T.I.M.E.
Triumphant Institute of
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Solutions for SM1002120

Exercise – 1
(Simple Equations)

Solutions for questions 1 to 50:

1. (a) $x + \frac{x}{2} + \frac{x}{3} + 4 = 26 \Rightarrow x + \frac{x}{2} + \frac{x}{3} = 22$

$\Rightarrow \frac{11x}{6} = 22 \Rightarrow x = 12$ Ans : (12)

(b) $\frac{4x}{5} + \frac{3x}{10} + ab = ab + a + \frac{6a}{5}$

$\frac{11x}{10} = \frac{11a}{5}$

$x = 2a$ Choice (B)

2. $5x – 4y – 5 = 0 \Rightarrow 5x – 4y = 5 \rightarrow (1)$
$3x – 5y + 10 = 0 \Rightarrow 3x – 5y = –10 \rightarrow (2)$
Multiplying equation (1) by 3 and equation (2) by 5 and subtracting the resultant equation (1) from resultant equation (2), we get, $13y = 65$
$\Rightarrow y = 5$
Substituting y in either equation (1) or (2) we get, x = 5
Choice (B)

3. $4x + y – 4z = 10 \rightarrow (1)$
$x + y – z = 4 \rightarrow (2)$
$5x – y + 3z = 16 \rightarrow (3)$
Adding equations (1) and (3) we get
$9x – z = 26 \rightarrow (4)$
Adding equations (2) and (3) we get $6x + 2z = 20 \rightarrow (5)$
Multiplying equation (4) by 2 and adding it to equation (5), we get, $24x = 72 \Rightarrow x = 3$
substituting x in equation (4) we get, z = 1
substituting x and z in equation (2) we get, y = 2.
Choice (B)

4. Let the cost of each banana and each guava be b and g respectively.
$8b + 9g = 43$
$5b + 7g = 31$
Subtracting the second equation from the first, we get,
$3b + 2g = 12$ Ans : (12)

5. Let the three consecutive integers be x, x + 1 and x + 2 respectively.
$x + x + 1 + x + 2 = 30$
$3x + 3 = 30 \Rightarrow x = 9$
Hence the three consecutive integers are 9, 10 and 11. The sum of their squares is 302. Ans : (302)

6. Let the number be a.

$2 + \frac{1}{2}\left(\frac{1}{3}\left(\frac{a}{5}\right)\right) = \frac{a}{15}$

$\Rightarrow 2 = \frac{a}{30} \Rightarrow a = 60$ Ans : (60)

7. Let the amount with R be r.

$r = \frac{2}{3}$(total amount with P and Q)

$r = \frac{2}{3}(6000 – r)$

$\Rightarrow 3r = 12000 – 2r$
$\Rightarrow 5r = 12000$
$\Rightarrow r = 2400$ Ans : (2400)

8. Let the three consecutive integers be x, x + 2 and x + 4, respectively.
$x + 4 + x + 2 = x + 13 \Rightarrow x = 7$
Hence the three consecutive odd integers are 7, 9 and 11.
Choice (D)

9. Let the number of rabbits and peacocks be r and p respectively. As each animal has only one head, so
$r + p = 60 \rightarrow (1)$
Each rabbit has 4 legs and each peacock has 2 legs. Total number of legs of rabbits and peacocks
$= 4r + 2p = 192 \rightarrow (2)$
Multiplying equation (1) by 2 and subtracting it from, equation (2), we get, $2r = 72 \Rightarrow r = 36$. Ans : (36)

10. Let the numerator and denominator of the fraction be n and d respectively.
$d = 2n – 1$

$\frac{n+1}{d+1} = \frac{3}{5}$

$5n + 5 = 3d + 3 \Rightarrow 5n + 5 = 3(2n – 1) + 3 \Rightarrow n = 5$
$d = 2n – 1 = 9$

Hence the fraction is $\frac{5}{9}$. Choice (C)

11. Let the two-digit number be 10a + b
$a + b = 17 \rightarrow (1)$
$a – b = 1 \rightarrow (2)$
Adding (1) and (2) we get $2a = 18 \Rightarrow a = 9$
Substitute a = 9 in equation (1) or (2), we get, b = 8
Hence the two-digit number is 98. Ans : (98)

12. Let the two digit number be 10a + b.
$a + b = 12 \rightarrow (1)$
If a > b, a – b = 6,
If b >a, b – a = 6
If a – b = 6, adding it to equation (1), we get
$2a = 18 \Rightarrow a = 9$
so b = 12 – a = 3
$\therefore$ Number would be 93.
if b – a = 6, adding it to equation (1),
$2b = 18 \Rightarrow b = 9$
a = 12 – b = 3.
$\therefore$ Number would be 39.Hence
$\therefore$ Number would be 39 or 93. Choice (C)

13. Let the three digit number be 100a + 10b + c.
$100c + 10b + a = 100a + 10b + c + 396$
$\Rightarrow 99(c – a) = 396 \Rightarrow c – a = 4$

$a = \frac{1}{2}c$, so $c – a = \frac{1}{2}c = 4$

$\Rightarrow c = 8 \Rightarrow a = \frac{1}{2}(8) = 4$

Hence the three-digit number is 428. Ans : (428)

14. Let the numerator and denominator of fraction be n and d respectively.

$\frac{n+4}{d+4} = \frac{2}{3} \Rightarrow 3n + 12 = 2d + 8$

$\Rightarrow 2d – 3n = 4 \rightarrow (1)$

$\frac{n}{d-1} = \frac{3}{5} \Rightarrow 5n = 3d – 3 \Rightarrow 3d – 5n = 3 \rightarrow (2)$

Multiply equation (1) by 3 and equation (2) by 2 and subtract it from resultant equation (1), we get, n = 6
Substitute n = 6 in equation (1) or (2),
we get, d = 11
So the fraction is 6/11. Choice (D)

15. Let x be the present age of the man.
20 years ago it was (x – 20) years.
Given x + 25 = 4(x – 20)
$\Rightarrow$ 3x = 105 $\Rightarrow$ x = 35 years. Ans : (35)

16. Let the present ages of Anand and Bala be a and b respectively.

$a - 10 = \frac{1}{3}(b - 10) \rightarrow \quad (1)$

and b = a + 12
Substituting b = a + 12 in the first equation,

$a - 10 = \frac{1}{3}(a + 2)$

3a – 30 = a + 2 $\Rightarrow$ 2a = 32 = a = 16 Ans : (16)

17. Let the present ages of P, Q and R be p, q, and r respectively.
p + q = r $\rightarrow$ (1)
k years later, the ages of P, Q, and R will be p + k, q + k and r + k respectively.
r + k + 20 = p + k + q + k
substituting r as p + q in the above equation, we get k = 20. Ans : (20)

18. Let the number of ₹2 coins with Amar be a.
Number of ₹5 coins with him = a + 10
5 (a + 10) + 2a = 190
$\Rightarrow$ 7a = 140 $\Rightarrow$ a = 20 Ans : (20)

19. Let the number of 25 paise coins in the bag be x. Number of 20 paise coins in the bag is 90 – x
Total value of coins = [25x + 20 (90 –x)] paise = 2100 paise
$\Rightarrow$ x = 60 Ans : (60)

20. Let the amounts to be received by P, Q and R be p, q and r respectively.
p + q + r = 1200

$p = \frac{1}{2}(q + r) \Rightarrow 2p = q + r$

Adding p both sides, 3p = p + q + r = 1200
$\Rightarrow$ p = ₹400

$q = \frac{1}{3}(p + r) \Rightarrow 3q = p + r$

Adding q both sides, 4q = p + q + r = 1200
$\Rightarrow$ q = 300
r = 1200 – (p + q) = ₹500 Ans : (500)

21. Let the fraction be $\frac{n}{d}$.

n = d + 5 $\Rightarrow$ d = n – 5
3n = d + 21 $\Rightarrow$ 3n = n – 5 + 21
$\Rightarrow$ 2n = 16 $\Rightarrow$ n = 8
d = n – 5 = 3

Hence the fraction is $\frac{n}{d} = \frac{8}{3}$. Choice (A)

22. Let the two digit number be 10a + b
a = b + 2 $\rightarrow$ (1)
10a + b = 7 (a + b) $\Rightarrow$ a = 2b
Substituting a = 2b in equation (1) we get
2b = b + 2 $\Rightarrow$ b = 2, Hence the units digit is 2.
Ans : (2)

23. Let the costs of each chair and each table be C and T respectively. 2C + 3T = 1300 $\rightarrow$ (1)
3C + 2T = 1200 $\rightarrow$ (2)
Subtracting the second equation from the first, we get – C + T = 100 $\Rightarrow$ T – C = 100. Choice (A)

24. Let the two-digit number be 10a + b.
9a + 8b = 10a + b
7b = a
0 < a $\le$ 9 and 0 < 7 b $\le$ 9 so b = 1
Hence b is 1, then a = 7, hence the number is 71.
Ans : (71)

25. Let the cost of each chair and each table be C and T respectively.
C + T = 500 $\rightarrow$ (1)
2C + 3T= 1300 $\rightarrow$ (2)
Multiplying equation (1) by 2 and subtracting it from equation (2) we get, T = 300. Ans : (300)

26. Let the ages of A, B and C be a, b and c respectively.

$a + b = \frac{4}{3}c$, $b = \frac{2}{3}c$

$a + b + c = 35 \Rightarrow \frac{4}{3}c + c = 35$

$\frac{7}{3}c = 35$

$\Rightarrow$ c = 15, b = $\frac{2}{3}$c = 10 Choice (B)

27. After x years the ages of the mother and the daughter would be 25 + x and 5 + x respectively.
25 + x = 2(5+ x)
$\Rightarrow$ 25 + x = 10 + 2x $\Rightarrow$15 = x
Age of the daughter x years later = 5 + x = 20 years.
Choice (D)

28. Let the distance that A and B can walk in 1 hour be a km and b km respectively.
5a = 9b – 2 $\rightarrow$ (1)
4a – 3b = 11 $\rightarrow$ (2)
Solving (1) and (2) we have a = 5 and b = 3.
Distance that A can walk in 2 hours = 2a = 10 km.
Ans : (10)

29. Let the cost of each pen, eraser and sharpener be p, e and s respectively.
3p + 4e + 10s = 75
6p + 7e + 20s = 146
multiplying the first of the above equations by 2 and subtracting the second equation from it, we get, e = 4.
Choice (B)

30. Let the number of marbles with Ajay and Vijay initially be A and V respectively. If Vijay gives x marbles to Ajay then Vijay and Ajay would have V – x and A + x marbles respectively.
V – x = A + x $\rightarrow$ (1)
If Ajay gives 2x marbles to Vijay, then Ajay and Vijay would have A – 2x and V + 2x marbles respectively.
V + 2x – (A – 2x) = 30 $\Rightarrow$ V – A + 4x = 30 $\rightarrow$ (2)
From (1) we have V – A = 2x
Substituting V – A = 2x in equation (2)
6x = 30 $\Rightarrow$ x = 5. Choice (B)

31. Given linear equations are 5x + 7y = 3 and 15x + 21y = 24.
Clearly given lines are parallel.
So, no common solution. Choice (A)

32. Total distance travelled = 1800 km
Distance travelled by plane = 600 km
Let distance travelled by bus = x

$\therefore$Distance travelled by train = $\frac{3x}{5}$

$\Rightarrow x + \frac{3x}{5} + 600 = 1800$

$\Rightarrow \frac{8x}{5} = 1200$

$\Rightarrow$ x = 750 km Choice (D)

33. Let the ages of Mohan and Sohan be 7x years and 3x years respectively.
3x + 25 = 40
$\Rightarrow$ x = 5
Required age = 7x + 25 = 60 years.
Choice (A)

34. P + Q = 12, Q + R = 18 and P + R = 24 $\rightarrow$ (1)

Adding these, we get 2(P + Q + R) = 54
$\therefore$ P + Q + R = 27.
(1) $\Rightarrow$ R = 15, P = 9 and Q = 3.
P.Q.R = 405. Choice (B)

35. Let the two digit number be 10a + b.
a + b = 4 + b – a
Cancelling b both sides, a + a = 4 $\Rightarrow$ a = 2
ab = 16
$b = \frac{16}{a} = \frac{16}{2} = 8$ when a = 2
Hence the number is 28. Choice (B)

36. Let the three-digit number be 100a + 10b + c.
100a + 10b + c = 10(a + b + c) + 9
$\Rightarrow$ 90a = 9c +9
$c = \frac{90a-9}{9} = 10a - 1$
As a = 1, c = 9 Ans : (9)

37. Let the four consecutive odd integers be 2x – 3, 2x – 1, 2x+1 and 2x +3 respectively.
$(2x-3)^2 + (2x-1)^2 + 64 = (2x+1)^2 + (2x+3)^2$
$4x^2 - 12x + 9 + 4x^2 - 4x + 1 + 64$
$= 4x^2 + 4x + 1 + 4x^2 + 12x + 9$
$\Rightarrow$ 32x = 64
$\Rightarrow$ x = 2
The four integers are 1, 3, 5 and 7 respectively.
Choice (D)

38. Let the three-digit number be 100a + 10b + c.
b = 2c, a = 2b $\Rightarrow$ a = 2 (2c) = 4c.
100a + 10b + c – (100c + 10b + a) = 594
99 (a – c) = 594 $\Rightarrow$ a – c = 6
As a = 4c, 4c – c = 3c = 6 $\Rightarrow$ c = 2
b= 2(2) = 4
a = 4(2) = 8
$\therefore$ The required number is 842. Ans : (842)

39. Let the two-digit number be 10 a + b.
10 a + b = 4 (a + b)
10a – 4a = 4b – b $\Rightarrow$ 6a = 3b $\Rightarrow$ b = 2a
a and b are single digit numbers. So the numbers satisfying the condition are 12, 24, 36, 48. Choice (D)

40. Let the three-digit number be 100a + 10b + c.
a = b + 2
c = b – 2
a + b + c = 3 b = 18 $\Rightarrow$ b = 6
so a = 8 and c = 4
Hence the three-digit number is 864. Ans : (864)

41. Let the numerator and denominator of the fraction be n and b respectively. 3n – d = 3 $\rightarrow$ (1)
$\frac{n}{d+3} = \frac{1}{3} \Rightarrow 3n = d + 3$ $\rightarrow$ (2)
As (1) and (2) represent the same, we cannot find the fraction. Choice (D)

42. Let the present ages of the father and the son be F and S respectively, F = 3S + 5
F + 15 = 2 (S + 15) Substituting F as 3S + 5 in the above equation we get 3S + 5 + 15 = 2S + 30
S = 10 years. Ans : (10)

43. Let the cost of each orange and each banana be a and b respectively.
12a + 18b = 84 $\rightarrow$ (1)

If the cost of each orange doubles, it becomes 2a
6(2a) + 16b = 80
12a + 16b = 80 $\rightarrow$ (2)
Subtracting equation (2) from (1) we get 2b = 4
$\Rightarrow$ b = 2 Choice (B)

44. $\frac{4x+6}{10x-6} = \frac{2x+9}{5x+5}$
(4x + 6) (5x + 5) = (2x + 9) (10x – 6)
$\Rightarrow 20x^2 + 30x + 20x + 30$
$= 20x^2 + 90x - 12x - 54 \Rightarrow 28x = 84$
$\Rightarrow$ x = 3. Choice (B)

45. Let the numbers be x and y.
Sum of the numbers x + y = 20 $\rightarrow$ (1)
Difference of the numbers x – y = 4 $\rightarrow$ (2)
Adding the two equations we get x = 12
So, 12 + y = 20 $\Rightarrow$ y = 8
$\therefore$ The numbers are 12, 8. Choice (D)

46. Let the price of book be y and the price of pen be x.
$\therefore$ x+ 2y = 70 $\rightarrow$ (1)
3x + 9y = 300 $\rightarrow$ (2)
By solving the equations we get x = 10 and y = 30
$\therefore$ The difference between the cost of a book and a pen is 30 – 10 = ₹20 Choice (B)

47. Let x years be the age of the youngest son. Then sum of the ages of three sons ages is x + x + 5 + x + 10 = 45
3x = 30 $\Rightarrow$ x = 10
Five years ago, the youngest son's age = 10 – 5 = 5 years.
= 5 years. Choice (C)

48. Let present ages of father and son be 'f' and 's' respectively.
If father's age is 's' years then son's age will be (2s – f).
$\therefore$ From given data
f = 7 (2s – f)
8f – 14s = 0
$\Rightarrow$ 4f – 7s = 0 $\rightarrow$ (1)
And also given that,
f + s = 110 $\rightarrow$ (2)
Multiplying equation (2) with 7 and adding it to equation (1)
$\Rightarrow$ 11f = 770 $\Rightarrow$ f = 70
$\Rightarrow$ s = 40
$\therefore$The present ages of the father and the son are 70 years and 40 years respectively. Choice (A)

49. Given that $\frac{1}{p} = \frac{x}{y+z}$; $\frac{1}{q} = \frac{y}{x+z}$; $\frac{1}{r} = \frac{z}{x+y}$
$\Rightarrow p = \frac{y+z}{x}$
$p + 1 = \frac{x+y+z}{x}$
Similarly, $q + 1 = \frac{x+y+z}{y}$
and $r + 1 = \frac{x+y+z}{z}$
$\therefore \frac{1}{p+1} + \frac{1}{q+1} + \frac{1}{r+1} = \frac{x+y+z}{x+y+z} = 1$ Choice (A)

50. If a is 0 or 1, both sides will be equal irrespective of the value of x.
Otherwise powers of a both sides must be equal i.e., x – 1 = 2x + 1 $\Rightarrow$ x = –2
Hence the value of x depends on a. Choice (D)

Exercise – 2
(Ratio, Proportion and Variation)

Solutions for questions 1 to 50:

1. Given that a : b = 4 : 3
$\frac{a}{b} = \frac{4}{3}$, $a = \frac{4}{3}b$
$$\frac{\left(\frac{a}{b}+\frac{b}{a}\right)}{5\left(\frac{a}{b}-\frac{b}{a}\right)} = \frac{\left(\frac{4}{3}+\frac{3}{4}\right)}{5\left(\frac{4}{3}-\frac{3}{4}\right)} = \frac{25}{5(7)} = \frac{5}{7}$$ Choice (A)

2. $\dfrac{(2a+3b)(3a-2b)}{a^2+b^2} = \dfrac{6a^2+5ab-6b^2}{a^2+b^2}$

Dividing both numerator and denominator by b^2, we get,

$$\frac{6\left(\frac{a}{b}\right)^2+5\frac{a}{b}-6}{\left(\frac{a}{b}\right)^2+1}$$

As $\frac{a}{b}=\frac{5}{4}$,

$$\frac{(2a+3b)(3a-2b)}{a^2+b^2} = \frac{6\left(\frac{5}{4}\right)^2+5\left(\frac{5}{4}\right)-6}{\left(\frac{5}{4}\right)^2+1} = \frac{154}{41}.$$

Choice (A)

3. $\frac{a}{b}=\frac{4}{5} \Rightarrow a=\frac{4}{5}b$

$\frac{b}{c}=\frac{3}{5} \Rightarrow b=\frac{3}{5}c$

$a = \frac{4}{5}\left(\frac{3}{5}c\right) = \frac{12}{25}c$

$a : b : c = \frac{12}{25}c : \frac{3}{5}c : c$

$= \frac{12}{25}c : \frac{3}{5}c : \frac{25}{25}c = 12 : 15 : 25$

$= 12 : 15 : 25.$ Choice (C)

4. There are four types of ratios : Duplicate ratio, triplicate ratio, sub-duplicate ratio and sub-triplicate ratio which are defined below.

i) Duplicate ratio : Duplicate ratio of a : b is defined as $a^2 : b^2$.

ii) Triplicate ratio : Triplicate ratio of a : b is defined as $a^3 : b^3$.

iii) Sub-duplicate ratio : sub-duplicate ratio of a : b is defined as $\sqrt{a} : \sqrt{b}$

iv) Sub triplicate ratio : Sub-triplicate ratio of a : b is defined as $\sqrt[3]{a} : \sqrt[3]{b}$.

(a) Duplicate ratio of 25:4 = $25^2 : 4^2$ = 625 : 16
Choice (C)

(b) Triplicate ratio of 8 :11= $8^3 : 11^3$ = 512 :1331.
Choice (B)

(c) Sub-duplicate ratio of 9 : 16 = $\sqrt{9} : \sqrt{16}$ = 3 : 4.
Choice (D)

(d) Sub-triplicate ratio of 343 : 729
$= \sqrt[3]{343} : \sqrt[3]{729}$ = 7 : 9. Choice (B)

5. Mean proportional of two numbers a and b is defined as $\sqrt{ab}$.

(a) a = 6, b = 24
Mean proportional $= \sqrt{6(24)}$ = 12. Ans : (12)

(b) a = 50, b = 512
Mean proportional $= \sqrt{(50)(512)}$ =160.
Ans : (160)

6. $\frac{a}{b}\times\frac{b}{c} = \frac{2}{3}\times\frac{4}{1}$

$\Rightarrow \frac{a}{c} = \frac{8}{3}$

a : c = 8 : 3 Choice (A)

7. Let the three numbers be x, 3x, 5x.
Given, x + 3x + 5x = 108 $\Rightarrow$ x = 12
$\therefore$ The largest number is 5x = 5 $\times$ 12 = 60 Ans : (60)

8. Let the ages of Arun, Brahma and Chari be 5x, 4x and 3x years respectively.
4x = 28 $\Rightarrow$ x = 7.
Sum of the ages of three persons
= 12x = 12 × 7 = 84 years. Ans : (84)

9. Let the two positive numbers be 5x and 8x respectively.
8x – 5x = 15
3x = 15 $\Rightarrow$ x = 5.
$\Rightarrow$ Smaller number = 5x = 25. Ans : (25)

10. Let the ratio of number of boys and girls in the class be b : g

Number of boys in the class = $\frac{b}{b+g}$ (70)

As the number of boys and the girls in the class must be integers, 70 must be divisible by b + g. This condition is not satisfied only by choice (D). Choice (D)

11. $\frac{p}{t} = \frac{p}{q}\times\frac{q}{r}\times\frac{r}{s}\times\frac{s}{t} = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)$

$= \frac{1}{5} \Rightarrow$ p : t = 1 : 5. Choice (C)

12. Let the shares of A, B and C be a, b and c respectively.

$a : b : c = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

Let us express each term with a common denominator of the least number divisible by the denominators of each term i.e.12.

$a : b : c = \frac{6}{12} : \frac{4}{12} : \frac{3}{12}$ = 6 : 4 : 3

Share of C = $\frac{3}{13}$ × 1560 = ₹360. Ans : (360)

13. Let the two numbers be 5x and 6x.
Let the number added to both so that their ratio becomes 7 : 8 be K.

$\frac{5x+K}{6x+K} = \frac{7}{8}$

$\Rightarrow$ 40x + 8K = 42x + 7K
$\Rightarrow$ K = 2x.
6x – 5x = 10 $\Rightarrow$ x =10
K = 2x = 20. Ans : (20)

14. Let the present ages of the father and the daughter be F and D respectively.

$\frac{F}{D} = \frac{3}{1} \Rightarrow$ F=3D.

$\frac{F+12}{D+12} = \frac{11}{5}$

$\Rightarrow$ 5F + 60 = 11D + 132
$\Rightarrow$ 5(3D) + 60 = 11D + 132
$\Rightarrow$ 4D = 72
$\Rightarrow$ D = 18
So, F = 3 × 18 = 54 years. Choice (D)

15. Given that,
a : b is 2 : 1,
b : c is 3 : 5
c : d is 4 : 5 and e : d is 6 : 5
a : b = 2 $\times$ 3 : 1 $\times$ 3
b : c = 3 : 5
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a : b : c = 6 $\times$ 4 : 3 $\times$ 4 : 5 $\times$ 4
c : d = 4 $\times$ 5 : 5 $\times$ 5
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a : b : c : d = 24 : 12 : 20 : 25 $\rightarrow$ (1)
$\Rightarrow$ d : e = 5 $\times$ 5 : 6 $\times$ 5
$\Rightarrow$ d : e = 25 : 30 $\rightarrow$ (2)
From (1) and (2)
a : b : c : d : e = 24 : 12 : 20 : 25 : 30 Choice (B)

16. Let the incomes of Chetan and Dinesh be 3x and 4x respectively. Let the expenditures of Chetan and Dinesh be 5y and 7y respectively. Savings is defined as (Income) –

(Expenditure). Hence the savings of Chetan and Dinesh are $3x - 5y$ and $4x - 7y$ respectively.
$3x - 5y = 2000 \quad \rightarrow \quad (1)$
$4x - 7y = 2000 \quad \rightarrow \quad (2)$
Multiplying (1) by 7 and (2) by 5 and subtracting the resultant equation (2) from resultant equation (1), we get, $x = 4000$
The incomes of Chetan and Dinesh are $3x$ = ₹12000 and $4x$ = ₹16000 respectively. Choice (A)

17. Let the monthly incomes of Amar and Bhuvan be 6x and 5x respectively. Let the monthly expenditures of Amar and Bhuvan be 3y and 2y respectively. Savings of Bhuvan every month = $\frac{1}{4}(5x)$
= (His income) – (His expenditure) = $5x - 2y$.
$\Rightarrow 5x = 20x - 8y$.
$y = \frac{15x}{8}$.
Ratio of savings of Amar and Bhuvan = $6x - 3y : \frac{1}{4}(5x)$
$= 6x - 3\left(\frac{15x}{8}\right) : \frac{5x}{4}$
$= \frac{3x}{8} : \frac{5x}{4} = 3 : 10$. Choice (B)

18. A gets thrice as much as the C.
$\Rightarrow A = 3C$
B gets twice as much as the C
$\Rightarrow B = 2C$
$A : B : C = 3C : 2C : C$
$\Rightarrow A : B : C = 3 : 2 : 1$ Choice (C)

19. Let the number of men and women in the conference hall be 5x and 4x respectively.
When 3 men and 6 women join in the conference, the ratio will become 7 : 6.
$\Rightarrow \frac{5x+3}{4x+6} = \frac{7}{6}$
$6(5x + 3) = 7(4x + 6)$
$\Rightarrow 30x + 18 = 28x + 42$
$\Rightarrow 30x - 28x = 42 - 18$
$\Rightarrow 2x = 24 \quad \Rightarrow x = 12$
$\therefore$ Number of men = $5x = 5 \times 12 = 60$
Number of women = $4 \times 12 = 48$. Choice (B)

20. Given ratio of the numbers is 2 : 5
When x is the number which is added to the actual numbers,
$\Rightarrow \frac{2k+x}{5k+x} = \frac{1}{2}$
$2(2k + x) = 5k + x$
$\Rightarrow 4k + 2x = 5k + x$
$\Rightarrow 2x - x = 5k - 4k$
$\Rightarrow x = k$
Given that $2k + x + 5k + x = 36$
$\Rightarrow 9x = 36 \Rightarrow x = 4$
Choice (C)

21. Let the two numbers be x and y.
$(5x + 2y) = 2(3x - 2y)$
$\Rightarrow 5x + 2y = 6x - 4y$
$\Rightarrow 6y = x \Rightarrow \frac{x}{y} = \frac{6}{1}$
or $(5x + 2y) = 2(2y - 3x)$
$\Rightarrow 5x + 2y = 4y - 6x$
$\Rightarrow 11x = 2y$
$\Rightarrow \frac{x}{y} = \frac{2}{11}$ Choice (D)

22. The ratio of basic salary and allowances of Srinivas is 5 : 4.
Let the salary be 5x and allowances be 4x.
Salary is increased by 40%,.
now salary = $5x \times \frac{140}{100} = 7x$
Allowance go up by 60%, now allowances
$= 4x \times \frac{160}{100} = \frac{32x}{5}$
Ratio of basic salary and allowances now = $7x : \frac{32}{5}x$
$= 35 : 32$. Choice (C)

23. Given that, $a : b = 1 : 2$, $b : c = 4 : 3$, $c : d = 4 : 5$, $d : e = x : 1.5x$
$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} \times \frac{d}{e} = \frac{1}{2} \times \frac{4}{3} \times \frac{4}{5} \times \frac{x}{1.5x} \Rightarrow \frac{a}{e} = \frac{16}{45}$.
Choice (A)

24. Let the weights of the three pieces into which the diamond breaks be 3x, 4x and 5x respectively. Let the total values of the pieces weighing 3x, 4x and 5x be denoted by V_1, V_2 and V_3 respectively.
$V_1 \propto (3x)^2 \Rightarrow V_1 = K(3x)^2$ where K is a constant
$V_2 \propto (4x)^2 \Rightarrow V_2 = K(4x)^2$
$V_3 \propto (5x)^2 \Rightarrow V_3 = K(5x)^2$
$V_1 + V_2 + V_3 = 50Kx^2$
Value of the diamond = $K(3x + 4x + 5x)^2$
$= 144Kx^2 = 1440000$
$\Rightarrow Kx^2 = 10000$.
Loss in the value of the diamond due to breakage
$= 144Kx^2 - 50Kx^2$
$= 94\ Kx^2 = 940000$
= ₹ 9.4lakhs. Ans : (9.4)

25. Let there be x fewer men.
We have, $M_1 W_1 C_1 = M_2 W_2 C_2$
$1000 \times 8 \times 10 = (1000 - x) \times 10 \times 16$
$\frac{1000-x}{1000} = \frac{8\times10}{10\times16}$
$\frac{1000}{1000} - \frac{x}{1000} = \frac{1}{2} = 1 - \frac{1}{2} = \frac{x}{1000} = \frac{1}{2} = \frac{x}{1000}$
x = 500 men.
So the difference between the two cases is 1000 – 500 = 500. Ans : (500)

26. The fourth proportional of a, b, c is given by $\frac{bc}{a} = \frac{1.6\times1.6}{0.8}$
$= 3.2$ Choice (C)

27. $x : \frac{1}{57} = \frac{3}{7} : \frac{5}{19}$ (or) $57x = \frac{3\times19}{5\times7}$
$x = \frac{1}{35}$ Choice (D)

28. $\frac{5x+10}{8x+10} = \frac{7}{10}$ By cross multiplying we get the value of x
$x = 5$
$\therefore$ The numbers are 25, 40 Choice (D)

29. Number of days left = 45 – 10 = 35
Number of men left = 150 – 25 = 125
$m_1 d_1 = m_2 d_2$
$150 \times 35 = 125 \times x$
x = 42 days Choice (D)

30. Let the three positive numbers be 6x, 7x and 8x respectively.
$(6x)^2 + (8x)^2 = 1600$
$\Rightarrow 36x^2 + 64x^2 = 1600$
$\Rightarrow 100x^2 = 1600$
$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$.
As $x > 0$, $x = 4$.
Hence the three numbers are 6(4), 7(4) and 8(4) i.e. 24, 28 and 32. Choice (D)

31. The number of boys in the class = $\frac{4}{7}(70) = 40$.

The number of girls in the class = $\frac{3}{7}$ (70) = 30.

