CHAPTER – 2

RATIO – PROPORTION – VARIATION

RATIO

If the values of two quantities A and B are 4 and 6 respectively, then we say that they are in the ratio 4 : 6 (read as "four is to six"). Ratio is the relation which one quantity bears to another of the same kind, the comparison being made by considering what multiple, part or parts, one quantity is of the other. The ratio of two quantities "a" and "b" is represented as a : b and read as "a is to b". Here, "a" is called antecedent, "b" is the consequent. Since the ratio expresses the number of times one quantity contains the other, it's an abstract quantity.

Ratio of any number of quantities is expressed after removing any common factors that ALL the terms of the ratio have. For example, if there are two quantities having values of 4 and 6, their ratio is 4 : 6, i.e., 2 : 3 after taking the common factor 2 between them out. Similarly, if there are three quantities 6, 8 and 18, there is a common factor among all three of them. So, dividing each of the three terms by 2, we get the ratio as 3 : 4 : 9.

If two quantities whose values are A and B respectively are in the ratio a : b, since we know that some common factor k(>0) would have been removed from A and B to get the ratio a : b, we can write the original values of the two quantities (i.e., A and B) as ak and bk respectively. For example, if the salaries of two persons are in the ratio 7 : 5, we can write their individual salaries as 7k and 5k respectively.

A ratio a : b can also be expressed as a/b. So if two items are in the ratio 2 : 3, we can say that their ratio is 2/3. If two terms are in the ratio 2, it means that they are in the ratio of 2/1, i.e., 2 : 1.

"A ratio is said to be a ratio of greater or less inequality or of equality according as antecedent is greater than, less than or equal to consequent". In other words,

- the ratio a : b where a > b is called a ratio of greater inequality (example 3 : 2)
- the ratio a : b where a < b is called a ratio of less inequality (example 3 : 5)
- the ratio a : b where a = b is called a ratio of equality (example 1 : 1)

From this we can find that a ratio of greater inequality is diminished and a ratio of less inequality is increased by adding same quantity to both terms, i.e., in the ratio a : b, when we add the same quantity x (positive) to both the terms of the ratio, we have the following results

if a < b then (a + x) : (b + x) > a : b
if a > b then (a + x) : (b + x) < a : b
if a = b then (a + x) : (b + x) = a : b

This idea can also be helpful in questions on Data Interpretation when we need to compare fractions to find the larger of two given fractions.

If two quantities are in the ratio a : b, then the first quantity will be a/(a + b) times the total of the two quantities and the second quantity will be equal to b/(a + b) times the total of the two quantities.

Examples

2.01. The sum of two numbers is 84. If the two numbers are in the ratio 4 : 3, find the two numbers.

Sol: As the two numbers are in the ratio 4 : 3, let their actual values be 4x and 3x.
As the sum of two numbers is 84, we have 4x + 3x = 84
$\Rightarrow$ 7x = 84
$\Rightarrow$ x = (84/7) = 12.
Hence 4x = 48 and 3x = 36.
Alternatively, the two numbers are (4/7) $\times$ 84 and (3/7) $\times$ 84 i.e., 48 and 36 respectively since the ratio of the two numbers is 4 : 3.

2.02. If 4a = 3b, find (7a + 9b) : (4a + 5b).

Sol: It is given that 4a = 3b,
Hence, (a/b) = (3/4)
$\Rightarrow$ a = 3k and b = 4k, where k is the common factor of a and b.
Required expression = (7a + 9b) : (4a + 5b)
= [(7 $\times$ 3k) + (9 $\times$ 4k)] : [(4 $\times$ 3k) + (5 $\times$ 4k)]
= (21k + 36k) : (12k + 20k)
= 57k : 32 k = 57 : 32

2.03. The number of red balls and green balls in a bag are in the ratio 16 : 7. If there are 45 more red balls than green balls, find the number of green balls in the bag.

Sol: Since the ratio of number of red and green balls is 16 : 7, let the number of red balls and green balls in the bag be 16x and 7x respectively. So, the difference of red and green balls is 9x.
16x – 7x = 9x = 45 $\Rightarrow$ x = 5
$\Rightarrow$ Hence the number of green balls
= 7x i.e., 35
Alternatively, 7x = (7/9) (9x)
= (7/9) (45) = 35.
Hence there are 35 green balls in the bag.

2.04. What least number must be added to each of a pair of numbers which are in the ratio 7 : 16 so that the ratio between the terms becomes 13 : 22?

Sol: Let the number to be added to each number be a. Let the actual values of the numbers be 7x and 16x, since their ratio is 7 : 16.
Given that,

$$\frac{7x+a}{16x+a} = \frac{13}{22}$$

$\Rightarrow$ 154x + 22a = 208x + 13a $\Rightarrow$ 9a = 54x
$\Rightarrow$ a = 6x. When x = 1, a is the least number required and is equal to 6.

2.05. A number is divided into four parts such that 4 times the first part, 3 times the second part, 6 times the third part and 8 times the fourth part are all equal. In what ratio is the number divided?

Sol: Let the first, second, third and fourth parts into which the number is divided be a, b, c and d respectively.
4a = 3b = 6c = 8d. Let the value of each of these be equal to e.

