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Prime CAT 02 2022 QA

Section-1

Sec 1

Q.1 [11831809]

Let p and q be the roots of the equation $x^2 + 7x - 11 = 0$. If roots of the equation $3x^2 + bx + c = 0$ are $p - 2$ and $q - 2$, then the value of $b - c$ is

Solution:

Correct Answer : 12

Answer key/Solution

Since p and q are the roots of the equation $x^2 + 7x - 11 = 0$.
Therefore, $p + q = -7/1 = -7$
and $pq = -11/1 = -11$
Roots of the equation $3x^2 + bx + c = 0$ are $p - 2$ and $q - 2$.
Therefore, $(p - 2) + (q - 2) = -b/3 \Rightarrow -7 - 4 = -b/3 \Rightarrow b = 33$
and $(p - 2)(q - 2) = c/3 \Rightarrow pq - 2(p + q) + 4 = c/3$
$\Rightarrow -11 - 2(-7) + 4 = c/3 \Rightarrow c = 21$
Hence, $b - c = 33 - 21 = 12$.

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Q.2 [11831809]

Two trains A and B running in opposite directions cross each other in 18 seconds and cross a tower in 21 seconds and 16 seconds respectively. The ratio of speed of train A to that of train B is

1 ◯ 3 : 2

2 ◯ 3 : 4

3 ◯ 4 : 3

4 ◯ 2 : 3

Solution:

Correct Answer : 4

Answer key/Solution

Let the speeds of trains A and B be x and y respectively.
Then, length of train A = 21x and length of train B = 16y.
So (21x + 16y)/(x + y) = 18
⇒ 21x + 16y = 18x + 18y
⇒ 3x = 2y ⇒ x/y = 2/3.

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Q.3 [11831809]

Rashi's house rent is 50% more than that of Ayushi and 75% more than that of Jyoti. Rashi's house rent is what percent of the total house rent of Ayushi and Jyoti together?

1 ◯ 77.01%

2 ◯ 80.77%

3 ◯ 67.17%

4 ◯ 90.33%

Solution:
Correct Answer : 2

Answer key/Solution

Let Rashi's house rent be Rs. P, which is 50% more than Ayushi's rent, means that if Ayushi's rent is Rs. y, then
$y + 50\%$ of $y = P \Rightarrow P = 1.5y \Rightarrow y = 2P/3$
Rashi's rent is 75% more than that of Jyoti ⇒ If Jyoti's rent is Rs. z, then $z + 75\%$ of $z = P$
$\Rightarrow 1.75z = P \Rightarrow z =$ Rs. $4P/7$
Total of Ayushi's and Jyoti's rents $= 2P/3 + 4P/7 = 26P/21$
Hence, Rashi's rent as a percentage of Ayushi's and Jyoti's rents
$= 21P/26P \times 100 \approx 80.77\%$.

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Q.4 [11831809]

Let N and M be number of integral solutions to $||x| - 2022| < 10$ and $|2030 - |y|| < 12$. What is the value of $|M - N|$?

Solution:
Correct Answer : 8

Answer key/Solution

$||x| - 2022| < 10.$
$\Rightarrow -10 < |x| - 2022 < 10$
$\Rightarrow 2012 < |x| < 2032$
$\therefore x = \pm 2013, \pm 2014, \dots, \pm 2031$
So, the total number of integer solutions = 38
Similarly, $|2030 - |y|| < 12$
$\Rightarrow -12 < 2030-|y| < 12$
$\Rightarrow -2042 < -|y| < -2018$
$\Rightarrow 2042 > |y| > 2018$
$\therefore x = \pm 2019, \pm 2020, \dots, \pm 2041$
So, the total number of integer solutions = 46
Hence, value of $|M - N| = 46 - 38 = 8$.