Among the girls, number of sports persons = $\frac{1}{5}$(30) = 6.

Among the girls, number of non-sports persons = $\frac{4}{5}$(30) = 24.

Number of sports persons in the class = $\frac{8}{35}$ (70) = 16.

As 6 girls are sports persons, the balance 10 sports persons must be boys.
Number of non sports persons among the boys in the class = 40 – 10 = 30
Number of non sports persons and non-sports persons among the boys are in the ratio 10 : 30 = 1 : 3. Choice (B)

32. Given that, $A \alpha r^2$

$\Rightarrow \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \Rightarrow A_2 = A_1 \left(\frac{r_2}{r_1}\right)^2$

Taking A_1 = 154 sq.units r_1 = 7 units and r_2 = 3.5 units

$\Rightarrow A_2 = 154 \left(\frac{3.5}{7}\right)^2$

$= 154 \left(\frac{1}{4}\right)$ = 38.5 sq.units. Ans : (38.5)

33. $P \propto Q^2$ when R is constant.

$P \propto \frac{1}{R}$ when Q is constant.

Hence P varies directly with Q^2 and inversely with R.

Hence $P \propto Q^2 (\frac{1}{R})$.

$\frac{P_1}{P_2} = \frac{Q_1^2 \; R_2}{Q_2^2 \; R_1}$

So, $\frac{36}{P_2} = \frac{12^2.16}{24^2.8}$

$\Rightarrow P_2 = 72$. Ans : (72)

34. Cost of article 6400.
The ratio of weight of broken pieces 3 : 5.
Let the weights of the broken pieces be 3x and 5x.
Total weight 3x + 5x = 8x
Cost of the article is proportional to the square of its weight.
$V \alpha w^2$
$V = Kw^2$
V = 6400
when weight is 8x
$6400 = k(8x)^2$
$6400 = k\,64x^2$
$kx^2 = 100$
Sum of the costs of the two broken pieces = $k(3x)^2 + k(5x)^2$
$= 34kx^2$
$= 34 \times 100 = 3400$
Loss incurred = 6400 – 3400 = ₹3000. Ans : (3000)

35. $m_1 d_1 = m_2 d_2$
$60 \times 27 = 18 \times d_2$
d_2 = 90 days. Choice (B)

36. Let the share of the first son be K.
Therefore the ratio of shares of the three sons is

K : K/2 : K/3 = 6 : 3 : 2

As 4 parts represent 12000, 3 parts will represent

$\frac{12000}{4} \times 3$ = ₹9000 Ans : (9000)

37. $\frac{3a^2 + 2b^2}{2b^2 - a^2} = \frac{15}{7} \Rightarrow 21a^2 + 14b^2 = 30b^2 - 15a^2$

$\rightarrow 36a^2 = 16b^2$

$\rightarrow \frac{a^2}{b^2} = \frac{16}{36} \rightarrow \frac{a}{b} = \frac{4}{6} = \frac{2}{3}$ Choice (B)

38. Let the two numbers be x and y. $3x = 2y \rightarrow \frac{x}{y} = \frac{2}{3}$

$\Rightarrow x = 2y/3 \Rightarrow \frac{3(x+y)}{5(x-y)} = \frac{15y}{5y} = 3$ Choice (D)

39. Let the two numbers be x and y.
$3x + 2y = 2(3x - 2y) \rightarrow 3x + 2y = 6x - 4y$
$\Rightarrow 3x = 6y \rightarrow x : y = 2 : 1$ Choice (B)

40. Let the numbers be 3x and 5x and 9x. Given,
$9x^2 + 25x^2 + 81x^2 = 460 \rightarrow x^2 = 4 \rightarrow x = 2$
Therefore the largest number = $9 \times 2 = 18$ Choice (D)

41. Let the numbers be 2x, 3x and 5x. Given,
$2x \times 5x = 10x^2 = 90 \rightarrow x^2 = 9 \rightarrow x = 3$
Therefore the difference between the largest and the smallest numbers.
= 5 x 3 – 2 x 3 = 9 Ans : (9)

42. Let the number of 50 paise, 25 paise and 1 rupee coins be 5x, 2x and 5x respectively.
Therefore the values of 50 paise and 25 paise coins is 5x(0.5), i.e., 2.5x and 2x(0.25) i.e., 0.5x respectively.
given $2.5x - 0.5x = 8 \Rightarrow 2x = 8 \Rightarrow x = 4$
number of 50 paise coins = $5x \Rightarrow = 20$. Ans : (20)

43. Let the number of 50 paise, 1 rupee, 5 rupee coins be 2x, 2x and x respectively. Then the values of 5 rupee and 1 rupee coins will be 5x and 2x respectively.
Given 5x – 2x = 3x.
$5x - 2x = 15 \Rightarrow 3x = 15 \Rightarrow x = 5$
Therefore the total value = 8x = 8 x 5 = ₹40 Ans : (40)

44. As $X \alpha Y^2 \Rightarrow X = kY^2$.
Given, $72 = k.6^2 \rightarrow k = 2$
$x = 2y^2$
$x = 2 \times 9^2 = 162$ Ans : (162)

45. $X \alpha Y$ and $X \alpha 1/Z^2 \Rightarrow X = \frac{KY}{Z^2}$

$2 = \frac{K.3}{64} \Rightarrow K = \frac{128}{3}$

$X = \frac{128}{3} \frac{Y}{Z^2} = \frac{128}{3} \times \frac{6}{4} = 64$ Ans : (64)

46. Let the radius and the area of the circle be represented by r and A respectively. As $A \alpha r^2$, $A = Kr^2 \Rightarrow 196 = K.7^2$
$\Rightarrow K = 4A = 4r^2 \Rightarrow A = 4 \times 8^2 = 4 \times 64 = 256$ sq.ft
Ans : (256)

47. As $X \alpha \sqrt[3]{Y}$, we have

$X = K.\sqrt[3]{Y}$; $12 = K.\sqrt[3]{8} \Rightarrow K = 6$

Therefore $X = 6.\sqrt[3]{Y} = 6 \times \sqrt[3]{27} = 18$ Choice (C)

48. Given, $F \alpha M_1 M_2$ and $F \alpha \frac{1}{D^2} \Rightarrow F = \frac{K M_1 M_2}{D^2}$

$\Rightarrow 18 = \frac{K.12}{4} \Rightarrow K = 6 \Rightarrow F = \frac{6 M_1 M_2}{D^2}$

$F = \frac{6 \, . \, 18}{9}$ = 12 Newtons Ans : (12)

49. Given $P \alpha D$ and $P \alpha \frac{1}{\sqrt{S}} \Rightarrow P = \frac{KD}{\sqrt{S}}$

$500 = \frac{K\ .\ 1000}{\sqrt{100}} \Rightarrow K = 5 \Rightarrow P = \frac{5\ x\ D}{\sqrt{5}}$

$P = \frac{5\ x\ 3000}{\sqrt{900}}$ = 500 units. Choice (C)

50. F = 2S; S = 3T; F : S : T = 6 : 3 : 1
Therefore ratio of the first, second and third son's shares = 6 : 3 : 1. As 2 parts represent ₹4000,
10 parts will represent
$\frac{4000}{2} \times 10$ = ₹20,000 Ans : (20,000)

Exercise – 3
(Percentages, Profit & Loss, Partnerships)

Solutions for questions 1 to 75:

1. Let x percent of 80 be 64.
$\therefore\ 80 \times \frac{x}{100} = 64 \Rightarrow x = \frac{64 \times 100}{80} \Rightarrow x = 80$
$\therefore$ 80% of 80 is 64. Choice (A)

2. $14\frac{2}{7}\% = \frac{100}{7} \times \frac{1}{100} = \frac{1}{7}$

$\Rightarrow 14\frac{2}{7}\% = \frac{1}{7}$

$\therefore$ The fractional value of $14\frac{2}{7}\%$ is $\frac{1}{7}$ Choice (B)

3. Given
g% of b + 1% of gb = 4% of (g + b)
$\Rightarrow \frac{g}{100} \times b + \frac{1}{100} \times gb = \frac{4}{100} \times (g + b)$
$\Rightarrow \frac{2gb}{100} = \frac{4}{100}(g + b)$
$\Rightarrow$ gb = 2 (g + b)
$\Rightarrow$ 2 (g + b) = gb
$\Rightarrow \frac{g+b}{gb} = \frac{1}{2} \Rightarrow \frac{1}{g} + \frac{1}{b} = \frac{1}{2}$

$\therefore$ The sum of reciprocals of g and b is $\frac{1}{2}$. Choice (C)

4. Let the original price be ₹100.
Price after increasing 20% is 120.
Price after decreasing 20% on 120 is ₹96.
Given latest price is ₹1440
The original price = $1440 \times \frac{100}{96}$ = ₹1500
Hence the original price is ₹1500 Ans : (1500)

5. Let the required percentage be x%.
$\therefore\ \frac{3}{20}$ = x% of $\frac{12}{25}$
$\Rightarrow \frac{3}{20} = \frac{x}{100} \times \frac{12}{25} \Rightarrow 12x = \frac{300 \times 25}{20} \Rightarrow x = \frac{25 \times 25}{20}$
$\Rightarrow$ x = 31.25 Choice (B)

6. Let the number be x.
$\therefore$ 72% of x = 468
$\Rightarrow \frac{72}{100} \times x = 468$
$\Rightarrow x = \frac{468 \times 100}{72} \Rightarrow x = 650$
Now, 80% of 650
$= \frac{80}{100} \times 650 = 520$ Ans : (520)

7. Let the price of the article three years ago be ₹100
In the 1st year, price of the article = 100 + 30 = ₹130.
In the 2nd year, price = 130 – 20% of 130
= 130 – 26 = ₹104.
In the 3rd year, price = 104 + 10% of 104
= 104 + 10.4 = ₹114.40.
But present price of the article = ₹2,288
$\therefore$ for 114.4 $\to$ 100
2288 $\to$?
$\therefore$ Required price = $\frac{2288 \times 100}{114.4} = 20 \times 100$ = ₹2000.
Ans : (2000)

8. Let the number be x. According to the problem.
x + 45% of x = 116.
$\Rightarrow x + \frac{45}{100}x = 116 \Rightarrow x = \frac{116 \times 100}{145} \Rightarrow x = 80.$
$\therefore$ The required number is 80. Ans : (80)

9. Given X : Y : Z = $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$
= 6 : 4 : 3
Required answer is $\frac{9-4}{4} \times 100$ = 125%. Choice (C)

10. Let the length and breadth of the rectangle be 10 units each.
Area = 10 $\times$ 10 = 100 sq. units.
Given length and breadth are increased by 10%.
$\therefore$ Their new length & breadth are 11 units each
Area = 11 $\times$ 11 = 121 sq. units
i.e., 21% more than 100
$\therefore$ The area of the rectangle is increased by 21%.
Choice (C)

11. Let C = 100
Given B is 20% less than C.
$\therefore$ B = 80
A is 30% more than B
$\therefore$ A = 80 + 24 = 104
A is '4' more than C
$\therefore$ A is 4% more than C. Choice (C)

12. If P is a% of R and Q is b% of R, then P is
$\frac{a}{b} \times 100\%$ of Q.
Here
First number = $\frac{25}{75} \times 100\,\%$
$= \frac{100}{3}\% = 33\frac{1}{3}\%$
$\therefore$ First number is $33\frac{1}{3}\%$ of the second. Choice (C)

13. 25% of x = $\frac{x}{4}$
15% of 1500 = $\frac{15}{100} \times 1500 = 225$
Given,
$\frac{x}{4} = 225 - 15$
$\Rightarrow \frac{x}{4} = 210 \Rightarrow x = 210 \times 4$
$\Rightarrow$ x = 840 Ans : (840)

14. A's marks = 80
B's marks = 60
Let x% of A = B
$\Rightarrow \frac{x}{100} \times 80 = 60 \Rightarrow x = \frac{60 \times 100}{80} = 75$
$\therefore$ B's marks is 75% of A's marks. Choice (D)

15. A's marks = 80
B's marks = 60
Difference = 80 – 60 = 20

Required percentage = $\frac{20}{60}\times 100$

= $\frac{100}{3} = 33\frac{1}{3}$%   Choice (B)

16. Given, P's salary = ₹5000
Q's salary = ₹6000
Let x% of P = Q

$\Rightarrow \frac{x}{100}\times 5000 = 6000$

$\Rightarrow x = \frac{6000\times 100}{5000} = 120$

∴ Q's salary is 120% of P's salary.   Choice (C)

17. P's salary = ₹5000
Q's salary = ₹6000
Difference = 6000 – 5000 = 1000

Required percentage = $\frac{1000}{6000}\times 100 = \frac{50}{3} = 16\frac{2}{3}$%

P's salary is $16\frac{2}{3}$% less than that of Q.   Choice (A)

18. Let Q gets 1 unit

25% of 1 unit = $\frac{1}{4} = 0.25$

∴ P gets (1 + 0.25) i.e., 1.25 units.
Difference = (1.25 – 1) = 0.25.

Required percentage = $\frac{0.25}{1.25}\times 100$

= $\frac{1}{5}\times 100 = 20\%$ Choice (C)

19. Given ratio = $\frac{1}{8}:\frac{1}{5}$ = 5 : 8

Let first number be 5x and the second number be 8x
The second number is more than first number by 3x.

Required percentage = $\frac{3x}{5x}\times 100$ = 60%   Choice (C)

20. Let the man's income be ₹x.

75% of x = $\frac{3x}{4}$

∴ Savings = x – $\frac{3x}{4} = \frac{4x-3x}{4} = \frac{x}{4}$

Given, $\frac{x}{4} = 2000$

⇒ x = ₹8000   Ans : (8000)

21. let the total number of votes polled be x.
Majority = 488.
i.e., 55% of x – 45% of x = 488
⇒ 10% of x = 488

$\Rightarrow \frac{10}{100}\times x = 488$

⇒ x = 4880
∴ The total number of votes polled were 4880.
Ans : (4880)

22. Given 6% of x = 24% of 380

$\Rightarrow \frac{6}{100}\times x = \frac{24}{100}\times 380$

⇒ 6x = 24 × 380
⇒ x = 4 × 380
⇒ x = 1520   Ans : (1520)

23. 40% of k = $\frac{40k}{100} = \frac{2}{5}k$

40% of 40 = $\frac{40}{100}\times 40 = 16$

According to the problem $\frac{2k}{5} - 16 = 40$

$\Rightarrow \frac{2k}{5} = 56 \Rightarrow$ k = 140.   Ans : (140)

24. Given 40% of 20 – 80% of 40 + 30% of 10 – 4% of 100

= $\frac{40}{100}\times 20 - \frac{80}{100}\times 40 + \frac{30}{100}\times 10 - \frac{4}{100}\times 100.$

= 8 – 32 + 3 – 4 = –25   Choice (C)

25. Let the original price of the article be ₹100
Price after 20% decrease = ₹80
27.5% of 80 = 22.
Price of article after increase of 27.5% = 80 + 22 = ₹102.
Difference of original and latest price = ₹2.
∴ Percentage Increase in price = 2.   Choice (A)

26. Let the cost price of an article be ₹x.
∴ (19% of x) – (12% of x) = 105

$\frac{19x}{100} - \frac{12x}{100} = 105$

⇒ 7x = 105 × 100 ⇒ x = 1500
∴ Cost Price = ₹1500   Ans : (1500)

27. Given that S.P. = ₹81
% of loss = 10%
Let C.P. be ₹x
∴ Loss = (x – 81)

We have, % of loss = $\frac{\text{loss}}{\text{C.P.}}(100)$

10 = $\frac{(x-81)}{x}100$

⇒ x = 10x – 810
⇒ 9x = 810 ⇒ x = 90
∴ C.P = ₹ 90   Ans : (90)

28. S.P. of each car is ₹3520, he gains 12% on first car and loses 12% on second car.
In this case, effect will be loss and percentage of loss is given

by = $\frac{[(\text{Profit}\%)(\text{loss}\%)]^2}{100} = \frac{(12)(12)}{100}$ = 1.44%

Choice (C)

29. Let the cost price of each article be ₹100.
Profit on the article sold at Profit = ₹20.
Loss on the article sold at loss = ₹10
As the cost price of both the articles are the same and profit on one article exceeds the loss on the other, the trader effectively makes a profit.
Effective profit made by the trader = ₹10
Total cost price = ₹200
∴ Overall profit percentage = 5%

Note:
If two articles are bought for the same price and one is sold at x% profit and the other is sold at y% loss, then in overall

(i) Profit of $\left(\frac{x-y}{2}\right)$% is made, if x > y

(ii) Neither profit nor loss is made, if x = y

(iii) Loss of $\left(\frac{y-x}{2}\right)$% is made, if y > x   Choice (D)

30. As the shopkeeper's is cost price is ₹5 less than his selling price, his profit is ₹5.
Let his cost price be ₹x

$\frac{12.5}{100}x = 5 \Rightarrow x = 40$ Ans : (40)

31. Let the cost price of Rohit be ₹x.

Selling price of bicycle = x + $\frac{4}{100}$x

$\therefore$ x + $\frac{4}{100}$x – 204 = x – $\frac{30}{100}$x

x = 600

Alternate method:
Rohit sold the bicycle at 4% more than his cost price. If he sold it at ₹204 less he would have sold it at 30% less than his cost price. Difference of these two selling prices = 34% of his cost price

$\frac{34}{100}x = 204 \Rightarrow x = 600$ Ans : (600)

32. Let the cost price of each article be ₹1.
Cost price of 12 articles = ₹12.
Selling price of 12 articles = Cost price of 15 articles = ₹15.
Profit is made in selling 12 articles since their selling price exceeds their cost price. Profit made in selling 12 articles = ₹3.
Profit percentage = 25%. Choice (A)

33. Given that cost price of 40 articles is equal to the selling price of 50 articles.
Let cost price of each article = Re.1
$\therefore$ Selling price of 50 articles = ₹40
But Cost price 50 articles = ₹50
Therefore, the trader incurred loss.

Percentage of loss = $\frac{10}{50} \times 100\% = 20\%$ Choice (A)

34. Let A purchased the article for S.100. Then the C.Ps of B and C are ₹110 and ₹93.5 respectively.
Given 6.5 ----------- 58.5
110 ----------- ?

? = $\frac{110 \times 58.5}{6.5}$ i.e. ₹990. Ans : (990)

35. Given that S.P. = ₹102 and loss = 15%

C.P.= $\frac{100\ (\text{S.P})}{(100 - \%\ \ell)} = \frac{100 \times 102}{85}$ = 20 × 6 = ₹120

To get 20% profit.

New S.P. = $\frac{(100 + \%\,\text{p})\text{C.P.}}{100} = \frac{120 \times 120}{100}$ = ₹144

Ans : (144)

36. Let his Cost Price be ₹x

Markup = ₹ $\frac{2}{5}$x

His marked price = ₹ $\frac{7}{5}$x

Discount = ₹ $\frac{14}{25}$x

His Selling price = ₹ $\frac{21}{25}$x

$\frac{21}{25}x = x - 640 \Rightarrow x = 4000$

His selling price = ₹3360 Ans : (3360)

37. Let the cost price of the suit be ₹x.

x + $\frac{25x}{100}$ = 320 $\Rightarrow$ x = 256

Required Percentage = 56¼% Choice (D)

38. Cost price of 1000 gms for the merchant = Selling price of 600 gms.
Let the cost price of each gm be ₹1.
Cost price of 600 gms = ₹600.
Selling price of 600 gms = ₹1000
Profit made in selling 600 gms = ₹400
Profit % = $66^2/_3$% Choice (D)

39. Let the cost price of Vijay be ₹100.
Vijay marked up by ₹50
Marked price of Vijay = ₹150
Profit of Vijay = ₹20
Selling price of Vijay = ₹120
Discount = ₹30
Discount% = 20% Choice (A)

40. Cost Price of 21 apples = ₹180
But $33^1/_3$% of apples are rotten.
$\therefore$ Number of apples to be sold = 21 – ($33^1/_3$% of 21) = 14
Selling price of 14 apples = 180 + ($16^2/_3$% of 180)
= 180 + (1/6 × 180) = ₹210

$\therefore$ Selling price of 1 apples = $\frac{210}{14}$ = ₹15. Ans : (15)

41. Let S.P. be ₹100 $\Rightarrow$ C.P. be ₹80
In the above case, if profit is calculated on selling price, then profit is 20%.
But profit percentage is to be calculated on cost price.

$\therefore$ Actual percentage of profit = $\frac{20}{80} \times 100$ = 25%

Choice (B)

42. Let the cost price be ₹100.
New selling price is ₹140.

Given, $\frac{7}{10}$ (Actual SP) = 140

$\Rightarrow$ Actual SP = ₹200.

Actual profit percentage = $\frac{200 - 100}{100} \times 100$ % = 100%

Choice (D)

43. Let the cost price of an article be ₹100
Given that successive discounts are 20% and 15%
$\therefore$ S.P. = 85% of 80% of 100

= $\left(\frac{85}{100}\right)\left(\frac{80}{100}\right)(100)$ $\Rightarrow$ S.P. = ₹68

Clearly, single discount is 32%. Choice (C)

44. Given S.P. = ₹6800

$\therefore$ Marked price = $\frac{\text{S.P.}(100)}{(100 - \text{d}\%)} = \frac{6800 \times 100}{100 - 15}$ = ₹8000

if S.P. = ₹8000, profit = 60%

$\therefore$ C.P. = $\frac{\text{S.P.}(100)}{(100 + 60)} = \frac{8000 \times 100}{160}$ = ₹5000

Ans : (5000)

45. Let the cost prices of A and B be a and b respectively.

$\therefore \frac{110}{100}a + \frac{80}{100}b = \frac{90}{100}a + \frac{120}{100}b = 3000$

$\Rightarrow \frac{110 - 90}{100}a = \frac{120 - 80}{100}b$

$\Rightarrow$ 20 a = 40 b $\Rightarrow$ a = 2b

$\therefore \frac{90a}{100} + \frac{120}{100}\left(\frac{a}{2}\right) = 3000$

$\Rightarrow$ 90a + 60a = 300000

$\Rightarrow a = \frac{300000}{150}$

$\Rightarrow$ a = 2000

Ans : (2000)

46. Let C.P. of an article be 'x'.

$\therefore$ S.P. = 120% of x = $\frac{6x}{5}$ {$\because$ Profit = 20%}

Now, C.P = Rs(x – 3000) and S.P. = ₹ $\left(\frac{6x}{5} - 3000\right)$

And new profit = 25%

$\therefore$ Profit = $\left(\frac{6x}{5} - 3000\right)$ – (x – 3000) = $\frac{x}{5}$

Percentage of profit $\Rightarrow \dfrac{\frac{x}{5}}{(x-3000)}$ × 100 = 25

$\Rightarrow$ 4x = 5x – 15000 $\Rightarrow$ x = 15000

$\therefore$ Original C.P. = ₹15000 Ans : (15000)

47. Let the total capital be ₹T. Let the entire period be x.
Ratio of the profit shares of P, Q and R

$= \left(\frac{1}{4}T\right)\left(\frac{1}{4}x\right) : \left(\frac{1}{9}T\right)\left(\frac{1}{9}x\right) : \left(T - \left(\frac{1}{4}T + \frac{1}{9}T\right)\right) x$

$\frac{1}{16}Tx : \frac{1}{81}Tx : \frac{23}{36}Tx = \frac{81:16:23\times36}{16.81}$ = 81 : 16: 828.