$a = \frac{e}{4}$, $b = \frac{e}{3}$, $c = \frac{e}{6}$, and $d = \frac{e}{8}$.

Hence $a : b : c : d = \frac{e}{4} : \frac{e}{3} : \frac{e}{6} : \frac{e}{8}$

$= \frac{6}{24} : \frac{8}{24} : \frac{4}{24} : \frac{3}{24}$

(Where 24 is the LCM of the denominators)
= 6 : 8 : 4 : 3.
Hence the ratio of the parts into which the number is divided is 6 : 8 : 4 : 3.

2.06. Divide 3150 into four parts such that half of the first part, a third of the second part, a fourth of the third part is equal to one-twelfth of the fourth part.

Sol: Let the four parts into which 3150 is divided be a, b, c and d.
Given that,

$\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{d}{12}$;

Let each of the above equal k.
Then, a = 2k, b = 3k, c = 4k and d = 12 k
As a + b + c + d = 3150, the equation becomes,
(2k + 3k + 4k + 12k) = 3150;
$\Rightarrow$ 21k = 3150 $\Rightarrow$ k = 150.
Hence, the four parts in the order, are:
300, 450, 600 and 1800.

2.07. If x : y = 4 : 3, y : z = 2 : 3, find x : y : z.

Sol: As y is common to both the ratios, make y in both ratios equal. This is done by making y have the value equal to the LCM of the two parts corresponding to y in the two ratios i.e., LCM (3, 2) = 6.
If y = 6, x = (4/3) $\times$ 6 = 8, z = (3/2) $\times$ 6 = 9
Hence x : y : z = 8 : 6 : 9.

2.08. If $\frac{a}{b} = \frac{4}{5}$, then find $\frac{2a^2+3b}{7a+6b^2}$.

Sol: It is given that (a/b) = (4/5).
Hence a and b can be taken as 4k and 5k, where k is the common factor of a and b.
Substituting the values in the given expression, the expression is $(2a^2 + 3b) : (7a + 6b^2)$
$[2(4k)^2 + 3(5k)] : [7(4k) + 6(5k)^2]$
$(32k^2 + 15k) : (28k + 150k^2)$
k(32k + 15) : k(28 + 150k)
(32k + 15) : (150k + 28)
As the value of k is not known, the value of the required expression cannot be determined.

2.09. Two numbers are in the ratio 4 : 5. If 7 is added to each, the ratio between the numbers becomes 5 : 6. Find the numbers.

Sol: Let the numbers be 4k and 5k, where k is the common factor.

$\frac{4k+7}{5k+7} = \frac{5}{6}$

24k + 42 = 25k + 35
$\Rightarrow$ k = 7.
The numbers are 4k = 28 and 5k = 35.

PROPORTION

When two ratios are equal, then the four quantities involved in the two ratios are said to be proportional i.e., if a/b = c/d, then a, b, c and d are proportional.

This is represented as a : b : : c : d and is read as "a is to b (is) as c is to d".

When a, b, c and d are in proportion, then a and d are called the EXTREMES and b and c are called the MEANS. We also have the relationship:
Product of the MEANS = Product of the EXTREMES i.e., b c = adv
If a : b = c : d then
b : a = d : c .... (1)
a : c = b : d .... (2)
(a + b) : b = (c + d) : d ... (3)
(obtained by adding 1 to both sides of the given relationship)
(a – b) : b = (c – d) : d ... (4)
(obtained by subtracting 1 from both sides of the given relationship)
(a + b) : (a – b) = (c + d) : (c – d) ...... (5)
{obtained by dividing relationship (3) above by (4)}

Relationship (1) above is called INVERTENDO
Relationship (2) is called ALTERNENDO;
Relationship (3) is called COMPONENDO;
Relationship (4) is called DIVIDENDO;
Relationship (5) is called COMPONENDO – DIVIDENDO.

The last relationship, i.e., COMPONENDO-DIVIDENDO is very helpful in simplifying problems. By this rule, whenever we know a/b = c/d, then we can write (a + b) / (a – b) = (c + d) / (c – d).
The converse of this is also true - whenever we know that (a + b) / (a – b) = (c + d)/(c – d), then we can conclude that a/b = c/d.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ ......, then each of these ratios is equal to $\frac{a+c+e+.....}{b+d+f+.....}$.

If three quantities a, b and c are such that a : b : : b : c, then we say that they are in CONTINUED PROPROTION. We also get $b^2 = ac$. In such a case, c is said to be the third proportional of a and b. Also, b is said to be the mean proportional of a and c.

VARIATION

Two quantities A and B may be such that as one quantities changes in value, the other quantity also changes in value **bearing certain relationship** to the change in the value of the first quantity.

DIRECT VARIATION

One quantity A is said to vary directly as another quantity B if the two quantities depend upon each other in such a

manner that if B is increased in a certain ratio, A also increases in the same ratio and if B is decreased in a certain ratio, A also decreases in the same ratio.
This is denoted as A α B (A varies directly as B).

If A α B then A = kB, where k is a constant. It is called the constant of proportionality.
For example, when the quantity of sugar purchased by a housewife doubles from the normal quantity, the total amount she spends on sugar also doubles, i.e., the quantity and the total amount increase (or decrease) in the same ratio.

