CHAPTER – 1

NUMBERS – I

Numbers is one of the most important topics required for competitive exams, particularly the MBA entrance exams. In this chapter, we have put together a number of models of problems – mainly based on the various problems that have been appearing in different exams.

BASIC ARITHMETIC OPERATIONS

Addition is the most basic operation. We have an intuitive understanding of the operation. It is the process of finding out the single number or fraction equal to two or more quantities taken together. The two (or more) numbers that are added are called addends and the result of the addition is called the sum. For two numbers A and B, this is denoted as A + B.

Subtraction is the process of finding out the quantity left when a smaller quantity (number or fraction) is reduced from a larger one. This is called the difference of the two numbers. The word difference is taken to mean a positive quantity, i.e., the difference of 10 and 8 is 2. The difference of 8 and 10 is also 2. This is also referred to as the remainder.

Multiplication is repeated addition. The number that is added repeatedly is the multiplicand. The number of times it is added is the multiplier. The sum obtained is the product.

For example, in the multiplication 3 x 4 = 12, 3 is the multiplicand, 4 is the multiplier and 12 is the product.

Division is repeated subtraction. From a given number, we subtract another repeatedly until the remainder is less than the number that we are subtracting. The number from which we are subtracting the second one is the dividend. The number that is subtracted repeatedly (the second one) is the divisor. The number of times it is subtracted is the quotient. The number that remains after we are done subtracting is the remainder. Division can also be thought of as the inverse of multiplication. A/B is that number with which B has to be multiplied to get A.
For example, in the division 32/5, 32 is the dividend, 5 is the divisor, 6 is the quotient and 2 is the remainder.

Involution (or raising to the power n) is repeated multiplication. Thus, a^n is the product of n a's. Here, a is the base, n is the index and a^n is the n^{th} power of a.
For example, a x a = a^2, which is the second power of a and a x a x a = a^3, which is the third power of a.

Evolution is the inverse of involution. The n^{th} root of a number is that number whose n^{th} power is the given number. The root of any number or expression is that quantity which when multiplied by itself the requisite number of times produces the given expression.

For example, the square root of a, $\sqrt{a}$ when multiplied by itself two times, gives a; similarly, the cube root of a, $\sqrt[3]{a}$ when multiplied by itself three times, gives a.

All the above operations are performed in Algebra also. Algebra treats quantities just as Arithmetic does, but with greater generality, for algebraic quantities are denoted by symbols which may take any value we choose to assign them as compared to definite values usually used in arithmetic operations.

Rule of Signs

The product of two terms with like signs is positive; the product of two terms with unlike signs is negative.
Example : –1 x –1 = +1 ;
+1 x –1 = –1 ;
+1 x +1 = +1 ;
–1 x +1 = –1;

CLASSIFICATION OF REAL NUMBERS

Real Numbers are classified into rational and irrational numbers.

Rational Numbers

A number which can be expressed in the form p/q where p and q are integers and q $\neq$ 0 is called a rational number.

For example, 4 is a rational number since 4 can be written as 4/1 where 4 and 1 are integers and the denominator 1 $\neq$ 0. Similarly, the numbers 3/4, –2/5, etc. are also rational numbers.

Recurring decimals are also rational numbers. A recurring decimal is a number in which one or more digits at the end of a number after the decimal point repeats endlessly (For example, 0.333....., 0.111111...., 0.166666...., etc. are all recurring decimals). Any recurring decimal can be expressed as a fraction of the form p/q and hence it is a rational number. We will study in another section in this chapter the way to convert recurring decimals into fractions.

Between any two numbers, there can be infinite number of other rational numbers.

Irrational Numbers

Numbers which are not rational but which can be represented by points on the number line are called irrational numbers. Examples for irrational numbers are $\sqrt{2}$, $\sqrt{3}$, $\sqrt[4]{5}$, $\sqrt[3]{9}$, etc.

Numbers like π, e are also irrational numbers.

Between any two numbers, there are infinite number of irrational numbers.

Another way of looking at rational and irrational numbers is

Terminating decimals and recurring decimals are both rational numbers.

Any non-terminating, non-recurring decimal is an irrational number.

Integers

All integers are rational numbers. Integers are classified into negative integers, zero and positive integers. Positive integers can be classified as Prime Numbers and Composite Numbers. In problems on Numbers, we very often use the word "number" to mean an "integer."

Prime Numbers

A number other than 1 which does not have any factor apart from one and itself is called a prime number.

Examples for prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, etc.

There is no general formula that can give prime numbers. **Every prime number greater than 3 can be written in the form of (6k + 1) or (6k – 1) where k is an integer.** For the proof of this, refer to 4th point under "Some important points to note" given later on in this chapter.

Composite Numbers

Any number other than 1, which is not a prime number is called a composite number. In other words, a composite number is a number which has factors other than one and itself.

Examples for composite numbers are 4, 6, 8, 9, 10, 14, 15, etc.

NOTE: The number 1 is neither prime nor composite.
The only prime number that is even is 2.

There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100. So, there are a total of 25 prime numbers between 1 and 100.

Even and odd numbers

Numbers divisible by 2 are called even numbers whereas numbers that are not divisible by 2 are called odd numbers.

Examples for even numbers are 2, 4, 6, 8, 10, etc. Examples for odd numbers are 1, 3, 5, 7, 9, etc.

NOTE: Every even number ends in 0, 2, 4, 6 or 8.
The sum of any number of even numbers is always even.

The sum of odd number of odd numbers (i.e., the sum of 3 odd numbers, the sum of 5 odd numbers, etc.) is always odd whereas the sum of even number of odd numbers (i.e., the sum of 2 odd numbers, the sum of 4 odd numbers, etc.) is always even.

The product of any number of odd numbers is always odd.

The product of any number of numbers where there is at least one even number is even.

Perfect Numbers

A number is said to be a perfect number if the sum of ALL its factors excluding itself (but including 1) is equal to the number itself.

For example, 6 is a perfect number because the factors of 6, i.e., 1, 2 and 3 add up to the number 6 itself.

Other examples of perfect numbers are 28, 496, 8128, etc.

HIERARCHY OF ARITHMETIC OPERATIONS

To simplify arithmetic expressions, which involve various operations like brackets, multiplication, addition, etc. a particular sequence of the operations has to be followed. For example, 2 + 3 x 4 has to be calculated by multiplying 3 with 4 and the result 12 added to 2 to give the final result of 14 (you should not add 2 to 3 first to take the result 5 and multiply this 5 by 4 to give the final result as 20). This is because in arithmetic operations, multiplication should be done first before addition is taken up.

The hierarchy of arithmetic operations are given by a rule called BODMAS rule. The operations have to be carried out in the order in which they appear in the word BODMAS, where different letters of the word BODMAS stand for the following operations:

B Brackets
O Of
D Division
M Multiplication
A Addition
S Subtraction

There are four types of brackets:

(i) Vinculum : This is represented by a bar on the top of the numbers. For example,
$2 + 3 - \overline{4+3}$; Here, the figures under the vinculum have to be calculated as 4 + 3 first and the "minus" sign before 4 is applicable to 7. Thus the given expression is equal to 2 + 3 – 7 which is equal to –2.

(ii) Simple Brackets: These are represented by ( )

(ii) Curly Brackets: These are represented by { }

(iv) Square Brackets: These are represented by [ ]

The brackets in an expression have to be opened in the order of vinculum, simple brackets, curly brackets and square brackets, i.e., [{ (¯) }] to be opened fom inside outwards.

After brackets is O in the BODMAS rule standing for "of" which means multiplication. For example, 1/2 of 4 will be equal to 1/2 x 4 which is equal to 2.

After O, the next operation is D standing for division. This is followed by M standing for multiplication. After Multiplication, A standing for addition will be performed. Then, S standing for subtraction is performed.

Two operations that have not been mentioned in the BODMAS rule are taking powers and extracting roots, viz, involution and evolution respectively. When these operations are also involved in expressions, there is never any doubt about the order in which the steps of the simplification should be taken. The sign for root extraction is a variant of the vinculum and for powers, brackets are used to resolve ambiguities in the order.

Examples:

1.01. Simplify [5 + 1/12 of {38 – (10 + $\overline{7-3}$) + 1/2 of 24} – 3].

Sol: By applying the BODMAS rule,
[5 + 1/12 of {38 – (10 + $\overline{7-3}$) + 1/2 of 24} – 3]
= [5 + 1/12 of {38 – (10 + 4) + 1/2 of 24} – 3]
= [5 + 1/12 of {38 – 14 + 12} – 3]
= [5 + 1/12 of {36} – 3] = [5 + 3 – 3] = 5

RECURRING DECIMALS

A decimal in which a digit or a set of digits is repeated continuously is called a recurring decimal. Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and the last of them, thus

$\frac{8}{3} = 2.666..... = 2.\dot{6}$ or $2.\overline{6}$;

$\frac{1}{7} = 0.142857142857142857... = 0.\overline{142857}$

In case of 1/7, where the set of digits 142857 is recurring, the dot is placed on top of the first and the last digits of the set or alternatively, a bar is placed over the entire set of the digits that recur.

A recurring decimal like $0.\overline{3}$ is called a pure recurring decimal because all the digits after the decimal point are recurring.

A recurring decimal like $0.1\overline{6}$ (which is equal to 0.16666...) is called a mixed recurring because some of the digits after the decimal are not recurring (in this case, only the digit 6 is recurring and the digit 1 is not recurring).

A recurring decimal is also called a "circulator". The digit, or set of digits, which is repeated is called the "period" of the decimal. In the decimal equivalent to 8/3, the period is 6 and in 1/7 it is 142857.

As already discussed, all recurring decimals are rational numbers as they can be expressed in the form p/q, where p and q are integers. The general rule for converting recurring decimals into fractions will be considered later. Let us first consider a few examples so that we will be able to understand the rule easily.

1.02. Express $0.\overline{6}$ in the form of a fraction.

Sol: $0.\overline{6} = 0.666…………$
Let x = 0.66 -------- (1)
As the period is of one digit, we multiply the given number by 10^1 i.e., 10
Therefore, 10x = 6.666 -------- (2)
(2) – (1) gives, ⇒ 9x = 6
⇒ x = 6/9 = 2/3

1.03. Express $0.\overline{81}$ in the form of a fraction.

Sol: $0.\overline{81} = 0.818181$ …………….
Let x = 0.8181 -------- (1)
As the period is containing 2 digits, we multiply by 10^2 i.e., 100
Therefore 100 x = 81.8181 -------- (2)
(2) – (1) gives, 99 x = 81
⇒ x = 81/99 = 9/11

1.04. Express the recurring decimal $0.\overline{024}$ in the form of a fraction.

Sol: $0.\overline{024} = 0.024024024$
Let x = 0.024024 --------- (1)
As the period contains 3 digits, we multiply with 10^3 i.e., 1000, therefore
1000 x = 24.024024 -------- (2)
(2) – (1) gives, 999x = 24
⇒ x = 24/999 = 8/333

We can now write down the rule for converting a pure recurring decimal into a fraction as follows:

A pure recurring decimal is equivalent to a vulgar fraction which has the number formed by the recurring digits (called the period of the decimal) for its numerator, and for its denominator the number which has for its digits as many nines as there are digits in the period.

Thus $0.\overline{37}$ can be written as equal to $\frac{37}{99}$; $0.\overline{225}$ can be written as equal to $\frac{225}{999} = \frac{25}{111}$;

$0.\overline{63} = \frac{63}{99} = \frac{7}{11}$.

A mixed recurring decimal becomes the sum of a whole number and a pure recurring decimal, when it is multiplied by suitable power of 10 which will bring the decimal point to the left of the first recurring figure. We can then find the equivalent vulgar fraction by the process as explained in case of a pure recurring decimal.

1.05. Express $0.2\overline{7}$ as a fraction.

Sol: Let x = $0.2\overline{7}$, then 10x = $2.\overline{7} = 2 + 0.\overline{7}$
= 2 + 7/9 (since $0.\overline{7}$ = 7/9)
⇒10x = 25/9
⇒ x = 25/90 = 5/18

1.06. Express $0.2\overline{79}$ in the form of a fraction.

Sol: Let x = $0.2\overline{79}$
10x = $2.\overline{79}$ = 2 + 79/99 = 277/99
x = 277/990

Now we can write the rule to express a mixed recurring decimal into a (vulgar) fraction as below:

In the numerator write the entire given number formed by the (recurring and non-recurring parts) and subtract from it the part of the decimal that is not recurring. In the denominator, write as many nines as the period (i.e., as many nines as the number of digits recurring) and then place next to it as many zeroes as there are digits without recurring in the given decimal.

i.e. $0.1\overline{56} = \frac{156-1}{990} = \frac{155}{990} = \frac{31}{198}$

$0.7\overline{3} = \frac{73-7}{90} = \frac{66}{90} = \frac{11}{15}$

INTEGERS

A number of problems are based on the operation of division and the relation between the quantities involved in division.

