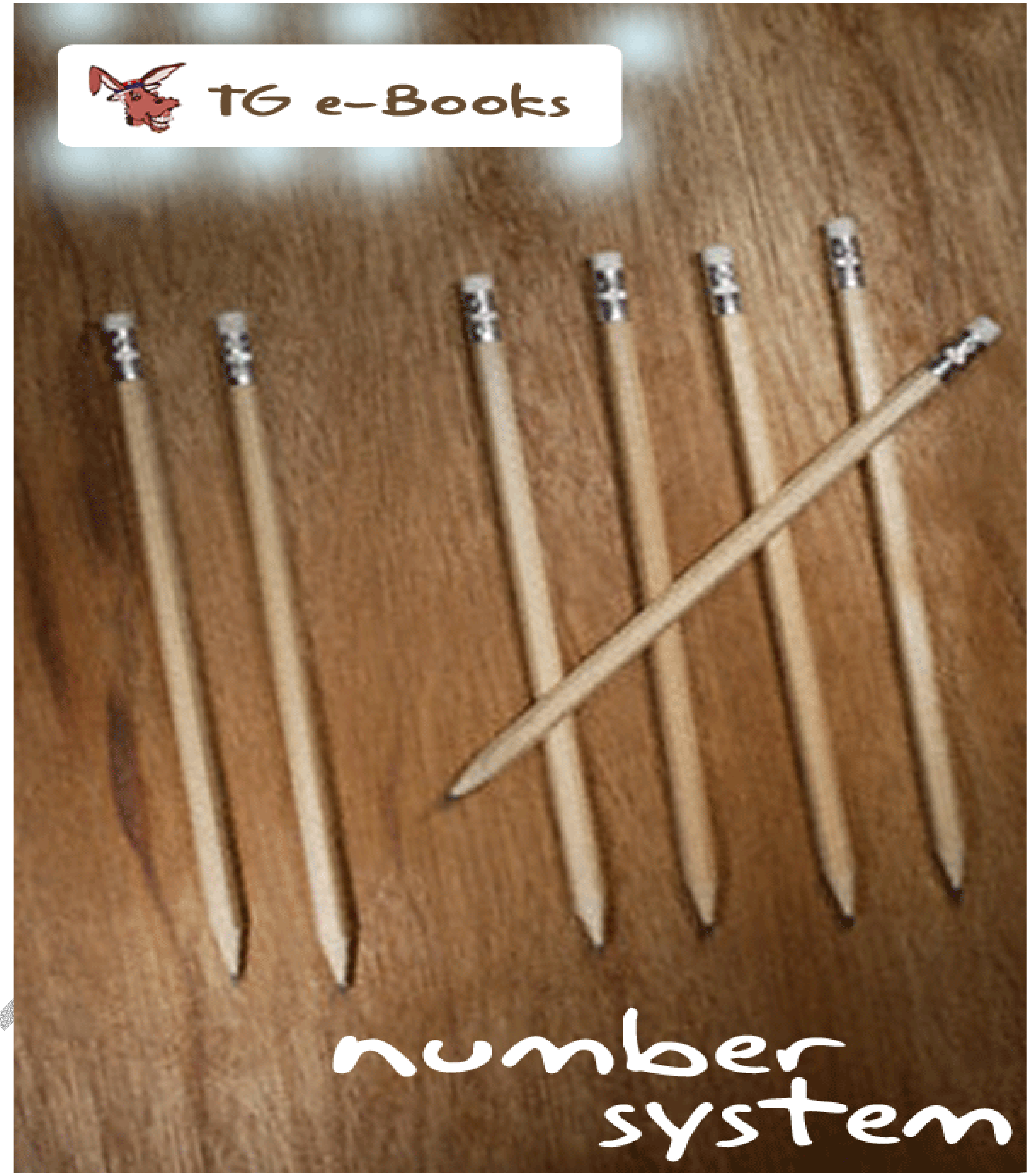
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TYPES OF NUMBERS

Natural Numbers

The group of numbers starting from 1 and including 1, 2, 3, 4, 5, and so on, are known as natural numbers. Zero, negative numbers, and decimals are not included this group.

If n is an odd natural number, what is the highest number that always divides $n \times (n^2 - 1)$?

Answer: $n \times (n^2 - 1) = (n - 1) \times n \times (n + 1)$, which is a product of three consecutive numbers. Since n is odd, the numbers $(n - 1)$ and $(n + 1)$ are both even. As they are two consecutive even numbers one of these numbers will be a multiple of 2 and the other will be a multiple of 4. Hence, their product is a multiple of 8. Since one out of every three consecutive numbers is a multiple of 3, one of the three numbers will be a multiple of three. Hence, the product of three numbers will be a multiple of $8 \times 3 = 24$.

Hence, the highest number that always divides $n \times (n^2 - 1)$ is 24.

The product of n consecutive natural numbers is always divisible by n!, where $n! = 1 \times 2 \times 3 \times 4 \times 5\ldots \times n$

For every natural number n, the highest number that $n \times (n^2 - 1) \times (5n + 2)$ is always divisible by is
(a) 6 (b) 24 (c) 36 (d) 48

Answer:

Case 1: If n is odd, $n \times (n^2 - 1)$ is divisible by 24 as proved in the earlier question.

Case 2: If n is even, both $(n - 1)$ and $(n + 1)$ are odd. Since product of three consecutive natural numbers is always a multiple of 3 and n is even, the product $n \times (n^2 - 1)$ is divisible by 6. Since n is even 5n is even. If n is a multiple of 2, 5n is a multiple of 2 and hence $5n + 2$ is a multiple of 4. If n is a multiple of 4, $5n + 2$ is a multiple of 2. Hence, the product $n \times (5n + 2)$ is a multiple of 8.

Hence, the product $n \times (n^2 - 1) \times (5n + 2)$ is a multiple of 24.

Prove that (2n)! is divisible by $(n!)^2$.

Answer: $(2n)! = 1 \times 2 \times 3 \times 4 \times \ldots \times (n - 1) \times n \times (n + 1) \times \ldots \times 2n = (n)! \times (n + 1) \times (n + 2) \times \ldots \times 2n$. Since $(n + 1) \times (n + 2) \times \ldots \times 2n$ is a product of n consecutive numbers, it is divisible by n!. Hence, the product $(n)! \times (n + 1) \times (n + 2) \times \ldots \times 2n$ is divisible by $n! \times n! = (n!)^2$.

Sum of first n natural numbers = $\frac{n(n+1)}{2}$

Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

Sum of cubes of first n natural numbers = $\left(\frac{n(n+1)}{2}\right)^2$

Number of single-digit natural numbers: 1- 9 = 9
Number of two-digit natural numbers: 10- 99 = 90
Number of three-digit natural numbers: 100- 999 = 900
Square of every natural number can be written in the form **3n** or **3n + 1**.
Square of every natural number can be written in the form **4n** or **4n + 1**.
Square of a natural number can only end in 00, 1, 4, 5, 6, and 9. No perfect square can end in 2, 3, 7, 8

or a single 0.
The tens digit of every perfect square is even unless the square is ending in 6 in which case the tens digit is odd.

If you write 1st 252 natural numbers in a straight line, how many times do you write the digit 4?

Answer: In the 1st 99 natural numbers, digit 4 comes 20 times. Similarly, from 100 to 199, digit 4 comes 20 times. Now from 200 to 299, digit 4 comes again 20 times out of which we need to subtract 5 numbers (254, 264, 274, 284 and 294). Therefore, total number of times that we write the digit 4 = 20 + 20 + 20 – 5 = 55.

If a book has 252 pages, how many digits have been used to number the pages?

Answer: From page number 1 to age number 9, we will use 1 digit per page $\Rightarrow$ digits used = 9.
From page number 10 to age number 99, we will use 2 digits per page $\Rightarrow$ digits used = $2 \times 90 = 180$.
From page number 100 to age number 252, we will use 3 digits per page $\Rightarrow$ digits used = $3 \times 153 = 459$.
Therefore, total number of digits used = 9 + 180 + 459 = 648

There are three consecutive natural numbers such that the square of the second minus twelve times the first is three less than twice the third. What is the largest of the three numbers?

Answer: Let the consecutive natural number be n, n + 1, n + 2.
$\Rightarrow (n + 1)^2 - 12n = 2(n + 2) - 3$. Solving, we get n = 12 and n + 2 = 14.

What is the smallest natural number which is cube of a natural number and fourth power of a **different** natural number?

Answer: Let $N = x^3$ and $N = y^4$. Therefore, N will contain 12th power (LCM of 3 and 4) of a natural number. Therefore, $N = a^{12} = (a^4)^3 = (a^3)^4$. The smallest such number is $2^{12} = 4096$.

1 and 8 are the first two natural numbers for which 1 + 2 + 3 + ... + n is a perfect square. Which number is the 4th such number?

Answer: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} = M^2$ (say) $\Rightarrow n(n + 1) = 2M^2$
Now n and n + 1 will have no factor in common. Since RHS is twice the square of a natural number, one of n and n + 1 will be twice of a perfect square and the other will be a perfect square. As twice of a perfect square will be even, the other square will be odd. We start investigating the odd squares and their neighbours. The fourth such numbers we get is 288×289.

Let N = 999 999 999 999 999 999. How many 9's are there in N^2?

Answer: $N^2 = (10^{18} - 1)^2 = 10^{36} + 1 - 2 \times 10^{18} = \underbrace{100000\ldots00001}_{\text{35 zeroes}} - \underbrace{2000..0000}_{\text{18 zeroes}} = \underbrace{99999..9998000000\ldots0001}_{\text{17 9's}}$

Find all values of n which satisfy $\left(\frac{n!}{4!}\right)^2 + \frac{7!5!}{4 \cdot 4!} = 240\left(\frac{n!}{4!}\right)$

Answer: Let n! = k. Taking 4! off both sides and simplifying, we get
$\Rightarrow k^2 - 5760k + 3628800 = 0 \Rightarrow k^2 - (6! + 7!)k + 6! \times 7! = 0 \Rightarrow k = 6!$ And $7! \Rightarrow n = 6$ and 7
$\Rightarrow$ Sum = 13

The number 1234567890123456789012 3…890 is four thousand digits long. First, you remove all of the digits in odd numbered places starting at the leftmost place. Next, you remove the digits in the odd numbered places in the remaining 2000 digits. You perform the same operation until no digits remain. What digit is the last to be removed?

Answer: After the first removal, the 2^{nd}, 4^{th}, 6^{th}, 8^{th}, 10^{th}, 12^{th}, 14^{th}, 16^{th},… will be left. After the second removal, 4^{th}, 8^{th}, 12^{th}, 16^{th}, 20^{th}, 24^{th}, digits will be left. After the third removal, 8^{th}, 16^{th}, 24^{th}, 32^{nd}, digits will be left… In short, after the nth removal, the first digit would be $(2^n)^{th}$ digit in the original number.

To reduce 4000 digits to a single digit, we need to perform the halving operation 11 times. Therefore, the digit left would be $(2^{11})^{th}$ digit or 2048^{th} digit which is 8.

Whole Numbers

All Natural Numbers plus the number 0 are called as Whole Numbers.

Integers

All Whole Numbers and their negatives are included in this group.

For how many integers n is $n^4 + 6n < 6n^3 + n^2$?

Answer: $n^4 + 6n - 6n^3 - n^2 < 0 \Rightarrow n(n^2 - 1)(n - 6) < 0$. n cannot be equal to 1 or 0 because LHS becomes 0. Now $n^2 - 1$ will always be positive, therefore, $n(n - 6)$ should be negative $\Rightarrow$ n = 2, 3, 4 and 5.

Find the sum of all two-digit positive integers which exceed the product of their digits by 12.

Answer: Let the two-digit integer be ab. Therefore, $10a + b = ab + 12 \Rightarrow (a - 1)(10 - b) = 2$ $\Rightarrow$ numbers are 28 or 39.

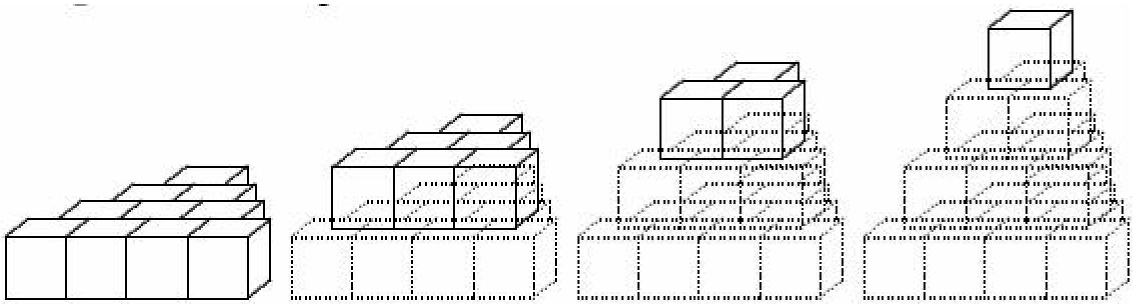
For which integer n is $2^8 + 2^{11} + 2^n$ is a perfect square?

Answer: In order to write the above expression in the form $(a + b)^2 = a^2 + 2ab + b^2$, we note that $2^8 = (2^4)^2$ and $2^{11} = 2 \times 2^4 \times 2^6$. Therefore, we need the square of $2^6 \Rightarrow 2^n = (2^6)^2 = 2^{12} \Rightarrow n = 12$.

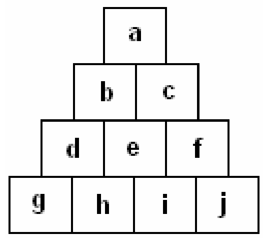
Find the smallest positive integer n for which $(2^2 - 1)(3^2 - 1)(4^2 - 1)\ldots(n^2 - 1)$ is a perfect square.

Answer: n_{th} term $= (n^2 - 1) = (n + 1)(n - 1) \Rightarrow$ series $= 1 \times 3 \times 2 \times 4 \times 3 \times 5 \ldots \times (n - 2) \times (n) \times (n - 1) \times (n + 1) = 2n(n + 1) \times k^2$ because all the other terms are squared. The fist value of n which $2n(n + 1)$ as perfect square is n = 8.

Twenty cubical blocks are arranged as shown. First, 10 are arranged in a triangular pattern; then a layer of 6, arranged in a triangular pattern, is centered on the 10; then a layer of 3, arranged in a triangular pattern, is centered on the 6; and finally one block is centered on top of the third layer. The blocks in the bottom layer are numbered 1 through 10 in some order. Each block in layers 2, 3 and 4 is assigned the number which is the sum of the numbers assigned to the three blocks on which it rests. Find the smallest possible number which could be assigned to the top block.



Answer: Let the numbers on the bottom 10 cubes be a, b, c, d, e, f, g, h, I, and j, as shown in the figure below:



The sum on the top cube will come out to be a + g + j + 3(b + c + d + f + h + i) + 6e. Giving the lowest value to highest occurring number (e = 1) and highest values to lowest occurring numbers (a = 8, g = 9, j = 10), we get the minimum value = 114.

A two-digit number is 18 less than the square of the sum of its digits. How many such numbers are there?
Answer: Let the two-digit number be AB, where A and B are single digits.
Therefore, $10A + B = (A + B)^2 - 18$. Now, the highest value of 10A + B can be 99, therefore the highest value of $(A + B)^2 - 18$ can also be 99. Also, $(A + B)^2$ will be greater than 18 to keep the R.H.S. positive.
$\Rightarrow (A + B)^2 = 25, 36, 49, 64, 81, 100$.
$\Rightarrow$ 10A + B = 7 (not possible), 18 (not possible), 31 (not possible), 46 (not possible), 63 , or 82. We see that two pairs (A, B) = (6, 3) and (8, 2) satisfy the above condition.

Donkey lives in a street with stable numbers 8 up to 100. Shrek wants 2 know at which number Donkey lives.Shrek asks him, "Is your number larger than 50"
Donkey answers, but lies.
Upon this Shrek asks: "Is your number a multiple of 4?"
Donkey answers, but lies again.
Then Shrek asks: "Is your number a square?"
Finally, Donkey answers truthfully
Upon this Shrek says: "I know your number if you tell me whether the first digit is a 3."
Now Donkey answers with a smirk on his face, which means now we don't know whether he lies or speaks the truth. Question: What is Donkey's real stable number?

Answer: For Shrek to pinpoint donkey's stable number, donkey must have answered the third question in a 'yes,' i.e. its stable number is a perfect square. Now, Shrek is looking for a square that is beginning with 3. The only such number is 36. Therefore, donkey must have said that the number is less than 50 and is a multiple of 4, both of which are lies. Therefore, the number is more than 50, NOT a multiple of 4 and is a perfect square. Only such number is 81.

Rational Numbers

Any number that can be expressed as a ratio of two integers is called a rational number.

This group contains decimal that either do not exist (as in 6 which is 6/1), or terminate (as in 3.4 which is 34/10), or repeat with a pattern (as in 2.333... which is 7/3).

Express 0.212121..in rational form.

Answer: Let A = 0.21212121 $\Rightarrow$ 100A = 21.212121.. = 21 + A $\Rightarrow$ A = $\frac{21}{99}$.

Rule: To express a recurring fraction in rational form, write the recurring digits once in the numerator and write as many 9s in the denominator as are the number of recurring digits. For example, 0.abcabcabc.. = $\frac{abc}{999}$ and 0.abcdabcdabcd.. = $\frac{abcd}{9999}$

Express 2.1434343…in rational form.

Answer: Let R = 2.1434343.. $\Rightarrow$ 10R = 21.43434343…= 21 + $\frac{43}{99}$ $\Rightarrow$ R = $\frac{2122}{990}$.

Rule: To write a fraction, which has both recurring and non-recurring parts, in a rational form, do the following steps:
Numerator: (Number formed by writing all the digits once) – (Number formed by writing all the non-recurring part once) = 2143 – 21 = 2122.
Denominator: Number of 9s equal to number of recurring digits followed by number of zeroes equal to non-recurring digits after the decimal.

What is the value of $\frac{2^{2004}+2^{2001}}{2^{2003}-2^{2000}}$?

Answer: $\frac{2^{2004}+2^{2001}}{2^{2003}-2^{2000}} = \frac{2^{2001}(2^3+1)}{2^{2000}(2^3-1)} = \frac{18}{7}$

Irrational Numbers

Any number that can not be expressed as the ratio of two integers is called an irrational number (imaginary or complex numbers are not included in irrational numbers).
These numbers have decimals that never terminate and never repeat with a pattern.
Examples include pi, e, and √2. 2 + √3, 5 - √2 etc. are also irrational quantities called **Surds.**

Express the value of $\frac{1}{\sqrt{5}+\sqrt{6}-\sqrt{11}}$ as a fraction whose denominator is rational.

Answer: $\frac{1}{\sqrt{5}+\sqrt{6}-\sqrt{11}} = \frac{\sqrt{5}+\sqrt{6}+\sqrt{11}}{(\sqrt{5}+\sqrt{6}-\sqrt{11})\times(\sqrt{5}+\sqrt{6}+\sqrt{11})}$

$$= \frac{\sqrt{5}+\sqrt{6}+\sqrt{11}}{[(\sqrt{5}+\sqrt{6})^2-(\sqrt{11})^2]} = \frac{\sqrt{5}+\sqrt{6}+\sqrt{11}}{2\sqrt{30}} = \frac{\sqrt{30}(\sqrt{5}+\sqrt{6}+\sqrt{11})}{60}$$

If p = $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}$ and q = $\frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}$, then the value of $p^2 + pq + q^2$ is

Answer: p = $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}$ = $\frac{(\sqrt{8}+\sqrt{7})^2}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})}$ = $(\sqrt{8}+\sqrt{7})^2$ = $15+2\sqrt{56}$

Similarly, q = $(\sqrt{8}-\sqrt{7})^2$ = $15-2\sqrt{56}$

$p^2 + pq + q^2$ = $(15+2\sqrt{56})^2 + (15)^2 - (2\sqrt{56})^2 + (15-2\sqrt{56})^2$ = 675 + 224 = 899

Real Numbers

This group is made up of all the Rational and Irrational Numbers. The ordinary number line encountered when studying algebra holds real numbers.

Imaginary Numbers

These numbers are formed by the imaginary number i (i = $\sqrt{-1}$). Any real number times i is an imaginary number.
Examples include i, 3i, −9.3i, and (pi)i. Now $i^2 = -1$, $i^3 = i^2 \times i = -i$, $i^4 = 1$.

What is the value of $\frac{i^4+i^6+i^8+i^{10}+i^{12}}{i^{14}+i^{16}+i^{18}+i^{20}+i^{22}}$?

Answer: $i^4 = 1$, $i^6 = i^4 \times i^2 = -1$, $i^8 = 1$, $i^{10} = -1$, and so on.

Hence, $\frac{i^4+i^6+i^8+i^{10}+i^{12}}{i^{14}+i^{16}+i^{18}+i^{20}+i^{22}} = \frac{1-1+1-1+1}{-1+1-1+1-1} = -1$

Complex Numbers

A Complex Numbers is a combination of a real number and an imaginary number in the form a + bi. a is called the real part and b is called the imaginary part.

Examples include 3 + 6i, 8 + (−5)i, (often written as 8 - 5i).

Note: a number in the form $\frac{1}{a+ib}$ is written in the form of a complex number by multiplying both numerator and denominator by the conjugate of a + ib, i.e. a − ib.

Hence, $\frac{1}{a+ib} = \frac{a-ib}{(a+ib)(a-ib)} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{ib}{a^2+b^2}$, which is in the form p + iq.

The value of $\left(\frac{1+i}{1-i}\right)^7$ is

Answer: $\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+i^2+2i}{1-i^2}$ = 2i/2 = i

Hence, $(\frac{1+i}{1-i})^7 = (i)^7 = -i$

Prime Numbers
All the numbers that have only two divisors, 1 and the number itself, are called prime numbers. Hence, a prime number can only be written as the product of 1 and itself. The numbers 2, 3, 5, 7, 11…37, etc. are prime numbers.
Note: 1 is not a prime number.

EXAMPLE

If $x^2 - y^2 = 101$, find the value of $x^2 + y^2$, given that x and y are natural numbers.

Answer: $x^2 - y^2 = (x + y)(x - y) = 101$. But 101 is a prime number and cannot be written as product of two numbers unless one of the numbers is 1 and the other is 101 itself. Hence, x + y = 101 and x − y = 1. → x = 51, y = 50.
→ $x^2 + y^2 = 51^2 + 50^2 = 5101$.

What numbers have exactly three divisors?

Answer: The squares of prime numbers have exactly three divisors, i.e. 1, the prime number, and the square itself.

For how many prime numbers p, is $p^2 + 3p - 1$ a prime number?

Answer: When p = 3, the expression gives a prime number (17). When p is not equal to 3, p^2 will be of the form 3k + 1 as every square number is of the form 3n or 3n + 1. Therefore, $p^2 + 3p - 1 = 3k + 1 + 3p - 1 \Rightarrow$ a multiple of 3. Therefore, for only p = 3, do we get a prime number (17) from the expression.

The number of positive integers n in the range $12 \le n \le 40$ such that the product (n – 1) (n – 2) (n – 3)..3·2·1 is not divisible by n is **(CAT 2003)**

Answer: The product (n – 1) (n – 2) (n – 3)..3·2·1 will not be divisible by n only when this product does not contain factors of n, i.e. n is a prime number. The prime numbers in the range given are 13, 17, 19, 23, 29, 31, and 37. 7 numbers in all.

To find whether a number N is prime or not
Find the root R (approximate) of the number N, i.e. R = $\sqrt{N}$. Divide N by every prime number less than or equal to R. If N is divisible by at least one of those prime numbers it is not a prime number. If N is not divisible by any of those prime numbers, it is a prime number.

Odd and Even Numbers

All the numbers divisible by 2 are called even numbers whereas all the numbers not divisible by 2 are called odd numbers. 2, 4, 6, 8… etc. are even numbers and 1, 3, 5, 7.. etc. are odd numbers.

Remember!	$(\text{Even})^{\text{Odd}}$ = Even
Odd + Odd = Even	Even × Odd = Even
Even + Even = Even	Even × Even = Even
Odd + Even = Odd	Odd × Odd = Odd
$(\text{Odd})^{\text{Even}}$ = Odd	$(\text{Odd})^{\text{Even}} \times (\text{Even})^{\text{Odd}}$ = Even
	$(\text{Odd})^{\text{Even}} + (\text{Even})^{\text{Odd}}$ = Odd

REMAINDERS

Suppose the numbers N_1, N_2, N_3… give quotients Q_1, Q_2, Q_3… and remainders R_1, R_2, R_3..., respectively, when divided by a common divisor D.

Therefore $N_1 = D \times Q_1 + R_1$,
$N_2 = D \times Q_2 + R_2$,
$N_3 = D \times Q_3 + R_3$.. and so on.

Let P be the product of N_1, N_2, N_3…

Therefore, $P = N_1N_2N_3.. = (D \times Q_1 + R_1)(D \times Q_2 + R_2)(D \times Q_3 + R_3)..$
$= D \times K + R_1R_2R_3...$ where K is some number ---- (1)

➔ In the above equation, since only the product $R_1R_2R_3$… is free of D, therefore the remainder when P is divided by D is the remainder when the product $R_1R_2R_3$… is divided by D.

Let S be the sum of N_1, N_2, N_3…
Therefore, $S = (N_1) + (N_2) + (N_3) +... = (D \times Q_1 + R_1) + (D \times Q_2 + R_2) + (D \times Q_3 + R_3)..$
$= D \times K + R_1 + R_2 + R_3$… where K is some number--- (2)

➔ Hence the remainder when S is divided by D is the remainder when $R_1 + R_2 + R_3$ is divided by D.

What is the remainder when the product 1998 × 1999 × 2000 is divided by 7?

Answer: the remainders when 1998, 1999, and 2000 are divided by 7 are 3, 4, and 5 respectively. Hence the final remainder is the remainder when the product 3 × 4 × 5 = 60 is divided by 7. Therefore, remainder = 4

What is the remainder when 2^{2004} is divided by 7?

Answer: 2^{2004} is again a product (2 × 2 × 2... (2004 times)). Since 2 is a number less than 7 we try to convert the product into product of numbers higher than 7. Notice that 8 = 2 × 2 × 2. Therefore we convert the product in the following manner- $2^{2004} = 8^{668} = 8 \times 8 \times 8...$ (668 times).
The remainder when 8 is divided by 7 is 1. Hence the remainder when 8^{668} is divided by 7 is the remainder obtained when the product 1 × 1 × 1... is divided by 7. Therefore, remainder = 1

What is the remainder when 2^{2006} is divided by 7?

Answer: This problem is like the previous one, except that 2006 is not an exact multiple of 3 so we cannot convert it completely into the form 8^x. We will write it in following manner- $2^{2006} = 8^{668} \times 4$. Now, 8^{668} gives the remainder 1 when divided by 7 as we have seen in the previous problem. And 4 gives a remainder of 4 only when divided by 7. Hence the remainder when 2^{2006} is divided by 7 is the remainder when the product 1 × 4 is divided by 7. Therefore, remainder = 4

What is the remainder when 25^{25} is divided by 9?

Answer: Again $25^{25} = (18 + 7)^{25} = (18 + 7)(18 + 7)...25$ times $= 18K + 7^{25}$
Hence remainder when 25^{25} is divided by 9 is the remainder when 7^{25} is divided by 9.
Now $7^{25} = 7^3 \times 7^3 \times 7^3$.. (8 times) $\times 7 = 343 \times 343 \times 343$... (8 times) $\times 7$.

The remainder when 343 is divided by 9 is 1 and the remainder when 7 is divided by 9 is 7.
Hence the remainder when 7^{25} is divided by 9 is the remainder we obtain when the product $1 \times 1 \times 1$... (8 times) $\times 7$ is divided by 9. The remainder is 7 in this case. Hence the remainder when 25^{25} is divided by 9 is 7.

What is the remainder when $32^{32^{32}}$ is divided by 7?

Let me put up the steps for finding remainder when X^{Y^z} is divided by D.

1. Divide X by D. Let the remainder be R. Therefore, you have to find the remainder when R^{Y^z} is divided by D. 32 gives a remainder 4 when divided by 7. Therefore, you are trying to find the remainder when $4^{32^{32}}$ is divided by 7.
2. Find a power of R that gives a remainder of + 1 or – 1 with D. If you find a power that gives a remainder – 1, twice of that power will give a remainder of + 1. Now I know that $4^3 = 64$ gives a remainder 1 when divided by 7.
3. Find the remainder when Y^z is divided by that power. Here, find the remainder when 32^{32} is divided by 3. The remainder is 1. Therefore, 32^{32} can be written as $3k + 1$ and $4^{32^{32}}$ can be written as 4^{3k+1} or $(4^3)^k \times 4$.
4. Now 4^3 gives a remainder 1 when divided by 7. Therefore, we need to find the remainder when 4 is divided by 7. Therefore, the remainder is 4.

