CHAPTER – 3

INEQUALITIES AND MODULUS

If 'a' is any real number, then 'a' is either positive or negative or zero. When ‘a’ is positive, we write $a > 0$ which is read 'a is greater than zero'. When ‘a’ is negative, we write $a < 0$ which is read 'a is less than zero'. If ‘a’ is zero, we write $a = 0$ and in this case, ‘a’ is neither positive nor negative.

Symbols and Notations:

‘>' means ‘greater than'
‘<' means ‘less than'
‘≥' means ‘greater than or equal to'
‘≤' means ‘less than or equal to'

For any two non-zero real numbers a and b,
(i) a is said to be greater than b when $a – b$ is positive.
(ii) a is said to be less than b when $a – b$ is negative.

These two statements are written as
(i) $a > b$ when $a – b > 0$ and
(ii) $a < b$ when $a – b < 0$.

For example,
3 is greater than 2 because $3 – 2 = 1$ and 1 is greater than zero. –3 is less than –2 because $–3 – (–2) = –1$ and –1 is less than zero.

Certain properties and useful results pertaining to inequalities are given below. A thorough understanding of these properties/results is very essential for being able to solve the problems pertaining to inequalities.

[In the following list of properties and results, numbers like a, b, c, d, etc. are real numbers]

- For any two real numbers a and b, either $a > b$ or $a < b$ or $a = b$.
- If $a > b$, then $b < a$.
- If $a \nless b$, then $a \geq b$ and if $a \ngtr b$, then $a \leq b$.
- If $a > b$ and $b > c$, then $a > c$.
- If $a < b$ and $b < c$, then $a < c$.
- If $a > b$, then $a \pm c > b \pm c$.
- If $a > b$ and $c > 0$, then $ac > bc$.
- If $a < b$ and $c > 0$, then $ac < bc$.
- If $a > b$ and $c < 0$, then $ac < bc$.
- If $a < b$ and $c < 0$, then $ac > bc$.
- If $a > b$ and $c > d$, then $a + c > b + d$.
- If $a < b$ and $c < d$, then $a + c < b + d$.
- The square of any real number is always greater than or equal to 0.
- The square of any non-zero real number is always greater than 0.
- If $a > 0$, then $–a < 0$ and if $a > b$, then $–a < –b$.
- If a and b are positive numbers and $a > b$, then
  (i) $1/a < 1/b$,
  (ii) $a/c > b/c$ if $c > 0$ and
  (iii) $a/c < b/c$ if $c < 0$.

- For any two positive numbers a and b,
  If $a > b$, then $a^2 > b^2$.
  If $a^2 > b^2$, then $a > b$.
  If $a > b$, then for any positive value of n, $a^n > b^n$.

- Let, A, G and H be the Arithmetic mean, Geometric mean and Harmonic mean of n positive real numbers. Then $A \geq G \geq H$, the equality occurring only when the numbers are all equal.

- If the sum of two positive quantities is given, their product is the greatest when they are equal; and if the product of two positive quantities is given, their sum is the least when they are equal.

- If $a > b$ and $c > d$, then we cannot say anything conclusively about the relationship between $(a – b)$ and $(c – d)$; depending on the values of a, b, c and d, it is possible to have
  $(a – b) > (c – d)$, $(a – b) = (c – d)$ or $(a – b) < (c – d)$

When two numbers a and b have to be compared, we can use one of the following two methods:

- If both a and b are positive, we can take the ratio a/b and depending on whether a/b is less than, equal to or greater than 1, we can conclude that a is less than, equal to or greater than b.

  In other words, for two positive numbers a and b,
  if $a/b < 1$ then $a < b$
  if $a/b = 1$ then $a = b$
  if $a/b > 1$ then $a > b$

- If one or both of a and b are not positive or we do not know whether they are positive, negative or zero, then we can take the difference of a and b and depending on whether $(a – b)$ is less than, equal to or greater than zero, we can conclude that a is less than, equal to or greater than b.

  In other words, for any two real numbers a and b,
  if $a – b < 0$, then $a < b$,
  if $a – b = 0$, then $a = b$,
  if $a – b > 0$, then $a > b$.

- For any positive number $x \geq 1$,

  $$2 \leq \left(1+\frac{1}{x}\right)^x < 2.8.$$

  The equality in the first part will occur only if $x = 1$.

- For any positive number, the sum of the number and its reciprocal is always greater than or equal to 2,

  i.e., $x+\frac{1}{x} \geq 2$ where $x > 0$. The equality in this relationship will occur only when $x = 1$.

Absolute Value:

(written as |x| and read as "modulus of x")

For any real number x, the absolute value is defined as follows:

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \text{ and} \\ -x, & \text{if } x < 0 \end{cases}$$

Properties of Modulus:

For any real number x and y,

1. $x = 0 \Leftrightarrow |x| = 0$
2. $|x| \geq 0$ and $-|x| \leq 0$
3. $|x + y| \leq |x| + |y|$
4. $||x| - |y|| \leq |x - y|$
5. $-|x| \leq x \leq |x|$
6. $|x \cdot y| = |x| \bullet |y|$
7. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$; $(y \neq 0)$
8. $|x|^2 = x^2$

In inequalities, the variables generally take a range of values unlike in the case of equations where the variables in general, take one value or a discrete set of values (In some specific cases, the variables may take only one value).

Interval Notation:

Generally, the solution set or the range of values satisfied by inequalities are not discrete.

So it is important to understand the "interval notation".
(a, b) read as "open interval a, b" means all real numbers between a and b excluding a and b (a < b).

[a, b] read as "closed interval a, b" means all real numbers between a and b including a and b (a < b). [a, b) means all numbers between a and b, with a being included and b excluded (a < b).
The problems on inequalities normally fall into 3 categories:

(a) consisting of first degree expressions in x.

(b) consisting of second degree expressions in x, directly in the problem or consisting single expression which reduces to quadratic expression.

(c) consisting of expressions including "modulus".

An example of each variety is taken below.

Examples:

3.01. Express all real numbers between 3 and 8 in the interval notation from where
(i) 3 and 8 are excluded.
(ii) 3 and 8 are included.
(iii) 3 is included and 8 is excluded.
(iv) 3 is excluded and 8 is included.

