

## CHAPTER – 7

# AVERAGES – MIXTURES – ALLIGATIONS

### AVERAGES

"Average is a very simple but effective way of representing an entire group by a single value."

"Average" of a group is defined as:

$$\text{Average} = \frac{\text{Sum of all items in the group}}{\text{Number of items in the group}}$$

"Sum of all the items in the group" means "sum of the values of all the items in the group".

A batsman's performance can be expressed as the average number of runs scored per innings rather than giving the scores in individual innings. For example, let us say a cricketer scored the following runs in 9 different innings in a year: 35, 56, 124, 29, 0, 87, 98, 45 and 75. Then his average score (per innings) for the year is

$$\frac{35 + 56 + 124 + 29 + 0 + 87 + 98 + 45 + 75}{9} = 61$$

Similarly, if there are 60 students in a class, instead of talking of the height of each individual student, we can talk of "average" height of the class. The average height of the class of students is equal to the sum of the heights of all the students of the class divided by the number of students in the class.

Average is also called the "mean" or mean value of all the values.

### POINTS TO REMEMBER

- If the value of each item is increased by the same value  $p$ , then the average of the group or items will also increase by  $p$ .
- If the value of each item is decreased by the same value  $p$ , then the average of the group or items will also decrease by  $p$ .
- If the value of each item is multiplied by the same value  $p$ , then the average of the group or items will also get multiplied by  $p$ .
- If the value of each item is divided by the same value  $p$  ( $p \neq 0$ ), then the average of the group or items will also get divided by  $p$ .
- The average of a group of items will always lie between the smallest value in the group and largest value in the group - i.e., the average will be greater than the smallest value and less than the largest value in the group.

### AN EASY METHOD TO CALCULATE AVERAGES

As already discussed, the average of a group of items whose values are given can be found out by the rule given at the beginning of this section. However, in most of the cases, we do not need to perform such elaborate additions and divisions. The calculation of averages can be simplified greatly by taking some arbitrary number ( $P$ ) as a starting point, take the deviations (differences) of the given items ( $Q_i$ ) from this arbitrary number, find the average of all these deviations ( $Q_i - P$ ) and algebraically add it to the arbitrary number ( $P$ ) to give the correct average of the given items.

If there are  $n$  items and they are denoted by  $Q_1, Q_2, Q_3, \dots, Q_n$ , then the average of these  $n$  items is given by

$$\text{Average} = P + \frac{1}{n} \sum_{i=1}^n (Q_i - P)$$

The extent to which this method will simplify the calculation will depend on the selection of the arbitrary value  $P$ . It should be selected in such a way that the positive and negative deviations cancel out each other to the extent possible. Then the final figure left for division will be relatively small making the division easier.

For example, the cricketer that we considered above scored the following runs in seven innings: 35, 56, 45, 43, 67, 70 and 48. Now, to find his average, we take an arbitrary figure, say 50 and first find the deviations of each of the scores from this figure. The deviations of the scores from 50 are  $-15, +6, -5, -7, +17, +20$  and  $-2$ . The sum of these deviations is  $+14$ .

Hence the average of the cricketer's scores is

$$50 + \frac{14}{7} = 52$$

Please note that the number  $P$  ( $= 50$  above) can be any value. Let us work out the same example taking a different value for  $P$ . Let us take  $P$  equal to 45. The deviations of the scores from  $P$  are  $-10, +11, 0, -2, +22, +25$  and  $+3$ . The sum of these deviations is 49. Hence the average is  $45 + 49/7 = 45 + 7 = 52$ .

### WEIGHTED AVERAGE

When two groups of items are combined together, then we can talk of the average of the entire group. However, if we know only the average of the two groups individually, we cannot find out the average of the combined group of items.

For example, there are two sections A and B of a class where the average height of section A is 150 cm and that of section B is 160 cm. On the basis of this information alone, we cannot find the average of the entire class (of the two sections). As discussed earlier, the average height of the entire class is

$$\frac{\text{total height of the entire class}}{\text{total number of students in the entire class}}$$

Since we do not have any information regarding the number of students in the two sections, we cannot find the average of the entire class. Now, suppose that we are given that there are 60 students in the section A and 40 students in section B, then we can calculate the average height of the entire class which, in this case will be equal to

$$\frac{60 \times 150 + 40 \times 160}{60 + 40} = 154 \text{ cm.}$$

This average height 154 cm of the entire class is called "**weighted average**" of the class.

The above step in calculating the weighted average of the class can be rewritten as below:

$$\begin{aligned} & \frac{60 \times 150 + 40 \times 160}{60 + 40} \\ &= \frac{60}{100} 150 + \frac{40}{100} 160 = \frac{3}{5} 150 + \frac{2}{5} 160 \end{aligned}$$

It is clear from the above step that we would have been able to calculate the average height of the entire class even if we had not been given the number of students in the individual sections but only the **ratio** of the number of students in the two sections (which in this case is 3 : 2).

Even if there are more than two groups of items to be combined, then also the weighted average can be calculated by the same method. For example, if three sections in a class have their average marks as 75, 76 and 79 respectively and their respective strengths are 30, 35 and 35, then the average mark of the entire class is given by

$$\frac{30 \times 75 + 35 \times 76 + 35 \times 79}{30 + 35 + 35} = 76.75$$

The method of deviations we used for calculating averages can be applied to calculate weighted average also. Here, that method will involve finding out deviations from the arbitrarily chosen number and calculating the weighted average of these deviations. In the above example, if we take 70 as the arbitrary figure, then the deviations of the three observed values given from 70 are +5, +6 and +9. The weighted average of these deviations is

$$\frac{30 \times 5 + 35 \times 6 + 35 \times 9}{30 + 35 + 35} = \frac{675}{100} = 6.75.$$

Hence, the weighted average will be  $70 + 6.75 = 76.75$

The arbitrary figure chosen can be any figure and if it is selected, as in the previous case, between the smallest and largest observed figures, some of the deviations will be positive and some negative making the final division relatively simpler. For example, in the above case, if we take with 76 as the arbitrary figure, the deviations are -1, 0 and +3. Then the weighted average will be

$$\frac{30 \times (-1) + 35 \times 0 + 35 \times (+3)}{30 + 35 + 35} = \frac{75}{100} = 0.75$$

Hence, the weighted average will be  $76 + 0.75 = 76.75$

"Weighted Average" can be defined or calculated for any MIXTURE.