Q's share = $\frac{16}{81+16+828}$ × 27750 = ₹480 Ans : (480)

48. Ratio of time periods for which Antony, Ben and Charles stayed = $\frac{\text{Profit}}{\text{Investment}}$ of respective partners. = $\frac{6}{3}:\frac{4}{4}:\frac{3}{6}$

= 4 : 2 : 1 Choice (B)

49. Investments of X, Y and Z are ₹20000, ₹25000 and ₹30000 respectively.
Let investment period of Z be x months.
$\therefore$ Ratio of annual investments of X, Y and Z is (20000 × 12) : (25000 × 12) : (30000 × x)
= 240 : 300 : 30x = 8 : 10 : x
The share of Z in the annual profit of ₹50000 is ₹14000.

$\Rightarrow \left(\frac{x}{18+x}\right)50000 = 14000 \Rightarrow \left(\frac{x}{18+x}\right)25 = 7$

$\Rightarrow$ 25x = 7x + (18 × 7)

$\Rightarrow$ x = 7 months.

$\therefore$ Z joined in the business after (12 – 7) months. i.e. 5 months. Choice (C)

50. The capital of Raheem is $\left(1-\frac{1}{3}-\frac{1}{4}\right)$ of total capital.

i.e. $\frac{5}{12}$ of total capital.

Total Profit = ₹240000

$\therefore$ Share of Raheem in the total profit = $\frac{5}{12}$ (240000)

= ₹100000. Ans : (1,00,000)

51. Increase in the price of mangoes per kg = 20 – 12 = ₹8.

$\therefore$Required % increase = $\frac{8}{12}\times100 = 66^2/_3\%$

Choice (C)

52. Let the price of the article be ₹100.
20% of 100 = 20.
New price 100 + 20 = ₹ 120

Required percentage = $\frac{120-100}{120}\times100$

$= \frac{20}{120}\times100 = \frac{50}{3} = 16\frac{2}{3}$ Choice (A)

53. Let length and breadth of the rectangle be each 1 unit.
Area of rectangle = l × b = 1 × 1 = 1 sq. unit
After changes length and breadth of rectangle are $\frac{1\times120}{100}=1.2$, $\frac{1\times90}{100}=0.9$ respectively.

Now area of rectangle = 1.2 × 0.9 = 1.08 sq.units

Percentage increase in area = $\frac{1.08-1}{1}\times100\% = 8\%$

Choice (A)

54. Let the first term be 1.

10% of 1 = $\frac{1}{10}=0.1$

Second term = 1 + 0.1 = 1.1

20% of 1 = $\frac{1}{5}=0.2$

Third term = 1 + 0.2 = 1.2
Let the required percentage be x.
$\therefore$ x% of 1.2 = 1.1

$\Rightarrow \frac{x}{100}\times1.2 = 1.1$

$\Rightarrow x = \frac{1.1\times100}{1.2} = \frac{1100}{12} = 91\frac{2}{3}\%$ Choice (C)

55. Let Mohan's salary be ₹100.
When increased by 20%,
Mohan's salary = ₹120
Again when decreased by 20%, Mohan's salary
= 120 – 24 = ₹96.
But present salary is ₹7,200
$\therefore$for, 96 $\to$ 100
7200 $\to$?

Required salary is $\frac{7200}{96}\times100$ i.e., ₹7500. Ans : (7500)

56. Number of four wheelers sold for 2 years = 1520.
Let 'x' two wheelers be sold in 2 years
According to the problem 40% of x = 1520

$\Rightarrow \frac{40}{100}\times x = 1520$

$\Rightarrow$ x = 3800

$\therefore$ 3800 two-wheelers are sold in 2 years. Ans : (3800)

57. The present population of the town = 5200

Population of the town after 2 years = 5200 $\left(1+\frac{5}{100}\right)^2$

= 5733 Ans : (5733)

58. Let the side of the square be 10 units.
Area = 10 × 10 = 100 sq. units.
If the side is decreased by 25%, then its new length = 7.5 units.
Area = 7.5 × 7.5 = 56.25 sq. units.

Decrease in the area of the square = $\frac{43.75}{100}\times100$

= 43.75% Choice (C)

59. Let's Anil's salary be ₹100.
Money spent on Rent = 40% of 100 = ₹40.
Money spent on medical grounds = 30% of (100 – 40)

$= \frac{3}{10}\times 60$ = Rs.18

Money spent on education = 20% of (60 – 18)

$= \frac{1}{5}\times 42$ = ₹8.40

Anil saves 100 – (40 + 18 + 8.40) i.e ₹33.60
$\therefore$for, 33.6 $\to$ 100
840 $\to$?

Required salary is $\frac{840}{33.6}\times100$.

i.e., ₹2500. Ans : (2500)

60. Let the maximum marks be M.
Method 1:

Ajay's marks = $\frac{30}{100}$M and Bala's marks = $\frac{40}{100}$M

Ajay failed by 15 marks and Bala got 5 marks more than the pass marks.

$\therefore$ Pass marks = $\frac{30}{100}$M + 15 = $\frac{40}{100}$M – 5

20 = $\frac{10}{100}$M

M = 200.

Also pass marks = 30% of 200 + 15 = 75

Method 2:
Let the pass marks be P
Bala's marks – Ajay's marks = P + 5 – (P – 15) = 20

$\frac{40}{100}M - \frac{30}{100}M = 20$

M = 200 Choice (A)

61. Let the selling price of Francis be ₹x.
50% of x = 5 $\Rightarrow$ x = 10
Cost price of Francis = x – 5 = ₹5

His actual profit percentage = $\frac{5}{5} \times 100 = 100\%$

Choice (A)

62. Total cost price of the vendor = 30 × 16 = ₹480
Total selling price of the vendor = 10 × 2 + 16 × 25 = ₹600.
Profit of the vendor = ₹120
Profit percentage of the vendor = 25% Choice (B)

63. Let the cost price of Ravi be ₹x.

Marked price of Ravi = ₹ $\frac{8}{5}x$

If he decreased his discount from 20% to 10%; the increase in the selling price =

₹ $\left(\frac{8}{5}x - \frac{10}{100}\left(\frac{8x}{5}\right)\right) - \left(\frac{8}{5}x - \frac{20}{100}\left(\frac{8x}{5}\right)\right) = \text{Rs.}\frac{4x}{25}$

This increase represents the extra profit = $\frac{4x}{25} = 80$

x = 500 Ans : (500)

64. Let the cost price of 20 m of the cloth be ₹20. The loss made in selling 20 m of the cloth = cost price of 5 m of that cloth = 5.

$\therefore$loss % = $\frac{20-15}{20} \times 100$

Loss % = 25% Choice (B)

65. Let the selling price of 25 m of the cloth be ₹25.
Profit made in selling 25 m of the cloth = selling price of 5 m of the cloth = 5.
Cost price of 25 m of the cloth = ₹25 – 5 = 20.
Profit percentage = 25%. Choice (A)

66. Let the selling price of 16 m of the cloth be ₹16
Loss made in selling 16 m of the cloth = selling price of 4 m of that cloth = 4.
Cost price of 16 m of the cloth = ₹16 + 4 = 20
Loss percentage = 20%. Choice (B)

67. CP of the DVD player = $\frac{5400}{1.2}$ = ₹4500.

CP of the watch = $\frac{4500}{3}$ = ₹1500

$\therefore$ Profit percentage on selling the watch = $\frac{500}{1500} \times 100$

= $33^1/_3$% Choice (D)

68. Let the marked price of the pair of trousers be ₹100.
First discount = ₹10
Price after discount = ₹90.
Second discount = ₹18.
Price after discount = Selling price = ₹72
Let the cost price of the pair of trousers be ₹x.

$x + \frac{44}{100}x = 72 \Rightarrow x = 50$

$\therefore$ The trouser was marked at 100% above the cost price.
Choice (B)

69. Ratio of profits of Anwar and Bhaskar
Ratio of product of investment and time of Anwar and Bhaskar
= (15000 × 4 + 10000 × 8) : (24000 × 4 + 16000 × 8) = 5 : 8.
Choice (D)

Note: Both Anwar and Bhaskar stayed for the same period and at any point of time the ratio of their capitals is 5 : 8, hence the ratio of their profits will be 5 : 8.

70. Ratio of investments of A and B is
(70000 × 12) : (120000 × 6) = 7 : 6
Total profit = ₹52000

$\therefore$ Share of 'B' = $\frac{6}{13}$ (52000) = ₹24000.

Ans : (24000)

71. Given that,
Ram's investment was thrice that of Shyam and period of investment was half that of Shyam.
Let the investment of Shyam = ₹x
The period of investment of Shyam = y monthly.

$\therefore$ Ratio of investments of Ram and Shyam is $(3x)\left(\frac{y}{2}\right)$: xy

= $\frac{3}{2}$: 1 = 3 : 2

Share of Ram = ₹19200

$\therefore$ Total profit = $\frac{5}{3}$(19200) = 5(6400) = ₹32000.

Ans : (32000)

72. Total investment = ₹60 lakhs
Let investment of B is x lakhs.
$\therefore$ Investment of A is (x – 10) lakhs
and investment of C is (x – 5) lakhs.
$\Rightarrow$ (x – 10) + x + (x – 5) = 60
$\Rightarrow$ 3x = 75
$\Rightarrow$ x = 25
$\therefore$ Ratio of investments of A, B and C is
(x – 10) : x : (x – 5)
= 15 : 25 : 20 = 3 : 5 : 4

$\therefore$ Share of 'C' in total profit = $\frac{4}{12}$ (18 lakhs) = 6 lakhs.

Ans : (6)

73. Let C.P. of Gold Coin (for Ratan) be ₹1000
S.P of Gold coin = 110% of 1000 = ₹1100
C.P. of Gold coin (for Kesav) = ₹1100
Again he sold it to Ratan for ₹1000

$\therefore$ Percentage of loss = $\frac{100}{1100} \times 100 = \frac{100}{11} = 9\frac{1}{11}\%$

Choice (C)

74. Let M.P. of article be ₹100
C.P. of article (for trader) = ₹80 {$\because$ 20% of discount allowed)
But trader desires to get 20% profit.
S.P. of article = 120% of 80 = ₹96
$\therefore$ Trader should allow 4% discount on the marked price.
Choice (B)

75. S.P. of Pen = ₹10.20 and loss = 15%

C.P. = $\frac{100\,(10.20)}{(100-15)} = \frac{1020}{85} = ₹12$

But he wants 12.5% profit.

$\therefore$ S.P. = $\left(\frac{100+12.5}{100}\right)12 = \frac{(112.5)(3)}{25}$ = ₹13.50

M.P. = $\frac{100\,(\text{S.P})}{(100-\%\text{d})}$ = $\frac{100\,(13.50)}{90}$ {$\because$ discount = 10%}

= $\frac{135}{9}$ = ₹15 Ans : (15)

Exercise – 4
(Simple Interest and Compound Interest)

Solutions for questions 1 to 25:

1. Simple interest $= \frac{18000 \times 4 \times 15}{100}$ = ₹10800

Amount = P + I = 18000 + 10800 = ₹28800

Ans : (28800)

2. $T = \frac{100 \times 16500}{15 \times 5000} = 22$ years Ans : (22)

3. Interest = 5476 – 3700 = ₹1776

so $r = \frac{100 \times 1776}{3700 \times 4} = 12\%$ p.a.

Now the rate of interest is (12 + 6)% p.a.

So, interest $= \frac{3700 \times 4 \times 18}{100}$ = ₹2664

Amount = 3700 + 2664 = ₹6364 Ans : (6364)

4. Let the sum be P.

$P = \frac{100 \times 1368}{3 \times 12}$ = ₹3800 Ans : (3800)

5. Let interest for 1 year be x.

As Amount = Principal + Interest, we have

P + 4x = 20720 → (1)

P + 6x = 24080 → (2)

(where P is the principal).

Solving the equations (1) and (2), we can get

∴ P = ₹14000 and x = ₹1680

Interest for 1 year on ₹14000 is ₹1680

So, $R = \frac{100 \times 1680}{14000 \times 1} = 12\%$ p.a. Choice (B)

6. Let the sum be ₹x, then it becomes ₹3x in 6 years

So, ₹2x is the interest on x for 6 years.

So $R = \frac{100 \times 2x}{x \times 6} = \frac{100}{3}\%$

If the sum becomes 8 times itself, then interest is 7x.

The required time period $= \frac{100 \times 7x}{x \times 100/3}$

$= \frac{100 \times 7x \times 3}{x \times 100} = 21$ years Ans : (21)

7. Let the principal be ₹x.

As amount = 4x, interest is 4x – x = 3x

$T = \frac{100 \times 3x}{x \times 25} = 12$ years Choice (A)

8. Let the sum be ₹x

$\frac{x \times 18 \times 2}{100} - \frac{x \times 12 \times 2}{100} = 840$

$\Rightarrow \frac{36x}{100} - \frac{24x}{100} = 840$

$\Rightarrow \frac{12x}{100} = 840 \Rightarrow x = 840 \times \frac{100}{12} = 7000$ Ans : (7000)

9. Let sum be ₹x

$8178 = \frac{x \times 1 \times 7}{100} + \frac{x \times 1 \times 10}{100} + \frac{x \times 1 \times 12}{100}$

$8178 = \frac{7x + 10x + 12x}{100}$

$8178 = \frac{29x}{100}$

$x = \frac{8178 \times 100}{29} = \text{Rs.}28200$ Ans : (28200)

10. Let the principal be ₹P.

Amount $= \frac{7P}{5}$

∴ Interest $= \frac{7P}{5} - P = \frac{2P}{5}$

$\therefore r = \frac{100 \times 2P/5}{P \times 3}$

$\Rightarrow r = \frac{100 \times 2}{5 \times 3} = 13^1/_3\%$ p.a. Choice (C)

11. $A = 18000\left(1 + \frac{15}{100}\right)^2$

$= 18000\left(1 + \frac{3}{20}\right)^2$

$= \frac{18000 \times 23 \times 23}{20 \times 20}$ = ₹23805

∴ Compound interest = 23805 – 18000 = ₹5805

Ans : (5805)

12. Let the principal be ₹x.

Given, $x\left(1 + \frac{20}{100}\right)^2 - x = 5544$

$\frac{36x}{25} - x = 5544$, $\frac{11x}{25} = 5544$

⇒ x = ₹12600. Ans : (12600)

13. ₹1440 – 1200 = ₹240 is the interest on ₹1200 for 1 year.

Rate of interest $= \frac{100 \times 240}{1200 \times 1} = 20\%$ p.a

If the sum is ₹100, then interest for second year is ₹24.

Given ₹24 ------- 1200,

100 ------- ?

? $= \frac{100 \times 1200}{24}$ = ₹5000 Choice (B)

14. Let A be the amount received at the end of three years.

$A = 4800\left(1 + \frac{10}{100}\right)\left(1 + \frac{20}{100}\right)\left(1 + \frac{25}{100}\right)$

$A = \frac{4800 \times 11 \times 6 \times 5}{10 \times 5 \times 4}$

∴ A = ₹7920

So, interest = 7920 – 4800 = ₹3120 Ans : (3120)

15. Let the principal be ₹x. Then, amount = ₹2x

So $2x = x\left(1 + \frac{R}{100}\right)^3$

$\Rightarrow \left(1 + \frac{R}{100}\right)^3 = 2$

If the sum becomes 8 times, therefore amount = ₹8x

Let it become 8 times itself in n years.

$\therefore 8x = x\left(1 + \frac{R}{100}\right)^n$

$\Rightarrow \left(1 + \frac{R}{100}\right)^n = 8 \Rightarrow \left(1 + \frac{R}{100}\right)^n = 2^3$

$\Rightarrow \left(1 + \frac{R}{100}\right)^n = \left(\left(1 + \frac{R}{100}\right)^3\right)^3$

$\Rightarrow \left(1 + \frac{R}{100}\right)^n = \left(1 + \frac{R}{100}\right)^9$

∴ n = 9 years Ans : (9)

16. (a) Here P = 2400

R = 10%, and n = 2

$A = P\left(1+\frac{R}{100}\right)^n = 2400\left(1+\frac{10}{100}\right)^2 = ₹2904$

Ans : (2904)

(b) Here R = 10% and n = 3

So A = $2000\left(1+\frac{10}{100}\right)^3$

$= \frac{2000 \times 11 \times 11 \times 11}{10 \times 10 \times 10} = ₹2662$

$\therefore$ Compound interest = 2662 – 2000 = ₹662

Ans : (662)

17. When compounded annually, interest

$= 12000\left(1+\frac{20}{100}\right)^1 - 12000 = ₹2400$

When compounded semi-annually, interest

$= 12000\left(1+\frac{10}{100}\right)^2 - 12000 = ₹2520$

Required difference = 2520 – 2400 = ₹120 Ans : (120)

18. P = d $\times$ $100^2/R^2$ $\Rightarrow d = \frac{PR^2}{100^2}$

$d = \frac{10000 \times 15 \times 15}{100 \times 100}$

d = ₹225 Ans : (225)

19. Using D = $P\left(\frac{R}{100}\right)^2$, we have 200 = $20000\left(\frac{R}{100}\right)^2$

$\Rightarrow R^2 = 100 \Rightarrow R = 10\%$ p.a. Ans : (10)

20. As simple interest for two years is ₹800, simple interest for each year is ₹400.
So compound interest for first year is ₹400 and for second year is 960 – 400 = ₹560
So, ₹560 – 400 = ₹160 is the interest on ₹400 for 1st year at the same rate of interest

$\therefore$ Rate of interest $= \frac{100 \times 160}{400 \times 1} = 40\%$ p.a.

The sum = $\frac{800 \times 100}{40 \times 2}$ = ₹1000. Choice (C)

21. Let the principle be ₹P

$24200 = P\left(1+\frac{r}{100}\right)^2 \rightarrow (1)$

$29282 = P\left(1+\frac{r}{100}\right)^4 \rightarrow (2)$

Dividing equation (2) by equation (1), we get

$\left(1+\frac{r}{100}\right)^2 = \frac{121}{100}$

$\left(1+\frac{r}{100}\right)^2 = \left(\frac{11}{10}\right)^2 \Rightarrow 1+\frac{r}{100} = \frac{11}{10}$

$\Rightarrow \frac{r}{100} = \frac{1}{10} \Rightarrow r = 10\%$ p.a. Choice (D)

22. $A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

$13695 = P\left(1+\frac{20}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{25}{100}\right)$

$\Rightarrow 13695 = P \times \frac{6}{5} \times \frac{11}{10} \times \frac{5}{4}$

$P = 13695 \times \frac{5}{6} \times \frac{10}{11} \times \frac{4}{5} = \text{Rs.}8300$ Ans : (8300)

23. Let the principal be ₹x

Amount $= x\left(1+\frac{20}{100}\right)^2$

Amount = ₹1.44x
So interest = 1.44x – x = 0.44x

Rate of interest $= \frac{0.44x \times 100\%}{x}$ = 44% Choice (C)

24. The simple interest for first and second years each will be ₹900
The compound interest for first year is ₹900 and for second year is ₹1980 – 900 = ₹1080

₹(1080 – 900) = ₹180 is the interest on ₹900 for 1 year

So $r = \frac{100 \times 180}{900 \times 1} = 20\%$ p.a.

As $P = \frac{d \times 100^2}{r^2}$,

$\Rightarrow P = \frac{180 \times 100 \times 100}{20 \times 20} = ₹4500$ Choice (B)

25. Principal = ₹8900
R = 9%p.a.
T = 1 year

$I = \frac{8900 \times 1 \times 9}{100} = \text{Rs.}801$

Principal = ₹8900 and R = 10% p.a. compounded semi-annually.

So $A = 8900\left(1+\frac{5}{100}\right)^2$ = ₹9812.25

Compound interest = 9812.25 – 8900 = ₹912.25
So profit made by A = ₹ (912.25 – 801)
= ₹111.25 Ans : (111.25)

Exercise – 5
(Time and Distance)

Solutions for questions 1 to 50:

1. (a) 54kmph = $54 \times \frac{5}{18} m/s = 15\ m/s$ Ans : (15)

(b) 108kmph = $108 \times \frac{5}{18} m/s = 30\ m/s$ Ans : (30)

(c) 21.6 kmph = $21.6 \times \frac{5}{18} m/s = 6\ m/s$ Ans : (6)

2. (a) $20 m/s = 20 \times \frac{18}{5}$ kmph = 72 kmph. Ans : (72)

(b) $45 m/s = 45 \times \frac{18}{5}$ kmph = 162 kmph Ans : (162)

(c) $12 \cdot 5 m/s = 12 \cdot 5 \times \frac{18}{5}$ kmph = 45 kmph Ans : (45)

(d) $\frac{13}{36} m/s = \frac{13}{36} \times \frac{18}{5} = 1.3$ kmph . Ans : (1.3)

3. 1 hour 40 min = $\frac{5}{3}$ hours

Distance = speed $\times$ time = $\left(\frac{11}{3} \times \frac{18}{5}\right) \times \frac{5}{3} = 22$ km

Choice (D)

4. Let Varma cover the remaining distance at x kmph

$\frac{10}{8} + \frac{(22 \cdot 5 - 10)}{x} = 3\frac{1}{3}$

$\frac{5}{4} + \frac{12 \cdot 5}{x} = \frac{10}{3} \Rightarrow \frac{12 \cdot 5}{x} = \frac{10}{3} - \frac{5}{4}$

$\frac{12 \cdot 5}{x} = \frac{40 - 15}{12} \Rightarrow x = 6$ Choice (D)

5. Let the distance travelled be x km.

Total time = $\frac{x/2}{30}+\frac{x/2}{25}=11$

$\Rightarrow \frac{x}{60}+\frac{x}{50}=11 \Rightarrow \frac{5x+6x}{300}=11$

$\Rightarrow$x = 300. Ans : (300)

6. Let the distance and original speed be d km and k kmph respectively.

$\frac{d}{0\cdot 8k}-\frac{d}{k}=\frac{20}{60}$

$\frac{5d}{4k}-\frac{d}{k}=\frac{1}{3}$

$\frac{5d-4d}{4k}=\frac{1}{3}\Rightarrow d=\frac{4}{3}k$

Time taken to cover the distance at original speed $=\frac{d}{k}=\frac{4}{3}$hours = 1 hour 20 minutes Choice (C)

7. Let the distance between school and house be x km.

$\frac{x}{6}-\frac{x}{8}=\frac{30}{60}$

$\Rightarrow \frac{4x-3x}{24}=\frac{1}{2}\Rightarrow x=12.$ Ans : (12)

8. Let the distance between house and office be x km

$\frac{x}{30}-\frac{x}{40}=\frac{20}{40}\Rightarrow \frac{x}{120}=\frac{1}{3}\Rightarrow x=40.$

Travelling at 40kmph, he reaches office in $\frac{40}{40}=1$ hour

Travelling at 40kmph, he is early by 15 min.
So, required speed to reach office on time

$=\frac{40}{\frac{5}{4}}=\frac{40\times 4}{5}$ = 32 kmph. Choice (B)

9. Total distance travelled = 2x km.

Total time taken = $\frac{x}{45}+\frac{x}{36}=\frac{x}{20}$ hours

Average speed = $\frac{2x}{x/20}=40$ kmph . Ans : (40)

10. Let the time taken reach the destination be 3x hours.
Total distance = 40 $\times$ 3x = 120x

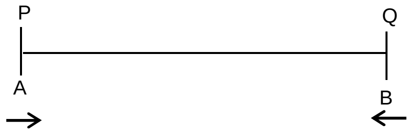
He covered $\frac{2}{3}\times 120x=80x$ km in $\frac{1}{3}\times 3x=x$ hours

So, the remaining 40x km, he has to cover in 2x hours

Required speed = $\frac{40x}{2x}=20$ kmph . Ans : (20)

11. Let the times taken to cover from A to B in car and cycle by x hours and y hours respectively.
x + y = 7 --- (1)
2x = 4 --- (2)
Solving the both equations we get y = 5
So, time taken to cover both ways by cycle = 2y hours = 10 hours. Ans : (10)

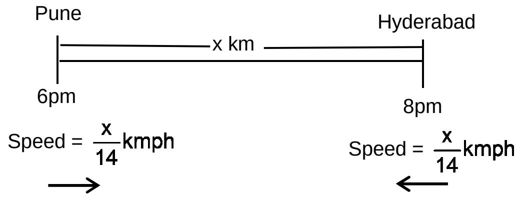
12.



Ratio of speeds of A and B is 3:2. So, by the time B covers 20 km, A covers 30km.
They totally cover (20 + 30) km to meet
So, distance between P and Q is 50 km. Ans : (50)

13. Let the distance between Pune and Hyderabad be x km. First bus takes 14 hours to cover x km and second bus also takes 14 hours to cover x km.

So speed of each bus is $\frac{x}{14}$kmph



Let both buses meet 't' hours after 8 pm

Then $\frac{x}{14}(t+2)+\frac{x}{14}(t)=x$

$\frac{t+2}{14}+\frac{t}{14}=1\Rightarrow \frac{2t+2}{14}=1$

$\Rightarrow$ 2t = 12 $\Rightarrow$ t = 6, 8.00 pm + 6 hrs = 2.00 am
So both buses meet at 2 a.m. Choice (D)

14. Distance = Relative speed $\times$ time
Number of kilometers apart they will be if they are moving in opposite direction = (25 + 20) 4 = 180
same direction = (25 – 20) 4 = 20. Choice (B)

15. To cross a person (standing), the train has to travel a distance equal to the length of the train.

Time $=\frac{\text{distance}}{\text{speed}}=\frac{600}{54\times\frac{5}{18}}=40$ sec . Choice (D)

16. Let length train be x m.
When a train crosses an electric pole, then the distance covered is its own length. So,

x = $12\times 36\times\frac{5}{18}$m $=120$

Time taken to cross the platform = $\frac{120+350}{36\times\frac{5}{18}}=47$ sec .

Ans : (47)

17. Distance to be covered to cross the tunnel
= (800 + 400) m = 1200 m

speed = $\frac{\text{distance}}{\text{time}}=\frac{1200}{120}=10$ m/s

$10\text{m/s}=10\times\frac{18}{5}$kmph =36 kmph Ans : (36)

18. Time taken to cross each other

$=\frac{\text{sum of the lengths of the two trains}}{\text{relative speed}}$

$=\frac{250+150}{(30+42)\frac{5}{18}}=20$ sec . Ans : (20)

19. Time taken to cross a moving person = $\frac{\text{length of train}}{\text{relative speed}}$

Time taken = $\frac{375}{(50+4)\frac{5}{18}}=\frac{375}{15}$ sec = 25 sec. Ans : (25)

20. When the second train leaves Mumbai the first train covers 40 $\times$ 1 = 40 km
So, the distance between first train and second train is 40 km at 10.00 am
Time taken by the trains to meet

$=\frac{\text{Distance}}{\text{relative speed}}=\frac{40}{50-40}=4$ hours

So the two trains meet at 2p.m. The two trains meet 4 $\times$ 50 = 200 km away from Mumbai. Choice (C)

21. Let the length of each train be x m

$$\frac{x+x}{(60+30)\frac{5}{18}} = 30 \Rightarrow 2x = 750$$

Time taken to cross each other when traveling in the same direction = $\frac{2x}{(60-30)\frac{5}{18}} = \frac{750 \times 18}{30 \times 5} = 90$ sec .