Sol. (i) (3, 8) i.e., $3 < x < 8$ and x is a real number.
(ii) [3,8] i.e., $3 \leq x \leq 8$ and x is a real number.
(iii) [3, 8) i.e., $3 \leq x < 8$ and x is a real number.
(iv) (3, 8] i.e., $3 < x \leq 8$ and x is a real number.

3.02. Express in the interval notation,
(i) all positive real numbers,
(ii) all negative real numbers,
(iii) all non-zero real numbers, and
(iv) all real numbers.

Sol. (i) $(0, +\infty)$ or simply $(0, \infty)$ i.e., $0 < x < \infty$
As we cannot write the greatest positive real number, we use the symbol ∞ (infinity),

(ii) $(-\infty, 0)$ i.e., $-\infty < x < 0$,

(iii) $(-\infty, 0) \cup (0, \infty)$
Here zero is excluded and the symbol "$\cup$" stands for the union of two sets and

(iv) $(-\infty, \infty)$ is the notation and this would include all real numbers.

3.03. Express in the interval notation,
(i) all real numbers greater than 3.
(ii) all real numbers less than or equal to –4.

Sol. (i) $(3, +\infty)$ or simply $(3, \infty)$ i.e., $3 < x < \infty$.
As we cannot write the greatest positive real number, we use the symbol ∞ (infinity).

(ii) $(-\infty, -4]$ i.e. $-\infty < x \leq -4$.
Here we use $-\infty$ to represent the least negative real number and we use the '] ' bracket to denote '$\leq$' relation.

3.04. If $11x - 21 \geq 2x + 15$, what is the range of values that x can take?

Sol. $11x - 21 \geq 2x + 15$
$11x - 2x \geq 15 + 21 \Rightarrow 9x \geq 36 \Rightarrow x \geq 4$
Interval notation is $[4, \infty)$.

3.05. Solve the inequalities which hold simultaneously:
$4x + 17 < 33$ and $8x - 12 < 36$.

Sol. $4x + 17 < 33$ – (A)
$8x - 12 < 36$ – (B)
(A) $\Rightarrow$ $4x < 33 - 17 \Rightarrow 4x < 16 \Rightarrow x < 4$ – (C)
(B) $\Rightarrow$ $8x < 36 + 12 \Rightarrow 8x < 48 \Rightarrow x < 6$ – (D)
We have to take the intersection of the range of values represented by the inequalities (C) and (D). i.e., $(x < 4) \cap (x < 6)$, which is $x < 4$.
Interval notation is $(-\infty, 4)$.

3.06. Which of these two numbers, 20^{21} and 21^{20} is greater?

Sol. Let $a = 21^{20}$ and $b = 20^{21}$.

Consider $\frac{a}{b} = \frac{21^{20}}{20^{21}} = \left(\frac{21}{20}\right)^{20} \times \frac{1}{20}$

$\Rightarrow \frac{a}{b} = \frac{(1+1/20)^{20}}{20}$

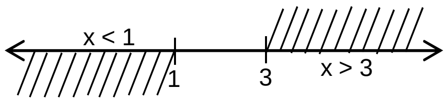
As $(1+1/x)^x$ lies between 2 and 2.8, for all positive values of x, $(1+1/20)^{20}$, cannot exceed 2.8.

So, $(1+1/20)^{20} < 2{\cdot}8$ i.e. $\frac{(1+1/20)^{20}}{20} < \frac{2 \cdot 8}{20} < 1$

As $\frac{a}{b} < 1$, $a < b$ i.e., $21^{20} < 20^{21}$.

3.07. Solve the simultaneous inequalities:
$9x + 8 > 2x + 29$ and $7x - 12 < 5x - 10$.

Sol. $9x + 8 > 2x + 29$ – (A)
$7x - 12 < 5x - 10$ – (B)
From (A), $9x - 2x > 29 - 8 \Rightarrow 7x > 21$
$\Rightarrow x > 3$ – (C)
From (B), $7x - 5x < -10 + 12$
$\Rightarrow 2x < 2 \Rightarrow x < 1$ – (D)
The intersection of (C) and (D) i.e., $(x > 3) \cap (x < 1)$ is empty.
This is because we cannot find a number less than 1 and at the same time greater than 3. The student can understand this on the number line.



Hence, the given inequalities have no solutions.

3.08. Solve for x if $3x^2 - 14x + 15 > 0$.

Sol. $3x^2 - 14x + 15 > 0$
$\Rightarrow 3x^2 - 9x - 5x + 15 > 0$
$\Rightarrow (3x - 5)(x - 3) > 0$
$\Rightarrow (3x - 5) > 0$ and $(x - 3) > 0$
(or) $(3x - 5) < 0$ and $(x - 3) < 0$
i.e., $x > \frac{5}{3}$ and $x > 3$
(or) $x < \frac{5}{3}$ and $x < 3$
i.e., $x > 3$ or $x < \frac{5}{3}$.
i.e., all values of $x < 5/3$ or all values of $x > 3$ satisfy the inequality.

The interval notation is $(-\infty, 5/3) \cup (3, \infty)$
In general, we can note the following rules for quadratic inequalities:

A quadratic inequality of the type $(x - p)(x - q) < 0$ (where $p < q$) is satisfied by all values of x that lie between p and q. In other words, $p < x < q$ will satisfy the inequality.

A quadratic inequality of the type $(x - p)(x - q) > 0$ (where $p < q$) is not satisfied by any value of x that lies between p and q. In other words, $p \le x \le q$ will not satisfy the inequality. So all values from $-\infty$ to $+\infty$, except those that lie between p and q will satisfy the inequality.

3.09. Solve for x : $4x^2 - 12x + 17 > 0$.

Sol. We try to resolve this expression into factors of first degree. As such it may not work out this way. So we identify the "hidden perfect square" in it.
We rewrite the expression as,
$((2x)^2 - 2 \cdot (2x) \cdot (3) + 3^2) + 8 \ge 0$
$\Rightarrow (2x - 3)^2 + 8 \ge 0$
As $(2x - 3)^2 \ge 0$ for all $x \in R$,
$(2x - 3)^2 + 8 \ge 8 + 0$
So $(2x - 3)^2 + 8 > 0$ for all $x \in R$
Hence all real numbers would satisfy this inequality. The interval notation is $(-\infty, \infty)$.

