CHAPTER – 8

LOGARITHMS

LOGARITHMS

In the equation $a^x = N$, we are expressing N in terms of a and x. The same equation can be re-written as, $a = N^{1/x}$. Here we are expressing a in terms of N and x. But, among a, x and N, by normal algebraic methods known to us, we cannot express x in terms of the other two parameters a and N. This is where logarithms come into the picture. When $a^x = N$, then we say x = logarithm of N to the base a, and write it as $x = \log_a N$. The definition of logarithm is given as: "the logarithm of any number to a given base is the index or the power to which the base must be raised in order to equal the given number."
Thus,

$$\boxed{\text{if } a^x = N \text{ then } x = \log_a N}$$

This is read as "log N to the base a".

In the above equation, N is a **POSITIVE NUMBER** and a is a **POSITIVE NUMBER OTHER THAN 1**.

This basic definition of logarithm is very useful in solving a number of problems on logarithms.

Example of a logarithm : $216 = 6^3$ can be expressed as $\log_6 216 = 3$.

Since logarithm of a number is a value, it will have an "integral" part and a "decimal" part. The integral part of the logarithm of a number is called the CHARACTERISTIC and the decimal part of the logarithm is called the MANTISSA.

Logarithms can be expressed to any base (positive number other than 1.) Logarithms from one base can be converted to logarithms to any other base. (One of the formulae given below will help do this conversion). However, there are two types of logarithms that are commonly used, on the basis of bases.

(i) Natural Logarithms or Napierian Logarithms: These are logarithms expressed to the base of a number called "e."

(ii) Common Logarithms: These are logarithms expressed to the base 10. For most of the problems under LOGARITHMS, it is common logarithms that we deal with. In examinations also, if logarithms are given without mentioning any base, it can normally be taken to be logarithms to the base 10.
The following should be remembered by the student regarding Common Logarithms.

I. The characteristic of the common logarithm of a number greater than unity is positive and is less by one than the number of digits in its integral part.

For example, the characteristic of log245 will be 2 (because the number has 3 digits and the characteristic should be one less than the number of digits in the number).

Similarly, the characteristic of log4758 will be 3.

II. The characteristic of the common logarithm of a number between 0 and 1 is negative and its magnitude is one more than the number of zeroes immediately after the decimal point.
For example, the characteristic of log0.0034 will be –3 or $\bar{3}$.

III. The mantissas are the same for the logarithms of all numbers which have the same significant digits in the same order. The values of mantissas, which are necessary to solve a problem, are usually given in the problem itself, as part of data.
The above points are helpful in using common logarithms in calculations.
Let us look at the value of log0.02 given that the value of log2 is 0.3010.

The characteristic of log 0.02 will be $\bar{2}$. The mantissa will be the same as that for log2. Hence the value of log0.02 is $\bar{2}.3010$. Here the mantissa 0.3010 is positive while the characteristic is negative. But the same can be written with a negative mantissa, in which case, the characteristic will be –1. Let us see how to do this conversion.

$\bar{2}.3010$ = –2 + 0.3010
= –2 + 1 – 1 + 0.3010 (by adding and subtracting 1)
= {–2 + 1} + {–1 + 0.3010}
= –1 + (–0.6990) = –1.6990

So, the value of log 0.02 can be written as $\bar{2}.3010$ or as –1.6990 and both are the same.
Similarly, given that the value of log3 is 0.4771, we can find out the value of log 0.003. Since there are two zeroes in this number immediately after the decimal point, the characteristic is $\bar{3}$ and the mantissa is positive and the same as that for log3. So the value of log 0.003 is $\bar{3}.4771$. This can also be written as –2.5229 (You should do this conversion of $\bar{3}.4771$ into –2.5229 in the same way we did for log 0.02).
Given below are some **important rules/ formulae** in logarithms:

(i) $\log_a a = 1$ (logarithm of any number to the same base is 1)
(ii) $\log_a 1 = 0$ (log of 1 to any base other than 1 is 0)
(ii) $\log_a (mn) = \log_a m + \log_a n$
(iv) $\log_a (m/n) = \log_a m - \log_a n$
(v) $\log_a m^p = p \times \log_a m$
(vi) $\log_a b = \dfrac{1}{\log_b a}$
(vii) $\log_a m = \dfrac{\log_b m}{\log_b a}$
(viii) $\log_{a^q} m^p = \dfrac{p}{q} \log_a m$
(ix) $a^{\log_a N} = N$
(x) $a^{\log b} = b^{\log a}$

You should memorize these rules/formulae because they are very helpful in solving problems.