Ans : (90)

22. $12 \text{ m/s} = 12 \times \frac{18}{5}$ kmph

3 hours 45 minutes = $3\frac{3}{4}$ hours $= \frac{15}{4}$ hours

distance = speed × time = $12 \times \frac{18}{5} \times \frac{15}{4}$ km $= 162$ km

Choice (B)

23. Speed = $8 \times \frac{18}{5}$ kmph

Time taken = $\frac{180}{8 \times \frac{18}{5}}$

$= 6\frac{1}{4}$ hours. = 6.25 hrs. Ans : (6.25)

24. Total time taken = $\left(\frac{20}{10} + \frac{20}{20} + \frac{20}{30}\right)$hours

$= \left(2 + 1 + \frac{2}{3}\right)$hours $= 3\frac{2}{3}$ hours . Choice (A)

25. Let the distance be x km.

Given that, $\frac{\frac{x}{2}}{30} + \frac{\frac{x}{2}}{75} = \frac{7}{5}$

$\Rightarrow \frac{x}{60} + \frac{x}{150} = \frac{7}{5} \Rightarrow \frac{x}{12} + \frac{x}{30} = 7$

$\frac{7x}{60} = 7 \Rightarrow x = 60.$ Ans : (60)

26. Let the distance the person has to travel to reach his office be x km.

$\frac{x}{40} - \frac{x}{50} = \frac{20}{60}$

$\Rightarrow \frac{5x - 4x}{200} = \frac{1}{3} \Rightarrow 3x = 200 \Rightarrow x = 66\frac{2}{3}$ Choice (B)

27. Let the distance be x km

$\frac{x}{50} - \frac{x}{60} = \frac{5}{60}$

$\frac{6x - 5x}{300} = \frac{1}{12} \Rightarrow x = \frac{300}{12} = 25.$ Ans : (25)

28. Let AB =x km.
Total distance travelled = 2AB = 2x km.

Total travel time = $\left(\frac{x}{40} + \frac{x}{60}\right)$ hours = $\frac{x}{24}$ hours.

Average speed = $\frac{2x}{\frac{x}{24}}$ = 48 kmph. Ans : (48)

29. Let the speeds of the cars be 5x, 6x and 7x.
Let the distance be d.

Ratio required = $\frac{d}{5x} : \frac{d}{6x} : \frac{d}{7x}$ = 42 : 35 : 30.

Choice (D)

30. Let the speed be x kmph and the time taken be y hours.
$(x + 3)(y - 2) = xy \Rightarrow -2x + 3y - 6 = 0$
$(x - 4)(y + 5) = xy \Rightarrow 5x - 4y - 20 = 0$

By solving the above equations, we get,
y = 10 and x = 12
So distance = xy = 12 × 10 = i.e 120 km. Ans : (120)

31. Time taken to meet = $\frac{\text{Distance}}{\text{relative speed}}$

$= \frac{180}{10 + 20} = 6$ hours . Choice (B)

32. Let speed of train be x kmph.
Decrease in speed equals 5m/s i.e., 18 kmph.

$\frac{160}{x - 18} - \frac{160}{x} = 2$

$\Rightarrow x(x - 18) = 1440 = 0 \Rightarrow x^2 - 18x - 1440 = 0$
$\Rightarrow x^2 - 48x + 30x - 1440 = 0 \Rightarrow (x - 48)(x + 30) = 0$
$\therefore x = 48$ (as x > 0) Ans : (48)

33. Let length of train be x m. The distance covered by the train to cross the platform = length of the train + length of the platform
So, to cover (x + 200) m the train takes 18 sec and to cover (x + 275) m it takes 27 sec.
So to cover 75 m it takes 27–18 = 9 sec

∴The speed of train = $\frac{75}{9}$ m/s

$= \frac{75}{9} \times \frac{18}{5} = 30$ kmph . Ans : (30)

34. For a train to cross a person standing, the distance to be covered is its own length
So, distance = 280 m and

speed = $27 \times \frac{5}{18} = \frac{15}{2}$ m/s

Time taken to cross the person =

$\frac{\text{Distance}}{\text{speed}} = \frac{280}{15} \times 2 = \frac{112}{3} = 37\frac{1}{3}$ sec . Choice (D)

35. Speed = $\frac{\text{Distance}}{\text{time}} = \frac{225 + 175}{10}$ m/s $= 40$ m/s .

Ans : (40)

36. Let the length of the second train be x m

$\frac{245 + x}{(60 - 38)\frac{5}{18}} = 90 \Rightarrow \frac{(245 + x)18}{(22 \times 5)} = 90$

$\Rightarrow 245 + x = 5 \times 22 \times 5$
$\Rightarrow x = 305$m. Ans : (305)

37. Time taken to cross the moving person

$= \frac{\text{length of train}}{\text{relative speed}} = \frac{600}{(45 + 9)\frac{5}{18}} = 40$ sec . Choice (B)

38. By the time A covers 500 m, B and C cover 480 m and 470 m respectively. In the time B covers 2880 m, C covers

$\frac{2880}{480} \times 470 = 2820$ m

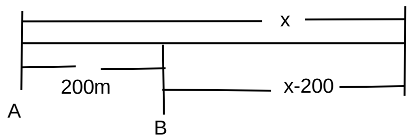
So B beats C by 2880 – 2820 = 60 m. Choice (B)

39. Speed of B = $\frac{160}{20}$ = 8 m per sec.

Time taken by B to cover 800m = $\frac{800}{8} = 100$ sec

∴ Time taken by A to cover 800m = 100 – 20 = 80 sec
∴ Speed of A = 800/8 = 10 m/s Ans : (10)

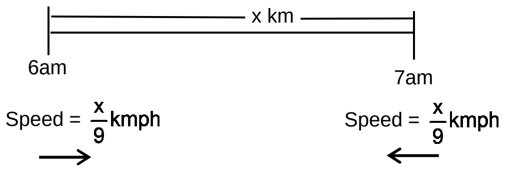
40. A's speed = $\frac{5}{3}$ times B's speed. Let the length of race be x m.
Let B's speed be 3s m/s, A's speed be 5s m/s



So $\frac{x}{5} = \frac{x-200}{3}$

$\Rightarrow 3x = 5x - 1000$
$\Rightarrow 2x = 1000$
$\Rightarrow x = 500.$ Choice (D)

41. Let distance between P and Q be x km.
The first train takes 9 hours and the second train also takes 9 hours to cover x km. So, the speed of each train is $\frac{x}{9}$ kmph .



Let both trains meet t hours after 7am.

$(t+1)\frac{x}{9} + t \cdot \frac{x}{9} = x$

$\frac{t+1}{9} + \frac{t}{9} = 1 \Rightarrow \frac{2t+1}{9} = 1$

$\Rightarrow t = 4.$
They meet 4 hours after 7am i.e., at 11.00am.
Choice (C)

42. Time taken to meet for the first time any where on the track
$= \frac{\text{length of track}}{\text{relative speed}}$

$= \frac{300}{(15+25)\frac{5}{18}} = \frac{300 \times 18}{40 \times 5} = 27$ seconds Choice (D)

43. Time taken to meet for the first time at the starting point $= \text{LCM}\left(\frac{\text{length of track}}{\text{speed of A}}, \frac{\text{length of track}}{\text{speed of B}}\right)$

$= \text{LCM}\left(\frac{600}{36 \times \frac{5}{18}}, \frac{600}{54 \times \frac{5}{18}}\right)$

= LCM (60, 40) = 120 sec. Choice (A)

44. Let the speed of the man in still water and speed of stream be x kmph and y kmph respectively.
Given x + y = 18 -- (1) and
x – y = 10 -- (2)
Solving, we get x = 14 and y = 4. Choice (D)

45. Let the distance between A and B be x km

Total time $= \frac{x}{9+1} + \frac{x}{9-1} = 4 \cdot 5$

$\Rightarrow \frac{x}{10} + \frac{x}{8} = \frac{9}{2} \Rightarrow \frac{4x+5x}{40} = \frac{9}{2} \Rightarrow x = 20..$ Ans: (20)

46. Let distance be d km
Let the speed of the man in still water and speed of stream be x kmph and y kmph respectively.

$\frac{d}{x+y} : \frac{d}{x-y} = 2:3$

$\frac{2}{x-y} = \frac{3}{x+y} \Rightarrow 2x + 2y = 3x - 3y$

$\Rightarrow x = 5y$ Also $xy = 20$
$\Rightarrow 5y^2 = 20 \Rightarrow y = 2 \Rightarrow x = 10.$ Ans: (10)

47. By the time A covers 1000 m, B covers (1000 – 50) = 950 m.
By the time B covers 1000 m, C covers 1000 – 100 = 900 m
So, the ratio of speeds of A and C =
$\frac{1000}{950} \times \frac{1000}{900} = \frac{1000}{855}$ So, by the time A covers 1000 m, C covers 855 m.
So in 1000 m race A beats C by 1000 – 855 = 145 m.
Choice (A)

48. When A covers 1000 m, then B and C covers 1000 – 100 = 900 m and 100 – 150 = 850 m respectively.

So, when B covers 2700 m, then C covers $\frac{2700}{900} \times 850$ = 2550m
So, in a 2700 m race B beats C by 2700 – 2550
= 150 m. Choice (B)

49. Time taken to meet for the first time any where on the track
$= \frac{\text{length of track}}{\text{relative speed}} = \frac{600}{15+10} = 24$ sec . Choice (B)

50. Time taken to meet at starting point

$= \text{LCM of} = \left(\frac{\text{length of track}}{\text{speed of first one}}, \frac{\text{length of track}}{\text{speed of second one}}\right)$

Time taken to meet = LCM $\left(\frac{720}{12} \frac{720}{18}\right)$

= LCM (60, 40) = 120 sec. Choice (D)

Exercise – 6
(Time and Work)

Solutions for questions 1 to 50:

1. One day work of A and B $= \frac{1}{9} + \frac{1}{18} = \frac{2+1}{18} = \frac{1}{6}$
So the time taken to complete the work is 6 days.
Ans: (6)

2. One day work of A, B and C
$= \frac{1}{90} + \frac{1}{30} + \frac{1}{45} = \frac{1+3+2}{90} = \frac{1}{15}$
$\therefore$ A, B and C together can do the work in 15 days.
Choice (A)

3. Work done by B in one day $= \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$
So, B alone can do the work in 24 days. Choice (D)

4. (A + B + C)'s two days work =
$\frac{1}{18} + \frac{1}{30} + \frac{2}{45} = \frac{5+3+4}{90} = \frac{2}{15}$

(A + B + C)'s one day work $= \frac{2}{15} \times \frac{1}{2} = \frac{1}{15}$

A's one day work $= \frac{1}{15} - \frac{1}{30} = \frac{1}{30}$

B's one day work $= \frac{1}{15} - \frac{2}{45} = \frac{1}{45}$

C's one day work $= \frac{1}{15} - \frac{1}{18} = \frac{1}{90}$

So, A, B and C can individually do the work in 30, 45 and 90 days. Choice (B)

5. A's one day work $= \frac{1}{15} - \frac{2}{75} = \frac{5-2}{75} = \frac{1}{25}$

A can do the work in 25 days. Choice (A)

6. The ratio of their working rates

$= \frac{1}{7} : \frac{1}{14} : \frac{1}{21} = 6:3:2$. Since, they work together,

the share of C = $\frac{2}{11} \times 220 = ₹40$. Ans : (40)

7. Let Ram complete the remaining work in x days.

Then, $\frac{x+7}{16} + \frac{7}{28} = 1$

$\Rightarrow \frac{x+7}{16} = \frac{3}{4}$

$\Rightarrow x + 7 = 12$

$\Rightarrow x = 5$. Choice (D)

8. B's one day work = $\frac{1}{20} - \frac{1}{40} - \frac{1}{60} = \frac{6-3-2}{120} = \frac{1}{120}$

So, B can do the work in 120 days. Ans: (120)

9. Let the work be completed in x days

$\frac{x-1}{15} + \frac{x-1}{30} + \frac{x}{45} = 1$

$\frac{6(x-1)+3(x-1)+2x}{90} = 1$

$6x - 6 + 3x - 3 + 2x = 90$

$x = 99 \Rightarrow x = 9$. Choice (D)

10. A can do the work in $\frac{30}{2}$ i.e.15 days

A and B's one day work = $\frac{1}{15} + \frac{1}{30} = \frac{2+1}{30} = \frac{1}{10}$

So A and B together can do the work in 10 days.

Ans : (10)

11. Let A can do the work in x days, then

B can do the work in 3x days.

$\frac{1}{x} + \frac{1}{3x} = \frac{1}{27}$

$\frac{3+1}{3x} = \frac{1}{27}$

$\Rightarrow x = 36$. Ans : (36)

12. Let Chandra can do the work in 2x days.

Bharat can do it in 6x days and Avinash can do it in 3x days.

$\frac{1}{2x} + \frac{1}{6x} + \frac{1}{3x} = \frac{1}{30}$

$\frac{3+1+2}{6x} = \frac{1}{30}$

$\Rightarrow x = 30$

So, the time taken to complete the work by Avinash, Bharat, Chandra is 90, 180, 60 days respectively.

Choice (B)

13. Let the total work be completed in x days.

$\frac{x}{12} + \frac{x-3}{24} + \frac{x-2}{48} = 1$

$\Rightarrow \frac{4x+2(x-3)+x-2}{48} = 1$

$\Rightarrow 4x + 2x - 6 + x - 2 = 48$

$\Rightarrow 7x = 56$

$\Rightarrow x = 8$. Ans: (8)

14. First 2 days work of Vinay and Varma = $\frac{1}{30} + \frac{1}{60}$

$= \frac{2+1}{60} = \frac{1}{20}$

2 days work = $\frac{1}{20}$

(2×20) days work = $\frac{1}{20} \times 20 = 1$

So, the work will be completed in 40 days. Ans : (40)

15. Part of work completed in first 2 days = $\frac{1}{8} + \frac{1}{16}$

$= \frac{2+1}{16} = \frac{3}{16}$

2 days work = $\frac{3}{16}$

(2×5) days work = $\frac{3}{16} \times 5 = \frac{15}{16}$

In 10 days $\frac{15}{16}$th of total work is completed, So the remaining work is $\frac{1}{16}$th of total work. On the 11th day, A works and he can do $\frac{1}{16}$th of total work in $\frac{1}{16} / \frac{1}{8} = \frac{1}{2}$ day. So total time taken is $10\frac{1}{2}$ days. Ans : (10.5)

16. We have $M_1D_1 = M_2D_2$

So $24 \times 35 = M_2 \times 21 \Rightarrow M_2 = 40$. Ans : (40)

17. We have $\frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$ (Variation rule)

$\frac{60 \times 30 \times 8}{200} = \frac{45 \times d_2 \times 6}{300}$ (Assume D_2 as d_2)

$d_2 = \frac{60 \times 30 \times 8 \times 300}{200 \times 45 \times 6}$

$d_2 = 80$. Choice (D)

18. We have $M_1D_1 = M_2D_2$

$120x = (x - 20)\,180$

$\Rightarrow 2x = (x - 20)\,3 \Rightarrow 2x = 3x - 60$

$\Rightarrow x = 60$. Ans : (60)

19. Let number of hours working per day initially be x

We have $M_1D_1H_1 = M_2D_2H_2$

$30 \times 24 \times x = 20 \times d_2 \times \frac{4x}{3}$

$\Rightarrow d_2 = \frac{30 \times 24 \times 3}{20 \times 4} = 27$ days. Choice (B)

20. Given that $12m = 20w$

$\Rightarrow 3m = 5w$

9 men + 12 women = 15 women + 12 women = 27 women

20 women can do the work in 54 days. So, 27 women can do it in $\frac{20 \times 54}{27} = 40$ days. Ans : (40)

21. 8 women can do the work in 15 days. So, to complete the work in 10 days number of women required = $\frac{8 \times 15}{10} = 12$

12 women = 4 women + 6 men

So, 6 men should work with 4 women. Choice (A)

22. $(5m + 9w)10 = (6m + 12w)\,8$

$\Rightarrow 50m + 90w = 48w + 96w$

$\Rightarrow 2m = 6w$

$\Rightarrow 1m = 3w$

$5m + 9w = 5m + 3m = 8m$

8 men can do the work in 10 days.

3m + 3w = 3m + 1w = 4m

So, 4 men can do the work in $\frac{10\times8}{4}=20$ days . Ans : (20)

23. (8m + 6b) 11 = (9m + 12b) 9
$\Rightarrow$ 88m + 66b = 81m + 108b
7m = 42b
$\Rightarrow$ 1m = 6b
8m + 6b = 8m + 1m = 9m
So 9 men can do the work in 11 days.
6m + 30b = 6m + 5m = 11m

$\therefore$11 men can do the work in $\frac{11\times9}{11}=9$ days .

Ans : (9)

24. Total provisions at the hostel = $800 \times 42 \times 2$ kg
For 600 men consuming at 4 kg per day per man,
the provision will last for $\frac{800 \times 42 \times 2}{600 \times 4} = 28$ days

Choice (A)

25. Part of tank filled by both pipes in 1 minute $= \frac{1}{30}+\frac{1}{20}=\frac{2+3}{60}=\frac{1}{12}$

$\therefore$Both the pipes can fill the tank in 12 minutes.

Choice (B)

26. Part of tank filled by all three pipes in one minute $= \frac{1}{18}+\frac{1}{15}-\frac{1}{45}=\frac{5+6-2}{90}=\frac{9}{90}=\frac{1}{10}$

So, the tank becomes full in 10 minutes. Ans: (10)

27. Let leak can empty the full tank in x hours $\frac{1}{6}-\frac{1}{x}=\frac{1}{9} \Rightarrow \frac{1}{x}=\frac{1}{6}-\frac{1}{9}=\frac{3-2}{18}=\frac{1}{18}$

$\therefore$ x = 18. Choice (C)

28. Let the pipe B be closed after x minutes.

$\frac{30}{16}-\frac{x}{24}=1 \Rightarrow \frac{x}{24}=\frac{30}{16}-1=\frac{14}{16}$

x = $\frac{14}{16}\times 24 = 21$. . Ans : (21)

29. Let the work be completed in x days after B joins.

$\therefore \frac{9}{30}+\frac{x}{30}+\frac{x}{40}=1$

$\Rightarrow \frac{7x}{120}=\frac{7}{10} \Rightarrow x = 12$

$\therefore$Share of A for his contribution in the

work = $\frac{12+9}{30} \times 210$ = ₹147 Choice (A)

30. Sravan's is one day work = $\frac{1}{25}-\frac{1}{75}=\frac{2}{75}$

Sravan worked for 8 days. So, his 8 days work $= 8\times\frac{2}{75}=\frac{16}{75}$

Sravan completed $\frac{16}{75}$th of total work.

So his share is $\frac{16}{75}\times225=$ Rs.48 . Choice (C)

31. A and B 's one day work = $\frac{2}{15}+\frac{1}{30}=\frac{4+1}{30}=\frac{1}{6}$

So, A and B together can do the work in 6 days.

Choice (C)

32. B and C can do the work in $30\times\frac{4}{3}=40$ days

A's one day work = $\frac{1}{22}-\frac{1}{40}=\frac{40-22}{22\times40}=\frac{18}{22\times40} = \frac{9}{440}$

A can do the work in $\frac{440}{9}=48\frac{8}{9}$ days . Choice (C)

33. A, B and C in one day can do $\frac{1}{24}+\frac{1}{18}=\frac{3+4}{72}=\frac{7}{72}$ of the work

A, B and C together can do the work in $\frac{72}{7}$ days

So they can do $3\frac{1}{2}$ times the work in $\frac{72}{7}\times\frac{7}{2}=36$ days.

Ans : (36)

34. B's one day work = $\frac{1}{30}+\frac{1}{50}-\frac{1}{20}=\frac{10+6-15}{300}=\frac{1}{300}$

B can do the work in 300 days

A's one day work = $\frac{1}{30}-\frac{1}{300}=\frac{9}{300}=\frac{3}{100}$

A can do the work in $33\frac{1}{3}$ days

C's one day work = $\frac{1}{50}-\frac{1}{300}=\frac{5}{300}=\frac{1}{60}$

C can do the work in 60 days. Choice (C)

35. Work done by (P + Q) + (P + R) + (R + Q) in one day $= \frac{1}{32}+\frac{1}{48}+\frac{1}{24}=\frac{3+2+4}{96}=\frac{9}{96}$

Work done by (P + Q + R) in one day = $\frac{9}{96\times2}=\frac{9}{192}$

P's one day work = $\frac{9}{192}-\frac{1}{24}=\frac{9-8}{192}=\frac{1}{192}$

P can do the work individually in 192 days. Choice (B)

36. Let A worked for x days.

$\frac{x}{21}+\frac{4}{28}=1$

$\Rightarrow \frac{x}{21}=\frac{6}{7} \Rightarrow x = 18$

A worked for 18 days. So A can complete the remaining work in 18 – 4 = 14 days. Choice (D)

37. Let A can do the work in x days.
B can do the same work in x +7 days.

Given, $\frac{1}{x}+\frac{1}{x+7}=\frac{5}{42}$

$\frac{x+7+x}{x(x+7)}=\frac{5}{42}$

$\Rightarrow 5x^2 + 35x = 84x + 294$
$\Rightarrow 5x^2 - 49x - 294 = 0$
$\Rightarrow 5x^2 - 70x + 21x - 294 = 0$
$\Rightarrow$ 5x (x – 14) + (as x > 0) 21 (x – 14) = 0
$\Rightarrow$(x –14) (5x +21) = 0
$\Rightarrow$ x = 14. Ans : (14)

Note: This problem can be answered easily by back substitution of the choices.

38. Time taken by B to complete the work $= 20\times\frac{3}{2}=30$days

Part of work completed in first 2 days $= \frac{1}{20}+\frac{1}{30}=\frac{3+2}{60}=\frac{1}{12}$

Work done in (2×12) days = $\frac{1}{12}\times12=1$

$\therefore$ Total work is completed in 24 days. Choice (D)

39. Let the work be completed in x days.

$$\frac{x-7}{30}+\frac{x-1}{24}+\frac{x}{15}=1$$
$$\frac{4x-28+5x-5+8x}{120}=1$$
17x = 153
$\Rightarrow$ x = 9. Choice (B)

40. Let the total work be completed in x days.
$$\frac{x}{25}+\frac{x-9}{60}+\frac{x-3}{40}=1$$
$$\frac{24x+10x-90+15x-45}{600}=1$$
$\Rightarrow$ 49x = 735 $\Rightarrow$ x = 15. Choice (B)

41. P's 9 days work = $\frac{9}{14}=\frac{9}{14}$
P alone completed 9/14 of work
So P gets 9/14 of 280 = 9/14 $\times$ 280 = 180
Remaining work is completed by P and Q together.
So, remaining balance = 280 – 180 = ₹100
P and Q divide this amount in the ratio of their one day work.
Ratio of one day work of P and Q is $\frac{1}{14}:\frac{1}{21}$=3 : 2
Share of P in ₹100 = $\frac{3}{5}\times 100$ =₹60
Total of P's share = 180 + 60 = ₹240. Ans : (240)

42. A's 6 days work + A's 10 days work
+B's 10 days work + B's 2 days work = 1
A's 16 days work + B's 12 days work = 1
(A + B)'s 12 days work + A's 4 days work = 1
$12\times\frac{5}{72}$ + A's 4 days work =1
A's 4 days work = $\frac{1}{6}$
So A's one day work = $\frac{1}{24}$
B's one day work = $\frac{5}{72}-\frac{1}{24}=\frac{2}{72}=\frac{1}{36}$
A and B can do the work in 24 and 36 days respectively.
Choice (D)

43. Let P and Q can do the work in x days.
(38 days work of P and Q) +
(6 days work of P,Q and R) = 1
$$\Rightarrow\frac{38}{x}+\frac{6}{25}=1$$
$$\frac{38}{x}=\frac{19}{25}\Rightarrow x = 50$$
R's one day work = $\frac{1}{25}-\frac{1}{50}=\frac{1}{50}$
So R can do the work in 50 days. Choice (A)

44. We have $\frac{M_1D_2}{W_1}=\frac{M_2D_2}{W_2}$ (Variation rule)
$$\frac{215\times 13.5}{677}=\frac{M_2\times\frac{13.5}{2}}{677\times 2}\Rightarrow M_2=215\times 4=860$$
Ans : (860)

45. One day work of 45 unskilled and 22 skilled workers
$$=\frac{1}{30}+\frac{1}{10}=\frac{3+1}{30}=\frac{2}{15}$$
So they take $\frac{15}{2}=7\cdot 5$ days to complete the work.
Choice (D)

46. 8 men can do a work in 36 days, so 16 men can do it in 18 days. In the same way, 5 boys can do the work in 72 days. One day work of 16 men and 5 boys
$$=\frac{1}{18}+\frac{1}{72}=\frac{5}{72}$$
So, they can do the work in $\frac{72}{5}=14\frac{2}{5}$ days.
Ans : (14.4)

47. (12m + 15b) 6 = (10m + 24b) 5
$\Rightarrow$ 72m + 90b = 50m + 120b
$\Rightarrow$ 22m = 30b
$\Rightarrow$ 11m = 15b
12m + 15b = 12m + 11m = 23m
8m + 30b = 8m + 22m = 30m
23 men can do the work in 6 days. So, 30 men can do it in
$\frac{23\times 6}{30}=4\frac{3}{5}$ days. Ans: (4.6)

48. 8m = 12w = 20b
$\Rightarrow$ 2m = 3w = 5b
6m + 12w + 10b = 15b + 20b + 10b = 45b
20 boys can do the work in 36 days. So, 45 boys can do it in
$\frac{20\times 36}{45}=16$ days. Choice (D)

49. In 5 days, Dharma completed $\frac{5}{17}$ of total work and so, he gets $\frac{5}{17}$ of the total earnings
i.e., $\frac{5}{17}\times 221$ =₹65 Ans : (65)

50. The work done in the first two days = $\frac{1}{12}+\frac{1}{36}=\frac{1}{9}$
So, 9 such two days are required to finish the work.
i.e., 18 days are required to finish the work.

