CHAPTER – 9

STATISTICS

Statistics deals with collection, classification, presentation, analysis and interpretation of numeric data (quantitative data). The quantitative data occurs in three forms namely (A) Individual series (B) Discrete series and (C) Continuous series.

This chapter presents various statistical constants and methods of computing them for an individual series. These statistical constants that concisely describe any given group of data fall into two categories.

(I) An average, or a measure of central tendency which indicates the central value of t he size of a typical member of the group.

(II) A measure of dispersion which indicates the extent to which the different items of the group are spread about the average.

I. Measures of Central Tendencies:

The measures we discuss here are
(A) Arithmetic Mean,
(B) Geometric Mean,
(C) Harmonic Mean,
(D) Median and
(E) Mode.

1. Arithmetic Mean (A.M.) $(\bar{x})$:

Given $x_1, x_2, \ldots\ldots\ldots, x_n$ (n individual items)

$$\boxed{\text{A.M.} = \bar{x} = \frac{x_1 + x_2 + \ldots. + x_n}{n}}$$

or $\bar{x} = \dfrac{\text{Sum of the observations}}{\text{The number of observations}}$

Examples:

(i) The arithmetic mean of (4, 7, 8, 14, –13) is
$= \dfrac{4+7+8+14+(-13)}{5} = \dfrac{20}{5} = 4$

(ii) The arithmetic mean of (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) is
$= \dfrac{1+2+3+4+5+6+7+8+9+10}{10} = \dfrac{55}{10} = 5.5$

Note:

(a) The algebraic sum of deviations about the mean is 0 or $\Sigma(x - \bar{x}) = 0$.

(b) The arithmetic mean of two numbers a, b is $\dfrac{a+b}{2}$.

(c) If b = AM of (a, c), then a, b and c are in arithmetic progression.

2. Geometric Mean (G.M.):

Given $x_1, x_2, \ldots\ldots\ldots, x_n$ (n individual items all being positive)

$$\boxed{\text{G.M.} = (x_1 \cdot x_2 \cdot \ldots\ldots, x_n)^{1/n}}$$

or G.M. = n^{th} root of the product of the numbers.

Examples:

(i) The geometric mean of (10, 20, 40) is
$= (10 \times 20 \times 40)^{1/3}$
$= (10 \times 20 \times 2 \times 20)^{1/3}$
$= (20 \times 20 \times 20)^{1/3} = 20$

(ii) The geometric mean of (3, 5, 15, 45, 75) is
$= (3 \times 5 \times 3 \times 5 \times 3^2 \times 5 \times 3 \times 5^2)^{1/5}$
$= (3^5 \times 5^5)^{1/5}$
$= 3 \times 5 = 15$

Note:

(a) Geometric mean is not very commonly used as it involves finding the n^{th} root, and hence requires complex calculations for higher values of n.

(b) The geometric mean of two positive numbers a, b is $\sqrt{ab}$.

(c) If b = GM of (a, c), then a, b and c are in geometric progression.

3. Harmonic Mean (H.M.):

Given $x_1, x_2, \ldots, x_n$ (n individual observations such that none of them is equal to 0),

$$\boxed{\text{H.M.} = \frac{n}{1/x_1 + 1/x_2 + \ldots\ldots\ldots. + 1/x_n}}$$

Examples:

(i) The harmonic mean of (1, 2, 3, 4, 5) is
$= \dfrac{5}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}} = \dfrac{5 \times 60}{60+30+20+15+12}$
$= \dfrac{300}{137}$

(ii) The harmonic mean of $\left(\dfrac{1}{1}, \dfrac{1}{3}, \dfrac{1}{9}, \dfrac{1}{27}\right)$
$= \dfrac{4}{1+3+9+27} = \dfrac{4}{40} = \dfrac{1}{10}$

Note:

(a) HM of two numbers a, b is $\dfrac{2ab}{a+b}$.

(b) If b = HM of (a, c), then a, b and c are in harmonic progression.

(c) For any two positive numbers a, b
(i) AM ≥ GM ≥ HM.
(ii) $(GM)^2 = (AM)(HM)$.

4. Median:

Median is the magnitude of the "middle-most" item in a series of values, when the values have been arranged in order of their magnitude. When there are odd number of observations, the middle number, when the values are arranged in ascending or descending order, is the median. When there are even number of observations, the average of the two numbers, at the middle when the values are arranged in ascending or in descending order is the median.