Like in the chapter on INDICES, in LOGARITHMS also there will be problems on
(i) Simplification using the formulae/rules listed above and
(ii) Solving for the value of an unknown given in an equation

In solving problems of the second type above, in most of the cases we take recourse to the basic definition of logarithms (which is very important and should be memorized).

The following examples will give problems of both the above types and also some problems on common logarithms.

The following rules also should be remembered while solving problems on logarithms:

Given an equation $\log_a M = \log_b N$,
(i) if M = N, then a will be equal to b; if $M \neq 1$ and $N \neq 1$.
(ii) if a = b, then M will be equal to N.

The examples that follow will explain all the above types of problems. Please note that unless otherwise specified, all the logarithms are taken to the base 10).

Examples

8.01. Evaluate $\log_{10} 200 + \log_{10} 40 + 2 \log_{10} 25$.

Sol. $\log_{10} 200 + \log_{10} 40 + 2\log_{10} 25$
$= \log_{10} (2 \times 100) + \log_{10} (4 \times 10) + 2 \log_{10} 5^2$
$= \log_{10} 2 + \log_{10} 10^2 + \log_{10} 2^2 + \log_{10} 10 + 4\log_{10} 5 = 3 \log_{10} 2 + 3 + 4 \log_{10} 5$
$= 3(\log_{10} 2 + \log_{10} 5) + 3 + \log_{10} 5$
$= 3(\log_{10} 10) + 3 + \log_{10} 5 = 6 + \log_{10} 5$

<u>Alternate method</u>:

As $\log_{10} 200 + \log_{10} 40 + \log_{10} 25^2$ is in the form of log m + log n + log p, it can be written as $\log (m \times n \times p)$
$\log_{10} (200 \times 40 \times 25 \times 25) = \log_{10} (5 \times 10^6)$
$= \log_{10}10^6 + \log_{10}5 = 6\log_{10}10 + \log_{10}5$
$= 6 + \log_{10}5$

8.02. Simplify the following expression and express the answer in terms of log5 : $\log_{10}300 + 2 \log_{10}243 - 3 \log_{10}81 + \log_{10}48 - \log_{10}800$.

Sol. As log m + log n = log mn and log m – log n = log (m/n), we have
$\log_{10}300 + \log_{10}243^2 - \log_{10}81^3 + \log_{10}(2^4 \times 3) - \log_{10}800$
$= \log_{10}\left[300 \times (3^5)^2 \times 2^4 \times 3\right] - \log_{10}[81^3 \times 800]$
$= \log_{10}\left[\dfrac{3 \times 100 \times 3^{10} \times 2^4 \times 3}{3^{12} \times 8 \times 100}\right] = \log_{10}2$
$= \log_{10}{}^{(10/5)} = \log_{10}10 - \log_{10}5 = 1 - \log_{10}5$

8.03. Solve for x: $\log_{10} 80x = 6$.

Sol. $\log_{10}80x = 6$
By the definition of logarithm,
$80x = 10^6 = 10 \times 10^5$
$\Rightarrow 8x = 10^5 = 1000 \times 10^2$
$\Rightarrow x = 125 \times 10^2 = 12500$

8.04. Solve for x: log4x – log8 = log12.

Sol. Since no base is given, we take the base as 10.
$\therefore \log_{10} 4x - \log_{10} 8 = \log_{10}12$
$\Rightarrow \log_{10} 4x = \log_{10} 8 + \log_{10} 12 = \log_{10} 96$
As the logarithms on both sides are to the same base and are equal, we have 4x = 96, Hence, x = 24

8.05. Solve for x: log(x + 1) + log(x – 1) = log224.

Sol. Given, $\log_{10}(x + 1) + \log_{10}(x - 1) = \log_{10}224$
As the logarithms on both sides are to the same base and they are equal, we have (x + 1) (x – 1) = 224 $\Rightarrow x^2 - 1 = 224$
$\Rightarrow x^2 = 225$
$\Rightarrow x = \pm 15$
As logarithms are not defined for negative values, the value of x is 15.

