –2/5, x = 1/2
Representing the above points on the number line, we have

$$\frac{x(x+2)(5x+2)(2x-1)}{x^2(x+2)^2} < 0$$



x = –1 satisfies the given inequality

∴ The union of the intervals marked with '√' is the required solution i.e., $x \in (-2, -2/5) \cup (0, 1/2)$ Choice (C)

10. $\frac{1}{2x-1} > \frac{3}{x}$

$\frac{2x-1}{(2x-1)^2} > \frac{3x}{x^2}$

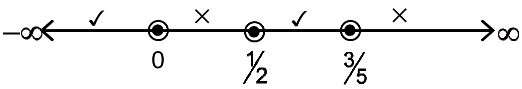
$x^2(2x-1) > 3x(2x-1)^2$

$x(2x-1)(x-6x+3) > 0$

$x(2x-1)(3-5x) > 0$

$x(2x-1)(5x-3) < 0$

Critical values are x = 0, $\frac{1}{2}, \frac{3}{5}$



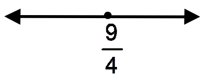
When x = – 1 the inequality is true.

∴ Solution is $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{3}{5}\right)$

Given x > 0, solution is $\left(\frac{1}{2}, \frac{3}{5}\right)$ Choice (D)

11. Given, $2x^2 + |4x - 9| = 7$ ------ (1)

Case (1); Let $x < \frac{9}{4}$



Then, $|4x - 9| = -(4x - 9)$.

So, (1) reduces to $2x^2 - 4x + 9 = 7$

$\Rightarrow 2x^2 - 4x + 2 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow$ x = 1which agrees with $x < \frac{9}{4}$

Case (2); Let $x > \frac{9}{4}$

Then, $|4x - 9| = 4x - 9$

So, (1) reduces to $2x^2 + 4x - 9 = 7$

$\Rightarrow 2x^2 + 4x - 16 = 0$

$\Rightarrow x^2 + 2x - 8 = 0$

$\Rightarrow (x + 4)(x - 2) = 0$

$\Rightarrow x = -4, 2$

These values not agree with $x > \frac{9}{4}$

Hence, the solution of the given equation is x = 1

Choice (A)

12. Given $\frac{5}{\sqrt{4-x}} - \sqrt{4-x} < 4$

Let $\sqrt{4-x} = a$,a > 0

$\Rightarrow \frac{5}{a} - a < 4$

$\frac{5-a^2}{a} < 4 \Rightarrow 5 - a^2 - 4a < 0$

$\Rightarrow a^2 + 4a - 5 > 0$

$(a + 5)(a - 1) > 0$

$\Rightarrow a < -5$ or $a > 1$

$a > 0 \therefore a \not< -5 \therefore a > 1$

$\sqrt{4-x} > 1$

$4 - x > 1$

$-x > -3$

$x < 3$

∴ Solution is $(-\infty, 3)$ Choice (C)

13. Given $2x^2 + 10x + 17 < 0$

$\Rightarrow x^2 + 5x + \frac{17}{2} < 0$

$\Rightarrow x^2 + 5x + \frac{25}{4} - \frac{25}{4} + \frac{17}{2} < 0$

$\Rightarrow \left(x + \frac{5}{2}\right)^2 + \frac{9}{4} < 0$

Since $\left(x + \frac{5}{2}\right)^2$ is always greater than or equal to 0, for any value of x.

There is no real value of x such that $2x^2 + 10x + 17 < 0$.

Choice (C)

14. $|x - 3| \le 9$

$\Rightarrow -9 \le x - 3 \le 9$

$-6 \le x \le 12 \rightarrow (1)$

$|4 - x| < 5$

$\Rightarrow -5 < 4 - x < 5$

$-9 < -x < 1$

$-1 < x < 9 \rightarrow (2)$

From (1) & (2) $-1 < x < 9$ Choice (C)

15. Given, x = |a| b and $|b| \ge 2$

$a + xb = a + |a|\, b^2$

and $a - xb = a - |a|b^2$

As $|-6| \ge 1$, the magnitude of $|a|\, b^2$ is greater than that of a. The sign of either expression is determined by the sign of this term.

i.e. $a + |a|\, b^2 \ge 0$ and $a - |a|\, b^2 \le 0$

The equality holds when a = 0. Choice (D)

16. We know that $|a| - |b| \le |a - b|$ and $|a - b| \le |a| + |b|$

$\therefore |a| - |b| \le |a - b| \le |a| + |b|$

i.e., $q \le r \le p$ Choice (C)

17. Given $|x| \ge |6 - x^2|$

Case (1) When $x \ge 0$ and $x^2 \le 6$ then

$x \ge 6 - x^2$

$x^2 + x - 6 \ge 0$

$(x + 3)(x - 2) \ge 0$

$x \notin (-3, 2)$

$\Rightarrow x \in [2, \sqrt{6}\,]$ ------(1)

Case (2) ;When $x \ge 0$,and $x^2 > 6$ then

$x \ge x^2 - 6$

$-x^2 + x - 6 \ge 0$ or $x^2 - x + 6 \le 0$

$\Rightarrow x \in [-2, 3]$

$x \in (\sqrt{6}, 3]$

Case (3) ; when x < 0 and $x^2 > 6$ then

$-x \ge x^2 - 6$

$-x^2 - x + 6 \ge 0$

$X^2 + x - 6 \le 0$

$(x+3)(x-2) \le 0$

$x \in [-3, 2]$

$x \in [-3, -\sqrt{6}\,)$

Case (4);x< 0, $x^2 < 6$

Then $-x \ge 6 - x^2$

$X^2 - x - 6 \ge 0$

$(x - 3)(x + 2) \ge 0$

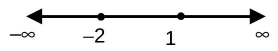
$x \notin (-2, 3)$

$\therefore x \in [-\sqrt{6}, -2]$

From the four cases, $x \in [-3,-2] \cup [2, 3]$ Choice (D)

18. $f(x) = |x| + |x-5| + |x+7|$
$\forall x \in R$, f (x) has no maximum value
The minimum value of f (x) is possible when x = 0, 5 or –7
$f(0) = 0+5+7 = 12$
$f(5) = 5+12 = 17$ and $f(-7) = 7+12+0 = 19$
Of these the minimum value of f (x) is 12 is at x = 0
$\therefore$ range of f (x) is $[12, \infty)$. Choice (D)

19.



When $x < -2$, $|x-1| > 3$
So, $|x+2| + |x-1| > 3$
When $x > 1$, $|x+2| > 3$ and so $|x+2| + |x-1| > 3$.
When $-2 \le x \le 1$, $|x+2| + |x-1| = 3$
Hence, there is no real number x such that $|x+2| + |x-1| < 2$. Choice (D)

20. Given, $x + y + z = 2$
We know that,
A.M.(x, y, z) $\ge$ G.M.(x, y, z)
$$\frac{x+y+z}{3} \ge \sqrt[3]{xyz}$$
$$\frac{2}{3} \ge \sqrt[3]{xyz}$$
$$\frac{8}{27} \ge xyz$$
$\therefore xyz \le \frac{8}{27}$ Choice (C)

21. We know that,
$$\frac{b}{a} + \frac{a}{b} \ge 2,$$
$$\frac{c}{a} + \frac{a}{c} \ge 2 \text{ and}$$
$$\frac{b}{c} + \frac{c}{b} \ge 2$$
Adding these inequalities, we get
$$\Rightarrow \frac{b}{a} + \frac{c}{a} + \frac{a}{b} + \frac{c}{b} + \frac{a}{c} + \frac{b}{c} \ge 6$$
i.e., $\frac{b+c}{a} + \frac{a+c}{b} + \frac{a+b}{c} \ge 6$
$\therefore$ The minimum value is 6. Choice (B)

22. $\left(p + \frac{1}{p}\right)^2 + \left(q + \frac{1}{q}\right)^2 = p^2 + \frac{1}{p^2} + 2 + q^2 + \frac{1}{q^2} + 2 = p^2 + (1-p)^2 + 4 + \frac{q^2 + p^2}{p^2q^2}$
$= 1 + 2p^2 - 2p + 4 + \frac{(p-q)^2 + 2pq}{p^2q^2} = 2p^2 - 2p + 5 + \frac{(p-q)^2 + 2pq}{p^2q^2}$
When the sum of two or more positive numbers is constant, their product will be maximum when the numbers are equal.
$p + q = 1$
$\therefore$ pq will be maximum when $p = q = \frac{1}{2}$
$\therefore$ Minimum value $\left(\frac{(p-q)^2 + 2pq}{p^2q^2}\right)$ will occur when p = q.
Its value is 8.
$2p^2 - 2p + 5$ will have a minimum value of $\frac{4(2)(5) - (-2)^2}{4(2)}$
= 4.5
$\therefore$ Minimum value is 12.5 Ans : (12.5)

23. We know that, AM(x, y) $\ge$ GM(x, y).
So, $AM\left(\frac{1}{x^2}, \frac{1}{y^2}\right) \ge GM\left(\frac{1}{x^2}, \frac{1}{y^2}\right)$
$$\frac{1}{x^2} + \frac{1}{y^2} \ge 2\sqrt{\frac{1}{x^2} \cdot \frac{1}{y^2}}$$
$\frac{1}{x^2} + \frac{1}{y^2} \ge \frac{2}{xy}$ ……………… (1)
Similarly, we get
$\frac{1}{y^2} + \frac{1}{z^2} \ge \frac{2}{yz}$ ……………… (2)
$\frac{1}{z^2} + \frac{1}{x^2} \ge \frac{2}{zx}$ ……………… (3)
Adding the three inequalities, we get
$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \ge \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$$
$\therefore$Option (A) is true.
We know that
A.M.(x, y, z) $\ge$ H.M.(x, y, z)
$$\frac{x+y+z}{3} \ge \frac{3}{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$
$$(x + y + z)\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) \ge 9$$
$\therefore (x + y + z)(yz + zx + xy) \ge 9xyz$
Option (B) is also true.
Concept A.M (x, y) $\ge$ G.M(x, y) we can prove $(x + y)(y + z)(x + z) \ge 8xyz$
$\therefore$ Option (C) is also true. Choice (D)

24. $3\ell^2 + \ell + 3 = \ell\{3(\ell + 1/\ell) + 1\} \ge 7\ell$
($\because \ell + 1/\ell \ge 2$ since AM$(\ell, 1/\ell) \ge$ GM$(\ell, 1/\ell)$)
Similarly, $5m^2 + m + 5 \ge 11m$ and $4n^2 + n + 4 \ge 9n$
$$\therefore \frac{(3\ell^2 + \ell + 3)(5m^2 + m + 5)(4n^2 + n + 4)}{11\ell mn} \ge \frac{7\ell(11m)(9n)}{11\ell mn} \ge 63$$
$\therefore$ Among the options, only (C) is possible Choice (C)

25. Given a, b, c, d are positive numbers
$\frac{a+b+c+d}{4} \ge \frac{4}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$ (AM $\ge$ HM)
$\Rightarrow \frac{12}{4} \ge \frac{4}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$ ($\because a + b + c + d = 12$)
$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \ge \frac{4}{3}$$
The equality holds when a = b = c = d = 3 Choice (A)

26. Given, $a^2 + b^2 = 6$; $c^2 + d^2 = 6$
$\Rightarrow a^2 + b^2 + c^2 + d^2 = 12$
We know that,
$$\frac{a^2 + c^2}{2} \ge ac; \frac{b^2 + d^2}{2} \ge bd$$
Adding the two inequalities, we get
$$\frac{a^2 + b^2 + c^2 + d^2}{2} \ge ac + bd$$
$6 \ge ac + bd$ (or) $ac + bd \le 6$
Similarly, we can show that $\frac{a^2 + b^2}{2} \ge ab; \frac{c^2 + d^2}{2} \ge cd$
$ab + cd \le 6$ and $\frac{a^2 + d^2}{2} \ge ad; \frac{b^2 + c^2}{2} \ge bc$
$\Rightarrow ad + bc \le 6.$ Choice (D)

27. Given a, b and c are positive real numbers.

$\therefore$ $a^2 + b^2 \ge 2ab$; $b^2 + c^2 \ge 2bc$; $c^2 + a^2 \ge 2ac$

$\Rightarrow a^2 + b^2 + b^2 + c^2 + c^2 + a^2 \ge 2ab + 2bc + 2ca$

$\Rightarrow \dfrac{a^2+b^2+c^2}{ab+bc+ca} \ge 1$

Since a, b and c are the lengths of the sides of triangles, $a - b < c$; $b - c < a$; $c - a < b$

$\Rightarrow a^2 + b^2 - 2ab < c^2$ $\quad b^2 + c^2 - 2bc < a^2$ and $c^2 + a^2 - 2ca < b^2$

$a^2 + b^2 - c^2 < 2ab$, $b^2 + c^2 - a^2 < 2bc$, $c^2 + a^2 - b^2 < 2ca$

Adding these inequalities, we get

$a^2 + b^2 + c^2 < 2ab + 2bc + 2ca$

$\dfrac{a^2+b^2+c^2}{ab+bc+ca} < 2$

Hence, $1 \le \dfrac{a^2+b^2+c^2}{ab+bc+ca} < 2$ Choice (B)

28. Given 3x + 4y = 34.

Objective: To maximize $E = x^3y^2 = (x.x.x)(y.y)$.

There are 5 factors – 3 x's and 2 y's. We write 3x + 4y as x + x + x + 2y + 2y. The sum of these 5 factors is constant.

$\therefore$ Their product, which is 4E is maximum, when all these 5 factors are equal, i.e., x = 2y.

This gives 5x = 34, i.e., x = 6.8 and y = 3.4

$\therefore$ Maximum value of $x^3.y^2 = (6.8)^3\,(3.4)^2 = (314.432)\,(11.56) = 3634.83392$

M = 3634.83392

M, when rounded to the nearest integer equals 3635.

Ans : (3635)

29. As $x^2y = 24$ (given),

$\left(\dfrac{3x}{2}\right)^2 (4y) = 9x^2y = 216$

$3x + 4y = \dfrac{3x}{2} + \dfrac{3x}{2} + 4y$

The product is constant;

So, the sum will be minimum, when all the factors are equal.

$\therefore\ \dfrac{3x}{2} = \dfrac{3x}{2} = 4y = \sqrt[3]{\dfrac{3x}{2}\,\dfrac{3x}{2}\,4y}$

$\dfrac{3x}{2} = \dfrac{3x}{2} = 4y = \sqrt{9x^2y}$

$= \sqrt[3]{9 \times 24} = \sqrt[3]{216} = 6.$

$\therefore \dfrac{3x}{2} = 6;$

$4y = 6 \Rightarrow y = \dfrac{6}{4}$

$x = 4,\ y = \dfrac{3}{2}$

hence the minimum value of $3x + 4y$

$= 3 \times 4 + 4 \times \dfrac{3}{2} = 12 + 6 = 18.$ Choice (A)

30. As $x^2y^3 = 108$,

$\left(\dfrac{x}{2}\right)^2 \left(\dfrac{y}{3}\right)^3 = \dfrac{x^2y^3}{108} = 1$

$x + y = \dfrac{x}{2} + \dfrac{x}{2} + \dfrac{y}{3} + \dfrac{y}{3} + \dfrac{y}{3}$

Consider the product of the all factors which is constant. When all these are equal the sum of the factors, i.e., x + y will be the minimum.

$\dfrac{x}{2} = \dfrac{x}{2} = \dfrac{y}{3} = \dfrac{y}{3} = \dfrac{y}{3}$

$= \sqrt[5]{\dfrac{x^2y^3}{2^2 3^3}} = \sqrt[5]{\dfrac{108}{108}} = 1$

$\therefore\ \dfrac{x}{2} = 1$ and $\dfrac{y}{3} = 1$, i.e., x = 2, y = 3

$\therefore$ The minimum value of $x + y$ is 5. Ans: (5)

Exercise – 3(b)

Solutions for questions 1 to 40:

1. (i) Given, $4x + 9 \ge 25$

$4x \ge 25 - 9$

$4x \ge 16$

$x \ge 4$ Choice (C)

(ii) Given, $13x - 5 \le 21$

$13x \le 26$

$x \le 2$

$x \in (-\infty, 2]$ Choice (B)

(iii) Given, $3 - 32x < 129 - 18x$

$3 - 129 < 32x - 18x$

$-126 < 14x$

$14x > -126$

$x > -\dfrac{126}{14}$

$x > -9$ Choice (C)

(iv) Given, $4x - 5 > 3x + 13$

$4x - 3x > 13 + 5$

$x > 18$

$x \in (18, \infty)$ Choice (A)

(v) Given, $8x - 43 \ge 5 + 10x$

$-43 - 5 \ge 10x - 8x$

$-48 \ge 2x$

$2x \le -48$

$x \le -24$

$\therefore$The solution set is $x \in (-\infty, -24]$. Choice (C)

2. Given, $3x + 4 \le 6$ and $4x + 3 \ge 6$

$3x \le 2$ and $4x \ge 3$

$x \le \dfrac{2}{3}$ and $x \ge \dfrac{3}{4}$ ……… (2)

There is no real value of x satisfying the above inequations simultaneously.

$\therefore$ The solution set is empty set. Choice (D)

3. Since $|3x - 9|$ is always positive, $27 - |3x - 9| \le 27$

$\therefore$ The maximum value of $27 - |3x - 9|$ is 27. Ans: (27)

4. Since $|4x - 5|$ is always positive.

$\therefore$ The expression $17 + |4x - 15|$ has a minimum when $|4x - 15| = 0$

i.e., at $x = \dfrac{15}{4}$ Choice (B)

5. $2 \le x \le 8$ and $4 \le y \le 12$.

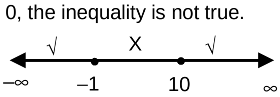
$\dfrac{x+y}{x} = 1 + \dfrac{y}{x}$. This is minimum when $\dfrac{y}{x}$ has its minimum value. This is when y is minimum and x is maximum.

$1 + \dfrac{4}{8} = \dfrac{3}{2}$ Choice (D)

6. Given, $x^2 - 9x - 10 > 0 \Rightarrow (x - 10)(x + 1) > 0$

The critical points of the above inequality are –1 and 10.

When x = 0, the inequality is not true.



So, when x < –1 or x > 10 the inequation will be true.

$\therefore$ The solution set is $(-\infty, -1) \cup (10, \infty)$. Choice (C)

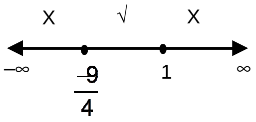
7. Given, $4x^2 + 5x - 9 \le 0$

$\Rightarrow 4x^2 + 9x - 4x - 9 \le 0$

$x(4x + 9) - 1(4x + 9) \le 0$

$(4x + 9)(x - 1) \le 0$

$\therefore$ The critical points are $-\frac{9}{4}$, 1



When x = 0, the given inequation is true.

Hence, the solution set is $x \in \left[-\frac{9}{4}, 1\right]$ Choice (D)

8. Given, $\frac{x-7}{x+5} > 3$

$\frac{x-7}{x+5} - 3 > 0$

$\Rightarrow \frac{x-7-3(x+5)}{x+5} > 0$

$\Rightarrow \frac{x-7-3x-15}{x+5} > 0$

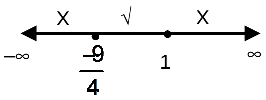
$\Rightarrow \frac{-2x-22}{x+5} > 0$

$\Rightarrow \frac{x+11}{x+5} < 0$

$\Rightarrow \frac{(x+11)(x+5)}{(x+5)^2} < 0$

$(x + 11)(x + 5) < 0$

The critical points are –11 and –5.



When x = 0, the inequality is not true.

$\therefore$ The solution set of the inequality is (–11, –5).

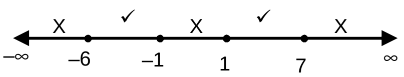
Choice (B)

9. Given, $(x^2 + 5x - 6)(x^2 - 6x - 7) < 0$

$(x^2 + 6x - x - 6)(x^2 - 7x + x - 7) < 0$

$(x + 6)(x - 1)(x + 1)(x - 7) < 0$

The critical points are –6, –1, 1 and 7.



When x = 0, the inequality is not true.

$\therefore$ The solution set is, $x \in (-6, -1) \cup (1, 7)$.

The integral values of x in the above solution set are –5, –4, –8, –3, –2, 2, 3, 4, 5 and 6.

$\therefore$ The required number of integral values is 9.

Choice (A)

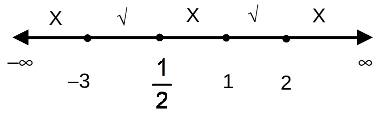
10. Given, $\frac{2x^2 + 5x - 3}{x^2 - 3x + 2} \le 0$

$\Rightarrow \frac{(x+3)(2x-1)}{(x-2)(x-1)} \le 0$

$\Rightarrow \frac{(x+3)(2x-1)(x-2)(x-1)}{(x-2)^2(x-1)^2} \le 0$

$\Rightarrow (x + 3)(2x - 1)(x - 1)(x - 2) \le 0$

$\Rightarrow$ The critical points are –3, 1/2, 1and 2



When x = 0, the inequality is true.

The solution set is $[-3, 1/2] \cup (1, 2)$ Choice (C)

11. Given, $|3x + 7| > 12$

$3x + 7 < -12$ or $3x + 7 > 12$

$3x < -19$ or $3x > 5$

$x < -\frac{19}{3}$ or $x > \frac{5}{3}$

$\therefore$ The solution set is $\left(-\infty, -\frac{19}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$ Choice (C)

12. Given, $|9 - x| < 2 - 3x$

Let x < 9

Then, $9 - x < 2 - 3x \Rightarrow 2x < -7$

$\Rightarrow x < -\frac{7}{2}$ which agrees with x< 9

Let x > 9

Then, $-(9 - x) < 2 - 3x$

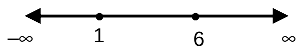
$-9 + x < 2 - 3x$

$4x < 11$

$x < \frac{11}{4}$ which does not agree with x > 9.

$\therefore$ The solution set is $\left(-\infty, -\frac{7}{2}\right)$. Choice (B)

13.



When x < 1, $|x - 6| > 5$

So, $|x - 1| + |x - 6| > 5$

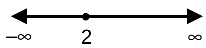
When x > 6, $|x - 1| > 5$ and so $|x - 1| + |x - 6| > 5$

When $1 \le x \le 6$, $|x - 1| + |x - 6| = 5$

Hence, there is no real number 'x' such that $|x - 1| + |x - 6| < 2$. Ans : (0)

14. Given, $x^2 + |3x - 6| = 4$ ——— (1)

Case (1); let x < 2



Then, $|3x - 6| = -(3x - 6)$

So, (1) reduces to $x^2 - (3x - 6) = 4$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x - 1)(x - 2) = 0$

$\Rightarrow x = 1, 2$

But x < 2

$\Rightarrow$ x = 1 is a solution

Case (2):

When x > 2 then $|3x - 6| = 3x - 6$.

So, (1) reduces to $x^2 + 3x - 6 = 4$

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow (x + 5)(x - 2) = 0$

$\Rightarrow x = -5, 2$

But $x \ge 2$

$\Rightarrow$ x = 2 is a solution

Hence, the solutions are x = 1, 2 Ans : (2)

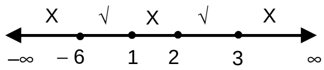
15. Given, $\frac{x^2 - 5x + 6}{x^2 + 5x - 6} < 0$

$\Rightarrow \frac{(x-3)(x-2)}{(x+6)(x-1)} < 0$

$\Rightarrow \frac{(x-3)(x-2)(x-1)(x+6)}{(x+6)^2(x-1)^2} < 0$

$\Rightarrow (x - 1)(x - 2)(x - 3)(x + 6) < 0$

The critical points are –6, 1, 2 and 3.



When x = 0, the inequality is true.
So, when $x \in (-6, 1)$ OR $x \in (2, 3)$ the inequality is true.
$\therefore$ The integral solutions are –5, –4, –3, –2, –1, 0.
Choice (B)

16. Given, $x^3 - 8 < x^2 - 2x$
$\Rightarrow (x - 2)(x^2 + 2x + 4) < x(x - 2)$
$\Rightarrow (x - 2 (x^2 + 2x + 4 - x) < 0$
$\Rightarrow (x - 2)(x^2 + x + 4) < 0$
$\Rightarrow (x - 2)\ [(x + \frac{1}{2})^2 + \frac{15}{4} < 0]$
$\Rightarrow x - 2 < 0\ [\because\ (x + \frac{1}{2})^2 + \frac{15}{4} > 0]$
$\Rightarrow x - 2 < 0$
$\Rightarrow x < 2$
$\therefore$ The solution set is $(-\infty, 2)$ Choice (D)

17. $|x - 5| > x^2 - 4x + 1$
Case 1:
If $x > 5$, then $|x - 5| = x - 5$
so, $x - 5 > x^2 - 4x + 1$
$x^2 - 4x + 1 < x - 5$
$x^2 - 5x + 6 < 0$
$(x - 3)(x - 2) < 0$
$x \in (2, 3)$ which does not agree with $x < 5$
Case 2:
If $x < 5$, then $|x - 5| = 5 - x$
So, $5 - x > x^2 - 4x + 1$
$x^2 - 4x + 1 < 5 - x$
$x^2 - 3x - 4 < 0$
$x^2 - 4x + x - 4 < 0$
$(x - 4)(x + 1) < 0$
$x \in (-1, 4)$
$\therefore$ The solution set is (–1, 4). Choice (C)

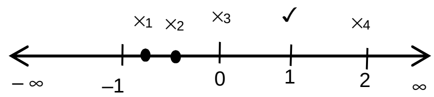
18. Given, $x^2 + y^2 + z^2 = 3$
We know that,
$x^2 + y^2 + z^2 \geq xy + yz + zx$
$\therefore\ xy + yz + zx \leq x^2 + y^2 + z^2$
i.e., $xy + yz + zx \leq 3$ Choice (A)

19. Given, $|b| \leq 1$
and $x = -|a|\ b$
$\therefore a - xb = a + |a|\ b^2$
When $a < 0$, $a - xb = a - ab^2 = a(1 - b^2) < 0$
When $a \geq 0$, $a - xb = a + ab^2 = a(1 + b^2) \geq 0$
a – xb can be less than zero or greater than zero.
Now, $a + xb = a - |a|b^2$
When $a < 0$, $a + xb = a + ab^2 = a(1 + b^2) < 0$
When $a \geq 0$, $a + xb = a - ab^2 = a(1 - b^2) \geq 0$
$(\because\ |b| < 1)$
$\therefore$a + xb can be less than zero or greater than zero.
Choice (D)

20. Given xy = 28 where x > 0, y > 0
$\Rightarrow (4x)(7y) = 28xy = 28^2$
Since the product of 4x and 7y is constant, the sum will be minimum when 4x and 7y are equal.
$\Rightarrow 4x = 7y = \sqrt{(4x)(7y)} = \sqrt{28xy} = 28.$
$\therefore 4x = 7y = 28.$
Hence the minimum value of
$4x + 7y$ is $= 28 + 28 = 56$.
Ans : (56)

21. $2p^2 + p + 2 = p\{2(p + \frac{1}{p}) + 1\} \geq p\{2 \times 2 + 1\} \geq 5p$
Similarly ,$7q^2 + q + 7 \geq 15q$ and $6r^2 + r + 6 \geq 13r$
$\therefore \frac{(2p^2 + p + 2)(7q^2 + q + 7)(6r^2 + r + 6)}{5pqr} \geq \frac{5p \times (15q) \times (13r)}{5pqr}$
≥ 195 Choice (D)

22. Given $\frac{7x}{5x + 4} - \frac{2}{x} < 0$
$\frac{7x^2 - 10x - 8}{x(5x + 4)} < 0$
$\Rightarrow \frac{7x^2 - 14x + 4x - 8}{x(5x + 4)} < 0$ (or) $\frac{(7x + 4)(x - 2)}{x(5x + 4)} < 0$
$\Rightarrow \frac{x(5x + 4)(7x + 4)(x - 2)}{x^2(5x + 4)^2} < 0$
$\Rightarrow x(5x + 4)(7x + 4)(x - 2) < 0$
The critical points are $x_1 = \frac{-4}{5}, x_2 = \frac{-4}{7}, x_3 = 0, x_4 = 2$
Representing the above points on the number line, we have



x = –1 does not satisfy the given inequality
$\therefore$ the region containing – 1 is not part of the solution
In the regions marked with '✓', the inequality (1) is satisfied.
$\therefore x \in (-4/5, -4/7) \cup (0, 2)$
$\therefore$ The only integral root is x = 1
$\therefore$ The number of integral roots is one. Choice (A)

23. Case (i): Let $x \geq -\frac{1}{6}$.
Then, $|6x + 1|$
$= 6x + 1$
So, $\left|x - |6x+1|\right| = 9$
$\Rightarrow |x - 6x - 1|$
$\Rightarrow |5x + 1| = 9$
$\Rightarrow x = \frac{8}{5}$ or –2
As $x \geq \frac{-1}{6}$, $x = \frac{8}{5}$ but not –2

Case (ii): Let $x < \frac{-1}{6}$
$|x - (-(6x + 1)| = 9$
$|7x + 1| = 9$
$x = \frac{8}{7}$ or $\frac{-10}{7}$
As $x \leq \frac{-1}{6}$, $x = \frac{-10}{7}$ but not $\frac{8}{7}$
Two solutions exist Choice (C)

24. $\frac{a + b + c}{3} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$ (AM ≥ HM)
$\Rightarrow \frac{9}{3} \geq \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$ $(\because a + b + c = 9)$
$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 1$ Choice (D)

25.

$$\left(\frac{a}{b}-\frac{b}{c}\right)^2+\left(\frac{c}{c^2}-\frac{b}{a}\right)^2+\left(\frac{c}{b}-\frac{a}{c}\right)^2+2\left(\frac{a}{c}+\frac{bc}{a^2}+\frac{a}{b}\right)$$

$$\Rightarrow \frac{a^2}{b^2}+\frac{b^2}{c^2}-\frac{2a}{c}+\frac{c^2}{a^2}+\frac{b^2}{a^2}-\frac{2bc}{a^2}+\frac{c^2}{b^2}+\frac{a^2}{c^2}-\frac{2a}{b}+\frac{2a}{c}+\frac{2bc}{a^2}+\frac{2a}{b}$$

$$\Rightarrow \left(\frac{a}{b}+\frac{b}{a}\right)^2-2+\left(\frac{b}{c}+\frac{c}{b}\right)^2-2+\left(\frac{a}{c}+\frac{c}{a}\right)^2-2$$

$$\Rightarrow \ge (2)^2+(2)^2+(2)^2-6\left(\therefore x+\frac{1}{x}\ge 2\right)$$

$\therefore \ge 6$ The minimum value of the given expression is 6

Choice (A)

26. $\frac{1}{2x-5} > \frac{4}{x}$

$\frac{2x-5}{(2x-5)^2} > \frac{4x}{x^2}$

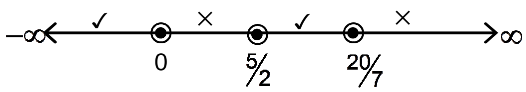
$\Rightarrow x^2(2x-5) > 4x(2x-5)^2$

$x(2x-5)(x-4(2x-5)) > 0$

$x(2x-5)(x-8x+20) > 0$

$x(2x-5)(7x-20) < 0$

Critical values of x are 0, $5/2, \frac{20}{7}$



When x = –1 the inequation holds

$\therefore$ solution region is $(-\infty,0)\cup\left(\frac{5}{2},\frac{20}{7}\right)$

Given x > 0, solution is $\left(\frac{5}{2},\frac{20}{7}\right)$

There is no integer between $\frac{5}{2}$ i.e. 2.5 and $\frac{20}{7}$ i.e. 2.85…

The number of integer solutions is zero. Ans : (0)

27. f (x) = |x| + |x+7|
As |x| and |x+7| are always non negative, f(x) has no maximum value exists
f (x) is minimum at x = 0 or x = –7
$\therefore$ minimum value of f (x) is 7
$\therefore$ range of f (x) is [7, ∞) Choice (B)

28. When 6x + 3 > 0

i.e, $x > \frac{-1}{2}$

$x^2 - (6x + 3) = -6$

$x^2 - 6x + 3 = 0$

x = $3+\sqrt{6}$ or x = $3-\sqrt{6}$

both solutions are greater than –1/2 and hence valid.

When 6x + 3 < 0, ie, $x < \frac{-1}{2}$

$x^2 + 6x + 3 = -6$

$x^2 + 6x + 9 = 0$

$(x + 3)^2 = 0$

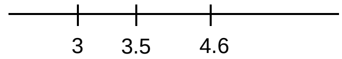
x = –3

Which is less than –1/2 and hence valid.

$\therefore$ No. of solutions = 3

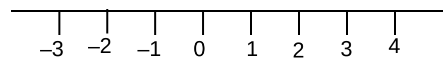
Choice (D)

29. |x – a| is the distance from x to a on the number line



f(x) = | x – 3| + |3.5 – x |+14.6 – x|
f(x) is the sum of the distances from x to 3, 3.5 and 4.6.
f(3) = 2.1
f(4.6) = 2.7
f(3 – k), where k is any positive number, is more than 2.1
f(4.6 + k), where k is any positive number, is more than 2.7.
If 3 < x < 4.6, | x – 3| + | 4.6 – x| = 1.6.
$\therefore$ f(x) = 1.6 + |3.5 – x|. This is minimum
when x is 3.5. $\therefore$ f(x) = 1.6 ($\because$ | 3.5 – x| = x – 3.5)
Minimum value of (x) i.e., M is 1.6. The value of x when f(x) = M is 3.5 Ans : (3.5)

30.



|x − 1| is the distance from x to a
|x + 3| + 7 > 2|x − 4| --------- (1)
For any point between − 3 and 4, both inclusive, |x + 3| + |x − 4| is 7
(1) ⇒ 7 − |x − 4| + 7 > 2|x − 4|

$|x-4| < \frac{14}{3} \Rightarrow 4 - x < \frac{14}{3}$ ($\because$x<4)

$x > -\frac{2}{3}$

We see that x = − 3 violates (1). Any left ward shift from − 3 by a units will increase both |x + 3| and |x − 4| by a units. $\therefore$ LHS of (1) increases by a while RHS of (1) increases by 2a.
As x = − 3 violates (1), the shifted point also violates (1)
For any point greater than 4,
|x − 4| = x − 4 and |x + 3| = x + 3
$\therefore$ x + 3 + 7 > 2(x − 4)
18 > x

$\frac{-2}{3} < x < 18$ Choice (D)

31. $\left(y^{\frac{1}{3}}-1\right)^2 + y^{\frac{1}{3}} - 13 \le 0 \Rightarrow y^{\frac{2}{3}} - 2y^{\frac{1}{3}} + 1\ y^{\frac{1}{3}} - 13 \le 0$

$\Rightarrow y^{\frac{2}{3}} - 2y^{\frac{1}{3}} - 12 \le 0.$

Let $y^{\frac{1}{3}} =$

$\Rightarrow a^2 - a - 12\ a \le 0$

$(a - 4)(a + 3) \le 0 \Rightarrow -3 \le a \le 4$

$\Rightarrow -3 \le y^{\frac{1}{3}} \le 4 \Rightarrow -27 \le y \le 64$ Choice (C)

32. Let f(x) = |x – 3| + | x – 7| + | x + 13|
f(x) attains minimum value at x = 3 or 7 or –13
f(3) = 4 + 16 = 20
f(7) = 4 + 20 = 24
f(–13) – 16 + 20 = 36
Of these, least value is 20 Choice (B)

33. Max (2x + 3, x – 4)
Cannot be determined because for x > –7
(2x + 3) > x – 4 and for x < –7 (2x + 3) < (x – 4).
Choice (D)

34. Consider, $\frac{(49)^{48}}{(48)^{51}}$

$= \frac{1}{(48)^3}\left(\frac{49}{48}\right)^{48}$

$= \frac{1}{(48)^3}\left(1+\frac{1}{48}\right)^{48}$

We know that, for x≥ 1 $\left(1+\frac{1}{x}\right)^x < 2.8$

$\therefore \frac{(49)^{48}}{(48)^{51}} < \frac{1}{(48)^3} \times 2.8 < 1$

$\Rightarrow (49)^{48} < (48)^{51}$

Similarly, we can show that $(105)^{20} < (100)^{21}$ and $41^{39} < 40^{40}$. Choice (D)

35. Given a, b, c are positive real numbers.

We know that, A.M $\left(\frac{bc}{a}, \frac{ca}{b}\right)$

$\geq$ G.M $\left(\frac{bc}{a}, \frac{ca}{b}\right)$

$\geq \frac{\frac{bc}{a}+\frac{ca}{b}}{2} \geq \sqrt{\frac{bc}{a}\cdot\frac{ca}{b}}$

$\frac{bc}{a}+\frac{ca}{b} \geq 2c$

Similarly, we can show that, $\frac{ca}{b}+\frac{ab}{c} \geq 2a$

and $\frac{ab}{c}+\frac{bc}{a} \geq 2b$

Adding these three inequalities, we get

$\frac{bc}{a}+\frac{ca}{b}+\frac{ab}{c} \geq a+b+c$

$a + b + c \leq \frac{bc}{a}+\frac{ca}{b}+\frac{ab}{c}$

$\therefore$ Option (A) is true

We know that,

$$\frac{\left(\frac{a}{bc}\right)^{\frac{1}{2}}+\left(\frac{b}{ca}\right)^{\frac{1}{2}}}{2} \geq \sqrt{\left(\frac{a}{bc}\right)^{\frac{1}{2}}\cdot\left(\frac{b}{ca}\right)^{\frac{1}{2}}}$$

$$\left(\frac{a}{bc}\right)^{\frac{1}{2}}+\left(\frac{b}{ca}\right)^{\frac{1}{2}} \geq 2\frac{1}{\sqrt{c}}$$

Similarly, we can show that, $\left(\frac{b}{ca}\right)^{\frac{1}{2}}+\left(\frac{c}{ab}\right)^{\frac{1}{2}} \geq \frac{2}{\sqrt{c}}$

and $\left(\frac{a}{bc}\right)^{\frac{1}{2}}+\left(\frac{c}{ab}\right)^{\frac{1}{2}} \geq \frac{2}{\sqrt{b}}$

Adding these inequalities, we get

$$\left(\frac{a}{bc}\right)^{\frac{1}{2}}+\left(\frac{b}{ca}\right)^{\frac{1}{2}}+\left(\frac{c}{ab}\right)^{\frac{1}{2}} \geq \frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}$$

$\therefore$ Option (B) is true.

Now, $\frac{(a^2)^2+(b^2)^2}{2} \geq \sqrt{a^4.b^4}$

$\Rightarrow \frac{a^4+b^4}{2} \geq a^2 b^2$

Similarly we can show that,

$\frac{b^4+c^4}{2} \geq b^2c^2$ and $\frac{c^4+a^4}{2} \geq c^2 a^2$

Adding these inequalities, we get $a^4 + b^4 + c^4 \geq a^2 b^2 + b^2 c^2 + c^2a^2$

$\therefore$ Option (C) is also true. Choice (D)

36. We know that,

A.M.(b + c, c + a, a + b) ≥ H.M.(b + c, c + a, a + b)

$$\Rightarrow \frac{b+c+c+a+a+b}{3} \geq \frac{3}{\frac{1}{b+c}+\frac{1}{c+a}+\frac{1}{a+b}}$$

$$\Rightarrow \frac{2}{9}(a+b+c) \geq \frac{1}{\frac{1}{b+c}+\frac{1}{c+a}+\frac{1}{a+b}}$$

$$\Rightarrow \frac{9}{2(a+b+c)} \leq \frac{1}{b+c}+\frac{1}{c+a}+\frac{1}{a+b}$$

$$\Rightarrow \frac{9}{2} \leq \frac{a+b+c}{b+c}+\frac{a+b+c}{c+a}+\frac{a+b+c}{a+b}$$

$$\Rightarrow \frac{9}{2} \leq \frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}+3$$

$\Rightarrow \frac{3}{2} \leq \frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ ………… (1)

Now, since a, b and c are the lengths of a triangle, b + c > a

2b + 2c > a + b + c

$\frac{1}{2(b+c)} < \frac{1}{a+b+c}$

$\frac{a}{b+c} < \frac{2a}{a+b+c}$

Similarly we have, $\frac{b}{c+a} < \frac{2b}{a+b+c}$

$\frac{c}{a+b} < \frac{2c}{a+b+c}$

Adding these inequalities, we get

$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} < 2$ .... (2)

$\therefore$ From (A) and (B), we have

$\frac{3}{2} \leq \frac{a}{b+c}+\frac{b}{b+c}+\frac{c}{a+b} < 2$ Choice (C)

37. Given, $(x^2 + x + 1)^x > 1$

Applying logarithm on both sides, we get

$\log(x^2 + x + 1)^x > \log 1$

$x\log(x^2 + x + 1) > 0$

$\Rightarrow x > 0$ and $\log(x^2 + x + 1) > 0$ (or) $x < 0$ and $\log(x^2 + x + 1) < 0$

Case (1): Let x > 0 and $\log(x^2 + x + 1) > 0$

$\Rightarrow x^2 + x + 1 > 1$ and $x > 0$

$\Rightarrow x^2 + x > 0$ and $x > 0$

$\Rightarrow x(x + 1) > 0$ and $x > 0$

$\Rightarrow x \in (-\infty, -1) \cup (0, \infty)$ and

$\Rightarrow x > 0$ ----------- (1)

Case (2):

Let x < 0 and $\log(x^2 + x + 1) < 0$

$\Rightarrow x^2 + x + 1 < 1$ and $x < 0$

$\Rightarrow x^2 + x < 0$ and $x < 0$

$\Rightarrow x(x + 1) < 0$ and $x < 0$

$\Rightarrow x \in (-1, 0)$ ………..(2)

$\therefore$ From (1) and (2)

$\therefore$ The solution set is $(-1, \infty)$ Choice (B)

38. Given, x + y = 5

Consider, $\left(\frac{x}{3}\right)^3+\left(\frac{y}{2}\right)^2$

$\frac{x}{3}+\frac{x}{3}+\frac{x}{3}+\frac{y}{2}+\frac{y}{2} = x+y = 5$ (constant)

$\therefore$ The product $\left(\frac{x}{3}\right)^3\left(\frac{y}{2}\right)^2$ is maximum,

when $\frac{x}{3}=\frac{y}{2}$

$\Rightarrow \frac{x}{3}=\frac{y}{2} = \frac{x+y}{3+2}$

$\frac{x}{3} = \frac{y}{2} = 1$

$\Rightarrow x = 3 ; y = 2$

$\therefore$ The maximum value of x^3y^2 is $(3)^3 (2)^2 = 108$.

Choice (D)

39. $\left|\frac{2x+1}{3x-8}\right| \geq 2$

$\Rightarrow \frac{2x+1}{3x-8} \leq -2$ or $\frac{2x+1}{3x-8} \geq 2$

$\frac{2x+1+2(3x-8)}{3x-8} \leq$ or 0 $\frac{2x+1-2(3x-8)}{3x-8} \geq 0$

$\frac{8x-15}{3x-8} \leq 0$ or $\frac{-4x+17}{3x-8} \geq 0$

$(8x–15)(3x–8) \leq 0$ or $(4x–17)(3x–8) \leq 0$

$x < \left[\frac{15}{8}, \frac{8}{3}\right)$, or $x < \left(\frac{8}{3}, \frac{17}{4}\right]$

$\therefore$ The required solution set is $\left[\frac{15}{8}, \frac{8}{3}\right) \cup \left(\frac{8}{3}, \frac{17}{4}\right]$

$= \left[\frac{15}{8}, \frac{17}{4}\right] - \left\{\frac{8}{3}\right\}$

Choice (A)

40. x is a positive integer

$\therefore \left|\frac{5x+124}{5(5x^2+1)}\right| = \frac{5x+124}{5(5x^2+1)}$

$\frac{5x+124}{5(5x^2+1)} < \frac{1}{50}$

$\Rightarrow 10(5x + 124) < 5x^2 + 1$

$\Rightarrow 0 < 5x^2 – 50x – 1239$

$\Rightarrow 0 < x^2 – 10x – 247.8$

$\Rightarrow 247.8 < x^2 – 10x$

Adding 5 both sides, $272.8 < (x – 5)^2$

$\therefore x – 5 > 16$ or $x > 21$.

$\therefore$ The least value of x satisfying the given inequality is 22.

Ans : (22)

Solutions for questions 41 to 50:

41. From statement I, we have

$– 5 < x < –1$ and $1 < y < 2$.

If $x = – 4$ and $y = 1.2$, $\frac{x^2}{y^3} > 5$.

But if $x = – 2$ and $y = 1.5$, $\frac{x^2}{y^3} < 5$

$\therefore$ Statement I alone is not sufficient.

From statement II, we have $x > 3$

and $0 < y < 1$.

As y is a proper fraction, $1/y^3$ is always be greater than 1.

Also as $x > 3$, $x^2 > 9$

$\therefore x^2 \times \frac{1}{y^3}$ will be $> 9 \times 1$ i.e., > 9.

$\therefore$ Statement II alone is sufficient.

Choice (A)

42. From statement I we have $y^2 > x^2$ and $y^3 < x^3$

Both x, y can't be +ve. Both can be –ve.

If they have opposite signs, y is –ve, x is +ve.

If both are –ve, $y^3 < x^3 < 0$ and $1/y > 1/x$

If they have opposite signs, $1/y < 1/x$.

Hence statement I alone is not sufficient.

From statement II we have $x^4 y^2 < x^2 y^4 \Rightarrow x^2 < y^2$

This is only part of I, clearly it is not sufficient and even the combination (which is the same as statement I) is not sufficient.

Choice (D)

43. $x^3.y^3$ will be positive, when x and y are either both positive or both negative.

From statement I, we have $y^2 = 16$ which means

$y = \pm 4$.

As x can be positive or negative, x^3y^3 is positive or negative.

$\therefore$statement I alone is not sufficient.

From statement II, we have $x + y = 0$ i.e., $x = –y$.

$\therefore$Between x and y, one is positive and the other is negative and $x^3y^3 \neq 0$. Even if $x = y = o$, x^3y^3

Hence $x^3y^3 \ngtr 0$. Even if $x = y = 0$, $x^3y^3 \ngtr 0$.

Statement II is sufficient

Choice (A)

44. The minimum value of the given expression is

$\frac{4(2)(20)-a^2}{4(2)} \Rightarrow 20 -- \frac{a^2}{8} > 0$

From statement I, for any value of a (i.e, $0 < a < 10$),

$20 – \frac{a^2}{8} > 0$ $\therefore$Statement I, alone is sufficient.

From statement II, we cannot say anything as far some values (say $a = –20$) of 'a' the expression can be positive or negative.

$\therefore$ Statement II is not sufficient

Choice (A)

45. We know that,

if n is odd, $x^n > 0 \Rightarrow x > 0$

if n is even, $x^n > 0 \Rightarrow$ x may be positive or negative.

So, statement I alone is sufficient but statement II alone is not sufficient.

Choice (A)

46. Clearly, either of the statements alone is not sufficient.

We have, $\frac{|y|}{|x|+|y|} = \frac{1}{1+\frac{|x|}{|y|}}$

So, $\frac{|y|}{|x|+|y|}$ will be minimum.

The maximum value of $\frac{|x|}{|y|}$ is $\frac{4}{2} = 2$

So, the minimum value of $\frac{|y|}{|x|+|y|}$ is $\frac{1}{1+2} = \frac{1}{3}$.

Hence, both I and II together are sufficient to answer the question.

Choice (C)

47. Clearly, either of the statements alone is not sufficient to answer the question.

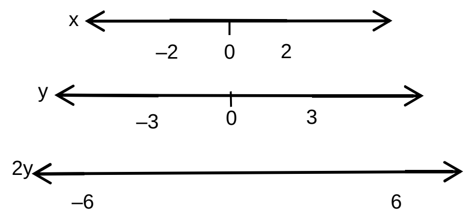
Now, $|x| < 2 \Rightarrow – 2 < x < 2$

$\Rightarrow x > –2$ and $x < 2$

and $|y| > 3 \Rightarrow y < – 3$ or $y > 3$

$\Rightarrow 2y < – 6$ or $2y > 6$

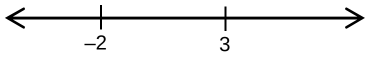
We can represent the possible values of x, y and 2y on three number lines as shown below.



