CHAPTER – 4

SEQUENCES AND SERIES

PROGRESSIONS

In this chapter, we will look at the problems on sequences or progressions of numbers, where the terms of the sequence follow a particular pattern either addition of a constant (Arithmetic Sequence or Arithmetic Progression) or multiplication by a constant (Geometric Sequence or Geometric Progression). A third type of progression - Harmonic Progression – has also been defined later.

ARITHMETIC PROGRESSION (A.P.)

An arithmetic progression is a sequence of numbers in which any number (other than the first) is more (or less) than the immediately preceding number by a constant value. This constant value is called the common difference. In other words, any term of an arithmetic progression can be obtained by adding the common difference to the preceding term.

Let a be the first term of an arithmetic progression; d the common difference and n the number of terms in the progression.
The n^{th} term is normally represented by T_n and the sum to n terms of an arithmetic progression is denoted by S_n
$T_n = n^{th}$ term $= a + (n – 1)d$
S_n = Sum of n terms $= \frac{n}{2} \times [2a + (n - 1)d]$, then the progression can be represented as a, a + d, a + 2d, …….., [a + (n – 1)d]. Here, quantity d is to be added to any chosen term to get the next term of the progression.

The sum to n terms of an arithmetic progression can also be written in a different manner.

Sum of first n terms $= \frac{n}{2} \times [2a + (n - 1)d]$

$= \frac{n}{2} \times [a + \{a + (n - 1)d\}]$

But, when there are n terms in an arithmetic progression, a is the first term and {a + (n – 1)d} is the last term. Hence,

$S_n = \frac{n}{2} \times$ [First Term + Last Term]

The average of all the terms in an arithmetic progression is called their Arithmetic Mean (A.M.). Since average is equal to {sum of all the quantities/number of quantities}, arithmetic progression must be equal to the Sum of the terms of the arithmetic progression divided by the number of terms in the arithmetic progression.
Arithmetic Mean of n terms in arithmetic progression.

$= \frac{Sn}{n} = \frac{1}{2}\{2a + (n - 1)d\}$

$= \frac{1}{2} \times (\text{First Term} + \text{Last Term})$

$= \frac{(\text{First Term} + \text{Last Term})}{2}$

i.e., A.M. is the average of the first and the last terms of the A.P.

Arithmetic Mean can also be obtained by taking the average of any two terms which are EQUIDISTANT from the two ends of the A.P. i.e.

- The average of the second term from the beginning and the second term from the end will be equal to the A.M.
- The average of the third term from the beginning and the third term from the end will also be equal to the A.M. and so on.

In general, the average of the k^{th} term from the beginning and the k^{th} term from the end will be equal to the A.M.

Conversely, if the A.M. of an A.P. is known, the sum to n terms of the series (S_n) can be expressed as
$S_n = n \times$ A.M

If three numbers are in arithmetical progression the middle number is called the Arithmetic Mean, i.e. if a, b, c are in A.P., then b is the A.M. of the three terms and $b = \frac{a+c}{2}$.

If a and b are in Arithmetic Progression (A.P.), then their A.M. $= \frac{(a+b)}{2}$.

If three numbers are in A.P., we can represent the three numbers as (a – d), a and (a + d).

If four numbers are in A.P., we can represent the four numbers as (a – 3d), (a – d), (a + d) and (a + 3d); (in this case, 2d is the common difference).

If five numbers are in A.P., we can represent the five numbers as (a – 2d), (a – d), a, (a + d) and (a + 2d).

Examples

4.01. Find the 10^{th} term of an arithmetic progression whose first term is 2 and the common difference is 3.

Sol. The nth term of an arithmetic progression is given by a + (n – 1)d, where a and d are the first term and the common difference of the arithmetic progression respectively.
As n = 10, a = 2 and d = 3, the 10^{th} term = 2 + (10 – 1) 3 = 29.

4.02. Find the number of terms in an arithmetic progression with the first term being 3 and the last term being 67, given that the common difference is 4.

Sol. The nth term = a + (n – 1) d
67 = 3 + (n – 1) 4
$\Rightarrow n - 1 = \frac{67 - 3}{4} = 16$
$\Rightarrow n = 17$.

4.03. Find the first term and common difference of an A.P., if the fourth term is 14 and the eleventh term is 42.

Sol. Fourth term = a + 3d = 14
Eleventh term = a + 10d = 42
Subtracting the first equation from the second, 7d = 28 $\Rightarrow$ d = 4.
Substituting d in any one of the two equations, we get a = 2.
Hence the first term and the common difference are 2 and 4 respectively.

4.04. Find number of terms in an A.P. whose sixth term is 19 and the twelfth term is 37 and the last term is 67.

Sol. Sixth term = a + 5d = 19 ----------- (1)
Twelfth term = a + 11d = 37 ----------- (2)
Subtracting equation (1) from equation (2),
6d = 18 $\Rightarrow$ d = 3.
Substituting d = 3 in equation (1) or (2), a = 4
The last term, if the A.P. has n terms, is
a + (n – 1) d
= 4 + (n – 1) 3 = 67
$\Rightarrow$ n – 1 = 21
$\Rightarrow$ n = 22.
Hence there are 22 terms in the A.P.

4.05. Find the sum of the first 24 terms of the A.P. given that the first term is 3 and the common difference is 5.

Sol. The sum of the first n terms of an arithmetic progression is given by $\frac{n}{2}[2a+(n-1)d]$

$\therefore$ Sum of the first 24 terms

$= \frac{24}{2}[(2\times3)+(23\times5)]$ = 1452.

4.06. Find the arithmetic mean of an A.P. with 33 terms if its first term is 1 and common difference is 2.

Sol. Arithmetic mean

$= \frac{1}{2}[2a+(n-1)d]$

$= \frac{1}{2}[2(1)+32(2)]$

= 33.

4.07. There are three numbers in A.P. whose sum is 24 and product is 480. Find the numbers.

Sol. Let the three numbers be a – d, a and a + d.
a – d + a + a + d = 24
$\Rightarrow$ 3a = 24
$\Rightarrow$ a = 8
(8 – d) (8) (8 + d) = 480
$\Rightarrow$ 64 – d^2 = 60
$\Rightarrow$ d^2 = 4
$\Rightarrow$ d = ± 2.
Hence the three numbers are 6, 8 and 10 when d = 2 and the same numbers in reverse order, when d = –2.

4.08. 148 is split into four parts which are in arithmetic progression such that the product of the second and third parts is 8 more than the product of the first and last part. What are the four parts?

Sol. Let the four parts be a – 3d, a – d, a + d and a + 3d.
Sum of the four parts = a – 3d + a – d + a + d + a + 3d = 148
$\Rightarrow$ 4a = 148
$\Rightarrow$ a = 37.
(a – d) (a + d) = (a – 3d) (a + 3d) + 8
$a^2 - d^2 = a^2 - 9d^2 + 8$
$\Rightarrow$ d^2 = 1
$\Rightarrow$ d = ±1.
Hence the four parts are 34, 36, 38 and 40 when d = 1. When d = –1, we get the same numbers in descending order.

