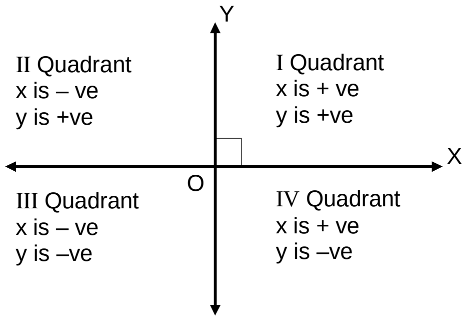
# CHAPTER – 6

# GRAPHS

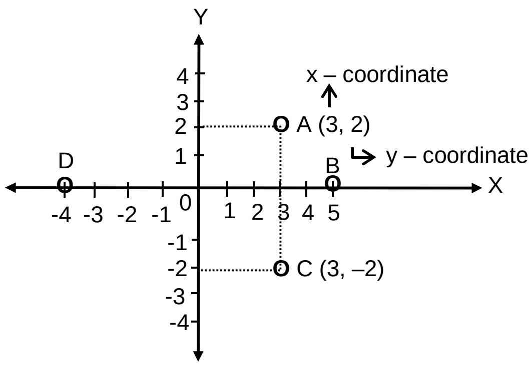
## GRAPHS:

By a graph, we mean the set of points (x, y) that satisfy a relation of the form $f(x, y) = 0$ or $f(x, y) \leq 0$ or $f(x, y) \geq 0$. The graphs in this section will be of those curves that can be drawn in a plane.
The graph sheet and the rectangular coordinate system is as shown below:



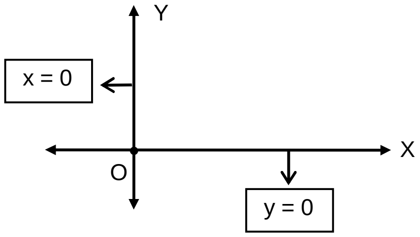

The graph sheet is divided into 4 quadrants by two mutually perpendicular lines called the coordinate-axes. The horizontal line is called the x-axis while the vertical line is called the y-axis.

Any point on the x-axis can be represented as (a, 0), while any point on the y-axis can be represented as (0, b). The point O, where the two lines meet is called the 'origin'.




The co-ordinates of the point A are (3, 2). If we want to specify the coordinates of the points, we can write A = (3, 2), B = (5, 0) etc and if we want to refer to the points we can write A (3, 2), B (5, 0) etc.

## Line Graphs:

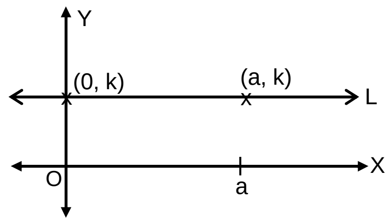



(1) We note that any point on the x-axis has the y-coordinate as 0.
So, y = 0 is the relation satisfied by any point on the line. The relation being an equation, is termed as the equation of the line.

(2) The equation of the y-axis is x = 0.

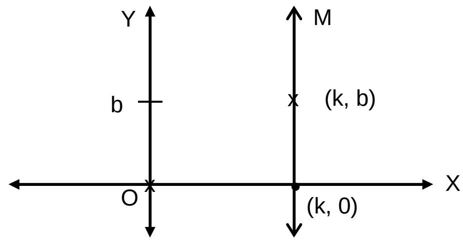
(3) The equation of a horizontal line or any line parallel to the x-axis is $\boxed{y = k \ \text{(constant)}}$

Reason: We note that every point on the line L has the y-coordinate fixed as k.



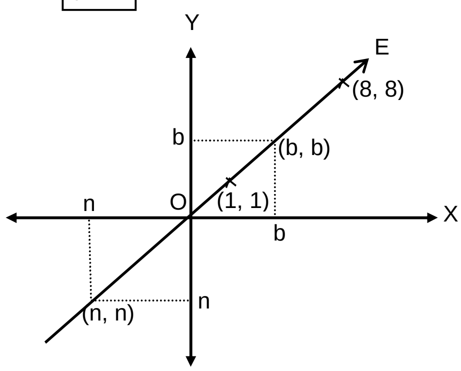

(4) The equation of a vertical line or any line parallel to the y-axis is $\boxed{x = k \ \text{(constant)}}$

Reason: We note that every point on the line M has the x-coordinate as k.



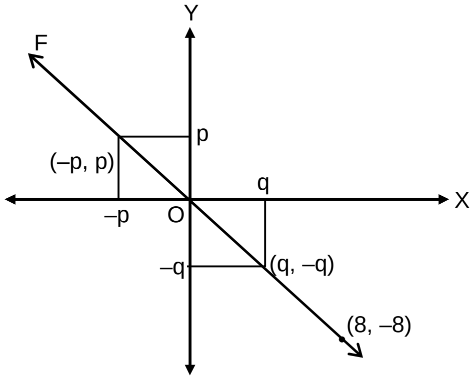

(5) On the line E shown in the figure, we find that x and y-co-ordinates are equal.

So, $\boxed{y = x}$ is the equation.

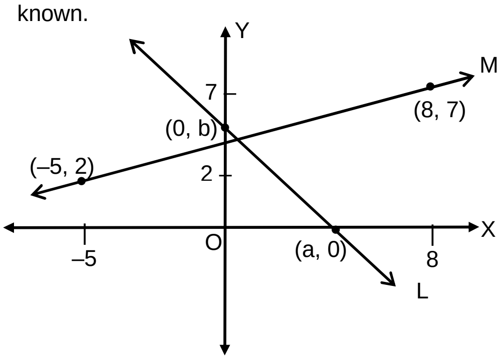



(6) On the line F, as shown in the figure, we find that x and y co-ordinates are equal in magnitude but opposite in sign.

So $\boxed{y = –x}$ is the equation.




(7) Obtaining the equation of a line if two points on it are known.




We apply the two-point formula for line equation $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ (Refer to the coordinate geometry chapter).

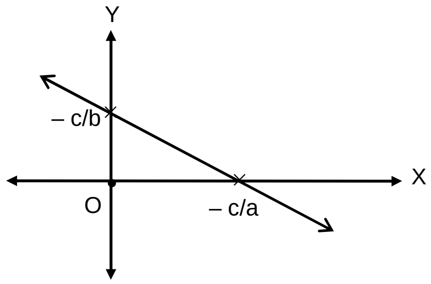
Accordingly, the equation of line L is

$y - 0 = \frac{b - 0}{0 - a}(x - a)$ or $\frac{x}{a} + \frac{y}{b} = 1$

The equation of M, is $y - 7 = \frac{2 - 7}{-5 - 8}(x - 8)$

or $5x - 13y + 51 = 0$.

(8) Plotting the graph of an inclined line:
For a line given as $ax + by + c = 0$; $ab \neq 0$
Step (i) Get the x-intercept as $x = -c/a$, by plugging $y = 0$.
Step (ii) Get the y-intercept as $y = -c/b$, by plugging $x = 0$.
Step (iii) Plot the points $(0, -c/b)$ and $(-c/a, 0)$ on the graph sheet.
Step (iv) Join these points by a line




(9) Graphing the regions bounded by lines
- i. $x \geq 0$ : The right half-plane (including the y-axis)
- ii. $y \geq 0$ : The upper half-plane (including the x-axis)
- iii. $x < 0$ : The left half-plane (excluding the y-axis)
- iv. $y < 0$ : The lower half-plane (excluding the x-axis)
- v. $x \geq 0$ and $y \geq 0$ : The first quadrant (with the boundaries)
- vi. $xy \leq 0$ : The second and fourth quadrants together (with the boundaries)
- vii. $xy > 0$ : The first and third quadrants together (without the boundaries)
- viii. $ax + by + c \geq 0$ OR $\leq 0$ : represents the half-planes demarcated by the line $ax + by + c = 0$ (with the line)

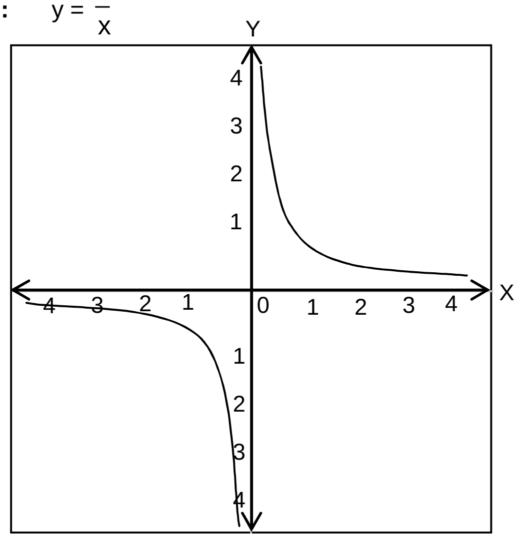
## Shifting of graphs:

Consider the graph of f(x). To get the graph of f(a + x) (a > 0) move the graph a units to the left. Similarly to get the graph of f(x – a) (where a > 0) move the graph a units to the right.