We can see that the minimum value of $|x – 2y|$ occurs when $x = 2$ and $2y = 6$ or when $x = –2$ when $2y = –6$ and this minimum value is 4. $\therefore$ $|x – 2y|$ is always greater than 4. By combining the 2 statements, we can answer the question.

Choice (C)

48.



When x < – 2, |x – 3| > 5
$\Rightarrow$ |x + 2| + |x – 3| > 5
When x > 3, |x + 2| > 5
$\Rightarrow$ |x + 2| + |x – 3| > 5
When –2 $\leq$x $\leq$3, |x + 2| + | x – 3| = 5
So, the minimum value of |x + 2| + | x – 3| is 5.
Clearly, either of the statements alone is sufficient.
Choice (B)

49. From statement I, $(x – 3)^2 + (y – 4)^2 = 0$
$\Rightarrow$ x = 3 and y = 4 $\Rightarrow$| x – y| = 1
From statement II,

$|x-3|+|3+4| = 0$

$\Rightarrow$ x = 3; y = 4 $\Rightarrow$ $|x-y|$ = 1.

$\therefore$Either of the statements alone is sufficient to answer the question. Choice (B)

50. From statement I, $x^2 – 2x – 15 < 0$
$\Rightarrow$ (x – 5) (x + 3) < 0 $\Rightarrow$ x $\in$ (–3, 5)
$\Rightarrow$ x – 1 < (–4, 4) $\Rightarrow$ |x – 1| < 4
$\therefore$ Statement I alone is sufficient.
From statement II, x(x – 4) > 0.
$\Rightarrow$ x < 0 or x > 4
$\Rightarrow$ x – 1 < – 1 or x – 1 > 3
$\Rightarrow$ |x – 1| > 1 or |x – 1| > 3
i.e., | x – 1| > 1
$\therefore$ Statement II alone is not sufficient. Choice (A)

Chapter – 4
(Sequences and Series)

Concept Review Questions

Solutions for questions 1 to 25:

1. Given that a = 2 and d = 5 – 2 = 3
t_n = a + (n – 1)d = 266
$\Rightarrow$ 2 + 3(n – 1) = 266
$\Rightarrow$ n – 1 = $\frac{266-2}{3}$ = 88
$\Rightarrow$ n = 89. Choice (B)

2. Geometric mean = $\sqrt[4]{(4)(8)(16)(32)}$ = 2 ($\sqrt{32}$) = $\sqrt{128}$
Choice (B)

3. The fourth term is equidistant from the first term and the seventh term. $\therefore$ Fourth term = arithmetic mean of 6 and 24 = 15 Choice (A)

4. Sum of the first 7 terms = $\frac{7}{2}$ [2 (1) + 6(3)] = 70
Ans : (70)

5. As we don't know the total number of terms nor the common difference, nor the ninth term, we cannot answer the question. Choice (D)

6. Arithmetic mean = middle term i.e. 9^{th} term = 11
Choice (B)

7. Sum of the first 25 terms = 25 (arithmetic mean)
= 25 (middle term) = 25 (20) = 500 Choice (C)

8. 8^{th} term = 7 + 7(5) = 42 Choice (C)

9. 4^{th} term = ar^3
2^{nd} term = ar
6^{th} term = ar^5
$(2^{nd}$ term) $(6^{th}$ term) = $(ar^3)^2$
$(ar^3)^2$ = (8) (32)
$ar^3 = \pm 16$
But as ar = 8, ar^3 must be positive i.e., 16. Choice (A)

10. Sum of the first 4 terms = $\frac{4(3^4-1)}{3-1} = 160$ Choice (B)

11. Sum of all the terms = $\frac{200}{2}$ [15 +45] = 6000

($\because$ Sum of the p^{th} term from the beginning and the p^{th} term from the end = sum of the first and the last terms).
Choice (B)

12. If p, q, r are in arithmetic progression the p^{th} term, q^{th} term and r^{th} term of an arithmetic progression will be in arithmetic progression Choice (A)

13. If p, q, r are in arithmetic progression, the p^{th} term, q^{th} term and the r^{th} term of a geometric progression will be in geometric progression. Choice (B)

14. $\frac{61}{2}$ [2a + 60 d] = 0
61 [a + 30 d] = 0
a + 30 d = 0, i.e., the 31^{st} term is 0. Choice (C)

15. If the sum of the first p terms of an arithmetic progression is q and the sum of its first r terms is also q (where r > p), the sum of its (p + 1)st term and the r^{th} term is always 0. Applying this concept for the given problem, the required sum = 0.
Ans : (0)

16. The quantities log a, log b and log c are in arithmetic progression if a, b and c are in geometric progression.
Choice (A)

17. Two distinct positive numbers have their arithmetic mean greater than their geometric mean. In the given problem, the arithmetic mean will exceed 9. Choice (C)

18. Eighth term = $ar^7 = 2\,(2)^7 = 256$ Ans: (256)

19. (i) In a geometric progression, the product of any odd number of consecutive terms is (middle term)$^{\text{number of terms}}$
$\therefore$ Required product = 2^{13} = 8192 Choice (D)

(ii) As we don't know the total number of terms, we cannot answer the question. Choice (D)

20. The series is a geometric progression
sum to infinity = $\frac{1}{1-\frac{1}{4}} = \frac{4}{3}$. Choice (C)

21. a = $\frac{1}{x}$ (sum of all the following terms)
= $\frac{1}{x}$ (sum to infinity – a)
a = $\frac{1}{x}$ $\left(\frac{a}{1-r} - a\right)$
(1 – r) ax = a – (a – ar) = ar
a (x – xr – r) = 0
As all the terms are positive, a $\neq$ 0.
r = $\frac{x}{x+1}$ Choice (B)

22. a = 1
$ar^3 = r^3 = 8$
r = 2
Sum of the first 7 terms
= $\frac{1(2^7-1)}{2-1} = 127$ Ans : (127)

23. The sum of the cubes of the first ten natural numbers

$= \left(\frac{(10)(11)}{2}\right)^2 = 3025$ Ans : (3025)

24. The 30^{th} term from the beginning is the 71^{st} term from the end. Choice (C)

25. $\frac{20}{2}[2a + 19d] = 210$
$\Rightarrow 2a + 19d = 21$
$\Rightarrow a + 9d + a + 10d = 21$
10^{th} term + 11^{th} term = 21
Note : In an arithmetic progression with N terms, the sum of all the terms is equal to
(i) N (middle term), if N is odd.
(ii) N (average of the 2 middle terms), if N is even.
Choice (A)

Exercise – 4(a)

Solutions for questions 1 to 40:

1. $t_{69} = 16 \times t_4 \Rightarrow a + 68d = 16(a + 3d)$
$\Rightarrow 15a = 20d \Rightarrow a = 4/3d$
Also, $a + 6d = 22$
$\Rightarrow 4/3d + 6d = 22 \Rightarrow d = 3$
$\therefore a = 4 \therefore t_{20} = 4 + 19 \times 3 = 61$ Ans : (61)

2. Let, the numbers be $a – d$, a and $a + d$
$a – d + a + a + d = 30 \Rightarrow a = 10$
Now $(10 – d) \times 10 \times (10 + d) = 840$
$\Rightarrow 100 – d^2 = 84 \Rightarrow d^2 = 16$
$\therefore d = \pm 4$
$\therefore$ The numbers are 10 – 4, 10 and 10 + 4 i.e., 6, 10 and 14. So the largest is 14. Choice (B)

3. $S_n = 2n^2 + 5n$
$S_{n-1} = 2(n – 1)^2 + 5(n – 1) = 2n^2 + n – 3$
$\therefore t_n = S_n – S_{n-1} = (2n^2 + 5n) – (2n^2 + n – 3) = 4n + 3$
$\therefore t_{10} = 4 \times 10 + 3 = 43$ Choice (B)

4. Let the numbers be $a – 3d$, $a – d$, $a + d$ and $a + 3d$.
$(a – 3d) + (a – d) + (a + d) + (a + 3d) = 56 \Rightarrow a = 14$
Also, $(a – d)(a + d) – (a – 3d)(a + 3d) = 8$
$\Rightarrow 9d^2 – d^2 = 8 \therefore d = 1$
$\therefore$ The smallest number is 14 – 3(1) = 11. Ans : (11)

5. Putting n = 33 (2 x 17 – 1)
$$\frac{Sn}{S'n} = \frac{n/2\{2a + 32d\}}{n/2\{2a' + 32d'\}} = \frac{a + 16d}{a' + 16d'} = \frac{t_{17}}{t'_{17}}$$
$$\frac{t_{17}}{t'_{17}} = \frac{3 \times 33 + 2}{4 \times 33 - 13} = \frac{101}{119}$$ Choice (C)

6. $S_n = \frac{n}{2}(a + l)$
$\Rightarrow \frac{n}{2}(4 + 241) = 9800$
$\Rightarrow n = \frac{2 \times 9800}{245} = 80$
$\therefore 4 + (80 – 1)d = 241$
$\Rightarrow d = \frac{237}{79} = 3$
$\therefore t_{35} = a + 34d = 4 + 34 \times 3 = 106$ Choice (C)

7. Let the last number be l.
$l = 10a$
Now $25/2\{a + l\} = 1100$
$\Rightarrow 25/2 \times 11a = 1100 \Rightarrow a = 8$ Choice (B)

8. The numbers would be in the form of 5k + 2. The smallest three-digit number would be 102 and the greatest would be 997. Now, let there be n such numbers
$102 + (n – 1) \times 5 = 997$
$\Rightarrow n = 180$
$\therefore$ Required sum $= \frac{180}{2}(102 + 997) = 98,910$
Choice (D)

9. Let the terms be a, ar and ar^2
$a \times ar \times ar^2 = 216 \Rightarrow a^3r^3 = 216 \Rightarrow ar = 6$
Now, $ar^4 = 162 \Rightarrow \frac{ar^4}{ar} = \frac{162}{6} \Rightarrow r^3 = 27$
$\therefore r = 3 \therefore a = 6/3 = 2$ Ans : (2)

10. Let a, b, c be $p – r$, p, $p + r$ respectively where $r > 0$
Then $(p – r) + p + (p – r) = 33 \Rightarrow 3p = 33 \Rightarrow p = 11$
$\therefore 11 – r, 11, 11 + r$ are in A.P.
$\Rightarrow 8 – r, 8, 12 + r$ are in G.P.
$\Rightarrow (12 + r)(8 – r) = 64 \Rightarrow r^2 + 4r – 32 = 0$
$\Rightarrow r = 4$ (r is positive and hence cannot be –8)
$\therefore c = 11 + r = 15$ Choice (A)

11. Let the numbers be a/r, a and ar
$a/r \times a \times ar = 13824 \Rightarrow a^3 = 13824 \Rightarrow a = 24$
Now $24/r + 24 + 24r = 84$
$\Rightarrow 24(r + 1/r) = 60 \Rightarrow r + 1/r = 5/2 \therefore r = 2$ or ½
$\therefore$ The numbers are 12, 24 and 48. Choice (B)

12. $t_1 + t_3 + t_8 + t_{12} + t_{17} + t_{19} = 555$
$\Rightarrow 6a + (2 + 7 + 11 + 16 + 18)d = 555$
$\Rightarrow 6(a + 9d) = 555 \Rightarrow 2a + 18d = 555/3$
$\Rightarrow a + (a + 18d) = 185$
$\therefore$ sum of the first 19 terms
$= 19/2\{2a + 18d\}$ {since $s_n = a/2(a + l)$}
$= 19/2 \times 185 = 1757.5$ Choice (C)

13. Let the smallest number be a and the common difference d.
$S_{40} = 3600$ and $S_{30} = 2/3 \times 3600 = 2400$
$40/2\{2a + 39d\} = 3600$
$\Rightarrow 2a + 39d = 180$ ------------------ (1)
Also $30/2\{2a + 29d\} = 2400$
$\Rightarrow 2a + 29d = 160$ ----------------- (2)
$(1) – (2) = 10d = 20 \Rightarrow d = 2$
$\therefore a = \frac{180 - (39 \times 2)}{2} = 51$ Ans : (51)

14. $3 = 2x\frac{3r}{1-r} \Rightarrow r = 1/3$
$\therefore t_5 = 3x(1/3)^4 = 1/27$ Choice (D)

15. The following diagram illustrates the path of the ball.
$\therefore$ total distance travelled by the ball before coming to rest
$= 12 + 2(6 + 3 + 1.5 + \ldots$ to $\infty)$
$= 12 + 2.\left(\frac{6}{1 - 1/2}\right) = 12 + 2(12) = 36$ m Ans : (36)

16. Side of the triangle t_2
= ½ (side of the first triangle) = 20 cm
Side of the triangle t_3
= ½ (side of the second triangle) = 10 cm and so on
$\therefore$ Sum of the perimeters of all the triangles
$= 3(40 + 20 + 10 + \ldots$ to $\infty)$
$= 3\left(\frac{40}{1 - 1/2}\right) = 3 \times 80 = 240$ cm Ans : (240)

17. Let the numbers be a/r, a and ar
$a/r \times a \times ar = 1000 \Rightarrow a^3 = 1000 \Rightarrow a = 10$
Now, $(10/r \times 10) + (10 \times 10r) + (10/r \times 10r) = 350$
$\Rightarrow 100(r + 1/r) = 250 \Rightarrow r + 1/r = 250/100$
$\Rightarrow r + 1/r = 5/2 \therefore r = 2$ or ½
$\therefore$ The greatest number = 10 x 2 = 20 Choice (A)

18. Required product = ax ar x ar^2 x …. X ar^{10}
$= a^{11} . r^{1+2+3+ \ldots +10} = a^{11} . r^{55} = (ar^5)^{11}$
$= (t_6)^{11} = 2^{11} = 2048$ Choice (B)

19. In a GP with common ratio r, if we take the sum of the first N terms, (in the question N =3) the sum of the next N terms and so on, we get a GP whose common ratio is r^n.
The sum of the first N terms is 9A
The sum of the first 2N terms is 6A. Therefore, the sum of the second set of N terms is –3A.
The sum of the third set of N terms has to be A
($\therefore$ 9A, –3A, A are in GP).
Therefore, the sum of the first 3N terms is 9A – 3A + A = 7A (or 6A + A) Choice (A)

20. $\frac{a}{1-r} = 3 \Rightarrow 3 - 3r = a$. Also, $\frac{a^2}{1-r^2} = 6$

$\Rightarrow \frac{a}{1-r} . \frac{a}{1+r} = 6 \Rightarrow \frac{a}{1+r} = 2$

$\Rightarrow 2 + 2r = a \;\; \therefore 3 - 3r = 2 + 2r$
$\Rightarrow r = 1/5 \therefore a = 3 - 3/5 = 12/5$ Choice (C)

21. $t_4 = 3r^3$
$t_7 = 3r^6$
$3r^3 - 3r^6 = 21/64 \Rightarrow r^3 (1-r^3) = 7/64$
Let $r^3 = y$
$64y^2 - 64y + 7 = 0$
$\Rightarrow y = 1/8$ or $7/8$
When y = 7/8, $r^3 = 7/8 \Rightarrow r = 3\sqrt{7/8}$
When, y = 1/8, $r^3 = 1/8 \Rightarrow r = ½$
r = ½ ($\because$ r is rational)

$S_\infty = \frac{3}{1-1/2} = 6$ Ans : (6)

22. Let the last three numbers be a – 12, a and a + 12
$\therefore$ The four numbers are a + 12, a –12, a and a + 12
Now, $(a - 12)^2 = a(a + 12)$
$\Rightarrow a^2 - 24a + 144 = a^2 + 12a \Rightarrow 36a = 144 \Rightarrow a = 4$
$\therefore$ The numbers are 16, –8, 4 and 16
$\therefore$ r = –8/16 = –1/2 Choice (D)

23. Since d = –2 the maximum sum of the series will be the sum of all the positive terms. The least positive term of the series is 2 or 0. (both giving equal sum)
Let, 2 be the n^{th} term
$2 = 60 + (n - 1) - 2 \Rightarrow n = 30$
$\therefore$ Required sum = 30/2 (60 + 2) = 930 Ans : (930)

24. The general term of the series $t_n = (101 - n).n$
$= 101n - n^2$
$\therefore S_n = 101\ \Sigma n - \Sigma n^2$
$= 101. \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$
But, there are 100 terms in the series.
$\therefore$ Required sum = 101 x $\frac{100 \times 101}{2} - \frac{100(101)(21)}{6}$
= 510050 – 338350 = 171700 Choice (B)

25. ½ + ¾ + 7/8 + 15/16 + ……
$= (1 - 1/2) + (1 - 1/2^2) + (1 - 1/2^3)$
$+ (1 - 1/2^4) + \ldots$ Upto 25 terms
$= 25 - (½ + 1/2^2 + 1/2^3 + \ldots + 1/2^{25})$
$= 25 - ½ . \frac{1-(1/2)^{25}}{1-1/2} = 25 - \frac{(2^{25}-1)}{2^{25}}$
$= \frac{24.2^{25}+1}{2^{25}}$ Choice (B)

26. $p (p + q) + p^2 (p^2 + q^2) + p^3(p^3 + q^3) + \ldots$
$= (p^2 + p^4 + p^6 + \ldots) + (pq + p^2q^2 + \ldots)$
$= \frac{p^2}{1-p^2} + \frac{pq}{1-pq}$
[–1< p < 1 and –1 < q < 1
$\Rightarrow$ – 1 < pq < 1 and 0 < p^2 <1]
$= \frac{p^2 - 2p^3q + pq}{(1-p^2)(1-pq)}$ Choice (D)

27. 2, 5, 8, 11, ….. 434
Let $434 = t_n \Rightarrow 2 + (n - 1)3 = 434 \Rightarrow n = 145$
3, 7, 11, 15, ….. 579
Let $579 = t_n$
$\Rightarrow 3 + (m - 1)4 = 579 \Rightarrow m = 145$
$\therefore$ each of the progressions has 145 terms
Let p^{th} term of the first A.P.
= q^{th} term of the second A.P.
$\Rightarrow 2 + (p - 1)3 = 3 + (q - 1) \Rightarrow 3p - 1 = 4q - 1$
$\Rightarrow 3p = 4q \Rightarrow p/q = 4/3 = k$ (say)
$\therefore$ p = 4k, q = 3k
Now, p ≤ 145 and q <145
$\Rightarrow$ 4k ≤ 145 and 3k < 145
$\Rightarrow$ k ≤ 36¼ and k < 48⅓
$\therefore$ From the two, we can say that k ≤ 36
$\therefore$ There would be 36 common terms in the two A.Ps.
Ans : (36)

28. If there are an odd number of rows, the average number of children per row is an integer. (This is equal to the number of children in the middle row)
If there are an even number of rows, the average number of children per row is the average for the middle two rows, which is an integer + 1/2.
We consider the choices.
(A) 810/6 = 135 ≠ integer + 1/2
(B) 810/8 = 101.25 ≠ integer + 1/2
(C) 810/5 = 162 = integer
(D) 810/10 = 81 ≠ integer + 1/2
$\therefore$ Only 5 is a possible value for the number of rows.
Choice (C)

29. Let the number of elements be denoted by n. Let the common difference be denoted by d.
1 + (n – 1) d = 400
(n – 1) d = 399 = 3(7) (19)
n – 1 as well as d must be factors of 399, but as n ≥ 3, n – 1 ≥ 2
$\therefore$ (n – 1, d) = (3, 133), (7, 57), (19, 21), (21, 19), (57, 7), (133, 3) or (399, 1)
$\therefore$ Number of arithmetic progressions, which can be formed = 7.
Ans : (7)

30. Number of members in C_1 on May 1, 2003 = x + 4a
Number of members in C_2 on May 1, 2003 = xb^4
x + 4a = xb^4 and a = 20x
$\therefore x (b^4 - 81) = 0$
As x ≠ 0, $b^4 - 81 = 0$
$\therefore$ b = 3 Ans : (3)

31. Distance covered by the athlete in the first $\frac{3}{2}$ minutes
$= \frac{\pi R}{3}\left(\frac{3}{2}\right) = \frac{\pi R}{2}$m
Distance covered by him in the next 3 minutes
$= \frac{\pi R}{6}$ (3) = $\frac{\pi R}{2}$ m
Distance covered by him in the next 6 minutes
$= \frac{\pi R}{12}$ (6) = $\frac{\pi R}{2}$ m
Distance covered by him in the next 12 minutes
$= \frac{\pi R}{24}$ (12) = $\frac{\pi R}{2}$ m
$\therefore$ Time taken by him to cover the first round
$= \left(\frac{3}{2} + 3 + 6 + 12\right)$ minutes.

Time taken by him to cover the next round
= (24 + 48 + 96 + 192) minutes

$= 16\left(\frac{3}{2} + 3 + 6 + 12\right)$ minutes

$\therefore$ Required ratio = 16 : 1 Choice (C)

32. Let the number of layers in the pile be N.
Number of balls in the Kth layer where K ≥ 1 = K (K + 1)

$$\sum_{K=1}^{N} K(K+1) = 3080$$

$$\sum_{K=1}^{N} K^2 + K = 3080$$

$$\frac{1}{6}N(N+1)(2N+1) + \frac{N(N+1)}{2} = 3080$$

N (N + 1) (N + 2) = 9240 = (20) (21) (22)
Comparing the two sides, N = 20 Ans : (20)

33. $1^2(4) + 2^2(7) + 3^2(10) + 4^2(13) + \ldots$

$$= \sum_{n=1}^{15} n^2(3n+1)$$

$$= 3\sum_{n=1}^{15} n^3 + \sum_{n=1}^{15} n^2$$

$$= 3(120)^2 + \frac{15(16)(31)}{6}$$

= 44440 Choice (A)

34. $\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \ldots$

$$= \frac{2^2-1}{2^2(1^2)} + \frac{3^2-2^2}{3^2(2^2)} + \frac{4^2-3^2}{4^2(3^2)} + \frac{5^2-4^2}{5^2(4^2)} + \ldots$$

$$= \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots + \frac{1}{19^2} - \frac{1}{20^2}$$

$= 1 - \frac{1}{400} = \frac{399}{400}$ Choice (C)

35. 2 + 13 + 28 + 47 + 70 + …..
= 4 – 2 + 15 – 2 + 30 – 2 + 49 – 2 + 72 – 2 + ….
= 1(4) – 2 + 3(5) – 2 + 5(6) – 2 + 7(7) – 2 + ….

$$= \sum_{n=1}^{20}\left((2n-1)(n+3)-2\right)$$

$$= \sum_{n=1}^{20}\left(2n^2+5n-5\right)$$

$$= 2\sum_{n=1}^{20} n^2 + 5\sum_{n=1}^{20} n - 5\sum_{n=1}^{20} 1$$

$$= \frac{2(20)(21)(41)}{6} + \frac{5(20)(21)}{2} - 5(20) = 6690$$

Ans : (6690)

36. In general, $\frac{2d^2}{a(a+d)(a+2d)} = \frac{1}{a} - \frac{2}{a+d} + \frac{1}{a+2d}$

In the given problem d = 1 and a= 2, 3, 4,….19 for the 18 terms respectively.

$\therefore \frac{2}{2(3)(4)} = \frac{1}{2} - \frac{2}{3} + \frac{1}{4}$ …. (1)

$\frac{2}{3(4)(5)} = \frac{1}{3} - \frac{2}{4} + \frac{1}{5}$ …. (2)

$\frac{2}{4(5)(6)} = \frac{1}{4} - \frac{2}{5} + \frac{1}{6}$ …………(3)

– –

$\frac{2}{17(18)(19)} = \frac{1}{17} - \frac{2}{18} + \frac{1}{19}$ …. (16)

$\frac{2}{18(19)(20)} = \frac{1}{18} - \frac{2}{19} + \frac{1}{20}$ …. (17)

$\frac{2}{19(20)(21)} = \frac{1}{19} - \frac{2}{20} + \frac{1}{21}$ …. (18)

When these 18 identities are added up (4 to 15 are not shown explicitly), only 6 terms remain on the RHS. If the given series is S, we get

$$2S = \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{3}\right) + \left(\frac{1}{20} - \frac{2}{20} + \frac{1}{21}\right) = \frac{1}{6} - \frac{1}{420} = \frac{69}{420} = \frac{23}{140}$$

$\therefore S = \frac{23}{280}$ Choice (D)

37. $T_n = (n-1) + 3\,T_{n-1}$
$= (n-1) + 3^1(n-2) + 3^2\,T_{n-2}$
$= (n-1) + 3^1(n-2) + 3^2(n-3) + 3^3 T_{n-3}$
$= (n-1) + 3^1(n-2) + \ldots + 3^{n-2}(1) + 3^{n-1}T_1$
(The sum of the index of 3 and the coefficient of the power of 3 is n – 1 for each term except the last)
Let $S_n = (n-1) + 3(n-2) + 3^2(n-3) + \ldots + 3^{n-2}(1)$ …….(1)
$3S_n = \quad + 3(n-1) + 3^2(n-2) + \ldots + 3^{n-2}(2) .. + 3^{n-1}(1)$ …(2)

(2) – (1) $2S_n = -(n-1) + 3 + 3^2 + \ldots\ldots 3^{n-2} \quad + 3^{n-1}$

$$= -(n-1) + 3\,\frac{3^{n-1}-1}{2}$$

$$S_n = -\frac{(n-1)}{2} + 3\,\frac{(3^{n-1}-1)}{4}.$$

$$\therefore T_n = \frac{3}{4}(3^{n-1}-1) - \left(\frac{n-1}{2}\right) + 3^{n-1}T_1$$

$$= \frac{3}{4}(3^{n-1}-1) - \left(\frac{n-1}{2}\right) + 3^n \qquad (\because T_1 = 3)$$

$$T_{100} = \frac{3}{4}(3^{99}-1) - \frac{99}{2} + 3^{100}$$

$$= \frac{3^{100}}{4} + 3^{100} - \frac{3}{4} - \frac{99}{2}$$

$= \frac{5.3^{100} - 201}{4}$ Choice (C)

38. $S = 2 + \frac{5}{5} + \frac{9}{5^2} + \frac{14}{5^3} + \frac{20}{5^4} + \ldots ------(1)$

$\frac{S}{5} = \frac{2}{5} + \frac{5}{5^2} + \frac{9}{5^3} + \frac{14}{5^4} + \ldots\ldots --------(2)$

$\frac{4S}{5} = 2 + \frac{3}{5} + \frac{4}{5^2} + \frac{5}{5^3} + \frac{6}{5^4} + \ldots..$ --[(1) minus (2) = (3)]

$\frac{4S}{5}\left(\frac{1}{5}\right) = \frac{2}{5} + \frac{3}{5^2} + \frac{4}{5^3} + \frac{5}{5^4} + \ldots\ldots ----(4)$

$\frac{16S}{25} = 2 + \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4} + \ldots\ldots$ [(3) minus (4)]

$$= 2 + \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{9}{4}$$

$S = \frac{225}{64}$ Choice (B)

39. The first 50 terms of the given series are −14, −18, ..., −210. These are in AP whose first term is −14 and common difference is −4

Sum of the first 50 terms = $\frac{50}{2}$ [−14 + (−210)] =−5600

Ans : (–5600)

40. $P = \frac{1}{(62)(122)} + \frac{1}{(63)(121)} + \frac{1}{(64)(120)} + . . . + \frac{1}{(122)(62)}$

$= \frac{1}{184}$

$\left[\frac{184}{(62)(122)} + \frac{184}{(63)(121)} + ... + \frac{184}{(92)(92)} + \frac{184}{(122)(62)}\right]$

$= \frac{1}{184}$

$\left[\frac{1}{62} + \frac{1}{122} + \frac{1}{63} + \frac{1}{121} + \frac{1}{64} + \frac{1}{120} + \ldots \frac{2}{92} \ldots + \frac{1}{122} + \frac{1}{62}\right]$

$= \frac{1}{184}\left[\frac{2}{62} + \frac{2}{63} + \frac{2}{64} + . . . + \frac{2}{122}\right]$

$= \frac{1}{92}\left[\frac{1}{62} + \frac{1}{63} + \frac{1}{64} + . . . + \frac{1}{122}\right]$

$Q = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\ . . . - \frac{1}{122}$

$=1+ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + + \frac{1}{122}$ - 2

$(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + ... + \frac{1}{122})$

$= \frac{1}{62} + \frac{1}{63} + \frac{1}{64} + ... \frac{1}{122.}$

$P = \frac{1}{92}$ Q i.e. $\frac{P}{Q} = \frac{1}{92}$ Choice (D)

Exercise – 4(b)

Solutions for questions 1 to 55:

1. We know that $S_n = n\frac{(\text{First term} + \text{Last term})}{2}$

First term = $(\ell - n)$

Last term = n^{th} term = ℓ.

$\therefore S_n = n\left[\frac{\ell - n + \ell}{2}\right] = \frac{n}{2}[2\ell - n]$

$= n\left[\ell - \frac{n}{2}\right]$ Choice (A)

2. $S_{30} = \frac{30}{2}[2a + 29d] = 150$

$2a + 29d = \frac{150}{15} = 10$

$2a + 29d = 10$

$(a + 29d) = (10 - a)$

$T_{30} = 10 - a$ Choice (C)

3. The n^{th} term of the series AP is $a + (n - 1)d$

So $15 (a + 14d) = 6 (a + 5d)$

$\Rightarrow 15a + 210d = 6a + 30 d$

$\Rightarrow 9a + 180d = 0$

$\Rightarrow a + 20d = 0$

The 21^{st} term is $a + 20d$

So the 21^{st} term is equal to 0 Choice (A)

4. $a + 38d = 4(a + 7d)$

$3a = 10d$

$a = \frac{10}{3}d$

But, given a = 10

$\therefore 10 = \frac{10}{3}d \Rightarrow d = 3$

25^{th} term is $T_{25} = a + 24d$

here a = 10, d = 3

$\Rightarrow 10 + 24 \times 3 = 82$ Ans : (82)

5. $T_2 + T_3 + T_6 + T_7 = 18$

$(a + d) + (a + 2d) + (a + 5d) + (a + 6d) = 18$

$4a + 14d = 18$

But $S_8 = \frac{8}{2}[2a + 7d]$

$= 4[2a + 7d] = 2[4a + 14d]$

$= 2 \times 18 = 36$ Choice (C)

6. Sum of all the cubes between 60 and 1000 is the sum of $4^3, 5^3, 6^3, \ldots 10^3$

We know that the sum of the cubes of the first 'n' natural numbers is $\left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{10(11)}{2}\right]^2 = (55)^2$

But, this includes the sum of the cubes of $1^3, 2^3, 3^3$

$= \left[\frac{3(4)}{2}\right]^2 = 6^2 = 36$

$\therefore 4^3 + 5^3 + ...10^3 = (55)^2 - 36 = 2989$

Ans : (2989)

7. $\log_2 2 + \log_2 2^2 + \log_2 2^3 + \ldots$up to n terms

$(\log_2 2)$ [1 + 2 + 3 + 4 + …up to n terms] = $\left[\frac{n(n+1)}{2}\right]$

Choice (A)

8. First series : 5, 10, 15 ….125

If we expand the series

5, 10, 15, 20, 25, 30……..120, 125

6, 11, 16, 21, 26, 31, 36…101, 106

From the two series we can see that there are no common terms at all.

The number of common terms is '0' Ans : (0)

9. $S_{100} = \frac{100}{2}[2a + 99d] = 5050$

$2a + 99d = \frac{5050}{50} = 101 \rightarrow (1)$

But $T_{10} = 10 = a + 9d$

$\Rightarrow 9d = 10 - a \rightarrow (2)$

$2a + (9 \times 11)d = 101$ ($\therefore$ 2a + 99d = 101)

$\Rightarrow 2a + 9d \times 11 = 101$

$\Rightarrow 2a + (10 - a)11 = 101$

$\Rightarrow -9a + 110 = 101$

$\Rightarrow -9a = -9 \Rightarrow a = 1$

Substitute a = 1 in (2) equation

$9d = 10 - 1 = 9 \Rightarrow d = 1$

$\therefore$ a = 1, d = 1.

Choice (D)

10. n^{th} terms are in the ratio of $\frac{2n + 3}{n - 11}$

If n = 1 in 2n + 3 series the first term is 2(1) + 3 = 5

$n = 2 \Rightarrow$ second term is 2(2) + 3 = 7

$T_2 = 7, d = T_2 - T_1 = 7 - 5 = 2$

$\therefore d_1 = 2$

If n = 1 in n – 11 series, the first term is (1) – 11 = –10

If $n = 2 \Rightarrow$ second term is 2 – 11 = –9

$\therefore d_2 = T_2 - T_1 = -9 - (-10) = 1$

$\therefore\ \dfrac{d_1}{d_2} = \dfrac{2}{1}$ (or) 2 : 1 Choice (C)

11. $S_{20} = \dfrac{20}{2}\left[2a + 19d\right] = 420$

$2a + 19d = 42 \rightarrow (1)$

$S_5 = \dfrac{5}{2}\left[2a + 4d\right] = 30$

$2a + 4d = 12 \rightarrow (2)$

Solving (1) and (2), we get

d = 2 and a = 2

The largest term will be the 20th term

$T_{20} = a + 19d = 2 + 19 \times 2 = 40$ Ans : (40)

12. $5^2 + 6^2 + 7^2 + \ldots\ldots + 44^2$

$= (1^2 + 2^2 + 3^2 + \ldots.. + 44^2) - (1^2 + 2^2 + 3^2 + 4^2)$

$= \dfrac{44(44+1)(2\times 44+1)}{6} - \dfrac{4(4+1)(2\times 4+1)}{6}$

= 29,340 Choice (B)

13. The smallest multiple of 6 greater than 200 is 204 and the largest multiple of 6 less than 1100 is 1098.

$204 + (n - 1)\,6 = 1098 \Rightarrow n = 150$

$\therefore$ Required sum $= \dfrac{150}{2}\{204 + 1098\} = 97{,}650$

Ans : (97650)

14. Let $S_n = 36$

n/2 { 2 x 12 + (n –1)–2} = 36

$\Rightarrow n(13 - n) = 36 \Rightarrow n^2 - 13n + 36 = 0$

$\Rightarrow (n - 4)(n - 9) = 0 \Rightarrow n = 4$ or 9 Choice (C)

15. $Sn = \leq 2500$

$\Rightarrow n/2\ \{14 + (n-1)3\} \leq 2500 \Rightarrow 3n^2 + 11n \leq 5000$

$\therefore$ The maximum value of n which would satisfy the given inequation is n = 39 Ans : (39)

16. $t_n = S_n - S_{n-1}$

$= 2n^2 + 4n - [2(n-1)^2 + 4(n-1)] = 4n + 2$

$\therefore$ sum of the squares of the terms $= \Sigma\,(4n + 2)^2$

$= 16\Sigma n^2 + 16\Sigma n + 4n$

$= \dfrac{16n(n+1)(2n+1)}{6} + \dfrac{16n(n+1)}{2} + 4n$

When n = 10,

required sum $= \dfrac{16\times10\times11\times21}{6}$

$\dfrac{+16\times10\times11}{2}$ + 4 x 10 = 6160 + 880 + 40 = 7080

Ans : (7080)

17. $\log_{2^{1/2}} x = \log_2 x$

Similarly $\log_{2^{1/3}} x = \log_2 x$

and so on $\therefore$ (1 + 2 + 3 + 4 + …. + 20) $\log_2 x = 420$

$\Rightarrow \log_2 x = \dfrac{420}{210} = 2$

$\therefore x = 2^2 = 4$ Choice (B)

18. Let the first term and the common difference of the progression be a and d respectively.

$\dfrac{13}{2}[2a + 12d] = \dfrac{27}{2}[2a + 26d]$

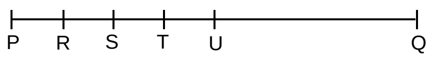
13a + 78 d = 27a + 351d

0 = 2a + 39d

$\therefore$ The sum of the first 40 terms $= \dfrac{40}{2}[2a + 39d] = 0$

Choice (A)

19.



Let P, R, S, T and U be the locations of the stones.

Let PR = RS = ST = TU = 4 m.

The distance covered when the first, second, third, fourth and fifth stones are moved to Q is

= 200 + 2 (196 + 192 + 188 + 184) = 1720 m. Ans : (1720)

20. $\dfrac{2}{3} + \dfrac{8}{9} + \dfrac{26}{27} + \dfrac{80}{81} + \ldots\ldots$ up to 20 terms

$= \left(1 - \dfrac{1}{3}\right) + \left(1 - \dfrac{1}{9}\right) + \left(1 - \dfrac{1}{27}\right) + \left(1 - \dfrac{1}{81}\right) + \ldots..$ up to 20 terms

$= 20 - \left(\dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \dfrac{1}{3^4} + \ldots\ldots + \dfrac{1}{3^{20}}\right)$

$= 20 - \dfrac{1}{3}\left(\dfrac{1 - \left(\dfrac{1}{3}\right)^{20}}{1 - \dfrac{1}{3}}\right) = 20 - \dfrac{3^{20} - 1}{2.3^{20}} = \dfrac{40.3^{20} - 3^{20} + 1}{2.3^{20}}$

$= \dfrac{39.3^{20} + 1}{2.3^{20}}$ Choice (D)

21. r = –b

$\therefore a = \dfrac{b}{1 - (-b)}$

$\Rightarrow a + ab = b$

$\Rightarrow b(1 - a) = a \Rightarrow b = a/(1 - a)$ Choice (B)

22. $5^{1/3}\,.\,5^{1/9}.5^{1/27}\ \ldots\ldots\infty$

(1/3 + 1/9 + 1/27 + …..∞)

$= 5 = 5^{(1/3/1-1/3)} = 51/2 = \sqrt{5}$ Choice (C)

23. Let the numbers be a/r, a and ar

Now a/r,4a and ar are in A.P.

$\Rightarrow 8a = a\,(r+1/r) \Rightarrow r^2 - 8r + 1 = 0$

$\Rightarrow \dfrac{r = 8 \pm \sqrt{64-4}}{2} = 4 \pm \sqrt{15}$

$\therefore r = 4 + \sqrt{15}$ Choice (B)

24. Let the three terms of the A.P be (a – d), (a), (a + d)

(a – d), (a), (a + d) $\dfrac{4}{3}$ are in G.P.

$\therefore\ \dfrac{a}{(a-d)} = \dfrac{\frac{4}{3}(a+d)}{a}$

$a^2 = \dfrac{4}{3}(a+d)(a-d)$

$3a^2 = 4(a^2 - d^2)$

$4d^2 = 4a^2 - 3a^2$

$d = \dfrac{a}{2} \Rightarrow a = \pm 2d$

Given, first term = a – d = 4.

a = ±2d

d = 4 or –4/3. Choice (D)

25. $3, \dfrac{6}{y}, \dfrac{12}{y^2}, \dfrac{24}{y^3} \ldots$

In the above series a = 3 and r = 2/y

So sum to infinite terms is $\dfrac{a}{1-r} = \dfrac{3}{1 - \dfrac{2}{7}} = \dfrac{3y}{y - 2}$

Choice (C)

26. Given series is $3.4^2 + 4.5^2 + 5.6^2 + \ldots$

In series $1\text{x}2^2 + 2 \text{ x } 3^2 + 3 \text{ x } 4^2 + 4 \text{ x } 5^2 + 5 \text{ x } 6^2$

The nth term will be of the form $T_n = n(n+1)^2$

$S_n = \sum T_n = \sum n(n^2 + 1 + 2n)$

$= \sum n^3 + \sum n + 2\sum n^2$

We are asked to find the sum of 15 terms, and first term is starting with '3'

$\therefore$ We have to take n = 17

$$\left[\frac{17(18)}{2}\right]^2 + \left[\frac{17 \times 18}{2}\right] + 2\left(\frac{17(18)(35)}{3 \times 2}\right)$$

$= (17 \times 9)^2 + (17 \times 9) + (17 \times 6 \times 35)$

$= 23409 + 153 + 3570 = 27132$

This includes the sum of 17 terms of the series which has 2 extra terms. They are

1. 2^2 and 2.3^2. So we have to subtract this total from the above result i.e subtract $1.4 + 2.9 = 22$

$\therefore 27132 - 22 = 27110$ Ans : (27110)

27. Let a_1, a_2 be the first terms of the two progressions. Let d_1, d_2 be the common differences of the two progressions.

We need to find $\frac{a_1 + 10d_1}{a_2 + 10d_2}$.

Given, $\dfrac{\frac{n}{2}\left[2a_1 + (n-1)\,d_1\right]}{\frac{n}{2}\left[2a_2 + (n-1)\,d_2\right]} = \dfrac{7n+4}{6n-5}$

$= \dfrac{2a_1 + (n-1)\,d_1}{2a_2 + (n-1)\,d_2} = \dfrac{7n+4}{6n-5}$

Take n = 21,

$\therefore \dfrac{2a_1 + 20d_1}{2a_2 + 20d_2} = \dfrac{7(21)+4}{6(21)-5}$

$\Rightarrow \dfrac{a_1 + 10d_1}{a_2 + 10d_2} = \dfrac{151}{121}$. Choice (D)

28. Let $s = 3 + 6x^2 + 9x^4 + 12x^6 + \ldots$

$x^2s = 3x^2 + 6x^4 + 9x^6 + 12x^8 + \ldots$

$s - x^2s = 3 + 3x^2 + 3x^4 + 3x^6 + \ldots$

$s(1 - x^2) = 3[1 + x^2 + x^4 + x^6 + \ldots]$

$s(1 - x^2) = 3\left[\dfrac{1}{1-x^2}\right]$ $[\because |x| < 1]$

$\therefore s = \dfrac{3}{(1-x^2)^2}$ Choice (C)

29. Option A:

$r = \frac{1}{2}$, $a = \frac{1}{2}$

$s_\infty = \dfrac{a}{1-r} = \dfrac{\frac{1}{2}}{1-\frac{1}{2}} = \dfrac{\frac{1}{2}}{\frac{1}{2}} = 1$

2nd option:

$r = \frac{3}{4}$, $a = \frac{1}{4}$

$s_\infty = \dfrac{\frac{1}{4}}{1-\frac{3}{4}} = \dfrac{\frac{1}{4}}{\frac{1}{4}} = 1$

Series is in the option C is not in G.P.

$\therefore$ Both (A) & (B) satisfy the given conditions.

3rd option:

$r = a = \frac{1}{4}$

$s_\infty = \dfrac{\frac{1}{4}}{1-\frac{1}{4}} = \dfrac{1}{3}$

$s_\infty \neq 1$

Only choices (A) and (B) satisfy the given conditions.

Choice (D)

30. Let b = a + d,

c = a + 2d

As a, a + 2d, a + d are in G.P,

$(a + 2d)^2 = a\,(a + d)$

$\Rightarrow a^2 + 4ad + 4d^2 = a^2 + ad$

$\Rightarrow 4d^2 = -3ad$

$\Rightarrow d\,(4d + 3a) = 0$

$\Rightarrow d = 0$ or $d = -\frac{3}{4}a$.

If d = 0, $\frac{a}{c} = 1$, which is violating the condition that $r \neq 1$.

$\therefore d = -\frac{3}{4}a$.

$\dfrac{a}{b} = \dfrac{a}{a - \frac{3}{4}a} = \dfrac{a}{\cancel{a}/4} = 4$. Ans : (4)

31. $S_{10} = \dfrac{a(1-r^{10})}{1-r} = 2$ .... (1)

$T_5 = a.r^4$

$T_6 = a.r^5$

But, given $T_5 = 2 . T_6$

$ar^4 = 2 . a . r^5$

$r = \frac{1}{2}$

Substitute $r = \frac{1}{2}$ in the equation (1)

$\dfrac{a\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}} = 2$

$a\left(1-\left(\frac{1}{2}\right)^{10}\right) = 1$

$a = \dfrac{1}{1-\left(\frac{1}{2}\right)^{10}}$

$a = \dfrac{1024}{1023}$ Choice (D)

32. $S_\infty = \dfrac{a}{1-r}$

$T_2 = a.r^{2-1} = ar$

$\therefore \dfrac{S_\infty}{T_2} = \dfrac{\frac{a}{1-r}}{ar} = \dfrac{9}{2}$

$\dfrac{a}{(1-r).ar} = \dfrac{9}{2}$

$\dfrac{1}{r(1-r)} = \dfrac{9}{2}$

$\Rightarrow 9r^2 - 9r + 2 = 0$

$\Rightarrow 9r^2 - 3r - 6r + 2 = 0$

$\Rightarrow 3r(3r - 1) - 2(3r - 1) = 0$

$\Rightarrow (3r - 1)(3r - 2) = 0$

$\Rightarrow r = \frac{1}{3}$ (or) $r = \frac{2}{3}$ Choice (C)

33. Let the terms be $\frac{a}{r}$, a, ar

Given, $\frac{a}{r} \times a \times ar = 343$

$a^3 = 343, \Rightarrow a = 7$

But given $\frac{a}{r}\times a + a\times ar + ar\times\frac{a}{r} = 171.5$

$a^2\left(\frac{1}{r}+r+1\right) = 171.5$

$7^2(1 + r^2 + r) = 171.5 \times r$

$1 + r^2 + r = \frac{7}{2}r$

$2 + 2r^2 + 2r - 7r = 0$

$2r^2 - 5r + 2 = 0$

$2r(r - 2) - 1(r - 2) = 0$

$(r - 2)(2r - 1) = 0$

$r = 2,\ r = \frac{1}{2}$

If we substitute r = 2

The series is $\frac{7}{2}$, 7, 14

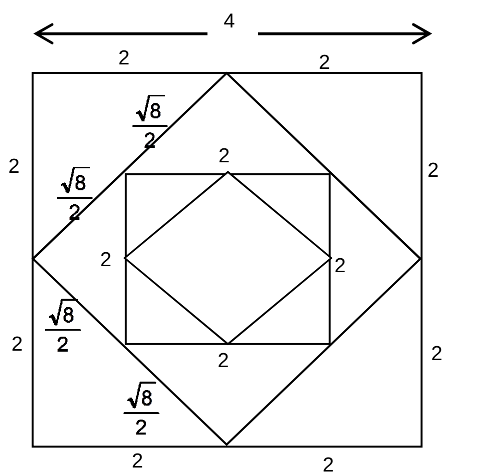
If $r = \frac{1}{2}$,

then the series is

14, 7, $\frac{7}{2}$

Any how the largest number is 14 Ans: (14)

34.



