CHAPTER – 1

SIMPLE EQUATIONS

There will be linear equations of one or two unknowns invariably in every problem. A linear equation is one where each variable occurs only in its first power and not in any higher powers. Some times we get three equations in three unknowns. In general, we need as many equations as the variables we will have to solve for. So, for solving for the values of two unknowns, we need two equations (or two conditions given in the problem) and for solving for the values of three unknowns, we need three equations (and hence the problem should give three conditions from which we can frame three equations). Solving the equations by itself is not a difficult task. The most important part of the problem is framing the equation/equations. Once the equations are framed, solving them is very easy. In this chapter, we will deal with problems involving as many equations (of first degree) as the number of unknowns. Later on, we will look at equations of second degree (Quadratic Equations) and linear equations where the number of equations will be less than that of the number of variables (under the chapter Special Equations).

ONE EQUATION IN ONE UNKNOWN

An equation like $2x + 4 = 26$ is an equation in one unknown. We have only one variable x whose value we have to find out. The steps in solving this are:

Step I: Take all quantities added to (or subtracted from) the x term (term with the unknown) to the right side with a change of sign.
i.e., $2x = 26 – 4 = 22$.

Step II: Take the co-efficient of x from left hand side and divide right hand side with this term to get the value of x.
i.e., $x = 22/2 = 11$. Therefore, $x = 11$.

TWO EQUATIONS IN TWO UNKNOWNS

A set of equations like
$2x + 3y = 8 \quad \to \quad (1)$
$5x + 4y = 13 \quad \to \quad (2)$
is called a system of simultaneous equations in two unknowns. Here, we have two variables (or unknowns) x and y whose values we have to find out. This can be done using the two given equations. The steps for this are as follows:

Step I: Using both the equations we first eliminate one variable (so that we can then have one equation in one unknown).
For this purpose, we multiply equation (1) with 5 (the co-efficient of x in the second equation) and multiply equation (2) with 2 (the co-efficient of x in the first equation) to eliminate x. Thus we have
$(1) \times 5 \Rightarrow 10x + 15y = 40 \quad \to \quad (3)$
$(2) \times 2 \Rightarrow 10x + 8y = 26 \quad \to \quad (4)$
Now, subtracting equation (4) from equation (3) we have $7y = 14 \quad \to \quad (5)$
This is one equation in one unknown.

Step II: Solve for the value of one variable from the equation (in one unknown) obtained from Step I above. Therefore, $y = 2$.

Step III: Substitute this value of the variable in one of the two equations to get the value of the second variable.

Substituting the value of y in equation (1) or equation (2), we get $x = 1$. Therefore the values of x and y that satisfy the given set of equations are $x = 1$ and $y = 2$.

THREE EQUATIONS IN THREE UNKNOWNS

A set of equations like
$x + 2y + 3z = 14 \quad \to \quad (1)$
$2x + y + 2z = 10 \quad \to \quad (2)$
$3x + 3y + 4z = 21 \quad \to \quad (3)$
is a system of three equations in three unknowns.

Here we have three unknowns x, y and z which we have to solve for from the three given equations. The procedure for the same is as follows:

Step I: Take two out of the three equations [say, eqn. (1) and (2)] and eliminate one variable (say x) so that we get an equation in two unknowns (y and z in this case).

For this purpose, take equations (1) and (2). Multiply equation (1) by 2 and subtract equation (2) from it.

$$\begin{aligned} \text{Equation (1)} \times 2 \Rightarrow \quad & 2x + 4y + 6z = 28 \\ & 2x + y + 2z = 10 \\ \hline & 3y + 4z = 18 \quad \to \quad (4) \end{aligned}$$

Step II: Repeat Step I for two other equations [say equations (2) and (3)] and eliminate the same variable (x in this case) so that we get one more equation in two unknowns (y and z).
For this purpose, take equations (2) and (3). Multiply equation (2) by 3 and from that subtract equation (3) multiplied by 2.

$$\begin{aligned} \text{Equation (2) x 3} \Rightarrow \quad & 6x + 3y + 6z = 30 \\ \text{Equation (3) x 2} \Rightarrow \quad & 6x + 6y + 8z = 42 \\ \hline & –3y – 2z = –12 \quad \to \quad (5) \end{aligned}$$

Step III: Now the equations in two unknowns that have been obtained from the above two steps have to be solved as discussed previously (in TWO EQUATIONS IN TWO UNKNOWNS) to get the values of two of the three variables (y and z in this case).
In this case, solving equations (4) and (5), we get $y = 2$ and $z = 3$.

Step IV: Substitute these values of the two variables in one of the three equations to get the value of the third variable.