From the above definition of direct variation, we can see that when two quantities A and B vary directly with each other, then A/B = k or **the ratio of the two quantities is a constant**. Conversely, **when the ratio of two quantities is a constant, we can conclude that they vary directly with each other.**

If X varies directly with Y and we have two sets of values of the variables X and Y – X_1 corresponding to Y_1 and X_2 corresponding to Y_2, then, since X α Y, we can write down

$$\frac{X_1}{Y_1} = \frac{X_2}{Y_2} \text{ or } \frac{X_1}{X_2} = \frac{Y_1}{Y_2}$$

INVERSE VARIATION

A quantity A is said to vary inversely as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A gets decreased in the same ratio and if B is decreased in a certain ratio, then A gets increased in the same ratio.

It is the same as saying that A varies directly with 1/B. It is denoted as A α 1/B i.e., A = k/B where k is k the constant of proportionality.

For example, as the number of men doing a certain work increases, the time taken to do the work decreases and conversely, as the number of men decreases, the time taken to do the work increases.

From the definition of inverse variation, we can see that when two quantities A and B vary inversely with each other, then AB = a constant, i.e., **the product of the two quantities is a constant**. Conversely, **if the product of two quantities is a constant, we can conclude that they vary inversely with each other**.

If X varies inversely with Y and we have two sets of values of X and Y – X_1 corresponding to Y_1 and X_2 corresponding to Y_2, then since X and Y are inversely related to each other, we can write down

$$X_1Y_1 = X_2Y_2 \text{ or } \frac{X_1}{X_2} = \frac{Y_2}{Y_1}$$

JOINT VARIATION

If there are three quantities A, B and C such that A varies with B when C is constant and varies with C when B is constant, then A is said to vary jointly with B and C when both B and C are varying. i.e., A α B when C is constant and Aα C when B is a constant; ⇒ A α BC
A α BC ⇒ A = kBC where k is the constant of proportionality.

Examples

2.10. Find x, if x + 2 : 4x + 1 : : 5x + 2 : 13x + 1.

Sol: In a proportion, product of means = product of extremes
$(x + 2)(13x + 1) = (4x + 1)(5x + 2)$
$\Rightarrow 13x^2 + x + 26x + 2 = 20x^2 + 8x + 5x + 2$
$\Rightarrow 13x^2 + 27x + 2 = 20x^2 + 13x + 2$
$\Rightarrow 7x^2 - 14x = 0 \quad \Rightarrow 7x(x - 2) = 0$
$\Rightarrow x = 0$ or 2.

2.11. If x varies directly as $y^4 + 9$ and x = 3 when y = 3, find x when y = 9.

Sol: $x \propto (y^4 + 9)$.
Hence $x = c(y^4 + 9)$ where c is a constant.
$c = \frac{x}{y^4 + 9}$.
When y = 3, x = 3 (given)
Hence $c = \frac{3}{3^4 + 9} = \frac{3}{90} = \frac{1}{30}$.

$x = \frac{1}{30}(y^4 + 9)$
When y = 9,
$x = \frac{1}{30}(y^4 + 9) = \frac{1}{30}(6561 + 9) = 219$.

In these types of problems on variation, there are typically three parts:
- the relationship between different variables is defined to frame an equation involving the variables and the constant of proportionality
- one set of values of all the values of all the variables is given to enable us find the value of the constant of proportionality
- the values of all but one variable of a second set are given and we are asked to find the value of the one variable whose value is not given.

Example

2.12. The curved surface area of a cylinder varies directly with the product of its height and the radius. When the height of the cylinder is 36 cm and its radius of the cylinder is 10 cm, the curved surface area of the cylinder is 720π sq.cm. Find the curved surface area of the cylinder when the height of the cylinder is 54 cm and the radius of the cylinder is 15 cm.

Sol: Let the curved surface area of the cylinder be denoted by s. Let the radius and the height of the cylinder be denoted by r and h.
$s \propto rh$. Hence s = c r h where c is a constant.
c = s/rh
When s = 720π sq.cm, r = 36 cm and h = 10 cm.
Hence, $c = \frac{720\pi}{36 \times 10} = 2\pi$.
Curved surface area of the cylinder when r = 48 cm and h = 15 cm is $2\pi \times 54 \times 15$
= 1620π sq.cm.

Alternate method:

As both radius and height become 3/2 times their original values, the curved surface area, being proportional to rh, becomes $\left(\frac{3}{2}\right)^2$ i.e., (9/4) times its original value.
Hence it is (9/4) $\times$ 720π = 1620π sq.cm.

Note that the there should be consistency of the units used for the variables, i.e., whatever be the units used to express the variables when the constant of proportionality is being calculated, the same units should be used for different variables later on also when finding the value of the variable which we are asked to find out.

2.13. The total monthly sales of two companies A and B are in the ratio 2 : 3 and their total monthly expenditures are in the ratio 3 : 4. Find the ratio of the profits of A and B given that company A's profit is equal to a fifth of its sales.

Sol: Let the total monthly sales of companies A and B be ₹2x and ₹3x and their total monthly expenditures be ₹3y and ₹4y.