Examples:

(i) The median of 4, 7, 12, 15, 20 is 12.

(ii) The median of 3, 5, 9, 11, 13, 15 is $\frac{9+11}{2}$ = 10.

Note:
(a) The median divides the distribution into two equal parts.
(b) Median is suitable for qualitative data as well.

5. Mode:

It is the item which is "most often" found in the given set of observations, i.e., the value occurring the highest number of times.

Examples:

(i) For the observations; 2, 1, 1, 2, 3, 4, 3, 2, 1, 2, 2, 1, 4, 6, 7.
Mode = 2
(ii) For the observations; 1, 2, 2, 3, 3, 3, 4, 4, 4, 4
Mode = 4
(iii) Consider the observations 5, 7, 11, 25, 36, 16.

Here no item occurs more than once. So, mode is ill-defined.

Empirical Formula:

Mode = 3Median – 2Mean.

This formula is valid for the distribution which are moderately symmetric. (symmetry being coincidence of mean, median and mode)

II. Measures of Dispersion:

The measures we discuss here are
(A) Range,
(B) Quartile Deviation,
(C) Mean Deviation and
(D) Standard Deviation/Variance.

1. Range:

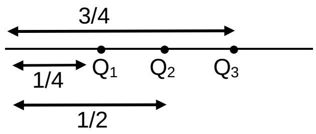
Given, $x_1, x_2, \ldots\ldots, x_n$ (n individual observations).
Range = maximum value – minimum value

Example:

(i) Range (2, 3, 5, 6, 8, 11, 13) = 13 – 2 = 11.
(ii) Range (7, 4, 8, 1, 6, 11, 15) = 15 – 1 = 14.

2. Quartile Deviation (Q.D.) or Semi Inter Quartile Range:

Quartiles are those values, which divide the distribution into four equal parts, when the values are arranged in ascending or descending order of magnitude.



Q_1 is called the first quartile, Q_2 is the middle quartile and Q_3 is the third quartile. The second quartile is also referred to as the median.

As the name semi-inter-quartile range itself suggests
Q.D. = $\frac{Q_3 - Q_1}{2}$ (one-half the range of quartiles)
<u>for calculation:</u>

Q_1 = size of $\left(\frac{n+1}{4}\right)^{th}$ item

Q_3 = size of $3\left(\frac{n+1}{4}\right)^{th}$ item

Examples:

(i) Find the Q.D. of the observations 5, 9, 13, 15, 21, 23, 25.

Sol: Since there are 7 terms; n = 7

$Q_1 = \left(\frac{7+1}{4}\right)^{th}$ item = 2nd item = 9.

$Q_3 = 3\left(\frac{7+1}{4}\right)^{th}$ item = 6th item = 23.

$\therefore$ Q.D = $\frac{Q_3 - Q_1}{2} = \frac{23-9}{2} = \frac{14}{2}$ = 7.

(ii) Find the Q.D. of the observations 16, 2, 8, 24, 4, 32, 18.

Sol: Arranging the numbers in order, we get
2, 4, 8, 16, 18, 24, 32; n = 7
$\therefore$ Q_1 = 2nd item = 4 and Q_3 = 6th item = 24
$\therefore$ Q.D = $\frac{Q_3 - Q_1}{2} = \frac{24-4}{2} = \frac{20}{2}$ = 10.

Note:
The data is not in the ascending order (or in the descending order). So we arrange it first and then proceed.

(iii) Find the Q.D. of 3, 7, 13, 25, 27, 31.

Sol: The observations 3, 7, 13, 25, 27, 31.

$Q_1 = \left(\frac{6+1}{4}\right)^{th}$ item = $1^3/_4$th item

= 1st item + $\frac{3}{4}$ (2nd item – 1st item)

$Q_1 = 3 + \frac{3}{4}(7-3)$ = 6 and

$Q_3 = 3\left(\frac{6+1}{4}\right)^{th}$ item = $5^1/_4$th item

$= 5^{th}$ item + $\frac{1}{4}$ (6^{th} item – 5^{th} item)

$Q_3 = 27 + \frac{1}{4}(31 - 27) = 28.$

$\therefore$ Q.D = $\frac{Q_3 - Q_1}{2} = \frac{28-6}{2} = 11$.