3.10. Solve for x: $\frac{x^2 + 5x - 14}{2x^2 + x - 6} < 0$

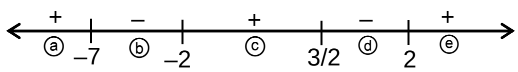
Sol. Let $f(x) = \frac{x^2 + 5x - 14}{2x^2 + x - 6} < 0$

We resolve the numerator and denominator into linear factors.

Now $f(x) = \frac{(x + 7)(x - 2)}{(2x - 3)(x + 2)} < 0$.
We multiply and divide the expression, with $(2x - 3)(x + 2)$
[Note: Cross multiplying should not be done as we are not sure of the sign of the expression in the denominator.]
i.e., $\frac{(x + 7)(x - 2)(2x - 3)(x + 2)}{(2x - 3)^2 (x + 2)^2} < 0$

$(x + 7)(x - 2)(2x - 3)(x + 2) < 0$

(as the denominator is positive).
We find the zeros (or roots) of the expression and place them on the number line in order.



We see that there are 5 sections on the number line indicated as ⓐ, ⓑ, ⓒ, ⓓ and ⓔ.
In region ⓐ, the expression is positive since each of $(x + 7)$, $(x + 2)$, $(2x - 3)$ and $(x - 2)$ is negative while in ⓑ, the expression is negative as $(x + 7)$ is positive while all the other 3 terms take a negative value.
Similarly the sign of expression can be evaluated in each region, and we conclude that in ⓒ it is positive, in ⓓ it is negative and in ⓔ it is positive.
As we want the expression to take a "negative sign", the solution set is the union of sections ⓑ and ⓓ.
The interval notation is $(-7, -2) \cup (3/2, 2)$.

3.11. Solve the inequality: $|2x + 7| > -5$.

Sol. As the modulus of any quantity is non-negative and any non-negative quantity is greater than –5, every real number x, would satisfy the inequality.
Symbolically, $|2x + 7| \ge 0 > -5$. So $|2x + 7| > -5$ for all $x \in R$. So all real numbers (R) form the solution set.
The interval notation is $(-\infty, \infty)$.

3.12. Solve for x : $|x^2 + x + 1| < -1$.

Sol. Any quantity less than –1 has to be negative. This means $|x^2 + x + 1|$ should be negative. But the modulus of a quantity cannot be negative. So no solution exists for this inequality.
The notation for this is that of empty set ϕ.

3.13. Solve for x : $|x - 3| = 7$

Sol. $x - 3 = 7$ or $-(x - 3) = 7$
$\Rightarrow x = 10$ or $x = -4$.
If $p > 0$, $|x - a| = p \Rightarrow x = a + p$ or $a - p$

3.14. Solve for x : $|2x + 5| < 13$.

Sol. By definition of modulus,

$$|2x + 5| = \begin{cases} 2x+5; \text{if } 2x+5 \geq 0 \text{ or } x \geq -5/2 \\ -(2x+5); \text{if } 2x+5 < 0 \text{ or } x < -5/2 \end{cases}$$

Hence we have to consider two cases $x \geq -5/2$ and $x < -5/2$.
Case (i): $x \geq -5/2$.
$|2x + 5| < 13$
$\Rightarrow 2x + 5 < 13 \Rightarrow x < 4$
Hence the admissible range of values of x is
$x \geq -5/2$ and $x < 4$ i.e., $x \in [-5/2, 4)$

Case (ii): $x < -5/2$
$|2x + 5| < 13 \Rightarrow -(2x + 5) < 13 \Rightarrow x > -9$
Hence the admissible range of values of x is
$x > -9$ and $x < -5/2$. i.e., $x \in (-9, -5/2)$
Hence all the values of x, for which $x \in (-9, -5/2) \cup [-5/2, 4) = (-9, 4)$, satisfy the given inequality.

3.15. Find the maximum value of
$f(x) = 8 - |-4 - x|; x \in R$.

Sol. $f(x) = 8 - |-4 - x|$ is the maximum when $|-4 - x|$ is the minimum. As the minimum value of $|-4 - x|$ is 0, the maximum value of f (x) is 8.

Some useful models:

Quite often, when dealing with positive real numbers we come across situations where the sum (or product) of certain variables is given and we are required to maximise (or minimise) the product (or sum) of the same. We illustrate the technique involved with a couple of examples.

Model 1: If $ax + by = k$ where a, b, x, y are all positive, maximise $x^m y^n$ where m and n are positive integers.

3.16. If $4x + 7y = 18$, then find the maximum value of x^4y^5.

Sol. Consider the expression $\left(\frac{4x}{4}\right)^4 \left(\frac{7y}{5}\right)^5$

Now the sum of all the factors of the above expression is

$4. \frac{4x}{4} + 5. \frac{7y}{5}$
$= 4x + 7y = 18$ (constant)
$\therefore$ Sum of the factors is constant.

$\left(\frac{4x}{4}\right)^4 \left(\frac{7y}{5}\right)^5$ is maximum when $\frac{4x}{4} = \frac{7y}{5}$

$\therefore \frac{4x}{4} = \frac{7y}{5} = \frac{4x + 7y}{9}$

$\Rightarrow \frac{4x}{4} = \frac{7y}{5} = 2; \Rightarrow x = 2$

and $y = \frac{10}{7}$

$\therefore$ The maximum value of $x^4y^5 = 24\left(\frac{10}{7}\right)^5$

Note: When the expression $ax + by$ is constant, the maximum value of $x^m y^n$ is realized when $\frac{ax}{m} = \frac{by}{n}$.

Model 2: If $x^m y^n = k$ where $x > 0$, $y > 0$ and m and n are positive integers, minimise $ax + by$ where $a > 0$, $b > 0$.