8.06. Express $\log\left(\dfrac{\sqrt[3]{a^4}}{c^2b^6}\right)$ in terms of loga, logb and logc.

Sol. $\log\dfrac{\sqrt[3]{a^4}}{c^2b^6} = \log a^{4/3} - \log c^2b^6$
$= 4/3 \log a - (\log c^2 + \log b^6)$
$= 4/3 \log a - 2 \log c - 6\log b$

8.07. Find the number of digits in $(14175)^{11}$ given that log 2 = 0.3010, log 3 = 0.477, log 7 = 0.845.

Sol. This is an example where common logarithms can be put to practical use.
Values of logarithms to the base 10 of 2, 3 and 7 are given.
Hence, the number 14175 needs to be expressed in terms of 2, 3, 7 and 10.
$14175 = 25 \times 567 = 25 \times 9 \times 63$
$= 5^2 \times 3^2 \times 3^2 \times 7 = 3^4 \times 5^2 \times 7$
$= 3^4 \times \left(\dfrac{10}{2}\right)^2 \times 7$
$= \dfrac{3^4 \times 7^1 \times 10^2}{2^2}$
Hence, $\log_{10}(14175)$
$= 4\log_{10}3 + \log_{10}7 + 2\log_{10}10 - 2\log_{10}2$
$= 4(0.477) + (0.845) + 2 - 2(0.3010) = 4.151$
Hence, $\log_{10}(14175)^{11}$
$= 11 . \log_{10}(14175)$
$= 11 \times 4.151 = 45.661$
As the characteristic is 45, the number $(14175)^{11}$ will have 45 + 1 = 46 digits.

8.08. Obtain an equation between x and y from the following equation without involving logarithms, 3log x = 4 logy + log5.

Sol. 3 logx = 4 logy + log5
$\Rightarrow \log x^3 = \log y^4 + \log 5$
$\Rightarrow \log x^3 = \log 5y^4$
$\therefore x^3 = 5y^4$

8.09. Find the logarithm of $3125 \times \sqrt[4]{125}$ to the base of $\sqrt[4]{5}$.

Sol. Let x be the required logarithm.
Hence x = $\log_{\sqrt[4]{5}} 3125\ \sqrt[4]{125}$
By replacing the logarithm, we get
$\left(\sqrt[4]{5}\right)^x = 3125\ \sqrt[4]{125}$
$\Rightarrow 5^{x/4} = 5^5\ 5^{3/4} \Rightarrow 5^{x/4} = 5^{23/4}$
$\Rightarrow x/4 = 23/4 \Rightarrow x = 23$

8.10. Find the number of zeroes between the decimal point and the first non zero digit in $(4/9)^{216}$, given log2 = 0.301 and log3 = 0.477.

Sol. In common logarithms of a number less than 1, if the mantissa is expressed as a positive figure, the magnitude of the characteristic is one more than the number of zeros after the decimal point. But if the mantissa is also taken as negative then the magnitude of the characteristic is EQUAL to the number of zeroes immediately after the decimal point.
$\therefore \log(4/9)^{216} = 216 \log 4/9 = 216 \log(2/3)^2$
$= 432(\log 2 - \log 3) = 432(0.301 - 0.477)$
$= 432(-0.176) = -76.032 = \overline{77}.968$

The characteristic of the above log is $\overline{77}$. Therefore, the number of zeroes after the decimal point but before the first non-zero digit is 76.

8.11. If log2 = 0.301, find the values of log1250, log 0.001250 and log125000.

Sol. Let us write down 1250 in terms of its prime factors.
$1250 = 2 \times 625 = 2 \times 5^4$
$\log 1250 = \log(2 \times 5^4)$
$= \log 2 + \log 5^4 = \log 2 + 4 \log 5$
log5 can be written as log(10/2) = 1 – log2
So log1250 = log2 + 4(1 – log2)
= 4 – 3 log2 = 4 – 3(0.301) = 3.097
log0.00125 and log125000 will have the same mantissa as log1250, but will have different characteristics.
log0.00125 has a characteristic of – 3 (as there are 2 zeroes after the decimal point) and log125000 will have a characteristic of 5 (as there are six digits in 125000)