Properties of Division

Before we take up the next area, the following simple points should be kept in mind.

1. A number when divided by d leaving a remainder of r is of the form dq + r where q is some integer from 0, 1, 2, ...

   For example, a number when divided by 4 leaving a remainder of 3 can be written in the form (4q + 3); a number when divided by 7 leaving a remainder of 4 can be written in the form (7q + 4)

2. When a number N is divided by divisor d if the remainder is r, then the number N – r is exactly divisible by d or in other words, when N – r is divided by d the remainder is 0.

   For example, when the number 37 is divided by 7, the remainder is 2; if this remainder 2 is subtracted from the number 37, the resulting number 35 is exactly divisible by 7.

3. When a number N is divided by a divisor d, if the remainder is r, then

   (a) the largest multiple of d which is less than or equal to N is obtained by subtracting r from N, i.e., N – r will be the largest multiple of d which is less than or equal to N.

   For example, when 27 is divided by 5, the remainder is 2; so 27 – 2, i.e., 25 is the largest multiple of 5 less than 27.

   (b) the smallest multiple of d which is greater than or equal to N is obtained by adding (d – r) to N, i.e., N + (d – r) will be the smallest multiple of d which is greater than N.

   For example, when 49 is divided by 8, the remainder is 1; hence the smallest multiple of 8 which is greater than 48 is 49 + (8 – 1) = 56

4. When a division is split into a sum of two divisions (with the same divisor as the original divisor), the original remainder will be equal to the sum of the remainders of the two individual divisions. Similarly, when a division is split into difference of two divisions, the original remainder will be equal to the difference of the remainders of the two divisions.

   For example, if we take the division 15/6 (where the remainder is 3), and write it as a SUM of two divisions 8/6 and 7/6 (where the remainders are respectively 2 and 1), the original remainder is equal to the SUM of the two remainders 2 and 1.

   $\frac{15}{6} = \frac{8}{6} + \frac{7}{6}$

   Remainder 3 = 2 + 1

   If we take the division 15/6 and write it as the DIFFERENCE of two divisions 29/6 and 14/6 (where the respective remainders are 5 and 2), the original remainder 3 is equal to the DIFFERENCE of the two remainders 5 and 2.

   $\frac{15}{6} = \frac{29}{6} - \frac{14}{6}$

   Remainder 3 = 5 – 2

5. If the remainder in a division is negative, then add the divisor repeatedly to the negative remainder till we get a positive remainder.

   For example, let us take the division 15/6 (where the remainder is 3) and split into difference of two divisions 25/6 and 10/6. The remainders of the two divisions are 1 and 4 respectively. The difference of these two remainders is 1 – 4 which is equal to –3 and this should be equal to the original remainder. Since this remainder is negative, add the divisor 6 to this negative remainder –3 to get the correct remainder 3.

   $\frac{15}{6} = \frac{25}{6} - \frac{10}{6}$

   Remainders are 3, 1, –4.

   Remainder 1 – 4 = –3 which is same as –3 + 6 = 3

6. In a division, if the dividend (the number which is being divided) is multiplied by a certain factor and then divided by the same divisor, then the new remainder will be obtained by multiplying the original remainder by the same factor with which the dividend has been multiplied.

   For example, when 11 is divided by 8, the remainder is 3. When the dividend 11 is multiplied by 2, we get 22 and when this number is divided by 8, the remainder is 6 which is same as the original remainder 3 multiplied by 2.

7. If the remainder is greater than the divisor, it means division is not complete. To get the correct remainder keep subtracting the divisor from the remainder till you obtain the positive remainder which is less than the divisor.

Factors, Multiples and Co-primes

Factors

If one number divides a second number exactly, then the first number is said to be a factor of the second number. For example, 5 is a factor of 15; 3 is a factor of 18. Factors are also called sub-multiples or divisors.

Multiples

If one number is divisible exactly by a second number, then the first number is said to be a multiple of the second number. For example, 15 is a multiple of 5; 24 is a multiple of 4.

Co-Primes

Two numbers are said to be relative primes or co-primes if they do not have any common factor other than 1. For example, the numbers 15 and 16 do not have any common factors and hence they are relative primes. Please note that none of the two numbers may individually be prime and still they can be relative primes. Unity is a relative prime to all numbers.

Rules for divisibility

In a number of situations, we will need to find the factors of a given number. Some of the factors of a given number can, in a number of situations, be found very easily either by observation or by applying simple rules. We will look at some rules for divisibility of numbers.

Divisibility by 2

A number divisible by 2 will have an even number as its last digit (For example 128, 246, 2346, etc)

Divisibility by 3

A number is divisible by 3 if the sum of its digits is a multiple of 3.

For example, take the number 9123, the sum of the digits is 9 + 1 + 2 + 3 = 15 which is a multiple of 3. Hence, the given number 9123 is divisible by 3. Similarly 342, 789 etc,. are all divisible by 3. If we take the number 74549, the sum of the digits is 29 which is not a multiple of 3. Hence the number 74549 is not divisible by 3.

Divisibility by 4

A number is divisible by 4 if the number formed with its last two digits is divisible by 4.

For example, if we take the number 178564, the last two digits form 64. Since this number 64 is divisible by 4, the number 178564 is divisible by 4.

If we take the number 476854, the last two digits form 54 which is not divisible by 4 and hence the number 476854 is not divisible by 4.

Divisibility by 5

A number is divisible by 5 if its last digit is 5 or zero (eg. 15, 40, etc.)

Divisibility by 6

A number is divisible by 6 if it is divisible both by 2 and 3 (18, 42, 96, etc.)

Divisibility by 7

If the difference between the number of tens in the number and twice the units digit is divisible by 7, then the given number is divisible by 7. Otherwise, it is not divisible by 7.

Take the units digit of the number, double it and subtract this figure from the remaining part of the number. If the result so obtained is divisible by 7, then the original number is divisible by 7. If that result is not divisible by 7, then the number is not divisible by 7.

For example, let us take the number 595. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 59. If 10 (which is the units digit doubled) is subtracted from 59 we get 49. Since this result 49 is divisible by 7, the original number 595 is also divisible by 7.

Similarly, if we take 967, doubling the units digit gives 14 which when subtracted from 96 gives a result of 82. Since 82 is not divisible by 7, the number 967 is not divisible by 7.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 7. For example, take 456745, We will write down the figures in various steps as shown below.

Col(1) Number	Col (2) Twice the units digit	Col (3) Remaining part of the number	Col(3) - Col(2)
456745	10	45674	45664
45664	8	4566	4558
4558	16	455	439
439	18	43	25

Since 25 in the last step is not divisible by 7, the original number 456745 is not divisible by 7.

Divisibility by 8

A number is divisible by 8, if the number formed by the last 3 digits of the number is divisible by 8.

For example, the number 3816 is divisible by 8 because the last three digits form the number 816, which is divisible by 8. Similarly, the numbers 14328, 18864 etc. are divisible by 8. If we take the number 48764, it is not divisible by 8 because the last three digits' number 764 is not divisible by 8.

In general, if the number formed by the last n digits of a number is divisible by 2^n, the number is divisible by 2^n.

Divisibility by 9

A number is divisible by 9 if the sum of its digits is a multiple of 9.

For example, if we take the number 6318, the sum of the digits of this number is 6 + 3 + 1 + 8 which is 18. Since this sum 18 is a multiple of 9, the number 6318 is divisible by 9. Similarly, the numbers 729, 981, etc. are divisible by 9. If we take the number 4763, the sum of the digits of this number is 20 which is not divisible by 9. Hence the number 4763 is not divisible by 9.

Divisibility by 10

A number divisible by 10 should end in zero.

Divisibility by 11

A number is divisible by 11 if the sum of the alternate digits is the same or they differ by multiples of 11 - that is, the difference between the sum of digits in odd places in the number and the sum of the digits in the even places in the number should be equal to zero or a multiple of 11.

For example, if we take the number 132, the sum of the digits in odd places is 1 + 2 = 3 and the sum of the digits in even places is 3. Since these two sums are equal, the given number is divisible by 11.

If we take the number 785345, the sum of the digits in odd places is 16 and the sum of the digits in even places is also 16. Since these two sums are equal, the given number is divisible by 11.

If we take the number 89394811, the sum of the digits in odd places is 8 + 3 + 4 + 1, which is equal to 16. The sum of the digits in even places is 9 + 9 + 8 + 1, which is equal to 27. The difference between these two figures is 11 (27 – 16), which is a multiple of 11. Hence the given number 89394811 is divisible by 11.

The number 74537 is not divisible by 11 because the sum of the digits in odd places is 19 and the sum of the digits in even places is 7 and the difference of these two figures is 12 and this is not a multiple of 11.

Divisibility by numbers like 12, 14, 15 can be checked out by taking factors of the number which are relatively prime and checking the divisibility of the given number by each of the factors. For example, a number is divisible by 12 if it is divisible both by 3 and 4.

The next number that is of interest to us from divisibility point of view is 19.

Divisibility by 19

If the sum of the number of tens in the number and twice the units digit is divisible by 19, then the given number is divisible by 19. Otherwise it is not.

Take the units digit of the number, double it and add this figure to the remaining part of the number. If the result so obtained is divisible by 19, then the original number is divisible by 19. If that result is not divisible by 19, then the number is not divisible by 19.

For example let us take the number 665. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 66. If 10 (which is the units digit doubled) is added to 66 we get 76. Since this result 76 is divisible by 19, the original number 665 is also divisible by 19.

Similarly, if we take 969, doubling the units digit gives 18 which when added to 96 gives a result of 114. Since 114 is divisible by 19, the number 969 is divisible by 19.

If we take 873, double the units digit (2 x 3 = 6) added to the remaining part of the number (87), we get 93 which is not divisible by 19. Hence the original number 873 is not divisible by 19.

If we take a larger number, the same rule may have to be repeatedly applied till the result comes to a number which we can make out by observation whether it is divisible by 19. For example, take 456760. We will write down the figures in various steps as shown below.

Col(1) Number	Col (2) Twice the units digit	Col (3) Remaining part of the number	Col(3) + Col(2)
456760	0	45676	45676
45676	12	4567	4579
4579	18	457	475
475	10	47	57

Since 57 in the last step is divisible by 19, the original number 456760 is divisible by 19.

Let us take another example, the number 37895. Let us follow the above process step by step till we reach a manageable number.

37895 Double the units digit 5 and add the 10 so obtained to 3789. We get
3799 Double the units digit 9 and add the 18 so obtained to 379. We get 397 Double the units digit 7 and add the 14 so obtained to 39. We get 53.

Since 53 is not divisible by 19, 37895 is not divisible by 19.

FACTORS AND CO-PRIMES OF A NUMBER

Number of Factors of a Number

If N is a composite number such that N = $a^p . b^q . c^r$ .... where a, b, c are prime factors of N and p, q, r .... are positive integers, then the number of factors of N is given by the expression
(p + 1) (q + 1) (r + 1) ...
For example 140 = $2^2 \times 5^1 \times 7^1$.
Hence 140 has (2 + 1)(1 + 1)((1 + 1), i.e., 12 factors.

Please note that the figure arrived at by using the above formula includes 1 and the given number N also as factors. So if you want to find the number of factors the given number has excluding 1 and the number itself, we find out (p + 1) (q + 1) (r + 1) and then subtract 2 from that figure.

In the above example, the number 140 has 10 factors excluding 1 and itself.

Number of ways of expressing a given number as a product of two factors

The given number N (which can be written as equal to $a^p. b^q. c^r$ ...... where a, b, c are prime factors of N and p, q, r..... are positive integers) can be expressed as the product of two factors in different ways.

The number of ways in which this can be done is given by the expression 1/2 {(p + 1)(q + 1)(r + 1).....}

So, 140 can be expressed as a product of two factors in 12/2 or 6 ways {because (p + 1) (q + 1) (r + 1) in the case of 140 is equal to 12}

If p, q, r, etc. are all even, then the product (p + 1) (q + 1) (r + 1) ……. becomes odd and the above rule will not be valid since we cannot take 1/2 of an odd number to get the number of ways. If p, q, r, ... are all even, it means that the number N is a perfect square. This situation arises in the specific cases of perfect squares because a perfect square can also be written as {square root x square root}. So, two different cases arise in case of perfect squares depending on whether we would like to consider writing the number as {square root x square root} also as one of the ways.

