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CDC 05 2022 QA

Section-1

Sec 1

Q.1 [11831809]

Two trains A and B were running in opposite directions with speeds in the ratio 6 : 5 respectively. Ram was running at a speed of 10.8 km/h parallel to the railway track in the same direction as that of train B. Train A and train B took equal time to cross Ram. Also, train A and train B crossed each other in 15 seconds. If the length of train B is 180 m, then the length, in meter, of train A is

1 ○ 300

2 ○ 315

3 ○ 295

4 ○ 345

Solution:
Correct Answer : 2

Answer key/Solution

Speed of Ram = 10.8 km/h = 10.8 × 5/18 = 3 m/s
Let the speed of train A and train B be 6x and 5x respectively and the length of train A be y.

Then, $\frac{y+180}{6x+5x}=15$

$\Rightarrow y+180=15\times 11x \qquad \dots (i)$

$\frac{y}{6x+3}=\frac{180}{5x-3} \Rightarrow 5xy-3y=6\times 180x+3\times 180$

$\Rightarrow x(5y-6\times 180)=3(y+180)$

$\Rightarrow x(5y-6\times 180)=3\times 15\times 11x$ (From (i))

$\Rightarrow y = 315$ m.

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Q.2 [11831809]
A triangle ABC of side 60 cm is an equilateral triangle. Points D and E are the midpoints of sides AB and AC respectively. If BE and CD intersect at O, then what is the area (in sq. cm) of triangle DEO?

1 150√3

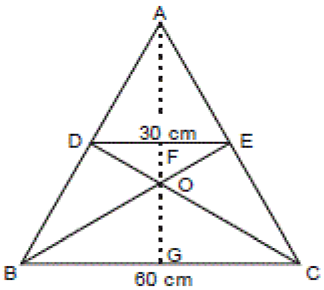
2 50√3

3 75√3

4 25√3

Solution:
Correct Answer : 3

Answer key/Solution



Since D and E are midpoints of AB and AC respectively.
Therefore, DE = 30 cm
AG = √(60² – 30²) = 30√3 cm
Since O is the centroid of the triangle ABC, AO = 2/3 × 30√3 = 20√3 cm
and AF = AG/2 = 15√3 cm
So FO = 20√3 – 15√3 = 5√3 cm
Hence, area of triangle DEO = 1/2 × 30 × 5√3 = 75√3 sq. cm.

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Q.3 [11831809]

If $f(x)=\sqrt{-x^2+16}+\frac{1}{x-3}$ is real, then x lies in which of these intervals?

1 ◯ $(-4, 3) \cup (3, 4)$

2 ◯ $(-\infty, -4) \cup (3, \infty)$

3 ◯ $[-4, 3) \cup (3, \infty)$

4 ◯ $[-4, 3) \cup (3, 4]$

Solution:
Correct Answer : 4

Answer key/Solution

The expression under the square root must not be negative and the denominator of $\frac{1}{x-3}$ must not be zero.

$-x^2+16 \geq 0$ or, $-4 \leq x \leq 4$

and $x \neq 3$ or, $x \in (-\infty,3) \cup (3,\infty)$

Since x must satisfy both conditions, the domain of f is the intersection of the sets $(-\infty,3) \cup (3,\infty)$ and $[-4, 4]$.

Hence, the domain of f is $[-4,3) \cup (3,4]$.

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Q.4 [11831809]

In 2001 the population of a town was a perfect square. Five years later, after an increase of 150 people, the population was 9 more than a perfect square. Now, in 2010, with an increase of another 150 people, the population is once again a perfect square. Which of the following is closest to the percent growth of the town's population during this 10-year period?

1 ◯ 62%

2 ◯ 58%

3 ◯ 54%

4 ◯ 64%

Solution:

Correct Answer : 1

Answer key/Solution

Let the population of the town in 2001 be p^2.
Let the population in 2006 be $q^2 + 9$.
Then, $p^2 + 150 = q^2 + 9$
$\Rightarrow 141 = q^2 - p^2 = (q - p)(q + p)$.
Since p and q are positive integers with $q > p$, possible choices of $(q - p, p + q) = (1, 141), (3, 47)$
If $(q - p, p + q) = (1, 141)$, then $p = 70$ and $q = 71$. But here $p^2 + 300$ is not a perfect square.
If $(q - p, p + q) = (3, 47)$, then $p = 22$ and $q = 25$. But here $p^2 + 300 = 28^2$ is a perfect square.
Now, we find the percent increase from $22^2 = 484$ to $28^2 = 784$.
Hence, percent increase $= (300/484) \times 100 \approx 62\%$.