Initially the square S_1 has the perimeter as $4 \times (4) = 16$

Then the square S_2 has the perimeter $4\sqrt{8} = 8\sqrt{2}$

Square S_3 has the perimeter $4 \times 2 = 8$

16, $8\sqrt{2}$, 8 … are in G.P. series with common ratio,

$r = \frac{8\sqrt{2}}{16} = \frac{1}{\sqrt{2}}$

$S_\alpha = \frac{a}{1-r} = \frac{16}{\left(1-\frac{1}{\sqrt{2}}\right)} = \frac{16\sqrt{2}}{\sqrt{2}-1}$

$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{(2+1)} = 32 + 16\sqrt{2}$ Choice (A)

35. $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+ \ldots 20\text{terms}$

$= \frac{4-3}{4\times3}+\frac{5-4}{5\times4}+\frac{6-5}{6\times5}+\ldots+\frac{23-22}{23\times22}$

$= \frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\ldots+\frac{1}{22}-\frac{1}{23}$

$= \frac{1}{3}-\frac{1}{23} = \frac{23-3}{23\times3} = \frac{20}{69}$

(All the terms will be cancelled except $\frac{1}{3}$ and $\frac{1}{23}$)

Choice (B)

36. Required sum

$= 1 \times 30 + 2 \times 29 + 3 \times 28 + \ldots + 30 \times 1$

$= \sum_{n=1}^{30} n(31-n)$

$= 31\sum_{n=1}^{30} n - \sum_{n=1}^{30} n^2$

$= \frac{31(30)(31)}{2} - \frac{30(31)(61)}{6}$

= 4960 Ans : (4960)

37. $\frac{8}{9}+\frac{12}{64}+\frac{16}{225}+\ldots$

$= \frac{3^2-1^2}{1^2(3^2)}+\frac{4^2-2^2}{2^2(4^2)}+\frac{5^2-3^2}{3^2(5^2)}+\frac{6^2-4^2}{6^2(4^2)}+\ldots$

$= \frac{1}{1^2}-\frac{1}{3^2}+\frac{1}{2^2}-\frac{1}{4^2}+\frac{1}{3^2}-\frac{1}{5^2}+\frac{1}{4^2}-\frac{1}{6^2}+\ldots+\frac{1}{17^2}-\frac{1}{19^2}$

$+\frac{1}{18^2}-\frac{1}{20^2}$

$= 1+\frac{1}{2^2}-\frac{1}{19^2}-\frac{1}{20^2} = 1-\frac{1}{19^2}+\frac{1}{2^2}-\frac{1}{20^2}$

$= \frac{360}{361}+\frac{99}{400}$ Choice (C)

38. $\frac{1}{3}+\frac{1}{\sqrt{2}+\sqrt{5}}+\frac{1}{\sqrt{3}+\sqrt{6}}+\frac{1}{\sqrt{4}+\sqrt{7}}+\ldots+\frac{1}{\sqrt{141}+\sqrt{144}}$

$= \frac{1}{3}+\frac{\sqrt{5}-\sqrt{2}}{3}+\frac{\sqrt{6}-\sqrt{3}}{3}+\frac{\sqrt{7}-\sqrt{4}}{3}+\ldots+\frac{\sqrt{144}-\sqrt{141}}{3}$

$= \frac{1+\sqrt{144}+\sqrt{143}+\sqrt{142}-\sqrt{2}-\sqrt{3}-\sqrt{4}}{3}$

$= \frac{11+\sqrt{143}+\sqrt{142}-\sqrt{2}-\sqrt{3}}{3}$ Choice (D)

39. $S = 1+\frac{3}{5}+\frac{6}{5^2}+\frac{10}{5^3}+\frac{15}{5^4}+\ldots\ldots ----(1)$

$\frac{S}{5} = \frac{1}{5}+\frac{3}{5^2}+\frac{6}{5^3}+\frac{10}{5^4}+\ldots\ldots -------(2)$

$\frac{4S}{5} = 1+\frac{2}{5}+\frac{3}{5^2}+\frac{4}{5^3}+\frac{5}{5^4}+\ldots$ ---------(3) [=(1) minus (2)]

$\frac{4S}{5}\left(\frac{1}{5}\right) = \frac{1}{5}+\frac{2}{5^2}+\frac{3}{5^3}+\frac{4}{5^4}+\ldots\ldots ------(4)$

$S\left(\frac{4}{5}\right)^2 = 1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\ldots$ [(3) min us (4)]

$= 1 + \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{5}{4}$

$S = \frac{125}{64}$. ∴ S lies between 1 and 2. Choice (B)

40. $S = 2+\frac{4}{5}+\frac{7}{5^2}+\frac{11}{5^3}+\frac{16}{5^4}+\ldots ----(1)$

$$\frac{S}{5}=\frac{2}{5}+\frac{4}{5^2}+\frac{7}{5^3}+\frac{11}{5^4}+\dots\ -----(2)$$

$$\frac{4S}{5}=2+\frac{2}{5}+\frac{3}{5^2}+\frac{4}{5^3}+\frac{5}{5^4}+\dots\dots\ ----(3)$$

[= (1) minus (2)]

$$\frac{4S}{5}\left(\frac{1}{5}\right)=\frac{2}{5}+\frac{2}{5^2}+\frac{3}{5^3}+\frac{4}{5^4}+\dots\ -----(4)$$

$$S\left(\frac{4}{5}\right)^2=2+0+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\dots\dots$$ [(3) minus (4)]

$$= 2 + \frac{\frac{1}{5^2}}{1-\frac{1}{5}}=\frac{41}{20}$$

Note:

$$2+\frac{4}{5}+\frac{7}{5^2}+\frac{11}{5^3}+\frac{16}{5^4}+\dots =$$

$$\left(1+\frac{3}{5}+\frac{6}{5^2}+\frac{10}{5^3}+\frac{15}{5^4}+\dots\right)\ +$$

$$\left(1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\dots\right.$$

$$=\frac{125}{64}+\frac{1}{1-\frac{1}{5}}=\frac{205}{64}\ \left(\because\ \left(\left(1+\frac{3}{5}+\frac{6}{5^2}+\frac{10}{5^3}+\frac{15}{5^4}+\dots\right)\right.\right.$$

equals $\frac{125}{64}$ from solution 4.5)) Choice (B)

41. Let the two numbers be a and b

$\frac{a+b}{2}$ = 20 ⇒ a + b = 40 ---------- (1)

$\sqrt{ab}$ = 16 ⇒ ab = 256 ---------- (2)

Now a – b = $\sqrt{(a+b)^2-4ab}$

= $\sqrt{1600}$ – 4 x 256 = $\sqrt{576}$ = 24 ---------- (3)

Solving (1) and (3)

a = 32, b = 8

So, the greater number is 32. Ans : (32)

42. If password length is one, there will be 24 such passwords

Total number of characters = 1(24)

Similarly with length two, three etc.

Total number of characters

= 1(24) + 2(23) + 3(22) + ….+ 24(1)

$$=\sum_{n=1}^{24}n(25-n)=25\sum_{n=1}^{24}n-\sum_{n=1}^{24}n^2$$

$$=25\,\frac{(24)(25)}{2}-\frac{24(25)(49)}{6}=7500-4900=2600.$$

Ans : (2600)

43. The data is tabulated below.

The position of the term	m	n
The value of the term	2n	2m

As m < n, this is a decreasing AP. The common difference is $\frac{2m}{n}-\frac{2n}{m}$ = -2.

If the first term is a, then a – 2(m – 1) = 2n

The $(n-m)^{th}$ term is a ± 2(n – m – 1)

= 2(m – 1) + 2n – 2n + 2m + 2 = 4m. Choice (C)

44. $S=\frac{1}{101}+\frac{1}{102}+\dots+\frac{1}{200}$

$$=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{200}\right)-\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{100}\right)$$

$$=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{200}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\dots+\frac{1}{200}\right)$$

$$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\dots-\frac{1}{200}$$

Also, $S=\frac{1}{101}+\frac{1}{102}+\dots+\frac{1}{200}$

$$=\frac{1}{2}\left[\frac{2}{101}+\frac{2}{102}+\dots+\frac{2}{200}\right]$$

$$=\frac{1}{2}\left[\frac{1}{101}+\frac{1}{200}+\frac{1}{102}+\frac{1}{199}+\dots\frac{1}{200}+\frac{1}{101}\right]$$

$$=\frac{301}{2}\left[\frac{1}{101(200)}+\frac{1}{102\,(199)}+\dots+\frac{1}{200\,(101)}\right]$$

Choice (D)

45. $1+\frac{1}{n^2}+\frac{1}{(n+1)^2}=\frac{(n+1)^2n^2+n^2+(n+1)^2}{n^2(n+1)^2}$

$$=\frac{n^4+2n^3+3n^2+2n+1}{(n(n+1))^2}=\frac{(n^2+n+1)^2}{(n(n+1))^2}$$

$$\therefore\ \sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}=\frac{n^2+n+1}{n(n+1)}$$

$$=1+\frac{1}{n}-\frac{1}{n+1}$$

$$S_1=1+\frac{1}{1}-\frac{1}{2}=\frac{3}{2}$$

$$S_2=S_1+1+\frac{1}{2}-\frac{1}{3}=\frac{8}{3}$$

$$S_3=\frac{8}{3}+1+\frac{1}{3}-\frac{1}{4}=\frac{15}{4}$$

$$S_4=\frac{15}{4}+1+\frac{1}{4}-\frac{1}{5}=\frac{24}{5}$$

$S_5=\frac{24}{5}+1+\frac{1}{5}-\frac{1}{6}=\frac{35}{6}$ and

$$S_6=\frac{35}{6}+1+\frac{1}{6}-\frac{1}{7}=\frac{48}{7}$$

$$\frac{S_5+S_6}{S_3+S_4}=\frac{\frac{35}{6}+\frac{48}{7}}{\frac{15}{4}+\frac{24}{5}}$$

$$\frac{\frac{533}{21}}{\frac{171}{10}}=\frac{5330}{3591}$$

Note: It can be observed that $S_N = N + 1 - \frac{1}{N+1}$.

Choice (C)

46. $Q = 1+\frac{1}{2^3}+\frac{1}{3^3}+\dots+\frac{1}{120^3}$

$$8Q = 8+1+\frac{8}{27}+\frac{1}{8}+\frac{8}{125}+\frac{1}{27}+\dots+\frac{8}{119^3}+\frac{1}{60^3}$$

$$= 1+\frac{1}{8}+\frac{1}{27}+\dots+\frac{1}{60^3}+8+\frac{8}{27}+\frac{8}{125}+\dots+\frac{8}{119^3}$$

$$8Q-P = 8+\frac{8}{27}+\frac{8}{125}+\dots+\frac{8}{119^3}$$

$$\frac{8Q-P}{64} = \frac{1}{8} + \frac{1}{216} + \frac{1}{1000} + \ldots + \frac{1}{238^3}$$

$$= \frac{1}{2^3} + \frac{1}{6^3} + \frac{1}{10^3} + \ldots + \frac{1}{238^3}$$

= R

Choice (D)

47. The values of n, t_n (the n^{th} term of the given sequence s(say) and t_{n-} t_{n-1} (the n^{th} term of the first order difference sequence D_1) are tabulated below.

n	1	2	3	4	5	6	7
s	−13	−7	0	8	17	27	38
D_1		6	7	8	9	10	11

The nth term of D_1 is n + 4. (The second term is 6, the third is 7 and so on)

t_n = −13 + (6 + 7 + 8 + 9 + . . . + n + 4) = −13 – (1 + 2 + 3 + 4 + 5) + (1 + 2 + 3 + 4 + 5 + 6 + 7 + . . . n + 4)

$$= -13 - 15 + \frac{(n+4)(n+5)}{2}$$

$$= -28 + \frac{(n+4)(n+5)}{2}$$

30^{th} term of the sequence is

$$t_{30} = -28 + \frac{(30+4)(30+5)}{2}$$

= 595 − 28 = 567 Ans : (567)

48. $1^2(2) + 2^2(7) + 3^2(12) + 4^2(17) + \ldots\ldots$

$$= \sum_{n=1}^{20} n^2(5n-3)$$

$$= 5\sum_{n=1}^{20} n^3 - 3\sum_{n=1}^{20} n^2$$

$$= 5\left[\frac{20(21)}{2}\right]^2 - 3\frac{(20)(21)(41)}{6}$$

= 211890 Choice (C)

49. $S = x + 5x^2 + 11x^3 + 21x^4 + 36x^5 + 57x^6 + \ldots$

$= (x + 4x^2 + 10x^3 + 20x^4 + 35x^5 + 56x^6 + \ldots) + (x^2 + x^3 + x^4 + x^5 + x^6 + \ldots)$ = (A polynomial whose coefficients have their second order differences in AP) + (An infinite GP whose first term is x^2 and common ratio is x) = (S_1 (say)) + $\frac{x^2}{1-x}$ (since $|x| < 1$).

$S_1 = x + 4x^2 + 10x^3 + 20x^4 + 35x^5 + 56x^6 + \ldots$ ------ (1)

$S_1x = x^2 + 4x^3 + 10x^4 + 20x^5 + 35x^6 + \ldots$ -------------(2)

$S_1(1-x) = x + 3x^2 + 6x^3 + 10x^4 + 15x^5 + 21x^6 + \ldots$

(3) = (1) minus (2)

$S_1(1-x)x = x^2 + 3x^3 + 6x^4 + 10x^5 + 15x^6 + \ldots$ ------(4)

$S_1(1-x)^2 = x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + 6x^6 + \ldots$ --(5)

[= (3) minus (4)

$S_1(1-x)^2x = x^2 + 2x^3 + 3x^4 + 4x^5 + 5x^6 + \ldots$ -------(6)

$S_1(1-x)^3 = x + x^2 + x^3 + x^4 + x^5 + x^6 + \ldots$ --[(5) minus (6)]

$$= \frac{x}{1-x}$$

$$S_1 = \frac{x}{(1-x)^4}$$

$$\therefore S_1 = \frac{x}{(1-x)^4} + \frac{x^2}{1-x}$$

Choice (B)

50. 2 + 15 + 32 + 53 + 78 + …..

= 5 – 3 + 18 – 3 + 35 – 3 + 56 – 3 + …..

= 1(5) – 3 + 3(6) – 3 + 5(7) – 3 + 7(8) – 3 + ….

$$= \sum_{n=1}^{20}\left[(2n-1)(n+4)-3\right]$$

$$= \sum_{n=1}^{20}\left(2n^2+7n-7\right)$$

$$= 2\sum_{n=1}^{20} n^2 + 7\sum_{n=1}^{20} n - 7\sum_{n=1}^{20} 1$$

$$= \frac{2\,(20)\,(21)\,(41)}{6} + \frac{7\,(20)\,(21)}{2} - 7(20) = 7070$$

Choice (D)

51. In general, $\frac{2d^2}{a(a+d)(a+2d)} = \frac{1}{a} - \frac{2}{a+d} + \frac{1}{a+2d}$

In the given problem, d= 2 and a =1, 3, 5,….19 for the given 10 terms

$$\therefore \frac{8}{1(3)(5)} = \frac{1}{1} - \frac{2}{3} + \frac{1}{5} \quad \ldots..\ (1)$$

$$\frac{8}{3(5)(7)} = \frac{1}{3} - \frac{2}{5} + \frac{1}{7} \quad \ldots\ldots(2)$$

$$\frac{8}{5(7)(9)} = \frac{1}{5} - \frac{2}{7} + \frac{1}{9} \quad \ldots..\ (3)$$

— —

$$\frac{8}{15(17)(19)} = \frac{1}{15} - \frac{2}{17} + \frac{1}{19} \quad \ldots..\ (8)$$

$$\frac{8}{17(19)(21)} = \frac{1}{17} - \frac{2}{19} + \frac{1}{21} \quad \ldots..\ (9)$$

$$\frac{8}{19(21)(23)} = \frac{1}{19} - \frac{2}{21} + \frac{1}{23} \quad \ldots..\ (10)$$

When these 10 identities (the 4 intermediate ones have not been shown explicitly) are added up, only 6 terms remain on the RHS - the first two from (1), the first from (2), the third from (9) and the second and the third from (10).We get

$$8S = 1 - \frac{2}{3} + \frac{1}{3} + \frac{1}{21} - \frac{2}{21} + \frac{1}{23} = \frac{2}{3} - \frac{2}{483} = \frac{2(480)}{3(483)}$$

$$\therefore S = \frac{40}{483}$$

Choice (B)

52. $T_n = (n-1) + 4\,T_{n-1}$

$= (n-1) + 4\,(n-2) + 4^2\,T_{n-3}$

$= (n-1) + 4\,(n-2) + 4^2\,(n-3) + 4^3\,T_{n-3}$

$= (n-1) + 4\,(n-2) + \ldots.. + 4^{n-2} + 4^{n-1}\,T_1$ (Given $T_1 = 4$)

The sum of the index of 4 and the coefficient of the power of 4 is n – 1 in each term except the last

Let $S_n = (n-1) + 4\,(n-2) + \ldots. + 4^{n-2}\,(1)$ $\therefore T_n = S_n + 4^{n-1}\,T_1$

$\therefore 4S_n = \ldots\ldots..4\,(n-1) + \ldots. + 4^{n-2}\,(2) + 4^{n-1} = S_n + 4^n$

$\therefore 3S_n = -(n-1) + 4 + 4^2 + \ldots. + 4^{n-2} + 4^{n-1}$

$$= -(n-1) + 4\frac{(4^{n-1}-1)}{3}$$

$$\therefore S_{200} = \frac{-199}{3} + \frac{4\,(4^{199}-1)}{9} \text{ and } T_{200}$$

$$= \frac{-199}{3} + \frac{4\,(4^{199}-1)}{9} + 4^{200}$$

$$= \frac{-601}{9} + \frac{10}{9}\left(4^{200}\right)$$

Choice (B)

53. Sum of all the natural numbers between 70 and 250

$$= \frac{249(249+1)}{2} - \frac{70 \times (70+1)}{2} = 31125 - 2485$$

= 28,890 – 250 = 28,640

Sum of all the natural numbers between 70 and 250 which are multiples of 6 = 72 + 78 + …. + 246 = 4770,

which are multiples of 8 = 72 + 80 + … + 248 = 3680

Sum of all the natural numbers between 70 and 250 which are divisible both by 6 and by 8 i.e.; divisible by 24 (lcm of 6

and 8) = 72 + 96 + ------ + 240 = 1248
$\therefore$ Required sum = 28640 – (4770 + 3680 – 1248)
= 21,438

Alternate method:
Sum of all natural numbers between 70 and 250
$= \frac{179}{2}(71 + 249) = 28640$
Sum of natural numbers between 70 and 250 divisible by 6 $= \frac{30}{2}(72 + 246) = 4770$
Sum of natural numbers between 70 and 250 divisible by 8 $= \frac{23}{2}(72 + 248) = 3680$
Sum of natural numbers between 70 and 250 divisible by both 6 and 8 i.e. divisible by 24
$= \frac{8}{2}(72 + 240) = 1248$
$\therefore$ Required sum = 28640 – (4770 + 3680 – 1248)
= 21,438 Choice (B)

54. $(1 + x)^{500} + (1 + x)^{499}x + \ldots.$

$$=(1+x)^{500}\frac{\left(1-\left(\frac{x}{1+x}\right)^{501}\right)}{1-\frac{x}{1+x}}$$

$= (1 + x)^{501} ((1 + x)^{501} – x^{501}) / (1 + x)^{501}$
$= (1 + x)^{501} – x^{501} = {}^{501}C_1x + .. + {}^{501}C_{199}x^{199} + {}^{501}C_{200}x^{200} + .. + {}^{501}C_{500}x^{500}$
Sum of the coefficients of x^{199} and x^{200} in the expansion $= {}^{501}C_{199} + {}^{501}C_{200} = {}^{502}C_{200}$
Choice (C)

55. S = 1(2)(3) + 2(3)(4) + 3(4)(5) + ….. + 30(31)(32)
$$S=\sum_{n=1}^{30} n(n+1)(n+2)$$
$$=\sum_{n=1}^{30} n^3+3\sum_{n=1}^{30} n^2+2\sum_{n=1}^{30} n$$
$$=\left[\frac{30(31)}{2}\right]^2+3\frac{(30)(31)(61)}{6}+30(31)$$
$$= \frac{(30)(31)}{12}((30)(31)(3) + 3\times 2\times 61 + 12)$$
= (930) (264) = 245520 Choice (A)

Solutions for questions 56 to 65:

56. Sum of n terms of an AP
$= \frac{n}{2}(2a + (n – 1) d)$
From statement I, we have $240 = \frac{n}{2}[138 + (n – 1)(– 6)]$
480 = n(138 – 6n + 6)
480 = n(144 – 6n)
$480 = 144n – 6n^2$
$\Rightarrow 6n^2 – 144n + 480 = 0$
$\Rightarrow n^2 – 24n + 80 = 0$
$\Rightarrow n^2 – 20n – 4n + 80 = 0$
$\Rightarrow$ n (n – 20) – 4(n – 20) = 0
$\Rightarrow$(n – 4) (n – 20) = 0.
So n = 4 or 20 so, the sum of first four terms or twenty terms is 240.
So statement I alone is not sufficient.
From statement II, we have
$\Rightarrow \frac{n}{2}(18 + (n –1)2) = 240$
$\Rightarrow$ n (18 + 2n –2) = 480
$\Rightarrow$ n (16 + 2n) = 480
$\Rightarrow 2n^2 + 16n – 480 = 0$
$\Rightarrow n^2 +8n – 240 = 0$
$\Rightarrow n^2 – 12n + 20n – 240 = 0$
$\Rightarrow$ n(n – 12) + 20 (n–12) = 0
$\Rightarrow$ n = 12
Statement II alone is sufficient Choice (A)

57. From statement I, a = 11.This alone is not sufficient.
From statement II, the sum of the first 3 terms is equal to the sum of the first 9 terms.
i.e. 3/2 (2a + 2d) = 9/2 (2a + 8d)
i.e. 2a + 2d = 6a + 24d
$\therefore$ 4a = – 22d
2a = –11d
But since there are 2 unknowns, statement II alone is insufficient.
Using both statements, we have a = 11
Hence $d = \frac{-2}{11}$ x (11) = – 2
As a and d are known we can find the sum of the first 10 terms of the series.
$\therefore$ I and II together are sufficient. Choice (C)

58. We know n = 11
From statement I, the first term is 15 i.e. a = 15.
But we do not know anything about d.
Hence statement I is insufficient.
From statement II, the last term is 25. Here we neither know d or a.
Hence statement II is insufficient
If we use both statements I and II, we still do not know about d.
Hence data is insufficient. Choice (D)

59. From statement I,
m^{th} term from the beginning and the $(m + 1)^{th}$ term from the end are the same which means they are the same terms unless d = 0.
Since we do not know a and d, data is insufficient.
From statement II first 2 terms are in the ratio
3 : 5 while first three terms are prime. This is possible only if first 3 terms are 3, 5, 7. Hence d = 2.
But we do not know the value of m we can't answer the question.
Using both the statements, Since we know $d \neq 0$, and m + (m + 1) – 1 = 100.
$\therefore$ m = 50
$\therefore m^{th}$ term is = 3 + 49 (2) = 101 Choice (C)

60. Let the three terms be a – d, a, and a + d.
From statement I ,the sum of the first 3 terms is 30.
$\therefore$ Sum of three terms is 3a = 30
$\Rightarrow$.a = 10
But we cannot solve for d. Hence statement I is insufficient.
From statement II, the product of the first 3 terms is 910. This is not useful as none of the terms are given.
Hence statement II is also insufficient.
Using both the statements, (10 – d) 10 (10 + d) = 910
So d = $\pm$ 3
Hence the 3 terms are 7, 10 and 13.or 13, 10 , 7
So we can't find the first term Choice (D)

61. Sum of the first n terms of a geometric progression
$= \frac{a(r^n-1)}{r-1}$
From statement I,
$3 + 3r^{11} = 246$
$\Rightarrow r^{11} = 81$
This equation have only one real value for r i.e., $r = 81^{1/11}$.
So we can find r and there by the sum of the first 12 terms
Statement I alone is sufficient.
From statement II, $3r^{11} = 123$
$r^{11} = 41$ This equation will have only one real value for r i.e., $r = 41^{1/11}$.

So we can find r and there by the sum of the first 12 terms.
Statement II alone is sufficient. Choice (B)

62. From statement I,

$$\frac{2a(1-r^k)}{1-r} = \frac{a}{(1-r)}$$

$$(1 - r^k) = \frac{1}{2}$$

$$r^k = \frac{1}{2}$$

$$r = \frac{1}{2^{\frac{1}{k}}}$$

Since k is unknown we cannot find r and a. So statement I alone is not sufficient. Statement II alone is not sufficient, as we don't know exact value of r. Using both statements also we cannot answer the question. Choice (D)

63. From statement I a, b, c are p^{th}, q^{th} and the r^{th} term, in a GP. Since we do not know anything about p, q, r, we cannot answer the question. Considering statement II, we know that p, q, r are in an AP.

Hence $q = \frac{p+r}{2}$

We do not know about a, b and c So we can't answer the question.

Using both statements, we have $x^q = x^{\frac{p+r}{2}}$

$\Rightarrow x^q = \sqrt{x^p\, x^r} \quad ax^{q-1}$

$= \sqrt{(ax^{p-1})(ax^{r-1})}$

Hence using both I and II together, we can answer the question. Choice (C)

64. From statement I, we have

$a = \frac{ar}{1-r}$

$\Rightarrow a = 2ar$
$\Rightarrow r = 1/2$
Hence statement I alone is sufficient.
From statement II, we have
$ar^{n-1} = 24$
$\Rightarrow ar^{n+2} = 81$
$\Rightarrow r^3 = \frac{81}{24} = \frac{27}{8}$
$\therefore r = 3/2$
Hence statement II alone is sufficient. Choice (B)

65. Given that r is greater than 1
From statement I,
We have a + ar = 15. We do not know r, so we can't find a.
From statement II,
$a^2 + a^2r^2 = 117$. We do not know r so we can't find a.
Using both the statements,
$a^2(1 + r)^2 = 225 \rightarrow (1)$
$a^2(1 + r^2) = 117 \rightarrow (2)$
$(1) - (2) \Rightarrow 2a^2r = 108$
$\Rightarrow a^2r = 54$
$a^2r = \frac{54}{r}$
$\therefore$ From (2),
$\frac{54}{r}(1 + r^2) = 117$

$\Rightarrow r + \frac{1}{r} = \frac{13}{6} = \frac{3}{2} + \frac{2}{3}$

$\therefore r = \frac{3}{2}$ or $\frac{2}{3}$ But r >1.

$\therefore r = 3/2$
Hence we can solve it using both the statements.
Choice (C)

Chapter – 5
(Functions)

Concept Review Questions

Solutions for questions 1 to 30:

1. A is the set of even numbers up to 20. Further, every even number takes the form x = 2n. So choice C is correct Choice (C)

2. P is a set of perfect squares less than 50. Choice D represents the same. Choice (D)

3. I: x has to be different from itself,
II: x has to be even as well as odd.
III: x has to be odd as well as even.
All the above represent the null set. Choice (D)

4. There are 4 distinct letters M,I,S,P. We do not repeat elements in a set. Thus, the cardinality of the set is 4. Choice (A)

5. n(A) = 1(We do not repeat elements in a set) Ans : (1)

6. The elements of the set are 2, {2, 3}, 3, {1, 2, 3} which are 4 in number. Choice (C)

7. Only the set mentioned in choice (B) is the subset of the given set. Choice A is an element of the given set. {2, 3} would be a subset. Choice (B)

8. As sets A and B have the same elements, the two sets are equal.
$\therefore A \subseteq B$ and $B \subseteq A$ are also true. Choice (D)

9. For any non-empty set, there exists a minimum of two subsets, One is the null set and the other is the set itself. Choice (B)

10. Statements 1 and 2 are true. Statement 3 is false. The null set has exactly one subset. Any other set has atleast 2 subsets. Choice (C)

11. The number of subsets of a set with n elements is 2^n. For the given set, n = 6. $\therefore$ Number of sub sets $2^6 = 64$. Ans : (64)

12. For a set with n elements, the number of proper subsets is $2^n - 1$. For the given set, this is $2^4 - 1$ or 15. Choice (B)

13. $A \cup B$ is the set containing the elements of A or B or both.
$\therefore A \cup B = \{1, 2, 3, 4, 5, 6\}$ Choice (B)

14. $A \cap B$ is the set containing the elements that are present in both A and B. (common elements make the intersection). i.e., $A \cap B = \{2, 3\}$ Choice (A)

15. The number of proper subsets of a set containing n elements is given by $2^n - 1$. Among the choices, only 31 can be expressed as $2^5 - 1$. Choice (B)

16. A – B is the set containing the elements of A that are not in B i.e. A – B = {b, d, e}. Choice (C)

17. B – A is the set containing the elements that are there in set B but not in A.
$\therefore$ B – A = {8, 10} Choice (A)

18. $A^c = \mu - A = \{2, 4, 6, 8, 9, 10\}$ Choice (C)

19. If $A \subseteq B$, then $A \cup B = B$ Choice (B)

20. If $A \subseteq B$, then $A \cap B = A$ Choice (A)

21. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
If A and B are disjoint, then $A \cup B$ will have the maximum number of elements, which is n(A) + n(B) = 5 + 8 = 13. Ans : (13)

22. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. If $A \cap B$ has the maximum number of elements, i.e., if $A \subseteq B$, then $A \cup B$ will have the minimum number of elements which is the number of elements of B itself, which is 9. Choice (B)

23. If $A \subseteq B$, then $A \cap B$ will have the maximum number of elements. If this is the case, $n(A \cap B) = n(A) = 4$.
Ans : (4)

24. $A \cup A' = \mu$ (Standard result) Choice (A)

25. $A \cap A' = \phi$ (Standard result) Choice (C)

26. $n(A \times B) = n(A)\, n(B) = 3(4) = 12$
(Standard Result) Choice (C)

27. $A \times B = \{(x, y)/x \in A, y \in B\}$ Choice (C)

28. The maximum number of elements in any relation from set A to set B is $n(A \times B)$, as $A \times B$ is the largest relation.
Choice (C)

29. The first coordinates should be from A and the second from B. Only choice A satisfies these conditions. Choice (A)

30. R^{-1} is a the set of reverse pairs corresponding to all the pairs in R.
$R^{-1} = \{(x, y)/(y, x) \in R\} = \{(2, 1), (3, 2), (4, 3), (1, 4)\}$
Choice (B)

Exercise – 5(a)

Solutions for questions 1 to 35:

1. $n(P(A)) = 2^n$ Here $n(A) = 5 \therefore n(P(A)) = 2^5 = 32$
Choice (C)

2. The number of subsets containing a but not b of a set A having n elements is given by 2^{n-2}
$\therefore 2^{n-2} = 16$
$\Rightarrow 2^{n-2} = 2^4$
$\Rightarrow n - 2 = 4$
$\Rightarrow n = 6$. Ans : (6)

3. $(n(B))^{n(A)} = 625 \Rightarrow n(B)^4 = 5^4$
$\Rightarrow n(B) = 5$ Choice (B)

4. The number of one-one functions that can be defined from set A with m elements to set B with n elements $= {}^nP_m$
${}^nP_m = 3360 = 16\,(210) = 16\,(15)\,(14)$
$= {}^{16}P_3 \therefore n(A) = 3$, Ans : (3)

5. The number of onto functions can be defined from set A to set B, where
$n(A) = m$ and $n(B) = n$ is $X = n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m + \ldots + [(-1)^{n-1}]\,{}^nC_{n-1}\,(1)^m$
Here n = 4, m = 6
i.e. $X = 4^6 - {}^4C_1(4-1)^6 + {}^4C_2(4-2)^6 - {}^4C_3(4-3)^6$
$= 4096 - 4(729) + 6(64) - 4(1) = 1560$ Choice (A)

6. The total number of functions that can be defined from set A to set B is $(n(B))^{n(A)} = 6^4 = 1296$
The number of one-one functions from set A to set B is
${}^{[n(B)]}P_{n(A)} = {}^6P_4 = 360$
$\therefore$ The number of functions which are not one-one
$= 1296 - 360 = 936$ Choice (C)

7. Given the number of bijections $= n! = 120 \therefore n = 5$
As bijections are possible from A to B, $n(A) = n(B) = 5$
Ans : (5)

8. $h(x) = \dfrac{x^2}{3x^2+1}$
$\forall x \in R, x^2 \geq 0$
$\therefore h(x) = 0$ when $x = 0$
The minimum value of $h(x) = 0$

$$\frac{x^2}{3x^2+1} = \frac{1}{3+\frac{1}{x^2}}$$

Since $3 + \dfrac{1}{x^2} > 3$

$$\therefore \frac{1}{3+\frac{1}{x^2}} < \frac{1}{3}$$

Hence upper limiting value of h(x) is $\dfrac{1}{3}$

$\therefore$ The range of h(x) is $\left[0, \dfrac{1}{3}\right)$

$h : R \longrightarrow B$ is an
$\therefore$ Range of h(x) = codomain of h(x)
$B = \left[0, \dfrac{1}{3}\right)$ Choice (B)

9. Given $f : N \longrightarrow N$
$f(n) = \dfrac{n}{2}$ if n is even
$= \dfrac{n+1}{2}$ if n is odd
If n = 1, $f(1) = \dfrac{1+1}{2} = 1$
n = 2 $f(2) = \dfrac{2}{2} = 1$
$\therefore f(1) = f(2)$
Hence f(n) is not one – one.
For all $m \in N$, there is same $n \in N$, such that f(x) = m.
$\therefore$ Codomain of f(n) = Range of f(n)
Hence f(n) is onto.
f(n) is onto but not one – one Choice (B)

10. Let the number of elements of set X be n
The number of proper subsets of X $= 2^n - 1$
The number of subsets that contain 2, 3, 4 is given by 2^{n-3}
The difference of the above two is $2^n - 1 - 2^{n-3} = 111$
$2^n - 2^{n-3} = 112 \Rightarrow 2^{n-3}(8-1) = 16\,(7) \Rightarrow n - 3 = 4$
$\therefore n = 7$ Ans : (7)

11. For any two functions f(x) and g(x) we know that
$(f + g)(x) = f(x) + g(x)$(By definition of sum of functions)
Choice (A)

12. $f(x) = 3x - 1$, $g(x) = 2x + 3$
$(gof)(x) = g[f(x)] = g(3x - 1) = 2(3x - 1) + 3 = 6x + 1$
$(gof)(-1) = 6(-1) + 1 = -5$ Ans : (5)

13. $(fof)(x) = f\left[\dfrac{3x+5}{4x-3}\right] = \dfrac{3\left(\frac{3x+5}{4x-3}\right)+5}{4\left(\frac{3x+5}{4x-3}\right)-3}$

$$= \frac{9x+15+20x-15}{12x+20-12x+9}$$

$= \dfrac{29x}{29} = x \therefore fof(x) = x$

$(fofof)(x) = \dfrac{3x+5}{4x-3} = f(x)$

$\therefore$ If there are an odd number of f's, the composite function is f(x).

∴ (fofofofof)(4) = f(4) = $\frac{3(4)+5}{4(4)-3}=\frac{17}{13}$ Choice (B)

14. f(x) = $\frac{2x-3}{5x+2}$ = y say

$\Rightarrow x = \frac{3+2y}{2-5y} \Rightarrow f^{-1}(y)=\frac{3+2y}{2-5y}$

or $f^{-1}(x) = \frac{3+2x}{2-5x}$ Choice (D)

15. Given f(x) = $(k - x^{1/n})^n$

$$(fof)(x) = \left[K - \left(\left(K - x^{\frac{1}{n}}\right)^n\right)^{\frac{1}{n}}\right]^n = (K - K + x^{1/n})^n = x$$

∴(fof)(2) = 2 Choice (B)

16. Given f(x) = px + q

f[f(x)] = f[px + q] = p(px + q) + q

= p^2x + pq + q

f[f(f(x))] = f[p^2x + pq + q]

= p[p^2x + pq + q] + q = $p^3x + p^2q$ + pq + q

= $p^3x + q(p^2 + p + 1)$ ….(1)

Given f[f(f(x))] = 105……(2)

Comparing (1) and (2), we get

$p^3 = 64$; $q(p^2 + p + 1) = 105$

∴p = 4 and $q(4^2 + 4 + 1) = 105$

⇒ 21q = 105 ⇒ q = 5

3p – 2q = 3(4) – 2(5) = 2 Ans : (2)

17. f(x) = $\sqrt{2x+3}$ + log (4 – x^2) + $\frac{1}{\sqrt{9-x^2}}$

(i) $\sqrt{2x+3}$ is real when 2 x + 3 ≥ 0 ⇒ x ≥ $\frac{-3}{2}$ or

$x \in \left[\frac{-3}{2},\infty\right)$ ------------------------- (i)

(ii) log(4 – x^2) is defined only when 4 – x^2 > 0 or

(x + 2) (x – 2) < 0

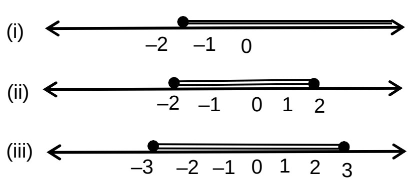
⇒ x ∈ (–2, 2) -------------------------- (ii)

(iii) $\frac{1}{\sqrt{9-x^2}}$ is real when 9 – x^2 > 0

⇒ (x + 3) (x – 3) < 0

⇒ x ∈ (–3, 3) ------------------------- (iii)

∴ The domain of f(x) is (i) ∩ (ii) ∩ (iii), which is [–3/2, 2)



Choice (A)

18. f(x) = $\sqrt{1-2x}$ + $\cos^{-1}\left(\frac{2x-1}{3}\right)$

(i) $\sqrt{1-2x}$ is real when 1 – 2 x ≥ 0 or x ≤ $\frac{1}{2}$

⇒ x ∈ (–∞, $\frac{1}{2}$] ------------------- (i)

(ii) We know that the domain of $\cos^{-1}$ (x) is –1 ≤ x ≤ 1

∴ ⇒ –1 ≤ $\frac{2x-1}{3}$ ≤ 1

– 3 ≤ 2 x –1 ≤ 3

⇔ – 3 + 1 ≤ 2 x ≤ 3 + 1

–2 ≤ 2 x ≤ 4

⇔ –1 ≤ x ≤ 2

∴ x ∈ [–1, 2] ---- (ii)

The domain of f(x) is (i) ∩ (ii), i.e., x ∈ $\left[-1, \frac{1}{2}\right]$

Choice (D)

19. Given $f\left(x-\frac{1}{x}\right) = x^3 - \frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3 + 3\left(x-\frac{1}{x}\right)$

⇒ f(t) = t^3 + 3t

$f\left(x+\frac{1}{x}\right)=\left(x+\frac{1}{x}\right)^3+3\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3} + 6\left(x+\frac{1}{x}\right)$

= $x^3 + 6x + \frac{6}{x}+\frac{1}{x^3}$ Choice (B)

20. [x] = x ∀ X ∈ Z

⇒ x – [x] = 0 ∀ x ∈ Z

∴ For only integer values of x, $\frac{5}{\sqrt[6]{x-[x]}}$ is not defined

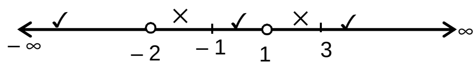
∴ Domain of f is R – Z Choice (B)

21. f(x) = $\sqrt{\frac{(x+1)(x-3)}{(x-1)(x+2)}}$

f is defined only when $\frac{(x+1)(x-3)}{(x-1)(x+2)}$ > 0

⇒ (x – 1) (x + 2) (x + 1) (x – 3) > 0 ------- (i)

critical points are x = –2, –1, 1, 3



When x = 0 statement (1) is true.∴a (–1, 1) is solution region

The regions marked '✓' form the required domain of f

∴ Domain of f is (–∞, –2) ∪ [–1, 1) ∪ [3, ∞)

Choice (D)

22. f(x) = $\frac{1}{3+2\cos x}$

As –1 ≤ cos x ≤ 1, –2 ≤ 2 cos x ≤ 2

or ⇒ 3–2 ≤ 3 + 2 cos x ≤ 3 +2

⇔ 1 ≤ 3 + 2 cos x ≤ 5

∴ Domain of f is R Choice (A)

23. Let g(x) = K^x and given g(4) = 2401

⇒ $K^4 = (7)^4$ ⇒K = ±7 ∴g(x) = $\pm 7^x$ and g(2) = $(\pm 7)^2$ = 49

Ans : (49)

24. Given $h\left(\frac{a}{b}\right)=\frac{h(a)}{h(b)}$

∴ $h\left(\frac{6}{1}\right) = \frac{h(6)}{h(1)}$ = h(6) = $\frac{h(6)}{h(1)}$

⇒ h(1) = 1

∴ $h\left(\frac{1}{6}\right) = \frac{h(1)}{h(6)}=\frac{1}{\frac{1}{12}}$ = 12 Ans : (12)

25. f (x) = kx

given f (3) = 18 = k(3)

⇒ k = 6

∴ f (x) = 6x

$f\left(\frac{1}{3}\right)$ = 6 × $\frac{1}{3}$ = 2 Ans : (2)

26. Given $f(x) = \frac{4^x}{4^x + 2}$

$f\left(\frac{1}{2}\right) = \frac{4^{1/2}}{4^{1/2}+2} = \frac{2}{2+2} = \frac{1}{2}$

$f\left(\frac{1}{4}\right) = \frac{4^{1/4}}{4^{1/4}+2}$

$f\left(\frac{3}{4}\right) = \frac{4^{3/4}}{4^{3/4}+2} = \frac{4^{1-\frac{1}{4}}}{4^{1-\frac{1}{4}}+2}$

$= \frac{4}{4+2.4^{1/4}}$

$= \frac{2}{2+4^{1/4}}$

$\therefore\ f\left(\frac{1}{2}\right) + f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$

$= \frac{1}{2} + \frac{4^{1/4}}{4^{1/4}+2} + \frac{2}{2+4^{1/4}}$

$= \frac{1}{2} + \frac{4^{1/4}+2}{4^{1/4}+2}$

$= \frac{1}{2} + 1 = 3/2$ Choice (A)

27. Given H(1) + H(2) + H(3) + ...+H(N) = $\frac{2NH(N)}{N-1}$ …. (1)

Given H(1) = 2460

Setting N = 2, H(1) + H(2) = $\frac{2(2)H(2)}{1}$

$\Rightarrow$ 3 H(2) = 2460
$\Rightarrow$ H(2) = 820

Setting N = 3, H(1) + H(2) + H(3) = $\frac{2(3)H(3)}{2}$

$\Rightarrow$2460 + 820 = 2 H(3)
$\Rightarrow$ H(3) = 1640

Setting N = 4, H(1) + H(2) + H(3) + H(4) = $\frac{2(4)H(4)}{3}$

2460 + 820 + 1640 = $\frac{5}{3}$ H(4)

$\therefore$ H(4) = 2952 Choice (A)

28. Given h(x + 1) = 3 h(x) – 2h(x –1) and h(0) = 1, h(1) = 2
setting x = 1, h(2) = 3 h(1) – 2 h(0) = 3(2)– 2(1) = 4
setting x = 2, h(3) = 3 h(2) – 2 h(1) = 3(4) – 2(2) = 8
setting x = 3, h(4) = 3 h(3) – 2 h(2) = 3(8) – 2(4) = 16
setting x = 4, h(5) = 3 h(4) – 2 h(3) = 3(16) – 2(8)= 32
setting x = 5, h(6) = 3 h(5) – 2 h(4) = 3(32) – 2(16) = 64
Ans : (64)

29. Given $g(x) = 3x^2 + 8$ and $h(x) = 3x^2 + 4x - 56$ and
$g(x - 2) = h(x + 2) \Rightarrow 3(x-2)^2 + 8 = 3(x+2)^2 + 4(x+2) - 56$
$3x^2 - 12x + 12 + 8 = 3x^2 + 12x + 12 + 4x + 8 - 56 - 28x = -56$

$\Rightarrow x = \frac{56}{28} = 2$ Ans : (2)

30. Given for x > 4, I(x) = I(I(x –1))
I(x + 1) = I(I(x))
I(5) = I(4 + 1) = I(I(4)) = I(2) = 4
I(6) = I(5 + 1) = I(I(5)) = I(4) = 2
I(7) = I(6 + 1) = I(I(6)) = I(2) = 4
I(8) = I(7 + 1) = I(I (7)) = I(4) = 2
We notice that the values of I(x) follow a cycle of two for x > 4
$\therefore$ I(6) = I(8) = I(10) =....=I(750) = 2 Choice (B)

31. Given f(x) = min (3 – 2 x, 3 x + 8)
3 – 2 x = 3 x + 8 $\Leftrightarrow$ x = – 1. For the value of x,
f(x) = min(3 – 2 x, 3 x + 8) = 5
If x < –1, then f(x) = 3 x + 8 and hence f (x) < 5
If x > –1 then f(x) = 3 – 2 x and hence f (x) < 5
$\therefore$ Maximum value of f(x) is 5 Ans : (5)

32. $2f(x) + 3f\left(\frac{1}{x}\right) = 3x + 2$

put x = 3, $2f(3) + 3f\left(\frac{1}{3}\right) = 11$ ……(1)

put x = $\frac{1}{3}$, $2f\left(\frac{1}{3}\right) + 3f(3) = 3$ …..(2)

(1) 2 – (2) $\times$3
$\Rightarrow$ – 5f(3) = 13
= $f(3) = \frac{-13}{5}$ Choice (A)

33. Given h(x, y, p, q) = x q – y p
Also h(z + 1, z, z + 2, 7) = h(5, 4, 3, 1)
$\Rightarrow$ 7(z + 1) – z(z + 2) = 5 – 12
$7z + 7 - z^2 - 2z = -7$
$\Rightarrow z^2 - 5z - 14 = 0$
(z – 7) (z + 2) = 0 $\Rightarrow$ z = – 2 or 7 Choice (A)

34. Given f(x) = 3 for x rational and f(x) = –3 for x irrational
$f(\sqrt{3}) = -3$, $f(\sqrt{4}) = 3$, $|f(\sqrt{5})| = 3$, $\sqrt{f(3)} = \sqrt{3}$

$\sqrt{|f(6)|} = \sqrt{3}$, $|f\sqrt{7}| = 3$

$\therefore f(\sqrt{3}) + f(\sqrt{4}) + |f(\sqrt{5})| + \sqrt{f(3)} + \sqrt{|f(6)|} + |f(\sqrt{7})|$

$= -3 + 3 + 3 + \sqrt{3} + \sqrt{3} + 3 = 6 + 2\sqrt{3}$

$= 2(3 + \sqrt{3})$ Choice (D)

35. f(4) = 11 – 16 + p = p – 5
g(2) = 6 + 4 + q = q + 10
f(4) $\times$ g(2) = (p – 5) (q + 10) < 0
Case $\boxed{1}$: p – 5 > 0 and q + 10 < 0
p > 5 and q < – 10
But p, q > 0
$\therefore$No solution
Case 2: p – 5 < 0 and q + 10 > 0
$\Rightarrow$ p < 5 and q > – 10
But p, q > 0
$\therefore$ The common range of p and q is (0, 5) Choice (B)

Exercise – 5(b)

Solutions for questions 1 to 45:

1. One-one functions from set A to set B are possible only if n(B) $\geq$ n(A).
Here n(B) < n(A).$\therefore$The number of one-one functions from A to B is zero. Choice (D)

2. In the formation of a subset every element is either included or excluded. For 1, 3, 11 the choice is made. The remaining elements may or may not be present.
The required number of subsets = $2^{n-3} = 2^3 = 8$.
Ans : (8)

3. The number of functions that can be defined from set A to set B is $(n(B))^{n(A)} = 3^4 = 81$. Ans : (81)

4. The number of one-one functions that can be defined from set A with m elements to set B with n elements is ${}^nP_m$. Here, m = 4, n = 7 .