## Examples:

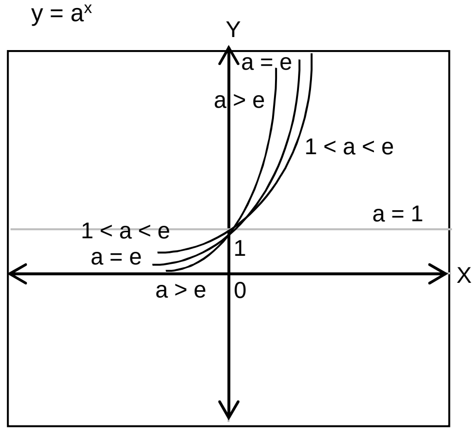
**6.01.** Draw the graph of $y = \frac{1}{x}$, $(x \neq 0)$

**Sol:** $y = \frac{1}{x}$



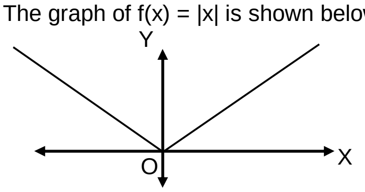

**6.02.** Draw the graph of $y = a^x$ for
(i) $1 < a < e$
(ii) $a > e$
(iii) $a = e$

**Sol:** $y = a^x$

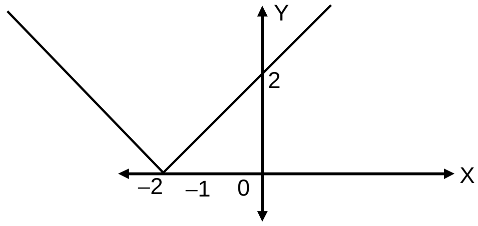


**6.03.** Draw the graph of y = |x + 2|

**Sol.** The graph of f(x) = |x| is shown below:



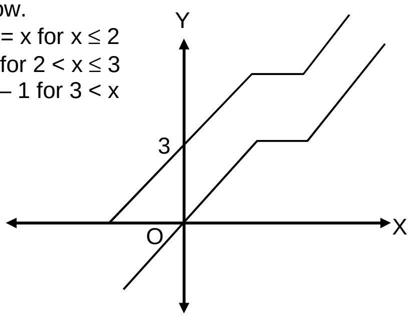

To get the graph of |x + 2|, move the above graph 2 units to the left. Then the graph is as follows:




**Note:** Similarly, the graph of |x – 2| is obtained by shifting the graph of |x|, 2 units to the right.
To get the graph of f(x) + k, shift the graph of f(x), k units up and similarly to get the graph of f(x) – k, shift the graph of f(x) k units down.

**6.04.** The graph of the following function is shown below.
f(x) = x for x ≤ 2
= 2 for 2 < x ≤ 3
= x – 1 for 3 < x

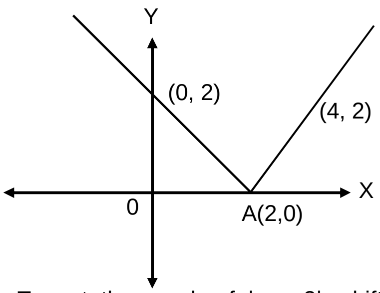
**Sol.**



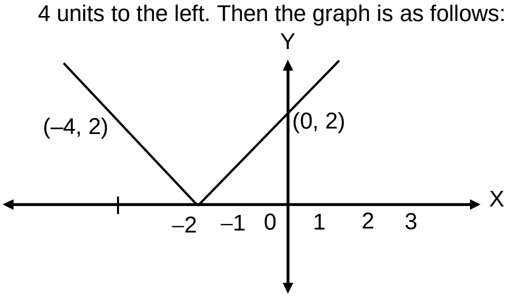

To get the graph of f(x) + 3, shift the graph of f(x), 3 units up.

**6.05.** Get the graph of |x + 2| from the graph of |x – 2|

**Sol.** The graph of f(x) = |x – 2| is as follows:

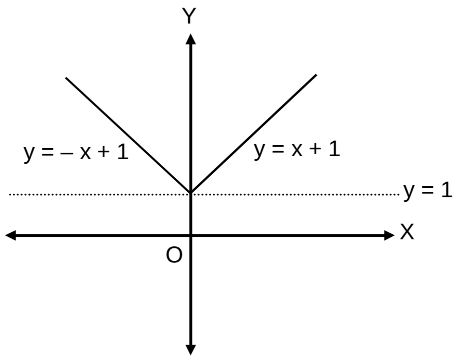



To get the graph of |x + 2|, shift this graph 4 units to the left. Then the graph is as follows:




**6.06.** Draw the graph of y = |x| + 1

**Sol.**




y = |x| + 1
Clearly the least value taken by y is 1, as no portion of the line is below y = 1.

The equation $\boxed{y = |x| + 1}$ can be written as

$$y = \begin{cases} x + 1; & x \geq 0 \\ -x + 1; & x < 0 \end{cases}$$

**Note:** The curve doesn't pass through the origin.

The table below shows how to get the graphs of some related functions from the graph of a given function, say y = f(x).

Table showing the changes made to y = f(x) and the corresponding effect in the graph.

| Related Function | To get the graph from the graph of y = f(x) |
|---|---|
| (1) y = \|f(x)\|. | (1) unchanged when f(x) ≥ 0 and reflected in the x-axis when f(x) < 0. |
| (2) y = –\|f(x)\|. | (2) unchanged when f(x) ≤ 0 and reflected in the x-axis when f(x) > 0. |
| (3) x = f(y). | (3) reflected in the line y = x. |
| (4) (i) y = cf(x), c > 1,<br>(ii) y = cf(x), 0 < c < 1. | (4) (i) vertical stretch,<br>(ii) vertical shrink. |
| (5) (i) f(cx); c > 1,<br>(ii) f(cx); 0 < c < 1. | (5) (i) horizontal shrink,<br>(ii) horizontal stretch. |
| (6) y – k = f(x – h). | (6) translates 'h' units horizontally to the right and 'k' units vertically down. |

## Floor and Ceiling (or ceil) Functions:

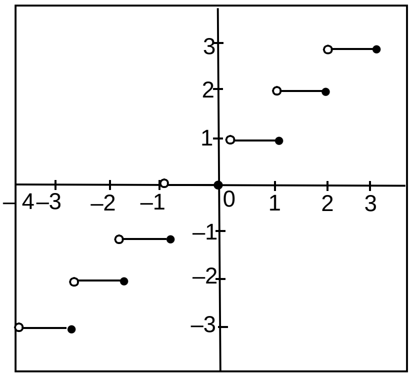
For any real number x, the greatest integer less than or equal to x is called the floor function. It is denoted as $\lfloor x \rfloor$.The least integer greater than or equal to x is called the ceiling (or ceil) function. It is denoted as $\lceil x \rceil$.

Thus $\lfloor 4.1 \rfloor = 4$, $\lfloor -1.2 \rfloor = -2$ and $\lfloor -7 \rfloor = -7$
while $\lceil 4.1 \rceil = 5$, $\lceil -1.2 \rceil = -1$ and $\lceil -7 \rceil = -7$

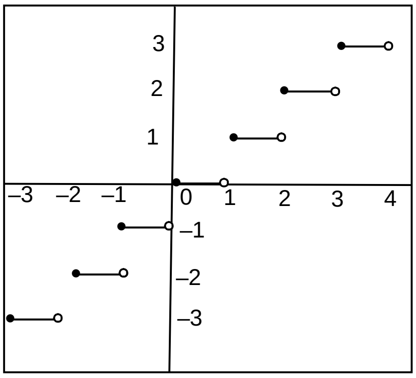
When the symbol [x] is used without any further explanation, it normally means $\lfloor x \rfloor$.

The graphs of these two functions are shown below

Ceil function




Floor function




We note that for any integer n, $\lfloor n \rfloor = n = \lceil n \rceil$ and for any non integer x, $\lfloor x \rfloor + 1 = \lceil x \rceil$

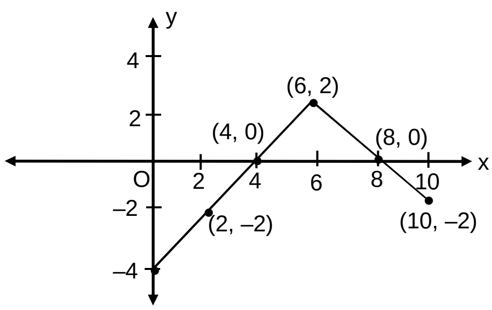
We also note that for the floor function, the endpoints on the left of each segment or 'step' is included, while the endpoint on the right is excluded. For the ceil function, it is vice-versa.

**6.07.** Sketch the graph of the function

$$y = \begin{cases} x - 4\ ; & 0 \le x < 6 \\ 8 - x\ ; & 6 \le x \le 10 \end{cases}$$

Also determine the maximum and the minimum values of y in the given domain.

