Sec 1

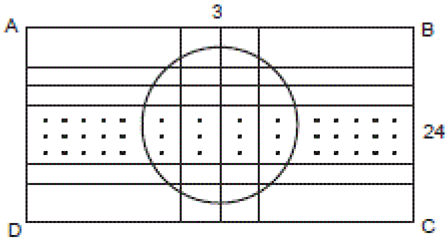
Q.1 [11831809]

A circle is drawn inside a rectangle ABCD and does not touch its sides. There are 24 parallel secants drawn in the circle, and three parallel secants are perpendicular to the first set of secants. All secants have their end points on the perimeter of the rectangle. The maximum number of regions that the rectangle is divided into is ________.

Solution:

Correct Answer : 154

Answer key/Solution



Required number of regions = 25 × 4 + 4 × 2 + 23 × 2 = 154.

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Q.2 [11831809]

A swimming pool is 120 feet long. John and Terry start from the opposite ends of the pool at speeds of 5 feet/s and 4 feet/s respectively and swim for 15 minutes. How many times do they cross each other?

1 ◯ 37

2 ◯ 35

3 ◯ 34

4 ◯ 33

Solution:

Correct Answer : 3

Answer key/Solution

John takes 24 s to complete one Lap.
Terry takes 30 s to complete one Lap.

In 15 minutes John completes $\frac{60 \times 15}{24}$ = 37.5 Laps.

Terry complete $\frac{60 \times 15}{30} = 30$ Laps

In the 1st 5 laps of John, they meet 5 times.
2nd 5 laps of John, they meet 4 times.
Both of them are back to their original positions.
So when John finishes 30 laps, they meet 27 times.
When John finishes more laps, they meet 5 more times.
In his 36, 37th and 38th laps they meet 0, 1, 1 times.
Hence, the answer is 34.

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Q.3 [11831809]

N is a natural number such that 300 < N < 600. P is the sum of N and the number formed by reversing its digits. The number formed by reversing the digits of N is also a three digit number. If P is equal to ‘k’ times the sum of the digits of N which is 9, find the minimum possible value of k.

1 ◯ 20

2 ◯ 47

3 ◯ 65

4 ◯ 56

Solution:

Correct Answer : 4

Answer key/Solution

Let the number N be denoted by ‘abc’. Therefore, the number formed by reversing the digits of N will be ‘cba’

Value of ‘abc’ + value of ‘cba’ = 101(a + c) + 20b

= k (a + b + c)

$$\Rightarrow k = \frac{101(a+c)+20b}{a+b+c} = \frac{20(a+b+c)}{(a+b+c)} + \frac{81(a+c)}{(a+b+c)} = 20 + \frac{81(a+c)}{9} = 20 + 9(a+c)$$

(Since a + b + c = 9)

k will be minimum when (a + c) is minimum and ‘b’ is maximum.

Minimum possible value of (a + c) = 3 + 1 = 4.

Hence, minimum value of k = 20 + 9 × 4 = 56.

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Q.4 [11831809]

Seven students appeared for an exam in which the maximum marks were 100. The marks scored by the students in the exam were found to be seven distinct integer values. The arithmetic mean of their marks was 85 whereas the median of their marks was 90. What could be the maximum difference between the marks of any two students?

Solution:

Correct Answer : 69

Answer key/Solution

The median of their marks was 90, therefore, one student must have scored 90, three must have scored more than 90 and three less than 90.

If the difference in the marks scored by any two students is to be maximized, then the sum of the marks of the top six students must be made as high as possible in order to arrive at the lowest possible marks for the seventh student.

Therefore, marks of the top six students can be 100, 99, 98, 90, 89 and 88

∴ To maximize the difference the marks of the seventh student = 85 × 7 – (100 + 99 + 98 + 90 + 89 + 88) = 31.

Hence, the required maximum difference = 100 – 31 = 69.

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Q.5 [11831809]

There are 10 boxes – B1, B2, B3, ..., B10 – with a certain number of balls each. The number of balls in the nth box is 'n' more than that in (n – 1)th box, for n > 1. If the number of balls in B4 is 15, what is the total number of balls in 10 boxes put together?