Given that A's profit = $\frac{1}{5}$ of sales = (2x/5).

$\therefore$ 2x – 3y = $\frac{1}{5}(2x)$

$\Rightarrow \frac{4}{5}(2x) = 3y$

$\Rightarrow y = \frac{8}{15}x$

Profit of company B

$= 3x - 4y = 3x - 4\left(\frac{8}{15}x\right) = \frac{13x}{15}$

Hence the ratio of the profits of the two companies are $\frac{2}{5}x : \frac{13x}{15}$ = 6 : 13

2.14. Given that x varies directly as y. Verify whether $(x + y)^3$ varies directly with $(x – y)^3$.

Sol: This is a model of a problem where a certain relationship is given and we are asked to check the relationship between different combinations of the two variables.
As x varies directly with y,
x = Ry where R is a constant.

$$\frac{(x+y)^3}{(x-y)^3} = \frac{(Ry+y)^3}{(Ry-y)^3}$$

$$= \frac{(R+1)^3}{(R-1)^3}$$

As R.H.S. of above equation is a constant, $(x + y)^3$ varies directly with $(x – y)^3$.

2.15. A part of the monthly expenses of Amar, a marketing executive are fixed and the remaining part varies with the distance travelled by him. If he travels 200 km in a month, his total expenditure is ₹3300. If he travels 500 km in a month, his total expenditure is ₹3900. Find his total expenditure, if he travels 800 km in a month.

Sol: Let the total expenses be ₹T, ₹F be the fixed part and ₹V be the variable part. Given that, T = F + V.
As V varies directly with the distance travelled, if distance travelled is denoted by d,
V = Rd where 'R' is the proportionality constant.
Hence T = F + Rd
From the given data,
3300 = F + 200 R $\rightarrow$ (1)
3900 = F + 500 R $\rightarrow$ (2)
Subtracting (1) from (2),
600 = 300 R. ($\Rightarrow$ R = 2)
Total expenditure if he travels 800 km
= F + 800 R = F + 500 R + 300 R
= 3900 + 600 = ₹4500

The problems involving ratio and proportion are just different forms of the models of the basic problems we saw above. For example, the problem we just solved above might be reframed bringing in Mangoes, Bananas, Baskets, etc. Here, practice and perseverance pay you a lot. In entrance exams, there will be either direct problems on ratio, proportion and variation or indirect problems of application of these concepts just discussed to areas like Time and Work or Time and Distance.

Concept Review Questions

Directions for questions 1 to 20: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Raja divided 35 sweets among his daughters Rani and Sita in the ratio 4 : 3. How many sweets did Rani get?

2. If p : Q = 3 : 4, find 5P : 7Q.

(A) $\frac{20}{37}$ (B) $\frac{3}{4}$ (C) $\frac{15}{28}$ (D) $\frac{20}{37}$

3. Ratio of two numbers is 3 : 5 and their sum is 40. Find the smaller of the two numbers.

4 If a : b = 7 : 3, find a + b : a – b.

(A) 5 : 2 (B) 2 : 5
(C) 7 : 3 (D) 3 : 7

5. If a + b : a – b = 3 : 2, find a : b.

(A) 5 : 1 (B) 1 : 5
(C) 3 : 5 (D) 5 : 3

6. If $\frac{x+y}{2x+y} = \frac{4}{5}$, then find $\frac{2x+y}{3x+y}$.

(A) $\frac{4}{5}$ (B) $\frac{5}{6}$ (C) $\frac{6}{7}$ (D) $\frac{3}{4}$

7. If x + y + z = 120 and $x = \frac{1}{2}y$ and $y = \frac{2}{3}z$, find z.

8. If a : b = 4 : 1, find $\frac{a-3b}{2a-b^2}$.

(A) $\frac{2}{7}$ (B) $\frac{1}{7}$
(C) $\frac{3}{7}$ (D) Cannot be determined

9. Find the following:

(a) Duplicate ratio of 3 : 4.
(A) 3 : 8 (B) 6 : 4
(C) 5 : 7 (D) 9 : 16

(b) Triplicate ratio of 2 : 3.
(A) 6 : 9 (B) 3 : 2
(C) 8 : 27 (D) 5 : 8

(c) Sub-duplicate ratio of 16 : 9.
(A) 2 : 3 (B) 4 : 3
(C) 4 : 9 (D) 8 : 3

(d) Mean proportional of 16 and 4.
(A) 64 (B) 16
(C) 8 (D) 14

10. Find the numbers which are in the ratio 3 : 2 : 4 such that the sum of the first and the second numbers added to the difference of the third and the second numbers is 21.

(A) 12, 8, 16 (C) 6, 4, 8
(C) 9, 6, 24 (D) 9, 6, 12

11. In a class of 30 students, which of the following can't be the ratio of boys and girls?

(A) 2 : 3 (B) 1 : 5 (C) 4 : 5 (D) 2 : 1

12. If a : b = 2 : 3 and b : c = 5 : 7, then find a : b : c.

(A) 10 : 15 : 21 (B) 10 : 21 : 15
(C) 9 : 12 : 14 (D) 12 : 7 : 18

13. At a party, there are a total of 28 adults. If x ladies join the party, the ratio of the number of ladies to that of gents will change from 3 : 4 to 5 : 4. Find x.