3. Mean Deviation (M.D.):

The mean deviation is calculated about mean or median or mode. But by default mean deviation is about mean. Mean deviation is the average of deviations of each item in the data set from the mean.

$$\boxed{M.D. = \frac{\sum_{i=1}^{n} | x_i - A |}{n}}$$

A = mean / median / mode; n = number of items.

Examples:

(i) Find the mean deviation of 2, 5, 9, 11, 13.

Sol: Mean of the observations $\bar{x}$ = 40/5 = 8
So the mean deviation is

$$M.D = \frac{|2-8|+|5-8|+|9-8|+|11-8|+|13-8|}{5}.$$

$$= \frac{6+3+1+3+5}{5} = \frac{18}{5} = 3.6$$

(ii) Find the mean deviation of 2, 3, 5, 9, 11.

Sol: Mean of the observations $\bar{x}$ = 30/5 = 6
So the mean deviation is

$$M.D. = \frac{4+3+1+3+5}{5} = \frac{16}{5} = 3.2$$

Note:

(A) Mean Deviation of two numbers a, b = $\frac{|a-b|}{2}$.

(B) Mean deviation is based on each and every observation.

4. Standard Deviation (S.D.):

It is the root mean squared deviation taken about the mean.

S.D. = $\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$, where $x_1, x_2, x_3, \ldots\ldots x_n$ are the n items given.

The expression $\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$ also equals to $\sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$

Examples:

(i) Find the S.D. of (2, 5, 8, 11, 14).

Sol: Mean of the observations = 40/5 = 8

$$S.D. = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{(2-8)^2 + (5-8)^2 + (8-8)^2 + (11-8)^2 + (14-8)^2}{5}}$$

$$= \sqrt{\frac{36+9+0+9+36}{5}}$$

$$= \sqrt{\frac{90}{5}} = \sqrt{18} = 3\sqrt{2}$$

(ii) Find the S.D. of (2, 5, 7, 10, 13, 17).

Sol: Mean of the observations = 54/6 = 9

$$S.D. = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{(-7)^2 + (-4)^2 + (-2)^2 + (1)^2 + (4)^2 + (8)^2}{6}}$$

$$= \sqrt{\frac{49+16+4+1+16+64}{6}} = \sqrt{\frac{150}{6}}$$

$$= \sqrt{25} = 5$$

Note:

(A) The square of the standard deviation is variance.

(B) The standard deviation is non-negative.

Examples:

9.01. Find the mean, median and mode of the observations 2, 5, 9, 11, 13, 13, 17, 18.

Sol: Mean = $\bar{x} = \frac{\sum x_i}{n} = \frac{2+5+\ldots\ldots\ldots+18}{8}$

= 88/8 = 11

Median: The given numbers are in ascending order, with 11 and 13 as the middle terms,

$\therefore$Median = $\frac{11+13}{2} = \frac{24}{2} = 12$

Mode: As the observation 13 has the greatest frequency, 13 is the mode.

9.02. If the arithmetic mean of 28, 31, 36, 43, 30 and K is 35, then find the value of K.

Sol: $\bar{x} = \frac{28+30+31+36+43+K}{6} = 35$

$\Rightarrow 168 + K = 210$

or K = 210 – 168 = 42.

9.03. Find the arithmetic mean and the median of the first seven natural numbers.

Sol: The numbers are 1, 2, 3, 4, 5, 6, 7.
As the numbers are in an A.P. the arithmetic mean is the average of first and last terms. i.e., 4.
Since, the middle term is 4, the value of the median is also equal to 4.

9.04. Find the geometric mean of 3, 6, 24, 48.

Sol: G.M. = (3, 6, 24, 48)

$= (3 \times 6 \times 24 \times 48)^{1/4}$

$= (3 \times 2 \times 3 \times 2^3 \times 3 \times 2^4 \times 3)^{1/4}$
$= (2^8 \times 3^4)^{1/4} = 2^2 \times 3 = 12.$

9.05. Find the geometric mean of the observation 1, 2, 4, 8, ……., 256.

Sol: G.M. = $(2^0 . 2^1 . 2^2 ……2^8)^{1/9}$
$= (2^{0+1+2+ .....8})^{1/9} = (2^{36})^{1/9} = 2^4 = 16$

Note: Since 1, 2, 4, 8, …., 256 are in G.P., the geometric mean can be calculated as
G.M. = $\sqrt{\text{first term} \times \text{last term}}$
$\therefore$ G.M. = $\sqrt{1 \times 256}$ = 16