Solution:

Answer key/Solution

Correct Answer : 270

Since the number of balls in the nth box is 'n' more than that in (n – 1)th box, for n > 1.

B4 = 15

So B3 = 11, B2 = 8, B1 = 6, ... , B10 = 60

Hence, the total number of balls in 10 boxes = 6 + 8 + 11 + 15 + ... + 60 = 270.

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Q.6 [11831809]

Let f(x) + f(2x) + f(1 + x) + f(2 – x) = x for all x. What is the value of 16 × f(0)?

1 ○ −8

2 ○ −4

3 ○ 0

4 ○ 4

Solution:

Answer key/Solution

Correct Answer : 2

We have,

$f(x) + f(2x) + f(1 + x) + f(2 - x) = x$ ... (i)

Putting x = 0 in (i), we get

$f(0) + f(0) + f(1) + f(2) = 0$

$\Rightarrow 2f(0) + f(1) + f(2) = 0$ ... (ii)

Putting x = 1 in (i), we get

$f(1) + f(2) + f(2) + f(1) = 1$

$\Rightarrow f(2) + f(1) = \frac{1}{2}$ ... (iii)

From (i) and (iii),

$2f(0) = -\frac{1}{2} \Rightarrow f(0) = \frac{-1}{4}$.

Hence, 16 × f(0) = 16 × (–1/4) = –4.

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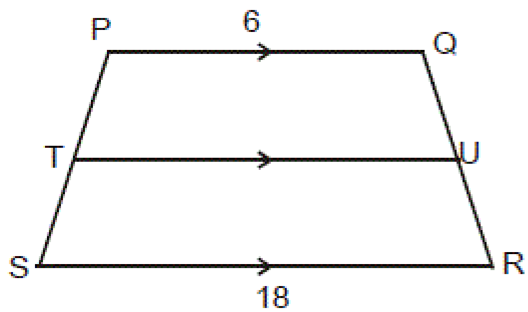
Q.7 [11831809]



PQRS is a trapezium with PQ || TU || SR. If the ratio of the area PQUT : TURS = 3 : 1. Find PT/TS.

1 ○ $\frac{3-\sqrt{7}}{\sqrt{7}-1}$

2 ○ $\frac{\sqrt{7}-1}{3-\sqrt{7}}$

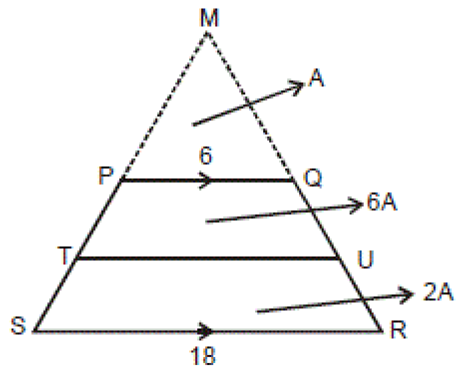
3 ○ 3

4 ○ 1/3

Solution:

Correct Answer : 2

Answer key/Solution



$\Delta MPQ \sim \Delta MSR$

So $\frac{Ar(MPQ)}{Ar(MSR)} = \left(\frac{6}{18}\right)^2 = \frac{1}{9}$

So Ar(MPQ = A, then Ar(PQUT) = 6A; Ar(TURS) = 2A

So $\frac{MP}{MT} = \left(\frac{A}{7A}\right)^{1/2} = \frac{1}{\sqrt{7}}$

Hence, $\frac{PT}{TS} = \frac{\sqrt{7}-1}{3-\sqrt{7}}$.

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Q.8 [11831809]

The volumes of two containers - A and B - are 36 liters and 24 liters, which contain milk and water solutions. If the entire quantity of these two solutions is mixed, the concentration of milk in the resulting solution is 40%. If 18 liters of each of the two solutions are mixed, the amount of Milk in the resulting solution is 15 liters. What is the concentration of milk of the 24 liters mixture?