Thus, to find out the number of ways in which a perfect square can be expressed as a product of 2 factors, we have the following 2 rules

(1) as a product of two DIFFERENT factors:½ {(p + 1) (q + 1)(r + 1) ...... –1 } ways (excluding $\sqrt{N} \times \sqrt{N}$).

(2) as a product of two factors (including $\sqrt{N} \times \sqrt{N}$) in 1/2 {(p + 1)(q + 1)(r + 1) .... +1} ways.

1.07. Find the number of factors of 1225.

Sol: If a number can be expressed as a product of prime factors like
$a^p \times b^q \times c^r$ x - - - -
where a, b, c, …..are the prime numbers, then the number of factors of the number is (p + 1) (q + 1) (r + 1)…………..

First express 1225 as a product of its prime factors. (Note that to express a given number as a product of its prime factors, we first need to identify the prime factors of the given number by applying the rules of divisibility.)
$1225 = 5 \times 7 \times 5 \times 7 = 5^2 \times 7^2$
Hence, the number of factors 1225 has is (2 + 1) (2 + 1) = 9

1.08. How many divisors excluding 1 and itself does the number 4320 have?

Sol: Note that the two terms factors and divisors are used interchangeably. First express 4320 in terms of its prime factors.
4320 = 18 x 24 x 10
= 3 x 3 x 2 x 3 x 2 x 2 x 2 x 2 x 5
$= 3^3 \times 2^5 \times 5^1$
Hence 4320 has (3 + 1) (5 + 1) (1 + 1) = 48 factors. Excluding 1 and itself, the number has (48 – 2) = 46 factors.

1.09. In how many ways can 3420 be written as a product of two factors?

Sol: By prime factorisation 3420
$= 2 \times 5 \times 2 \times 19 \times 3^2 = 2^2 \times 3^2 \times 5^1 \times 19^1$
If a number is expressed as product of prime factors, like $a^p \times b^q \times c^r \times \ldots\ldots$ where a, b, c,….. are prime numbers, then the number of ways in which the number can be expressed as a product of two factors = 1/2 [(p + 1) (q + 1) (r + 1)………]
Hence, 3420 can be written as product of two factors in ½[(2 + 1) (2 + 1) (1 + 1) (1 + 1)] = 18 ways

1.10. In how many ways can the number 52900 be written as a product of two different factors?

Sol: First expressing 52900 as a product of its prime factors, we get $52900 = 23^2 \times 2^2 \times 5^2$. Since all the powers are even, the given number is a perfect square. (Remember we can look at writing the number as a product of two factors either including or excluding the "square root x square root". Since we have to find the number of ways of writing the number as a product of two "different" factors, we cannot consider square root x square root)
So, required number of ways is
1/2 {(2 + 1) (2 + 1) (2 + 1) – 1}= 1/2{27 – 1} = 13

Sum of all the factors of a number

If a number $N = a^p \cdot b^q \cdot c^r$ ………. where a, b, c….. are prime numbers and p, q, r ….. are positive integers, then, the sum of all the factors of N (including 1 and the number itself) is:

$$\left(\frac{a^{p+1}-1}{a-1}\right) \cdot \left(\frac{b^{q+1}-1}{b-1}\right) \cdot \left(\frac{c^{r+1}-1}{c-1}\right) \ldots\ldots\ldots\ldots$$

The above can be verified by an example.

Consider the number 48, when resolved into prime factors, $48 = 2^4 \times 3^1$. Here a = 2, b = 3, p = 4, q = 1.
Hence, sum of all the factors

$$= \left(\frac{2^{4+1}-1}{2-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right) = \frac{31}{1} \times \frac{8}{2} = 124$$

The list of factors of 48 is
1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.

Product of all the factors of a number

The following examples explain the method of finding the product of all the factors of a number.

1.11. What is the product of all the factors of 180?

Sol: $180 = 4(45) = 2^2 3^2 5^1$. There are (2 +1) (2 + 1) (1 + 1) or 18 factors.
If the given number is not a perfect square, at least one of the indices is odd and the number of factors is even. We can form pairs such that the product of the two numbers in each pair is the given number (180 in this example).

$\therefore$ The required product is $180^{9.}$

In general, if $N = p^a\, q^b\, r^c$ (where at least one of a, b, c is odd), the product of all the factors of N is $N^{\frac{d}{2}}$, where d is the number of factors of N and is given by (a+1)(b+1)(c+1).

1.12. Let us see what happens when N is a perfect square. Find the product of all the factors of 36.

Sol: $36 = 2^2 3^2$ (there are 9 factors)
1(36) = 2(18) = 3(12) = 4(9) = 6(6)
$\therefore$ The product of all the factors is 36^4 (6).

In general, let $N = p^a\, q^b\, r^c$ where each of a, b, c is even.
There are (a + 1)(b + 1)(c + 1) say d factors. We can form $\frac{d-1}{2}$ pairs and we would be left with one lone factor, i.e., $\sqrt{N}$. The product of all these factors is $N^{\frac{d-1}{2}}(\sqrt{N}) = N^{\frac{d}{2}}$
$\therefore$ Whether or not N is a perfect square, the product of all its factors is $N^{\frac{d}{2}}$, where d is the number of factors of N.

1.13. What is the product of all the factors of 1728?

Sol: The product of the factors of a positive integer N is $N^{k/2}$, where k is the number of factors of N.
Now $1728 = 12^3 = 2^6 3^3$ and k = (6 + 1) (3 + 1) = 28
$\therefore$ The product of all the factors of 1728 = 1728^{14}

Number of ways of writing a number as product of two co-primes

Using the same notation and convention used earlier. If $N = a^p \cdot b^q \cdot c^r$……………, then, the number of ways of writing N as a product of 2 co-primes is 2^{n-1}, where 'n' is the number of distinct prime factors of the given number N.

Taking the example of 48, which is $2^4 \times 3^1$, the value of 'n' is 2 because only two distinct prime factors (i.e. 2 and 3 only) are involved.

Hence, the number of ways = $2^{2-1} = 2^1 = 2$ i.e. 48 can be written as product of 2 co-primes, in two different ways. They are (1, 48) and (3, 16).

Number of co-primes to N, that are less than N

If N is a number that can be written as $a^p \cdot b^q \cdot c^r$ ………., then, the number of co-primes of N, which are less than N, represented by ϕ (N) is,

$$N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)..........$$

For example if, 48 is considered,
$N = a^p \cdot b^q \cdot c^r$ ……….. i.e. $48 = 2^4 \cdot 3^1$.
Hence, a = 2, b = 3, p = 4, q = 1.

$$\phi(48) = 48\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 48 \times \frac{1}{2} \times \frac{2}{3} = 16.$$

Note : If numbers less than 48 are listed, and co-primes to 48 are spotted, the count of co-primes will be 16.

Sum of co-primes to N that are less than N

The sum of the co-primes of N, that are less than N is $\frac{N}{2} \cdot \phi(N)$. If we consider the above example, already we have ϕ (48) = 16.
Hence, sum of co-primes of 48 that are less than 48

$$= \frac{N}{2} \cdot \phi(N) = \frac{48}{2} \times 16 = 384$$

Note: After listing the co-primes of 48 that are less than 48, they can be added and the sum can be verified.

1.14. Find the largest four digit multiple of 31.

Sol: We take the largest four-digit number possible i.e., 9999 and divide it by 31. We get a remainder of 17. This remainder 17 is then subtracted from 9999 giving 9982 which is a multiple of 31. Therefore 9982 is the largest four digit multiple of 31.

1.15. Find the smallest five digit multiple of 17.

Sol: First we consider 10,000, the smallest five- digit number. Dividing 10,000 by 17 we get the remainder 4. We take the difference between the divisor 17 and the remainder 4 which is 13 and add this 13 to 10,000. We get 10,013 which is the smallest five-digit multiple of 17.

FACTORS AND MULTIPLES OF TWO OR MORE NUMBERS

Least Common Multiple (LCM) and Highest Common Factor (HCF)

Least Common Multiple (LCM) of two or more numbers is the least number which is divisible by each of these numbers (i.e. leaves no remainder; or remainder is zero). The same can be algebraically defined as "LCM of two or more expressions is the expression of the lowest dimension which is divisible by each of them i.e. leaves no remainder; or remainder is zero."

Highest Common Factor (HCF) is the largest factor of two or more given numbers. The same can be defined algebraically as "HCF of two or more algebraical expressions is the expression of highest dimension which divides each of them without remainder.

HCF is also called GCD (Greatest Common Divisor).

Product of two numbers = LCM x HCF

LCM is a multiple of HCF

For finding **LCM and HCF of fractions,** first reduce each fraction to its simplest form i.e., cancel out any common factors between the denominator and numerator and then apply appropriate formula from the following :

$$\textbf{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

$$\textbf{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

LCM and HCF can each be found by either one of two methods :
(1) Factorization (2) Division

We will look at both the methods.

LCM by Factorization

Resolve the numbers into prime factors. Then multiply the product of all the prime factors of the first number by those prime factors of the second number, which are not common to the prime factors of the first number.

This product is then multiplied by those prime factors of the third number, which are not common to the prime factors of the first two numbers.

In this manner, all the given numbers have to be dealt with and the last product will be the required LCM.

In other words, take the product of ALL the prime factors of all the numbers except where a factor is occurring in more than one number, it is taken only ONCE in the product. This product is the LCM of all the numbers.

1.16. Find the LCM of 144, 504 and 720.

Sol: Write each number in terms of its prime factors.
$144 = 3^2 \times 2^4$
$504 = 3^2 \times 2^3 \times 7$
$720 = 2^4 \times 3^2 \times 5$
LCM is $2^4 \times 3^2 \times 7 \times 5 = 5040$

LCM by Division

Select any one prime factor common to at least two of the given numbers. Write the given numbers in a line and divide them by the above prime number. Write down the quotient for every number under the number itself. If any of the numbers is not divisible by the prime factor selected, write the number as it is in the line of quotients. Repeat this process for the line of quotients until you get a line of quotients, which are prime to each other (i.e., no two "quotients" should have a common factor).

The product of all the divisors and the numbers in the last line will be the required LCM.

1.17. Find the LCM of 12, 18 and 27.

Sol: By division method

$$\begin{array}{r|l} 2 & 12, 18, 27 \\ \hline 3 & 6, 9, 27 \\ \hline 3 & 2, 3, 9 \\ \hline & 2, 1, 3 \end{array}$$

LCM = 2 x 3 x 3 x 2 x 1 x 3 = 108

HCF by Factorization

Resolve the given number into prime factors. The product of the prime factors common to all the numbers will be the required HCF.

1.18. Find the HCF of 324, 576 and 784.

Sol: First of all resolve all the numbers into their prime factors

$324 = 3^4 \times 2^2$

$576 = 3^2 \times 2^6$

$784 = 7^2 \times 2^4$

Then take the product of the factors common to all the numbers.

HCF here will be $2^2 = 4$

HCF by Long Division

Take two numbers. Divide the greater by the smaller; then divide the divisor by the remainder; divide the divisor of this division by the next remainder and so on until the remainder is zero. The last divisor is the HCF of the two numbers taken.

By the same method find the HCF of this HCF and the third number. This will be the HCF of the three numbers.

1.19. Find the HCF of 2223 and 3762.

Sol:

```
2223)3762(1
     2223
     1539)2223(1
          1539
           684)1539(2
               1368
                171)684(4
                    684
                      0
```

Hence HCF of 2223 and 3762 is 171.

LCM and HCF Models

LCM - Model 1

In this model of problem, you will need to find out the smallest number (or number in a specified range like the largest five-digit number) which when divided by 2 or more other numbers (i.e., divisors) leaves the **same** remainder in all cases.

The basic distinguishing feature of this model of problems is that the remainder will be the **same** in all the cases (and that remainder will also be given).

The smallest such number will be the remainder itself. The next higher number that satisfies the given conditions is the LCM of the given numbers (i.e., divisors) plus the remainder given, i.e., add the remainder (which is the same in all cases) to the LCM of the given numbers (i.e., divisors).

To find any larger number that satisfies a given condition, we will first need to find out a multiple of the LCM in that range and add the remainder to this multiple of the LCM.

The general rule can be written as follows:

> Any number which when divided by p, q or r leaving the same remainder s in each case will be of the form
> **k (LCM of p, q and r) + s**
> where k = 0, 1, 2, ...
> If we take k = 0, then we get the smallest such number.

1.20. Find the smallest number which when divided by 5 or 8, leaves a remainder of 2 in each case and the number being greater than the two divisors.