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Q.5 [11831809]

A two-digit number when reversed becomes 75% greater than the original. By how much percentage is the units digit greater or lesser than the tens' place digit?

Solution:

Correct Answer : 100

Answer key/Solution

Let the units digit be x and the tens digit be y.
The number is $(10y + x)$.
The reversed number is $(10x + y)$.
According to the question, $(10x + y) = 1.75(10y + x)$
$\Rightarrow 8.25x = 16.5y \Rightarrow x = 2y$
Hence, this means that x is 100% greater than y.

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Q.6 [11831809]

How many even integers are there between 200 and 600 whose digits are all different and come from the set {1, 2, 4, 6, 7, 8}?

Solution:
Correct Answer : 24
Note that the last digit of the 3-digit number 'abc' must be even to obtain an even number. Therefore, 'c' must either be 2, 4, 6 or 8.
Case 1: Let c = 2. Then, a must be 4 and b can be 1, 6, 7 or 8.
So, 4 possible numbers are there.
Case 2: Let c = 4. Then, a must be 2 and b can be 1, 6, 7 or 8.
So, 4 possible numbers are there.
Case 3: Let c = 6. Then, a can be 2 or 4 and b can be 1, 7, 8 and one out of 2 or 4.
So, 8 possible numbers are there.
Case 4: Let c = 8. Then, a can be 2 or 4 and b can be 1, 6, 7 and one out of 2 or 4.
So, 8 possible numbers are there.
Hence, total possible numbers = 4 + 4 + 8 + 8 = 24.

Answer key/Solution

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Q.7 [11831809]
If the equation $x^2 + mx + 96 = 0$ has two distinct integer roots, then how many distinct values are possible for |m|?

1 ◯ 6

2 ◯ 5

3 ◯ 12

4 ◯ 10

Solution:
Correct Answer : 1
Let r1 and r2 be the two distinct integer roots of the equation.
Then, sum of roots = r1 + r2 = −m and product of roots = r1.r2 = 96
Possible combinations of integers whose product equal to 96 are: (1, 96), (2, 48), (3, 32), (4, 24), (6, 16) and (8, 12) where r1 and r2 are positive. 6 combinations
For each of these combinations, both r1 and r2 could be negative and their product will still be 96.
i.e., r1 and r2 can take following values too: (−1, −96), (−2, −48), (−3, −32), (−4, −24), (−6, −16) and (−8, −12). 6 combinations
Hence, |m| will take 6 distinct possible values.

Answer key/Solution

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Q.8 [11831809]

Ten inlet pipes of same capacity fill a tank in same time in which 'a' outlet pipes of same capacity can empty it. Alternatively, 4 inlet and 2 outlet pipes is opened for first minute and 8 inlet and 4 outlet pipes open for 2nd minute and process continues till tank is completely filled in 30 minutes. If the time in which 2 outlet pipes can empty the completely filled tank is 72 minutes, then what is the value of 'a'?

Solution:

Correct Answer : 13

Answer key/Solution

Let efficiency of one inlet pipe = x, then efficiency of 10 inlet pipes will be 10x.
Efficiency of 1 outlet pipe = –10x/a
Therefore, 15(4x – 20x/a) + 15(8x – 40x/a) = Tank capacity
⇒ 15x(12 – 60/a) = Tank capacity
Now, 2 outlet pipes can empty the completely filled tank is 72 minutes.
Therefore, 15x(12 – 60/a)/(20x/a) = 72
⇒ a = 13.

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Q.9 [11831809]

The total age of a joint family of 9 persons was 232 years in 2010. After 5 years, a 70-year old member expired and a baby was born in the same year. In 2020, a 52-year old member expired whereas a person got married and brought home a bride who was 22 years old. If a baby is born to the newlyweds in 2022, then what is the average age of the family in 2022?

1 ◯ 22 years

2 ◯ 25 years

3 ◯ 24 years

4 ◯ 28 years

Solution:

Correct Answer : 3

Answer key/Solution

Age of the family before death and birth in 2015 = 232 + 45 = 277 years
Age of the family after birth and death in 2015 = 277 – 70 = 207 years
Age of the family before death and wedding in 2020 = 207 + 45 = 252 years
Age of the family after death and wedding in 2020 = 252 – 52 + 22 = 222 years
Since there are 10 members in the family in year 2022.
Hence, the average age of the family in 2022 = (222 + 18)/10 = 24 years.