Therefore, log0.00125 = $\bar{3}$.097 and log12500 = 5.097

Concept Review Questions

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Simplify : $\log_{(81)(343)}(189)(147)$
(A) 0.5 (B) 1 (C) 1.5 (D) 2

2. If $\log_p r = \log_q r$, which of the following holds true?
(A) p = q
(B) p ≠ q
(C) p need not be equal to q
(D) p ≥ q

3. If $\log_3 4 + \log_3 24 = \log_3 m$, find m.

4. Simplify : $\log_2 96 - \log_2 3$.

5. Find the value of $\log_5 m^0$ where m ≠ 0.

6. Simplify : $\log_{16} 8^4$.
(A) 4 (B) 3 (C) 2 (D) 5

7. log 2 + log 4 + log 6 = ________.
(A) log 12 (B) log 48
(C) log 24 (D) log 36

8. Simplify : $\dfrac{\log_5 27}{\log_5 64}$.
(A) $\log_4 3$ (B) $\log_3 4$
(C) $\log_{64} 27$ (D) $\log_{27} 64$

9. If $5^{\log_5 7^2}$ = k, find k.

10. If 5 = $\log_3$ p, find p.

11. If $\log_{36} 49 = \dfrac{\log_{36} 7}{\log_x 6}$, then find x.
(A) 6 (B) $\frac{1}{6}$ (C) $\frac{1}{36}$ (D) 36

12. If $\log_z x = \log_z y$ where z ≠ 1 and x and y are positive, then which of the following is necessarily true?
(A) x = y
(B) x ≠ y
(C) x need not be equal to y
(D) x ≥ y

13. If $\log_{81} 25 = k \log_3 5$, find k.
(A) 2 (B) 4 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

14. Find the integral part of $\log_2 10000$.

15. N is a 15-digit number. Find the integral part of $\log_{10}$N.
(A) 14 (B) 15
(C) 16 (D) 17

Exercise – 8(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Simplify: $\log\frac{15}{8} + 2\log\frac{8}{5} - 3\log\frac{2}{3} - \log 2.7$.
(A) log 2 + log 3 (B) 1 – log 2
(C) 1 + log 3 (D) 1 + log 2

2. Simplify: $\log_2 \log_2 \log_2 \log_{\sqrt{3}} 6561$.

3. Simplify:
$\log_{b^2} a \times \log_{c^2} b \times \log_{d^2} c \times \log_{e^2} d \times \log_{a^2} e$.
(A) 1 (B) 1/8 (C) 1/16 (D) 1/32

4. If $\log_2 \log_3 \log_2 \log_x 2^{1024} = 1$, then find x.

5. If $5 + \log_{10}x = 5\log_{10}y$, then express x in terms of y.
(A) $x = 5y^5$ (B) $x = (y/10)^5$
(C) $x^2 = 10y^5$ (D) $x = (10y)^5$

6. If $x = y^2 = z^3 = w^4 = u^5$, then find the value of $\log_x xyzwu$.
(A) $2\frac{17}{60}$ (B) $1\frac{11}{120}$ (C) 32 (D) $2\frac{7}{30}$

7. Simplify:
$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} + \frac{1}{\log_6 x} + \frac{1}{\log_7 x}.$$
(A) $\frac{1}{\log_{27} x}$ (B) $\log_{27}x$
(C) $\log_{5040}x$ (D) $\log_x 5040$

8. If $\log_y x = 5$ and $\log_{2y}8x = 4$, then find the value of x.

9. If $36^{\{\log_6 1/2 + 2\log_x \sqrt{2}\}} = 1/2$, then x is

10. If $\log_{y^3} x$ is the same as $\log_{x^3} y$, then find the value of $\log_{x^3} y$.
(A) 1/3 (B) –1/3
(C) 2/3 (D) Either (A) or (B)

11. If 2 [log(x + y) – log 5] = logx + logy, then find the value of $\frac{x^2 + y^2}{xy}$.

12. If $\frac{\log x}{\log 4} = \frac{\log 343}{\log 49} = \frac{\log y}{\log 64}$, then find x + y.

13. Arrange in ascending order;
$A = \log_7 2401$, $B = \log_{7\sqrt{7}} 343$, $C = \log_{\sqrt{6}} 216$,
$D = \log_2 32$.
(A) ACBD (B) BDCA
(C) DCAB (D) BADC

