CHAPTER – 4

GEOMETRY

In CAT and other similar MBA Entrance Exams, the problems relating to Geometry cover mostly triangles, quadrilaterals and circles. Even though polygons with more than four sides are also covered, the emphasis on such polygons is not as much as it is on triangles and circles. In this chapter, we will look at some properties as well as theorems and riders on parallel lines, angles, triangles (including congruence and similarity of triangles), quadrilaterals, circles and polygons.

ANGLES AND LINES

An angle of 90° is a right angle; an angle less than 90° is acute angle; an angle between 90° and 180° is an obtuse angle; and angle between 180° and 360° is a reflex angle.

The sum of all angles made on one side of a straight line AB at a point O by any number of lines joining the line AB at O is 180°. In Fig. 4.01 below, the sum of the angles u, v, x, y and z is equal to 180°.

When any number of straight lines join at a point, the sum of all the angles around that point is 360°. In Fig. 4.02 below, the sum of the angles u, v, w, x, y and z is equal to 360°.

Fig.4.01

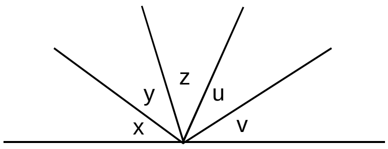
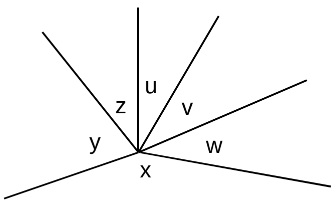


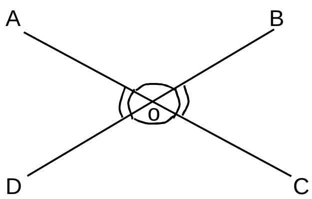
Fig. 4.02



Two angles whose sum is 90° are said to be complementary angles and two angles whose sum is 180° are said to be supplementary angles.

When two straight lines intersect, vertically opposite angles are equal. In Fig. 4.03 given below, ∠AOB and ∠COD are vertically opposite angles and ∠BOC and ∠AOD are vertically opposite angles. So, ∠AOB = ∠COD and ∠BOC = ∠AOD.

Fig. 4.03



Two lines which make an angle of 90° with each other are said to be PERPENDICULAR to each other.

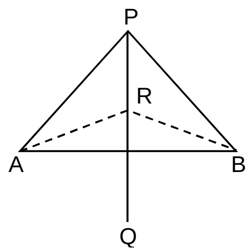
If a line l passes through the mid-point of a line segment AB, then the line l is said to be the BISECTOR of segment AB.

If a line l is drawn passing through the vertex of an angle dividing the angle into two equal parts, then the line l is said to be the ANGLE BISECTOR of the angle. Any point on the angle bisector of an angle is EQUIDISTANT from the two arms of the angle.

If a line l is perpendicular to a line segment and passes through the mid-point of AB, then the line l is said to be the PERPENDICULAR BISECTOR of the segment AB.

Any point on the perpendicular bisector of a line segment is EQUIDISTANT from the two end points of the segment.

Fig. 4.04

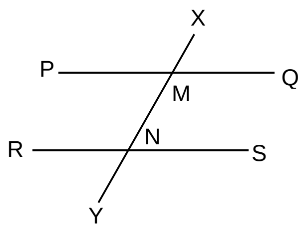


In Fig. 4.04, line PQ is the perpendicular bisector of the line segment AB. Any point R on the perpendicular bisector of AB will be equidistant from A and B, i.e., RA = RB. In particular, PA = PB, QA = QB.

PARALLEL LINES

When a straight line cuts two or more lines at distinct points, then the cutting line is called the TRANSVERSAL. When a straight line XY cuts two parallel lines PQ and RS [as shown in Fig. 4.05], the following relations hold between the various angles that are formed. [M and N are the points of intersection of XY with PQ and RS respectively].

Fig. 4.05



(a) Alternate angles are equal, i.e.
∠PMN = ∠MNS and ∠QMN = ∠MNR

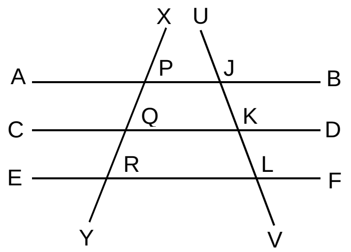
(b) Corresponding angles are equal, i.e.
∠XMQ = ∠MNS; ∠QMN = ∠SNY;
∠XMP = ∠MNR; ∠PMN = ∠RNY

(c) Sum of the interior angles on the same side of the transversal is equal to 180°, i.e.
$\angle QMN + \angle MNS = 180°$ and $\angle PMN + \angle MNR = 180°$

(d) Sum of the exterior angles on the same side of the transversal is equal to 180°, i.e.
$\angle XMQ + \angle SNY = 180°$, and $\angle XMP + \angle RNY = 180°$

If three or more parallel lines make intercepts on a transversal in a certain proportion, then they make intercepts in the same proportion on any other transversal as well. In Fig. 4.06, the lines AB, CD and EF are parallel and the transversal XY cuts them at the points P, Q and R. If we now take a second transversal, UV, cutting the three parallel lines at the points J, K and L, then we have PQ/QR = JK/KL.

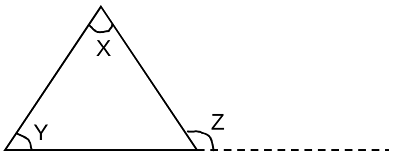
Fig. 4.06



If three or more parallel lines make equal intercepts on one transversal, they make equal intercepts on any other transversal as well.

TRIANGLES

Fig. 4.07



The sum of the three angles of a triangle is 180°

The exterior angle of a triangle at each vertex is equal to the sum of the interior angles at the other two vertices. (Exterior angle is the angle formed at any vertex, by one side and the extended portion of the second side at that vertex).

A line perpendicular to a side and passing through the midpoint of the side is said to be the perpendicular bisector of the side. It is not necessary that the perpendicular bisector of a side should pass through the opposite vertex in a triangle in general.

The perpendicular drawn to a side from the opposite vertex is called the altitude to that side.

The line joining the midpoint of a side with the opposite vertex is called the median drawn to that side. A median divides the triangle into two equal halves as far as the area is concerned.

An equilateral triangle is one in which all the sides are equal (and hence, all angles are equal, i.e., each of the angles is equal to 60°). A triangle in which two si des are equal is called an isosceles triangle. In an isosceles triangle the angles opposite the equal sides are equal. A scalene triangle is one in which no two sides are equal, and hence no two angles are equal.

In an isosceles triangle, the unequal side is called the BASE. The angle where the two equal sides meet is called the VERTICAL ANGLE. In an isosceles triangle, the perpendicular drawn to the base from the vertex opposite the base (i.e., the altitude drawn to the base) bisects the base as well as the vertical angle. That is, the altitude drawn to the base will also be the perpendicular bisector of the base as well as the angle bisector of the vertical angle. It will also be the median drawn to the base.

In an equilateral triangle, the perpendicular bisector, the median and the altitude drawn to a particular side coincide and that will also be the angle bisector of the opposite angle. If a is the side of an equilateral triangle, then its altitude is equal to $\sqrt{3}a/2$

Sum of any two sides of a triangle is greater than the third side. Therefore, the difference of any two sides of a triangle is less than the third side.

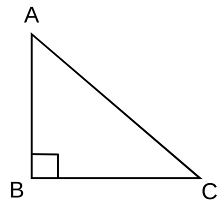
If the sides are arranged in the ascending order of their length, then the angles opposite the sides (in the same order) will also be in ascending order (i.e., greater side has the greater angle opposite to it). Therefore, if the sides are arranged in descending order of their length, the angles opposite the sides (in the same order) will also be in descending order (i.e., smaller angle has smaller side opposite to it).

There can be only one right angle or only one obtuse angle in any triangle. There can also not be one right angle and an obtuse angle both present in the same triangle.

The hypotenuse is the side opposite the right angle in a right-angled triangle. In a right-angled triangle, the hypotenuse is the longest side. In an obtuse angled triangle, the side opposite the obtuse angle is the longest side.

Fig. 4.08

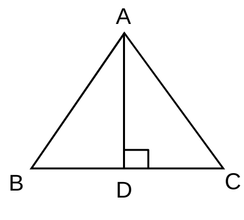
(Right-angled triangle)



In a right-angled triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. In Fig. 4.08, $AC^2 = AB^2 + BC^2$. This result is known as Pythagoras theorem.

Fig. 4.09

(Acute-angled triangle)

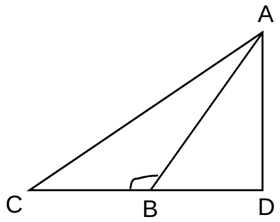


In an acute angled triangle, the square of the side opposite the acute angle is less than the sum of the squares of the other two sides by a quantity equal to twice the product of one of these two sides and the projection of the second side on the first side.

In Fig. 4.09, $AC^2 = AB^2 + BC^2 - 2\,BC.BD$

Fig. 4.10

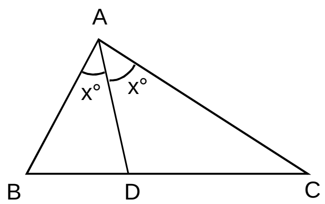
(Obtuse angled triangle)



In an obtuse angled triangle, the square of the side opposite the obtuse angle is greater than the sum of the squares of the other two sides by a quantity equal to twice the product of one of the sides containing the obtuse angle and the projection of the second side on the first side.

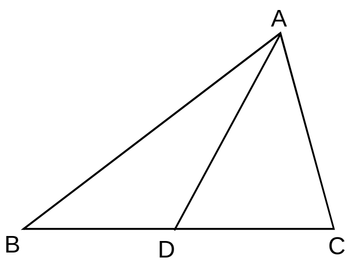
In Fig. 4.10, $AC^2 = AB^2 + BC^2 + 2BC.BD$

Fig. 4.11



In any triangle, the internal bisector of an angle bisects the opposite side in the ratio of the other two sides. In triangle ABC, if AD is the angle bisector of angle A, then BD/DC = AB/AC. This is called the Angle Bisector Theorem (refer to Fig. 4.11).

Fig. 4.12

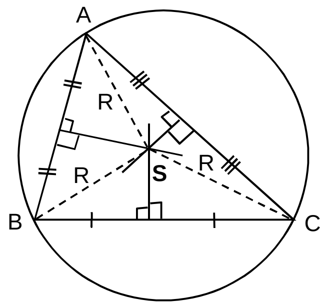


In ΔABC, if AD is the median from A to side BC (meeting BC at its mid point D), then $2(AD^2 + BD^2) = AB^2 + AC^2$. This is called the **Apollonius Theorem**. This is useful in calculating the lengths of the three medians given the lengths of the three sides of the triangle (refer to Fig. 4.12).

GEOMETRIC CENTRES OF A TRIANGLE

CIRCUMCENTRE

Fig 4.13



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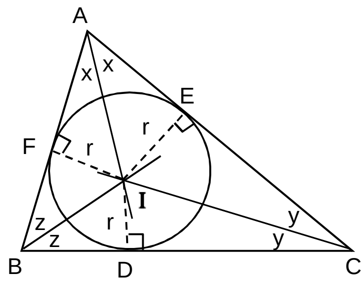
The three perpendicular bisectors of a triangle meet at a point called Circumcentre of the triangle. This is normally denotes as S. The circumcentre of a triangle is equidistant from its vertices and the distance of circumcentre from each of the three vertices is called circumradius (represented by R) of the triangle. The circle drawn with the circumcentre as centre and circumradius as radius is called the cirumcircle of the triangle. It passes through all three vertices of the triangle. (refer to Fig. 4.13)

INCENTRE AND EXCENTRES

Fig. 4.14



The bisectors of the three angles of a triangle meet at a point called incentre of the triangle. This is normally denoted as I. The incentre is equidistant from the three sides of the triangle i.e., the perpendiculars drawn from the incentre to the three sides are equal in length and this length is called the inradius (represented by r) of the triangle. The circle drawn with the incentre as centre and the inradius as radius is called the incircle of the triangle. It touches all three sides of the triangle.

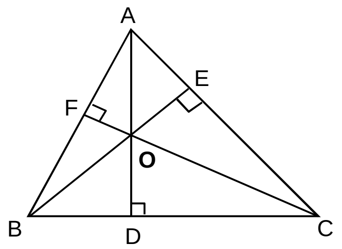
In Fig. 4.14, $\angle BIC = 90° + \frac{1}{2}\angle A$ where I is the incentre.

$\angle CIA = 90° + \frac{1}{2}\angle B$; and $\angle AIB = 90° + \frac{1}{2}\angle C$.

If the bisector of one angle and the bisectors of the external angles at the other two vertices are drawn, they meet at a point called Excentre. There are three excentres for any triangle - one corresponding to the bisector of each angle.