3.17. If $x^2y^3 = 2^2 3^6 5^2$, find the minimum value of $3x + 5y$.

Sol. Given $x^2y^3 = 2^2 3^6 5^2$
$\Rightarrow 3^2x^2\, 5^3y^3 = 2^2 3^8 5^5$

$\Rightarrow \left(\frac{3x}{2}\right)^2 \left(\frac{5y}{3}\right)^3 = 3^5 5^5$

The LHS is the product of 5 factors the sum of these factors, i.e., $3x + 5y$ will have its minimum value, when all these factors are equal.

i.e., when $\frac{3x}{2} = \frac{5y}{2}$

$\Rightarrow y = \frac{9x}{10}$

Given $x^2y^3 = 2^2 3^6 5^2$

$\Rightarrow x^2\left(\frac{9x}{10}\right)^3 = 2^2 3^6 5^2$

$\Rightarrow x^5 = 2^5 5^5$ or $x = 10$

$\therefore y^3 = \frac{2^2 3^6 5^2}{2^5 5^5} = 3^6$

$\Rightarrow y = 9$
$\therefore 3x + 5y = 3(10) + 5(9) = 75$

Note: When the expression $x^m y^n$ is constant, the minimum value of $ax + by$ is realized when $\frac{ax}{m} = \frac{by}{n}$.

Model 3: The greatest value of $(a - x)^m (b + x)^n$, for any real value of x numerically less than a, b and $m, n \in Z^+$, occurs when $\frac{a-x}{m} = \frac{b+x}{n}$ or at $x = \frac{an - bm}{m + n}$.

3.18. Find the maximum value of $(x - 6)^2 (11 - x)^3$ for $6 \leq x \leq 11$.

Sol. If $E = (x - a)^m (b - x)^n$ and $a \leq x \leq b$, then the maximum value of E occurs when $\frac{x - a}{m} = \frac{b - x}{n}$
Here a = 6, b =11, m = 2 and n= 3. The given expression is maximum when
$\frac{x-6}{2} = \frac{11-x}{3} \Rightarrow x = 8$
$\therefore$ The maximum value is $2^2\, 3^3 = 108$

Model 4: If a, b are two positive numbers, the mean of their m^{th} powers (say M) and the m^{th} power of their mean (say P) are related as follows, depending on the value of M.

(i) If m < 0 or m > 1, then P is less than or equal to M.

i.e., $\left(\frac{a+b}{2}\right)^m \le \left(\frac{a^m+b^m}{2}\right)$

(For example, $\left(\frac{a+b}{2}\right)^2 < \left(\frac{a^2+b^2}{2}\right)$

The equality holds if a = b.

(ii) If m = 0 or 1, then P = M

i.e.,$\left(\frac{a+b}{2}\right)^0 = \frac{a^0+b^0}{2}$ =1

and $\left(\frac{a+b}{2}\right)^1 = \frac{a^1+b^1}{2}$

$= \frac{a+b}{2}$

(iii) If 0 < m < 1, then M is less than or equal to P.

i.e., $\frac{a^m+b^m}{2} \le \left(\frac{a+b}{2}\right)^m$

(For example, $\frac{\sqrt{a}+\sqrt{b}}{2} < \sqrt{\frac{a+b}{2}}$) . The equality holds iff a = b.

This rule can be extended for 3 or more quantities.
If a > 0, b > 0, c > 0. then

(i) If m < 0 or m > 1, $\left(\frac{a+b+c}{3}\right)^m \le \frac{a^m+b^m+c^m}{3}$.

The equality holds iff a = b = c.

(ii) if m = 0 or 1, $\left(\frac{a+b+c}{3}\right)^m = \frac{a^m+b^m+c^m}{3}$

(iii) if 0 < m < 1, $\frac{a^m+b^m+c^m}{3} \le \left(\frac{a+b+c}{3}\right)^m$.

The equality holds if a = b = c.

Model 5: The minimum value of $\frac{(x+a)(x+b)}{(x+c)}$, where x > – a, x > – b, x > – c and c does not lie between a and b, is a – c +b – c +2 $\sqrt{(a-c)(b-c)}$ and the corresponding value of x is $\sqrt{(a-c)(b-c)} - c$.

3.19. Find the minimum value of $\frac{(6+x)(12+x)}{(4+x)}$ $(x > -4)$

Sol. $\frac{(6+x)(12+x)}{4+x} = \frac{(4+x+2)(4+x+8)}{4+x}$

$= \frac{(4+x)^2+10(4+x)+16}{4+x}$

$= 4 + x + 10 + \frac{16}{4+x} = 10 + (4 + x) + \left(\frac{16}{4+x}\right)$

As x > – 4, 4 + x > 0. ∴ $\frac{16}{4+x}$ > 0 and hence both 4 + x and $\frac{16}{4+x}$ are greater than 0.

∴ A.M (4 + x, $\frac{16}{4+x}$) ≥ GM (4 + x, $\frac{16}{4+x}$)

∴ (4 + x + $\frac{16}{4+x}$) ≥ 2 $\sqrt{(4+x)\left(\frac{16}{4+x}\right)}$

i.e. (4 + x + $\frac{16}{4+x}$) ≥ 8

∴ 10 + (4 + x) + ($\frac{16}{4+x}$) ≥ 18 i.e., the minimum value of the given expression is 18.

Alternative method:

Let E_m be the required minimum value
For x > – 4,
$E_m = a - c + b - c + 2\sqrt{(a-c)(b-c)}$
Here a = 6, b = 12 and c = 4
∴ $E_m = 6 - 4 + 12 - 4 + 2\sqrt{(6-4)(12-4)}$
$= 2 + 8 + 2\sqrt{2(8)} = 18$

3.20. If x > –6, then find the minimum value of the expression $\frac{(9+x)(18+x)}{6+x}$.

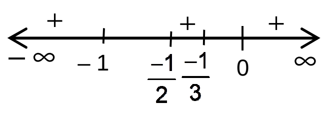
Sol. The minimum value of the expression for x > –6 is $a - c + b - c + 2\sqrt{(a-c)(b-c)}$
here a = 9, b= 18, and c = 6
$= 9 - 6 + 18 - 6 + 2\sqrt{(9-6)(18-6)} = 27$

3.21. If $\frac{2x}{3x+1} + \frac{1}{x} > 0$, then find the range of real values of x.

Sol. $\frac{2x^2+3x+1}{x(3x+1)} > 0$

$\Rightarrow \frac{(2x+1)(x+1)}{x(3x+1)} > 0$

⇒ x(2x + 1) (3x + 1)(x + 1) > 0
∴ The critical points, (points on the number line for which the expression is 0) are $\frac{-1}{2}, \frac{-1}{3}, -1, 0$



The expression is positive in the ranges marked with a positive sign
∴ solution is (–∞, –1) ∪ $\left(\frac{-1}{2}, \frac{-1}{3}\right)$ ∪ (0, ∞)