**Sol.**




The graph is made of 2 line segments.

For $0 \le x < 6$ we take the points (0, –4), (2, –2), (4, 0).

For $6 \le x \le 10$, we take the points (6, 2), (8, 0), (10, –2).

The maximum value of y is 2 and the minimum is –4.

**6.08.** Sketch the graph of $y = \dfrac{|x-7| - |x-3|}{2}$ and hence $x = \dfrac{|y-7| - |y-3|}{2}$.

**Sol.** We first write the function y in terms of x, more explicitly.

Case (i): $x \le 3$:

$$y = \frac{(-(x-7)) - (-(x-3))}{2} = \frac{7 - x + x - 3}{2} = 2.$$

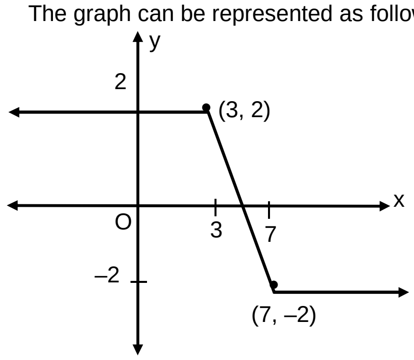
Case (ii): $3 \le x \le 7$:

$$y = \frac{(-(x-7)) - (x-3))}{2} = \frac{-2x + 10}{2} = -x + 5.$$

Case (iii): $x \ge 7$:

$$y = \frac{(x-7) - (x-3)}{2} = \frac{-4}{2} = -2.$$

The graph can be represented as follows:

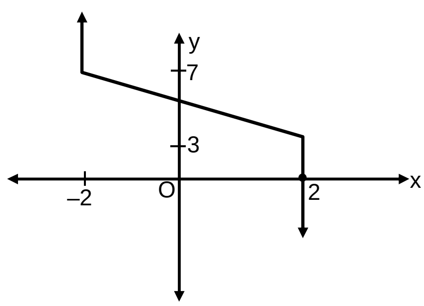



$$\text{Thus } y = \begin{cases} 2\ ; & \text{for } x \le 3 \\ -x + 5\ ; & \text{for } 3 \le x < 7 \\ -2\ ; & \text{for } x \ge 7 \end{cases}$$

To get the graph of

$x = \dfrac{|y-7| - |y-3|}{2}$, we need to reflect the graph discussed above in the line y = x.

The graph is as follows:

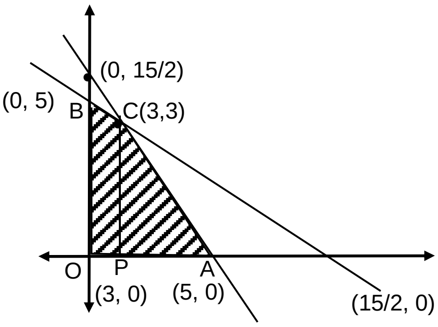



The expression being

$$x = \begin{cases} 2 & ;\text{for } y \le 3 \\ -y + 5 & ;\text{for } 3 \le y \le 7 \\ -2 & ;\text{for } y > 2 \end{cases}.$$

**6.09.** Find the area enclosed by the region bounded by the relations $2x + 3y \le 15$, $3x + 2y \le 15$, $x \ge 0$, $y \ge 0$.

**Sol.** The demarcating lines for the region are $2x + 3y = 15$, $3x + 2y = 15$, $x = 0$, $y = 0$. The area of the region required is that of the quadrilateral OACB. We break it as trapezium OPCB and triangle PAC.



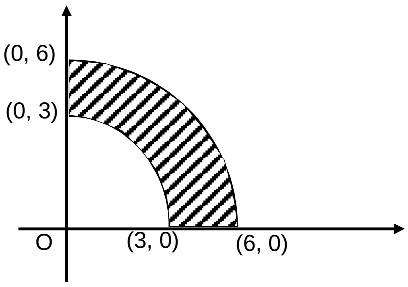

The sum of the areas

$= \frac{1}{2}((5+3)3) + \frac{1}{2} \times 2 \times 3 = 12 + 3$

= 15 sq. units.

**6.10.** Two points P and Q move in such a way that the distance of P from the origin is 6 units and that of Q is 3 units. Find the area enclosed between the truncated annular region in the first quadrant.

**Sol.**

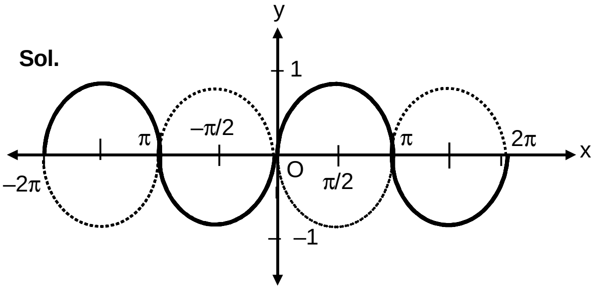



The set of points traversed by P will be the points on the arc of the circle with radius 6 centred at the origin and that of Q will be on the arc of the circle with radius 3 centred at the origin. The area enclosed equals

$= \frac{1}{4}\left(\pi(6)^2 - \pi(3)^2\right) = \frac{27\pi}{4}$ sq. units .

**6.11.** Sketch the graph of
(i) $y = |\sin x|$
(ii) $y = -|\sin x|$ on $[-2\pi, 2\pi]$

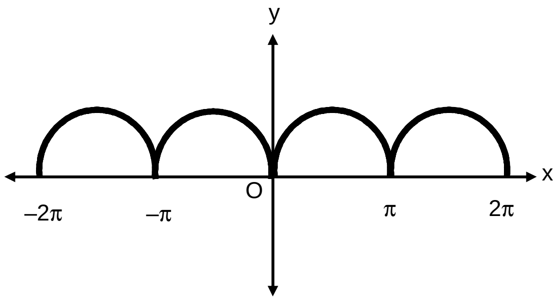
**Sol.**



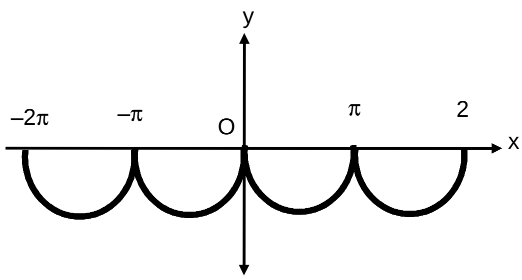

We first recall the graph of $y = \sin x$ on $[-2\pi, 2\pi]$

(i) For the graph of $y = |\sin x|$, we need to reflect in the x-axis, the portion of the graph defined between $(-\pi, 0)$ and $(\pi, 2\pi)$.
The dotted portion above x-axis is to be traced back. Analytically, $y = |\sin x| \ge 0$
$\Rightarrow y \ge 0$ (no portion is below the x-axis)

Accordingly we get the graph as




(ii) $y = -|\sin x|$. The graph can be obtained by reflecting in the x-axis, the portion of the graph defined between $(-2\pi, -\pi)$ and $(0, \pi)$.
Accordingly, we get the graph as

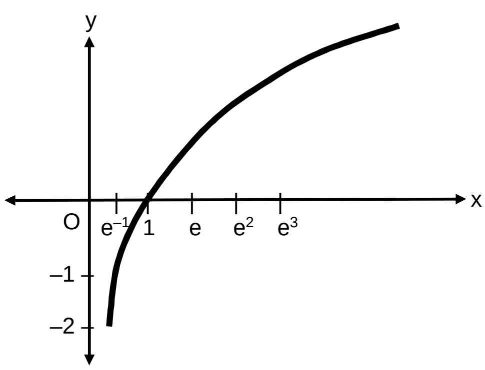



The portion that is above in $y = \sin x$ moves down. Analytically, $y = -|\sin x| \le 0 \Rightarrow y \le 0$ (no portion is above x – axis)

**6.12.** Sketch the graph of
(i) $y = \log_e x$,
(ii) $y = \log(-x)$,
(iii) $y = |\log x|$ and
(iv) $y = -|\log x|$.
(taking the natural base as 'e')

**Sol.** (i) $y = \log x$ (given)
We know that y is defined only for $x > 0$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | 1 | e | $e^2$ | $e^3$ | $e^{-1}$ | $e^{-2}$ | $e^{-3}$ |
| y | 0 | 1 | 2 | 3 | –1 | –2 | –3 |



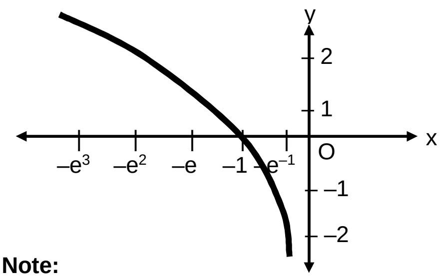
**Note:**

(A) No portion of the graph lies in the second and the third quadrants as y is defined only for positive values of x.