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Q.10 [11831809]

The area of a rhombus ABCD is 192 sq. cm. If AC : BD = 2 : 3, then find the side length, in cm, of the rhombus ABCD.

1 ◯ $12\sqrt{3}$

2 ◯ $4\sqrt{13}$

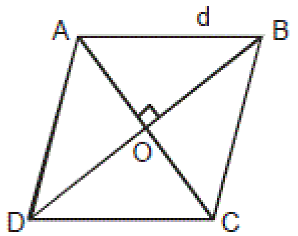
3 ◯ $6\sqrt{3}$

4 ◯ $8\sqrt{13}$

Solution:

Correct Answer : 2

Answer key/Solution



Let AC = 2x and BD = 3x.
Then, $1/2 \times 2x \times 3x = 192 \Rightarrow x = 8$
So AC = 16 cm and BD = 24 cm.
Let d be the side length of the rhombus.
Then, $8^2 + 12^2 = d^2 \Rightarrow d = 4\sqrt{13}$ cm.

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Q.11 [11831809]

How many integer values of x satisfy the equation $|2 + \log_{1/7} x| = 1 + |1 + \log_{1/7} x|$?

Solution:

Correct Answer : 7

Answer key/Solution

$|2 + \log_{1/7} x| = 1 + |1 + \log_{1/7} x|$
Or, $|2 - \log_7 x| = 1 + |1 - \log_7 x|$
$\log_7 x \le 1$ for $1 \le x \le 7$
For $x \ge 8$
$\log_7 x > 1$
When $\log_7 x > 1$,
$2 - \log_7 x < 1$ and $1 + |1 - \log_7 x| > 1$
Hence, possible integral values are 1, 2, 3, 4, 5, 6 and 7.

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Q.12 [11831809]

The marked price of an article is Rs. 5,000. If two successive discounts of x% and (x + 1)%, on the marked price is equal to a single discount of Rs. 1,430, then what will be the selling price (in Rs.) of the article if a single discount of x% is given on the marked price?

1 ◯ 4,500

2 ◯ 4,250

3 ◯ 3,750

4 ◯ 3980

Solution:

Correct Answer : 2

Answer key/Solution

Total discount percentage = 1430/5000 × 100 = 28.6%

$\therefore$ x + x + 1 – x(x + 1)/100 = 28.6

$\Rightarrow x^2 - 199x + 2760 = 0$

$\Rightarrow$ x = 184, 15

Therefore, the successive discounts offered were 15% and 16%.

Hence, required selling price = 5000 × 0.85 = Rs.4,250.

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Q.13 [11831809]

What is the number of ways of distributing 20 identical apples, 8 identical oranges and 4 identical kiwis in 4 baskets, so that each basket contains at least four apples and one orange?

1 ◯ 1225

2 ◯ 64

3 ◯ 42875

4 ◯ 343000

Solution:

Correct Answer : 3

Answer key/Solution

Each basket definitely contains 4 apples and 1 orange.

So we have to distribute 4 identical apples, 4 identical oranges and 4 identical kiwis among 4 baskets.

The number of ways to distributing 4 identical apples among 4 baskets

$= {}^{4+4-1}C_{4-1} = {}^{7}C_{3} = 35$

Hence, total number of ways = 35 × 35 × 35 = 42875.

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Q.14 [11831809]

Let w > x > y > z and w + x + y + z = 44, where w, x, y, and z are integers. The pairwise positive differences of these numbers are 1, 3, 4, 5, 6 and 9. What is the sum of the possible values for w?

Solution:

Correct Answer : 31

Answer key/Solution

Let w − x = a, w − y = b, w − z = c.

As above, we know that c = 9.

Thus, a < b < c, so w + x + y + z = w + (w − a) + (w − b) + (w − 9) = 4w − a − b − 9 = 44.

This means a + b + 9 is a multiple of 4.

Testing values of a and b, we find (a, b, c) = (1, 6, 9), (3, 4, 9) and (5, 6, 9). The corresponding (w, x, y, z) = (15, 14,

9, 6), (15, 12, 11, 6) and (16, 11, 10, 7). The first set does not satisfy the given conditions, but the other two do.