9.06. Find the harmonic mean of the observation 10, 20, 30, 40, 50.

Sol: H.M. (10, 20, 30, 40, 50)
$$= \frac{5}{\frac{1}{10}+\frac{1}{20}+\frac{1}{30}+\frac{1}{40}+\frac{1}{50}}$$
$$= \frac{5 \times 600}{60+30+20+15+12} = \frac{3000}{137}.$$

9.07. Show that A.M., G.M. and H.M. of two positive numbers a and b are in geometric progression.

Sol: For two positive numbers a and b,
A.M. = $\frac{a+b}{2}$; G.M. = $\sqrt{ab}$, H.M. = $\frac{2ab}{a+b}$
Now $(\text{G.M.})^2 = ab = \frac{a+b}{2} \times \frac{2ab}{a+b}$
$\therefore (\text{G.M.})^2 = \text{A.M.} \times \text{H.M.}$
Since $(\text{G.M.})^2 = \text{A.M.} \times \text{H.M.}$;
A.M., G.M. and the H.M. of two positive numbers forms a geometric progression.

9.08. Find the range, quartile deviation and mean deviation for the data 7, 11, 12, 14, 18, 24, 26.

Sol: Range = Maximum value – Minimum value
= 26 – 7 = 19

Quartile deviation:

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ item = 2^{nd} item = 11

$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ item = 6^{th} item = 24

$\therefore$ Q.D. = $\frac{Q_3 - Q_1}{2} = \frac{24-11}{2} = 6.5$

Mean deviation:
Mean of the observations = 112/7 = 16
$$\text{M.D.} = \frac{\sum |x_i - \bar{x}|}{n}$$
$$= \frac{|7-16|+|11-16|+|12-16|+|14-16| + |18-16|+|24-16|+|26-16|}{7}$$
$$= \frac{9+5+4+2+2+8+10}{7} = \frac{40}{7}$$

9.09. Find the standard deviation and variance of the observation 8, 12, 20, 24, 36.

Sol: Mean of the observation
$\bar{x} = \frac{\sum x}{n} = \frac{8+12+20+24+36}{5} = 20$
$\frac{\sum x^2}{n} = \frac{64+144+400+576+1296}{5} = \frac{2480}{5} = 496$
$\therefore$(S.D.) = $\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$
$= \sqrt{496 - 400} = \sqrt{96} = 4\sqrt{6}$
and variance = $(\text{S.D.})^2 = (4\sqrt{6})^2 = 96$

9.10. Find the range and quartile deviation for the following prices of 11 items in a super market: 13, 21, 42, 19, 23, 64, 56, 71, 85, 25, 9.

Sol: Arranging the prices in ascending order;
9, 13, 19, 21, 23, 25, 42, 56, 64, 71, 85
Range = maximum price – minimum price
= 85 – 9 = 76
$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ item = $\left(\frac{11+1}{4}\right)^{th}$ item
= 3^{rd} item = 19
$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ item = $3\left(\frac{11+1}{4}\right)^{th}$ item
= 9^{th} item = 64
Q.D. = $\frac{64-19}{2} = \frac{45}{2} = 22.5$

Concept Review Questions

Directions for questions 1 to 20: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The mid-value of the class 45 – 65 is $\boxed{}$.

2. The size of the class 12 – 22 is (the previous class is 2 – 12) $\boxed{}$.

3. The arithmetic mean of 3a, 3b, 3c is _____.
(A) 3 (a + b + c) (B) a + b + c
(C) $\frac{a+b+c}{3}$ (D) $a^3 + b^3 + c^3$

4. For a symmetric distribution, which of the following is true?
(A) mode = 2 median – 3 mean
(B) mode = 3 median + 2 mean
(C) mean = median = mode
(D) mean = mode + median

5. The empirical relation between mean, median and mode is
(A) mean + median = mode _____.
(B) mean = 3 median – 2 mode
(C) mode = 3 median – 2 mean
(D) None of these

6. The mode of the data 1, 2, 1, 5, 6, 7, 8, 7, 1 is $\boxed{}$.

7. The mode of the data 2, 5, 11, 12, 15 and 32 is _____.
(A) 2 (B) 32
(C) 30 (D) ill-defined

8. The geometric mean of a, b and c is _____.
(A) $(abc)^3$ (B) abc
(C) $\sqrt[3]{abc}$ (D) None of these

9. Find the geometric mean of the data 5, 75 and 9.
$\boxed{}$

10. If A, G and H are the arithmetic mean, geometric mean and harmonic mean respectively of two positive numbers a and b, then which of the following is true?
(A) $A \cdot G = H^2$ (B) $A^2 H^2 = G$
(C) $G \cdot H = A^2$ (D) $A \cdot H = G^2$

11. The range of the data 10, 8, 12, 23, 18, 35, 56, 82, 49, 76 is $\boxed{}$.

12. If the range and the maximum value of a set of numbers are 15 and 101 respectively, then the minimum value of the set is $\boxed{}$.