4.09. The first term and the last term of an A.P. are 7 and 51 respectively. If the sum of the terms of the A.P. is 348, find the common difference.

Sol. Let the number of terms be n.

$\frac{n}{2}[7+51]$ = 29n = 348

n = 12.
Let common difference be d.
7 + (12 – 1) d = 51
$\Rightarrow$ 11d = 44
$\Rightarrow$ d = 4.

4.10. The 5^{th} term and the 21^{st} term of a series in an A.P. are 10 and 42 respectively. Find the 31^{st} term.

Sol. a + 4d = 10 -------------- (1)
a + 20d = 42 ------------- (2)
By subtracting equation (1) from equation (2), 16d = 32
$\Rightarrow$ d = 2.
Substituting d = 2 in either (1) or (2), a = 2.
31^{st} term = a + 30d = 2 + 30(B) = 62.

4.11. Find the values of three numbers in arithmetic progression such that their sum is 30 and the sum of their squares is 308.

Sol. Let the three numbers in A.P. be a – d, a and a + d.
a – d + a + a + d = 30
$\Rightarrow$ 3a = 30
$\Rightarrow$ a = 10.
$(a - d)^2 + a^2 + (a + d)^2$
$= a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2$
$3a^2 + 2d^2 = 308$
$2d^2 = 308 - 3a^2 = 8$, as a = 10.
$\Rightarrow$ d^2 = 4 $\Rightarrow$ d = ± 2.
Hence the numbers are 8, 10 and 12 when d = 2 and the same numbers in reverse order when d = –2.

HARMONIC PROGRESSION (H.P)

If the reciprocals of the terms of a sequence are in arithmetic progression, the sequence is said to be a harmonic progression, For example, 1, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$,........ is a harmonic progression. In general, the sequence $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$,....... is a harmonic progression.

For two numbers a and b, their Harmonic Mean (H.M) is given by $\frac{2ab}{a+b}$.
For any two positive numbers a and b,

$$\boxed{A.M \geq G.M. \geq H.M}$$

If a, b, c are in harmonic progression, b is said to be the harmonic mean of a and c. In general, if $x_1, x_2, \ldots\ldots x_n$ are in harmonic progression $x_2, x_3, \ldots. x_{n-1}$ are the n – 2 harmonic means between x_1 and x_n.

GEOMETRIC PROGRESSION (G.P.)

Numbers taken in a certain order, are said to be in Geometrical Progression, if the ratio of any (other than the first number) to the preceding one is the same. This ratio is called the Common Ratio. In other words, any term of "a geometric progression can be obtained by multiplying the preceding number by the common ratio.

The common ratio is normally represented by r. The first term of a geometric progression is denoted by a.

A geometric progression can be represented as a, ar, ar^2, ... where a is the first term and r is the common ratio of the geometric progression.
n^{th} term of the geometric progression is ar^{n-1}.

Sum to n terms :

$$\frac{a\left(1-r^n\right)}{1-r} \text{ or } \frac{a\left(r^n-1\right)}{r-1}$$

$$= \frac{xar^{n-1}-a}{r-1} = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

Thus the sum to n terms of a geometric progression can also be written as

$$S_n = \frac{r \times \text{Last term} - \text{First term}}{r-1}$$

If n terms a_1, a_2, a_3, ... a_n are in G.P., then the Geometric Mean (G.M.) of these n terms is given by $= \sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots\ldots\ldots \cdot a_n}$

If three terms are in geometric progression then the middle term is a Geometric Mean of the other two terms, i.e., if a, b and c are in G.P., then b is the geometric mean of the three terms and $b^2 = ac$.

If there are two terms a and b, their geometric mean (G.M.) is given by $\sqrt{ab}$.

For any two unequal positive numbers a and b, their Arithmetic Mean is always greater than their Geometric Mean, i.e.

For any two unequal positive numbers a and b $\frac{a+b}{2} > \sqrt{ab}$; (a + b) > $2\sqrt{ab}$

When there are three terms in geometric progression, we can represent the three terms to be a/r, a and ar

When there are four terms in geometric progression, we can represent the four terms as $\frac{a}{r^3}$, $\frac{a}{r}$, ar and ar^3.

(In this case r^2 is the common ratio)

INFINITE GEOMETRIC PROGRESSION

If $-1 < r < +1$ or $| r | < 1$, then the sum of a geometric progression does not increase infinitely; it "converges" to a particular value. Such a G.P. is referred to as an infinite geometric progression. The sum of an infinite geometric progression is represented by S_∞ and is given by the formula

$$S_\infty = \frac{a}{1-r}.$$

SOME IMPORTANT RESULTS

The results of the sums to n terms of the following series are quite useful and hence should be remembered by students.

Sum of the first n natural numbers

$$= \sum n = \frac{n(n+1)}{2}$$

Sum of squares of the first n natural numbers

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first n natural numbers

$$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n+1)^2}{4} = \left[\sum n\right]^2$$

Examples

4.12. If the 7^{th} term of a G.P, having all positive terms, is (729/1024) and the first term is 4, find the common ratio.

Sol. The nth term of a geometric progression is ar^{n-1} where a is the first term, r is the common ratio and n is the number of terms.

$$\Rightarrow 4r^6 = \frac{729}{1024}$$

$$\Rightarrow r^6 = \frac{729}{4096}$$

$$\Rightarrow r^6 = \left(\frac{3}{4}\right)^6$$

$\Rightarrow r = \frac{3}{4}$.

(positive value only is considered because all the terms are positive)
Hence the common ratio is 3/4.

4.13. The sum of the first five terms of a G.P. is 363. If the common ratio is 1/3 find the first term.

Sol. Sum of the first n terms of a G.P.

$= \frac{a(1-r^n)}{1-r}$ where a is the first term, r is the common ratio and n is the number of terms.

$$363 = \frac{a\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}}$$

$$a = \frac{363\times\left(1-\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^5} = 243.$$

4.14. Find the sum of the terms of a G.P. if the first term is 4 and the last term is (1/64) and the common ratio is (1/2).

Sol. The sum of the terms of a G.P. is

$$\frac{r\times\text{last term}-\text{first term}}{r-1}$$

Since the last term is less than the first term, for the sake of convenience, we rewrite the above expression as

$$\frac{\text{first term}-r\times\text{last term}}{1-r}$$

$$= \frac{4-\frac{1}{2}\left(\frac{1}{64}\right)}{1-\frac{1}{2}} = \frac{\frac{511}{128}}{\frac{1}{2}} = \frac{511}{64}.$$

4.15. Find the last term of a G.P. whose first term is 9 and common ratio is (1/3) if the sum of the terms of the G.P. is (40/3).

Sol. Sum of the G.P.

$$= \frac{\text{first term }-r\times\text{last term}}{1-r}$$

$$\frac{40}{3} = \frac{9-\frac{1}{3}\text{ (last term)}}{\frac{2}{3}}$$

$$\Rightarrow \text{last term} = \left(\frac{-40}{3}\times\frac{2}{3}+9\right)\times 3$$

$$= \frac{-80}{3}+27 = \frac{1}{3}$$

4.16. Find three numbers in G.P. having a sum of 21 and a product of 216.

Sol. Let the three numbers be $\frac{a}{r}$, a and ar.