Substitute the value of y and z in equation (1) to get the value of x = 1.
Thus the values of the three variables x, y and z that satisfy the three given equations are x = 1; y = 2 and z = 3

Examples

1.01. If 3 tables and 4 chairs together cost ₹2800 and 2 tables and 3 chairs together cost ₹1950, then find the cost of each table and each chair respectively.

Sol: Let the cost of each table be x and the cost of each chair be y, then we have the following equations from the given data.
$3x + 4y = 2800 \quad \rightarrow \quad (1)$
$2x + 3y = 1950 \quad \rightarrow \quad (2)$
To solve these two equations, multiply equation (1) by 2 and equation (2) by 3 and then subtracting one from the other, we get y = ₹250
Substituting the value of y in equation (1) we get 3x + 1000 = 2800 $\Rightarrow$ x = ₹600
Therefore, the cost of each table is ₹600 and the cost of each chair is ₹250.

1.02. Arjun, Balu and Charu went to a shop to purchase pencils, sharpeners and erasers. Arjun bought 5 pencils, 2 sharpeners and 3 erasers for ₹13.50. Balu bought 4 pencils, 3 sharpeners and 2 erasers for ₹12. Charu bought 6 pencils, 2 sharpeners and 4 erasers for ₹16. Find the cost of 3 pencils, 4 sharpeners and 8 erasers.

Sol: Let the price of each pencil, sharpener and eraser be p, s and e respectively. From the data given we get,
$5p + 2s + 3e = 13.50 \quad \rightarrow \quad (1)$
$4p + 3s + 2e = 12 \quad \rightarrow \quad (2)$
$6p + 2s + 4e = 16 \quad \rightarrow \quad (3)$
Let us take equations (1) and (3) and eliminate the variable s by subtracting (1) from (3)
$p + e = 2.50 \quad \rightarrow \quad (4)$
Then take equations (1) and (2), multiply (1) with 3 and (2) with 2 and subtract one from the other.
We get
$7p + 5e = 16.50 \quad \rightarrow \quad (5)$
(5) – [5 $\times$ (4)] gives 2p = 4
p = 2; substituting the value of p in (4),
e = 0.50; substituting these values in (1)
s = 1
Using these values we find that 3 pencils, 4 sharpeners and 8 erasers cost
($3 \times 2 + 4 \times 1 + 8 \times 0.5$) = ₹14.

1.03. The sum of the digits of a two-digit number is 12. If the digits are interchanged, the resulting number is 18 more than the original number. Find the original number.

Sol: Let us consider the two-digit number as xy, where x is the tens digit and y is the units digit. Hence the number itself is equal to 10x + y.
Since the sum of the digits is given as 12,
$x + y = 12 \quad \rightarrow \quad (1)$
When the digits are interchanged y becomes the tens digit and x the units digit. The number then becomes (10y + x).
Since this number is 18 more than the original number, we have (10y + x) – (10x + y) = 18.
$\Rightarrow 9y - 9x = 18$
$\Rightarrow y - x = 2 \quad \rightarrow \quad (2)$
On adding (1) and (2), we get y = 7
and substituting y in (2), we get x = 5
Hence the number is 57.

1.04. Ten years from now, the age of Raja's father will be twice Raja's age. Ten years ago, the age of Raja's father was thrice Raja's age. Find the present age of Raja and his father.

Sol: Let the present ages of Raja and his father be x years and y years respectively.
Ten years from now, Raja's age will be x + 10 and his father's age will be y + 10.
$y + 10 = 2(x + 10)$
$\Rightarrow y = 2x + 10 \quad \rightarrow \quad (1)$
Ten years ago, Raja's age was x – 10
and his father's age was y – 10
Given, $y - 10 = 3(x - 10)$
$y = 3x - 20 \quad \rightarrow \quad (2)$
Equating the values of y in (1) and (2),
we have $2x + 10 = 3x - 20$
$\Rightarrow x = 30.$
By substituting x = 30 in (1), we get y = 70
$\therefore$Raja's present age is 30 years and his father's present age is 70 years.

1.05. The present age of a father is thrice the age of his son. Fifteen years hence, the father's age will be twice the son's age. How many years ago was the age of the father six times the age of the son?

Sol: Let the present ages of the son and father be x years and y years respectively.
Given, $y = 3x \quad \rightarrow$ (1) and
$\Rightarrow y + 15 = 2(x + 15) = 2x + 30 \quad \rightarrow \quad (2)$
Substituting y = 3x in (2), we get
$3x + 15 = 2x + 30,$
$\Rightarrow x = 15,$
Substituting x = 15 in (1), we get y = 3x
$= 3 \times 15 = 45.$
Let us say p years ago, age of the father was six times the age of his son.
$45 - p = 6(15 - p)$
Solving we get, p = 9
$\therefore$9 years ago, father's age was six times the son's age.