14. The monthly salaries of X and Y are in the ratio 3 : 4. The monthly expenditures of X and Y are in the ratio 4 : 5. Find the ratio of the monthly savings of X and Y.

(A) 5 : 3 (B) 4 : 7
(C) 3 : 5 (D) Cannot be determined

15. The present ages of Rohit and Sunil are in the ratio 3 : 5. 10 years hence, the ratio of their ages will be 4 : 5. Find the present age of Rohit. (in years)

16. x varies directly as the square of y. When y = 8, x = 192. Find x when y = 10.

(A) 100 (B) 30 (C) 300 (D) 200

17. Quantities a and b are inversely proportional to each other. When a = 8, b = 240. Find b when a = 6.

18. Quantity A varies directly with the sum of the quantities B and C. If B increases by 2 and C increases by 4, by how much does A increase?

(A) 2 (B) 4
(C) 6 (D) Cannot be determined

19. Quantity P varies inversely with the product of Q and R. When Q = 6 and R = 12, P = 75. Find P when Q = 5 and R = 10.

20. A varies directly with B when C is constant and inversely with C when B is constant. A is 16, when B is 28 and C is 7. Find the value of A, when B is 9 and C is 6.

(A) 6 (B) 7 (C) 8 (D) 9

Exercise – 2(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If $\frac{a}{b}=\frac{2}{3}$ and $\frac{p}{q}=\frac{3}{4}$, then find $\frac{3ap-bq}{2ap+3bq}$.

(A) $\frac{3}{7}$ (B) $\frac{5}{8}$ (C) $\frac{1}{8}$ (D) $\frac{2}{3}$

2. If $\frac{23x^3-11y^3}{10x^3+6y^3}=\frac{3}{4}$, find $\frac{x+y}{2x-y}$.

(A) 2 : 1 (B) 3 : 2 (C) 4 : 3 (D) 5 : 4

3. 2x + y – 5z = 0 and 3x – 2y – 4z = 0. Find x : y : z

(A) 1 : 2 : 1 (B) 1 : 1 : 1
(C) 1 : 1 : 2 (D) 2 : 1 : 1

4. What must be added to both the terms a and b so that their ratio becomes equal to p : q?

(A) $\frac{aq+bp}{p+q}$ (B) $\frac{ap+bq}{p+q}$

(C) $\frac{aq-bp}{p-q}$ (D) $\frac{ap-bq}{p-q}$

5. For which of the following values of a : b is $(10a^2 + ab) : (3ab - b^2) = 10 : 1$?

(A) 1 : 2 (B) 3 : 5 (C) 5 : 2 (D) 5 : 3

6. In a school there are 650 students. The ratio of the number of boys to that of the girls is 8 : 5. How many more girls should join the school so that the ratio becomes 4 : 3?

7. If $\frac{a}{b}=\frac{c}{d}$, then the value of which of the following is equal to the value of $(a^2 + b^2)/(c^2 + d^2)$?

(A) $\frac{1}{2}$ (B) $\frac{a+b}{c+d}$

(C) $\frac{ab}{cd}$ (D) $\frac{a-b}{c-d}$

8. There are two persons A and B and A's present age is greater than B's present age. Which of the following will be the relation between the ratio of the squares of their ages after 7 years (X) and the ratio of their ages after 12 years (Y)?

(A) X < Y (B) X = Y
(C) X > Y (D) X ≤ Y

9. The ratio of the present ages of a man and his wife is 5 : 4. Which of the following can't be a possible ratio of their ages 20 years ago?

(A) 7 : 5 (B) 3 : 2
(C) 13 : 10 (D) 6 : 5

10. The income and the expenditure of a person are in the ratio 5 : 4. If the income of the person is ₹18000, then his savings (in ₹) are $\boxed{}$.

11. A certain sum is divided among A, B and C in a manner that for every rupee that A gets, B gets 75 paise and for every rupee that B gets, C gets 50 paise. If C's share in the total sum is ₹420, then find the share of A (in ₹).

12. A bag contains one rupee, 50 paise and 25 paise coins in the ratio 1 : 2 : 4. If the total amount in the bag is ₹75, then find the number of 50 paise coins in the bag.

13. The sum of the present ages of a woman and her daughter is 60 years. When the woman attains her husband's present age, the ratio of the ages of her husband and her daughter will be 2 : 1. Find the present age (in years) of her daughter.

(A) 10 (B) 15 (C) 20 (D) 25

14. A certain sum of money is divided among A, B and C such that A gets half of what B and C together get. B gets one-third of what A and C together get. If A got ₹500 more than B, then how much money was divided?

15. There are five identical glasses containing milk in the ratio 3 : 4 : 5 : 6 : 7. How many glasses are at least half full of milk if the total volume of milk in the glasses is three-fifth of the total volume of the five glasses?

16. P, Q, R, S and T are five positive integers satisfying P = 3Q = 4R and 2Q = 5S = 12T. Which of the following pairs contain a number that can never be an integer?