Sol: The LCM of 5 and 8 is 40

Hence the required number is 40 + 2 = 42

1.21. Find the largest three-digit number which when divided by 4 or 7 leaves a remainder of 3 in each case.

Sol: The LCM of 4 and 7 is 28. Since we are interested in the largest three-digit number, we should first find out the largest three-digit multiple of 28. This can be obtained by first dividing 999 by 28, which leaves a remainder of 19. Hence 999 – 19 = 980 is the largest three-digit multiple of 28. Now add the remainder 3 to the number to get 983, which is the required number.

1.22. Find the smallest seven-digit number which when divided by 8 or 13 leaves a remainder of 5 in each case.

Sol: We need to find the smallest seven-digit multiple of 104 (104 is the LCM of 8 and 13) and add the remainder of 5 to that multiple to get the required number. Take the smallest seven-digit number 1000000 and divide by 104. We get a remainder of 40.

Take the difference between the divisor 104 and the above remainder 40, which is 64. This is added to 1000000 to give 1000064, which is the smallest seven-digit multiple of 104.

Now add the remainder 5 to get the required number as 1000069

1.23. Find the smallest number greater than the divisors, which when divided by 6, 13 and 17 leaves a remainder of 7 in each case.

Sol: The smallest number that satisfies the above condition is obtained by taking the LCM of the numbers 6, 13 and 17 and adding the remainder of 7 to it. LCM of 6, 13 and 17 is 1326.

Hence, the required number is 1326 + 7

= 1333

LCM - Model 2

In this model, the remainders in the divisions given will not be the same but the difference between the divisor and the remainder (i.e. the complement of the remainder) will be the same in each case. For example, you may be asked to find out "the smallest number which when divided by 4 or 6 gives respective remainders of 3 and 5." Here, the remainders are not the same as in LCM - Model 1; but the difference between the divisor and the remainder is same in each case. In the first case the difference between the divisor and the remainder is 1(= 4 – 3). In the second case also the difference between the divisor and the remainder is 1(= 6 – 5).

The smallest such number is LCM minus constant difference (the constant difference being the difference between the divisor and the corresponding remainder in all cases).

Similarly, any multiple of the LCM minus the constant remainder also will satisfy the same condition.

In the example considered above, the LCM of 4 and 6 is 12 and hence the required number is 11 (which is equal to 12 – 1).

The general rule can be written as follows:

Any number which when divided by p, q or r leaving respective remainders of s, t and u where (p – s) = (q – t) = (r – u) = v (say), will be of the form **k (LCM of p, q and r) –** v The smallest such number will be obtained by substituting k = 1.

1.24. Find the smallest number which when divided by 19 and 23 leaves remainders of 13 and 17 respectively.

Sol: The LCM of 19 and 23 is 437. The difference between the divisor and the remainder in each case is the same i.e. 19 – 17 = 6 and 23 – 17 = 6. Hence, the number that satisfies the given conditions will be equal to 437 – 6 = 431.

1.25. Find the largest four-digit number which when divided by 7 and 12 leaves remainders 5 and 10 respectively.

Sol: The difference between the divisor and the remainder is the same in each case i.e. 7 – 5 = 2 and 12 – 10 = 2. The LCM of 7 and 12 is 84. We will first find the largest four digit multiple of 84 and subtract 2 from it. The largest four-digit number 9999 when divided by 84 leaves a remainder of 3. The largest four-digit number divisible by 84 is hence 9999 – 3 = 9996. Hence the required number is 9996 – 2 = 9994

1.26. Find the smallest number which, when divided by 7, 13 and 23 leaves respective remainders of 5, 11 and 21.

Sol: The difference between the divisor and the remainder is the same in each case, it is 2. The smallest number satisfying the given condition can be obtained by subtracting 2 from the LCM of the given divisors. The LCM of 7, 13 and 23 is 2093. Hence the required number is 2093 – 2 = 2091

1.27. Find the smallest six-digit number which when divided by 8 leaves a remainder of 3 and when divided by 14 leaves a remainder of 9.

Sol: Here again the difference between the divisor and the remainder in each case is 5, (8 – 3 = 5 and 14 – 9 = 5).
First find the smallest six-digit multiple of 56, the LCM of 8 and 14. 100000 leaves a remainder of 40 when divided by 56. Take the difference between 56 and 40, which is 16 and add it back to 100000 to give us the smallest six-digit multiple of 56 which is 100016. Hence the required number is 100016 – 5 = 100011.

LCM - Model 3

In this model the remainders will not be the same and even the differences between each of the given divisors and the corresponding remainders also will not remain the same.

Let us take an example and see how to solve this type of problem.

Find out the smallest number which when divided by 7 gives a remainder of 3 and when divided by 5 gives the remainder of 2.

Solution: Here, the remainders are not the same. The difference between the divisor and the remainder in the first case is 4 and in the second case, is 3.

Take the larger of the two given divisors –7 in this case. The required number, when divided by 7 gives a remainder of 3. We know that a number when divided by 7 giving a remainder of 3 is of the form 7k + 3, which means we are looking for a number of the form 7k + 3.

Since the same number, when divided by 5 gives a remainder of 2, this number (7k + 3) when divided by 5 gives a remainder of 2. We know that if there is a remainder in a division, by subtracting the remainder from the given number, the resulting number will then be exactly divisible by the divisor. This means, if 2 is subtracted from (7k + 3), the resulting number, i.e., 7k + 1 will be exactly divisible by 5. We should now give values of 0, 1, 2, ... to k and find out for what value of k, 7k + 1 will be divisible by 5.

The smallest value of k which satisfies the above condition, we notice, is 2 and hence k = 2 will give us a number that we are looking for. Since the number, we said, is 7k + 3 the number is 7 x 2 + 3 i.e. 17. So 17 is the smallest number which satisfies the two given conditions.

The next higher number which satisfies this condition is obtained by adding LCM of 7 and 5 to the smallest number 17 found above. In this manner by adding multiples of 35 (which is LCM of the two given numbers) to 17, we get a series of numbers that satisfy the given conditions. In other words any number of the form (35 m + 17) will satisfy the given conditions.
From this, we can also find out the smallest 4 digit number, largest 5 digit number, etc. that will satisfy the given conditions.

For example, let us find out the largest five-digit number that satisfies the conditions that the remainders are 3 and 2 respectively when divided by 7 and 5.

Since we know that any number that satisfies the above condition will be of the form (35 m + 17) and we want the largest 5-digit such number, we need to find a number close to 99999, i.e., 35 m + 17 ≤ 99999 ⇒ 35 m ≤ 99982 ⇒ we need to find a multiple of 35 which less than or equal to 99982 (and we have already learnt how to find the multiple of a given number which is less than or equal to another given number). A multiple of 35 less than or equal to 99982 is 99960 (i.e., 35 m = 99960). Hence the required number which is 35 m + 17 will then be equal to 99960 + 17, i.e., 99977.

1.28. Find the smallest number which, when divided by 6 leaves a remainder of 2 and when divided by 13 leaves a remainder of 6.

Sol: The required number will be in the form of (6k + 2) because when divided by 6 it leaves a remainder of 2. The same number when divided by 13 leaves a remainder of 6. Subtracting this remainder from the number (6k + 2), the resulting number (6k – 4) should be divisible by 13.
Trying out values of 0, 1, 2,…… for k, when k = 5, (6k – 4) will be 26 which is divisible by 13. Hence the required number is 6k + 2 = 6(5) + 2 = 32

HCF - Model 1

In this model, we have to identify the largest number that exactly divides the given dividends (which are obtained by subtracting the respective remainders from the given numbers).

> The largest number with which the numbers p, q or r are divided giving remainders of s, t and u respectively will be the **HCF of the three numbers (p – s), (q – t) and (r – u).**

Let us understand this model with an example.

1.29. Find the largest number with which when 425 and 373 are divided, respective remainders of 2 and 4 are left.

Sol: Since 425 when divided by the number gives a remainder of 2 it means 425 – 2 = 423 is exactly divisible by that number. Similarly 373 – 4 = 369 is also exactly divisible by that number. This means that the number we are looking for is the largest number which will divide 369 and 423 exactly. That will be the HCF of 369 and 423, which is 9.

1.30. Find the largest number with which when 394 and 658 are divided, respective remainders of 1 and 3 are left.

Sol: As discussed in the previous example, the required number is the HCF of (394 – 1) and (658 – 3) i.e., HCF of 393 and 655. The HCF of 393 and 655 is 131.

HCF - Model 2
In this model, the problem will be as follows:

"Find the largest number with which if we divide the numbers p, q and r, the remainders are the same."

Take the difference between any two pairs out of the three given numbers. Let us say we take the two differences (p – q) and (p – r). The HCF of these numbers will be the required number.

> Here, the required number = HCF of (p – q) and (p – r) = HCF of (p – q) and (q – r) = HCF of (q – r) and (p – r)

Let us take an example and look at this model.

1.31. Find the largest number with which when 472, 832 and 1372 are divided, the remainders are the same.

Sol: Take the difference between any two numbers out of the three given numbers
832 – 472 = 360
1372 – 832 = 540
The required number is the HCF of these two differences i.e., HCF of 360 and 540 which is 180.

1.32. Find the largest number with which when 247, 457 and 1087 are divided, the remainder in each case is the same.

Sol: Taking the difference of two of the numbers at a time, we get 457 – 247 = 210 and 1087 – 457 = 630
The required number is the HCF of the two differences i.e., HCF of 210 and 630 which is 210.

Successive Division

If the quotient of a division is taken and this is used as the dividend in the next division, such a division is called "successive division." A successive division process can continue upto any number of steps – until the quotient in a division becomes zero for the first time. i.e., the quotient in the first division is taken as dividend and divided in the second division; the quotient in the second division is taken as the dividend in the third division; the quotient in the third division is taken as the dividend in the fourth division and so on.

If we say that 2479 is divided successively by 3, 5, 7 and 2, then the quotients and remainders are as follows in the successive division.

Dividend	Divisor	Quotient	Remainder
2479	3	826	1
826	5	165	1
165	7	23	4
23	2	11	1

Here we say that when 2479 is successively divided by 3, 5, 7 and 2 the respective remainders are 1, 1, 4 and 2.

1.33. A number when divided successively by 13 and 3 gives respective remainders of 5 and 1. What will be the remainder when the largest such two-digit number is divided by 12?

Sol: We write down the divisors one after the other and their respective remainders below them.

Divisors	13	3
	+ ↓	×
Remainders	5	1

Then starting from the last remainder, we go diagonally left upwards to the first row multiplying and then come down directly adding the figure already obtained. We continue this process till we reach the figure on the extreme left in the second row, so we get (1 x 13) + 5 = 18. So the number is of the form (d_1 . d_2 . k + 18) where d_1, d_2 are divisors. In this case, it is = 39k + 18, for k = 0, 1, 2, …… So the largest two-digit number is 39(2) + 18 = 96.
This when divided by 12 leaves a remainder of 0.

1.34. A number when successively divided by 2, 3 and 5 leaves respective remainders of 1, 2 and 3. How many such numbers are there below 1000?

Sol: Let us write down all the divisors and their respective remainders as shown below:

Divisors	2	3	5
	+ ×	+ ×	
Remainders	1	2	3

We start at the bottom right corner 1 and go from 2nd row to 1st row diagonally to the left, multiplying. We get 3 x 3 = 9, then we come down to the 2nd row adding we get, 9 + 2 = 11. Again multiplying diagonally left upwards, we get 11 x 2 = 22 and coming down to 2nd row, adding we get 22 + 1 = 23.
∴The smallest number that satisfies the given condition is 23. The general form of the numbers that satisfy the given condition is got by adding the multiples of the PRODUCT of the divisors (which may be greater than or equal to the LCM) which is 30, to the smallest value obtained, which is 23.

Therefore the general form is 30k + 23.
For k = 0, 1, 2, ……32 the number is less than 1000. Hence there are 33 numbers less than 1000 that satisfy this condition.

1.35. A number when successively divided by 9, 5 and 4 leaves respective remainders of 2, 1 and 3. What will be the remainders when the least such number is divided successively by 7, 3 and 4?

Sol: Here again, we will first find the smallest number which satisfies the given condition.

Divisors	9	5	4
Remainders	2	1	3

The smallest number is
[{(3 x 5) + 1} x 9 + 2] = 146.

When 146 is successively divided by 7, 3 and 4, the results are

Dividend	Divisor	Quotient	Remainder
146	7	20	6
20	3	6	2
6	4	1	2

The remainders are 6, 2 and 2 respectively.

1.36. A number when successively divided by 9, 5 and 4 leaves respective remainders of 2, 1 and 3. What will be the remainders when the largest such three-digit number is divided successively by 7, 3 and 4?