ORTHOCENTRE

Fig 4.15

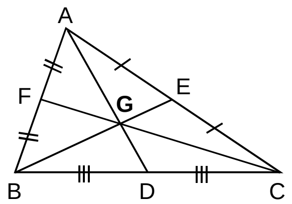


The three altitudes of a triangle meet at a point called Orthocentre. This is normally denoted as O (refer to Fig. 4.15).

$\angle BOC = 180° - A$; $\angle COA = 180° - B$;
$\angle AOB = 180° - C$.

CENTROID

Fig. 4.16



The three medians of a triangle meet at a point called the Centroid. This is normally denoted by G (refer to Fig. 4.16).

Important points about geometric centres of a triangle

Please note the following important points pertaining to the geometric centres of a triangle ABC. In an acute angled triangle, the circumcentre lies inside the triangle. In a right-angled triangle, the circumcentre lies on the hypotenuse of the triangle (it is the midpoint of the hypotenuse). In an obtuse angled triangle, the circumcentre lies outside the triangle.

In an acute angled triangle, the orthocentre lies inside the triangle. In a right-angled triangle, the vertex where the right angle is formed (i.e., the vertex opposite the hypotenuse) is the orthocentre. In an obtuse angled triangle, the orthocentre lies outside the triangle.

In a right-angled triangle the length of the median drawn to the hypotenuse is equal to half the hypotenuse. This median is also the circumradius of the right-angled triangle.

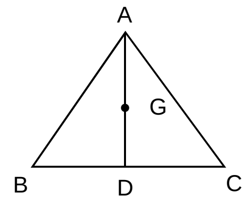
The centroid divides each of the medians in the ratio 2 : 1, the part of the median towards the vertex being twice in length to the part towards the side.

The inradius is less than half the smallest altitude of the triangle.

In an isosceles triangle, the centroid, the orthocentre, the circumcentre and the incentre, all lie on the median to the base.

In an equilateral triangle, the centroid, the orthocentre, the circumcentre and the incentre, all coincide.

Fig. 4.17



Hence, in equilateral triangle ABC shown in Fig. 4.17, AD is the median, altitude, angle bisector and perpendicular bisector. G is the centroid which divides the median in the ratio of 2 : 1. Hence, AG = 2/3 AD and GD = 1/3 AD.

But since AD is also the perpendicular bisector and angle bisector and since G is the circumcentre as well as the incentre, AG is the circumradius and GD is the inradius of the equilateral triangle ABC. Since AD is also the altitude, its length is equal to $\sqrt{3}a/2$ where a is the side of the equilateral triangle. Hence, the circumradius of the equilateral triangle,

$$R = \frac{2}{3}\left(\frac{\sqrt{3}}{2}\right).\ a = a/\sqrt{3}$$

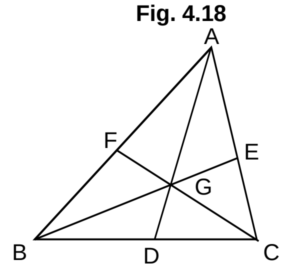
and the inradius (r) = $\frac{1}{3}\left(\frac{\sqrt{3}}{2}.a\right)$

$= a/2\sqrt{3}$

Since the radii of the circumcircle and the incircle of an equilateral triangle are in the ratio 2 : 1, the areas of the circumcircle and the incircle of an equilateral triangle will be in the ratio of 4 : 1.

When the three medians of a triangle (i.e., the medians to the three sides of a triangle from the corresponding opposite vertices) are drawn, the resulting six triangles are equal in area and hence the area of each of these triangles is equal to one-sixth of the area of the original triangle.

Fig. 4.18

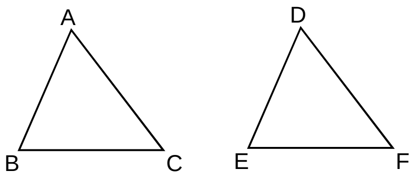


In Fig 4.18, AD, BE and CF are the medians drawn to the three sides. The three medians meet at the centroid G. The six resulting triangles AGF, BGF, BGD, CGD, CGE and AGE are equal in area and each of them is equal to $1/6^{th}$ of the area of triangle ABC.

SIMILARITY OF TRIANGLES

Two triangles are said to be similar if the three angles of one triangle are equal to the three angles of the second triangle. Similar triangles are alike in shape. The corresponding angles of two similar triangles are equal but the corresponding sides are only proportional and not necessarily equal.

Fig. 4.19



For example, in Fig 4.19, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. Therefore ΔABC is similar to ΔDEF. We write $\Delta ABC \sim \Delta DEF$. (Note: We should not write any of the following $\Delta ABC \sim DFE$, $\Delta ABC \sim \Delta$ EFD. $\Delta ABC \sim \Delta EDF$, $\Delta ABC \sim \Delta FDE$ or $\Delta ABC \sim \Delta FED$. The relation should indicate which pair of angles are equal.) The ratios of the corresponding sides are equal.

i.e. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

By "corresponding sides", we mean that if we take a side opposite to a particular angle in one triangle, we should consider the side opposite to the equal angle in the second triangle. In this case, since AB is the side opposite to $\angle C$ in $\angle ABC$, and since $\angle C = \angle F$, we have taken DE which is the side opposite to $\angle F$ in ΔDEF.

Two triangles are similar if,

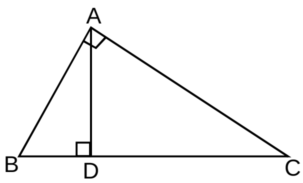
- the three angles of one triangle are respectively equal to the three angles of the second triangle, or
- two sides of one triangle are proportional to two sides of the other and the included angles are equal, or
- if the three sides of one triangle are proportional to the three sides of another triangle.

In two similar triangles,

(a) Ratio of corresponding sides = Ratio of heights (altitudes) = Ratio of the lengths of the medians = Ratio of the lengths of the angle bisectors = Ratio of inradii = Ratio of circumradii = Ratio of perimeters.

(b) Ratio of areas = Ratio of squares of corresponding sides

Fig. 4.20



In a right-angled triangle, the altitude drawn to the hypotenuse divides the given triangle into two similar triangles, each of which is in turn similar to the original triangle. In triangle ABC in Fig. 4.20, ABC is a right-angled triangle where $\angle A$ is a right angle. AD is the perpendicular drawn to the hypotenuse BC. The triangles ABD, CAD and CBA are similar because of the equal angles given below.

In triangle ABC, $\angle A = 90°$. If $\angle B = \theta$, then $\angle C = 90° - \theta$.
In triangle ABD, $\angle ADB = 90°$. We already know that $\angle B = \theta$; hence $\angle BAD = 90° - \theta$.
In triangle ADC, $\angle ADC = 90°$. We already know that $\angle C = 90° - \theta$; hence $\angle CAD = \theta$.
We can write down the relationship between the sides in these three triangles. The important relationships that emerge out of this exercise are :

1. $AD^2 = BD \cdot DC$;
2. $AB^2 = BC \cdot BD$;
3. $AC^2 = CB \cdot CD$.

CONGRUENCE OF TRIANGLES

Two triangles will be congruent if at least one of the following conditions is satisfied:

– Three sides of one triangle are respectively equal to the three sides of the second triangle (normally referred to as the S-S-S rule, i.e., the side-side-side congruence).

– Two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the second triangle (normally referred to as the S-A-S rule, i.e., side-angle-side congruence).

– Two angles and the included side of a triangle are respectively equal to two angles and the included side of the second triangle (normally referred to as the A-S-A rule, i.e., angle-side-angle congruence). (Note: Hence AAS is also a sufficient condition for congruence.)

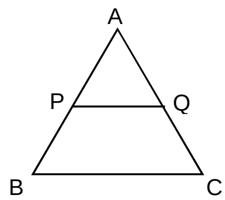
– Two right-angled triangles are congruent if the hypotenuse and one side of one triangle are respectively equal to hypotenuse and one side of the second triangle.

In two congruent triangles,

- the corresponding sides (i.e., sides opposite to corresponding angles) are equal.
- the corresponding angles (angles opposite to corresponding sides) are equal.
- the areas of the two triangles will be equal.

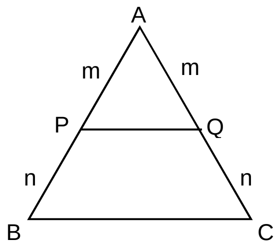
Some more useful points about triangles

Fig. 4.21



A line drawn parallel to one side of a triangle divides the other two sides in the same proportion. For example, in Fig. 4.21, PQ is drawn parallel to BC in ΔABC. This will divide the other two sides AB and AC in the same ratio, i.e., AP/PB = AQ/QC.

Fig. 4.22

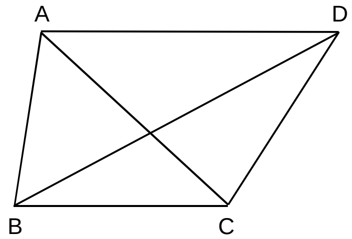


Conversely, a line joining two points lying on different sides of a triangle, dividing the respective sides in the same ratio, is parallel to the third side. In Fig. 4.22, P divides AB in the ratio m : n and Q divides AC in the ratio m : n. Now, the line joining P and Q is parallel to the third side BC. Also, the length of PQ will be equal to $\frac{m}{m+n}$ times the length of BC.

We can say that a line drawn through a point on a side of the triangle parallel to a second side will cut the third side in the same ratio in which the first side is divided.

The line joining the midpoints of two sides of a triangle is parallel to the third side and it is half the third side.

Fig. 4.23



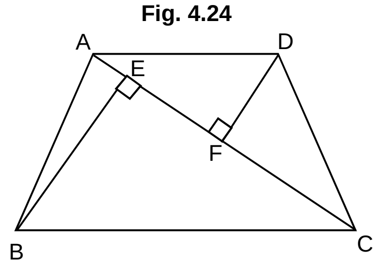
Two triangles having the same base and lying between the same pair of parallel lines have their areas equal (Fig. 4.23).

AD is parallel to BC. Hence, area of ΔABC = area of ΔDBC

QUADRILATERALS

Any four-sided closed figure is called a quadrilateral. By imposing certain conditions on the sides and/or angles of a quadrilateral, we can get the figures trapezium, parallelogram, rhombus, rectangle, square.

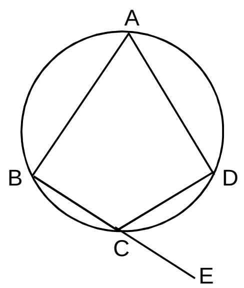
Fig. 4.24



The sum of the four angles of a quadrilateral is equal to 360°. The perpendiculars drawn to a diagonal (in a quadrilateral) from the opposite vertices are called "offsets". In Fig. 4.24, BE and DF are the offsets drawn to the diagonal AC.

If the four vertices of a quadrilateral lie on the circumference of a circle (i.e., if the quadrilateral can be inscribed in a circle) it is called a cyclic quadrilateral (refer to Fig. 4.25). In a cyclic quadrilateral, the sum of opposite angles = 180° i.e., in Fig. 4.25, A + C = 180° and B + D = 180°.

Fig. 4.25

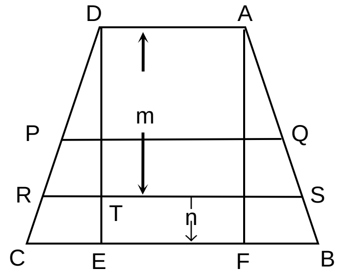


Also, in a cyclic quadrilateral, any exterior angle is equal to the interior angle at the opposite vertex i.e., in Fig. 4.25, $\angle$DCE is equal to $\angle$BAD.

Now, we will look at different quadrilaterals and their properties.

TRAPEZIUM

Fig. 4.26



If one side of a quadrilateral is parallel to the opposite side, then it is called a trapezium. The two sides other than the parallel sides in a trapezium are called the oblique sides.

In Fig. 4.26, ABCD is a trapezium, in which AD is parallel to BC.

If the midpoints of the two oblique sides are joined, it is equal in length to the average of the two parallel sides, i.e., in Fig. 4.26, PQ = 1/2 [AD + BC]

In general, if a line is drawn in between the two parallel sides of the trapezium such that it is parallel to the parallel sides and also divides the distance between the two parallel sides in the ratio m : n (where the portion closer to the shorter of the two parallel sides is m), the length of the line is given by :

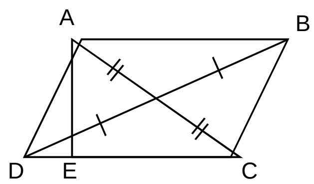
$\frac{m}{m+n}$ (Longer side) + $\frac{n}{m+n}$ (Shorter side), where shorter side and longer side refer to the shorter and longer of the two parallel sides of the trapezium.

In Fig. 4.26, RS is the line parallel to AD and BC and the ratio of the distances DT and TE is m : n.

The length of RS is given by $\frac{mBC}{m+n} + \frac{nAD}{m+n}$

PARALLELOGRAM

Fig. 4.27



A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram.
In a parallelogram

- both pairs of opposite sides are equal
- both pairs of opposite angles are equal
- Sum of any two adjacent angles is 180°.
- Each diagonal divides the parallelogram into two congruent triangles.
- The diagonals bisect each other.