13. The second quartile of a distribution is equal to its
(A) Mean (B) Median
(C) Mode (D) Range

14. If the arithmetic mean of a set of 15 observations is 12, then the sum of the observations is _____.
(A) 180 (B) 150 (C) 240 (D) 160

15. If the standard deviation of 10, 20, 30, 40 and 50 is S, then the standard deviation of 20, 30, 40, 50 and 60 is _____.
(A) S (B) S + 10 (C) S – 10 (D) 10 S

16. The mean deviation of 30 and 40 is $\boxed{}$.

17. If the standard deviation of $x_1, x_2, \ldots, x_n$ is S, then the variance of $x_1 + c, x_2 + c, \ldots, x_n + c$ is _____.
(A) S (B) S^2 (C) 2 S (D) $\sqrt{S}$

18. If the range of the set $S = \{x_1, x_2, x_3, \ldots, x_n\}$ is R, then the range of the set $\{x_1 - 2, x_2 - 2, \ldots, x_n - 2\}$ is _____.
(A) R – 2 (B) R + 2 (C) R (D) 2 R

19. If the arithmetic mean of $x_1, x_2, \ldots, x_n$ is A, then the arithmetic mean of $x_1 + a, x_2 + a, \ldots, x_n + a$ is
(A) A + a (B) A – a (C) a . A (D) A

20. The arithmetic mean of the first 'n' natural numbers is 8 n = $\boxed{}$.

Exercise – 9(a)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The arithmetic mean of 24, 4, 12, 8, 36, 48, 20, 16, 28, 32, 40 and 44 is _____.
(A) 52 (B) 26 (C) 28 (D) 56

2. The first term and the common difference of an arithmetic progression are 3 and 4 respectively. Find the arithmetic mean of the first 15 terms of the arithmetic progression.

3. The arithmetic mean of a set of 17 observations is 20. If two observations 13 and 27 are discarded, then the arithmetic mean of the remaining observations is .

4. The arithmetic mean of the cubes of the first n natural numbers is _____.
(A) $\frac{(n+1)(2n+1)}{6}$ (B) $\frac{(n+1)^2}{4}$
(C) $\frac{n(n+1)^2}{4}$ (D) $\frac{n+1}{2}$

5. If the arithmetic mean of the series $a_1, a_2, \ldots a_n$ is A and the arithmetic mean of the series $b_1, b_2, \ldots b_n$ is B, then the arithmetic mean of the series $a_1 + b_1$, $a_2 + b_2$, ……, $a_n + b_n$ is _____.
(A) $\frac{A+B}{2}$ (B) A + B
(C) 2(A + B) (D) $\frac{A+B}{n}$

6. The arithmetic mean of the multiples of 7 between the numbers 100 and 200 is .

7. The arithmetic mean of the series $x_1, x_2, \ldots x_n$ is A. If the observation x_i is replaced by x^1, then the arithmetic mean of the new series is
(A) x^1 (B) $\frac{nA - x_i + x^1}{n}$
(C) $\frac{nA + x_i + x^1}{n}$ (D) $\frac{nA + x^1}{n}$

8. In calculating the arithmetic mean of a set of 15 observations as 25, four observations 4, 12, 19 and 35 were misread as 1, 3, 8 and 13 respectively. Find the correct mean.

9. The average of a set of 120 observations is 20. If the average of 80 of these observations is also 20, then the average of the remaining 40 observations is .

10. The geometric mean of 45, 245, 21 and 525 is _____.
(A) 135 (B) 105 (C) 75 (D) 95

11. The median of the values 12.12346, 12.12355, 12.12382, 12.13245, 12.15632, 12.12345, 12.12432, 12.18932, 12.12344, 12.12346 and 12.12349 is .