(A) $\left(\frac{2P}{15},\frac{Q}{T}\right)$ (B) $\left(\frac{P}{T},\frac{4R}{T}\right)$

(C) $\left(\frac{P}{4},\frac{RS}{180}\right)$ (D) $\left(\frac{P}{8},\frac{S}{R}\right)$

Directions for questions 17 and 18: These questions are based on the information given below.

There are two colleges in the town – college A and college B. There are 500 more students in college A than in college B. The ratio of the boys to that of the girls in college A is 3 : 2 and that in college B is 4 : 1. The ratio of the number of Science, Humanities and Commerce students in college A and college B are 2 : 5 : 3 and 2 : 3 : 3 respectively. The number of commerce students in both the colleges is the same.

17. How many students are there in college A?

18. How many girls are there in the two colleges together?

19. Ram has four times as much money with him as Shyam does. Each day, Ram spends a constant amount of money while Shyam earns a fourth of the amount that Ram spends. After 10 days, the ratio of the amounts with Ram and Shyam is 12:13. After how many more days will the ratio of the amounts with them be 4:31 ?
(A) 20 (B) 15 (C) 5 (D) 10

20. If a, b and c are in continued proportion, then which of the following is equal to a : c?
(A) $a^2 : b^2$ (B) $(a^2 + b^2) : (b^2 + c^2)$
(C) $b^2 : c^2$ (D) All of these

21. If $\frac{ka}{b+c} = \frac{kb}{c+a} = \frac{kc}{a+b} = \ell$ and $k \neq 0$, $a + b + c \neq 0$, then what is the value of 'ℓ'?
(A) k (B) $\frac{k}{3}$ (C) $\frac{k}{2}$ (D) $\frac{k}{4}$

22. If (x + y) varies directly as (x – y), then $(x^2 + y^2)$ will vary as _______.
(A) $x^2 - y^2$ (B) xy
(C) Both (A) and (B) (D) None of these

23. The distance travelled by a freely falling body is directly proportional to the square of the time taken. If a body falls 144 m in 6 seconds, then find the distance that the body fell in the 7th second.

24. A writer gets a fixed amount for his book apart from the royalty that he gets per book sold. He gets ₹30000 and ₹50000 for 1000 books sold and 2000 books sold respectively. What is his income per book, when 5000 books are sold?

25. The monthly telephone bill has a fixed tariff for upto 50 outgoing calls. Outgoing calls in excess of 50 are charged at a certain fixed rate per call. The monthly bills of Ramesh and Suresh who made 98 outgoing calls and 218 outgoing calls respectively were ₹300 and ₹450 respectively. Find the monthly bill of a person who has made 160 outgoing calls (in ₹)

26. A varies directly as the sum of the two quantities B and C. B in turn varies directly as x and C varies inversely as x. When x = 2, A = 6 and when x = 4, A = 9. Find the value of A when x = 16.
(A) 2½ (B) 1 (C) 8½ (D) 32¼

27. The total surface area of a special cylinder having a certain height is the sum of 2 parts. One of the parts varies directly with its radius. The other part varies directly with the square of its radius. The total surface areas of two cylinders having the same height whose radii are 10 cm and 20 cm are 720 sq.cm and 2640 sq.cm respectively. Find the total surface area (in sq.cm) of a cylinder of the same height whose radius is 15 cm.

28. A quantity p equals the sum of three other quantities, the first of which is a constant, the second varies directly as x and the third varies directly as x^2. When x = 1; p = 13, when x = 2, p = 36 and when x = 3, p = 79. Find the constant.

29. The cost of a precious stone varies as the cube of its weight. A certain precious stone broke into three pieces whose weights are in the ratio 1 : 2 : 3, as a result of which its cost reduces by ₹80280. What was the cost of the stone before breaking?
(A) ₹88840 (B) ₹96336
(C) ₹102400 (D) ₹112880

30. The consumption of petrol per hour of my car varies directly as the square of its speed. When the car is travelling at 50 kmph its consumption is 2 litres. If each litre costs ₹30 and other expenses per hour are ₹60, then what would be the minimum expenditure required to cover a distance of 500 km?
(A) ₹800 (B) ₹1200
(C) ₹1500 (D) None of these

Exercise – 2(b)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If a : b = 3 : 7, find the value of (5a + b) : (4a + 5b).
(A) 15 : 44
(B) 22 : 35
(C) 15 : 49
(D) 22 : 47

2. The ratio of the number of students in 3 sections A, B and C is 3 : 7 : 8. If there are a total of 180 students in these sections, find the number of students in section A.

3. A purse contains 72 coins comprising one rupee, 50 paise and 25 paise coins, their values being in the ratio 10 : 15 : 8. Find the number of 50 paise coins.

4. In a class, $\frac{2}{5}$th of the students are girls and $\frac{3}{4}$th of them travel to school by bus. 12 girls of that class do not travel to school by bus. Find the strength of the class.

5. The ratio of the present ages of Geetha and Sita is 9 : 5. 8 years hence, Sita would be as old as Geetha is now. Find the present ages (in years) of Sita and Geetha respectively.
(A) 10, 18 (B) 18, 10 (C) 9, 5 (D) 5, 9

6. The ratio of two numbers is 3 : 5 and their sum is 40. Find the larger of the two numbers.

7. The present ages of two persons are in the ratio 7 : 8. Twenty years ago the ratio of their ages was 9 : 11. Find the present age of the older person.
(A) 64 years (B) 72 years
(C) 56 years (D) 40 years

8. Seventy eight is divided into two parts such that five times the first part and four times the second part are in the ratio 15 : 14. Find the first part.