Concept Review Questions

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Which of the following sets represents the set of all real numbers lying between 2 and 5 excluding 2 and 5?
(A) [2, 5] (B) [2, 5)
(C) $(-\infty, 2) \cup (5, \infty)$ (D) (2, 5)

2. The set consisting of all real numbers lying between – 5 and 2 including 2 but excluding – 5 is _______.
(A) [–5, 2] (B) (–5, 2)
(C) (–5, 2] (D) [–5, 2)

3. If a and b are two positive real numbers, then which of the following is not always positive?
(A) a + b (B) a – b (C) ab (D) $\frac{a}{b}$

4. For two real numbers 'a' and 'b', which of the following is true?
(A) ab is always positive.
(B) a + b is always positive.
(C) a – b is always positive.
(D) None of these

5. For real numbers a, b, c and d, which of the following is/are always true?
(A) If a > b and b > c, then a > c.
(B) If a > b and c > d, then ac > bd
(C) If a > b and c > d then a – c > b – d
(D) All the above

6. If 'a' and 'b' are real numbers, then which of the following is/are always true?
(A) If $\frac{a}{b} > 1$, then a > b
(B) If $\frac{a}{b} < 1$, then a < b
(C) If a > 0, b > 0 and $\frac{a}{b} > 1$, then a > b
(D) All the above

7. The range of real values of x satisfying the inequation x + 2 ≥ 5 is _______.
(A) $[3, \infty)$ (B) $(3, \infty)$
(C) $(-\infty, -3]$ (D) $(-\infty, -3)$

8. If 3 – x ≤ 4, then _______.
(A) x ≤ – 1 (B) x ≥ 1
(C) x ≥ – 1 (D) x ≤ 1

9. The solution set of the inequation 2x + 3 > 4 is _______.
(A) $[1/2, \infty)$ (B) $(-\infty, 1/2)$
(C) $(-\infty, 1/2]$ (D) $(1/2, \infty)$

10. The number of integer values of x satisfying the inequation $x^2 - 5x - 14 \le 0$ is _______.
(A) 10 (B) 9 (C) 8 (D) 6

11. If 'a' is a positive real number, then the minimum value of $a + \frac{1}{a}$ is $\boxed{}$

12. Which of the following expressions is always positive?
(A) $x^2 - 3x + 2, x \in R$ (B) $x^2 + 2x - 35, x \in R$
(C) $x^2 + 4x + 5, x \in R$ (D) $x^2 - x - 6, x \in R$

13. Which of the following is/are always true?
(A) If a > b, then $a^2 < b^2$
(B) If a > b, then $a^2 > b^2$
(C) If a > 0, b > 0 and a > b, then $a^2 > b^2$
(D) All the above

14. The solution set of the in equations in –5 ≤ x < 7 is _______.
(A) (–5, 7) (B) [–5, 7] (C) [–5, 7) (D) (–5, 7]

15. The equation which has a solution as x ≤ –3 or x ≥ 5 is _______.
(A) (x+3) (x–5) ≥ 0 (B) (x – 3) (x+5) ≤ 0
(C) (x+3) (x–5) ≤ 0 (D) (x–3) (x+5) ≥ 0

16. The solution set of 2x–5 ≥ 7x +10 is _______.
(A) $(-3,\infty)$ (B) $(-\infty,3)$
(C) $(-\infty,-3]$ (D) R

17. If 3x +4 ≤ –5x +12, then which of the following is true?
(A) x ≥ 1 (B) x ≤ 1 (C) x ≥ –1 (D) x ≤ –1

18. The solution set of (x + 13) (x – 15) ≥ 0 is _______.
(A) [–13, 15] (B) [–15, 13]
(C) x ≤ –13 or x ≥ 15 (D) none of these

19. The minimum value of 10 + |x| is $\boxed{}$

20. The value of x at which the expression 5 – |2 – x| has the maximum value is $\boxed{}$

21. The number of values of x satisfying the equation |x + 2| = 2x – 1 is $\boxed{}$

22. The solution set of the inequation |x – 1| > – 2 is _______.
(A) ϕ (B) $[0, \infty)$
(C) $(-\infty, \infty)$ (D) $(-\infty, 0]$

23. The solution set of the inequation |x + 2| < – 3 is _______.
(A) ϕ (B) $[0, \infty)$
(C) $(-\infty, \infty)$ (D) $(-\infty, 0]$

24. The number of integer values of x satisfying the inequation |x + 3| ≤ 5 is $\boxed{}$

25. The solution set of the inequation |x – 2| ≤ 3 is
(A) $(-\infty, -1] \cup [5, \infty)$ (B) [– 5, 1]
(C) [– 1, 5] (D) $(-\infty, -5] \cup [1, \infty)$

Exercise – 3(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the range of the values of x satisfying each of the following inequalities.

(i) $4x + 3 > 6x + 7$

(A) $x > 2$ (B) $x > -2$
(C) $x < -2$ (D) $x < 2$

(ii) $7x - 5 > 4x + 13$

(A) $(-6, \infty)$ (B) $(6, \infty)$
(C) $(-6, 6)$ (D) $(-\infty, 6)$

(iii) $6x - 9 \geq 3x + 5$

(A) $x \geq \frac{14}{3}$ (B) $x \geq 14$
(C) $x \leq \frac{14}{3}$ (D) $x \leq 14$

(iv) $7x + 5 \leq 3x - 11$

(A) $[-4, 4]$ (B) $(-\infty, 4)$
(C) $[-4, \infty)$ (D) $(-\infty, -4]$

(v) $10x - 13 > 7x + 9$

(A) $(-22/3, \infty)$ (B) $(22/3, \infty)$
(C) $(14/3, 22/3)$ (D) $(7, \infty)$

2. What are the real values of x that satisfy the simultaneous inequations $6x + 9 < 3x + 5$ and $4x + 7 > 2x - 5$?

(A) $(-6, \infty)$ (B) $(4/3, 6)$
(C) $(-6, -4/3)$ (D) $R - (-6, -4/3)$

3. What are the real values of x that satisfy the inequality $5x + 7 - 2x^2 > 0$?

(A) $(-1, 7/2)$ (B) $(-2, -1)$
(C) $(7/2, \infty)$ (D) $(-\infty, -1)$

4. If $\frac{x-7}{x+8} > 4$, $(x \neq -8)$ then the range of the real values of x is ________.

(A) $R - [-13, -8]$ (B) $R - (8, 13)$
(C) $(-13, -8)$ (D) $(8, 13)$

5. If $\frac{x^2+5x+4}{x^2-7x+12} \leq 0$ (where $x \neq 3, 4$), then $x \in$ _______.

(A) $[-4, -1] \cup (3, 4)$ (B) $(-4, 3) \cup (4, \infty)$
(C) $[-1, 4) \cup (3, 7]$ (D) $(-5, -4)$