## MIXTURES

Mixing of two or more qualities of things produces a mixture. When two items of different qualities are thus mixed, the quality of the resultant mixture lies in between the qualities of the original constituent items, i.e., it will be higher than the lowest quality and lower than the highest quality of the items being mixed.

In the above example that we took, the "quality" that we looked at was the height of the students. We could also have taken their weights or the marks scored by them or any other "quality" or "parameter" and calculated the "weighted average" value of that particular "quality" for the entire group.

Similarly, if two types of a product of different prices per unit are mixed, the unit price of the resultant mixture will lie between the prices of the two types that form the mixture.

Here, the average quality is essentially the **weighted average** of the two constituent items.

If  $q_1$  is the quantity (or number of items) of one particular item of quality  $p_1$ , and  $q_2$  be the quantity (or number of items) of the second item of quality  $p_2$  are mixed together to give a new mixture, then the **weighted average** value ( $p$ ) of the quality of the mixture is given by

$$p = \frac{p_1 q_1 + p_2 q_2}{q_1 + q_2}$$

Even if there are more than two groups of items mixed, the weighted average rule can be applied. We will only have to take figures (as shown in the formula for the two groups) for all the groups in the numerator as well as the denominator and calculate the weighted average. For example, if there are four groups of quantities  $q_1, q_2, q_3$  and  $q_4$  whose respective qualities are  $p_1, p_2, p_3$  and  $p_4$ , then the weighted average quality of the group can be written as

$$p = \frac{p_1 q_1 + p_2 q_2 + p_3 q_3 + p_4 q_4}{q_1 + q_2 + q_3 + q_4}$$

A mixture can also be a solution - that is, a liquid mixed with another liquid which is normally water. The concentration of the solution is expressed as the proportion (or percentage) of the liquid in the total solution.

For example, if 10 litres of pure alcohol is mixed with 40 litres of water, then in a total solution of 50 litres, there is 10 litres of alcohol. Hence the concentration of this solution is  $0.2 (= 10/50)$  or 20%.

Similarly, if 30 litres of pure milk is mixed with 10 litres of water, the concentration of this solution can be expressed as 75% ( $= 30/40$ ) milk or 25% water.

We can also have two solutions mixed together to give a new solution. Such problems can also be handled in the same manner as other mixtures. In the weighted average rule, the quality of the constituents ( $p_1, p_2$ , etc.) will then be the concentrations of various solutions mixed together.

## Examples

**7.01.** In exams held on eight subjects, Shyam scored 85, 85, 90, 91, 92, 94, 95 and 96 marks. Find his average score.

**Sol:** The average score of Shyam

$$= \frac{\text{Sum of the scores in the subjects}}{\text{Number of subjects}}$$

$$= \frac{85 + 85 + 90 + 91 + 92 + 94 + 95 + 96}{8}$$

$$= \frac{728}{8} = 91 \text{ marks.}$$

### Alternate method:

The problem can be solved using assumed average/mean also.

Let us take 90 as the assumed mean, then the actual average score

$$= 90 + \frac{-5 + -5 + 0 + 1 + 2 + 4 + 5 + 6}{8}$$

$$= 90 + \frac{8}{8} = 91 \text{ marks.}$$

**7.02.** The monthly incomes of Raja for four months of a year are ₹15,000, ₹16,000, ₹17,000 and ₹18,000. What is Raja's average monthly income for these four months?

**Sol:** Average monthly income  

$$= \frac{\text{Total income for the given months}}{\text{Number of months}}$$

$$= \frac{66,000}{4} = ₹16,500$$

**7.03.** During mid-summer, the average daily temperatures for five consecutive days are 36°C, 39°C, 40°C, 41°C and 44°C. What is the average temperature recorded per day for the five days?

**Sol:** Average temperature =  

$$\frac{\text{Total temperature during the given period}}{\text{Number of days}}$$

$$= \frac{36+39+40+41+44}{5} = \frac{200}{5} = 40^\circ\text{C}.$$

**7.04.** Chetan purchased two dozen mangoes of one variety at ₹6 per dozen, 4 dozen mangoes of second variety at ₹7 per dozen and one dozen mangoes of third variety at ₹9 per dozen. What is the average cost per dozen of the mangoes purchased by Chetan?

**Sol:** Cost of 2 dozen mangoes of first variety =  $2 \times 6 = ₹12$   
 Cost of 4 dozen mangoes of second variety =  $4 \times 7 = ₹28$   
 Cost of 1 dozen mangoes of third variety =  $1 \times 9 = ₹9$   
 Total cost of 7 dozen mangoes = ₹49  
 Therefore average cost per dozen of mangoes  

$$= \frac{49}{7} = \text{Rs. } 7$$

**7.05.** The average age of a group of 10 friends is 15 years. A new person joins them and their average age increases to 16 years. What is the age of the new person?

**Sol:** Total age of 10 friends =  $(10) \times (15) = 150$  years  
 Total age of the 11 persons after the person joins =  $(11) \times (16) = 176$  years  
 Age of the new person =  $176 - 150 = 26$  years

**7.06.** Eight kgs of rice costing ₹12 per kg is mixed with seven kgs of rice costing ₹9.60 per kg. What is the average cost per kg of the mixture?

**Sol:** Cost of 8 kg of rice =  $8 \times 12 = ₹96$   
 Cost of 7 kg of rice =  $7 \times 9.60 = ₹67.20$   
 Total cost of 15 kg of rice = ₹163.20  
 Average cost of the mixture =  $₹ \frac{163.20}{15}$   

$$= ₹10.88 \text{ per kg.}$$

**7.07.** A batsman scored an average of 54 runs in the first 6 tests. If the first test is not counted, and the seventh is counted, then his average score goes up to 58 runs. If his score in the first test was 50 runs, find his score in the seventh test.

**Sol:** Total score in the first 6 tests =  $6 \times 54 = 324$  runs  
 Total score in the last six tests =  $6 \times 58 = 348$  runs.  
 Score in the seventh test = Total score in all the 7 tests – Total score in the first 6 tests =  $348 + 50 - 324 = 398 - 324 = 74$

**7.08.** The average weights of all the students in three sections A, B and C of a class are 50 kg, 60 kg and 40 kg respectively. If the number of students in these three sections is 40, 70 and 90 respectively, then find the average weight of the entire class.