(B) As $x \to 0$ we note that $y \to -\infty$.

(ii) We note that x is replaced with (–x) in logx. So it just means reflection in y-axis; here, we note that log(–x) is defined only for x < 0.

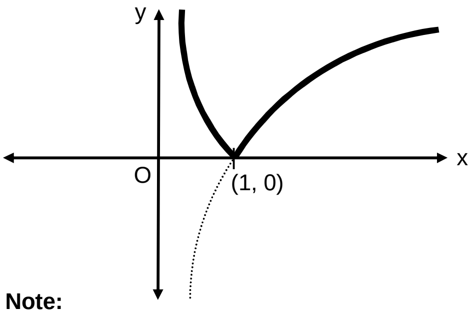
Accordingly the graph becomes,




**Note:**
No portion of the graph is in the first and the fourth quadrants as x < 0.

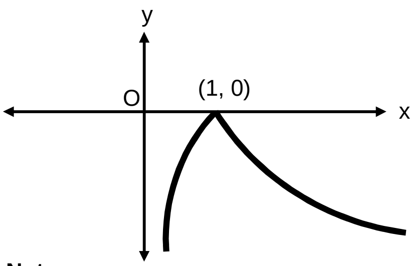
(iii) y = | log x |
We know that y is defined for x > 0. Also y is non-negative: The portion in log x, x > 0 that is below the x-axis is reflected in x-axis.

Accordingly, the graph becomes




**Note:**
The dotted line indicates the original form y = logx. The introduction of modulus into logx makes the portion below to move above.

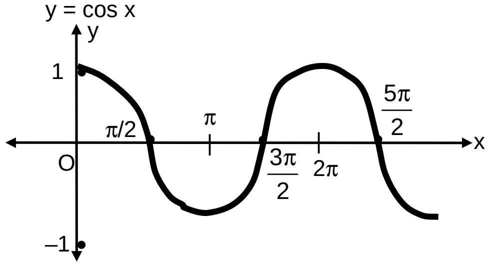
(iv) y = –|log x|;
We know that the graph is defined for x > 0. As it is of the form y = –|f(x)|, it first means reflection of |f(x)| in the x-axis. Here f(x) = logx. Accordingly, the graph becomes



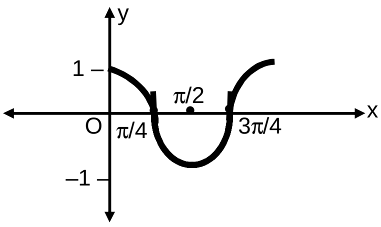

**Note:**
No portion of the graph is above the x-axis.

**6.13.** Sketch the graph of y = cosx and hence sketch the graph of
(i) y = cos 2x (ii) y = cos x/2
(iii) y = 2 cos x (iv) y = 1/2 cos x, on [0, 2π]

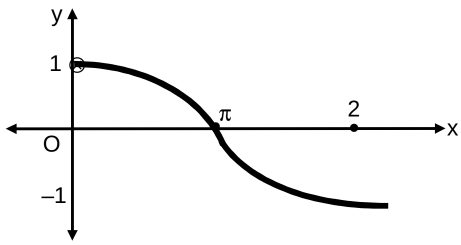
**Sol.** y = cos x



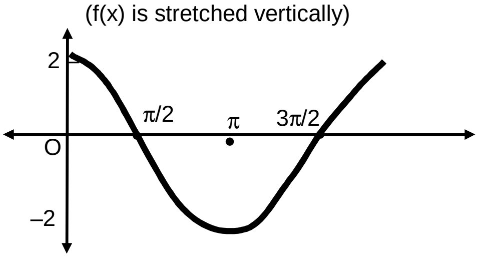

(i) y = cos2x
$-1 \le y \le 1$
(f(x) is shrunk horizontally)



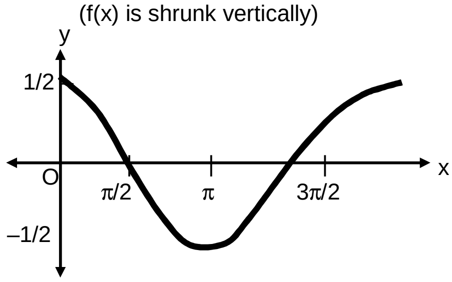

(ii) y = cos(x/2)
$-1 \le y \le 1$
(f(x) is stretched horizontally)




(iii) y = 2cosx
$-2 \le y \le 2$
(f(x) is stretched vertically)

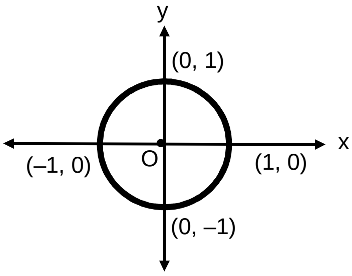



(iv) y = 1/2 cosx
$-1/2 \le y \le 1/2$
(f(x) is shrunk vertically)



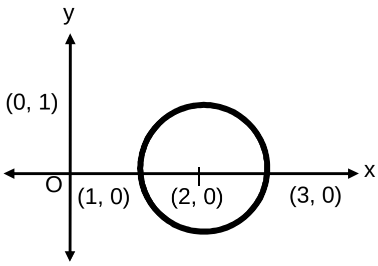

**6.14.** The graph of $x^2 + y^2 = 1$ is a circle centered at the origin whose radius = 1. Sketch the graph of
(i) $(x-2)^2 + y^2 = 1$
(ii) $x^2 + (y-2)^2 = 1$
(iii) $(x-2)^2 + (y-2)^2 = 1$

**Sol.**

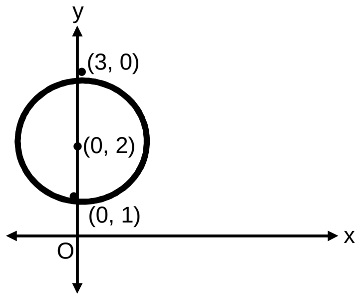



The graph of $x^2 + y^2 = 1$ is as shown above.

(i) The graph of $(x - 2)^2 + y^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$, horizontally 2 units to the right



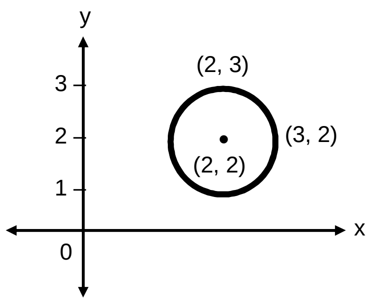

(ii)




The graph of $x^2 + (y - 2)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$, vertically upwards by 2 units.

(iii) The graph of $(x - 2)^2 + (y - 2)^2 = 1$ is obtained by translating the graph of $x^2 + y^2 = 1$ vertically upwards by 2 units and 2 units to the right horizontally.

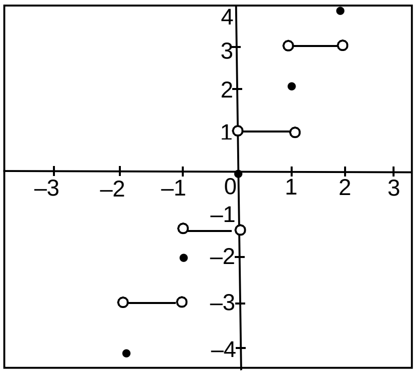
The graph is as shown below:




**6.15.** Sketch the graph of $y = \lceil x \rceil + \lfloor x \rfloor$

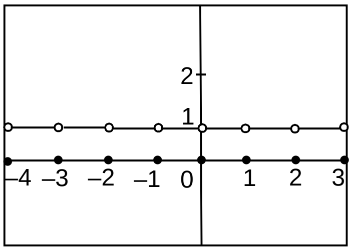
For integral values of x, $\lfloor x \rfloor = \lceil x \rceil = x$ and $y = 2x$, an even number. For other values of x, y is odd. Endpoints on both ends of the steps are excluded.

$Y = \lceil x \rceil + \lfloor x \rfloor$



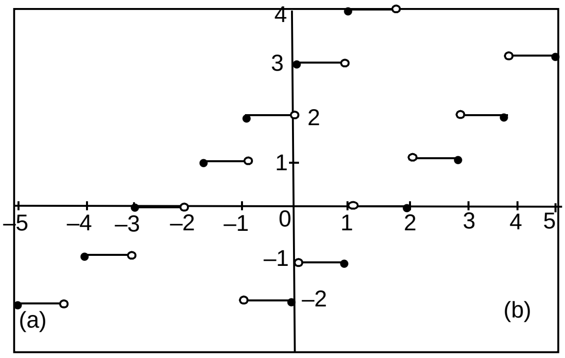

**6.16.** Sketch the graph of $y = \lceil x \rceil - \lfloor x \rfloor$.

$y = \lceil x \rceil - \lfloor x \rfloor$



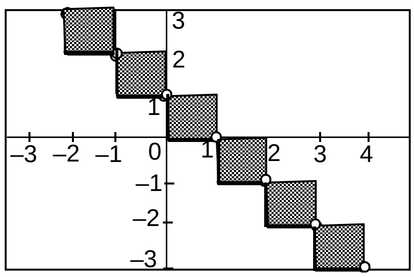

The graph is the line $y = 1$ (with missing points) for nonintegral values of x and the isolated points given by $y = 0$ for integral values of x.