∴ The number of one-one functions is ⇒ ${}^7P_4 = 840$
Choice (D)

5. Given h(x) = h[h(x – 1)] for x > 4
⇒ h(x + 1) = h[h(x)]
h(5) = h(4 + 1) = h[h(4)] = h (4) = 4
h(6) = h(5 + 6) = h[h(5)] = h (4) = 4
h(7) = h(6 + 1) = h[h(6)] = h (4) = 4
We notice that the value of h(x) is 4 for all x > 4
∴ h[796] = 4 Ans : (4)

6. The number of onto functions that can be defined from set A to set B is $2^n - 2$ (∵ n(A) = n, n(B) = 2)
$2^n - 2 = 2046$
⇒ $2^n = (2048) = 2^{11}$ ⇒ n = 11 Choice (B)

7. The number of functions that can be defined from set A to set B = $(n(B))^{n(A)} = 5^6 = 15625$
Of these, the number of onto functions
$5^6 - {}^5C_1 (4)^6 + {}^5C_2 (3)^6 - {}^5C_3 (2)^6 + {}^5C_4 (1)^6 = 1800$
The required number of into functions = 15625 – 1800 = 13825 Choice (D)

8. The number of bijections that can be defined is 6! = 720
Ans : (720)

9. If n(P) = x then the number of proper subsets of P is $2^x - 1$. The number of subsets containing a, c, f but not e is 2^{x-4}
∴ $(2^x - 1) + 2^{x-4} = 33 \Rightarrow 2^{x-4}(17) = 2(17)$
∴x = 5. Ans : (5)

10. (gof)[x] = g[f(x)] = g(2 x – 1) = 3(2 x – 1) + 4 = 6 x + 1
Let y = 6 x + 1 ⇒ x = $\frac{y-1}{6}$
∴ $(gof)^{-1}(x) = \frac{x-1}{6} \Rightarrow (gof)^{-1}(0) = \frac{-1}{6}$ Choice (B)

11. f(x) = $\frac{2x}{\sqrt{x^2-2x-15}}$
f(x) is real ⇒ $x^2 - 2x - 15 > 0$
(x – 5) (x + 3) > 0
⇒ x ∈ R – [–3, 5]
∴ The domain of f(x) is (–∞, –3) ∪ (5, ∞) Choice (C)

12. Given $f(2x + 1) = 4x^2 + 4x + 7 = (2x + 1)^2 + 6$
∴ $f(t) = t^2 + 6$
$f(1 - 2x) = (1 - 2x)^2 + 6 = 4x^2 - 4x + 7$ Choice (C)

13. The function $\frac{1}{\log|x-1|}$ is not defined if |x – 1| = 0 or |x – 1| = 1
i.e., if x = 1 or x = 0, 2
i.e., if x = 0, 1 or 2
The function $\frac{1}{x+1}$ is not defined if x + 1 = 0, i.e. x = –1.
Hence, the domain of f(x) is R – {–1, 0, 1, 2}
Choice (C)

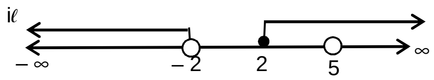
14. Every real number 'x' lies between two integers.
Given, [x] is the greatest integer less than or equal to x.
So, 0 ≤ x – [x] < 1 Choice (C)

15. f(x) = $\frac{2}{1-[x]}$
For the function f(x) to be defined, the denominator should not be zero.
1 – [x] ≠ 0 ⇒ [x] ≠ 1
From the definition of [x]. [x] = 1, for x ∈ [1, 2)
Therefore, the domain of f(x) is (– ∞, 1) ∪ [2, ∞)
Choice (A)

16. f(x) = $\sqrt{x+5} + \frac{1}{\sqrt{|x|-3}}$.
For this function to be defined,
x + 5 ≥ 0 and |x| – 3 > 0
x + 5 ≥ 0 ⇒ x ≥ –5 …….. (i)
|x| – 3 > 0 |x| > 3
x ∉ [–3, 3] …….(ii)
x ∈ (– ∞, –3) ∪ (3, ∞).
From (1) and (2) x ∈ [–5, – 3] ∪ (3, ∞)
∴The domain of the function is
x ∈ [–5, –3) ∪ (3, ∞) Choice (C)

17. We want the range of the function y = 2 + |sinx| |sinx| will take the values from 0 to 1.
Therefore, the range of the function is [2, 3]
Choice (B)

18. f(x) = $\sqrt{x^2-4}$ + log|x – 5|
(i) $\sqrt{x^2-4}$ is real if $x^2 - 4 \geq 0$
⇒ (x – 2) (x + 2) ≥ 0 ⇒ x ∈ (–∞, –2] ∪ [2, ∞)
(ii) log|x – 5| is defined for all x ≠ 5
∴ x ∈ R – {5}
∴ Domain of f(x) is (i) ∩ (ii)

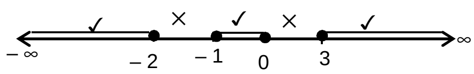


∴ Domain of f(x) is (–∞, –2] ∪ [2, 5) ∪ (5, ∞)
Choice (D)

19. Range of a cos x + b sin x + c is
$\left[c - \sqrt{a^2+b^2}, c + \sqrt{a^2+b^2}\right]$
Here a = 8, b = 15 and c = 20
∴ Range of f is $\left[20 - \sqrt{8^2+15^2}, 20 + \sqrt{8^2+15^2}\right]$ = [3, 37]
Choice (C)

20. f(x) = $\frac{1}{4-\sin 2x}$
We know – 1 ≤ sin 2 x ≤ 1
– 1 ≤ – sin 2 x ≤ 1
⇔ 3 ≤ 4 – sin 2 x ≤ 5
⇒ $\frac{1}{5} \leq \frac{1}{4-\sin 2x} \leq \frac{1}{3}$
∴ Range of f(x) is $\left[\frac{1}{5}, \frac{1}{3}\right]$ Choice (D)

21. f(x) = $\sqrt{x(x+1)(x+2)(x-3)}$
f(x) is real if x(x+ 1) (x + 2) (x – 3) ≥ 0
∴ Critical values of x are 0, –1, –2, 3



When x = 1, the inequation is not satisfied
⇒ The interval (0, 3) is not a solution
By observation, we notice that the interval with '✓' marks represents the solution regions
⇒ Domain of f is (–∞, –2] ∪ [–1, 0] ∪ [3, ∞) Choice (C)

22. Given $f(x) = K^x$
Option A: $f(x) f(-x) = K^x K^{-x} = K^0 = 1$ (true)
Option B: $f(x + 3) f(x - 3) = K^{x+3} K^{x-3} = K^{2x} = (K^x)^2 = (f(x))^2$ (true)
Option C: f(x + 3) – 3f(x + 2) + 3 f(x + 2) f(–1) – f(x) f(6)

$= K^{x+3} - 3\,K^{x+2} + 3\,K^{x+2}K^{-1} - K^x K^0 = K^x (K^3 - 3\,K^2 + 3\,K - 1)$
$= K^x(k-1)^3 = (K-1)^3 f(x)$ – (true) Choice (D)

23. Given $f_1(x) = f_2(x-1)$
$\Rightarrow 5\,x^2 + 7 = 3(x-1)^2 - 3(x-1) + 12$
$\Rightarrow 5\,x^2 + 7 = 3\,x^2 - 6\,x + 3 - 3\,x + 3 + 12$
$\Rightarrow 2\,x^2 + 9\,x - 11 = 0 \Leftrightarrow (2\,x + 11)(x-1) = 0$
$\Rightarrow x = \dfrac{-11}{2}$, $x = 1$ [$x > 0 \Rightarrow x = 1$] Ans : (1)

24. Given $g(x) = \min(1 - 3\,x, 2\,x + 5)$
$1 - 3\,x = 2\,x + 5, \Leftrightarrow -5\,x = 4 \Leftrightarrow x = \dfrac{-4}{5}$
$g(x) = \min(1 - 3x, 2\,x + 5) = \dfrac{17}{5}$
For $x < \dfrac{-4}{5}$, we see that $2\,x + 5 < 1 - 3\,x$ so $g(x) = 2\,x + 5$
and $g(x) < \dfrac{17}{5}$
For $x > \dfrac{-4}{5}$, $g(x) = 1 - 3x$ and $g(x) < \dfrac{17}{5}$
$\therefore$ Maximum value of $g(x) = \dfrac{17}{5}$ Choice (C)

25. Given $f(a, b, c, d) = a\,b - c\,d$
Also $f(x-1, 2\,x, x+3, 2) = f(3, 9, 5, -3)$
$\therefore 2\,x(x-1) - 2(x+3) = 3(9) - (5)(-3)$
$\Leftrightarrow 2\,x^2 - 2\,x - 2\,x - 6 = 42 \Leftrightarrow x^2 - 2\,x - 3 - 21 = 0$
$\Leftrightarrow x^2 - 2\,x - 24 = 0 \Leftrightarrow (x-6)(x+4) = 0$
$\therefore$ The positive integral value that satisfies the given function is $x = 6$
$\therefore$ The number of positive integral values of x is 1.
Ans : (1)

26. Given $h(x) = 2\,x - 3$ and $g(x) = x^2 - 3$
$(hog)(2) = h[g(2)] = h(4-3) = 2(1) - 3 = -1$
$(goh)(3) = g[h(3)] = g(6-3) = g(3) = 9 - 3 = 6$
$\therefore \dfrac{hog(2)}{goh(3)} = \dfrac{-1}{6}$ Choice (B)

27. $f(x) = 2\,x^2 - 1$, $g(x) = 2\,x + 3$
$(fog)(x) = f[g(x)] = f(2\,x+3) = 2(2\,x+3)^2 - 1 = 8\,x^2 + 24\,x + 17$
$(gof)(x) = g[f(x)] = g(2\,x^2 - 1) = 2(2\,x^2 - 1) + 3 = 4\,x^2 + 1$
$(fog)(x) - (gof)(x) = 8\,x^2 + 24\,x + 17 - 4\,x^2 - 1$
$= 4\,x^2 + 24\,x + 16$. This is a quadratic function. Choice (B)

28. Consider $f\left(\dfrac{1}{150}\right) + f\left(\dfrac{2}{150}\right) + f\left(\dfrac{3}{150}\right) + \ldots + f\left(\dfrac{297}{150}\right) + f\left(\dfrac{298}{150}\right)$
$+ f\left(\dfrac{299}{150}\right)$
$\left[f\left(\dfrac{1}{150}\right) + f\left(\dfrac{299}{150}\right)\right] + \left[f\left(\dfrac{2}{150}\right) + f\left(\dfrac{298}{150}\right)\right] + \left[f\left(\dfrac{3}{150}\right) + f\left(\dfrac{297}{150}\right)\right]$
+.....149 pairs of terms + $f\left(\dfrac{150}{150}\right)$
The pairs are in the form of $f(x) + f(2-x)$
$\therefore$ The value of each of the above pairs of terms is 6. The required value is149 (6)+ f(1)
Given $f(x) + f(2-x) = 6$
setting $x = 1$, $f(1) + f(1) = 6$
$\Rightarrow f(1) = 3$
$\therefore$ The required value is 894 + 3 = 897 Ans : (897)

29. $f(x) = \dfrac{x}{\sqrt{1+x^2}}$
$fof(x) = f[f(x)]$

$= f\left[\dfrac{x}{\sqrt{1+x^2}}\right]$

$= \dfrac{\dfrac{x}{\sqrt{1+x^2}}}{\sqrt{1+\dfrac{x^2}{1+x^2}}}$

$fof(x) = \dfrac{x}{\sqrt{1+2x^2}}$

Similarly $fofof(x) = \dfrac{x}{\sqrt{1+3x^2}}$

$\therefore$ $fofofofof(x) = \dfrac{x}{\sqrt{1+5x^2}}$

$\therefore$ $fofofofof(3) = \dfrac{3}{\sqrt{1+5.(9)}} = \dfrac{3}{\sqrt{46}}$ Choice (C)

30. Given $f(x+2) + f(x) = 0$ ……(1)
Put x = x + 2
$f(x+2) + f(x+4) = 0$
$\Rightarrow f(x+4) = -f(x+2)$
$f(x+4) = f(x)$ ………….(1)
$\therefore$ f(1) = f(5) = f(9) = f(13)…….
f(2) = f(6) = f(10) = f(14) …..
f(3) = f(7) = f(11) = f(15) ……
f(4) = f(8) = f(12) = f(16) ……
f(4) = -f (2) = -8
$\therefore$ f(15) + f(16) = f(3) + f(4)
= –13 Ans : (–13)

31. Given S = {x|x is a composite number}
S = {4, 6, 8, 9, 10, …..}
A_n = {x|x is a factor of n}
A_4 = {x|x is a factor of n}
= {1, 2, 4}
A_6 = {1, 2, 3, 6}
A_8 = {1, 2, 4, 8}
A_9 = {1, 3, 9}
$\bigcap_{s\in S} A_s = A_4 \cap A_6 \cap A_8 \cap A_9 \ldots = \{1\}$
Choice (B)

32. Let n(A) = m and n (B) = n. number of functions that can be defined from A to B is n^m and the number of one – one functions that can be defined from A to B is ${}^nP_m$
$\therefore$ Given m = 4 ,n = 7
The number of functions which are not one – one = total number of functions – total number of one – one functions.
$= 7^4 - {}^7P_4 = 2401 - 840 = 1561$ Ans : (1561)

33. $f(x) + 3f\left(\dfrac{1}{x}\right) = 3x + 1$ ------- (1)
Put $x = \dfrac{1}{x}$ in (1) $f\left(\dfrac{1}{x}\right) + 3f(x) = \dfrac{3}{x} + 1$ ------ (2)
$(2) \times 3 - (1)$ $f(x) = \dfrac{1}{8}\left(\dfrac{9}{x} + 2 - 3x\right)$
$\therefore f(3) = \dfrac{1}{8}\left[\dfrac{9}{3} + 2 - 3.3\right] = \dfrac{-1}{2}$ Choice (A)

34. From option (A) : $f(x) = 9^{x(x-1)}$ let $y = 9^{x(x-1)}$
Clearly it cannot be an inverse of itself
Option (B) $g(x) = a^{\log x^2}$ let $y = a^{\log x^2}$ or log y = 2 log x loga.

$\text{Log } x = \dfrac{\log y}{2\log a}$ $\therefore$ it is also not inverse of itself.

Option (C): $I(x) = \dfrac{a-x}{a+x}$ let $y = \dfrac{a-x}{a+x}$

$ay + yx = a - x$ or $x(y+1) = a(1-y)$

$x = \dfrac{a(1-y)}{1+y}$

$\therefore$ I (x) is also not inverse of itself Choice (D)

35. $f(x) = px + q$
$f(f(x)) = p(px + q) + q = p^2x + pq + q$
$f[f(f(x))] = p^2(px +) + pq + q$
$= p^3x + p^2q + pq + q$
given $f(f(f(x))) = 8x - 56 \Rightarrow p^3x + p^2q + pq + q$
$= 8x - 56 \Rightarrow p^3 = 8$
$\Rightarrow p = 2$
$p^2q + pq + q = -56$
$4q + 2q + q = -56$
$q = -8$
$\therefore f(x) = 2x - 8$

$f^{-1}(x) = \dfrac{x+8}{2}$

$\Rightarrow f^{-1}(-6) = \dfrac{-6+8}{2} = 1$ Choice (B)

36. $f(x) = x^{x-3}$
$f(2) = 2^{-1}$
$2f(2) = 2 \times 2^{-1} = 1$
$f(2f(2)) = f(1) = 1$
$2f[2f(2)] = 2 \times 1 = 2$
$f[2f(2f(2))] = 2^{2-3} = 2^{-1}$
$2f[2f(2f(2))] = 2 \times 2^{-1} = 1$ Ans : (1)

37. Given $f(x) = \dfrac{x-3}{|3x-9|}$, $x \neq 3$; $f(x) = \dfrac{x-3}{3|x-3|}$

When $x > 3$ then $|x-3| = x-3$

Then $f(x) = \dfrac{x-3}{3(x-3)} = \dfrac{1}{3}$

When $x < 3$ then $|x-3| = -(x-3)$

Then $f(x) = \dfrac{x-3}{-3(x-3)} = \dfrac{-1}{3}$

$\therefore$ Range of f (x) is $\left\{\dfrac{-1}{3}, \dfrac{1}{3}\right\}$ Choice (D)

38. $h(x) = \max(x + 2, x - 3) = x + 2$
$g(x) = \min(3x - 1, 3x + 5) = 3x - 1$
$f(x) = g(x) - h(x) = 3x - 1 - x - 2 = 2x - 3$
$f(x) \geq 9 \Rightarrow 2x - 3 \geq 9 \Rightarrow x \geq 6 \Rightarrow x \in [6, \infty)$
Choice (D)

39. As $f(x) = |x - 2| + |x + 3| + |x - 1|$, (the sum of the distances of the point x from 1, 2 and –3 on the number line) when x increases f(x) also increases. Hence f(x) has a minimum but not a maximum value. The minimum value is obtained when x = 1. This value is f(1) = 1+ 4 = 5
$\therefore$ The range of the function is $[5, \infty)$ Choice (C)

40. $f^2 = f(x) f(x)$ i.e., $f^2(2) = f(2) f(2) = 3(3) = 9$
$g(2) = 4$, $f(2) = 3$

$\therefore \dfrac{f^2(2)+g(2)}{f(2)-g(2)} = \dfrac{9+4}{3-4} = -13$ Ans : (–13)

41. Given $f(x) = \min(x - 2, x + 3) = x - 2$
and $g(x) = \max(x - 4, x - 5) = x - 4$
Also, $h(x) = f(x) + g(x) = x - 2 + x - 4 = 2x - 6$
$h(x) \leq 6 \Rightarrow 2x - 6 \leq 6 \Rightarrow x \leq 6$ Choice (D)

42. $f(x) = \min(x - 4, x + 2) = x - 4$
$g(x) = \max(x - 2, x + 3) = x + 3$
$h(x) = f(x) + g(x) = x - 4 + x + 3 = 2x - 1$
$h(x) \leq 5$
$\Rightarrow 2x - 1 \leq 5$
$\Rightarrow x \leq 3$
$\Rightarrow x \in (-\infty, 3]$ Choice (A)

43. Given $f(xy) = f(x) f(y)$ which is true only when
$f(x) = x^k$
so $f(3) = 27$
$\Rightarrow 3^k = 27 = 3^3 \Rightarrow k = 3$
$\therefore f(x) = x^3$

$\sum_{n=1}^{12} f(x) = 1^3 + 2^3 + 3^3 + \ldots + 12^3 = \left(\dfrac{12(12+1)}{2}\right)^2$

$= 6084$ Ans : (6084)

44. $f(2x) + f(3x) + f(x + 2) + f(3 - x) = x$
Let x = 0 $\therefore$ $f(0) + f(0) + f(2) + f(3) = 0$ ……(1)
Let x = 1 $\therefore$ $f(2) + f(3) + f(3) + f(2) = 1$

$\Rightarrow f(2) + f(3) = \dfrac{1}{2}$ -------- (2)

(1), (2) $\Rightarrow 2f(0) + \dfrac{1}{2} = 0 \Rightarrow f(0) = -\dfrac{1}{4}$ Choice (D)

45. $f(x) = 125x^3 + \frac{1}{x^3}$ also β is the root of $5x + \frac{1}{x} = 3 \Rightarrow 5\beta + \frac{1}{\beta} = 3$
cubing on both sides

$\left(5\beta + \frac{1}{\beta}\right)^3 = 3^3$

$125\beta^3 + \frac{1}{\beta^3} + 3.5\beta \,.\, \frac{1}{\beta}\left(5\beta + \frac{1}{\beta}\right) = 27$

$125\beta^3 + \frac{1}{\beta^3} = 27 - 45 = -18$

$\therefore f(\beta) = 125\beta^3 + \frac{1}{\beta^3} = -18$ Choice (C)

Chapter – 6
(Graphs)

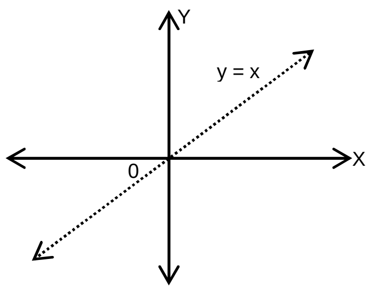
Concept Review Questions

Solutions for questions 1 to 20:

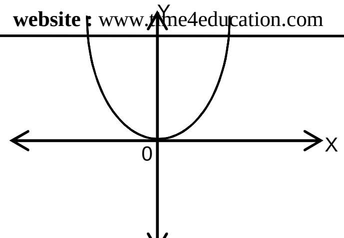
1. Put y = 0 in the equation $x + y - 1 = 0$.
Then, $x + 0 - 1 = 0$
$\Rightarrow x = 1$
$\therefore$ The curve $x + y - 1 = 0$ meets the x-axis at the point (1, 0).
Choice (B)

2. Put x = 0 in the equation $x^2 + y^2 = 1$.
Then, $0 + y^2 = 1 \Rightarrow y^2 = 1$
$\Rightarrow y = \pm 1$
$\therefore$ The graph of the equation $x^2 + y^2 = 1$ meets the y-axis at the points (0, –1) and (0, 1). Choice (A)

3. The graph of the function y = x is:



Hence, the graph of y = x lies in the I and III quadrants.
Choice (C)



4. The graph of $y = x^2$ is :

Hence, the graph of $y = x^2$ belongs to I and II quadrants.
Choice (A)

5. The given equations are:
$y = 2x + 1$ ------ (1)
and $y = x^2 - 2$ ------ (2)
Solving (1) and (2), we get
$x^2 - 2 = 2x + 1$
$\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x - 3)(x + 1) = 0$
$\Rightarrow x = 3$ or $x = -1$
$\therefore y = 7$ or $y = -1$
Hence, the required points of intersection are (3, 7) and (–1, –1) Choice (B)

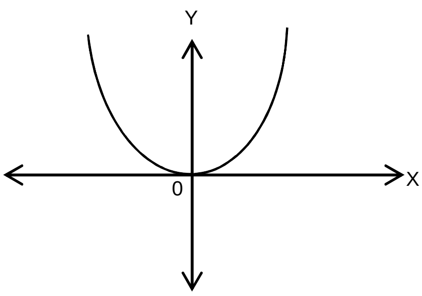
6. Given equations are:
$y = x - 1$ -------- (1)
and $x^2 = 2y$ ------ (2)
Solving (1) and (2) ; we get
$x^2 = 2(x - 1)$
$\Rightarrow x^2 - 2x + 2 = 0$
$\Rightarrow (x - 1)^2 + 1 = 0$
For no real value of x, is $(x - 1)^2 + 1 = 0$
Hence, the curves $y = x - 1$ and $x^2 = 2y$ do not intersect.
Ans : (0)

7. $ac = 0 \Rightarrow a = 0$ or $c = 0$
case (i) : If $a = 0$ and $c \neq 0$, then $ax + by + c = 0$
$\Rightarrow by + c = 0$ which represents a horizontal line.
case (ii) : If $a \neq 0$ and $c = 0$, then $ax + by + c = 0$
$\Rightarrow ax + by = 0$
which represents a line passing through the origin
case (iii) : If $a = 0$ and $c = 0$, then $ax + by + c = 0$
$\Rightarrow by = 0$
$\Rightarrow y = 0$ ($\because b \neq 0$), which represents the x-axis.
Hence, $ax + by + c = 0$, $ac = 0$ represents either a horizontal line or an inclined line. Choice (D)

8. Given, $a^2 + b = 0$ and $a \neq 0$
$\Rightarrow a \neq 0$ and $b \neq 0$
Hence, the equation $ax + by + c = 0$ represents an inclined line. (i.e., a line that in not parallel to either axis)
Choice (C)

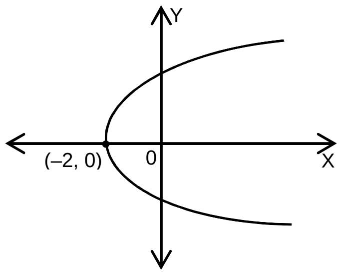
9. The point (1, 0) alone satisfy the equation
$x^2 + y^2 - 2x + 2y + 1 = 0$ Choice (C)

10. Graph of the curve $x^2 = y$ is:



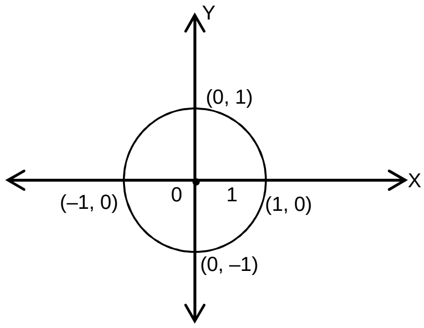
Hence, the curve $x^2 = y$ is symmetric about the y-axis.
Choice (B)

11. Graph of the curve $x = y^2 - 2$ is:



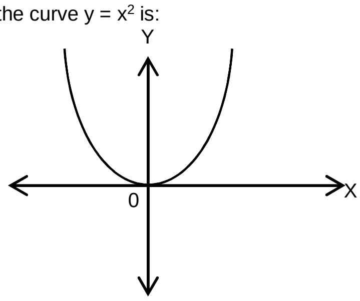
Hence, the curve $x = y^2 - 2$ is symmetric about the x-axis.
Choice (A)

12. Graph of the curve $x^2 + y^2 = 1$ is:



Hence, a line parallel to the x-axis meets the graph at exactly either one or two points. Choice (D)

13. Graph of the curve $y = x^2$ is:



Hence, a line parallel to y-axis meets the graph at exactly one point. Choice (A)

14. Choice (C) is the graph of $x \geq -3$. Choice (C)

15. Given $y < x + 2$ -----(1)
Consider the graph $y = x + 2$

x	0	–2	1	–1
y	2	0	3	1

Plotting these points we get the line shown in the graph, As (0, 0) Satisfies the inequality (1), (1) represents the shaded region in the graph. Choice (B)

16. The equation $2x + 3y = 0$ is satisfied by (0, 0). This represents a line that passes through the origin.
Choice (C)

17. The graph of $y = mx + c$ meets the y – axis at (0, c)
Choice (B)

18. The given equation is in the form of x = k which represents a line parallel to the y – axis.

Choice (B)

19. y = –|x| Choice (A)

20. The graph represents the equation y = |x – 1|

Choice (B)

Exercise – 6(a)

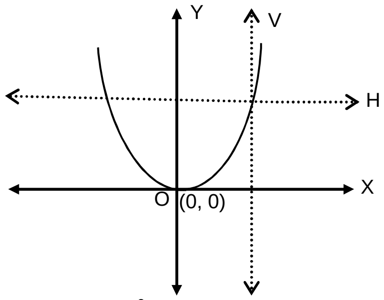
Solutions for questions 1 to 25:

1. Both options (2) and (3) satisfy the point (2, 0).
But $\log_e(x - 1)$ is not defined for x < 1.

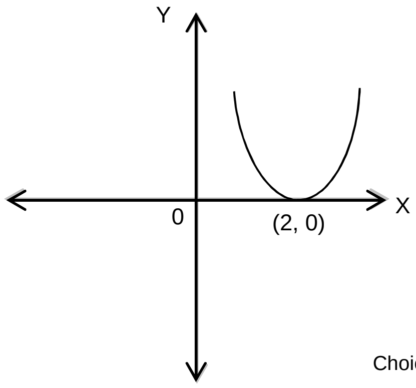
So, y = $\log_{0.5}\left(\frac{x}{2}\right)$ is the required relation.

Choice (C)

2. The graph of y = x^2 is a parabola as shown below



The graph of y = $(x - 2)^2$ after shifting this 2 units horizontally towards right i.e, as shown below.



Choice (A)

3. The graph represents x = – (|y| + 2) Choice (C)

Solutions for questions 4 to 7:

4. From the given choices, the relation y = | log (–x) |, x < 0 best describes the given graph. Choice (D)

5. The shaded region lies between the lines x = ±1 and y = ±1.
Hence, –1 ≤ x ≤ 1 and –1 ≤ y ≤ 1 is the required relation.

Choice (D)

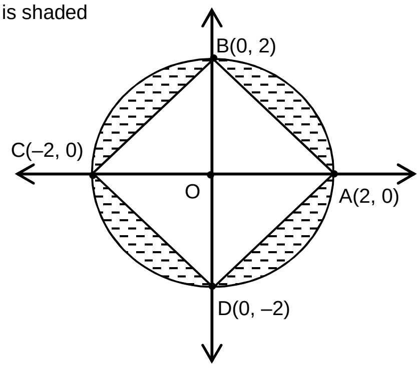
6. Both options (1) and (2) satisfy the points (–1, 1), (0, 0) and (1, –1)
The point (2, –1) lies on the given graph and Choice (B) only passes through this point.

Hence, y = $\frac{|x-1| - |x+1|}{2}$ is the required relation.

Choice (B)

7. The relation |x| + |y| = 2 represents four lines x + y = 2, – x + y = 2, x – y = 2 and – x – y = 2
These four lines form a square with vertices (2, 0), (0, 2), (–2, 0) and (0, –2).
Hence, the shaded region is described by the relation | x | + | y | ≤ 2. Choice (D)

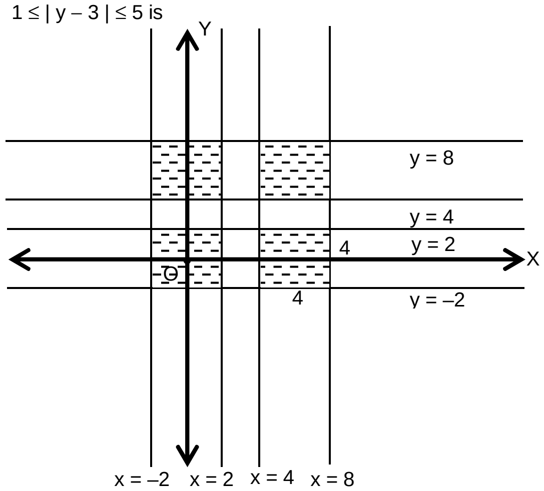
8. The relation | x | + | y | = 2 represents a rhombus and the relation $x^2 + y^2 = 4$ represents a circle.
The region described by the relations | x | + | y | ≥ 2 and $x^2 + y^2 \le 4$ is shaded



The shaded region is the required region.
∴ Required Area = Area of the circle – Area of rhombus ABCD = $\pi(2)^2 - \frac{1}{2}\,4 \times 4 = 4\pi - 8 = 4(\pi - 2)$ sq units.

Choice (B)

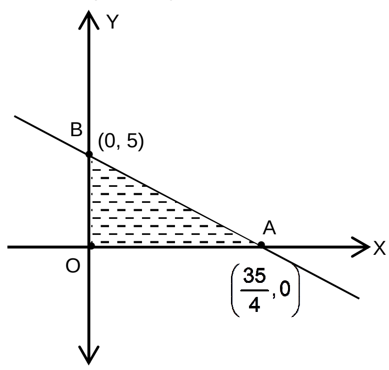
9. | x – 3 | = 1 ⇒ x = 2, 4
| x – 3 | = 5 ⇒ x = –2, 8
| y – 3 | = 1 ⇒ y = 2, 4
| y – 3 | = 5 ⇒ y = –2, 8
∴ The region described by the relations 1 ≤ | x – 3 | ≤ 5 and 1 ≤ | y – 3 | ≤ 5 is



The shaded region is the required region. The region consists of 4 squares each of side 4 units.
Hence, the required area = 4(4 × 4) = 64 sq units.

Ans : (64)

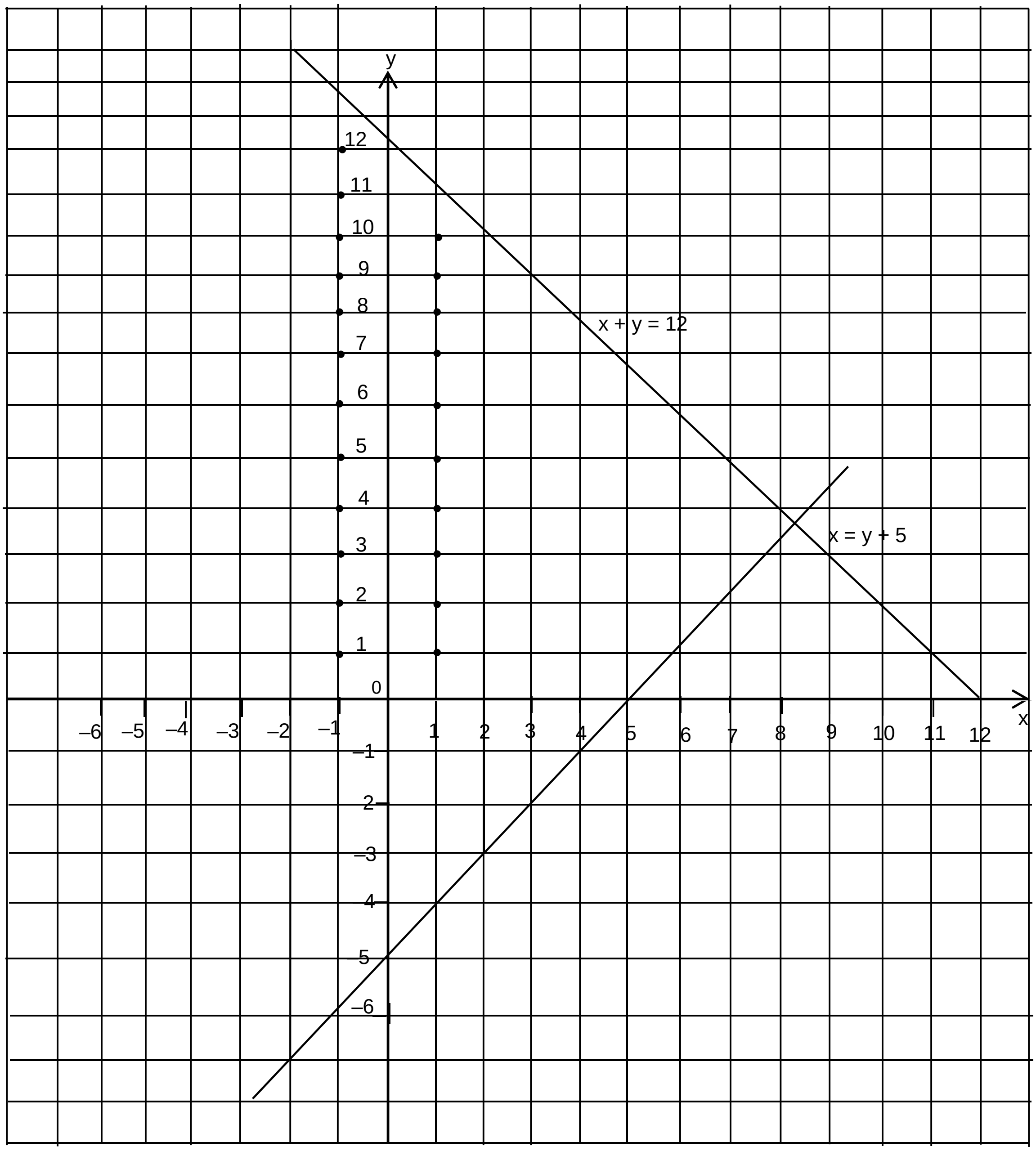
10. The relations x ≥ 0, y ≥ 0 and 4x + 7y ≤ 35
Describe the required region OAB.



The possible values of y satisfying the relation $4x + 7y \le 35$ are:
y = 1, 2, 3 and 4 ($\because$ for $y \ge 5$, $7y \ge 35$)
When y = 1, $4x \le 28$
$\Rightarrow x \le 7$
$\Rightarrow x = 1, 2, 3, \ldots\ldots 7$
When y = 2, $4x \le 21$
$\Rightarrow x \le \frac{21}{4}$
$\Rightarrow x = 1, 2, 3, 4, 5$
When y = 3, $4x \le 14$
$\Rightarrow x \le \frac{14}{4}$
$\Rightarrow x = 1, 2, 3$
When y = 4, $4x \le 7$
$\Rightarrow x \le \frac{7}{4}$
$\Rightarrow x = 1$
Hence, the required number of points is 7 + 5 + 3 + 1 = 16.

Ans : (16)

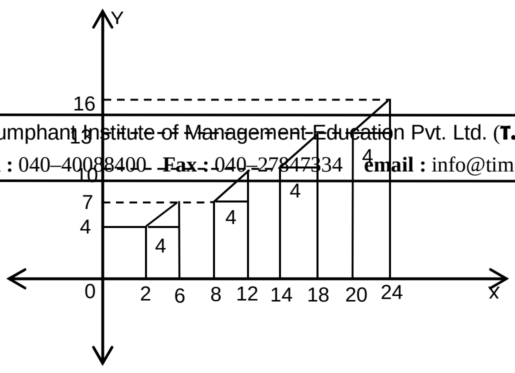
11. The given lines are x = +2, x = – 2, x + y = 12 and x – y = 5.



We see that we have a total of 48 points, lying inside the region bounded by the given lines, which have integral coordinates.

Ans: (48)

12.



We know that, the area of trapezium = $\frac{1}{2}$ × (sum of the lengths of parallel sides) × distance between the parallel sides.

∴ The required area = $\frac{1}{2} \times (4 + 7) \times 4 + \frac{1}{2} \times (7 + 10) \times 4 + \frac{1}{2} \times (10 + 13) \times 4 + \frac{1}{2} \times (13 + 16) \times 4$

= 22 + 34 + 46 + 58 = 160 sq units. Choice (B)

13. g(x) is obtained from f(x) by shifting the graph horizontally towards right 3 units and translate the graph vertically (up) by 2 units.
∴ g(x) = f(x – 3) + 2. Choice (D)

Solutions for questions 14 to 18:

14. Clearly, we can observe that f(x) = –g(–x) Choice (A)

15. g(x) can be obtained by reflecting f(x) only in x-axis OR by reflecting f(x), first in y-axis and then in x-axis. Choice (D)

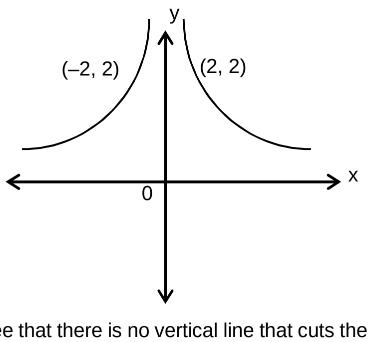
16. g(x) can be obtained from f(x) in the following ways.
(i) Reflect the graph f(x) in x – axis then in y – axis.
i.e., f(x) = –g(–x)
(ii) Reflect the graph f(x) in y – axis.
f(x) = g(–x). Choice (D)

17. g(x) is a straight line passing through the origin and lying in the IInd and IVth quadrants.
∴ g(x) ≡ x + y = 0.
The graph f(x) is always positive.
∴ f(x) = |g(x)|. Choice (C)

18. The graph g(x) can be obtained only by reflecting the graph f(x) in the y-axis.
f(x) = g(–x). Choice (A)

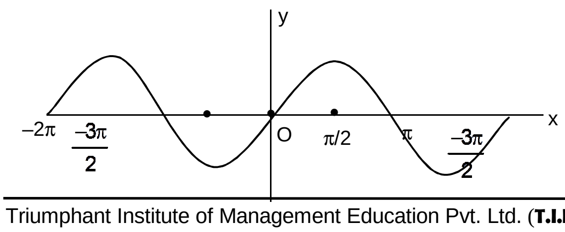
Solutions for questions 19 to 23:

19. The graph of lxly = 2 is as follows.



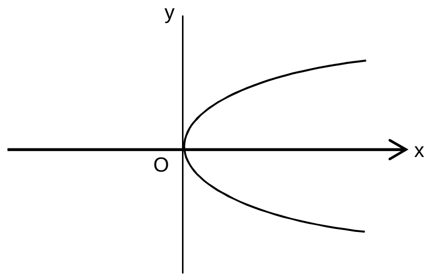
We see that there is no vertical line that cuts the graph more than once and there exists horizontal lines that cut the graph more than once. Choice (A)

20. The graph of sinx in the interval [0, 2π] is as follows.



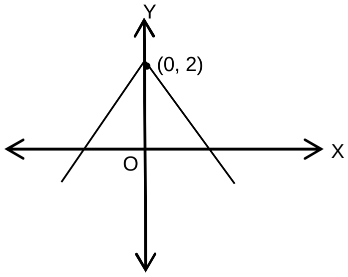
There exists horizontal lines that cut the graph more than once, but no vertical lines. Choice (A)

21. The graph y^2 = 16x is as follows.



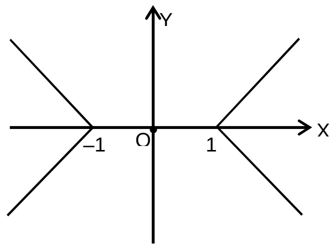
There is a vertical line that cuts the graph more than once but no horizontal line. Choice (B)

22. The graph of y = 2 – | x | is:



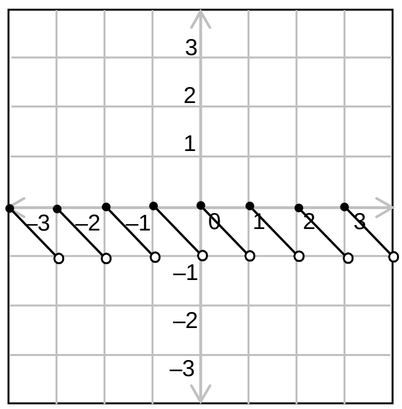
There exists horizontal lines intersecting the graph more than once but no vertical lines. Choice (A)

23. The graph of |x| + |y| = 1 is

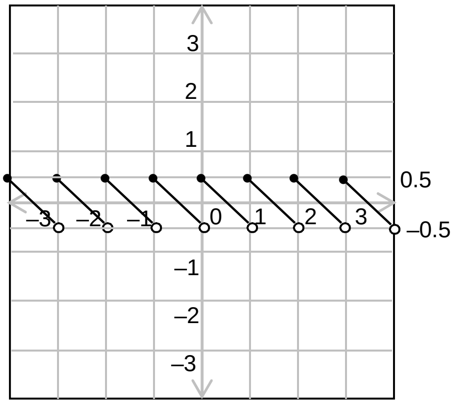


There exists horizontal and vertical lines intersecting the graph more than once. Choice (C)

24. The graph of y = [x] – x is



By adding $\frac{1}{2}$ to y we get



That is

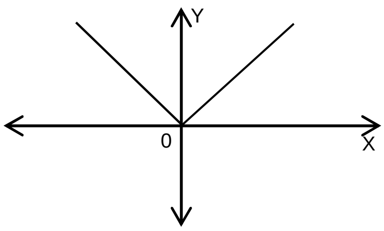
$y = [x] - x + \frac{1}{2}$ Choice (C)

25. From graph we can observe that
For $0 \le x < 1$, the range of y is $0 \le y < 1$
For $1 \le x < 2$, the range of y is $-1 \le y < 0$
For $-1 \le x < 0$, the range of y is $1 \le y < 2$
The equation of the graph is $[x] + [y] = 0$ Choice (D)

Exercise – 6(b)

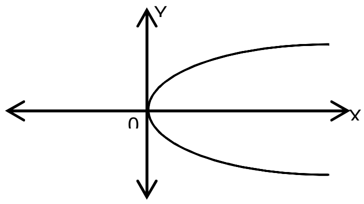
Solutions for questions 1 to 3:

1. Graph of the equation $y = |x|$ is:



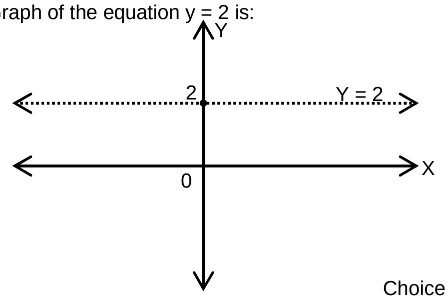
Choice (C)

2. The graph of the equation $y^2 = x$ is:



Choice (A)

3. Graph of the equation y = 2 is:



Choice (B)

Solutions for questions 4 to 7:

4. Given ab > 0 and bc = 0
ab > 0
$\Rightarrow$ a > 0 and b > 0 or a < 0 and b < 0 ------ (1)
also bc = 0
$\Rightarrow$ b = 0 or c = 0 or both may be zero
but from (1) $b \neq 0$
$\Rightarrow$ c = 0.
$\therefore$ i.e. $ab \neq 0$ and c = 0
$\Rightarrow$ the given equation ax + by + c = 0 when $ab \neq 0$ and c = 0 represents an inclined line passing through origin.
Choice (C)

5. Given $\frac{c^2}{ab} = 24$

Clearly $c \neq 0$ and ab > 0 $\left(\because \frac{c^2}{ab} = 24\right)$.

i.e. $a \neq 0$, $b \neq 0$ and $c \neq 0$
$\Rightarrow$ ax + by + c = 0 always represents an inclined line not passing through origin. Choice (B)

6. $|a| + |b| = 2$,

When a = 0 then b = ± 2; the line represents a horizontal line.
When a = ± 2 then b = ± 0, the line represents a vertical line
When a = ± 1 then b = ± 1, the line represents an inclined line. Choice (D)

7. $bc \neq 0$; a = 0;
$\therefore$ The line represents a horizontal line. Choice (A)

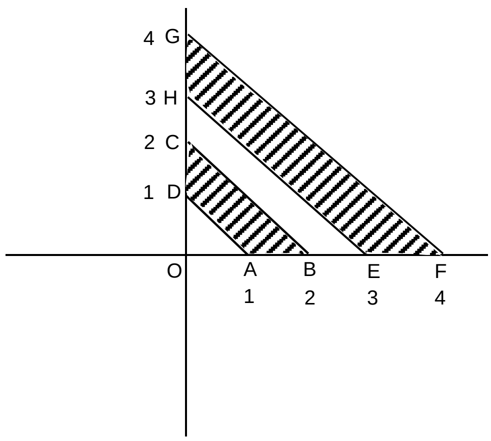
Solutions for questions 8 to 10:

8. The graph f(x) is shifted towards right by 3 units and the graph g(x) is obtained
$\therefore$ g(x) = f(x – 3) Choice (A)

9. The given points satisfy the relation $y = \frac{|x-3| + |x-5|}{2}$
Therefore, the function which represents the figure given is
$y = \frac{|x-3|+|x-5|}{2}$ Choice (C)

10. Substituting the points in the given graph in all the options. We notice only choice C satisfies. Choice (C)

11.



Area of the shaded region = Area of ABCD + Area of EFGH.
Area of ABCD = Area of Δ OBC – Area of Δ OAD.

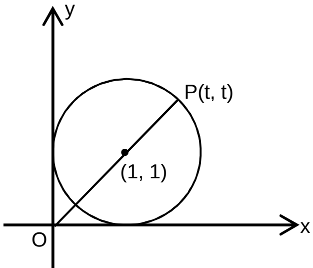
$= \frac{(2\times2)}{2} - \frac{(1\times1)}{2} = \frac{3}{2}$

Similarly, Area of EFGH = $8 - \frac{9}{2} = \frac{7}{2}$

Total area of shaded region = 5 sq. units. Ans: (5)

12. g(x) is obtained from f(x) by translating the graph horizontally left by 4 units and vertically (down) by 3 units.
Choice (B)

13. The graph represents a circle centred at (1, 1) with radius as 1 unit. The farthest point on the circle should lie on the line y = x.
Let P(t, t) be that point.



$(t-1)^2 + (t-1)^2 = 1$ (Using distance formula).

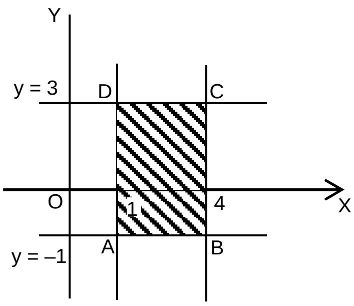
$(t-1)^2 = \frac{1}{2}$

$t = \left(1+\frac{1}{\sqrt{2}}\right)$ or $\left(1-\frac{1}{\sqrt{2}}\right)$

As we want the farthest point on the circle, it is $\left(1+\frac{1}{\sqrt{2}}, 1+\frac{1}{\sqrt{2}}\right)$. Choice (A)

14. $1 \le x \le 4$ would represent an infinite vertical band in the first quadrant and similarly $0 \le |y-1| \le 2$.
or $-2 \le y - 1 \le 2$
$-1 \le y \le 3$ represents an infinite horizontal band.

The intersection is shown as the shaded region in the figure below.



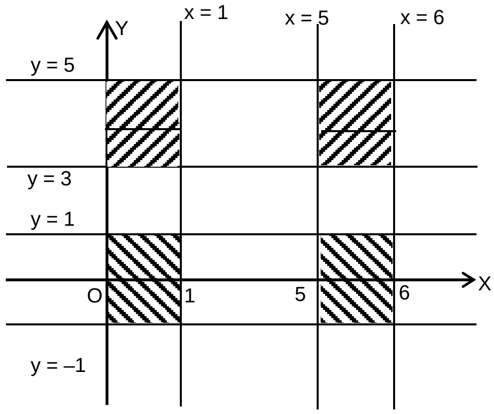
The area of the shaded region is = AB × BC = 3 × 4 = 12 units.
Ans: (12)

15. Firstly, we will simplify $2 \le |x-3| \le 3$

Case (i) (x > 3)
$2 \le x - 3 \le 3$
$5 \le x \le 6$

Case (ii) (x < 3)
$2 \le 3 - x \le 3$
$-2 \ge x - 3 \ge -3$
$\Rightarrow 1 \ge x \ge 0$
$\Rightarrow 0 \le x \le 1$

Similarly, we simplify $1 \le |y-2| \le 3$
Case (i) (y > 2)
$1 \le y - 2 \le 3$
$3 \le y \le 5$
Case (ii) (y < 2)
$1 \le 2 - y \le 3$
$-1 \ge y - 2 \ge -3$
$1 \ge y \ge -1$
$\Rightarrow -1 \le y \le 1$.
Now observing the graph obtained



Area enclosed, is equal to the area of the shaded regions i.e 2 + 2 + 2 + 2 = 8 Choice (A)

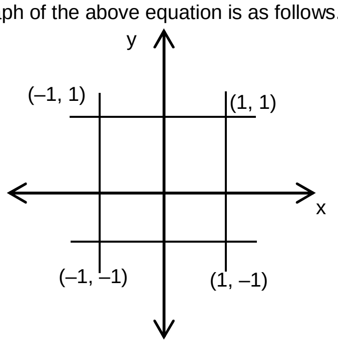
16. $4x + 20y \le 100$
$\Leftrightarrow x + 5y \le 25$
The possible values of y, the corresponding highest value of x and the total number of values of x are tabulated below.

