CHAPTER – 1

SPECIAL EQUATIONS

We have learnt about simultaneous equations in two and three unknowns. When we have two independent equations in two unknowns or three independent equations in 3 unknowns, we can solve for the unknowns. This type of equations are called "Determinate Equations". The unknowns in Determinate Equations have unique values. However, if we have only one equation in two unknowns, or two equations in three unknowns, such equations are called "Indeterminate Equations". The variables here do not have unique values but take more than one value - in general, infinite number of values.

If we impose certain other conditions on these unknowns, then such indeterminate equations also can yield unique values for the unknowns. We take such conditions also into account while solving such equations.

Consider 4x + y = 15. This being one equation in two unknowns, is indeterminate. Suppose we impose a condition that both x and y are positive integers. With his condition the possibilities are reduced to a finite number x = 1, y = 11; x = 2, y = 7; x = 3, y = 3. If we further impose the condition that x should be equal to y, then there is a unique solution x = 3, y = 3. So, even though we have one equation, because of additional conditions, it may have finite or sometimes even a unique solution. The conditions that we have, could be explicitly mentioned as above or could be in built into the problem as we see in the following examples.

Examples:

1.01. Abhinav purchased a certain number of pencils and a certain number of pens spending ₹15 on the whole. If each pen costs him ₹4 and each pencil ₹1, then how many pens and pencils could he have bought?

Sol. Solving this problem is basically solving the equation 4x + y = 15, where x and y respectively denote the number of pens and pencils purchased, here from the context we know that x and y should be both positive integers and hence this equation has exactly the three solutions given by (x, y) = (1, 11) (or) (2, 7) (or) (3, 3), as discussed earlier.

Hence we see that when certain special conditions are imposed on the unknowns, the indeterminate equations also can yield finite number of solutions and sometimes even a unique solution. The conditions we normally come across are

(a) minimum values of the unknowns.
(b) maximum values of the unknowns.
(c) unknowns being positive integers.
(d) limits on difference in the values of unknowns, etc.

1.02. Sachin asked Anil for his birthday. Anil replied, "Take the date and the month, of my birthday. Multiply the date with 12 (which is the number of months in a year) and the month with 31 (which is the number of days in a month) and add up the two products. The sum is 452. Find Anil's birthday.

Sol. If we denote the date as D and month as M, we have 12D + 31M = 452. Here we have only one equation with two unknowns. This is an indeterminate equation. However, we have the following additional information.

(i) D is a positive integer less than or equal to 31.
(ii) M is a positive integer less than or equal to 12.

To solve this equation and infact to solve any equation in two variables, we shall

(a) first divide the entire equation by the least coefficient.
(b) get all the fractional values on to one side, say left and all the integral values on to the other side of the equation.
(c) replace the whole of right-hand side by k, where k is an integer.

The following is the sequence of steps:
12D + 31M = 452.

(i) $\frac{12D + 31M}{12} = \frac{452}{12}$

(ii) $D + 2M + \frac{7M}{12} = 37 + \frac{8}{12}$

(iii) $\frac{7M - 8}{12} = 37 - 2M - D = k$ (k an integer)

$\Rightarrow M = \frac{12k + 8}{7}$

By trial we find out the value of k which gives a proper value of M (M being a positive integer less than or equal to 12). We see that k = 4 gives M = 8. Further values of k are not feasible. Using this we now find D which is 17. Thus the date is 17th August. Here we get a unique solution for the equation.

We can write the same solution concisely, by focussing only on the remainder.
12D + 31M = 452

We select the smaller of the two coefficients, i.e., 12, as a divisor, because the number of possible remainders is less. When the LHS is divided by 12, the remainder is equal to the remainder when 7M is divided by 12.

$\therefore \text{Rem}\left(\frac{7M}{12}\right) = \text{Rem}\left(\frac{452}{12}\right) = 8$

$\Rightarrow 7M = 12x_1 + 8 \Rightarrow \text{Rem}\left(\frac{5x_1}{7}\right) = 6$

$\Rightarrow 5x_1 = 7x_2 + 6 \Rightarrow \text{Rem}\left(\frac{2x_2}{5}\right) = 4$

$\Rightarrow 2x_2 = 5x_3 + 4 \Rightarrow \text{Rem}\left(\frac{x_3}{2}\right) = 0$

$\therefore x_3$ can be 0, 2, 4 etc or –2, –4, ……

$x_3 = 0 \Rightarrow x_2 = 2 \Rightarrow x_1 = 4 \Rightarrow M = 8$

Substituting in the original equation we get D = 17

The other values for D are obtained by adding (or subtracting) the coefficient of M (i.e., 31) from this value of D.