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Q.5 [11831809]

ABC is a triangle whose $\angle A = 120°$. The angle bisector AD of $\angle A$ meets BC at D. If AB = 2AC and AD = 20 cm, then the value of BC (in cm) is

1 ○ 30

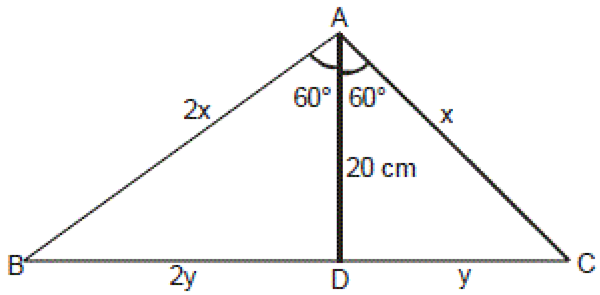
2 ○ $20\sqrt{7}$

3 ○ 60

4 ○ $30\sqrt{7}$

Solution:
Correct Answer : 4

Answer key/Solution



In triangle ABC, AB = 2AC ⇒ AB : AC = 2 : 1.
Let AC = x, then AB = 2x.
AD is the angle bisector of ∠A, therefore, BD : DC = 2 : 1.
Let DC = y, then BD = 2y.
Using cosine rule in triangle ADC,

$$y^2 = 20^2 + x^2 - 2x \times 20 \times \cos 60^\circ$$

$$\Rightarrow y^2 = 400 + x^2 - 20x \qquad \dots \text{(i)}$$

Similarly, using cosine rule in triangle ADB,

$$(2y)^2 = 20^2 + (2x)^2 - 2 \times 2x \times 20 \times \cos 60^\circ$$

$$\Rightarrow 4y^2 = 400 + 4x^2 - 40x \qquad \dots \text{(ii)}$$

From (i) and (ii), x = 30 cm and y = $10\sqrt{7}$ cm
Hence, BC = 2y + y = 3y = 3 × $10\sqrt{7}$ = $30\sqrt{7}$ cm.

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Q.6 [11831809]
All page numbers of a book are added up, starting with page 1. However, two consecutive page numbers were mistakenly added twice. The sum obtained was 1250. The page numbers added twice were ________________.

1 ○ 11, 12

2 ○ 13, 14

3 ○ 12, 13

4 ○ 10, 11

Solution:
Correct Answer : 3
Since two consecutive pages were added twice, so 1250 is not the actual sum but it is an increased sum.
Let the total number of pages be 'n'.
Then, $n(n + 1)/2 = 1250$
By hit and trial, $n = 49$
So sum = $49 \times 50/2 = 1225$
Since the given sum is 1250, so we can say that the consecutive page numbers 12 and 13 were added twice.

Answer key/Solution

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Q.7 [11831809]

It takes 11 workers a total of 5 hours to complete a project, with each working at the same rate. If five workers start at 8.00 AM, and one worker per hour joins them from 1:00 PM, then at what time will the project get over?

1 ◯ 5:00 PM

2 ◯ 5:30 PM

3 ◯ 6:00 PM

4 ◯ 7: 00 PM

Solution:
Correct Answer : 1

Answer key/Solution

Given that 11 workers complete the work in 5 hours.
∴ 1 worker completes the same work in 55 hours.
Let the total work be equivalent to 55 man-hours.
From 8:00 AM to 1.00 PM, five workers work together for 5 hours, so work done is equivalent to 25 man-hours.
From 1:00 PM to 2:00 PM, 6 man-hours of work will be done. From 2:00 PM to 3:00 PM, 7 man-hours of work will be done. From 3:00 PM to 4:00 PM, 8 man-hours of work will be done. From 4:00 PM to 5:00 PM, 9 man-hours of work will be done. Thus, the total amount of work done till 5:00 PM is = 25 + 6 + 7 + 8 + 9 = 55 man-hours.
Thus, the project will be over at 5:00 PM.

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Q.8 [11831809]

If $T_n = 5T_{n-1} - 6T_{n-2}$ for $n \geq 2$ such that $T_0 = 2$, and $T_1 = 5$, then find the last digit of T_{2003}.