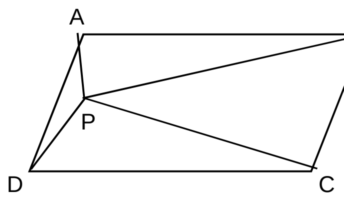
Conversely, if in a quadrilateral, if any one of the following conditions holds, then it is a parallelogram.

- both pairs of opposite sides are equal
- both pairs of the opposite angles are equal
- the diagonals bisect each other
- a pair of opposite sides are parallel and equal

If two adjacent angles of a parallelogram are equal, then all four angles will be equal and each in turn will be equal to 90°. Then the figure will be a rectangle.

If any two adjacent sides of a parallelogram are equal, then all four sides are equal to each other and the figure is a rhombus.

Fig. 4.28



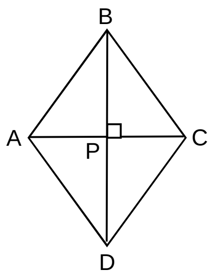
If any point inside a parallelogram is taken and is joined to all the four vertices the four resulting triangles will be such that the sum of the areas of opposite triangles is equal. In Fig. 4.28, P is a point inside the parallelogram ABCD and it is joined to the four vertices of the parallelogram by the lines PA, PB, PC and PD respectively. Then Area of triangle PAB + Area of triangle PCD = Area of triangle PBC + Area of triangle PAD = Half the area of parallelogram ABCD.

If there is a parallelogram and a triangle with the same base and between the same parallel lines, then the area of the triangle will be half that of the parallelogram.

If there is a parallelogram and a rectangle with the same base and between the same parallel lines, then the areas of the parallelogram and the rectangle will be the same. The figure formed by joining the midpoints of the sides of any quadrilateral taken in order, is a parallelogram.

RHOMBUS

Fig. 4.29



A rhombus is a parallelogram in which all sides are equal. (Note: If in a parallelogram, one pair of adjacent sides are equal, it is a rhombus).

Since a rhombus is a parallelogram, all the properties of a parallelogram apply to a rhombus. Further, in a rhombus, the diagonals bisect each other **perpendicularly**.

Conversely, any quadrilateral where the two diagonals bisect each other at right angles is a rhombus.

The four triangles that are formed by the two bisecting diagonals with the four sides of the rhombus are all congruent. In Fig. 4.29, the four triangles PAB, PBC, PCD and PAD are congruent.

Side of a rhombus

$= \sqrt{1/4\,(\text{Sum of squares of the diagonals})}$

RECTANGLE

A rectangle is also a special type of parallelogram and hence all properties of a parallelogram apply to rectangles also. A rectangle is a parallelogram in which two adjacent angles are equal. Therefore, each of the angles is equal to 90°.

The diagonals of a rectangle are **equal** (and, of course, bisect each other).

When a rectangle is inscribed in a circle, the diagonals become diameters of the circle.

If a and b are the two adjacent sides of a rectangle, then the diagonal is given by $\sqrt{a^2+b^2}$.

If a rectangle and a triangle are on the same base and between the same parallels, then the area of the triangle will be equal to half the area of the rectangle.

SQUARE

A square is a rectangle in which all four sides are equal (or a rhombus in which all four angles are equal, i.e., all are right angles) Hence, the diagonals are equal and they bisect at right angles. So, all the properties of a rectangle as well as those of a rhombus hold good for a square.

Diagonal = $\sqrt{2} \times$ Side

When a square is inscribed in a circle, the diagonals become the diameters of the circle.

When a circle is inscribed in a square, the side of the square is equal to the diameter of the circle.

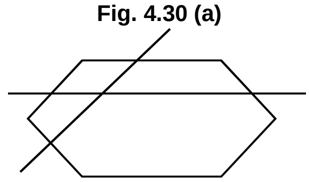
The largest rectangle that can be inscribed in a given circle will be a square.

POLYGON

Any closed figure with three or more sides is called a polygon.

A polygon, which is such that for the line say ℓ containing any one side, all the other sides are on the same side of ℓ is called a convex polygon. It there is at least one side for which the other sides lie on both sides of ℓ, the polygon is said to be a concave polygon. A convex polygon is one in which each of the interior angles is less than 180°. It can be noticed that any straight line drawn cutting a convex polygon passes through only two sides of the polygon, as shown in the figure below.

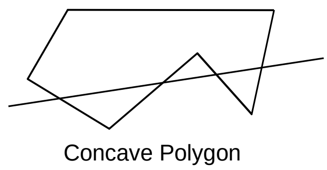
Fig. 4.30 (a)



Convex Polygon

In a concave polygon, it is possible to draw lines passing through more than two sides, as shown in the figure below.

Fig. 4.30 (b)



Concave Polygon

A regular polygon is a convex polygon in which all sides are equal and all angles are equal. A regular polygon can be inscribed in a circle. The centre of the circumscribing circle (the circle in which the polygon is inscribed) of a regular polygon is called the centre of the polygon.

The names of polygons with three, four, five, six, seven, eight, nine and ten sides are respectively triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon and decagon.

The sum of interior angles of a convex polygon is equal to (2n – 4) right angles where n is the number of the sides of the polygon.

If each of the sides of a convex polygon is extended, the sum of the external angles thus formed is equal to 4 right Angles (i.e., 360°).

In a regular polygon of n sides, if each of the interior angles is d°, then d = $\frac{2n-4}{n}(90^\circ)$ and each exterior angle = $\frac{360^\circ}{n}$.

It will be helpful to remember the interior and exterior angles of the following regular polygons:

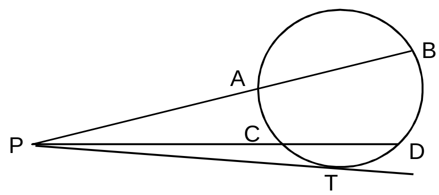
	Interior	Exterior
Regular pentagon	108°	72°
Regular hexagon	120°	60°
Regular octagon	135°	45°

If the centre of a regular polygon (with n sides) is joined with each of the vertices, we get n identical triangles inside the polygon. In general, all these triangles are isosceles triangles. Only in case of a regular hexagon, all these triangles are equilateral triangles, i.e., in a regular hexagon, the radius of the circumscribing circle is equal to the side of the hexagon.

A line joining any two non-adjacent vertices of a polygon is called a diagonal. A polygon with n sides will have $\frac{n(n-3)}{2}$ diagonals.

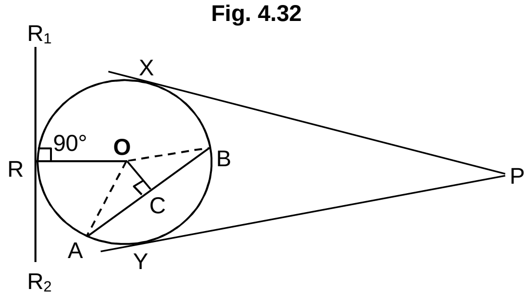
CIRCLES

Fig. 4.31



A circle is a closed curve drawn such that any point on the curve is equidistant from a fixed point. The fixed point is called the centre of the circle and the distance from the centre to any point on the circle is called the radius of the circle.

Fig. 4.32



A diameter of a circle is a straight line passing through the centre of the circle and joining two points on the circle. A circle is symmetric about any diameter.

A chord (e.g. AB in Fig. 4.32) is a point joining any two points on the circumference of a circle. A diameter is the longest chord in a circle.

A secant is a line intersecting a circle in two distinct points and extending outside the circle also.

A line that touches the circle at only one point is a tangent to the circle (R_1R_2 is a tangent touching the circle at the point R in Fig. 4.32).

If PAB and PCD are two secants (in Fig. 4.31), then PA.PB = PC.PD

If PAB and PCD are secants and PT is a tangent to the circle at T (in Fig. 4.31), then PA.PB = PC.PD = PT^2.

Two tangents can be drawn to the circle from any point outside the circle and these two tangents are equal in length. In Fig. 4.32, P is the external point and the two tangents PX and PY are equal.

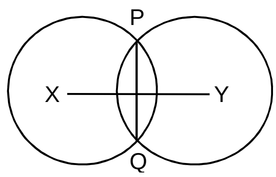
A tangent is perpendicular to the radius drawn at the point of tangency (In Fig. 4.32, $R_1R_2 \perp OR$). Conversely, if a perpendicular is drawn to the tangent at the point of tangency, it passes through the centre of the circle.

A perpendicular drawn from the centre of the circle to a chord bisects the chord (In Fig. 4.32, OC, the perpendicular from O to the chord AB bisects AB) and conversely, the perpendicular bisector of a chord passes through the centre of the circle.

Two chords that are equal in length will be equidistant from the centre, and conversely two chords which are equidistant from the centre of the circle will be of the same length.

One and only one circle passes through any three given non-collinear points.

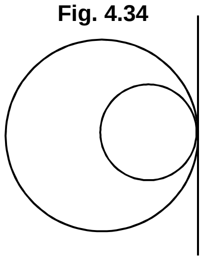
Fig. 4.33



When there are two intersecting circles, the line joining the centres of the two circles will perpendicularly bisect the line segment joining the points of intersection. In Fig. 4.33, the two circles with centres X and Y respectively intersect at the two points P and Q. The line XY (the line joining the centres) bisects PQ (the line segment joining the two points of intersection).

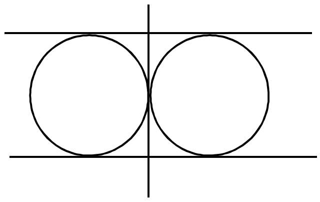
Two circles are said to touch each other if a common tangent can be drawn touching both the circles at the same point. This is called the point of contact of the two circles. The two circles may touch each other internally (as in Fig. 4.34) or externally (As in Fig. 4.35). When two circles touch each other, then the point of contact and the centres of the two circles are collinear, i.e., the point of contact lies on the line joining the centres of the two circles.

Fig. 4.34



If two circles touch internally, the distance between the centres of the two circles is equal to the difference in the radii of the two circles. When two circles touch each other externally, then the distance between the centres of the two circles is equal to the sum of the radii of the two circles.

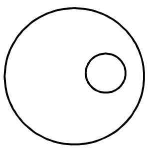
Fig. 4.35



A line which is tangent to two circles is called a common tangent. In general, for two circles, there can be anywhere from zero to four common tangents drawn depending on the position of one circle in relation to the other.

A common tangent which cuts the line joining the centres of the two circles, not between the centres but rather beyond either centre, or if the centres of the two circles lie on the same side of the common tangent, the tangent is called a direct common tangent. Instead, if they lie on opposite sides, it is called a transverse common tangent.

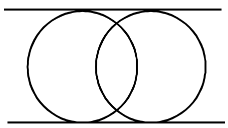
Fig. 4.36



If two circles are such that one lies completely inside the other (without touching it), then there will not be any common tangent to these circles (refer to Fig. 4.36).

Two circles touch each other internally (i.e., one circle lies inside the other), then there is only one common tangent possible and it is drawn at the point of contact of the two circles (refer to Fig. 4.34).

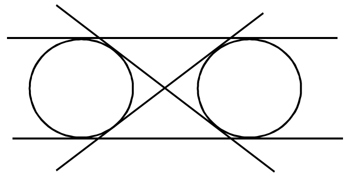
Fig. 4.37



Two intersecting circles have two common tangents. Both these are direct common tangents and the two intersecting circles do not have a transverse common tangent (refer to Fig. 4.37).

Two circles touching each other externally have three common tangents. Out of these, two are direct common tangents and one is a transverse common tangent. The transverse common tangent is at the point of contact (Refer to Fig. 4.35).

Fig. 4.38



Two circles which are non-intersecting and non-enclosing (i.e. one does not lie inside the other) have four common tangents - two direct and two transverse common tangents (Refer to Fig. 4.38).

If r_1 and r_2 are the radii of the two non-intersecting non-enclosing circles,

Length of the direct common tangent

$= \sqrt{(\text{Distance between centre})^2 - (r_1 - r_2)^2}$

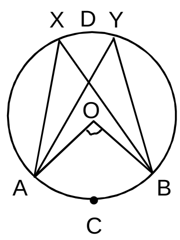
Length of transverse common tangent

$= \sqrt{(\text{Distance between centre})^2 - (r_1 + r_2)^2}$

Two circles are said to be concentric if they have the same centre. For two concentric circles, the circle with the smaller radius lies completely within the circle with the bigger radius.

ARCS AND SECTORS

Fig. 4.39

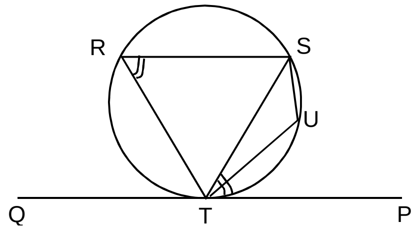


An arc is a connected part of a circle. In Fig. 4.39, ACB is called minor arc and ADB is called major arc. In general, if we talk of an arc AB, we refer to the minor arc. AOB is called the angle formed by the arc AB (at the centre of the circle).