Sol: From the previous example, we know that the smallest number which satisfies the given conditions is 146. The number itself is of the form 146 + (9)(5)(4)k, viz 146 + 180k where k = 0, 1, 2, ... The largest three-digit number is 866 when k = 4. When 866 is successively divided by 7, 3 and 4 the results are:

Dividend	Divisor	Quotient	Remainder
866	7	123	5
123	3	41	0
41	4	10	1

The remainders are 5, 0 and 1 respectively.

Factorial

Factorial is first defined for positive integers. It is denoted by /__ or !. Thus "Factorial n" is written as n! or /n. n! is defined as the product of all the integers from 1 to n.
Thus n! = 1.2.3. ... n(n – 1) n.
0! is defined to be equal to 1.
0! = 1 and, 1! is also equal to 1.

IGP of a Divisor in a Number

Very often we would like to know how many times we can divide a given number by another and continue to get integral quotients. We first consider prime divisors and then other divisors.

If a single number is given we simply represent it in its canonical form (the simplest and must convenient form). For example, consider N = 258,048.

By trial, we express N = $2^{12}3^{2}7^{1}$. We see immediately that N can be divided by 2 a total of 12 times, by 3 two times and by 7 just once. In other words the index of the greatest power (IGP) of 2 in N is 12, of 3 is 2 and of 7 is 1.

IGP of a number in N!

This model involves finding the index of the greatest power (**IGP**) of a divisor that divides the factorial of a given number (say N). (The statement ‘a divides b’ means the remainder of b divided by ‘a’ is 0. In this case, we also say ‘b is divisible by a’.) Let us understand this type of problem with the help of an example.

1.37. Find the IGP of 7 that can divide 256!, without leaving any remainder. (This can be concisely stated as find the IGP of P in N!)

Sol: First we shall take a look at the detailed explanation and then look at a simple method for solving the problem. When we write N = 256! in its expanded form, we have 256 x 255 x 254 x ……. x 3 x 2 x 1
When we divide 256! by a power of 7, we have the first 256 natural numbers in the numerator. The denominator will have only 7’s. The 256 numbers in the numerator have 36 multiples of 7 which are 7, 14, 21 ……. 252. Corresponding to each of these we can have a 7 in the denominator which will divide N completely without leaving any remainder i.e., 7^{36} can definitely divide 256! Further, every multiple of 49 after cancelling out 7 as above, will still have one more 7 left. Hence for

every multiple of 49 N we can have an additional 7 in the denominator. There are 5 multiples of 49 in 256! Hence we can have a 7^5 in the denominator. As $7^{36+5} = 7^{41}$, 41 is the IGP.

The above calculation is summarised below. Successively dividing 256 by 7, we get:

$$\begin{array}{r|l} 7 & 256 \\ 7 & 36 \\ & 5 \end{array}$$

Add all the quotients to get 36 + 5 = 41.
So the IGP of 7 contained in 256! is 41.

Please note that this method is applicable only if the number whose greatest power is to be found out is a prime number.

1.38. Find the IGP of 3 in 599!

Sol: Divide 599 successively by 3

$$\begin{array}{r|l} 3 & 599 \\ 3 & 199 \rightarrow \text{quotient} \\ 3 & 66 \rightarrow \text{quotient} \\ 3 & 22 \rightarrow \text{quotient} \\ 3 & 7 \rightarrow \text{quotient} \\ & 2 \end{array}$$

Add all the quotients,
199 + 66 + 22 + 7 + 2 = 296
Hence, 296 is the largest power of 3 that divides 599! without leaving any remainder.

1.39. Find the IGP of 10 that can divide 890!.

Sol: Here we cannot apply the successive division method as 10 is not a prime number. We know 10 can be written as 2 x 5 and these are prime numbers. So we find the largest powers of 2 and 5 respectively that can divide 890! and the smaller of the two indices is the index of the required power.

$$\begin{array}{r|l} 2 & 890 \\ 2 & 445 \\ 2 & 222 \\ 2 & 111 \\ 2 & 55 \\ 2 & 27 \\ 2 & 13 \\ 2 & 6 \\ 2 & 3 \\ & 1 \end{array}$$

Sum of the quotients = 883

$$\begin{array}{r|l} 5 & 890 \\ 5 & 178 \\ 5 & 35 \\ 5 & 7 \\ & 1 \end{array}$$

Sum of the quotients = 221
Since the largest power of 5 is the smaller, the largest power of 10 (i.e., 2 x 5) is 221.

If the divisor (say D) is not a prime number, we resolve it into its prime factors. Let $D = p^m q^n$ (where p, q are primes and m, n are positive integers). We first determine the IGP of p that divides N and the IGP of q that divides N. Let these be a and b respectively. Therefore, the IGP of p^m that divides N is $\left\lfloor \frac{a}{m} \right\rfloor$ and the IGP of q^n that divides N is $\left\lfloor \frac{b}{n} \right\rfloor$. Finally, the IGP of D that divides N is the smaller of $\left\lfloor \frac{a}{m} \right\rfloor$ and $\left\lfloor \frac{b}{n} \right\rfloor$. [$\lfloor x \rfloor$ is the greatest integer less than or equal to x.]

1.40. Find the IGP of 12 in 50!

Sol: $12 = 2^2 3$.
The IGP of 2 in 50! is obtained by successive division as shown below.

Number/Quotient	50	25	12	6	3	1
Divisor	2	2	2	2	2	

The IGP of 2 in 50! is 25 + 12 + 6 + 3 + 1 = 47

The IGP of 2^2 in 50! is $\left\lfloor \frac{47}{2} \right\rfloor = 23$

The IGP of 3 in 50! is 16 + 5 + 1 = 22
$\therefore$ The IGP of 12 in 50! is the smaller of 23 and 22, viz 22.

The following two results will prove to be extremely useful in problems on IGPs.

Let the IGP of p in A and B be m and n respectively.
(1) The IGP of p in AB is m + n.
(2) (a) If $m \neq n$, the IGP of p in A + B is the smaller of m and n.
(b) If m = n, the IGP of p in A + B is at least m. It could be more. (For example the IGP of 2 in 58 is 1 and the IGP of 2 in 6 is also 1. But the IGP of 2 in 58 + 6 is 6.)

1.41. Find the IGP of 2 in 31! + 32! + 33! + ... + 40!.

Sol: The IGP of 2 in 31! is 15 + 7 + 3 + 1, viz 26.
The IGP of 2 in 32! is 16 + 8 + 4 + 2 + 1, viz 31.
The IGP of 2 in the other terms is 31 or more.
$\therefore$ The IGP of 2 in the given expression is 26.

ALGEBRAIC IDENTITIES

There are a number of identities that we have studied in lower classes. We consolidate them here. We can classify them on two criteria – the number of symbols that are used and the degree of each term in the identity.

Identities with two symbols (degree 2)
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^2 - b^2 = (a + b)(a - b)$

Identities with two symbols (degree 3)
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Identities with three symbols (degree 2)
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2ac - 2bc$
$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$

$(x + a)(x + b) = x^2 + x(a+b) + ab$

Identities with three symbols (degree 3)
$(a + b)(b + c)(c + a) = a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2) + 2abc$
$= a^2(b + c) + b^2(c + a) + c^2(a + b) + 2abc$
$= ab(a + b) + bc(b + c) + ca(c + a) + 2abc$
$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc)$

Examples:

112^2	=	$(100 + 12)^2$
	=	$100^2 + (2 \times 100 \times 12) + 12^2 = 12544$
89^2	=	$(100 - 11)^2$
	=	$100^2 - (2 \times 100 \times 11) + 11^2 = 7921$
17×23	=	$(20 - 3)(20 + 3) = 20^2 - 3^2 = 391$
17^2	=	$(17 + 3)(17 - 3) + 3^2$
	=	$20 \times 14 + 9 = 289$
39^2	=	$(39 + 1)(39 - 1) + 1^2 = 40 \times 38 + 1^2$
	=	$1520 + 1 = 1521$
13^3	=	$(10 + 3)^3$
	=	$10^3 + 3^3 + (3 \times 10 \times 3)(10 + 3) = 2197$

SOME IMPORTANT POINTS

Please note the following points also which will be very useful in solving problems on Numbers.

1. When any two consecutive integers are taken one of them is odd and the other is even. Hence the product of any two consecutive integers is always even i.e. divisible by 2.

   Two consecutive integers can be written in the form of n and n – 1 or n and n + 1. Hence, any number of the form n(n – 1) or n(n + 1) will always be even.

2. Out of any 3 consecutive integers, one of them is divisible by 3 and at least one of the three is definitely even. Hence, the product of any 3 consecutive integers is always divisible by 6.

   Three consecutive integers can be of the form (n – 1), n and (n + 1). The product of 3 consecutive integers will be of the form $(n - 1)n(n + 1)$ or $n(n^2 - 1)$ or $(n^3 - n)$. Hence any number of the form $(n - 1)n(n + 1)$ or $n(n^2 - 1)$ or $(n^3 - n)$ will always be divisible by 6.

3. Out of any n consecutive integers, exactly one number will be divided by n and the product of n consecutive integers will be divisible by n!

4. Any prime number greater than 3 can be written in the form of 6k + 1 or 6k – 1. The explanation is:

   Let p be any prime number greater than 3. Consider the three consecutive integers (p – 1), p and (p + 1). Since p is a prime number greater than 3, p CANNOT be even. Since p is odd, both (p – 1) and (p + 1) will be even, i.e., both are divisible by 2.
   Also, since, out of any three consecutive integers, one number will be divisible by 3, one of the three numbers (p – 1), p or (p + 1) will be divisible by 3. But, since p is prime number – that too greater than 3 – p cannot be divisible by 3. Hence, either (p – 1) or (p + 1), one of them – and only one of them – is definitely divisible by 3.

   If (p – 1) is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e., it will be of the form 6k.
   If (p – 1) is of the form 6k, then p will be of the form (6k + 1).
   If (p + 1) is divisible by 3, since it is also divisible by 2, it will be divisible by 6, i.e., it will be of the form 6k.
   If (p + 1) is of the form 6k, then p will be of the form (6k – 1).

   Hence any prime number greater than 3 will be of the form (6k + 1) or (6k – 1).

Some Solved Examples:

1.42. Find the HCF of 1311 and 1653.

Sol: By division method,

```
1311 ) 1653 ( 1
       1311
       342 ) 1311 ( 3
             1026
             285 ) 342 ( 1
                   285
                   57 ) 285 ( 5
                        285
                         0
```

Therefore HCF of 1653 and 1311 is 57

1.43. Find the HCF of 1891 and 2257.

Sol: By division method,

```
1891 ) 2257 ( 1
       1891
       366 ) 1891 ( 5
             1830
             61 ) 366 ( 6
                  366
                   0
```

Therefore, the HCF is 61

1.44. Find the LCM of 2/9, 5/8 and 7/10.

Sol: Each fraction is in its simplest form.
Hence, LCM of the fractions

$= \frac{\text{LCM of the numerators}}{\text{HCF of the denominators}} = 70/1 = 70$

1.45. Find the HCF of 3/7, 5/9 and 11/10.

Sol: As each fraction is in its simplest form,

HCF of fractions $= \frac{\text{HCF of the numerators}}{\text{LCM of the denominators}}$

= 1/630

1.46. Arrange the following in ascending order: 2/5, 6/11, 5/13.

Sol: Take the LCM of the denominators and then compare the numerators

$$\frac{2}{5} = \frac{2 \times 143}{5 \times 143} = \frac{286}{715}$$

$$\frac{6}{11} = \frac{6 \times 65}{11 \times 65} = \frac{390}{715}$$

$$\frac{5}{13} = \frac{5 \times 55}{13 \times 55} = \frac{275}{715}$$

Comparing, we get 6/11 > 2/5 > 5/13
Ascending order is 5/13, 2/5, 6/11

Alternate method:

$\frac{2}{5} = 0.40$

$\frac{6}{11} = 0.54$

$\frac{5}{13} = 0.38$

The ascending order is $\frac{5}{13}$, $\frac{2}{5}$ and $\frac{6}{11}$

1.47. Test for divisibility of 2, 3, 4, 5, 6, 9, 10, 11 and 19 on the following numbers
(a) 672,
(b) 703 and
(c) 2310

Sol: (a) 672 → It is even. Hence it is divisible by 2. Sum of the digits = 15, which is divisible by 3. Hence, the number is divisible by 3. The last two digits form the number 72. Hence the number is divisible by 4. It does not end with 5 or 0 hence is not divisible by 5 or 10
Number is divisible by 2 as well as 3. Hence it is divisible by 6. Sum of the digits is not divisible by 9 hence the number 672 is not divisible by 9.
Difference between the sums of the alternate digits of the number = 1, hence not divisible by 11. Number of tens in the number + twice the unit's digit = 67 + 4 = 71, 7 + 2(1) = 9, hence it is not divisible by 19.