**6.17.** Sketch the graph of (a) $\lfloor x \rfloor + 3$ or $\lfloor x+3 \rfloor$ and (b) $\lceil x \rceil - 2$ or $\lceil x-2 \rceil$




We see that for all x, $\lfloor x+3 \rfloor = \lfloor x \rfloor + 3$ and for all x, $\lceil x-2 \rceil = \lceil x \rceil - 2$

**6.18.** Sketch the graph of $\lfloor x \rfloor + \lfloor y \rfloor = 0$




We get a sequence of squares, in each of which the left and lower sides are included, which is indicated by the solid line while the right and upper sides are excluded, which is indicated by the broken line.

## *Concept Review Questions*

***Directions for questions 1 to 20:*** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

**1.** The graph of the equation $x + y - 1 = 0$ meets the x – axis at the point _______.
(A) (0, 1) (B) (1, 0) (C) (–1, 0) (D) (1, 1)

**2.** At what point(s) does the graph of the equation $x^2 + y^2 = 1$ meet the y – axis?
(A) (0, 1), (0, –1) (B) (0, 1), (0, 2)
(C) (0, 2), (0, –2) (D) None of these

**3.** The graph of the function $y = x$ lies in the quadrants _______.
(A) I, II (B) II, III (C) I, III (D) II, IV

**4.** To which quadrants does the graph of the curve $y = x^2$ belong?
(A) I, II (B) II, III (C) III, IV (D) I, IV

**5.** At what points do the graphs of $y = 2x + 1$ and $x^2 - 2 = y$ intersect?
(A) (3, 7), (1, 1) (B) (3, 7), (–1, –1)
(C) (–3, 7), (1, –1) (D) (3, –7), (–1, 1)

**6.** At how many points do the curves $y = x - 1$ and $x^2 = 2y$ intersect?

$\boxed{\phantom{00}}$

**7.** When $ac = 0$ and $b \neq 0$, the equation $ax + by + c = 0$ always represents
(A) a horizontal line (B) a vertical line
(C) an inclined line (D) Either (A) or (C)

**8.** If $a^2 + b = 0$, where a and b are real, and $a \neq 0$, then the equation $ax + by + c = 0$ always represents
(A) a horizontal line
(B) a vertical line
(C) an inclined line
(D) None of these

**9.** The curve $x^2 + y^2 - 2x + 2y + 1 = 0$ passes through
(A) (0, 0) (B) (0, 1)
(C) (1, 0) (D) None of these

**10.** The curve $x^2 = y$ is symmetric about _______.
(A) x – axis
(B) y – axis
(C) a line parallel to x – axis
(D) a line parallel to y – axis

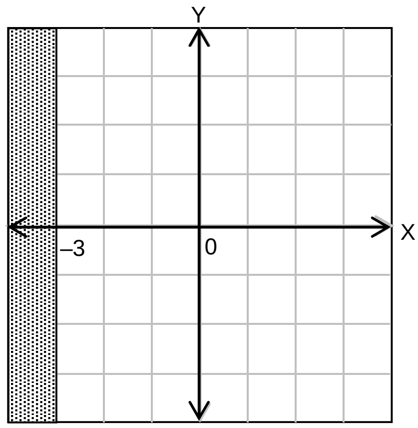
**11.** The curve $x = y^2 - 2$ is symmetric about _______.
(A) x – axis
(B) y – axis
(C) a line parallel to x – axis
(D) a line parallel to y – axis

**12.** A line drawn parallel to x-axis meets the graph of the curve $x^2 + y^2 = 1$ at _______ point(s)?
(A) only one (B) only two
(C) more than two (D) None of these

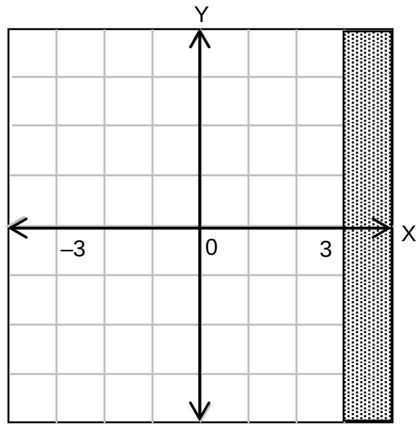
13. A line drawn parallel to y-axis meets the graph of the curve $y = x^2$ at _______ point(s)?
(A) only one (B) only two
(C) more than two (D) None of these

**14.** The graph of $x \geq -3$ is

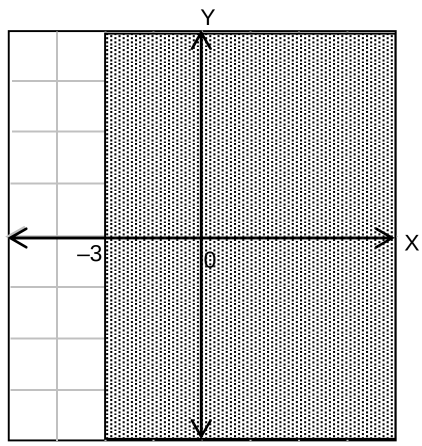
(A)



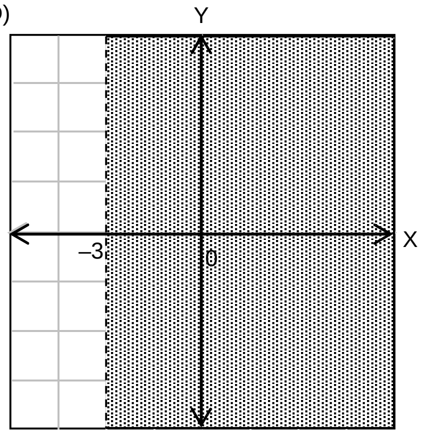

(B)



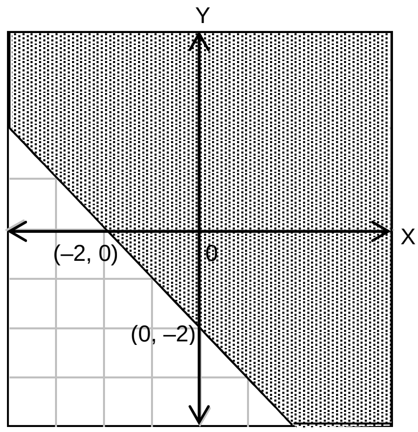

(C)



(D)

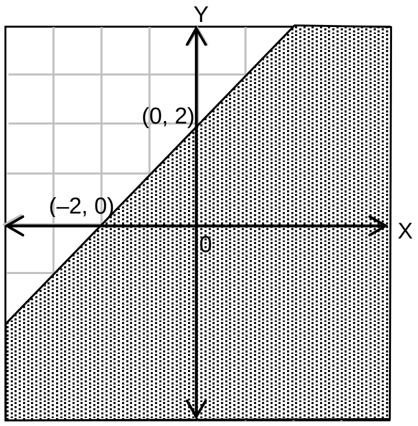



(D)

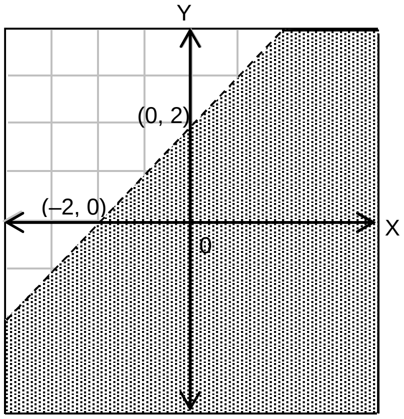



**15.** The graph of $y < x + 2$ is

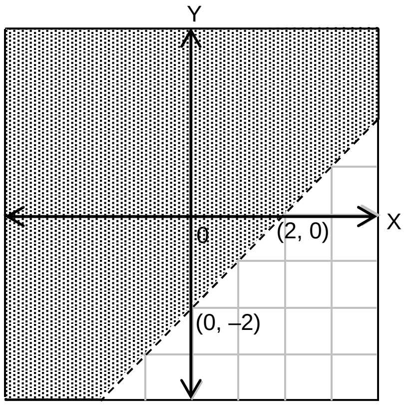
(A)




(B)




(C)

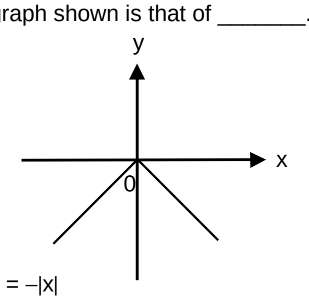



**16.** The graph of the line $2x + 3y = 0$
- (A) is parallel to the x– axis
- (B) is parallel to the y – axis
- (C) passes through the origin
- (D) lies along the x – axis.