Thus, possible values of w are 15 and 16.

Hence, sum is 15 + 16 = 31.

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Q.15 [11831809]

An alloy of aluminum, copper and zinc contains 75% aluminum, 8% copper and 17% zinc. A second alloy of aluminum and zinc melted with the first and the new alloy then contains 70% aluminum, 5% copper and 25% zinc. Find the percentage of aluminum in the second alloy.

1 ◯ 61.67%

2 ◯ 53.33%

3 ◯ 67.33%

4 ◯ 56.67%

Solution:

Correct Answer : 1

Answer key/Solution

Let x and y be the mass of 1st alloy and 2nd alloy.
Aluminum in the 1st alloy = 0.75x
Copper in the 1st alloy = 0.08x
Zink in the 1st alloy = 0.17x
According to the question, for copper 0.08x/(x + y) = 0.05
⇒ 0.08x = 0.05x + 0.05y ⇒ 3x = 5y ⇒ x = 5 and y = 3
Let p be the percentage of aluminum in the 2nd alloy
According to the question, 5 × 0.75 + 3 × p/100 = (5 + 3) × 0.7 ⇒ 3p = 185 ⇒ p = 61.67%.

Alternate Solution:
Using allegation rule:
For copper:

```
8     0          75    x
 \   /             \  /
   5                70
 /   \             /  \
5  :  3      (70 – x) : 5
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Hence, (70 – x)/5 = 5/3
⇒ 210 – 3x = 25
⇒ 3x = 185
⇒ x = 61.67%.

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Q.16 [11831809]

Let p be a common root of the quadratic equations $x^2 - 8x + c = 0$ and $x^2 - bx + 8 = 0$. If the other roots of the first and second equations are integers in the ratio 3 : 2, then the absolute difference between the other roots of the equations is

1 ◯ 1

2 ◯ 2

3 ◯ 4

4 ◯ 8

Solution:

Correct Answer : 2

Answer key/Solution

Let 3q and 2q be the other roots of the first and second equations respectively.
Then, p + 3q = 8 and p × 3q = c ... (i)
p + 2q = b and p × 2q = 8 ... (ii)
From (i) and (ii), pq = 4 giving c = 12

The first equation is $x^2 - 8x + 12 = 0 \Rightarrow x = 2, 6$
For p = 6, 3q = 2 ⇒ 2q = 4/3 (Not an integer)
So common root is p = 2, so the other roots of the equations are 6 and 4.
Hence, required difference is 6 – 4 = 2.

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Q.17 [11831809]

Citizens visiting a park can be categorised as, children, adults, and elderly who run an average of 8, 10, and 5 minutes per day, respectively. There are 50% more children than adults, and 50% more adults than elderly. The average number of minutes run per day by all of them lies in the interval ___.

1 ◯ (5, 6)

2 ◯ (6.5, 8)

3 ◯ (5, 7)

4 ◯ (7.5, 9.5)

Solution:

Correct Answer : 4

Answer key/Solution

Let us say that there are '4a' elderly.
According to the given information, there must be 6a adults and 9a children.
The average time run by each of them is equal to the total amount of time run divided by the number of citizens.

This gives us $\frac{9a \times 8 + 6a \times 10 + 4a \times 5}{9a + 6a + 4a} = \frac{152}{19} = 8$.

Hence, the required interval is (7.5, 9.5).

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Q.18 [11831809]

Let a_n be a sequence of integers such that $a_{p+q} = a_p + a_q + pq$ and $a_1 = 1$ for all positive integers p and q. Then a_{20} is divisible by how many prime numbers?

Solution:
Correct Answer : 4

Answer key/Solution

$a_{p+q} = a_p + a_q + pq$
For p = 1,
$a_{1+q} = a_1 + a_q + q = 1 + a_q + q$
$a_2 = 1 + a_1 + 1$
$a_3 = 1 + a_2 + 2$
$a_4 = 1 + a_3 + 3$
⋮
$a_{20} = 1 + a_{19} + 19$
Adding all the equations, we get
$\Rightarrow a_2 + a_3 + \dots + a_{20} = 19 + (a_1 + a_2 + \dots + a_{19}) + (1 + 2 + \dots + 19)$

$\Rightarrow a_{20} = 19 + 1 + \frac{19 \times 20}{2} = 210 = 2 \times 3 \times 5 \times 7$