Note: Here, the number of days in which they together can finish the work is a whole number. So, irrespective of the person starting the work the work will take exactly 18 days to complete. Choice (B)

Exercise – 7
(Averages – Mixtures – Alligations)

Solutions for questions 1 to 40:

1. The first ten two digit natural numbers are the natural numbers from 10 to 19.
Their sum is 145.
Their average is $\frac{145}{10}$ = 14.5 Ans : (14.5)

2. If each students height was 1 cm more, the average height of the students, increases by 1 cm and becomes 156 cm.
Choice (C)

3. Given M + P + C = 80 × 3 = 240 — (1)
M + P = 90 × 2 = 180 ——— (2)
P + C = 70 × 2 = 140 ——— (3)

where M, P and C are marks obtained by the student in Mathematics, Physics and Chemistry.
P = (2) + (3) – (1) = 180 + 140 – 240 = 80 Ans : (80)

4. Let the number of boys in the group originally be x.
Total weight of the boys = 30x
After the boy weighing 35 kg joins the group, total weight of boys = 30x + 35
So 30x + 35 = 31(x + 1)
x = 4 Choice (B)

5. Total score of the batsman in 20 matches = 800
Total score of the batsman in the next 10 matches = 130.
Total score of the batsman in the 30 matches = 930
Average score of the batsman = $\frac{930}{30}$ = 31 Ans : (31)

6. Let the number of students who wrote the exam be x.
Total marks of students = 80 x.
Total marks of (x – 5) students = 90(x – 5)
$80x - (5 \times 40) = 90(x - 5)$
250 = 10x
25 = x Ans : (25)

7. Let the number of students who wrote the exam be x.
Calculated total marks of the students = 20x
Correct value of the total marks of students =
20x – 70 – 85 + 60 + 77 = 20x – 18
$20x - 18 = 18x \Rightarrow x = 9$ Ans : (9)

8. Let the number of students in section A and section B be 9x and x respectively.
Average weight of students of both the sections put together
$= \frac{\text{Total weight of students of both sections}}{\text{Total number of students of both sections}}$
$= \frac{30(9x) + 45x}{9x + x}$ = 31.5 kg Ans : (31.5)

9. The average of all even natural numbers less than 50 equals
$\frac{\text{Sum of all even natural numbers less than 50}}{\text{Number of even natural numbers less than 50}}$
Sum of all even natural numbers = 2 + 4 + 6 + 8 + …. + 48
This represents the sum of the first 24 even natural numbers.
The sum of the first n even natural numbers is given by n(n + 1)
Their average = $\frac{n(n+1)}{n}$ = n + 1.
Hence average of first 24 even natural numbers = 25(Taking n = 24). An s: (25)

10. The average of 11 consecutive numbers is equal to the middle term which is 6^{th} term. Ans: (12)

11. Total score of the batsman in x innings = 36x
Total score of batsman in (x + 1) innings = 36x + 80
Average score of batsman in (x + 1) innings = 40
$\therefore$ 36x + 80 = 40(x + 1)
$\Rightarrow$ x = 10
Hence x + 1 = 11 Ans : (11)

12. Total age of the students of the class = 20 × 20 = 400 years
Total age of the members of the class including their class teacher = 21 × 21 = 441 years.
$\therefore$ Age of the class teacher = 441 – 400 = 41 years.
Choice (B)

13. Let the average age of the 11 men in the original group be A.
So 11A = 29 + 31 + (sum of ages of rest)
Let the ages of the two men replacing the two men aged 29 years and 31 years be x and y respectively.
11(A – 1) = x + y + (sum of ages of rest)
Subtracting the above equation from the previous equation,
11 = 60 – (x + y)
x + y = 49
Average age of Amar and Bhavan
$= \frac{x+y}{2}$ = 24.5 years Ans : (24.5)

14. Total score of Ajay in the seven examinations = 90 + 330 = 420.
Total score of Ajay in the first six examinations
$= (6 \times 60) = 360$.
Score of Ajay in the last exam = 420 – 360 = 60.
Ans : (60)

15. Total number of runs scored by Anil Kumble
$= (29 \times 36) + 30 = 1074$
Average runs scored by Anil Kumble in that season
$= \frac{1074}{30}$ = 35.8 runs/match Ans : (35.8)

16. Quantity of alcohol in vessel P = $\frac{50}{100}(2)$ = 1 litre
Quantity of alcohol in vessel Q = $\frac{75}{100}(4)$ = 3 litres
Quantity of alcohol in the mixture formed
(1 + 3) litres = 4 litres
As 6 litres of the mixture is formed, ratio of alcohol and water in the mixture formed = 4 : (6 – 4) = 2 : 1
Choice (D)

17. Let us say the ratio of the quantities of cheaper and dearer varieties = x : y
By the rule of alligation, $\frac{x}{y} = \frac{8.75 - 7.50}{7.50 - 6} = \frac{5}{6}$
Choice (D)

18. Let x litres of water be mixed with 1 litre of wine solution.
Quantity of wine in 1 litre of wine solution
$= \frac{80}{100}(1)$ = 0.8 litres
Quantity of wine in (1 + x) litres of mixture
$= \frac{60}{100}(1+x)$ litres
As quantity of wine before and after mixing remains the same, 0.8 = $\frac{60}{100}(1+x)$ = 0.6 + 0.6x
1/3 = x
Required ratio = x : 1 = 1 : 3

Alternative method

Water 80% milk
0% 80%
60%
20 60
Required ratio = 20 : 60 = 1 : 3 Choice (D)

19. Quantity of water in the mixture = $\frac{10}{100}(70)$ = 7 litres
If x litres of water is added, (7 + x) litres of water would be present in (70 + x) litres of mixture.
Given that $\frac{7+x}{70+x} \times 100 = 12\frac{1}{2}$
$\frac{7+x}{70+x} = \frac{\frac{25}{2}}{100} = \frac{1}{8}$
8(7 + x) = 70 + x
56 + 8x = 70 + x
7x = 14
x = 2

Alternative method:

Quantity of milk in the mixture = $\frac{90}{100}(70)$ = 63 litres
After adding water, milk would form $87\frac{1}{2}$% of the mixture.
Hence, if quantity of mixture after adding water is x litres,
$\frac{87\frac{1}{2}}{100}x = 63$
x = 72
Hence 72 – 70 = 2 litres of water must be added. Ans : (2)

20. Let the number of boys be x.

Number of girls is 120 – x
Total number of chocolates received by boys and girls = 2x + 3(120 – x)
= 360 – x = 300
x = 60. So, the number of boys is 60. Choice (B)

21. Let the amount lent at 7% be ₹x
Amount lent at 10% is ₹6000 – x
Total interest for one year on the two sums lent

$= \frac{7}{100}x + \frac{10}{100}(6000 - x) = 600 - \frac{3x}{100}$

$600 - \frac{3}{100}x = 450$

x = 5000
Amount lent at 10% = 1000
Required ratio = 5000 : 1000 = 5 : 1 Choice (A)

22. Let the cost prices of the colour televisions sold at 30% profit and 40% profit be ₹x and ₹ (35,000 – x) respectively.
Total selling price of televisions =

$x + \frac{30}{100}x + (35{,}000 - x) + \frac{40}{100}(35{,}000 - x)$

$= \frac{130}{100}x + \frac{140}{100}(35{,}000 - x)$

$\Rightarrow \frac{130x}{100} + \frac{140}{100}(35000 - x) = 35{,}000 + \frac{32}{100}(35{,}000)$

x = 28,000
35,000 – x = 7000
Difference in the cost prices of televisions = ₹21,000
Choice (A)

23. Let the quantities of A and B mixed be 3x kg and 7x kg
Cost of 3x kg of A = 9(3x) = ₹27x
Cost of 7x kg of B = 15(7x) = ₹105x
Cost of 10x kg of the mixture = 27x + 105x = ₹132x

Cost of the 5 kg of mixture = $\frac{132x}{10x}(5)$ = ₹66

Profit made in selling 5 kg of the mixture = $\frac{25}{100}$ (cost of 5 kg of the mixture) = $\frac{25}{100} \times 66$ = ₹16.50 Ans : (16.50)

24. Let the average number of chocolates eaten by the boys be x.
Number of chocolates eaten by Bhavan = x + 18.
Total number of chocolates eaten by the others = (5) (18) = 90 ⇒ 19x = x + 18 + 90
x = 6
So Bhavan ate 6 + 18 = 24 chocolates Ans : (24)

25. Weight of Amar = Weight of 20 students of the class – Weight of other 19 students of the class = (20) (25) – (19) (24.8) = 28.80 kg Ans : (28.80)

26. Sum of the 20 numbers = 20 × 40 = 800
Sum of the last 19 numbers = 19 × 42 = 798
First number = sum of the 20 numbers – sum of the last 19 numbers = 800 – 798 = 2. Choice (B)

27. Let the ratio of quantities of A and B be x : y.
By alligation rule, $\frac{x}{y} = \frac{80 - 50}{50 - 30} = \frac{3}{2}$ Choice (D)

28. Let the cost price of the dilute milk per litre be ₹x

$x + \frac{25}{100}x = 12.50 \Rightarrow x = 10$

Let the quantity of water added be y litres. Total cost of water and milk before diluting = ₹120.
Total cost of mixture after diluting = 10(y + 10).
As total cost of water and milk before diluting = Total cost of mixture after diluting
120 = 10(y + 10) ⇒ 2 = y Ans : (2)

29. Let us say the trader mixed x litres of water with 1 litre of milk.
Assuming cost price of 1 litre of milk is ₹1, cost price of (1 + x) litres of solution = ₹1.
Selling price of (1 + x) litres of solution

= cost price + profit = $1 + \frac{25}{100}(1)$ = ₹1.25

The trader would have charged the milk solutions at ₹1 per litre
So, to realize ₹1.25 he must sell 1.25/1
i.e. 1.25 litres of the milk solution.
So, there is 1 litre of milk in 1.25 litres of the milk solution.
∴ Concentration of milk in the solution = 1/1.25 × 100 = 80% Choice (D)

30. Let the number of boys be b.
Number of girls = 40 – b.
Method 1:
Total amount (in ₹) = (8)(b) + (5.50)(40 – b) = 250
8b = 220 – 5.50b = 250
b = 12
Method 2:

Average value received/student = ₹ $\frac{250}{40}$ = ₹$\frac{25}{4}$

Applying alligation, we get $\frac{40-b}{b} = \frac{8 - \frac{25}{4}}{\frac{25}{4} - \frac{11}{2}} = \frac{7}{3}$

∴ b = $\frac{3}{7}(40)$ = 12. Ans : (12)

31. Let the quantities of P and Q mixed be 4x kg and 3x kg. Let the cost of Q be ₹y per kg.
Cost of P would then be ₹(y + 7) per kg.
Cost of the 7x kg of mixture formed on mixing P and Q
= 4x (y + 7) + 3xy
= ₹(7xy + 28x)

Cost of each kg of the mixture
= ₹ $\frac{(7xy + 28x)}{7x}$
= y + 4 = 23
y = 19 Choice (B)

32. Cost of 1 dozen mangoes = (1) (6) = ₹6
Cost of 2 dozen mangoes = (2) (10) = ₹20
Cost of 5 dozen mangoes = (5) (6) = ₹30
Total cost of 8 dozen mangoes = ₹56
Average cost per dozen of mangoes
= ₹ $\frac{56}{8}$ = ₹7 Ans : (7)

33. Total weight of the 4 girls = (4) (25) = 100 kg
Total weight of 5 girls after the girl joins them = (5) (26) = 130 kg
Weight of the girl who joined = 130 – 100 = 30 kg
Choice (A)

34. Cost of 4 kg of rice = (4) (6) = ₹24
Cost of 8 kg of rice = (8) (12) = ₹96
Total cost of 12 kg of rice = 24 + 96 = ₹120
Cost of the mixture = 120/12 = ₹10 per kg. Ans : (10)

35. By alligation rule,
$\frac{\text{Quantity of tea costing ₹9 per kg}}{\text{Quantiy of tea costing ₹12 per kg}} = \frac{12 - 10.2}{10.2 - 9} = \frac{3}{2}$

Quantity of tea costing ₹12 per kg = $\frac{2}{3}(18) = 12\text{kg}$
Choice (D)

36. Total cost of 16 items = ₹(16 × 59).
Total cost of the final 9 items
= ₹[(16 × 59) – (4 × 60) – (39 + 49 + 40)] = ₹576
New average = 576/9 = ₹64 Ans : (64)

37. Let the cost price of the mixture be ₹x per kg

$x + \frac{25}{100}x = 30$

$5x/4 = 30$

$x = 24$

Whenever two varieties (measured in kg) are mixed in the ratio 1 : 1, cost of the mixture is given by average of the costs of the two varieties (in Rs per kg)

$= \frac{\text{Cost of variety A} + \text{Cost of variety B}}{2} = 24$

Cost of variety B = 2(24) – Cost of variety A = ₹26 per kg. Choice (B)

38. Sum of the three numbers = (20) (3) = 60
Third number = 60 – (sum of the other two numbers)
= 60 – 42 = 18 Ans : (18)

39. Let the initial quantity of milk in the vessel be T litres.
Let us say y litres of the mixture is taken out and replaced by water for n times, iteratively.
Quantity of milk finally in the vessel is then given by

$\left(\frac{T-y}{T}\right)^n T$

For the given problem, T = 90, y = 9 and n = 2.
Hence quantity of milk finally in the vessel

$= \left(\frac{90-9}{90}\right)^2 (90) = 72.9$ litres Ans : (72.9)

40. With reference to the above solution if the number of times the procedure is repeated is n, final quantity of milk in the vessel $= \left(\frac{200-20}{200}\right)^n (200) = 0.9^n (200) = 145.8$

$0.9^n = \frac{148.8}{200} = (0.9)^3$

Equating powers of 0.9 both sides, n = 3. Ans : (3)

Exercise – 8
(Numbers)

Solutions for questions 1 to 100:

1. The units digit of $6054 \times 216 \times 312$ is the same as the units digit of the product of the units digits of each of the numbers multiplied above. The units digit of the product of units digits of the three numbers is 8. Ans : (8)

2. (a) For a number to be divisible by 4, the number formed by the last two digits of that number must be divisible by 4.
For a number to be divisible by 9, the sum of its digits must be divisible by 9.
Among the choices, choices (A) and (B) are divisible by 4 but of the first two choices, only Choice (B) is divisible by 9. Hence Choice (B) is divisible by both 4 and 9. Choice (B)

(b) For a number to be divisible by 8, number formed by its last three digits must be divisible by 8. This condition is satisfied only by choices (C) and (D) of these two choices, only Choice (C) is divisible by 9. Hence, Choice (C) is divisible by both 8 and 9. Choice (C)

(c) For a number to be divisible by 6, it must be divisible by 2 and 3.
A number is divisible by 2, if its last digit is divisible by 2.
A number is divisible by 3, if the sum of its digits is divisible by 3.
Among the choices, choices (A) and (D) are divisible by 2. Of these choices, only Choice (D) is divisible by 3. Hence Choice (D) is divisible by 6. Choice (D)

(d) For a number to be divisible by 11, difference of the sum of its digits in the odd places and the sum of its digits in the even places must be 0 or divisible by 11. This condition is satisfied by choices (B) and (D) only.
For a number to be divisible by 12 it must be divisible by its factors 4 and 3.
Both choices (B) and (D) are divisible by 3 but only Choice (B) is divisible by 4. Hence Choice (B) is divisible by both 11 and 12. Choice (B)

(e) A number is divisible by 15, if it is divisible by its co-prime factors i.e. 3 and 5.
All the choices are divisible by 3. For a number to be divisible by 5, it must end with 0 or 5. All the choices end with 0 and hence all choices are divisible by 5. Hence all choices are divisible by 15.
A number is divisible by 24, if it is divisible by its co-prime factors 8 and 3. Only Choice (A) is divisible by 8. Hence Choice (A) is divisible by 15 and 24. Choice (A)

3. (a) A number is divisible by 36, if it is divisible by both 4 and 9. By using the same method as given in solution 2(a), Choice (B) follows. Choice (B)

(b) Given four options are divisible by 2.
A number is divisible 33, if it is divisible by both 3 and 11.
Both choices (A) and (B) are divisible by 3. Of these, only
Choice (B) is divisible by 11. Choice (B)

(c) A number is divisible by 18, if it is divisible by 9 and 2.
Both choices (A) and (B) are divisible by both 9 and 2.
A number is divisible by 12, if it is divisible by 4, 3.
Choice (A) is divisible by 4. Choice (A)

(d) An odd number divisible by 25 must end with the last two digits of 25 or 75. An even number divisible by 25 must end with 00 or 50.
All the first three choices are divisible by 25. Choice (D)

4. The least number to be added to the numbers to make them divisible by 9 is equal to the difference of the least multiple of 9 greater than the sum of the digits and sum of the digits.

(a) Sum of digits = 15. Nearest multiple of 9 greater than sum of digits = 18.
Hence 3 has to be added. Choice (C)

(b) Sum of digits = 28. Nearest multiple of 9 greater than sum of digits = 36.
Hence 8 has to be added. Choice (A)

(c) Sum of digits = 26. Nearest multiple of 9 greater than the sum of digits = 27.
Hence 1 has to be added. Choice (C)

(d) Sum of digits = 21. Nearest multiple of 9 greater than the sum of digits = 27.
Hence 6 has to be added. Choice (B)

(e) Sum of digits = 14. Nearest multiple of 9 greater than the sum of the digits = 18.
Hence 4 has to be added. Choice (A)

5. Divisibility of 11: If the difference of the sum of the digits at odd places and the sum of the digits at even places of a number is zero or a multiple of 11, then that number is divisible by 11.

(a) Given number is 945678 $\underline{x}$
As per divisibility rule of 11, difference
= (x + 21) – 18 = x + 3
$\Rightarrow$ x + 3 = 11 $\Rightarrow$ x = 8 Choice (C)

(b) Given number is 37679 $\underline{x}$.
As per divisibility rule of 11, difference
= (x +14) – 18 = x – 4
$\Rightarrow$ x – 4 = 0 $\Rightarrow$ x = 4 Choice (A)

(c) Given number is 4 x 56369
As per divisibility rule of 11, difference
$= 21 - (12 + x) = 9 - x$
$\Rightarrow 9 - x = 0 \Rightarrow x = 9$ Choice (D)

(d) Given number is 65428 x 1
As per divisibility rule of 11, difference
$= 19 - (x + 7) = 12 - x$
$\Rightarrow 12 - x = 11 \Rightarrow x = 1$ Choice (A)

(e) Given numbers is 129x
As per divisibility rule of 11, difference
$= (x + 2) - 10 = x - 8$
$\Rightarrow x - 8 = 0$
$\Rightarrow x = 8$ Choice (D)

6. (a) $1680 = 168 \times 10$
$= 24 \times 7 \times 10 = 2^3 \times 3 \times 7 \times (2 \times 5)$
$= 2^4 \times 3 \times 5 \times 7$ Choice (A)

(b) $676 = 26^2 = (2 \times 13)^2$
$= 2^2 \times 13^2$ Choice (A)

(c) $2860 = 10 \times 286$
$= 10 \times 2 \times 143$
$= 2 \times 5 \times 2 \times 13 \times 11$
$= 2^2 \times 5 \times 13 \times 11.$ Choice (A)

(d) $2160 = 10 \times 216$
$= 10 \times 6^3 = 2 \times 5 \times (2 \times 3)^3$
$= 2^4 \times 5 \times 3^3$ Choice (B)

7. (a) $765 = 5 \times 153$
$= 5 \times 17 \times 3^2$ Choice (A)

(b) $1521 = 9 \times 169$
$= 3^2 \times 13^2$ Choice (A)

(c) Given number is 2244.
$2244 = 2^2 \times 561 = 2^2 \times 3 \times 17 \times 11$ Choice (A)

(d) Given number is 5776
$5776 = 2^4 \times 361 = 2^4 \times 19^2$ Choice (B)

(e) As the sum of the digits of 11979 is 27, it is divisible by 9. Dividing 11979 by 9, the quotient obtained is 1331.
We have $1331 = 11^3$, $11979 = 9 \times 11^3 = 3^2 \times 11^3$.
Choice (A)

8. (a) $56 \times 445 + 77 \times 555 + 21 \times 445$
$= (56 + 21) \times 445 + 77 \times 555$
$= 77 \times 445 + 77 \times 555$
$= 77\,(445 + 555) = 77000$ Ans : (77000)

(b) $6\frac{1}{6} + 4\frac{5}{6} - 3\frac{3}{4} - 6\frac{1}{4}$
$= \frac{37}{6} + \frac{29}{6} - \frac{15}{4} - \frac{25}{4}$
$= 11 - 10 = 1$ Ans : (1)

(c) $\frac{3.36 - 2.34}{3} \times \frac{2.79 + 4.34 + 4.77}{3.4}$
$\frac{1.02}{3} \times \frac{11.9}{3.4} = 0.34 \times \frac{7}{2} = 1.19.$ Ans : (1.19)

9. (a) $221 \times 650 + 442 \times 175$
$221 \times 650 + 221 \times 350$
$= 221(1000) = 221000$ Ans : (221000)

(b) $3\frac{5}{6} + 6\frac{1}{7} - 2\frac{1}{3} - 1\frac{1}{2}$
$= (3 + 6 - 2 - 1) + \left(\frac{5}{6} + \frac{1}{7} - \frac{1}{3} - \frac{1}{2}\right)$
$= 6 + \left(\frac{5}{6} + \frac{1}{7} - \frac{5}{6}\right) = 6\frac{1}{7}$ Choice (A)

(c) $(2.45)^3 + 7.35(1.55)^2 - 4.65(2.45)^2 - (1.55)^3$
$= (2.45)^3 + 3(2.45)(1.55)^2$
$- 3(1.55)(2.45)^2 - (1.55)^3$
$= (2.45 - 1.55)^3.$
The denominator of the fraction to be simplified is $(2.45 - 1.55)^2$
The fraction is 0.9 Ans : (0.9)

10. (a)

1 +1	1 8 7 6 9 1 (–)	137
23 + 3	87 (–) 69	
267	1 8 6 9 (–) 1 8 6 9	
	0	

$\therefore \sqrt{18769} = 137$ Choice (C)

(b)

1 +1	222.01 1 (–)	14.9
24 + 4	122 (–) 96	
289	2601 (–) 2601	
	0	

$\therefore \sqrt{222.01} = 14.9$ Choice (D)

(c)

9 +9	8 8 7 6 8 1 (–)	94
184	736 (–) 736	
	0	

$\therefore \sqrt{8836} = 94$ Choice (B)

Note: All the questions can also be answered by going back from the choices.

11. (a)

	179
1	32041
1	1
27	220
7	189
349	3141
	3141
	0

(or) As $175^2 < 32041 < 180^2$, $\sqrt{32041}$ is 179.
Ans : (179)

(b)

	77
7	5929
7	49
147	1029
	1029
	0

$\therefore \sqrt{5929} = 77.$

(OR) As $75^2 < 5929 < 80^2$, $\sqrt{5929} = 77.$
Ans : (77)

(c)

$$\begin{array}{r|l}
 & 12.3 \\
1 & 151.29 \\
1 & 1 \\
\hline
22 & 51 \\
2 & 44 \\
\hline
243 & 729 \\
 & 729 \\
\hline
 & 0
\end{array}$$

$\therefore \sqrt{151.29} = 12.3$

(OR) As $12.0^2 < 151.29 < 13.0^2$, $\sqrt{151.29} = 12.3$

Ans : (12.3)

(d)

$$\begin{array}{r|l}
 & 251 \\
2 & 63001 \\
2 & 4 \\
\hline
45 & 230 \\
5 & 225 \\
\hline
501 & 501 \\
 & 501 \\
\hline
 & 0
\end{array}$$

$\therefore \sqrt{63001} = 251$

(OR) As $250^2 < 63001 < 255^2$, $\sqrt{63001} = 251$

Ans : (251)

(e)

$$\begin{array}{r|l}
 & 214 \\
2 & 45796 \\
2 & 4 \\
\hline
41 & 57 \\
1 & 41 \\
\hline
424 & 1696 \\
 & 1696 \\
\hline
 & 0
\end{array}$$

$\therefore \sqrt{45796} = 214$

(OR) As, $210^2 < 45796 < 215^2$, $\sqrt{45796} = 214$

Ans : (214)

12. (a) $a^n + b^n$ is divisible by a + b only when n is odd.
Taking a = 7 and b = 6, $7^n + 6^n$ is divisible by 13 only when n is odd. Choice (C)

(b) $11^n - 23^n = 11^n - (2^3)^n$
$11^n - 8^n$
$a^n - b^n$ is divisible by a – b for all values of n.
Taking a = 11 and b = 8, $11^n - 8^n$ is divisible by 3 for all values of n. Choice (A)

(c) $2^{4n} + 3^n = (2^4)^n + 3^n$
$= 16^n + 3^n$
$16^n + 3^n$ is never divisible by (16 –3) i.e.,13
Choice (D)

(d) $a^n - b^n$ is divisible by a + b only when n is even.
Taking a = 7 and b = 6, $7^n - 6^n$ is divisible by 13 only when n is even. Choice (B)

13. (a) $5^{2n} - 1 = 25^n - 1$ is divisible by 24, hence, divisible by 8.
$(a^n - b^n)$ is always divisible by a – b Choice (C)

(b) $8^n + 1 = 2^{3n} + 1^{3n}$.
A number in the form $a^p + b^p$ where p is a natural number is divisible by a + b only when p is odd.
Taking a = 2, b = 1 and p = 3n, the number $8^n + 1$ is always divisible by 3 only when 3n is odd.
If 3n is odd, then n is odd. Choice (A)

14. Let the number be N.
The quotient when N is divided by 162 is denoted by Q_1
$N = 162Q_1 + 29$.
$\therefore$ When N is divided by 27, the quotient is $6Q_1 + 1$ and the remainder is 2. Ans : (2)

15. Let the number be N. When it is divided by 204 let us say the quotient is Q.
N = 204Q + 60.
When N is divided by 34, the quotient is 6Q + 1 and the remainder is 26. Ans : (26)

16. Let $x = 0.46\bar{7}$.
$100x = 46.\bar{7} \rightarrow (1)$
$1000x = 467.\bar{7} \rightarrow (2)$
Subtracting (1) from (2), 900x = 421
$\rightarrow x = \frac{421}{900}$. Choice (B)

17. Let x be 1.27878……….
10 x = 12.78………
and 1000x = 1278.78……….
$\therefore$ 990x = 1266
$\Rightarrow x = \frac{1266}{990} = \frac{211}{165}$ Choice (A)

18. Statement (a)
Numbers which are relative primes must have a H.C.F. of 1. Consecutive numbers are always relative prime.
Statement (b)
$133 = 19 \times 7$
$285 = 19 \times 15$.
As 19 is the HCF of 133 and 285, they are not relative primes.
Statement (c)

As both 210 and 255 are divisible by 3, they are not relative primes.
Statement (d)
$15 = 3 \times 5$
$91 = 7 \times 13$
As 15 and 91 have no common factor other than 1, they are relative primes.
Statement (e)
123 = 3 X 41
164 = 4 X 41
As 41 is a common factor of 123 and 164, 123 and 164 cannot be relatively prime. Choice (C)

19. HCF (77, 85) = 1
HCF (29, 203) = 29 [$\therefore 203 = 7 \times 29$]
HCF (103, 205) = 1
HCF (109, 242) = 1
Hence 29 and 203 are not relatively prime to each other.
Choice (B)

20. $\frac{a^3+b^3}{a^2-ab+b^2} = a + b$
$\therefore$required quantity = 89 + 11 = 100.
Ans : (100)

21. Given divisors are 3, 5, 9, 11 and 19
Required number = L.C.M (3, 5, 9, 11, 19) $\times$ n, where 'n' is any natural number
$= 9405 \times n$.
As the options (b), (c), (d) and (e) are not the multiples of 9405, Choice (B) follows. Choice (B)