6. Max [min $(x - 1, x + 2)$, max $(x + 3, x + 5)$] is equal to

(A) $x + 3$ (B) $x + 5$
(C) $x + 2$ (D) $x - 1$

7. For which of the following range of values of x is $x^3 + 1$ greater than $x^2 + x$?

(A) $(-1, 1) \cup (1, \infty)$ (B) $(-2, \infty)$
(C) $(-\infty, 1)$ (D) $(-2, 1)$

8. For a positive integer n, $2 - \frac{1}{n} < x \leq 4 + \frac{1}{n}$. The range of x is ______.

(A) $(1, 5]$ (B) $[3/2, 9/2]$
(C) $[0, 4]$ (D) $(2, 4)$

9. If $\frac{5x}{x+2} - \frac{1}{2x} < 0$, then the range of x is _______.

(A) $(-\infty, -2) \cup (1/2, \infty)$
(B) $(-2, -2/5) \cup (1/2, \infty)$
(C) $(-2, -2/5) \cup (0, 1/2)$
(D) $(-2/5, 0) \cup (1/2, \infty)$

10. If $\frac{1}{2x-1} > \frac{3}{x}$ where $x > 0$, find the range of x.

(A) $(0, 5)$ (B) $(1, 3)$
(C) $(2, 5)$ (D) $(1/2, 3/5)$

11. The number of solutions of the equation $2x^2 + |4x - 9| = 7$ is _______.

(A) 1 (B) 4 (C) 2 (D) 0

12. Solve for x : $\frac{5}{\sqrt{4-x}} - \sqrt{4-x} < 4$.

(A) $(4,\infty)$ (B) $(3,\infty)$
(C) $(-\infty, 3)$ (D) $(-3,\infty)$

13. What are the real values of x that satisfy the inequation $2x^2 + 10x + 17 < 0$?

(A) $(-4, -1)$ (B) $(4, \infty)$
(C) { } (D) $(-\infty, 0)$

14. If $|x-3| \leq 9$ and $|4 - x| < 5$ then _______.

(A) $-6 < x < 9$ (B) $-1 < x < 12$
(C) $-1 < x < 9$ (D) $-1 \leq x < 10$

15. If $|b| \geq 2$ and $x = |a|\,b$, then which of the following is always true?

(A) $a + xb \geq 0$
(B) $a + xb \leq 0$
(C) $a - xb \leq 0$
(D) Both (A) and (C)

16. For two real numbers a and b, let $p = |a| + |b|$, $q = |a| - |b|$, and $r = |a - b|$. Which of the following is true?

(A) $r \leq p \leq q$ (B) $p \leq q \leq r$
(C) $q \leq r \leq p$ (D) $q \leq p \leq r$

17. If $|x| \geq |6 - x^2|$ then find the range of x.

(A) $[-3,2] \cup [3,\infty)$
(B) $(-\infty,-3] \cup [3, \infty)$
(C) $(-\infty, -3] \cup [2,3]$
(D) $[-3,-2] \cup [2,3]$

18. The range of $f(x) = |x| + |x - 5| + |x + 7|$ is _______.

(A) $[-5, 7]$ (B) $[-7, 5]$
(C) $[-7, \infty)$ (D) $[12, \infty)$

19. The solution set for the inequation $|x - 1| + |x + 2| < 2$ is _______.

(A) $(-\infty,-2) \cup (1, \infty)$
(B) $[2, 1)$
(C) $(-1, 2)$
(D) an empty set

20. If x, y and z are positive real numbers such that x + y + z = 2, then which of the following is always true?

(A) $xyz \le \frac{1}{9}$ (B) $xyz \ge \frac{2}{3}$

(C) $xyz \le \frac{8}{27}$ (D) $xyz \ge \frac{8}{27}$

21. The minimum value of $\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}$, where a, b and c are positive real numbers, is _______.

(A) 4 (B) 6
(C) 5 (D) 3

22. Let p and q be positive numbers having a sum of 1. $\left(p+\frac{1}{p}\right)^2+\left(q+\frac{1}{q}\right)^2$ will have a minimum value of $\boxed{}$

23. If x, y and z are positive real numbers, then which of the following holds?

(A) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2} \ge \frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}$

(B) (x + y + z) (xy + yz + zx) ≥ 9xyz
(C) (x + y) (y + z) (x + z) ≥ 8xyz
(D) All the above

24. If ℓ, m and n are positive numbers, $\frac{(3\ell^2+\ell+3)(5m^2+m+5)(4n^2+n+4)}{11\ell mn}$ can be _______.

(A) 55 (B) 62 (C) 67 (D) 49

25. If a, b, c, d are four positive numbers such that a + b + c + d = 12, which of the following is the least value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}$?

(A) $\frac{4}{3}$ (B) $\frac{5}{3}$

(C) 1 (D) $\frac{2}{3}$

26. If $a^2 + b^2 = c^2 + d^2 = 6$, then which of the following is/are always true?
(A) ac + bd ≤ 6 (B) ab + cd ≤ 6
(C) ad + bc ≤ 6 (D) (A), (B) and (C)

27. If a, b and c are the lengths of the sides of a triangle, then the range of the expression $\frac{a^2+b^2+c^2}{ab+bc+ca}$ is _______.
(A) (1, 2) (B) [1, 2)
(C) (1, 3) (D) (–1/2, 3)

28. If x and y are positive numbers satisfying 3x + 4y = 34 then the maximum value of $x^3 y^2$ is M. M, when rounded to the nearest integer, equals $\boxed{}$.

29. If x and y are positive real numbers and $x^2y = 24$, then the minimum value of 3x + 4y is _______.
(A) 18 (B) 16
(C) 24 (D) 32

30. If x and y are positive real numbers and $x^2y^3 = 108$, then the least value of x + y is $\boxed{}$.

Exercise – 3(b)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. Find the range of the values of x satisfying each of the following inequalities.