The angle subtended by an arc at the centre is double the angle subtended by the arc at any point X (or Y) on the remaining part of the circle.

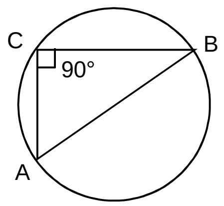
In Fig. 4.39, $\angle AOB = 2\angle AXB = 2\angle AYB$
Angles in the same segment are equal. In Fig. 4.39,
$\angle AXB = \angle AYB$.

Fig. 4.40



The angle between a tangent and a chord through the point of contact of the tangent is equal to the angle made by the chord in the alternate segment (i.e., segment of the circle on the side other than the side of location of the angle between the tangent and the chord). This is normally referred to as the "alternate segment theorem." In Fig. 4.40, PQ is a tangent to the circle at the point T and TS is a chord drawn at the point of contact. Considering $\angle PTS$ which is the angle between the tangent and the chord, the angle TRS is the angle in the "alternate segment". So, $\angle PTS = \angle TRS$.
Similarly, $\angle QTS = \angle TUS$.

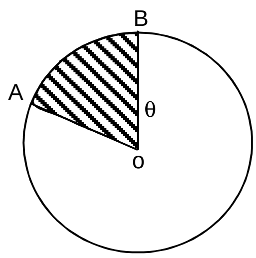
Fig. 4.41



We have already seen in quadrilaterals, that the opposite angles of a cyclic quadrilateral are supplementary and that the external angle of a cyclic quadrilateral is equal to the interior opposite angle.

The angle in a semicircle (or the angle the diameter subtends in a semicircle) is a right angle. The converse of the above is also true and is very useful in a number of cases - in a right angled triangle, a semi-circle with the hypotenuse as the diameter can be drawn passing through the third vertex (Refer to Fig. 4.41).

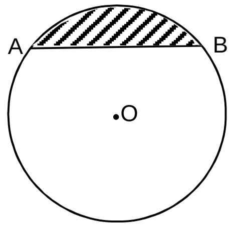
Fig. 4.42



The region bounded by an arc and the two radii at the two end points of the arc is called sector.

In Fig. 4.42, the shaded figure AOB is called the minor sector. The rest of the circle is the major sector.

Fig. 4.43



The region bounded by an arc and the line segment joining the endpoints of the arc is called a segment of a circle (or circular region). In fig 4.43, the shaded region is a minor segment. The rest of the circular region is a major segment.

AREAS OF PLANE FIGURES

Mensuration is the branch of geometry that deals with the measurement of length, area and volume. We have looked at properties of plane figures till now. Here, in addition to areas of plane figures, we will also look at surface areas and volumes of "solids." Solids are objects, which have three dimensions (plane figures have only two dimensions).

Let us briefly look at the formulae for areas of various plane figures and surface areas and volumes of various solids.

TRIANGLES

The area of a triangle is represented by the symbol Δ. For any triangle, the three sides are represented by a, b and c and the angles opposite these sides represented by A, B and C respectively.

(i) For any triangle in general,

(a) When the lengths of three sides a, b, c are given,
Area = $\sqrt{s(s-a)(s-b)(s-c)}$, where
$s = \frac{a+b+c}{2}$
This is called Hero's formula.

(b) When base (b) and altitude (height) to that base are given,
Area = ½ (base)(altitude) = ½ bh

(c) Area = ½ absinC = ½ bcsinA = ½ casinB

(d) Area = $\frac{abc}{4R}$, where R is the circumradius of the triangle.

(e) Area = rs, where r is the inradius of the triangle and s, the semi-perimeter.

Out of these five formulae, the first and the second are the most commonly used and are also more important from the examination point of view.

(ii) For a right angled triangle,
Area = (½) Product of the sides containing the right angle

(iii) For an equilateral triangle
Area = $\frac{\sqrt{3}a^2}{4}$, where a is the side of the triangle
The height of an equilateral triangle $= \frac{\sqrt{3}a}{2}$.

(iv) For an isosceles triangle
Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$, where a is length of either of the two equal sides and b is the third side.

QUADRILATERALS

(i) For any quadrilateral
Area of the quadrilateral = ½ (One diagonal) (Sum of the offsets drawn to that diagonal)

For example, for the quadrilateral ABCD shown in Fig. 4.24, area of quadrilateral ABCD = ½ x AC x (BE + DF)

(ii) For a cyclic quadrilateral where the lengths of the four sides are a, b, c and d,
Area = $\sqrt{(s - a)\ (s - b)\ (s - c)\ (s - d)}$, where s is the semi-perimeter, i.e., s = (a + b + c + d)/2

(iii) For a trapezium
Area = ½(Sum of parallel sides) Distance between them
= (½) (AD + BC) (AF) (refer to Fig. 4.26)

(iv) For a parallelogram
(a) Area = Base (Height)
(b) Area = Product of two sides (Sine of included angle)

(v) For a rhombus
Area = ½ (Product of the diagonals)
Perimeter = 4 x Side of the rhombus

(vi) For a rectangle
Area = Length (Breadth)
Perimeter = 2(ℓ + b), where ℓ and b are the length and the breadth of the rectangle respectively

(vii) For a square
(a) Area = Side^2
(b) Area = (½)(Diagonal^2)
[diagonal = $\sqrt{2}$ (side),
Perimeter = 4(Side)]

(viii) For a polygon
(a) Area of a regular polygon = ½ (Perimeter) (Perpendicular distance from the centre of the polygon to any side)
(The centre of a regular polygon is equidistant from all its sides)
(b) For a polygon which is not regular, the area has to be found out by dividing the polygon into suitable number of quadrilaterals and triangles and adding up the areas of all such figures present in the polygon.

CIRCLE

(i) Area of the circle = πr^2, where r is the radius of the circle
Circumference = $2\pi r$

(ii) Sector of a circle
Length of arc = $\frac{\theta}{360^\circ}(2\pi r)$
Area = $\frac{\theta}{3\quad^\circ}$ (πr^2), where θ is the angle of the sector in degrees and r is the radius of the circle.
Area = (1/2)lr; l is length of arc and r is radius.

(iii) Ring : Ring is the space enclosed by two concentric circles.

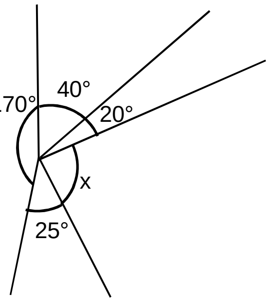
Area = $\pi R^2 - \pi r^2 = \pi(R + r)\ (R - r)$, where R is the radius of the outer circle and r is the radius of the inner circle.

ELLIPSE

Area = π ab where "a" is semi-major axis and "b" is semi-minor axis.
Perimeter = π (a + b)

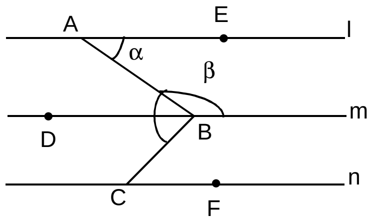
Examples:

4.01. Calculate the value of x in the figure.



Sol: The angles at a point add up to 360°.
Hence (20° + 40° + 25° + 170°) + x = 360°
∴ x = 360° − 255° = 105°

4.02. If l, m and n are lines parallel to each other, calculate the measures of the angles α and β in the given figure. Given that angle ABC is 100° and angle DBC is 75°. Also A, E; D, B; and C, F are points on l, m and n respectively.



Sol: As ∠DBC = 75°
∠DBA = 100° −∠DBC = 100° − 75° = 25°
α = ∠ DBA = 25°
(As α and DBA are alternate angles, they are equal)
As interior angles on the same side of the transversal are supplementary,
α + β = 180° ⇒ ∴β = 180° − 25 = 155°

4.03. How many degrees are there in an angle which is equal to one-fourth of its supplement?

Sol: Let the angle be x. Then its supplement is 180°– x.
Given, x = $\frac{1}{4}$ (180° – x) ⇒ 5x = 180° ⇒ x = 36°.

4.04. How many degrees are there in an angle which is equal to one-fifth of its complement?

Sol: Let the angle be x. Then its complement is 90° – x.
Given that, x = $\frac{1}{5}$ (90° – x)
⇒ 6x = 90° ⇒ x = 15°

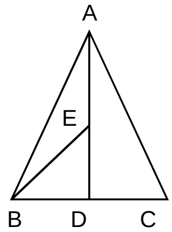
4.05. In a right-angled triangle ABC, find the length of the altitude BD drawn from B to the hypotenuse AC, given AB = 9 cm and BC = 12 cm.

Sol: Since the sides containing the right angle are 9 cm and 12 cm, the area of the triangle = $\frac{1}{2}(9)(12)$ = 54 cm^2.
As AB = 9 cm, BC = 12 cm, AC will be 15 cm. Considering AC as the base and BD as the altitude, the area of ΔABC

$= \frac{1}{2}(AC)(BD) = \frac{1}{2}(15)(BD)\text{ cm} = 54\text{ cm}^2$

$BD = \frac{54(2)}{15}\frac{\text{cm}^2}{\text{cm}} = \frac{108}{15}\text{ cm} = 7.2\text{ cm.}$

4.06. In the figure given below, ABC is a triangle in which D is the midpoint of the side BC and E is the midpoint of AD. What is the ratio of the areas of ΔBED and ΔABC?



Sol: Since AE = ED, BE is a median on AD and the median divides ΔABD into halves, such that area of ΔBED = Area of ΔAEB = $\frac{1}{2}$ (Area of ΔABD).

Since AD is the median, it divides ΔABC into halves such that area of ΔABD = area of ΔADC = $\frac{1}{2}$ (Area of ΔABC).

Hence the area of ΔBED: Area of ΔABC

$= \frac{1}{2}\left(\frac{1}{2}(\text{Area of }\Delta ABC)\right)$: Area of ΔABC

$= \frac{1}{4} : 1 = 1 : 4.$

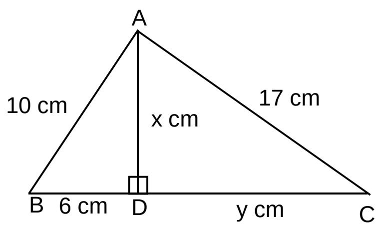
4.07. Triangles ABC and DEF are similar. If $\angle A = \angle D$, $\angle C = \angle F$, BC = 3.4 cm, DE = 4.8 cm and AB = 1.6 cm, then find the length of EF.

Sol: Given that triangles ABC and DEF are similar. Hence the corresponding sides are proportional.

$\therefore$ We have, $\frac{AB}{DE} = \frac{BC}{EF}$

Hence $EF = \frac{(BC)(DE)}{(AB)} = \frac{(3.4)(4.8)}{(1.6)}\ \frac{\text{cm}^2}{\text{cm}} = 10.2\text{ cm.}$

4.08. In the figure given below, AD is perpendicular to BC in triangle ABC. Find x and y.



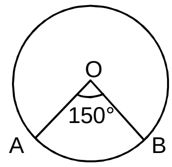
Sol: From ΔABD, $x^2 = AB^2 - BD^2$
(By Pythagoras theorem)
$x^2 = 10^2 - 6^2 = 64 \Rightarrow x = 8.$
From ΔADC $y^2 = AC^2 - x^2$
(By Phythagoras theorem)
$y = \sqrt{17^2 - 8^2} = \sqrt{225} = 15$

4.09. A and B leave a point at the same time. A travels North at 18 km/hr and B travels West at 24 km/hr. Find the distance between A and B after two hours.

Sol: Let the starting point be O. After 2 hours, let A be at X, which will be $2 \times 18 = 36$ km from O and B be at Y, which will be $2 \times 24 = 48$ km from O in the direction perpendicular to OX. Using Pythagoras theorem we can find the third side XY. $XY = \sqrt{36^2 + 48^2} = 60\text{ km}$. Hence A and B will be 60 km away from each other.

4.10. In the figure given below, if the length of major arc AB = 154 cm, find the length of the radius OA and find the area of the minor sector AOB.

Sol:



Angle of the major arc = 360° − 150° = 210°

Length of the major arc = $\frac{\text{Angle of the arc}}{360^\circ}(2\pi r)$

$\therefore \frac{210^\circ}{360^\circ}(2)\left(\frac{22}{7}\right)(OA) = 154\text{ cm} \Rightarrow OA = 42\text{ cm.}$

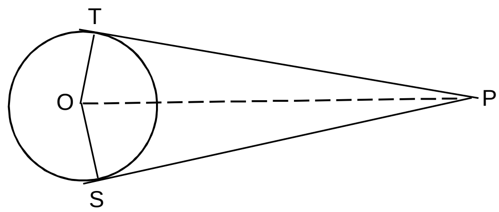
Area of the minor sector

$= \frac{\text{Angle of the sec tor}}{360^\circ} \times \pi r^2$

$= \frac{150}{360}\left(\frac{22}{7}\right)(42^2)\text{ cm}^2 = 2310\text{ cm}^2.$

4.11. In a circle with centre O, PT and PS are tangents drawn to it from point P. If PT = 24 cm and OT = 10 cm, then find the length of PO.