1.06. If both the numerator and the denominator of a fraction are decreased by 3, the fraction becomes 2/3. If both the numerator and the denominator are increased by 7, the fraction becomes 3/4. Find the fraction.

Sol: Let the fraction be $\frac{x}{y}$
When both the numerator and the denominator are decreased by 3, we have, $\frac{x-3}{y-3} = \frac{2}{3}$
$\Rightarrow 3x - 2y = 3 \quad \rightarrow \quad (1)$

When both the numerator and the denominator are increased by 7, we have $\frac{x+7}{y+7} = \frac{3}{4}$

$\Rightarrow 4x - 3y = -7 \quad \to \quad (2)$

Multiplying equation (2) by 3 and equation (1) by 4 and subtracting one from the other, we have y = 33, putting y = 33 in (1) we get x = 23.

$\therefore$ The required fraction is $\frac{23}{33}$

1.07. Find the values of x and y from the following equations.

$$\frac{20}{x+y} + \frac{12}{x-y} = 8 \text{ and } \frac{30}{x+y} - \frac{4}{x-y} = 1$$

Sol: Let $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$.

Substituting these in the given equations,

we get $20p + 12q = 8 \quad \to \quad (1)$

and $30p - 4q = 1 \quad \to \quad (2)$

Multiplying equation (2) by 3 and adding to equation (1), we get 110p = 11

$\Rightarrow p = \frac{11}{110} = \frac{1}{10}$

Substituting p in (1) we get, 12q = 6

$\Rightarrow q = \frac{1}{2}$

$\therefore \frac{1}{x+y} = \frac{1}{10}$ and $\frac{1}{x-y} = \frac{1}{2}$

$\Rightarrow x + y = 10 \quad \to \quad (3)$

$\Rightarrow x - y = 2 \quad \to \quad (4)$

Adding (3) and (4), we get

$2x = 12 \quad \Rightarrow x = 6$

Substituting x = 6 in (3), we get y = 4

ADDITIONAL CASES IN LINEAR EQUATIONS

1) If the number of equations is less than the number of unknowns, then we say the variables are 'indeterminate' or we have an "indeterminate" system of equations. Here, we cannot uniquely determine the values of all the variables. There will be infinite sets of solutions that satisfy the equations.

   For example, if we take the following two equations in three unknowns,

   $x + y + 2z = 8$

   $2x - y + 3z = 13$

   this system of equations have infinite number of solutions and no unique solution is possible. For any value we take for x, we can find a corresponding set of values for y and z.

2) However, even in case of indeterminate equations, say, of three variables, it is possible that the value of one of the variables may be uniquely determined, i.e., if we have two equations and three unknowns, we may be still able to determine the value of one variable uniquely but the other two variables will have infinite number of values. This will happen if the ratio of the coefficients of two variables in one equation is the same as the ratio of the coefficients of the same two variables in the second equation.

   This depends on the equations given. Example 1.08 will clarify this aspect.

1.08. Two books, four pens and five files cost ₹50. Three books, six pens and seven files cost ₹70. Find the cost of each file.

Sol: Let x, y and z be the cost of each book, pen and file respectively.

Then we have

$2x + 4y + 5z = 50 \quad \to \quad (1)$

$3x + 6y + 7z = 70 \quad \to \quad (2)$

Here, the coefficients of x and y in equation (1) are in the ratio 1 : 2 which is the same as that of the ratio in equation (2).

As the ratio of these coefficients are same, we can find the value of variable z.

If we multiply (1) by 3 and (2) by 2 and subtract one from the other the variables x and y are eliminated and we get the value of z as 10.

$\therefore$ The cost of each file is ₹10

3) Even in case of indeterminate equations, when some additional conditions are either implicitly built into the problem or explicitly imposed by specifying some constraints on the values of the variables, we may some times be able to determine the values of the variables uniquely or find out a finite set of values that the variables may take. Such problems are separately considered under the chapter "SPECIAL EQUATIONS."

4) Sometimes, even if we have equations less in number than the number of variables (i.e., indeterminate equations), while we cannot find out the values of ALL the variables uniquely, it may be possible to find out the value of some specific combination of the variables.

1.09. If Ramesh eats 5 vadas, 4 idlies and 5 kachories the bill amounts to ₹131. If he eats 8 vadas, 6 idlies and 10 kachories, the bill amounts to ₹210. If he eats 6 vadas, 4 idlies and 10 kachories, then what should Ramesh pay?

Sol: Let the cost of each vada, idlie and kachori be V, I, and K respectively, then we have

$5V + 4I + 5K = 131 \quad \to \quad (1)$

$8V + 6I + 10K = 210 \quad \to \quad (2)$

While there are only two equations in three unknowns we can see that by taking the difference of the two equations

we get 3V + 2I + 5K = 79

The cost of 6 vadas, 4 idlies and 10 kachories is 6V + 4I + 10K

= 2(3V + 2I + 5K) = 2(79) = ₹158.