(b) 703 → The number is not even, hence it is not divisible by 2, 4, or 6. Sum of the digits is 10, hence it is not divisible by 3 or 9. It does not end with 5 or 0, hence it is not divisible by 5 or 10, difference between the sums of the alternate digits of the number is 10, hence it is not divisible by 11. Number of tens in the number + twice the units digit = 70 + 6 = 76, 7 + 12 = 19.
Hence it is divisible by 19.

(c) 2310 → The number is even, so it is divisible by 2. Sum of the digits is 6. So it is divisible by 3, but not by 9. Ends in 0, so divisible by both 5 and 10. The last two digits of the number which is 10 is not divisible by 4 hence the number is not divisible by 4. Difference between the alternate digits = 0.
Hence it is divisible by 11
Number of tens + twice the unit's digit
= 231 + 0 = 231
23 + 2 = 25
Therefore it is not divisible by 19.

1.48. Simplify the expression [2/3 of 4/5 {(9 x 3) – (6 x 2)} + 1/4 – 1/12}] using BODMAS rule.

Sol: [2/3 of 4/5 {(9 x 3) – (6 x 2)} + 1/4 – 1/12]
Applying BODMAS rule,
= [2/3 of 4/5 {27 – 12} + 1/4 – 1/12]
= [2/3 of 12 + 1/4 – 1/12]
= 8 + 1/4 – 1/12 = 49/6

1.49. Arrange 12/7, 14/9 and 9/5 in descending order.

Sol: LCM of the denominators = 315

$$\frac{12}{7} = \frac{12 \times 45}{7 \times 45} = \frac{540}{315}$$

$$\frac{14}{9} = \frac{14 \times 35}{9 \times 35} = \frac{490}{315}$$

$$\frac{9}{5} = \frac{9 \times 63}{5 \times 63} = \frac{567}{315}$$

Descending order is 567/315, 540/315, 490/315
∴ Descending order is 9/5, 12/7, 14/9

1.50. Simplify: $\dfrac{5\frac{4}{9} \times \frac{19}{7}}{2\frac{5}{7} - \frac{9}{14}}$.

Sol: $5^4/_9 = 49/9$
$2^5/_7 = 19/7$
Hence the numerator = 49/9 x 19/7
= 133/9
The denominator = 19/7 – 9/14
= 38/14 – 9/14 = 29/14

Given fraction = $\dfrac{\frac{133}{9}}{\frac{29}{14}}$

$$= \frac{133 \times 14}{9 \times 29} = \frac{1862}{261}$$

1.51. Simplify: 3.56 x 3.56 x 3.56 – 1.06 x 1.06 x 1.06 – 3 x 3.56 x 3.56 x 1.06 + 3 x 3.56 x 1.06 x 1.06

Sol: The given expression is in the form of $a^3 - b^3 - 3a^2 b + 3ab^2$ where a = 3.56 and b = 1.06.
The above expression is equal to $(a – b)^3$.
Hence, the simplified value is
$(3.56 – 1.06)^3 = (2.50)^3 = 15.625$

1.52. Simplify: 3.66^3 + 3 x 3.66 x 1.34 x 1.34 + 3 x 3.66 x 3.66 x 1.34 + 1.34^3

Sol: The given expression is in the form of $a^3 + 3ab^2 + 3a^2b + b^3$ where a = 3.66 and b = 1.34
The above expression is equal to $(a + b)^3$.
Hence, the simplified value is
$(3.66 + 1.34)^3 = 5^3 = 125$

Concept Review Questions

Directions for questions 1 to 75: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. $3^6 = \boxed{}$

2. $2^{15} = \boxed{}$

3. The sum of 50 odd numbers is _____.
(A) even (B) odd (C) Can't say

4. The product of 45 even numbers is _____.
(A) even (B) odd (C) Can't say

5. The sum of 30 composite numbers is _____.
(A) even (B) odd (C) Can't say

6. The product of 20 composite numbers is _____.
(A) even (B) odd (C) Can't say

7. The product of 20 prime numbers is _____.
(A) even (B) odd (C) Can't say

8. If the sum of 35 distinct prime numbers is even, then one of them has to be $\boxed{}$

9. The number of distinct prime factors of 13013 is $\boxed{}$

10. $0.2\overline{7}$ = _____.
(A) $\frac{5}{18}$ (B) $\frac{277}{999}$ (C) $\frac{152}{333}$ (D) $\frac{3}{10}$

11. $0.4\overline{56}$ = _____.
(A) $\frac{152}{300}$ (B) $\frac{152}{330}$ (C) $\frac{152}{333}$ (D) $\frac{226}{495}$

12. $0.12\overline{3}$ = _____.
(A) $\frac{37}{333}$ (B) $\frac{37}{330}$ (C) $\frac{37}{300}$ (D) $\frac{41}{333}$

13. Which of the following are not twin primes?
(A) 239 and 241 (B) 149 and 151
(C) 179 and 181 (D) 229 and 231

14. Which of the following is a prime number?
(A) 437 (B) 323 (C) 567 (D) 241

15. Which of the following numbers is divisible by 11?
(A) 1111111 (B) 77777
(C) 246642 (D) 7654321

16. The eight-digit number 7654321A is divisible by 9. Find A.
$\boxed{}$

17. The six-digit number 24687X is divisible by 9. Find X.
(A) 0 (B) 7
(C) 9 (D) Cannot be determined

18. Is the ten-digit number PQRSTU6736 divisible by 32?
(A) Yes (B) No (C) Can't say

19. Is the nine-digit number PQRST9875 divisible by 625?
(A) Yes (B) No (C) Can't say

20. When the sum of the digits of a number is subtracted from the number, the result will always be divisible by _____.
(A) 6 (B) 9 (C) 11 (D) 5

21. The LCM and HCF of two numbers are 264 and 2 respectively. If one of them is 22, the other is $\boxed{}$.

22. If LCM (x, y, z) = (x)(y)(z), HCF (y, z) = _____.
(A) 1 (B) 2
(C) x (D) Can't say

23. HCF (x, y, z) = 1. Is LCM (x, y, z) = (x)(y)(z)?
(A) Yes (B) No (C) Can't say

24. Find the number of factors of $3^8 \times 6^4$.
$\boxed{}$

25. Is (3^3) (7^7) (21^5) a perfect square?
(A) Yes (B) No (C) Can't say

26. A number has an odd number of factors. Is it a perfect cube?
(A) Yes (B) No (C) Can't say

27. A number has an even number of factors. Is it a perfect square?
(A) Yes (B) No (C) Can't say

28. $11111^2 = \boxed{}$

29. Find the number of ways in which (5^8) (7^{10}) can be written as a product of 2 distinct natural numbers.
(A) 40 (B) 45 (C) 49 (D) 50

30. Find the number of ways in which (3^6) (7^3) can be written as a product of 2 distinct natural numbers.
$\boxed{}$

31. Find the number of ways in which (2^6) (3^{10}) can be written as a product of two co-primes.
(A) 2 (B) 4 (C) 6 (D) 11

32. Find the number of ways in which (2^3) (3^4) (5^6) (7^8) can be written as a product of two co-primes.
$\boxed{}$

33. Find the sum of the factors of (2^4) (3^3).
(A) 930 (B) 1240 (C) 1085 (D) 808

34. Find the sum of the factors of 437.
$\boxed{}$

35. If N is a perfect number, the sum of all the factors of N equals _____.
(A) N (B) 2N (C) 3N (D) 4N

36. Find the number of numbers less than 2^{14} and co-prime to it.
(A) 2^{12} (B) 2^{22} (C) 2^{23} (D) 2^{13}

37. If $N = 3^p \times 2^q \times 3^{2r}$, find the number of numbers less than N and co-prime to it (Express your answer in terms of N).
(A) $\frac{N}{4}$ (B) $\frac{N}{3}$ (C) $\frac{2N}{3}$ (D) $\frac{N}{2}$

38. Find the number of numbers co-prime to 289 and less than it.

$\boxed{}$

39. Find the sum of the numbers which are co-prime to 48 and less than 48.

$\boxed{}$

40. If $x + \frac{1}{x} = 4$, $x^2 + \frac{1}{x^2} =$ ______.
(A) 8 (B) 10 (C) 12 (D) 14

41. If $y - \frac{1}{y} = 3$, $y^2 + \frac{1}{y^2} = \boxed{}$

42. Which of the following is/are a factor(s) of $x^4 - 3x^2 + 1$?
(A) $x^2 - 2x + 1$
(B) $x^2 - x - 1$
(C) $x^2 + x - 1$
(D) More than one of the above

43. If x + y = 12 and xy = 18, $x^3 + y^3 = \boxed{}$

44. If $p^3 + q^3 + r^3 = 3pqr$, then which of the following is true?
(A) p = q = r
(B) p + q + r = 0
(C) p + q + r = 0 or p = q = r
(D) None of these

45. $\frac{p^3 + r^3 - q^3 + 3pqr}{p^2 + q^2 + r^2 + pq + qr - rp} =$ _____.
(A) p + q – r (B) p + r – q
(C) r + q – p (D) p – r

46. If p + q + r = 0, $p^3 + q^3 + r^3 =$ _____.
(A) 3pqr (B) 6pqr (C) 0 (D) pqr

47. LCM (480, 360, 320) = $\boxed{}$

48. HCF (63, 84, 147) = $\boxed{}$

49. $LCM\left(\frac{5}{6}, \frac{9}{10}, \frac{8}{9}\right) = \boxed{}$

50. $HCF\left(\frac{7}{12}, \frac{21}{5}, \frac{14}{18}\right) =$ _____.
(A) $\frac{7}{360}$ (B) $\frac{7}{180}$ (C) $\frac{7}{1080}$ (D) $\frac{7}{2160}$

51. Is HCF [HCF (p, q), HCF (r, s)] = HCF[HCF (p, r), HCF (q, s)]?
(A) Yes (B) No (C) Can't say

52. Is LCM $[LCM(p, q), LCM(r, s)] =$ LCM $[LCM(p, r), LCM(q, s)]$?
(A) Yes (B) No (C) Can't say

53. Find the value of $\frac{10.59^3 - 4.78^3}{10.59^2 + 4.78^2 + (10.59)(4.78)}$.

$\boxed{}$

54. Find the index of the greatest power of 2 which divides 256!.

$\boxed{}$

55. Find the least natural number that should be added to 54321677 to make it divisible by 8.

$\boxed{}$

56. Any 6 consecutive natural numbers will have their product divisible by _____.
(A) 600 (B) 2160 (C) 480 (D) 720

57. The product of 10 consecutive even natural numbers is always divisible by _____.
(A) $2^{10} \times 11!$ (B) $2^{10} \times 12!$
(C) $2^{10} \times 10!$ (D) None of these

58. Find the least natural number by which (3^8) (5^{13}) (7^{19}) must be multiplied so that the product is a perfect square.

$\boxed{}$

59. Find the least natural number by which (5^{13}) (7^{17}) must be divided so that the quotient is a perfect square and the remainder is zero.

$\boxed{}$

60. Find the least natural number by which (2^{10}) (3^{14}) (5^9) must be divided so that the quotient is a perfect cube and the remainder is zero.
(A) 6 (B) 12 (C) 36 (D) 18

61. Find the least natural number to be added to 395 so that the sum is a perfect cube.

$\boxed{}$

62. Find the least natural number to be subtracted from 500 so that the result is a perfect square.

$\boxed{}$

63. A number when divided by 54 leaves a remainder of 31. Find the remainder when the number is divided by 27.

$\boxed{}$

64. Find the least natural number which, when divided by 7 or 8, leaves a remainder of 3 in either case.
(A) 3 (B) 59 (C) 31 (D) 25

65. Find the least natural number which when divided by 24 and 18 leaves remainders of 18 and 12 respectively.

$\boxed{}$

66. Find the least natural number which when divided by 5 and 6 leaves remainders of 3 and 5 respectively.

$\boxed{}$

67. Find the area of the smallest square which can be formed with rectangles of dimensions 7 cm $\times$ 5 cm (in sq.cm).
(A) 1225 (B) 4900
(C) 11025 (D) None of these

68. Find the largest number which divides 107 and 78 leaving remainders of 17 and 18 respectively.

$\boxed{}$

69. Find the largest number which divides 34, 58 and 94 leaving the same remainder in each case.
(A) 6 (B) 9 (C) 12 (D) 8

70. Find the least natural number which when successively divided by 5, 6 and 7 leaves respective remainders of 3, 4 and 5.

