CHAPTER – 8

CUBES

A cube is a three dimensional solid having 6 faces, 12 edges and 8 corners. All the edges of a cube are equal and hence all the faces are square in shape.

In competitive exams a few questions may be asked based on cubes.

The questions on cubes may belong to any one of the following categories.

I. A cube is cut by making certain specified number of cuts. The directions in which the cuts are made may or may not be given. We are to find the number of identical pieces resulting out of the given cuts.

II. The number of identical pieces, into which a cube is cut, is given and we need to find the number of cuts.

III. A cube could be painted on all or some of its faces with the same colour or different colours and then cut into a certain specified number of identical pieces. Then questions of the form — "How many small cubes have 2 faces painted?". "How many smaller cubes have only one face painted?" etc. could then be framed.

Exercise – 8(a)

Directions for questions 1 to 10: Select the correct alternative from the given choices.

1. What is the maximum number of identical pieces a cube can be cut into by 3 cuts?
(A) 9 (B) 8 (C) 7 (D) 6

2. What is the maximum number of identical pieces a cube can be cut into by 4 cuts?
(A) 10 (B) 12 (C) 16 (D) 5

3. A cube is cut parallel to one face by making 10 cuts [such that all the resulting pieces are identical]. What is the maximum number of identical pieces that can be obtained by now making two more cuts (in any direction)?
(A) 33 (B) 40 (C) 55 (D) 44

4. What is the maximum number of identical pieces a cube can be cut into by 13 cuts?
(A) 120 (B) 140 (C) 180 (D) 150

5. What is the least number of cuts required to cut a cube into 24 identical pieces?
(A) 2 (B) 4 (C) 6 (D) 8

6. What is the maximum number of identical pieces a cube can be cut into by 7 cuts?
(A) 36 (B) 49 (C) 25 (D) 56

7. What is the least number of cuts required to divide a cube into 120 identical pieces?
(A) 6 (B) 8 (C) 15 (D) 12

8. What is the maximum number of identical pieces into which a cube can be divided by 12 cuts?
(A) 100 (B) 144
(C) 150 (D) 125

9. What is the maximum number of identical pieces a cube can be cut into by 6 cuts?
(A) 12 (B) 36 (C) 18 (D) 27

10. What is the maximum number of identical pieces a cube can be cut into by 5 cuts?
(A) 25 (B) 20 (C) 18 (D) 16

Directions for questions 11 to 13: These questions are based on the following information.

A large cube painted on all six faces is cut into 27 smaller but identical cubes.

11. How many of the smaller cubes have no faces painted at all?
(A) 0 (B) 1 (C) 3 (D) 4

12. How many of the smaller cubes have exactly one face painted?
(A) 3 (B) 6 (C) 12 (D) 15

13. How many of the smaller cubes have exactly two faces painted?
(A) 36 (B) 6 (C) 12 (D) 15

Directions for questions 14 to 16: These questions are based on the following information.

A large cube is painted on all six faces and then cut into a certain number of smaller but identical cubes. It was found that among the smaller cubes, there were eight cubes which had no face painted at all.

14. How many smaller cubes was the original large cube cut into?
(A) 27 (B) 48 (C) 64 (D) 125

15. How many small cubes have exactly one face painted?
(A) 12 (B) 24 (C) 16 (D) 32

16. How many small cubes have exactly two faces painted?
(A) 6 (B) 12 (C) 18 (D) 24

Directions for questions 17 to 24: Select the correct alternative from the given choices.