**Sol:** Applying the weighted average rule, the average weight of all the students in three sections of the class is  

$$\frac{(50 \times 40) + (60 \times 70) + (40 \times 90)}{40 + 70 + 90}$$

$$= \frac{9800}{200} = 49 \text{ kg.}$$
 Alternatively, if we take the arbitrary figure 50 kg as the assumed mean, then the average  

$$= 50 + \frac{(0 \times 40 + 10 \times 70 + (-10 \times 90))}{40 + 70 + 90}$$

$$= 50 + \frac{700 - 900}{200}$$

$$= 50 - \frac{200}{200} = 49 \text{ kg}$$

**7.09.** The rainfalls recorded by a city during the seven days of a week were as follows: 5 cm per day on the first 3 days, 8 cm on the fourth day, 10 cm on the fifth day, 12 cm on the sixth day and 7 cm on the seventh day. Find the average rainfall recorded per day during the week.

**Sol:** Average rainfall  

$$= \frac{[(5 \times 3) + 8 + 10 + 12 + 7]}{7}$$

$$= \frac{52}{7} = 7\frac{3}{7} \text{ cm per day.}$$

**7.10.** Suhas earned an average of ₹1500 per month from January through May. Then he earned ₹1600, ₹1700, ₹1800, ₹1900 and ₹2000 during the months June through October. During November he earned 50% of what he earned in December. If his average earnings for the entire year is ₹1500, then find his earnings in the month of November.

**Sol:** Let the earnings for the month of November be x. Then the earnings in December will be 2x. Now the total annual earnings will be  $(1500 \times 5) + 1600 + 1700 + 1800 + 1900 + 2000 + x + 2x = ₹16,500 + 3x$ .  
 Total earnings for the given year =  $12 \times 1500 = ₹18,000$  which is equal to  $16,500 + 3x$ .  

$$\Rightarrow 16,500 + 3x = 18,000$$

$$\Rightarrow 3x = 1500 \Rightarrow x = ₹500$$
 Hence, the earnings in the month of November are ₹500.

- 7.11.** The average age of employees of a company is 35 years. If five new persons with an average age of 32 years join the company, the average age of the company becomes 34 years. How many people are there in the company?

**Sol:** Let there be  $n$  persons in the company. The total age of the persons after the five new persons join the company is  $35n + (5 \times 32)$ . This is equal to  $34(n + 5)$ .  
Therefore,  $34(n + 5) = 35n + 160$   
 $\Rightarrow 34n + 170 = 35n + 160 \Rightarrow n = 10$ .  
So, there are 10 persons in the company.

- 7.12.** If 30 kg of rice priced at ₹20 a kg is mixed with 10 kg of a second variety of rice priced at ₹24 a kg, then what is the average cost of the resulting mixture?

**Sol:** Applying the weighted average rule, the average cost of the mixture per kg is  
$$\frac{(30 \times 20) + (10 \times 24)}{(30 + 10)} = ₹21$$

- 7.13.** If two kinds of grapes which cost ₹18 a kg and ₹24 a kg are mixed in the ratio of 3 : 2, then find the cost of the mixture per kg.

**Sol:** As the two kinds of grapes are mixed in the ratio 3 : 2, let the actual quantities mixed be  $3k$  and  $2k$ . Applying the weighted average rule, the average cost of the mixture can be calculated as  
$$\frac{(18 \times 3k) + (24 \times 2k)}{(3k + 2k)} = \frac{102}{5}$$
  
 $= ₹20.40$  per kg

- 7.14.** If 20 litres of 30% milk solution is mixed with 25 litres of 20% milk solution, then find the concentration of milk in the resulting solution.

**Sol:** We can apply the weighted average rule directly taking the concentrations of the 2 mixtures as the parameters in the weighted average rule. Concentration of milk in the resulting solution  
$$= \frac{\frac{30}{100} \times 20 + \frac{20}{100} \times 25}{(20 + 25)} \times 100$$
  
$$= \frac{11}{45} \times 100 = 24\frac{4}{9}\%$$

## ALLIGATIONS

We will take the Weighted Average rule discussed in the previous section and rewrite the formula such that the quantity terms come on one side and the price terms come on the other side. If we do this we get the rule

$$\frac{q_1}{q_2} = \frac{p - p_2}{p_1 - p}$$

This is called the RULE OF ALLIGATION. This rule connects quantities and prices in mixtures. This can also

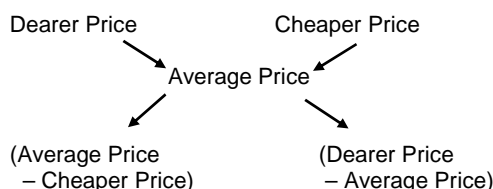
be written as 
$$\frac{q_1}{q_2} = \frac{p_2 - p}{p - p_1}$$

In a descriptive manner, the Rule of Alligation can be written as 
$$\frac{\text{Quantity of Cheaper}}{\text{Quantity of Dearer}} =$$

$$\frac{\text{Rate of Dearer} - \text{Average Rate}}{\text{Average Rate} - \text{Rate of Cheaper}}$$

This rule is a very powerful rule and is useful in problems on weighted averages and mixtures. This rule is also useful in a number of problems which can be treated as mixtures and applied to parameters other than price also. We will take examples where alligation rule can be applied.

In actual practice, to apply alligation rule, we do not need to remember the above formula at all. It can be made very simple by representing the rule pictorially. The above formula can be represented as follows:



{We write the dearer and cheaper prices in one line with some gap in between them. Then, we write the average price in between these two but in the line below the line in which dearer and cheaper prices are written. Then, take the differences of quantities as shown along the arrows and write along the same direction of the arrows continued, i.e., in a diagonally opposite corner. The difference between the top left hand quantity (Dearer Price) and Average Price has to be written at the bottom right hand corner. Similarly, the difference between the top right hand corner (Cheaper Price) and the Average Price has to be written at the bottom left hand corner. Now the ratio of the two quantities in the bottommost line will give us the ratio of the quantities of Dearer and Cheaper varieties. Please note that since we took Dearer Price on the top left corner, the ratio of the bottom left figure to that of the bottom right figure will give the ratio of Dearer Quantity to Cheaper Quantity}

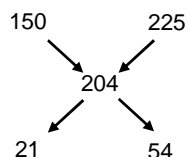
## Examples

- 7.15.** How many kilograms of wheat costing ₹20 per kg must be mixed with 28 kg of wheat costing ₹15 per kg, so that the resultant mixture costs ₹18 per kg?