Given that $\frac{a}{r}\times a\times ar = 216$

$\Rightarrow a^3 = 216 \quad \Rightarrow a = 6$

Sum of the numbers $= \left(\frac{a}{r}\right) + a + ar = 21$

$a + ar + ar^2 = 21r$
but a = 6
$6 + 6r + 6r^2 = 21r$
$6r^2 - 15r + 6 = 0$
$2r^2 - 5r + 2 = 0$
$\Rightarrow (2r - 1)(r - 2) = 0$
$\Rightarrow r = (1/2)$ or 2
Hence the numbers are 3, 6 and 12 when r = 2 and the same numbers in reverse order when r = (1/2).

4.17. The sum to infinity of a G.P. is 27 and the sum of the squares of the terms is 243. Find the common ratio.

Sol. Let the first term be a and the common ratio be r.
Given that,

$\frac{a}{1-r} = 27$ ------------ (1) and

$\frac{a^2}{1-r^2} = 243$ -------- (2)

Dividing (2) by (1),

$\frac{a}{1+r} = 9$ ------------ (3)

Dividing (3) by (1),

$\frac{1-r}{1+r} = \frac{1}{3}$

$\Rightarrow 3 - 3r = 1 + r$

$\Rightarrow 4r = 2 \Rightarrow r = \frac{1}{2}$

4.18. Given that $|y| < 1$, find the value of
$3 + 6y + 9y^2 + 12y^3 + \ldots\ldots\ldots\ldots$

Sol. Let $S = 3 + 6y + 9y^2 + 12y^3 + \ldots\ldots\ldots$
Multiplying the equation by y, we get
$yS = 3y + 6y^2 + 9y^3 + 12y^4 + \ldots\ldots\ldots$
Subtracting second equation from the first,

$S(1 - y) = 3 + 3y + 3y^2 + \ldots\ldots\ \infty)$

$$S = \frac{(3+3y+3y^2+\ldots..\infty)}{1-y}$$

Again $3 + 3y + 3y^2 \ldots\ldots\ \infty$ is a geometric progression of infinite terms.

$$\therefore S = \frac{\frac{3}{1-y}}{1-y} = \frac{3}{(1-y)^2}$$

4.19. Find the sum of the terms of the series

$1, \frac{4}{5}, \frac{16}{25}, \frac{64}{125}, \ldots\ldots\ldots\ldots\infty$

Sol. It can be noticed that that the given series is a G.P. with infinite terms. The common ratio is (4/5) and this is < 1.

The sum to infinity of a series with first term a and the common ratio $r = [a/(1-r)]$

As a = 1 and r = 4/5,

the sum to infinity = $\dfrac{1}{1-(4/5)} = 5$

4.20. Find the sum of the following series,

$2\sqrt{2}\ ,\ \dfrac{4}{\sqrt{3}},\dfrac{4\sqrt{2}}{3},..........\infty$

Sol. The ratio of any term to its previous term starting from the 2nd term is $\sqrt{\dfrac{2}{3}}$. Hence the series is in G.P. The sum to infinity of the series = $\dfrac{2\sqrt{2}}{1-\sqrt{\dfrac{2}{3}}}$

$= \dfrac{2\sqrt{6}}{\sqrt{3}-\sqrt{2}}$

$= \dfrac{2\sqrt{6}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$

(multiplying both numerator and denominator by $\sqrt{3}+\sqrt{2}$)

$= \dfrac{2\sqrt{6}\ \left(\sqrt{3}+\sqrt{2}\right)}{3-2} = 2\sqrt{6}\left(\sqrt{3}+\sqrt{2}\right)$

4.21. If $(2^3 - t_1) + (4^3 - t_2) + (6^3 - t_3) + (8^3 - t_4) + (10^3 - t_5) + \ldots\ \{(2n)^3 - t_n)\}$ $= \dfrac{n^2(n+1)^2}{4}$, find the general expression for t_n.

Sol. $2^3 + 4^3 + 6^3 + \ldots\ldots\ldots(2n)^3$

$= 8\ (1^3 + 2^3 + 3^3 + \ldots\ldots + n^3)$

$= 8\left(\dfrac{n(n+1)}{2}\right)^2 = \dfrac{8n^2(n+1)^2}{4}$

$= 2n^2(n+1)^2$ ---------- (1)

Substituting (A) in the given equation and transposing the terms, the equation becomes:

$t_1 + t_2 + t_3 + t_4 + \ldots\ldots + t_n$

$= 2n^2(n+1)^2 - \dfrac{n^2(n+1)^2}{4}$

$= \dfrac{7}{4}n^2(n+1)^2$ ---------- (2)

If this sum is denoted by S_n, then

$S_{n-1} = \dfrac{7}{4}(n-1)^2(n)^2$ ---------- (3)

But, $t_n = S_n - S_{n-1}$

$= \dfrac{7}{4}n^2(n+1)^2 - \dfrac{7}{4}(n-1)^2 n^2$

$= \dfrac{7}{4}n^2\ \left((n+1)^2-(n-1)^2\right)$

$= \dfrac{7}{4}n^2(4n)$

$= 7n^3$

SERIES

We shall look at some useful models on series which have appeared in management entrances. The series could include AP, GP or other patterns of summations which involve concepts of progressions. There could also be other series which appear to be related to progressions, but actually involve techniques of mathematical manipulation. These techniques are best illustrated or learned using examples. But for most questions observing the pattern proved to be a useful method for arriving at the answer.

4.22. Find the value of the expression

$\dfrac{1(2)(3)+2(3)(4)+3(4)(5)+\ldots+8(9)(10)}{1^2(2)+2^2(3)+3^2(4)+\ldots+8^2(9)}$

Sol. $\dfrac{1(2)(3)+2(3)(4)+3(4)(5)+\ldots..+8(9)(10)}{1^2(2)+2^2(3)+3^2(4)+\ldots.+8^2(9)}$

$= \dfrac{\sum_{n=1}^{8} n(n+1)(n+2)}{\sum_{n=1}^{8} n^2.(n+1)}$

$= \dfrac{\sum_{n=1}^{8} n^3 + 3\sum_{n=1}^{8} n^2 + 2\sum_{n=1}^{8} n}{\sum_{n=1}^{8} n^3 + \sum_{n=1}^{8} n^2}$

$= \dfrac{36^2 + \dfrac{3(8)(9)(17)}{6} + \dfrac{2(8)(9)}{2}}{36^2 + \dfrac{8(9)(17)}{6}} = \dfrac{33}{25}$

4.23. If $A = \dfrac{1}{35(68)} + \dfrac{1}{36(67)} + \dfrac{1}{37(66)} + .. + \dfrac{1}{68(35)}$ and $B = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots + \dfrac{1}{67} - \dfrac{1}{68}$ then the value of $\dfrac{A}{B}$ is