17. What is the least number of identical cuboids, each of dimensions 2 cm $\times$ 4 cm $\times$ 5 cm, that are required to form a cube?
(A) 160 (B) 240 (C) 220 (D) 200

18. 125 small but identical cubes have been put together to form a large cube. How many more such small cubes will be required to cover this large cube completely?
(A) 208 (B) 212 (C) 218 (D) 224

19. 64 smaller but identical cubes are placed on a table to form a large cube. How many more such smaller cubes are now required to enclose this large cube placed on the table completely?
(A) 125 (B) 116 (C) 100 (D) 132

20. A cube of side 6 cm has been cut into 64 smaller but identical cubes. If it was estimated that it would take 4 litres of paint to paint all the faces of the original cube, then how much paint is required to paint all the faces of all the smaller cubes?
(A) 16 litres (B) 12 litres
(C) 20 litres (D) 4 litres

21. 125 small but identical cubes are put together on a table to form one large cube. A knife is passed through this cube starting along one edge of the top face to the diagonally opposite edge on the bottom face. How many of the small cubes are cut by this knife?
(A) 25 (B) 36 (C) 64 (D) 16

22. Each face of a cube is painted either white or black. In how many different ways can the cube be painted?
(A) 8 (B) 10 (C) 12 (D) 16

23. A cube is cut into smaller but identical cubes such that the edges of each small cube are integers. It was found that a particular cube X could be cut into 27 identical cubes or 64 identical cubes. What is the largest number of small, but identical cubes, that can be cut from X, if X has the least possible dimensions?
(A) 1331 (B) 729 (C) 1728 (D) 2179

24. It was found that a cube can be cut into certain number of identical cuboids each measuring 1 cm $\times$ 2 cm $\times$ 5 cms. What is the side of the smallest such cube? How many such cuboids can be formed from such a cube?
(A) 10 cm, 100 (B) 5 cm, 50
(C) 20 cm, 800 (D) 20 cm, 200

Directions for questions 25 to 27: These questions are based on the following information.

There is a cube in which one pair of opposite faces is painted red, the second pair of opposite faces is painted blue and the third pair of opposite faces is painted green. This cube is now cut into 216 smaller but identical cubes.

25. How many small cubes are there with no red paint at all?
(A) 121 (B) 144 (C) 169 (D) 100

26. How many small cubes are there with at least two different colours on their faces?
(A) 49 (B) 64 (C) 56 (D) 81

27. How many small cubes are there with only red and green on their faces?
(A) 9 (B) 16 (C) 27 (D) 18

Directions for questions 28 to 30: These questions are based on the following information.

There is a cube in which one pair of adjacent faces is painted red, the second pair of adjacent faces is painted blue and a third pair of adjacent faces is painted green. This cube is now cut into 216 smaller but identical cubes.

28. How many small cubes are there with one face painted red?
(A) 64 (B) 81 (C) 60 (D) 120

29. How many small cubes are with both red and green on their faces?
(A) 8 (B) 12 (C) 16 (D) 32

30. How many small cubes are there showing only green or only blue on their faces?
(A) 64 (B) 72 (C) 81 (D) 96

Exercise – 8(b)

Directions for questions 1 to 3: These questions are based on the following information.

A cube has all the six faces painted in six different colours – White, Blue, Red, Yellow, Green and Pink in such a way that Pink and Green are on two opposite faces. The cube is placed on a table with the Pink face touching the top of the table. Red is facing you, whereas White and Blue faces are opposite to each other. The cube is cut into 120 identical pieces by making the least number of cuts possible where all the cuts are parallel to the faces of the cube. Least number of possible cuts are made in the horizontal direction and maximum number of possible cuts are made parallel to the Red face.

1. How many small pieces have White colour on their faces?
(A) 36 (B) 42 (C) 30 (D) 24

2. How many small pieces have at least two different colours on their faces?
(A) 44 (B) 28 (C) 38 (D) 30

3. How many small pieces have no colour on their faces?
(A) 42 (B) 24 (C) 36 (D) 27

Directions for questions 4 to 6: These questions are based on the following information.

Some smaller and identical cubes are taken. Each cube is painted in red colour on all of its faces. 27 such cubes are taken to make a bigger cube and that cube is painted in blue on all of its faces. Such 27 cubes are made and joined to make a much bigger cube and this bigger cube is painted in green on all of its faces. (Assume that we have sufficient number of smaller cubes.)