9. If a is 3/4th of b, b is 3/2 times of c and d is 1/4th of c, then find a : d.
(A) 9 : 1 (B) 9 : 2 (C) 8 : 3 (D) 8 : 1

10. What should be subtracted from both the numbers which are in the ratio 3 : 4 so that the ratio becomes 2 : 3?
(A) 4 (B) 6
(C) 10 (D) Cannot be determined

11. When 1 is subtracted from both the numerator and the denominator of a fraction $\frac{P}{Q}$, it becomes 2. Which of the following can be the sum of P and Q?
(A) 8 (B) 9 (C) 12 (D) 16

12. If $a \neq b$, what must be subtracted from both the terms a and b so that their ratio reverses?
(A) ab (B) $\sqrt{ab}$ (C) a + b (D) $\sqrt{a+b}$

13. What number must be subtracted from both the numerator and denominator of the fraction $\frac{22}{37}$ so that it becomes 2 : 7?

14. 3x + y – 5z = 0 and 4x + 5y – 14z = 0. Find x : y : z.
(A) 1 : 1 : 1 (B) 2 : 1 : 1
(C) 1 : 2 : 1 (D) 1 : 1 : 2

15. In a class there are a total of 80 boys and girls. Which of the following can't represent the ratio of the number of boys and girls in the class?
(A) 3 : 5 (B) 1 : 3 (C) 2 : 3 (D) 1 : 6

16. In an exam, the ratio of the marks scored by A and B is 6 : 5. The total marks scored by A and B and the maximum marks in the exam are in the ratio 11 : 8. If the difference of their marks is 25, what is the maximum marks for which the exam was conducted?

17. The marks scored by a student in three subjects are in the ratio 4 : 5: 6. If the candidate scored an overall aggregate of 60% of the sum of the maximum marks and the maximum marks in all three subjects is the same, in how many subjects did he score more than 60%?

18. The ratio of the present ages of A and B is 1 : 3. After six years, the sum of their ages would be twice the sum of their present ages. Find the present ages of A and B (in years) respectively.
(A) 3, 9 (B) 3, 6 (C) 9, 12 (D) 9, 3

19. The earnings of Mr. A in three months are in the ratio 5 : 4 : 7. The difference between the product of his earnings in the first and the third months to the product of his earnings in the first and the second months is ₹600000. Find his earnings in the third month.

20. P and Q are two workers. For every hour of work, P earns ₹3 less than Q. The hourly earnings of P and Q are in the ratio 4 : 5. Find the total earnings of both on a day on which both work for 9 hours.

21. The ratio of the present ages of A and B is 11 : 4 and 15 years ago it was 8 : 1. If five years ago the ratio of the ages of B and C was 3 : 2, then what is C's present age?
(A) 15 years (B) 10 years
(C) 22 years (D) 25 years

22. In a bag the number of ten rupee notes, five rupee notes and two rupee notes is in the ratio of their denominations. The total value of five rupee notes is ₹84 more than that of two rupee notes. Find the total value of ten rupee notes (in ₹).

23. If ℓ, m, n are non zero $\ell^2 + 8m^2 + 9n^2 = 4m(\ell + 3n)$, then $\ell : m : n$ = ____.
(A) 1 : 2 : 3 (B) 6 : 3 : 2
(C) 1 : 4 : 5 (D) 3 : 2 : 1

24. There are two numbers a^2 and 9. Their mean proportion is b, then find the value of $\left(a^2+b^2\right)/\left(b^2-a^2\right)$.

25. If a, b, c and d are in proportion, then which of the following is equal to $(a-b)(a-c)/a$?
(A) a + b + d – c (B) a – b – c – d
(C) a + d – b – c (D) a + b – c + d

26. If a, b and c are in continued proportion, then which of the following equals c : a?
(A) $b^2 : a^2$ (B) $(c^2 – b^2) : (b^2 – a^2)$
(C) $c^2 : b^2$ (D) All of these

27. The volume of a sphere varies directly as the cube of its radius. If three cubes of radii 3 cm, 4 cm and 5 cm are melted and recast into one sphere, then find the radius of the sphere.

28. The volume of a pyramid varies directly as its height and also as the area of its base. When the height is 15 m and the area of the base is 32 sq.m, the volume is 160 cu.m. Find the height of a pyramid of volume 240 cu.m, when the area of the base is 50 sq.m.
(A) $14^2/_5$ m (B) $14^3/_5$ m (C) $15^2/_3$ m (D) $17^1/_5$ m

29. The area of a circle varies directly with the square of its radius. A circle is drawn such that its area is equal to the sum of the areas of the three circles of radii 9 cm, 12 cm and 20 cm. Find the radius (in cm) of the circle drawn.

30. The distance travelled by a freely falling body is directly proportional to the square of the time for which it falls. A body fell 95 m in the 10^{th} second. Find the distance (in m) it fell in the 14^{th} second.