14. If $\log_{10} [1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2} = 1$, then which of the following is the value of x?
(A) 1 (B) 2 (C) 10 (D) 0

15. Solve for x : $4^{\log_2 8} + 27^{\log_{27} 81} = 144 + \log_{10} x$.

16. If $(\log_x 3)(\log_{x/81} 3) = \log_{x/729} 3$, then the value of x is _______.
(a) 3 (b) 9 (c) 27 (d) 81
(A) a or b (B) a or c
(C) b or c (D) a, c or d

17. If $3\log x = \frac{\log_2 8}{\log_9 16 . \log_4 10}$, then find the value of x.

18. If $\log_y x = 8$ and $\log_{10y}16x = 4$, then find the value of y.
(A) 1 (B) 2 (C) 3 (D) 5

19. If $2\log_5 P + 1/2\log_5 Q = 1$, then express Q in terms of P.
(A) $Q = 25P^2$ (B) $Q = 25/P^2$
(C) $Q = 25P^4$ (D) $Q = 25/P^4$

20. Given log 2542 = 3.4052, find log 0.0000002542.
(A) $\bar{5}.4052$ (B) $\bar{6}.4052$
(C) $\bar{7}.4052$ (D) $\bar{8}.4052$

21. If $\log_{10}24 = x$, $\log_{10}80 = y$ and $\log_{10}25 = z$, then find $\log_{10}48$ in terms of x, y and z.
(A) x + y + z + 3 (B) x + y + z – 3
(C) $x^2 + y^2 + z^2 + 4$ (D) $x^2 + y^2 + z^2 - 4$

22. If $\log_{25}225 = p$ and $\log_{17}25 = q$, then find $\log_{2025}1275$.
(A) $\frac{pq - 2}{q(4p - 1)}$ (B) $\frac{pq + 2}{q(4p - 2)}$
(C) $\frac{pq + 2}{q(4p + 2)}$ (D) $\frac{pq + q + 2}{q(4p - 2)}$

23. If $\log_2 \log_2 \log_2 \left(\sqrt{x - 13} + \sqrt{x - 45}\right) = 1$, then how many values can x take?

24. Find $\sqrt[8]{0.004225}$, given log4225 = 3.6258 and log 50492 = 4.70322.
(A) 0.46755 (B) 0.50492
(C) 0.70322 (D) 0.40592

25. How many digits are there in 25^{50}, given that log2 = 0.3010?
(A) 69 (B) 70 (C) 71 (D) 75

26. Find the number of zeros after the decimal in $(2/3)^{500}$; given log2 = 0.3010 and log3 = 0.4771.

27. If $3(\log_{10} y - \log_{10} \sqrt[3]{y}) = 8 \log_y 10$, find y.
(A) 100
(B) 1/100
(C) 10
(D) Either (A) or (B)

28. If $\log_p q = m$, $\log_q r = n$, $\log_r p = mn$, (m, n) then cannot be _______.
(A) (2, 1/2)
(B) (e, –1/e)
(C) (3, 3)
(D) (1, 1)

29. If $p \ge q$ and $q > 1$, the value of $\log_p \frac{p^3}{q^2} + \log_q \frac{q^3}{p^2}$ cannot be _______.
(A) 1 (B) 2 (C) 2.5 (D) 1.5

30. N_1, N_2,…… N_{15} are positive integers such that $\log N_1$ $\log N_2$,…….., $\log N_{15}$ are in arithmetic progression. If $N_8 - N_4 = 3600$ and $N_7 - N_5 = 1440$, the sum of N_1, N_2, …. N_{15} is _______.
(A) $15(4^{15} - 1)$ (B) $60(4^{15} - 1)$
(C) $30(2^{15} - 1)$ (D) $60(2^{15} - 1)$

Exercise – 8(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. Find the logarithm of $64 \times \sqrt[3]{512}$ to the base $\sqrt[3]{8}$.