Sol. $A = \dfrac{1}{35(68)} + \dfrac{1}{36(67)} + \ldots. + \dfrac{1}{68(35)}$

$= \dfrac{1}{103}\left[\dfrac{103}{35(68)} + \dfrac{103}{36(67)} + \ldots + \dfrac{103}{68(35)}\right]$

$= \dfrac{1}{103}\left[\dfrac{1}{35} + \dfrac{1}{68} + \dfrac{1}{36} + \dfrac{1}{67} + \ldots. + \dfrac{1}{68} + \dfrac{1}{35}\right]$

$= \dfrac{2}{103}\left[\dfrac{1}{35} + \dfrac{1}{36} + \ldots. + \dfrac{1}{68}\right]$

$B = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots. + \dfrac{1}{67} - \dfrac{1}{68}$

$= 1 + \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + .. + \dfrac{1}{67} + \dfrac{1}{68} - 2\left[\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{6} + .. \dfrac{1}{68}\right]$

$= 1 + \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + \ldots. + \dfrac{1}{67} + \dfrac{1}{68} - \left[1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots. \dfrac{1}{34}\right]$

$= \dfrac{1}{35} + \dfrac{1}{36} + \dfrac{1}{37} + \ldots. + \dfrac{1}{68}$

$\dfrac{A}{B} = \dfrac{2}{103}$

4.24. Find the sum of the terms

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots+\sqrt{1+\frac{1}{14^2}+\frac{1}{15^2}}$$

Sol. $$1+\frac{1}{n^2}+\frac{1}{(n+1)^2}=\frac{(n+1)^2n^2+n^2+(n+1)^2}{n^2(n+1)^2}$$

$$=\frac{n^4+2n^3+3n^2+2n+1}{(n(n+1))^2}=\frac{(n^2+n+1)^2}{(n(n+1))^2}$$

$$\therefore \sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}=\frac{n^2+n+1}{n(n+1)}=1+\frac{1}{n(n+1)}$$

$$=1+\frac{1}{n}-\frac{1}{n+1}$$

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots\ldots\sqrt{1+\frac{1}{14^2}+\frac{1}{15^2}}$$

$$=1+\frac{1}{1}-\frac{1}{2}+1+\frac{1}{2}-\frac{1}{3}+\ldots.+1+\frac{1}{14}-\frac{1}{15}$$

$$=14+1-\frac{1}{15}=14\frac{14}{15}$$

4.25. In a certain series, for n ≥ 2, T_n equals $5T_{n-1}-3$. If $T_1 = 5$, then T_{100} =

Sol. It is given
$T_n = 5T_{n-1} - 3$
$= 5(5T_{n-2} - 3) - 3$
$= 5^2T_{n-2} - 5\,(3) - 3$
$= 5^2(5T_{n-3} - 3) - 5\,(3) - 3$
$= 5^3\,T_{n-3} - 3\,(5^2) - 3\,(5) - 3\,(1)$
So we can get
$T_n = 5^{n-1}\,T_1 - 3\,(5^{n-2}) - 3\,(5^{n-1}) - \ldots.\ 3\,(1)$
$= 5^n - 3(1 + 5^1 + \ldots.+5^{n-2})$

$$=5^n-3\frac{(5^{n-1}-1)}{5-1}$$

$$=\frac{4(5^n)-3(5^{n-1})+3}{4}=\frac{17(5^{n-1})+3}{4}$$

$$T_{100}=\frac{17\,(5^{99})+3}{4}$$

Concept Review Questions

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Which term of the series 2, 5, 8, 11, …… is 266?
(A) 88th (B) 89th
(C) 90th (D) 91st

2. Find the geometric mean of 4, 8, 16 and 32.
(A) 128 (B) $\sqrt{128}$
(C) 15 (D) 16

3. Find the fourth term of the arithmetic progression whose first term is 6 and the seventh term is 24.
(A) 15
(B) 7.5
(C) 9
(D) Cannot be determined

4. Find the sum of the first seven terms of an arithmetic progression whose first term is one and the common difference is 3.

5. Find the sum of the first 9 terms of an arithmetic progression whose first term is 5 and the last term is 33.
(A) 85.5
(B) 171
(C) 342
(D) Cannot be determined

6. Find the arithmetic mean of an arithmetic progression with 17 terms whose ninth term is 11.
(A) 22
(B) 11
(C) 33
(D) Cannot be determined

7. Find the sum of the first 25 terms of an arithmetic progression whose 13th term is 20.
(A) 250
(B) 400
(C) 500
(D) Cannot be determined

8. Find the eighth term of an arithmetic progression whose first term is 7 and the common difference is 5.
(A) 24
(B) 47
(C) 42
(D) Cannot be determined

9. Find the fourth term of a geometric progression whose second term is 8 and sixth term is 32.
(A) 16
(B) –16
(C) Either (A) or (B)
(D) Neither (A) or (B)

10. Find the sum of the first four terms of a geometric progression whose first term is 4 and the common ratio is 3.
(A) 320
(B) 160
(C) 80
(D) Cannot be determined

11. An arithmetic progression has 200 terms. The pth term from the beginning is 15. The pth term from the end is 45. Find the sum of all the terms.
(A) 12000
(B) 6000
(C) 9000
(D) Cannot be determined

12. If p, q and r are in arithmetic progression, the pth term, qth term and the rth term of an arithmetic progression are in _______.
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) None of these

13. If p, q and r are in arithmetic progression, then the p^{th} term, q^{th} term and the r^{th} term of a geometric progression are in _______.
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) Both (A) and (B)

14. The sum of the first 61 terms of an arithmetic progression is 0. Which of the following terms is necessarily 0?
(A) 15^{th} (B) 30^{th}
(C) 31^{st} (D) None of these

15. The sum of the first 30 terms of an arithmetic progression is 40. The sum of its first 80 terms is also 40. Find the sum of its 31^{st} and 80^{th} terms.

16. If a, b and c are distinct positive numbers in geometric progression, log a, log b and log c will be in _______.
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) None of these

17. Two distinct positive numbers have their geometric mean equal to 9. Their arithmetic mean will be _______.
(A) < 9 (B) = 9
(C) > 9 (D) Either (B) or (C)

18. Find the eighth term of a geometric progression whose first term as well as common ratio is 2.

19. (i) A geometric progression has its seventh term equal to 2. Find the product of its first 13 terms.
(A) 4 (B) 16
(C) 2048 (D) 8192

(ii) Find the geometric mean of the progression in (i).
(A) 2 (B) –2
(C) 2 or –2 (D) None of these

20. Find the sum to infinity of 1, 1/4, 1/16,…….
(A) $\frac{3}{4}$ (B) $\frac{5}{4}$ (C) $\frac{4}{3}$ (D) $\frac{4}{5}$

21. Each term of a geometric progression is $\frac{1}{x}$ th of the sum of all the terms of the progression following it. Find the common ratio of the progression in terms of x.
(A) $\frac{x-1}{x}$ (B) $\frac{x}{x+1}$
(C) $\frac{x+1}{x+2}$ (D) None of these

22. Find the sum of the first 7 terms of a geometric progression whose first term is 1 and the fourth term is 8.

23. Find the sum of the cubes of the first ten natural numbers.

24. The 30^{th} term from the beginning in a 100 term series is the ________ term from the end.
(A) 69^{th} (B) 70^{th} (C) 71^{st} (D) 72^{nd}

25. The sum of the first 20 terms in an arithmetic progression is 210. Find the sum of the 10^{th} and the 11^{th} terms of the progression.
(A) 21
(B) 10.5
(C) 42
(D) Cannot be determined.