**Sol:** Applying the rule of alligation,  
$$\frac{\text{Quantity costing ₹15 per kg used for mixing}}{\text{Quantity costing ₹20 per kg used for mixing}} = \frac{\text{Dearer price} - \text{Average price}}{\text{Average price} - \text{cheaper price}}$$
  
$$= \frac{20 - 18}{18 - 15} = \frac{2}{3}$$
 Let the quantity of wheat costing ₹20 per kg to be mixed be  $x$ .  
$$\frac{28}{x} = \frac{2}{3} \Rightarrow x = 28 \times \frac{3}{2} = 42 \text{ kg.}$$

- 7.16.** A total amount of ₹204 was collected from boys and girls of a class of 150 students. If each boy contributes ₹1 and each girl contributes ₹1.50, then find the number of boys and girls in the class.

**Sol:** Assuming the class is full of girls total amount collected =  $(150)(1.50) = ₹225$ . Assuming the class is full of boys, total amount collected =  $(150)(1) = ₹150$ .  
Ratio of girls to boys can be found by applying the alligation rule.



Ratio of girls and boys = 54 : 21

$$\text{Number of girls} = \frac{54}{75} \times 150 = 108$$

$$\text{Number of boys} = \frac{21}{75} \times 150 = 42$$

Alternatively the ratio of boys and girls can be found as shown below.

Average amount contributed by each student

$$= \frac{204}{150} = ₹34/25$$

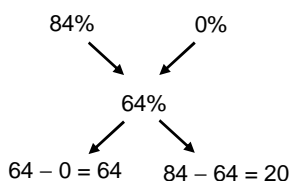
Ratio of boys and girls



i.e., 7 : 18.

- 7.17.** There is a 60 litre solution of milk and water in which milk forms 84%. How much water must be added to this solution to make it a solution in which milk forms 64%?

**Sol:** We apply alligation rule here taking 84% milk solution (of milk and water) mixed with a 0% milk solution (or pure water) to give 64% milk solution.



The ratio of the initial solution and water is 64 : 20 = 16 : 5. Since there was 60 litres of solution initially, water to be added is

$$\frac{60}{16} \times 5 = \frac{75}{4} = 18.75 \text{ litres}$$

- 7.18.** Two vessels contain milk solutions, with milk and water in the ratio of 2 : 11 in the first vessel and in the ratio of 5 : 9 in the second. In what ratio

should the contents of these two vessels be mixed such that the resultant mixture has milk and water in the ratio 3 : 8?

**Sol:** Here, we can apply the alligation rule taking the concentration of the mixtures. The concentration of the milk in the first vessel is (2/13) and that in the second vessel is (5/14).

Concentration of milk in the resulting mixture =  $\frac{3}{11}$

The ratio in which the two mixtures should be

$$\text{mixed is } \frac{5}{14} - \frac{3}{11} : \frac{3}{11} - \frac{2}{13} \text{ i.e. } \frac{13}{154} : \frac{17}{143}$$

$$= 13 \times 143 : 17 \times 154 = 169 : 238$$

If there is P volume of pure liquid initially and in each operation, Q volume is taken out and replaced by Q volume of water, then at the end of n such operations, the concentration (k) of the liquid in the

$$\text{solution is given by } \left\{ \frac{P-Q}{P} \right\}^n = k$$

This gives the concentration (k) of the liquid as a PROPORTION of the total volume of the solution.

If the concentration has to be expressed as a percentage, then it will be equal to 100k.

If the volume of the liquid is to be found out at the end of n operations, it is given by kP, i.e., the concentration k multiplied by the total volume P of the solution.

- 7.19.** A vessel has 500 ml of alcohol. 50 ml of alcohol is removed and 50 ml of water is poured into the vessel (bringing the volume of the mixture in the vessel back to 500 ml). If this operation is repeated another 2 times, what is the percentage of alcohol in the vessel at the end?

**Sol:** Using the formula discussed above, concentration of alcohol in the resultant solution =

$$\left( \frac{500 - 50}{500} \right)^3 = \left( \frac{450}{500} \right)^3 = \left( \frac{9}{10} \right)^3$$

$$= 0.729$$

Therefore, percentage of alcohol in the vessel is 72.9%.