$\boxed{}$

71. Find the number of three-digit natural numbers divisible by 8, 12 and 15.

$\boxed{}$

72. The product of a seven-digit, ten-digit and a twelve-digit number is a ______ digit number.
(A) 27 (B) 28
(C) 28 or 29 (D) 27 or 28 or 29

73. The number of digits in the square root of a thirteen-digit number is $\boxed{}$.

74. The number of digits in $(2PQR)^4$ where 2PQR is a four-digit number is _____.
(A) 13 (B) 14
(C) 15 (D) Can't say

75. The number of digits in the cube root of a 29 digit number is _____.
(A) 10 (B) 9
(C) 8 (D) Cannot say

Exercise – 1(a)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. A number when divided by a divisor D leaves a remainder of 13. When thrice the number is divided by D, the remainder obtained is 2. Find the number of possibilities for D.

2. C is a composite number with an even number of factors. Consider the following statements.
(a) C has a factor lying between 1 and $\sqrt{C}$.
(b) C has a factor lying between $\sqrt{C}$ and C.

Which of the following can be concluded?
(A) Both (a) and (b) are true.
(B) Both (a) and (b) are false.
(C) (a) is true but (b) is false.
(D) (a) is false but (b) is true.

3. X is a number formed by the first 100 digits of the number N which is formed by writing the first 100 natural numbers one after another as follows: 1234567891011…….
Find the remainder when X is divided by 16.

4. P is a three-digit number. Upon reversing P, another three digit number Q is obtained. Q > P and Q – P is divisible by 5. Which of the following is always true?
(A) 100 < P < 499 (B) 105 < P < 505
(C) 110 < P < 510 (D) 115 < P < 515

5. (i) What is the product of the factors of 18^3?
(A) 18^{84} (B) 18^{42}
(C) 18^{314} (D) None of these

(ii) What is the product of the factors of 12!?
(A) $2^{10}3^5 5^2(77)$ (B) $12!^{396}$
(C) $12!^{782}$ (D) None of these

6. What is the number of ways in which 11025 can be expressed as the product of a pair of co-primes?

7. How many odd natural numbers up to 1400 are divisible neither by 5 nor by 7?

8. Find the sum of the numbers which are co-prime to 840 and are less than it.
(A) 40320 (B) 80640
(C) 120960 (D) 60480

9. From a certain city, buses start for four different places every 15, 20, 25 and 30 minutes starting from 8:00 a.m. At what time, for the first time after 8:00 a.m., would all the buses start together again?
(A) 10:00 a.m. (B) 12:00 noon
(C) 1:00 p.m. (D) 2:00 p.m.

10. I have less than 200 sweets with me. If I distribute them equally among 10, 16 or 20 children, I would be left with 1 sweet in each case. If I distribute the sweets equally among 23 children, I would not have any sweets left with me. How many sweets do I have?

11. What is the smallest five-digit number which when divided by 7, 11 and 21 leaves a remainder of 5 in each case?
(A) 10164 (B) 10169 (C) 10118 (D) 10123

12. Find the smallest number which when divided by 3, 5, 7 and 9 leaves respective remainders of 2, 4, 6 and 8.

13. Find the largest five-digit number which when divided by 8 leaves a remainder of 3 and when divided by 7 leaves a remainder of 1.
(A) 99948 (B) 99953 (C) 99960 (D) 99947

14. Find the greatest number which divides 565, 847 and 1551 leaving remainders of 5, 7 and 11 respectively.

15. The signboard outside the department store 'Ram and Shyam' lights up as described below. When the switch is turned on, all the three words light up and remain lighted for 3 seconds. After that, the first word is switched off for $7^5/_6$ seconds, the second word is switched off for $1^1/_3$ seconds and the third word is switched off for $5^2/_3$ seconds. Then each word is again switched on for 3 seconds and then switched off for the time duration mentioned above. This process continues repeatedly. After how many seconds of switching on the signboard will the entire board be switched on for the second time for 3 seconds?
(A) $40\frac{1}{3}$ (B) $41\frac{2}{3}$ (C) $42\frac{2}{3}$ (D) $43\frac{1}{3}$

16. The HCF and LCM of a pair of numbers are 11 and 1001 respectively. If the sum of the two numbers is 220, find the smaller of the two numbers.

17. In finding the HCF of two numbers using the division method the last divisor is 8 and the quotients are 1, 1, 14 and 2 in that order. Find the two numbers.
(A) 256, 496 (B) 248, 480
(C) 280, 344 (D) 320, 464

18. What is the HCF of all numbers of the form $n(n^2 + 20)$, n being any even number?
(A) 24 (B) 12 (C) 8 (D) 48

19. A number when divided successively by 6 and 7 leaves remainders of 4 and 5 respectively. Find the remainder when the number is divided by 21.

20. Find the largest four-digit number which when successively divided by 3, 4 and 5 leaves respective remainders of 2, 3 and 4.
(A) 9985 (B) 9995 (C) 9996 (D) 9959

21. Find the index of the greatest power of 20 which divides 200!

22. Find the number of zeros at the end of 175!.

23. If k(N) denotes the number of ways of expressing N as a difference of two perfect squares, then which of the following is the least?
(A) k(187) (B) k(120) (C) k(110) (D) k(105)

24. How many positive integers are factors of exactly one of 1125 and 1800?

25. The sum of the all the factors of 33333333 is _____.
(A) 49775912 (B) 48833116
(C) 47699184 (D) 49997952

26. If n is an integer greater than 3, which of the following is always not a factor of $n(n^2 - 4)(n^4 - 10n^2 + 9)$?
(A) 126 (B) 72 (C) 52 (D) 144

27. $\frac{A}{3}$ is an integer but $\frac{A}{6}$ is not. $\frac{B}{5}$ is an integer but $\frac{B}{10}$ is not. Which of the following may not be an integer?
(A) $\frac{5A - 3B}{15}$ (B) $\frac{5A - 3B}{30}$
(C) $\frac{5A - B}{10}$ (D) $\frac{5A - B}{20}$

28. Find the smallest natural number to be multiplied with 72000 so that the product is a 5th power of an integer.
(A) 10800 (B) 21600 (C) 5400 (D) 32400

29. There were N houses in a colony, numbered 1 to N. All but 5 of the houses were destroyed in an earthquake. The numbers on the houses which were not destroyed are consecutive. If the sum of the numbers on the destroyed houses is 1085, then the least of the numbers on the houses which were not destroyed can be _____.
(A) 36 (B) 26
(C) 25 (D) Either (36) or (26)

30. $N = 2^{25}3^{15}5^{16}$. Find the number of factors of N which are perfect cubes.
(A) 324 (B) 200 (C) 288 (D) 225

31. $X = \left\{\frac{7}{128}, \frac{7}{64}, \frac{7}{32}, \ldots\ldots 3584\right\}$

Y is a subset of X such that the product of no two elements of Y is 196. The maximum number of elements that Y can have is _____.

32. Let x, y and z be three natural numbers such that x + y + z = 9m + 10, where m is a natural number. For any m, which of the following holds true?
(A) The minimum possible value of $x^2 + y^2 + z^2$ is $27m^2 - 60m + 34$.
(B) The maximum possible value of $x^2 + y^2 + z^2$ is $27m^2 + 60m + 34$.
(C) The minimum possible value of $x^2 + y^2 + z^2$ is $27m^2 + 60m + 34$.
(D) The maximum possible value of $x^2 + y^2 + z^2$ is $27m^2 - 60m + 34$.

33. Find the index of the greatest power of 2 in 64! + 65! + ……… + 120!.

34. If p, q and r are prime numbers satisfying p = q + 2 = r + 4, how many combinations exist for p, q, r?

35. $\lceil a \rceil$ is the least integer greater than or equal to a

$F(b, a) = \left\lceil \frac{a}{b} \right\rceil$.

F (10, F(9, F(8, F(7, F(6, i))))) = 1, where i is an integer.
Consider the following statements.
I. i is not less than 9.
II. i is not more than 30000.

Which of the following is/are true?
(A) Only I (B) Only II
(C) Both I and II (D) Neither I nor II

36. The sets P_p are defined to be {p, p + 1, p + 2, p + 3, p + 4, p + 5} where p = 1, 2, 3, …. 88. How many of these sets contain a multiple of 8?

37. If $25 \le x \le 49$ and $y = \frac{x^2 + 3\sqrt{x}(x + 9) + 81}{x + 6\sqrt{x} + 9}$, then y satisfies
(A) $18 \le y < 36$ (B) $19 \le y \le 38$
(C) $20 \le y < 45$ (D) $23 \le y < 20$

38. A pair of numbers is formed with the digits 1, 2, 3, 4, ...9 such that all the digits are used and no digit occurs more than once in any number. Find the minimum difference between the two numbers of any such pair.

39. Find the number of four-digit numbers, that have distinct digits whose product is 540.

40. If $1 \le P \le 40$, how many values of P exist such that (P – 1)! is not divisible by P?
(A) 11 (B) 10 (C) 12 (D) 13

Exercise – 1(b)

Directions for questions 1 to 60: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. If p is a prime number greater than 3, (p^2 – 1) is divisible by which of the following?
(A) 6 (B) 24
(C) 2 (D) All of these

2. What is the least natural number that should be subtracted from 478185 so that the resulting number is a multiple of 19?

3. X denotes the set of all the prime numbers less than 50. How many zeroes does the product of the elements of X end with?

4. Find the smallest natural number which must be added to 1648 so that a remainder of 10 is left when the resulting number is divided by 14 or 21.
(A) 28 (B) 42 (C) 32 (D) 18

5. The product of the remainders obtained when (N^3 + N) and (N^3 – N) are divided by 6, N being a positive integer greater than 1, is

6. Express $10.0\overline{43}$ as a fraction.
(A) 10043/999 (B) 9943/999
(C) 10043/990 (D) 9943/990

7. The difference of a four-digit number and any number formed by permuting its digits would always be divisible by
(A) 18 (B) 11 (C) 10 (D) 9

8. If n is a natural number, which of the following numbers always divides $n^7 – n$?
(A) 13 (B) 9 (C) 7 (D) 5

Moderate

9. If a and b are two positive integers then (a + b)! is divisible by _____.
(A) a! (B) b!
(C) a!b! (D) All of these

10. If $1 \le k \le 40$, how many prime numbers are of the form 5k + 1?

11. Two numbers when divided by a certain divisor leave remainders 238 and 342 respectively. When the sum of the two numbers is divided by the same divisor, the remainder is 156. Find the divisor.
(A) 384 (B) 404
(C) 424 (D) Cannot be determined

12. S is a set of positive integers such that each element x of S satisfies the following conditions.
(i) Each digit of x is even.
(ii) $2100 \le x \le 2300$.
How many elements of S are divisible by 3?

13. (i) Find the number of divisors of 900 excluding 1 and itself.
(A) 25 (B) 27 (C) 23 (D) 21
(ii) In how many ways can 1500 be expressed as a product of two of its factors?
(A) 18 (B) 12 (C) 24 (D) 36

14. What is the number of different ways in which the number 784 can be expressed as a product of two different factors?

15. What is the product of the factors of 4! ×5!?
(A) $120^8 \times 24^4$ (B) 2880
(C) 2880^{21} (D) 2880^{42}

16. What is the product of the factors of ($8^9 \times 9^8$)?
(A) $2^{3213}\,3^{1904}$ (B) $2^{1904}\,3^{3213}$
(C) 6^{238} (D) $2^{6426}\,3^{3808}$

17. How many numbers are co-prime to 8640 and are less than it?

18. What is the minimum number of identical square tiles required to cover a rectangular floor of dimensions 3 m 78 cm by 4 m 80 cm?
(A) 3200 (B) 5040 (C) 7600 (D) 8100

19. The index of the greatest power of 11 in $\frac{900!}{450!}$ is

20. When the numbers 5, 7 and 11 divide a multiple of 17, the remainders left are 4, 6 and 10 respectively. Which multiple of 17 is the least number that satisfies the given condition?
(A) 384th (B) 317th (C) 385th (D) 325th

21. Find the smallest number which when divided by 31 leaves a remainder of 7 and when divided by 25 leaves a remainder of 6.

22. Find the greatest number which divides 6155 and 4935 leaving remainders of 5 and 15 respectively.

23. Three cakes have weights $5\frac{1}{4}$ lb, $7\frac{3}{4}$ lb and $8\frac{1}{5}$ lb. Each cake is to be cut into pieces, such that all the pieces have equal weight, and each piece must have the maximum possible weight. Find the number of guests who can be served, if each piece is served to a different guest.

24. A number when successively divided by 7, 11 and 5 leaves respective remainders of 5, 1 and 1. Find the smallest such number.