22. Given divisors are 18, 27, 36 and 48.
Required numbers are of the form of L.C.M(18, 27, 36 and 48) $\times$ n, where n is any i.e., $432 \times n$.
Except 3556, options (a), (b), (c) and (e) can be written in the form of $432 \times n$. Choice (A)

23. (a) The sum of the digits of 24151 is 13. The least number to be added to the number to make the number multiple of 9 is the nearest multiple of 9 greater than the sum of the digits – sum of the digits = 18 – 13 = 5.
Choice (A)

(b) The sum of the digits of 335672 is 26. Hence 1 is the least natural number to be added to it to make it a multiple of 9. Choice (D)

(c) Given number is 765413.
Sum of the digits of 765413 is 26.
Hence 1 is the least natural number to be added to the number to make it a multiple of 9. Choice (A)

(d) Sum of digits of 567491 is 32. Hence 4 has to be added to the number to make it multiple of 9.
Choice (C)

(e) Sum of digits of 765436 is 31. Hence 5 has to be added to the number to make it multiple of 9.
Choice (B)

24. (a) Sum of the digits at odd places = 8
Sum of the digits at even places = 8. The difference of the sums of alternate digits is 0. Hence the number is divisible by 11. The least natural number that to be added to given number is 11. Choice (D)

(b) Given number is 896656.
For a number to be divisible by 11, the difference of the sum of its digits at odd places and the sum of the digits at even places, should be either '0' or a number divisible by 11.
From the choices if 9 is added to 896656, the result is 896665

Here sum of the digits at odd places = 8 + 6 + 6 = 20
Sum of the digits at even places = 9 + 6 + 5 = 20
Since, 20 – 20 = 0, Choice (C) follows. Choice (C)

(c) Given number is 584560.
From Choice (A), 584560 + 2 = 584562
Here, (5 + 4 + 6) – (8 + 5 + 2) = 15 – 15 = 0
∴ Test of Divisibility by 11 satisfies. Choice (A)

(d) Given number is 504215.
From Choice (A), 504215 + 3 = 504218.
Here, (5 + 4 + 1) – (0 + 2 + 8) = 10 – 10 = 0.
Choice (A)

25. $9000 = 3^2 \times 5^3 \times 2^3$
The smallest natural number to be multiplied with to make it a perfect square = $5 \times 2 = 10$ Ans : (10)

26. $1080 = 108 \times 10$
$= 18 \times 6 \times 10 = 2 \times 3^2 \times 2 \times 3 \times 2 \times 5 = 2^3 \times 3^3 \times 5$
The least natural number to be multiplied to make it a perfect cube = $5 \times 5 = 25$ Ans : (25)

27. $520 = 26 \times 20 = 2 \times 13 \times 2^2 \times 5$
$= 2^3 \times 13 \times 5$
Required smallest number = $2 \times 13 \times 5 = 130$
∴ 130 is the smallest number which should be multiplied with 520 to make it a perfect square. Ans : (130)

28. $16200 = 2^3 \times 3^4 \times 5^2$
A perfect cube has a property of having the indices of all its prime factors divisible by 3.
Required number = $3^2 \times 5 = 45$ Choice (C)

29. H.C.F. of two numbers = $\dfrac{\text{product of two numbers}}{\text{L.C.M. of two numbers}}$

$= \dfrac{2432}{608} = 4.$ Ans : (4)

30. LCM of two numbers is given by
$\dfrac{\text{Product of the two numbers}}{\text{HCF of the two numbers}}$

$= \dfrac{2560}{16} = 160.$ Choice (C)

31. (L.C.M.) (H.C.F) = Product of the numbers
(20) (HCF) = 80; HCF = 4 Ans : (4)

32. Let the two numbers be hk and hl.
Sum = hk + hl = 98 ….. (1)
LCM = hkl = 168 ………(2)
(1) ÷ (2)
$\dfrac{K + \ell}{K \times \ell} = \dfrac{7}{12}$
⇒ k = 4 or 3 and l = 3 or 4
hkl = 168
h x 4 x 3 = 168 ⇒ h = 14
∴ the numbers are 14 x 3 and 14 x 4.
The difference of the numbers is 14(4 – 3) = 14.
Choice (D)

33. L.C.M. of $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{4}{7}, \dfrac{9}{20}$
$= \dfrac{\text{L.C.M of } 1, 2, 4, 9}{\text{H.C..F of } 2, 3, 7, 20}$
= 36/1 = 36 Choice (A)

34. HCF of $\dfrac{2}{5}, \dfrac{4}{3}, \dfrac{11}{6} = \dfrac{\text{H.C.F of } 2, 4, 11}{\text{L.C.M of } 5, 3, 6} = 1/30$
Choice (D)

35. LCM of fractions = $\dfrac{\text{LCM of numerators}}{\text{HCF of deno min ators}}$
$= \dfrac{\text{LCM }(2,4,6)}{\text{HCF }(7,9,11)} = \dfrac{12}{1} = 12$ Ans : (12)

36. The three bells toll together after every LCM of (30, 45, 60) = 180 minutes = 3 hours
After 9:30 a.m., they toll together again at 12:30 p.m.
Choice (C)

37. The time interval between simultaneous tolling of the bells
= LCM (10, 15, 20, 30) seconds = 60 seconds
= 1 minute.
Hence the bells will toll together again for the first time after 10:00 a.m. at 10:01 a.m. Choice (A)

38. The time interval for them to meet at the starting point for the next time = LCM (10, 8, 12,18) = 360 minutes.
Choice (C)

39. The least number which when divided by different divisors leaves the same remainder in each case is given by L.C.M (different divisors) + remainder.
Hence the required least number = L.C.M (11, 24, 26) + 4
= 3432 + 4 = 3436 Choice (A)

40. Required number = L.C.M (13, 15 and 23) – 9.
= 4485 – 9. = 4476. Ans : (4476)

41. Let the smallest number be N. Let quotients obtained when N is divided by 22 and 16 be q_1 and q_2 respectively.
$22q_1 + 7 = 16q_2 + 3$
$\Rightarrow 22q_1 + 4 = 16q_2$
Let us substitute values of q_1 which are natural numbers starting from $q_1 = 1$ and check when q_2 is a positive integer for the first time. So the least integral value of q_2 so obtained is 3
when $q_1 = 2$. We have, N = 22(2) + 7 = 51 Ans : (51)

42. The least number which when divided by different divisors leaving the same remainder in each case
= LCM (different divisors) + remainder left in each case.
Hence the required least number
= LCM (35, 11) + 1 = 386. Ans : (386)

Note: 1 is the least positive integer that satisfies the given condition. But look at the choices, we go with 386.

43. L.C.M. of 9, 27 and 15 is 135
∴ Required number = 135k – 6
When k = 1
⇒ 135 k – 6 = 135 – 6 = 129,
the smallest three digit number.
When k = 7
⇒ 135k – 6 = 945 – 6 = 939, the greatest 3-digt number.
Choice (C)

44. Let 'N' is the smallest number which when divided by 13 and 16 leaves respective remainders of 2 and 5.
$\therefore$ Required number = (L.C.M. of 13 and 16) – (common difference of any divisor and corresponding remainder)
= (208) – (11) = 197 Choice (B)

45. The required number is of the form 13k + 7.
13k + 7, when divided by 8 leaves a remainder of 3.
$\therefore$ 13k + 7 – 3 = 13k + 4 is divisible by 8.
Least value of k satisfying the above condition is 4.
$\therefore$ required number is $13 \times 4 + 7 = 59$. Choice (C)

46. Required number is the LCM of 12 and 17 – (12 – 5)
i.e., 204 – 7
i.e., 197 Ans : (197)

47. Required number is the (LCM of 8, 7) + 3
i.e., 56 + 3
i.e, 59 Choice (A)

48. Required number is the HCF of (407 – 5), (327 – 3)
i.e., HCF of 402, 324.
i.e., HCF of $2 \times 3 \times 67$, $2^2 \times 3^4$.
i.e., $2 \times 3 = 6$ Ans : (6)

49. HCF of 470 – 350, 890 – 470, 890 – 350
i.e., HCF of 120, 420, 540
i.e., HCF of $2^3 \times 3 \times 5$, $2^2 \times 3 \times 5 \times 7$, $2^2 \times 3^3 \times 5$
i.e., $2^2 \times 3 \times 5$
i.e., 60 Ans : (60)

50. Required number is HCF of (149 – 5), (261 –9).
i.e., HCF of 144, 252
i.e., HCF of $2^4 \times 3^2$, $2^2 \times 3^2 \times 7$
i.e., $2^2 \times 3^2$
i.e., 36 Ans : (36)

51. Least natural number = L.C.M $(2.3.5^2, 3.5.7^2, 5.7^2.11)$ = $2.3.5^2.7^2.11$ which has 5 distinct prime factors. $\therefore$ x = 5 ($\because$ L.C.M of two or more numbers in their prime factorised form is the product of all the prime numbers with each being raised to the maximum power). Ans : (5)

52. The greatest number which divides three dividends p, q, and r leaving the same remainder in each case is given by H.C.F (any two of (q – p), (r – p) and (r – q) where r > q > p.
in the given problem, p = 106, q = 241 and r = 286.
Hence the greatest number which divides these three numbers leaving the same remainder in each case is H.C.F [any two of (241 – 106, 286 – 241, 286 – 106)] =
i.e., H.C.F. of any two of 135,45 and 180 = 45
Ans : (45)

53. Required number = H.C.F. of (2053 – 3) and (3909 –9)
i.e., H.C.F. of 2050 and 3900

```
2050) 3900 (1
      2050
      1850) 2050 (1
            1850
             200) 1850 ( 9
                  1800
                    50) 200 (4
                        200
                          0
```

$\therefore$Required Number = 50 Ans : (50)

54. The greatest number which divides three dividends p, q and r leaving the same remainder in each case is given by HCF(any two of (q – p), (r – p) and (r – q)).
where r > q > p.
For the given problem,
p = 83, q = 125 and r = 209.
Hence the greatest number which divides these three leaving the same remainder in each case is HCF (any two of (125 – 83, 209 – 83, 209 – 125)) = 42. Choice (C)

55. Take the LCM of the denominators and then compare the numerators.
LCM of 9, 7 and 2 = 126
$\frac{5}{9} = \frac{70}{126}$
$\frac{4}{7} = \frac{72}{126}$
$\frac{1}{2} = \frac{63}{126}$
Comparing the numerators, 72 > 70 > 63
Hence $\frac{4}{7} > \frac{5}{9} > \frac{1}{2}$ Choice (A)

56. Substituting 2.25 = a and 2.75 = b
the expression to be simplified becomes
$a^3 + 3ab^2 – 3a^2b – b^3 = (a – b)^3$
As a = 2.25 and b = 2.75
$(a – b)^3 = (–0.5)^3 = –0.125$ Choice (C)

57. Substituting 5.71 = a and 3.21 = b,
the expression to be simplified becomes
$\frac{a^3 - b^3}{a^2 + ab + b^2}$ which is equal to a – b
As a = 5.71 and b = 3.21,
a – b = 2.5 Ans : (2.5)

58. All the five bells ring together after [LCM of 2, 3, 4, 5, 6] seconds.
i.e., after 60 seconds
i.e., after 1 minute
$\therefore$ in 2 hours, they ring together 2×60 i.e., 120 times
Ans : (120)

59. Required value = Maximum value of x = H.C.F (lengths of wires) = H.C.F(784 cm, 812 cm) = H.C.F (28.28, 28.29) cm = 28. H.C.F (28, 29) cm = 28. Ans : (28)

60. Let the two numbers be 8x and 8y where x and y are relatively prime.
LCM (8x, 8y) = 8 LCM (x, y) = 8xy.
8xy = 1960.
xy = 245.
245 can be written as a product of two factors in three ways i.e.(1, 245), (5, 49) and (7, 35).
As x and y are relatively prime, (x,y) can be (1, 245) or (5, 49) but not (7, 35).
Hence two possible pairs exist. Choice (B)

61. There are fourteen 7's in 100!, two $7^{2'}$ s in 100! Hence the highest power of 7 in 100! is 16. Ans : (16)

62. 120 is successively divided by 3, then the sum of quotients is the largest power of 3.
$\therefore$ Largest power = 40 + 13 + 4 + 1 = 58 Ans : (58)

63. There are 40 5s in 200!, 8 5^2s in 200! 5^3s and no 5^4s or any higher power in 200!.So, there are forty 5 s, eight 5^2s and one 5^3 in 200!.
$\therefore$Highest power of 5 in 200! = 40 + 8 + 1 = 49
Hence the highest power of 5 in 200! is 49. Ans : (49)

64. Six is not a prime number.
$\therefore$ write 6 as 2×3
Highest power of 2 in 100 ! is
$\frac{100}{2} + \frac{100}{2^2} + \frac{100}{2^3} + \frac{100}{2^4} + \frac{100}{2^5} + \frac{100}{2^6}$
= 50 + 25 + 12 + 6 + 3 + 1 = 97
Highest power of 3 in 100! is
$\frac{100}{3} + \frac{100}{3^2} + \frac{100}{3^3} + \frac{100}{3^4}$ = 33 + 11 + 3 + 1= 48
Now $2^{97} = 2^{48} \times 2^{49}$ and 48 is the highest power of 3
$\therefore$ highest power of 6 is 48.
$(2^{48} \times 3^{48})$ Choice (C)

65. Highest power of 5 in 167 !

$= \text{Integral part of } \frac{167}{5} + \text{Integral part of } \frac{167}{5^2}$

$+ \text{Integral part of } \frac{167}{5^3} = 33 + 6 + 1 = 40$ Choice (C)

66. The sum of the digits of 56x4y2 should be divisible by 9. i.e. 17 + x + y should be divisible by 9.
The least value of x + y is 1. Ans : (1)

67. Number is divisible by 33 means, it is divisible by both 3 and 11.
Choice A : 356974.
Sum of the digits = 34, which is not divisible by 3.
Choice B : 548672.
Sum of the digits = 32, (not divisible by 3)
Choice C : 237698
Sum of the digits = 35, (not divisible by 3)
Choice D : 568722.
Sum of the digits = 30 (divisible by 3)
Also 5 + 8 + 2 = 6 + 7 + 2.
The number is divisible by 11. Choice (D)

68. For 1 4 3 x 5 to be divisible by 9,
1 + 4 + 3 + x + 5 = 13 + x should be divisible by 9.
$\therefore$ least value of x is 5. Ans : (5)

69. $10^n + 1$ is not divisible by 9 for any value of n.
(put n = 1, 2 and check) Choice (D)

70. Required number is of the form 60x.
$60x = 4 \times 15x$
For 60x to be a perfect square, x should be 15.
$\therefore$ the number is $= 60 \times 15 = 900$. Ans : (900)

71. LCM of 7, 9, 16
$= 7 \times 9 \times 16 = 1008 > 1000$. Ans : (0)

72. Least six digit number is 100000.
Remainder, when 100000 is divided by 323 is 193.
$\therefore$ required number is 100130. Ans : (100130)

73. The number of zeroes at the end of 400! is given by the largest power of 5 in 400!.
$\therefore$ Number of zeroes = The sum of the quotients when 400 is successively divided by 5
= 80 + 16 + 3 = 99 Ans : (99)

74. $1200 = 12 \cdot 100 = (3.2^2)(2^2.5^2) = 2^4.3^1.5^2$.
Number of divisors of a number of the form $a^p.b^q.c^r$……. where a, b, c, …… are prime and p, q, r,…… are natural numbers is given by (p + 1) (q + 1) (r + 1)……
Number of divisors of 1200 = (4 + 1) (1 + 1) (2 + 1) = 30.
Ans : (30)

75. Let us say a number N has two prime factors a and b. Suppose $N = a^m \times b^n$ where m and n are positive integers.
The number of factors of N is given by (m + 1) (n + 1).
For the given problem, taking N = 1764
$= 42^2 = (7 \times 2 \times 3)^2$
$= 7^2 \times 2^2 \times 3^2$
Number of factors of N = (2 + 1) (2 + 1) (2 + 1) = 27.
$\therefore$ Number of factors excluding 1 and itself = 27 – 2 = 25
Choice (C)

76. $3663 = 3^2 \times 11 \times 37$
Number of factors = (2 + 1) (1 + 1) (1 + 1) = 12
Number of ways that 3663 can be resolved into factors = 12/2 = 6
($\because$ Given number is not a perfect square) Choice (A)

77. $1476 = 2^2 \times 3^2 \times 41$
$\therefore$ number of factors = (2 + 1) (2 + 1) (1 + 1) = 18
Ans : (18)

78. The prime factors of $2^{13} \times 3^{14} \times 5^{15}$ are 2, 3 and 5.
Choice (A)

79. $960 = 2^6 \times 3 \times 5$
Total number of factors = (6 + 1) (1 + 1) (1 + 1) = 28
960 is not a perfect square.
$\therefore$ required number of ways $= \frac{28}{2} = 14$ Choice (D)

80. Remainder, when 3^1 is divided by 11 = 3
Remainder, when 3^2 is divided by 11 = 9
Remainder, when 3^3 is divided by 11 = 5
Remainder, when 3^4 is divided by 11 = 4
Remainder, when 3^5 is divided by 11 = 1
Remainder, when 3^6 is divided by 11 = 3
$\therefore$ Remainder, when 3^1 is divided by 11 = Remainder, when 3^6 is divided by 11 = Remainder, when 3^{5x1+1} is divided by 11.
$\therefore$ Remainder, when 3^{91} is divided by 11 = Remainder, when 3^{5x18+1} is divided by 11 = Remainder, when 3^1 is divided by 11 = 3. Choice (A)

81. Remainder, when 5^1 is divided by 6 = 5
Remainder, when 5^2 is divided by 6 = 1
Remainder, when 5^3 is divided by 6 = 5
Remainder, when 5^4 is divided by 6 = 1
37 is odd
$\therefore$ remainder when 5^{37} is divided by 6 = 5. Ans : (5)

82. It can be verified that for any value of p we have Rem $\left(\frac{p^2}{24}\right)$ = 1. Ans : (1)

83. The remainder of 3^Ndivided by 4 where N is any natural number is always 3 or 1. The remainder is 3 when N is odd and is 1 when N is even.
21465 is odd. $\therefore$ Rem $\left(\frac{3^{21465}}{4}\right) = 3$. Choice (D)

84. $2^{2400} = (2^3)^{800}$
$9 = 2^3 + 1$
Taking $x = 2^3$ and $f(x) = 2^{2400}$,
$f(x) = x^{800}$
$9 = x + 1$
When f(x) is divided by x – a where a is any real number, the remainder is given by f(a).
The remainder for the problem is $f(-1) = (-1)^{800} = +1$.
Choice (D)

85. Let the divisor be D. Let the number when divided by D leaving a remainder of 13 be N and the quotient obtained be Q_1.
$N = Q_1D + 13$ ————— (1)
Let the quotient when thrice the number i.e. 3N is divided by D be Q_2
$3N = Q_2D + 1$ ————— (2)
Multiplying (1) by 3,
$3N = 3Q_1D + 39 = D\left(3Q_1 + \frac{38}{D}\right) + 1$.
Comparing the above equation with (2),
Q_2 and $3Q_1$ are integers, $\frac{38}{D}$ is also an integer.
Hence D must be a factor of 38. As any divisor is greater than its remainder, D > 13. Hence D can be 38 or 19. Choice (C) follows. Choice (C)

86. Any number when divided by 5 will leave a remainder of its last digit divided by 5.
3^n ends with 1 or 3 or 9 or 7. Hence the possible remainders when 3^n is divided by 5 are 1 or 3 or 4 or 2. The sum of these remainders equals 10. Ans : (10)

87. (a) The last digit of 8452 × 7156 × 2143 × 1567 is the last digit of the product of last digits of the four numbers which are multiplied. The last digit of the product of last digits of these four numbers is 2.
($\because 2 \times 6 \times 3 \times 7 = 252$) Ans : (2)

(b) Given number is $2^{48} \times 7^{40} \times 9^{48}$
We have, 2^n ends with 2, 4, 8 or 6.
$\therefore$ 2^{48} ends with 6, since 48 is multiple of '4'.
7^n ends with 7, 9, 3 or 1.
$\therefore$ 7^{40} ends with 1, since 40 is multiple of 4.
4^n ends with 9, 1.
4^{48} ends with 6, since 48 is multiple of 2.
$\therefore$ The units digit of
$2^{48} \times 7^{40} \times 4^{48}$ is 6.
($\because$ 6 × 1 × 6 ends with 6) Ans : (6)

(c) 3^{4n} and 9^{2n} end with 1. 6^m always ends with 6.
Hence, $3^{4n} \times 9^{2n} \times 6^n$ ends with 6. Ans: (6)

88. We know that,
6^{25} ends with 6,
9^{16} ends with 1 and
5^{40} ends with 5.
Hence $6^{25} + 9^{16} + 5^{40}$ ends with the units digit of 6 + 1 + 5 i.e.2. Choice (A)

89. Last digit of 4^{99} :
99 is odd
$\therefore$ last digit of 4^{99} is 4
Last digit of 7^{99}.
Last digit of $7^1 = 7$
Last digit of $7^2 = 9$
Last digit of $7^3 = 3$
Last digit of $7^4 = 1$
Last digit of $7^5 = 7$
$\therefore$ last digit of 7^1 = Last digit of 7^5
= Last digit of $7^{4\times 1+1}$
$\therefore$ last digit of 7^{99} = Last digit of $7^{24\times 4+3}$
= Last digit of $7^3 = 3$
$\therefore$ last digit of $4^{99} + 7^{99}$ is 4 + 3 = 7. Ans: (7)

90. $1296 = 6.216 = 6.6^3 = 6^4$.
$\therefore$ $\sqrt[4]{1296}$ = 6 which is rational. Choice (D)

91. Let the number be x. Let the quotient when x is divided by 7 be q_1
$x = 7q_1 + 6$
Let the remainder when q_1 is divided by 5 be q_2.
$q_1 = 5q_2 + 4$.
$x = 7(5q_2 + 4) + 6$.
$= 35q_2 + 34$.
The smallest value of x is obtained when $q_2 = 0$ as 34.
The remainder when 34 is divided by 20 is 14. Choice (B)

92. Suppose the number is x

$$\begin{array}{r|l} 3 & x \\ \hline 8 & y-2 \\ \hline & z-7 \end{array}$$

where y and z are respective quotients.
$\therefore$ y = 8z + 7 and x = 3y + 2 = 3(8z + 7) + 2
= 24z + 23 = 6 × 4z + 3 × 6 + 5 = 6(4z + 3) + 5
$\therefore$ remainder, when x is divided by 6 is 5. Ans: (5)

93. (i) Suppose the required number = x

$$\begin{array}{r|ll} 4 & x & \\ \hline 5 & y & -3 \\ \hline 6 & z & -4 \\ \hline & w & -2 \end{array}$$

$\therefore$x = 4y + 3
y = 5z + 4
z = 6w + 2
$\therefore$ x = 120w + 59.
Least value of x is 59. Ans: (59)

(ii)
$$\begin{array}{ccc} 4 & 5 & 6 \\ \downarrow\nwarrow & \downarrow\nwarrow & \\ 3 & 4 & 2 \end{array}$$
= (2 x 5 + 4) 4 + 3 = 59 is the least possible number.

94. 31.32.33………. 39 = 31.32.33.34.35…….39
= 32.35 (31.32.33.3.36……39).
32.35 ends with 2.5 = 0.
$\therefore$ The product ends with 0. $\therefore$ X ends with 0.
Ans: (0)

95. 7.999… is a recurring decimal which is a rational number.
[Suppose x = 7.999…….
10x = 79.999…….
$\therefore$ 9x = 72 $\Rightarrow$ x = 8, a rational number.] Choice (A)

96. Let x be 5.555…….
$\Rightarrow$ 10x = 55.555…….. and 9x = 50
$\Rightarrow x = \dfrac{50}{g}$ which is in the form of $\dfrac{p}{q}$
It is a rational number. Choice (D)

97. $\dfrac{a^3-b^3}{a-b} = a^2 + ab + b^2$

$\dfrac{24^3-12b^3}{24-12} = (24)^2 + (24)(12) + (12)^2$
$= (12)^2 [4 + 2 + 1] = 7 \times 144 = 1008$ Ans: (1008)

98. $\sqrt{1681} = 41$
$\therefore$ 1681 is a perfect square. Choice (B)

99. (a).

$$\begin{array}{r|l} 2 & 54,\ 72,\ 48 \\ \hline 2 & 27,\ 36,\ 24 \\ \hline 2 & 27,\ 18,\ 12 \\ \hline 3 & 27,\ 9,\ 6 \\ \hline 3 & 9,\ 3,\ 2 \\ \hline & 3,\ 1,\ 2 \end{array}$$

L.C.M = $2^4 \times 3^3$
= 16 × 27 = 432.
Now,

$$\begin{array}{l} 48)\ 54\ (1 \\ \quad\ \ \underline{48} \\ \quad\ \ 6)\ 48\ (8 \\ \qquad\ \ \underline{48} \\ \qquad\ \ \ 0 \end{array}$$

H.C.F. of 6 and 72 is 6, since 6 is the factor of 72.
Choice (B)

(b) 51 = 3 × 17
119 = 7 × 17
187 = 11 × 17
H.C.F (51, 119, 187) = 17.
L.C.M (51, 119, 187) = 3 × 7 × 11 × 17 = 3927.
Choice (B)

(c) H.C.F ($2^2 \times 3 \times 7$, $2 \times 3^2 \times 5 \times 7^2$)
= 2 × 3 × 7 = 42
L.C.M ($2^2 \times 3 \times 7$, $2 \times 3^2 \times 5 \times 7^2$)
$= 2^2 \times 3^2 \times 5 \times 7^2$
= 8820. Choice (A)

(d) We have,
H.C.F of fractions
$= \dfrac{\text{H.C.F.of numerators}}{\text{L.C.M .of deno min ators}}$
L.C.M of fractions
$= \dfrac{\text{L.C.M. of numerators}}{\text{H.C.F. of deno min ators}}$

H.C.F $\left(\dfrac{7}{18}, \dfrac{2}{15}, \dfrac{8}{9}\right) = \dfrac{\text{H.C.F }(7,2,8)}{\text{L.C.M}(18,15,9)} = \dfrac{1}{90}$

L.C.M $\left(\dfrac{7}{18}, \dfrac{2}{15}, \dfrac{8}{9}\right) = \dfrac{\text{L.C.M}(7,2,8)}{\text{H.C.F}(18,15,9)} = \dfrac{56}{3}$.