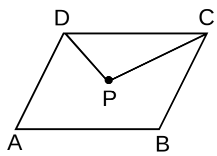
Sol:



From the above figure
$\angle OTP = 90^\circ$, (Radius makes an angle of 90°with the tangent at the point of tangency).
In triangle OTP,
$PO^2 = PT^2 + OT^2 = 24^2 + 10^2\text{ cm}^2$
$PO = \sqrt{24^2 + 10^2}\text{ cm} = 26\text{ cm}$

4.12. In parallelogram ABCD, DP and CP are the angle bisectors of $\angle$CDA and $\angle$DCB respectively. Find $\angle$CPD.

Sol:



Given DP is the angle bisector of $\angle$CDA

$\Rightarrow \angle CDP = \frac{1}{2} \angle CDA$ ----- (1)

Given CP is the angle bisector of $\angle DCB$

$\Rightarrow \angle DCP = \frac{1}{2} \angle DCB$ ---- (2).

In a parallelogram the sum of adjacent angles = 180°.
Therefore, $\angle CDA + \angle DCB = 180°$.
From (1) and (2),
$2\angle CDP + 2\angle DCP = 180°$
$\Rightarrow \angle CDP + \angle DCP = 90°$
In triangle $\angle PDC$,
$\angle CPD = 180° - (\angle PDC + \angle PCD)$
$= 180° - 90° = 90°$

4.13. The number of sides of a regular polygon is 18. Find the interior angle of the polygon.

Sol: Exterior angle of a regular polygon
$= 360°/n = 360°/18 = 20°$.
The sum of the interior angle and the exterior angle is 180°.
Interior angle of this polygon = 180° – 20°
= 160°.

4.14. For a polygon of 12 sides, find the sum of the interior angles of the polygon.

Sol: Sum of the interior angles of a polygon.
$= (2n - 4) \times 90°$ where n is the number of sides.
$\therefore$ Sum of the interior angles of this polygon
$= [2(12) - 4]\ 90° = 1800°$.

4.15. If one angle of a pentagon is 40º and all the other angles are equal, find the measure of each of the other angles.

Sol: Sum of the interior angles of a pentagon.
$= [(2\,(5) - 4)]\ 90° = 540°$
One angle is 40°, the sum of the remaining four equal angles
$= 540° - 40° = 500°$

Each of the other four angles $= \frac{500°}{4} = 125°$.

Concept Review Questions

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the circumradius of a triangle whose sides are 6 cm, 8 cm and 10 cm. (in cm)

2. The sides of a triangle are 12 cm, 16 cm and 20 cm. Find the sum of the distances from the orthocentre to the vertices of the triangle. (in cm)

3. In a triangle, the smallest altitude is 18 cm. The inradius is always

(A) > 9 cm (B) 9 cm
(C) < 9 cm (D) Either (A) or (B)

4. In an equilateral triangle, the orthocentre divides each median into 2 parts. The larger part and the smaller part are in the ratio of

(A) 3 : 2
(B) 3 : 1
(C) 2 : 1
(D) None of these

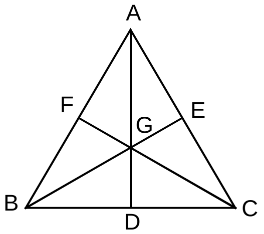
5. An equilateral triangle has a side of 18 cm. Find the inradius of the triangle (in cm).

(A) $3\sqrt{3}$ (B) $4\sqrt{3}$ (C) $6\sqrt{3}$ (D) $8\sqrt{3}$

6. In the above question, find the cirumradius of the triangle (in cm).

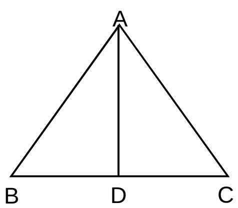
(A) $3\sqrt{3}$ (B) $4\sqrt{3}$ (C) $6\sqrt{3}$ (D) $8\sqrt{3}$

7.



In the figure, AD, BE and CF are the medians and G is the centroid. If the area of triangle ABC is 24 cm^2, find the area of triangle GFB (in cm^2).

8.



In the figure, AB = 10 cm, AC = 12 cm and DC = 8 cm. If AD bisects ∠BAC, find BD (in cm).

(A) 5 (B) $5\frac{2}{3}$ (C) $6\frac{1}{3}$ (D) $6\frac{2}{3}$

9. In triangle PQR, if ∠P = 50°, find ∠QIR, where I is the incentre of the triangle. (in degrees)

10. In ΔABC, $AB^2 + AC^2 < BC^2$ and $\angle BAC = x°$, then which of the following is true?

(A) x < 90° (B) x = 90°
(C) x > 90° (D) None of these

11. The sides of a triangle are 4 cm, 6 cm and 8 cm. Its circumcentre lies _____.

(A) inside the triangle
(B) on the triangle
(C) outside the triangle
(D) either inside or on the triangle

12. The point of concurrence of angle bisectors of a triangle is called the ______ of the triangle.

(A) incentre (B) circumcentre
(C) centroid (D) orthocentre

13. In triangle PQR, S and T are points on PQ and PR respectively, such that ST is parallel to QR. If PS = 8 cm, SQ = 4 cm and TR = 6 cm, find PT. (in cm)

14. PQRS is a cyclic quadrilateral ∠P = 50° and ∠Q = 70°. Find the respective measures of ∠R and ∠S.

(A) 50°, 70° (B) 70°, 50°
(C) 110°, 130° (D) 130°, 110°

15. Triangle PQR is right angled at Q. QS is an altitude to PR. Find QS if PS = 32 cm and SR = 18 cm. (in cm)

16. ABCD is a trapezium. AB is parallel to CD. AB = 12 cm and CD = 24 cm. Find the length of the line EF which is parallel to AB and CD and divides the distance between them in the ratio 2 : 3. (in cm)

17. In triangle PQR, S and T are points on PQ and PR respectively such that ST is parallel to QR. PS = 4 cm, SQ = 12 cm and ST = 3 cm. Find QR (in cm).

(A) 9.6 (B) 8.4 (C) 10.8 (D) 12

18. The number of diagonals of a regular decagon is ______.

19. The radius of a circle is 15 cm. Find the length of a chord which is 9 cm from the centre of the circle. (in cm)

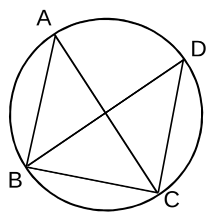
20. P and Q are points on the circumference of a circle with centre O. R is a point on the major arc PQ. ∠POQ = 100°. Find ∠PRQ.

(A) 40° (B) 45° (C) 50° (D) 55°

21. XY is the diameter of a circle. Z is a point on the circumference of the circle. XY = 26 cm and XZ = 24 cm. Find YZ (in cm).

22. Among a parallelogram, a rhombus and an isosceles trapezium, how many are necessarily cyclic quadrilaterals?

23.



In the figure, $\angle BCA = 45°$ and $\angle BDC = 50°$. Find $\angle ABC$ (in degrees).

24. A regular hexagon has a side of 4 cm. Find its area (in cm^2).

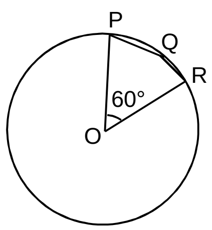
(A) $24\sqrt{3}$ (B) $18\sqrt{3}$

(C) $36\sqrt{3}$ (D) None of these

25. A right circular cone is cut parallel to its base at one-third of its height from the top. Find the ratio of the volume of the smaller piece to that of the original cone.

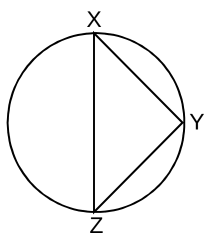
(A) 1 : 26 (B) 1 : 27 (C) 2 : 25 (D) 1 : 8

26.



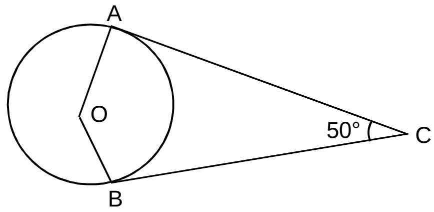
In the figure, O is the centre of the circle and $\angle POR = 60°$. Find $\angle PQR$ (in degrees).

27.



In the figure, XZ is the diameter of the circle. $\angle XZY = 35°$. Find $\angle YXZ$ (in degrees).

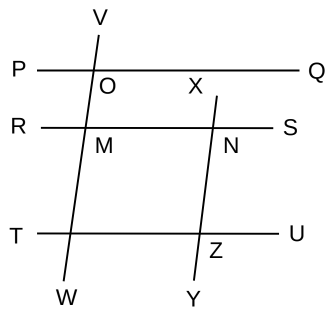
28.



In the figure, O is the centre of the circle. AC and BC are tangents to the circles. If $\angle ACB = 50°$, find $\angle AOB$.

(A) 130° (B) 80° (C) 100° (D) 90°

29.



In the figure, $\angle XZT = 130°$. PQ, RS and TU are parallel. VW and XY are parallel. Find $\angle VOP$.

(A) 125° (B) 130° (C) 120° (D) 135°

30. The midpoints of the sides of a quadrilateral of area 200 sq.cm are joined. What type of quadrilateral is formed and what is its area?

(A) Parallelogram, 100 sq.cm.
(B) Rectangle, 100 sq.cm.
(C) Rectangle, 50 sq.cm.
(D) Parallelogram, 50 sq.cm.

31. Find the maximum number of common tangents that can be drawn to two non-intersecting and non-enclosing circles.

(A) 1 (B) 2 (C) 3 (D) 4

32. Find the maximum number of common tangents that can be drawn to two circles which touch each other externally.

33. Find the maximum number of common tangents that can be drawn to two circles which intersect each other.

34. A triangle has its circumcentre on one of its sides. It can be _____.

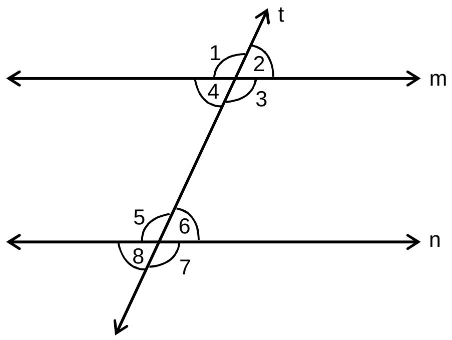
(A) acute–angled
(B) obtuse–angled
(C) right–angled
(D) acute, obtuse or right-angled

35. An equilateral triangle has a side of 12 cm. Find the area (in cm^2) of the triangle formed by the incentre, centroid and the circumcentre.

Exercise – 4(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. (i) In the given figure, lines m and n are parallel and t is the transversal.



$\angle 8$ is less than $\angle 3$ by 90°. Consider the following statements:

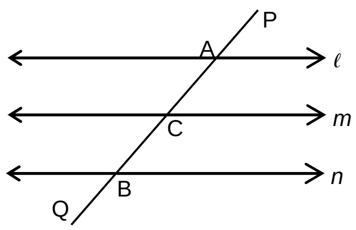
I. $\angle 1 = \angle 3 = \angle 5 = \angle 7 = 135°$

II. $\angle 2 = \angle 4 = \angle 6 = \angle 8 = 45°$

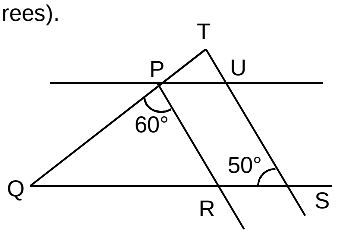
Which of the following statements is/are true?

(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

(ii) In the given figure, ℓ, m and n are parallel lines. PQ is a transversal intersecting the three lines. The perpendicular distances between (ℓ and m) and (m and n) are in the ratio 3 : 4. If AC = 12 cm, what is the length of AB (in cm)?



2. In the given figure, PR || TS and PU || RS. Find $\angle$TPU (in degrees).



3. ABC is a right-angled triangle, right-angled at B. An equilateral triangle ABD is constructed on side AB. A line is drawn from D parallel to BC, meeting AB at the point E. Find the length of DE, if AC = 41 cm and BC = 9 cm.

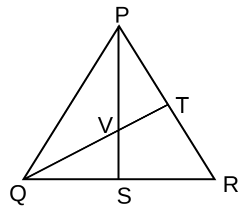
(A) $10\sqrt{3}$ cm

(B) $15\sqrt{3}$ cm

(C) $20\sqrt{3}$ cm

(D) $24\sqrt{3}$ cm

4. In the given figure, QS : SR = PV : VS = 4 : 3, PT = 8 cm. Find PR (in cm).



5. The total number of diagonals in a regular polygon is 20. What is the interior angle of the polygon?

(A) 90° (B) 115° (C) 120° (D) 135°

6. If each angle of a regular polygon of n sides is 144°, find the value of n.

7. An equilateral triangle has a side of 36 cm. Find the circumradius of the triangle (in cm).

(A) $6\sqrt{3}$ (B) $8\sqrt{3}$ (C) $12\sqrt{3}$ (D) $16\sqrt{3}$

8. How many scalene triangles with integral sides (in cm) have a perimeter of 20 cm?

(A) 3 (B) 4 (C) 5 (D) 6

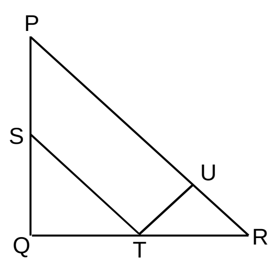
9. In ΔABC, $\angle$ABC = 80°, $\angle$ACB = 40°, AP is the bisector of $\angle$BAC and AQ $\perp$ BC. Find $\angle$PAQ.