1 ◯ 33.33%

2 ◯ 50%

3 ◯ 66.66%

4 ◯ 75%

Solution:

Correct Answer : 2

If the entire volumes of A and B are mixed, milk constitutes 40% of 60 liters which is 24 liters.

Note that the resultant solution on mixing 18 liters each of A and B will have the same concentration of milk as mixing 24 liters of A and 24 liters of B.

If 24 liters of A are mixed with the entire 24 liters of B, milk constitutes 15/36 i.e., 5/12 of 48 liters, which is 20 liters.

Therefore, the extra 12 liters of A contain the extra 4 liters of milk.

So 36 liters of A contain 12 liters of milk and hence 24 liters B contains 12 liters of milk.

Hence, concentration of milk of the 24 liters mixture is 12/24 i.e., 50%.

Answer key/Solution

Bookmark FeedBack

Q.9 [11831809]

In a school there are 30 Quiz teams of 5 each and N cultural teams of 4 each. Every student in the school is a member of exactly 6 quiz teams and exactly 8 cultural teams. Find the value of N.

Solution:

Correct Answer : 50

In a school there are 30 quiz teams of 5 each, therefore, the total number of students will be 30 × 5 = 150.

Also, every student is a part of exactly 6 quiz teams, so, 150/6 = 25 distinct students.

Now, these 25 students are part of exactly 8 cultural teams.

Hence, N = 25 × 8/4 = 50.

Answer key/Solution

Bookmark FeedBack

Q.10 [11831809]

How many points (a, b) where a and b are whole numbers, are on or inside the region bounded by x-axis; x = 5 and the graph $y = x^2$?

1 ◯ 54

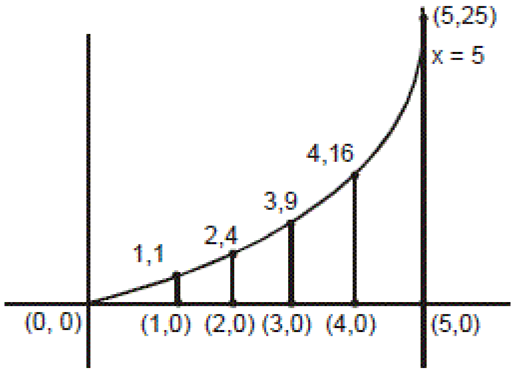
2 ◯ 55

3 ◯ 60

4 ◯ 61

Solution:

Correct Answer : 4



All the points (a, b) are on the lines shown in the figure.
So the total number of points are
1 + 2 + 5 + 10 + 17 + 26 = 61.

Answer key/Solution

Bookmark FeedBack

Q.11 [11831809]

Ashish had Rs. 100 which he completely used up to buy a, b, c quantities of toffees, chocolates and packets of chips each of which cost him Re.1, Rs. 15 and Rs. 25 respectively. If he bought at least one unit of each item, the number of triplets (a, b, c) that is possible is

1 ◯ 14

2 ◯ 8

3 ◯ 12

4 ◯ 4

Solution:
Correct Answer : 2
Given: a + 15b + 25c = 100
We can deduce from the equation that 1 = c < 4 as a, b, c ≥ 1.
When c = 1:
The equation reduces to a + 15b = 75. 'b' can acquire the values 1, 2, 3 and 4.
Hence, the possible triplets are (60, 1, 1); (45, 2, 1); (30, 3, 1), (15, 4, 1).
When c = 2:
The equation reduces to a + 15b = 50. 'b' can acquire the value of 1, 2 and 3.
Hence, the possible triplets are (35, 1, 2), (20, 2, 2), (5, 3, 2).
When c = 3:
The equation reduces to a + 15b = 25. 'b' can acquire the value of 1.
Hence, the only possible triplet is (10, 1, 3).
Hence, in all there are 8 ordered triplets (a, b, c) possible.

Answer key/Solution

Bookmark FeedBack

Q.12 [11831809]

In a triangle ABC right-angled at B, the bisector of the external ∠CAF, when produced, intersects the base CB (extended) at E. If AB = 6 cm and AC = 10 cm, then find the length (in cm) of the line segment AE.