The corresponding value of M is obtained by subtracting (or adding) the coefficient of M to the, value of M, i.e.,

(D, M) could be (17, 8) (48, –4) (79, –16) and (–14, 20) (–45, 32) (–76, 44) etc.

Alternative method:

12D + 31M = 452 -------- (1)

We notice that 4 is a factor of 12D and 452

$\therefore$ It is a factor of 31M (or M) i.e., M = 4, 8 or 12.

We also notice that

$\text{Rem}\left(\frac{31M}{12}\right) = \text{Re m}\left(\frac{7M}{12}\right) = \text{Re m}\left(\frac{452}{12}\right) = 8$

If M = 4, $\text{Rem}\left(\frac{7M}{12}\right) = 4$

If M = 8, $\text{Rem}\left(\frac{7M}{12}\right) = 8$

If M = 12, $\text{Rem}\left(\frac{7M}{12}\right) = 0$

$\therefore$ M = 8. Substituting in (1), we get D = 17, i.e., the date is 17 August.

1.03. Samir bought two varieties of pens, the first variety costing ₹17 each and the second variety costing ₹10 each, spending ₹207 in total. In how many different combinations could he have purchased the pens?

Sol. Let x be the number of pens of the first variety and y be the number of pens of the second variety. Then, 17x + 10y = 207.

Proceeding as discussed in the last example,

$x + \frac{7x}{10} + y = 20 + \frac{7}{10}$

$\frac{7x - 7}{10} = k$ (integer)

$x = \frac{10k + 7}{7}$

As x is a positive integer, the values of k can be 0, 7, 14, ……. when, k = 0, x = 1 and y = 19; when k = 7, x = 11 and y = 2, when k = 14 x = 21, which is not possible as the amount spent on the first variety (17×21) in this case will be more than the total amount spent.

Hence Samir can buy these pens in exactly two different combinations.

We can write this solution briefly by focussing only on the remainders.

17x + 10y = 207

$7x = 10x_1 + 7 \Rightarrow \text{Rem}\left(\frac{3x_1}{7}\right) = 0$

x_1 can be 0, 7, 14, 21 etc or –7, –14 etc.

$x_1 = 0 \Rightarrow x = 1 \Rightarrow y = 19$

The other values of x are obtained by adding (or subtracting) 10 successively and those of y are obtained by subtracting (or adding) 17 successively.

i.e., (x, y) could be (1, 19), (11, 2), (21, –15) or (–9, 36) (–19, 53) etc.

As x > 0, y > 0 only (1, 19) and (11, 2) are acceptable solutions.

Let us take another example and look at the most general method of solving such problems (please note that some of the steps given in the example below may not be required in some problems).

1.04. I bought two different varieties of ice-creams, the first variety costing ₹7 per ice-cream and the other costing ₹13 per ice-cream. If I had paid a total amount of ₹134 for both varieties of ice-creams together, how many of each variety did I buy?

Sol. If p is the number of ice-creams costing ₹7 per piece and q is the number of ice-creams costing ₹13 per piece, we have 7p + 13q = 134 – (1)

The various steps involved in solving this equation are explained below.

Step 1:

Dividing the equation with the least coefficient separating all fractions on to the left and integers on to the right and then denoting the right hand side by k, we get

$p + q + \frac{6q}{7} = 19 + \frac{1}{7}$

$\frac{6q - 1}{7} = k$ (where k is an integer)

Step 2:

Rewrite this relationship such that the coefficient of the variable (q in this case) is 1. To do this, multiply both sides of the relationship with a constant, such that the coefficient of q will then be 1 more than a multiple of the denominator (the denominator being 7 in this case). By observation we can see that 36 (which is a multiple of the coefficient 6) is 1 more than 35 (which is the multiple of the denominator 7). Hence to get 36, we multiply both sides of the relationship with 6.