Highest value of x	Possible value of y	Total number of values of x
25 20	0 1	26 21
15 10	2 3	16 11
5 0	4 5	6 1
		81

We can see that in all there are 81 non-negative solutions, i.e. there are 81 points of the required kind. Ans : (81)

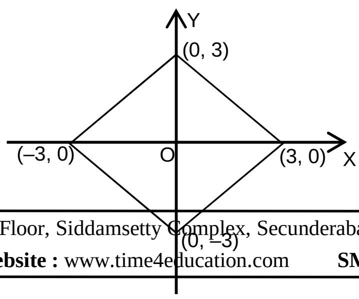
Solutions for questions 17 to 21:

17. $|x+y| + |x-y| + 2 = 2$
Graph of the above equation is as follows.



We can have both horizontal and vertical lines intersecting the graph more than once. Choice (C)

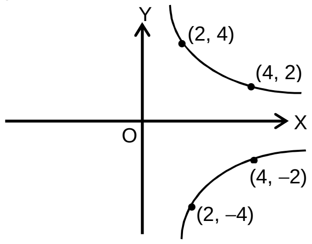
18. $|x| + |y| = 3$
The graph of this equation is as follows.



In this figure also, we can have both vertical and horizontal lines which intersect the graph more than once.
Choice (C)

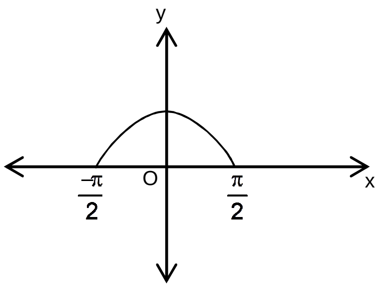
19. $x = \frac{8}{|y|}$, $y \neq 0$

This means x is positive and, neither x nor y is zero.
This graph is as follows



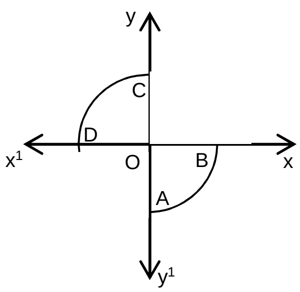
We can have vertical lines intersecting the graph more than once. For example, x = 2, but we have no horizontal lines intersecting the graph at exactly two points. Choice (B)

20. $y = \cos x, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



From the graph, we can see that, there are no vertical lines which intersect the graph more than once but there are many horizontal lines which intersect the graph more than once.
Choice (A)

21.



Points which are 3 units from the origin represent the points on a circle of radius 3 units centred at origin. Since xy < 0 only points in the quadrants II and IV have to be considered.
∴ There are no horizontal and no vertical lines which intersect the graph more than once. Choice (D)

Solutions for question 22:

22. We have,
$1 = (1)^2 - 1 + 1$
$3 = (2)^2 - 2 + 1$
$7 = (3)^2 - 3 + 1$
$13 = (4)^2 - 4 + 1$
$21 = (5)^2 - 5 + 1$ and
$31 = (6)^2 - 6 + 1$

Hence, the relation between x and y is $y = x^2 - x + 1$.
Choice (B)

23. For the graph of the function f(x, y) = 0 to be symmetric about x-axis.
f(x, y) = f(x, – y)

From the options given only Choice (A) has this symmetric property. Choice (A)

24. From the graph we can observe that, it is a set of straight lines with slope 1.
From 0 to 1, y = x
From 1 to 2, y = x – 1
From 2 to 3, y = x – 2 etc;
We can say that y = x – [x] Choice (A)

25. From graph we can observe that
For $0 \leq x < 1$, the range of y is $0 \leq y < 1$
For $1 \leq x < 2$, the range of y is $1 \leq y < 2$
The equation of the graph is [x] = [y] Choice (B)

Chapter – 7
(Indices & surds)

Concept Review Questions

Solutions for questions 1 to 15:

1. $4^{2x} = 4^4$
Equating powers of 4 on both sides, 2x = 4.
x = 2 Ans : (2)

2. Conjugate of the surd $\sqrt{13} - 2 = -(\sqrt{13}+2)$
Choice (C)

3. $\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \sqrt{5}-\sqrt{2}$

$\frac{1}{3+\sqrt{8}} = \frac{3-\sqrt{8}}{(3+\sqrt{8})(3-\sqrt{8})} = 3-2\sqrt{2}$

$\frac{3}{\sqrt{5}+\sqrt{2}} + \frac{1}{3+\sqrt{8}} = 3+\sqrt{5}-3\sqrt{2}$ Choice (B)

4. $\sqrt{108} = 6\sqrt{3}$
$\sqrt{75} = 5\sqrt{3}$
$\sqrt{108} - \sqrt{75} = \sqrt{3}$
Its positive square root = $\sqrt[4]{3}$ Choice (D)

5. $\sqrt{128} - \sqrt{56} = 8\sqrt{2} - 2\sqrt{14} = \sqrt{2}(8-2\sqrt{7})$

Its positive square root = $\sqrt[4]{2}(\sqrt{7+1-2\sqrt{7\times1}})$
∴Positive square root = $\sqrt[4]{2}(\sqrt{7}-1)$ Choice (A)

6. Let $\sqrt{14+6\sqrt{5}} = \sqrt{a}+\sqrt{b}$ squaring on both sides,
$14 + 6\sqrt{5} = a + b + 2\sqrt{ab}$
a + b = 14
ab = 45
Solving for a and b, a = 9 and b = 5 or vice versa positive square root = $\sqrt{9}+\sqrt{5} = 3+\sqrt{5}$ Choice (A)

7. The given expression is

$$x^{\frac{(b+c-a)+(c+a-b)+(a+b-c)}{a+b+c}} = x^{\frac{a+b+c}{a+b+c}} = x$$

($\because$ As a, b, c are positive numbers, a + b + c cannot be zero.)
Choice (B)

8. $\dfrac{3}{5^{\frac{2}{3}}+10^{\frac{1}{3}}+2^{\frac{2}{3}}}$

$= \dfrac{3(5^{\frac{1}{3}}-2^{\frac{1}{3}})}{(5^{\frac{2}{3}}+10^{\frac{1}{3}}+2^{\frac{2}{3}})(5^{\frac{1}{3}}-2^{\frac{1}{3}})} = \dfrac{3(5^{\frac{1}{3}}-2^{\frac{1}{3}})}{5^1-2^1}$

($\therefore$ $(a^2 + ab + b^2)(a - b) = a^3 - b^3$)

$= 5^{\frac{1}{3}} - 2^{\frac{1}{3}}$ Choice (C)

9. Rationalising factor of $a^{\frac{1}{3}}+b^{\frac{1}{3}}$ is $a^{\frac{2}{3}} - (ab)^{\frac{1}{3}} + b^{\frac{2}{3}}$

Rationalising factor of $4^{\frac{1}{3}} + 3^{\frac{1}{3}}$ is $4^{\frac{2}{3}} - 12^{\frac{1}{3}} + 3^{\frac{2}{3}}$
Choice (D)

10. $\dfrac{2}{\sqrt{27}+\sqrt{11}} = \dfrac{2(\sqrt{27}-\sqrt{11})}{(\sqrt{27}+\sqrt{11})(\sqrt{27}-\sqrt{11})} = \dfrac{2(\sqrt{27}-\sqrt{11})}{16}$

$= \dfrac{\sqrt{27}-\sqrt{11}}{8}$

$\dfrac{1}{\sqrt{11}+\sqrt{3}} = \dfrac{\sqrt{11}-\sqrt{3}}{(\sqrt{11}+\sqrt{3})(\sqrt{11}-\sqrt{3})} = \dfrac{\sqrt{11}-\sqrt{3}}{8}$

The greater among $\sqrt{27}-\sqrt{11}$

And $\sqrt{11}-\sqrt{3}$ decides the greater of the surds

$\sqrt{27}-\sqrt{11} - (\sqrt{11}-\sqrt{3})$

$= 3\sqrt{3}-\sqrt{11}-\sqrt{11}+\sqrt{3} = 4\sqrt{3}-2\sqrt{11}$

as $(4\sqrt{3})^2 = 48$ and $(2\sqrt{11})^2 = 44$

$(4\sqrt{3})^2 > (2\sqrt{11})^2$

$\therefore 4\sqrt{3} > 2\sqrt{11}$

$\therefore \dfrac{\sqrt{27}-\sqrt{11}}{8}$ is greater i.e., $\dfrac{2}{\sqrt{27}+\sqrt{11}}$ is greater.

Choice (A)

11. $2^{3^3} = 2^{27}$

$3^{3^2} = 3^9$

$2^{27} = (2^3)^9$

$2^3 > 3$

$\therefore (2^3)^9 > 3^9$

$\therefore 2^{3^3}$ is greater. Choice (A)

12. $(12 + 4\sqrt{7} = 4(3+\sqrt{7})$ its rationalizing factor is $4(3-\sqrt{7})$ or any rational multiple of this. Choice (C)

13. $2^{x^x} = 2^4 = 2^{2^2}$. Comparing both sides, x = 2. Ans : (2)

14. $2^p\,3^q = 2(216) = 2(6^3) = 2\,(2\times3)^3$

$2^p\,3^q = (2^4)\,(3^3)$

Comparing both sides, p = 4 and q = 3

p + q = 7 Ans : (7)

15. $2^{\frac{1+3+5+7+9+11}{2}} = (2^3)^x$

$2^{18} = 2^{3x}$

$\Rightarrow x = 6$ Choice (A)

Exercise – 7(a)

Solutions for questions 1 to 35:

1. $\left[(64)^{-1/6}\times(81)^{3/4}\times(64)^{2/3}\right]^{1/3}$

$= \left[(2^6)^{-1/6}\times(3^4)^{3/4}\times(2^6)^{2/3}\right]^{1/3}$

$= [2^{-1}\times3^3\times2^4]^{1/3}$

$= \left[(2\times3)^3\right]^{1/3} = 6$ Ans : (6)

2. $\left[\dfrac{x^4y^{-5}z^2}{x^{-3}yz^4}\right]\div\left[\dfrac{x^2yz^3}{x^{-8}y^2z^5}\right]^7$

$= [x^7\,y^{-6}\,z^{-2}]^{11} \div [x^{10}\,y^{-1}\,z^{-2}]^7 = [x^{77}\,y^{-66}\,z^{-22}]\times[x^{-70}\,y^7\,z^{14}]$

$= x^7\,y^{-59}\,z^{-8} = \dfrac{x^7}{y^{59}z^8}$ Choice (A)

3. $(1.761)^x = (0.1761)^y = 10^z \Rightarrow 1.761 = 10^{z/x}$

Also, $(0.1761)^y = 10^z$

$\Rightarrow 0.1761 = 10^{z/y}$

$\Rightarrow 1.761 = 10\,.10^{z/y} = 10^{z/y+1}$

$\therefore 10^{z/x} = 10^{z/y+1}$

$\Rightarrow \dfrac{z}{x} = \dfrac{z}{y} + 1 \Rightarrow z\left[\dfrac{1}{x}-\dfrac{1}{y}\right] = 1$

$\Rightarrow \dfrac{1}{z} = \dfrac{1}{x} - \dfrac{1}{y}$ Choice (A)

4. $x - 4 = 4^{1/3} + 4^{2/3}$

Cubing both sides, we get $(x-4)^3 = \left[4^{1/3}+4^{2/3}\right]^3$

$\Rightarrow x^3 - 12x^2 + 48x - 64$

$= 4 + 4^2 + 3\,.\,4^{1/3}\,.\,4^{2/3}\left[4^{1/3}+4^{2/3}\right]$

$\Rightarrow x^3 - 12x^2 + 48x - 64 = 20 + 12(x-4)$

$\Rightarrow x^3 - 12x^2 + 48x - 64 = 20 + 12x - 48$

$\Rightarrow x^3 - 12x^2 + 36x = 64 + 20 - 48$

$\Rightarrow x^3 - 12x^2 + 36x = 36$

$\Rightarrow x^3 - 12x^2 + 36x + 8 = 44$ Choice (D)

5. $(2\sqrt{3})^{3x-4} = (2\sqrt{3})^8 \Rightarrow 3x - 4 = 8 \Rightarrow 3x = 12$

$\therefore x = 4$ Ans : (4)

6. $x^y = y^x \quad \rightarrow \quad (1)$

$y = x^2 \quad \rightarrow \quad (2)$

Substituting the value of y from (2) to (1)

$x^{x^2} = (x^2)^x \Rightarrow x^{x^2} = x^{2x} \Rightarrow x^2 = 2x$

Either x = 0 or x = 2

If x = 0, y = 0, which is not possible, since it won't satisfy (1).

When x = 2, $y = x^2 = 2^2 = 4$ Choice (D)

7. $2^x - 3^y = 23 \quad \rightarrow \quad (1)$

$2^{x-4} - 3^{2y-3} = 5 \Rightarrow \dfrac{2^x}{16} - \dfrac{3^2y}{27} = 5$

$\Rightarrow 27.\,2^x + 16.\,3^{2y} = 2160 \quad \rightarrow \quad (2)$

From (1) : $2^x = 23 + 3^y$

$\Rightarrow 27(23 + 3^y) + 16.3^{2y} = 2160$

$\Rightarrow 16.3^{2y} + 27.\,3^y = 1539 = 0$

Let $3^y = z$

$16z^2 + 27z - 1539 = 0$

$\Rightarrow 16z^2 - 144z + 171z - 1539 = 0$

$\Rightarrow 16z(z-9) + 171(z-9) = 0 \Rightarrow (16z + 171)(z-9) = 0$

either, z = 9 or z = –171/16

But 3^y cannot be negative

$\therefore 3^y = 9 \Rightarrow y = 2 \therefore 2^x = 23 + 3^y = 23 + 9 = 32$

$\therefore x = 5$ Choice (D)

8. $2^x.\,3^{2y} = 144 \Rightarrow 2^x\,.\,3^{2y} = 2^4\,.\,3^2$

$\therefore$ x = 4 and 2y = 2 or y = 1
$\therefore$ x + y = 4 + 1 = 5 Ans : (5)

9. $\sqrt[qr]{p^2}\,.\,\sqrt[rp]{q^2}\,.\,\sqrt[pq]{r^2}$

$= a^{\frac{p^2}{qr}}\,.\,a^{\frac{q^2}{rp}}\,.\,a^{\frac{r^2}{pq}}$

$= a^{\frac{p^2}{qr}+\frac{q^2}{rp}+\frac{r^2}{pq}} = a^{\frac{p^3+q^3+r^3}{pqr}} = a^{\frac{3pqr}{pqr}} = a^3$

(since, $p^3 + q^3 + r^3 = 3pqr$, when p + q + r = 0)
Choice (B)

10. $\frac{6}{2\sqrt{3}+\sqrt{6}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{4}{\sqrt{6}-\sqrt{2}}$

$= \frac{6(2\sqrt{3}-\sqrt{6})}{(2\sqrt{3})^2-(\sqrt{6})^2} - \frac{1(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2} + \frac{4(\sqrt{6}+\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}$

$= 2\sqrt{3}-\sqrt{6}-\sqrt{3}-\sqrt{2}+\sqrt{6}+\sqrt{2}=\sqrt{3}$ Choice (B)

11. LCM of 2, 3 and 4 is 12.
$\sqrt{5} = (5^6)^{1/12} = (15625)^{1/12}$
$\sqrt[4]{7} = (7^3)^{1/12} = (343)^{1/12}$
$\sqrt[3]{11} = (11^4)^{1/12} = (14641)^{1/12}$
Since 343 < 14641 < 15625
$\sqrt[4]{7} < \sqrt[3]{11} < \sqrt{5}$ Choice (A)

12. Raising each number to the power 12, we get
$\left(3^{\frac{1}{2}}\right)^{12}, \left(4^{\frac{1}{3}}\right)^{12}$ and $\left(5^{\frac{1}{4}}\right)^{12}$ i.e. 3^6, 4^4 and 5^3
i.e. 729, 256 and 125.
125 < 256 < 729
$\therefore 5^{\frac{1}{4}} < 4^{\frac{1}{3}} < 3^{\frac{1}{2}}$ Choice (D)

13. $5^a = 7^b = 35^c = k$
$a = \log_5 k$
$b = \log_7 k$
$c = \log_{35} k$
$\frac{1}{a}+\frac{1}{b} = \log_k 5 + \log_k 7 = \log_k 35 = \frac{1}{c}$ Choice (D)

14. We note that 2 + 3 + 4 = 9, 2(3) = 6, 2(4) = 8 and 3(4) = 12.
$\therefore\ \sqrt{9+2\sqrt{6}+2\sqrt{8}+2\sqrt{12}} = \sqrt{2}+\sqrt{3}+\sqrt{4}.$

$\frac{(\sqrt{2}+\sqrt{3}+\sqrt{4})+(-\sqrt{2}+\sqrt{3}+\sqrt{4})}{(\sqrt{2}+\sqrt{3}+\sqrt{4})-(-\sqrt{2}+\sqrt{3}+\sqrt{4})}$

$= \frac{2(\sqrt{3}+\sqrt{4})}{2\sqrt{2}} = \sqrt{\frac{3}{2}}+\sqrt{2}$ Choice (D)

15. $\sqrt{3-2\sqrt{2}} = \sqrt{2}-1$

and $\sqrt{5-\sqrt{24}} = \sqrt{5-2\sqrt{6}} = \sqrt{3}-\sqrt{2}$

$\therefore \frac{\sqrt{3-2\sqrt{2}}}{\sqrt{2}+1} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{5-2\sqrt{6}}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}+1} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)} + \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$

$= 3-2\sqrt{2}+5+2\sqrt{6} = 8-2\sqrt{2}+2\sqrt{6}$ Choice (C)

16. $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$

$= \frac{(\sqrt{1+x}+\sqrt{1+x})^2}{(\sqrt{1+x}+\sqrt{1-x})(\sqrt{1+x}-\sqrt{1-x})}$

$= \frac{1+x+1-x+2\sqrt{(1+x)(1-x)}}{1+x-1+x} = \frac{1+\sqrt{1-x^2}}{x}$

$\therefore$ Required value = $\sqrt{\frac{1+\sqrt{1-\frac{1}{4}}}{\frac{1}{2}}}$

$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}} = \sqrt{2+\sqrt{3}}$

$= \sqrt{\frac{1}{2}(4+2\sqrt{3})} = \frac{\sqrt{3}+1}{\sqrt{2}} = \frac{1}{2}(\sqrt{2}+\sqrt{6})$ Choice (B)

17. $\frac{12}{3+\sqrt{5}-2\sqrt{2}}$

$= \frac{12(3+\sqrt{5}+2\sqrt{2})}{(3+\sqrt{5})^2-(2\sqrt{2})^2} = \frac{36+12\sqrt{5}+24\sqrt{2}}{14+6\sqrt{5}-8}$

$= \frac{6(6+2\sqrt{5}+4\sqrt{2})}{6(\sqrt{5}+1)} = \frac{(6+2\sqrt{5}+4\sqrt{2})(\sqrt{5}-1)}{(\sqrt{5})^2-1^2}$

$= \frac{4(1-\sqrt{2}+\sqrt{5}+\sqrt{10})}{4} = 1-\sqrt{2}+\sqrt{5}+\sqrt{10}$

$\Rightarrow$ x = 1, a = –1, b = 1, c = 1
x + a + b + c = 1 – 1 + 1 + 1 = 2 Ans : (2)

18. $\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$

$= \sqrt{\frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-1^2}} = \sqrt{\frac{(\sqrt{5}+1)^2}{4}} = \frac{\sqrt{5}+1}{2}$

$x^2 = \frac{(\sqrt{5})^2+2\sqrt{5}+1}{4}$

$= \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}$

$\therefore 7x^2 - 7x = \frac{21+7\sqrt{5}}{2} - \frac{7\sqrt{5}+7}{2} = 14/2 = 7$ Ans : (7)

19. $\sqrt{x}+\sqrt{2-x} = \sqrt{y}+\sqrt{2-y}$
Squaring both sides
$x + 2 - x + 2\sqrt{x(2-x)} = y + 2 - y + 2\sqrt{y(2-y)}$
$\Rightarrow \sqrt{x(2-x)} = \sqrt{y(2-y)} \Rightarrow 2x - x^2 = 2y - y^2$
$\Rightarrow x^2 - y^2 = 2(x-y) = 0 \Rightarrow (x-y)(x+y-2) = 0$
either x – y = 0 $\Rightarrow$ x = y
or x + y – 2 = 0
$\Rightarrow$ x + y = 2
Similarly, $\sqrt{y}+\sqrt{2-y} = \sqrt{z}+\sqrt{2-z}$
$\Rightarrow$ Either, y = z or y + z = 2 and
$\sqrt{x}+\sqrt{2-x} = \sqrt{z}+\sqrt{2-z}$
$\Rightarrow$ Either z = x or x + z = 2
Combining all the three, we find that
Either x = y, y = z and z = x or x + y = 2, y + z = 2 and x + z = 2. Choice (C)

20. $\dfrac{\left[\sqrt{(\sqrt{3}+\sqrt{2})}+\sqrt{(\sqrt{3}-\sqrt{2})}\right]^2}{\sqrt{3}-1}$

$= \dfrac{\left[\sqrt{\sqrt{3}+\sqrt{2}}\right]^2+\left[\sqrt{(\sqrt{3}-\sqrt{2}}\right]^2+\dfrac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}}{\sqrt{3}-1}$

$=\dfrac{2\sqrt{3}+2}{\sqrt{3}-1}=\dfrac{2(\sqrt{3}+1)}{\sqrt{3}-1}=\dfrac{2(\sqrt{3}+1)^2}{(\sqrt{3})^2-1}=(\sqrt{3}+1)^2$

$\therefore \sqrt{x} = (\sqrt{3}+1)$ Choice (A)

21. $\dfrac{(x-\sqrt{3})(\sqrt{12}+\sqrt{8})}{(\sqrt{12}-\sqrt{8})}=1$

$\Rightarrow \dfrac{(x-\sqrt{3})(12+8+2\sqrt{12\times 8})}{12-8}=1$

$\dfrac{(x-\sqrt{3})(12+8+2\sqrt{12\times 8})}{12-8}=1$

$\Rightarrow (x-\sqrt{3})(5+2\sqrt{6})=1$

$\Rightarrow x-\sqrt{3}=\dfrac{1}{5+2\sqrt{6}}$

$\Rightarrow x-\sqrt{3}=\dfrac{5-2\sqrt{6}}{(5)^2-(2\sqrt{6})^2}=5-2\sqrt{6}$

$\Rightarrow x = 5+\sqrt{3}-2\sqrt{6}$ Choice (C)

22. $\dfrac{3+\sqrt{20}-\sqrt{45}}{7+\sqrt{320}-\sqrt{125}}$

$=\dfrac{3+2\sqrt{5}-3\sqrt{5}}{7+8\sqrt{5}-5\sqrt{5}}$

$=\dfrac{3-\sqrt{5}}{7+3\sqrt{5}}=\dfrac{(3-\sqrt{5})(7-3\sqrt{5})}{49-45}$

$=\dfrac{36-16\sqrt{5}}{4}$

$\therefore x = 9-2\sqrt{20}$

$\therefore \sqrt{x}=\sqrt{5}-2$ Choice (D)

23. Let $z=\left[\sqrt{x^2+2y\sqrt{x^2-y^2}}+\sqrt{x^2-2y\sqrt{x^2-y^2}}\right]$

$\Rightarrow z^2=\left[\sqrt{x^2+2y\sqrt{x^2-y^2}}\right]^2+\left[\sqrt{x^2-2y\sqrt{x^2-y^2}}\right]^2$

$+2\sqrt{\left\{x^2+2y\sqrt{x^2-y^2}\right\}\times\left\{\sqrt{x^2-2y\sqrt{x^2-y^2}}\right\}}$

$\Rightarrow z^2 = x+2y\sqrt{x^2-y^2}+x^2-2y\sqrt{x^2-y^2}$

$+2\sqrt{x^4-4y^2(x^2-y^2)}$

$=2x^2+2\sqrt{(2y^2)^2-2(x^2)(2y^2)+(x^2)^2}$

$=2x^2+2(2y^2-x^2)=4y^2=(2y)^2$

$\therefore z = 2y$ Choice (C)

24. $\sqrt{(4+\sqrt{15})^3}+\sqrt{(4-\sqrt{15})^3}=a\sqrt{5}$

$\Rightarrow\left[\sqrt{\frac{1}{2}(8+2\sqrt{15})}\right]^3+\left[\sqrt{\frac{1}{2}(8-2\sqrt{15})}\right]^3$

$\Rightarrow \dfrac{1}{2\sqrt{2}}\left\{(\sqrt{5}+\sqrt{3})^3+(\sqrt{5}-\sqrt{3})^3\right\}=a\sqrt{5}$

$\Rightarrow \dfrac{1}{2\sqrt{2}}\left\{2(\sqrt{5})^3+18\sqrt{5}\right\}=a\sqrt{5}$

$\Rightarrow \dfrac{1}{2\sqrt{2}}28\sqrt{5}=a\sqrt{5}$

$\Rightarrow \dfrac{14\sqrt{5}}{\sqrt{2}}=a\sqrt{5}\Rightarrow a\sqrt{5}=7\sqrt{2}\times\sqrt{5}$

$\therefore a = 7\sqrt{2}$ Choice (B)

25. $\left\{\dfrac{2^{1/2}.4^{3/4}.8^{5/6}.16^{7/8}.32^{9/10}.64^{11/12}.128^{13/14}.}{256^{15/16}}\right\}^{5/16}$

$=\left[\dfrac{2^{1/2}.(2^2)^{3/4}.(2^3)^{5/6}.(2^4)^{7/8}.(2^5)^{9/10}.(2^6)^{11/12}.}{(2^7)^{13/14}(2^8)^{15/16}}\right]^{5/16}$

$=\left[2^{1/2+3/4+5/6+7/8+9/10+11/12+13/14+15/16}\right]^{5/16}$

$=\left[2^{64/2}\right]^{5/16}=2^{10}=1024$ Ans: (1024)

26. $3^{2x+2}=3^8\Rightarrow 2x+2=8$
$x = 3$
Also, $2^{3y-1}=2^{11}$
$3y-1=11\Rightarrow y=4$
$\therefore \dfrac{2x+3y}{y-x}=\dfrac{6+12}{4-3}=18$ Ans: (18)

27. $\left(11^{1/p}\right)^2-11^{1/p}-72=0$ and $\left(11^{1/q}\right)^2-11^{1/q}-90=0$

$\left(11^{1/p}-9\right)\left(11^{1/p}+8\right)=0$ and $\left(11^{1/q}-10\right)\left(11^{1/q}+0\right)=0$

$11^{1/p}=9$ and $11^{1/q}=10$ ($\because$ 11x is never negative)

$11^{1/p-1/q}=\dfrac{11^{1/p}}{11^{1/q}}=\dfrac{9}{10}$ Choice (B)

28. Let $\sqrt{11\sqrt{11\sqrt{11\ldots\ldots\infty}}}=x$
Squaring both sides, we get
$=11\sqrt{11\sqrt{11\sqrt{11\ldots\ldots\infty}}}=x^2 \quad\Rightarrow 11x=x^2\Rightarrow x(x-11)=0$
Either, x = 0, (which is not possible) or x = 11
Now $11=14641^x\Rightarrow 11=(11^4)^x\Rightarrow 11^{4x}=11$
$\Rightarrow 4x=1$ or $x=1/4$ Choice (C)

29. Let $a=\sqrt{7}+\sqrt{17}\Rightarrow a^2=24+2\sqrt{119}$

$b=\sqrt{8}+\sqrt{14}\quad\Rightarrow b^2=22+2\sqrt{112}$

$c=\sqrt{10}+\sqrt{14}\Rightarrow c^2=24+2\sqrt{140}$

$d=\sqrt{3}+\sqrt{22}\Rightarrow d^2=25+2\sqrt{66}$

Taking ranges
a^2 = 24 + (21 to 22) =
45 to 46
b^2 = 22 + (21 to 22) =
43 to 44
c^2 = 24 + (23 to 24) =
47 to 48
d^2 = 25 + (16 to 17) =

41 to 42
Thus we get d < b < a < c. Choice (D)

30. $\frac{1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2}=\sqrt{2}-1$

Similarly, $\frac{1}{\sqrt{2}+1}=\sqrt{3}-\sqrt{2}$

$\frac{1}{\sqrt{4}+\sqrt{3}}=\sqrt{4}-\sqrt{3}$

- - - - -
- - - - -

$\frac{1}{\sqrt{100}+\sqrt{99}}=\sqrt{100}-\sqrt{99}$

$\therefore$ The given expression is equal to

$(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+........(\sqrt{100}-\sqrt{99})$

$=\sqrt{100}-1=10-1=9$ Ans : (9)

31. $\left(\frac{2}{7}\right)^{\frac{3x+1}{4}}=\left[\left(\left(\frac{2}{7}\right)^{-2}\right)^{1/3}\right]^{\frac{5x-37}{2}}$

$\Rightarrow \frac{3x+1}{4}=\frac{37-5x}{3}$

$\Rightarrow 9x+3=148-20x \Rightarrow 29x=145 \Rightarrow x=5$ Ans: (5)

32. $\frac{x-y}{\sqrt{x}+\sqrt{y}}+\frac{x-y}{\sqrt{x}-\sqrt{y}}=4$

$(x-y)\left\{\frac{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})}{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}\right\}=4$

$\Rightarrow \sqrt{x}-\sqrt{y}+\sqrt{x}+\sqrt{y}=4 \Rightarrow 2\sqrt{x}=4$

$\Rightarrow \sqrt{x}=2 \Rightarrow x=4$ Ans: (4)

33. $\frac{3}{2}(x+4)+\sqrt{2x^2}+19x+35$

$=\frac{1}{2}\left[3x+12+2\sqrt{(x+7(2x+5)}\right]$

$=\frac{1}{2}\left[(x+7)+(2x+5)+2\sqrt{(x+7\,(2x+5)}\right]$

$=\frac{1}{2}\left[(\sqrt{x+7})^2+(\sqrt{2x+5})^2+2\sqrt{(x+7\,(2x+5)}\right]$

$=\frac{1}{2}\left[\sqrt{x+7}+\sqrt{2x+5}\right]^2$

$\therefore \sqrt{\frac{3}{2}(x+4)+\sqrt{2x^2+19x+35}}$

$=\frac{1}{\sqrt{2}}\left[\sqrt{x+7}+\sqrt{2x+5}\right]$ Choice (C)

34. $5^x+4^y=141 \rightarrow (1)$

$5^{x-1}+4^{y-1}=69$

$\Rightarrow \frac{5^x}{5^2}+\frac{4^x}{4}=69 \Rightarrow 4.5^x+25.4^y=6900 \rightarrow (2)$

Solving equations (1) and (2), we get

$5^x=125=5^3 \Rightarrow x=3$ and $4^y=16=4^2 \Rightarrow y=2$

$\therefore x-y=3-2=1$ Ans : (1)

35. $xyz=1 \Rightarrow x^2y^2z^2=1$

Multiplying the numerator and the denominator of $\frac{1}{1+y^2+x^{-2}}$ by x^2 it becomes $\frac{x^2}{x^2+x^2y^2+1}$.

Multiplying the numerator and the denominator of $\frac{1}{1+z^2+y^{-2}}$ by x^2y^2 it becomes $\frac{x^2y^2}{x^2y^2+1+x^2}$

$\frac{1}{1+x^2+z^{-2}}=\frac{1}{x^2+x^2y^2+1}$

$\therefore$ The given expression equals $\frac{x^2+x^2y^2+1}{x^2+x^2y^2+1}=1$

Choice (A)

Exercise – 7(b)

Solutions for questions 1 to 35:

1. $\left[\frac{8a^6}{64a^{12}}\right]^{-2/3}\times\left[\frac{216a^9}{512a^{-3}}\right]^{2/3}$

$=\left[8a^6\right]^{2/3}\times\left[\frac{6^3a^{12}}{8^3}\right]^{2/3}$

$=4a^4\times\frac{6^2}{8^2}a^8=\frac{9}{4}a^{12}$ Choice (D)

2. The given expression

$=\frac{2^{(m+2)}.2^{(m-2)}.3^{(m-2)}.2^{2m}.5^{2m}.3^{(m-1)}.5^{(m-1)}.5^{2m-2n}}{(2^2)^{2m}.3^{2m-3}.5^{5m-2n+5}.5^{-3}}$

$=2^{(m+2)+(m-2)+2m-4m}\,.\,3^{(m-2)+(m-1)-2m+3}.$
$5^{2m+(m-1)+(2m-2n)-(5m--2n+5)+8}$

$=2^0.3^0.5^2=25$ Choice (D)

3. $5^x+5^{x-1}=5^{x+2}=5^x(1+5+5^2)=31.5^x$

The least value that x can take is 1.

$\therefore$ The greatest factor of 31.5^x is 31×5

i.e. 155 Choice (B)

4. $\sqrt{\sqrt[3]{\sqrt[4]{\sqrt[6]{25^{72}}}}}=\sqrt{\sqrt[3]{\sqrt[4]{25^{72/6}}}}=\sqrt{\sqrt[3]{25^{12/4}}}$

$=\sqrt{25^{3/3}}=\sqrt{25}=5$ Ans : (5)

5. Let $2^x=7^y=14^z=k$

$\Rightarrow 2=k^{1/x}$

$7=k^{1/y}$

and $14=k^{1/z}$

But $14=2\times7$

$\Rightarrow k^{1/z}=k^{1/x}.k^{1/y} \Rightarrow k^{1/z}=k^{1/x+1/y}$

$\Rightarrow \frac{1}{z}=\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$

$\therefore z=\frac{xy}{x+y}$ Choice (B)

6. $\frac{5^x}{5^y}=125 \Rightarrow 5^{x-y}=5^3 \Rightarrow x-y=3 \rightarrow (1)$

$\frac{2^x}{4^y}=2 \Rightarrow \frac{2^x}{2^{2y}}=2 \Rightarrow 2^{x-2y}=2$

$\Rightarrow x-2y=1 \rightarrow (2)$

Solving (1) and (2), we get

x = 5, y = 2

$\therefore \frac{3^{2x}}{27^y}+4=\frac{3^{10}}{3^{3\times2}}+4=3^4+4=85$ Ans : (85)

7. $\frac{1+4x}{1+\sqrt{1+4x}}+\frac{1+4x}{1-\sqrt{1+4x}}$

$=(1+4x)\left[\frac{1}{1+\sqrt{1+4x}}+\frac{1}{1-\sqrt{1+4x}}\right]$

$= (1 + 4x)\left[\dfrac{2}{1-(1+4x)}\right] = -\dfrac{1+4x}{2x}$

when x = 1/4, the given expression becomes

$-\dfrac{1+4(1/4)}{2(1/4)} = -4$ Ans : (–4)

8. $\dfrac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}} = \dfrac{\sqrt{x^2-1}\,(x+\sqrt{x^2-1})}{x^2-(\sqrt{x^2-1})}$

$= x\sqrt{x^2-1} + x^2 - 1$

Now $x = \dfrac{1}{2}\left[\sqrt{5}+\dfrac{1}{\sqrt{5}}\right] = \dfrac{3}{\sqrt{5}}$ $x^2 = 9/5$

$\Rightarrow x^2 - 1 = \dfrac{9}{5} - 1 = \dfrac{4}{5} = \sqrt{x^2-1} = \dfrac{2}{\sqrt{5}}$

$\therefore x\sqrt{x^2-1} + (x^2-1)$

$= \dfrac{3}{\sqrt{5}} \times \dfrac{2}{\sqrt{5}} + \dfrac{4}{5} = \dfrac{10}{5} = 2$ Choice (D)

9. $\dfrac{\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}} = \dfrac{x}{32}$

$= \dfrac{(\sqrt{2}+1)-(\sqrt{2}-1)}{(\sqrt{2}+1)+(\sqrt{2}-1)} = \sqrt{\dfrac{x}{32}} = \dfrac{1}{\sqrt{2}} = \sqrt{\dfrac{x}{32}}$

Squaring both sides, we get

$\dfrac{x}{32} = \dfrac{1}{2} \Rightarrow x = 16$ Choice (D)

10. $\dfrac{10\sqrt{2}}{\sqrt{18}-\sqrt{3+\sqrt{5}}} + \sqrt{30-10\sqrt{5}}$

$= \dfrac{10\sqrt{2}}{3\sqrt{2}-\sqrt{\dfrac{6+2\sqrt{5}}{2}}} + \sqrt{30-2\sqrt{125}}$

$= \dfrac{(10\sqrt{2})\sqrt{2}}{(3\sqrt{2})\sqrt{2}-(\sqrt{5}+1)} + \sqrt{(5-\sqrt{5})^2}$

$= \dfrac{20}{6-\sqrt{5}-1} + 5 - \sqrt{5} = \dfrac{20}{5-\sqrt{5}} + 5 - \sqrt{5}$

$= \dfrac{20(5+\sqrt{5})}{(5)2-(\sqrt{5})^2} + 5 - \sqrt{5} = 5+\sqrt{5}+5-\sqrt{5} = 10$

Ans : (10)

11. $\dfrac{2^a}{32^a} = \dfrac{50b}{25b} \Rightarrow 2^{a-5a} = 2 \Rightarrow 2^{-4a} = 2$

$\Rightarrow -4a = 1 \Rightarrow a = -1/4$ Choice (C)

12. $2(3+\sqrt{2}+\sqrt{3}+\sqrt{6}) = 6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}$

$= 1^2 + (\sqrt{2})^2 + (\sqrt{3})^2 + 2.1.\sqrt{2} + 2.1.\sqrt{3} + 2.\sqrt{2}.\sqrt{3}$

$= (1+\sqrt{2}+\sqrt{3})^2$

$\therefore \sqrt{3+\sqrt{2+\sqrt{3+\sqrt{6}}}}$

$= \sqrt{1/2(6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6})} = \dfrac{1}{2}(1+\sqrt{2}+\sqrt{3})$

$\therefore$ The given expression becomes

$\dfrac{\sqrt{2}(1+\sqrt{2}+\sqrt{3})}{\dfrac{1}{\sqrt{2}}(1+\sqrt{2}+\sqrt{3})} = 2$ Choice (B)

13. $\sqrt{4+2\sqrt{3}} = \sqrt{(\sqrt{3}+1)^2} = (\sqrt{3}+1)$

$= \dfrac{1}{\sqrt{\sqrt{4+2\sqrt{3}}+\sqrt{3}(3+2\sqrt{3})}}$

$= \dfrac{1}{\sqrt{\sqrt{3}+1+3\sqrt{3}+6}} = \dfrac{1}{\sqrt{7+4\sqrt{3}}} = \dfrac{1}{\sqrt{7+2\sqrt{12}}}$

$= \dfrac{1}{\sqrt{(2+\sqrt{3})^2}} = \dfrac{1}{2+\sqrt{3}} = \dfrac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= 2-\sqrt{3}$ Choice (A)

14. $\dfrac{1-\sqrt{2}}{3+2\sqrt{2}} = \dfrac{(1-\sqrt{2})(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})}$

$= \dfrac{3+4-5\sqrt{2}}{9-8} = 7 - 5 \times 1.414 = 7 - 7.07 = -0.07$

$\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \dfrac{4-2\sqrt{3}}{2}$

$= 2-\sqrt{3} = 2 - 1.732 = 0.268$

$\therefore \dfrac{1-\sqrt{2}}{3+2\sqrt{2}} + \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$= -0.07 + 0.268 = 0.198$ Choice (C)

15. $\left(24-16\sqrt{2}\right)^{\frac{1}{6}}\left(32+16\sqrt{2}\right)^{\frac{1}{3}}$

$= \left[8\left(3-2\sqrt{2}\right)\right]^{\frac{1}{6}} \times \left[16\left(2+\sqrt{2}\right)\right]^{\frac{1}{3}}$

$= 8^{\frac{1}{6}}\left[\left(\sqrt{2}-1\right)^2\right]^{\frac{1}{6}} \times \left[16\left(2+\sqrt{2}\right)\right]^{\frac{1}{3}}$

$8^{\frac{1}{6}}16^{\frac{1}{3}}\left[\left(\sqrt{2}-1\right)\left(2+\sqrt{2}\right)\right]^{\frac{1}{3}}$

$= 2^{\frac{1}{2}}.\ 2^{\frac{4}{3}}.2^{\frac{1}{6}} = 2^{\frac{1}{2}+\frac{4}{3}+\frac{1}{6}} = 2^2 = 4$ Choice (B)

16. $\left[2^{9\times\frac{2}{9}} \times 3^{7\times\frac{3}{7}} \times 15^{3\times\frac{1}{3}} \times 5\right]^{\frac{1}{2}}$

$= \left[2^2 \times 15 \times 3^3 \times 5\right]^{\frac{1}{2}} = 2 \times 15 \times 3 = 90$ Choice (B)

17. $\dfrac{2^{a+a+2+6a} \times 3^{a+2+4a+4b}}{2^{4a+2b-4+6} \times 3^{a-b+2a+b-2+2}}$

$= 2^{8a+2-4a-2b-2} \times 3^{5a+4b+2-3a}$

$= 2^{4a-2b} \times 3^{2a+4b+2} = 4^{2a-b} \times 9^{a+2b+1}$ Choice (D)

18. Given $(2.56)^a = (0.00256)^b = 10^c$

$2.56 = 10^{\frac{c}{a}} \to$ (I)

$0.00256 = 10^{\frac{c}{b}} \to$ (II)

$\dfrac{(I)}{(II)} = \dfrac{2.56}{0.00256} = \dfrac{10^{\frac{c}{a}}}{10^{\frac{c}{b}}}$

$10^3 = 10^{\frac{c}{a}-\frac{c}{b}} \Rightarrow 3 = \dfrac{c}{a} - \dfrac{c}{b}$

$\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{3}{c}$ $\Rightarrow \dfrac{1}{a} = \dfrac{3}{c} + \dfrac{1}{b}$ Choice (C)

19. Given terms are $\sqrt[3]{4}, \sqrt[4]{7}$ and $\sqrt[6]{17}$ i.e.,

$(4^4)^{\frac{1}{12}}, (7^3)^{\frac{1}{12}}, (17^2)^{\frac{1}{12}}$

$(256)^{\frac{1}{12}}, (343)^{\frac{1}{12}}$ and $(289)^{\frac{1}{12}}$

$\therefore$ The descending order is $\sqrt[4]{7}, \sqrt[6]{17}$ and $\sqrt[3]{4}$

Choice (C)

20. $\dfrac{8}{2+2\sqrt{2}+2\sqrt{3}} = \dfrac{4}{1+\sqrt{2}+\sqrt{3}}$

$= \dfrac{4\sqrt{2}}{2+\sqrt{2}+\sqrt{6}} = \dfrac{4\sqrt{2}\left(2+\sqrt{2}-\sqrt{6}\right)}{\left(2+\sqrt{2}\right)^2-\left(\sqrt{6}\right)^2}$

$= \dfrac{4\sqrt{2}\left(2+\sqrt{2}-\sqrt{6}\right)}{6+4\sqrt{2}-6} = 2+\sqrt{2}-\sqrt{6}$

$\therefore\ 2+\sqrt{2}-\sqrt{6} = x+y\sqrt{2}+z\sqrt{6}$

$x+y+z = 2+1-1 = 2$

Ans : (2)

21. $\dfrac{\sqrt{10}}{\sqrt{2-\sqrt{3}}}+\sqrt{20-10\sqrt{3}}$

$= \dfrac{\sqrt{2}\times\sqrt{10}}{\sqrt{4-2\sqrt{3}}}+\sqrt{20-2\sqrt{5\times 15}}$

$= \dfrac{2\sqrt{5}}{\sqrt{3}-1}+\sqrt{15}-\sqrt{5} = \sqrt{5}\left(\sqrt{3}+1\right)+\sqrt{15}-\sqrt{5}$

$= \sqrt{15}+\sqrt{5}+\sqrt{15}-\sqrt{5} = 2\sqrt{15}$

Choice (C)

22. Let $a = 4+\sqrt{8}$ and $b = 4-\sqrt{8}$

$x = a^{\frac{1}{3}}+b^{\frac{1}{3}}$

Cubing on both sides, $x^3 = a+b+3(ab)^{1/3}\left(a^{\frac{1}{3}}+a^{\frac{1}{3}}\right)$

$\Rightarrow x^3 = 8+3(8)^{\frac{1}{3}}(x)$

$\Rightarrow x^3 - 6x = 8$

Ans: (8)

23. Consider $\sqrt{17+2\sqrt{72}} = \sqrt{17+2\sqrt{9\times 8}}$

$= \sqrt{9}+\sqrt{8} = 3+\sqrt{8}$

$\sqrt{6+4\sqrt{2}} = \sqrt{6+2\sqrt{4\times 2}} = \sqrt{2}+2$

$\therefore$ The given expression $= \dfrac{\left(3+\sqrt{8}\right)^3+\left(3-\sqrt{8}\right)^3}{\left(2+\sqrt{2}\right)^3+\left(2-\sqrt{2}\right)^3}$

$= \dfrac{2[(3)^3+3(3)(8)]}{2[(2)^3+3(2)(2)]} = \dfrac{99}{20}$

Choice (A)

24. **(a)** $2^{300} = (2^3)^{100} = 8^{100}$

$3^{200} = (3^2)^{100} = 9^{100}$

$6 < 8 < 9$.

$\therefore\ 6^{100} < 8^{100} < 9^{100}$ i.e. $6^{100} < 2^{300} < 3^{200}$

Choice (B)

(b) $5^{1/6} = 5^{2/12} = 25^{1/12}$

$3^{1/4} = 3^{3/12} = 27^{1/12}$

$4^{1/3} = 4^{4/12} = 256^{1/12}$

$2^{1/2} = 2^{6/12} = 64^{1/12}$

$\therefore$ Smallest is $5^{1/6}$

Choice (A)

25. Given $x = 3+2\sqrt{2}$

Now $\dfrac{1}{x} = \dfrac{1}{3+2\sqrt{2}}+\dfrac{3-2\sqrt{2}}{3-2\sqrt{2}} = \dfrac{3-2\sqrt{2}}{9-8} = 3-\sqrt{2}$

Now $x^2 - \dfrac{1}{x^2} = (3+2\sqrt{2})^2-(3-2\sqrt{2})^2$

$= 17+12\sqrt{2} - 17+12\sqrt{2}$

$= 24\sqrt{2} = 12\times 2\sqrt{2} = 24\sqrt{2}$

Choice (A)

26. Given $\sqrt{p+\sqrt{q}+\sqrt{r}+\sqrt{s}} = \sqrt{a}+\sqrt{b}+\sqrt{c}$

Squaring on both sides,

$p+\sqrt{q}+\sqrt{r}+\sqrt{s} = (\sqrt{a}+\sqrt{b}+\sqrt{c})^2$

$p+\sqrt{q}+\sqrt{r}+\sqrt{s} = a+b+c+2\sqrt{ab}+2\sqrt{bc}+2\sqrt{ac}$

Equating rational parts on both sides,

$a+b+c = p$

Choice (A)

27. $\sqrt{15+2\sqrt{35}-2\sqrt{21}-2\sqrt{15}}$

$= \sqrt{15+2\sqrt{7}\sqrt{5}-2\sqrt{7}\sqrt{3}-2\sqrt{5}\sqrt{3}}$

$= \sqrt{(\sqrt{7})^2+(\sqrt{5})^2+(-\sqrt{3})^2+2(\sqrt{7})(\sqrt{5})+2(\sqrt{7})(-\sqrt{3})+2(\sqrt{5})(-\sqrt{3})}$

$= \sqrt{(\sqrt{7}+\sqrt{5}-\sqrt{3})^2}$

($\because$ $a^2+b^2+c^2+2ab+2bc+2ca = (a+b+c)^2$)

$= \sqrt{7}+\sqrt{5}-\sqrt{3}$.