1 ◯ 10

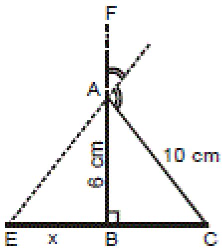
2 ◯ $8\sqrt{3}$

3 ◯ $10\sqrt{2}$

4 ◯ $6\sqrt{5}$

Solution:
Correct Answer : 4

Answer key/Solution



Let, the length of BE be 'x' cm.

$BC = \sqrt{AC^2 - AB^2} = 8$ cm

Using the external angle bisector theorem in ΔABC, we get

$$\frac{AB}{AC} = \frac{BE}{CE} \Rightarrow \frac{6}{10} = \frac{x}{x+8} \Rightarrow x = 12 \text{ cm}$$

$$\Rightarrow AE = \sqrt{AB^2 + BE^2} = \sqrt{180} \text{ cm} = 6\sqrt{5} \text{ cm.}$$

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Q.13 [11831809]

If xy + yz + zx = 0, then the value of $\left(\frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy}\right)$ is

1 ◯ 0

2 ◯ 1

3 ◯ 3

4 ◯ x + y + z

Solution:
Correct Answer : 1

Answer key/Solution

Given, $xy + yz + zx = 0$
$\Rightarrow yz = -xy - zx$
Similarly, $xy = -yz - zx$ and $zx = -xy - yz$
Substituting these values in the original equation,

$$\frac{1}{x^2-(-xy-zx)}+\frac{1}{y^2-(-xy-yz)}+\frac{1}{z^2-(-yz-zx)}$$

$$\Rightarrow \frac{1}{x(x+y+z)}+\frac{1}{y(x+y+z)}+\frac{1}{z(x+y+z)}$$

$$\Rightarrow \frac{yz+xz+xy}{xyz(x+y+z)}=0.$$

Alternate method:
Substituting $x = z = 2$
$\Rightarrow y = -1$

On substituting the value of x, y and z we get $\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}=\frac{1}{6}-\frac{1}{3}+\frac{1}{6}=0$

The result would be the same for all values of x, y and z.

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Q.14 [11831809]
A = $\log_{10}(1 + 2 + 3 + ... + n) + \log_{10}2$, where n is a natural number. Find the number of possible values of n for which 2 < A < 3.

Solution:
Correct Answer : 22

Answer key/Solution

$A = \log_{10}(1 + 2 + 3 + \ldots + n) + \log_{10}2$

$= \log_{10}[2 \times (1 + 2 + 3 + \ldots + n)] = \log_{10}\left[2\times\frac{n(n+1)}{2}\right]$

$= \log_{10} n(n + 1)$.
By the question, $2 < A < 3 \Rightarrow 100 < n(n + 1) < 1000$.
It is possible only for values of n from 10 to 31, i.e. there are (31 – 10) + 1 = 22 possible values of n.

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Q.15 [11831809]
In a quadrilateral ABCD, AC and BD intersect at O, where AB = 32 cm, AO = 30 cm, OB = 10 cm, BC = 26 cm, OC = 24 cm, OD = 18 cm and CD = 30 cm. If it is given that one of the dimensions out of AB, BC and CD is incorrect while all the other dimensions are correct, then the incorrect dimension is of side ___.

1 ◯ AB

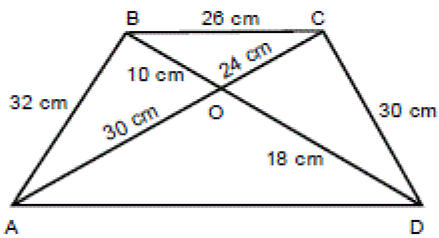
2 ◯ BC

3 ◯ CD

4 ◯ Cannot be determined

Solution:
Correct Answer : 1

🔑 Answer key/Solution



Note that $24^2 + 10^2 = 26^2$ and $24^2 + 18^2 = 30^2$

∴ At least one of the triangles BOC and COD will be a right angled triangle with the right angle at O. But if one of them is a right angled triangle, then all the other triangles must also be right angled triangles.