31. A writer gets a fixed amount for his book apart from the royalty he gets per book sold. He gets ₹22000 and ₹46000 for 6000 books sold and 18000 books sold respectively. Find his income per book when 25000 books are sold.

32. The lift (upward force) exerted by air on an airfoil varies jointly as the area of the airfoil and the square of the wind velocity. When the wind velocity is 3 mph and the area of the airfoil is 4 sq.ft, then the lift exerted on it is 128 lb. Find the lift exerted on an airfoil of area 4 sq.yards when the wind velocity is 4 mph.
(A) $2227^5/_9$ lb (B) $512^2/_9$ lb
(C) 1024 lb (D) 2048 lb

33. Quantity X varies directly with the product of the quantities Y and Z. When Y = 45 and Z = 20, X = 200. When Y = 90 and Z = 30, find X.

34. Ashok's monthly salary varies directly with the number of working days in that month. His salary would be ₹9000 if there are 20 working days in a month. Find his monthly salary if there are 21 working days in that month.
(A) ₹9550 (B) ₹9450 (C) ₹9650 (D) ₹9850

35. A variable x varies directly with the cube of another variable y. When x = 4, y = 2. Find y when x = 32.

36. The cost of a precious stone varies directly as the square root of its weight. A certain precious stone broke into 3 pieces whose weights are in the ratio 1 : 4 : 4. As a result, its value went up by ₹12000. Find its initial value.
(A) ₹9000 (B) ₹12000 (C) ₹15000 (D) ₹18000

37. The volume of a gas is inversely proportional to the pressure acting on it when the temperature is constant and directly proportional to the temperature when the pressure acting on it is constant. When the temperature is 40 and the pressure acting on it is 64, its volume is 200. Find the pressure when the temperature and its volume are 50 and 400 respectively.
(A) 32 (B) 40
(C) 44 (D) 50

38. The weight of a solid right circular cone of a certain material varies directly as the square of its radius when its height is constant and varies directly as its height when its radius is constant. The weight of one such cone is 12 kg, its radius is 2 cm and its height is 4 cm. Find the weight of another such cone whose radius is 4 cm and whose height is 3 cm.

39. The kinetic energy of a moving body varies directly with its mass when its velocity is constant and with the square of its velocity when its mass is constant. A body has a mass of 7.2 kg and a velocity of 0.2 m/sec and a kinetic energy of 0.144 joules. Find the kinetic energy of a body having a mass of 3.6 kg and a velocity of 0.8 m/sec (in joules).

40. The expenses of a hostel are partly fixed and partly variable, varying with the number of occupants. If there are 20 occupants, then each of the occupants has to bear ₹650 per month and if there are 5 more occupants, then the share of each of the occupants comes down by ₹50 per month. How many occupants were there if the share of each occupant is ₹500?
(A) 35 (B) 40
(C) 45 (D) 50

Directions for questions 41 to 45: Each question is followed by two statements, I and II. Indicate your responses based on the following directions:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using statements I and II together but not using I or II alone.
Mark (D) if the question cannot be answered even using statements I and II together.

41. Companies X and Y earned profits in 2015. Is the profit of X less than that of Y in 2015?
I. The ratio of the sales of X and Y is 4 : 5.
II. The ratio of the expenditures of X and Y is 3 : 4.

42. What is the ratio of the savings of A and B?

I. The incomes of A and B are in the ratio 4 : 5.

II. The ratio of the expenditures of A and B is 4 : 5.

43. What is the value of $\frac{7a + 9b}{4a + 5b}$?

I. $b + a = \frac{1}{2}(6a - b)$

II. $3a + 4b = 5$

44. Find x : y : z.

I. $\frac{x}{y} = \frac{y}{z}$ II. $\frac{y}{z} = \frac{z}{x}$

45. ₹510 was spent on diesel. What was the total money spent on petrol, diesel and CNG?

I. The amounts required to buy 1 litre of petrol, 2 litres of diesel and 3 litres of CNG are equal.

II. Equal quantities of each fuel were purchased.

Key

Concept Review Questions

1. 20
2. C
3. 15
4. A
5. A
6. B
7. 60
8. D
9. (a) D
   (b) C
   (c) B
   (d) C
10. D
11. C
12. A
13. 8
14. D
15. 6
16. C
17. 320
18. D
19. 108
20. A

Exercise – 2(a)

1. C
2. A
3. D
4. C
5. C
6. 50
7. C
8. C
9. D
10. 3600
11. 1120
12. 50
13. C
14. 6000
15. 3
16. D
17. 2500
18. 1400
19. C
20. D
21. C
22. C
23. 52
24. 22
25. 377.50
26. D
27. 1530
28. 10
29. B
30. B

Exercise – 2(b)

1. D
2. 30
3. 30
4. 120
5. A
6. 25
7. A
8. 36
9. B
10. D
11. A
12. C
13. 16
14. C
15. D
16. 200
17. 1
18. A
19. 1400
20. 243
21. A
22. 400
23. B
24. 1.25
25. C
26. D
27. 6
28. A
29. 25
30. 135
31. 2.40
32. D
33. 600
34. B
35. 4
36. D
37. B
38. 36
39. 1.152
40. D
41. D
42. C
43. A
44. C
45. C