Solution:

Correct Answer : 5

Answer key/Solution

$T_0 = 2 = 2^0 + 3^0$
$T_1 = 5 = 2^1 + 3^1$
$T_2 = 13 = 2^2 + 3^2$
$T_3 = 35 = 2^3 + 3^3$
And so on.

Therefore, $T_n = 2^n + 3^n$.
$\Rightarrow T_{2003} = 2^{2003} + 3^{2003}$
Hence, last digit of T_{2003} = Last digit (8 + 7) = 5.

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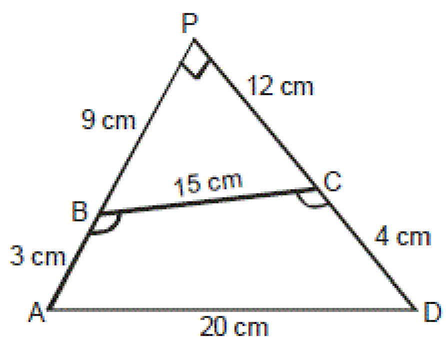
Q.9 [11831809]

ABCD is a quadrilateral such that AB = 3 cm, BC = 15 cm, CD = 4 cm and AD = 20 cm. If ∠ABC + ∠BCD = 270°, then what is the area (in sq. cm) of quadrilateral ABCD?

Solution:

Correct Answer : 42

Answer key/Solution



Since ∠ABC + ∠BCD = 270°, so ∠PBC + ∠PCB = 180° + 180° – 270° = 90°
So ∠P = 90°.
9 12 15 (Pythagorean triples)
12 16 20 (Pythagorean triples)
Hence, area of quadrilateral ABCD
= Area of triangle PAD – Area of triangle PBC
= 1/2 × 12 × 16 – 1/2 × 9 × 12 = 96 – 54 = 42 sq. cm.

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Q.10 [11831809]

Let R be the range of the function $f(x) = \dfrac{x^2 - 4x + 9}{3x^2 - 12x + 28}$. Then, what is the value of R for all real values of x?

1 ○ 1/4 ≤ R < 1/2

2 ◯ 5/16 ≤ R < 1/3

3 ◯ 5/8 ≤ R < 1/2

4 ◯ 5/16 ≤ R ≤ 1/3

Solution:
Correct Answer : 2

Answer key/Solution

Given function is $f(x) = \frac{x^2 - 4x + 9}{3x^2 - 12x + 28}$.

$\Rightarrow f(x) = \frac{(x-2)^2 + 5}{3(x-2)^2 + 16}$

$\Rightarrow f(x) = \frac{(x-2)^2 + 5}{3\left((x-2)^2 + 5\right) + 1}$

Let $k = (x - 2)^2 + 5$

$\therefore f(x) = \frac{k}{3k+1}$

The minimum value of k = 5.

So minimum value of $f(x) = \frac{5}{3 \times 5 + 1} = \frac{5}{16}$.

The maximum value of f(x) approaches to 1/3 i.e., less than 1/3.
Hence, the range R of the function f(x) is 5/16 ≤ R < 1/3.

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Q.11 [11831809]

In a business, Anil invests two-third of the capital for 8 months, Sunil invests one-sixth of the capital for one-third of the total time and Kamal invests the rest of the capital for the whole time, that is 12 months. Find the difference between the profit (in Rs.) of Sunil and Kamal if Anil's share is Rs.9,276.

Solution:
Correct Answer : 2319

Answer key/Solution

Let total capital be Rs.x, total time = 12 months.
According to the question, 2x/3 × 8 : x/6 × 4 : (1– 2/3 – 1/6)x × 12 = 16 : 2 : 6 = 8 : 1 : 3
Anil's share = 8/12 of the total profit = Rs.9,276
Hence, required difference between the shares of Kamal and Sunil = 1/4 × 9,276 = Rs.2,319.