However, in ΔABO, $10^2 + 30^2 \neq 32^2$

Hence, AB is written incorrectly.

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Q.16 [11831809]

In 2020 the ratio of tariff to number of existing users was 1 : 40 for vfone. In 2020, when vfone increased the tariff by 20%, the number of users decreased by 20%. In 2021, vfone again increased its tariff pack by 10% and number of users further decreased by 15%. Which of the following could be the current tariff among the choices, if tariff is always a natural number? (in Rs.)

1 ◯ 770

2 ◯ 660

3 ◯ 682

4 ◯ 894

Solution:
Correct Answer : 2
Let 'x' and 'y' be the amount of tariff pack and number of existing users.
Given:
$x \times 1.2 \times 1.1$ = current amount of tariff pack
$y \times 0.8 \times 0.85$ = current number of existing users
Let $x = a$ and $y = 40a$.
Then,
$x \times 1.2 \times 1.1 = 1.32a$
$y \times 0.8 \times 0.85 = 27.2a$
Ratio of current amount of tariff pack and number of existing users is 33 : 680. Therefore, current amount of tariff pack must divisible by 3 and 11 both, which is possible only in option (2).
Hence, the current amount of tariff pack can be Rs. 660.

Answer key/Solution

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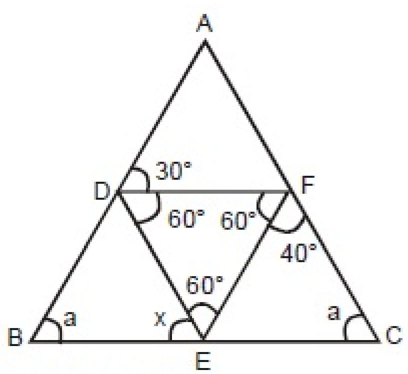
Q.17 [11831809]

ABC is an isosceles triangle with AB = AC. DEF is an equilateral triangle with its vertices D, E, F on AB, BC, AC respectively. If $\angle CFE = 40°$ and $\angle ADF = 30°$, then what is the angle $\angle DEB$ equal to (in degrees)?

Solution:
Correct Answer : 35



$a + 40° = x + 60°$
$40° - x = x - 30°$
$a + x = 60° + 30°$
$\Rightarrow x = 35°$.

Answer key/Solution

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Q.18 [11831809]

If $|x - 1| - |x| + |2x + 3| \geq 2x + 4$, where $x \geq -10$, then the possible integer values of x is

1 ○ 10

2 ○ 11

3 ◯ 9

4 ◯ 8

Solution:

Correct Answer : 2

Answer key/Solution

There are four possibilities.

Case I: When $x \geq 1$:

$(x - 1) - x + 2x + 3 \geq 2x + 4 \Rightarrow 2 \geq 4$ (not possible)

Case II: When $0 < x < 1$:

$1 - x - x + 2x + 3 \geq 2x + 4$

$\Rightarrow 4 - 2x \geq 4$, or $-2x \geq 0$ (not possible)

Case III: When $-\frac{3}{2} \leq x \leq 0$:

$1 - x + x + 2x + 3 \geq 2x + 4$

$4 \geq 4$ (possible)

Case IV: When $x \leq -\frac{3}{2}$:

$1 - x + x - 2x - 3 \geq 2x + 4$

$\Rightarrow -6 \geq 4x \Rightarrow x \leq -\frac{3}{2}$ (possible)

From the above cases, it can be concluded that the required range is $x \leq 0$.

So $-10 \leq x \leq 0$.

Hence, 11 integer values of x are possible.

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Q.19 [11831809]

From the set of the first 9 natural numbers, three distinct prime numbers a, b and c are selected to form a quadratic equation of the form $ax^2 + bx + c = 0$, having real roots. How many such equations can be formed?

Solution:

Correct Answer : 6

Answer key/Solution

For distinct real roots of a quadratic equation, we must have $b^2 > 4ac$.

As a, b and c are distinct prime numbers from 1 to 9, the possible values of ordered triplets (a, b, c) are

(2, 5, 3), (3, 5, 2), (2, 7, 3), (3, 7, 2), (2, 7, 5), (5, 7, 2).