**17.** The graph of $y = mx + c$ meets the y – axis at the point
- (A) (c, 0)
- (B) (0, c)
- (C) (0, m)
- (D) (0, –c)

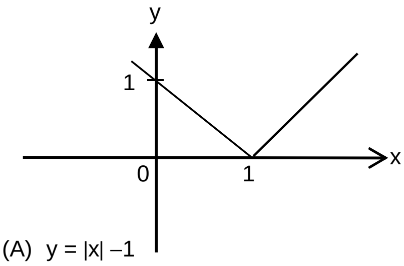
**18.** The graph of $2x – 3 = 0$ is a line
- (A) parallel to the x – axis
- (B) parallel to the y – axis
- (C) passing through the origin
- (D) None of the these

**19.** The graph shown is that of _______.




- (A) $y = –|x|$
- (B) $y = |x|$
- (C) $x = |y|$
- (D) None of these

**20.** The graph shown is that of _______.

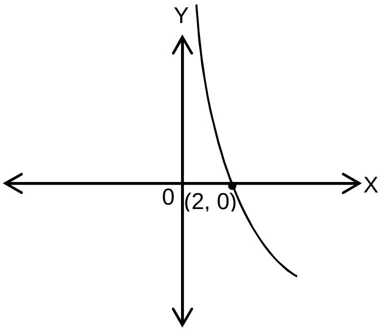



- (A) $y = |x| –1$
- (B) $y = |x – 1|$
- (C) $y = |x + 1|$
- (D) $y = |x| + 1$

## *Exercise – 6(a)*

***Directions for questions 1 to 25:*** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

**1.** Which of the following equations best describes the graph given below?

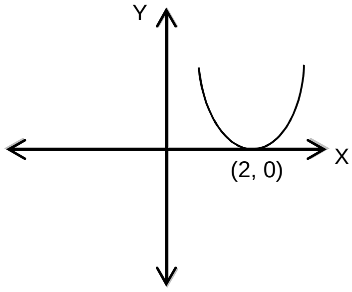



(A) $y = \log_{0.5}x$ (B) $y = \log_{0.5}(x - 1)$

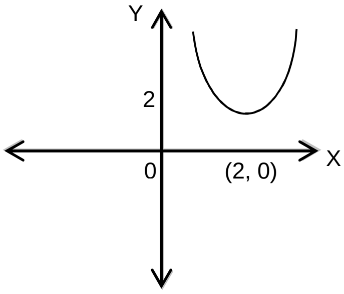
(C) $y = \log_{0.5}\left(\frac{x}{2}\right)$ (D) $y = \log_{0.5}2x$

**2.** The graph of the function $y = (x - 2)^2$ is

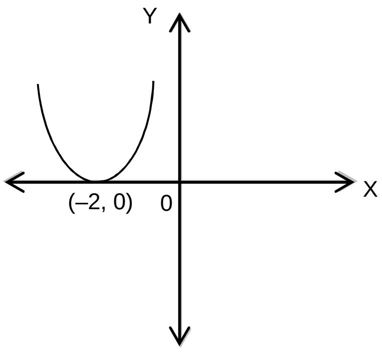
(A)



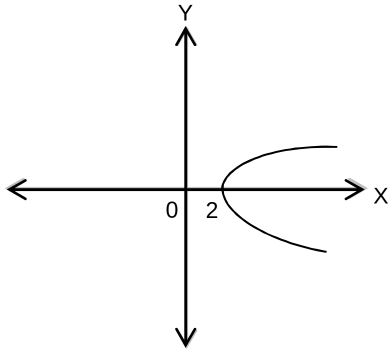

(B)



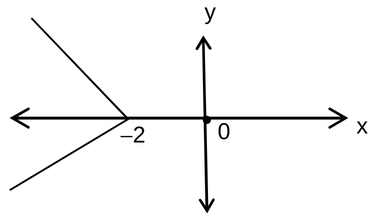

(C)




(D)



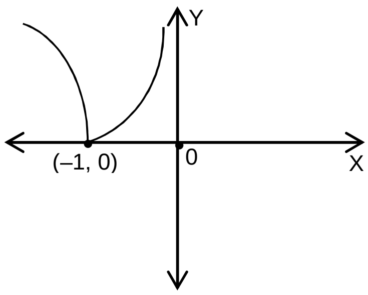

**3.** Which of the following equations best describes the graph given below?




(A) $y = -|x + 2|$ (B) $x = |y + 2|$
(C) $x = -(|y| + 2)$ (D) $x = 2 - |y|$

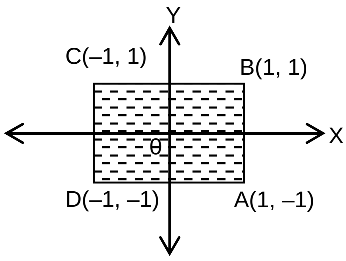
***Directions for questions 4 to 7:*** In each of these questions a graph is given. Choose the relation that best describes the graph from the choices given.

**4.**



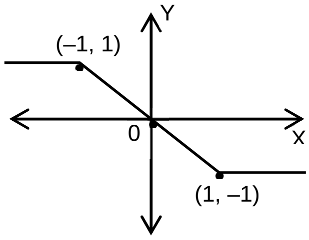

(A) $y = |\log x|, x > 0$ (B) $y = -|\log x|, x < 0$
(C) $y = |\log(-x)|, x > 0$ (D) $y = |\log(-x)|, x < 0$

**5.**




(A) $0 \le x \le 1$ and $0 \le y \le 1$
(B) $|x| \le 1$ and $|y| \ge 1$
(C) $|x| \ge 1$ and $|y| \ge 1$
(D) $|x| \le 1$ and $|y| \le 1$

**6.**



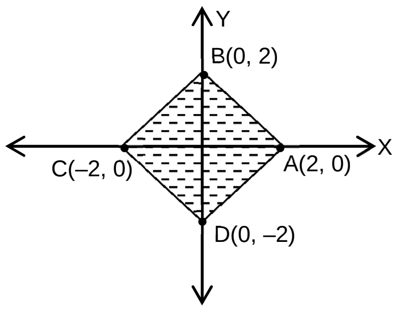

(A) $x = \frac{|y-1| - |y+1|}{2}$

(B) $y = \frac{|x-1| - |x+1|}{2}$

(C) $y = \frac{|x+1| - |x-1|}{2}$

(D) $x = \frac{|y+1| - |y-1|}{2}$

**7.**




(A) $|x| + |y| \le 1$
(B) $|x| + |y| \ge 1$
(C) $|x| + |y| \le 3$
(D) $|x| + |y| \le 2$

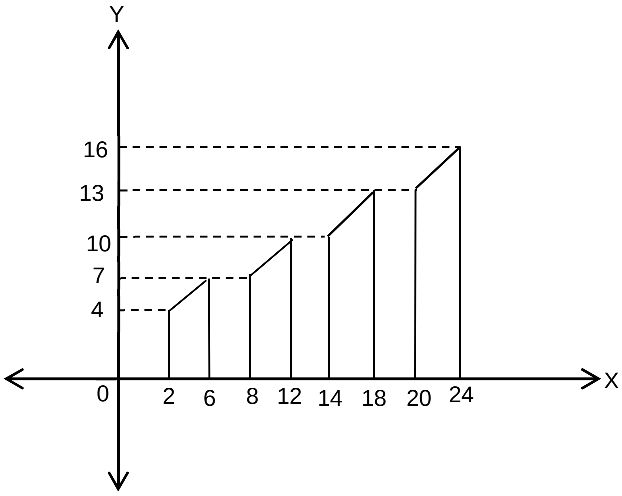
**8.** Find the area of the region described by the relations $|x| + |y| \ge 2$ and $x^2 + y^2 \le 4$ in square units.
(A) $4(\pi - 1)$
(B) $4(\pi - 2)$
(C) $4(4 - \pi)$
(D) $2(4 - \pi)$

**9.** Find the area of the region described by the relations $1 \le |x - 3| \le 5$ and $1 \le |y - 3| \le 5$ in square units.

**10.** How many points in the region enclosed by $x \ge 0$, $y \ge 0$ and $4x + 7y \le 35$ have both the coordinates as positive integers?