(A) 10° (B) 15° (C) 20° (D) 35°

10. In triangle PQR, PA, QB and RC are the medians. The centroid is at point G. What is the ratio of the area of quadrilateral CQAG and the area of triangle PQR?

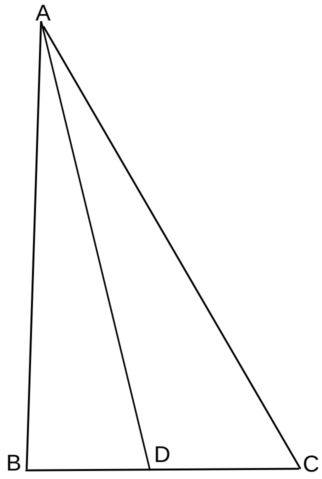
(A) 1 : 4 (B) 1 : 3 (C) 1 : 2 (D) 1 : 6

11. In the given figure, QS = QT and RU = RT. $\angle$QPR = 36°. Find $\angle$STU.



(A) 96° (B) 84° (C) 72° (D) 60°

12. (i)



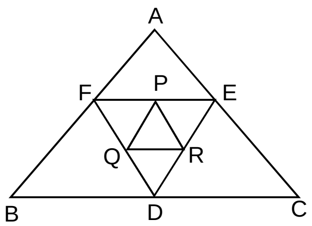
In the given triangle ABC, AD is the bisector of $\angle$A. If AB = 12 cm, AC = 16 cm and BD = 3 cm, what is the length of BC?

(A) 8 cm (B) 15 cm (C) 6 cm (D) 7 cm

(ii) ABC is a right-angled triangle, right angled at B. The bisector of the external angle at A meets CB produced at D. If AC = 5 cm, BC = 4 cm, find the length of CD.

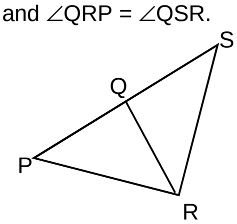
(A) 5 cm (B) 7 cm (C) 8 cm (D) 6 cm

13.



In the given figure, if the area of triangle ABC is 64 cm^2, then what is the area of the triangle PQR, where D, E and F are the midpoints of the sides of ΔABC and P, Q and R are the midpoints of the sides of ΔDEF (in cm^2)?

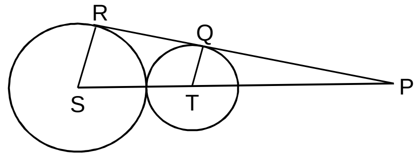
14. In the given figure, PS = 32 cm, PQ = 18 cm, SR = 25 cm and $\angle$QRP = $\angle$QSR.



Find the ratio of the perimeters of the triangle PQR and PRS.

(A) 3 : 2 (B) 3 : 1 (C) 2 : 1 (D) 3 : 4

15. In the given figure radii of two circles with centres S and T respectively are in the ratio 5 : 3 and PT = 12 cm. If R and Q are points of contacts of the tangent drawn from P to the circles with centres S and T respectively, then find the areas of the quadrilateral RQTS and the triangle PQT.



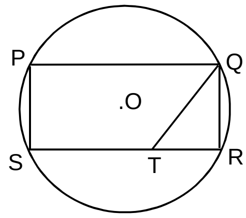
(A) $16\sqrt{15}$ cm^2, $9\sqrt{15}$ cm^2

(B) $8\sqrt{15}$ cm^2, $4.5\sqrt{15}$ cm^2

(C) $24\sqrt{15}$ cm^2, $13.5\sqrt{15}$ cm^2

(D) None of these

16. In the given figure (not to scale), PQRS is a rectangle inscribed in a circle with centre O. PQ > PS. Areas of the rectangle PQRS and the circle are in the ratio $2\sqrt{5} : 3\pi$ and $\angle$OQP = $\angle$TQR. Find QR : RT.



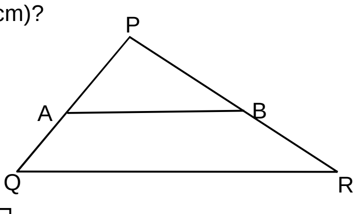
(A) $1:\sqrt{5}$ (B) $2:\sqrt{5}$ (C) $\sqrt{5}:1$ (D) $\sqrt{5}:2$

17. In trapezium ABCD, AB and CD are the parallel sides. The diagonals AC and BD intersect at X. The ratio of the area of Δ AXB to the area of Δ CXD is 25 : 49. Find the ratio of the area of Δ AXD and the area of ABCD.

(A) $\frac{35}{144}$ (B) $\frac{5}{7}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

18. ABC is a triangle. D and E are points on AB and AC respectively. F is a point on BC. DF is parallel to AC and EF is parallel to AB. The areas of the triangles EFC and ABC are 64 and 400 respectively. Find the area of the quadrilateral ADFE.

19. In the given figure, AB is parallel to QR and the ratio of the areas of triangles PAB and PQR is 1 : 4. If the perimeter of PQR is 24 cm, what is the perimeter of PAB (in cm)?



20. PQR is a triangle in which PQ = PR = 26 cm. S is a point on segment QR such that SR = 3 cm and PS = 25 cm. Find the length of QS.

(A) 16 cm (B) 17 cm (C) 18 cm (D) 20 cm

21. In the right-angled triangle ABC, the right angle is at B and BD is perpendicular to AC. The lengths of AD and CD are 5 and 6 respectively. Find AB^2.

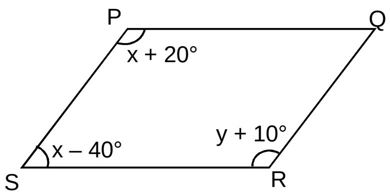
(A) 144 (B) 121

(C) 132 (D) None of these

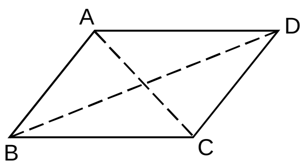
22. In triangle ABC, AD and BE are medians and G is the centroid. $\angle AGE = 30°$, AD = 12, BE = 18. Find the area of the triangle.

23. ABCD is a trapezium, in which AD and BC are parallel. If the four sides AB, BC, CD and DA are 8, 10, 12 and 16 respectively, then find the magnitude of the sum of the squares of the two diagonals.

24. In the given figure given, PQRS is a parallelogram. Find y (in degrees).



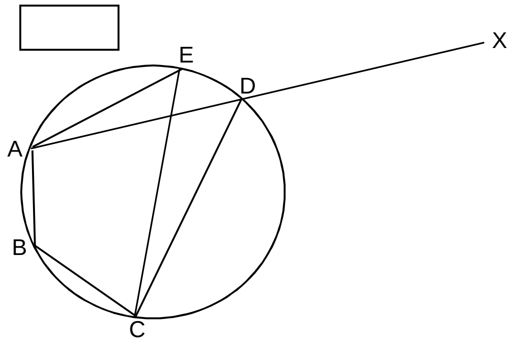
25.



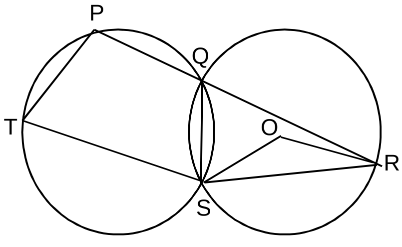
In the given figure, ABCD is a parallelogram and the lengths of AC and BD are 10 units and 12 units respectively. The sum of the squares of AB and BC is _____.

(A) 120 (B) 140 (C) 122 (D) 128

26. (i) In the given figure, ABCD is a cyclic quadrilateral. If $\angle CDX = 130°$, then what is $\angle AEC$ (in degrees)?

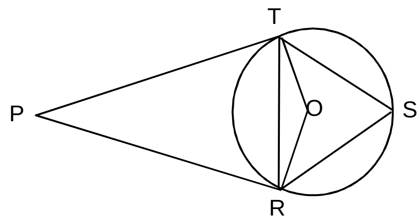


(ii)



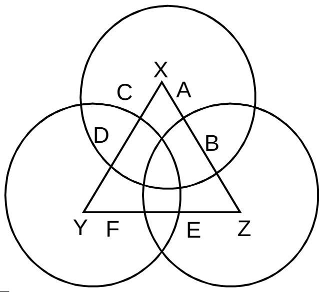
In the given figure, O is the centre of the right circle. $\angle OSR = 20°$. Find $\angle PTS$ (in degrees).

27. In the given figure, PT and PR are two tangents to the circle at T and R respectively. If O is the centre of the circle and $\angle TPR = 60°$, find $\angle TSR$.



(A) 40° (B) 60°
(C) 120° (D) None of these

28. Three circles with centres X, Y and Z with radius 30 cm each intersect each other as shown. AB = 12 cm, CD = 6 cm and EF = 18 cm. Find the perimeter of triangle XYZ (in cm).



29. Two parallel chords of a circle of radius 25 cm have lengths of 30 cm and 48 cm. Find the distance between the two chords (in cm).

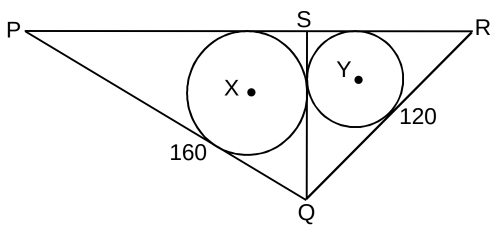
(A) 6.5 or 13.5 (B) 13 or 27
(C) 19.5 or 40.5 (D) None of these

30. A, B, C and D are points on the circumference of a circle. ABD is equilateral and AC is the diameter of the circle. Find the ratio of the perimeter of ABCD and the length of AC.

(A) $1+\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}+\sqrt{3}$

(C) $1+\sqrt{3}$ (D) $2+\sqrt{3}$

31.



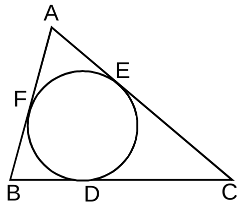
In the given figure, triangle PQR is right-angled at Q. QS is the altitude to side PR. Circles with centres X and Y are inscribed in ΔPQS and ΔRQS respectively. Find XY^2.

32. Triangle PQR has a perimeter of 24 cm. X is a point inside PQR which is equidistant from each vertex of PQR. $\angle QPX = 35°$ and $\angle PRX = 25°$
Consider the following statements
I. PQ is less than 8 cm.
II. PR is more than 8 cm.
Which statement(s) is/are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

33. The sides of a cyclic quadrilateral are 10 cm, 14 cm, 12 cm and 15 cm. If one of its diagonals is 15 cm, then find the other diagonal (in cm).

34. There are 12 uniformly spaced points on a circle with radius 10. Each of these points is joined with the two adjacent points by straight lines. Find the perimeter of the polygon so formed.
(A) $30(\sqrt{6}-\sqrt{2})$ (B) $60(\sqrt{6}-\sqrt{2})$
(C) $60(\sqrt{6}+\sqrt{2})$ (D) $30(\sqrt{6}+\sqrt{2})$

35.



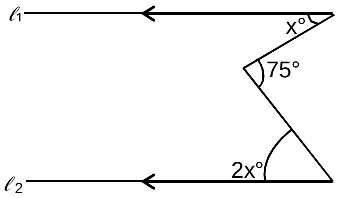
In the given figure, AB = 5 cm, BC = 6 cm, CA = 7 cm. Find DC (in cm).

Exercise – 4(b)

Directions for questions 1 to 55: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

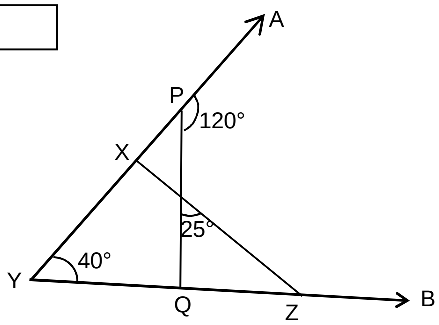
Very Easy / Easy

1.



In the given figure, $\ell_1 \parallel \ell_2$. What is the value of x?