4. How many smaller cubes are painted in exactly one colour?
(A) 120 (B) 100 (C) 27 (D) 96

5. How many smaller cubes are painted in green?
(A) 362 (B) 332 (C) 386 (D) 278

6. How many smaller cubes are painted in only red and blue?
(A) 296 (B) 324 (C) 316 (D) 356

Directions for questions 7 to 9: These questions are based on the following information.

Three different faces of a cube are painted in three different colours - red, green and blue. This cube is now cut into 216 smaller but identical cubes.

7. What are the least and the largest numbers of small cubes that have exactly one face painted?
(A) 75 and 86 (B) 64 and 81
(C) 64 and 72 (D) 75 and 84

8. What is the maximum number of small cubes that have one face painted green and one face blue and no other face painted?
(A) 2 (B) 4 (C) 6 (D) 8

9. What are the least and the maximum numbers of cubes that have no face painted at all?
(A) 125 and 130 (B) 120 and 125
(C) 115 and 120 (D) 100 and 125

Directions for questions 10 to 13: These questions are based on the following information.

Each face of a cube is painted in green, red or blue.

10. Totally in how many different ways can the cube be painted?
(A) 49 (B) 56 (C) 64 (D) 81

11. In how many different ways can the cube be painted with at least two faces blue?
(A) 24 (B) 30 (C) 34 (D) 42

12. In how many different ways can the cube be painted such that all three colours are there on the cube?
(A) 32 (B) 29 (C) 25 (D) 30

13. In how many different ways can the cube be painted such that no two adjacent faces have the same colour?
(A) 3 (B) 1 (C) 2 (D) 4

Directions for questions 14 to 16: These questions are based on the following information.

Two colours, red and blue, are used to paint a cube. Red is painted on three faces, each of which is adjacent to the other two and blue is painted on the remaining faces. Assume that one can see exactly three faces when the cube is kept on a plane.

14. What is the total number of ways in which the blue colour is not seen at all when the cube is kept on a table?
(A) 4 (B) 3 (C) 2 (D) 1

15. What is the total number of ways in which exactly one face painted blue is seen?
(A) 2 (B) 4 (C) 3 (D) 5

16. What is the total number of ways in which exactly two faces painted blue are seen?
(A) 3 (B) 2 (C) 5 (D) 1

Directions for questions 17 to 19: These questions are based on the following information.

A cube is painted red, blue and green in such a way that each face is painted with a single colour and each colour is painted on two adjacent faces. The cube is placed on a table and one can see exactly three faces of the cube.

17. What is the total number of distinct corners from where red and blue colours are visible?
(A) 5 (B) 4 (C) 6 (D) 8

18. What is the total number of ways in which all three colours can be seen?
(A) 2 (B) 3 (C) 1 (D) 5

19. What is the total number of distinct possible combinations of three colours that can be seen?
(A) 8 (B) 9 (C) 7 (D) 6

Directions for questions 20 to 22: These questions are based on the following information.

Each face of a die is marked with a different number from 1 to 6. The numbers on the faces of the die are marked in such a way that the sum of the numbers on any pair of opposite faces is seven. Two such dice are thrown. Assume that one can always see exactly three faces of each die.

20. What is the total number of distinguishably different ways in which the sum of the numbers on the visible faces of both the cubes together is 20?
(A) 2 (B) 6 (C) 3 (D) 5

21. What is the total number of distinguishably different ways in which the sum of numbers on visible faces is exactly 10 on at least one die?
(A) 12 (B) 17 (C) 15 (D) 19

22. What is the total number of ways in which a specified number is visible on both the dice?
(A) 32 (B) 16 (C) 14 (D) 18

Directions for questions 23 to 26: These questions are based on the following information:

125 small and identical cubes are numbered using only odd numbers from 1 to 249 (in that order) and are assembled together to form a larger cube.
(i) The front face is laid out first from the bottom row left to right, then the row above it left to right and so on.
(ii) This process in continued until the rear face is laid out in a similar manner.