Choice (B)

(e) $72 = 2^3 \times 3^2$
$96 = 2^5 \times 3$
$64 = 2^6$
H.C.F (72, 96, 64) = $2^3 \times 3^0 = 8$
L.C.M (72, 96, 64) = $2^6 \times 3^2$
= 576. Choice (D)

100. (a) L.C.M. of 18, 27 and 36.

$$\begin{array}{r|l} 2 & 18, 27, 36 \\ \hline 3 & 9, 27, 18 \\ \hline 3 & 3, 9, 6 \\ \hline & 1, 3, 2 \end{array}$$

L.C.M = $2^2 \times 3^3 = 108$
H.C.F of 18, 27 and 36

18) 36(2
36

0
∴ 18) 27 (1
18
-----
9) 18 (2
18
------
(0)

∴ H.C.F. = 9 Choice (A)

(b) L.C.M. of 81 and 216

$$\begin{array}{r|l} 3 & 81, 216 \\ \hline 3 & 27, 72 \\ \hline 3 & 9, 24 \\ \hline & 3, 8 \end{array}$$

∴ L.C.M. = $3^4 \times 8 = 648$
H.C.F of 81 and 216
81) 216(2
162
----
54) 81(1
54
----
27) 54 (2
54

0

∴ H.C.F = 27 Choice (B)

(c) HCF (fractions)
$= \frac{\text{HCF(numerators of the fractions)}}{\text{LCM(denominators of the fractions)}}$
LCM (fractions)
$= \frac{\text{LCM(numerators of the fractions)}}{\text{HCF(denominators of the fractions)}}$

HCF $\left(\frac{9}{13}, \frac{7}{11}, \frac{3}{8}\right)$
$= \frac{\text{HCF}(9,7,3)}{\text{LCM}(13,11,8)} = \frac{1}{1144}$.

LCM $\left(\frac{9}{13}, \frac{7}{11}, \frac{3}{8}\right)$
$= \frac{\text{LCM}(9, 7, 3)}{\text{HCF}(13, 11, 8)} = 63$. Choice (B)

(d) HCF $\left(\frac{5}{6}, \frac{13}{9}, \frac{7}{12}\right) = \frac{\text{HCF}(5, 13, 7)}{\text{LCM}(6, 9\ 12)} = \frac{1}{36}$

LCM $\left(\frac{5}{6}, \frac{13}{9}, \frac{7}{12}\right) = \frac{\text{LCM}(5, 13, 17)}{\text{HCF}(6, 9\ 12)} = \frac{455}{3}$

Choice (A)

(e) $14 = 2 \times 7$
$25 = 5^2$
HCF (14, 25) = 1
LCM (14, 25) = 350 Choice (A)

Exercise – 9
(Number Systems)

Solutions for questions 1 to 15:

1. The number of bits in the given number is 11 and so we introduce one leading 0.
Then forming blocks of 4 bits each, we get
$\overline{0110}\ \overline{0111}\ \overline{1010}$
Now converting the number in each into its decimal equivalent, we get
$(\overline{0110}\ \overline{0111}\ \overline{1010})_2 = (67A)_{16}$ Choice (A)

2. We have,
$(5126)_7 = 6 \times 7^0 + 2 \times 7^1 + 1 \times 7^2 + 5 \times 7^3 = (1784)_{10}$

$$\begin{array}{r|l} 12 & 1784 \\ \hline 12 & 148 - 8 \\ \hline 12 & 12\ - 4 \\ \hline & 1\ - 0 \end{array}$$

∴$(5126)_7 = (1048)_{12}$ Ans : (1048)

3. $(101111011111)_2$
Decimal equivalent of 111 = 7
Decimal equivalent of 011 = 3
Decimal equivalent of 111 = 7
Decimal equivalent of 101 = 5
∴ $(1011\ 1101\ 1111)_2 = (5737)_8$ Ans : (5737)

4. We have,
$(254)_7 = 2 \times 7^2 + 5 \times 7 + 4 = (137)_{10}$
$(B5)_{12} = B \times 12 + 5 = 11 \times 12 + 5 = (137)_{10}$
$(211)_8 = 2 \times 8^2 + 1 \times 8 + 1 \times 8^0 = (137)_{10}$ Choice (D)

5. $(BAD)_{16} = D + (A) \times 16 + (B) \times 16^2$
$= 13 + 10 \times 16 + 11 \times 16^2$
$= 173 + 256 \times 11 = 173 + 2816 = (2989)_{10}$ Ans : (2989)

6.
$$\begin{array}{r|l} 2 & 575 \\ \hline 2 & 287 - 1 \\ \hline 2 & 143 - 1 \\ \hline 2 & 71 - 1 \\ \hline 2 & 35 - 1 \\ \hline 2 & 17 - 1 \\ \hline 2 & 8 - 1 \\ \hline 2 & 4 - 0 \\ \hline 2 & 2 - 0 \\ \hline & 1 - 0 \end{array}$$

∴ $(575)_{10} = (1000111111)_2$ Choice (C)

7.
$$\begin{array}{r|l} 16 & 2571 \\ \hline 16 & 160 - B \\ \hline & 10 - 0 \end{array}$$

∴$(2571)_{10} = (A0B)_{16}$ Choice (A)

8. $(2EFA)_{16} = 2\ (16^3) + E\ (16^2) + F\ (16) + A\ (A)$
$= 2\ (4096) + 14\ (16^2) + 15\ (16) + 10$
$= 8192 + 3584 + 240 + 10$
$= 12026$ Choice (C)

9. $(3AC)_{13} = 3\ (13^2) + A\ (13^1) + C\ (13^0)$
$= 3\ (169) + 10\ (13) + 12\ (A)$
$= 507 + 130 + 12$
$= (649)_{10}$

$$\begin{array}{r|l} 6 & 649 \\ \hline 6 & 108 - 1 \\ \hline 6 & 18 - 0 \\ \hline & 3 - 0 \end{array}$$

$\therefore (649)_{10} = (3001)_6$ Choice (B)

10.

12 12
A 1 2
8 3 9
----
1 9 5

$\therefore$ $(A12)_{12} – (839)_{12} = (195)_{12}$ Choice (B)

11. $(325)_8 = 3 \times 8^2 + 2 \times 8^1 + 5 \times 1 = (213)_{10}$
$(12)_8 = 1 \times 8^1 + 2 \times 1 = 10$
$(355)_8 \times (12)_8 = 213 \times 10 = (2130)_{10}$

Now,
$$\begin{array}{r|l} 8 & 2130 \\ \hline 8 & 266 – 2 \\ \hline 8 & 33 – 2 \\ \hline & 4 – 1 \end{array}$$

$\therefore$ $(2130)_{10} = (4122)_8$ Ans : (4122)

12. We have, $(121)_9 = 1 \times 9^2 + 2 \times 9 + 1 \times 1$
$= 81 + 18 + 1 = (100)_{10}$
$\therefore \sqrt{(121)_9} = \sqrt{(100)_{10}} = (10)_{10} = (11)_9$ Ans : (11)

13. $(312)_6 = 3 \times 6^2 + 1 \times 6 + 2 \times 1$
$= 108 + 6 + 2 = (116)_{10}$
$(116)^2 = 13456$

Now,
$$\begin{array}{r|l} 6 & 13456 \\ \hline 6 & 2242 – 4 \\ \hline 6 & 373 – 4 \\ \hline 6 & 62 – 1 \\ \hline 6 & 10 – 2 \\ \hline 6 & 1 – 4 \\ \hline & 0 – 1 \end{array}$$

$\therefore$ $(312)_6 \times (312)_6 = (142144)_6$ Ans : (142144)

14. We have, $(44)_8 = (36)_{10}$
$(12)_7 = (9)_{10}$
$\therefore$ LCM of $(36)_{10}$ and $(9)_{10}$ is $(36)_{10}$. Choice (B)

15. Consider $(452)_8$
Binary equivalent of 4 is 100
Binary equivalent of 5 is 101
Binary equivalent of 2 is 010
$\therefore$ $(452)_8 = (100101010)_2$
Similarly, we can show that,
$(AE2)_{16} = (101011100010)_2$.
$\therefore$ $(452)_8 + (AE2)_{16} =$
100101010
101011100010

11000 0001100

$\therefore$ $(452)_8 + (AE2)_{16} = (110000001100)_2$ Choice (B)

Exercise – 10
(Geometry)

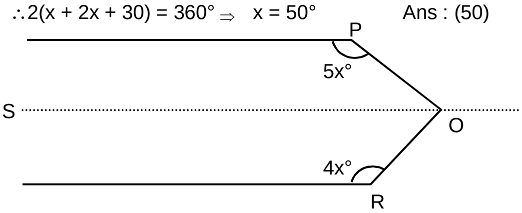
Solutions for questions 1 to 50:

1. $\angle 2 = \angle 4$ ($\because$ vertically opposite angles)
$\angle 2 = \angle 8$ ($\because$ alternate interior angles)
$\angle 8 = \angle 6$ ($\because$ vertically opposite angles)
$\Rightarrow$ $\angle 2 = \angle 4 = \angle 6 = \angle 8$
Let $\angle 2 = \angle 4 = x°$
Given that, $x + x = 80°$ $\Rightarrow x = 40°$
$\therefore \angle 1 + \angle 4 = 180°$
$\Rightarrow$ $\angle 1 = 140°$
$\therefore \Rightarrow$ $\angle 2 = \angle 4 = \angle 6 = \angle 8 = 40°$ Choice (B)

2. Since ℓ_1 and ℓ_2 are perpendicular to m, ℓ_1 is parallel to ℓ_2.
l_4 is parallel to l_3 $\therefore x + 50 = 90°$ $\Rightarrow$ $x = 40°$ Choice (B)

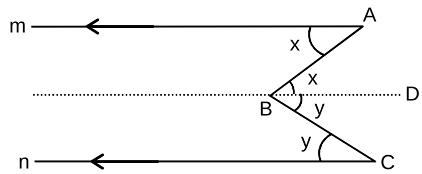
3. The angles opposite to x°, 2x° and 30° are also correspondingly x°, 2 x° and 30°.
$\therefore 2(x + 2x + 30) = 360° \Rightarrow x = 50°$ Ans : (50)

4.



By construction, draw a line parallel to the given two lines.
$\angle PQS = 180° – 5x$
($\because$ Sum of the interior angles on the same side of the transversal) and $\angle RQS = 180° – 4x$.
Therefore, $180° – 5x + 180° – 4x = 90°$
$\Rightarrow x = 30°$ Choice (C)

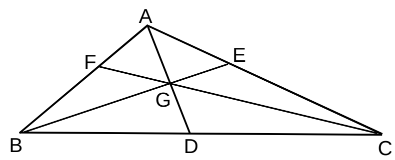
5. By construction draw a line parallel to m and n.
$\angle ABD = x$ and $\angle DBC = y$ ($\because$ Alternate angles)
$\therefore x + y = 80°$



Given that $y – x = 20°$.
By solving the two equations, we get,
$x = 30°$ Ans : (30)

6. In a triangle if one angle is equal to the sum of the other two angles, then it is a right angled triangle. Choice (B)

7.



The three medians AD, BE and CF divide the triangle into six triangles of equal areas. Quadrilateral FBDG consists of two such equal triangles. Triangle ADC consists of three such triangles. The required ratio is 2 : 3. Choice (D)

8. Since AD is the median so the hypotenuse, BC = 20 cm.
$AC = \sqrt{20^2 - 12^2} = 16$ cm. Ans : (16)

9. By apollonius theorem,
$AB^2 + AC^2 = 2(AD^2 + BD^2)$
$(5)^2 + (7)^2 = 2(AD^2 + 4^2)$
$\Rightarrow$ $AD = \sqrt{21}$cm.
AD lies between $\sqrt{16}$ cm and $\sqrt{25}$ cm, i.e. between 4 cm and 5 cm. Choice (B)

10. Let AD be the median to base BC.
BC = 40 cm $\Rightarrow$ BD = 20 cm
$AD = \sqrt{25^2 - 20^2} = 15$ cm. Ans : (15)

11. Only in an obtuse angled triangle,
$AB^2 + AC^2 < BC^2$ $\Rightarrow$ $\angle BAC > 90°$
i.e. $x > 90°$ Choice (C)

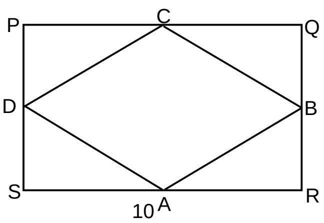
12. $\angle ADC = 180° – 105° = 75°$
$\angle ACD = 60°$
In triangle ADC,
$\angle DAC = 180° – (75° + 60°) = 45°$ Ans : (45)

13. In trapezium ABCD,
$\angle DAB + 70° = 180°$
$\Rightarrow \angle DAB = 110°$.
($\because$ Adjacent angles on the same side of the transversal).

Similarly, $\angle ADC + 60° = 180°$
$\Rightarrow \angle ADC = 120°$.
$\angle A + \angle C = 110° + 60° = 170°$
$\angle B + \angle D = 70° + 120° = 190°$
$\angle A + \angle C < \angle B + \angle D$ Choice (C)

14. $\angle ADE = \angle DAE = \angle AED = 60°$ (angle of an equilateral triangle) and 'E' is on produced $\overline{CD}$, so in rhombus ABCD, $\angle BAD = \angle BCD = 60°$.
In triangle BAE, $\angle BAE = 60° + 60° = 120°$.
AB = AD (sides of rhombus)
AD = AE (sides of equilateral triangle)
$\Rightarrow$ AB = AE.
$\therefore \angle ABE = \angle AEB$
In triangle ABE, $\angle ABE + \angle AEB + \angle BAE = 180°$
$\Rightarrow 2\angle AEB + 120° = 180° \Rightarrow \angle AEB = 30°$. Choice (B)

15.



AR = 10 cm
BR = 5 cm
$AB = \sqrt{10^2 + 5^2} = \sqrt{125} = 5\sqrt{5}$
Perimeter $= 4 \times 5\sqrt{5} = 20\sqrt{5}$ cm Choice (D)

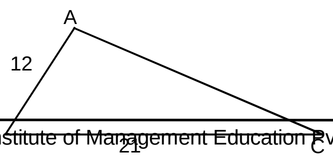
16. A rhombus is a parallelogram.
$\therefore$ Its area = product of adjacent sides $\times$ sine of the included angle.
Let the side of the square or the rhombus be 'a' cm.
The ratio of their areas = $a^2 : a^2 \sin 30°$
$= 1 : \frac{1}{2} = 2 : 1$ Choice (C)

17. The interior angle of a regular polygon of n sides $= \frac{(2n-4)90°}{n}$
For a hexagon, interior angle $= \frac{(2 \times 6 - 4) \times 90°}{6} = 120°$
In triangle CDE,
CD = DE (sides of a regular hexagon)
$\therefore \angle CED = \angle DEC = 30°$ Ans : (30)

18. (a) The ratio of sides AB, BC and CA is
$3 : 3\sqrt{3} : 6 = 1 : \sqrt{3} : 2$ so the angles opposite to these sides are 30°, 60°, 90°.
$\therefore$ The angle opposite $\overline{AC}$ is 90° i.e. $\angle ABC = 90°$
Ans : (90)

(b) In triangle ABC,
$4x = 180° \Rightarrow x = 45°$ and $2x = 90°$
ABC is an isosceles right angled triangle.
$\therefore AB : BC : CA = 1 : 1 : \sqrt{2}$
If CA = 10,
$AB = BC = 5\sqrt{2}$ cm
Area of triangle ABC
$= \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} = 25$ cm^2 Choice (C)

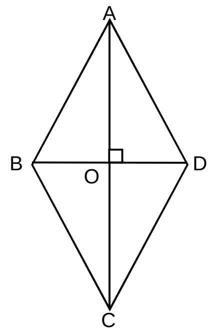
(c)



In a triangle, sum of the sides is greater than the third side and third side is greater than the difference between the two sides.
(21 – 12) < AC < (21 + 12)
9 < AC < 33
(21 + 12 + 9) < perimeter < (21 + 12 + 33)
$\Rightarrow$ 42 cm < p < 66 cm Choice (D)

19. Let the length of the rectangle and the base of the triangle be l units.
Given that,
$lb : \frac{1}{2}lh = 1 : 1 \Rightarrow \frac{b}{h} = \frac{1}{2} \Rightarrow b : h = 1 : 2$ Choice (C)

20.



Given that AD = 13 cm, OA $= \frac{24}{2} = 12$ cm.
$\Rightarrow OD = \sqrt{13^2 - 12^2} = 5$ cm.
So, BD = 10 cm
Area of the rhombus $= \frac{1}{2} \times AC \times BD$
$= \frac{1}{2} \times 24 \times 10 = 120$ cm^2 Ans : (120)

21. Since AC = BC
$\angle A = \angle B$ and $\angle A + \angle B = 140°$
($\because$ exterior angle is equal to the sum of the interior opposite angles)
$\therefore \angle A = \angle B = 70°$
$\angle C = 40°$ Choice (B)

22. $\angle QIR = 90° + \frac{1}{2}\angle P = 90 + \frac{1}{2}(80°) = 130°$ Choice (A)

23. In triangles ADE and ABC
$\angle A = \angle A$ (common)
$\angle ABC = \angle ADE$, $\angle AED = \angle ACB$ ($\because$ corresponding angles)
Triangle ADE is similar to triangle ABC. In similar triangles, areas are proportional to the squares of corresponding sides.
$$\frac{\text{Area of triangle ADC}}{\text{Area of triangle ABC}} = \frac{1^2}{4^2}$$
Area of triangle ADC $= \frac{1}{16} \times 432 = 27$ cm^2 Ans : (27)

24. $AC = \sqrt{12^2 + 5^2} = 13$ cm
BD is the median drawn to the hypotenuse
$\therefore BD = \frac{AC}{2} = \frac{13}{2} = 6.5$ cm Ans : (6.5)

25. In a parallelogram, sum of the adjacent angles is 180°
x + x – 40 = 180
$\Rightarrow 2x = 220 \Rightarrow x = 110°$
Opposite angles are equal
$\therefore y + 20 = x - 40$

$\Rightarrow$ y = 110 – 40 – 20 $\Rightarrow$ y = 50° Ans : (50)

26. Since BC = 80 cm, BE = 40 cm and AD = DC = 9 cm

AE $= \sqrt{40^2 + 9^2} = 41$ cm, AE = DE = 41 cm

Perimeter of triangle AED

= 80 + 41 + 41 = 162 cm Choice (C)

27. Given that BE = CE

In triangles ABE and DCE

AB = DC (sides of square),

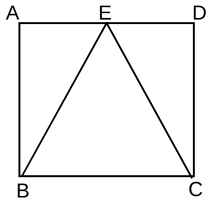
$\angle$BAE = $\angle$CDE

BE = CE (given)

$\therefore$ Triangles AEB and DEC are congruent.

Hence AE = ED

$\Rightarrow$ AE = ED = 5 cm



BE $= \sqrt{10^2 + 5^2} = 5\sqrt{5}$ cm {$\because$ AB = 10 cm} Choice (B)

28. Area of a rectangle = l $\times$ b

Area of the parallelogram = l $\times$ b $\times$ sin θ

where θ is the angle between sides l and b

$\therefore$The required ratio is

(l $\times$ b) : (l $\times$ b sin 30°)

$= 1 : \frac{1}{2} = 2 : 1 \quad = 2$ Ans : (2)

29. The sum of the exterior angles of a polygon is 360°. External angle of a regular polygon with n sides is given by 360°/n.

E = $\frac{360°}{12}$ = 30°.

Also I + E = 180° for any regular polygon.

$\therefore$ I = 180° – 30° = 150° Choice (C)

30. Measurement of each interior angle of a regular polygon of n sides

$= \frac{(2n-4)}{n} \times 90° \Rightarrow \frac{(2 \times 8 - 4)}{8} \times 90 = 135°$

$\therefore\angle$DAB = $\angle$CDA

Let $\angle$CDA = x

x + x + 135° + 135° = 360°

($\therefore$Sum of the angles in a quadrilateral)

$\Rightarrow$ x = 45° Choice (B)

31. Let r and R be the inradius and the circumradius of the circles respectively.

$2\pi r = 154$

r = $\frac{49}{2}$ cm

R : r = 2 : 1 $\Rightarrow$ R = 49 cm

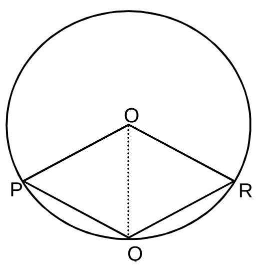
Circumference of the circumcircle

$= 2 \times \frac{22}{7} \times 49 = 308$ cm Ans : (308)

32. AC, the diagonal = 10 $\sqrt{2}$ cm {$\because$ AB = 10 cm}

AO = $\frac{1}{2} \times 10\sqrt{2}$ cm = $5\sqrt{2}$ cm Choice (B)

33.



Join OQ, POQ and ROQ are equilateral triangles.

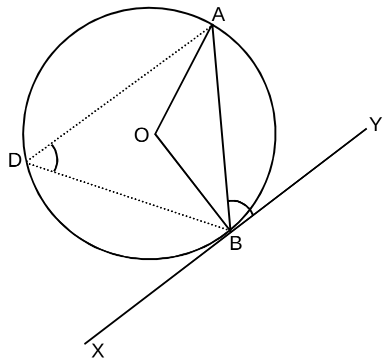
$\therefore\angle$POQ = 60° and $\angle$ROQ = 60°

$\angle$POR = 60° + 60° = 120° Choice (C)

34. Bigger arc makes an angle of 220° at the centre of the circle. It makes half the angle in the segment ACB.

$\therefore\angle$ACB = 110° Ans : (110)

35.



By construction take a point D on the circumference of the circle. Join AD and BD.

$\angle$ADB = $\angle$ABY = 30°

Angle made by the chord with the tangent is equal to the angle made in the alternate segment.

$\angle$AOB = 2($\angle$ADB)

$\therefore\angle$AOB = 60° Choice (C)

36. $\angle$ACD = $\angle$ABD = 80° (Angles in the same segment).

In triangle ABE,

$\angle$A + $\angle$ABD + $\angle$AEB = 180°

30° + 80° + $\angle$AEB = 180°

$\Rightarrow$ $\angle$AEB = 70° Ans : (70)

37. Since 'O' is the centre of the circle, BOD is its diameter.

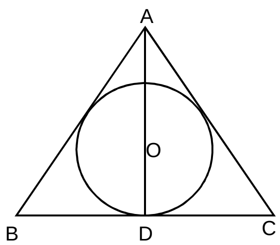
Therefore $\angle$BCD = 90°

$\angle$OBA = $\angle$OCD = 40°

(angles in the same segment are equal).

$\angle$BCA = $\angle$BCD – $\angle$OCD = 90° – 40° = 50° Choice (C)

38.



AD is the median of the triangle.

Given that OD = $3\sqrt{3}$ cm, so OA = $6\sqrt{3}$ cm.

$\therefore$ AD = $9\sqrt{3}$ cm.

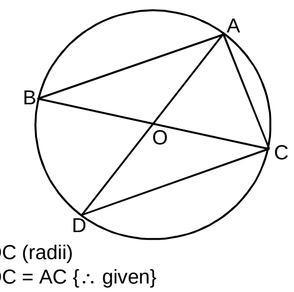
But AD is the altitude of the triangle ABC.

$\frac{\sqrt{3}}{2} \times s = 9\sqrt{3}$ cm. $\Rightarrow s = 18$cm.

Perimeter of triangle ABC = 3 $\times$ 18 = 54 cm.

Ans : (54)

39.



OA = OC (radii)

OA = OC = AC {$\therefore$ given}

OAC is an equilateral triangle.
Hence $\angle AOC = 60°$

$\angle ADC = \frac{1}{2} \times 60° = 30°$ Choice (A)

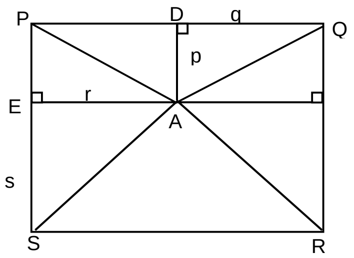
40. $\angle OQP = 90°$
($\therefore$ Radius makes an angle of 90° with the tangent).
$\angle QOP = 60° \{\because \angle OPQ = 30°\}$

$OQ : QP : PO = 1 : \sqrt{3} : 2.$

$OQ = \frac{12}{2} = 6$ cm $\{\because OP = 12$ cm$\}$ Choice (C)

41. The perpendicular distance from ℓ_1 to ℓ_2 is equal to that from ℓ_2 to ℓ_3
$\therefore \frac{PQ}{QR} = \frac{1}{1}$
PQ = QR = 6 cm Ans : (6)

42.



Let AD = P, DQ = q, AE = r and ES = s
$AQ^2 + AS^2 = p^2 + q^2 + r^2 + s^2 = p^2 + r^2 + q^2 + s^2$
$= AP^2 + AR^2$ Choice (D)

43. Let the perimeter of the square or perimeter of equilateral triangle be x cm.
$4s = x \Rightarrow s = x/4$

$3a = x \Rightarrow a = \frac{x}{3}$

Where 's' is the side of the square and 'a' is the side of the triangle.

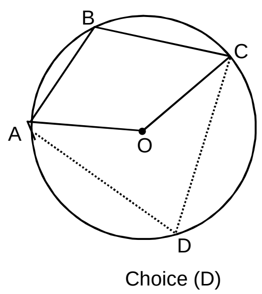
The required ratio is $s^2 : \frac{\sqrt{3}}{4} a^2$

$= \frac{x^2}{4^2} : \frac{\sqrt{3}}{4} \times \frac{x^2}{3^2} = 3\sqrt{3} : 4$ Choice (A)

44. Number of diagonals in a regular polygon of n sides $= \frac{n(n-3)}{2}$

Number of diagonals in a nonagon $= \frac{9(9-3)}{2} = 27$
Ans : (27)

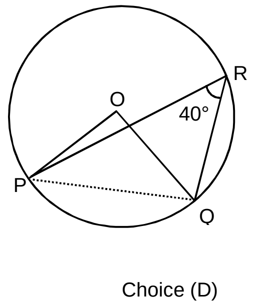
45. By construction, take any point D on arc AC, join AD and CD. Arc AC makes an angle of 150° with the centre O. Hence it makes 75° at point D.
$\therefore \angle ADC = 75°$
ABCD is a cyclic quadrilateral
$\therefore \angle B + \angle D = 180°$
$\angle B + 75° = 180°$
$\Rightarrow \angle B = 105°$



Choice (D)

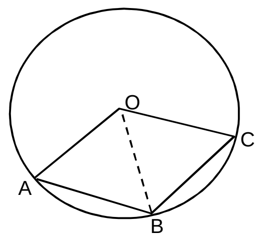
46. $\angle PAC = \angle ABC$ (By alternate segment theorem)
$\therefore \angle ABC = 80°$
$\angle AOC = 2\angle ABC = 2 \times 80 = 160°$ Choice (A)

47. Given that, $\angle PRQ = 40°$, $\angle POQ = 80°$
($\therefore$ The angle subtended by an arc at the centre is double the angle subtended by the same arc at a point on the remaining part of the circles).
In triangle OPQ, OP = OQ (radii)
$\therefore \angle OPQ = \angle OQP$ ($\because$ Angles opposite to equal sides)
$\angle OPQ + \angle OQP + 80 = 180°$
$\Rightarrow \angle OPQ = 50°$



Choice (D)

48.