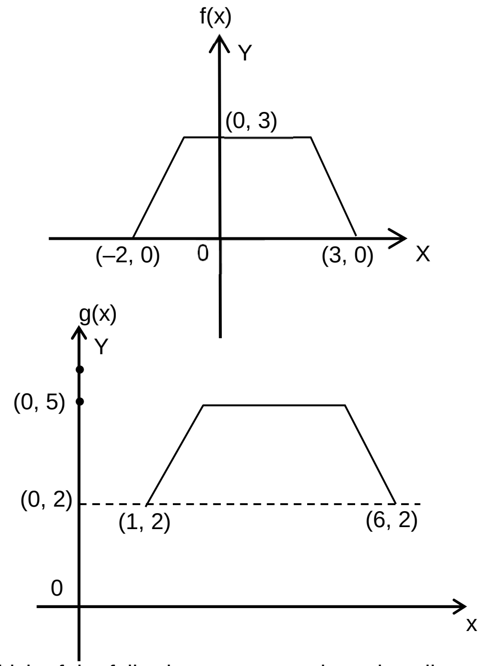
**11.** How many points with integral coordinates lie inside the region bounded by the lines $|x| = 2$, $x + y = 12$ and $x - y = 5$?

**12.** Find the sum of the areas of the trapeziums that are shown in the figure below. (in square units)




(A) 80 (B) 160
(C) 120 (D) 320

**13.**



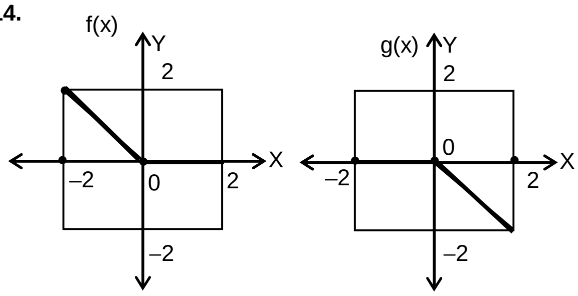

Which of the following statements best describes the relation between the two graphs?

(A) $g(x) = f(x - 2) - 1$
(B) $g(x) = f(x - 2) + 2$
(C) $g(x) = f(x + 3) + 2$
(D) $g(x) = f(x - 3) + 2$

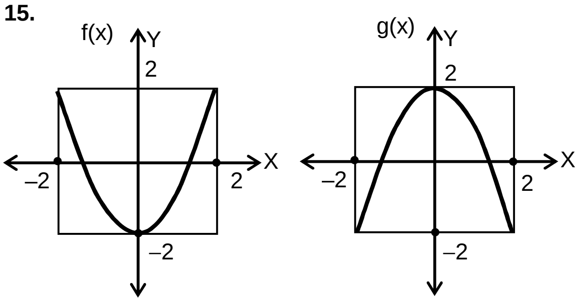
***Directions for questions 14 to 18:*** In each of these questions, a pair of graphs f(x) and g(x) is given. The graphs are shown as thick curves in the interval [–2, 2]. The following relations may hold between the graphs. Mark your answer as

(A) if $f(x) = \pm g(-x)$
(B) if $f(x) = -g(x)$
(C) if $f(x) = |g(x)|$
(D) None of the above relation holds.

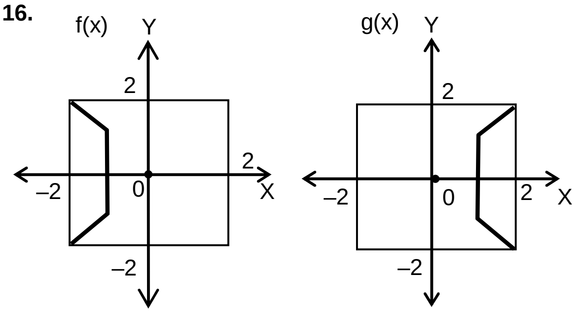
**14.**



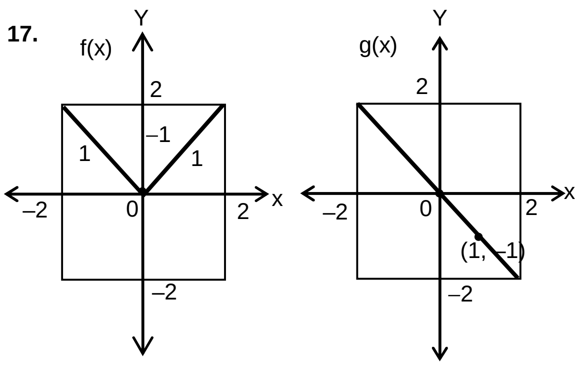
**15.**



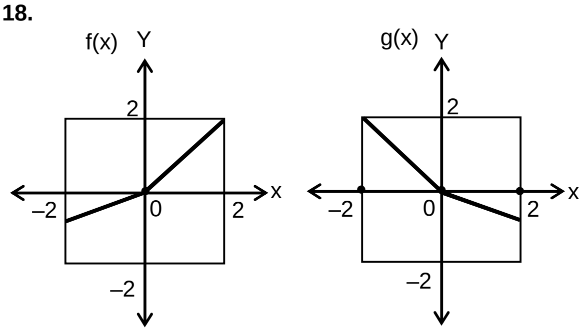
**16.**



**17.**



**18.**



***Directions for questions 19 to 23:*** In each of these questions the relation satisfied by the points on a graph is given.

Give your response as

(A) If a horizontal line can intersect the graph more than once.
(B) If a vertical line can intersect the graph more than once.
(C) Both (A) and (B)
(D) Neither (A) nor (B)

**19.** $|x|y = 2$

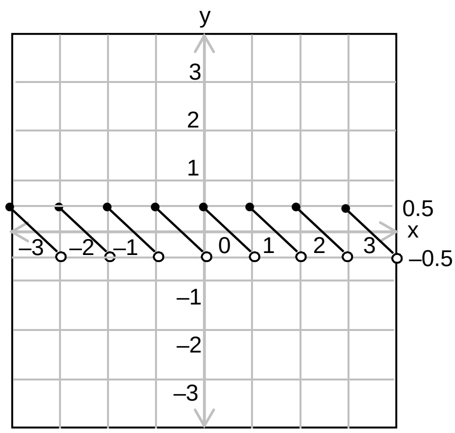
**20.** $y = \sin x$ where $x \in [-2\pi, 2\pi]$

**21.** $y^2 = 16x$.

**22.** $y = 2 - |x|$

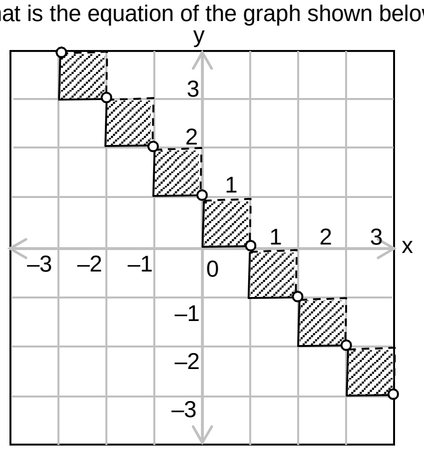
**23.** $|x| - |y| = 1$

**24.** Which of the following relations represents the graph shown below?



(A) $y = [x] - x$ (B) $y = x - [x] - \frac{1}{2}$
(C) $y = [x] - x + \frac{1}{2}$ (D) $y = \frac{x - [x]}{2}$

**25.** What is the equation of the graph shown below?



(A) $[x] = [y]$ (B) $[x] + [y] = 1$
(C) $[x] - 1 = [y]$ (D) $[x] + [y] = 0$

## *Exercise – 6(b)*

***Directions for questions 1 to 25:*** For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

### Very Easy / Easy

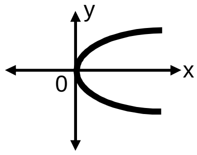
***Directions for questions 1 to 3:*** Match the equations with their graphs.

**1.** The graph of the equation $y = |x|$.

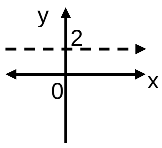
**2.** The graph of the equation $y^2 = x$.

**3.** The graph of the equation $y = 2$.

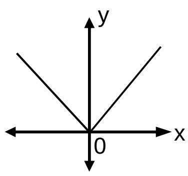
(A)



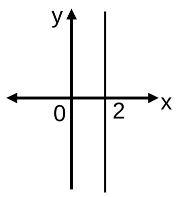

(B)




(C)




(D)




### Moderate

***Directions for questions 4 to 7:*** These questions are based on the following data.

For the equation ax + by + c = 0 some conditions are given in each of these questions.
Study the conditions and decide the nature of the graph.

Mark your answer as:

(A) for a vertical line or a horizontal line not passing through the origin
(B) for an inclined line not passing through the origin
(C) for a line passing through the origin.
(D) otherwise.