5) Sometimes, even if we have three equations in three unknown, we may not be able to uniquely determine the values of the variables if the equations are not "INDEPENDENT," i.e., one of the given equations can be written as a "linear combination" of the other two equations.

   For example, let us take the following system of three equations in three unknowns.

   $3x + 5y + 7z = 12 \quad \to \quad (1)$

   $x - 3y + 9z = 16 \quad \to \quad (2)$

   $9x + 8y + 31z = 54 \quad \to \quad (3)$

If we try to solve these equations, we will find that we cannot get a unique solution. That is because these equations are not independent. In this case, equation (3) can be obtained by multiplying equation (1) by 2.5 and equation (2) by 1.5 and adding them.

If there are three equations l_1, l_2 and l_3 in three unknowns, we say that they are linearly dependent if one of the three equations can be written as a linear combination of the other two, i.e., **$l_3 = l_1 + kl_2$ where k is any constant**.

In such a case, the system of equations will have infinite number of solutions.
If it is not possible to write the three equations in the form above, then they are linearly independent and the system of equations will have a unique solution.

6) Sometimes, we can have "inconsistent" equations. For example, if we know that $x + 2y = 4$, then the value of $2x + 4y$ has to be 8. The expression $(2x + 4y)$ cannot take any other value. If it is given any other value, there will be inconsistency in the data because then we will effectively be saying that $x + 2y = 4$ and at the same time $x + 2y \neq 4$.

So, if we have the system of equations
$x + 2y = 4$ and $2x + 4y = k$, this system of equations will be consistent ONLY If the value of $k = 8$. For any other value of k, the system of equations will be INCONSISTENT.

In the above system of equations, when $k = 8$, there will be infinite number of solutions (and not a unique solution).

1.10. For what value of k will the following system of equations be consistent.
$2x – 3y = 4$; $6x – 9y = k$

Sol: In the two equations, the ratio of the coefficients of x terms is same as the ratio of the coefficients of y terms (which is 1 : 3). Hence the ratio of the constant terms should be the same, for the equations to be consistent. For the constant terms to be in the ratio 1 : 3, the value of k has to be 3(4) = 12. Hence the value of k for which the equations are consistent is 12.

Concept Review Questions

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Solve for x.
$3(x + 4) + 8 = 5x$

2. Solve the following pair of equations for x and y respectively.
$12x – 10y –2 = 0$ and $10x – 10y + 20 = 0$
(A) 13, 11 (B) 11, 13 (C) 12, 10 (D) 10, 12

3. Thrice a number exceeds three-fourth of it by 36. Find the number.

4. Ashok's age, 30 years hence, will be twice his age five years ago. Find his present age. (in years)

5. Three pens and four erasers cost ₹18. Four pens and three erasers cost ₹17. Find the cost of 14 pens and 14 erasers.
(A) ₹70 (B) ₹60 (C) ₹50 (D) ₹40

6. The cost of two dosas and three idlis is ₹46. The cost of a dosa and two idlis is ₹26. Find the cost of four dosas and four idlis (in ₹).

7. The digits of a two-digit number differ by six. Find the difference of the number and the number formed by reversing its digits.

8. The difference between a three-digit number and the number formed by reversing its digits is divisible by ______.
(A) 9
(B) 11
(C) Both 9 and 11
(D) Neither 9 nor 11

9. The first and the last digits of a three-digit number differ by 4. Find the difference of the number and the number formed by reversing its digits.

10. Five sharpeners and six erasers cost ₹28. Six sharpeners and five erasers cost ₹27. Find the cost (in ₹) of each sharpener and each eraser respectively.
(A) 3, 2 (B) 2, 3 (C) 1, 4 (D) 4, 1

11. How many pairs of x and y satisfy $3x + 6y = 18$ and $9x + 18y = 57$?
(A) 2
(B) 1
(C) 0
(D) None of these

12. How many pairs of x and y satisfy the equations $4x + 6y = 16$ and $6x + 9y = 24$?
(A) 0
(B) 1
(C) ∞
(D) None of these

13. How many pairs of x and y satisfy the equations $6x + 5y = 16$ and $8x + 7y = 22$?
(A) 0
(B) 1
(C) ∞
(D) None of these

14. Three chocolates, four biscuits and five cakes cost ₹34. Six chocolates and eight biscuits cost ₹38. Find the cost of each cake (in ₹).