2. If the logarithm of 19683 to a base is 6, then what is the base?
(A) $\sqrt{3}$ (B) $3\sqrt{3}$ (C) 9 (D) 3

3. If the logarithm of a number to the base $\sqrt{3}$ is 2, then find the logarithm of the same number when the base is $3\sqrt{3}$.
(A) 0 (B) 1 (C) 1/3 (D) $\frac{2}{3}$

4. Which of the following is true, if $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{xz} = \frac{1}{xyz}$?
(A) $\log(xy + yz + xz) = xyz$
(B) $\log\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = xyz$
(C) $\log(x + y + z) = 0$
(D) $\log\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \log 1$

5. If $(abc)^x = a + b + c$ and $abc \ne 0$, then $\frac{1}{x}$ is _______.
(A) $\log_{a+b+c}(abc)$
(B) $\log_{a+b+c}(ab + bc + ca)$
(C) $\log_{abc}(a + b + c)$
(D) $\log_{ab+bc+ca}(a + b + c)$

6. Simplify: $\log_{17} 6^{\log_7 [\log_{20} 25 \times \log_{25} 20]}$.
(A) 0 (B) ½ (C) 3/4 (D) 1

Moderate

7. Find the value of x, if
$\frac{x}{\log 2} + \frac{x}{\log 4} + \frac{x}{\log 16} + \ldots \infty = \log 2$.
(A) log2 (B) $\frac{\log 2}{2}$
(C) $\frac{(\log 2)^2}{2}$ (D) $\left(\frac{\log 2}{2}\right)^2$

8. Find the value of x, if
$2^{\frac{1}{\log_x 4}} \times 2^{\frac{1}{\log_x 16}} \times 2^{\frac{1}{\log_x 256}} \ldots \infty = 2$.

9. Simplify: $\log a + \log a^2 + \log a^3 + \ldots\ldots\ldots + \log a^{20}$.
(A) $\log a^{210}$ (B) log a
(C) 1 (D) $\log a^{1/210}$

10. If $\log(x + y) - \log 2 = \frac{1}{2}(\log x + \log y)$ and y = 2, then find the value of x.

11. If $\log_x 5 + \log_x 8 + \log_x 12 - \log_x 60 = 3 \log_x A$, then find the value of A.

12. If $\log(x^5 y^2) = 5a + 2b$ and $\log(x^2 y^5) = 2a + 5b$, find log (xy) in terms of a and b.
(A) a + b (B) ab (C) $a^2 + b^2$ (D) a/b

13. Find x, if $\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = \frac{25}{4}$.

14. The logarithm of a number to a certain base is 9. The logarithm of 64 times the number to a base which is 11 times the original base is 6. Find the original base.
(A) 11/6 (B) 33/7 (C) $11^1/_4$ (D) 121/4

15. If $a = \log_4 2$, $b = \log_6 4$, $c = \log_8 6$, then find abc.
(A) 1/3 (B) 1 (C) log 48 (D) 3

16. Find $\frac{x}{y} + \frac{y}{x}$, if $\log(x + y) + \log(x - y) = 2\log(x + y) - \log x - \log y - \log 2$.
(A) $x^2 - y^2 - 1$
(B) $\frac{x^2 - y^2 - 1}{2}$
(C) $2(x^2 - y^2 - 1)$
(D) $x^2 + y^2 + 1$

17. Find the value of $\sqrt{\frac{a}{b}}$, if

$\log_2\left(\log_2 2^{(a-b)}\right) = 2\log_2\left(\sqrt{a}-\sqrt{b}\right)+1.$

$\boxed{}$

18. If $2\log_4 y + 2\log_4 z + 2\log_4 x = 1$, then find the value of xyz.

$\boxed{}$

19. Which of the following is true,
if $\frac{a}{\log_a abc} + \frac{b}{\log_b abc} + \frac{c}{\log_c abc} = 1$?
(A) $a^a + b^b + c^c = 1$
(B) $a^a\, b^b\, c^c = 1$
(C) $a^{a-1}\, b^{b-1}\, c^{c-1} = abc$
(D) $a^{a-1}\, b^{b-1}\, c^{c-1} = 1$

20. Find the value of b, if $\log_{a\sqrt{y}} x^a = 1$, $\log_{a\sqrt{y}} x^{a^{b-1}}$ = 1 and $a^2 \neq 1$.
(A) 2
(B) b – a = 1
(C) b = 1 – a
(D) b = a

21. Find m, if $\log_2 [3\log_2 2^{2m} + 2\log_2 4] = 4$.

$\boxed{}$

22. If $\log_{(x-y)}(x + y) = \frac{1}{2}$, then find the value of $\log_{\left(x^2-y^2\right)}\left(x^2 - 2xy + y^2\right)$.