Choice (C)

28. Given $(0.abc) = (abc)^{\frac{1}{p}}$

$(a.bc) = (abc)^{\frac{1}{q}}$

$(ab.c) = (abc)^{\frac{1}{r}}$

We know that $\dfrac{(ab.c)(a.bc)}{(0.abc)} = abc$

$\therefore \dfrac{(abc)^{\frac{1}{r}}(abc)^{\frac{1}{q}}}{(abc)^{\frac{1}{p}}} = abc$

$(abc)^{\frac{1}{r}+\frac{1}{q}} = abc\times(abc)^{\frac{1}{p}}$

$\dfrac{1}{r}+\dfrac{1}{q} = 1+\dfrac{1}{p}$

Choice (D)

29. $\left(6+\sqrt{35}\right)^{x/2} = a \Rightarrow \left(6-\sqrt{35}\right)^{x/2} = \dfrac{1}{a}$

$\Rightarrow a+\dfrac{1}{a} = 12 \Rightarrow a^2-12a+1 = 0$

$\Rightarrow a = \dfrac{12\pm\sqrt{144-4}}{2} = \dfrac{12\pm\sqrt{140}}{2} = 6\pm\sqrt{35}$

$(6+\sqrt{35})^{x/2} = 6\pm\sqrt{35} = (6+\sqrt{35})^{\pm 1}$

$x/2 = \pm 1$ $\therefore$ Two solutions for x.

Choice (B)

30. $x-6 = \sqrt{7}$

Cubing on both sides

$x^3-18x^2+108x-216 = 7\sqrt{7} = 7(x-6)$

$\Rightarrow x^3-18x^2+101x-132 = 42$

Choice (A)

31. $x = 2+\sqrt{3}+\sqrt{5} \Rightarrow x-2 = \sqrt{3}+\sqrt{5}$

Squaring both sides'

$x^2-4x+4 = 3+5+2\sqrt{15} \Rightarrow x^2-4x-4 = 2\sqrt{15}$

Squaring both sides again;

$x^4+16x^2+16-8x^3+32x-8x^2 = 60$

$\Rightarrow x^4-8x^3+8x^2+32x = 44$

Choice (B)

32. Let x = $\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}+\sqrt{2-\sqrt{3}}}$

$=\sqrt{2+\sqrt{3}}=\sqrt{1/2(4+2\sqrt{3})}=\frac{\sqrt{3}+1}{\sqrt{2}}$

and, $\sqrt{2-\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{2}}$

$\therefore x=\frac{2+\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}+1}{\sqrt{2}}}+\frac{2-\sqrt{3}}{\sqrt{2}+\frac{\sqrt{3}-1}{\sqrt{2}}}$

$=\frac{2+\sqrt{3}}{\frac{3+\sqrt{3}}{\sqrt{2}}}+\frac{2-\sqrt{3}}{\frac{\sqrt{3}+1}{\sqrt{2}}} \quad =\frac{\sqrt{2}(2+\sqrt{3})}{\sqrt{3}(\sqrt{3}+1)}+\frac{(2-\sqrt{3})\sqrt{2}}{\sqrt{3}+1}$

$=\frac{\sqrt{2}}{(\sqrt{3}+1)}\left\{\frac{2+\sqrt{3}+\sqrt{3}(2-\sqrt{3})}{\sqrt{3}}\right\}$

$=\frac{\sqrt{2}}{\sqrt{3}(\sqrt{3}+1)}(3\sqrt{3}-1)$ Choice (D)

33. $\frac{\sqrt{8}+\sqrt{6}-\sqrt{10}}{\sqrt{6+2\sqrt{3}-2\sqrt{5}-\sqrt{15}}}$

$=\frac{\sqrt{8}+\sqrt{6}-\sqrt{10}}{\sqrt{6+\sqrt{12}-\sqrt{20}-\sqrt{15}}}$

$=\frac{\sqrt{2}\left(2\sqrt{2}+\sqrt{6}-\sqrt{10}\right)}{\sqrt{12+2\sqrt{12}-2\sqrt{20}-2\sqrt{15}}}$

$=\frac{4+2\sqrt{3}-2\sqrt{5}}{\sqrt{4}+\sqrt{3}-\sqrt{5}}=2$ Choice (C)

34. x = $\frac{13}{\sqrt{19-8\sqrt{3}}}=\frac{13}{\sqrt{19-2\sqrt{16(3)}}}=\frac{13}{4-\sqrt{3}}=4+\sqrt{3}$

x = 4 + $\sqrt{3}$

$\Rightarrow x-4=\sqrt{3}$

Cubing on both sides,

$(x-4)^3=\left(\sqrt{3}\right)^3$

$\Rightarrow x^3-64-12x^2+48x=3\sqrt{3}$

$\Rightarrow x^3-12x^2+48x-64=3(x-4)$

$\Rightarrow x^3-12x^2+48x-64=3x-12$

$\Rightarrow x^3-12x^2+45x-52=0$

$\Rightarrow x^3-12x^2+45x=52$

$\Rightarrow x^3-12x^2+45x+50=102$ Ans : (102)

35. Consider $\frac{13}{\sqrt{19-8\sqrt{3}}}=\frac{13}{\sqrt{19-2\sqrt{16\times3}}}$

$=\frac{13}{4-\sqrt{3}}=4+\sqrt{3}\rightarrow(1)$

Consider $\frac{23}{\sqrt{31+12\sqrt{3}}}=\frac{23}{\sqrt{31+2\sqrt{36\times3}}}$

$=\frac{23}{\sqrt{31+2\sqrt{27\times4}}}$

$=\frac{23}{\sqrt{27}+2}=\sqrt{27}-2=3\sqrt{3}-2\rightarrow(2)$

Consider $\frac{24}{\sqrt{48+24\sqrt{3}}}=\frac{24}{\sqrt{48+2\sqrt{144\times3}}}$

$=\frac{24}{\sqrt{48+2\sqrt{36\times12}}}=\frac{24}{6+\sqrt{12}}=6-2\sqrt{3}\rightarrow(3)$

$\therefore$ (1) – (2) – (3)

$=4+\sqrt{3}-3\sqrt{3}+2-6+2\sqrt{3}=0$ Choice (B)

Solutions for questions 36 to 40:

36. x is an integer if the denominator is a factor of the numerator. Using statement I, simplifying the expression, we get $x=\frac{b^6}{a^1.c^{35}}$. Since we do not know the value of c or its relationship with b we cannot find whether x is an integer or not.
Using statement II and simplifying the expression, we get $x=(a^{51})(b^{55})(c^{57})$ which is always an integer.
Hence statement II alone is sufficient. Choice (A)

37. From statement I, we have $4^{x+2}+(729)^{y-4}=1025$
This is possible only for $2^{10}+3^0$ i.e. 4^5+729^0
$\therefore y-4=0$
$\Rightarrow y=4$ and $x=3$
$\therefore$ statement I alone is sufficient.
From statement II, we have $3^{2x}2^y=324=2^2\,3^4$
y = 2, x = 2
Hence statement II alone is sufficient. Choice (B)

38. Simplifying $\frac{\left(a^3\right)^2 b}{a^2}$, we get $\frac{a^6b^3}{a^2}=a^4b^3$
From statement I, we have $(ab)^3\,a=3/4a$
$\Rightarrow (ab)^3=3/4a \Rightarrow a^4b^3=3/4$
$\Rightarrow a^4b^3=3/4$
Hence statement I alone is sufficient.
From statement II, we have
$b=\frac{1}{2a^{\frac{4}{3}}}\Rightarrow a^{4/3}=1/2b\Rightarrow a^4b^3=1/8$
$\therefore$ Statement II alone is also sufficient. Choice (B)

39. Statement I alone doesn't give any new information as the value of AB which is given in I can be calculated from the information given in the question. $\therefore$ We cannot find the value of A.
From statement II alone,
$\frac{1}{A}+\frac{1}{B}=K$ (say the given number)
A + B = KAB
[AB can be calculated from the data given in the questions]
So, we have the sum and product of the two numbers.
We 'll get a quadratic equation for a (or b) and normally two possible values for each of a and b. If the sum of two number (a, b) is s and the product is P, a
$=\frac{s+\sqrt{s^2-4p}}{2}$, $b=\frac{s-\sqrt{s^2-4p}}{2}$ or vice versa. We would expect that a is not uniquely determined. But we have to make sure that $s^2-4p\neq0$. Otherwise a would be uniquely determined. $\therefore$We need to compute s^2-4p

$s=A+B=7+3\sqrt{2}-\frac{2}{3}\sqrt{3}$

and $4p=4AB=\left(\frac{1}{7}\right)\left[15-3\sqrt{6}-9\sqrt{2}+5\sqrt{3}\right]$

The rational part of $(A+B)^2=49+18+\frac{4}{3}$. As this is not equal to the rational part of 4AB, $(A+B)^2\neq4AB$. and consequently A is not uniquely determined.
Therefore II alone is not sufficient. As I is redundant and II is insufficient, I and II together also are not sufficient.

Choice (D)

40. $\sqrt[x]{\sqrt[4]{\sqrt[3]{\sqrt{2^{9(216)}}}}} = \left(2^{9\times216}\right)^{\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\left(\frac{1}{x}\right)} = 2^{\frac{81}{x}}$

If x is a factor of 81, then $2^{\frac{81}{x}}$ is an integer.
From statement I, x is a multiple of 10.
Hence $2^{\frac{81}{x}}$ is not an integer.
Hence statement I alone is sufficient.
From statement II, $x = 3^n$. Since it is known that x < 100, the possible values of n are 1, 2, 3 and 4.
∴ x = 3, 9, 27, 81 only.
Hence $2^{\frac{81}{x}}$ is definitely an integer.
Hence statement II alone is also sufficient. Choice (B)

Chapter – 8
(Logarithms)

Concept Review Questions

Solutions for questions 1 to 15:

1. $\text{Log}_{(81)(343)}(189)(147) = \log(3^4)(7^3)(3^3)(7)(7^2)(3)$
$= \log_{(3^4.7^3)}\left(3^4.7^3\right) = 1$ Choice (B)

2. If r = 1, $\log_p r = \log_q r = 0$
values of p and q need not be equal Choice (C)

3. $\text{Log}_3 4 + \log_3 24 = \log_3 96 = \log_3 m \Rightarrow m = 96$
($\because \log_m a + \log_m b = \log_m ab$) Ans : (96)

4. $\log_2 96 - \log_2 3 = \log_2 96/3$
$= \log_2 32 = 5$ Ans : (5)

5. As, $m^0 = 1$,
$\log_5 m^0 = 0$ (∵ log 1 to any base is zero.) Ans : (0)

6. $\text{Log}_{16} 8^4 = \log_{2^4} 8^4 = \log_{2^4} (2^4)^3$
$= 3\log_{2^4} 2^4 = 3$ Choice (B)

7. Log 2 + log 4 + log 6
∴ log 2 + log 4 + log 6 = log 8+ log 6
= log 8.6 = log 48. Choice (B)

8. $\frac{\log_5 27}{\log_5 64} = \frac{\log_5 3^3}{\log_5 4^3} = \frac{3\log_5 3}{3\log_5 4} = \log_4 3$ Choice (A)

9. $5^{\log_5 7^2} = 7^2$
∴ $k = 7^2 = 49$ Ans : (49)

10. $p = 3^5 = 243$ Ans : (243)

11. $\log_{36}49 = \frac{\log 7^2}{\log 6^2} = \frac{2\log 7}{2\log 6} = \frac{\log_{36} 7}{\log_{36} 6} = \frac{\log_{36} 7}{\log_x 6}$ (given)
Comparing the two sides, x = 36 Choice (D)

12. Only if x = y, $\log_z x = \log_z y$ Choice (A)

13. $\log_{3^4} 5^2 = \frac{2\log 5}{4\log 3} = (1/2)\log_3 5$
∴ $(1/2)\log_3 5 = k\log_3 5$ or k = ½ Choice (C)

14. $2^{13} = 8192$ and $2^{14} = 16384$
∴ integral part of $\log_2 10000 = 13$ Ans : (13)

15. A 15 digit number is of the form $k.\,10^{14}$ where $1 \le k < 10$
$\log_{10}(k.10^{14}) = \log_{10} k + \log_{10} 10^{14}$ As $0 \le \log_{10} k.10^{14} < 1$,
$14 \le \log_{10}(k.\,10^{14}) < 15$
∴ integral part of $\log_{10}$ N is 14

<u>Alternate method :</u>

Integral part of the logarithm of any number greater than 1 to the base 10 is always 1 less than the number of digits in the number. In the given problem, as N has 15 digits, integral part of $\log_{10}$: N is 14. Choice (A)

Exercise – 8(a)

Solutions for questions 1 to 30:

1. $\log\frac{15}{8} + 2\log\frac{8}{5} - 3\log\frac{2}{3} - \log\frac{27}{10}$
$= \log\frac{15}{8}\times\frac{64}{25}\times\frac{27}{8}\times\frac{10}{27}$
log6 = log2 + log 3 Choice (A)

2. $\log_{\sqrt{3}} 6561 = \log_{\sqrt{3}}\left(\sqrt{3}\right)^{16} = 16$
$\log_2 16 = \log_2 2^4 = 4$
$\log_2 4 = \log_2 2^2 = 2$
$\log_2 2 = 1$
∴ $\log_2 \log_2 \log_2 \log_2 \log_{\sqrt{3}} 6561 = 1$ Ans : (1)

3. $\log_{b^2} a \text{ x } \log_{c^2} b \text{ x } \log_{d^2} c \text{ x } \log_{e^2} d \text{ x } \log_{a^2} e$
$= \frac{\log a}{2\log b} \text{x} \frac{\log b}{2\log c} \text{x} \frac{\log c}{2\log d} \text{x} \frac{\log d}{2\log e} \text{x} \frac{\log e}{2\log a}$
$= (1/2)^5 = 1/32$ Choice (D)

4. $\log_3 \log_2 \log_x 2^{1024} = 2^1 \Rightarrow \log_2 \log_x 2^{1024} = 3^2$
$\Rightarrow \log_x 2^{1024} = 2^9 \Rightarrow 2^{1024} = x^{512} \Rightarrow 4^{512} = x^{512}$
∴ x = 4 Ans : (4)

5. $5 + \log_{10}x = 5\log_{10}y$
$5\log_{10}10 + \log_{10}x = \log_{10}y^5 \Rightarrow \log_{10}x = \log_{10}y^5 - \log_{10}10^5$
$\Rightarrow \log_{10}x = \log_{10}\frac{y^5}{10^5} \Rightarrow x = y^5/10^5 = (y/10)^5$
Choice (B)

6. Let, $x = y^2 = z^3 = w^4 = u^5 = k$
$x = k, y = k^{½}, z = k^{1/3}, w = k^{¼}, u = k^{1/5}$
∴ $xyzwu = k^{1+1/2+1/3+1/4+1/5} = k^{137/60}$
∴ $\log_x{}^{xyzwu} = \log_k k^{137/60} = 2\frac{17}{60}$ Choice (A)

7. $\frac{1}{\log_2{}^x} = \frac{1}{\frac{\log x}{\log 2}} = \frac{\log 2}{\log x}$

Similarly, $\frac{1}{\log_3 x} = \frac{\log 3}{\log x}$ and so on

∴ $\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}+\frac{1}{\log_5 x}+\frac{1}{\log_6 x}+\frac{1}{\log_7 x}$

$= \frac{\log 2}{\log x}+\frac{\log 3}{\log x}+\frac{\log 4}{\log x}+\frac{\log 5}{\log x}+\frac{\log 6}{\log x}+\frac{\log 7}{\log x}$

$= \frac{\log 2+\log 3+\log 4+\log 5+\log 6+\log 7}{\log x}$

$= \frac{\log\left(2 \text{ x } 3 \text{ x } 4 \text{ x } 5 \text{ x } 6 \text{ x } 7\right)}{\log x} = \log_x 5040$ Choice (D)

8. $\log_y x = 5 \Rightarrow x = y^5$ → (1)
$\log_{2y} 8x = 4 \Rightarrow 8x = (2y)^4$ → (2)
$(1)/(2) \Rightarrow \frac{1}{8} = \frac{y}{16} \Rightarrow y = 2$
∴ $x = 2^5 = 32$ Ans : (32)

9. $36^{\log_6 1/2} \times 36^{\log_x (\sqrt{2})^2} = 1/2$

$\Rightarrow (6^2)^{\log_6 1/2} \times 36^{\log_x 2} = 1/2$

$\Rightarrow 6^{\log_6 (1/2)^2} \times 36^{\log_x 2} = 1/2 \Rightarrow 36^{\log_x 2} = 2$

$\Rightarrow$ Applying log on both sides, $\log_x 2 \times \log 36 = \log 2$

i.e. $\log_x 2 = \log_{36} 2 \Rightarrow x = 36$ Ans : (36)

10. Let, $\log_{y^3} x = k \Rightarrow x = (y^3)^k = y^{3k}$

Also, $\log_{x^3} y = k \Rightarrow y = (x^3)^k = x^{3k}$

$\therefore y = x^{3k} = (y^{3k})^{3k} = {}_y9k^2 \ \therefore 9k^2 = 1$

$\Rightarrow k = \pm 1/3$ Choice (D)

11. $2[\log(x + y) - \log 5] = \log xy$

$\Rightarrow \log (x + y/5) = 1/2 \log xy \Rightarrow \log (x + y/5) = \log \sqrt{xy}$

$\Rightarrow \frac{x+y}{5} = \sqrt{xy} \Rightarrow x + y = 5\sqrt{xy}$

Squaring both sides; $x^2 + 2xy + y^2 = 25xy$

$\Rightarrow x^2 + y^2 = 23xy \quad \Rightarrow \frac{x^2 + y^2}{xy} = 23$ Ans : (23)

12. $\frac{\log 343}{\log 49} = \frac{\log 7^3}{\log 7^2} = \frac{3}{2}$

$\therefore \frac{\log x}{\log 4} = \frac{3}{2} \Rightarrow \log x = 3/2 \log 4 = \log 4^{3/2} = \log 8$

$\therefore x = 8$

and, $\frac{\log y}{64} = \frac{3}{2} \Rightarrow \log y = 3/2 \log 64 = \log 64^{3/2} = \log 512$

$\therefore y = 512$

$\therefore x + y = 8 + 512 = 520$ Ans : (520)

13. $A = \log_7 2401 = \log_7 7^4 = 4$

$B = \log_{7\sqrt{7}} 343 = \log_{7\sqrt{7}} (7\sqrt{7})^2 = 2$

$C = \log_{\sqrt{6}} 216 = \log_{\sqrt{6}} (\sqrt{6})^6 = 6$

$D = \log_2 32 = \log_2 2^5 = 5$

$\therefore$The arrangement in ascending order would be BADC.

Choice (D)

14. $[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-1/2} = [1- \{1- 1/1-x^2\}^{-1}]^{-1/2}$

$= [1 -\{-x^2/1-x^2\}^{-1}]^{-1/2} = [1 + 1-x^2/x^2]^{-1/2} = (1/x^2)^{-1/2} = x$

$\therefore$LHS = $\log x \Rightarrow \log x = 1 \ \therefore x = 10$ Choice (C)

15. $\log_2{}^8 = 3; \Rightarrow {}_4\log_2{}^8 = 4^3 = 64$

also, ${}_{27}\log_{27}{}^{81} = 81$

$\therefore 144 + \log_{10}{}^x = 64 + 81$

$\Rightarrow \log_{10}{}^x = 1 \Rightarrow x = 10$ Ans : (10)

16. Let, $\log_3 x = a$

$\therefore \log_x 3 = 1/\log_3 x = 1/a$

$\log_{x/81} 3 = \frac{1}{\log_3 x81}$

$= \frac{1}{\log_3 x - \log_3 81} = \frac{1}{\log_3 x - 4} = \frac{1}{a-4}$

and $\log_{x/729} 3 = \frac{1}{\log_3 x/729}$

$= \frac{1}{\log 3x - \log_3 729} = \frac{1}{\log_3 x - 6} = \frac{1}{a-6}$

Now, $\frac{1}{a} \times \frac{1}{a-4} = \frac{1}{a-6}$

$\Rightarrow a^2 - 4a = a - 6$

$\Rightarrow a^2 - 5a + 6 = 0 \Rightarrow (a - 2)(a - 3) = 0$

either a = 2

$\Rightarrow \log_3 x = 2 \Rightarrow x = 3^2 = 9$

or, $a = 3 \Rightarrow \log_3 x = 3 \Rightarrow x = 3^3 = 27$ Choice (C)

17. $\log_9 16 \times \log_4 10 = \frac{\log 16}{\log 9} x \frac{\log 10}{\log 4}$

$= \frac{2\log 4}{2\log 3} x \frac{1}{\log 4} = \frac{1}{\log 3}$

Also, $\log_2 8 = 3 \log_2 2 = 3$

$\therefore 3 \log x = \frac{3}{\frac{1}{\log 3}} \Rightarrow 3\log x = 3 \log 3 \Rightarrow \log x = \log 3$

$\therefore x = 3$ Ans : (3)

18. $\log_y x = 8 \Rightarrow x = y^8$ $\rightarrow$ (1)

and, $\log_{10y} 16x = 4 \Rightarrow 16x = (10y)^4$

$\Rightarrow 16x = 2^4 \times 5^4 \times y^4 \Rightarrow x = 5^4 \text{ x } y^4$ $\rightarrow$ (2)

Combining (1) and (2), we get;

$Y^8 = 5^4 \times y^4$

$\Rightarrow y^4 (y^4 - 5^4) = 0$

$Y^4 = 0 \Rightarrow y = 0$ which is not possible

or, $y^4 - 5^4 = 0 \Rightarrow y = 5$ Choice (D)

19. $2 \log_5 P + 1/2 \log_5 Q = 1 \Rightarrow \log_5 P^2 + \log_5 \sqrt{Q} = 1$

$\Rightarrow \log_5 P^2 \sqrt{Q} = 1 \ P^2 \sqrt{Q} = 5 \Rightarrow Q = 25/P^4$ Choice (D)

20. Since, $0.0000002542 = 2.542 \times 10^{-7}$

$\therefore \log 0.0000002542 = \bar{7}.4052$ Choice (C)

21. $\log_{10} 24 + \log_{10} 80 + \log_{10} 25 = x + y + z$

$\Rightarrow \log_{10}(24 + 80 + 25) = x + y + z$

$\Rightarrow \log_{10} 48000 = x + y + z$

$\Rightarrow \log_{10} 48 + \log_{10} 1000 = x + y + z$

$\therefore \log_{10} 48 = x + y + z - 3$ Choice (B)

22. $\log_{25} 225 = p$

$\log_{25} (25)(9) = p$

$1 + \log_{25} 9 = p$

$\log_{25} 9 = p - 1$

$\log_{2025} 1275 = \log_{81 \times 125} 51 \times 25 = \frac{\log_{25} 51 + \log_{25} 25}{\log_{25} 81 + \log_{25} 25}$

$\frac{\log_{25} 3 + \log_{25} 17 + 1}{4\log_{25} 3 + 1} = \frac{\frac{p-1}{2} + \frac{1}{q} + 1}{4\left(\frac{p-1}{2}\right) + 1}$

$= \frac{pq - q + 2 + 2q}{2q[2(p-1)+1]} = \frac{pq + q + 2}{q(4p-2)}$ Choice (D)

23. $\log_2 \log_2 \log_2 \left(\sqrt{x-13} + \sqrt{x-45}\right) = 1$

$\Rightarrow \log_2 \log_2 \left(\sqrt{x-13} + \sqrt{x-45}\right) = 2$

$\Rightarrow \log_2 \left(\sqrt{x-13} + \sqrt{x-45}\right) = 4$

$\Rightarrow \sqrt{x-14} + \sqrt{x-45} = 16$

Squaring both sides, $2x - 58 + 2\sqrt{x^2 - 58x + 585} = 256$

$\Rightarrow 2\sqrt{x^2 - 58x + 585} = 314 - 2x$

$\Rightarrow 4(x^2 - 58x + 585) = 98596 - 1256x + 4x^2$

$\Rightarrow 1024x = 96256 \Rightarrow x = 94$

$\sqrt{x-13} + \sqrt{x-45}$ is defined when x is 94.

$\therefore$ x can be 94

x has only one possible value. Ans : (1)

24. $\log (0.004225)^{1/8} = \frac{1}{8} (\bar{3}.6258) = \frac{1}{8} (-2.3742)$

$= -0.296775 = \bar{1}.703225 = \bar{1}.70322$

Since, log 50492 = 4.70322

$\therefore$Antilog $\bar{1}$ .70322 = 0.50492

$\therefore 8\sqrt{0.004225} = 0.50492$ Choice (B)

25. $\log 25^{50} = 50 \log 25 = 50 \log \frac{100}{4}$

$= 50[\log 100 - \log 2^2] = 50[\log 100 - 2 \log {}^2]$

$= 50 [2 - 2 \times 0.3010] = 50 \times 1.398 = 69.9$

$\therefore$The number of digits in 25^{50} is (69 + 1) i.e., 70

Choice (B)

26. Let $k = (2/3)^{500}$, applying log on both sides

logk = 500 [log2 – log3]

= 500 [0.3010 – 0.4771] = 500 (–0.1761) = –88.05

Number of zeroes after decimal is 88. Ans : (88)

27. $3 (\log_{10} y - \log_{10} \sqrt[3]{y}) = 8 \log_y 10$

$\Rightarrow 3 (\log_{10} y - \frac{1}{3} \log_{10} y) = \frac{8}{\log_{10} y}$

$\Rightarrow 2 (\log_{10} y)^2 = 8$

$\Rightarrow \log_{10} y = \pm 2$

$\therefore$ y = 100 or $\frac{1}{100}$ Choice (D)

28. $\log_p q = mn \Rightarrow p^{mn} = q$ ---(1)

$m \log_q r = mn \Rightarrow q^n = r$

$n \log_r p = mn \Rightarrow r^m = p$

$r^m = q^{mn} = p$ ----(2)

From (1) and (2), $q^{mn} = p^{(mn)^2} = p$

Equating the indices, $(mn)^2 = 1$.

$\therefore$ mn = ± 1. Only (3, 3) violates this condition.

Choice (C)

29. $\log_p \frac{p^3}{q^2} + \log_q \frac{q^3}{p^2} = \log_p p^3 - \log_p q^2 + \log_q q^3 - \log_q p^2$

$= 3 - 2 \log_p q + 3 - 2 \log_q p$

$= 6 - 2 (\log_q p + \log_p q)$

For any two positive numbers x and y where $x \geq y > 1$, $\log_y x + \log_x y \geq 2$

$\therefore \log_q p + \log_p q \geq 2$.

$\therefore$ The given expression cannot exceed 2. Only 2.5 violates this condition. Choice (C)

30. log N_1, log N_2, …. log N_{15} are in arithmetic progression. This is only possible if N_1, N_2 ……N_{15} are in geometric progression

Let N_1 = a and $\frac{N_2}{N_1} = r$

$N_2 = ar$, $N_3 = ar^2$, …….. $N_{15} = ar^{14}$

$N_8 - N_4 = ar^3 (r^4 - 1) = 3600$ and $N_7 - N_5 = ar^4 (r^2 - 1) = 1440$

$\frac{ar^3(r^4-1)}{ar^4(r^2-1)} = \frac{3600}{1440}$ i.e. $\frac{r^2+1}{r} = \frac{5}{2}$

$2r^2 - 5r + 2 = 0$

r = 2 or $\frac{1}{2}$

If r is $\frac{1}{2}$, a will be negative. This is not possible.

$\therefore$ r is 2

(r, a) = (2, 30)

$N_1 + N_2 + …….N_{15} = \frac{a(r^{15}-1)}{r-1} = 30(2^{15} - 1)$.

Choice (C)

Exercise – 8(b)

Solutions for questions 1 to 35:

1. $\log_{\sqrt[3]{8}} \sqrt[3]{512}\ (64) = \log_{8^{1/3}} (8^3)^{1/3} (8^2)$

$= \log_{8^{1/3}} 8^1(8^2) = 3 \log_8 8^3$ ($\therefore \log_{b^m} a = \frac{1}{m}\log_b a$)

$= (3)(3)\log_8 8 = 9(1) = 9$ Ans : (9)

2. Let the base be x

$\log_x 19683 = 6 \Rightarrow x^6 = 19683 = (3\sqrt{3})^6$

$\therefore x = 3\sqrt{3}$ Choice (B)

3. Let the number be x; $\log_{\sqrt{3}} x = 2$

Now $3\sqrt{3} = (\sqrt{3})^3$

So $\log_{3\sqrt{3}} x = \log_{(\sqrt{3})^3} x = \frac{1}{3} \cdot 2 = \frac{2}{3}$ Choice (D)

4. Given $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{xz} = \frac{1}{xyz}$

$\Rightarrow \frac{x+y+z}{xyz} = \frac{1}{xyz}$

$\therefore x + y + z = 1$

Taking log on both sides

log(x + y + z) = log1

log(x + y + z) = 0 Choice (C)

5. $(abc)^x = a + b + c$

Taking log on both sides

xlog(abc) = log(a + b + c)

$\Rightarrow \frac{1}{x} = \frac{\log abc}{\log a + b + c} = \log_{a+b+c} (abc)$ Choice (A)

6. $\log_{20}25 \times \log_{25}20 = \frac{\log 25}{\log 20} \times \frac{\log 20}{\log 25} = 1$

$\therefore \log_7 1 = 0$

$\therefore$The given expression becomes $\log_{17}6^0$

$= \log_{17}1 = 0$ Choice (A)

7. Given $\frac{x}{\log 2} + \frac{x}{\log 4} + \frac{x}{\log 16} + ...\infty = \log 2$

$\Rightarrow x \left(\frac{1}{\log 2}\right)\left(1 + \frac{1}{2} + \frac{1}{4} + ...\right) = \log 2$

$\Rightarrow 2x = (\log 2)^2$

$\Rightarrow x = (\log 2)^2/2$ Choice (C)

8. $2^{\frac{1}{\log_x 4}} \times 2^{\frac{1}{\log_x 16}} \times 2^{\frac{1}{\log_x 256}} ... \infty = 2$

Let $\log_x 4 = a$

$2^{\frac{1}{a} + \frac{1}{2a} + \frac{1}{4a} + ...} = 2^1$

Equating the indices on the two sides, we get

$\frac{1}{a} + \frac{1}{2a} ...\infty = 1$

$\frac{1}{a} [1 + \frac{1}{2} + \frac{1}{4} + ….\infty] = 1$

$\Rightarrow \frac{1}{a}(2) = 1$ ($\because$ sum of infinite terms)

$\Rightarrow a = 2$

$\therefore \log_x 4 = 2 \Rightarrow 4 = x^2$

$\Rightarrow x = 2$ Ans : (2)

9. Log a + $\log a^2 + \log a^3 + \log a^4 + ……….\log a^{20}$

= (1 + 2 + 3 + ……+20)loga

$= \frac{20 \times 21}{2} \log a = 210 \log a = \log a^{210}$ Choice (A)

10. $\log (x + y) - \log 2 = 1/2 (\log x + \log y)$

$\Rightarrow \log (x+y/2) = 1/2 \log xy \Rightarrow \log (x+y/2) = \log \sqrt{xy}$

$\Rightarrow \frac{x+y}{2} = \sqrt{xy}$

Squaring both sides;

$x^2 + y^2 + 2xy = 4xy$

$\Rightarrow x^2 + y^2 - 2x\,y = 0 \Rightarrow (x - y)^2 = 0 \Rightarrow x = y$

$\therefore x = 2$ Ans : (2)

11. $\log_x 5 + \log_x 8 + \log_x 12 - \log_x 60 = 3\log_x A$

$\Rightarrow \log_x(5x8x12/60) = 3\log_x A \Rightarrow \log_x A^3 = \log_x 8$

$\Rightarrow A^3 = 8 \Rightarrow A = 2$ Ans : (2)

12. $\log (x^5 y^2) = 5a + 2b$

$\Rightarrow 5 \log x + 2\log y = 5a + 2b \quad \rightarrow \quad (1)$

Similarly, $\log(x^2 y^5) = 2a + 5b$

$\Rightarrow 2 \log x + 5 \log y = 2a + 5b \quad \rightarrow \quad (2)$

$\therefore \log(xy) = \log x + \log y$

Adding (1) and (2) $7 \log x + 7 \log y = 7a + 7b$

$\log x + \log y = a + b$ Choice (A)

13. $\log_9 x = \log_{3^2} x = 1/2 \log_3 x$

Similarly, $\log_{27} x = 1/3 \log_3 x$

And $\log_{81} x = 1/4 \log_3 x$

$(1 + 1/2 + 1/3 + 1/4) \log_3 x = \frac{25}{4} \Rightarrow \frac{25}{12} \log_3 x = \frac{25}{4}$

$\Rightarrow \log_3 x = 3 \Rightarrow x = 3^3 = 27$ Ans : (27)

14. Let the number be x and the base y

$\log_y x = 9 \Rightarrow x = y^9 \quad \rightarrow \quad (1)$

also, $\log_{11y} 64x = 6 \Rightarrow 64x = 11^6 \,.\, y^6 \quad \rightarrow \quad (2)$

$(2) \div (1) \Rightarrow 64 = \frac{11^6}{y^3}$

$\Rightarrow y^3 = \frac{\left(11^2\right)^3}{(4)^3} = (121/4)^3$

$\therefore y = \frac{121}{4}$ Choice (D)

15. $a = \log_4 2 = \frac{\log 2}{\log 4}$

Similarly, $b = \frac{\log 4}{\log 6}$ and $c = \frac{\log 6}{\log 8}$

$\therefore abc = \frac{\log 2}{\log 4} x \frac{\log 4}{\log 6} x \frac{\log 6}{\log 8} = \frac{\log 2}{\log 8} = \frac{\log 2}{3\log 2} = 1/3$

Choice (A)

16. Given $\log(x + y) + \log(x - y) =$

$2\log(x + y) - \log x - \log y - \log 2$

$\Rightarrow \log(x + y) + \log(x - y) = \log(x + y)^2 - [\log x + \log y + \log 2]$

$\Rightarrow \log(x^2 - y^2) = \log\left[\frac{(x+y)^2}{2xy}\right]$

$\therefore x^2 - y^2 = \frac{(x+y)^2}{2xy}$

$\Rightarrow x^2 - y^2 = \frac{x^2 + y^2 + 2xy}{2xy}$

$\Rightarrow x^2 - y^2 = \frac{x^2}{2xy} + \frac{y^2}{2xy} + \frac{2xy}{2xy}$

$\Rightarrow x^2 - y^2 - 1 = \frac{x}{2y} + \frac{y}{2x} \Rightarrow x^2 - y^2 - 1 = \frac{1}{2}\left[\frac{x}{y} + \frac{y}{x}\right]$

$\Rightarrow 2 (x^2 - y^2 - 1) = \frac{x}{y} + \frac{y}{x}$ Choice (C)

17. Given $\log_2\left(\log_2 2^{(a-b)}\right) = 2\log_2\left(\sqrt{a} - \sqrt{b}\right) + 1$

$\Rightarrow \log_2(a-b) = \log_2\left(\sqrt{a} - \sqrt{b}\right)^2 + 1$

$\Rightarrow \log_2(a-b) - \log_2\left(\sqrt{a} - \sqrt{b}\right)^2 = 1$

$\Rightarrow \log_2\left[\frac{a-b}{\left(\sqrt{a} - \sqrt{b}\right)^2}\right] = 1 \Rightarrow 2 = \frac{a-b}{\left(\sqrt{a} - \sqrt{b}\right)^2}$

$\Rightarrow 2 = \frac{\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right)}{\left(\sqrt{a} - \sqrt{b}\right)^2} \Rightarrow \frac{2}{1} = \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}$

$\Rightarrow 2\left(\sqrt{a} - \sqrt{b}\right) = \sqrt{a} + \sqrt{b} \Rightarrow \sqrt{a} = 3\sqrt{b} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = 3$

$\Rightarrow \sqrt{\frac{a}{b}} = 3$ Ans : (3)

18. Given $2\log_4(y) + 2\log_4(z) + 2\log_4(x) = 1$

$\log_4(xyz)^2 = 1$

$\Rightarrow (xyz)^2 = 4$

$\therefore xyz = 2$

($\because$ xyz is positive) Ans : (2)

19. Given $\frac{a}{\log_a abc} + \frac{b}{\log_b abc} + \frac{c}{\log_c abc} = 1$

$\Rightarrow a\log_{abc} a + b\log_{abc} b + c\log_{abc} c = 1$

$\Rightarrow \log_{abc} a^a + \log_{abc} b^b + \log_{abc} c^c = 1$

$\Rightarrow \log_{abc} a^a\, b^b\, c^c = 1$

$\Rightarrow (abc)^1 = a^a\, b^b\, c^c$

$\Rightarrow \frac{a^a b^b c^c}{abc} = 1 \Rightarrow a^{a-1}\, b^{b-1}\, c^{c-1} = 1$ Choice (D)

20. Given $\log_{\sqrt[a]{y}} x^a = 1$

$\Rightarrow x^a = y^{1/a} \quad \rightarrow \quad (1)$

$\log_{\sqrt[a]{y}} x^{a^{b-1}} = 1$

$\Rightarrow x^{a^{b-1}} = y^{1/a} \quad \rightarrow \quad (2)$

Form (1) and (2)

$\therefore x^a = x^{a^{b-1}}$

As x is not -1, 0 or 1 $a = a^{b-1} \quad \rightarrow \quad (3)$

As the expression $\sqrt[a]{y}$ occurs in the question, $a \neq 0$,

and as $a^2 \neq 1$, a is not 1or -1 $\therefore$ equating the indices on the two sides of (3), b = 2 Choice (A)

21. $\log_2 [\,3\log_2 2^{2m} + 2 \log_2 4\,] = 4$

$\Rightarrow 3(2m) + 4 = 2^4$

$\Rightarrow 6m + 4 = 16$

$\Rightarrow 6m = 12 \Rightarrow m = 2$ Ans : (2)

22. Given $\log_{(x-y)}(x + y) = \frac{1}{2}$

Consider $\log_{x^2-y^2} x^2 - 2xy + y^2 = \log_{(x-y)(x+y)}(x - y)^2$

$= 2 \log_{(x-y)(x+y)}(x - y) = \frac{2}{\log_{(x-y)}(x-y)(x+y)}$

$= \frac{2}{\log_{(x-y)}(x-y) + \log_{(x-y)}(x+y)} = \frac{2}{1 + \frac{1}{2}} = \frac{4}{3}$

Choice (C)

23. Given $\log_p 8 = b \Rightarrow \log_p 2 = b/3 \quad \rightarrow \quad (1)$

$\log_p 12 = a$

$\log_p 3 + 2\log_p 2 = a$

$\log_p 3 = a - 2\left(\frac{b}{3}\right)$ (∵ from (1))

$\Rightarrow \log_p 3 - 1 = a - \frac{2b}{3} - 1$

$\Rightarrow \log_p 3 - \log_p p = a - \frac{2b}{3} - 1$

$\Rightarrow \log_p\left(\frac{3}{p}\right) = \frac{(3a - 2b - 3)}{3}$

$\Rightarrow 3\log_p\left(\frac{3}{p}\right) = 3a - 2b - 3$

$\Rightarrow \log_p\left(\frac{3}{p}\right)^3 = 3a - 2b - 3$ Choice (B)

24. Given $(x + 1)\log y + \log x + \log z = 2\log z + \log y + \log x$
$\Rightarrow x \log y = \log z \Rightarrow y^x = z$ Choice (D)

25. $2\log x + \log(x^4 + 1 + 2x^2) = \log(x^2 + 1) + \log x^2 + 1$
$\Rightarrow \log(x^2 + 1)^2 - \log(x^2 + 1) = 1$
$\Rightarrow \log\left[\frac{(x^2+1)^2}{(x^2+1)}\right] = 1 \Rightarrow \log(x^2 + 1)] = 1$
$\Rightarrow x^2 + 1 = 10$
$\Rightarrow x^2 = 9 \Rightarrow x = 3$
We have to reject the negative value as logx appears in the equation. Ans : (3)

26. $\frac{1}{3}\log\left[(a^3 + b^3)(a + b)\right] = \log(a + b)$
$\Rightarrow \log[(a^3 + b^3)(a + b)] = 3\log(a + b)$
$\Rightarrow (a^3 + b^3)(a + b) = (a + b)^3 \Rightarrow a^3 + b^3 = (a + b)^2$
But given a = 2b
$\therefore (2b)^3 + b^3 = (2b + b)^2 \Rightarrow 8b^3 + b^3 = (3b)^2$
$\Rightarrow 9b^3 = 9b^2 \Rightarrow b = 1$ and $a = 2$ Choice (A)

27. Given $\log_{x+y} x\text{-}y = a$
Now $\frac{1+a}{a} = \frac{1}{a} + 1 = \frac{1}{\log_{x+y} x - y} + 1 = \log_{(x\text{-}y)}(x + y) + 1$
$= \log_{(x\text{-}y)}(x + y) + \log_{(x\text{-}y)}(x - y) = \log_{(x\text{-}y)}[(x + y)(x - y)]$
$= \log_{(x\text{-}y)}(x^2 - y^2)$ Choice (A)

28. $\bar{3}.0838 = -3 - 0.0838 = -4 + 1 - 0.0838 = \bar{4}.9162$
$\therefore$ Antilog $(\bar{3}.0838)$ = Antilog $(\bar{4}.9162)$
= 0.0008246 Choice (C)

29. $\log\frac{8}{25} + 2\log\frac{15}{16} + 5\log\frac{24}{49}$
$= \log 2^3 - \log 5^2 + 2\log(3 \times 5)$
$-2\log 2^4 + 5\log(2^3 \times 3) - 5\log 7^2$
$= (3 - 8 + 15)\log 2 + (2 + 5)\log 3 + (2 - 2)\log 5 - 10\log 7$
$= 10\log 2 + 7\log 3 - 10\log 7$
$= 10 \times 0.3010 + 7 \times 0.4771 - 10 \times 0.8451$
$= 3.01 + 3.3397 - 8.451 = -2.1013$ Choice (B)

30. Let $N = (2205)^{25} = [(5)(441)]^{25} = [(3)^2(5)(7^2)]^{25} = 3^{50}\,5^{25}\,7^{50}$
$\therefore \log N = 50\log 3 + 25\log 5 + 50\log 7$
$= 50(0.4771) + 25(1 - 0.301) + 50(0.845) = 83.58$
The characteristic is 83
∴The given number has (83 + 1) or 84 digits
Ans : (84)

31. $\frac{1}{2}\log_2 P + 2\log_2 Q = 1 + \log_{\left(\frac{5}{10}\right)^4}\frac{10}{5}$

The second term on the RHS is
$\log_{(5/10)^4}(5/10)^{-1} = \frac{-1}{4}$

$\therefore \log_2 P^{\frac{1}{2}} + \log_2 Q^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \log_2 P^{\frac{1}{2}}Q^2 = \frac{3}{4}$

$\Rightarrow P^{\frac{1}{2}}Q^2 = 2^{\frac{3}{4}}$

$\Rightarrow (P^{\frac{1}{2}}Q^2)^4 = \left(2^{\frac{3}{4}}\right)^4$

$\Rightarrow P^2Q^8 = 8$ Choice (C)

32. $4^x \cdot 81^{1-x} = 50$
$\Rightarrow 2^{2x} \cdot 3^{4-4x} = 50$
Taking log on both sides, we get
$\log 2^{2x} + \log 3^{4-4x} = \log 50$
$\Rightarrow 2x(\log 2) + (4 - 4x)\log 3 = \log(100/2)$
$\Rightarrow (2x0.3010)x + (4 - 4x)(0.4771) = 2 - 0.3010$
$\Rightarrow 0.6020x + 1.9084 - 1.9084x = 1.699$
$\Rightarrow (1.9084 - 0.6020)x = 1.9084 - 1.699$
$\Rightarrow 1.3064x = 0.2094$
$\Rightarrow x = \frac{0.2094}{1.3064} = 0.16$ Choice (A)

33. $\log x \geq \log(x - 12) + \text{lgo}1.05$
$\log_{10}x \geq \log_{10}(x - 12) + \text{lgo}_{10}1.05$
$\log_{10}x - \log_{10}(x - 12) \geq \text{lgo}_{10}1.05$

$\log_{10}\frac{x}{x - 12} \geq \text{lgo}_{10}1.05$

$\frac{x}{x - 12} \geq 1.05$ (∵ If b is any strong base and $\log_b c \geq \log_b d$, it follows that $c \geq d$) - (1)
log x and log(x – 12) are only defined when their arguments are positive.
x and x – 12 must be positive. ∴ x > 12
Multiplying x – 12 to both sides of (1),
we get by x -12 x ≥ 1.05(x – 12)
(1.05)(12) ≥ 1.05x – x
252 ≥ x
The range of x is (12, 252] Choice (D)

34. $0 < \log_{x+3}(3x - 1) + \log_{3x-1}(x + 3) \leq 2$
Let $\log_{x+3}(3x - 1)$ be a.

$0 < a + \frac{1}{a} \leq 2$

$a + \frac{1}{a}$ is positive and at most 2. ∴ a must be positive.