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Q.12 [11831809]

In an examination, Shiv wrote 6 papers having equal maximum possible marks. The marks he secured in these papers are in the ratio 4 : 6 : 7 : 8 : 11 : 12. The average of his highest and lowest scores is 48%. Find the number of papers in which he scored not less than 60%.

1 ◯ 0

2 ◯ 1

3 ◯ 2

4 ◯ 3

Solution:

Correct Answer : 3

Answer key/Solution

Let the marks be 4x, 6x, 7x, 8x, 11x and 12x.
The average of highest and lowest score is 8x.

So, $8x = \frac{48}{100}$, $x = \frac{6}{100}$

Therefore, marks are:
24/100, 36/100, 42/100, 48/100, 66/100, 72/100.
In two subjects he scored more than 60%.

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Q.13 [11831809]

Three equal circles are placed inside an equilateral triangle such that any circle is tangential to two sides of the equilateral triangle and to two other circles. If the total area of the three circles is 3π sq. cm, then what is the area of the equilateral triangle outside the three circles?

1 ◯ $\sqrt{3}(4 + \sqrt{3}) - 3\pi$

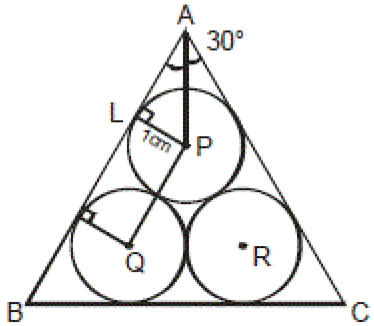
2 ◯ $2\sqrt{3}(2 + \sqrt{3}) - 3\pi$

3 ◯ $2(4 + \sqrt{3}) - 3\pi$

4 ◯ $\sqrt{3}(4 + 3\sqrt{3}) - 3\pi$

Solution:
Correct Answer : 2

Answer key/Solution



Let r be the radius of the equal circles.
Then, area of three circles = 3π
$\Rightarrow 3\pi \times r \times r = 3\pi$
$\Rightarrow r = 1$ cm
In triangle ALP,
PL/AL = tan 30°
$\Rightarrow$ 1/AL = $1/\sqrt{3}$
$\Rightarrow$ AL = $\sqrt{3}$ cm
So side length of the equilateral triangle ABC
= AB = $\sqrt{3} + 1 + 1 + \sqrt{3} = 2(\sqrt{3} + 1)$ cm
So area of the equilateral triangle
$= \frac{\sqrt{3}}{4} \times 2(\sqrt{3}+1) \times 2(\sqrt{3}+1) = 2\sqrt{3}(2 + \sqrt{3})$ sq. cm.
Hence, area of the equilateral triangle outside the three circles
= $[2\sqrt{3}(2 + \sqrt{3}) - 3\pi]$ sq. cm.

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Q.14 [11831809]

Four friends A, B, C and D live in a shared accommodation. At the end of a particular month A paid 40% of the rent. Of the remaining rent, B paid 50%. Of the remaining rent, C paid 70%. D paid the remaining Rs. 918. What is the average monthly rent (in Rs.) paid by the four friends?

1 ◯ 2,250

2 ◯ 2,450

3 ◯ 2,550

4 ◯ 3,250

Solution:

Correct Answer : 3

Answer key/Solution

Let the total rent be Rs. x, so A paid Rs. 0.4x.
Since B paid 50% of the balance, B paid Rs. 0.5(0.6x) = Rs. 0.3x
Remaining rent = x – 0.4x – 0.3x = Rs. 0.3x
Since C paid 70% of this, D would have paid the remaining 30% of it.
$\therefore$ C paid 0.7(0.3x) = Rs. 0.21x and D paid Rs. 0.09x.
Given that: 0.09x = 918 $\Rightarrow$ x = Rs.10,200
Hence, required average = 10200/4 = Rs.2,550.

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Q.15 [11831809]

If $\log_2 x + \log_2 y \geq 6$, then what is the least value of x + y?