Exercise – 4(a)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The 69^{th} term of an arithmetic progression is 16 times the fourth term of the progression. If the 7^{th} term of the progression is 22, find the 20^{th} term of the progression.

2. Find the largest of three numbers in arithmetic progression whose sum is 30 and whose product is 840.
(A) 10 (B) 14 (C) 17 (D) 21

3. Find the 10^{th} term of an arithmetic progression, if the sum upto n terms of the arithmetic progression is $2n^2 + 5n$.
(A) 41 (B) 43 (C) 47 (D) 49

4. The sum of four numbers in arithmetic progression is 56. The difference between the product of the two numbers in the middle and that of the two numbers in the extreme ends is 8. Find the smallest of the four numbers.

5. Find the ratio of the 17^{th} terms of two arithmetic progressions if the ratio of the sums upto n terms of the two progressions is (3n + 2) : (4n – 13).
(A) 4 : 7 (B) 7 : 6
(C) 101 : 119 (D) 36 : 55

6. Find the 35^{th} term of an arithmetic progression whose first and the last terms are 4 and 241 respectively and the sum of all the terms is 9800.
(A) 101 (B) 104 (C) 106 (D) 112

7. There are 25 numbers in arithmetic progression such that the largest of the numbers is 10 times the smallest number. If the sum of these numbers is 1100, then find the smallest of the numbers.
(A) 7 (B) 8 (C) 10 (D) 9

8. Find the sum of all the three digit numbers which when divided by 5 leaves a remainder of 2.
(A) 45,570 (B) 68,780
(C) 88,170 (D) 98,910

9. The product of the first three terms of a geometric progression is 216 and its 5^{th} term is 162. Find the first term of the geometric progression.

10. The sum of a, b, c which are in arithmetic progression is 33. If a and b are decreased by 3 and c is increased by 1, the resulting numbers would be in geometric progression. If a < b < c, find the value of c.
(A) 15 (B) 18 (C) 19 (D) 16

11. The sum of three positive integers is 84 and their product is 13,824. If the three numbers are in geometric progression, then find the largest of the three numbers.
(A) 36 (B) 48 (C) 60 (D) 72

12. Find the sum of the first 19 terms of an arithmetic progression, if the sum of the first, the third, the eighth, the twelfth, the seventeenth and the nineteenth terms is 555.
(A) 1285.5 (B) 14555.75
(C) 1757.5 (D) 2358.25

13. The sum of 40 numbers in arithmetic progression is 3600. If the numbers are arranged in ascending order then the sum of the last 10 numbers is $1/3^{rd}$ of the sum of the 40 numbers. Find the smallest of the numbers.

14. The first term of an infinite geometric progression is 3 and any term is equal to twice the sum of all the succeeding terms. Find the fifth term of the geometric progression.
(A) 1/9 (B) 1/18 (C) 1/22 (D) 1/27

15. A ball is dropped from a height of 12 m and it rebounds 1/2 of the distance it falls. If it continues to fall and rebound in this way, how far will it travel before coming to rest? (in m)

16. An equilateral triangle T_2 is formed by joining the midpoints of the sides of another equilateral triangle T_1. A third equilateral triangle T_3 is formed by joining the midpoints of T_2 and this process of forming equilateral triangles is continued indefinitely. If each side of T_1 is 40 cm, find the sum of the perimeters of all the triangles.(in cm)

17. The product of three numbers in geometric progression is 1000 and the sum of the products of the numbers taken two at a time is 350. Find the greatest of the three numbers.
(A) 20 (B) 10 (C) 15 (D) 25

18. What is the product of the first 11 terms of a geometric progression if the sixth term is 2?
(A) 1008
(B) 2048
(C) 2164
(D) Cannot be determined

19. The sum of the first 3 terms of a geometric progression (GP) is 9A, while the sum of the first 6 terms of the GP is 6A. The sum of the first 9 terms of the GP is _______.
(A) 7A (B) 8A (C) 9A (D) 10A

20. The sum of the terms of an infinite geometric series G is 3 and the sum of the terms of an infinite geometric series whose terms are the squares of the terms of G is 6. Find the first term of G.
(A) 11/5 (B) 16/5 (C) 12/5 (D) 22/5

21. The first term of an infinite geometric series is 3. The common ratio of the series is a rational number. The difference between the fourth and the seventh terms of the series is $\frac{21}{64}$. Find the sum of the terms of the series.

22. In a set of four numbers, the first three are in geometric progression while the last three are in arithmetic progression. The first and the fourth numbers are the same and the common difference of the arithmetic progression is 12. Find the common ratio of the geometric progression.
(A) 2 (B) –2 (C) 1/2 (D) –1/2

23. Find the maximum sum of the series 60, 58, 56, 54, 52, …..

24. Find the value of $(100 \times 1) + (99 \times 2) + (98 \times 3) + \ldots\ldots + (2 \times 99) + (1 \times 100)$
(A) 12850 (B) 171700 (C) 23250 (D) 50500

25. Find the sum upto 25 terms of the series 1/2 + 3/4 + 7/8 + 15/16 + ……..
(A) $\frac{25 \cdot 2^{25} - 1}{2^{25}}$ (B) $\frac{24 \cdot 2^{25} + 1}{2^{25}}$
(C) $\frac{25.225 + 1}{2^{25}}$ (D) $\frac{2.2^{24} - 1}{2^{25}}$

26. Find the sum of the series $p(p + q) + p^2(p^2 + q^2) + p^3(p^3 + q^3) + \ldots\ldots$upto infinite terms if $-1 < p < 1$ and $-1 < q < 1$.
(A) $\frac{2 + p^2 + q^2}{(1 - p^2)(1 - q^2)}$ (B) $\frac{p^2 - q^2 - 2}{(1 - p^2)(1 - q^2)}$
(C) $\frac{p^2 - q^2 + 2}{(1 - p^2)(1 - q^2)}$ (D) None of these

27. Find the number of terms which are common to the progressions 2, 5, 8, 11, ……… 434 and 3, 7, 11, 15, …….. 579.

28. A group of 810 children arranged themselves in N rows for a group photograph. In each row except the last, there were 3 more children than the row behind it. Which of the following can be the value of N?
(A) 6 (B) 8 (C) 5 (D) 10

29. S = {1, 2, 3, ….., 400}. Find the number of arithmetic progressions, which can be formed from the elements of S, which begin with 1 and end with 400 and have at least 3 elements.