SOME SPECIAL CASES:

When both the dividend and the divisor have a factor in common.

Let N be a number and Q and R be the quotient and the remainder when N is divided by the divisor D.
Hence, $N = Q \times D + R$.
Let $N = k \times A$ and $D = k \times B$ where k is the HCF of N and D and $k > 1$. Hence $kA = Q \times kB + R$.
Let Q_1 and R_1 be the quotient and the remainder when A is divided by B. Hence $A = B \times Q_1 + R_1$.
Putting the value of A in the previous equation and comparing we get-

$k(B \times Q_1 + R_1) = Q \times kB + R \Rightarrow R = kR_1$.

Hence to find the remainder when both the dividend and the divisor have a factor in common,
- Take out the common factor (i.e. divide the numbers by the common factor)
- Divide the resulting dividend (A) by resulting divisor (B) and find the remainder (R_1).
- The real remainder R is this remainder R1 multiplied by the common factor (k).

What the remainder when 2^{96} is divided by 96?

The common factor between 2^{96} and 96 is $32 = 2^5$.
Removing 32 from the dividend and the divisor we get the numbers 2^{91} and 3 respectively.
The remainder when 2^{91} is divided by 3 is 2.
Hence the real remainder will be 2 multiplied by common factor 32.
Remainder = 64

The concept of negative remainder

$15 = 16 \times 0 + 15$ or $15 = 16 \times 1 - 1$.

The remainder when 15 is divided by 16 is 15 the first case and −1 in the second case. Hence, the remainder when 15 is divided by 16 is 15 or −1.

➔ When a number N < D gives a remainder R (= N) when divided by D, it gives a negative remainder of R − D.

For example, when a number gives a remainder of −2 with 23, it means that the number gives a remainder of 23 − 2 = 21 with 23.

Find the remainder when 7^{52} is divided by 2402.

Answer: $7^{52} = (7^4)^{13} = (2401)^{13} = (2402 - 1)^{13} = 2402K + (-1)^{13} = 2402K - 1$.

Hence, the remainder when 7^{52} is divided by 2402 is equal to −1 or 2402 − 1 = 2401.
Remainder = 2401.

When dividend is of the form $a^n + b^n$ or $a^n - b^n$

Theorem1: $a^n + b^n$ is divisible by a + b when n is **ODD**.
Theorem 2: $a^n - b^n$ is divisible by a + b when n is **EVEN**.
Theorem 3: $a^n - b^n$ is **ALWAYS** divisible by a − b.

What is the remainder when $3^{444} + 4^{333}$ is divided by 5?

Answer: The dividend is in the form $a^x + b^y$. We need to change it into the form $a^n + b^n$.
$3^{444} + 4^{333} = (3^4)^{111} + (4^3)^{111}$. Now $(3^4)^{111} + (4^3)^{111}$ will be divisible by $3^4 + 4^3 = 81 + 64 = 145$. Since the number is divisible by 145 it will certainly be divisible by 5. Hence, the remainder is 0.

What is the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7?

Answer: The remainders when 5555 and 2222 are divided by 7 are 4 and 3 respectively. Hence, the problem reduces to finding the remainder when $(4)^{2222} + (3)^{5555}$ is divided by 7.

Now $(4)^{2222} + (3)^{5555} = (4^2)^{1111} + (3^5)^{1111} = (16)^{1111} + (243)^{1111}$. Now $(16)^{1111} + (243)^{1111}$ is divisible by 16 + 243 or it is divisible by 259, which is a multiple of 7. Hence the remainder when $(5555)^{2222} + (2222)^{5555}$ is divided by 7 is zero.

$20^{2004} + 16^{2004} - 3^{2004} - 1$ is divisible by:
(a) 317 (b) 323 (c) 253 (d) 91

Answer: $20^{2004} + 16^{2004} - 3^{2004} - 1 = (20^{2004} - 3^{2004}) + (16^{2004} - 1^{2004})$. Now $20^{2004} - 3^{2004}$ is divisible by 17 (Theorem 3) and $16^{2004} - 1^{2004}$ is divisible by 17 (Theorem 2). Hence the complete expression is divisible by 17.

$20^{2004} + 16^{2004} - 3^{2004} - 1 = (20^{2004} - 1^{2004}) + (16^{2004} - 3^{2004})$. Now $20^{2004} - 1^{2004}$ is divisible by 19 (Theorem 3) and $16^{2004} - 3^{2004}$ is divisible by 19 (Theorem 2). Hence the complete expression is also divisible by 19.

Hence the complete expression is divisible by $17 \times 19 = 323$.

When $f(x) = a + bx + cx^2 + dx^3 + \ldots$ is divided by x − a

The remainder when $f(x) = a + bx + cx^2 + dx^3 + ..$ is divided by $x - a$ is $f(a)$. Therefore, If $f(a) = 0$, $(x - a)$ is a factor of $f(x)$.

What is the remainder when $x^3 + 2x^2 + 5x + 3$ is divided by $x + 1$?

Answer: The remainder when the expression is divided by $(x - (-1))$ will be $f(-1)$. Remainder $= (-1)^3 + 2(-1)^2 + 5(-1) + 3 = -1$

If $2x^3 - 3x^2 + 4x + c$ is divisible by $x - 1$, find the value of c.

Answer: Since the expression is divisible by $x - 1$, the remainder $f(1)$ should be equal to zero $\Rightarrow 2 - 3 + 4 + c = 0$, or $c = -3$.

The remainders when F(x) is divided by x - 99 and x - 19 are 19 and 99, respectively. What is the remainder when F(x) is divided by (x - 19) (x - 99)?

Answer: $F(x) = (x - 99)a + 19$ --- (1) and $F(x) = (x - 19)b + 99$ --- (2). Multiplying (1) by $(x - 19)$ and (2) by $(x - 99)$ we get
$(x - 19)F(x) = (x - 99)(x - 19)a + 19(x - 19)$ and
$(x - 99)F(x) = (x - 99)(x - 19)b + 99(x - 99)$

Subtracting, we obtain:

$80F(x) = (x - 99)(x - 19)k - 80x + 9440 \Rightarrow F(x) = \frac{(x-19)(x-99)k}{80} - x + 118$. We know from (1) and (2) that F(x) does not have fractional coefficients. Therefore, k will be divisible by 80. Therefore, the remainder when F(x) is divided by $(x - 99)(x - 19)$ is $118 - x$.

Euler's Theorem

If M and N are two numbers coprime to each other, i.e. $HCF(M,N) = 1$ and $N = a^p b^q c^r ...$, $\text{Remainder}[\frac{M^{\phi(N)}}{N}] = 1$, where $\phi(N) = N(1 - \frac{1}{a})(1 - \frac{1}{b})(1 - \frac{1}{c})...$ and is known as Euler's Totient function.. $\phi(N)$ is also the number of numbers less than and prime to N.

Find the remainder when 5^{37} is divided by 63.

Answer: 5 and 63 are coprime to each other, therefore we can apply Euler's theorem here.
$63 = 32 \times 7 \Rightarrow \phi(63) = 63(1 - \frac{1}{3})(1 - \frac{1}{7}) = 36$

Therefore, $\text{Remainder}[\frac{5^{37}}{63}] = \text{Remainder}[\frac{5^{36} \times 5}{63}] = 5$

Find the last three digits of 57^{802}.

Answer: Many a times (not always), the quicker way to calculate the last three digits is to calculate the remainder by 1 000. We can see that 57 and 1 000 are coprime to each other. Therefore, we can use Euler's theorem here if it's useful.

$1\,000 = 2^3 \times 5^3 \Rightarrow \phi(1000) = 1000(1-\frac{1}{2})(1-\frac{1}{4}) = 400$

Therefore,

$$\text{Remainder}[\frac{57^{400}}{1000}] = 1 \Rightarrow \text{Remainder}[\frac{57^{400} \times 57^{400}}{1000}] = \text{Remainder}[\frac{57^{800}}{1000}] = 1$$

$$\Rightarrow \text{Remainder}[\frac{57^{802}}{1000}] = \text{Remainder}[\frac{57^{800} \times 57^{2}}{1000}] = 249$$

Hence, the last two digits of 57^{802} are 249.

Fermat's Theorem

> If p is a prime number and N is prime to p, then $N^p - N$ is divisible by p.

What is the remainder when $n^7 - n$ is divided by 42?

Answer: Since 7 is prime, $n^7 - n$ is divisible by 7. $n^7 - n = n(n^6 - 1) = n(n + 1)(n - 1)(n^4 + n^2 + 1)$. Now $(n - 1)(n)(n + 1)$ is divisible by 3! = 6. Hence $n^7 - n$ is divisible by 6 x 7 = 42. Hence the remainder is 0.

Fermat's Little Theorem

> If N in the above Euler's theorem is a prime number, then $\phi(N) = N(1-\frac{1}{N}) = N - 1$. Therefore, if M and N are coprime to each other and N is a prime number, $\text{Remainder}[\frac{M^{N-1}}{N}] = 1$

Find the remainder when 52^{60} is divided by 31.

Answer: 31 is a prime number therefore $\phi(N) = 30$. 52 and 31 are prime to each other. Therefore, by Fermat's theorem:

$$\text{Remainder}[\frac{52^{30}}{31}] = 1 \Rightarrow \text{Remainder}[\frac{52^{60}}{31}] = 1$$

Wilson's Theorem

> If P is a prime number then $\text{Remainder}[\frac{(P-1)!+1}{P}] = 0$. In other words, (P – 1)! + 1 is divisible by P if P is a prime number. It also means that the remainder when (P – 1)! Is divided by P is P – 1 when P is prime.

Find the remainder when 40! is divided by 41.

Answer: By Wilson's theorem, we can see that 40! + 1 is divisible by 41 $\Rightarrow \text{Remainder}[\frac{40!}{41}] = 41 - 1 = 40$

Find the remainder when 39! is divided by 41.

Answer: In the above example, we saw that the remainder when 40! is divided by 41 is 40.

$\Rightarrow$ 40! = 41k + 40 $\Rightarrow$ 40 × 39! = 41k + 40. The R.H.S. gives remainder 40 with 41 therefore L.H.S. should also give remainder 40 with 41. L.H.S. = 40 × 39! where 40 gives remainder 40 with 41. Therefore, 39! should give remainder 1 with 41.

Chinese Remainder Theorem

This is a very useful result. It might take a little time to understand and master Chinese remainder theorem completely but once understood, it is an asset.

If a number N = a × b, where a and b are prime to each other, i.e., hcf(a, b) = 1, and M is a number such that $\text{Remainder}[\frac{M}{a}] = r_1$ and $\text{Remainder}[\frac{M}{b}] = r_2$ then $\text{Remainder}[\frac{M}{N}] = ar_2x + br_1y$, where $ax + by = 1$

Confused?

Following example will make it clear.

Find the remainder when 3^{101} is divided by 77.

Answer: 77 = 11 × 7.

By Fermat's little theorem, $\text{Remainder}[\frac{3^6}{7}] = 1$ AND $\text{Remainder}[\frac{3^{10}}{11}] = 1$

$$\text{Remainder}[\frac{3^{101}}{7}] = \text{Remainder}[\frac{3^{96} \times 3^5}{7}] = \text{Remainder}[\frac{(3^6)^{16} \times 3^5}{7}] = \text{Remainder}[\frac{1 \times 3^5}{7}] = 5 = r_1$$

$$\text{Remainder}[\frac{3^{101}}{11}] = \text{Remainder}[\frac{3^{100} \times 3}{11}] = \text{Remainder}[\frac{(3^{10})^{10} \times 3}{11}] = \text{Remainder}[\frac{1 \times 3}{11}] = 3 = r_2$$

Now we will find x and y such that 7x + 11y = 1. By observation we can find out, x = –3 and y = 2.

Now we can say that $\text{Remainder}[\frac{3^{101}}{77}] = 7 \times 3 \times -3 + 11 \times 5 \times 2 = 47$

We can also solve this problem by Euler's theorem and this is the method I follow most of the time. No confusion remains thereby.

Find the remainder when 3^{101} is divided by 77.

Answer: $\phi(77) = 77(1 - \frac{1}{7})(1 - \frac{1}{11}) = 60$

$$\text{Remainder}[\frac{3^{60}}{77}] = 1 \Rightarrow \text{Remainder}[\frac{3^{101}}{77}] = \text{Remainder}[\frac{3^{60} \times 3^{41}}{77}] = \text{Remainder}[\frac{1 \times 3^{41}}{77}] = \text{Remainder}[\frac{3^{41}}{77}]$$

$$\text{Remainder}[\frac{3^4}{77}] = \text{Remainder}[\frac{81}{77}] = 4$$

$$\Rightarrow \text{Remainder}[\frac{3^{41}}{77}] = \text{Remainder}[\frac{(3^4)^{10} \times 3}{77}]\text{Remainder}[\frac{4^{10} \times 3}{77}] = \text{Remainder}[\frac{4^4 \times 4^4 \times 4^2 \times 3}{77}] = \text{Remainder}[\frac{256 \times 256 \times 48}{77}]$$

$$= \text{Remainder}[\frac{25 \times 25 \times 48}{77}] = \text{Remainder}[\frac{9 \times 48}{77}] = 47$$

Find the smallest number that when divided by 7, 8 and 9 leave a remainder of 5, 4 and 2.

Answer: Let N be the number. Therefore, N = 7a + 5, N = 8b + 4, N = 9c + 2.

To solve these equations, learn an important rule first

In the equation **ax + by = c**, the integer values of x lie at a common difference of b and the integer values of y lie at a common difference of b.

Now N = 7a + 5 and N = 8b + 4 $\Rightarrow$ 8b – 7a = 1. One pair of values of a and b that satisfy this equation is (9, 8). Now, the values of a and b will lie at a common difference of 8 and 7, respectively. Therefore, the other solution sets are: (17, 15), (25, 22) etc. The values of b can be expressed in the form
b = 8 + 7k, where k = 0, 1, 2, 3…

Now N = 8b + 4 and N = 9c + 2 $\Rightarrow$ 9c – 8b = 2. One pair of values of b and c that satisfy this equation is (2, 2). Now, the values of b and c will lie at a common difference of 9 and 8, respectively. Therefore, the other solution sets are: (11, 10), (20, 18) etc. The values of b can be expressed in the form
b = 2 + 9m, where m = 0, 1, 2, 3…

Now b = 8 + 7k and b = 2 + 9m $\Rightarrow$ 9m – 7k = 6. One pair of value is (3, 3).
Therefore, b = 2 + 9m = 2 + 9 $\times$ 3 = 29
$\Rightarrow$ N = 8b + 4 = 234.

Find the remainder when $32^{32^{32}}$ is divided by 9.

Answer: Notice that 32 and 9 are coprime. $\phi(9) = 9(1 - \frac{1}{3}) = 6$

Hence by Euler's theorem, $\text{Remainder}[\frac{32^6}{9}] = 1$. Since the power is 32^{32}, we will have to simplify this power in terms of 6k + r. Therefore, we need to find the remainder when 32^{32} is divided by 6.

$$\text{Remainder}[\frac{32^{32}}{6}] = \text{Remainder}[\frac{2^{32}}{6}] = \text{Remainder}[\frac{(2^8)^4}{6}] = \text{Remainder}[\frac{256 \times 256 \times 256 \times 256}{6}] = \text{Remainder}[\frac{256}{6}] = 4$$

Therefore, $32^{32^{32}} = 32^{6k+4} = (32^6)^k \times 32^4$

$$\Rightarrow \text{Remainder}[\frac{(32^6)^k \times 32^4}{9}] = \text{Remainder}[\frac{32^4}{9}] = \text{Remainder}[\frac{5 \times 5 \times 5 \times 5}{9}] = \text{Remainder}[\frac{625}{9}] = 4$$

What will be the remainder when N = $10^{10} + 10^{100} + 10^{1000} + ... + 10^{10000000000}$ is divided by 7?

Answer: By Fermat's Little Theorem 10^6 will give remainder as 1 with 7.

$$\text{Remainder}[\frac{10^{10}}{7}] = \text{Remainder}[\frac{10^6 \times 10^4}{7}] = \text{Remainder}[\frac{10^4}{7}] = \text{Remainder}[\frac{3^4}{7}] = 4$$

Similarly, all the other terms give remainder of 4 with 7. Therefore, total remainder = 4 + 4 + 4… (10 times) = 40.

Remainder of 40 with 7 = 5

What is the remainder when N = $2222^{5555} + 5555^{2222}$ is divided by 7?

2222^6 will give remainder 1 when divided by 7.

$5555 = 6K+5 \Rightarrow 2222^{5555} = 2222^{6k+5} \Rightarrow \text{Remainder}[\frac{2222^{5555}}{7}] = \text{Remainder}[\frac{2222^5}{7}] = \text{Remainder}[\frac{3^5}{7}] = 5$

Also 55556 will give remainder 1 when divided by 7.

$5555^{2222} = 5555^{6k+2} \Rightarrow \text{Remainder}[\frac{5555^{2222}}{7}] = \text{Remainder}[\frac{5555^2}{7}] = \text{Remainder}[\frac{4^2}{7}] = 2$

So final remainder is (5 + 2) divided by 7 = 0

Find the remainder when 8^{643} is divided by 132.

Answer: Note that here 8 and 132 are not coprime as HCF (8, 132) = 4 and not 1. Therefore, we cannot apply Euler's theorem directly.

$\text{Remainder}[\frac{8^{643}}{132}] = \text{Remainder}[\frac{2^{1929}}{132}] = 4 \times \text{Remainder}[\frac{2^{1927}}{33}]$. Now we can apply Euler's theorem.

$$\phi(33) = 33(1-\frac{1}{3})(1-\frac{1}{11}) = 20 \Rightarrow \text{Remainder}[\frac{2^{20}}{33}] = 1 \Rightarrow \text{Remainder}[\frac{2^{1927}}{33}] = \text{Remainder}[\frac{2^7}{33}] = 29$$

$$\Rightarrow \text{Real remainder} = 4 \times 29 = 116$$

To find the number of numbers that are less than or equal to a certain natural number n, and that are divisible by a certain integer

To find the number of numbers, less than or equal to n, and that are divisible by a certain integer p, we divide n by p. The quotient of the division gives us the number of numbers divisible by p and less than or equal to n.

How many numbers less than 400 are divisible by 12?

Answer: Dividing 400 by 12, we get the quotient as 33. Hence the number of numbers that are below 400 and divisible by 12 is 33.

How many numbers between 1 and 400, both included, are not divisible either by 3 or 5?

Answer: We first find the numbers that are divisible by 3 or 5. Dividing 400 by 3 and 5, we get the quotients as 133 and 80 respectively. Among these numbers divisible by 3 and 5, there are also numbers which are divisible both by 3 and 5 i.e. divisible by 3 x 5 = 15. We have counted these numbers twice. Dividing 400 by 15, we get the quotient as 26.

Hence the number divisible by 3 or 5 = 133 + 80 – 26 = 187
Hence, the numbers not divisible by 3 or 5 are = 400 – 187 = 213.

How many numbers between 1 and 1200, both included, are not divisible by any of the numbers 2, 3 and 5?

Answer: as in the previous example, we first find the number of numbers divisible by 2, 3, or 5. from set theory we have

$n(AUBUC) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$n(2U3U5) = n(2) + n(3) + n(5) - n(6) - n(15) - n(10) + n(30)$
$\rightarrow n(2U3U5) = 600 + 400 + 240 - 200 - 80 - 120 + 40 = 880$
Hence number of numbers not divisible by any of the numbers 2, 3, and 5 = 1200 – 880 = 320.

Some Special Problems:

Find the remainder when 123456789101112…….40 is divided by 36.

Answer: $36 = 9 \times 4$. Therefore, we first find the remainders when this number is divided by 9 and 4. The remainder by 9 would be the remainder when the sum of digits is divided by 9. Sum of digits $= 4 \times (1 + 2 + 3 + 4 + \ldots + 9) + 10 \times (1 + 2 + 3) + 4 = 180 + 60 + 4 = 244 \Rightarrow$ remainder by 9 = 1.
The remainder by 4 would be the remainder when the last two digits are divided by 4 $\Rightarrow$ remainder by 4 = 0.

Therefore, to find the remainder we need to find the smallest multiple of 4 that gives remainder 1 with 9. The smallest such number = 28. Therefore, remainder = 28.

Find the remainder when 112123123412345…12345678 is divided by 36.

Answer: $36 = 9 \times 4$. Therefore, we first find the remainders when this number is divided by 9 and 4. The remainder by 9 would be the remainder when the sum of digits is divided by 9. Sum of digits $= 1 \times 8 + 2 \times 7 + 3 \times 6 + \ldots + 8 \times 1 = 120 \Rightarrow$ remainder by 9 = 3.
The remainder by 4 would be the remainder when the last two digits are divided by 4 $\Rightarrow$ remainder by 4 = 2.

The overall remainder would be the smallest number that gives remainder 3 with 9 and remainder 2 with 4. Therefore, the number would satisfy the equation $9a + 3 = 4b + 2 \Rightarrow 4b - 9a = 1 \Rightarrow (a, b) = (3, 7)$ and the number = 30. Therefore, remainder = 30.

Let $n! = 1 \times 2 \times 3 \times \ldots \times n$ for integer $n \geq 1$. If $p = 1! + (2 \times 2!) + (3 \times 3!) + \ldots + (10 \times 10!)$, then $p+2$ when divided by 11! leaves a remainder of **(CAT 2005)**
1. 10 2. 0 3. 7 4. 1

Answer: Nth term of the series $= n \times n! = (n + 1 - 1) \times n! = (n + 1)! - n!$

Therefore, $p = 2! - 1! + 3! - 2! + 4! - 3! + \ldots + 11! - 10! = 11! - 1! \Rightarrow p + 2 = 11! + 1 \Rightarrow$ remainder by $11! = 1$

Find the remainder when $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + 98 \times 99 + 99 \times 100$ is divided by 101.

Answer: Nth term of the series $= n \times (n + 1) = n^2 + n$.

Therefore, sum of the series $\sum(n^2 + n) = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6} = \frac{99 \times 100 \times 101}{6} \Rightarrow$ remainder by 101 $= 0$.

A number when divided by 8 leaves remainder 3 and quotient Q. The number when divided by 5 leaves remainder 2 and quotient Q + 8. What is the number?

Answer: Let the number be $N \Rightarrow N = 8Q + 3$ and $N = 5(Q + 8) + 2 = 5Q + 42$
$8Q + 3 = 5Q + 42 \Rightarrow Q = 13 \Rightarrow N = 107$

Find the largest natural number that divides 364, 414, and 539 and leaves the same remainder in each case.

Answer: Let the divisor be D and the remainder be R. Therefore, $364 = Da + R$, $414 = Db + R$, $539 = Dc + R$
Subtracting first equation from the second and the second equation from the third we get

$50 = D(b - a)$ and $125 = D(c - b)$. As D is the common factor in RHS of both the equation, it should be the common factor on the LHS of both the equation. The HCF of 50 and 125 is 25. Therefore, the highest number can be 25.

What is the remainder when $\underbrace{11111\ldots11111}_{243 \text{ times}}$ is divided by 243?

Answer: It can be proved that a number formed by writing any single digit 3^n times will be divisible by 3^n. This is left to students to check it out.

How many numbers between 1 and 1000 are there such that $n^2 + 3n + 5$ is divisible by 121?

Answer: 0 values. $n^2 + 3n + 5 = (n - 4)(n + 7) + 33$. Now, 33 is divisible by 11 but not 121. n + 7 and n - 4 are two numbers with a difference of 11, therefore either both are divisible by 11 or both are not divisible by 11. If both are divisible by 11, their product is divisible by 121 but 33 is divisible only by 11 therefore the expression is not divisible by 121. If both are not divisible by 11, the expression is again not divisible by 121.

Find the remainder when $1^{39} + 2^{39} + 3^{39} + 4^{39} + \ldots + 12^{39}$ is divided by 39.

Answer: $1^p + 2^p + 3^p + \ldots n^p$ is divisible by $1 + 2 + 3 + \ldots + n$ if p is odd. Therefore, remainder = 0 as $1 + 2 + 3 + \ldots + 12 = 78$ which is a factor of 13.

DIVISORS OF A NUMBER

Divisors:
For a natural number N, all the numbers, including 1 and N itself, which divide N completely are called divisors of N.

Example: The number 24 is divisible by 1, 2, 3, 4, 6, 8, 12, and 24. Hence all these numbers are divisors of 24.

How to find the number of divisors of a number:

Let us the find the number of divisors of 60.

$60 = 2^2 \times 3 \times 5$.
Any divisors of 60 will have powers of 2 equal to either 2^0 or 2^1 or 2^2.
Similarly, any divisor of 60 will have powers of 3 equal to either 3^0 or 3^1, and powers of 5 equal to either 5^0 or 5^1.

To make a divisor of 60, we will have to choose a power of 2, a power of 3 and a power of 5. A power of 2 can be chosen in 3 ways out of 2^0 or 2^1, or 2^2. Similarly, a power of 3 can be chosen in 2 ways and a power of 5 can be chosen in 2 ways.

Therefore, the number of divisors = $3 \times 2 \times 2 = 12$.

Notice that we have added 1 each to powers of 2, 3 and 5 and multiplied.

Now for the formula:

Let N be a composite number such that $N = (x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors. Then, the number of divisors of $N = (a + 1)(b + 1)(c + 1)$..

Find the number of divisors of 21600.
Answer: $21600 = 2^5 \times 3^3 \times 5^2 \Rightarrow$ Number of divisors = $(5 + 1) \times (3 + 1) \times (2 + 1) = 6 \times 4 \times 3 = 72$.

How many divisors of 21600 are odd numbers?
Answer: An odd number does not have a factor of 2 in it. Therefore, we will consider all the divisors having powers of 3 and 5 but not 2. Therefore, ignoring the powers of 2, the number of odd divisors = $(3 + 1) \times (2 + 1) = 4 \times 3 = 12$.

How many divisors of 21600 are even numbers?
Answer: Total number of divisors of 21600 = 72.
Number of odd divisors of 21600 = 12.
$\Rightarrow$ Number of even divisors of 21600 = $72 - 12 = 60$.

How many divisors of 360 are not divisors of 540 and how many divisors of 540 are not divisors of 360?
Answer: The best option here is to find the number of common divisors of 360 and 540. For that we find the highest common powers of all the common prime factors in 360 and 540.
Now, $360 = 2^3 \times 3^2 \times 5$ and $540 = 2^2 \times 3^3 \times 5$.

The number of common factors would be made by $2^2 \times 3^2 \times 5$. The number of factors made by this = $3 \times 3 \times 2 = 18$. Therefore, the two numbers will have 18 factors in common.

Number of factors of 360 = $4 \times 3 \times 2 = 24 \Rightarrow$ Number of factors of 360 which are not factors of 540 = $24 - 18 = 6$.
Number of factors of 540 = $3 \times 4 \times 2 = 24 \Rightarrow$ Number of factors of 540 which are not factors of 360 = $24 - 18 = 6$.

How many divisors of the number $2^7 \times 3^5 \times 5^4$ have unit digit equal to 5?
Answer: For unit digit equal to 5, the number has to be a multiple of 5 and it should not be a multiple of 2 otherwise the unit digit will be 0. To be a multiple of 5, the powers of 5 that it can have is 5^1, 5^2, 5^3 or 5^4. The powers of 3 can be 3^0, 3^1, 3^2, 3^3, 3^4 or 3^5.

Therefore, the number of divisors which have a unit digit of 5 = 4 × 6 = 24.

How many divisors of 36^{36} are perfect cubes?

Answer: $36^{36} = 2^{72}3^{72}$. To find the divisors which are perfect cubes, we need to take those powers of prime factors which are multiples of 3. Therefore, powers of 2 will be $2^0, 2^3, 2^6, 2^9, \dots 2^{72}$ and similarly, powers of 3 will be $3^0, 3^3, 3^6, 3^9, \dots 3^{72}$. Both are 25 in number. Therefore, number of divisors = 25 × 25 = 625.

Reverse Operations on Divisors:

Find all the numbers less than 100 which have exactly 8 divisors.

Answer: To find the number of divisors of a number, we used to add 1 to powers of all the prime factors and then multiply them together. Now, given the number of divisors, we will express this number as a product and then subtract 1 from every multiplicand to obtain the powers.

8 = 2 × 2 × 2 = (1 + 1) × (1 + 1) × (1 + 1). Therefore, the number is of the form $a^1b^1c^1$, where a, b and c are prime. The numbers can be 2 × 3 × 5 = **30**, 2 × 3 × 7 = **42**, 2 × 3 × 11 = **66**, 2 × 3 × 13 = **78**, 2 × 5 × 7 = **70**.