Solution:

Correct Answer : 16

Answer key/Solution

$\log_2 x + \log_2 y \geq 6$

or, $\log_2 xy \geq 6$

or, $xy \geq 2^6$

or, $\sqrt{xy} \geq 2^3$

Since AM $\geq$ GM

Therefore, $\frac{x+y}{2} \geq \sqrt{xy}$

or, $x + y \geq 2\sqrt{xy} \geq 16$

Hence, the least value of x + y is 16.

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Q.16 [11831809]

Sanjeev brought two mobile phones having different prices for a total cost of Rs. 60,000. By selling one for 3/4 of its cost price and another for a profit of 25%, he earned a profit of Rs. 5,000 on the whole transaction. Find his % profit if he had sold the phone with the lower price at no profit and no loss.

1 ◯ 15%

2 ◯ 16.67%

3 ◯ 11.11%

4 ◯ 20%

Solution:
Correct Answer : 2

Answer key/Solution

Let the CP of one phone be Rs.x and the CP of the other be Rs.(60000 – x).
According to the question,
5x/4 + 3/4(60000 – x) = 65000
⇒ x/2 = 20000 ⇒ x = Rs.40,000
The CP of the other phone was Rs.20,000.
If he sold this at CP, then total SP = 50000 + 20000 = Rs.70,000
Hence, required percentage profit = 10000/60000 × 100 ≈ 16.67%.

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Q.17 [11831809]
For how many integer values of c, the equation $|x^2 - 6x - c| = 10$ has exactly two different real roots?

1 ◯ 2

2 ◯ 18

3 ◯ 19

4 ◯ 20

Solution:
Correct Answer : 3

Answer key/Solution

The equation $|x^2 - 6x - c| = 10$ has exactly two different real roots.
So $|(x - 3)^2 - 9 - c| = 10$
Therefore, $(x - 3)^2 - 9 - c = 10$ and $(x - 3)^2 - 9 - c = -10$
⇒ $(x - 3)^2 = c + 19$ and $(x - 3)^2 = c - 1$
For values of c from –18 to 0 integers, the given equation has exactly two different real roots.
Hence, the number of values of c is 19.

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Q.18 [11831809]
Let ‘abc’ and ‘pqr’ be two 3-digit positive integers such that all six digits are distinct. The sum of these two numbers is a 3-digit number S. What is the smallest possible value for the sum of the digits of S?

1 ◯ 3

2 ◯ 4

3 ◯ 5

4 ◯ 7

Solution:
Correct Answer : 2

Answer key/Solution

Given: abc + pqr = S. Assume abc < pqr. The hundred's digits of these numbers i.e., a and p must be at least 1 and 2 respectively, so abc ≥ 100 and pqr ≥ 200. (Since smallest possible sum of digits of S has to be found.)
bc + qr = 100 such that b, c, q and r have different digits which are not 1 or 2.
There are many solutions, but b = 0, c = 3, q = 9 and r = 7, the numbers will be 103 + 297 = 400. The minimum possible sum of digits is 4.

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Q.19 [11831809]
Find the number of common terms between the two sequences S1 = {21, 25, 29, ..., 421} and S2 = {16, 21, 26, ..., 471}.

Solution:
Correct Answer : 21

Answer key/Solution

For S1, the first term 'a1' = 21 ; common difference 'd1' = 25 – 21 = 4; last term 'l1' = 421
Number of terms in S1 = (421 – 21)/4 + 1 = 101 terms
For S2, the first term 'a2' = 16 ; common difference 'd2' = 21 – 16 = 5; last term 'l2' = 471
n th term of the first sequence 21 + (n – 1)4 = 4n + 17
m th term of the second sequence 16 + (m – 1)5 = 5m + 11
Common terms between the two sequences = 4n + 17 = 5m + 11 ⇒ 5m = 4n + 6
Possible integral values of n that satisfy 5m = 4n + 6 are {1, 6, 11...101}
where a = 1; d = 6 – 1 = 5; l = 101
Number of common terms = (101 – 1)/5 + 1 = 21.