$\frac{a^2+1}{a} \leq 2$
$a^2 + 1 \leq 2a$
$a^2 - 2a + 1 \leq 0$ i.e. $(a - 1)^2 \leq 0$
But $(a - 1)^2$ cannot be negative. ∴ $(a - 1)^2 = 0$
a = 1
∴ 3x − 1 = x + 3
x = 2. There is only 1 possible value for x. Ans : (1)

35. $\log_2 3, \log_x 3, \log_y 3$ are in HP
$\therefore \log_3 2, \log_3 x, \log_3 y$ are in Ap, i.e., 2, x, y are in GP
$\therefore 2(2^a\text{ -}7/2) = (2^a - 5)^2$
$\Rightarrow 2^{2a} - 10.2^a + 25 = 2.2^a - 7$
$\Rightarrow 2^{2a} - 12(2^a) + 32 = 0$
$\Rightarrow (2^a - 8)(2^a - 4) = 0$
$\Rightarrow 2^a = 8$ or $2^a = 4$
a = 3 or = 2
If a = 2 $2^a - 5$ would be negative
∴ a = 3 Choice (C)

Chapter – 9
(Permutations and Combinations)

Concept Review Questions

Solutions for questions 1 to 30:

1. $^{12}C_2 = \dfrac{12!}{10!\ 2!} = \dfrac{12\ (11)}{2\ (1)} = 66$ Choice (B)

2. $^{10}C_7 = \dfrac{10!}{3!\ 7!} = \dfrac{10\ (9)\ (8)}{3\ (2)\ (1)} = 120\ \left(\because {}^nC_r = \dfrac{n!}{(n-r)!}\right)$

Choice (C)

3. Given, $^nC_{12} = {}^nC_{18}$

$\Rightarrow n = 12 + 18 = 30\ (\because {}^nC_r = {}^nC_s \Rightarrow n = r + s)$

$\therefore {}^{30}C_3 = \dfrac{30!}{27!\,3!} = \dfrac{30\ (29)\ (28)}{3\ (2)\ (1)} = 4060$ Choice (A)

4. $^8P_3 = \dfrac{8!}{5!} = 8\ (7)\ (6) = 336.\ \left(\because {}^nP_r = \dfrac{n!}{(n-r)!}\right)$

Choice (B)

5. Given, $^nP_4 = {}^nP_5 \Rightarrow \dfrac{n!}{(n-4)!} = \dfrac{n!}{(n-5)!} \Rightarrow \dfrac{1}{n-4} = 1$

$\Rightarrow n - 4 = 1 \Rightarrow n = 5$ Ans : (5)

6. $^{12}C_3 = \dfrac{12\ (11)\ (10)}{3\ (2)\ (1)} = 220$ Choice (B)

7. $^{10}P_4 = \dfrac{10!}{6!} = 10\ (9)\ (8)\ (7) = 5040$ Ans : (5040)

8. $^nP_r = r!\ {}^nC_r$. Choice (C)

9. $^nC_{n-1} = {}^nC_1 \Rightarrow {}^{240}C_{239} = {}^{240}C_1 = 240.$ Ans : (240)

10. $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^8C_7 + {}^8C_6 = {}^9C_7.$ Choice (C)

11. We know if $^nC_r = {}^nC_s \Rightarrow r = s$ or $n = r + s$

$\therefore {}^nC_7 = {}^nC_3 \Rightarrow n = 7 + 3 = 10.$ Choice (A)

12. $^nP_r = 5040 = 10 \times 98 \times 7 = {}^{10}P_4$

$\Rightarrow n = 10, r = 4$ also $^{10}C_4 = 210 \quad \therefore r = 4$ Choice (C)

13. $^8P_3$. Choice (A)

14. $^7C_5$. Choice (B)

15. We can arrange the letters of the given word as shown below.

S						L

$\therefore$ The required number of ways = 5!. Choice (A)

16. All the given digits are even. Hence no odd number can be formed. Ans : (0)

17. There is only one even digit i.e., 6 fixing it at the units place and arrange the remaining four digits.

				6

This can be done in 4! ways. Choice (C)

18. The number of ways of preparing a garland with n different flowers is $\dfrac{(n-1)!}{2}$

$\therefore$ The required arrangements are $\dfrac{(21-1)!}{2}$. Choice (D)

19. In the given word letter A is repeated for two times and R also repeated for two times.

Hence the required number of arrangements possible are $\dfrac{7!}{2!\ 2!}$. Choice (B)

20. The required number of arrangements is 7! = 5040

Ans : (5040)

21. Every chocolate can be dealt with in 2 ways (Neha may give away the chocolate to one friend or the other).

$\therefore$ The required number of ways is 2^{10}. Choice (B)

22. Every element can be dealt with in 2 ways. It may or may not be included in a subset.

$\therefore$ The required number of non-empty subsets of A is $2^8 - 1 = 255$. Ans : (255)

23. Each letter can be posted in 4 ways.

$\therefore$ The required number of ways is 4^6. Choice (C)

24. A decagon has 10 vertices.

$\therefore$ The number of diagonals in a regular decagon is

$^{10}C_2 - 10 = 35$ Ans : (35)

25. We know that, n persons can be seated around a circular table in (n – 1) ! ways.

$\therefore$ 6 persons can be seated around a circular table in (6 – 1) ! = 5 ! = 120 ways. Ans : (120)

26. Out of 12 scholars, we can select 4 of them in $^{12}C_4$ ways.

$^{10}C_4 = \dfrac{12\times11\times10\times9}{1\times2\times3\times4} = 495$ ways. Choice (B)

27. An octagon has 8 vertices. Any 2 vertices can be selected in $^8C_2$ ways.

$= \dfrac{8\times7}{1\times2} = 28$ ways Ans : (28)

28. Number of ways of arranging '11' delegates around a table.

= (11 – 1)! = 10! = 3628800 Ans : (3628800)

29. The number of diagonals in a regular polygon of n sides is $\dfrac{n(n-3)}{2}$.

An duo decagon has 12 sides.

Hence number of diagonals is

$= \dfrac{12(12-3)}{2} = 54$ Ans : (54)

30. n persons can be arranged in a row in n! ways.

$\therefore$ 5 persons can be arranged in 5! Ways. Choice (C)

Exercise – 9(a)

Solutions for questions 1 to 35:

1. In the word THUNDER the vowels are E, U and the consonants are T, H, N, D, R. Of the seven places available, we have 4 odd places.

$\therefore$ The two vowels E and U can be arranged in these 4 places in $^4P_2 = 12$ ways.

Now, in the remaining 5 places the consonants T, H, N, D and R can be arranged in 5 ! or 120 ways.

Hence the required number of ways = 12 (120) = 1440

Ans : (1440)

2. In the word LIMPET, the vowels are I, E and the consonants are L, M, P, T.

Assume the two vowels I and E as a single unit.

This unit together with the 4 consonants can be arranged in 5! ways.

The two vowels can be arranged internally in 2! ways.

Hence, the required number of ways.

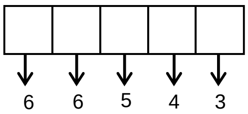
= 5! (2!) = 120 (2) = 240 Ans : (240)

3. In the word COUPLE the vowels are O, U, E and the consonants are C, P, L
–C–C–C–
First we arrange the three consonants. This can be done in 3! ways.
Now there are 4 slots in between them.
The 3 vowels can be arranged in these 4 slots in $^4P_3$ ways
$\therefore$Total number of ways = 3! $^4P_3$
= 3! 4! Choice (C)

4. (i) Number of ways of selecting at least one variety of ice-cream = $2^{12} - 1$
= 4096 – 1 = 4095 Choice (B)

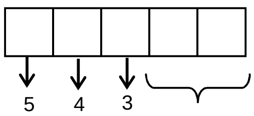
(ii) Number of ways of selecting at least 2 of them
= Total ways – number of ways of selecting at most one variety of ice-cream.
= $2^{12} - (^{12}C_0 + {}^{12}C_1)$ = 4096 – 13 = 4083
Choice (D)

5. The first place cannot be filled with 0

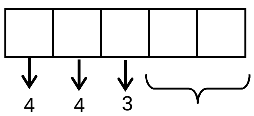


So, the first place can be filled in 6 ways.
Now, the remaining 4 places can be filled in 6, 5, 4 and 3 ways respectively.
Hence the number of five-digit numbers
= (6)(6)(5)(4)(3) = 2160 Ans : (2160)

6. If a number is divisible by 4, the possible combinations for the last two digits are.
04, 12, 16, 20, 24, 32, 36, 40, 52, 56, 60 and 64.
(i) There are 4 combinations in which the digit 0 occupies the fourth or fifth places.
Each such combination gives (5)(4)(3) = 60 five-digit numbers that are divisible by 4.



(ii) There are 8 combinations in which the digit 0 does not occur as either of the last two digits. Each such combination gives (4)(4)(3) = 48 five-digit numbers that are divisible by 4.



Hence, the required number of five-digit numbers
= (4)(60) + (8)(48)
= 240 + 384 = 624 Ans : (624)

7. Every post-card can be posted into any of the 6 letter boxes.
$\therefore$Each post card can be posted in 6 ways.
Hence, 10 post cards, can be posted into the 6 letter boxes in (6)(6)(6) …… (10 times) = 6^{10} ways. Choice (B)

8. Bow Side Stroke Side
2 2
Of the 10 persons available, 2 must be placed on the bow side and 2 others must be placed on the stroke side.

Now, of the remaining 6 persons, 3 must be accommodated on the bow side and the other 3 must be accommodated on the stroke side.
This can be done in $^6C_3$ ways.
Also, the five persons on the bow side can be arranged in 5! ways and the five persons on the stroke side can be arranged in 5! ways.
Hence, the required number of ways of arranging the crew is

$^6C_3\, 5!.5! = \dfrac{6!\,(5!)^2}{3!.3!}$ Choice (A)

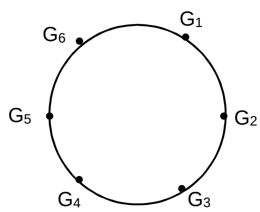
9. Of the 100 persons, 10 must be seated in the balcony and 15 other persons must be placed on the ground floor. Now, of the remaining 75 persons, 30 must be accommodated in the balcony and the other 45 persons must be accommodated on the ground floor.
This can be done in $^{75}C_{30}$ ways i.e., $\dfrac{75!}{45!30!}$ ways
Choice (D)

10. Assume the two Ns form a single unit
This unit together with the remaining 10 letters can be arranged in $\dfrac{11!}{2!\ 2!\ 2!\ 2!}$ ways Choice (B)

11. (i) The 20 distinct books can be divided equally among 4 girls in $\dfrac{20!}{(5!)^4}$ ways Choice (D)

(ii) The 20 distinct books can be divided equally into 4 parcels in $\dfrac{20!}{(5!)^4\,4!}$ ways. Choice (B)

12.



Six girls can be arranged around a circle in 5! ways. Now, the six boys can be arranged in the six places in between the girls in 6! ways.
Hence, the required number of ways = 5!6! Choice (C)

13. Of the 10 places available, there are 5 even places.
The digits 2,4,6,4 and 2 can be placed in these 5 even places in $\dfrac{5!}{2!2!}$ ways.
The remaining digits 1,3,1,1 and 9 can be placed in the remaining 5 places in $\dfrac{5!}{3!}$ ways.

Hence, the required number of ways = $\dfrac{5!}{2!2!}\dfrac{5!}{3!}$

$= \dfrac{(5!)^2}{(2!)^2\,3!}$ Choice (D)

14. (i) There are 3 Is, 2 Ts and one each of the letters A, N, B, O, C
The following are the ways of selecting 4 letters from these letters :
(1) All four are distinct letters: There are $^7C_4$ such selections.
(2) Three are 3 similar letters and one letter is of another kind: There are $^3C_3\,{}^6C_1$ such selections
(3) Two are similar and the other two letters are dissimilar: There are $^2C_1\,{}^6C_2$ such selections

(4) Two are similar of one kind, two others are of another kind. There are $^2C_2$ such selections.

Hence, the required number of selections
$= {}^7C_4 + {}^6C_1 + {}^2C_1\,{}^6C_2 + {}^2C_2 = 35 + 6 + 30 + 1$
= 72 Choice (D)

(ii) Number of arrangements that can be made by taking 4 letters.

$$= \left({}^7C_4\right)4! + {}^6C_1\left(\frac{4!}{3!}\right) + {}^2C_1\,{}^6C_2\left(\frac{4!}{2!}\right) + {}^2C_2\frac{4!}{2!2!}$$

= (35) (24) + (6) (4) + (30) (12) + (1) (6) = 1230
Choice (B)

15. digits are 3, 4, 5, 6. The number of digits and the corresponding number of numbers are tabulated below.

No. of digits	No. of numbers
1	4
2	$4^2 = 16$
3	$4^3 = 64$
4	$4^4 = 256$
Total	340

Ans : (340)

16. Given

Number of diagonals = $\frac{3}{2}$ (number of the sides)

$$\Rightarrow \frac{n(n-3)}{2} = \frac{3}{2}n$$

$\Rightarrow n = 6$ Choice (D)

17. The alphabetical order of the letters of the word TINSEL is E, I, L, N, S, T.
The word LISTEN is preceded by the words which begin with E, I, LE, LIE, LIN, LISE and LISN.
These are 5!, 5!, 4!, 3!, 3!, 2! and 2! in number respectively.
Hence, the rank of the word
LISTEN = 2(5!) + 4! + 2(3!) + 2(2!) + 1
= 240 + 24 +12 + 4 + 1 = 281 Ans : (281)

18. Sum of all the numbers
= (n – 1) ! (Sum of the digits)(111.. ……1)
(n times)
= (5 – 1) ! (1 + 2 + 5 + 8 + 9)(11111)
= 4! (25)(11111) = 6666600 Choice (D)

19. Suppose the digit in the right most position of a number is 1 or 5. Then, the two odd positions in the remaining 4 positions must be occupied by two even digits.
This can be done in $^3C_2 \times 2! \times 2! = 12$ ways.
So, the sum of the digits in the right most position of the numbers in this case is 12 × 1 + 12 × 5 = 72.
Now, let the digit in the right most position of a number be 2, 4 or 8. Then, the remaining two even digits must occupy an even and an odd position.
This can be done in $({}^2C_1 \times {}^2C_1 \times 2!) \times 2! = 16$ ways.
So, the sum of the digits in the right most position of the numbers in this case is 16 × 2 + 16 × 4 + 16 × 8 = 224.
Hence, the required sum is 72 + 224 = 296.
Ans : (296)

20. The maximum possible number of points of intersection of diagonals inside a convex polygon of n sides is given by $^nC_4$.
Here n = 9,

and $^9C_4 = \frac{9\,(8)\,(7)\,(6)}{2\,(3)\,(4)} = 126$ Ans : (126)

21. The required number of triangles = $T - N_1 - N_2$
Where
T = Total number of triangles.
N_1= Number of triangles with 1 side in common with the polygon.
N_2 = Number of triangles with 2 sides in common with the polygon.
$= {}^{15}C_3 - ({}^{15}C_1\,{}^{11}C_1) - 15 = 275.$ Ans : (275)

22. Each question can be attempted in 4 ways.
$\therefore$ the number of ways of attempting the entire paper = 4^{12}
Choice (C)

23. For each problem, the candidate can either answer the question or leave it out, i.e., there are 3 ways of dealing with each problem and hence 3^{12} ways of dealing with the 12 problems. This includes the case that the candidate leaves out all the problems. Therefore, the number of ways of attempting one or more problems is $3^{12} - 1$.
Choice (D)

24. The number of ways of dealing with the 6 copies of a biography = 7 (one way of not giving away and 6 ways of giving away). Similarly, the number of ways of dealing with the 5 copies of the autobiography and the 4 copies of the novel are 6 and 5 respectively. Hence, the number of ways of giving away one or more books = (6 + 1) (5 + 1) (4 + 1) – 1 = 210 – 1 = 209. Ans : (209)

25. The number of direct lines required between any two villages belonging to the same zone is
$3 \times \left({}^4C_2 \times 3\right) = 54$
The number of direct lines required between any two villages belonging to the different zones is
${}^3C_2\left({}^4C_2 \times {}^4C_1 \times 2\right) = 96$
Hence the required number of direct lines is 54 + 96 = 150.
Choice (D)

26. Let m and n be the number of girls and the number of boys respectively, then

$^mC_2 = 300 \Rightarrow \frac{m(m-1)}{2} = 300 \Rightarrow m(m-1) = 600$

$\Rightarrow m = 25.$

Also $^nC_2 = 105$

$\frac{n(n-1)}{2} = 105 \Rightarrow n(n-1) = 210 \Rightarrow n = 15$

Hence the number of games in which one player is a boy and the other is a girl is
$^{25}C_1 \times {}^{15}C_1 = 25\ (15) = 375$ Choice (C)

27. The first stripe can be of any colour. Every stripe there after must be of a different colour from that of the preceding stripe.

4	3	3	3	3	3	3

$\therefore$ The advertisement board can be designed is $4 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2916.$ Choice (B)

28. Given 1,000,000 < K < 10,000,000
The sum of the digits in K is 3. The possibilities for K are tabulated below

First digit	Other digits	No.of numbers
1	Two is, four 0s	15
1	One 2, five 0s	6
2	One 1, five 0s	6
3	Six 0s	1
		28

Ans : (28)

29. We have to fill the boxes with blue and green colour balls such that no two adjacent boxes contain green coloured balls. So, the number of ways of filling up the boxes with all blue balls is 1. viz., BBBBBB.

The number of ways of filling up the boxes with 5 blue and 1 green balls is 6. viz., BBBBBG, BBBBGB, BBBGBB, BBGBBB, BGBBBB, GBBBBB.

The number of ways of filling up the boxes with 4 blue and 2 green balls is ${}^5C_2$ = 10. (By arranging the 4 blue balls in a row we can generate 5 positions between them and the 2 green balls can be placed in these 5 positions in ${}^5C_2$ ways).

The number of ways of filling up the boxes with 3 blue and 3 green balls is ${}^4C_3$ = 4.

(By arranging the 3 blue balls in a row, we can generate 4 positions between them and the 3 green balls can be placed in these 4 positions in ${}^4C_3$ ways).

In the remaining combinations it is not possible to arrange the balls in the desired pattern.

Hence, the required number of ways of filling up the boxes is 1 + 6 + 10 + 4 = 21. Ans : (21)

30. (i) 2 points determine a line.

$\therefore$ 12 points can give ${}^{12}C_2$ straight lines.

But 4 points are collinear. They give only one line.

$\therefore$ The number of straight lines that can be formed

$= {}^{12}C_2 - {}^4C_2 + 1$

= 66 – 6 + 1= 61 Choice (B)

(ii) Three points determine a triangle.

$\therefore$12 non-collinear points can give ${}^{12}C_3$ triangles.

But 4 points are collinear. So, these 4 points do not give any triangles.

Hence, the number of triangles.

= ${}^{12}C_3$ - ${}^4C_3$ = 216 Choice (B)

31. We know that, the number of non-negative integral solutions of the equation $x_1 + x_2 + \ldots.. + x_k = n$ is ${}^{(n+k-1)}C_{k-1}$.

$\therefore$The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 = 10$ is ${}^{(10+4-1)}C_{4-1} = {}^{13}C_3$ = 286. Choice (A)

32. We know that if there are m horizontal lines and n vertical lines the number of ways of travelling from one corner to another corner is ${}^{m+n}C_n$

Here m = 5; n = 5

Required number of paths ${}^{5+5}C_5 = {}^{10}C_5 = 252$

Ans : (252)

33. The number of positive integral solutions of the equation of the form $x_1 + x_2 + \ldots.. + x_n = n$ is ${}^{n-1}C_{k-1}$.

$\therefore$The number of positive integral solutions of the equation x + y + z + t = 25 is ${}^{25-1}C_{4-1} = {}^{24}C_3$ = 2024. Choice (A)

34. We need to select 3 rows and 3 columns out of the 5 rows and 6 columns. This can be done in ${}^5C_3{}^6C_3$ or 10(20) or 200 ways. Each such selection P corresponds to 6 ways of placing the coins. Say we choose R_1, R_2, R_3 and C_1, C_2, C_3. To get the cell, we need to fix both the row and column. R_1 can be associated with any of the 3 columns. Then R_2 can be associated with either of the other 2 columns. (R_3 would pair up with the remaining columns)

$\therefore$ The total number of ways of placing the coins is 200(6) = 1200.

Alternative Method

The first coin can be placed in any of the 30 cells. (5×6)

The next one can be placed in any of the 20 cells (4×5) that remain after we delete the row and column in which the first coin is placed.

The third coin can be placed in any of the 12 cells (3×4)

$\therefore$ The 3 coins can be placed in 30(20) (12) ways. But as the coins are identical, each distinguishable way of placing the coins has been counted 3! or 6 times in this expression. The number of ways is 30(20)(12)/6 or 1200.

Choice (A)

35. Let there are be n lines in a plane of which no two lines are parallel and no three lines are concurrent. The number of regions that these x lines divide the plane is is $(\sum_1^n i) + 1$

$= \frac{8.(9)}{2} + 1 = 37$ Ans : (37)

Exercise – 9(b)

Solutions for questions 1 to 40:

1. ${}^{30}C_{10}$ (Direct result). Choice (B)

2. Each prize can be distributed in 3 ways.

So, the 6 prizes can be distributed in 3^6 ways.

Choice (A)

Note: The statement "eligible to receive one or more prizes" can be treated on par with "each boy can receive any number of prizes". Just because someone is eligible to receive one or more prizes, doesn't mean that they are entitled to receive at least one prize. It is not to be confused with "each boy should receive at least one prize".

3. First, the persons can be divided into two groups consisting of 4 persons and 6 persons in $\frac{10!}{4!.6!}$ ways.

Then, the two groups can be arranged around the two circular tables in 3! and 5! ways. Hence, the required number of ways is $\frac{10!}{4!.6!} \times (3! \times 5!) = \frac{10!}{24} = \frac{10!}{4!}$ Choice (B)

4. The required number of ways is $\frac{(n-1)!}{2} = \frac{14!}{2}$

Choice (D)

5. Every question can be attempted in 2 ways.

So, the entire paper can be attempted in 2^{15} ways.

Choice (C)

6. There are 3 vowels (I, EA) and 4 consonants (S, P, C, L) in the word SPECIAL.

Of the 7 places required to arrange the 7 letters, there are 4 odd places (1st, 3rd, 5th and 7th) and the 3 vowels can be arranged in these 4 places in ${}^4P_3$ ways.

The remaining 4 consonants can be arranged in the remaining 4 positions in 4! ways.

Hence, the required number of ways is ${}^4P_3 \times 4!$ = 576.

Ans : (576)

7. (i) The number of ways of inviting at least one friend is $2^{12} - 1 = 4095$. Choice (B)

(ii) The number of ways of inviting at least 10 friends is ${}^{12}C_{10} + {}^{12}C_{11} + {}^{12}C_{12}$ = 66 + 12 + 1 = 79.

Choice (B)

8. There are four vowels viz., A, I, O, U and three consonants viz., V, R, S in the word "VARIOUS".

The possible ways of arranging the letters so that the vowels and consonants appear alternately is VCVCVCV. This can be done in $4! \times 3!$ = 144 ways. Ans: (144)

9. There are 3 E's, 2 D's, 2 S's and 1 each of the letters A, R in the word ADDRESSEE.

The possible combinations of selecting 4 letters are as follows:

(1). <u>All the four letters are distinct:</u> The number of distinct letters in the word is 5.

So, 4 letters can be selected in ${}^5C_4$ = 5 ways.

(2) 3 similar letters and 1 different letter: In this case, we select all the three E's and one out of the remaining 4 letters.

This can be done in $^4C_1$ = 4 ways.

(3) 2 similar letters and 2 distinct letters : In this case, we can select 2E's, 2D's or 2S's and 2 distinct letters from the remaining 4 letters.
This can be done in 3 × $^4C_2$ = 18 ways.

(4) 2 pairs of similar letters : In this case, we can select 2E's and 2D's or 2D's and 2S's or 2E's and 2S's.
So, there are 3 ways in this case.
Hence, the total number of ways of selecting 4 letters is 5 + 4 + 18 + 3 = 30. Ans : (30)

10. In each of the cases in the above problem, we have to arrange the 4 letters selected.
This can be done in,

$$^5C_4 \times 4! + {}^4C_1 \times \frac{4!}{3!} + 3 \times {}^4C_2 \times \frac{4!}{2!} + 3 \times \frac{4!}{2!2!}$$

= 120 + 16 + 216 + 18 = 370. Choice (B)

11. We know that a number will be divisible by 3 only when the sum of the digits of the number is divisible by 3.
Now, the possible combinations of four digits that can be made by using the digits 0, 1, 2, 3 and 8 are: 0123, 0128, 0138, 0238 and 1238. The sum of the digits of the combinations 0123 and 0138 are only divisible by 3.
The number of 4-digit numbers that can be formed with each of these combinations is 3 × 3 × 2 × 1 = 18.
Hence, the required number is 18 + 18 = 36. Ans : (36)

12. (i) The number of ways of dividing 16 books equally among 4 boys is $\frac{16!}{4!.4!.4!.4!} = \frac{16!}{(4!)^4}$ Choice (A)

(ii) The number of ways of dividing 16 books equally into 4 parcels is $\frac{16!}{4!.(4!)^4} = \frac{16!}{(4!)^5}$ Choice (C)

13. The alphabetical order of the letters of the word ARISE is A, E, I, R, S.
The number of words which begin with A, E, I, RAE, RAIE are respectively. 4!, 4!, 4!, 2! and 1!. The next word is the required word RAISE.
Hence, the rank of the word RAISE is 24 + 24 + 24 + 2 + 1 + 1 = 76. Ans : (76)

14. We know that, the sum of all n-digit numbers that can be formed using n-digits is

$$(n-1)!\ (\text{sum of the digits}) \times \underbrace{11....1}_{\text{n times}}$$

∴ The required sum is (1 + 3 + 4 + 6 + 8) × 11111
= 5866608 Choice (C)

15. The maximum possible number of points of intersection of diagonals inside the polygon is 495

$\therefore {}^nC_4 = 495 \Rightarrow \frac{n\,(n-1)\,(n-2)\,(n-3)}{2\,(3)\,(4)} = 495$
= 5 (9) (11)
$\Rightarrow$ n (n –1) (n –2) (n –3) = 12 (11) (10) (9)
$\Rightarrow$ n = 12. Ans : (12)

16. The required number of triangles = $N_1 + N_2$.
= $({}^{10}C_1\ {}^6C_1) + 10 = 70$. Ans : (70)

17. Prof. Balamurali and Asst. B of Sheshadri cannot be included together in the delegation.
Hence, the number of ways of forming the delegation is $^{10}C_5 . {}^6C_3 - {}^9C_4 . {}^5C_2$ Choice (C)

18. The number of direct lines required between any two villages belonging to the same zone is
4 × ($^4C_2$ × 2) = 48.
The number of direct lines required between any two villages belonging to different zones is
$^4C_2 \times ({}^4C_1 \times {}^4C_1) \times 1 = 96$.
Hence, the required number of direct lines is 48 + 96 = 144.
Ans : (144)

19. Let m, n be the number of girls and the number of boys respectively.

Then, $^mC_2 = 66$

$\Rightarrow \frac{m(m\ 1)}{2!} = 66. \quad \Rightarrow m = 12$

Also, $^mC_1 \times {}^nC_1 = 240$
$\Rightarrow$ n = 20.
Hence, the number of games in which both the players were boys is $^{20}C_2 = \frac{20(19)}{2} = 190$. Ans : (190)

20. The number of 5–digit numbers which begin with 1 and 2134 are 4!, and1! respectively
Hence, the rank of the word 21354 = 4! + 1! + 1
= 24 + 1 +1 = 26 Choice (A)

21. The total number of matches played is $\frac{m(m-3)}{2}$

Given, the total time taken for all the matches is 30 minutes.

$\therefore \frac{m(m-3)}{2} \times 1\frac{1}{2} = 30$

$\Rightarrow \frac{m(m-3)}{2} \times \frac{3}{2} = 30$

$\Rightarrow$ m (m – 3) = 40 = 8 x 5
∴ m = 8. Ans : (8)

22. The possibilities for k are: 1100000000, 1010000000, 1001000000, 100100000, 1000010000, 1000001000, 1000000100, 1000000010, 1000000001 and 2000000000 i.e., there are 10 possible values of k. Choice (A)

23. We know that the points lying on a straight line do not form triangles.
Hence, the number of triangles that can be formed by using the given points is
$^{19}C_3 - ({}^9C_3 + {}^{10}C_3) = 765$. Ans : (765)

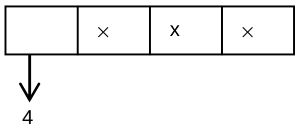
24. The number of line segments that can be drawn by joining 4 points with each of the remaining 6 points is 4 × 6 = 24.
Since each of the remaining 6 points are joined to exactly 5 points, the 6 points must form 3 pairs and the points in each pair must be joined by a line.
Hence, the total number of lines drawn in the plane were 24 + 3 = 27. Choice (A)

25. To travel from the point P to the Q, one must use 5 horizontal roads and 3 vertical roads. This can be done in $\frac{8!}{5!3!} = 56$ ways. Choice (C)

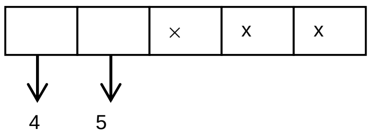
26. There can be at most 2 Hs in each string. Suppose there is no H in the strings. Then, the number of strings that can be formed is 9 × 9 × 9 × 9 = 6561. Suppose there is exactly one H in each string.
Then, the letters H and A must occupy the 1st, 2nd or 2nd, 3rd or 3rd, 4th positions. So, the number of strings in this case is 3 × (9 × 9) = 243. Now, let there be two H's in each string.

Then, the only string that is possible in this case is HAHA. Hence, the required number of strings is 6561 + 243 + 1 = 6805. Ans : (6805)

27. A number is divisible by 125 if and only if the number formed by its last three digits is divisible by 125. So, the possible combinations for the last three digits are: 000, 125, 250 and 500. Now, the number of three-digit numbers divisible by 125 is 3.
The number of four-digit numbers divisible by 125 is 4 (4) = 16.



the number of five-digit numbers divisible by 125 is 4 (4) (5) = 80.



Hence, the required number of numbers is 3 + 16 + 80 = 99. Choice (C)

28. The possible ways of arranging the pencils are: RBWBWBR, WBRBRBW, BWBRWRB, BRBWRWB, BWRBWRB, BWRRBRWB, BRWBRWB, BRWBWRB, BWRWBRB and BRWBBWB
i.e. a total of 10 possible arrangements. Choice (D)

29. As the number is an even number, the last digit can be 0, 2, 4, 6 or 8.

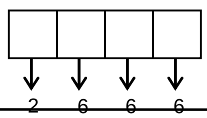
<u>The number of 8-digit telephone numbers which begin with 270 and end with 0</u>:
In this case, 270 _ _ _ _ 0 of the remaining four places, two places must be occupied by the digits 0 and 9 and the remaining places can be occupied by any of the digits other than 0 and 9. This can be done in $({}^4C_2 \times 2!) \times 8 \times 8 = 768$ ways.

<u>The number of 8-digit telephone numbers which begin with 270 and end with 2, 4, 6 or 8</u>:

In this case, of the remaining four places, 2 places must be 270 _ _ _ _ _ occupied by '0', one place by '9' and the remaining place by any one of the remaining 8 digits.
This can be done in $4 \times \left({}^4C_3 \times \frac{3!}{2!}\right) \times 8 = 384$ ways.
Similarly, the number of 8-digit telephone numbers which begin with 279 and end with 0 is 279 _ _ _ _ 0.
${}^4C_2 \times 8 \times 8 = 384$.
The number of 8-digit telephone numbers which begin with 279 and end with 2, 4, 6 or 8 is $4 \times {}^4C_3 \times 8 = 128$.
Hence, the required number of telephone numbers is 768 + 384 + 384 + 128 = 1664. Choice (B)

30. Let the number of red, blue and green balls selected from the supply be x_1, x_2 and x_3 respectively.
Then, $x_1 + x_2 + x_3 = 12$. The number of non-negative integral solutions of the above equation is
${}^{(n+k-1)}C_{k-1} = {}^{14}C_2 = 91$. Ans : (91)

31.



The first place can be filled with 4 or 5 in 2 ways.
Hence the required number of 4-digit numbers that lie between 4000 and 6000 is (2)(6)(6)(6) i.e., 432
Choice (B)

32. Of the 15 persons, 7 must be arranged around one table and the other 8 must be arranged around the other table.
This can be done in ${}^{15}C_7\, 6!7!$ ways.
i.e., $\frac{15!}{7!\,8!} 6!\,7!$ or $\frac{15!\,6!}{8!}$ ways. Choice (B)

33. (i) The number of positive integral solutions of the equation $x_1 + x_2 + \ldots.. + x_n = n$ is ${}^{n-1}C_{k-1}$.
$\therefore$The number of positive integral solutions of the equation $a + b + c = 15$ is ${}^{15-1}C_{3-1}$
$= {}^{14}C_2 = 91$. Choice (C)

(ii) The number of non-negative integral solutions of the equation $p + q + r + s = 30$ is ${}^{30+4-1}C_{4-1} = {}^{33}C_3$
$= \frac{33 \times 32 \times 31}{3 \times 2 \times 1} = 5456$. Choice (D)

(iii) Given that $A + B + C + D \le 15$ ----- (1) and A, B, C, D are non negative integers.
Let $A + B + C + D = 15 - E$ where E is an integer satisfying $0 \le E \le 15$
$A + B + C + D + E = 15$ ----- (2)
Here A, B, C, D, E are non negative integers.
The solutions of (2) is ${}^{(15+5-1)}C_{(5-1)} = {}^{19}C_4$
Choice (D)

34. Let B_1, B_2, B_3 be the three boxes 5 letters can be posted in these boxes in the following way.

B_1	B_2	B_3	No. ways
1	1	3	${}^5C_1\,{}^4C_1\,{}^3C_3 = 20$
1	3	1	${}^5C_1\,{}^4C_3\,{}^1C_1 = 20$
3	1	1	${}^5C_3\,{}^2C_1\,{}^1C_1 = 20$
1	2	2	${}^5C_1\,{}^4C_2\,{}^2C_2 = 30$
2	1	2	${}^5C_2\,{}^3C_1\,{}^2C_2 = 30$
2	2	1	${}^5C_2\,{}^1C_1\,{}^1C_1 = 30$

Hence, the total number of ways = 3(20) + 3(30) = 150
Ans : (150)

35. Given that $x + y + z = 20$ where $x \ge 2$, $y \ge 3$, $z \ge 4$.
Let $x = x + 2$, $y = y + 3$, $z = z + 4$
$\Rightarrow x + y + z = 20 - (2 + 3 + 4) = 11$ ------ (1)
and $x \ge 0$, $y \ge 0$, $z \ge 0$. We know that the number of non-negative integral solutions of the equation of the form $x_1 + x_2 + \ldots.. + x_n = n$ is ${}^{n+k-1}C_{k-1}$.
The number of non-negative integral solutions of (1) is ${}^{11+3-1}C_{3-1} = {}^{13}C_2 = 78$. Ans : (78)

36. The number of handshakes between group A and group B is = 8(6) = 48
Similarly the number of handshakes between group B and group C is 6n
The number of handshakes between group C and group A is 8n
Given total number of handshakes = 104
$\therefore$ 48 + 6n + 8n = 104 $\Rightarrow$14n = 56 $\Rightarrow$ n = 4 Ans : (4)

37. The lines, the number of bounded and unbounded regions they create are tabulated below.

Lines	B	U
1 to 4	0	5
5^{th}	0	5
6^{th}	4	2
–	–	–
15	13	2

The 8 rows for lines 7 to 14 have not been shown explicitly.
$\therefore$ The total number of bounded regions is 4 + 5 + … + 13 = 91 – 6 = 85
The total number of unbounded regions is 5 + 5 + 2(10) = 30
The total number of regions is 85 + 30 or 115
Choice (C)

38. 6 prizes have to be distributed among 4 athletes, such that each gets at least one. Either the split (of the prizes) is 3, 1, 1, 1 or 2, 2, 1, 1. There are 4 cases corresponding to the first split (Any one of the 4 athletes could get 3 prizes) and 6 cases corresponding to the second split (Any two of the 4 athletes could get 2 prizes each) consider the split 3, 1, 1, 1. The number of ways of selecting the prizes to be given to one athlete is ${}^6C_3$. The other 3 prizes can be distributed among the other 3 athletes in ${}^3P_3$ or 3! ways.

$\therefore$ There are ${}^6C_3\ {}^3P_3$ or 20(6) or 120 ways.
Consider the second split 2, 2, 1, 1. The number of ways of selecting the prizes for each of this splits is ${}^6C_2\ {}^4C_2\ {}^2C_1$ (or ${}^6C_2\ {}^4C_2\ {}^2P_2$) which is 15(6)(2) or 180.
$\therefore$ The total number of ways in which the prizes might get distributed is 4(120) + 6(180) = 1560. Ans : (1560)

39. Out of 6 letters any two letters can be placed into their corresponding envelopes in ${}^6C_2$ ways.

Of the remaining 4 letters, no letter should be placed into its corresponding envelope. This can be done in $4!\left[\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right]$, viz 9 ways

$\therefore$ The total number of ways that 6 letters are placed into envelopes such that exactly two letters are placed into their corresponding envelopes is $9{}^6C_2 = 9\ (15) = 135$
Ans : (135)

40. The greatest number is 19. We select the remaining seven numbers from 8 to 18. This can be done in ${}^{11}C_7$ways. These 8 numbers can be permuted in 8! ways
$\therefore$ Total number of arrangements is ${}^{11}C_7\ .8! = {}^{11}C_4\ .8! = (330)8!$ Choice (B)

Chapter – 10
(Probability)

Concept Review Questions

Solutions for questions 1 to 30:

1. When three dice are rolled the possibilities that the total score is greater than 16 are as follows.
(5, 6, 6), (6, 6, 5) (6, 5, 6) and (6, 6, 6)
Hence total number of possible cases are 4. Ans : (4)

2. When 'n' dice are rolled total possible outcomes are 6^n.
There is only one favourable case that all dice shows the number '1'.

$\therefore$Required probability is $\frac{1}{6^n}$ Choice (C)

3. When five coins are tossed the possible outcomes are $2^5 = 32$. Ans : (32)

4. If A and B are independent events, then P (A $\cap$ B) = P (A) . P (B). = $\frac{1}{3}\times\frac{2}{7}=\frac{2}{21}$. Choice (D)

5. When four dice are rolled, out comes are $6^4 = 1296$.
If total score on the dice is maximum, then all the dice shows 6 only.
This is occur only once.
$\therefore$Required probability is = $\frac{1}{1296}$. Choice (A)

6. Since A and B are mutual exclusive and exhaustive events, the non occurrence of A is equal to the occurrence of B.
$P(B) = P(\overline{A}) = \frac{3}{4}$. Ans : (0.75)

7. Total number of balls is 5 + 2 + 3 = 10.
One ball can be drawn from 10 balls in ${}^{10}C_1$ ways.
One green ball can be drawn from 3 green balls in ${}^3C_1$ ways.

$\therefore$ The required probability is $=\frac{3}{10}$. Ans : (0.3)

8. When a dice is rolled, the n dice shows either an even or an odd number. It is a sure event
$\therefore$ Required probability = 1. Ans : (1)

9. We know that
$P(A \cup B) = P(A) + P(B) – P(A \cap B)$
$= \frac{3}{4}+\frac{7}{10}-\frac{3}{5}=\frac{15+14-12}{20} = \frac{17}{20}$. Choice (A)

10. One card can be drawn from 52 cards in ${}^{52}C_1$ ways.
One red card can be drawn from 26 red cards in ${}^{26}C_1$ ways.

$\therefore$ The required probability $=\frac{26}{52}=\frac{1}{2}$. Ans : (0.5)

11. When 7 coins are tossed the total out comes are 2^7.
There are only two favourable out comes that all are shows head or all shows tail.

$\therefore$ Required probability = $\frac{2}{2^7}=\frac{1}{2^6}$ Choice (D)

12. The probability of occurrence of A is $\frac{3}{7}$. i.e., $P(A) = \frac{3}{7}$

$\therefore P(\overline{A}) = 1 – P(A) = 1 – \frac{3}{7} = \frac{4}{7}$. Choice (A)

13. The sum of all probabilities in an experiment is 1.
Ans : (1)

14. In a non – leap year February month have 28 days.
i.e., every day occur 4 times.
$\therefore$ 5 Sundays is not possible.
Probability = 0. Ans : (0)

15. Letters can arranged in 4 envelops in 4! Ways but all the letters can be inserted into their respective envelops in only one way.

$\therefore$ The required probability = $\frac{1}{4!}$ Choice (C)

16. In the above problem if one letters goes to a wrong addressed envelop then at least some other letter also goes to a wrongly addressed envelop.
$\therefore$ It is an impossible event hence its probability = 0
Ans : (0)

17. The possible outcomes are HH, HT, TH and TT of which the outcomes HT and TH are favourable.

$\therefore$ The required probability is $\frac{2}{4}=\frac{1}{2}$ Ans : (0.5)

18. 4 and 6 are the composite numbers from 1 to 6.

$\therefore$ The required probability is $\frac{2}{6}=\frac{1}{3}$ Choice (B)

19. There are 21 consonants in the English alphabet.

$\therefore$ The required probability is $\frac{21}{26}$ Choice (C)

20. There are 2 black kings in a pack of cards.

$\therefore$ The required probability is $\frac{2}{52}=\frac{1}{26}$ Choice (D)

21. P (both the dice show even numbers) = $\frac{3}{6}\times\frac{3}{6}=\frac{1}{4}$

Ans : (0.25)

22. Four cards can be drawn from a pack in ${}^{52}C_4$ ways.
Let 'E be the event of each card being an ace.
This can be done in ${}^{4}C_4$ i.e., 1 way.

So $P(E)=\frac{1}{{}^{52}C_4}$ Choice (C)

23. In a pack of 52 cards, there are two black jacks.
$\therefore n(E) = {}^{2}C_1$
$n(S) = 52$

$\therefore$ Probability = $\frac{{}^{2}C_1}{52}=\frac{1}{26}$ Choice (B)

24. The possible pairs whose product equals 12 are (2, 6) (3, 4), (4, 3) (6, 2). These are 4 in number.
The total number of outcomes when 2 dices are rolled is $6 \times 6 = 36$.
Hence the probability = 4/36 = 1/9 Choice (A)

25. Given set = {1, 2, 3, 4, 5, 6, 7, 8, 9}
From this set multiples of 3 are {3, 6, 9}
n(E) = 3
n (S) = 9

Hence probability = $\frac{3}{9}=\frac{1}{3}$ Choice (A)

26. Total cases $6 \times 6 = 36$
E = {(1, 2) (2, 1)}
$\therefore$ Favourable case = 2

$\therefore$ Probability = $\frac{2}{36}=\frac{1}{18}$ Choice (B)

27. There are 52 complete weeks in a year.
The remaining one day (365 – 52 x 7) can be any day of the week

$\therefore$P(a non-leap year has 53 Mondays) = $\frac{1}{7}$ Choice (D)

28. We know that, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\therefore P(A\cap B) = \frac{1}{2}+\frac{5}{12}-\frac{7}{12}=\frac{6+5-7}{12}=\frac{4}{12}=\frac{1}{3}$

Choice (D)

29. Since A, B and C are mutually exclusive and exhaustive events, $P(A\cup B \cup C) = 1$ $P(A) + P(B) + P(C) = 1$

$\therefore P(C) = 1 - 0.2 - 0.6 = 0.2$ Ans : (0.2)

30. P(both A and B are short listed) = $P(A \cap B) = P(A).\ P(B)$

[$\because$ Since A and B are independent events]

$= \frac{3}{7}\times\frac{1}{7}=\frac{3}{49}$ Choice (D)

Exercise – 10(a)

Solutions for questions 1 to 35:

1. Three consecutive letters can be selected from the English alphabet in 24 ways.
The possible ways that the three consecutive letters selected are all consonants are BCD, EGH, JKL, KLM, LMN, PQR, QRS, RST, VWX, WXY, XYZ
i.e. there are 11 possible ways.

$\therefore$ Required probability = $\frac{11}{24}$ Choice (B)

2. Three letters can be selected from the 26 letters in ${}^{26}C_3$ ways

(i) 3 consonants can be selected from the 21 consonants in ${}^{21}C_3$ ways.
$\therefore$ the probability that the three letters are consonants
$= \frac{{}^{21}C_3}{{}^{26}C_3}=\frac{133}{260}$ Choice (D)

(ii) One consonant can be selected from the 21 consonants in ${}^{21}C_1$ ways and two vowels can be selected from the 5 vowels in ${}^{5}C_2$ ways.
$\therefore$ The number of ways of selecting one consonant and two vowels = ${}^{21}C_1\,{}^{5}C_2 = 210$

$\therefore$ Required probability = $\frac{210}{{}^{26}C_3}=\frac{21}{260}$

Choice (C)

3. When 'n' coins are tossed together, the probability of getting exactly 'r' tails = $\frac{{}^{n}C_r}{2^n}$

$\therefore$The probability of getting at least two tails
= 1 – (probability of getting no tail + probability of getting exactly one tail)

$= 1 - \left\{\frac{{}^{6}C_0}{64}+\frac{{}^{6}C_1}{64}\right\} = 1-\frac{7}{64}=\frac{57}{64}$ Choice (D)

4. When 8 coins are tossed together, the total number of outcomes = 2^8
In order to have the number of heads more than the number of tails, we must get 5 heads, 6 heads, 7 heads or 8 heads.
$\therefore$Required probability
= p(5 heads) + p(6 heads) + p(7 heads) + p(8 heads)
$= \frac{{}^{8}C_5}{2^8}+\frac{{}^{8}C_6}{2^8}+\frac{{}^{8}C_7}{2^8}+\frac{{}^{8}C_8}{2^8} = \frac{56+28+8+1}{2^8}=\frac{93}{256}$
Choice (C)

5. Total possible outcomes = $2^4 = 16$
Favourable cases are, {HTHT, THTH}

$\therefore$ Required probability = $\frac{2}{16}=\frac{1}{8}$ Ans : (0.125)

6. Total possible outcomes = $6 \times 6 = 36$
Let E be the event that the number in the first trial is a factor of that obtained in the second trial.
Then, favourable cases for the event E are
{(1,1), (1, 2) (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)} i.e., there are 14 favourable cases

$\therefore$Required probability = $\frac{14}{36}=\frac{7}{18}$ Choice (C)

7. Total possible outcomes = $6^4 = 1296$
In the following cases the total score on the four dice is greater than or equal to 22, 6664, 6655, 6665, 6666

Now, the combination 6664 appears in $\frac{4!}{3!}$ = 4 ways.

the combination 6655 appears in $\frac{4!}{2!\,2!}$ = 6 ways

the combination 6665 appears in $\frac{4!}{3!}$ = 4 ways and the combination 6666 appears in 1 way

$\therefore$Total number of unfavourable cases = 4 + 6 + 4 + 1 = 15

Hence the required probability = $1 - \frac{15}{1296} = \frac{1281}{1296} = \frac{427}{432}$

Choice (C)

8. A number can be selected from the natural numbers 1 to 30 in $^{30}C_1$ ways

Let E be the event that the number is divisible by 4 or 7 then, favourable cases for E are

= {4, 7, 8, 12, 14, 16, 20, 21, 24, 28}

$\therefore$ Required probability = $\frac{10}{30} = \frac{1}{3}$ Choice (D)

9. Total number of squares in a 8×8 chess board

$= 1^2 + 2^2 + 3^2 + \ldots\ldots + 8^2 = 204$

Number of squares of size $3 \times 3 = 6 \times 6 = 36$

Hence, required probability = $\frac{36}{204} = \frac{3}{17}$ Choice (C)

10. (i) There is no way of placing exactly one letter wrongly

$\therefore$Its probability is 0 Ans : (0)

(ii) If seven letters are placed into seven right envelopes, the eighth letter is also placed into right envelope.

$\therefore$ (at least seven letters are placed in to right envelopes) = P(all the 8 letters are placed into right envelopes) = $\frac{1}{8!}$ Choice (A)

(iii) P(at least two letters are placed wrongly)

= 1 – [P(exactly one letter is placed in a wrong envelope) + P (all the letters are placed into the right envelops)] = $1 - \left(0 + \frac{1}{8!}\right) = 1 - \frac{1}{8!}$ Choice (C)

(iv) P(none of the right letters are placed into their corresponding envelopes)

$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \frac{1}{8!}$ Choice (D)

11. (i) Two cards can be drawn from 52 cards in $^{52}C_2$ ways

Two spades can be drawn from the 13 spades in $^{13}C_2$ ways.

Two diamonds can be drawn from 13 diamonds in $^{13}C_2$ ways.

$\therefore$ P(both are spades or both are diamonds)

$= \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} = 2.\frac{^{13}C_2}{^{52}C_2} = \frac{2}{17}$ Choice (D)

(ii) P(both are queens) = $\frac{^{4}C_2}{^{52}C_2}$

P (both are red coloured cards) = $\frac{^{26}C_2}{^{52}C_2}$

Now, there are two red queens in the pack of cards.

$\therefore$ P(both are red queens) = $\frac{^{2}C_2}{^{52}C_2}$

Hence, P(both are queens or both are red)

$= \frac{^{4}C_2}{^{52}C_2} + \frac{^{26}C_2}{^{52}C_2} - \frac{^{2}C_2}{^{52}C_2} = \frac{330}{1326} = \frac{55}{221}$

Choice (B)

(iii) P (both are diamonds) = $\frac{^{13}C_2}{^{52}C_2}$

P (neither is a king) = $\frac{^{48}C_2}{^{52}C_2}$

Now there are 12 diamonds that are not kings.

$\therefore$ P(both are diamonds and neither is a king)

$= \frac{^{12}C_2}{^{52}C_2}$

Hence, P (both are diamonds or neither is a king)

$= \frac{^{13}C_2}{^{52}C_2} + \frac{^{48}C_2}{^{52}C_2} - \frac{^{12}C_2}{^{52}C_2}$

$= \frac{78}{1326} + \frac{1128}{1326} - \frac{66}{1326} = \frac{1140}{1326} = \frac{190}{221}$ Choice (D)

12. (i) There are 16 honours in a pack of cards.