30. Two new charity organizations C_1 and C_2 were formed, with x members each, on January 1, 2003. On the first day of each subsequent month in C_1, the number of members increases by a certain number **a**, while in C_2, the number of members increases in such a way that the ratio of the number of members in a month to that in the preceding month bear a ratio equal to **b**. On May 1, 2003, both organizations had the same number of members. If **a** = 20x, find **b**.

31. An athlete started running on a circular path of radius R m. His average speed (in m/min) was πR/3 during the first 3/2 minutes, πR/6 during the next 3 minutes, πR/12 during the next 6 minutes, πR/24 during the next 12 minutes and so on. What is the ratio of the times taken for going round the circular path for the nth time and the $(n-1)^{th}$ time?
(A) 8 : 1 (B) 4 : 1 (C) 16 : 1 (D) 32 : 1

32. 3080 balls, each having a radius of 2 cm, are stocked in a pile. There are 2 balls in the topmost layer, 6 balls in the second, 12 balls in the third, 20 in the fourth and so on. Find the number of layers in the pile.

33. Find the sum of the first 15 terms of the series $1^2(4) + 2^2(7) + 3^2(10) + 4^2(13)$……
(A) 44440 (B) 44400
(C) 44404 (D) 44000

34. Find the sum of the first 19 terms of the series
$\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\frac{9}{400}+.....$
(A) $\frac{199}{200}$ (B) $\frac{19}{20}$ (C) $\frac{399}{400}$ (D) $\frac{299}{300}$

35. Find the sum of the first 20 terms of the series 2 + 13 + 28 + 47 + 70 + …..

36. Find the value of $\frac{1}{2(3)(4)}+\frac{1}{3(4)(5)}+...+\frac{1}{19(20)(21)}$.
(A) $\frac{23}{208}$ (B) $\frac{23}{290}$ (C) $\frac{23}{217}$ (D) $\frac{23}{280}$

37. In a certain series, the n^{th} term T_n equals $3T_{n-1} + n - 1$. If $T_1 = 3$, then the value of T_{100} is _______.
(A) $\frac{3^{100}-201}{2}$ (B) $3^{100} - 201$
(C) $\frac{5\left(3^{100}\right)-201}{4}$ (D) $\frac{3^{101}-201}{2}$

38. If $S=2+\frac{5}{5}+\frac{9}{5^2}+\frac{14}{5^3}+\frac{20}{5^4}+.....$, then the value of S is _______.
(A) $\frac{205}{64}$ (B) $\frac{225}{64}$
(C) $\frac{175}{64}$ (D) $\frac{305}{64}$

39. Find the sum of the first 50 terms of the series 1 − 3 − 5 − 7, 3 − 5 − 7 − 9, 5 − 7 − 9 − 11 . . .

40. $P = \frac{1}{(62)(122)}+\frac{1}{(63)(121)}+\frac{1}{(64)(120)}+ . . . . + \frac{1}{(122)(62)}$
$Q = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}....-\frac{1}{122}$
Find $\frac{P}{Q}$.
(A) $\frac{1}{184}$ (B) $\frac{1}{46}$ (C) $\frac{3}{92}$ (D) $\frac{1}{92}$

Exercise – 4(b)

Directions for questions 1 to 55: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. The first term of an arithmetic progression is n less than the nth term, ℓ of the progression. Find the sum to n terms of the progression.
(A) $n\left[\ell-\frac{n}{2}\right]$ (B) $n\left[\frac{n+\ell}{2}\right]$
(C) $n\left[\frac{\ell}{2}+n\right]$ (D) $n\left[\frac{\ell}{2}-n\right]$

2. The sum to 30 terms of a series in arithmetic progression is 150. If the first term of the series is a, the 30^{th} term in the series will be _______.
(A) 10 + a (B) a – 10 (C) 10 – a (D) $\frac{a}{10}$

3. In an arithmetic progression, 15 times the 15^{th} term is equal to 6 times the 6^{th} term. Find the 21^{st} term of the progression.
(A) 0
(B) 14
(C) 26
(D) Cannot be determined

4. The 39^{th} term of a series in arithmetic progression is 4 times the 8^{th} term of the series. If the first term of the series is 10, then find the 25^{th} term of the series.

5. If the sum of the second term, the third term, the sixth term and the seventh term of an arithmetic progression is 18, find the sum of the first 8 terms of the progression
(A) 28 (B) 24
(C) 36 (D) Cannot be determined

6. Find the sum of all the perfect cubes from 60 to 1000.

7. Find the sum to n terms of the series
$\log_2 2, \log_2 4, \log_2 8, \log_2 16$ ………
(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(2n+1)(n+1)}{6}$
(C) $\left[\frac{n(n+1)}{2}\right]^2$ (D) $\frac{n^2(n+1)^2}{2}$

8. Find the number of common terms in the two progressions 5, 10, 15 ….125 and 6, 11, 16 ….106.

Moderate

9. The sum to 100 terms of a series in arithmetic progression is 5050 and the 10^{th} term is 10. What are the values of the first term and the common difference respectively?
(A) 2, 2 (B) 2, 1 (C) 1, 2 (D) 1, 1

10. Find the ratio of the common differences d_1 and d_2 of two arithmetic progressions whose respective n^{th} terms are in the ratio of $2n + 3 : n - 11$.
(A) 1 : 2 (B) 2 : 3 (C) 2 : 1 (D) 1 : 3

11. The sum of the terms of an arithmetic progression of 20 terms is 420. The sum of the least 5 terms of the progression is 30. Find the largest term of the progression.

12. Find the sum of all the perfect squares between 20 and 2000.
(A) 28750 (B) 29340 (C) 29370 (D) 29750

13. Find the sum of all the multiples of 6 between 200 and 1100.

14. Find the number of terms, in the series 12, 10, 8, 6, 4, ……. to which the sum is 36.
(A) 4 (B) 9
(C) Either (A) or (B) (D) Neither (A) nor (B)

15. Find the maximum value of n for which the sum to n terms of the progression 7, 10, 13, ….. is less than 2500.

16. Find the sum of the squares of the first 10 terms of the series whose sum upto n terms is given by $2n^2 + 4n$.

17. If $\log_2 x + \log_{2^{1/2}} x + \log_{2^{1/3}} x + \log_{2^{1/4}} x +$ …... upto 20 terms is 420, find the value of x.
(A) 2 (B) 4 (C) 16 (D) 8

18. The sum of the first 13 terms of an arithmetic progression equals the sum of its first 27 terms. Find the sum of its first 40 terms.
(A) 0 (B) –1
(C) 1 (D) Cannot be determined

19. Five heavy stones are placed, on the road connecting P and Q, at intervals of 4m, with the first stone at P itself. Ajay started from P and began to move all the stones to Q by carrying one stone at a time. If the distance PQ is 200 m find the minimum distance he has to travel (in m).