(i) 4x + 9 ≥ 25
(A) x > 5 (B) x < 4
(C) x ≥ 4 (D) x ≤ 4

(ii) 13x – 5 ≤ 21
(A) (2, ∞) (B) (–∞, 2]
(C) [2, ∞) (D) (–0, 2)

(iii) 3 – 32x < 129 – 18x
(A) x > 9 (B) x < 9
(C) x > –9 (D) x < –9

(iv) 4x – 5 > 3x + 13
(A) (18, ∞) (B) (–18, ∞)
(C) (–18, 18) (D) (–∞, 18)

(v) 8x – 43 ≥ 5 + 10x
(A) [–24, 0) (B) [–24, 24]
(C) (–∞, –24] (D) [–24, ∞)

2. Find the range of the values of x satisfying the inequalities 3x + 4 ≤ 6 and 4x + 3 ≥ 6.

(A) $\left[\frac{3}{4},\frac{3}{2}\right]$ (B) (–∞, 2/3]

(C) [3/4, ∞) (D) ϕ

3. What is the maximum value of the expression 27 – |3x – 9|?

$\boxed{}$

4. At what value of x is 17 + |4x – 15| minimum?

(A) $\frac{4}{15}$ (B) $\frac{15}{4}$ (C) $-\frac{15}{4}$ (D) $-\frac{4}{15}$

5. If 2 ≤ x ≤ 8 and 4 ≤ y ≤ 12, the minimum value of $\frac{x+y}{x}$ is _______.

(A) 4 (B) 5

(C) $\frac{1}{2}$ (D) None of these

6. What is the range of the values of 'x' that satisfy the inequality $x^2 - 9x - 10 > 0$?
(A) (–1, 9)
(B) (–1, 10)
(C) $(-\infty, -1) \cup (10, \infty)$
(D) $(-\infty, 1) \cup (9, \infty)$

Moderate

7. What is the range of the values of 'x' that satisfy the inequality
$4x^2 + 5x - 9 \le 0$?
(A) [–9/4, –1] (B) [–5/2, 1]
(C) [–1, 9/4] (D) [–9/4, 1]

8. If $\frac{x-7}{x+5} > 3$ and $x \neq -5$, then $x \in$ _______.
(A) $(-\infty, -5)$ (B) (–11, –5)
(C) (–11, –8) (D) (5, 11)

9. Find the number of integers satisfying the inequality $(x^2 + 5x - 6)(x^2 - 6x - 7) < 0$.
(A) 9 (B) 8 (C) 13 (D) 10

10. The solution set for the inequality $\frac{2x^2+5x-3}{x^2-3x+2} \le 0$ is _______.
(A) $(-4, -3) \cup (2, \infty)$ (B) R – [–1/2, 1]
(C) $[-3, 1/2] \cup (1, 2)$ (D) (–3, 2)

11. If $|3x + 7| > 12$, then the range of the values of x is _______.
(A) [–19/3, 5/3]
(B) $(-\infty.-19/3]$
(C) $(-\infty,-19/3) \cup (5/3, \infty)$
(D) $[-19/3, \infty)$

12. Find the range of the values of x if $|9 - x| < 2 - 3x$.
(A) $[7/2, \infty)$ (B) $(-\infty,-7/2)$
(C) $(-\infty, 7/2)$ (D) $[-7/2, \infty)$

13. How many solutions exist for the inequality $|x - 1| + |x - 6| < 2$?
$\boxed{}$

14. How many solutions does the equation $x^2 + |3x - 6| = 4$ have?
$\boxed{}$

15. Find the number of integer values of x satisfying the inequality $\frac{x^2-5x+6}{x^2+5x-6} < 0$ and $x \neq -6$, $x \neq 1$.
(A) 4 (B) 6 (C) 8 (D) 9

16. For which of the following range of the values of x is $x^3 - 8 < x^2 - 2x$?
(A) (–2, 2) (B) $(-2, \infty)$
(C) $(2, \infty)$ (D) $(-\infty, 2)$

17. For what range of values of x does the inequality $|x - 5| > x^2 - 4x + 1$ hold true?
(A) (2, 3) (B) (1, 4)
(C) (–1, 4) (D) (0, 3)

18. If x, y and z are positive real numbers such that $x^2 + y^2 + z^2 = 3$, then which of the following is always true?
(A) $xy + yz + zx \le 3$
(B) $xy + yz + zx \ge 1$
(C) $xy + yz + zx \le 2$
(C) $xy + yz + zx \ge 3$

19. If $|b| \le 1$ and $x = -|a|b$, then which of the following is/are always true?
(A) $a - xb \ge 0$
(B) $a + xb \ge 0$
(C) Both (A) and (B)
(D) Neither (A) nor (B)

20. If x and y are positive real numbers such that xy = 28, then the least value of 4x + 7y is $\boxed{}$.

21. For the positive numbers, p, q, r, $\frac{(2p^2+p+2)(7q^2+q+7)(6r^2+r+6)}{5pqr}$ can be _______.
(A) 225
(B) 200
(C) 215
(D) All of the previous choices

22. The number of integral values that satisfy the inequation $\frac{7x}{5x+4} - \frac{2}{x} < 0$ is _______.
(A) 1 (B) 2
(C) 4 (D) 0

23. The number of distinct solutions of the equation $\big|x - |6x + 1|\big| = 9$ is _______.
(A) 0 (B) 1 (C) 2 (D) 3

24. If a, b, c are three positive numbers such that a + b + c = 9, which of the following is true?
(A) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \le 2$ (B) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \le 1$
(C) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge 2$ (D) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge 1$

25. The minimum value of the expression $\left(\frac{a}{b}-\frac{b}{c}\right)^2 + \left(\frac{c}{a}-\frac{b}{a}\right)^2 + \left(\frac{c}{b}-\frac{a}{c}\right)^2 + 2\left(\frac{a}{c}+\frac{bc}{a^2}+\frac{a}{b}\right)$ where a, b, c are positive real numbers is _______.
(A) 6 (B) 4 (C) 8 (D) 2

26. The number of positive integer solutions of $\frac{1}{2x-5} > \frac{4}{x}$ is $\boxed{}$.

27. The range of $f(x) = |x| + |x + 7|$ is _______.
(A) $[0, \infty)$ (B) $[7, \infty)$ (C) [0, 7] (D) [–7, 7]

28. How many values of x satisfy the equation $x^2 - |6x + 3| = -6$?
(A) 0 (B) 2
(C) 4 (D) None of these

29. The function f(x) = |x – 3| + |3.5 – x| + |4.6 – x| where x is a real number and f(x) has a minimum value of M. When f (x) = M, x = $\boxed{}$.