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Q.20 [11831809]
A vessel contains a mixture of liquid A and liquid B in the ratio 11 : 14 respectively. Some of the mixture is withdrawn and some amount of liquid C is added and then the ratio of liquids A, B and C becomes 22 : 28 : 19 respectively. After adding liquid C, the total amount of mixture in the vessel is 15 liters less than the initial amount of mixture. If the amount of liquid A taken out from vessel is 153 liters less than the amount of liquid C added to the vessel, then find the amount of liquid B (in liters) initially in the vessel.

1 ◯ 420

2 ◯ 528

3 ◯ 680

4 ◯ 588

Solution:
Correct Answer : 4

Answer key/Solution

Let, the amount of liquid A and liquid B initially in the container = 11x liters and 14x liters, respectively.
And, the amount of liquid taken out from the container = y liters
Also, the amount of liquid A, liquid B and liquid C after adding liquid C in the container = 22z liters, 28z liters and 19z liters, respectively.
So, $25x = 69z + 15$
And $25x - 50z = y \Rightarrow y + 50z = 25x$
$\Rightarrow y + 50z = 69z + 15 \Rightarrow 19z = y - 15$
Also, $19z = 11y/25 + 153 \Rightarrow y - 15 = 11y/25 + 153$
$\Rightarrow 14y/25 = 168 \Rightarrow y = 300$
So, $300 = 19z + 15 \Rightarrow z = 15$
So $25x = 300 + 750 \Rightarrow x = 1050/25 = 42$
Hence, the amount of liquid B initially in the vessel = $42 \times 14 = 588$ liters.

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Q.21 [11831809]

Let S be the set of all fractions p/q such that p and q are relatively prime positive integers. How many fractions in S are such that if both p and q are increased by 1, the value of the fraction increases by 10%?

Solution:
Correct Answer : 1

Answer key/Solution

Given: $\frac{p+1}{q+1} = \frac{11}{10} \times \frac{p}{q}$
Cross multiply and combining like terms to get: $pq + 11p - 10q = 0$.
This can be further factored into $(p - 10)(q + 11) = -110$.
Since $p, q > 0$, so $p - 10 > -10$ and $q + 11 > 11$.
Using the factors of -110, $(-1, 110)$, $(-2, 55)$, $(-5, 22)$.
For $(-1,110)$, $p = 9$ and $q = 99$, they are not relatively prime.
For $(-2, 55)$, $p = 8$ and $q = 44$, they are not relatively prime.
For $(-5, 22)$, $p = 5$ and $q = 11$, they are relatively prime.
Therefore, only one such fraction is there.

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Q.22 [11831809]

The capacity of four taps B, C, D and E is 2, 3, 4 and 5 times of tap A respectively. If A, C, and E act as input pipes and B and D act as output pipes, time required to fill the tank is ‘m’. If C, D, E act as input pipes and A and B act as output pipes, then time required to fill the tank is ‘n’. If A and B working together as input pipes can fill the tank in 4 hours, then what is the value of $|m - n|$?

1 ◯ 4 hours 40 minutes

2 ◯ 3 hours 20 minutes

3 ◯ 2 hours 40 minutes

4 ◯ 4 hours 20 minutes

Solution:

Correct Answer : 3

Answer key/Solution

Let tap B takes x hours to fill the tank. So A = 2x, B = x, C = 2x/3, D = 2x/4, E = 2x/5.
Since A and B together can fill the tank in 4 hours, therefore,
1/2x + 1/x = 1/4 , which implies x = 6.
Calculate time 'm' and 'n'.

$1/m = 1/12 + 1/4 + 5/12 - 1/6 - 1/3 \Rightarrow m = 4$ hours

and $1/n = 1/4 + 1/3 + 5/12 - 1/12 - 1/6 \Rightarrow n = 4/3$ hours

Hence, $|m - n| = 8/3$ = 2 hours 40 minutes.

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