## Concept Review Questions

**Directions for questions 1 to 25:** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The current average age of a family of four members A, B, C and D is 15 years. Find the average age (in years) of the family after 5 years.  
(A) 25      (B) 20      (C) 15      (D) 19
2. The average age of a class of 20 students is 20 years. If a new student, whose age is 20 years, joins the class, find the new average age of the class (in years).
3. The average weights of the boys and the girls in a class are 30 kg and 40 kg respectively. Find the average weight of all the students in the class (in kg).  
(A) 30      (B) 35  
(C) 40      (D) Cannot be determined
4. In an office, the average height of female employees is 150 cm and the average height of male employees is 160 cm. If there are 300 employees in the office, find the average height (in cm) of all the employees.  
(A) 150      (B) 160  
(C) 155      (D) Cannot be determined
5. The average age of all the girls in a class of 42 students is 13 years. The average age of all the boys as well as the average age of 23 of the boys of the same class is 16 years. Which of the following could be the average age of the students in the class? (in years)  
(A) 13.5      (B) 14      (C) 14.5      (D) 15
6. The average marks scored by the girls in a class of 45 students in an exam is 90. If the number of boys in the class and their average marks in the same examination are 15 and 60 respectively, what is the average marks of all the students in the class?  
(A) 85      (B) 80  
(C) 75      (D) Cannot be determined
7. In an office, the average salary of the male employees as well as the average salary of the female employees is ₹15000 per month. Find the average salary (in ₹) per month of all the employees.  
(A) 1400      (B) 15000  
(C) 16000      (D) Cannot be determined
8. The average marks scored by Mahesh in five out of six subjects in an exam is 60. If Mahesh scored an average of 75 marks in the six subjects, find his score in the sixth subject.
9. The ratio of the number of boys to the number of girls in a class is 3 : 5. If the average weights of the boys and the girls in the class are 20 kg and 16 kg respectively, find the average weight of the students in the class (in kg).  
(A) 17  
(B) 18  
(C) 17.5  
(D) Cannot be determined
10. Find the average of the numbers 41, 45, 49, 53, 57, 61, 65, 69 and 73.
11. The average number of sweets with six children is six. If another child joined them the average number of sweets with the seven children will become 7. Find the number of sweets with the seventh child.
12. A bag contains 3 kg of potatoes and 5 kg of tomatoes. If the cost of potatoes is ₹6 per kg and the cost of tomatoes is ₹8 per kg, find the average price (in ₹) per kilogram of the vegetables in the bag.
13. A total of 57 sweets were distributed among 10 children such that each girl gets 5 sweets and each boy gets 6 sweets. Find the number of girls.
14. The average monthly income of a man for the first 4 months, next 4 months and the last 4 months of a year are ₹4000, ₹5600 and ₹6420 respectively. Find his average monthly income (in ₹) for the year.  
(A) 5310      (B) 5320      (C) 5340      (D) 5380
15. Find the average of  $(5 + 5 + \dots \text{upto } 100 \text{ times})$  and  $(8 + 8 + \dots \text{upto } 200 \text{ times})$ .
16. The average monthly income of a man for the first 9 months of a year is ₹12000. His average monthly income for the last 3 months of that year is ₹16000. Find his average monthly income (in ₹) for that year.
17. The average monthly income per person in a family of 5 members is ₹9000. In a certain month, each member got ₹900 less than usual. Find the average monthly income (in ₹) per person in that month.
18. A vessel contains 20 litres of a mixture of milk and water containing 60% milk. 5 litres of pure milk is added to it. Find the percentage of milk in the new mixture.  
(A) 50%      (B) 52%  
(C) 62%      (D) 68%
19. A vessel contains 25 litres of a mixture of milk and water containing 40% milk. Find the quantity (in litres) of pure milk to be added to the vessel so that the ratio of milk and water in the vessel becomes 1 : 1.  
(A) 2.5      (B) 5      (C) 7.5      (D) 10

20. A vessel contains 70 litres of a mixture of milk and water containing 90% milk. Find the quantity (in litres) of water to be added to the vessel so that the percentage of milk in the new solution is 87.5%  
(A) 1 (B) 2 (C) 3 (D) 4
21. A milkman has 20 litres of pure milk. Find the quantity (in litres) of water to be added to it so that he gets 50% profit by selling it at its cost price.  
(A) 5 (B) 10 (C) 15 (D) 20
22. A vessel contains 10 litres of pure milk. 1 litre of milk is taken out and replaced by an equal amount of water. 1 litre of mixture is then taken out and replaced by an equal amount of water. Find the ratio of milk and water in the final mixture.  
(A) 81 : 100  
(B) 81 : 19  
(C) 19 : 81  
(D) 19 : 100
23. In question 22, find the final quantity of milk (in litres).  
(A) 9 (B) 8.1  
(C) 7.29 (D) None of these
24. 6 kg of sugar costing ₹10 per kg is added to 9 kg of sugar costing ₹15 per kg. At what price (in ₹) should this mixture be sold so that there is no loss or gain?
25. Two solutions of sulphuric acid are mixed in the ratio of 3 : 7. The first solution contains 20% sulphuric acid and the second solution contains 30% sulphuric acid. Find the concentration (in %) of sulphuric acid in the final mixture.

### Exercise – 7(a)

**Directions for questions 1 to 30:** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The average age of a group went up by 2 years when a man aged 34 years was replaced by an old man aged 58 years. How many members were there in the group?  
(A) 10 (B) 18 (C) 14 (D) 12
2. In a class of 64 students, the average of the marks obtained is 88. If the top 10 students got on an average 142 marks, find the average of the marks obtained by the remaining students.
3. The average age of 30 students of a class is 16 years. If the age of the class teacher is also included, then average age of the class, increases by 1 year. Find the age (in years) of the class teacher.
4. The average runs scored by a batsman, who has played 52 innings, increased by one after an innings of 126. Find the average runs of the batsman before this innings.  
(A) 71 (B) 73 (C) 64 (D) 58
5. Sixteen men went to a hotel. Fifteen of them paid ₹80 each and the sixteenth man paid ₹75 more than the average bill of all the sixteen men. Find the total bill.  
(A) ₹1020 (B) ₹1280  
(C) ₹1360 (D) Cannot be determined
6. The average of  $n$  numbers is 32. If three-fourth of the numbers are increased by 4 and the remaining are decreased by 6, what is the new average?
7. The ace leg spinner Bhayanak Singh Bedi had 1200 runs taken of him in a certain number of matches. In the next match he took 4 wickets conceding 144 runs as a result of which his average (runs conceded / number of wickets taken) increased by 1. How many wickets did he take (in the matches considered) before the last match?  
(A) 40 (B) 48 (C) 60 (D) 77
8. The average age of a family of five members is the same today as it was five years ago. There is no change in the family except that the elder daughter was replaced by the daughter-in-law. If the age of the elder daughter is 48 years, how old (in years) is the daughter-in-law?
9. The average marks obtained by 40 students of a class is 85. The difference between the marks obtained by the students who got the highest mark and the lowest mark is 108. If both these students are removed, the average falls by 1 mark. Find the highest mark.  
(A) 144 (B) 158  
(C) 172 (D) Cannot be determined
10. There are three classes – P, Q and R. The number of students in P, Q and R are 18, 24 and 30 respectively. Which of the following is not true?  
(A) If a student is shifted from Q to R and the average weight of both the classes increased, then his weight must be between the average weights of the classes.  
(B) If the total weight of the students of each class is the same, then the average weight of the three classes is the same as the average weight of Q.  
(C) If a student is shifted from P to R and the average weight of both the classes decreased, then his weight must be between the average weights of these classes.  
(D) None of these
11. A salesman earned ₹8500 on an average for the first 10 months of a year. How much should he earn (in ₹) on an average in the last two months, so that the average monthly earnings for the year becomes ₹11000?
12. In a hockey tournament, the average number of goals scored by India in the first six matches was 2 while that in the last six matches was 4. If 3 goals were scored in the sixth match, find the average number of goals scored by India in the tournament if a total of 11 matches were played.
13. There are 55 persons. The age of each person (in years) is a two-digit number. The average age of the persons is  $A$  years. Had one of the 55 persons, who is  $ab$  years old, been  $ba$  years old, the average age of the 55 persons would be  $1.2A$  years. Find the maximum value of  $A$ .  
(A) 12.1 (B) 13.2  
(C) 11 (D) Data inconsistent
14. In an office, the average salary of all the employees is ₹7000. If the average salary of the 22 executives is ₹12000 and that of the others is ₹5000, find the total number of employees in the office.  
(A) 68 (B) 77 (C) 78 (D) 84
15. Find the average weight of the students of the secondary block of Mc William High School, given the number of students and the average weight of each of the classes are as follows:

Class	Number of students	Average Weight (in kg)
VI	250	43
VII	240	46
VIII	275	46
IX	200	55
X	220	52



16. In a school there are a total of 50 students in class X, who are divided into three sections A, B and C. Sections A and B have an equal number of students. All the students of the class wrote a test. The average marks obtained by the students of sections A and B together is 52.5. The average marks obtained by the students of sections A and C together is 60. The average marks obtained by the students of sections B and C together is 70. The average marks obtained by the students of sections A, B and C together is 60. How many students are there in section C?  
 (A) 10 (B) 20  
 (C) 15 (D) Cannot be determined
17. Find the average of all the three-digit numbers whose units digit is half the hundreds digit.  
 (A) 547.5 (B) 550 (C) 500 (D) 524.5
18. How many litres of water should a milkman add to 35 litres of milk costing ₹560, so that by selling the milk at ₹14 per litre, he just recovers his cost?
19. How many litres of water should be added to 25 litres of milk costing ₹12 per litre so that by selling the mixture at the cost price a profit of 20% is made?  
 (A) 2 litres (B) 5 litres (C) 8 litres (D) 10 litres
20. A vessel had 20 ml of a mixture of milk and water. The mixture has 40% milk. X ml of milk was added to the mixture. Y ml of water was then added to the mixture. Each of the two additions reverses the ratio of milk and water in the vessel. Find the difference of X and Y.
21. Two varieties of rice are mixed in the ratio 2 : 5 and the mixture is sold at ₹12 per kg at a profit of 20%. If the first variety costs ₹7 per kg more than the second variety, find the cost per kg of the first variety.  
 (A) ₹7 (B) ₹15  
 (C) ₹16 (D) None of these
22. Two vessels contain water and alcohol in the ratios 1 : 2 and 3 : 4. The two solutions are then mixed by taking 6 litres from the first vessel and 35 litres from the second. Find the ratio of alcohol to water in the resulting solution.  
 (A) 15 : 22 (B) 22 : 15 (C) 24 : 17 (D) 17 : 24
23. Two vessels have petrol, diesel and kerosene mixed in the ratios 1 : 2 : 4 and 3 : 5 : 6. If the quantities in the two vessels are mixed in the ratio 1 : 1, what is the ratio of petrol, diesel and kerosene in the resultant mixture?  
 (A) 5 : 9 : 14 (B) 3 : 6 : 8  
 (C) 4 : 8 : 11 (D) 4 : 7 : 10
24. Two containers contain petrol and diesel in the ratios 4 : 3 and 3 : 1. How many litres from the first container should be mixed with 16 litres from the second so that the ratio in the resultant mixture is 32 : 19?
25. A shopkeeper mixed 3 varieties of wheat costing ₹12 per kg, ₹18 per kg and ₹21 per kg and sold the mixture at ₹15 per kg at 20% profit. Which of the following represents a possible ratio of the varieties mixed?  
 (A) 60 : 1 : 3 (B) 44 : 1 : 2  
 (C) 39 : 2 : 1 (D) 27 : 1 : 1
26. Fresh grapes contain 84% water while raisins contain 20% water. How many kg of raisins can be prepared from 80 kg of fresh grapes?
27. From 800 litres of alcohol, 80 litres are drawn and replaced with water. 80 litres of this mixture is then drawn and replaced with water and this process is repeated one more time. Find the final quantity (in litres) of alcohol in the solution.
28. Vessel A contains 5 litres of milk and vessel B contains 5 litres of water. One litre of milk is taken from A and is poured into B. One litre of the mixture in B is then poured into A. If the present quantities of milk in B and water in A are  $V_M$  and  $V_W$  respectively, then which of the following holds true?  
 (A)  $V_M > V_W$  (B)  $V_M = V_W$   
 (C)  $V_M < V_W$  (D) Cannot be determined
29. Two containers A and B contain equal volumes of water and alcohol respectively. 3 litres of water is taken from A and poured into B. From the resulting solution in B, 3 litres is taken out and poured into A. If the quantity of water in both the containers is the same after the two transfers find the volume of alcohol in B initially (in litres).  
 (A) 12 (B) 9 (C) 6 (D) 3
30. I lent a sum of money at 10% p.a. interest and another sum of money at 12% p.a. interest. The total money I lent equals ₹25000. At the end of one year I received ₹2900 as total interest. Find the amount (in ₹) lent at 12% p.a. interest.

### Exercise – 7(b)

**Directions for questions 1 to 40:** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the average of the first 10 multiples of 50.  
 (A) 85  
 (B) 125  
 (C) 215  
 (D) 275
2. The average weight of three men and two women is 60 kg. If the average weight of the women is 45 kg, find the average weight (in kg) of the men.