$\frac{36q - 6}{7} = 6k \Rightarrow \frac{35q}{7} + \frac{q}{7} - \frac{6}{7} = 6k$

$\Rightarrow \frac{q - 6}{7} = 6k - 5q.$

Here again the R.H.S. is the sum or the difference of integers and hence will be an integer. We call this k_1 and thus we get

$\frac{q - 6}{7} = k_1$

(Note that this step has to be done mentally. However we completely skipped this step in the two examples we took earlier in this chapter).

Step 3:

Write the variable in the equation in terms of k_1. In this case the variable q is written in terms of k_1 as $q = 7k_1 + 6$.

Step 4:

Substitute this value of one variable, in terms of k, in the original equation to express the other variable also in terms of k.
Here, substituting $q = 7k_1 + 6$ in equation (1) we get $p = 8 - 13k_1$.

Step 5:
On the basis of the values of the two variables (expressed in terms of k_1), identify what values k_1 can take to ensure that the variables are positive integers.
$q = 7k_1 + 6$ and $p = 8 - 13k_1$
To ensure that p is a positive integer, $k_1 \le 0$
To ensure that q is a positive integer, $k_1 \ge 0$
The only value of k_1 that satisfies both these conditions is $k_1 = 0$.
For $k_1 = 0$, q = 6 and p = 8. Thus this problem has a unique solution.

Note:
In step 5, if k_1 can have more than one possible value, then the problem has more then one solution. The problem will have as many solutions as the number of values k_1 can take.
This solution can be written compactly by focussing only on the remainders.

$7p + 13q = 134 \Rightarrow \text{Rem}\left(\frac{6q}{7}\right) = 1$

$\Rightarrow 6q = 7x_1 + 1 \Rightarrow \text{Rem}\left(\frac{x_1}{6}\right) = 5$

$\Rightarrow x_1 = 6x_2 + 5$
x_2 can be any integer.
$x_2 = 0 \Rightarrow x_1 = 5 \Rightarrow q = 6 \Rightarrow p = 8$
$\therefore$ (p, q) could be (8, 6), (21, –1) etc.
or (–5, 13), (–18, 20) etc
$p > 0, q > 0$, (p, q) = (8, 6)

1.05. $9x - 7y = 17$

$\text{Rem}\left(\frac{2x}{7}\right) = \text{Rem}\left(\frac{17}{7}\right) = 3$

Sol: $2x = 7x_1 + 3 \Rightarrow \text{Rem}\left(\frac{x_1}{2}\right) = 1$

$\therefore x_1 = 1, 3, 5$ etc or –1, –3, –5 etc.
$x_1 = 1 \Rightarrow x = 5 \Rightarrow y = 4$
$\therefore$ In this example, because there is a negative sign, values of both x and y will increase together or decrease together.
i.e., (x, y) = (5, 4), (12, 13) (19, 22) etc or (–2, –5) (–9, –14) etc.

Concept Review Questions

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If the remainder obtained when 4S is divided by 7 is 3 then a possible value of S where S is a positive integer is _____.
(A) 5 (B) 6 (C) 7 (D) 4

2. A person purchased three items a, b and c spending a total amount of ₹101. Each of the items a, b and c cost ₹80, ₹3 and ₹2 respectively. Find the maximum number of items altogether he could have purchased.

3. If $4x + 5y = 27$ where $x, y \in Z^+$, then possible values of x and y are _____ respectively.
(A) 2 and 3 (B) 3 and 5
(C) 3 and 4 (D) 3 and 3

4. If $3x + 7y = 37$ where $x, y \in Z^+$ then the number of combinations of x and y is _____.
(A) 1 (B) 2 (C) 3 (D) 0

5. If $5x + 16y = 100$ where $x, y \in Z^+$, the values that y can take are _____.
(A) multiples of 10 (B) multiples of 4
(C) multiples of 5 (D) None of these

6. If Remainder $\left(\frac{4Q}{5}\right) = 4$, Q is of the form _____.
(A) $4k + 1, k \in Z$ (B) $5k, k \in Z$
(C) $5k + 1, k \in Z$ (D) $5k - 1, k \in Z$

7. The number of solutions of the equation $2x + 3y = 14$ in which both x and y are positive integers is

8. The number of non-negative integral solutions of the equation $3x + 5y = 20$ is

9. A and B are positive integers such that $5A + 8B = 78$. Which of the following is a possible value of A?
(A) 3 (B) 9
(C) 14 (D) 11

10. X and Y are positive integers such that $5X + 7Y = 135$. Which of the following is a possible value of Y?
(A) 7 (B) 12
(C) 15 (D) 20

11. Ramesh bought some pencils and erasers spending an amount of ₹15. If a pencil cost him ₹4 and an eraser ₹3, then how many pencils did he buy?
(A) 3
(B) 4
(C) 1
(D) Cannot be determined

12. Shakuntala has some parrots and rabbits with her. If 4 times the number of parrots added to 7 times the number of rabbits is equal to 29, then how many parrots does she have?