25. A number when successively divided by 4, 5 and 6 leaves remainders of 3, 4 and 5 respectively.
(i) Find the hundreds digit of the smallest such number.
(A) 1 (B) 2 (C) 3 (D) 4

(ii) Find the hundreds digit of the second smallest such number.
(A) 2 (B) 3 (C) 4 (D) 5

26. If A, B and C are distinct digits such that the square of the two-digit number AB equals the three-digit number CCB, then the sum of all possible values of CCB equals

27. If a + b = 11 and ab = 7, then find the value of $a^3 + b^3$.

28. If S is the sum of the first 15828 prime numbers, then S is divisible by _____.
(A) 6 (B) 4
(C) 8 (D) None of these

29. M is the sum of 10 consecutive natural numbers. Which of the following is a possible value of M?
(A) 785 (B) 780 (C) 45 (D) 100

30. The n^{th} element of a series, denoted by Y_n, is given by $Y_n = (-1)^n Y_{n-1}$.
If $Y_0 = -y$, where y is a positive number, which of the following is always true?
(A) Y_n is positive when n is even and is not divisible by 4.
(B) Y_n is positive when n is odd.
(C) Y_n is negative when n is even and is not divisible by 4.
(D) More than one of the above

31. Find the value of the expression below.

$$\left[\frac{1}{6^2-1}+\frac{1}{8^2-1}+\frac{1}{10^2-1}+.......+\frac{1}{16^2-1}\right]\frac{20}{3}$$

(A) $\frac{8}{17}$ (B) $\frac{9}{16}$ (C) $\frac{21}{32}$ (D) $\frac{27}{64}$

32. The n^{th} triangular number is the sum of the first n natural numbers. Which of the following is a triangular number?
(A) 262 (B) 515 (C) 824 (D) 903

33. A teacher gave a student the task of adding N natural numbers starting from 1. After a while, the student reported his result as 700. The teacher then replied that his result was wrong. The student then realized that he had added one number twice by mistake. Find the sum of the digits of the number which the student had added twice.

34. A three-digit number 'xyz' is such that the number equals x! + y! + z!. Find the difference of the number formed by reversing its digits and the original number.

35. (i) If $x^2 - 4x + 1 = 0$, find the value of $x^4 + \frac{1}{x^4}$.

(ii) If x > 1 and $x + \frac{1}{x} = 4$, find the value of $x^4 - \frac{1}{x^4}$.
(A) 112 (B) $112\sqrt{3}$
(C) 224 (D) $224\sqrt{3}$

36. The 600-digit number 1223334444…..is divided by 16. Find the remainder.
(A) 4 (B) 8 (C) 0 (D) 12

37. X is any even natural number such that $Y = X^2 - 2X$. Find the greatest natural number which always divides $Y^2 - 8Y$.
(A) 192 (B) 96 (C) 384 (D) 144

38. Find the sum of all the factors of 324 which are multiples of 3.

39. Let X be the set of integers {8, 14, 20, 26, 32, …. 350, 356, 362. 368, 374} and Y be a subset of X such that no two elements of Y have a sum of 382. Find the maximum number of elements Y can have.

40. Five two-digit numbers less than 25 have a product of 838695. Find the sum of the five numbers.
(A) 79 (B) 83 (C) 89 (D) 73

41. N is a four-digit number having distinct digits. Each digit of N is one of 5, 6, 7, 8, 9. The number of values of N which are divisible by 8 is ______.

42. $E = (3p + 6q - 9r)^3 + (3p - 6q + 9r)^3 + (-3p + 6q + 9r)^3 - 3(3p + 6q - 9r)(3p - 6q + 9r)(-3p + 6q + 9r)$, where p, q, r are such that p + 2q = –3r.
Consider the following statements:
I. E is at least zero.
II. E is at most zero.
Which of the statement(s) can be concluded?
(A) Only I
(B) Only II
(C) Both I and II
(D) Neither I nor II

43. If $T_n = n$, where n is a natural number, for how many values of n up to 150 is $T_n + T_{n+1}$ a perfect square?

44. The nine-digit number 7543299p6 is divisible by 36. Find the value of p.

45. How many times the HCF of the fractions $\frac{3}{14}, \frac{6}{35}$ and $\frac{16}{21}$ is their LCM?
(A) 1440 (B) 144 (C) 210 (D) 48

46. S is a set of three-digit numbers which satisfy the following conditions.
(i) No element of S exceeds 500.
(ii) Each element of S is divisible by 7 but not by 11.
How many elements does S contain?

47. N is a four-digit number. It exceeds the number formed by reversing its digits by M, where M is a multiple of 45. The first and the last digits of N can be _____ respectively.
(A) 6, 2 (B) 8, 5 (C) 9, 3 (D) 5, 0

48. P = 1! + 2! + 3! + ……… + 60!
Which of the following can be concluded?
(A) P is a perfect square but not a perfect cube.
(B) P is a perfect cube but not a perfect square.
(C) P is a perfect square as well as a perfect cube.
(D) P is neither a perfect square nor a perfect cube.

49. The Oxford University Press designed a 1200 page dictionary on a computer. Just before the dictionary was supposed to go for printing, it was spotted that none of the pages of the dictionary were numbered. A typist then numbered the pages of the dictionary. How many times did the typist press the number keys (0 to 9) in order to number the pages of the dictionary?

50. P is the product of four consecutive even natural numbers. Q = P + 16.
How many of the following statements are true?
(1) $\frac{Q}{16}$ is an odd perfect square.
(2) Q is divisible by 8.
(3) Q is divisible by 16.
(4) Q is divisible by 32.

Difficult / Very Difficult

51. N is an integer and the product of all its factors is equal to N^2. The sum of all the factors of N excluding N is 57. How many values of N are there?

52. The integers 39276 and 38304 leave the same remainder when divided by a three-digit natural number N. How many possible values can N assume?
(A) 8 (B) 7 (C) 6 (D) 5

53. When a three-digit number divides 64484 and 62767, the remainder is the same in both the cases. What is the number?
(A) 101
(B) 458
(C) 767
(D) Cannot be determined

54. P and Q are two two-digit numbers. Their product equals the product of the numbers obtained on reversing them. None of the digits in P or Q is equal to the other digit in it or any digit in the other number. The product of tens digits of the two numbers is a composite single digit number. How many ordered pairs (P, Q) satisfy these conditions?
(A) 8 (B) 16 (C) 12 (D) 4

55. The HCF of $3^p\ 5^{q+4}\ 7^7\ 11^5$ and $3^{q+5}\ 5^{p+4}\ 7^x\ 11^x$ is $3^x 5^x 7^x 11^x$ where p and q are natural numbers. Find the value of p.

56. If a, b, c and d are natural numbers such that $a^d + b^d = c^d$, which of the following is always true?
(A) d is never more than the minimum of a, b and c.
(B) d is never less than the maximum of a, b and c.
(C) d lies between the minimum of a, b and c and the maximum of a, b and c.
(D) None of these

57. Given that $a \# b = \frac{HCF(a,b)}{LCM(a,b)}$
If (p # q) # (r # s) = 1, then p # q = _____.
(A) 1 (B) q (C) p (D) r # s

58. Let $\lceil x \rceil$ denote the least integer greater than or equal to x.
Let $A(x, y) = \lceil 2x \rceil + \lceil 2y \rceil$ and $B(x, y) = \lceil x \rceil + \lceil y \rceil + \lceil x + y \rceil$. Which of the following is false?
(A) A(x, y) = B(x, y) (B) A(x, y) > B(x, y)
(C) A(x, y) < B(x, y) (D) A(x, y) ≠ B(x, y)

59. S is the sum of four consecutive two-digit numbers. When S is divided by 10, the quotient is q and the remainder is 0. If q is a perfect square, the number of combinations which exist for the four numbers is ______.

60. How many natural numbers upto 2050 are neither perfect squares nor perfect cubes?
(A) 1966 (B) 1978 (C) 1996 (D) 1990

Data Sufficiency

Directions for questions 61 to 75: Each question is followed by two statements, I and II. Answer each question using the following instructions:

Choice (A) if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.
Choice (B) if the question can be answered by using either statement alone.
Choice (C) if the question can be answered by using both statements together, but cannot be answered by using either statement alone.
Choice (D) if the question cannot be answered even by using both the statements together.

61. x, y, z are three consecutive prime numbers. What are the values of x, y, z?
I. y – z = 4, x – y = 6
II. x < 60

62. A number k has three prime factors 2, 5 and 3. What is the value of k?
I. Number of factors of k is 12
II. k is a multiple of 4

63. If k and ℓ are odd numbers, is $7\frac{1}{2}+\frac{k}{3}+\frac{\ell}{6}$ an integer?
I. $k = \ell$
II. $k = 5\ell$

64. If a and b are natural numbers, is a + b < ab?
I. a = b
II. a = 1

65. N is a two digit integer greater than 70. What is the value of N?
I. N is a product of two different single digit integers.
II. N can be expressed as a product of three distinct prime numbers

66. x is a two digit number whose square is a three digit number. What is value of x?
I. The units digit of x and x^2 is the same.
II. The digit in the units place and the hundreds place of x^2 is same.

67. Is the sum of (2a – b) and (2a + 5b – 4c) divisible by 3?
I. c is negative, while a > 0 and b > 0
II. a, b, |c| are successive integers.

68. Is N the HCF of two numbers x, y?
I. Both x, y are multiples of N
II. x – y = 2N

69. A number K is multiplied with $0\cdot\overline{ab}$. Is the product an integer?
I. K is a multiple of 9
II. K is a multiple of 11

70. Both y and z are prime numbers and x is a natural number. If x + y = z and z < 50 what is the value of x?
I. x is a multiple of 14.
II. LCM of y, z is 527.

71. x and y are integers. Is y an odd number?
I. xy is an odd number.
II. x + y is an odd number

72. 810A4B6C is a number where A, B, C represent distinct digits. Find A + B + C.
I. The number is divisible by 5, 8, 9
II. Both A and B are non-prime.

73. Is the natural number X, greater than 189, a prime number?
I. The number has only five multiples less than 1000.
II. The number is odd and does not end in 5.

74. If the product of three positive integers is 40, how many of these are odd?
I. The sum of three positive integers is odd.
II. If the three integers are a, b and c then, ab + bc + ca is odd.

75. What is the remainder when $N^2 - 30N + 200$ is divided by 50?
I. N is the smallest natural number which leaves a remainder of 4 when divided by 12, 13, 14.
II. N is the smallest natural number which leaves a remainder of 6, 7 respectively when divided by 16 and 17.

Key

Concept Review Questions

1. 729	16. 8	31. A	46. A	61. 117
2. 32768	17. D	32. 8	47. 2880	62. 16
3. A	18. C	33. B	48. 21	63. 4
4. A	19. B	34. 480	49. 360	64. A
5. C	20. B	35. B	50. B	65. 66
6. C	21. 24	36. D	51. A	66. 23
7. C	22. A	37. B	52. A	67. A
8. 2	23. C	38. 272	53. 5.81	68. 30
9. 3	24. 65	39. 384	54. 255	69. C
10. A	25. A	40. D	55. 3	70. 173
11. D	26. C	41. 11	56. D	71. 8
12. C	27. B	42. D	57. C	72. D
13. D	28. 123454321	43. 1080	58. 35	73. 7
14. D	29. C	44. C	59. 35	74. B
15. C	30. 14	45. B	60. D	75. A

Exercise – 1(a)

1. 1
2. A
3. 9
4. B
5. (i) B
   (ii) B
6. 4
7. 480
8. B
9. C
10. 161
11. B
12. 314
13. D
14. 140
15. D
16. 77
17. B
18. D
19. 13
20. D
21. 49
22. 43
23. C
24. 30
25. D
26. C
27. D
28. A
29. D
30. A
31. 9
32. C
33. 64
34. 1
35. D
36. 66
37. B
38. 2469
39. 48
40. D

Exercise – 1(b)

1. D
2. 12
3. 1
4. B
5. 0
6. D
7. D
8. C
9. D
10. 10
11. C
12. 9
13. (i) A
    (ii) B
14. 7
15. C
16. D
17. 2304
18. B
19. 45
20. B
21. 131
22. 1230
23. 424
24. 89
25. (i) A
    (ii) A
26. 666
27. 1100
28. D
29. A
30. A
31. A
32. D
33. 7
34. 396
35. (i) 194
    (ii) B
36. A
37. C
38. 840
39. 31
40. A
41. 14
42. C
43. 8
44. 9
45. A
46. 52
47. D
48. D
49. 3693
50. 3
51. 4
52. C
53. A
54. B
55. 5
56. A
57. D
58. B
59. 2
60. C
61. D
62. C
63. A
64. A
65. B
66. D
67. C
68. D
69. C
70. A
71. A
72. C
73. C
74. B
75. B