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{4}{3}$ (D) $\frac{1}{3}$

23. If $\log_p 8 = b$ and $\log_p 12 = a$, express $\log_p\left(\frac{3}{p}\right)^3$ in terms of a and b.
(A) a – b – 1
(B) 3a – 2b – 3
(C) a – 2b – 3
(D) 2a – 2b – 3

24. If $(x+1)\log y + \log x + \log z = 2\log z + \log y + \log x$, which of the following is true?
(A) $x^{y^z} = xyz$
(B) $\left(x^y\right)^z = xyz$
(C) $y = z^x$
(D) $y^x = z$

25. Solve for x, if $2\log x + \log(x^4 + 1 + 2x^2) = \log(x^2 + 1) + \log x^2 + 1$(All the log expressions are to the base 10).

$\boxed{}$

26. If $\frac{1}{3}\log\left[\left(a^3+b^3\right)(a+b)\right] = \log(a + b)$ and a = 2b, find the respective values of a and b.

(A) 2, 1 (B) 1, 2
(C) 4, 3 (D) 4, 2

27. If $\log_{(x+y)}(x - y) = a$, $\frac{1+a}{a}$ is equal to ________.
(A) $\log_{(x-y)}(x^2 - y^2)$
(B) $\log_{(x+y)}(x^2 - y^2)$
(C) $\log_{(x+y)}\left(\frac{x^2}{y^2}\right)$
(D) $\log_{(x+y)}(x^2 + y^2)$

28. If log 8246 = 3.9162, then find the number whose logarithm is –3.0838.
(A) 0.08246 (B) 0.008246
(C) 0.0008246 (D) 0.8246

29. If log2 = 0.3010, log3 = 0.4771, and log7 = 0.8451, find the value of log 8/25 + 2 log 15/16 + 5 log 24/49.
(A) –1.7825 (B) –2.1013
(C) –2.2257 (D) –2.4767

30. How many digits are there in $(2205)^{25}$, given that log2 = 0.3010, log3 = 0.4771 and log7 = 0.8450?

$\boxed{}$

31. If $1/2 \log_2 P + 2\log_2 Q = 1 + \log_{0.0625} 2$, which of the following is always true?

(A) $\frac{P^2}{Q^8} = 8$ (B) $\frac{P^8}{Q^2} = 8$

(C) $P^2 Q^8 = 8$ (D) $P^8 Q^2 = 8$

Difficult / Very Difficult

32. Given log 2 = 0.3010 and log 3 = 0.4771, solve the equation $(4^x)(81^{1-x}) = 50$ and find an approximate value of x.
(A) 0.16 (B) 0.24
(C) 0.44 (D) 0.52

33. If $\log x \geq \log(x - 12) + \log 1.05$, the range of x is _______.
(A) (0, 252] (B) (12, 240]
(C) [252, ∞) (D) (12, 252]

34. If $0 < \log_{x+3}(3x - 1) + \log_{3x-1}(x + 3) \leq 2$, then the number of possible values of x is $\boxed{}$.

35. Let $2^a - 5 = x$ and $2^a - \frac{7}{2} = y$.
If $\log_2 3$, $\log_x 3$, $\log_y 3$ are in HP, then a = _____
(A) 2 (B) 1 (C) 3 (D) 5

Key

Concept Review Questions

1. B	4. 5	7. B	10. 243	13. C
2. C	5. 0	8. A	11. D	14. 13
3. 96	6. B	9. 49	12. A	15. A

Exercise – 8(a)

1. A	7. D	13. D	19. D	25. B
2. 1	8. 32	14. C	20. C	26. 88
3. D	9. 36	15. 10	21. B	27. D
4. 4	10. D	16. C	22. D	28. C
5. B	11. 23	17. 3	23. 1	29. C
6. A	12. 520	18. D	24. B	30. C

Exercise – 8(b)

1. 9	8. 2	15. A	22. C	29. B
2. B	9. A	16. C	23. B	30. 84
3. D	10. 2	17. 3	24. D	31. C
4. C	11. 2	18. 2	25. 3	32. A
5. A	12. A	19. D	26. A	33. D
6. A	13. 27	20. A	27. A	34. 1
7. C	14. D	21. 2	28. C	35. C