13. Ritish has some coins in denominations of two rupees and five rupees. If the total amount with him is ₹13, then how many coins does he have?

14. Satwick purchased some scales and charts. Each scale cost him ₹7 and each chart ₹5. If he has spent a total amount of ₹42, then how many items did he purchase?
(A) 8 (B) 7 (C) 9 (D) Cannot be determined

15. Kashyap purchased some cricket balls and shuttle cocks. The price of each cricket ball is ₹10 and the price of each cock is ₹11. If Kashyap spends a total amount of ₹130, then how many cocks did he purchase?

Exercise – 1(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Supriya asked Rajesh "What is the date of your birth"? Rajesh replied, "25 times the date of my birth added to 9 times the month of my birth is 563". In which month was Rajesh born?
(A) September (B) July
(C) February (D) November

2. The "Friends Club" bought some shuttlecocks and cricket balls. Each shuttlecock cost ₹8 and each cricket ball cost ₹15. In how many different ways could the club have bought the items if it spent a total amount of ₹769?

3. Kushal takes up an assignment, wherein each working day he is given a target. For each day he meets the target, he is paid ₹105 and for each day he does not meet the target he is paid ₹18 less. If by the end of a month, he is paid a total of ₹2988, then for how many days does he meet the target?
(A) 21 (B) 9 (C) 18 (D) 19

4. John and Rosy have a collection of less than 90 coins together. If Rosy gives John a certain number of coins, then John would have 4 times as many coins as Rosy. Instead if John gives the same number of coins to Rosy, then John would have 3 times as many coins as Rosy. The number of coins with John and Rosy together can be _____.
(A) 80 (B) 29 (C) 35 (D) 85

5. The average weight of a certain group of n men is 72 kg. When two men whose weights are 73 kg and 67 kg join the group, and one man whose weight is more than 58 kg leaves the group, the average weight of the group goes up by 3 kg. If the initial number of men in the group is an even number, then the weight of the man who leaves the group is ____ kg.

6. The number 284 has to be divided into two positive parts such that one part is a multiple of 9 and the other is a multiple of 17. In how many ways can the number be divided?

7. Jasmine wants to buy some apples, oranges and jack fruits. Each apple costs ₹8, each orange costs ₹3 and each jack fruit costs ₹13. If she wants to spend an amount of ₹112 and if she wants to buy a minimum of 4 fruits of each variety, then in how many different combinations can she buy the fruits?
(A) 1 (B) 2 (C) 3 (D) 4

8. A question paper contains a certain number of 10-mark, 5-mark and 2-mark questions with a total of 35 questions in it. The maximum marks that a student can secure in the paper are 100. How many 5-mark questions are there in the paper?
(A) 30 (B) 2 (C) 3 (D) 4

9. On a certain winter day, a dealer sold some woollen jackets, sweaters and gloves. He sold each jacket at ₹300, each sweater at ₹175 and each pair of gloves at ₹100. If the dealer sold at least one piece of each variety and made a total collection of ₹1175 by the end of the day, then how many sweaters did he sell?
(A) 1 (B) 5 (C) 4 (D) 2

10. An organization has 14 employees belonging to three different sections A, B and C. The monthly salaries of each of the employees of the sections A, B and C are ₹11000, ₹6000 and ₹2000 respectively. If the total salaries paid to the employees of all the three sections is ₹84000, then the number of employees in the section A is ____

11. In an acute triangle, the angles are all positive integers, when measured in degrees. 13 times one of the angles is equal to 17 times another. What is the measure in degrees of the least possible angle in the triangle?

Directions for questions 12 and 13: These questions are based on the data given below.

Ajay spent an amount of ₹1000 for buying black boards and white chalk boxes for his school. Each black board cost him ₹25 and each chalk box cost him ₹5. He noted that if he had bought as many chalk boxes as the black boards and as many black boards as the chalk boxes he had actually bought, he would have spent less than half of what he actually spent.