8 = 4 × 2 = (3 + 1) × (1 + 1). Therefore, the number is of the form a^3b, where a and b are prime. The numbers can be $2^3 \times 3$ = **24**, $2^3 \times 5$ = **40**, $2^3 \times 7$ = **56**, $2^3 \times 11$ = **88**, $3^3 \times 2$ = **54**.

The number can also be of the form a^7, but there is no such number less than 100.

Find the smallest number with 15 divisors.

Answer: 15 = 3 × 5 = (2 + 1)(4 + 1) ⇒ The number is of the form a^2b^4, where a and b are prime. To find the smallest such number, we give the highest power to smallest rime factor, i.e. 2, and the next highest power to next smallest prime number, i.e. 3, and so on. Therefore, the smallest number = $2^4 \times 3^2$ = 144.

If N is Natural number, N has 4 factors, and summation of factors excluding N is 31, how many values for N are possible?
Answer:

Let N be a composite number such that $N = (2)^a(y)^b(z)^c$.. where y, z.. are prime factors. Then, the number of even divisors of N = (a)(b + 1)(c + 1) and number of odd divisors of N = (b + 1)(c + 1)

How many divisors of 21600 are perfect squares?
Answer: In a perfect square, all the prime factors have even powers. For example, $2^5 \times 6^8$ will not be a perfect square as the power of 2 is odd whereas $2^4 \times 6^8$ will be a perfect square because all the prime factors have even powers. 21600 = $2^5 \times 3^3 \times 5^2$ therefore, all the divisors made by even powers of 2, 3 and 5 will be perfect squares.
The even powers of 2 are $2^0, 2^2, 2^4$, even powers of 3 are 3^0 and 3^2, and even powers of 5 are 5^0 and 5^2. We can select an even power of 2 in 3 ways, even power of 3 in 2 ways, and even power of 5 in 2 ways. Therefore, the number of combinations = 3 × 2 × 2 = 12.

Let N be a composite number such that $N = (x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors. Then, the sum of divisors of N = $\frac{x^{a+1}-1}{x-1} \times \frac{y^{b+1}-1}{y-1} \times \frac{z^{c+1}-1}{z-1} \dots$

What is the sum of divisors of 60?

Answer: 60 = $2^2 \times 3 \times 5 \Rightarrow$ Sum of the divisors = $\frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1} = 168$

Find the sum of even divisors of $2^5 \times 3^5 \times 5^4$

Answer: All the even divisors of the number will have powers of 2 equal to one of 2, 2^2, 2^3, 2^4, or 2^5. Therefore, sum of even divisors = $(2 + 2^2 + 2^3 + 2^4 + 2^5) \times (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5) \times (1 + 5 + 5^2 + 5^3 + 5^4)$

$= \frac{2(2^5-1)}{2-1} \times \frac{3^6-1}{3-1} \times \frac{5^5-1}{5-1} = 17625608$

A positive integer is *bold* if it has 8 positive divisors that sum up to 3240. For example, 2006 is bold because its 8 positive divisors, 1, 2, 17, 34, 59, 118, 1003 and 2006, sum up to 3240. Find the smallest positive bold number.

Answer: If a number has 8 divisors it can be of the forms a^7, ab^3, or abc, where a, b and c are prime numbers. The sum of the divisors in each case is given below:

Type of number	$\mathbf{a^7}$	$\mathbf{ab^3}$	**abc**
Sum = 3240	$\frac{a^8-1}{a-1}$	$\frac{a^2-1}{a-1} \times \frac{b^4-1}{b-1} = (a+1)(b^3+b^2+1)$	$\frac{a^2-1}{a-1} \times \frac{b^2-1}{b-1} \times \frac{c^2-1}{c-1} = (a+1)(b+1)(c+1)$
Examples	None	None	1614 = (2 + 1)(3 + 1)(269 + 1), 1790 = (2 + 1)(5 + 1)(179 + 1), 1958 = (2 + 1)(11 + 1)(89 + 1)

Therefore, 1614 is the smallest **bold** number.

Let N be a composite number such that N = $(x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors. Then, the product of divisors of N = $(N)^{\frac{(a+1)(b+1)(c+1)}{2}} = \left(x^a y^b z^c\right)^{\frac{(a+1)(b+1)(c+1)}{2}}$

What is the product of divisors of 60?

Answer: 60 = $2^2 \times 3 \times 5 \Rightarrow$ product of divisors of 60 = $(60)^{\frac{3\times2\times2}{2}} = 60^6 = 2^{12} \times 3^6 \times 5^6$

Let A = set of all divisors of 8100 and B = set of all divisors of 21600. What is the product of the elements of AUB?

Answer: 8100 = $2^2 \times 3^4 \times 5^2$ and 21600 = $2^5 \times 3^3 \times 5^2$. AUB will have all the divisors of 8100 and 21600 with the common divisors written only once. Therefore, these common divisors will be multiplied only once. The common divisors will come from $2^2 \times 3^3 \times 5^2$ and are 36 in number. Their product will be $(2^2 \times 3^3 \times 5^2)^{18} = 2^{36} \times 3^{54} \times 5^{36}$

Required product=

$$\frac{\text{product of divisors of 8100} \times \text{product of divisors of 21600}}{\text{product of common divisors}} = \frac{(2^2 \times 3^4 \times 5^2)^{\frac{45}{2}} \times (2^5 \times 3^3 \times 5^2)^{36}}{2^{36} \times 3^{54} \times 5^{36}} = 2^{189} \times 3^{144} \times 5^{81}$$

Let N be a composite number such that N = $(x)^a(y)^b(z)^c$.. where x, y, z.. are prime factors.

If N is not a perfect square, then, the number of ways N can be written as a product of two numbers $= \frac{(a+1)(b+1)(c+1)}{2} = \frac{\text{Number of divisors}}{2}$

If N is a perfect square, then, the number of ways N can be written as a product of two numbers $= \frac{(a+1)(b+1)(c+1)+1}{2} = \frac{\text{Number of divisors}+1}{2}$

REMEMBER! A perfect square has odd number of factors. In other words, any number which has odd number of factors is a perfect square.

For example, the divisors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60. Now, $60 = 1 \times 60 = 2 \times 30 = 3 \times 20 = 4 \times 15 = 5 \times 12 = 6 \times 10$. Therefore, **divisors occur in pairs for numbers which are not perfect squares**.

The divisors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.
$36 = 1 \times 36 = 2 \times 18 = 3 \times 12 = 4 \times 9 = 6 \times 6$. Therefore, **divisors occur in pairs except for the square root for numbers which are perfect squares.**

N is a composite number with an even number of factors. Consider the following statement

I: N has a factor lying between 1 and $\sqrt{N}$

II: N has a factor lying between $\sqrt{N}$ and N

Which of the following options is true?

A. Both I and II are true
B. I is true but II is false
C. I is false but II is true
D. Both I and II are false

Answer: Since N is a composite number, it has more than two factors. Since N has even number of factors, it is NOT a perfect square and therefore it has at least one factor lying between 1 and $\sqrt{N}$ and one factor lying between $\sqrt{N}$ and N. Therefore, option [A].

How many ordered pairs of integers, (x, y) satisfy the equation xy = 110?

Answer: $110 = 2 \times 5 \times 11$. Hence, the number of divisors of 110 is $= 2 \times 2 \times 2 = 8$. Hence, the number of positive ordered pairs of x and y = 8 (as (2, 55) is not same as (55, 2)). Also, since we are asked for integers, the pair consisting of two negative integers will also suffice. Hence the total number of ordered pairs $= 2 \times 8 = 16$.

The number of ways in which a composite number can be resolved into two factors which are prime to each other $= 2^{n-1}$, where n is the number of different prime factors of the number.

For example, let the number $N = 2^{10} \times 3^7 \times 5^6 \times 7^4$. We have to assign these prime factors and their powers to one of the two factors. As the two factors will be prime to each other, we will have to assign a prime factor with its power (for example 210) completely to one of the factors. For every prime factor, we have two ways of assigning it. Therefore, the total number of ways $= 2 \times 2 \times 2 \times 2 = 16$. As we are not looking for ordered pairs, the required number of ways $= \frac{16}{2} = 8$.

Number of numbers less than or prime to a given number:

If N is a natural number such that $N = a^p \times b^q \times c^r$, where a, b, c are different prime factors and p, q, r are positive integers, then the number of positive integers less than and prime to N = $N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$. Therefore, $N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$ numbers have **no** factor in common with N.

The above formula is extremely versatile as it lets us find not only the numbers which do not contain any of the prime factors of N but also the numbers which do not contain some selected prime factors of N. The following examples will make it clear:

How many of the first 1200 natural numbers are not divisible by any of 2, 3 and 5?
Answer: 1200 is a multiple of 2, 3 and 5. Therefore, we need to find the number of numbers which are less than and prime to 1200. Number of numbers = $1200\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)=320$

How many of the first 1200 natural numbers are not divisible by any of 2 and 5?
Answer: Unlike the previous problem, this problem only asks for number not divisible by only 2 factors of 1200, i.e. 2 and 5. Therefore, in the formula we remove the part containing the factor of 3 and calculate the numbers of numbers prime to 1200 with respect to prime factors 2 and 5. The required number = $1200\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)=480$

How many of the first 1200 natural numbers are either prime to 6 or to 15?
Answer: Number of numbers prime to 6 are $1200\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)=400$ and numbers prime to 15 are $1200\left(1-\frac{1}{5}\right)\left(1-\frac{1}{3}\right)=640$ Out of these numbers, we will have to subtract numbers which are prime to both 6 and 15 (the question asks for either..or). These are 320 in numbers in all (we have already calculated it).
Therefore, the required number = 400 + 640 – 2 × 320 = 400

The number of divisors of every natural number from 1 to 1000 is calculated. Which natural number has the highest number of divisors?

Answer: The number less than 1000 which can incorporate highest number of prime factor is = 2 × 3 × 5 × 7 = 210. Now we are looking for highest multiple of 210 that is less than 1000. The multiple is 210 × 4 = 840 which has 32 divisors.

UNIT'S DIGIT

To find the units digit of x^y we only consider the units digits of the number x.
To calculate units digit of 237^{234} we only consider the units digit of 237. Hence, we find the units digit of 7^{234}.
To find the units digit of a × b, we only consider the units digits of the numbers a and b.
To calculate units digit of 233 × 254, we only consider the units digit of 233 and 254 i.e. 3 and 4, respectively. Hence, we find the units digit of 3 x 4, respectively.

To calculate units digit of x^y where x is a single digit number

To calculate units digit of numbers in the form x^y such 7^{253}, 8^{93}, 3^{74} etc.

Case 1: When y is NOT a multiple of 4
We find the remainder when y is divided by 4. Let y = 4q + r where r is the remainder when y is divided by 4, and
0 < r < 4. **The units digit of x^y is the units digit of x^r.**

Case 2: When y is a multiple of 4
We observe the following conditions:
Even numbers 2, 4, 6, 8 when raised to powers which are multiple of 4 give the units digit as 6.
Odd numbers 3, 7, and 9 when raised to powers which are multiple of 4 give the units digit as 1.

Find the units digit of 7^{33}.

Answer: The remainder when 33 is divided by 4 is 1. Hence the units digit of 7^{33} is the unit digit of 7^1 = 7

Find the units digit of 43^{47}.

Answer: The units digit of 43^{47} can be found by finding the units digit of 3^{47}. 47 gives a remainder of 3 when divided by 4. Hence units digit = units digit of 3^3 = 7

Find the units digit of $28^{28} - 24^{24}$.

Answer: We have to find the units digit of $8^{28} - 4^{24}$. Since 28 and 24 are both multiples of 4, the units digits of both 8^{28} and 4^{24} will be 6. Hence the units digit of the difference will be 0.

Find the units digit of $43^{43} - 22^{22}$.

Answer: Units digit of 43^{43} is 7 and units digit of 22^{22} is 4. Hence the units digit of the expression will be 7 – 4 = 3.

Find the units digit of 3^{3^3}

Answer: Again, we find the remainder when the power is divided by 4. Therefore, we find the remainder when 3^3 is divided by 4. Now, 3^3 = 27, remainder by 4 = 3.

Therefore, units digit of 3^{3^3} = units digit of 3^3 = 7.

Find the units digit of $7^{11^{13^{17}}}$.

Answer: Again, we find the remainder when the power is divided by 4. Therefore, we find the remainder when $11^{13^{17}}$ is divided by 4. Now 11 = 12 – 1 $\Rightarrow$ Remainder $[11^{\text{Odd}}]$ = Remainder$[(-1)^{\text{Odd}}] = -1 = 3$.

Therefore, units digit of $7^{11^{13^{17}}}$ = units digit of 7^3 = 3.

Find the units digit of $1^3 + 2^3 + 3^3 + \ldots + 98^3 + 99^3$

Answer: Unit digit of $1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, 8^3, 9^3$ are 1, 8, 7, 4, 5, 6, 3, 2, and 9, respectively. The sum of these units digits gives a unit digit of 5. Now these units digit will repeat 10 times each. Therefore, units digit of the sum = 5 $\times$ 10 = 0.

LAST TWO DIGITS

Before we start, let me mention binomial theorem in brief as we will need it for our calculations.

$$\boxed{(x + a)^n = {}^nC_0a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + \ldots \text{ where } {}^nC_r = \frac{n!}{r!(n-r)!}}$$

Last two digits of numbers ending in 1

Let's start with an example.

What are the last two digits of 31^{786}?

Solution: $31^{786} = (30 + 1)^{786} = {}^{786}C_0 \times 1^{786} + {}^{786}C_1 \times 1^{785} \times (30) + {}^{786}C_2 \times 1^{784} \times 30^2 + \ldots$, Note that all the terms after the second term will end in two or more zeroes. The first two terms are ${}^{786}C_0 \times 1^{786}$ and ${}^{786}C_1 \times 1^{785} \times (30)$. Now, the second term will end with one zero and the tens digit of the second term will be the product of 786 and 3 i.e. 8. Therefore, the last two digits of the second term will be 80. The last digit of the first term is 1. So the last two digits of 31^{786} are 81.

Now, here is the shortcut:

Multiply the tens digit of the number (3 here) with the last digit of the exponent (6 here) to get the tens digit. The units digit is equal to one.

Here are some more examples:

Find the last two digits of 41^{2789}
In no time at all you can calculate the answer to be 61 ($4 \times 9 = 36$. Therefore, 6 will be the tens digit and one will be the units digit)

Find the last two digits of 71^{56747}
Last two digits will be 91 (7×7 gives 9 and 1 as units digit)

Now try to get the answer to this question within 10 s:

Find the last two digits of $51^{456} \times 61^{567}$
The last two digits of 51^{456} will be 01 and the last two digits of 61^{567} will be 21. Therefore, the last two digits of $51^{456} \times 61^{567}$ will be the last two digits of $01 \times 21 = 21$

Last two digits of numbers ending in 3, 7 or 9

Find the last two digits of 19^{266}.

$19^{266} = (19^2)^{133}$. Now, 19^2 ends in 61 ($19^2 = 361$) therefore, we need to find the last two digits of $(61)^{133}$.
Once the number is ending in 1 we can straight away get the last two digits with the help of the previous method. The last two digits are 81 ($6 \times 3 = 18$, so the tens digit will be 8 and last digit will be 1)

Find the last two digits of 33^{288}.

$33^{288} = (33^4)^{72}$. Now 33^4 ends in 21 ($33^4 = 33^2 \times 33^2 = 1089 \times 1089 =$ xxxxx21) therefore, we need to find the last two digits of 21^{72}. By the previous method, the last two digits of $21^{72} = 41$ (tens digit $= 2 \times 2 = 4$, unit digit $= 1$)

So here's the rule for finding the last two digits of numbers ending in 3, 7 and 9:

Convert the number till the number gives 1 as the last digit and then find the last two digits according to the previous method.

Now try the method with a number ending in 7:

Find the last two digits of 87^{474}.

$87^{474} = 87^{472} \times 87^2 = (87^4)^{118} \times 87^2 = (69 \times 69)^{118} \times 69$ (The last two digits of 87^2 are 69) $= 61^{118} \times 69 = 81 \times 69 = 89$

If you understood the method then try your hands on these questions:

Find the last two digits of:

1. 27^{456}
2. 79^{83}
3. 583^{512}

Last two digits of numbers ending in 2, 4, 6 or 8

There is only one even two-digit number which always ends in itself (last two digits) - 76 i.e. 76 raised to any power gives the last two digits as 76. Therefore, our purpose is to get 76 as last two digits for even numbers. We know that 24^2 ends in 76 and 2^{10} ends in 24. Also, 24 raised to an even power always ends with 76 and 24 raised to an odd power always ends with 24. Therefore, 24^{34} will end in 76 and 24^{53} will end in 24.

Let's apply this funda:

Find the last two digits of 2^{543}.

$2^{543} = (2^{10})^{54} \times 2^3 = (24)^{54}$ (24 raised to an even power) $\times 2^3 = 76 \times 8 = 08$

(NOTE: Here if you need to multiply 76 with 2^n, then you can straightaway write the last two digits of 2^n because when 76 is multiplied with 2^n the last two digits remain the same as the last two digits of 2^n. Therefore, the last two digits of 76×2^7 will be the last two digits of $2^7 = 28$. Note that this funda works only for powers of $2 \geq 2$)

Same method we can use for any number which is of the form 2^n. Here is an example:

Find the last two digits of 64^{236}.
$64^{236} = (2^6)^{236} = 2^{1416} = (2^{10})^{141} \times 2^6 = 24^{141}$ (24 raised to odd power) $\times 64 = 24 \times 64 = 36$

Now those numbers which are not in the form of 2n can be broken down into the form $2n \times$ odd number. We can find the last two digits of both the parts separately.

Here are some examples:

Find the last two digits of 62^{586}.
$62^{586} = (2 \times 31)^{586} = 2^{586} \times 3^{586} = (2^{10})^{58} \times 2^6 \times 31^{586} = 76 \times 64 \times 81 = 84$

Find the last two digits of 54^{380}.
$54^{380} = (2 \times 3^3)^{380} = 2^{380} \times 3^{1140} = (2^{10})^{38} \times (3^4)^{285} = 76 \times 81^{285} = 76 \times 01 = 76$.

Find the last two digits of 56^{283}.
$56^{283} = (2^3 \times 7)^{283} = 2^{849} \times 7^{283} = (2^{10})^{84} \times 2^9 \times (7^4)^{70} \times 7^3 = 76 \times 12 \times (01)^{70} \times 43 = 16$

Find the last two digits of 78^{379}.
$78^{379} = (2 \times 39)^{379} = 2^{379} \times 39^{379} = (2^{10})^{37} \times 2^9 \times (39^2)^{189} \times 39 = 24 \times 12 \times 81 \times 39 = 92$

POWERS OF A NUMBER CONTAINED IN A FACTORIAL

Highest power of prime number p in n! = $\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\left[\frac{n}{p^4}\right]+\ldots$ where [x] denotes the greatest integer less than or equal to x.

Find the highest power of 2 in 50!

The highest power of 2 in 50! = $\left[\frac{50}{2}\right]+\left[\frac{50}{4}\right]+\left[\frac{50}{8}\right]+\left[\frac{50}{16}\right]+\left[\frac{50}{32}\right]$ = 25 + 12 + 6 + 3 + 1 = 47

Find the highest power of 30 in 50!

30 = 2 × 3 × 5. Now 5 is the largest prime factor of 30, therefore, the powers of 5 in 50! will be less than those of 2 and 3. Therefore, there cannot be more 30s than there are 5 in 50! So we find the highest power of 5 in 50! The highest power of 5 in 50! = $\left[\frac{50}{5}\right]+\left[\frac{50}{25}\right]$ = 10 + 2 = 12. Hence the highest power of 30 in 50! = 12

Find the number of zeroes present at the end of 100!

We get a zero at the end of a number when we multiply that number by 10. So, to calculate the number of zeroes at the end of 100!, we have to find the highest power of 10 present in the number. Since 10 = 2 × 5, we have to find the highest power of 5 in 100! The highest power of 5 in 100! = $\left[\frac{100}{5}\right]+\left[\frac{100}{25}\right]$ = 20 + 4 = 24
Therefore, the number of zeroes at the end of 100! = 24

Find the number of divisors of 15!

Answer: To find the number of divisors of 15!, we will have to first find out powers of every prime factor in 15!. The prime factors in 15! Are , 3, 5, 7, 11 and 13.

Powers of 2 in 15! = $\left[\frac{15}{2}\right]+\left[\frac{15}{4}\right]+\left[\frac{15}{8}\right]=7+3+1=11$

Powers of 3 in 15! = $\left[\frac{15}{3}\right]+\left[\frac{15}{9}\right]=5+1=6$

Powers of 5 in 15! = $\left[\frac{15}{5}\right]=3$

Powers of 7 in 15! = 2
Powers of 11 in 15! = 1
Powers of 13 in 15! = 1

Therefore, 15! = $2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13 \Rightarrow$ Number of divisors = 12 × 7 × 4 × 3 × 2 × 2 = 4032.

What is the rightmost non-zero digit in 15!?

Answer: We saw that 15! = $2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11 \times 13$. Now $2^3 \times 5^3$ will give 10^3 or 3 zeroes at the end. Removing $2^3 \times 5^3$, we will be left with $2^8 \times 3^6 \times 7^2 \times 11 \times 13$. Calculating units digit of each prime factor separately, the units digit of the product $2^8 \times 3^6 \times 7^2 \times 11 \times 13$ = units digit of 6 × 9 × 9 × 1 × 3 = 8.
Therefore, rightmost non-zero digit = 8

To find the powers of p^a in n! where p is a prime number and a is a natural number.

Highest power of prime number p^a in n! = $\left[\frac{\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\left[\frac{n}{p^4}\right]+\ldots}{a}\right]$ where p is a prime number, a is a natural number and [x] denotes the greatest integer less than or equal to x.

Find the highest power of 72 in 100!

72 = 8 × 9. Therefore, we need to find the highest power of 8 and 9 in 72!.

$8 = 2^3 \Rightarrow$ highest power of 8 in 100! = $\left[\frac{\left[\frac{100}{2}\right]+\left[\frac{100}{4}\right]+\left[\frac{100}{8}\right]+\left[\frac{100}{16}\right]+\left[\frac{100}{32}\right]+\left[\frac{100}{64}\right]}{3}\right]=32$

$9 = 3^2 \Rightarrow$ highest power of 9 in 100!= $\left[\frac{\left[\frac{100}{3}\right]+\left[\frac{100}{9}\right]+\left[\frac{100}{27}\right]+\left[\frac{100}{81}\right]}{2}\right]=24$

As powers of 9 are less, therefore, powers of 72 in 100! = 24

Find the highest power of 24 in 150!

24 = 8 × 3. Therefore, we need to find the highest power of 8 and 3 in 150!

$8 = 2^3 \Rightarrow$ highest power of 8 in 150! = $\left[\frac{\left[\frac{150}{2}\right]+\left[\frac{150}{4}\right]+\left[\frac{150}{8}\right]+\left[\frac{150}{16}\right]+\left[\frac{150}{32}\right]+\left[\frac{150}{64}\right]+\left[\frac{150}{128}\right]}{3}\right]=48$

Highest power of 3 in 150! = $\left[\frac{150}{3}\right]+\left[\frac{150}{9}\right]+\left[\frac{150}{27}\right]+\left[\frac{150}{81}\right]=72$

As the powers of 8 are less, powers of 24 in 150! = 48.

n! has x number of zeroes at the end and (n + 1)! has x + 3 zeroes at the end. Find the number of possible values of n if n is a three digit number.

Answer: We can see that increasing the natural number by 1, we are gathering 3 more powers of 5. Therefore, n + 1 is a multiple of 125 but not a multiple of 625 as it would result in 4 powers of 5. Therefore, n + 1 will be equal to all the multiples of 125 minus 625.

DIVISIBILITY

Divisibility by 2, 4, 8, 16, 32..

A number is divisible by 2, 4, 8, 16, 32,.. 2^n when the number formed by the last one, two, three, four, five...n digits is divisible by 2, 4, 8, 16, 32,..2^n respectively.

Example: 1246384 is divisible by 8 because the number formed by the last three digits i.e. 384 is divisible by 8. The number 89764 is divisible by 4 because the number formed by the last two digits, 64 is divisible by 4.

A 101 digit number is formed by writing first 55 natural numbers next to each other. Find the remainder when the number is divided by 16.

Answer: to find remainder by 16 we only divide the number formed by the last 4 digits by 16. The last 4 digits would be 5455 $\Rightarrow$ remainder by 16 = 15.

Divisibility by 3 and 9

A number is divisible by 3 or 9 when the sum of the digits of the number is divisible by 3 or 9 respectively.

Example: 313644 is divisible by 3 because the sum of the digits- 3 + 1 + 3 + 6 + 4 + 4 = 21 is divisible by 3.

The number 212364 is divisible by 9 because the sum of the digit- 2 + 1 + 2 + 3 + 6 + 4 = 18 is divisible by 9.

The six-digit number 73A998 is divisible by 6. How many values of A are possible?

Answer: Since the number is ending in an even digit, the number is divisible by 2. To find divisibility by 3, we need to consider sum of the digits of the number. The sum of the digits = 7 + 3 + A + 9 + 9 + 8 = 36 + A.

For the number to be divisible by 3, the sum of the digits should be divisible by 3. Hence A can take values equal to 0, 3, 6, and 9. Therefore, number of values possible = 4

Divisibily by 6, 12, 14, 15, 18..

Whenever we have to check the divisibility of a number N by a composite number C, the number N should be divisible by all the prime factors (the highest power of every prime factor) present in C .

divisibility by 6: the number should be divisible by both 2 and 3.
divisibility by 12: the number should be divisible by both 3 and 4.
divisibility by 14: the number should be divisible by both 2 and 7.
divisibility by 15: the number should be divisible by both 3 and 5.
divisibility by 18: the number should be divisible by both 2 and 9.

Divisibility by 7, 11, and 13

Let there be a 6- digit number abcdef. We can write

abcdef = abc000 + def = abc $\times$ 1000 + def = abc $\times$ 1000 + abc $-$ abc + def = abc $\times$ 1001 + def $-$ abc.

1001 = 7 $\times$ 11 $\times$ 13 $\Rightarrow$ abc $\times$ 1001 is divisible by 7, 11 and 13. Therefore,
$\Rightarrow$ If the number def $-$ abc is divisible by 7, the number abcdef is divisible by 7.
$\Rightarrow$ If the number def $-$ abc is divisible by 11, the number abcdef is divisible by 11.
$\Rightarrow$ If the number def $-$ abc is divisible by 13, the number abcdef is divisible by 13.

Now let's see the application

Let a number be ....kjlhgfedcba where a, b, c, d, are respectively units digits, tens digits, hundreds digits, thousands digits and so on.

Starting from right to left, make groups of three digit numbers successively and continue till the end. It is not necessary that the leftmost group has three digits. Grouping of the above number in groups of three, from right to left, is done in the following manner→ kj,ihg,fed,cba

Add the alternate groups (1^{st}, 3^{rd}, 5^{th} etc.. and 2^{nd}, 4^{th}, 6^{th}, etc..) to obtain two sets of numbers, N_1 and N_2. In the above example, N_1 = cba + ihg and N_2 = fed + kj

Let D be difference of two numbers, N_1 and N_2 i.e. D = $N_1 - N_2$.
→ If D is divisible by 7, then the original number is divisible by 7.
→ If D is divisible by 11, then the original number is divisible by 11

→ If D is divisible by 13 then the original number is divisible by 13.

Corollary:
Any six-digit, or twelve-digit, or eighteen-digit, or any such number with number of digits equal to multiple of 6, is divisible by ***EACH*** of 7, 11 and 13 if all of its digits are **same**.

For example 666666, 888888888888 etc. are all divisible by 7, 11, and 13.

Find if the number 29088276 is divisible by 7.

Answer: We make the groups of three as said above- 29,088,276. $N_1 = 29 + 276 = 305$ and $N_2 = 88$. $D = N_1 - N_2 = 305 - 88 = 217$. We can see that D is divisible by 7. Hence, the original number is divisible by 7.

Find the digit A if the number 888…888A999…999 is divisible by 7, where both the digits 8 and 9 are 50 in number.

Answer: We know that 888888 and 999999 will be divisible by 7. Hence 8 written 48 times in a row and 9 written 48 times in a row will be divisible by 7. Hence we need to find the value of A for which the number 88A99 is divisible by 7. By trial we can find A is = 5.

Find a four-digit number **abcd** with distinct digits which is divisible by 4, such that **bacd** is divisible by 7, **acbd** is divisible by 5, and **abdc** is divisible by 9.