**4.** $ab > 0$ and $bc = 0$.

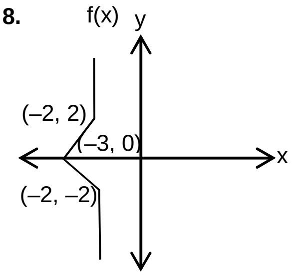
**5.** $\dfrac{c^2}{ab} = 24$

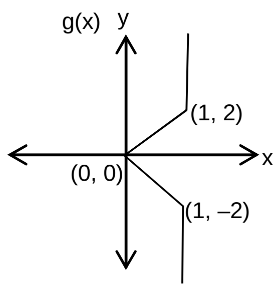
**6.** $|a| + |b| = 2$

**7.** $bc \neq 0$, $a = 0$.

***Directions for questions 8 to 10:*** Select the correct alternative from the given choices.

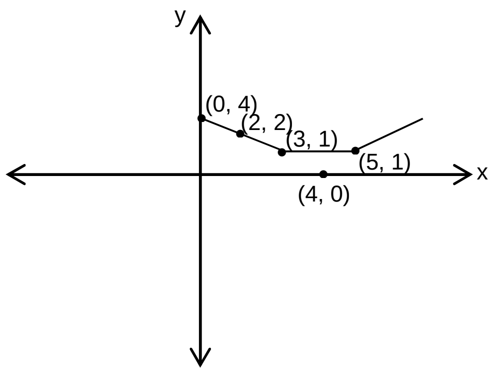
**8.**




(A) $g(x) = f(x - 3)$
(B) $g(x) = f(x + 3)$
(C) $g(x - 3) = f(x)$
(D) $g(x) + 3 = f(x)$

**9.**



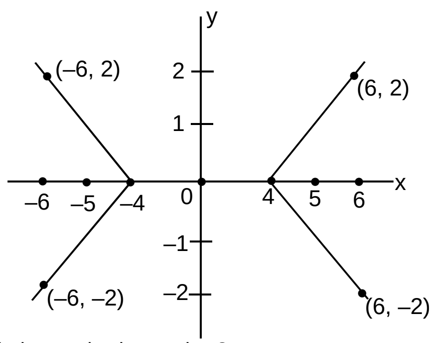

(A) $y = \dfrac{|x-3| + |5-x|}{3}$

(B) $y = \dfrac{|x-1| + |x-5|}{2}$

(C) $y = \dfrac{|x-3| + |x-5|}{2}$

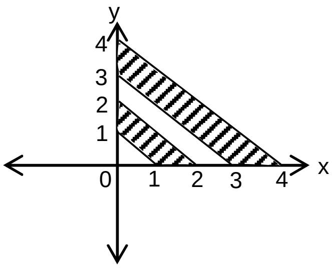
(D) $y = \dfrac{|x-1| - |x-5|}{2}$

**10.**

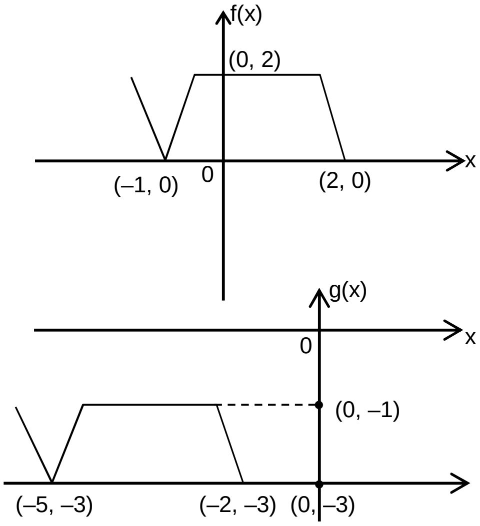



(A) $|x + y| + |x - y| = 8$
(B) $|x + y| - |x - y| = 0$
(C) $|x| - |y| = 4$
(D) $|x + y| - |x - y| = 4$

**11.** In the figure given below, find the area of the shaded region (in sq units).

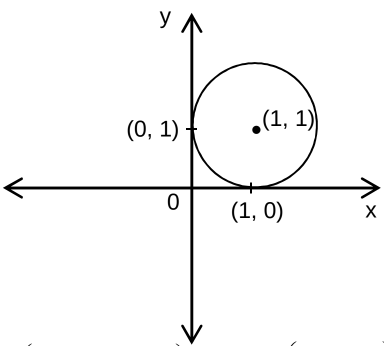



**12.** Which of the following statements best describes the relation between the two graphs?




(A) $g(x) = f(x - 4) + 3$ (B) $g(x) = f(x + 4) - 3$
(C) $g(x) = f(x + 3) - 4$ (D) $g(x) = f(x - 3) + 4$

**13.** On the given circle, find the coordinates of the point that is farthest from the origin.




(A) $\left(1+\frac{1}{\sqrt{2}}, 1+\frac{1}{\sqrt{2}}\right)$ (B) $\left(1+\frac{1}{\sqrt{2}}, 1\right)$

(C) $\left(1+\frac{1}{\sqrt{2}}, 1-\frac{1}{\sqrt{2}}\right)$ (D) (1, 1)

**14.** Find the area enclosed by the region described by $1 \le x \le 4$ and $0 \le |y-1| \le 2$ in square units.

**15.** Find the area enclosed by the region described by $2 \le |x - 3| \le 3$ and $1 \le |y - 2| \le 3$ in square units.
(A) 8 (B) 10 (C) 12 (D) 16

**16.** The number of points with non-negative integral coordinates satisfying the inequation $4x + 20y \le 100$ is

***Directions for questions 17 to 21:*** In each of the following questions, the relation represented by a graph is given. Use this relation and decide your response as:
(A) if a horizontal line can intersect the graph more than once.
(B) if a vertical line can intersect the graph more than once.
(C) if both (A) and (B) are possible.
(D) otherwise

**17.** $|x + y| + |x - y| = 2$

**18.** $|x| + |y| = 3$

**19.** $x|y| = 8$

**20.** $y = \cos x,\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

**21.** All the points (x, y) that are at a distance of 3 units from the origin such that $xy < 0$.

***Directions for question 22:*** Select the correct alternative from the given choices.

**22.** Consider the following table:

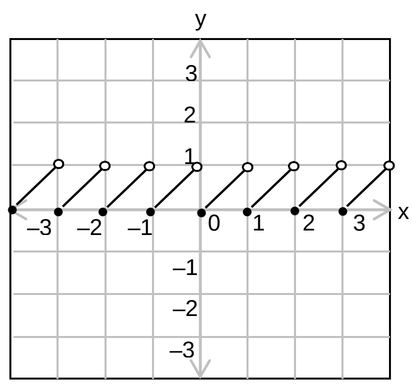
| x | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| y | 1 | 3 | 7 | 13 | 21 | 31 |

Which of the following can be the relation between x and y (a, b are constants)?
(A) $y = a + bx$ (B) $y = a + bx + cx^2$
(C) $y = a\log bx$ (D) $y = ae^{bx}$

## Difficult / Very Difficult

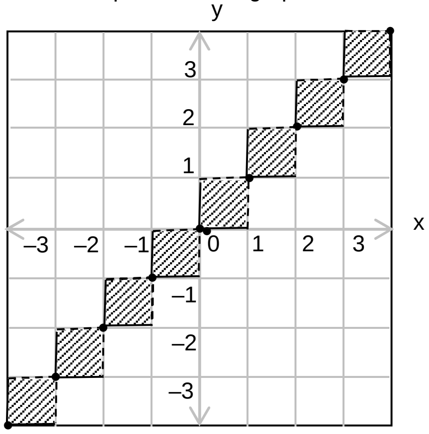
**23.** Which of the following relations are symmetrical about x-axis?
(A) $y^2 - |y| + x - 7 = 0$ (B) $y^2 + 2y + x - 7 = 0$
(C) $y^2 - yx + x - 7 = 0$ (D) Both (A) and (C)

**24.** The equation of the graph given below is




(A) $y = x - [x]$. (B) $y = [x] - x$.
(C) $y = [x]$. (D) $y = -[x]$.

**25.** What is the equation of the graph shown below?




(A) [x] + [y] = 0 (B) [x] – [y] = 0
(C) [x] + [y] = 1 (D) [x] – [y] = 1

# Key

## *Concept Review Questions*

| | | | | |
|---|---|---|---|---|
| 1. B | 5. B | 9. C | 13. A | 17. B |
| 2. A | 6. 0 | 10. B | 14. C | 18. B |
| 3. C | 7. D | 11. A | 15. B | 19. A |
| 4. A | 8. C | 12. D | 16. C | 20. B |

## *Exercise – 6(a)*

| | | | | |
|---|---|---|---|---|
| 1. C | 6. B | 11. 48 | 16. D | 21. B |
| 2. A | 7. D | 12. B | 17. C | 22. A |
| 3. C | 8. B | 13. D | 18. A | 23. C |
| 4. D | 9. 64 | 14. A | 19. A | 24. C |
| 5. D | 10. 16 | 15. D | 20. A | 25. D |

## *Exercise – 6(b)*

| | | | | |
|---|---|---|---|---|
| 1. C | 6. D | 11. 5 | 16. 81 | 21. D |
| 2. A | 7. A | 12. B | 17. C | 22. B |
| 3. B | 8. A | 13. A | 18. C | 23. A |
| 4. C | 9. C | 14. 12 | 19. B | 24. A |
| 5. B | 10. C | 15. A | 20. A | 25. B |