12. In how many different combinations could he have bought the items?
(A) 1 (B) 2
(C) 3 (D) 4

13. If Ajay bought at least 10 of each of the items, then how many black boards did he buy?
(A) 38 (B) 39
(C) 40 (D) 37

Directions for questions 14 and 15: These questions are based on the following data.

Lokesh purchased a total of 38 CDs, DVDs and cassettes spending a total amount of ₹3540. Each CD cost ₹80, each DVD cost ₹150 and each cassette cost ₹30.

14. Among all the possible combinations of the 3 items that he could have bought, if Lokesh decides to select the combination in which the number of CDs is as great as it can be, how many DVDs and cassettes together did he buy?
(A) 21 (B) 23 (C) 14 (D) 12

15. If Lokesh bought the 3 items, in that combination of numbers in which the number of DVDs was as small as it could be, how many cassettes did he buy?
(A) 4 (B) 12
(C) 10 (D) 6

16. How many 3-digit numbers are there which leave a remainder of 5 when divided by 17 and a remainder of 6 when divided by 11?

17. Meghana bought packs of ice-cream in two flavours – vanilla and straw-berry – spending a total of ₹710. If each vanilla pack costs ₹55 and each straw–berry pack costs ₹80, then how many packs did she buy in all?

Directions for questions 18 and 19: These questions are based on the data given below:

In a certain cricket match, Mithali Raj scored 130 runs. She scored all the runs only in sixes, twos and singles. The total number of scoring shots was 42. The number of sixes hit is at least 8 more than the number of twos taken and the number of singles taken is at most 10 more than the number of twos taken.

18. How many sixers did Mithali Raj hit in the match?
(A) 16 (B) 18 (C) 8 (D) 17

19. Which of the following statements is/are true?
(A) The number of sixers hit is twice the number of twos taken and the total number of runs scored in sixers is 96.
(B) The number of singles taken is 2 more than the number of sixers hit.
(C) The total number of runs scored in twos and singles is eight less than the number of scoring shots.
(D) All the above.

20. A potter makes more than 100 but less than 300 pots and arranges them in rows, with each row consisting of the same number of pots, for drying. He finds that if he places 6 pots more per row, he can arrange the pots in 10 less rows. How many pots does the potter make?

21. Pallavi has some coins in the denominations of five rupees, two-rupees and one rupee. The number of two-rupee coins is four times the number of one rupee coins and the total amount with Pallavi is ₹135. If Pallavi has at least 10 five-rupee coins, then how many coins does she have in all?

Directions for questions 22 and 23: These questions are based on the data given below.

Anoop Bhandari of Anmol Jewellers bought three types of precious stones – diamonds, rubies and sapphires spending a total of ₹75000. The price of each diamond, ruby and sapphire was ₹1800, ₹2700 and ₹1200 respectively. He bought a total of 42 stones and a minimum of 5 stones of each variety.

22. If Anoop Bhandari bought two varieties of stones in equal number, then how many rubies did he buy?
(A) 12 (B) 10 (C) 8 (D) 6

23. If the number of sapphires bought was more than the number of diamonds and rubies together bought, then how many diamonds did he buy?
(A) 6 (B) 12 (C) 14 (D) 10

24. There are two kinds of insects living in a certain anthill. The number of insects of the first kind triples every day while that of the second kind becomes 6 times every day. Initially, there are more than 25 of the first kind and more than 2 of the second kind. After 4 days the total number of insects is 6804. Find the total number of insects at the beginning.
(A) 54 (B) 24 (C) 39 (D) 69

25. Arpitha goes to a stationery shop to buy some pens, rulers and refills. She decides to buy twice as many refills as pens and at least 10 pens more than rulers. She finds that refills are sold at half the price of pens and rulers are thrice as expensive as refills. If each refill costs ₹5 and Arpitha spends a total of ₹350, then how many items did she buy in all?

26. How many ordered pairs of positive integers (x, y) satisfy the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{12}$?