12. The median of the first 100 natural numbers is .

13. Which of the following can be the range of the values of the median for the values 5, 11, 8, 3, 25, 39, x, 14, 23, 4, 25, 6, 9, 15 and 29?
(A) [14, 15] (B) [9, 11]
(C) [11, 14] (D) [23, 25]

14. The median of a set of 20 observations is 100. If each observation is divided by 4, then the median of the new set of observations is _____.
(A) 100 (B) 25 (C) 12.5 (D) 50

15. The mode of the data 2, 5, 3, 2, 6, 2, 7, 8, 2, 7, 7, 3, 7, 8, 7 and 10 is .

16. The range of a set of 30 observations is 50. If 2 is added to each of the observations, then the range of the new set of observations is .

17. The quartile deviation of the series 1, 8, 4, 32, 19, 23, 25, 14, 12, 35 and 15 is
(A) 11 (B) 4 (C) 8.5 (D) 12

18. The mean deviation of the series 1, 4, 12, 18, 13, 16, 25, 3, 5 and 3 is .

19. The standard deviation of a series of 'n' observations is σ. If each observation is multiplied by 3, then the standard deviation of the new series is _____.
(A) σ (B) 9σ (C) $\frac{\sigma}{3}$ (D) 3σ

20. If the standard deviation of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 is M, then the standard deviation of 101, 102, 103, 104, ….. and 111 is _____.
(A) M (B) 100 + M
(C) 100 – M (D) M – 100

21. The algebraic sum of the squares of the deviations of 11 observations about their arithmetic mean is 275. The standard deviation of the 11 observations is .

22. The standard deviation of the first 13 natural numbers is _____.

(A) $\sqrt{7}$ (B) $\frac{\sqrt{14}}{2}$ (C) $\sqrt{14}$ (D) 4

23. If 3, 2 and 9 occur with frequencies 2, 5 and 3 respectively, then their arithmetic mean is $\boxed{}$.

24. A man travels a distance of 200 km on a motorcycle and covers the first 100 km at an average speed of V_1 km/hr and the second 100 km at an average speed of V_2 km/hr ($V_1 \neq V_2$). The average speed of the bike for the entire journey is equal to _____.
(A) arithmetic mean of V_1 and V_2.
(B) geometric mean of V_1 and V_2.
(C) harmonic mean of V_1 and V_2.
(D) Arithmetic mean of $\frac{1}{V_1}$ and $\frac{1}{V_2}$

25. The arithmetic mean (AM) of the series $x_1, x_2, \ldots x_n$ is M. Let y_i = AM (x_i, x_{i+1}, x_{i+2}) for $1 \leq i \leq n - 2$ and y_{n-1} = AM (x_{n-1}, x_n, x_1), y_n = AM (x_n, x_1, x_2).
The arithmetic mean of $y_1, y_2, \ldots.. y_n$ is _____.
(A) $\frac{2}{3}M$ (B) M
(C) $\frac{3}{2}M$ (D) 3M

Exercise – 9(b)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. The arithmetic mean of 2, 12, 8, 16, 17, 18, 23 and 40 is _____.
(A) 19.5 (B) 17 (C) 16 (D) 17.5

2. The arithmetic mean of the first 100 natural numbers is $\boxed{}$.

3. The arithmetic mean of a set of 22 observations is 25. If two observations 23 and 47 are discarded, then the arithmetic mean of the remaining observations is _____.
(A) 24 (B) 24.5 (C) 23.5 (D) 25

4. If the arithmetic mean of the series $x_1, x_2, x_3, \ldots.. x_n$ is M, then the arithmetic mean of the series $\frac{2x_1-3}{5}, \frac{2x_2-3}{5}, \ldots\ldots, \frac{2x_n-3}{5}$ is _____.
(A) $\frac{2M+3}{5}$ (B) $\frac{2M-3}{5}$ (C) $M - \frac{3}{5}$ (D) $\frac{2M}{5}$

5. In calculating the arithmetic mean of a set of 10 observations as 30, three observations 18, 12 and 21 were misread as 38, 6 and 22 respectively. Find the actual mean of the set of observations.
(A) 28 (B) 28.5 (C) 31.7 (D) 30

6. There are 100 employees in an organization. The average wage of 40 of these employees is ₹2000 per month and the average wage of the remaining employees is ₹3000 per month. The average wage of all the 100 employees per month (in ₹) is $\boxed{}$.

Moderate

7. In an increasing arithmetic progression of 20 terms, the 10th term is 48 and the common difference is 4. The arithmetic mean of the 20 terms is $\boxed{}$.