Answer: acbd is divisible by 5, therefore, d is either 5 or 0. As abcd is divisible by 4, d cannot be 5 ⇒ d = 0.
As abcd is divisible by 4 and d = 0 ⇒ c = 2, 4, 6 or 8. Now, sum of the digits should be equal to 9, 18 or 27 as the number abdc is divisible by 9.
Case 1: c = 2, d = 0 ⇒ (a, b) = (1, 6), (3, 4), (7, 9). ⇒ No result.
Case 2: c = 4, d = 0 ⇒ (a, b) = (2, 3), (6, 8), (5, 9) ⇒ No result.
Case 3: c = 6, d = 0 ⇒ (a, b) = (1, 2), (5, 7), (4, 8), (3, 9). Checking for divisibility by 7, **abcd = 2160 and 5760**.
Case 4: c = 8, d = 0 ⇒ (a, b) = (4, 6), (3, 7), (1, 9). ⇒ No result.

A number consisting entirely of the digit one is called a repunit; for example, 11111. Find the smallest repunit that is divisible by 63.

Answer: 333333. A number formed by repeating a single digit 6 times is divisible by 7. Also, the sum of digits is divisible by 9.

The product of a two-digit number by a number consisting of the same digits written in the reverse order is equal to 2430. Find the lower number?

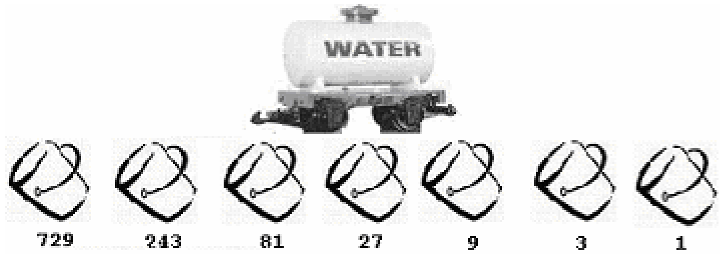
The product of the number and its reciprocal is ending in 0, therefore the number is to be of the form x5 where x is even, such that x5 × 5x = 2430. As the R.H.S. is a multiple of 9, the L.H.S. should also be a multiple of 9. Only x = 4 yields the result. Therefore, the number is 45.

The number 523abc is divisible by 7, 8 and 9. Then a × b × c is equal to

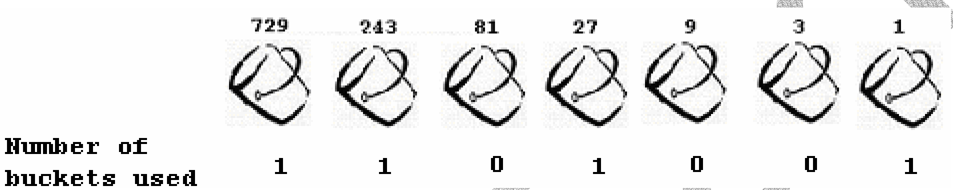
Answer: The LCM of 7, 8, and 9 is 504. Therefore, 523abc should be divisible by 504. Now 523abc = 504000 + 19abc. Therefore, 19abc should be divisible by 504.
19abc = 19000 + abc = 18648 + 352 + abc. 18648 is divisible by 504 ⇒ 352 + abc should be divisible by 504. Therefore, abc = 504 – 352 or 2 × 504 – 352 = 152 or 656. Therefore, a × b × c = 1 × 5 × 2 = 10 or 6 × 5 × 6 = 180.

BASE SYSTEM

Suppose you have a 1 000 L tank to be filled with water. The buckets that are available to you all have sizes that are powers of 3, i.e. 1, 3, 9, 27, 81, 243, and 729 L. Which buckets do you use to fill the tank in the minimum possible time?



You will certainly tell me that the first bucket you will use is of 729 L. That will leave 271 L of the tank still empty. The next few buckets you will use will 243 L, 27 L and 1 L. The use of buckets can be shown as below



We can say that 1 000 = 729 + 243 + 27 + 1
$= 1 \times 3^6 + 1 \times 3^5 + 0 \times 3^4 + 1 \times 3^3 + 0 \times 3^2 + 0 \times 3^1 + 0 \times 3^0$.

The number 1 000 has been written in increasing powers of 3. Therefore, 3 is known as the 'base' in which we are expressing 1 000.

For example, The number 7368 can be written as $8 + 6 \times 10 + 3 \times (10)^2 + 7 \times (10)^3$.
The number 10 is called the 'base' in which this number was written.

Let a number abcde be written in base p, where a, b, c, d and e are single digits less than p. The value of the number abcde $= e + d \times p + c \times p^2 + b \times p^3 + a \times p^4$

For example, if the number 7368 is written in base 9,
The value of $(7368)_9 = 8 + 6 \times 9 + 3 \times 9^2 + 7 \times 9^3 = 5408$ (this value is in base 10)

There are two kinds of operations associated with conversion of bases:

Conversion from any base to base ten

The number $(pqrstu)_b$ is converted to base 10 by finding the value of the number. i.e. $(pqrstu)_b = u + tb + sb^2 + rb^3 + qb^4 + pb^5$.

Convert $(21344)_5$ to base 10.

Answer: $(21344)_5 = 4 + 4 \times 5 + 3 \times 25 + 1 \times 125 + 2 \times 625 = 1474$

Conversion from base 10 to any base

A number written in base 10 can be converted to any base 'b' by first dividing the number by 'b', and then successively dividing the quotients by 'b'. The remainders, written in reverse order, give the equivalent number in base 'b'.

Write the number 25 in base 4.

4	25	
	6	1
	1	2
	0	1

Writing the remainders in reverse order the number 25 in base 10 is the number 121 in base 4.

Addition, subtraction and multiplication in bases:

Add the numbers $(4235)_7$ and $(2354)_7$

Answers: The numbers are written as

$$\begin{array}{cccc} 4 & 2 & 3 & 5 \\ 2 & 3 & 5 & 4 \end{array}$$

The addition of 5 and 4 (at the units place) is 9, which being more than 7 would be written as $9 = 7 \times 1 + 2$. The Quotient is 1 and written is 2. The Remainder is placed at the units place of the answer and the Quotient gets carried over to the ten's place. We obtain

$$\begin{array}{cccc} & +1 & +1 & \\ 4 & 2 & 3 & 5 \\ 2 & 3 & 5 & 4 \\ \hline 6 & 6 & 2 & 2 \\ \hline \end{array}$$

At the tens place: 3 + 5 + 1 (carry) = 9

Similar procedure is to be followed when multiply numbers in the same base

Multiply $(43)_8 \times (67)_8$

Answer:

$7 \times 3 = 21 = 8 \times 2 + 5 \Rightarrow$ we write 5 and carry $2 \times$ base (8)
$7 \times 4 + 2$ (carry) $= 30 = 8 \times 3 + 6$ we write 6 and carry $3 \times$ base (8)
$6 \times 3 = 18 = 8 \times 2 + 2 \Rightarrow$ we write 2 and carry $2 \times$ base (8)
$6 \times 4 + 2$ (carry) $= 26 = 8 \times 3 + 2 \Rightarrow$ we write 2 and carry $3 \times$ base (8)

$$\begin{array}{r} (4\quad 3)_8 \\ (6\quad 7)_8 \\ \hline 365 \\ 322 \\ \hline 3605 \\ \hline \end{array}$$

For subtraction the procedure is same for any ordinary subtraction in base 10 except for the fact that whenever we need to carry to the right we carry the value equal to the base.

Subtract 45026 from 51231 in base 7.

Answer:

$$\begin{array}{r} 51231 \\ -\;45026 \\ \hline 3202 \\ \hline \end{array}$$

In the units column since 1 is smaller than 6, we carry the value equal to the base from the number on the left. Since the base is 7 we carry 7. Now, 1 + 7 = 8 and 8 – 6 = 2. Hence we write 2 in the units column. We proceed the same way in the rest of the columns.

Important rules about bases

A number in base N is divisible by N – 1 when the sum of the digits of the number in base N is divisible by N – 1.

When the digits of a k-digit number N_1, written in base N are rearranged in any order to form a new k-digit number N_2, the difference $N_1 - N_2$ is divisible by N – 1.

If a number has even number of digits in base N, the number is divisible by base N + 1 if the digits equidistant from each end are the same, i.e. the number is a palindrome.

The number 35A246772 is in base 9. This number is divisible by 8. Find the value of digit A.

Answer: The number will be divisible by 8 when the sum of the digits is divisible by 8.
Sum of digits = 3 + 5 + A + 2 + 4 + 6 + 7 + 7 + 2 = 36 + A. The sum will be divisible by 8 when A = 4.

A four-digit number N_1 is written in base 13. A new four-digit number N_2 is formed by rearranging the digits of N_1 in any order. Then the difference $N_1 - N_2$ is divisible by

(a) 9 (b) 10 (c) 12 (d) 13

Answer: The difference is divisible by 13 – 1 = 12.

In what base is the equation 53 × 15 = 732 valid?

Answer: Let the base be b. $(53)_b = 3 + 5b$, $(15)_b = b + 5$, $(732)_b = 7b^2 + 3b + 2$
$\Rightarrow (3 + 5b) \times (b + 5) = 7b^2 + 3b + 2 \Rightarrow b = 13$.

A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals **(CAT 2003)**

Answer: Whenever we change a number from base 10 to any other base, the units digit is the first remainder when the number is divided by that base. Therefore, M when divided by 2, 3 and 5 gives remainder 1 in each case. LCM of 2, 3 and 5 is 30. Therefore, M = 30k + 1 = 31, 61 and 91. Out of these 3 numbers, only the number 91 satisfies the second criterion of leading digit (last remainder).

A palindromic number reads the same forward and backward. A 10-digit palindromic number in base 16 will always be divisible by

Answer: If a number has even number of digits in base N, the number is divisible by base N + 1 if the digits equidistant from each end are the same. Therefore, the number will be divisible by 16 + 1 = 17.

If n is a natural number, find the possible terminating digits of $n^2 + n$ in base 5.
Answer: For a natural number n, $n^2 + n$ will always end in either 0, or 2 or 6. Therefore, when written in base 5, the first remainders will be 0, 1 or 2. Therefore, the units digit of $n^2 + n$ written in base 5 will be 0, 1 or 2.

Solve in base 7, the pair of equations 2x - 4y = 33 and 3x + y = 31, where x, y and the coefficients are in base 7.

Answer: Working in base 7:

2x - 4y = 33 --- (1)

3x + y = 31 --- (2)

Multiply (2) by 4, noting that $3 \times 4 = 15$ in base 7 and $31 \times 4 = 154$ in base 7. We obtain

15x + 4y = 154 --- (3)

Adding (1) and (3) we get $20x = 220 \Rightarrow x = 11 \Rightarrow y = -2$.

My ABN AMRO ATM Pin is a four-digit number. My HDFC BANK ATM Pin is also a four-digit number using the same digits, in a different order, as those in my ABN AMRO Pin. When I subtract the two numbers, I get a four-digit number whose first three digits are 2, 3 and 9. What is the unit digit of the difference?
Answer: When the digits of a k-digit number N_1, written in base N are rearranged in any order to form a new k-digit number N_2, the difference $N_1 - N_2$ is divisible by N − 1. Therefore, the difference of the two numbers would be divisible by 9. Hence, the sum of the digits of the difference should be divisible by 9. Therefore, unit digit = 4.

A number N written in base b is represented as a two-digit number A2, where A = b - 2. What would N be represented as when written in base b - 1?
Answer: $N = (A2)_b = 2 + A \times b = 2 + (b - 2)b = b^2 - 2b + 2 = (b - 1)^2 + 1 = (101)_{b-1}$.

HCF AND LCM

What is highest common factor (HCF) and least common multiple (LCM)? How do you calculate HCF and LCM of two or more numbers? Are you looking for problems on HCF and LCM? This chapter will answer all these questions.

Highest Common Factor (HCF)

The largest number that divides two or more given numbers is called the highest common factor (HCF) of those numbers. There are two methods to find HCF of the given numbers:

Prime Factorization Method- When a number is written as the product of prime numbers, the factorization is called the prime factorization of that number. For example, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

To find the HCF of given numbers by this method, we perform the prime factorization of all the numbers and then check for the **common** prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given number. The HCF is product of all such prime factors with their respective least indices.

Find the HCF of 72, 288, and 1080

Answer: $72 = 2^3 \times 3^2$, $288 = 2^5 \times 3^2$, $1080 = 2^3 \times 3^3 \times 5$
The prime factors common to all the numbers are 2 and 3. The lowest indices of 2 and 3 in the given numbers are 3 and 2 respectively.
Hence, HCF = $2^3 \times 3^2 = 72$.

Find the HCF of $36x^3y^2$ and $24x^4y$.

Answer: $36x^3y^2 = 2^2 \cdot 3^2 \cdot x^3 \cdot y^2$, $24x^4y = 2^3 \cdot 3 \cdot x^4 \cdot y$. The least index of 2, 3, x and y in the numbers are 2, 1, 3 and 1 respectively. Hence the HCF = $2^2 \cdot 3 \cdot x^2 \cdot y = 12x^2y$.

Division method- To find HCF of two numbers by division method, we divide the higher number by the lower number. Then we divide the lower number by the first remainder, the first remainder by the second remainder... and so on, till the remainder is 0. The last divisor is the required HCF.

Find the HCF of 288 and 1080 by the division method.

Answer:

```
288|1080|3
     864
     216|288|1
         216
          72|216|3
             216
               0
```

Hence, the last divisor 72 is the HCF of 288 and 1080.

Three company of soldiers containing 120, 192, and 144 soldiers are to be broken down into smaller groups such that each group contains soldiers from one company only and all the groups have equal number of soldiers. What is the least number of total groups formed?

Answer: The least number of groups will be formed when each group has number of soldiers equal to the HCF. The HCF of 120, 192 and 144 is 24. Therefore, the numbers of groups formed for the three companies will be 5, 8, and 6, respectively. Therefore, the least number of total groups formed = 5 + 8 + 6 = 19.

The numbers 2604, 1020 and 4812 when divided by a number N give the same remainder of 12. Find the highest such number N.

Answer: Since all the numbers give a remainder of 12 when divided by N, hence (2604 – 12), (1020 – 12) and (4812 – 12) are all divisible by N. Hence, N is the HCF of 2592, 1008 and 4800. Now $2592 = 2^5 \times 3^4$, $1008 = 2^4 \times 3^2 \times 7$ and $4800 = 2^6 \times 3 \times 5^2$. Hence, the number N = HCF = $2^4 \times 3 = 48$.

The numbers 400, 536 and 645, when divided by a number N, give the remainders of 22, 23 and 24 respectively. Find the greatest such number N.

Answer: N will be the HCF of (400 – 22), (536 – 23) and (645 – 24). Hence, N will be the HCF of 378, 513 and 621. → N = 27.

The HCF of two numbers is 12 and their sum is 288. How many pairs of such numbers are possible?

Answer: If the HCF if 12, the numbers can be written as 12x and 12y, where x and y are co-prime to each other. Therefore, $12x + 12y = 288 \rightarrow x + y = 24$.

The pair of numbers that are co-prime to each other and sum up to 24 are (1, 23), (5, 19), (7, 17) and (11, 13). Hence, only four pairs of such numbers are possible. The numbers are (12, 276), (60, 228), (84, 204) and (132, 156).

The HCF of two numbers is 12 and their product is 31104. How many such numbers are possible?

Answer: Let the numbers be 12x and 12y, where x and y are co-prime to each other. Therefore, $12x \times 12y = 31104 \rightarrow xy = 216$. Now we need to find co-prime pairs whose product is 216.

$216 = 2^3 \times 3^3$. Therefore, the co-prime pairs will be (1, 216) and (8, 27). Therefore, only two such numbers are possible.

Find the HCF of $2^{100} - 1$ and $2^{120} - 1$

Answer: $2^{100} - 1 = (2^{20})^5 - 1 \Rightarrow$ divisible by $2^{20} - 1$ ($a^n - b^n$ is always divisible by $a - b$)
Similarly, $2^{120} - 1 = (2^{20})^6 - 1 \Rightarrow$ divisible by $2^{20} - 1$ ($a^n - b^n$ is always divisible by $a - b$)
$\Rightarrow$ HCF $= 2^{20} - 1$

Least Common Multiple (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

To calculate the LCM of two or more numbers, we use the following two methods:

Prime Factorization Method: After performing the prime factorization of the numbers, i.e. breaking the numbers into product of prime numbers, we find the highest index, among the given numbers, of all the prime numbers. The LCM is the product of all these prime numbers with their respective highest indices.

Find the LCM of 72, 288 and 1080.

Answer: $72 = 2^3 \times 3^2$, $288 = 2^5 \times 3^2$, $1080 = 2^3 \times 3^3 \times 5$. The prime numbers present are 2, 3 and 5. The highest indices (powers) of 2, 3 and 5 are 5, 3 and 1, respectively.
Hence the LCM $= 2^5 \times 3^3 \times 5 = 4320$.

Find the LCM of $36x^3y^2$ and $24x^4y$.

Answer: $36x^3y^2 = 2^2 \cdot 3^2 \cdot x^3 \cdot y^2$ $24x^4y = 2^3 \cdot 3 \cdot x^4 \cdot y$. The highest indices of 2, 3, x and y are 3, 2, 4 and 2 respectively.
Hence, the LCM $= 2^3 \cdot 3^2 \cdot x^4 \cdot y^2 = 72x^4y^2$.

Division Method: To find the LCM of 72, 196 and 240, we use the division method in the following way:

$$
\begin{array}{r|l}
2 & 72,\ 240,\ 196 \\ \hline
2 & 36,\ 120,\ 98 \\ \hline
2 & 18,\ 60,\ 49 \\ \hline
3 & 9,\ 30,\ 49 \\ \hline
 & 3,\ 10,\ 49
\end{array}
$$

L.C.M. of the given numbers = product of divisors and the remaining numbers = $2 \times 2 \times 2 \times 3 \times 3 \times 10 \times 49 = 72 \times 10 \times 49 = 35280$.

Remember!
For **TWO** numbers, HCF x LCM = product of the two numbers

For example, the HCF of 288 and 1020 is 72 and the LCM of these two numbers is 4320. We can see that
72 x 4320 = 288 x 1080 = 311040.

Note- This formula is applicable only for two numbers.

PROPERTIES OF HCF AND LCM

- The HCF of two or more numbers is smaller than or equal to the smallest of those numbers.
- The LCM of two or more numbers is greater than or equal to the largest of those numbers
- If numbers N_1, N_2, N_3, N_4 etc. give remainders R_1, R_2, R_3, R_4, respectively, when divided by the same number P, then P is the HCF of $(N_1 - R_1)$, $(N_2 - R_2)$, $(N_3 - R_3)$, $(N_4 - R_4)$ etc.
- If the HCF of numbers N_1, N_2, N_3 … is H, then N_1, N_2, N_3... can be written as multiples of H (Hx, Hy, Hz..). Since the HCF divides all the numbers, every number will be a multiple of the HCF.
- If the HCF of two numbers N_1 and N_2 is H, then, the numbers $(N_1 + N_2)$ and $(N_1 - N_2)$ are also divisible by H. Let $N_1 = Hx$ and $N_2 = Hy$, since the numbers will be multiples of H. Then, $N_1 + N_2 = Hx + Hy = H(x + y)$, and
$N_1 - N_2 = Hx - Hy = H(x - y)$. Hence both the sum and differences of the two numbers are divisible by the HCF.
- If numbers N_1, N_2, N_3, N_4 etc. give an equal remainder when divided by the same number P, then P is a factor of $(N_1 - N_2)$, $(N_2 - N_3)$, $(N_3 - N_4)$…
- If L is the LCM of N_1, N_2, N_3, N_4.. all the multiples of L are divisible by these numbers.
- If a number P always leaves a remainder R when divided by the numbers N_1, N_2, N_3, N_4 etc., then P = LCM (or a multiple of LCM) of N_1, N_2, N_3, N_4.. + R.

Find the highest four-digit number that is divisible by each of the numbers 24, 36, 45 and 60.

Answer: $24 = 2^3 \times 3$, $36 = 2^2 \times 3^2$, $45 = 3^2 \times 5$ and $60 = 2^3 \times 3^2 \times 5$. Hence, the LCM of 24, 36, 45 and 60 = $2^3 \times 3^2 \times 5 = 360$. The highest four-digit number is 9999. 9999 when divided by 360 gives the Remainder 279. Hence, the number (9999 – 279 = 9720) will be divisible by 360. Hence the highest four-digit number divisible by 24, 36, 45 and 60 = 9720.

Find the highest number less than 1800 that is divisible by each of the numbers 2, 3, 4, 5, 6 and 7.

Answer: The LCM of 2, 3, 4, 5, 6 and 7 is 420. Hence 420, and every multiple of 420, is divisible by each of these numbers. Hence, the number 420, 840, 1260, and 1680 are all divisible by each of these numbers. We can see that 1680 is the highest number less than 1800 which is multiple of 420. Hence, the highest number divisible by each one of 2, 3, 4, 5, 6 and 7, and less than 1800 is 1680.

Find the lowest number which gives a remainder of 5 when divided by any of the numbers 6, 7, and 8.

Answer: The LCM of 6, 7 and 8 is 168. Hence, 168 is divisible by 6, 7 and 8. Therefore, 168 + 5 = 173 will give a remainder of 5 when divided by these numbers.

What is the smallest number which when divided by 9, 18, 24 leaves a remainder of 5, 14 and 20 respectively?

Answer: The common difference between the divisor and the remainder is 4 (9 – 5 = 4, 18 – 14 = 4, 24 – 20 = 4). Now the LCM of 9, 18, and 24 is 72. Now 72 – 4 = 72 – 9 + 5 = 72 – 18 + 14 = 72 – 24 + 20. Therefore, if we subtract 4 from 72, the resulting number will give remainders of 5, 14, and 20 with 9, 18, and 24.

Hence, the number = 72 – 4 = 68.

A number when divided by 3, 4, 5, and 6 always leaves a remainder of 2, but leaves no remainder when divided by 7. What is the lowest such number possible?

Answer: the LCM of 3, 4, 5 and 6 is 60. Therefore, the number is of the form 60k + 2, i.e. 62, 122, 182, 242 etc. We can see that 182 is divisible by 7. Therefore, the lowest such number possible = 182.

For how many pairs (***a***, ***b***) of natural numbers is the LCM of ***a*** and ***b*** is $2^3 5^7 11^{13}$?

Let's solve for the powers of 2. One of the number will have 2^3 in it, as the LCM has 2^3. Now the other number can have the powers of 2 as 2^0, 2^1, 2^2, and 2^3. Therefore, number of pairs will be 4: (2^3, 2^0), (2^3, 2^1), (2^3, 2^2), and (2^3, 2^3) and the number of ordered pairs will be 2 × 4 - 1 = 7 (we cannot count the pair (2^3, 2^3) twice.

Similarly ordered pairs for powers of 5 = 2 × 8 - 1 = 15.
Number of ordered pairs for powers of 7 = 2 × 14 - 1 = 27.
Total ordered pairs (a, b) = 7 × 15 × 27 = 2835

DIGIT-SUM RULE

Given a number N_1, all the digits of N_1 are added to obtain a number N_2. All the digits of N_2 are added to obtain a number N_3, and so on, till we obtain a single digit number N. This single digit number N is called the digit sum of the original number N_1.

What is the digit sum of 123456789?

Answer: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 → 4 + 5 = 9. Hence, the digit sum of the number is 9.

Note: In finding the digit-Sum of a number we can ignore the digit 9 or the digits that add up to 9. For example, in finding the digit-sum of the number 246819, we can ignore the digits 2, 6, 1, and 9. Hence, the digit-sum of 246819 is = 4 + 8 = 12 = 1 + 2 = 3.

Digit-Sum Rule of Multiplication: The digit-sum of the product of two numbers is equal to the digit sum of the product of the digit sums of the two numbers!

Example: The product of 129 and 35 is 4515.
Digit sum of 129 = 3 and digit sum 35 = 8. Product of the digit sums = 3 × 8 = 24 → Digit-sum = 6.
Digit-sum of 4515 is = 4 + 5 + 1 + 5 = 15 = 1 + 5 = 6. Digit-sum of the product of the digit sums = digit sum of 24 = 6 → Digit sum of the product (4515) = Digit-sum of the product of the digit sums (24) = 6

Applications of Digit-Sum

Rapid checking of calculations while multiplying numbers

Suppose a student is trying to find the product 316 × 234 × 356, and he obtains the number 26525064. A quick check will show that the digit-sum of the product is 3. The digit-sums of the individual numbers (316, 234 and 356) are 1, 9, and 5. The digit-sum of the product of the digit sum is 1 × 9 × 5 = 45 = 4 + 5 = 9. ⇒ the digit-sum of the product of the digit-sums (9) ≠ digit-sum of the 26525064 (3). Hence, the answer obtained by multiplication is not correct.

Note: Although the answer of multiplication will not be correct if the digit-sum of the product of the digit-sums is not equal to digit-sum of the product, but the reverse is not true i.e. the answer of multiplication **may or may not be** correct if the digit-sum of the product of the digit-sums is equal to digit-sum of the product

Finding the sum of the digits of a number raised to a power

The digits of the number $(4)^{24}$ are summed up continually till a single digit number is obtained. What is that number?

Answer: 4^3 = 64. Digit sum of 64 is = 1. $4^{24} = 4^3 \times 4^3 \times 4^3 \ldots \times 4^3$ (8 times). Digit sums on both sides will be the same. ⇒ digit sum of 4^{24} = digit sum of 1 × 1 × 1 × 1… (8 times) = 1

Remember!
If one of the multiplicand is 9, the digit sum is always 9. 235 × 9 = 2115 → digit sum = 9
6224 × 9 = 56016 → digit sum = 9

Find the sum of the sum of the sum of the digits of 25!

Answer: 25! = 1 × 2 × 3 × … × 24 × 25. As one of the multiplicands is 9, the digit sum will be 9.

Determining if a number is a perfect square or not

S. No.	Number	Square	Digit-Sum of the square		S. No.	Number	Square	Digit-Sum of the square
1	1	1	**1**		10	10	100	**1**
2	2	4	**4**		11	11	121	**4**
3	3	9	**9**		12	12	144	**9**
4	4	16	**7**		13	13	169	**7**
5	5	25	**7**		14	14	196	**7**
6	6	36	**9**		15	15	225	**9**
7	7	49	**4**		16	16	256	**4**
8	8	64	**1**		17	17	289	**1**
9	9	81	**9**		18	18	324	**9**

It can be seen from the table that the digit-sum of the numbers which are perfect squares will always be 1, 4, 9, or 7.

Note: A number will **NOT** be a perfect square if its digit-sum is **NOT** 1, 4, 7, or 9, but it **may or may not** be a perfect square if its digit-sum is 1, 4, 7, or 9.

Is the number 323321 a perfect square?

Answer: the digit-sum of the number 323321 is 5. Hence, the number cannot be a perfect square.

A 10-digit number *N* has among its digits one 1, two 2's, three 3's, and four 4's. Is *N* be a perfect square?

Answer: We can see that the digit sum of a perfect square is always 1, 4, 7, or 9. As the digit sum of the number is 3, it cannot be a perfect square.

If a five-digit number N is such that the sum of the digits is 29, can N be the square of an integer?

GREATEST INTEGER FUNCTION

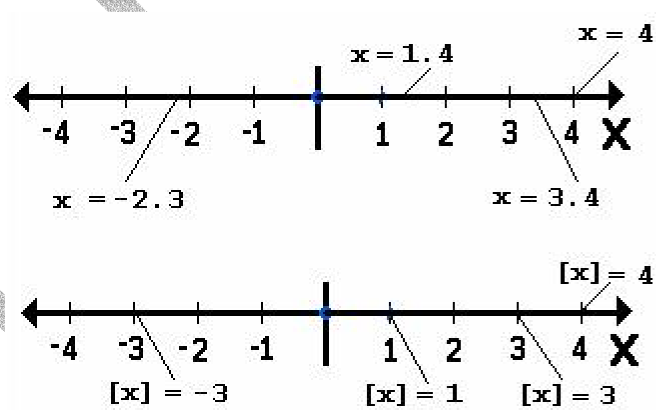
The greatest integer function, denoted by [x], gives the greatest integer less than or equal to the given number x.

To put it simply, if the given number is an integer, then the greatest integer gives the number itself, otherwise it gives the first integer towards the left of the number of x on the number line.

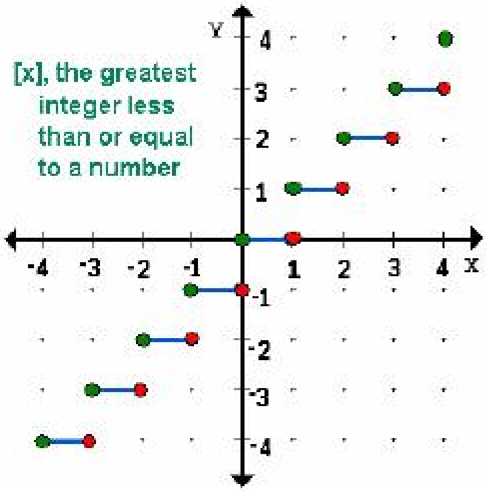
For example, [1.4] = 1, [4]= 4, [3.4] = 3, [- 2.3] = -3, [- 5.6] = - 6, and so on.