15. The sum of a two-digit number and its reverse is k times the sum of its digits. Find the value of k.
(A) 9 (B) 10
(C) 11 (D) Cannot be determined

Exercise – 1(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Solve: 199x + 201y = 1001; 201x + 199y = 999.
(A) x = 2, y = 3 (B) x = 3, y = 2
(C) x = 3, y = 4 (D) x = 4, y = 3

2. Solve: $\frac{16}{2x+3y} - \frac{7}{3x+2y} = 1$,
$\frac{8}{3(2x+3y)} + \frac{21}{3x+2y} = \frac{10}{3}$.
(A) x = 2, y = 1 (B) x = 1, y = 2
(C) x = –2, y = –1 (D) x = –1, y = –2

3. A rope of 77 meters is cut into two pieces such that the length of one piece is $4/7^{th}$ of the other. What is the length of $3/14^{th}$ of the longer piece? (in m)

4. Anand has only 10 paise and 25 paise coins with him. If he has 70 coins in all worth ₹10 with him, how many 25 paise coins does he have?
(A) 20 (B) 25 (C) 40 (D) 50

5. Find the greater of the two numbers such that their sum is 200 and the difference of their squares is 8000.
(A) 80 (B) 100 (C) 120 (D) 140

6. A fraction becomes 1/2, if its numerator is increased by 1 and the denominator by 3. It becomes 2/5 if the numerator is increased by 2 and the denominator by 7. Find the fraction.
(A) 1/2 (B) 4/7 (C) 1/5 (D) 2/3

7. There is some money with Ajay and some with Vijay. If Ajay gives ₹30 to Vijay, then the amounts with them would be interchanged. Instead, if Vijay gives ₹20 to Ajay, then Ajay would have ₹70 more than Vijay would have. Find the amount that Ajay has.
(A) ₹40 (B) ₹50
(C) ₹70 (D) Cannot be determined

8. How many two-digit numbers with their tens digit greater than their units digit, have the sum of their digits equal to twice their difference?

9. A two-digit number is such that the sum of its digits is thrice the difference of its digits. If the number exceeds the number formed by reversing its digits by 36, find the number.

10. The difference between a three-digit number and the number formed by reversing its digits is 297. The sum of the units and the tens digits is the same as the difference of the hundreds and the units digits. Also, the hundreds digit is twice the units digit. Find the number.
(A) 242 (B) 342 (C) 603 (D) 884

11. Four years ago, a man was thrice as old as his son. Eight years hence, the man will be twice as old as his son. What is the present age (in years) of the son?

12. The present average age of Ram and his wife Sita and their daughter is 35 years. Fifteen years from now, the age of Sita will be equal to the sum of the present ages of Ram and the daughter. Find the present age (in years) of Sita.

13. X says to Y, "I am twice as old as you were when I was as old as you are." The sum of their present ages is 63 years. Find the present age of X.
(A) 24 years (B) 39 years
(C) 36 years (D) 42 years

14. Six years ago, the age of a person was two years more than five times the age of his son. Four years hence, his age will be two years less than three times the age of his son. After how many years from now will their combined age be 100 years?
(A) 48 years (B) 14 years
(C) 19 years (D) 38 years

15. **(a)** If the following three equations form a system of dependent equations, what is the value of p?
I. 3x + 2y – 7z = 56
II. 5x + 3y + z = 16
III. px + 12y – 19z = 200

(b) Find k if the given system of equations has infinite solutions.
2x + ky = 1 + 2y and kx + 12y = 3

(c) Find the value of k if the equations 4x + 5y = 32 and 12x + 15y = 2k are not inconsistent.

16. The cost of two balls, three bats and eight pairs of gloves is ₹2500, while the cost of four balls, five bats and ten pairs of gloves is ₹4000. Find the cost of each bat.
(A) ₹350 (B) ₹500
(C) ₹800 (D) Cannot be determined

17. The cost of three pens, four rulers and five refills is ₹75 while that of ten refills, six pens and seven rulers is ₹138. Find the cost of three pens, one ruler and five refills.
(A) ₹39
(B) ₹42
(C) ₹44
(D) Cannot be determined

18. The cost of two pencils, one eraser and three sharpeners is ₹23. The cost of six pencils, three erasers and one sharpener is ₹29. The cost of 14 pencils, seven erasers and seven sharpeners is ₹91. Find the cost of each pencil.
(A) ₹3 (B) ₹5
(C) ₹4 (D) Cannot be determined

19. A bag has a total of 120 notes in denominations of ₹2, ₹5 and ₹10. The total value of the notes in the bag is ₹760. If there were twice as many ₹5 notes, the total value of the notes would be ₹960. Find the number of ₹10 notes in the bag.

Directions for questions 20 and 21: These questions are based on the information given below.

Rohan went to a stationery shop to purchase pens, erasers and rulers. He purchased more number of pens than erasers and more number of erasers than rulers. He purchased at least 10 items of each type. The total number of items purchased is 35.