8. $a_1, a_2, \ldots\ldots, a_n$ are 'n' distinct real numbers such that $a_1 < a_2 < a_3 < \ldots.. < a_n$ and the arithmetic mean of $a_1, a_2, \ldots.., a_n$ is M. Let b_i = max [$a_1. a_2. \ldots., a_i$]. The arithmetic mean of $b_1, b_2 \ldots\ldots, b_n$ is _____.
(A) 2M (B) M (C) $\frac{M}{2}$ (D) M – 2

9. Which of the following can be the range of the values of the median for the series 14, 12, 23, x, 15, 29, 5?
(A) [12, 14] (B) [14, 15]
(C) [15, 29] (D) [13, 18]

10. The median of the series 1, $\frac{2}{3}, \frac{12}{5}, \frac{13}{6}, \frac{1}{2}$ and 2 is $\boxed{}$.

11. The median of a set of 21 observations is 50. If 2 is subtracted from each of the observations, then the median of the new set of observations is $\boxed{}$.

12. Consider the non-decreasing series of the numbers, 1, 8, 8, 13, 14, 14, x, y, 18, 20, 31, 34, 38 and 40. If the median of the series is 15, then the mode of the series is _____.
(A) 14 (B) 16
(C) 18 (D) Cannot be determined

13. The geometric mean of 1, 4, 4^2, …. and 4^{101} is _____.
(A) 2^{101} (B) 4^{50} (C) $2^{101}\sqrt{2}$ (D) 4^{51}

14. The geometric mean of the numbers 75, 80, 225, 20 and 144 is $\boxed{}$.

15. The arithmetic mean of the first 'n' even natural numbers is _____.
(A) n(n + 1) (B) n + 1 (C) $\frac{n+1}{2}$ (D) n

16. The harmonic mean of 1, 2, 4, 7, 14 and 28 is $\boxed{}$.

17. The median of the first 20 prime numbers is $\boxed{}$.

18. The algebraic sum of the deviations of 12 numbers about 9 is 60. Find the arithmetic mean of the numbers.
(A) 14 (B) 12 (C) 13 (D) 15

19. The range of a set of 25 observations is 100. If each observation is divided by 4, then the range of the new set of observations is $\boxed{}$.

20. The mean deviation of the series 12, 5, 9, 15, 31, 20, 4, 17 and 22 about the mean is _____.

(A) $\frac{58}{9}$ (B) $\frac{56}{9}$ (C) $\frac{20}{3}$ (D) 6

21. If the standard deviation of the series $x_1 + 2$, $x_2 + 2$, ….. $x_n + 2$ is σ, then the standard deviation of $x_1 - 2$, $x_2 - 2$, …… $x_n - 2$ is _____.

(A) σ (B) σ – 2 (C) σ + 2 (D) $\frac{\sigma}{2}$

22. The variance of the first 11 natural numbers is $\boxed{}$.

23. The standard deviation of 7, 7, 7, 7, 7, 7, 7, 7, 7, 7 and 18 is _____.

(A) 10 (B) $2\sqrt{2}$ (C) $\sqrt{10}$ (D) $2\sqrt{5}$

Difficult / Very Difficult

24. The median of the series sin1º, sin2º, sin3º, …. sin179º is _____.

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) sin 46º (D) sin 89º

25. The quartile deviation of 2, 12, 19, 17, 52, 11, 32, 23, 25, 12 and 39 is $\boxed{}$.

Key

Concept Review Questions

1.	55	5.	C	9.	15	13.	B	17.	B
2.	10	6.	1	10.	D	14.	A	18.	C
3.	B	7.	D	11.	74	15.	A	19.	A
4.	C	8.	C	12.	86	16.	5	20.	15

Exercise – 9(a)

1.	B	6.	150.5	11.	12.12355	16.	50	21.	5
2.	31	7.	B	12.	50.5	17.	C	22.	C
3.	20	8.	28	13.	C	18.	6.8	23.	4.3
4.	C	9.	20	14.	B	19.	D	24.	C
5.	B	10.	B	15.	7	20.	A	25.	B

Exercise – 9(b)

1.	B	6.	2600	11.	48	16.	3	21.	A
2.	50.5	7.	50	12.	D	17.	30	22.	10
3.	A	8.	B	13.	A	18.	A	23.	C
4.	B	9.	B	14.	60	19.	25	24.	B
5.	B	10.	1.5	15.	B	20.	C	25.	10