NOTE: We can see that [1.4] = 1 + 0.4 or x = [x] + {x}, where {x} is the fractional part of x. For x = -2.3, [x] = - 3 and {x} = 0.7

In the following figure, we can see that the greatest integer function gives the number itself (when the given number is an integer) or the first integer to the left of the number on the number line.



The graph of greatest integer function is given below. Note that the **red** dot indicates that integer value on the number line is not included while the **green** dot indicates that the integer value is included.



Examples:

- What is the value of $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+\cdots+[\sqrt{49}]+[\sqrt{50}]$ where [x] denotes the greatest integer function?
  Answer: It can be seen that
  $[\sqrt{1}] = 1, [\sqrt{2}] = [1.41] = 1, [\sqrt{3}] = [1.73] = 1, [\sqrt{4}] = 2$ and so on..

  Therefore, from $[\sqrt{1}]$ to $[\sqrt{3}]$, the value will be 1, from $[\sqrt{4}]$ to $[\sqrt{8}]$ the value will be 2, from $[\sqrt{9}]$ to $[\sqrt{15}]$ the value will be 3 and so on..
  Therefore, the total value = 3 × 1 + 5 × 2 + 7 × 3 + ... + 13 × 6 + 2 × 7
  = 217.

- If $[\sqrt{x}] = 5$ and $[\sqrt{y}] = 6$, where x and y are natural numbers, what can be the greatest possible value of x + y?
  Answer: It is clear that $[\sqrt{25}] = 5, [\sqrt{26}] = 5, [\sqrt{27}] = 5$ and so on. The highest value of x that we can take is 35, since $[\sqrt{35}] = 5$ but $[\sqrt{36}] = 6$.
  Similarly, the highest value of y we can take is 48, since
  $[\sqrt{48}] = 6$ but $[\sqrt{49}] = 7$
  Therefore, the greatest value of x + y = 35 + 48 = 83.

- What is the value of x for which x[x] = 28?
  Answer: If the value of x is 5, x[x] = 25, and if the value of x is 6 x[x]=36.
  Therefore, the value of x lies between 5 and 6.
  If x lies between 5 and 6, [x] = 5.
  $\Rightarrow x = \frac{28}{[x]} = \frac{28}{5} = 5.6$

- Let x be number and [x] and {x} denote the greatest integer less than x and fractional part of x. If $[x]^3 + \{x\}^2 = -63.64$, then the value of x is
  Answer: Since the value of $[x]^3 + \{x\}^2$ is negative, the value of [x] is negative. The cube lying near –63.64 is –64. Therefore, $[x]^3 = -64$ or [x] = –4. Therefore, $\{x\}^2 = 0.36$ or {x} = 0.6
  Hence, x = [x] + {x} = –4 + 0.6 = –3.4

1. The product of three consecutive odd numbers is 531117. What is the sum of the three numbers?

A. 183
B. 213
C. 243
D. 273

2. N is the smallest number such that $\frac{N}{2}$ is a perfect square and $\frac{N}{3}$ is a perfect cube. Then, the number of divisors of N is

A. 12
B. 16
C. 20
D. 24

3. N is a number such that 200 < N < 300 and it has exactly 6 positive divisors. How many different values of
N are possible?

A. 12
B. 13
C. 14
D. 15

4. In 1936, my age was equal to the last two digits of my birth year. My grandfather said that it was true for
him also. Then, the sum of my age and my grandfather's age in 1936 was

A. 84
B. 86
C. 90
D. 94

5. If n is a natural number such that $10^{12} < n < 10^{13}$ and the sum of the digits of n is 2, then the number of values n can take is

A. 13
B. 12
C. 11
D. 10

6. N is the sum of the squares of three consecutive odd numbers such that all the digits of N are the same. If
N is a four-digit number, then the value of N is

A. 3333
B. 5555
C. 7777
D. 9999

Read the information given below and answer the question that follows.

While driving on a straight road, Jason passed a milestone with a two-digit number. After exactly an hour, he passed a second milestone with the same two digits written in reverse order. Exactly one more hour after that, he passed a third milestone with the same two digits reversed and separated by a zero.

7. What is the sum of the two digits?

A.5
B.6
C.7

D.8

8. The squares of the natural numbers are written in a straight line 149162536… to form a 200-digits number. What is the 100th digit from the left?

A.2
B.5
C.6
D.9

9. $18^{2000} + 12^{2000} - 5^{2000} - 1$ is divisible by

A.323
B.221
C.299
D.237

10. Let n be the smallest positive number such that the number S = $(8^n)(5^{600})$ has 604 digits. Then the sum of the digits of S is

A. 19
B.8
C.10
D.11

11. Swadesh threw five standard dice simultaneously. He found that the product of the numbers on the top faces was 216. Which of the following could not be the sum of the numbers on the top five faces?

A.17
B.18
C.19
D.20

12. If $S = \dfrac{(10^4+324)(22^4+324)(34^4+324)(46^4+324)(58^4+324)}{(4^4+324)(16^4+324)(28^4+324)(40^4+324)(52^4+324)}$, then the value of *S* is

A.373
B.256
C.504
D.588

13. For how many integers S is $\dfrac{S}{20-S}$ square of an integer?

A.5
B.4
C.3
D.2

14. How many natural numbers between 1 and 900 are NOT multiples of any of the numbers 2, 3, or 5?

A.240
B.250
C.270
D.300

15. The numbers 123 456 789 and 999 999 999 are multiplied. How many times does digit '9' come in the product?

A.0
B.1

C.2
D.3

16. All the divisors of 72 are multiplied. The product can be written in the form 2^a*3^b. Then the value of a + b is
A.28
B.30
C.34
D.40

17. The number A4531B, where A and B are single-digit numbers, is divisible by 72. Then A + B is equal to
A.5
B.7
C.8
D.4

18. In the nineteenth century a person was X years old in the year X^2. How old was he in 1884?
A.43
B.58
C.68
D.78

19. How many natural numbers less than 65 have odd number of divisors (including 1 and those numbers themselves)?
A.8
B.12
C.15
D.21

Read the information below and answer the question that follows.

In a mathematical game, one hundred people are standing in a line and they are required to count off in fives as 'one, two, three, four, five, one, two, three, four, five,' and so on from the first person in the line. The person who says 'five' is taken out of the line. Those remaining repeat this procedure until only four people remain in the line.

20. What was the original position in the line of the last person to leave?
A.93
B.96
C.97
D.98

21. An Indian king was born in a year that was a square number, lived a square number of years and died in a year that was also a square number. Then the year he could have been born in was
A.1936
B.1764
C.1600
D.1444

22. To number the pages of a book, exactly 300 digits were used. How many pages did the book have?
A.136
B.137
C.138
D.139

23. Let $S = p^2 + q^2 + r^2$, where p and q are consecutive positive integers and $r = p \times q$. Then $\sqrt{S}$ is

A. an even integer
B. an odd integer
C. always irrational
D. sometimes irrational

24. The number (2n)! is divisible by
I. $(n!)^2$
II. $((n-1)!)^2$
III. $n! \times (n+1)!$

A.I and II only
B.II and III only
C.I and III only
D.I, II and III

Use the following information to answer the next question.

Baghira, the oldest inmate of Tihar Jail is learning mathematics. He notices the following facts about his prisoner number:
I. It is a three digit number not bigger than 500.
II. If you sum the cube of the digits of the number, you get the number itself.
III. The number is the sum of consecutive factorials.

25. Then, the sum of the digits of Baghira's prisoner number is
A.12
B.10
C.9
D.6

Read the information given below and answer the question that follows.

Kallu Kallan Kalia's nightclub number is a three-digit perfect square. This number when written in the reverse order also gives a perfect square and is telephone extension of Kallu's office. His Mercedes registration number is a four-digit perfect square formed by repeating the rightmost digit of his nightclub number.

26. The sum of the digits of Kallu's nightclub number is
A. a perfect square
B. an even number
C. a prime number
D. a perfect number

Read the information given below and answer the question that follows.

The teacher of Confusius, the confused soul, told him: "my age is not prime but odd and if you reverse the digits of my age and add that number to my age, you obtain a number that is a perfect square. If you reverse the digits of my age and subtract the number from my age you again have a perfect square."

27. The age of Confucius' teacher was a number divisible by
A.7
B.9
C.13
D.15

28. How many positive integers less than or equal to 120 are relatively prime to 120?
A.24
B.32
C.36

D. 40

29. If $3^{27^{x}} = 27^{3^{x}}$, then x is equal to

A. -1

B. $\frac{1}{2}$

C. 1

D. 2

30. If S = $\frac{1}{\sqrt{2}-\sqrt{1}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}-\sqrt{3}} - \ldots - \frac{1}{\sqrt{121}-\sqrt{120}}$, then the value of S is

A. 12

B. 11

C. 10

D. –10

31. The difference between the cubes of two consecutive positive integers is 1027. Then the product of these integers is

A. 552

B. 342

C. 306

D. 132

32. How many integers between 1 and 1000, both inclusive, can be expressed as the difference of the squares of two non negative integers?

A. 750

B. 748

C. 300

D. 250

33. The product P of three positive integers is 9 times their sum, and one of the integers is the sum of the other two. The sum of all possible values of P is

A. 621

B. 702

C. 540

D. 336

34. Let [x] = greatest integer less than or equal to x. Let A = [$2x$], B = 2[x] and

C = [$x + \frac{1}{2}$] + [$x - \frac{1}{2}$]. Then

I. A, B and C can be equal for some value of *x.*

II. A, B and C can all take different values for some value of *x.*

A. I is true but II is false

B. II is true but I is false

C. Both I and II are true

D. Both I and II are false

Use the information given below and answer the question that follows.

It is given that m and n are two smallest natural numbers satisfying following conditions

I. $5 < n < m$

II. both $mn - 3$ and $mn + 3$ are prime numbers

35. Then $m + n$ is equal to

A. 15

B.17
C.19
D.21

36. S is a six digit number beginning with 1. If the digit 1 is moved from the leftmost place to the rightmost place the number obtained is three times of S. Then the sum of the digits of S is
A21
B.24
C.26
D.27

37. Which of the following numbers can be written as the sum of the squares of three odd natural numbers?
A.5021
B.4445
C.3339
D.1233

38. If S = $\frac{10^{33}+2}{3}$, then, the sum of digits of S is
A.97
B.100
C.103
D.106

Read the information given below and answer the question that follows.

In a bag, some slips of paper are kept with the numbers thirteen or fourteen written on them. The slips with number thirteen written on them are five more than the slips with number fourteen written on them.

39. Which of the following can be the sum of the numbers in the bag?
A.254
B.300
C.327
D.353

40. How many two-digit positive integers are there which are one and a half times larger than the product of their digits?
A.0
B.1
C.2
D.3

41. For how many values of k is 12^{12} the least common multiple of 6^6, 8^8, and k?
A.1
B.12
C.24
D.25

42. The value of $x + y$ such that $x^2 - y^2 = 343$, is
A.343
B.49
C.7
D.A or B

43. All the divisors of 360, including 1 and the number itself, are summed up. The sum is 1170. What is the sum of the reciprocals of all the divisors of 360?

A.3.25
B.2.75
C.2.5
D.1.75

44. Let $S = \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$ and $T = \frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{d}{|d|} + \frac{abcd}{|abcd|}$, where a, b, c, and d are not equal to zero. Then the set of intersection of all values of *S* and *T* is

A.{0}
B.{ϕ}
C.{1}
D.{4}

45. What is the sum of the real values of *x* satisfying the equation $4 \times 3^{2x+2} - 9^{2x} = 243$?

A.1
B.$\frac{3}{2}$
C.$\frac{5}{2}$
D.3

46. A 10-digit number *N* has among its digits one 1, two 2's, three 3's, and four 4's. Is *N* be a perfect square?

A. Yes
B. No
C. Maybe
D. Can't say

47. There were 90 questions in an exam. If 1 mark was awarded for every correct answer and 1/3rd mark was deducted for every wrong answer, how many different net scores were possible in the exam?

A.120
B.358
C.359
D.360

48. For how many ordered pairs (*x*, *y*), where x and y are non-negative integers, is the equation $\sqrt{x} + \sqrt{y} = \sqrt{1332}$ satisfied?

A.3
B.5
C.7
D.9

49. For how many prime number *p*, is $p^2 + 15p - 1$ also a prime number?

A.0
B.1
C.2
D.3

50. If 11 sweets are distributed among four boys, then, which of the following is true?

A. Two boys each received more than 1 sweet

B. One of the boys received more than 3 sweets
C. One of the boys received fewer than 3 sweets
D. One of the boys received exactly 2 sweets

51. The least common multiple of $2^6 - 1$ and $2^9 - 1$ is

A. $2^{12} + 27 \times 2^9 - 217$
B. $2^{12} + 63 \times 2^3 - 1$
C. $2^{12} + 5 \times 2^9 - 1$
D. $2^{12} + 9 \times 2^8 - 1$

52. All possible pairs are formed from the divisors of 21600. How many such pairs have HCF of 45?

A.8
B.276
C.34
D.49

53. Let H(x, y) = Highest common factor of x and y, and L(x, y) = Least common multiple of x and y. For three numbers, a, b and c it is given that

H(a, b) = 6	L(a, b) = 180
H(b, c) = 30	L(b, c) = 90
H(a, c) = 6	L(c, a) = 60

The value of a × b × c is

A. 32 400
B. 97 200
C. 16 200
D. 97 200

54. How many natural numbers between 1 and 100 have exactly four factors?

A.30
B.32
C.300
D.33

55. 'Privileged Number' is a natural number which has two prime numbers as its neighbors on the number line. For example, 4 and 12 are privileged numbers. What is the mean of all privileged numbers less than 100?

A.30.5
B.24
C.28.5
D.26

56. If $R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then **(CAT 2005)**

A.$R > 1.0$
B.$0 < R \le 0.1$
C.$0.1 < R \le 0.5$
D.$0.5 < R \le 1.0$

57. The digits of a three-digit number ***A*** are written in the reverse order to form another three-digit number ***B***. If ***B*** > ***A*** and ***B***−***A*** is perfectly divisible by 7, then which of the following is necessarily true? **(CAT 2005)**

A.$112 < A < 311$

B. $100 < A < 299$
C. $106 < A < 305$
D. $118 < A < 317$

58. The total number of integer pairs (x, y) satisfying the equation $x + y = xy$ is **(CAT 2004)**
A. 0
B. 1
C. 2
D. none of the above

59. Let *x* and *y* be positive integers such that *x* is prime and *y* is composite. Then,
A. $y - x$ cannot be an even integer.
B. $\frac{x+y}{x}$ cannot be an even integer.
C. xy cannot be an even integer.
D. None of the other statements is true.

60. If $|b| > 1$ and $x = -|a|\, b$, then which one of the following is necessarily true? **(CAT 2003)**
A. $a - xb \le 0$
B. $a - xb > 0$
C. $a - xb \ge 0$
D. $a - xb < 0$

61. If *a*, *a* + 2, and *a* + 4 are prime numbers, then the number of possible solutions for *a* is **(CAT 2003)**
A. 1
B. 2
C. 3
D. more than 3

62. A faulty odometer of a car always jumps from digit 4 to digit 6, always skipping the digit 5, regardless of the position. For example, after traveling for one kilometer the odometer reading changed from 000149 to 000160. If the odometer showed 000000 when the car was bought and now it shows 001000, how many kilometers has the car traveled?

63. A number N when divided by a divisor D gives a remainder of 52. The number 5N when divided by D gives a remainder of 4. How many values of D are possible?
A. 1
B. 3
C. 6
D. 7

64. Which one of the following numbers is a perfect square?
A. $35! \times 36!$
B. $37! \times 38!$
C. $34! \times 37!$
D. $36! \times 37!$

65. In how many ways can 7^{13} be written as product of 3 natural numbers?

66. N has f factors, 2N has 2f factors, 6N has 4f factors, 15N has 3f factors. How many factors 30N has?

67. N is a number such that the ratio of sum of its digit to product of its digits is 3: 40. If N is divisible by 37 and N is the smallest such number, how many factors does N have?

A.8
B.16
C.30
D.20

68. The sum of 20 numbers (may or may not be distinct) is 801. What is their minimum LCM possible?
A.36
B.42
C.56
D.60

69. The sum of 20 distinct numbers is 801. What is their minimum LCM possible?
A.480
B.360
C.840
D.420

70. What is the smallest positive composite number generated by the expression $p^2 - p - 1$ where p is a prime number?
A.13
B.155
C.40
D.270

71. N the least positive integer that is eleven times the sum of its digits. Then N is divisible by
A.4
B.7
C.15
D.9

72. The value of A + B that satisfies $(6^{30} + 6^{-30})(6^{30} - 6^{-30}) = 3^A8^B - 3^{-A}8^{-B}$ is
A.20
B.60
C.80
D.40

73. The value of $\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$ is
A.2
B.1
C. $\sqrt{7}$
D. $\sqrt{2}$

74. Consider the set S = {1, 2, 3, ..., 1000}. How many arithmetic progressions can be formed from the elements of S that start with 1 and end with 1000 and have at least 3 elements? **(CAT 2006)**
A.4
B.3
C.7
D.6
E.8

76. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers? **(CAT 2006)**
A.25

B.67
C.41
D.73
E.21

77. The number of employees in Obelix Menhir Co. is a prime number and is less than 300. The ratio of the number of employees who are graduate and above, to that of employees who are not, can possibly be **(CAT 2006)**
A.97: 84
B.87: 100
C.85: 98
D.101: 88
E.110: 111

78. When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed? **(CAT 2006)**
A.6
B.8
C.10
D.5
E.7

79. The four numbers x, y, x + y and x – y are all prime numbers. Then the sum of these four prime numbers is
A. divisible by 7
B. divisible by 5
C. even
D. prime
E. divisible by 3

80. The single digits *a* and *b a*re neither both nine nor both zero. The repeating decimal 0.abababab... is expressed as a fraction in lowest terms. How many different denominators are possible?
A.4
B.6
C.5
D.3

81. What is the value of n such that n! = 3! × 5! × 7!
A.10
B.11
C.8
D.9

82. What is the value of N such that N × [N] = 27, where [N] represents the greatest integer less than or equal to N?
A.5.4
B.5.8
C.6.1
D.5.6

83. A gadha never lives up to 100 years because its stupidity gets it killed. Dhondu and Bhondu are the cutest gadhas in Donkeyland. When you write Dhondu's age followed by Bhondu's age, you get a four-digit perfect square. After 31 years, if you write their ages in the same order you again obtain a four-digit perfect square. How old is Dhondu?
A.20

B.12
C.14
D.10

84. Let N = $2^{15} \times 3^{12}$. How many factors of N^2 are less than N but do not divide N completely?
A.387
B.180
C.208
D.310

85. The product of the ages of some teenagers is 10584000. The sum of their ages is equal to
A.86
B.88
C.85
D.89
E.87

86. In a village of 2029 inhabitants, at least x villagers have the same English initials for their first name and their surname. The least possible value of x is
A.4
B.3
C.6
D.5
E.2

87. A Number N is divisible by 10, 90, 98 and 882 but it is not divisible by 50 or 270 or 686 or 1764. It is also known that N is a factor of 9261000. What is N?
A.4410
B.22050
C.13230
D.8820

88. My grandfather said he was 84 years old but he was not counting the Sundays. How old my grandfather really was?
A.64
B.97
C.96
D.98

89. Let S = $\frac{1630}{4542}$. If you swap two digits of the numerator of S with two digits of the denominator, the fraction is equal to $\frac{1}{3}$. What is the sum of the digits of the denominator of the new fraction?
A.12
B.8
C.9
D.15

90. Number S is obtained by squaring the sum of digits of a two-digit number D. If the difference between S and D is 27, then the two-digit number D is **(CAT 2002)**
A.24
B.54
C.34
D.45

91. A child was asked to add first few natural numbers (i.e. 1 + 2 + 3 + …) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered he had missed one number in the sequence during addition. The number he missed was **(CAT 2002)**

A. less than 10
B.10
C.15
D. more than 15

92. Suppose for any real number x, [x] denotes the greatest integer less than or equal to x. Let L (x, y) = [x] + [y] + [x + y] and R (x, y) = [2x] + [2y]. Then it is impossible to find any two positive real numbers x and y for which **(CAT 2002)**

A.L(x, y) = R(x, y)
B.L(x, y) $\neq$ R(x, y)
C.L(x, y) < R(x, y)
D.L(x, y) > R(x, y)

93. At a bookstore, 'MODERN BOOK STORE' is flashed using neon lights. The words are individually flashed at the intervals of $2\frac{1}{2}$ s, $4\frac{1}{4}$ s and $5\frac{1}{8}$ s respectively, and each word is put off after a second. The least time after which the full name of the bookstore can be read again is **(CAT 2002)**

A.49.5 s
B.73.5 s
C.1744.5 s
D.855 s

94. Three pieces of cakes of weights $4\frac{1}{2}$ lb, $6\frac{3}{4}$ lb and $7\frac{1}{5}$ lb respectively are to be divided into parts of equal weight. Further, each part must be as heavy as possible. If one such part is served to each guest, then what is the maximum number of guests that could be entertained? **(CAT 2002)**

A.54
B.72
C.20
D. None of the above

95. The number of positive integers n in the range $12 \leq n \leq 40$ such that the product (n – 1)(n – 2)…3.2.1 is not divisible by n is **(CAT 200)**

A.5
B.7
C.13
D.14

96. Let T be the set of integers {3, 11, 19, 27,… 451, 459, 467} and S be the subset of T such that the sum of no elements of S is 470. The maximum possible number of elements in S is **(CAT 2002)**

A.32
B.28
C.29
D.30

97. Let x, y and z be distinct integers. X and y are odd and positive, and z is even and positive. Which one of the following statements cannot be true? **(CAT 2001)**

A. $y(x - z)^2$ is even
B. $y^2(x - z)$ is odd

C. $y(x - z)$ is odd
D. $z(x - y)^2$ is even

98. Let b be a positive integer and $a = b^2 - b$. If $b \geq 4$, then $a^2 - 2a$ is divisible by **(CAT 2001)**
A. 15
B. 20
C. 24
D. all of the above

99. All the page numbers from a book are added, beginning at page 1. However, one page number was added twice by mistake. The sum obtained was 1 000. Which page number was added twice? **(CAT 2001)**
A. 44
B. 45
C. 10
D. 12

100. If $x = \sqrt{2(1+\sqrt{2})}$, then the value of $x^3 + x^2 - 2x - 2$ is
A. 0
B. $6\sqrt{1+\sqrt{2}} + 3(1+\sqrt{2})$
C. $2\sqrt{2} + 4\sqrt{1+\sqrt{2}}$
D. $1+\sqrt{2}$

101. For how many integers n is $\frac{5n+23}{n-7}$ also an integer?
A. 2
B. 4
C. 6
D. 8

102. In the value of the number 30!, all the zeroes at the end are erased. Then, the unit digit of the number that is left is
A.2
B.4
C.6
D.8

103. Let $S = (3 + 3^2 + 3^3 + \ldots + 3^{400}) - (7 + 7^2 + 7^3 + \ldots + 7^{201})$. The last two digits of S are
A. 00
B. 07
C. 43
D. 93

104. How many natural numbers between 1 and 900 are NOT multiples of any of the numbers 2, 3, or 5?
A. 240
B. 250
C. 270
D. 300

105. If $N = (63)^{1! + 2! + 3! + ..+ 63!} + (18)^{1! + 2! + 3!..+ 18!} + (37)^{1! + 2! + 3! + ...+ 37!}$, then the unit digit of N is
A. 2

B. 4
C. 6
D. 8

106. The last two digits of 4^{1997} are
A. 96
B. 36
C. 84
D. 24

107. The remainder when 7^{7^7} is divided by 9 is
A. 2
B. 4
C. 6
D. 8

Read the information below and answer the question that follows.

In a mathematical game, one hundred people are standing in a line and they are required to count off in fives as 'one, two, three, four, five, one, two, three, four, five,' and so on from the first person in the line. The person who says 'five' is taken out of the line. Those remaining repeat this procedure until only four people remain in the line.

108. What was the original position in the line of the last person to leave?
A. 93
B. 96
C. 97
D. 98

109. If S = $5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ where n is any whole number, then S is always divisible by
A. 7
B. 17
C. 19
D. 33

110. $53^{53} - 27^{27}$ is certainly divisible by
A. 7
B. 9
C. 10
D. 11

111. $43^{444} + 34^{333}$ is divisible by
A. 2
B. 5
C. 9
D. 11

112. The remainder when 2^{2005} is divided by 13 is
A. 2
B. 3
C. 4
D. 8

113. How many positive integers less than or equal to 120 are relatively prime to 120?
A. 24

B. 32
C. 36
D. 40

114. What is the remainder when S = 1! + 2! + 3! +... + 19! + 20! is divided by 20?
A. 0
B. 1.
C. 33
D. 13

115. How many perfect squares are the divisors of the product $1! \cdot 2! \cdot 3! \cdots 8!$?
A. 120
B. 240
C. 360
D. 720

116. $25^{25^{25}}$ when divided by 9 gives the remainder
A. 2
B. 4
C. 5
D. 7

117. The unit digit of $7^{7^{7}}$ is
A. 1
B. 3
C. 7
D. 9

118. Given that x and y are integers and $5x^2 + 2y^2 = 5922$, what can be the unit digit of y?
A. 3
B. 5
C. 7
D. 9

119. What is the remainder when $6^{83} + 8^{83}$ is divided by 49?
A. 0
B. 14
C. 35
D. 42

120. What is the remainder when $(81)^{21} + (27)^{21} + (9)^{21} + (3)^{21} + 1$ is divided by $3^{20} + 1$?
A. 0
B. 1
C. 61
D. 121

121. What is the remainder when the number $\underbrace{123123123.....123123}_{300 digits}$ is divided by 99?

A. 18
B. 27
C. 33
D. 36

122. What is the remainder when $1+(11)^{11}+(111)^{111}+(1111)^{1111}+...\underbrace{(111...111)}_{10\,digits}{}^{\overbrace{111...111}^{10\,digits}}$ is divided by 100?

A. 40
B. 30
C. 10
D. 0

123. The remainder when $x^{200}-2x^{199}+x^{50}-2x^{49}+x^2+x+1$ is divided by $(x-1)(x-2)$ is
A. 7
B. $2x - 3$
C. $6x - 5$
D. 0

124. If the remainder when x^{100} is divided by $x^2 - 3x + 2$ is $ax + b$, then the values of a and b are
A. 2^{100} and 1
B. 2^{100} and $1 - 2^{100}$
C. 2^{100} and $2 - 2^{100}$
D. $2^{100} - 1$ and $2 - 2^{100}$

125. If $x = (16^3 + 17^3 + 18^3 + 19^3)$, then x divided by 70 leaves a remainder of **(CAT 2005)**
A. 69
B. 35
C. 0
D. 1

126. The rightmost non-zero digit of the number 30^{2720} is
A. 1
B. 3
C. 7
D. 9

127. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is
A. 4
B. 15
C. 0
D. 18

128. What is the remainder when 4^{96} is divided by 6? **(CAT 2003)**
A. 0
B. 3
C. 2
D. 4

129. A number N when divided by a divisor D gives a remainder of 52. The number 5N when divided by D gives a remainder of 4. How many values of D are possible?
A. 2
B. 3
C. 4
D. 6

130. What is the remainder when $(17)^{36} + (19)^{36}$ is divided by 111?
A. 0
B. 2

C. 36
D. 110

131. How many zeroes are present at the end of 25! + 26! + 27! + 28! + 30!?
A. 5
B. 6
C. 7
D. 8

132. How many zeroes does 5^5! End in?
A. 781
B. 100
C. 50
D. 3906

133. 10000! = $(100!)^K \times P$, where P and K are integers. What can be the maximum value of K?
A. 105
B. 102
C. 103
D. 104

134. What is the remainder when 7^{7^7} is divided by 13?
A. 10
B. 6
C. 7
D. 1

135. What is the value of n such that n! = 3! × 5! × 7!
A. 10
B. 11
C. 8
D. 9

136. What are the last two digits of $\frac{1}{5^{903}}$?

A. 08
B. 48
C. 36
D. 18

137. What are the last two digits of $2^{2^{2003}}$?
A. 56
B. 36
C. 76
D. 16

138. M and N are integers such that $0 \le N \le 9$ and $\frac{M}{810} = \overline{0.9N5} = 0.9N59N59N5....$ Then the value of M + N is equal to
A. 752
B. 789
C. 853

D. 927

139. Let M and N be single-digit integers. If the product 2M5 $\times$ 13N is divisible by 36, how many ordered pairs (M, N) are possible?