20. How many rulers did Rohan purchase?

21. If each pen cost ₹20, each ruler cost ₹2 and each eraser cost ₹5, find the minimum amount (in ₹) that Rohan spent for purchasing the items.

Directions for questions 22 and 23: These questions are based on the information given below.

A shopkeeper sold a certain number (a two-digit number) of toys all priced at a certain value (also a two-digit number when expressed in rupees). By mistake he reversed the digits of both, the number of items sold and the price of each item. In doing so, he found that his stock left at the end of the day showed 72 items more than what it actually was.

22. What could be the actual number of toys sold?
(A) 19 (B) 49 (C) 91 (D) 94

23. If the faulty calculations show a total sale of ₹1577, what was the actual selling price of each toy?
(A) ₹38 (B) ₹57 (C) ₹75 (D) ₹83

24. A so-called great gambler started playing a card game with a certain amount of money. In the first round he tripled his amount and he gave away ₹p to his friend. In the second round he doubled the amount with him and gave away ₹3p to his friend. In the third round he quadrupled the amount with him and gave away ₹2p to his friend and was finally left with no money. If he gave away a total of ₹360 to his friend, then what was the amount of money that he started with (in ₹)?

25. In t minutes, the time would be 8:00 a.m. If 40 minutes ago, the time was 3t minutes past 2:00 a.m., then find the present time.
(A) 6:20 a.m. (B) 6:40 a.m.
(C) 5:20 a.m. (D) 5:40 a.m.

26. An exam has 120 questions. Each correct answer carries 1 mark. Each wrong answer is penalized by $\frac{1}{3}^{rd}$ of a mark and each unanswered question is penalized by $\frac{1}{6}^{th}$ of a mark. A student who attempted the exam scored 60 marks. The minimum number of answers that the student could have got wrong is ______ .

27. A shopkeeper had a weighing balance with uneven pans. The left and the right pans of the balance weighed 0.6 kg and 0.95 kg respectively. When the shopkeeper placed some rice in the left pan and standard weights in the right pan until the pans leveled, he had to use 'ab' kg (where 'ab' is a two-digit number) of the standard weights. If instead, he placed the rice in the right pan and standard weights in the left pan until the pans leveled, he had to use ('ba' +18.7) kg of the standard weights. The actual weight of rice placed in the pans can be _______.
(A) 76.35 kg (B) 85.35 kg
(C) 35.35 kg (D) 53.35 kg

28. Bala had three sons. He had some chocolates which he distributed among them. To his eldest son, he gave 3 more than half the number of chocolates with him. To his second eldest son he gave 4 more than one-third of the remaining chocolates with him. To his youngest son he gave 4 more than one-fourth of the remaining chocolates with him. He was left with 11 chocolates. How many chocolates did he initially have?
(A) 180 (B) 78 (C) 144 (D) 120

29. Prakash, Sameer, Ramesh and Tarun have a total of ₹240 with them. Prakash has half the total amount of what the others have. Sameer has one-third of the total amount of what the others have. Ramesh has one-fourth of the total amount of what the others have. Find the amount with Tarun (in ₹)

30. There are ten children standing in a line, not all of whom have the same number of chocolates with them. If the first child distributes his chocolates among the remaining nine such that he doubles their respective number of chocolates then he will be left with one chocolate. If the tenth child takes away one chocolate from each of the remaining nine then he will have four chocolates less than the first child initially had. What is the total number of chocolates with the second child to the ninth child?

Exercise – 1(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Solve: $\frac{x}{2} + 2y = 14$, $3x + \frac{y}{6} = 13$.

(A) x = 2, y = 6 (B) x = 4, y = 6
(C) x = 4, y = 12 (D) x = 2, y = 12

2. Solve : $8(x + 5) + 7(y – 2) = – 5$,

$$2(x + 6) - \frac{4 - y}{5} = 5.$$

(A) x = 2, y = –1 (B) x = –2, y = 1
(C) x = –3, y = –1 (D) None of these

3. Solve: $\frac{x}{4} + 2y = 7$; $\frac{19}{x + (y/4)} = 4$.

(A) x = 2, y = 3 (B) x = –2, y = 6
(C) x = 4, y = 3 (D) x = –4, y = 6

4. The number of pencils with P is 5/3 times the number of pencils with Q. If P has 18 pencils more than Q, then find the total number of pencils with them.

5. A question paper consists of 50 questions. Each correct answer fetches three marks and one mark is deducted for each wrong answer. A student who attempted all the questions scored 90 marks. Find the number of questions answered correctly by him.

6. The sum of the ages of two friends A and B 18 years ago was half of the sum of their ages today. Presently, A is twice as old as B. What is the present age of A (in years)?