27. How many ordered pairs of integers (x, y) where y > 0 satisfy the equation $\frac{5}{x}+\frac{1}{y}=\frac{1}{31}$?
(A) 2 (B) 4 (C) 6 (D) 8

28. How many ordered pairs of integers satisfy the equation $\frac{11}{x}-\frac{2}{y}=\frac{1}{9}$?
(A) 16 (B) 32 (C) 15 (D) 40

29. How many positive integral values of (x, y) satisfy the equation $x^2 - y^2 = 1155$?
(A) 16 (B) 32 (C) 8 (D) 4

30. If $a^2 - b^2 = 132$, how many integral values can (a, b) take?

Exercise – 1(b)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. Ramu told Ravi "8 times the date of my birth added to 15 times the month of my birth is 240". In which month was Ramu born?
(A) January (B) March
(C) September (D) August

2. A fruit-seller wants to sell some fruits that he has and get exactly ₹100. He has some mangoes which cost ₹5 each and some apples which cost ₹6 each. In how many possible combinations can he sell the fruits to obtain the desired amount if he sells at least one fruit of each kind?

3. In the month of June, a craftsman takes up an assignment on a daily–wage basis. If he completes the day's task he earns ₹50 or else he earns only ₹30 per day. At the end of the month if he earns ₹1430, on how many days did he not complete the task given to him?
(A) 28 (B) 1 (C) 15 (D) 5

4. In the previous question, for how many days did the craftsman report for work?
(A) 29 (B) 24 (C) 28 (D) 30

5. In how many ways can 149 be divided into two positive parts such that one part is divisible by 5 and the other part is divisible by 8?

6. How many 2–digit numbers leave a remainder of 1 when divided by 17 and a remainder of 2 when divided by 3?

Moderate

7. A and B together have a collection of less than 60 chocolates. If A gives a certain number of chocolates to B, then B would have thrice the number of chocolates left with A. Instead, if B gives the same number of chocolates to A, then both will have the same number of chocolates. The number of chocolates with A can be
(A) 6 (B) 16 (C) 19 (D) 14

8. The average percentage of a group of 'n' students is 75. Three students who got 75%, 85% and 99% join the group and one student whose percentage is between 94 and 100 leaves the group. The average percentage of the group now goes up by 2. If initially the number of students is a multiple of 5, then what is the number of students in the group now?

9. Ravi wants to buy some stationery items. He wants to buy pencils, pens and sheets of paper spending a total of ₹50. If each pen costs ₹6, each pencil cost ₹5 and each sheet cost ₹3, and if Ravi wants to buy atleast 3 items of each kind, then in how many combinations can he buy the items?
(A) 4 (B) 1 (C) 2 (D) 3

10. On a certain day, a book dealer sold some books on art, some books on science and some magazines. The cost of each book on art is ₹100, the cost of each book on science is ₹120 and the cost of each magazine is ₹25. If at the end of the day the total sales are ₹685, then what is the maximum possible number of books and magazines he could have sold that day, if he had sold at least one of each kind?

Directions for questions 11 and 12: These questions are based on the information given below.

A gift shop owner buys some gifts of two kinds A and B, spending a total of ₹4000. Each piece of the kind A costs ₹100 and each piece of the kind B costs ₹20. Instead, if he buys as many pieces of B as he actually bought of A and as many pieces of A as he actually bought of B, he would spend less than half of what he actually spent.

11. In how many different combinations could he have bought the gifts?

12. If he buys at least 10 pieces of each variety, then how many gifts did he buy in total?

13. In a triangle, all the angles are acute and are an integral number of degrees. 19 times one angle is equal to 21 times another. What is a possible value of the least angle in the triangle, in degrees?
(A) 19 (B) 20 (C) 57 (D) 49

14. Ravi bought a total of 40 pens of three varieties A, B and C. Each pen of variety A, B and C costs ₹24, ₹50 and ₹30 respectively. He spent an amount of ₹1420. If he has bought the greatest number of pens of variety B, then how many pens of varieties A and C together did he buy?
(A) 23 (B) 26 (C) 24 (D) 25

15. Ram bought 2 kinds of books – arts and science. Each art book costs ₹72 and each science book costs ₹110. If he has tendered exactly ₹1020 for the purchase, how many books could he have bought in total?
(A) 10 (B) 9 (C) 11 (D) 14

16. Eat–well Candies sells three types of chocolates – eclairs, caramels and mints. Each eclair costs ₹3, each caramel costs ₹2 and each mint costs ₹1. Ravi

purchased chocolates worth a total of ₹45. He bought twice as many mints as caramels. How many eclairs could he have bought?
(A) 4 (B) 7 (C) 6 (D) 10

Directions for questions 17 and 18: These questions are based on the information given below.