A. 2
B. 3
C. 4
D. 5

140. Let x and y be two four-digit palindromes (numbers that read the same forwards and backwards) and z be a five-digit palindrome. If x + y = z, how many values of z are possible?

A. 2
B. 3
C. 4
D. 5

141. Suppose you write the numbers 0, 1, 2, 3,…, 20 on the board. You perform following operation 20 times- You pick up two numbers at random (call them a and b), erase them, and write down $|a - b|$. Then, the final number left on the board is

A. odd
B. even
C. more than 10
D. less than 10

142. Find the last two digits of $11^{10} - 9$.

A. 52
B. 72
C. 92
D. 02

143. If $S = 1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \ldots + 20 \times 2^{20}$, find the remainder when S is divided by 19.

A. 0
B. 2
C. 9
D. 18

144. Find the remainder when $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + 98 \times 99 + 99 \times 100$ is divided by 101.

A. 0
B. 1
C. 99
D. 100

145. Which is larger $A = 99^{11} + 100^{11}$ or $B = 101^{11}$?

A. A
B. B
C. Both are equal
D. Cannot be determined

146. The number 523abc is divisible by 7, 8 and 9. Then the value of a $\times$ b $\times$ c

A. is 10
B. is 60
C. is 180
D. cannot be determined

147. How many numbers between 1 and 1000 are there such that $n^2 + 3n + 5$ is divisible by 121?
2. How many numbers between 1 and 1000 are there such that $n^2 + 3n + 5$ is divisible by 121?
A. 0
B. 5
C. 7
D. 10

148. Which of the following natural numbers can be expressed as the sum of the squares of six odd integers?
A. 1996
B. 1997
C. 1998
D. 1999

149. The number of integer solutions of the equation $x^2 + 12 = y^4$ is
A. 2
B. 4
C. 6
D. 8

150. For how many positive integers does n! end with exactly 100 zeros?
A. 0
B. 3
C. 4
D. 5

151. How many ordered pairs (a, b) satisfy $a^2 = b^3 + 1$, where a and b are integers?
A. 2
B. 3
C. 4
D. 5

152. Let $N = \underbrace{111...111}_{73 \text{ times}}$. When N is divided by 259, the remainder is R_1 and when N is divided by 32, the remainder is R_2. Then $R_1 + R_2$ is equal to
A. 6
B. 8
C. 253
D. none of these

153. What are the last two non-zero digits of 36! – 24!?
A. 08
B. 36
C. 64
D. 88

154. For how many integers m is $m^3 - 8m^2 + 20m - 13$ a prime number?
A. 1
B. 2
C. 3
D. more than 3

155. What is the unit digit of $7^{15^{16^{17}}}$?

A. 1
B. 3
C. 7
D. 9

156. A number has exactly 32 factors out of which 4 are not composite. Product of these factors is 30.How may such numbers are possible?
A. 0
B. 3
C. 4
D. 6

157. What is the unit digit of the expression 1! + 2! + 3! +... + 99! + 100!?
A. 2
B. 3
C. 4
D. 6

158. P and Q are two distinct whole numbers and P + 1, P + 2, P + 3, ..., P + 7 are integral multiples of Q + 1, Q + 2, Q + 3,..., Q + 7, respectively. What is the minimum value of P?
A. 0
B. 240
C. 420
D. 1080

159. What is the remainder when 128^{500} is divided by 153?
A. 67
B. 76
C. 85
D. 94

160. In how many zeroes does $\frac{2002!}{(1001!)^2}$ end in?
A. 0
B. 1
C. 2
D. 3

161. How many zeroes will be at the end of $27!^{27!}$?
A. 36
B. $6^{27!}$
C. $6 \times 27!$
D. $6! \times 27!$

162. What is the remainder when 5^{26} is divided by 15?
A. 1
B. 2
C. 5
D. 10

163. We define a number 44...48...89 with the following procedure. First, we insert the number 48 between the two digits of 49. Next, 48 is inserted between the digits 4 and 8 of the resulting number. Finally, we repeat this second step a few times. Every number obtained this way is
A. a prime number

B. a perfect square
C. a perfect cube
D. of the form 3n + 1

164. Given that $X_1 = 5$, $X_2 = 25$ and $X_{n+2} = GCD(X_{n+1}, X_n) + X_n$. What is the LCM of ($X_{19}$ and X_{20})?
A. 1840
B. 2560
C. 2160
D. 5120

165. M and N are two natural numbers such that M + N = 949. LCM of M and N is 2628. What is the HCF of M and N?
A. 23
B. 73
C. 69
D. none of these

166. Given a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5, then the value of a + b + c + d is
A. –2
B. $-\frac{10}{3}$
C. $\frac{1}{2}$
D. 6

167. If $N = 539 \times 2^{18}$ and $M = 9 \times 2^{13}$, then the remainder when N is divided by M is
A. 8
B. 8×2^{13}
C. 2^{15}
D. none of these

168. If $N = 2^{2999} \times 5^{3002}$, then the sum of the digits of N is equal to
A.7
B.8
C.10
D.13

169. If n is a positive integer such that 2n has 28 positive divisors and 3n has 30 positive divisors, then how many positive divisors does 6n have?
A.32
B.34
C.35
D.36

170. Find the highest natural number N, less than 400, such that N can be written as sum of consecutive natural numbers in 11 ways but cannot be written as sum of 11 consecutive natural numbers.

171. What is the unit digit of $1^{2!} + 2^{3!} + 3^{4!} + \ldots + 49^{50!} + 50^{51!}$?
A.1
B.3
C.5
D.7

172. If {x} denotes fractional part of x then $\{\frac{5^{200}}{8}\}$ is

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{5}{8}$

173. The remainder when $\underbrace{888222888222888222....}_{9235\ \text{digits}}$ digits is divided by 5^3 is

A.1
B.38
C.47
D.103

174. The number A4531B, where A and B are single-digit numbers, is divisible by 72. Then A + B is equal to

A. 5
B. 7
C. 8
D. 4

175. N is the smallest natural number which when multiplied by 7 gives a product P. Every digit of P is one. The product when N is multiplied by 8 is

A.127 784
B.125 384
C.126 984
D.181 384

176. If $[3(150 + S)]^2 = 22752S$ then the value of single-digit number S is

A.0
B.3
C.5
D.9

177. If $S = 5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ where n is any whole number, then S is always divisible by

A.7
B.17
C.19
D.33

178. The highest power of 12 that can divide $5^{36} - 1$ is

A. 1
B. 2
C. 3
D. 4

179. The number 6162 in base 10 is written as $(222)_b$. Then, the base b is equal to

A. 60

B. 55
C. 45
D. 42

180. If in some base x, $(563)_x + (544)_x + (433)_x = (2203)_x$ then x is equal to
- **A.** 6
- **B.** 7
- **C.** 9
- **D.** 11

181. Let M be the greatest number divisible by 8, such that no digit from 0 to 9 is repeated in M. What is the remainder when M is divided by 1000?
- **A.** 320
- **B.** 120
- **C.** 104
- **D.** 96

182. The number 2006! is written in base 22. How many zeroes are there at the end?
- **A.** 500
- **B.** 450
- **C.** 200
- **D.** 199

183. For how many positive integers x between 1 and 1000, both inclusive, is $4x^6 + x^3 + 5$ is divisible by 7?
- **A.** 0
- **B.** 4
- **C.** 11
- **D.** 36

Read the information given below and answer the question that follows.

Two natural numbers a and b are given in base 10. The number a can be written as 212 in base b and 128 in base $b + 2$.

184. The value of $a + b$ in base 10 is
- **A.** 219
- **B.** 125
- **C.** 114
- **D.** 107

185. A three-digit number *abc* is divisible by 7 if
- **A.** 3a + b + c is divisible by 7
- **B.** a + 2b + c is divisible by 7
- **C.** 2a + 3b + c is divisible by 7
- **D.** 2a + 2b + c is divisible by 7

Read the information given below and answer the question that follows.

For single digit numbers a, b and c, $(abc)_7 = (cba)_9$

186. The value of $a + b + c$
- **A.** = 8
- **B.** = 11
- **C.** = 16
- **D.** cannot be determined

187. The digits 1, 2, 3, 4, and 5 are each used once to compose a five-digit number ***abcde*** such that the three-digit number ***abc*** is divisible by 4, ***bcd*** is divisble by 5, and ***cde*** is divisble by 3. Find the digit **a**.

A. 1
B. 2
C. 3
D. 4

188. How many one's will you write in binary representation of $2^{90} - 1$.

A. 88
B. 89
C. 90
D. 91

189. A = $25^{86} - 5^{171}$. How many digits would be there in A when it is represented in base 5?

A. 172
B. 171
C. 87
D. 86

190. Let A = 288^{30}, then the number of factors of A is

A. 1891
B. 9211
C. 8959
D. 7333

191. How many factors of 10^{10} end with a zero?

A. 21
B. 90
C. 100
D. 121

192. Out of first 300 even natural numbers, how many numbers have even number of factors?

A. 276
B. 288
C. 290
D. 294

193. A number has exactly 1024 factors. What can be the maximum number of prime factors of this number?

A. 10
B. 8
C. 6
D. 5

194. How many three-digit natural numbers have less than 5 prime factors?

A. 900
B. 890
C. 871
D. 843

195. The remainder when 25! is divided by 10^7 is

A. 2
B. 4×10^6
C. 6×10^6

D. 2×10^6

196. For how many values of natural number A, where A < 500, is the HCF of A and 2500 equal to 1?

A. 400
B. 200
C. 100
D. 50

197. If A = 10! + 12! + 14! + 16! +…………100!, then the highest power of 2 in A is

A. 7
B. 8
C. 9
D. 10

198. 123456789123456789………….. up to 180 digits when divided by 11 will leave a remainder of

A. 4
B. 5
C. 6
D. 0

199. When a 4 digit number is multiplied by N, the 4 digit no. repeats itself to give an 8 digit no. if the 4 digit no. has all distinct digits then N is a multiple of

A. 11
B. 27
C. 7
D. 73

200. For any natural number n, then which of the following could represent the exact number of zeroes that n! could end for some values of n.

A. 17
B. 29
C. 30
D. 32

201. What is the largest factor of 11! that is one bigger than a multiple of 6 ?

202. If the number 2718AB6 is divisible by 72 (A and B are single digits), then the number of possible ordered pairs (A, B) is

203. The smallest natural number n such that $\frac{16!}{n}$ is a perfect square is

204. In a writing code d_n stands for single digit d coming n times consecutively. For example, $3_5 6_2 7_4$ would mean the number 33333667777. For what ordered triplet (x, y, z) $2_x 3_y 5_z + 3_p 5_q 2_r = 5_3 7_2 8_3 5_1 7_3$?

205. Find the largest positive integer *n* such that $n^3 + 100$ is divisible by (n + 10)

206. Find the smallest number n such that n! is divisible by 990.

207. Let P be a prime number greater than 3. Then, when $P^2 + 17$ is divided by 12, the remainder is

208. A 101 digit number is formed by writing first 55 natural numbers next to each other. Find the remainder when the number is divided by 16.

209. If n is a integer, how many values of n will give an integral value of $\frac{11n^2 + 5n + 6}{n}$?

210. The inhabitants of planet ZORCON use a number system, which is similar to the decimal system used here on EARTH, except that it has been eleven distinct digits instead of usual ten digits. The extra digit is an alien digit theta (θ) which is inserted between the digits 5 and 6. Now 6 – 5 = 2 and theta is the digit which is equidistant from 5 and 6 i.e. $\theta - 5 = 6 - \theta = 1$. All the algebraic signs and operations carry the same meaning as in the usual sense. What is the decimal equivalent of the two digit number "9θ"?

211. Find the smallest natural number n which satisfies the equation $n^3 + 2n^2 = b$, where b is the square of an odd integer.

212. The digits a, b and c of the three-digit natural number abc satisfy the condition 49a + 7b + c = 286. Find the digits a, b and c.

213. A natural number N is 100 times the number of its factors. Find the sum of the digits of N, given that N has only 2 prime factors.

A. 1
B. 2
C. 3
D. 4
E. 5

214. Let p be a prime number such that $p \geq 23$. Let n = p! + 1. The no. of primes in the list (n + 1),
(n + 2), (n + 3), … (n + p - 1)

215. N is a natural number. How many values of N exist, such that $N^2 + 24N + 21$ has exactly three factors?

216. What is the sum of digits of a two digit number, which is 32 less than the square of the product of its digits?

217. abcd is a four-digit number in base 7 such that 2(abcd) = bcda. Find a

218. How many three digit numbers in base 11 are possible such that when they are expressed in base 9, have their digit reversed?

219. Find the smallest number which when divided by 7, 8, 9 gives the remainders as 2, 4, respectively?

220. What should be the values of a and b so that 30a0b03 is divisible by 13?

221. Let x, y and z be natural numbers such that $x > y > z \geq 2$ and xyz = 2002. Let A and B be the maximum and minimum values, respectively, of x + y + z, find A –B.

SOLUTIONS

1. C.

The unit digits of three consecutive odd numbers would be (1, 3, 5), (3, 5, 7), (5, 7, 9), (7, 9, 1) and (9, 1, 3). As the unit digit of the product is 7, only the last triplet of units digits, i.e. (9, 1, 3) will qualify. Therefore, we need to find three consecutive odd numbers ending in 9, 1 and 3 such that there product is 531117. We can see that $80^3 = 512000$. Therefore, the numbers would be lying around 80. The numbers are 79, 81 and 83 and the sum is 243.

2. C.

$\frac{N}{2}$ is a perfect square $\Rightarrow \frac{N}{2} = x^2$. $\frac{N}{3}$ is a perfect cube $\Rightarrow \frac{N}{3} = y^3$. $\frac{N}{2} \times \frac{N}{3} = x^2y^3 \Rightarrow N = \sqrt{6x^2y^3} = xy\sqrt{6y}$. For N to be a natural number, 6y has to be a perfect square $\Rightarrow y = 6 \Rightarrow N = 2^3 \times 3^4 \Rightarrow$ number of divisors = 20.

3. B

The number which has 6 divisors will be of the form ab^2 or c^5, where a, b and c are prime numbers. The numbers are 2×11^2, 5×7^2, 11×5^2, 23×3^2, 29×3^2, 31×3^2, 53×2^2, 59×2^2, 61×2^2, 67×2^2, 71×2^2, 73×2^2 and 3^5.

4. B

The ages will be lying on either side of 1900 for us to have two solutions to same situation. Let the birth year of grandfather be 18ab. Therefore, his age in 1936 = 1936 − 18ab = 36 + 1900 − 18ab = 36 + 100 − ab. This should be equal to ab ⇒ ab = 36 + 100 − ab ⇒ ab = 68. Similarly, the birth year for grandson is 1918. The sum of ages = 68 + 18 = 86

5. A

10^{12} = 1 000 000 000 000. Since the number is greater than 10^{12} and the sum of the digits is 2, one of the zeroes in
1 000 000 000 000 will be replaced by 1. Therefore, there will be 12 numbers generated this way. Apart from this, the number 2 000 000 000 000 also satisfies the criterion. Hence, 13 numbers are possible.

6. B

Every odd square can be written in the form 4k + 1. Let the three squares be 4a + 1, 4b + 1, 4c + 1. Therefore, sum = 4(a + b + c) + 3 = 4k + 3. Therefore, subtracting 3 from these options we should get a multiple of 4. Only B and D fulfill the criterion. Also, every square will be of the form 3n or 3n + 1. Out of three consecutive odd numbers, one will be a multiple of 3.

7. C

The numbers are ab, ba and a0b and the difference is constant between two consecutive numbers. $10b + a - (10a + b) = 100a + b - (10b + a) \Rightarrow b - a = 11a - b \Rightarrow b = 6a$. Therefore, the numbers are 16, 61 and 106. The sum of digits = 7

8. D

Single digit square = 3 (1, 3, 9) Digits written = 3
Two-digit squares = 6 (16, 25, … 81) Digits written = 2 × 6 = 12.
Three-digit squares = 22 (100, 121, … 961) Digits written = 3 × 22 = 66
Total digits written so far = 3 + 12 + 66 = 81. Digits left = 100 − 81 = 19.
After 961, we will start writing four-digit squares – 1024, 1089… with every square we cover four digits. Since we need to cover 19 digits, we will have to write 4 four-digit squares and then we will see the 3rd digit of the fifth four-digit square. The fifth four-digit square = $36^2 = 1296 \Rightarrow$ 3rd digit = 9. Therefore, 100th digit = 9.

9. B

Rule: $a^n - b^n$ is divisible by both a + b and a − b when n is even.

$18^{2000} + 12^{2000} - 5^{2000} - 1 = 18^{2000} - 5^{2000} + 12^{2000} - 1^{2000} \Rightarrow 18^{2000} - 5^{2000}$ is divisible by 13 and 23. Similarly, $12^{2000} - 1^{2000}$ is divisible by 11 and 13. As 13 is the common factor, the whole expression is divisible by 13.

$18^{2000} + 12^{2000} - 5^{2000} - 1 = 18^{2000} - 1 + 12^{2000} - 5^{2000} \Rightarrow 18^{2000} - 1$ is divisible by 17 and 19. Similarly, $12^{2000} - 5^{2000}$ is divisible by 7 and 17. As 17 is the common factor, the whole expression is divisible by 17.
As the expression is divisible by both 13 and 17, it is divisible by 221.

10. A
$2^{600} \times 5^{600}$ will have 601 digits (1 followed by 600 zeroes). To get 604 digits we will have to increase the power of 2 such that we get a four digit number followed by 600 zeroes. The power should also be a multiple of 3 as $8 = 2^3$. The smallest such power is $2^{12} = 8^4$. Therefore, $S = 8^{204} \times 5^{600}$ = 4096000000… (604 digits). The sum of the digits of S = 19.

11. C
$216 = 6 \times 6 \times 6 \times 1 \times 1 \Rightarrow$ Sum = 6 + 6 + 6 + 1 + 1 = 20
$216 = 3 \times 6 \times 6 \times 2 \times 1 \Rightarrow$ Sum = 3 + 6 + 6 + 2 + 1 = 18
$216 = 3 \times 6 \times 3 \times 2 \times 2 \Rightarrow$ Sum = 3 + 6 + 3 + 2 + 2 = 16
$216 = 3 \times 6 \times 3 \times 4 \times 1 \Rightarrow$ Sum = 3 + 6 + 3 + 4 + 1 = 17

We can check that the sum of 19 cannot be obtained.

12. A
$10^4 + 324 = (10^2 + 18)^2 - 2 \times 10^2 \times 18 = (10^2 + 18)^2 - 36 \times 10^2 = (10^2 + 18)^2 - (6 \times 10)^2 = (10^2 + 18 + 60) \times (10^2 + 18 - 60) = 178 \times 58$.
Simplify the rest similarly. Many terms will cancel out.

13. B
4 integers- 0, 10, 16, and 18.

14. A
The number of numbers prime to and less than 900 = $900(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5}) = 240$

15. A

$123456789 \times 999999999 = 123456789 \times (1000000000 - 1) = 123456789000000000 - 123456789 = 123456788876543211$

16. B

When the divisors of $N = X^aY^bZ^c$ are multiplied the product is equal to $(N)^{\frac{(a+1)(b+1)(c+1)}{2}}$. When the divisors of $72 = 2^3 \times 3^2$ are multiplied, the product is equal to $(2^3 \times 3^2)^{\frac{(2+1)(3+1)}{2}} = 2^{18} \times 3^{12}$. Therefore, a + b =30

17. A
If a number is divisible by 72, it is divisible by 8 and 9. To be divisible by 8, the number formed by the last three digits should be divisible by. Therefore, 31B should be divisible by 8 $\Rightarrow$ B = 2. To be divisible by 9, the sum of the digits of the number should be divisible by 9 $\Rightarrow$ A = 3. Therefore, A + B = 5

18. D
There is only one perfect square in the nineteenth century 432 = 1849. Therefore, the man was 43 years old in 1849. Therefore, he was 78 years old in 1884.
19.A

Only perfect divisors have odd number of divisors. Number of perfect divisors less than 65 = 8 (1, 4, 9, 16, 25, 36, 49, 64)

20. D

After the first cycle, 80 people will remain and 99th person will be the last. After the second cycle 64 people will remain and 98th person will be the last. Now, even after the subsequent cycles, the 98th person will not leave the queue as the number of remaining people will never be a multiple of 5. Note that the last person leaves the queue only if the number of remaining people is a multiple of 5.

21. C

The king was born in 1600 (40^2) and died in 1681 (41^2). He lived for 81 years.

22. A

For page number 1 to 9, single digits will be used per page. Total digits used = 9
From page number 10 to 99, two digits will be used per page. Total digits used = 2×90 = 180.
Total digits used till page number 99 = 9 + 180 = 189.
Digits left = 300 − 189 = 111.
From page 100 onwards, three digits will be used per page. Therefore, number of pages required to cover 111 digits = 111/3 = 37.
Therefore, the book has 136 pages.

23. B

Let p = n, q = n + 1 and r = n(n + 1) $\Rightarrow p^2 + q^2 + r^2 = n^2 + n^2 + 1 + 2n + n^2(n^2 + 2n + 1) = n^4 + 2n^3 + 3n^2 + 2n + 1 = (n^2 + n + 1)^2 \Rightarrow S = n^2 + n + 1 = n(n + 1) + 1$ = even number + odd number = odd number.

24. D

25. C

The number is the sum of consecutive factorial and it is less than 500. Therefore, the number cannot have 6! Or above in the sum of consecutive factorial as 6! = 720 > 500. As the number is a three-digit number it will certainly have 5! in the sum of consecutive factorial. Now the following possibilities are there for the number-

5! + 4! = 144
5! + 4! + 3! = 150
5! + 4! + 3! + 2! = 152
5! + 4! + 3! + 2! + 1! = 153.

Now the number is the sum of the cube of its digits. Only 153 satisfies the criterion. Sum of the digits of 153 = 9.

26. A

The number is 144 which is a perfect square. 441 is also a perfect square. 1444 is also a perfect square.

27. C

Let the two digit number be ab. The value of this number = 10a + b. By reversing the digits, the number becomes ba, with value 10b + a.
10a + b + 10b + a = 11(a + b) = a perfect square $\Rightarrow$ a + b = 11 $\Rightarrow$ The pairs are (2, 9), (3, 8), (4, 7), (5, 6).
10a + b − (10b + a) = 9(a − b) = a perfect square $\Rightarrow$ a − b = 1, 4 or 9. Only the pair (5, 6) satisfies one of the three differences. Therefore, 65 is the age of the teacher and it is divisible by 13.

28. B

The number of numbers less than and prime to 120 = $120\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)=32$

29. B

$$3^{27^{x}} = 3^{(3^{3})^{x}} = 3^{3^{3x}}$$

$$27^{3^{x}} = (3^{3})^{3^{x}} = 3^{3 \times 3^{x}} = 3^{3^{x+1}}$$

$\Rightarrow 3x = x + 1 \Rightarrow x = 1/2.$

30. D

$$\frac{1}{\sqrt{2}-\sqrt{1}} = \frac{1}{\sqrt{2}-\sqrt{1}} \times \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}+\sqrt{1}} = \sqrt{2}+\sqrt{1}$$

$$\frac{1}{\sqrt{2}-\sqrt{1}} - \frac{1}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}-\sqrt{3}} - \ldots - \frac{1}{\sqrt{121}-\sqrt{120}} \; \sqrt{1}+\sqrt{2}-\sqrt{2}-\sqrt{3}+\sqrt{3}+\sqrt{4}-\sqrt{4}-\sqrt{5}+\ldots-\sqrt{121}-\sqrt{120} = -11 + 1 = -10$$

31. B

$(n + 1)^3 - n^3 = 1027 \Rightarrow 3n^2 + 3n - 1026 = 0 \Rightarrow n^2 + n - 342 = 0 \Rightarrow n = 18.$

32. A

All multiples of 4 and all odd numbers can be written as difference of squares of non-negative integers. Odd numbers in first 1000 natural numbers = 500. Multiples of 4 = 250. Therefore, total numbers = 750.

33. B

P = abc = 9(a + b + c). Let a = b + c $\Rightarrow$ abc = 18a $\Rightarrow$ bc = 18 $\Rightarrow$ b, c = (1, 18), (2, 9), (3, 6) $\Rightarrow$ a = 19, 11, 9 $\Rightarrow$ abc = 342, 198, 162 $\Rightarrow$ Sum = 702

34. D

Keep the values. Neither of the statements are true.

35. A

m and n are 7 and 8. Therefore, m + n = 15

36. D

$$\begin{array}{r} 1ABCDE \\ \times 3 \\ \hline ABCDE1 \\ \hline \end{array}$$

Going from right to left, E = 7, D = 5, C = 8, B = 2, A = 4. The sum of the digits = 27

37. C

The sum of the squares of three odd numbers will be in the form 4k + 3. Only C satisfies the criterion.

38. B

S = 333…3334 (33 digits). Sum of digits = 3 × 32 + 4 = 100

39. A

Let the number of slips with 14 written on them be x. Therefore, sum = 14x + 13(x + 5) = 27x + 65. Therefore, the number should be of the form 27k + 65. Only A satisfies the option.

40. B

41. D

$12^{12} = 2^{24} \times 3^{12}$, $6^6 = 2^6 \times 3^6$, $8^8 = 2^{24}$. The LCM is $12^{12} = 2^{24} \times 3^{12}$. 2^{24} will come from 8^8 but 3^{12} will have to come from k. Therefore, k will contain 3^{12}. Also, the powers of 2 in k can be from 0 to 24. Therefore, k can be equal to 3^{12}, 2×3^{12}, $2^2 \times 3^{12}$, …$2^{24} \times 3^{12}$. In total there 25 values.

42. D

(x + y)(x – y) = 343

43. A

The divisors of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180 and 360. We can see that 360 = 1 × 360 = 2 × 180 = 3 × 120 = … = 18 × 20.

Therefore, $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{180}+\frac{1}{360}=\frac{360}{360}+\frac{180}{360}+\frac{120}{360}+\ldots+\frac{2}{360}+\frac{1}{360}=\frac{1+2+3+4+5\ldots+180+360}{360}=\frac{1170}{360}=3.25$

44. B

$\frac{x}{|x|}$ is equal to either 1 or –1, depending on whether x is positive or negative. Taking zero negative, one negative, two negative, three negative or four negative out of a, b, c, and d, we obtain the values of $S=\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{abc}{|abc|}$ to be {4, 0, –4} and the values of $T=\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{d}{|d|}+\frac{abcd}{|abcd|}$ equal to {5, 1, –1, –3}. Hence the intersection of S and T is a null set.

45. C

46. B

The digit sum of the number = 1 + 2 × 2 + 3 × 3 + 4 × 4 = 30 = 3 + 0 = 3. Therefore, the number cannot be a perfect square as the digit sum of perfect squares are always 1, 4, 7 or 9.

47. B

The maximum marks that can be achieved is 90 and the minimum marks that can be achieved is –30. Technically, from –30 to 90, we should have all the scores at a difference of $\frac{1}{3}$, i.e. $-30, -29\frac{2}{3}, -29\frac{1}{3}, -29, \ldots 0, \ldots 89, 89\frac{1}{3}, 89\frac{2}{3}, 90$. Therefore, there should be 90 + 1 + 270 = 361 scores possible. But we cannot achieve scores of $89\frac{2}{3}, 89\frac{1}{3}$ and $88\frac{1}{3}$ (think why). You can obtain rest of the scores. Therefore, total number of scores possible = 361 – 3 = 358.

48. C

$\sqrt{1332} = 6\sqrt{37}$. Therefore, $\sqrt{x}$ and $\sqrt{y}$ should be of the form $a\sqrt{37}$ and $b\sqrt{37}$ where a + b = 6. Therefore, pairs of (a, b) = (0, 6), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0).

49. B

Every square can be written in the form 3k or 3k + 1. $p^2 + 15p - 1 = (3k + 1) + 15p - 1$ = a multiple of 3. Therefore, it cannot be a prime number. If p = 3, $p^2 + 15p - 1 = 53$. Therefore, the expression is prime for only one value of p.

50. C

51. $2^6 - 1 = (2^3)^2 - 1$ which is a multiple of $2^3 - 1$ as an – bn is always divisible by a – b. similarly $2^9 - 1$ is also a multiple of $2^3 - 1$. Therefore, HCF = $2^3 - 1$. Now HCF × LCM = product of two numbers.
$\Rightarrow (2^3 - 1) \times \text{LCM} = (2^6 - 1) \times (2^9 - 1) \Rightarrow \text{LCM} = 2^{12} + 63 \times 2^3 - 1$.