30. If |x + 3| + 7 > 2|x – 4|, the range of x is ______.

(A) $\frac{-2}{3} < x < 16$ (B) $\frac{-3}{4} < x < 18$

(C) $\frac{-5}{6} < x < 18$ (D) $\frac{-2}{3} < x < 18$

31. Find the values of y satisfying the inequation $\left(y^{\frac{1}{3}} - 1\right)^2 + y^{\frac{1}{3}} - 13 \le 0$.

(A) $-3 \le y \le 4$ (B) $27 \le y \le 64$
(C) $-27 \le y \le 64$ (D) $-64 \le y \le -27$

32. The minimum value of
|x – 3| + |x – 7| + |x + 13|, where x is a real number is _______.
(A) 24 (B) 20 (C) 36 (D) 22

33. What is the value of
max [max(2x + 3, x – 4), min (2x + 4, 2x + 1)]?
(A) 2x + 3
(B) x – 9
(C) 2x + 4
(D) Cannot be determined

Difficult

34. Which of the following is/are true?
(A) $49^{48} < 48^{51}$
(B) $(105)^{20} < (100)^{21}$
(C) $(41)^{39} < (40)^{40}$
(D) All the previous choices

35. Which of the following is/are true for a, b, c > 0?

(A) $a + b + c \le \frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c}$.

(B) $\left(\frac{a}{bc}\right)^{\frac{1}{2}} + \left(\frac{b}{ca}\right)^{\frac{1}{2}} + \left(\frac{c}{ab}\right)^{\frac{1}{2}} \ge a^{-\frac{1}{2}} + b^{-\frac{1}{2}} + c^{-\frac{1}{2}}$,

(C) $a^4 + b^4 + c^4 \ge a^2.b^2 + b^2.c^2 + c^2.a^2$
(D) All the previous choices

36. If a, b and c are the lengths of the sides of a triangle then the range of the values of $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is _______.
(A) [1, 2) (B) [3/2, 3] (C) [3/2, 2) (D) [1, 3)

37. Solve for x: $(x^2 + x + 1)^x > 1$.
(A) (–∞, 1) (B) (–1, ∞) (C) (0, ∞) (D) (1, ∞)

38. If x and y are positive real numbers such that x + y = 5, then the maximum value of x^3y^2 is _______.
(A) 144 (B) 72 (C) 96 (D) 108

39. Solve for x: $\left|\frac{2x+1}{3x-8}\right| \ge 2$.

(A) $\left[\frac{15}{8}, \frac{17}{4}\right] - \left\{\frac{8}{3}\right\}$ (B) $\left[\frac{15}{8}, \frac{17}{4}\right]$

(C) $\left(-\infty, \frac{15}{8}\right] \cup \left[\frac{17}{4}, \infty\right)$ (D) none of these

40. If x is a positive integer satisfying $\frac{5x+124}{5(5x^2+1)} < \frac{1}{50}$, the least value of x is $\boxed{}$.

Data Sufficiency

Directions for questions 41 to 50: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A)	if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B)	if the question can be answered using either statement alone.
Mark (C)	if the question can be answered using I and II together but not using I or II alone
Mark (D)	if the question cannot be answered even using I and II together.

41. Is $\frac{x^2}{y^3} > 5$?
I. $-5 < x < -1$and $1 < y < 2$
II. $x > 3$ and $0 < y < 1$

42. If x and y are both real numbers, is $\frac{1}{x} > \frac{1}{y}$?
I. $y^2 > x^2$ and $y^3 < x^3$
II. $x^4y^2 < x^2y^4$

43. If x > y, is $x^3y^3 > 0$?
I. $y^2 = 16$
II. x + y = 0

44. Is $2x^2 - ax + 20$ greater than 0?
I. 10 > a > 0
II. a < 10

45. Is x > 0?
I. $x^7 > 0$
II. $x^6 > 0$

46. What is the minimum value of $\frac{|y|}{|x| + |y|}$?
I. $-4 \le x \le 2$.
II. $2 \le y \le 5$

47. Is |x – 2y| < 4?
I. |x| < 2
II. |y| > 3

48. What is the minimum value of |x + 2| + |x – 3|?
I. $-2 \le x \le 3$
II. x is a real number

49. What is the value of |x – y|?
I. $(x - 3)^2 + (y - 4)^2 = 0$
II. |x – 3| + |y – 4| = 0

50. Is |x – 1| < 4?
I. $x^2 - 2x - 15 < 0$
II. x(x – 4) > 0

Key

Concept Review Questions

1. D	6. C	11. 2	16. C	21. 1
2. C	7. A	12. C	17. B	22. C
3. B	8. C	13. C	18. C	23. A
4. D	9. D	14. C	19. 10	24. 11
5. A	10. A	15. A	20. 2	25. C

Exercise – 3(a)

1. (i) C	3. A	9. C	15. D	21. B	27. B
(ii) B	4. C	10. D	16. C	22. 12.5	28. 3635
(iii) A	5. A	11. A	17. D	23. D	29. A
(iv) D	6. B	12. C	18. D	24. C	30. 5
(v) B	7. A	13. C	19. D	25. A	
2. C	8. A	14. C	20. C	26. D	

Exercise – 3(b)

1. (i) C	8. B	19. D	30. D	41. A
(ii) B	9. A	20. 56	31. C	42. D
(iii) C	10. C	21. D	32. B	43. A
(iv) A	11. C	22. A	33. D	44. A
(v) C	12. B	23. C	34. D	45. A
2. D	13. 0	24. D	35. D	46. C
3. 27	14. 2	25. A	36. C	47. C
4. B	15. B	26. 0	37. B	48. B
5. D	16. D	27. B	38. D	49. B
6. C	17. C	28. D	39. A	50. A
7. D	18. A	29. 3.5	40. 22	