Kishan Lal, a jeweller, bought a total of 27 precious stones of types A, B and C for a total of ₹30,000. The prices of each stone of types A, B and C are ₹750, ₹1000 and ₹1250 respectively. He bought more than 3 stones of each variety and he did not buy the same number of stones of any two varieties.

17. How many stones of the types A and B together did he buy?
(A) 11 (B) 12 (C) 13 (D) 14

18. Which variety of stones did he buy the most?
(A) A (B) B (C) C (D) A or B

Directions for questions 19 and 20: These questions are based on the information given below.

A cashier collects a sum of ₹700 in 27 notes in the denominations of ₹50, ₹10 and ₹5. The number of 5 rupee notes is less than the number of 50 rupee notes, and the difference is at most 2.

19. How many ₹10 notes does the cashier have?

20. If he loses two ₹50 notes, then what would be the total number of ₹50 notes with him?

21. A fruit vendor packs less than 21 dozen fruits into more than 15 boxes. He finds that if he packs 5 fruits less per box, he needs 10 boxes more to pack the fruits. How many fruits does he pack?
(A) 208 (B) 168 (C) 132 (D) 100

22. Madhavi has 5 times as many one-rupee stamps as three-rupee stamps. She also has some five-rupee stamps with her. The total value of these stamps is ₹75. If she has at least 20 one-rupee stamps, then how many stamps does she have in all?
(A) 36 (B) 37 (C) 38 (D) 39

23. Ram and Ravi, two magicians, present a magic-show together. Ram triples every red flower in 10 seconds and Ravi doubles every yellow flower in 10 seconds. If after a minute there were 1049 red and yellow flowers, how many flowers were there initially?

24. How many ordered pairs of positive integers (x, y) satisfy the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{7}$?
(A) 1 (B) 2
(C) 3 (D) more than 3

25. If $a^2 - b^2 = 1001$, how many positive integral values can (a, b) take?
(A) 4 (B) 8 (C) 5 (D) 7

Difficult / Very Difficult

26. In an organisation, there are a total of 36 employees belonging to three different departments – A, B and C with each department having a minimum of 10 employees. The organisation decides to pay a bonus of ₹5000, ₹3000, and ₹2000 to each of the employees of the departments A, B and C respectively. If the total amount paid to the employees as bonus is ₹1,22,000, then the maximum possible number of employees in department C is

27. Sangamithra goes to a stationery shop to buy some scales, pencils and crayons. She decides to buy twice as many crayons as pencils and at least one pencil more than scales. Pencils are thrice as expensive as scales, and scales are sold at half the price of crayons. If each scale costs ₹2 and the total amount spent is ₹94, how many items did she purchase in all?
(A) 23 (B) 25 (C) 27 (D) 29

28. How many ordered pairs of integers (x, y) where x > 0, satisfy the equation $\frac{3}{x}+\frac{4}{y}=\frac{1}{5}$?

29. How many ordered pairs of positive integers (x, y) satisfy the equation $\frac{37}{x}-\frac{4}{y}=\frac{1}{13}$?
(A) 18 (B) 11 (C) 9 (D) 20

30. How many integral values of (x, y) satisfy the equation $4x^2 - 9y^2 = 2100$?
(A) 72 (B) 36
(C) 18 (D) None of these

Key

Concept Review Questions

1. B	3. D	5. C	7. 2	9. C	11. A	13. 5	15. 10
2. 10	4. B	6. C	8. 2	10. C	12. 2	14. A	

Exercise – 1(a)

1. B	4. A	7. B	10. 4	13. A	16. 6	19. D	22. B	25. 50	28. D
2. 6	5. 59	8. B	11. 30	14. C	17. 12	20. 225	23. A	26. 15	29. C
3. A	6. 2	9. A	12. B	15. A	18. A	21. 43	24. C	27. D	30. 8

Exercise – 1(b)

1. D	4. A	7. A	10. 13	13. B	16. B	19. 5	22. B	25. A	28. 26
2. 3	5. 4	8. 7	11. 2	14. A	17. A	20. 10	23. 6	26. 12	29. B
3. B	6. 2	9. B	12. 48	15. C	18. C	21. A	24. C	27. A	30. D