52. The pair will be (45a, 45b) where a and b will be co-prime to each other. Now 21600 = $2^5 \times 3^3 \times 5^2$. To find a and b, we first take the factor of 45 from 21600, which leaves $2^5 \times 3 \times 5$. Now we need to find the number of co-prime pairs (a, b) that we can make out of $2^5 \times 3 \times 5$. Let's write down the powers of the prime factors in order to find the co-prime factors: $(2, 2^2, 2^3, 2^4, 2^5)$, 3, 5

Therefore, the number of co-prime pairs is found by various combinations of these prime factors:

(Prime factor, Prime factor) – $(2, 3), (2^2, 3), (2^3, 3)\ldots (2^4, 5), (2^5, 5), (3, 5)$ ----- 11 in number
(Two prime factors, prime factor) – $(2 \times 3, 5), (2^2 \times 3, 5),\ldots (2^4 \times 5, 3), (2^5 \times 5, 3), (3 \times 5, 2), \ldots (3 \times 5, 2^5)$ --- 15 in number
(1, prime factor) – $(1, 2), (1, 2^2) \ldots (1, 2^5), (1, 3), (1, 5)$ --- 7 in number
(1, two prime factors) – $(1, 2 \times 3), (1, 2^2 \times 3), \ldots (1, 2^4 \times 5), (1, 2^5 \times 5), (1, 3 \times 5)$ --- 11 in number
(1, three prime factors)- $(1, 2 \times 3 \times 5), (1, 2^2 \times 3 \times 5), \ldots (1, 2^5 \times 3 \times 5)$ --- 5 in number.

Therefore, total number of co-prime pairs (a, b) = 49.

53. **A**
54. **B**
The number will be of the form $a \times b$ or c^3, where a, b and c are prime factors. Count all the numbers below 100 of the above form.
55. **A**
56. **A**
$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + b^{n-1})$
$\Rightarrow 30^{65} - 29^{65} = (30 - 29)(30^{64} + 30^{63}29 + 30^{62}29^2 + \ldots + 29^{64}) > 30^{64} + 29^{64}$.

57. **C**
Let A = abc and B = cba. Therefore, B – A = 100c + 10b + a – (100a + 10b + c) = 99(c – a). B – A is a multiple of 7 $\Rightarrow$ c – a = 7 $\Rightarrow$ (a, c) = (1, 8) or (2, 9). Therefore, the number can be from 108 to 19 or from 209 to 299.

58. **C**
59. **D**
60. $a - xb = a + |a|\, b^2$. $|a| \geq a$ and $b^2 > 1$. Therefore, $a - xb > 0$.

61. **A**
62. **D**
The number of kilometers that the car has traveled = 1 000 – number of numbers which contain the digit 5 = 1000 – (1 + 18 + 252) = 729.

63. **A**

64. 111 = 37 × 3. Therefore we find the remainder when the expression is divided by both 3 and 37. Now we know that if x is prime to p, where p is a prime number, $(x)^{p-1} - 1$ is divisible by p. In other words, $(x)^{p-1}$ gives remainder 1 when divided by p. Therefore, both 17^{36} and 19^{36} will give remainder 1 with 37. Therefore, total remainder with 37 = 1 + 1 = 2. Also, $17^{36} = (17^{18})^2 \Rightarrow$ will give remainder 1 with 3. Therefore, both 17^{36} and 19^{36} will give remainder 1 with 3. Therefore, total remainder with 37 = 1 + 1 = 2. Therefore, remainder with 111 = 2.
65. Let $7^{13} = 7^a \times 7^b \times 7^c$. The powers should add upto 13 $\Rightarrow$ (a, b, c)
= (0, 0, 13), (0, 1, 12), (0, 2, 11), (0, 3, 10)… (0, 6, 7),
(1, 1, 11), (1, 2, 10),… (1, 6, 6),
(2, 2, 9), (2, 3, 8)…. (2, 5, 6),

(3, 3, 7), (3, 4, 6), (3, 5, 5)
(4, 4, 5)
Total number of ways= 7 + 6 + 4 + 3 + 1 = 21

66. It can be seen that N is not a multiple of 2 and 3 as introducing a factor of 2 or 6 doubles and quadruples the number of factors. But N is a multiple of 5 as introducing 15 does not quadruples the number of factors. We see that taking N = 5 satisfies the given options. Now $30N = 2 \times 3 \times 5^2$ $\Rightarrow$ factors = 12 = 6f.

67. C

$$\frac{3}{40} = \frac{6}{80} = \frac{9}{120} = \frac{12}{160} = \frac{15}{200} = \frac{18}{240} = \ldots$$

The numerator is going to be the sum of the digits and the denominator is going to be the product. To know the digits of the number, we can factorize the denominator. We can quickly check and see that we can find single digit numbers satisfying the conditions only for the ratio $\frac{18}{240}$ (2, 3, 8, 5).

Therefore, we need to find a four-digit number made by 2, 3, 8, and 5, and which is divisible by 37.

The rule for divisibility of 37 is that we make groups of 3 from right to left and keep adding them. The number thus obtained should be divisible by 37. Mentally I got the number as 5328 because (328 + 5 = 333 --> divisible by 37).

$5328 = 2^4 \times 3^2 \times 37 \Rightarrow$ number of factors = $5 \times 3 \times 2 = 30$

68. A

69. B

Find a number with more than 20 divisors, maybe 21, 22, 24 divisors. Then, remove some of the divisors such the sum of the rest of the divisors was 801. And Voila! I got my answer!

360 is the number with 24 divisors. The sum of these divisors is 1170. If you remove 360, 2, 3 and 4 from these divisors, you have 20 numbers whose sum is 801. You cannot find a number smaller than this.

Therefore, the smallest LCM is 360 and the 20 numbers are all the divisors of 360 minus 360, 2, 3, and 4.

70. B

71. C Let N be a two-digit number ab $\Rightarrow$ 10a + b = 11(a + b) $\Rightarrow$ no solution for positive a and b. Let N be a three-digit number abc $\Rightarrow$ 100a + 10b + c = 11(a + b + c) $\Rightarrow$ 89a = b + 10c $\Rightarrow$ (a, b, c) = (1, 9, 8). Therefore, the number is 198 and it is divisible by 9.

72. C

73. B

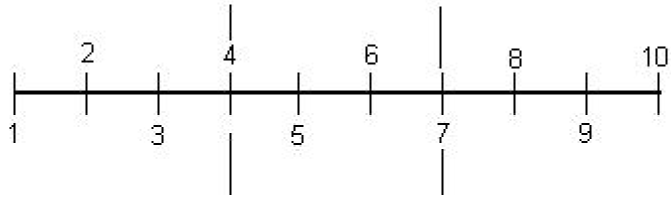
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}} = \sqrt{2^2-\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)^2} = \sqrt{2-\sqrt{2+\sqrt{3}}}$$

$$\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}} = \sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2-\sqrt{2+\sqrt{3}}} = \sqrt{2-\sqrt{3}}$$

$$\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}} = \sqrt{2+\sqrt{3}}\sqrt{2-\sqrt{3}} = 1$$

74. C

Let's start small in this case. In place of 1 000 numbers, let us take the first 10 natural numbers. They are shown below:



Notice that two consecutive numbers have a common difference of 1 between them. If you take this common difference as an interval, you have 9 such intervals. Now we choose 3 or more numbers in AP with first and last terms being 1 and 10, respectively, by putting partitions, as shown. The partitions will have to be put such that there are equal number of intervals between any two consecutive partitions (since the numbers are to be in AP, equal number of intervals mean same common difference).

In essence we are trying to find the number of ways of dividing 9 intervals equally. This will be nothing but the number of divisors of 9 i.e. 1, 3, and 9. Since we cannot take 1, we can only take 3 and 9.

Similarly, we are trying to find the number ways of dividing 999 intervals equally, or the number of divisors of 999. Now $999 = 3^3 \times 37$ therefore number of divisors $= 4 \times 2 = 8$. And not counting 1, the number of ways = 7.

75. C

Let the 4 odd numbers be $2n + 1$, $2n + 3$, $2n + 5$, and $2n + 7$. The sum of these four numbers is equal to $8n + 16$ or $8(n + 2)$. This is a multiple of 8. Since the sum is divisible by 10, it is also a multiple of 10. Therefore, the sum is a multiple of 40 (LCM of 8 and 10).

Let the sum = 40k. Now 40k when divided by 10, leaves 4k which is a perfect square. 4 is a square therefore k also has to be a perfect square.

Therefore $k = 1, 4, 9, 16, 25 \Rightarrow 40k = 40, 160, 360, 1000$..etc.

$8n + 16 = 40, 160, 360, 1000 \Rightarrow n = 3, 18, 43, 123$.

$\Rightarrow 2n + 1 = 7, 37, 87…$

numbers are (7, 9, 11, 13) or (37, 39, 41, 43), (87, 89, 91, 93) etc..

Only 41 is given in the options.

76. A

77. A

$10b + a - (10a + b) = 18 \Rightarrow 9(b - a) = 18 \Rightarrow b - a = 2$. Apart from 13 there are 6 such numbers which satisfy this criterion. 24, 35, 46, 57, 68 and 79.

78. D

x = 5, y = 2. the four numbers are 2, 3, 5 and 7. There sum is 17 which is prime.

79. C

Denominator can be 99, 33, 11, 9 and 3.

80. D

The integer, the last three digits of whose square are x25, will have units digit as 5. Let the last three digit of the number be ab5. Now the last three digits of the square will come from the square of the last three digits of the numbers only. Therefore, the last three digits of the square = last three digits of ab5 $\times$ ab5. Multiplying, we can see that digit x = units digit of $b^2 + b = 0, 2$ or 6.

81. Since R.H.S. has 7! Multiplied by a positive quantity, L.H.S will be greater than 7!. As R.H.S. does not have the prime factor of 11, L.H.S. will be less than 11!. As R.H.S. has only one power of 5 (in

5!), L.H.S. will be less than 10!. Therefore, only possibilities are 8! And 9!. Considering powers of 3 on both sides, n = 9.

82. We can see that $5 \le N \le 6$ as $25 \le N^2 \le 36$. Therefore, N is between 5 and 6 $\Rightarrow$ [N] = 5. Now N $\times$ [N] = 27

$\Rightarrow N = \frac{27}{5} = 5.4$

83. Let the ages of Dhondu and Bhondu be **ab** and **cd**, respectively. Therefore, **abcd** = m^2. Also **pqrs** = n^2, **pq** = **ab** + 31 and **rs** = **cd** + 31. **pqrs** – **abcd** = 3131 = $n^2 - m^2 \Rightarrow 31 \times 101 = (n + m)(n - m) \Rightarrow n = 66$ and m = 35. Therefore, abcd = 35^2 = 1225 $\Rightarrow$ Dhondu's age = 12.

84. Number of factors of N = 16 $\times$ 13 = 208. Number of factors of N^2 = 31 $\times$ 25 = 775. Now, for N^2, leaving aside N, half of the factors of N^2 will be lying below N and the other half will be lying above N. Therefore, number of factors of N^2 lying below N = 774/2 = 387. Therefore, numbers which are multiples of N but not multiples of N^2 = 387 – 207 (not considering N) = 180.

85. **C**

86. The number of combinations of Initial names and surnames = 26 $\times$ 26 = 676. Therefore, a group of 676 people can be given sets of different initial names and surnames. Then another group of 676 people can be given same sets of initial names and surnames. In all, three groups of 676 people each can be given combinations of different names and surnames such that three persons, one from each group, will have the same combination. This makes 2028 people. Then the remaining one person can be given any one of the 676 combinations. Therefore, at least 4 people will have the same combination.

87. **D**

88. **D**

For every 7 days the grandfather is counting only 6 days. Therefore, the actual number of years =

$\frac{84 \times 7}{6} = 98$

89. **D**

We note that the denominator in the new fraction has to be a multiple of 3, i.e. the sum of the digits has to be a multiple of 3. The sum of the digits in the current denominator (4542) = 4 + 5 + 4 + 2 = 15. Therefore, if we need to shift 2 digits from the denominator, they should be multiple of 3 to keep denominator as multiple of 3. Also, from the numerator, we need to shift two digits which are a multiple of 3.

Also, we keep in mind that unit digit of the denominator = 3 $\times$ unit digit of the numerator. Working this way we find the new fraction to be $\frac{1352}{4506}$.

90. B

Work with options.

91. D

The sum of the numbers has to be more than 575 as one number has been missed out. The nearest sum greater than 575 is the sum of first 34 natural numbers, i.e. $\frac{34 \times 35}{2} = 595$. Therefore, the missing number was 20. Note that the sum of first 35 natural numbers will not work. (why?)

92. We can check that L(x, y) = R(x, y) for x, y being integer

We can now put non integral values and check.

Let x, y = (1, 1.5) $\Rightarrow$ L(x, y) = 4 R(x, y) =5.

Now only c can be valid.

93. Because each word is lit for a second,

$$\text{LCM}\left(\frac{5}{2}+1, \frac{17}{4}+1, \frac{41}{8+1}\right) = \text{LCM}\ \frac{7}{2}, \frac{21}{4}, \frac{49}{8}$$

$$\frac{LCM(7,21,49)}{HCF(2,4,8)} = \frac{49\times 3}{2} = 73.5 \text{ s}$$

94. $\text{HCF}\left(\frac{9}{2}, \frac{27}{4}, \frac{36}{5}\right) = \frac{HCF(9,27,36)}{LCM(2,4,5)} = \frac{9}{20}$ Ib

= Weight of each piece

Total weight= 18.45 Ib

Maximum number of guests = $\frac{18.45\times 20}{9}$ = 41

95. If n is a composite number, all the factors of n (except x itself) will be present amongst n-1, n-2, n-3,…. 3, 2, 1.

Therefore the product (n-1) (n-2) (n-3)…. 3.2. I will be divisible by n. The product will not be divisible by n only if n is a prime number. There are 7 such prime number between 12 and 40.

96. Let's start picking up numbers in increasing order and see how many numbers we can take. If we pick up 3, we cannot pick up 467. If we pick up 1) we cannot pickup 459,…. And so on. The highest number we can pick up this way is 235. This makes it 30 numbers.

97. x is and is even $\Rightarrow$ x-z is old $\Rightarrow$ $(x-z)^2$ is odd

y is odd $\Rightarrow$ y $(x-z)^2$ is odd. Statement A is wrong.

98. a = b (b-1) $\Rightarrow$ a is product of two consecutive numbers

$\Rightarrow$ a is even, a^2-2a = a(a-2) $\Rightarrow$ product of two consecutive even numbers $\Rightarrow$ a is a multiple of 8. a-2 = b^2-b-2 = (b-2) (b+1)

$\Rightarrow$ a^2-2a = b(b-1) (b-2) (b+1) $\Rightarrow$ (b-2) (b-1) (b) (b+1) = product of 4 consecutive natural numbers

$\Rightarrow$ divisible by 41 = 24

99. The actual sum would be less then 1000. Let's find out the nearest value of the sum $\frac{n(n+1)}{2} \approx 1000$ $\Rightarrow$ n(n+1)= 2000. We take the roots both sides to arrive at x as n and n+1 are two consecutive numbers and their product $\approx x^2$ =2000 $\Rightarrow$ n $\approx$ 44. For n=44, sum = $\frac{44X45}{2}$ = 990. Therefore, page number 10 was added twice.

100. x= $\sqrt{2(1+\sqrt{2})}$ $x^2 = 2+2\sqrt{2} \Rightarrow (x^2-2)= 2\sqrt{2}$

$x^3+x^2-2x-2 = x(x^2-2)+x^2-2 = (x^2-2)(x+1)$

$= 2\sqrt{2}(\sqrt{2(1+\sqrt{2})}+1) = 2\sqrt{2} + 4\sqrt{1+\sqrt{2}}$

101. $\frac{5x+23}{x-7} = \frac{5(x-7)+58}{x-7} = 5+\frac{58}{x-7}$. For this to be integer $\frac{58}{x-7}$ should be an integer $\Rightarrow$ 58 is divisible by x-7 $\Rightarrow$ x-7 = -58, -29, -2, -1, 1, 2, 29, 58 $\Rightarrow$ We have corresponding 8 value for n.

102. D

Finding the powers of all the prime factors in 30! We obtain, 30! = $2^{26} \times 3^{14} \times 5^7 \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29$. Removing 27×57 for the number of zeroes, we obtain $2^{19} \times 3^{14} \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29$. We need to find the units digit of this product to find the rightmost non-zero digit. The units digit of the product = units digit of $8 \times 9 \times 1 \times 1 \times 9 \times 7 \times 9 \times 3 \times 9 = 8$.

103. D

Let's see the last two digits of summation in the groups of 4.

$3 + 3^2 + 3^3 + 3^4 = 03 + 09 + 27 + 81 = 20$

$3^5 + 3^6 + 3^7 + 3^8 = 43 + 29 + 87 + 61 = 20$

$3^9 + 3^{10} + 3^{11} + 3^{12} = 83 + 49 + 47 + 41 = 20$

Similarly, for 7,

$7 + 7^2 + 7^3 + 7^4 = 07 + 49 + 43 + 01 = 00$

$7^5 + 7^6 + 7^7 + 7^8 = 07 + 49 + 43 + 01 = 00$

$(3 + 3^2 + 3^3 + 3^4) + (3^5 + (3^6 + 3^7 + 3^8) + \ldots + 3^{400} = (20) + (20) + \ldots + 20 = 00$

$(7 + 7^2 + 7^3 + 7^4) + (7^5 + 7^6 + 7^7 + 7^8) + \ldots + 7^{200}) + 7201 = 00 + 00 + 00 + \ldots + 00 + 07 = 07$

Therefore, difference = 00 – 07 = 93.

104. A

The number of numbers prime to and less than 900 = $900(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5}) = 240$

105. D

The units digit is found by finding the remainder of the exponent with 4. 4! and onwards the remainder with 4 will be 0. Therefore, we need to find the remainder of 1! + 2! + 3! with 4 $\Rightarrow$ remainder = 1. Therefore, units digit of the expression = units digit of $63^1 + 18^1 + 37^1 = 8$

106. C

Let's see the cycle of the last two digits for powers of 4

$4^2 = 16$
$4^3 = 64$
$4^4 = 56$
$4^5 = 24$
$4^6 = 96$
$4^7 = 84$
$4^8 = 36$
$4^9 = 44$
$4^{10} = 76$
$4^{11} = 04$.

Therefore, 4^1 and 4^{11} have the same last two digits, i.e. the last two digits of 4 repeat after every increase of 10 in the exponent. Therefore, 4^{1991} will have the same last two digits. Therefore, 4^{1997} will have the same last two digits as $4^7 = 84$.

107. B

$7^3 = 343$ gives remainder 1 with 9. $7^{77} = 7^{75} \times 7^2$ = remainder with 9 $(1 \times 49) = 4$.

108. D

After the first cycle, 80 people will remain and 99th person will be the last. After the second cycle 64 people will remain and 98th person will be the last. Now, even after the subsequent cycles, the 98th person will not leave the queue as the number of remaining people will never be a multiple of 5. Note that the last person leaves the queue only if the number of remaining people is a multiple of 5.

109. D

$a^n + b^n$ is divisible by a + b when n is **odd** $\Rightarrow 5^{2n+1} + 17^{2n+1}$ is divisible y 17 + 5 = 22 $\Rightarrow 5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ is divisible by 11. The expression is also divisible by 3. Therefore, it is divisible by 33. **NOTE:** Keeping n = 0 will give the answer straightaway.

110. C

The unit digit of 53^{53} = 3. Unit digit of 27^{27} = 3. Therefore, unit digit of $53^{53} - 27^{27}$ = 0. Therefore, the expression is divisible by 10.

111. B

Units digit of 43^{444} = 1. Units digit of 34^{333} = 4. Therefore, units digit of expression = 5 $\Rightarrow$ the expression is divisible by 5.

112. A

The number of numbers less than and prime to 13 = 12. Therefore, 2^{12} gives remainder 1 with 13. $2^{2005} = 2^{2004} \times 2 = (2^{12})^{167} \times 2 \Rightarrow$ Remainder $(1 \times 2) = 2$.

113. B

The number of numbers less than and prime to 120 = $120\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 32$

114. D

Numbers from 5! and onwards are divisible by 20. Therefore we need to find the remainder when 1! + 2! + 3! + 4! is divided by 20. 1! + 2! + 3! + 4! = 33 $\Rightarrow$ remainder = 13.

115. C

We first do the prime factorization of the product by finding the powers of the prime factors in all the factorials.

Powers of 2: 2! = 1, 3! = 1, 4! = 3, 5! = 3, 6! = 4, 7! = 4, 8! = 7, Total = 23

Powers of 3: 3! = 1, 4! = 1, 5! = 1, 6! = 2, 7! = 2, 8! = 2. Total = 9

Powers of 5: 5! = 1, 6! = 1, 7! = 1, 8! = 1. Total = 4

Powers of 7: 7! = 1, 8! = 1. Total = 2.

Therefore, the product = $2^{23} \times 3^9 \times 5^4 \times 7^2$.

Number of divisors that are perfect square = (number of even powers of 2) $\times$ (number of even powers of 3) $\times$ (number of even powers of 5) $\times$ (number of even powers of 7) = $12 \times 5 \times 3 \times 2$ = 360.

116. D

Number of numbers less than and prime to 9 = $9\left(1-\frac{1}{3}\right) = 6$. Therefore, 256 will give remainder 1 with 6. $25^{25^{25}} = 25^{(24+1)^{25}} = 25^{6k+1} = (25^6)^k \times 25 \Rightarrow$ Remainder = $1 \times 7 = 7$

117. B

7^4 gives a unit digit of 1. $7^{7^7} = 7^{(8-1)^7} = 7^{4k+3} = (7^4)^k \times 7^3$ = units digit $1 \times$ units digit $3 = 3$

118. D

$5x^2 + 2y^2 = 5922$, the R.H.S. is even and therefore, L.H.S. should also be even $2y^2$ is even therefore, $5x^2$ should also be even, therefore x will be even $\Rightarrow 5x^2$ will end in 0. As the unit digit of R.H.S. is 2, the unit digit of $2y^2$ should also be 2 $\Rightarrow$ unit digit of y^2 should be 1 $\Rightarrow$ unit digit of y can be 9.

119. C

$6^{83} + 8^{83} = (7 + 1)^{83} + (7 - 1)^{83} = (1 + 83 \times 7 + \ldots + 7^{83}) + (-1 + 83 \times 7 - \ldots + 7^{83}) = 2 \times 83 \times 7$ + (a term divisible by 49). Remainder = 35.

120. C

Let 320 = x. Therefore the given expression is $f(x) = 81x^4 + 27x^3 + 9x^2 + 3x + 1$. To find the remainder by x + 1, keep x = –1.

121. C

$\underbrace{123123123.....123123}_{300\text{digits}} = 123\underbrace{1001001....1001}_{298\text{ digits}} = 123(10^{297} + 10^{294} + 10^{291} + ... + 10^3 + 1)$. 10^{odd} gives a remainder of 10 with 99 whereas 10^{even} gives a remainder of 1 with 99. Therefore, in the expression $10^{297} + 10^{294} + 10^{291} + ... + 10^3 + 1$, we will get 50 10 and 50 ones when we divide by 99. Also, 123 gives remainder 24 with 99.

Therefore, remainder = 24(50 × 10 + 50) = remainder when 24 × 550 is divided by 99 = 33.

122. D

We can solve this by finding the last two digits of every number. The last two digits of every term (except the first one) will be 11. Therefore, last two digits of the expression = 1 + 9 × 11 = 1 + 99 = 00. Therefore, the expression is divisible by 100.

123. Let $f(x) = x^{200} - 2x^{199} + x^{50} - 2x^{49} + x^2 + x + 1$.
Remainder by x – 1 = 1 ⇒ f(x) = G(x)(x – 1) + 1 --- (1)
Similarly, f(x) = H(x)(x – 2) + 7. --- (2)
Multiply 1 by (x – 2) and 2 by (x – 1), and subtract.

124. $x^2 - 3x + 2 = (x - 1)(x - 2)$
Remainder when x^{100} is divided by x – 1 = 1. Remainder when x^{100} is divided by x – 2 = 2^{100}. Therefore, $x^{100} = Q(x)(x - 1) + 1$ and $x^{100} = R(x)(x - 2) + 2^{100}$. Multiply first equation by x – 2 and the second equation by x – 1 and subtract. The result is
$X^{100} = H(x)(x - 1)(x - 2) + x(2^{100} - 1) + 2 - 2^{100}$. Therefore, we get the values of a and b.

125. C

$16^3 + 19^3$ is divisible by 16 + 19 = 35, ($a^n + b^n$ is divisible by a + b is n is **ODD**). Similarly, $17^3 + 18^3$ is divisible by 35. Also, the expression contains two even and two odd terms. Therefore, the sum is even. Therefore, the whole expression is also divisible by 2 ⇒ it is divisible by 70.

126. A
127. C
128. D
129. N = D × Q + 52 ⇒ 5N = 5D × Q + 260. 5N gives remainder 4 with D. As 5DQ is divisible by D, 260 should give remainder 4 with D. Therefore, 260 – 4 = 256 should be divisible by D. Now D can have values equal to divisors of 256 which are greater than 52 (as D gives a remainder 52 it has to be greater than 52). The only values are 64, 128 and 256.
130. 111 = 37 × 3. Therefore we find the remainder when the expression is divided by both 3 and 37. Now we know that if x is prime to p, where p is a prime number, $(x)^{p-1} - 1$ is divisible by p. In other words, $(x)^{p-1}$ gives remainder 1 when divided by p. Therefore, both 17^{36} and 19^{36} will give remainder 1 with 37. Therefore, total remainder with 37 = 1 + 1 = 2. Also, $17^{36} = (17^{18})^2$ ⇒ will give remainder 1 with 3. Therefore, both 17^{36} and 19^{36} will give remainder 1 with 3. Therefore, total remainder with 37 = 1 + 1 = 2. Therefore, remainder with 111 = 2.
131. 25! + 26! + 27! + 28! + 30! = 25! (1 + 26 + 27 × 26 + 28 × 27 × 26) + 30! = 25! × a number ending in 5 + 30! ⇒ 6 zeroes.
132. A
The number of zeroes = the highest power of 5 =
$$\left[\frac{5^5}{5}\right] + \left[\frac{5^5}{25}\right] + \left[\frac{5^5}{125}\right] + \left[\frac{5^5}{625}\right] + \left[\frac{5^5}{3125}\right] = 625 + 125 + 25 + 5 + 1 = 781$$
133. In this case, we check the powers of both the highest and the lowest prime number contained in 100!. The highest and the lowest prime numbers in 100! are 97 and , respectively, and their highest power in 100! are 1 and 97 respectively. The highest powers of 2 and 97 in 10000! are

9995 and 104, respectively. Therefore, the values of K according to 2 and 9 are $\left[\frac{9995}{97}\right] = 103$ and

$\left[\frac{104}{1}\right] = 104$, respectively. As the value of 2 cannot exceed 103, K = 103.

134. The number of numbers less than and prime to 13 = 12. Therefore, 712 will give remainder 1 when divided by 13.

$$7^{7^7} = 7^{7\times7\times7\times7\times7\times7\times7} = 7^{49\times49\times49\times7} = 7^{(48+1)\times(48+1)\times(48+1)\times7} = 7^{12k+7}$$

$$\Rightarrow \text{Remainder by } 13 = \text{Remainder when } 7^7 \text{ is divided by } 13.$$

$$7^7 = 49\times49\times49\times7 = \text{Remainder}[-4\times-4\times-4\times7] = 6$$

135. Since R.H.S. has 7! Multiplied by a positive quantity, L.H.S will be greater than 7!. As R.H.S. does not have the prime factor of 11, L.H.S. will be less than 11!. As R.H.S. has only one power of 5 (in 5!), L.H.S. will be less than 10!. Therefore, only possibilities are 8! And 9!. Considering powers of 3 on both sides, n = 9.

136. The last two digits of $\frac{1}{5^{903}}$ = The last two digits of 2^{903} = 08.

137. Since we look for 210 to determine the last two digits in case of 2, we first need to find the last two digits of the power, i.e. 2^{2003}. Now, $2^{2003} = (2^{10})^{200} \times 2^3 = 76 \times 08 = 08$. Therefore 2^{2003} = N08 where N is some large number.

$$2^{2^{2003}} = 2^{N08} = 2^{N00+8} = (2^{10})^{N0} \times 2^8 \Rightarrow \text{Last two digits} = 76\times56 = 56$$

138. $\frac{M}{810} = \frac{9N5}{999} \Rightarrow \frac{M}{90} = \frac{9N5}{111} \Rightarrow \frac{M}{30} = \frac{9N5}{37} \Rightarrow M = \frac{9N5\times30}{37}$

As 30 is not divisible by 37 and M is a whole number 9N5 is a multiple of 37 ⇒ 9N5 = 925 ⇒ N = 2 ⇒ M = 750 ⇒ M + N = 752

139. 2M5 is odd therefore it cannot be divisible by 2.