3. A man bought 2 dozen apples for ₹24 and  $x$  dozen apples for ₹120. The average amount (per dozen) spent on the two lots was the same. Find the total number of apples with the man.
4. A metal weighs 1500 kg per cubic metre and another metal weighs 2500 kg per cubic metre. Find the weight (in kg) of 5 cubic metres of an alloy formed by mixing 40% of the first metal and 60% of the second metal.
5. A milkman dilutes 36 litres of pure milk with water. The percentage of milk in the solution is now 75%. How many litres of water did he add?
6. A's capacity to do a work is the average of C's capacity and twice B's capacity. B's capacity is the average of C's capacity and half of A's capacity. Find the ratio of capacities of A and B.  
 (A) 5 : 8 (B) 8 : 5 (C) 4 : 5 (D) 5 : 4
7. Six boys and six girls went to a hotel to have lunch. Each boy paid an equal amount and it is ₹20 more than what each girl paid. The bill amount was ₹1380. Half of the girls as well as half of the boys left after paying their share of the bill. Find the average amount (in ₹) paid by the others.
8. There are 48 students in a class. The age of one of them is twice that of another. If these 2 are replaced by 2 others whose ages are 16 years and 11 years, the average age of the class increases by 1.5 months. Find the age of the younger of the 2 students (in years).
9. In a company of 20 employees, each person gets a salary of ₹8000 per month. Ten employees were promoted and got 22.5% increase in their salaries. Two other employees left the company. Find the average income (in ₹) of the remaining 18 employees.  
 (A) 9125 (B) 9250  
 (C) 9500 (D) 9000
10. In a section of class V, the average marks of 45 students is 75. If 5 other students, (from another section) join this section, the average marks increases by 2. Find the average marks of the 5 new students.
11. In a town during a certain week, every day there was a  $1^\circ\text{C}$  increase in temperature over the previous day. If the average temperature for the first and the last days (i.e., Monday and Sunday) was  $37^\circ\text{C}$ , what was the average temperature (in  $^\circ\text{C}$ ) for Monday, Tuesday and Wednesday?
12. In an office, the average salary of the men is ₹3000. The average salary of all the employees is ₹8000. There are 80 men in the office and the average salary of the women employees is ₹10000. Find the number of women in the office.  
 (A) 100 (B) 120 (C) 150 (D) 200
13. If 10 cu.cm of ink can write, on an average, 340 words, how many words can one write with a quarter pint (1 pint = 0.568 litres) of ink?  
 (A) 4,828 (B) 8,484 (C) 19,312 (D) 24,467
14. Roja attempted to add 10 two-digit numbers. One of them, P, was the reverse of one of the others. If P was replaced by another two-digit number Q and its reverse was replaced by the reverse of Q, the average of the numbers would be 6.6 more. (The sum of the digits of Q) – (The sum of the digits of P) = .
15. The average weight of N boys in a group is 30 kg. If 5 boys with an average weight of 12 kg join the group, the average weight would be the same as if 5 boys with an average weight of 36 kg leave the group. Find N.  
 (A) 15 (B) 20 (C) 25 (D) 10
16. In a three match test series India scores an average of 325 runs in the first and the second test matches and an average of 300 runs in the second and the third test matches and an average of 350 runs in third and the first test matches. Find the overall average score for the test series.
17. The average height of 50 students in a class is 165 cm. Five students whose average height is 169 cm leave the class and ten students whose average height is 167 cm join the class. Find the new average height (in cm) of the class.
18. For 25 consecutive odd integers taken in ascending order, the average of the last 15 is  $n$ . Find the average of all the integers in terms of  $n$ .  
 (A)  $n + 15$  (B)  $n + 10$   
 (C)  $n - 10$  (D)  $n - 15$
19. A salt manufacturing company produced a total of 5000 tonnes of salt in January of a particular year. Starting from February, its production increased by 100 tonnes every month over the previous month until the end of that year. Find its average monthly production for that year. (in tonnes)
20. Ajay spent ₹3,000 on an average in the first 5 months of an year. His average spending for the next 4 months and the last 3 months of that year were ₹3,600 and ₹4,000 respectively. If he saved a total of ₹15,000 that year, find his average monthly income that year (in ₹)

21. In a shooting tournament, a group of 70 archers initially participated. Their average number of points is 36. If 20 archers whose average number of points is 40 left and 30 other archers whose average number of points is 30 join the group, find the average points of the new group.
- 
22. The total cost of producing a certain number of units of an article is partly fixed and partly varies directly with the number of units of that article produced. The average cost of producing 15,000 units of that article is ₹2 while that of producing 25,000 units of that article is ₹1.5 Find the variable cost of each unit of the article produced.
- (A) ₹1.25 (B) ₹1.50 (C) ₹0.75 (D) ₹2.50
23. A milkman bought 10 litres of pure milk at ₹10/litre and 10 litres of adulterated milk at ₹6/litre. He mixed both and sold the mixture at ₹10 / litre. Find his profit percentage.
- 
24. A man bought 10 litres of pure milk and added 5 litres of water to it but spilt 2 litres of the mixture. Of the remaining mixture, he sold 3 litres, and again added 2 litres of water to the remaining quantity. Find the percentage of milk in the final mixture.
- (A) 56% (B) 55.55%  
(C) 56.5% (D) Cannot be determined
25. A shopkeeper mixes two varieties of rice costing ₹8 per kg and ₹12 per kg in the ratio 1 : 3. If the cost of the second variety drops to ₹11.50 per kg, in what ratio should he now mix the two so that the cost of the mixture remains unchanged?
- (A) 2 : 3 (B) 1 : 6 (C) 3 : 5 (D) 2 : 5
26. Two varieties of wheat are mixed together in the ratio 3 : 4. The cost price per kg of the first variety is ₹7 more than that of the second variety. The mixture is sold at 20% profit at ₹36 per kg. Find the cost price of the second variety, in rupees per kg.
- 
27. A shopkeeper mixes three varieties of rice costing ₹10, ₹12 and ₹17 per kg. Which of the following can represent a possible ratio in which the three varieties are mixed, if the trader makes a profit of 20% by selling the mixture at ₹15.60 per kg?
- (A) 9 : 14 : 36 (B) 11 : 14 : 25  
(C) 14 : 36 : 43 (D) 2 : 6 : 3
28. In fresh grapes, 80% of the weight is water while in dry grapes only 50% of the weight is water. How many kg of dry grapes can be obtained from 20 kg of fresh grapes?
- 
29. Two vessels contain petrol and kerosene in the ratios 2 : 5 and 4 : 3. In what ratio should the solution in the first vessel be mixed with the solution in the second, so as to get a solution with petrol and kerosene in the ratio 3 : 4?
- (A) 1 : 1 (B) 1 : 2 (C) 2 : 5 (D) 3 : 5
30. The concentrations of spirit in three vessels A, B and C are 45%, 30% and 25% respectively. If 4 litres from vessel A, 5 litres from vessel B and 6 litres from vessel C are mixed, find the concentration of spirit in the resultant solution.
- (A) 25% (B) 31.25% (C) 37.5% (D) 32%
31. How many litres of 27% alcohol solution are to be added to 20 litres of 36% alcohol solution to obtain a 30% alcohol solution?
- 
32. Two varieties of rice A and B priced at ₹6.75 per kg and ₹9.75 per kg respectively were mixed and sold at ₹10.80 per kg at a profit of 20%. Find the ratio in which A and B are mixed.
- (A) 1 : 4 (B) 1 : 3 (C) 1 : 2 (D) 1 : 5
33. A vessel contains 80% alcohol solution. 20% of the solution was removed and replaced with water. If this process is repeated, find the percentage of alcohol which remains in the solution.
- (A) 64% (B) 51.2%  
(C) 40.96% (D) None of these
34. A sum of ₹18000 was invested partly at 10% p.a. simple interest and partly at 18% p.a. simple interest. If the total interest on the sum was ₹2160 at the end of one year, find the sum invested (in ₹) at 10% p.a.
- 
35. In a company there are 252 employees, in which the ratio of the number of men and women is 2 : 1. Some more women are employed and the ratio of men and women becomes 1 : 1. The average age of all the employees is now 22 years and the average age of the women is 2 years less than the average age of the men. Find the average ages of men and women. (both in years)
- (A) 22, 20 (B) 23, 21 (C) 24, 22 (D) 21, 23
36. The average age of x persons is 60 years. If two persons of 52 years and 68 years leave the group and two new persons of y years and 72 years join the group, then the average age of the group increases by 1 year. If x is a perfect square and  $54 < y < 64$ , find the value of y.
- (A) 59 (B) 55 (C) 57 (D) 61
37. A vessel is completely filled with milk. 20 litres is withdrawn from the vessel and is replaced with water. 20 litres of the mixture is then replaced with water. The vessel now has 18 litres of milk. Find the capacity of the vessel (in litres)
- (A) 70 (B) 50 (C) 60 (D) 40
38. There are two containers with mixtures of Pepsi and Coke. In container 1, Pepsi and Coke are in the ratio 3 : 2 and in container 2 Pepsi and Coke are in the ratio 2 : 3. How many litres of the mixture should be taken from container 1 and mixed with an appropriate quantity of the mixture from container 2 in order to make 20 litres of a mixture containing Pepsi and Coke in the ratio 9 : 11?
- (A) 10 (B) 15 (C) 20 (D) 5