Hence, a_{20} is divisible by 4 prime numbers.

Alternate Solution:

$a_1 = 1$

$a_2 = a_{1+1} = 1 + a_1 + 1 = 3$

$a_3 = a_{1+2} = 1 + a_2 + 2 = 6$

⋮

$a_n = \frac{n(n+1)}{2}$

So, $a_{20} = \frac{20(20+1)}{2} = 210 = 2 \times 3 \times 5 \times 7$

Hence, required prime numbers are 4.

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Q.19 [11831809]

A rectangular field 65 m long and 45 m wide is to be covered with grass, leaving 5 m on all four sides. If the cost of laying grass in the field is Rs. 16 per square meter, then the cost (in Rs.) of laying grass on the field is

Solution:
Correct Answer : 30800

Answer key/Solution

Required cost = (65 – 2 × 5) × (45 – 2 × 5) × 16
= 55 × 35 × 16 = Rs.30,800.

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Q.20 [11831809]

Dev and Anand invest Rs. 50,000 and Rs. 40,000 respectively in a business at the start of a year. In each of the next six months after the first month Dev keeps on adding Rs. 1,000 while Anand keeps on removing Rs. 1,000. In the remaining months Dev keeps on removing Rs. 1,000 per month while Anand keeps on adding Rs. 1,000 per month. Manoj joined them with Rs. 60,000 three months after the start and continued till the end of the year. What will be the difference in the shares of Dev and Manoj after a year if the total profit at the end of the year is Rs. 7,77,600?

1 ◯ Rs. 54,880

2 ◯ Rs. 48,600

3 ◯ Rs. 39,100

4 ◯ Rs. 46,080

Solution:
Correct Answer : 4
Dev's investment = 1000 × [50 + (51 + 52 + 53 + 54 + 55 + 56) + (55 + 54 + 53 + 52 + 51)]
= 1000 × [50 + 56 + 2 (51 + 52 + 53 + 54 + 55)] = 1000 × 636 = Rs. 6,36,000
Anand's investment = 1000 × [40 + (39 + 38 + 37 + 36 + 35 + 34) + (35 + 36 + 37 + 38 + 39)]
= 1000 × [40 + 34 + 2(35 + 36 + 37 + 38 + 39)] = 1000 × 444 = Rs. 4,44,000
Manoj's investment = 60000 × 9 = Rs. 5,40,000
Ratio of profit = 636000 : 444000 : 540000 = 636 : 444 : 540 = 159 : 111 : 135
Hence, difference in the shares of Dev and Manoj = (159 – 135)/405 × 777600 = Rs.46,080.

Answer key/Solution

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Q.21 [11831809]

A completes 2/3 of a certain job in 12 days. B can complete 1/3 of the same job in 10 days and C can complete 3/4 of the work in 15 days. All of them work together for 6 days and then B and C quit. How long (in days) will it take for A to complete the remaining work alone?

Solution:
Correct Answer : 3
A takes 18 days to complete the work, B takes 30 days to complete the work and C takes 20 days to complete the work.
Let the total work be 180 units.
In one day, A completes 10 units, B completes 6 units and C completes 9 units.
In 6 days, work done by all three would be = 6 × (10 + 6 + 9) = 150 units
Remaining work = 30 units
Hence, time taken by A to complete the work = 30/10 = 3 days.

Answer key/Solution

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Q.22 [11831809]

If A, B, C are three distinct real numbers in Geometric Progression (GP) and A + B + C = X × B, then what is the sum of all integral values of X in the interval (–5, 5)?

1 ◯ 5

2 ◯ −5

3 ◯ −9

4 ◯ −3

Solution:

Correct Answer : 2

Answer key/Solution

$A + B + C = X \times B$

$\Rightarrow A + Ar + Ar^2 = X \times Ar$ (A, B, C are in GP)

$\Rightarrow 1 + r + r^2 = Xr$

$\Rightarrow r^2 + (1 - X)r + 1 = 0$

Since A, B, C are distinct real numbers, $D > 0$.

$(1 - X)^2 - 4 > 0$

Or, $(1 - X)^2 - 2^2 > 0$

Or, $(1 - X + 2)(1 - X - 2) > 0$

Or, $(X - 3)(X + 1) > 0$

Therefore, $X < -1$ or $X > 3$

Hence, required sum of integral values $= 4 - 4 - 3 - 2 = -5$.

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