20. Find the sum upto 20 terms of the series $\frac{2}{3} + \frac{8}{9} + \frac{26}{27} + \frac{80}{81} + \ldots\ldots$
(A) $\frac{19.3^{20} - 1}{2.3^{20}}$ (B) $\frac{19.3^{20} + 1}{2.3^{20}}$
(C) $\frac{39.3^{20} - 1}{2.3^{20}}$ (D) $\frac{39.3^{20} + 1}{2.3^{20}}$

21. If $a = b - b^2 + b^3 - b^4 +$ ……….., and $|b| < 1$, express b in terms of a.
(A) $\frac{a}{1+a}$ (B) $\frac{a}{1-a}$ (C) $\frac{a^2}{1-a}$ (D) $\frac{a^2}{1+a}$

22. Find $5^{1/3} \cdot 5^{1/9} \cdot 5^{1/27} \cdot 5^{1/81}$……..
(A) 1/5 (B) 5 (C) $\sqrt{5}$ (D) 25

23. Three numbers are in an increasing geometric progression. If the second number is multiplied by 4, then the numbers would be in arithmetic progression. Find the value of the common ratio.
(A) $4 - \sqrt{15}$ (B) $4 + \sqrt{15}$
(C) 8 (D) Either (A) or (B)

24. Three numbers are in an arithmetic progression. If the third number is multiplied by 4/3, then the numbers would be in geometric progression. If the first term of the arithmetic progression is 4, find the common difference of the arithmetic progression.
(A) 2
(B) 4
(C) 8
(D) Cannot be determined

25. Find the sum to infinite terms of the following series $3, \frac{6}{y}, \frac{12}{y^2}, \frac{24}{y^3}$ …, given that y > 2.
(A) $\frac{2y}{(y-2)}$ (B) $\frac{4y}{(y-2)}$
(C) $\frac{3y}{(y-2)}$ (D) $\frac{y}{(y-2)}$

26. Find the sum to 15 terms of the series $3.4^2 + 4.5^2 + 5.6^2 + \ldots$

$\boxed{}$

27. If the ratio of the sums up to n terms of the two arithmetic progressions is (7n + 4) : (6n – 5), find the ratio of the 11^{th} terms of the two progressions

(A) $\frac{81}{61}$ (B) $\frac{74}{55}$ (C) $\frac{144}{115}$ (D) $\frac{151}{121}$

28. If | x | < 1, find the sum to infinity of the series $3 + 6x^2 + 9x^4 + 12x^6 + \ldots$

(A) $\frac{3}{(1+x^2)}$ (B) $\frac{3}{(1+x^2)^2}$

(C) $\frac{3}{(1-x^2)^2}$ (D) $\frac{3}{(1-x^2)}$

29. Find a series in geometric progression for which the sum to infinity is 1 and |r| < 1.

(A) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots.$ (B) $\frac{1}{4}, \frac{3}{16}, \frac{9}{64} \ldots.$

(C) $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots.$ (D) Either (A) or (B)

30. There are three numbers a, b and c such that a, b and c are in arithmetic progression and a, c and b are in geometric progression. If the common ratio of the progression a, c and b is not equal to 1, find the value of a : b.

$\boxed{}$

31. The sum of the first ten terms of a geometric progression is 2. Find the first term if the fifth term is twice the sixth term.

(A) $\frac{511}{512}$ (B) $\frac{512}{511}$

(C) $\frac{256}{255}$ (D) $\frac{1024}{1023}$

32. If the sum to infinite terms and the 2^{nd} term of a geometric progression are in the ratio of 9 : 2, then find the common ratio.

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) Either (A) or (B) (D) Neither (A) nor (B)

33. There are three numbers in geometric progression. Their product is 343 and the sum of the products of the numbers taken two at a time is 171.5. Find the largest of the three numbers.

$\boxed{}$

34. The side of a square s_1 is 4 cm. If the midpoints of each side are joined, another square s_2 is formed. If the midpoints of s_2 are joined, another square s_3 is formed. If this process is continued infinitely, then the sum of the perimeters (in cm) of all the squares is _______.

(A) $32 + 16\sqrt{2}$ (B) 32

(C) 48 (D) $32 - 16\sqrt{2}$

35. Find the sum to 20 terms of the series

$\frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \ldots\ldots.$

(A) $\frac{5}{96}$ (B) $\frac{20}{69}$ (C) $\frac{20}{96}$ (D) $\frac{20}{23}$

36. Mohan wrote 30 ones, followed by 29 twos, 28 threes …, one thirty. He added all the numbers. What is the sum he got?

$\boxed{}$

37. Find the sum of the first 18 terms of the series

$\frac{8}{9} + \frac{12}{64} + \frac{16}{225} + \ldots. + \frac{(n+2)^2 - n^2}{n^2(n+2)^2}.$

(A) $\frac{360}{361} - \frac{99}{400}$

(B) $\frac{288}{289} - \frac{80}{324}$

(C) $\frac{360}{361} + \frac{99}{400}$

(D) $\frac{288}{289} + \frac{80}{324}$

38. Find the value of

$\frac{1}{3} + \frac{1}{\sqrt{2}+\sqrt{5}} + \frac{1}{\sqrt{3}+\sqrt{6}} + \frac{1}{\sqrt{4}+\sqrt{7}} + \ldots + \frac{1}{\sqrt{141}+\sqrt{144}}.$

(A) $\frac{11+\sqrt{143}+\sqrt{142}+\sqrt{2}+\sqrt{3}}{2}$

(B) $\frac{11+\sqrt{143}-\sqrt{142}-\sqrt{2}+\sqrt{3}}{2}$

(C) $\frac{11-\sqrt{143}+\sqrt{142}+\sqrt{2}-\sqrt{3}}{2}$

(D) $\frac{11+\sqrt{143}+\sqrt{142}-\sqrt{2}-\sqrt{3}}{3}$

39. If $S = 1 + \frac{3}{5} + \frac{6}{5^2} + \frac{10}{5^3} + \frac{15}{5^4} + \ldots$ **,** then S lies in which of the following ranges?

(A) between 0 and 1 (B) between 1 and 2

(C) between 2 and 3 (D) between 3 and ∞

40. If $S = 2 + \frac{4}{5} + \frac{7}{5^2} + \frac{11}{5^3} + \frac{16}{5^4} + \ldots$, then the value of S is

(A) $\frac{225}{64}$ (B) $\frac{205}{64}$

(C) $\frac{215}{64}$ (D) $\frac{235}{64}$

41. Find the greater of the two numbers whose arithmetic mean is 20 and whose geometric mean is 16.

$\boxed{}$

42. Mohith forgot his password for a special application. His password was either a list of successive characters from 'ABCD…X' or a single character from the same. What is the total number of characters in all possible passwords he could have had?

$\boxed{}$

43. The mth term of an arithmetic progression is 2n, while the nth term of the progression is 2m, where n > m. Find the (n – m)th term.
(A) 2(n – m) (B) 4n
(C) 4m (D) 4(n – m)

44. If $S=\frac{1}{101}+\frac{1}{102}+.....\frac{1}{200}$, then S is equal to

(A) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+.....-\frac{1}{200}$

(B) $\frac{301}{2}\left[\frac{1}{101(200)}+\frac{1}{102(199)}+...+\frac{1}{200(101)}\right]$

(C) $\frac{1}{101(200)}+\frac{1}{102(199)}+....+\frac{1}{200(101)}$

(D) Both (A) and (B)

45. Given

$$S_n=\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+...+\sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}}$$

Find the value of $\frac{S_5+S_6}{S_3+S_4}$.