39. From a vessel containing only milk, 10 litres are drawn and replaced with water. 10 litres of the mixture is now taken out and replaced with water again. The ratio of milk to water now is 25 : 24. How many litres of milk was there initially?  
(A) 20 litres (B) 25 litres (C) 32 litres (D) 35 litres
40. A vessel contains 2.5 litres of water and 10 litres of milk. 20% of the contents of the vessel are removed. To the remaining contents, a certain quantity of water is added to reverse the ratio of water and milk. A certain quantity of milk is added again to reverse the ratio of water and milk. Find the quantity of milk added (in litres).  
(A) 100 (B) 110 (C) 120 (D) 130
- Directions for questions 41 to 45:** Each question is followed by two statements, I and II. Indicate your responses based on the following directions:
- Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.  
Mark (B) if the question can be answered using either statement alone.  
Mark (C) if the question can be answered using statements I and II together but not using I or II alone.  
Mark (D) if the question cannot be answered even using statements I and II together.
41. Variety A of rice costs ₹32 per kg. Variety B of rice costs ₹22 per kg. The two varieties of rice are mixed in a certain ratio. What is the ratio in which they are mixed?
- I. The mixture is sold at ₹36 per kg at 20% profit.  
II. Weight of the mixture is 12 kg.
42. Is the average age of the students taking SAT greater than 22?  
I. Out of the 100 test takers, 92 average 14.  
II. The oldest test taker is 24 years old.
43. The average weight of Amar, Bhanu, Chetan and Dinesh is 80 kg. Find the average weight of Amar and Bhanu.  
I. The weight of Amar is the average of the weights of the others.  
II. The weight of Bhanu is the average of the weights of the others.
44. Can A contains milk and water solution and can B contains orange juice and water solution. These solutions are mixed. What is the ratio of water, milk and orange juice in the resultant solution?  
I. Can A contains 40% water and can B contains 35% orange juice.  
II. Half of the resultant solution is water
45. There are N men in a group. Their average age is 20 years. Find N  
I. If 2 men aged 22 years and 28 years join the group, the average age of the group increases by a prime number.  
II. If 2 men aged 22 years and 28 years leave the group, the average age of the group decreases by a prime number

## Key

### Concept Review Questions

- |       |        |          |           |        |
|-------|--------|----------|-----------|--------|
| 1. B  | 6. B   | 11. 13   | 16. 13000 | 21. B  |
| 2. 20 | 7. B   | 12. 7.25 | 17. 8100  | 22. B  |
| 3. D  | 8. 150 | 13. 3    | 18. D     | 23. B  |
| 4. D  | 9. C   | 14. C    | 19. B     | 24. 13 |
| 5. D  | 10. 57 | 15. 7    | 20. B     | 25. 27 |

### Exercise – 7(a)

- |         |           |        |        |           |
|---------|-----------|--------|--------|-----------|
| 1. D    | 7. C      | 13. D  | 19. B  | 25. C     |
| 2. 78   | 8. 23     | 14. B  | 20. 5  | 26. 16    |
| 3. 47   | 9. B      | 15. 48 | 21. B  | 27. 583.2 |
| 4. B    | 10. D     | 16. A  | 22. C  | 28. B     |
| 5. C    | 11. 23500 | 17. A  | 23. A  | 29. D     |
| 6. 33.5 | 12. 3     | 18. 5  | 24. 35 | 30. 20000 |

### Exercise – 7(b)

- |          |         |           |           |       |
|----------|---------|-----------|-----------|-------|
| 1. D     | 10. 95  | 19. 5550  | 28. 8     | 37. B |
| 2. 70    | 11. 35  | 20. 4700  | 29. A     | 38. D |
| 3. 144   | 12. D   | 21. 32.75 | 30. D     | 39. D |
| 4. 10500 | 13. A   | 22. C     | 31. 40    | 40. C |
| 5. 12    | 14. 6   | 23. 25    | 32. B     | 41. A |
| 6. B     | 15. D   | 24. B     | 33. B     | 42. C |
| 7. 115   | 16. 325 | 25. B     | 34. 13500 | 43. C |
| 8. 7     | 17. 165 | 26. 27    | 35. B     | 44. C |
| 9. D     | 18. C   | 27. D     | 36. C     | 45. A |