2. In the following figure, find $\angle BZX$ (in degrees).



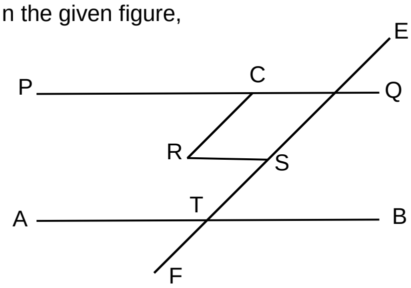
Moderate

3. In triangle ABC, $\angle B - \angle A = \angle C - \angle B$ and $\angle A : \angle C = 3 : 5$. Find the difference of the largest and the smallest angles.
(A) 20° (B) 25° (C) 30° (D) 35°

4. A rhombus is divided into two triangles by its longer diagonal. The perimeter of either of these triangles is 100 cm. It is divided into four triangles by its diagonals. The perimeter of any of these triangles is 60 cm. Find the length of the shorter diagonal (in cm).

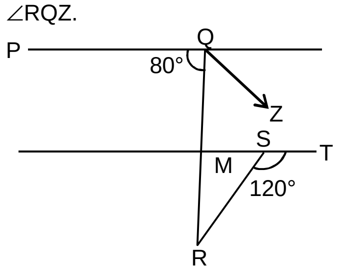
5. AB is a chord of the circle with centre O. M is the midpoint of AB and P is a point on BA produced. If PB is 8 cm and PA is 3 cm, then what is the difference of PM^2 and AM^2 (in cm^2)?
(A) 20 (B) 40 (C) 24 (D) 26

6. In the given figure,



PQ || AB, AB || RS and RC || EF. If $\angle CRS = 50°$, find $\angle STA$ (in degrees).

7. In the following figure, $\angle RQZ = 2\angle QRS$ and PQ || ST. Find $\angle RQZ$.

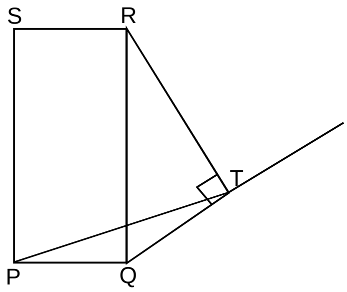


(A) 20° (B) 30° (C) 40° (D) 60°

8. If X is any point inside the triangle ABC and P is the perimeter of triangle ABC, then AX + BX + CX is greater than _______.
(A) 3P (B) 2P (C) $\frac{3P}{2}$ (D) $\frac{P}{2}$

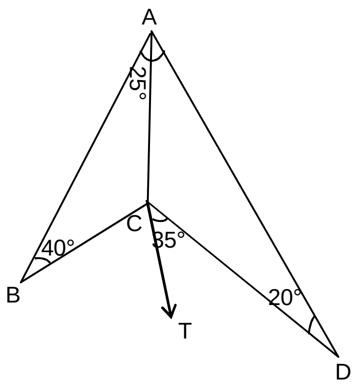
9. In triangle ABC, AB = 12 cm and BC = 21 cm. What is the range of the perimeter (p) of ABC?
(A) 3 cm < p < 21 cm (B) 9 cm < p < 33 cm
(C) 33 cm < p < 66 cm (D) 42 cm < p < 66 cm

10. In the figure below, PQRS is a rectangle. QR = 2PQ = 2QT and $\angle$RTQ = 90°. Find $\angle$QTP.

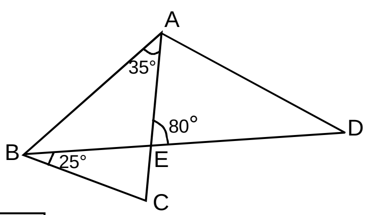


(A) 15° (B) 20° (C) 25° (D) 30°

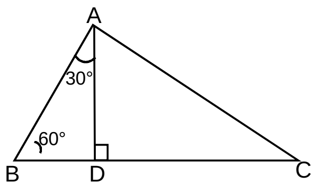
11. In the figure AC = CD. Find $\angle$BCT (in degrees).



12. In the figure, $\angle$EBC = 25°, $\angle$BAC = 35° and $\angle$AED = 80°. Find ($\angle$ABC + $\angle$EAD + $\angle$ADE) (in degrees).



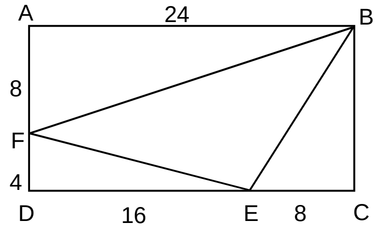
13.



In the given figure, the lengths of BD and DC are in the ratio of 2 : 3 and BC = 5 cm. Find AC.
(A) $\sqrt{17}$ cm (B) $\sqrt{23}$ cm
(C) $\sqrt{29}$ cm (D) $\sqrt{21}$ cm

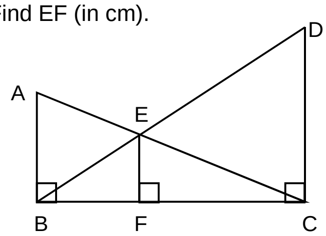
14. ABCD is a trapezium in which AB is parallel to DC. The diagonals AC and BD intersect at the point E and AB : CD = 2 : 3. If the area of triangle AEB = 64 sq.cm, find the area of triangle CED (in cm^2).

15. In the given figure ABCD is a rectangle. Find the length of the line joining the midpoints of BF and EF.



(A) $2\sqrt{13}$
(B) $3\sqrt{3}$
(C) $\sqrt{13}$
(D) None of the previous choices

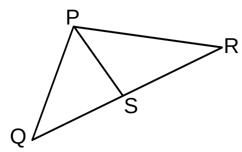
16. In the given figure, AB = 30 cm, CD = 45 cm and BC = 15 cm. Find EF (in cm).



17. The sides of triangle ABC are a cm, b cm, and c cm. The sides of triangle DEF are d cm, e cm and f cm.
$a^2 + b^2 + c^2 = 50$
$d^2 + e^2 + f^2 = 50$
ad + be + cf = 50
What can be said about the two triangles?
(A) They have the same perimeter.
(B) They have the same area.
(C) Both (A) and (B)
(D) Neither (A) nor (B)

18. The perimeters of similar triangles ABC and PQR are 32 cm and 40 cm respectively. If the area of triangle PQR is 100 cm^2, find the area of triangle ABC (in cm^2).

19. In the given figure, PS is the median drawn to the side QR of triangle PQR. If PQ = 11 cm, PR = 13 cm and PS = 8 cm, what is the length of QS?



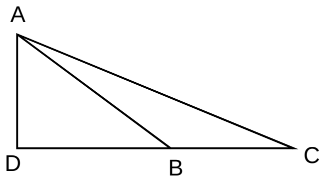
(A) $4\sqrt{14}$ cm (B) 9 cm
(C) $3\sqrt{15}$ cm (D) 10 cm

20. In triangle ABC, AD ⊥ BC. D divides BC in the ratio 1 : 3 internally. If AB = 9 cm and AC = 21 cm, find BC.

(A) $12\sqrt{5}$ cm (B) $15\sqrt{5}$ cm
(C) $16\sqrt{5}$ cm (D) $18\sqrt{5}$ cm

21. In triangle ABC, AB = AC and D is a point on BC. If AB = 65 cm, AD = 63 cm and CD = 8 cm, find BD (in cm).

22.



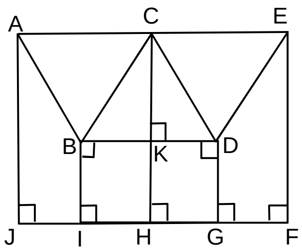
In triangle ABC, the lengths of AB, BC and AC are 5 cm, 3 cm and 7cm respectively. D is a point on CB extended such that AD is perpendicular to BC. What is the length of AD?

(A) 6.25 cm (B) $5\sqrt{3}$ cm
(C) $4\sqrt{3}$ cm (D) None of these

23. D and E are two points on the sides AB and AC of triangle ABC respectively such that triangle ADE is similar to triangle ABC. DE divides triangle ABC into two parts of equal area. If AB = 12 cm, find BD.

(A) $(6 - \sqrt{2})$ cm
(B) $(12 - 6\sqrt{2})$ cm
(C) $(12 - 3\sqrt{2})$ cm
(D) None of these

Directions for questions 24 and 25: These questions are based on the information given below.

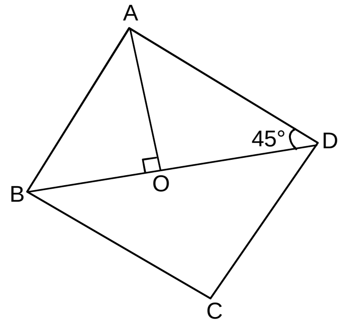


In the figure above, AJ = 2IB
= 3JI = CH = 3IH
= 3HG = EF = 3GF.

24. Find ∠CDK.
(A) 60°
(B) 45°
(C) 30°
(D) None of these

25. Find the ratio of the areas of quadrilaterals ABIJ and EFHC.
(A) 5 : 12 (B) 7 : 16
(C) 3 : 8 (D) 11 : 24

26.



In the parallelogram given, AB = 8 cm and BO = 6 cm Find BC.

(A) 12 cm (B) $2\sqrt{14}$ cm
(C) $2\sqrt{7}$ cm (D) None of these

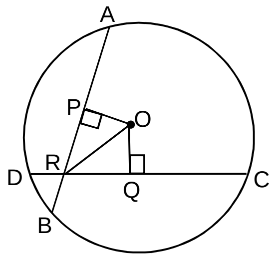
27. The diagonal of a square ABCD is $22\sqrt{2}$ cm. An equilateral triangle BCP is drawn on side BC. There is a point M on BC such that PM is parallel to AB. Find the ratio of PM and AC.

(A) $\sqrt{3} : 4\sqrt{2}$ (B) $\sqrt{3} : 2\sqrt{2}$
(C) $2\sqrt{3} : 3\sqrt{2}$ (D) None of these

28. In a quadrilateral ABCD, AB is parallel to CD. AB = 8 cm, BC = 10 cm, CD = 16 cm and AD = 10 cm. Find the sum of the lengths of the diagonals.

(A) $4\sqrt{57}$ cm (B) $5\sqrt{53}$ cm
(C) $6\sqrt{47}$ cm (D) $7\sqrt{41}$ cm

Directions for questions 29 and 30: These questions are based on the information given below.

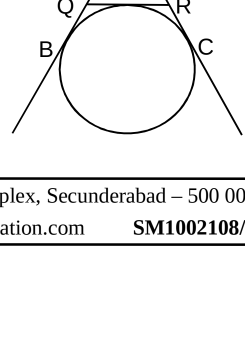


In the given figure, AB and DC are equal chords of the circle with centre O and OP = 3 cm, AP = 8 cm, OR = 5 cm.

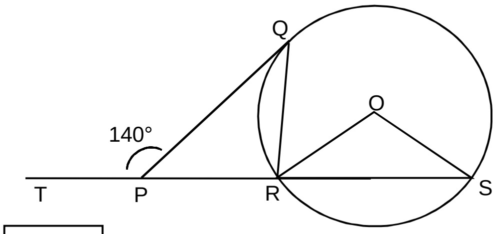
29. What is the length of CR (in cm)?

30. Find the length of QC (in cm).

31. In the given figure, PB, PC and QR are tangents to the circle at B, C and A respectively. If PB = 8 cm, find the perimeter (in cm) of the triangle PQR.

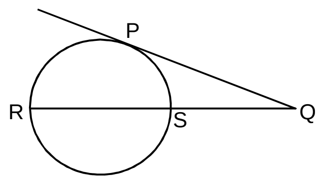


32. In the figure below, $\angle QPT = 140^o$, PQ is tangent to the circle at Q and PR = QR. O is the centre of the circle. Find $\angle ROS$ (in degrees).



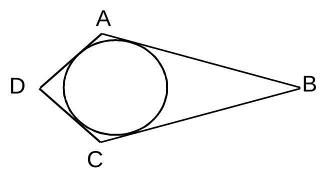
33. In an isosceles triangle ABC, AC = AB, BD is the foot of the perpendicular from B to AC and the lengths of BD and CD are 5 cm and 4 cm respectively. Find the length of AD (in cm).

34. In the figure below, RS is a diameter, which when extended meets the tangent drawn at P at the point Q. If $\angle PQR = 30°$, then ______.



(A) PQ < RQ (B) PQ = RQ
(C) PQ > RQ (D) PQ ≥ RQ

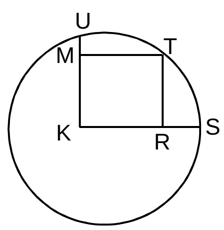
35.



In the given figure, ABCD is a quadrilateral. If AB = 10 cm, AD = 3 cm and CD = 4 cm, then what is the length of BC?
(A) 11 cm (B) 12 cm
(C) 10 cm (D) Cannot be determined

Directions for questions 36 and 37: These questions are based on the information given below.

In the figure, K is the centre of the circle, T is a point on the circumference of the circle and MKRT is a square.