7. A student was asked to find $3/7^{th}$ of a number and he instead multiplied it by 7/3. As a result, he got an answer, which was more than the correct answer by 1680. What was the number?

(A) 882 (B) 273 (C) 840 (D) 1684

8. A boy has a total of ₹14 in denominations of 25 paise and 20 paise coins. If the numbers of coins of the two denominations were swapped, the total value of coins would be ₹1 less. Find the total number of coins.

9. Govind is four times as old as Ganesh is. 20 years hence, Govind's age will be twice that of Ganesh's age. Find Ganesh's present age. (in years)

(A) 20 (B) 10 (C) 15 (D) 30

10. Five years ago, Alok's age was five times Bharan's age. Five years hence, Alok's age will be thrice that of haran's age. Find Bharan's present age. (in years)

11. Praveen's present age is twice that of Mahesh's age four years ago. Eight years hence, Praveen would be twice as old as Mahesh is today. Find the sum of their present ages.

(A) 36 years
(B) 44 years
(C) 64 years
(D) Cannot be determined

12. Alok's age is 5/3 times of Alakhnanda's age. Alakhnanda is now 3 times as old as she was, when Alok was as old as Alakhnanda is today. Find Alok's age when Alakhnanda was half as old as Alok is now.

(A) 60 (B) 50
(C) 40 (D) Data insufficient

13. The sum of the ages of Ajay and Bala, 20 years ago was five-ninth the sum of their present ages. Ajay's present age exceeds that of Bala by 20 years. Find the present age of Ajay. (in years)

14. Ten years ago, the age of a man was 20 years less than 6 times his son's age. Ten years hence, his age will be 30 years less than thrice his son's age. After how many years from now will their combined age be 90 years?

(A) 5 (B) 10 (C) 15 (D) 20

15. A two-digit number is formed by either subtracting 16 from eight times the sum of the digits or by adding 20 to 22 times the difference of the digits. Find the number.

(A) 24 (B) 48 (C) 64 (D) 82

16. If x/4 years ago, Alok was 14 years old and x/4 years from now he will be 4x years old, how old will he be 5x years from now? (in years)

17. Two boys and two girls went to a movie. They found that there were only two tickets available in the counter and they bought them. For purchasing the remaining two tickets (in black), they spent ₹50 more for each ticket than the actual price. At the end they found that each person had spent ₹60 for the ticket as his/her share. Find the actual price (in ₹) of each ticket.

18. There are two two-digit numbers such that the tens digit of the first number is 3/2 times the tens digit of the second number, while the sum of the two numbers is 158. Which of the following can be the difference between them?

(A) 58 (B) 71 (C) 36 (D) 40

19. In a three-digit number, the difference between hundreds digit and the tens digit is equal to the difference between the tens digit and the units digit. If the sum of the digits is 9, how many numbers satisfy the given condition?

20. The difference between a three-digit number and the number formed by reversing its digits is 792. The sum of its digits is 18 and the hundreds digit is 9 times its units digit. Find the number.

21. Mr. Ram distributed a total of 225 chocolates among his sons - A, B, C and D. The number of chocolates he gave to A and D together is twice the number of chocolates he gave to B and C together. If B received 15 more chocolates than C, find the number of chocolates C received.

22. If the numerator of a fraction is increased by two and the denominator by one, the fraction becomes 13/15. If the numerator and the denominator are each decreased by four, the fraction becomes 4/5. Find the fraction.

(A) 9/24 (B) 13/19
(C) 24/29 (D) None of these

23. Ajay and Sita are two of Mr.Sharma's children. Ajay has half as many brothers as sisters. Sita has as many brothers as sisters. Find the number of children Mr.Sharma has.

24. Rohan went to the market to buy 10 kg of each of oranges, mangoes, bananas and grapes. The cost of 5 kg oranges and 2 kg mangoes together was ₹310. The cost of 3 kg mangoes and 3.5 kg bananas together was ₹230. The cost of 1.5 kg bananas and 5 kg grapes together was ₹160. Find the total amount spent by Rohan (in ₹).

Directions for questions 25 and 26: These questions are based on the information given below.

Arjun went to a market to buy apples, bananas and oranges. He bought more bananas than apples and more oranges than bananas. He bought a total of 45 fruits and at least 13 of each.

25. If he bought less than 15 bananas, how many oranges did he buy?

26. The cost of an apple, a banana and an orange is ₹5, ₹4 and ₹3 respectively. What is the minimum possible expenditure (in ₹) that Arjun could have incurred?

27. If a, b, c and d satisfy the equations

$a + 7b + 3c + 5d = 0$,
$8a + 4b + 6c + 2d = -16$,
$2a + 6b + 4c + 8d = 16$ and
$5a + 3b + 7c + d = -16$, then $(a + d)(b + c)$ equals _____.