(A) $\frac{5303}{3519}$ (B) $\frac{5303}{3591}$

(C) $\frac{5330}{3591}$ (D) $\frac{5330}{3519}$

46. $P = 1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+....+\frac{1}{60^3}$

$Q = 1+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{4^3}+....+\frac{1}{120^3}$

$R = \frac{1}{2^3}+\frac{1}{6^3}+\frac{1}{10^3}+....+\frac{1}{238^3}$

R = ________.

(A) $\frac{8Q+P}{64}$ (B) $\frac{7Q+P}{64}$

(C) $\frac{7Q-P}{64}$ (D) $\frac{8Q-P}{64}$

47. The 30th term of the sequence −13, −7, 0, 8, 17, ... is $\boxed{}$.

Difficult

48. Find the sum of the first 20 terms of the series $1^2(2)+2^2(7)+3^2(12)+4^2(17)+.....$
(A) 289110 (B) 298110
(C) 211890 (D) 211089

49. If $S = x + 5x^2 + 11x^3 + 21x^4 + 36x^5 + 57x^6 + ...$, where $|x|<1$, then S is equal to ________.

(A) $\frac{x}{1-x}+\frac{x^2}{(1-x)^4}$ (B) $\frac{x^2}{1-x}+\frac{x}{(1-x)^4}$

(C) $\frac{x}{1-x}+\frac{x^2}{(1-x)^3}$ (D) $\frac{x^2}{1-x}+\frac{x}{(1-x)^3}$

50. Find the sum of the first 20 terms of the series 2 + 15 + 32 + 53 + 78 + ……
(A) 6060 (B) 6090
(C) 9060 (D) 7070

51. Find the value of $\frac{1}{1(3)(5)}+\frac{1}{3(5)(7)}+...+\frac{1}{19(21)(23)}$.

(A) $\frac{80}{483}$ (B) $\frac{40}{483}$ (C) $\frac{139}{1932}$ (D) $\frac{193}{1932}$

52. In a certain series, the n^{th} term T_n equals $4T_{n-1} + n - 1$. If $T_1 = 4$, then find the value of T_{200}.

(A) $\frac{10\left(4^{200}\right)-601}{3}$ (B) $\frac{10\left(4^{200}\right)-601}{9}$

(C) $\frac{10\left(4^{100}\right)+601}{3}$ (D) $\frac{10\left(4^{100}\right)-301}{9}$

53. Find the sum of all the natural numbers between 70 and 250 which are neither divisible by 6 nor by 8.
(A) 20,440 (B) 21,438
(C) 22,550 (D) 25,250

54. Find the sum of coefficients of x^{199} and x^{200} in the expansion of $(1 + x)^{500} + (1 + x)^{499}x + (1 + x)^{498}x^2 + ...+ x^{500}$.

(A) ${}^{501}C_{200}$ (B) ${}^{501}C_{201}$

(C) ${}^{502}C_{200}$ (D) ${}^{502}C_{201}$

55. S = 6 + 24 + 60 + 120 + …. (30 terms). Find the value of S.
(A) 245520 (B) 204640
(C) 286480 (D) 304960

Data Sufficiency

Directions for questions 56 to 65: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone
Mark (D) if the question cannot be answered even using I and II together.

56. If the sum of first n terms of an arithmetic progression is 240, then find n.
I. The first term and the common difference of the progression are 69 and – 6 respectively.
II. The first term and the common difference of the progression are 9 and 2 respectively.

57. What is the sum of the first ten terms of an arithmetic progression?
I. The first term of the series is 11.
II. The sum of the first three terms is equal to the sum of the first nine terms.

58. What is the sum of the first 11 terms of an arithmetic progression?
I. The first term is 15.
II. The last term is 25.

59. In an arithmetic progression having 100 terms, what is the value of the m^{th} term?
   I. The values of the m^{th} term from the beginning and the $(m + 1)^{th}$ term from the end are equal.
   II. The ratio of the first two terms is 3 : 5 and the first 3 terms are prime numbers.

60. What is the first term of an arithmetic progression?
   I. Sum of the first three terms is 30.
   II. Product of the first three terms is 910.

61. The first term of a geometric progression is 3. What is the sum of the first 12 terms of the geometric progression?
   I. The sum of the first term and the 12^{th} term of the geometric progression is 246.
   II. The 12^{th} term of the geometric progression is 120 more than the first term.

62. What is the sum to infinity of the descending geometric progression?
   I. Sum to infinity is twice the sum of the first k terms.
   II. Common ratio of the progression is greater than zero.

63. Are the terms a, b, c in geometric progression?
   I. a, b, c are the p^{th}, q^{th} and the r^{th} terms respectively of a geometric progression.
   II. p, q, r are in an arithmetic progression.

64. What is the common ratio of an infinite geometric progression?
   I. Each term of the progression is distinct and is equal to the sum of the terms that follow it.
   II. n^{th} term of the geometric progression is 24, while the $(n + 3)^{th}$ term is 81.

65. If the common ratio of a geometric progression is greater than one, what is the first term of the series?
   I. Sum of first two terms is 15.
   II. Sum of the squares of the first two terms is 117.

Key

Concept Review Questions

1. B	7. C	13. B	19. (i) D	24. C
2. B	8. C	14. C	(ii) D	25. A
3. A	9. A	15. 0	20. C	
4. 70	10. B	16. A	21. B	
5. D	11. B	17. C	22. 127	
6. B	12. A	18. 256	23. 3025	

Exercise – 4(a)

1. 61	9. 2	17. A	25. B	33. A
2. B	10. A	18. B	26. D	34. C
3. B	11. B	19. A	27. 36	35. 6690
4. 11	12. C	20. C	28. C	36. D
5. C	13. 51	21. 6	29. 7	37. C
6. C	14. D	22. D	30. 3	38. B
7. B	15. 36	23. 930	31. C	39. –5600
8. D	16. 240	24. B	32. 20	40. D

Exercise – 4(b)

1. A	14. C	27. D	40. B	53. B
2. C	15. 39	28. C	41. 32	54. C
3. A	16. 7080	29. D	42. 2600	55. A
4. 82	17. B	30. 4	43. C	56. A
5. C	18. A	31. D	44. D	57. C
6. 2989	19. 1720	32. C	45. C	58. D
7. A	20. D	33. 14	46. D	59. C
8. 0	21. B	34. A	47. 567	60. D
9. D	22. C	35. B	48. C	61. B
10. C	23. B	36. 4960	49. B	62. D
11. 40	24. D	37. C	50. D	63. C
12. B	25. C	38. D	51. B	64. B
13. 97650	26. 27110	39. B	52. B	65. C