$36 = \begin{bmatrix} 1\times36 \\ 3\times12 \\ 9\times4 \end{bmatrix}$ ⇒ Case I : 1×36 ⇒ 13N is divisible by 36, i.e. 4 and 9 ⇒ not possible because N = 5 for 13N to be divisible by 9.

Case II : 3×12 ⇒ 13N is divisible by 12, i.e. 3 and 4 ⇒ N = 2. Also 2M5 is divisible by 3 ⇒ M = (2, 5, 8) corresponding pairs are (2, 12), (5, 2) (8, 2)

Case III : 9×4 ⇒ 13N is divisible by 4 ⇒ N = 2, 6. Also 2M5 is divisible by 9 ⇒ M = 2. Pairs are (2, 2) and (2, 6) Therefore, there are 4 pairs in all ⇒ (2, 2), (5, 2), (8, 2) and (2, 6)

140. Putting the addition in symbolic form we get

abba Now e=1, because the carry can be at most 1. Therefore, a + c = 11,
<u>+cddc</u> for 1 to be at the unit s place also
<u>efgfe</u>

⇒abba as a + c = 11, f = 1 or 2, depending on when there b + d gives a <u>cddc</u> carry of 0 or 1. If f = 1 or 2, b + d= 0 or 1 for f to be at the tens <u>1 fgf1</u> place. Proceeding in this number, 2= 110011 or 12221.

141. The concept can be understood easily in this way :- There are 10 odd numbers and 11 even numbers. The 10 odd numbers can be reduced to an even number only and the 11 even numbers can also be reduced to an even number. The final reduced number would be even.

142. The last two digits of 11^{10} are 01. Therefore the last two digits of $11^{10} - 9$ are 92.

143. Let $5 = 1\times 2 + 2\times 2^2 + 3\times 2^3 + 4\times 2^4 + ... + 20\times 2^{20}$

$25 = 1\times 2 + 2\times 2^2 + 3\times 2^3 + 4\times 2^4 + ... + 20\times 2^{21}$

Subtracting

$5 = 20\times 2^{21} - (1\times 2 + 1\times 2^2 + 1\times 2^3 + ... + 1\times 2^{20})$

$= 20\times 2^{21} - (2^{21} - 2) = 19\times 2^{21} + 2$

$\Rightarrow$ Remainder by 19= 2

144. N^{th} term = $n(n+1) = n^2+n$

$$(n^2+n) = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)}{2}\left[\frac{2n+1}{3}+1\right] = \frac{n(n+1)(n+2)}{3}$$

reeping n=99, we get the sum= $\frac{99\times 100\times 101}{3}$

$\Rightarrow$ Remainder by 101= 0

145. $(101)^{11} - 99^{11} = (100+1)^{11} - (100-1)^{11}$

$= (100^{11} + 100^{10}\times 11 + 55\times 100^9 + ... +1) - (100^{11} - 11\times 100^{10} + 55\times 100^9 ....1)$

$= 2(11\times 100^{10} + 165\times 100^9 + ... +1)$

$\frac{101^{11} - 99^{11}}{100^{11}} = \frac{22}{100} + \frac{330}{1000} + ... + \frac{1}{100^{11}} < 1$ (We can check that rest of the1st terms will be negligible.)

$\Rightarrow 101^{11} < 100^{11} + 99^{11}$

146. LCM of 7, 8, and 9 is 504. 523 abc when divided by 504 gives 19 abc= 19000 + abc. The remainder when 19000 is divided by 504 is 352

$\Rightarrow$ 352 **+** abc is divided by 504 is 352.

$\Rightarrow$ 352 + abc = 504 or 504×2

$\Rightarrow$ abc = 152 or 656 $\Rightarrow a\times b\times c$ = 10 or 180

147. $n^2 + 3n + 5 = (n+7)(n-4) + 33$. $(n+7)$ and $(n-4)$ are two numbers at a difference of 11, therefore either are divisible 11 or both are not divisible by 11.

Case 1: both $(n+7)$ and $(n-4)$ are divisible by 11 $\Rightarrow (n+7)(n-4)$ is a multiple of 121. But 33 is not a multiple of 121

$\Rightarrow (n+7)(n-4) + 33$ is not divisible by 121.

Case 2: both $(n+7)$ and $(n-4)$ are not divisible by 11. 33 is divisible by 11. $\Rightarrow (n-7)(n+4)+33$ is not divisible by 11 $\Rightarrow$ it is not divisible by 121.

148. Squares of odd numbers are of the form 4k+1

$\Rightarrow$ sum of six squares = (4a+1 +4b+1 + 4c+1 + 4d+1 + 4e+1 + 4f+1)

$\Rightarrow$ f the form 4k+b. Only 1998 satifis this criterion.

149. $y^4 - x^2 = 12 \Rightarrow (y^2+x)(y^2-x) = 12$. y^2-x and y^2+x are two numbers at a difference of 2x, i.e an even number. Therefore we need to break 12 into product of two number the difference between then being even

$\Rightarrow (y^2 + x)(y^2 - x) = 2\times 6 \Rightarrow x = +2 \Rightarrow y = \pm 2$

Therefore, the solutions are (-2, -2), (-2, 2), (2, -2), (2,2).

150. 405! To 409! All and in 100 zeroes $\Rightarrow$ 5 numbers.

151. $b^3 = a^2 - 1 = (a + 1)(a - 1)$ $\Rightarrow$ product of two numbers at a difference of -2. $\Rightarrow$ = $b^3 = -2 \times -4$ or 2×4
$\Rightarrow$ a = -3 or 3 $\Rightarrow$ b = 2. Also b = 0 $\Rightarrow$ a = 1 or –1. When a = 0. = –1.
Therefore pairs are (-3, 2), (3,2), (1, 0), (–1, 0), (0, –1)

152. For a prime number P not equal to 2, 3 or 5 single digit ($\neq$P) written P-1 times is divisible by P. 259 = 7×37 $\underbrace{111.....111}_{72 times}$ is divisible by both 7 and 37 $\Rightarrow \underbrace{111....111}_{73 times}$ leaves remainder 1 with both 7 and 37 $\Rightarrow$ remainder by 259 = 1

153. 36! Ends in 8 zeroes and 24! Ends in 4 zeroes $\Rightarrow$ last two digits of 36! – 24! = 100 – last two digits of 24! $24! = 2^{22} \times 3^{10} \times 5^4 \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23$ =36 $\therefore$last two digits of 36!- 24! = 100-36 = 64

154. $m^3 - 8m^2 + 20m - 13 = (m-1)(m^2 - 7m + 13)$. If this product has to yield a prime number, one of the multiplicands has to be equal to 1 and the other one has to be equal to a prime number.
m-1 =1 $\Rightarrow$ m=2 $\Rightarrow$ $m^2 - 7m + 13 = 3$. Therefore m=2
$(m^2 - 7m + 13) = 1$ $\Rightarrow$ (m-3) (m-4)=0 $\Rightarrow$ m = 3 or 4 $\Rightarrow$ m-1 = 2 or 3
$\therefore$ therefore m = 2, 3, or 4

155. Remainder of 15, 16, 17 with 4=1
Unit digit = 7

156. 4 factors are not composite $\Rightarrow$ removing 1 as a factor, number has 3 prime factors. Since the product of prime factors = 30, prime factors are 2, 3 and 5.
32 = $2 \times 2 \times 8$ or $2 \times 4 \times 4$. Therefore, the number will be of the form abc^7 or ab^3c^3. 6 such numbers are possible, 3 for each case.

157. We only need to add till 4! To get the unit digit.
$\Rightarrow$ unit digit = 3

158. To keep P as minimum we take 8= 0
$\therefore$P+1 is divisible by 1, P+1 is divisible by 2,…, P+7 is divisible by 7 $\Rightarrow$ P is divisible 1, 2, 3, 4,…., 6, and 7 $\Rightarrow$ P = LCM of (1, 2, 3,.., 7) = 420

159. 153= 9×17, The number $(1200)^{500}$ gives remainder 16 with 17 and 4 with 9…. Therefore the remainder will be the smallest such number giving these remainders. Let the number be N 9a+4 = 17b+16 $\Rightarrow$ 9a= 9b+8b+12. L.H.S is a multiple of 9 therefore R.H.S should also be a multiple of 9
$\Rightarrow$ 8b+12 is a multiple of 9= b =3
$\Rightarrow$ number = 67

160. 2002! Ends in 499 zeroes. $(1001!)^2$ ends in 498 zeroes $\frac{2002!}{(1001!)^2}$ Ends in a single zero.

161. $6 \times 27!$

162. We take out the common term form 5^{26} and 15, i.e. 5 remainder = $5 \times$remainder when 5^{25} is divided by 3 = 5×2= 10

163. A perfect square.

164. $X_1 = 5$, $X_2 = 25$, $X_3 = 10$, $X_4 = 30$, $X_5 = 20$, $X_6 = 40$, $X_7 = 40$, $X_8 = 80$, $X_9 = 80$, $X_{10} = 160$, $X_{11} = 160$ ….. $\Rightarrow X_{18} = X_{19} = 2560$ AND $X_{20} = X_{21} = 5120$ LCM $(X_{19}, X_{20}) = 5120$

165. $949 = 73\times13$, $2628 = 73\times9\times4$
$\therefore$ M and can be 73×9, and 73×4.
$\Rightarrow$ HCF of M and N = 73

166. (a+1) +(b+2) +(c+3) +(d+4) = 4(a+b+c+d+5)
$\Rightarrow$ a+ b+ c+ d= 10= 4(a+ b+ c+ d) +20
$\Rightarrow$ a+ b+ c+ d= $-\frac{10}{3}$

167. $N = 539\times2^{18}$ and $M = 9\times2^{13}$
Taking out common factor 2^{13} we get Remainder
$= 2^{13}\times$ Remainder when 539 is divided by 9
$= 2^{13}\times8 = 2^{16}$

168. $N = 2^{2999}\times5^{3002} = 10^{2999}\times5^3 = 125000000….,$
Sum of digits = 8

169. $28 = 7\times4 = (6+1)(3+1) \Rightarrow 2n = a^3b^6$
$30 = 6\times5 = (5+1)(4+1) \Rightarrow 3n = x^4y^5$
Comparing we can see $n = 2^5\times3^3$

170. Number of ways that a number can be written as a sum of consecutive natural numbers = Number of odd factors -1 $= 2\times2\times3 = (1+1)(1+1)(2+1)$
$\Rightarrow$ Number of odd factors = $12 = 3\times4 = (2+1)(3+1)$
$\therefore$ the number is of the form a^2b^3 of abc^2
Sum of 11 consecutive numbers= (n) + (n+1), + (n+2) +…. + (n+10)
= 11n+55 = 11(n+5) = 11n $\Rightarrow$ The numbers is not a multiple of 11.
The highest such number less then is 400 $5\times7\times3^2 = 315$

171. After 4! And beyond, the powers will be multiples of 4. having aside $1^{2!}$ **+**$2^{3!}$ + $3^{4!}$ **+….** + $19^{20!}$, the pattern $11^{12!} + 12^{13!} + ….. + 19^{20!}$ Will repeat 4 times unit digit = 1+6+1+6+5+6+1+6+1= 1
Therefore unit digit= 2+1 = 3

172. $\frac{5^{200}}{8} = \frac{(5^2)^{100}}{8}$ = Remainder $\rightarrow$ 1

173. The remainder by 125 is the remainder when the last three digits is divided by 125. Let there digits = 228 $\rightarrow$ Remainder = 103

174. A
If a number is divisible by 72, it is divisible by 8 and 9. To be divisible by 8, the number formed by the last three digits should be divisible by. Therefore, 31B should be divisible by 8 $\Rightarrow$ B = 2. To be divisible by 9, the sum of the digits of the number should be divisible by 9 $\Rightarrow$ A = 3. Therefore, A + B = 5

175. C
A digit when repeated 6 times is divisible by 7. Therefore, 111 111 would be a multiple of 7 $\Rightarrow$ = 111 111. Therefore, N = P/7 = 15873 $\Rightarrow$ 8P = 126984.

176. D

As the L.H.S. is a multiple of 32 = 9, R.H.S. will also be a multiple of 9. Therefore, 2 + 2 + 7 + 5 + 2 + S = 18 + S will be divisible by 9. Therefore, S = 0 or 9. But if S is zero 3(150 + S) will end in a zero and its square will end in two zeroes. But the tens digit of R.H.S is not zero. Therefore, S = 9.

177. D

$a^n + b^n$ is divisible by a + b when n is **odd** $\Rightarrow 5^{2n+1} + 17^{2n+1}$ is divisible y 17 + 5 = 22 $\Rightarrow 5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ is divisible by 11. The expression is also divisible by 3. Therefore, it is divisible by 33. **NOTE:** Keeping n = 0 will give the answer straightaway.

178. B

$5^{36} - 1 = (4 + 1)^{36} - 1 = 4^{36} + 36 \times 4^{35} + \ldots + 36 \times 4 + 1 - 1 = 4^2(4^{34} + 9 \times 4^{34} + \ldots + 9)$.
Therefore, the highest power of 4 is 2.
$5^{36} - 1 = (6 - 1)^{36} - 1 = 6^{36} - 36 \times 6^{35} + \ldots - 36 \times 6 + 1 - 1 = 6^3(633 - 6^{34} + \ldots - 1)$. Therefore, the highest power of 3 is 3. Therefore, the highest power of 12 is 2.

179. B

$(222)_b = 6162 \Rightarrow 2b^2 + 2b + 2 = 6162 \Rightarrow b = 55$

180. B

Adding the unit digit of the numbers on L.H.S., we get 3 + 4 + 3 = 10. But the unit digit is 3. Therefore, base x is 3 less than 10 (the sum of units digit) or x = 7.

181. B

As every digit is used only once, the highest number would have 9876543 as the first 7 digits. Now we need to use 0, 1, 2 at the end such that the number is divisible by 8. The only number we can form is 120. Therefore, M = 9876543120 ⇒ remainder by 1000 = 120

182. D

When we write 2006! In base 22, we successively divide 2006! by 22 and keep writing down the remainders. The first remainder will become the units digit, the second remainder will become the tens digit, the third remainder will become the hundreds digit and so on. Therefore, the number of zeroes that 2006! written in base 22 will have will be equal to the number of times 22 divides 2006! completely. The number of times 22 divides 2006! completely is equal to the highest power of 22 in 2006! or equal to highest power of 11 in 2006!.

The highest power of 11 in 2006! = $\left[\frac{2006}{11}\right] + \left[\frac{2006}{121}\right] + \left[\frac{2006}{1331}\right] = 182 + 16 + 1 = 199$

183. A

For any natural number x, x^6 will give remainder 1 with 7 whereas x^3 will give remainder 1 or 6 with 7. The given expression will never give remainder 0 with 7 and hence will never be divisible by 7.

184. C

$(212)_b = (128)_{b+2} \Rightarrow 2b^2 + b + 2 = (b + 2)^2 + 2(b + 2) + 8 \Rightarrow b^2 - 5b - 14 = 0 \Rightarrow b = 7 \Rightarrow 114$

185. C

a b c = 100a + 10b + c = 98a + 2a + 7b + 3b + c = 7k + 2a + 3b + c.

186. A

$(a\ b\ c)_7 = (c\ b\ a)_9 \Rightarrow 49a + 7b + c = 81c + 9b + a \Rightarrow b = 24a - 40c = 8(3a - 5c)$. The solution to these equations are a , c = (2, 1) and (5, 3). Therefore b = 8 or 0. But b = 8 is not possible in base 7. Therefore, a + b + c = 5 + 0 + 3 = 8

187. **bcd** is divisible by 5 ⇒ d = 5. **abc** is divisible y 4 ⇒ bc = 12, 32, 52 or 24. As b cannot be equal to 5, only options are 12, 32, or 24 ⇒ c can be 2 or 4. **cde** is divisible by 3 ⇒ **c5e** is divisible by 3 ⇒ (c, e) = (1, 3),
(3, 1), (3, 4) or (4, 3). Only the last value satisfies the condition that c can be 2 or 4. Therefore, c = 4 and
e = 3. Also, this gives b = 2. Therefore, a = 1

189. In binary $2^{90} = \left(\underbrace{10000....000}_{91 digits}\right)$

Therefore $(2^{90}-1)$ in base 2= $\left(\underbrace{100000...000}_{91 digits}-1\right)_2 = \left(\underbrace{11111...111}_{90 digits}\right)_2$

190. $25^{86} - 5^{171}.\ 5^{172} - 5^{171} = 5^{171} \times 4$

$= \left(\underbrace{100000....000}_{172 digits}\right)_5 \times (4)_5 = \left(\underbrace{40000...000}_{172}\right)_5$

191. $A = 288^{30} = (2^5 \times 3^2)^{30} = 2^{150} \times 3^{60}$
Number of factors = $151 \times 61 = 9211$

192. $10^{10} = 2^{10} \times 5^{10}$ A factor will end in a zero if it has both 2 and 5 in it. Therefore we cannot take 2^0 or 5^0 in fornins the factors. Therefore, total number of factors having both 2 and 5 in it = 10×10 = 100

193. Only perfect square have odd number of factors. Therefore we first count perfect squares in the first 300 even natural numbers.

194. Since 1024= $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
So this no. can be, N= $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9 \times n_{10}$
Where n_1, n_2, n_3,----- n_{10} are prime no.
So, this number N can have 10 prime factors.

195. The smallest number which can be for m by the five prime factors is
N= $2 \times 3 \times 5 \times 7 \times 11$
= 2310
It means all three digit number have less then 5 prime factors
$\therefore$ so that there are 300 three digit numbers have less then 5 prime factors.

196. $\Rightarrow 25! = 2^{22} \times 3^{10} \times 5^6 \times 7^3 \times 11^2 \times 13^1 \times 17^1 \times 19^1 \times 23^1$
$= 2^{16} \times 3^{10} \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23 \times 10^6$

$$\frac{25!}{10^7} = \frac{2^{16} \times 3^{10} \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23 \times 10}{10^7}$$

$$\frac{2^{16} \times 3^{10} \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23 \times 10}{10}$$

When we divide the above numerator to by 10 the remainder in same as the unit digit of the numerator.
$\therefore$ Unit digit of $2^{16} \times 3^{10} \times 7^3 \times 11^2 \times 13 \times 17 \times 19 \times 23$
= $6 \times 9 \times 3 \times 1 \times 3 \times 7 \times 9 \times 3$
= 4
The remainder when 25! Is divided by $10^7 = 4 \times 10^6$

197. $2500 = 2^2 \times 5^4$
Since the HCF A and 2500 is equal to 1 it means A can not have any even value and A can not be multiple of 5
$\therefore$ number of even number from 1 to 2500 = 1250
$\therefore$ number of multiple of 5 from 1 to 2500 = 500

$\therefore$ number of values of A = 2500- 1250-500 = 750

198. A= (10! + 12! + 14!+ 16!+ …..100!)
=10! [1 + e_1 +e_2 +e_3 +….. e_{99}]
= 10! × k
Since e_1, e_2, e_3,……e_{99} each of them is even
$\therefore$ 1+ e_1+ e_2+ e_3+….e_{99} = k = odd
The highest power of 2 is A in same as reighest power of 2 in 10!
$\frac{10}{2}$ = 5, 10
$\therefore$ highest power of 2 in 10! In = 5+2+1= 8

199. If we look at the last 18 digit of the sequence:-
1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9
The sum of odd positioned number = 45
The sum of even positioned number = 45
The same will continue upto 180 digits from right
$\therefore$ the same of all odd positioned number = 90×45
The same of all even positioned number= 90×45
$\therefore$ the difference of sum of all odd positioned number with all even positioned number = 0
$\therefore$remainder= 0

200. Lets assume that this 4 digit number is abcd if we multiply it with 10001 the condition would be satisfied
now 10001 is not divisible by11
since 10001= 73×137

201. The power of 5 in n; increases by one at the interval of 5
It increases by another one at the interval of 25
And similarly it increases by another extra one at the interval 125
I In 70! Highest power of 5 = 14+2 = 16
I In 75! Highest power of 5 = 15+3 = 18
I In 120!highest power of 5 = 24+4+1= 28
I In 125!highest power of 5 = 25+5+1= 31
I In 130! Highest power of 5= 26+5+1= 32

202. 11! = $2^8 \times 3^4 \times 5^2 \times 7^1 \times 11$
Since the factor is not a multiple of 6, so we should remove all pairs of 2×3
$\therefore$n = $2^4 \times 5^2 \times 7 \times 11$
Now, the number in one more than a multiple of 6, which means it in an odd number, So we should remove all factors of 2
$\therefore$ n= $5^2\ 7 \times 11$
Now since the number in of (6k+1) form, which means when we divide it by 6 it lower a remainder of 1,
$\therefore$Rem$\left(\frac{5}{6}\right)$ = 5, Rem =1, Rem$\left(\frac{7}{6}\right)$ = 1, Rem$\left(\frac{11}{6}\right)$ = 5
$\therefore$ That largest factor which on being divided by 6 leaes the remainder of 1 = $5 \times 7 \times 11$
= 385

203. 72= 8×9 (co- prime factors of 72)

Now, 2718 AB6 should be divisible by 8 and 9 individualy. Sum of the digits of the number should by divisible by 9 and the last three digit of the number should by divisible by 8 the possible values which satisfies the above two conditions are. 2718216, 2718576, 2718936 So the number of possible ordered pairs of (A, B) =3

204. 16! = $2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11^1 \times 13^1$

Since $\frac{16!}{n}$ has to be a perfect square, which means all prime factors should have even power.

$\therefore$ n = $2 \times 5 \times 11 \times 13$

= 1430

205. If we check in revers order.

$$\begin{array}{r} 555778885777 \\ \hline 2222233335555 \\ +333555552222 \\ \hline \end{array}$$

From the above summation we can say

X= 5, y= 4, z= 3

P= 3, q= 5, r= 4

206. We know that (n^3 + 10^3) which is nothing but (n^3 + 1000) in divisible by (n+10) now, (n^3 + 1000) = (n^3 + 100 + 900)

Since (n^3 + 100 + 900) is divisible by (n +10) for any value of n which means. (n^3+100) & 900 should be divisible by (n+10) now the largest possible value of (n+10) which can satisfy this in 900

$\therefore$ largest possible value of n = 900-10

= 890

207. 990= $2 \times 3^2 \times 5 \times 11$

We have to select n! in such a way that it should consist of 11, 5, 3^2, and 2

$\therefore$ n= 11

And 11! = $2^8 \times 3^4 \times 5^2 \times 7^1 \times 11$

$\therefore$ the smallest possible value of n = 11

208. We know that all prime number greater than 3 can be expressed as 6n=1

$\therefore \frac{p2+17}{12} = \frac{(6n+1)^2+17}{12} = \frac{36n^2+1+12n+17}{12}$

$36n^2$ in divisible by 12

$\pm$ 12n in divisible by 12

$\therefore$Remainder for the above exp= Rem$\left(\frac{1+17}{12}\right)$

= Rem $\left(\frac{18}{12}\right)$

= 6

209. The last 4 digit of the given sequence in 5455 the remainder of the given sequence would be same as the remainders when we divide 5455 by 16

$\therefore$Remainder = Rem $\left(\frac{5455}{10}\right)$

= 15

210. To get the integeral value ($11n^2 + 5n + 6$) have to be divisible by n which means 6 should be divisible by 4

$\therefore$ n can be equal to 1, 2, 3 and 6

So there four such values of n

211.

Zorcon value	Decimal value
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
	10

$\therefore (9\theta) = (11 \times 9 + 11^o \times 6)$
$= 99+6$
$= 105$

211. b in the square of an odd integer

$n^3 + 2n^2 = b$ = odd integer.

Since $2n^2$ will always be an even number

$\therefore$ n^3 would an odd number

$\therefore$ n has to be an odd number

now $n^3 + 2n^2 = n^2 (n+2)$

to make ($n^3 + 2n^2$) a perfect square (n+2) has to perfect square.

$\therefore$n = 2,7, 14, 23

So the smallest possible odd value of n = 7

212. Since 49a + 7b+c = 286

Or $7^2a + 7^1b + 7^oc = 286$

Or $(abc)_7 = (286)$

A which means abc in the equivalent of 286 in base 7= $(abc)_7 = (556)$

213. Since N = 100 $\times$ number of factors, N is a multiple of 100 and its two prime factors are 2 and 5.

Let N = $2^a \times 5^6$ $\Rightarrow$ $2^a \times 5^6$ = 100 $\times$ (a+1) (b+1) $\Rightarrow$ $2^{a-2} \times 5^{6-2}$ = (a +1) (b + 1). As minimum one 5 is requited on the L. H. S. minimum value of b = 3. Keeping b = 3 we get $2^{a-2} \times 5$ = (a + 1) $\times$ 4. To balance powers of 2 on to the sides a = 4 $\Rightarrow$ $2^2 \times 5$ = 5 $\times$ 4. As both sides are equal now $\Rightarrow$ a = 4 and b = 3 $\Rightarrow$ N = $2^4 \times 5^3 = 2000$

214. n = $p!+1$ n + 1 = $p!+2$, n + 2 = $p!+3$, n + 3 = $p!+4$, … n+p-1 = $p!+p$ in $p!+2$, 2 would be a common factor, in $p!+3$, 3 would be a common factor, and so on….

$\therefore$ none of the given numbers would be a prime number.

215. Only squares of prime numbers have exactly three factors

Let $N^2 + 24N + 21 = p^2$ $\Rightarrow$ $(N+12)^2 - p^2 = 123$

$\Rightarrow$ (N+ 12 + p) (N + 12 – p) = 123 123 = 1 $\times$ 123 or 3 $\times$ 41

$\Rightarrow$ p = 61 or 19 $\Rightarrow$ two values of N are possible.

216. 10a + 6 = $(ab)^2 - 32$ now the maximum value of 10a + b would be 99 $\Rightarrow$ the maximum value of $(ab)^2$ would be 131. as $(ab)^2$ would be 36 $\Rightarrow$ $(ab)^2$ = 36, 49, 64, 81, 100, 121

10a+b = 4, 17, 32, 49, 68, 89 $\Rightarrow$ only 17 satisfies the criterion.

217.

$$\begin{array}{r} abcd \\ \times 2 \\ \hline bcda \\ \hline \end{array}$$

Since the operation is in base 7 a would be lass than 4 otherwise we would have a 5 digit number. Therefore a=1, 2 or 3. We can see that.
a = 1 or 2

$$\begin{array}{r} 1254 \\ \times 2 \\ \hline 2541 \\ \hline \end{array} \qquad \begin{array}{r} 2541 \\ \times 2 \\ \hline 5412 \\ \hline \end{array}$$

218. $(abc)_{11} = (cba)_9$ $\Rightarrow$ 121a + 116 + c = 81c + 9b + a

120a + 2b = 80c $\Rightarrow$ 60a + b = 40c $\Rightarrow$ b = 0 or 10 as R.H.S. is a multiple of 10.
$\Rightarrow$ 60a = 40c $\Rightarrow$ 3a = 2c $\Rightarrow$ c = 3 or 6 (c Cannot be 9)
$\Rightarrow$ numbers are (203) and (406)

219. Since the remainders when we divide the number by 7, 8, 9 are 2, 4, 6 $\Rightarrow$

7k + 2 = $8k_1 + 4 = 9k_2 + 6$

Solving the last 2. $\Rightarrow$ $8k_1 + 4 = 9k_2 + 6$

$$8k_1 = 9k_2 + 2$$

$$k_2 = \frac{9k_2 + 2}{8}$$

As k_1 & k_2 are natural numbers, so value of k_2 should be such that k_1 is a natural no. i.e. k_2 =6
$\Rightarrow$ 9 × 6 + 6 = 60 is the smallest number which when divided by 8 & 9, the remainder are 4 & 6 respect.
$\Rightarrow$ L.C.M (8, 9) + 60 also satisfies the same. So 72k_3 + 60 also satisfies the same conditions .
Now 72k_3 + 60 = 7k + 2. Solving as above we get k_3 =6 $\Rightarrow$ smallest number = 492.

220. Using the divisibility test of 13. $\underbrace{30a0}\underbrace{bo3}$ i.e. ['b03' + 3] – ['0a0'] should be divisible 13.

$\Rightarrow$ [100b + 0 + 3 + 3] – [0 + 10a +0]
= 100b – 10a + 6 should be divisible by 13.
$\Rightarrow$ the ordered pairs (a, b) that satisfy it are :- (2, 3), (3, 7), (5, 2), (6, 6), (8, 1), (9, 5)

221. x > y > z $\geq$ 2 , xyz = 2002.
Now 2002 = 2 × 7 × 11 × 13
If product of 3 numbers is given their product would be least when they are equal. So to make the sum
Minimum the numbers are
x = 14, y = 13, z = 11
$\Rightarrow$ B = 14 + 13 + 11 = 38
11 by to make the sum maximum
x = 143, y = 7, z = 2
$\Rightarrow$ A = 143 + 7 + 2 = 152
So, A – B = 152 – 38 = 114.