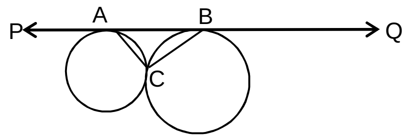


36. Find the ratio of the perimeter of the square MKRT and the circumference of the circle.
(A) $7\sqrt{2} : 11$ (B) $7 : 11\sqrt{2}$
(C) $7 : \sqrt{2}$ (D) 7 : 22

37. Find the ratio of the area of the square MKRT and the area of the circle.
(A) 1 : 4 (B) 7 : 22 (C) 7 : 44 (D) 14 : 33

38. Two parallel chords PQ and RS are drawn on opposite sides of the centre of a circle, of radius 17 cm. If RS is at a distance of 22 cm from PQ and PQ is 16 cm long, find RS.
(A) $4\sqrt{15}$ cm (B) $10\sqrt{10}$ cm
(C) $8\sqrt{15}$ cm (D) $9\sqrt{10}$ cm

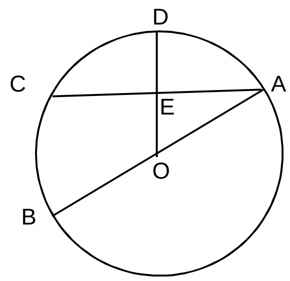
39. In the following figure, C is the point of contact of the two circles and A, B are the points of contact of the common tangent PQ with the two circles. Find $\angle ACB$.



(A) 60° (B) 90°
(C) 120° (D) Cannot be determined

40. In a polygon, the sum of all its interior angles equals the sum of all its exterior angles. Each interior angle is x°or its supplement where $x \neq 90$. How many of the angles must be x°?

41.



In the above figure, O is the center of the circle, OD is perpendicular to AC. AB = 25 cm, CE = 12 cm. Find the length of DE (in cm).

42. ABCD is a cyclic quadrilateral in which AC is a diameter of the circumcircle of the quadrilateral and AB= AD = $\frac{\sqrt{3}}{2}$(AC). Find $\angle BCD$.
(A) 60° (B) 120°
(C) 90° (D) 150°

43. What is the sum of the squares of the sides of triangle ABC (in cm^2), if the sum of the squares of its medians is 36 cm^2?

44. In triangle PQR, $\angle R$ is an obtuse angle. The perpendicular drawn from P meets QR produced at S. If QR = RS, QR = 8 cm and PQ = 20 cm, then find PR.
(A) $3\sqrt{13}$ cm (B) $4\sqrt{13}$ cm
(C) $6\sqrt{13}$ cm (D) $7\sqrt{13}$ cm

45. In triangle ABC, what is the distance between the midpoint of BC and the foot of the perpendicular from A to BC, if the lengths BC, CA and AB are 5 cm, 7 cm and 6 cm respectively? (in cm)

46. In triangle ABC, S is the point equidistant from A, B and C. $\angle BSC = 120°$, and $\angle BSA = 140°$. Find $\angle ABC$ (in degrees).

Directions for questions 47 and 48: These questions are based on the information given below.

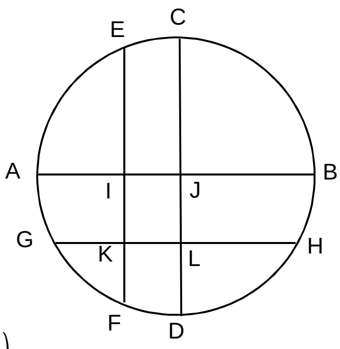
ABCD is a rectangle divided into two rectangles AEFD and EBCF by drawing a line segment EF. AE = 28 cm, EB = 12 cm, and BC = 10 cm. G, H and I lie within AEFD. J, K and L lie within EBCF. The distances from each point in one rectangle to each of the three points in other were measured. The shortest of these distances was found to be 41 cm.

47. Which of the following is always true?
(A) Distance between G and H is less than that between J and I.
(B) (I, J) can represent the closest pair of points among the 6 given points.
(C) The closest pair of points among the 6 given points must be a pair of points in EBCF.
(D) None of the above

48. Which of the following can be the measure of GI?
(A) 0.5 cm (B) 1.5 cm (C) 2.5 cm (D) 3.5 cm

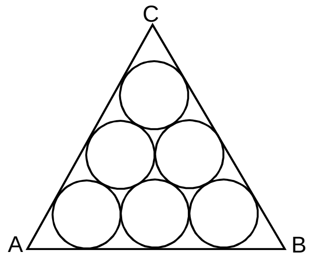
49. The centres of three circles which touch externally form a triangle of sides 15 cm, 16 cm and 17 cm. Find the radius of the largest circle (in cm).

50. In the figure below, AB and CD are two diameters of the circle with centre J perpendicular to each other. Chords EF and GH are perpendicular to AB and CD respectively at I and L respectively. Chords EF and GH intersect at K. AI : IB = DL : LC = 1 : 3 and AB = 8 cm. Find GK (in cm).



(A) $2(\sqrt{3}-1)$
(B) $2(2\sqrt{3}-1)$
(C) $\sqrt{3}-1$
(D) $3(\sqrt{3}-1)$

51.

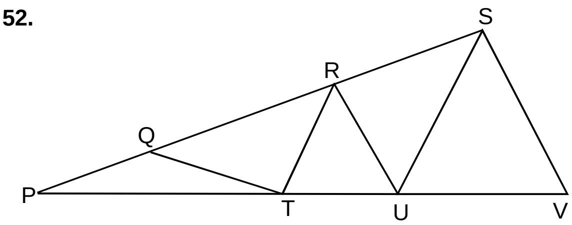


In the figure, ABC is an equilateral triangle of side a and each circle is of radius r. Find r in terms of a.

(A) $\frac{a(2-\sqrt{3})}{4}$ (B) $\frac{a(2-\sqrt{3})}{2}$
(C) $\frac{a(2+\sqrt{3})}{16}$ (D) $\frac{a(2+\sqrt{3})}{32}$

Difficult / Very Difficult

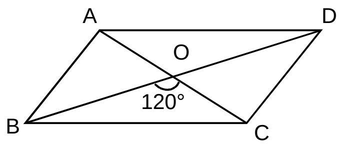
52.



In the figure above, PQ = QU = US = SV = VR = RT = TP. Find $\angle P$.

(A) 30° (B) $25\frac{2}{7}^{\circ}$
(C) $25\frac{5}{7}^{\circ}$ (D) Cannot be determined

53.



In the given parallelogram, AO is 5 units, BO is 7 units and angle BOC = 120°. The area of quadrilateral ABCD is _____.
(A) 35 sq.units (B) 70 sq.units
(C) $35\sqrt{3}$ sq.units (D) $70\sqrt{3}$ sq.units.

54. If ABCD is a cyclic quadrilateral and AD = 3 units, CD = 4 units, BC = 5 units and AB = 6 units, then the product of AC and BD is _____.
(A) 29 sq.units (B) 35 sq.units
(C) 39 sq.units (D) 41 sq.units

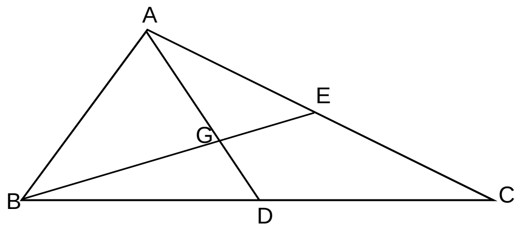
55. A, B, C and D are points on a circle, in that order such that $\angle CAB = 20°$ The angle bisectors of $\angle CBD$ and $\angle BCA$ intersect at E and F is a point on BC such that EF is perpendicular to EC. If $\angle BEF = \angle ABD$, find $\angle ACD$ (in degrees).

Data Sufficiency

Directions for questions 56 to 65: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

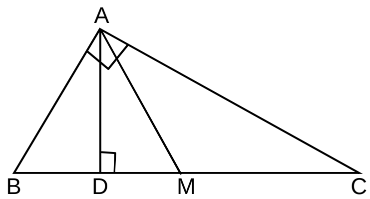
Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone
Mark (D) if the question cannot be answered even using I and II together.

56. In the figure below, ABC is a triangle, AD and BE are two of the medians. What is the area of the quadrilateral GDCE?



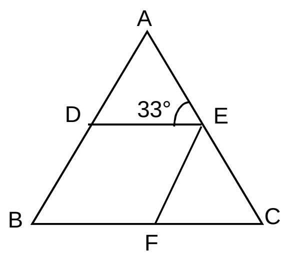
I. AG : GE = 2 : 1
II. AB = 10 cm and BC = 20 cm

57. What is the length of the median AM of the triangle ABC, if $\angle BAC = 90°$?



I. BC = 21 cm.
II. AD = 14 cm.

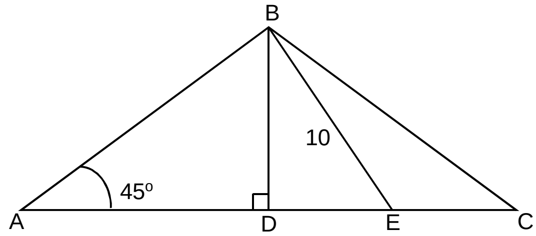
58. In the figure below, $\angle AED = 33°$. If D, E and F are the mid-points of the sides ΔABC, what is the measure of $\angle EFB$?



I. $\angle FEC = 78°$
II. $\angle BAC = 78°$

59. Find the area of the triangle ABC.
I. $\angle ABC = 60°$
II. AB = 10 cm and AC = 10 cm.

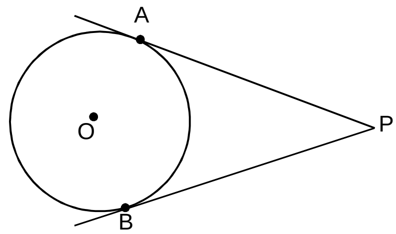
60. In the figure below, if BE = 10 cm, and $\angle A = 45°$, then what is the length of AB?



I. $\angle BCA = 45°$.
II. $\angle BEA = 60°$.

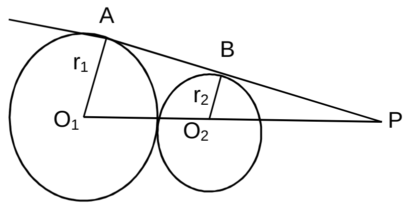
61. PQRS is a rectangle and points A, B, C lie on PQ, QR, RS, respectively. What is the ratio of the area of the triangle ABC to the area of rectangle PQRS?
I. Point A divides PQ in the ratio 1 : 2.
II. AC is parallel to PS

62. In the figure below two tangents from P meet the circle at A and B. What is the length of PO?



I. O is the centre of the circle whose radius is 10 cm.
II. The circumradius of the triangle AOP is 14 cm.

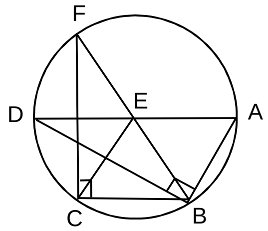
63. In the figure below, A and B are the points of contact of the direct common tangent from P to the circles with centres O_1 and O_2 respectively. If r_1, r_2 are the radii of the circles, find the length of AB.



I. r_1 = 15 cm, r_2 = 9 cm
II. $\angle BPO = 30°$

64. The points O_1 and O_2 are the centres of two circles having radii 10 cm and 20 cm respectively. What is the distance between the points O_1 and O_2?
I. The two circles have only one common tangent
II. The two circles have three common tangents.

65.



What is the measure of ∠BEA in the figure above if ∠ABD = ∠BCF = 90°?

I. ∠BDA = 30° II. ∠CFB = 30°

Key

Concept Review Questions

1. 5	6. C	11. C	16. 16.8	21. 10	26. 150	31. D
2. 28	7. 4	12. A	17. D	22. 1	27. 55	32. 3
3. C	8. D	13. 12	18. 35	23. 85	28. A	33. 2
4. C	9. 115	14. D	19. 24	24. A	29. B	34. C
5. A	10. C	15. 24	20. C	25. B	30. A	35. 0

Exercise – 4(a)

1. (i) C	8. B	15. B	23. 528	30. C
(ii) 28	9. C	16. C	24. 110	31. 3200
2. 70	10. B	17. A	25. C	32. C
3. C	11. C	18. 192	26. (i) 50	33. 22
4. 18.5	12. (i) D	19. 12	(ii) 70	34. B
5. D	(ii) D	20. B	27. B	35. 4
6. 10	13. 4	21. D	28. 144	
7. C	14. D	22. 72	29. B	

Exercise – 4(b)

1. 25	14. 144	27. B	40. 2	53. C
2. 125	15. A	28. A	41. 9	54. C
3. C	16. 18	29. 12	42. B	55. 20
4. 20	17. C	30. 8	43. 48	56. C
5. C	18. 64	31. 16	44. B	57. A
6. 130	19. B	32. 120	45. 1.3	58. B
7. C	20. A	33. 1.125	46. 50	59. C
8. D	21. 32	34. A	47. A	60. A
9. D	22. D	35. A	48. A	61. C
10. A	23. B	36. B	49. 9	62. A
11. 70	24. D	37. C	50. A	63. A
12. 170	25. C	38. C	51. B	64. B
13. D	26. B	39. B	52. C	65. A