$\therefore$ P(all the four cards are honours) = $\frac{^{16}C_4}{^{52}C_4}$

Choice (B)

(ii) There are four different suits each of which contains four honours.

$\therefore$ P(the cards are honours of four different suits)

$= \frac{^4C_1\,^4C_1\,^4C_1\,^4C_1}{^{52}C_4} = \frac{256}{^{52}C_4}$ Choice (C)

(iii) We know that, there are 36 numbered cards of which 18 are red and 18 are black. 3 red number cards, and 1 black number card can be drawn in $^{18}C_3\,^{18}C_1$ ways. Similarly 3 black number cards, and 1 red number card can be drawn in $^{18}C_3\,^{18}C_1$ ways.

$\therefore$Total number of favourable cases = 2 ($^{18}C_3\,^{18}C_1$)

$\therefore$Required probability = $\frac{2.\,^{18}C_1\;^{18}C_3}{^{52}C_4}$ Choice (D)

13. The number of five digit numbers that can be formed using the digits 0, 1, 2, 3, 4 and 5 = $^6P_5 - {}^5P_4$ = 720 – 120

A number is divisible by 5 if the digit in its units place is either 0 or 5.

Now, the number of five digit numbers that end with '0' are $^5P_4$ = 120 the number of five digit numbers that end with '5' are $^5P_4 - {}^4P_3$ = 120 – 24 = 96

$\therefore$ The total number of five digit numbers which are divisible by 5 = 120 + 96

$\therefore$Required probability = $\frac{216}{600} = \frac{9}{25}$ Ans : (0.36)

14. Let A and B be the two events.

Given, P ($\overline{A}$) :P (A) = 4 : 5 $\Rightarrow$ P($\overline{A}$) = $\frac{4}{9}$; P(A) = $\frac{5}{9}$

Also, P(B) : P($\overline{B}$) = 3 : 7 $\Rightarrow$ P(B) = $\frac{3}{10}$ and P($\overline{B}$) = $\frac{7}{10}$

(i) $A \cap \overline{B}$ and $\overline{A} \cap B$ denote events that exactly one of them will occur.

P($A \cap \overline{B}$) = P(A) P($\overline{B}$) = $\frac{5}{9}.\frac{7}{10} = \frac{7}{18}$ P($\overline{A} \cap B$)

= P($\overline{A}$) .P(B) = $\frac{4}{9}.\frac{3}{10} = \frac{2}{15}$

$\therefore$ Required probability is

P($A \cap \overline{B}$) + P ($\overline{A} \cap B$) = $\frac{7}{18} + \frac{2}{15} = \frac{35+12}{90} = \frac{47}{90}$

Choice (A)

(ii) P(neither of them occurs) = P $\left(\overline{A} \cap \overline{B}\right)$

= P($\overline{A}$). P($\overline{B}$) = $\frac{7}{10}\left(\frac{4}{9}\right) = \frac{14}{45}$ Choice (C)

15. Total number of coins in the bag = 4 + 7 + 9 = 20

3 coins can be drawn from 20 coins in $^{20}C_3$ ways

If all the coins chosen are Re. 1 coins then the amount will be minimum.

3 one rupee coins can be drawn from the 9 one rupee coins in $^9C_3$ ways.

If E denotes the event of drawing the minimum possible amount, then

$P(E) = \frac{^9C_3}{^{20}C_3} = \frac{9(8)(7)}{20(19)(18)} = \frac{7}{95}$

$P(\bar{E}) = 1 - P(E) = 1 - \frac{7}{95} = \frac{88}{95}$

$\therefore$ The odds against the event E are $P(\bar{E}) : P(E)$

$\frac{88}{95} : \frac{7}{95} = 88 : 7$ Choice (D)

16. Five bulbs can be chosen from the 18 bulbs in $^{18}C_5$ ways.
Bad bulbs = 6; good bulbs = 12
If we select at least one good bulb, then the room will be lighted.

The probability that the room is not lighted is $\frac{^6C_5}{^{18}C_5}$

$\therefore$Required probability = $1 - \frac{^6C_5}{^{18}C_5}$ Choice (A)

17. Given, P(A U B) = 0.6 and P(A) = 0.3

(i) If A and B are mutually exclusive events, then $P(A \cap B) = 0$
$\therefore$P(A U B) = P(A) + P(B)
$\Rightarrow$ 0.6 = 0.3 + P(B) $\Rightarrow$ 0.3 = P(B)
Hence, P(B) = 0.3 Ans : (0.3)

(ii) If A and B are independent events, then
$P(A \cap B) = P(A). P(B)$
$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
= P(A) + P(B) – P(A). P(B)
P(A U B) = P(A) + P(B) [1 – P(A)]
$\Rightarrow$ 0.6 = 0.3 + P(B) (1 – 0.3) 0.3 = P(B) (0.7)

$\frac{0.3}{0.7} = P(B) \therefore P(B) = \frac{3}{7}$ Choice (D)

18. Let A, B and C be the events that Shiva, Jagan and Rohit hit the target respectively.

$\therefore P(A) = \frac{2}{3}$; $P(\bar{A}) = \frac{1}{3}$ $P(B) = \frac{5}{7}$; $P(\bar{B}) = \frac{2}{7}$

$P(C) = \frac{3}{8}$; $P(\bar{C}) = \frac{5}{8}$

(i) P(None of them hit the target)
$= P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}).P(\bar{B}).P(\bar{C})$

$= \frac{1}{3}.\left(\frac{2}{7}\right).\left(\frac{5}{8}\right) = \frac{5}{84}$ Choice (C)

(ii) The event $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$ denotes that exactly two persons hit the target, and $A \cap B \cap C$ is the event that all the three persons hit the target.

$P(A \cap B \cap \bar{C}) = P(A)\,P(B)\,P(\bar{C}) = \frac{2}{3}.\left(\frac{5}{7}\right).\left(\frac{5}{8}\right). = \frac{25}{84}$

$P(A \cap \bar{B} \cap C) = P(A).\,P(\bar{B}).P(C) = \frac{2}{3}.\left(\frac{2}{7}\right).\left(\frac{3}{8}\right). = \frac{1}{14}$

$P(\bar{A} \cap B \cap C) = P(\bar{A}).\,P(B).\,P(C) = \frac{1}{3}.\left(\frac{5}{7}\right).\left(\frac{3}{8}\right).$

$= \frac{5}{56}$

$P(A \cap B \cap C) = P(A)\,.\,P(B).\,P(C) = \frac{2}{3}.\left(\frac{5}{7}\right).\left(\frac{3}{8}\right). = \frac{5}{28}$

$\therefore$Required probability
$= P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) + P(A \cap B \cap C)$

$= \frac{25}{84} + \frac{1}{14} + \frac{5}{56} + \frac{5}{28} = \frac{50+12+15+30}{168} = \frac{107}{168}$
Choice (B)

(iii) Exactly one of them hits the target.
$(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$ denotes that exactly one of them hits the target.
$P(A \cap \bar{B} \cap \bar{C}) = P(A)\,P(\bar{B}).P(\bar{C})$

$= \frac{2}{3}.\left(\frac{2}{7}\right).\left(\frac{5}{8}\right) = \frac{5}{42}$

$P(\bar{A} \cap B \cap \bar{C}) = P(\bar{A}).P(B).\,P(\bar{C})$

$= \frac{1}{3}.\left(\frac{5}{7}\right).\left(\frac{5}{8}\right) = \frac{25}{168}$

$P(\bar{A} \cap \bar{B} \cap C) = P(\bar{A})\,.\,P(\bar{B})\,.\,P(C) = \frac{1}{3} \cdot \frac{2}{7} \cdot \frac{3}{8}$

$= \frac{1}{28}$

$\therefore$Required probability is
$P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$

$= \frac{5}{42} + \frac{25}{168} + \frac{1}{28} = \frac{20+25+6}{168} = \frac{51}{168} = \frac{17}{56}$
Choice (A)

19. A card can be drawn from the 52 cards in $^{52}C_1$ ways. A spade card can be drawn from 13 spades cards in $^{13}C_1$ ways.

$\therefore$ The probability that the cards drawn is a spade is $\frac{13}{52} = \frac{1}{4}$

$\therefore$The probability that the card drawn is not a spade is

$1 - \frac{1}{4} = \frac{3}{4}$

Let p denote the probability that the card is a spade and q denote the probability that the card is not a spade.

Then $p = \frac{1}{4}$; $q = \frac{3}{4}$

In the following cases B may win the game.

Case 1
In the first draw A loses the game and B wins the game.
The probability that B wins the game in the first draw is q.p

Case 2
In the first draw both A and B lose the game. In the second drawn A loses the game and B wins the game.
The probability that B wins the game in the second draw is q q q p.

Case 3
In the first two draws both A and B lose the game, in the third draw A loses the game and B wins the game.
The probability that B wins the game in the third draw is q q q q q p.

This process continues till B wins the game.
$\therefore$Required probability is
$q\,p + q^3p + q^5p + q^7p + \ldots. =$

$q\,p\,[1 + q^2 + q^4 + q^6 + \ldots.] = q\,p\left[\frac{1}{1-q^2}\right]\left(\because S_\infty = \frac{a}{a-r}\right)$

$= \frac{3}{4} \times \frac{1}{4}\left[\frac{1}{1-(3/4)^2}\right] = \frac{3}{16} \times \frac{16}{7} = \frac{3}{7}$ Choice (C)

20. (i) When the first ball drawn is not replaced:
Number of balls in the bag = 8 + 6 = 14
One pink ball can be drawn from the 8 pink balls in $^8C_1$ ways.

$\therefore$Probability of drawing a pink ball is $\frac{8}{14}$

Since the first ball is not replaced in the bag, second ball can be drawn from the remaining 13 balls only.

Now, the probability of drawing an orange ball is $\frac{6}{13}$

Hence, the probability of drawing a pink ball in the first draw and an orange ball in the second draw

$= \frac{8}{14}\times\frac{6}{13}=\frac{24}{91}$ Choice (A)

(ii) When the <u>first ball drawn is replaced</u>
If first ball is replaced in the bag,
The probability of drawing an orange ball in the second draw is $\frac{6}{14}$

$\therefore$ Required probability is $\frac{8}{14}\times\left(\frac{6}{14}\right)=\frac{12}{49}$

Choice (C)

21. Total number of balls in the bag = 6 + 5 + 4 = 15.
Three balls can be drawn from the 15 balls in ${}^{15}C_3$ ways.

(i) To draw three balls of different colours,
we have to select one ball from each colour.
one white ball can be drawn from 6 white balls in ${}^{6}C_1$ ways and
one green ball can be drawn from 5 green balls in ${}^{5}C_1$ ways.
one red ball can be drawn from 4 green balls in ${}^{4}C_1$ ways.
$\therefore$The number of ways of drawing balls of three different colours is ${}^{6}C_1\,{}^{5}C_1\,{}^{4}C_1 = 120$
$\therefore$ The probability that the three balls are of different colours is $\frac{120}{{}^{15}C_3}=\frac{24}{91}$ Choice (D)

(ii) 3 white balls can be drawn from 6 white balls in ${}^{6}C_3$ ways.

$\therefore$Probability of drawing 3 white balls is $\frac{{}^{6}C_3}{{}^{15}C_3}$

3 green balls can be drawn from 5 green balls in ${}^{5}C_3$ ways.

$\therefore$Probability of drawing 3 green balls is $\frac{{}^{5}C_3}{{}^{15}C_3}$

3 red balls can be drawn from 4 red balls in ${}^{4}C_3$ ways

$\therefore$Probability of drawing 3 red balls is $\frac{{}^{4}C_3}{{}^{15}C_3}$

Hence the probability of drawing three balls of same colour $= \frac{{}^{6}C_3}{{}^{15}C_3}+\frac{{}^{5}C_3}{{}^{15}C_3}+\frac{{}^{4}C_3}{{}^{15}C_3}=\frac{20+10+4}{455}=\frac{34}{455}$

Choice (C)

(iii) The possible combinations are as follows:
2 white balls and 1 ball of another colour (green or red) can be drawn in ${}^{6}C_2.{}^{9}C_1$ ways
2 green balls and 1 ball of another colour (white or red) can drawn in ${}^{5}C_2\,.\,{}^{10}C_1$ ways
2 red balls and 1 ball of another colour (green or white) can be drawn in ${}^{4}C_2\,.\,{}^{11}C_1$ ways
$\therefore$The total number of ways of drawing two balls of one colour and the third ball of a different colour is
${}^{6}C_2\,{}^{9}C_1 + {}^{5}C_2\,{}^{10}C_1 + {}^{4}C_2\,{}^{11}C_1 = 15\,(9) + 10\,(10)\,6\,(11)$
= 135 + 100 + 66 = 301

$\therefore$Required probability $= \frac{301}{{}^{15}C_3}=\frac{301}{455}$ Choice (A)

22. Let A and B be the two bags.
The number of balls in the bag A = 9 + 5 = 14
The number of balls in the bag B = 6 + 8 = 14

The probability of selecting a bag is $\frac{1}{2}$

<u>Case 1</u>
Suppose bag A is selected.
2 balls can be drawn from the 14 balls in ${}^{14}C_2$ ways.
2 white balls can be drawn from 9 white balls in ${}^{9}C_2$ ways.
$\therefore$ The probability of drawing two white balls from bag A is $\frac{{}^{9}C_2}{{}^{14}C_2}$

Hence, the probability of selecting the bag A and then drawing two white balls from it $= \frac{1}{2}\left(\frac{{}^{9}C_2}{{}^{14}C_2}\right)$

<u>Case 2</u>
Suppose bag B is selected

Probability of drawing two white balls from bag B is $\frac{{}^{6}C_2}{{}^{14}C_2}$

So, the probability of selecting the bag B and then drawing two white balls from it $= \frac{1}{2}\left(\frac{{}^{6}C_2}{{}^{14}C_2}\right)$

$\therefore$ The required probability $= \frac{1}{2}\left(\frac{{}^{9}C_2}{{}^{14}C_2}\right)+\frac{1}{2}\left(\frac{{}^{6}C_2}{{}^{14}C_2}\right)$

$= \frac{36}{182}+\frac{15}{182}=\frac{51}{182}$ Choice (B)

23. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.7 = 0.5 + P(B) - 0.2$
$P(B) = 0.4$
$P(\overline{B}) = 1 - 0.4 = 0.6$ Ans : (0.6)

24. $P(A \cap B)$
$= P(A \cup B) - P(B)$
$= \frac{1}{2}-\frac{2}{5} = 0.1$ Ans : (0.1)

25. Let p be the probability of getting an even number and q be the probability of getting an odd number. Then, p + q = 1
Given, p = 4q

$\therefore$ $4q + q = 1 \Rightarrow q = \frac{1}{5} \Rightarrow p = \frac{4}{5}$

$\therefore$For each even number, the probability of getting it is $\frac{4}{15}$

and for each odd number, probability is $\frac{1}{15}$

The favourable cases for getting the total score more than 16 are: 566, 656, 665 and 666.
Probability of the combination 566, 656 or 665 is

$\frac{1}{15}\left(\frac{4}{15}\right)\left(\frac{4}{15}\right)=\frac{16}{3375}$

$\therefore$Probability of the combination 666 $= \frac{4}{15}\left(\frac{4}{15}\right)\left(\frac{4}{15}\right)=\frac{64}{3375}$

$\therefore$ Required probability $= 3\left(\frac{16}{3375}\right)+\frac{64}{3375}=\frac{112}{3375}$

Choice (D)

26. Let X be the event that A and B agree and T be the event of the statement is true.

We require $P(T/X) = \frac{P(X \cap T)}{P(X)}$

$(X \cap T)$ is the event that the statement is true and they agree. i.e they both speak truth.

$\therefore$ $P(X \cap T) = \frac{2}{4}\times\frac{7}{10}=\frac{21}{40}$

X is the event that they agree. They agree either when both speak truth or both lie.

$\therefore P(X) = \frac{3}{4}\times\frac{7}{10}+\frac{1}{4}\times\frac{3}{10}=\frac{24}{40}$

$\therefore P(T/X) = \frac{21}{24}=\frac{7}{8}$ Ans : (0.875)

27. Total number of balls = 8 + 11 + 12 = 31
20 balls can be selected from 31 balls in = $^{31}C_{20}$
There are 23 non green balls
20 balls can be selected from 23 non green balls in = $^{23}C_{20}$ ways

$\therefore$ Required probability = $\frac{^{23}C_{20}}{^{31}C_{20}}$

$= \frac{23\times22\times21\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{31\times30\times29\times28\times27\times26\times25\times24\times23\times22\times21\times3\times2\times1}$

$= \frac{^{11}P_8}{^{31}P_8}$ Choice (D)

28. The total outcomes = 365 × 365
Favourable cases that the two persons are not born on the same day= 365 × 364
$\therefore$ Required probability

$= \frac{365\times364}{365\times365} = \frac{364}{365}$ Choice (D)

29. A regular hexagon has 6 sides and $\frac{6(6-3)}{2}$ = 9 diagonals i.e., totally 15 line segments. Hence, the probability of a line segment a diagonal = $\frac{9}{15}=\frac{3}{5}$

Ans : (0.6)

30. P(4 heads) = P(7 heads)
$^nC_4(½)^4\ (½)^{n-4} = {}^nC_7\ (½)^7\ (½)^{n-7}$
$^nC_4 = {}^nC_7 \Rightarrow n = 11$

P(2 heads) = $^{11}C_2\ (½)^{11} = \frac{55}{2048}$ Choice (C)

31. Let A-be the event that Federer qualifies for the final.
Let B-be the event that Nadal qualifies for the semifinal
It is given
P(A) = 0.7, P(B) = 0.5 P(A ∪ B) = 0.8
P(A ∩ B) = P(A) + P(B) –P(A ∪ B) = 0.7 + 0.5 – 0.8 = 0.4

$P\left(A/B\right) = \frac{P(A\cap B)}{P(B)} = \frac{0.4}{0.5} = 0.8$ Ans : (0.8)

32. The possible time (M minutes) and the corresponding probabilities are tabulated below.

Time	4	3	2
Prob	1/4	1/4	1/4

Time	7	6	5
Prob	$1/4^2$	$1/4^2$	$1/4^2$

etc.

$\therefore$ The expected value of the time that the frog takes (in minutes) to come out is E = $E_1 + E_2 + E_3$ where

$E_1 = 4\left(\frac{1}{4}\right)+7\left(\frac{1}{4^2}\right)+10\left(\frac{1}{4^3}\right)+....$

$E_2 = 3\left(\frac{1}{4}\right)+6\left(\frac{1}{4^2}\right)+9\left(\frac{1}{4^3}\right)+....$

$E_3 = 2\left(\frac{1}{4}\right)+5\left(\frac{1}{4^2}\right)+8\left(\frac{1}{4^3}\right)+....$

This is the sum of 3 AGP, in which |r| < 1.
Consider the AP a, a + d, a + 2d,……
the GP b, br, br^2,……
and the AGP, ab, (a + d) br, (a + 2d) br^2,…..
The sum to infinity of the AGP, is

$\frac{ab}{1-r}+\frac{dbr}{(1-r)^2} = \frac{\text{First term of AGP}}{1-r}+\frac{d(\text{sec ond term of GP})}{(1-r)^2}$

$E_1 = \frac{1}{3/4}+\frac{3/4^2}{3^2/4^2} = \frac{4}{3}+\frac{1}{3}$

$E_2 = \frac{3/4}{3/4}+\frac{3/4^2}{3^2/4^2} = \frac{3}{3}+\frac{1}{3}$

$E_3 = \frac{2/4}{3/4}+\frac{3/4^2}{3^2/4^2} = \frac{2}{3}+\frac{1}{3}$

$\therefore E = \frac{12}{3} = 4$ Ans : (4)

33. When a die is rolled, for each number the probability of getting it is $\frac{1}{6}$.
When the number on the dice is 1, 3 or 5 Raju receives ₹ 2, ₹6 and ₹10 respectively.
When the number on the die is 2, 4 or 6, then Raju receives ₹6, ₹12 and ₹18 respectively.

$\therefore$ Expected value = $\frac{1}{6}[2+6+10+6+12+18] = \frac{54}{6}$ = ₹9

$\therefore$To make an average profit of ₹7 per throw, Raju must pay ₹2 (9 – 7) for each time to throw the die. Ans : (2)

34. The number of multiples of 7 from 201 to 350 is 22
The number of multiples of 13 from 201 to 350 is 11.
The number multiples of both 7 and 13 between 201 and 350 is 1
The probability that the card is a multiple of 7 (but not 13) is $\frac{21}{150}$
The probability that the card is a multiple of 13 (but not 7) is $\frac{10}{150}$
The probability that the card is a multiple of both 7 and 13 is $\frac{1}{150}$

$\therefore$Expected value is = $\frac{21}{150}(25)+\frac{10}{150}(60)+\frac{1}{150}(100)$

$= \frac{7}{2}+4+\frac{2}{5} = \frac{21+24+4}{6} = \frac{49}{6}$ = ₹8.16

$\therefore$ Gain = 8.16 – 4 = ₹4.16 Ans : (4.16)

35. Probability of getting heads = $\frac{70}{100}$

Probability of getting tails = $\frac{30}{100}$

$\therefore$Expected value = $\frac{70}{100}$ (20) – $\frac{30}{100}$ (25)

$= 14 - \frac{15}{2} = \frac{13}{2} = 6.50$

$\therefore$ Average profit = ₹6.50 Ans : (6.50)

Exercise – 10(b)

Solutions for questions 1 to 35:

1. (i) 7 letters can be arranged in 7 addressed envelopes in 7! ways. $\therefore$n(S) = 7!
We can arrange the 7 letters into the corresponding 7 addressed envelopes in only one-way.

$\therefore$ Required probability is $\frac{1}{7!}$ Choice (C)

(ii) If six letters can be placed in their corresponding addressed envelopes, then the seventh letter is also in the correct envelope.
$\therefore$ The required probability is 0. Ans : (0)

2. Three cards can be drawn from 52 cards in $^{52}C_3$ ways n(S) = $^{52}C_3$

(i) Since we have 4 suits i.e., Diamonds, Spades, Clubs and Hearts and each suit contains 13 cards. Three cards can be selected from 13 cards in $^{13}C_3$ ways.
$\therefore$Number of favourable outcomes = 4 . $^{13}C_3$

Required probability = $4.\dfrac{^{13}C_3}{^{52}C_3}$ Choice (A)

(ii) From the four suits we can select 3 suits in $^4C_3$ ways. One card can be selected from each suit in $^{13}C_1$.
$^{13}C_1 \, . \, ^{13}C_1$
$\therefore$ Number of favourable outcomes
= $^4C_3 \, . \, ^{13}C_1 \, . \, ^{13}C_1 \, . \, ^{13}C_1$

$\therefore$ Required probability = $\dfrac{4(13)^3}{^{52}C_3}$ Choice (B)

(iii) We know that, each suit contains 9 number cards. 2 number cards can be drawn from 9 cards in $^9C_2$ ways.
Third card can be selected from the remaining 27 number cards in $^{27}C_1$ ways.
$\therefore$ Total number of favourable outcomes is 4($^9C_2$) ($^{27}C_1$)

Required probability = $\dfrac{4\left(^9C_2\right)\left(^{27}C_1\right)}{^{52}C_3}$ Choice (C)

3. The total number of balls in the bag is = 6 + 4 = 10.

(i) The probability of drawing a blue ball is $\dfrac{6}{10}$. Since the ball is not replaced remaining number of balls in the bag is 9. The probability of drawing an orange ball from 9 balls is 4/9.
Hence, the probability of drawing the blue ball first and orange ball second is $\dfrac{6}{10}\times\dfrac{4}{9}=\dfrac{4}{15}$ Choice (B)

(ii) If the first ball is replaced the probability of drawing an orange ball from 10 balls is 4/10.
$\therefore$ Required probability = $\dfrac{6}{10}\times\dfrac{4}{10}=\dfrac{6}{25}$

Ans : (0.24)

4. (i) Two black cards can be selected in $^{26}C_2$ ways.
A pack contain 8 black honours.
$\therefore$ The number ways of selecting two same honours is 4.
$\therefore$ Required probability = $\dfrac{4}{^{26}C_2}=\dfrac{4}{325}$ Choice (B)

(ii) Suppose one card is king, then the second card can be any of the other 3 cards from the other suit and it can be selected in 4 × 3 = 12 ways. Since there are four honour cards.

$\therefore$ Required probability = $\dfrac{12}{325}$. Choice (D)

5. Let A-be the event that the sum of the digits is ten.
Let B-be the event that the number is divisible by five.
The numbers for which the sum of the digits is ten are 19, 28, 37….91 Out of these, the only number divisible by five is 55.

P(A) = $\dfrac{9}{90}$ P(A∩B) = $\dfrac{1}{90}$

$P\left(B/A\right)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{\frac{1}{90}}{\frac{1}{90}}=\dfrac{1}{9}$ Choice (A)

6. Three consecutive letters can be selected from the English alphabet in 24 ways.
$\therefore$ n(S) = 24
Since, no two consecutive letters are vowels, the number of ways that among three letters one letter is vowel is 13.

$\therefore$Required probability = $\dfrac{13}{24}$ Choice (B)

7. Two numbers can be selected from 50 natural numbers in $^{50}C_2$ ways.
$\therefore$ n(S) = $^{50}C_2$

(i) Let E be the event that the two numbers selected are even, then n(E) = $^{25}C_2$

$\therefore P(E)=\dfrac{n(E)}{n(S)}=\dfrac{^{25}C_2}{^{50}C_2}=\dfrac{12}{49}$ Choice (A)

(ii) Let E_1 be the event that one number selected is odd then $\therefore$ n(E_1) = $^{25}C_1 \, . \, ^{25}C_1$

$\therefore P(E_1)=\dfrac{^{25}C_1 . ^{25}C_1}{^{50}C_2}=\dfrac{25}{49}$ Choice (C)

8. Eight coins are tossed, so n(S) = 2^8
Let E be the favourable event. It is required to get 4 heads and 4 tails which is possible in the favourable out come HHHHTTTT. This can be arranged in $\dfrac{8!}{4!4!}$ ways.

$n(E)=\dfrac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}=70$

$\therefore$Required probability = $\dfrac{n(E)}{n(S)}=\dfrac{70}{2^8}=\dfrac{35}{128}$ Choice (D)

9. When two fair dice are thrown, the total number of out comes is n(S) = 6 × 6 = 36
Let E be a favourable event.
E = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)}
$\therefore$ n(E) = 14

$\therefore$ Required probability = $\dfrac{n(E)}{n(s)}=\dfrac{14}{36}=\dfrac{7}{18}$ Choice (D)

10. When four dice are thrown, the total number of outcomes is n(s) = 6 × 6 × 6 × 6 = 6^4.
Let E be the event, that the product of the numbers showing up is a prime number.
$\therefore$ The possible events are (1, 1, 1, 2) and (1, 1, 1, 3), (1, 1, 1, 5)

(1, 1, 1, 2) can be permuted in $\dfrac{4!}{3!}$ = 4 ways

(1, 1, 1, 3) can be permuted in $\dfrac{4!}{3!}$ = 4 ways

(1, 1, 1, 5) can be permuted in $\dfrac{4!}{3!}$ = 4 ways

$\therefore$Total number of favourable outcomes is 12.

$\therefore$ Required probability = $\dfrac{12}{6^4}=\dfrac{1}{108}$ Choice (B)

11. Two cards can be drawn from 52 cards in $^{52}C_2$ ways.

(i) Probability of drawing 2 spades from 13 spades is $\dfrac{^{13}C_2}{^{52}C_2}$

Probability of drawing 2 hearts from 13 hearts is $\dfrac{^{13}C_2}{^{52}C_2}$

$\therefore$ Required probability = $\dfrac{^{13}C_2}{^{52}C_2}+\dfrac{^{13}C_2}{^{52}C_2}=2.\dfrac{^{13}C_2}{^{52}C_2}$

$=\dfrac{6}{51}=\dfrac{2}{17}$ Choice (B)

(ii) Probability of drawing 2 number cards from 36 number cards is P(N) = $\frac{^{36}C_2}{^{52}C_2}$

Probability of drawing 2 red cards from 26 red cards is P(R) = $\frac{^{26}C_2}{^{52}C_2}$

Probability of drawing 2 red number cards from 18 red number cards is P(R ∩ N) = $\frac{^{18}C_2}{^{52}C_2}$

∴ Required probability

$P(R \cup N) = P(R) + P(N) - P(R \cap N)$

$= \frac{^{26}C_2}{^{52}C_2} + \frac{^{36}C_2}{^{52}C_2} - \frac{^{18}C_2}{^{52}C_2} = \frac{401}{663}$ Choice (C)

(iii) Probability of drawing 2 kings from 4 kings is P(K) $= \frac{^{4}C_2}{^{52}C_2}$

Probability of drawing 2 diamonds from 13 diamonds is P(D) = $\frac{^{13}C_2}{^{52}C_2}$

Since in a pack there is only one diamond kings, so these events are mutually exclusive.

∴ Required probability = $\frac{^{4}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} = \frac{14}{221}$

Choice (D)

12. The number of 5 digit numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 is

$5 \times {^5P_4} = 5 \times 120 = 600$

We know that, if the unit place in a number is either '0' or 5 the number is divisible by 5.

The number of numbers such that the unit place contain '0' is $^5P_4 = 120$.

The number of numbers such that the unit place contain '5' is $4.{^4P_3} = 4 \times 24 = 96$

Favourable 5 digit numbers which are divisible by 5 is 120 + 96 = 216

∴ Required probability = $\frac{216}{600} = \frac{9}{25}$ Ans : (0.36)

13. Let A and B be the two independent events.

Given, $P(\bar{A}) : P(A) = 4 : 5$ $P(\bar{A}) = \frac{4}{9}$; $P(A) = \frac{5}{9}$

$P(B) : P(\bar{B}) = 3 : 7$ $P(B) = \frac{3}{10}$; $P(\bar{B}) = \frac{7}{10}$

(i) $P(A \cap \bar{B}) = P(A).\ P(\bar{B}) = \frac{5}{9}.\frac{7}{10} = \frac{35}{90}$

$P(\bar{A} \cap B) = P(\bar{A}).\ P(B) = \frac{4}{9}.\frac{3}{10} = \frac{12}{90}$

∴ Required probability

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= \frac{35}{90} + \frac{12}{90} = \frac{47}{90}$ Choice (B)

(ii) The probability that none of the events occur is denoted by $P\ \bar{A} \cap \bar{B}$

$P(\bar{A} \cap \bar{B}) = P(\bar{A})\ .\ P(\bar{B})$ (∴A, B are independent)

$= \frac{4}{9}.\frac{7}{10} = \frac{14}{45}$ Choice (A)

(iii) The Probability of at least one of them occurs is $1 - P(\bar{A} \cap \bar{B})$ ∴ $1 - \frac{14}{45} = \frac{31}{45}$

Choice (C)

14. Probability of drawing a queen card is $\frac{1}{13}$. Probability of drawing a card which is not a queen is $\frac{12}{13}$.

The winning sequence of A can be A, $\bar{A}\bar{B}A$, $\bar{A}\bar{B}\ \bar{A}\bar{B}A$, …

where $\bar{A}$ is the event of A not drawing a queen card, $\bar{B}$ is the event of B not drawing a queen card. According the above sequence A may pick the card in 1st, 3rd, 5th … trials. The probability of A winning the game is

$P(A) + P(\bar{A}).P(\bar{B}).P(A) + P(\bar{A}).P(\bar{B}).P(\bar{A}).P(\bar{B}).P(A) + \ldots$

$= P(A)(1 + P(\bar{A}).P(\bar{B}) + P(\bar{A}).P(\bar{B}).P(A) + \ldots)$

$= \frac{1}{13}\left(1 + \frac{12}{13}.\frac{12}{13} + \frac{12}{13}.\frac{12}{13}.\frac{12}{13}.\frac{12}{13} + \ldots\right)$

$= \frac{1}{13}(1 + \left(\frac{12}{13}\right)^2 + \left(\frac{12}{13}\right)^4 + \ldots)$

$= \frac{1}{13}.\left(\frac{1}{1 - \left(\frac{12}{13}\right)^2}\right)$ ($\because\ S_\infty = \frac{a}{1-r}$)

$= \frac{1}{13} \times \frac{169}{169-144} = \frac{13}{25}$

∴ Probability that B wins the game is P(B) = 1 – P(A)

$= 1 - \frac{13}{25} = \frac{12}{25}$ Ans : (0.48)

15. The total number of balls = 7 + 6 + 2 = 15.

Three balls can be drawn from 15 balls in $^{15}C_3$ ways.

(i) Let E be the event that the three balls are of same colour. The three balls must be either green or black.

Probability of drawing 3 green balls = $\frac{^{7}C_3}{^{15}C_3}$.

Probability of drawing 3 black balls = $\frac{^{6}C_3}{^{15}C_3}$

∴ Required probability P(E) = $\frac{^{7}C_3}{^{15}C_3} + \frac{^{6}C_3}{^{15}C_3}$

$= \frac{35+20}{^{15}C_3} = \frac{55}{455} = \frac{11}{91}$ Choice (D)

(ii) Let E, be the event that two balls are green and the third is of another colour.

Let E_2 be the event that two balls are black and the third is of another colour, and let E_3 be the event that two balls are pink and the third is of another colour.

∴ $P(E_1) = \frac{^{7}C_2.{^8C_1}}{^{15}C_3}$ $P(E_2) = \frac{^{6}C_2.{^9C_1}}{^{15}C_3}$

$P(E_3) = \frac{^{2}C_2.{^{13}C_1}}{^{15}C_3}$

∴ Required probability = $P(E_1) + P(E_2) + P(E_3)$

$= \frac{^{7}C_2.{^8C_1}}{^{15}C_3} + \frac{^{6}C_2.{^9C_1}}{^{15}C_3} + \frac{^{2}C_2.{^{11}C_1}}{^{15}C_3} = \frac{316}{455}$

Choice (C)

(iii) If three balls are of different colour, then we have to select one ball from each colour.

∴ Number of favourable cases are = $^7C_1\ .\ ^6C_1\ .\ ^2C_1$

$\therefore$ Required probability = $\dfrac{{}^7C_1.{}^6C_1.{}^2C_1}{{}^{15}C_3} = \dfrac{12}{65}$

Choice (D)

16. Total number of coins in the bag = 6 + 5 + 4 = 15.
5 coins can be drawn from 15 coins in ${}^{15}C_5$ ways.
When 4 coins are one rupee coins and fifth coin is a two-rupee coin the amount will be minimum.
$\therefore$ Number of favourable cases = ${}^4C_4 . {}^5C_1 = 5$

The probability that the amount will be minimum is $\dfrac{5}{{}^{15}C_5}$

$P(E) = \dfrac{5}{3003}$ $\therefore P(\bar{E}) = 1 - P(E)$

$= 1 - \dfrac{5}{3003} = \dfrac{2998}{3003}$

$\therefore$Odds in favour of the required event = 5 : 2998

Choice (B)

17. Let B_1 and B_2 be two bags. B_1 contains 7 red and 3 blue balls, B_2 contains 6 blue and 4 red balls.

The probability of selecting any one of the bag is $\dfrac{1}{2}$.

If B_1 is selected, then the probability of drawing 2 red balls from it is $\dfrac{{}^7C_2}{{}^{10}C_2}$.

$\therefore$ The probability of drawing 2 blue balls from it is $\dfrac{{}^3C_2}{{}^{10}C_2}$.

The probability of drawing two balls of same colour from B_1 is $\dfrac{1}{2}\left(\dfrac{{}^7C_2}{{}^{10}C_2} + \dfrac{{}^3C_2}{{}^{10}C_2}\right) = \dfrac{1}{2}\dfrac{24}{45}$.

Similarly the probability of drawing two balls of same colour from bag B_2 is

$\dfrac{1}{2}\left(\dfrac{{}^6C_2}{{}^{10}C_2} + \dfrac{{}^4C_2}{{}^{10}C_2}\right) = \dfrac{1}{2}\left(\dfrac{21}{45}\right) = \dfrac{21}{90}$

$\therefore$ Required probability = $\dfrac{24}{90} + \dfrac{21}{90} = \dfrac{45}{90} = \dfrac{1}{2}$. Ans : (0.5)

18. Given, $P(A \cup B) = \dfrac{3}{4}$; $P(A) = \dfrac{7}{20}$.

(i) Given, A and B are mutually exclusive, then $A \cap B = \phi$
$P(A \cup B) = P(A) + P(B)$ $P(B) = P(A \cup B) - P(A)$
$= \dfrac{3}{4} - \dfrac{7}{20} = \dfrac{15-7}{20} = \dfrac{8}{20} = \dfrac{2}{5}$. Ans : (0.4)

(ii) If A and B are equally likely, then P(A) = P(B)
$\therefore P(B) = \dfrac{7}{20}$. Ans : (0.35)

(iii) If A and B are independent events, then
$P(A \cap B) = P(A) . P(B)$
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$P(A\cup B) = P(A) + P(B) - P(A).P(B)$
$P(A\cup B) = P(A) + P(B)\,(1 - P(A))$
$\dfrac{3}{4} = \dfrac{7}{20} + P(B)\left(1 - \dfrac{7}{20}\right)$ $\dfrac{3}{4} - \dfrac{7}{20} = P(B)\left(\dfrac{13}{20}\right)$
$\dfrac{8}{20} = P(B)\dfrac{13}{20}$ $P(B) = \dfrac{8}{20} \times \dfrac{20}{13} = \dfrac{8}{13}$ Choice (B)

19. Given P(H) = 3P(T)

We know that P(H) + P(T) = 1 3P(T) + P(T) = 1 $P(T) = \dfrac{1}{4}$

$P(H) = \dfrac{3}{4}$

Since the coin is tossed 3 times, two heads may occur in 1st and 2nd trial, 2nd and 3rd trial or 1st and 3rd trial.
$\therefore$ The required probability
$= \dfrac{3}{4} \times \dfrac{3}{4} \times \dfrac{1}{4} + \dfrac{1}{4} \times \dfrac{3}{4} \times \dfrac{3}{4} + \dfrac{3}{4} \times \dfrac{1}{4} \times \dfrac{3}{4} = \dfrac{9}{64} + \dfrac{9}{64} + \dfrac{9}{64} = \dfrac{27}{64}$.

Choice (D)

20. A square can be selected from a chess board in
$1^2 + 2^2 + \ldots.. + 8^2 = 204$ ways.
We have 64 squares of size 1 x 1.
$\therefore$ Required probability = $\dfrac{64}{204} = \dfrac{16}{51}$ Choice (C)

21. When five coins are tossed, the total possible out comes are $2^5 = 32$.
When head occurs for three are more times then the number of heads exceeds the number of tails.
$\therefore$ Required probability = P(getting 3 heads) + P(getting 4 heads + P(getting 5 heads)
$= \dfrac{{}^5C_3}{2^5} + \dfrac{{}^5C_4}{2^5} + \dfrac{{}^5C_5}{2^5} = \dfrac{10+5+1}{2^5} = \dfrac{16}{32} = \dfrac{1}{2}$

Ans : (0.5)

22. Between 1 to 25 total numbers are 23.
$n(S) = {}^{23}C_1$
The number of even numbers between 1 to 25 is 12.
The number of numbers which are divisible by 7 is 3.
The number of numbers which are divisible by 2 or 7 is 1.
$\therefore$ Required probability = $\dfrac{14}{23}$. Choice (C)

23. Three bulbs can be selected from 24 bulbs in ${}^{24}C_3$ ways. $n(S) = {}^{24}C_3$.
The number of good bulbs = 18
The number of defective bulbs = 6.
If we select at least one good bulb the room will be lighted.
The probability that the all the bulbs are defective
$= \dfrac{{}^6C_3}{{}^{24}C_3} = \dfrac{5}{506}$
$\therefore$ The probability that at least one bulb is good is
$1 - \dfrac{5}{506} = \dfrac{501}{506}$. Choice (D)

24. The probability that Akil reach the summit = P(A) = 2/3
$\therefore P(\bar{A}) = 1/3$
The probability that Nikil reach the summit is P (N) = 5/8
$\therefore P(\bar{N}) = 3/8$

The probability that Sunil reach the summit is $P(S) = \dfrac{4}{7}$

$\therefore P(\bar{S}) = \dfrac{3}{7}$

(i) Probability that none of them reaches the summit
$= P(\bar{A} \cap \bar{N} \cap \bar{S}) = P(\bar{A})P(\bar{N})P(\bar{S})$
$= \dfrac{1}{3} \times \dfrac{3}{8} \times \dfrac{3}{7} = \dfrac{3}{56}$. Choice (B)

(ii) Probability that exactly two reaches the summit
$= P(A \cap S \cap \bar{N}) + P(A \cap N \cap \bar{S}) + P(N \cap S \cap \bar{A})$
$= P(A)\,P(S)\,P(\bar{N}) + P(A)\,P(N)\,P(\bar{S}) + P(N)\,P(S)\,P(\bar{A}) = \dfrac{2}{3} \cdot \dfrac{4}{7} \cdot \dfrac{3}{8} \cdot \dfrac{2}{3} \cdot \dfrac{5}{8} \cdot \dfrac{3}{7} + \dfrac{1}{3} \cdot \dfrac{5}{8} \cdot \dfrac{4}{7} = \dfrac{37}{84}$

Choice (A)

(iii) Probability that at least two of them reaches the summit = Exactly two of them reach the summit + three of them reach the summit

$= \frac{37}{84} + \frac{5}{21} = \frac{57}{84} = \frac{19}{28}$ Choice (D)

25. $P(x = x_i)$ is the probability of typing x_i characters wrong.
Required probability

$$= \frac{P(x=1)}{P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)+P(x=6)}$$

$$= \frac{(0.7)^5(0.7)(0.7)(0.7)(0.7)(0.7)\ (0.3)}{1-P(x=0)} = \frac{(0.7)^5(0.3)}{1-(0.7)^6}$$

Choice (A)

26.

Number	1	2	3	4	5	6
Amount	2	6	6	12	10	18

When a dice is rolled the corresponding amount is shown in the above table.
We know that the probability of each number is 1/6.
$\therefore$ Expected amount is

$$2 \times \frac{1}{6} + 6x\frac{1}{6} + 6x\frac{1}{6} + 12x\frac{1}{6} + 10x\frac{1}{6} + 18x\frac{1}{6}$$

$= \frac{1}{6}$ (54) = ₹9

Given that he wants to make an average profit of ₹6.
$\therefore$ Each time he will be willing to pay ₹3 to throw the dice.
Ans : (3)

27. Probability of getting head $= \frac{70}{100} = \frac{7}{10}$

Probability of getting tail $= \frac{30}{100} = \frac{3}{10}$

$\therefore$ Expected amount $= 20x\frac{70}{100} - 25x\frac{30}{100}$

$= \frac{1400-750}{100} = \frac{650}{100}$

$\therefore$ Profit of ₹6.50. Ans : (6.50)

28. Ten people can be seated in a row in 10! ways
Let E be event that two boys are never together, $\overline{E}$ be the event that the two boys, are always together.
Let the two particular persons be always together. Considering the two people as a unit, the total number of persons = 9
9 persons can be arranged in 9! ways and the two people can be arranged among themselves in 2! ways

$n(\overline{E}) = 2! \times 9!$

$P(\overline{E}) = \frac{n(\overline{E})}{n(S)} = \frac{9! \times 2!}{10!} = \frac{1}{5}$

$\therefore$ Required probability $P(E) = 1 - P(\overline{E})$

$= 1 - \frac{1}{5} = \frac{4}{5}$. Ans : (0.8)

29. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{1}{4} + \frac{2}{3} - \frac{3}{4}$

$= \frac{3+8-9}{12} = \frac{2}{12} = \frac{1}{6}$

$P(A).P(B) = \frac{1}{4}.\frac{2}{3} = \frac{1}{6}$

$P(A \cap B) = P(A) . P(B)$
$\therefore$ A and B are independent events and are not equally likely.
Choice (B)

30. $P(A) = \frac{2}{3+2} = \frac{2}{5}$

$P(\overline{A}) = 1 - \frac{2}{5} = \frac{3}{5}$

$P(B) = \frac{2}{2+5} = \frac{2}{7}$

$P(\overline{B}) = 1 - \frac{2}{7} = \frac{5}{7}$

P(atleast one of A, B occurs)
= 1 – P(neither occurs)

$1 - P(\overline{A} \cap \overline{B})$

$= 1 - \frac{3}{5}.\frac{5}{7} = \frac{4}{7}$ Choice (C)

31. Probability of Arun speaking truth $= P(A) = \frac{70}{100} = \frac{7}{10}$

and that of Bhargav speaking truth $= P(B) = \frac{65}{100} = \frac{13}{20}$

They contradict when one speaks truth and the other lies ie

$P(\overline{A}\ B \text{ or } A\overline{B}) = \frac{7}{10} \times \frac{7}{20} + \frac{3}{10} \times \frac{13}{20}$

$= \frac{88}{200} = 0.44$ Ans : (0.44)

32. If a coin is flipped 7 times, then $n(S) = 2^7$.
The favourable out comes are case (1) let 4 consecutive out comes be head it is possible in the following ways.
(HHHH) THH, (HHHH) THT, (HHHH) TTH, (HHHH) TTT, TTT (HHHH), HHT(HHHH),THT(HHHH), HTT
(HHHH)
T(HHHH)TH,T(HHHH)TT, HT(HHHH)T,TT(HHHH)T Which are 12 similarity 4 consecutive out comes to be tails also possible in 12 ways.
$\therefore$ Number of favourable events are = 12 + 12 = 24

Required probability $= \frac{24}{2^7} = \frac{3}{2^4} = \frac{3}{16}$ Choice (C)

33. The multiples of 8 in between 201 to 350 is 18.
The multiples of 13 in between 201 to 350 is 11.
The multiple of 8 and 13 in between 201 and 350 is 2.
$\therefore$ Probability of getting a number which is a multiple of 8 (and not 13) is $\frac{16}{150}$
Probability of getting a number which is a multiple of 13 (and not 8) is $\frac{9}{150}$
Probability of getting a number which is a multiple of both 8 and 13 is $\frac{2}{150}$
$\therefore$ Expected amount

$= \frac{16}{150}x15 + \frac{9}{150}x40 + \frac{2}{150}x80 = \frac{240+360+160}{150}$

$= \frac{760}{150} = 5.06$

Since he pays each time ₹2, he will make a profit of ₹5.06 – 2 = ₹3.06 Ans : (3.06)

34. $P(X \cap Y) = P(X) + P(Y) - P(X \cup Y)$

$= -\frac{3}{5}+\frac{1}{3}+\frac{2}{5}=\frac{-9+5+6}{15}=\frac{2}{15}$

$= \frac{1}{3}\times\frac{2}{5}$ = P (X). P (Y)

$\Rightarrow$ X, Y are independent but not equally likely.

Choice (B)

35. Set of letters of word "PLASMA" say X is:
X = { A, L, M, P, S }
Set of letters of word 'MANASA' say Y is
Y = { A, M, N, S}
$X \cap Y$ = { A, M, S}
We can first consider the probability of selecting the same latter (i.e. A M or S)

Probability of selecting A from 'PLASMA' = $\frac{2}{6}=\frac{1}{3}$

Probability of selecting M from 'PLASMA' = $\frac{1}{6}$

Also probability of selecting 'S' is $\frac{1}{6}(\because 1.\ S)$

Consider the word 'MANASA'

Probability of selecting 'A' is $=\frac{3}{6}=\frac{1}{2}$ $(\because 3-\text{A's})$

Probability of selecting 'M' is $=\frac{1}{6}$ $(\because 1.\ M)$

Probability of selecting 'S' is $=\frac{1}{6}$ $(\because 1.\ S)$

Probability that selected letters from the two words are the same.

$=\frac{1}{3}\left(\frac{1}{2}\right)+\frac{1}{6}\times\left(\frac{1}{6}\right)+\frac{1}{6}\times\left(\frac{1}{6}\right)=\frac{8}{36}=\frac{2}{9}$

Required probability that the letters selected from the two words are not the same,

$1-\frac{2}{9}=\frac{7}{9}$

Choice (D)