(A) 0 (B) 16 (C) –16 (D) –64

28. Considering the equations $2x - 3y = 8$ and $px - qy = 66$, answer the following questions:

(i) Find $4(p + q)$ if the equations above have infinite solutions.

(ii) Find p if q = 9 and the equations above have no solution.

29. In a four-digit number with distinct digits, the sum of the middle digits equals the sum of the extreme digits. The sum of its second and fourth digits equals five times the sum of its other two digits. If the sum of its digits is 18, what is the sum of all the possible values of the hundreds digit?

(A) 21 (B) 24 (C) 27 (D) 30

30. A two-digit number is obtained by either subtracting 12 from four times the sum of its digits or by adding 6 to twice the difference of its digits. Find the number.

(A) 16
(B) 28
(C) 39
(D) Cannot be determined

31. Ramu has some chocolate boxes with him to sell. He sells them either as full boxes or half boxes. The first customer buys half a box more than half the number of boxes with Ramu. The second customer buys half a box more than half the remaining number of boxes with him. Ramu continues to sell in this manner to eight other customers. He is left with no boxes to sell after that. How many chocolate boxes did Ramu have in the beginning?

(A) 511 (B) 513 (C) 1023 (D) 1025

32. Alok went to a casino to play a card game. He played 10 rounds of that game. In each round, he doubled his amount and then gave ₹x to his friend. After 10 rounds, he had ₹1023. Find the sum of the digits of the least possible value of x. (All the amounts involved (in rupees) are integers)

Directions for questions 33 and 34: These questions are based on the information given below.

A shopkeeper sold a two-digit number of toys all priced at a certain value (also a two-digit number when expressed in rupees). By mistake he reversed the digits of both, the number of items sold and the price (in ₹) of that item while entering in the computer. So, the stock which was shown as left at the end of the day in the computer showed 81 items more than what it actually was.

33. How many possibilities exist for the actual number of toys sold?

34. If the faulty calculations show a total sale of ₹486, what was the actual selling price (in ₹) of each toy?

35. Raja went to a casino to play a card game. He played 3 rounds of the game. In each round he doubled the amount he had with him and gave ₹X to his friend at the end of the round. The amount he had with him at the end of the third round after giving ₹X to his friend was ₹140 more than the sum of the amounts with him at the end of the previous rounds after giving ₹X to his friend. The amount with him at the end of the second round after giving ₹X to his friend was ₹160 more than the amount he had with him at the end of the first round after giving ₹X to his friend. Find the value of X.

(A) 10 (B) 20
(C) 30 (D) 40

Directions for questions 36 to 40: Each question is followed by two statements, I and II. Indicate your responses based on the following directions:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using statements I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using statements I and II together.

36. If $x + 2y + 3z = 14$, then find the value of z.
I. $2x + 3y + z = 14$
II. $3x + y + 2z = 11$

37. Guru had some one-rupee coins, 50-rupee notes and 100-rupee notes. He exchanged all his coins for 50-rupee and 100-rupee notes (not by value, only by number). After the exchange, Guru has ₹500. How many 50-rupee notes does he have after the exchange?
I. He has not more than 6 notes after the exchange.
II. He has not less than 6 notes after the exchange.

38. What is my age?
I. Five years ago, my sister's age was half of my age.
II. Five years from now, my sister's age will be three – fourths of my age.

39. How many questions did I attempt in a maths test having 25 questions?
I. I scored 16 marks.
II. For every correct answer I got 1 mark while for every incorrect answer I lost $^1/_4$ mark.

40. If $3x + 7y = 19$, then find the value of y.
I. $6x + 14y = 38$
II. $9x – 20y = 16$

Key

Concept Review Questions

1. 10
2. B
3. 16
4. 40
5. A
6. 80
7. 54
8. C
9. 396
10. B
11. C
12. C
13. B
14. 3
15. C

Exercise – 1(a)

1. A
2. B
3. 10.5
4. A
5. C
6. D
7. D
8. 3
9. 84
10. C
11. 16
12. 45
13. C
14. C
15. (a) 19
(b) 6
(c) 48
16. D
17. A
18. D
19. 50
20. 10
21. 340
22. C
23. A
24. 55
25. B
26. 3
27. D
28. B
29. 52
30. 12

Exercise – 1(b)

1. B
2. C
3. C
4. 72
5. 35
6. 48
7. A
8. 60
9. B
10. 15
11. D
12. D
13. 55
14. B
15. C
16. 35
17. 35
18. C
19. 9
20. 981
21. 30
22. C
23. 7
24. 1400
25. 18
26. 175
27. C
28. (i) 165
(ii) 6
29. B
30. A
31. C
32. 6
33. 1
34. 45
35. B
36. C
37. A
38. C
39. D
40. A