23. What is the sum of the series of numbers starting from the bottom left cube on the rear face to the top right cube of the rear face?
(A) 1000 (B) 1025 (C) 1125 (D) 1250

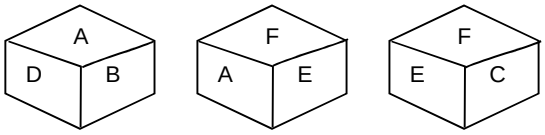
24. What is the sum of the series of numbers forming the body diagonal starting from the top right corner of the front face to the bottom left corner of the rear face?
(A) 625 (B) 525 (C) 645 (D) 650

25. What is the sum of numbers starting from the cube at the center of the front face to the cube at the center of the rear face?
(A) 625 (B) 525 (C) 645 (D) 650

26. What is the sum of the series of numbers forming the face diagonal on the right side, starting from top right corner of the front face to the bottom right corner of the rear face?
(A) 625 (B) 525 (C) 645 (D) 650

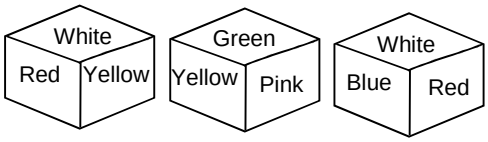
Directions for questions 27 and 28: These questions are based on the different faces of a dice.

27.



Which letter is on the opposite face of letter C?
(A) D (B) A (C) B (D) C

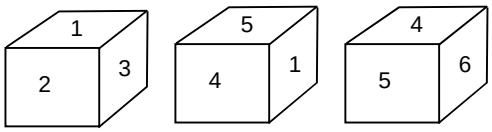
28.

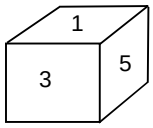


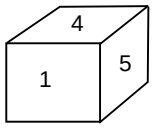
Which colour is opposite to Yellow colour?
(A) White (B) Green
(C) Blue (D) Pink

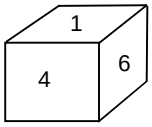
Directions for questions 29 and 30: In each of the questions, three views of a cube are given. In the options another 4 views of the same cube are given, one of which is wrong. Identify the choice which contains the wrong view and mark it as your answer. The letters/numbers shown on the faces in the diagrams are used only to identify the respective faces in the diagrams, but are not printed or painted on the faces of the cubes.

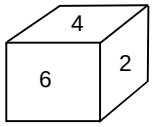
29.



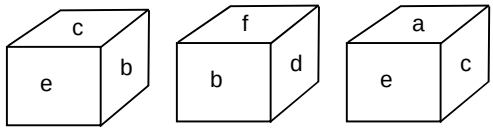
(A) 

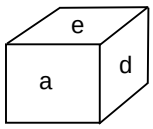
(B) 

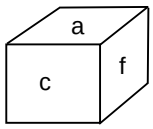
(C) 

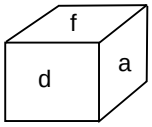
(D) 

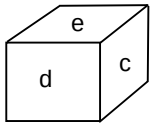
30.



(A) 

(B) 

(C) 

(D) 

Key

Exercise – 8(a)

1.	B	6.	A	11.	B	16.	D	21.	A	26.	C
2.	B	7.	D	12.	B	17.	D	22.	B	27.	B
3.	D	8.	D	13.	C	18.	C	23.	C	28.	C
4.	D	9.	D	14.	C	19.	B	24.	A	29.	C
5.	C	10.	C	15.	B	20.	A	25.	B	30.	B

Exercise – 8(b)

1.	D	6.	C	11.	C	16.	A	21.	C	26.	C
2.	A	7.	D	12.	B	17.	B	22.	B	27.	B
3.	B	8.	C	13.	B	18.	A	23.	C	28.	C
4.	C	9.	B	14.	D	19.	C	24.	A	29.	C
5.	C	10.	B	15.	C	20.	D	25.	A	30.	D