Hence, there are six such quadratic equations.

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Q.20 [11831809]

Aman, Bhaskar takes a, b days to complete a job independently. If they take turns to work independently on the job, on alternate days the task is completed in 8 days. If a + b > 20 days, find the time taken by Bhaskar to finish the task if he is the slower among the two.

Solution:
Correct Answer : 20
Since they finish the task in 8 days.
1/a + 1/b = 1/4
Since in 2 days they finish 1/4th of the task.
So we get, 4a + 4b = ab or ab – 4a – 4b = 0
Adding to both side
We get
ab – 4a – 4b + 16 = 16
(a – 4)(b – 4) = 16
= 1 × 16 (a = 5, b = 20)
= 2 × 8 (a = 6; b = 12)
= 4 × 4 (a = 8; b = 8)
Hence, Bhaskar takes 20 days to finish the task.

Answer key/Solution

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Q.21 [11831809]
A group comprising some women and men can complete a piece of work in 96 days. If a man in the group is replaced by a woman, the resulting group can complete the same work in 72 days. If two men in the original group are replaced by two women, then in how many days can the resulting group complete the work?

1 ◯ 48

2 ◯ 57.6

3 ◯ 60

4 ◯ 67.6

Solution:
Correct Answer : 2

Answer key/Solution

Let, a woman and a man can complete the work alone in 'W' and 'M' days respectively and the group initially has 'n' women and 'm' men.

$\therefore \frac{n}{W}+\frac{m}{M}=\frac{1}{96}$ ...(i)

$\Rightarrow \frac{(n+1)}{W}+\frac{(m-1)}{M}=\frac{1}{72}$ ...(ii)

Assume that after two men have been replaced by two women, the group can complete the same work in 'p' days.

$\Rightarrow \frac{(n+2)}{W}+\frac{(m-2)}{M}=\frac{1}{p}$ ...(iii)

Now, (ii) – (i) = (iii) – (ii)

$\Rightarrow \frac{1}{72}-\frac{1}{96}=\frac{1}{p}-\frac{1}{72} \Rightarrow p = 57.6.$

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Q.22 [11831809]

The function f(x) is defined for all real values of x as f(x) = min (2x + 1, x + 2, −4x + 8). What is the maximum value of f(x)?

1 ◯ 3

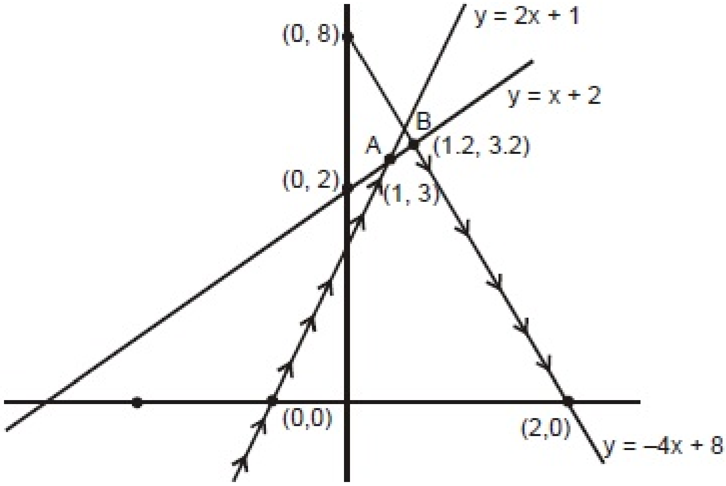
2 ◯ 3.2

3 ◯ 3.4

4 ◯ 4

Solution:
Correct Answer : 2

Answer key/Solution



Finding point of intersection A of y = 2x + 1 and y = x + 2
We get (1, 3)
f(x) at this point = 3
Finding point of intersection B of y = x + 2 and y = –4x + 6

We get, 5x = 6; $x = \frac{6}{5}; y = \frac{16}{5}$

f(x) at this point = 3.2
Hence, maximum value of the function is at B = 3.2.

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