Given that, $\angle OAB = 80°$ and $\angle OCB = 70°$
By construction, join OB, In triangle OAB, OA = OB (radii)
$\angle OAB = \angle OBA$ ($\because$ Angles opposite to equal sides)
$\therefore \angle AOB = 180° - (80 + 80) = 20°$ …… (1)
In triangle OBC, OC = OB (radii)
$\therefore \angle OCB = \angle OBC = 70°$
($\therefore$ Angles opposite to equal sides)
$\angle BOC = 180° - (70 + 70) = 40°$……… (2)
$\angle AOC = 20 + 40 = 60°$ (from (1) & (2)) Ans : (60)

49. By construction, join AB. $\angle BAC = 90°$
(Radius makes an angle of 90° with the tangent)
Since ABC is a right angled triangle and AB = 18 cm, AC = 24 cm, BC $= \sqrt{18^2 + 24^2} = 30$ cm
Ans : (30)

50. AC $= \sqrt{15^2 + 20^2} = 25$ cm
As radius of the semicircle is 12.5 cm, perimeter of the given figure $= \pi(12.5) + 15 + 20 = (12.5\pi + 35)$ cm
Choice (B)

Exercise – 11
(Mensuration)

Solutions for questions 1 to 50:

1. The triangle with sides 26 cm, 24 cm and 10 cm is right angled, where the hypotenuse is 26 cm.
Area of the triangle
$= \frac{1}{2} \times 24 \times 10 = 120$ cm^2 Ans : (120)

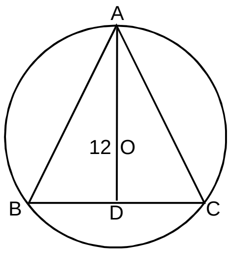
2. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times S^2$

where S is the side of triangle $= \frac{\sqrt{3}}{4} \times 16 \times 16$

$= 64\sqrt{3}$ cm^2 Choice (A)

3. Area of a triangle = r × s
where r is the inradius and s is the semi perimeter of the triangle
$\therefore$ Area of triangle $= 2.5 \times \frac{28}{2} = 35$ cm^2
Choice (B)

4.



In triangle ABC, AD is the perpendicular bisector as well as the median and height.
AO : AD = 2 : 3

$\therefore$ AD $= \frac{3}{2} \times 12 = 18\text{cm}, \frac{\sqrt{3}}{2} \times s = 18$

$\Rightarrow s = \frac{36}{\sqrt{3}}$ cm (where s is the side)

Area of the triangle $= \frac{\sqrt{3}}{4} \times s \times s$

$= \frac{\sqrt{3}}{4} \times \frac{36}{\sqrt{3}} \times \frac{36}{\sqrt{3}} = 108\sqrt{3}$ cm^2 Choice (D)

5. By basic proportionality theorem, $ST = \frac{PR}{2}$

Height of the triangle SQT $= \frac{1}{2}$ (height of triangle PQR)

$\therefore$ Area of the triangle SQT

$= \frac{1}{4}$ (Area of triangle PQR)

$= \frac{1}{4}(48) = 12$ cm^2 Ans : (12)

6. Area of the triangle $= \frac{1}{2} \times 3 \times 4 = 6$ cm^2

Semi perimeter (S) of the triangle $= \frac{3+4+5}{2} = 6$ cm

rs = Area of the triangle, where r is the inradius.
6r = 6 $\Rightarrow$ r = 1 cm
Area of the incircle = $\pi(1)^2 = \pi$ cm^2 Choice (A)

7. Area of the square = s $\times$ s = 5(125 $\times$ 64)
$\Rightarrow$ s = 25 $\times$ 8 = 200 cm
$\therefore$ Perimeter of the square = 4 $\times$ 200 = 800 cm
Ans : (800)

8. Area of the trapezium

$= \frac{1}{2} \times$ (Sum of the parallel sides)

(Distance between them) $= \frac{1}{2}(x^2 - y^2)$

$= \frac{1}{2}(x+y)(x-y)$

x and y are the parallel sides, sum of the sides = (x + y), so distance between the sides = (x – y)
Choice (D)

9. When a rectangle is inscribed in a circle, the diagonal of the rectangle will be diameter of the circle.

$\therefore$ Diameter $= \sqrt{18^2 + 24^2}$ = 30 cm

$\Rightarrow$ radius = 15 cm
Area of the circle = $\pi r^2 = \pi(15)^2 = 225\pi$ cm^2 Choice (D)

10. The sides of the biggest square in the rectangle will be 35 cm.
Area unused from the rectangle
= (Area of the rectangle) – (Area of the square)
= (40 $\times$ 35) – (35 $\times$ 35)
= 35 (40 – 35) = 35 $\times$ 5 = 175 cm^2 Ans: (175)

11. (a) Area of the rectangle = lb = 20 $\times$ 16 = 320 cm^2
Choice (A)

(b) Area of a square $= \frac{\text{Product of diagonals}}{2}$

$\frac{50}{2} = 25\text{cm}^2$ Choice (D)

(c) Area of rhombus $= \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 15 \times 10 = 150$ cm^2
Choice (A)

(d) Area of a parallelogram = base $\times$ height
= 24 $\times$ 16 = 384 cm^2 Choice (B)

12. Perimeter of the rectangle = 2(16 + 6) = 44 cm
$\therefore$ Circumference of the circle = $2\pi r$ = 44
$\Rightarrow$ r = 7 cm

Area of the circle $= \frac{22}{7} \times 7 \times 7 = 154\,\text{cm}^2$ Choice (A)

13. The diagonal (d) of the square is equal to the diameter of the circle = 40 cm

Area of the circle $= \frac{d^2}{2}$ = 800 cm^2 Choice (B)

14. The circumference of the circle is equal to the perimeter of the rectangle.
Let ℓ = 6x and b = 5x

$2(6x + 5x) = 2 \times \frac{22}{7} \times 3.5 \Rightarrow x = 1$

$\therefore \ell$ = 6 cm and b = 5 cm
Area of the rectangle = 6 $\times$ 5 = 30 cm^2 Ans : (30)

15. Since the circle and the square are symmetrical figures, there will be four equal areas in the circle that are not covered by the square. Therefore, area of shaded region

$= \frac{\text{Area of the circle} - \text{Area of the square}}{4}$

$= \frac{\pi(10)^2 - (20^2)/2}{4}$

$= (25\pi - 50)\text{cm}^2$ Choice (D)

16. Diameter of each circle = 7 cm

$\therefore$ Radius $= \frac{7}{2}$ cm

Total area of 4 circles $= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 154$ cm^2

Total area of the square = 14 $\times$ 14 = 196

The required fraction $= \frac{196 - 154}{196} = \frac{3}{14}$

Choice (B)

17. The hexagon can be divided into six equilateral triangles of equal areas.

Area of the hexagon $= 6 \times \frac{\sqrt{3}}{4} \times 4 \times 4 = 24\sqrt{3}$ cm^2

Choice (C)

18. Given that, r : h = 3 : 4, h $= \frac{4}{3}$r ,

where r is the radius of the sphere or cylinder and h is the height of the cylinder.
Ratio of the volumes of sphere and cylinder

$= \frac{4}{3}\pi r^3 : \pi r^2 h = \frac{4}{3} r^3 : r^2\left(\frac{4}{3}\right) r = 1 : 1$ Choice (C)

19. The total surface are a of a cube
= 6(S)^2, where S is the side

$\therefore$ Surface area = 6 $(8)^2$ = 384 cm^2 Ans : (384)

20. The length of the cuboid formed will be 16 cm its breadth and height remains 8 cm each.
Total surface area = 2lb + 2bh + 2hl
= 2 (16) (8) + 2 (8) (8) + 2 (8) (16)
= 640 cm^2 Choice (D)

21. Volume of a cuboid = ℓbh
$20 \times 8 \times 15 = 2400$ cm^3 Ans : (2400)

22. The volume of a cone $= \frac{1}{3}\pi r^2 h$

Only radius (r) and height (h) are varying. Hence $\frac{1}{3}\pi$ may be ignored.

$$\frac{V_1}{V_2} = \frac{r_1^2 h_1}{r_2^2 h_2} \Rightarrow \frac{1}{10} = \frac{(1)^2 h_1}{(2)^2 h_2} \Rightarrow \frac{h_1}{h_2} = \frac{2}{5}$$

i.e., $h_1 : h_2 = 2 : 5$ Choice (A)

23. Total area of the paths
= (Area of path I) + (Area of path II) – (common area)
= $(100 \times 2) + (80 \times 2) - (2 \times 2) = 356$ m^2 Choice (A)

24. In one revolution, the distance covered by the wheel is its circumference. Distance covered in 500 revolutions

$= 500 \times 2 \times \frac{22}{7} \times 28 = 88000$ cm = 880 m

Ans : (880)

25. Volume of the wire (Cylinder) is equal to the volume of the sphere.

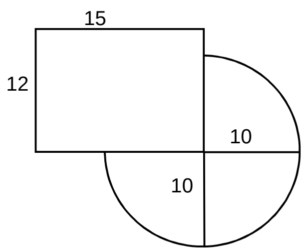
$\pi(16)^2 \times h = \frac{4}{3}\pi(12)^3 \Rightarrow h = 9$ cm Ans : (9)

26. Area of base = πr^2 = 154
$\Rightarrow$ r = 7 cm
Slant height $= \sqrt{h^2 + r^2}$
$= \sqrt{24^2 + 7^2}$ = 25 cm
$\therefore$ Curved surface area = πrl
$= \frac{22}{7} \times 7 \times 25 = 550$ cm^2 Choice (C)

27. Let the thickness of the sheet be t m.
(1600) (t) = 20.
t = $\frac{1}{80}$.
$\frac{1}{80}$ m = 1.25 cm Ans : (1.25)

28. Let the required maximum number be N_1.
Length of the rope = N_1 (circumference of the first cylinder's base) = (200) (circumference of the second cylinder's base)
$N_1(2.\pi.30) = (200)(2.\pi.42)$
$N_1 = \frac{200.42}{30} = 280$ Ans : (280)

29.



The area that can be grazed by the cow is in the form of three quadrants.
$\therefore$ Total area that can be grazed
$= \frac{3}{4} \times \pi(10)^2 = 75\pi$ m^2 Choice (C)

30. Given r = 35 m
where r is the radius of the garden.
R = r + width = 35 + 7, R = 42
Area of the path
= $\pi R^2 - \pi r^2$
$\frac{22}{7} \times (42)^2 - \frac{22}{7}(35)^2$
= 1694 m^2 Ans : (1694)

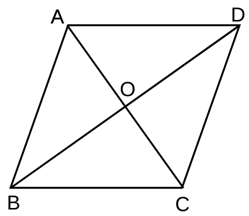
31. The angle in a semi circle is 90°.
Therefore $\angle$ACB = 90°
AB = 26 cm
AC = $\sqrt{(26)^2 - (10)^2}$ = 24 cm

Area of the shaded region = Area of the semi circle – Area of the triangle
$= \frac{1}{2} \cdot \pi \times 13^2 - \frac{1}{2} \times 10 \times 24$
$= \frac{1}{2}(169\pi - 240)$ cm Choice (C)

32. The volume of the cube = $12 \times 12 \times 12 = 1728$ cm^3.
Choice (A)

33. Area of a trapezium = 1/2 (sum of parallel sides) $\times$ perpendicular distance between them
$= \frac{1}{2}(20 + 10)(15)$
= 225 cm^2 Ans : (225)

34.



Given that
AC = 8 cm, BD = 6 cm
Let 'O' be the mid point of AC and BD.
BO = $\frac{1}{2}$BD = $\frac{1}{2} \times 6 = 3$
AO = $\frac{1}{2}$AC = $\frac{1}{2} \times 8 = 4$
In triangle AOB, $\angle$O = 90°
$AB^2 = AO^2 + OB^2$
$AB^2 = 4^2 + 3^2$
AB = $\sqrt{25}$ = 5
Perimeter of rhombus
= $4 \times AB = 4 \times 5 = 20$ cm. Ans : (20)

35. Volume of the cylinder = $\pi r^2 h$
$= \frac{22}{7} \times 7 \times 7 \times 12 = 1848$ cm^3. Choice (C)

36. The side of the cubical wooden block is 42 cm.
Diameter of the sphere is 42 cm.
Volume of sphere $= \frac{4}{3}\pi r^3$
$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 2 = 38808$ cm^3 Ans : (38808)

37. External length of the picture (with frame)
= internal length + 2 $\times$ width = 80 + 2 $\times$ 10 = 100 cm
and external breadth

= 50 + 2 × 10 = 70 cm
∴ Area of the frame = external area – internal area
= (100 × 70) – (80 × 50) = 3000 cm^2 Ans: (3000)

38. Let the radius and the length of the cylinder be r and h respectively.

$\frac{2\pi rh}{\pi r^2} = \frac{4}{3}$.

$\frac{h}{r} = \frac{2}{3}$.

$\therefore \frac{r}{h} = \frac{3}{2}$. Choice (A)

39. Let the radius of the circle be r. Let the side of the triangle be a.

$2\pi r = 3a$

$a = \frac{2\pi r}{3}$

$c = \pi r^2$

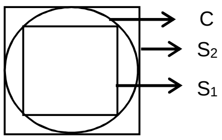
$E = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\left(\frac{2}{3}\pi r\right)^2 = \frac{\sqrt{3}}{4}.\frac{4}{9}.\pi^2 r^2 = \pi r^2\left(\frac{\sqrt{3}}{9}\pi\right)$

$\frac{\sqrt{3}\pi}{9} \simeq \frac{(1.73)(3.14)}{9}$ which is less than $\frac{(2)(4)}{9} = \frac{8}{9}$

$\therefore \frac{\sqrt{3}\pi}{9} < 1$.

∴ C > E. Choice (D)

40.



Side of S_2 = Diameter of C = Diagonal of S_1.
Let the radius of C be r.

Side of S_2 = 2r and $\sqrt{2}$ (side of S_1) = 2r i.e. side of S_1 = $\sqrt{2}$ r

Required ratio = $\left(\sqrt{2}r\right)^2 : (2r)^2 = 1 : 2$.

Choice (A)

41. Area of the field $= \frac{1}{2} \times 25 \times 20 = 250\ m^2$

∴ Cost of tilling the field = 250 × 15 = ₹3750

Ans : (3750)

42. Height of the triangle

$= \frac{\sqrt{3}}{2} \times 18 = 9\sqrt{3}$ cm

In an equilateral triangle height and median are equal.

So inradius = $\frac{1}{3} \times 9\sqrt{3} = 3\sqrt{3}$ cm

Area of the incircle = $\pi(3\sqrt{3})^2 = 27\pi$ cm^2

Choice (C)

43. The ratio of the sides of the two triangles = $\frac{4}{3} : \frac{1}{3}$ or 4 : 1

The ratio of their areas = $4^2 : 1^2$ = 16 : 1 Choice (C)

44. Area of a triangle = $\frac{abc}{4R}$

Where a, b, c are the sides and R is the circumradius of the triangle.

$\therefore$Area $= \frac{1500}{4 \times 5} = 75\ cm^2$ Choice (B)

45. Perimeter of the sector = length of the arc + 2(radius)

$= \left(\frac{135}{360} \times 2 \times \frac{22}{7} \times 21\right) + 2(21)$

= 49.5 + 42 = 91.5 cm Ans : (91.5)

46. $\frac{1}{3}\pi r^2 h = 196\pi$

$\Rightarrow r^2h = 196 \times 3$ --------- (1)

Area of the base = $\pi r^2 = 154 \Rightarrow r^2 = 49$

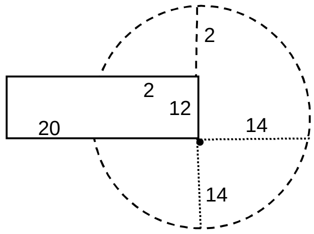
$\Rightarrow$ r = 7cm $\Rightarrow h = \frac{196 \times 3}{49}$ = 12 {from (1)} Ans : (12)

47. Volume of the cubical metallic piece
= volume of the water column

$\Rightarrow 3 \times 3 \times 3 = \pi(r^2)(3) \Rightarrow \pi(r^2)(3) = 3 \times 3 \times 3$

$\Rightarrow r = 3/\sqrt{\pi}$ cm Choice (B)

48.



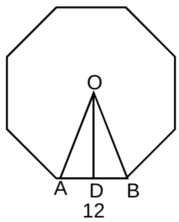
The area that can be covered by the cow is shown in the above figure. It consists of three quadrants of radius 14ft and one quadrant of radius 2 ft

∴The required area

$= 3 \times \frac{1}{4} \times \pi(14)^2 + \frac{1}{4} \times \pi(2^2)$

= 147 π + π = 148 π sq. feet Choice (A)

49.



AB is the side of the octagon and OD is the perpendicular distance from the centre to AB, in the triangle AOB DB = 6 cm

$\angle AOB = \frac{360^\circ}{8} = 45^\circ$

$\angle DOB = \frac{45^\circ}{2}$

$OD = \frac{6}{\tan\frac{45^\circ}{2}}$

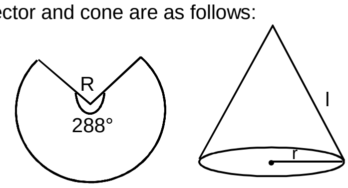
$\therefore$ OD = $6\cot\frac{45^\circ}{2}$

Area of Δ AOB = $\frac{1}{2} \times AB \times OD = \frac{1}{2} \times 12 \times 6\cot\frac{45^\circ}{2}$

Area of the octagon = 8 × Area of Δ AOB

$= 8 \times \frac{1}{2} \times 12 \times 6\cot\frac{45^\circ}{2} = 288\cot\frac{45^\circ}{2}$ Choice (A)

50. The sector and cone are as follows:



The radius (r) of the sector will be the slant height (l) of the cone.

The length of the arc of the sector is circumference of the base of the cone.

$\therefore$ l = 15 cm and

$\frac{288}{360} \times 2 \times \pi \times 15 = 2 \times \pi r \Rightarrow$ r = 12 cm

$h = \sqrt{15^2 - 12^2} = 9\text{cm}$

$\therefore$ Volume of the cone

$= \frac{1}{3}\pi(r^2)h = \frac{1}{3} \times \pi(12)^2 \times 9 = 432\pi\text{cm}^3$ Choice (C)

Exercise – 12
(Coordinate Geometry)

Solutions for questions 1 to 40:

1. Distance = $\sqrt{(2+4)^2 + (3-5)^2}$

$= \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = 2\sqrt{10}$ Choice (B)

2. Let (x, 0) be the centre

$\Rightarrow \sqrt{(x-4)^2 + 5^2} = 5$

$\Rightarrow (x - 4)^2 = 0$

$(x - 4) = 0 \Rightarrow x = 4.$

Centre = (4, 0) Ans : (4)

3. Proceeding from choices, Choice (C) satisfies the given condition. Choice (C)

4. d = $\sqrt{1+9} = \sqrt{10}$

AB = 3d = $3\sqrt{10}$ Choice (D)

5. The ratio in which (x, y) divides the line joining (x_1, y_1) and (x_2, y_2) is $x_1 - x : x - x_2$ or $y_1 - y : y - y_2$

$\Rightarrow$ ratio = –2 – 2 : 2 – 0 = –4 : 2 = –2 : 1

i.e. 2 : 1 externally Choice (B)

6. Area = $\frac{1}{2}\begin{vmatrix} 3+1 & 2+2 \\ -1+1 & -2-4 \end{vmatrix} = 12\text{sq.units}$

Ans : (12)

7. Area of triangle = 4(Area of mid point triangle)

Area of mid point triangle = $\frac{1}{2}\begin{vmatrix} -6+\frac{5}{2} & -4+\frac{1}{2} \\ -\frac{5}{2}+\frac{9}{2} & -\frac{1}{2}-\frac{3}{2} \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -\frac{7}{2} & \frac{-7}{2} \\ 2 & -2 \end{vmatrix} = \frac{1}{2}|7 + 7| = 7$

$\therefore$ Area of triangle = 4 $\times$ 7 = 28 Ans : (28)

8. Equation of the line through (2, 2) and (5, 5) is x = y. This passes through (a, 4)

$\Rightarrow$ a = 4 Ans : (4)

9. Let A(0, 0), B(3, 3) and C(k, 0) be the given points.

From the choices, when k = 6

AB = $\sqrt{9+9} = \sqrt{18}$ units

BC = $\sqrt{(3-6)^2 + (3-0)^2} = \sqrt{18}$ units

And CA = $\sqrt{(6-0)^2 + (0-0)^2} = \sqrt{36}$ units

$\Rightarrow AB^2 + BC^2 = 18 + 18 = 36 = CA^2 \Rightarrow \angle B = 90°$.

Also, AB = BC

So, when k = 6, ΔABC is a right angled isosceles triangle.

Choice (D)

10. The two given vertices are rational. In an equilateral triangle if two vertices are rational, the third vertex should be irrational. The irrational vertex is given in Choice (D).

Choice (D)

11. Centroid of the mid point triangle is the centroid of the triangle

$\therefore$ centroid = $\left(\frac{3+5+4}{3}, \frac{6+4+2}{3}\right)$ = (4, 4) Choice (A)

12. Centroid = (1, 1)

$\Rightarrow \left(\frac{2+4+x}{3}, \frac{5+6+y}{3}\right) = (1, 1)$

x = –3, y = –8

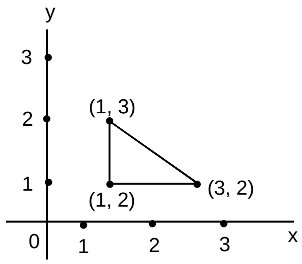
(x, y) = (–3, –8) Choice (A)

13. Centroid divides AD median in the ratio 2 : 1.

$\Rightarrow$ Centroid = $\left(\frac{2(3)+1(-1)}{2+1}, \frac{2(5)+1(7)}{2+1}\right)$

= (5/3, 17/3) Choice (B)

14.



Plotting the given points we find that the triangle is a right angled triangle. By observation, since the x-coordinate of (1, 2) and (1, 3) is the same and the y-coordinate of (1, 2) and (3, 2) is same, the triangle is right angled triangle, right angled at the common point (1, 2). This is the orthocentre.

Choice (B)

15. Let A(0, 0), B(0, 6) and C(6, 0)

$AB^2 = 6^2$

$BC^2 = 6^2 + 6^2$

$AC^2 = 6^2$

$BC^2 = AB^2 + AC^2$

$\Rightarrow$ Given triangle is right angle at A.

Hence orthocentre is A (0,0)

Circumcentre is mid point of hypotenuse.

$\Rightarrow$ Circumcentre = (6/2, 6/2) = (3, 3)

$\Rightarrow$ S = (3, 3)

OS = $\sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$ Choice (C)

16. A (5, 6), B (7, 8), C (4, 10) and D (x, y)

Mid point of AC = Mid point of BD

$\Rightarrow \left(\frac{9}{2}, 8\right) = \left(\frac{7+x}{2}, \frac{y+8}{2}\right) \Rightarrow$ x = 2 and y = 8

$\therefore$ (x, y) = (2, 8). Choice (D)

17. Do not intersect $\Rightarrow$ parallel. $\Rightarrow a_1/a_2 = b_1/b_2$

$\Rightarrow$ 3/k = 4/1 $\Rightarrow$ k = 3/4 = 0.75 Ans : (0.75)

18. Condition of perpendicularity is

$\Rightarrow a_1 a_2 + b_1 b_2 = 0$

$\Rightarrow$ 1(7) + (–1)k = 0

$\Rightarrow$ k = 7 Ans : (7)

19. 4(x – 2) + 3(y – 1) = 0

4x – 8 + 3y – 3 = 0

4x + 3y – 11 = 0 Choice (D)

20. 5(x – 2) – 2(y – 1) = 0

5x – 2y – 10 + 2 = 0 $\Rightarrow$ 5x – 2y – 8 = 0 Choice (D)

21. Slope of the line passing through the points (2, 3) and (–4, 1) = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$

Since, the required line is perpendicular to the line joining the points (2, 3) and (–4, 1), the slope of the required line = –3

Also, required line passes through the point (1, –2).
$\therefore$ Equation of the required line is $y - y_1 = m(x - x_1)$
i.e., $y - (-2) = -3(x - 1)$
i.e., $y + 2 = -3x + 3$
i.e., $3x + y - 1 = 0$ Choice (B)

22. The line parallel to x-axis, is y = c
$\Rightarrow y = 8$ Ans : (8)

23. (1, 1) satisfies the both equations. Choice (C)

24. $x + y + a = 0$ passes through (1, 2)
$\Rightarrow 1 + 2 + a = 0$
$\Rightarrow a = -3$. Equation is $x + y - 3 = 0$.
$x + y = 3 \Rightarrow x/3 + y/3 = 1$
Intercepts are 3, 3. Ans : (6)

25. Let the line be $x/a + y/a = 1$
$\Rightarrow x + y = a$
As the above line passes though (–2, 1/2), a = –3/2
$\therefore$ Required line is $2(x + y) + 3 = 0$ Choice (A)

26. $7x + 4y = 28 \Rightarrow x/4 + y/7 = 0$
Area = $\frac{1}{2}(4 \times 7) = 14$ Ans : (14)

27. $-\left[\frac{3(1)+2(2)-12}{3(4)+2(5)-12}\right] = -\left[\frac{\text{less than zero}}{\text{greater than zero}}\right] = +\text{ve}$
Hence both the points lie on the opposite side of the line.
Choice (B)

28. $x^2 - 3x + 2 = 0$
$x_1 = 1 \Rightarrow (x - 1)(x - 2) = 0$
$x_2 = 2$. Intercepts may be 1, 2 or 2, 1
$\Rightarrow$ Equations are $x/1 + y/2 = 1$ or $x/2 + y/1 = 1$
Choice (C)

29. Distance = $\frac{|c|}{\sqrt{a^2 + b^2}}$
$\therefore$ Distance = $\frac{|-9|}{\sqrt{4^2 + 3^2}} = \frac{9}{5}$ Ans : (1.8)

30. Distance between parallel lines is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$
$\Rightarrow \frac{|-5 + 11|}{\sqrt{4^2 + 3^2}} = \frac{6}{5}$ Ans : (1.2)

31. Let 'a' be the side of square. Length of diagonal is
$\sqrt{2}a = \sqrt{(1 - 3)^2 + (2 - 4)^2}$
$\Rightarrow 2a^2 = 2^2 + 2^2 \Rightarrow 2a^2 = 8$
$a^2 = 4$ Area is a^2
$\therefore$ Area = 4sq. units Ans : (4)

32. $x + y + 3 = 0$, $x - y - 1 = 0$
$\Rightarrow x = -1, y = -2$
It passes through $ax + y + 4 = 0$
$\Rightarrow -a - 2 + 4 = 0 \Rightarrow a + 2 = 4$
a = 2 Ans : (2)

33. Point of intersection of lines $3x + y = 2$ and $x + 4y + 3 = 0$ is (1, –1)
$\therefore$ Required line = line perpendicular to $x + 2y + 4 = 0$ and passing through (1, –1)
$2(x - 1) - (y + 1) = 0$
$\Rightarrow 2x - y = 3$ Choice (C)

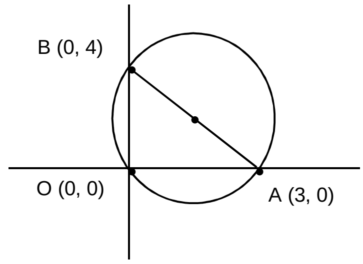
34. Given lines are,
$(ax-2y+1 = 0)b \Rightarrow abx - 2by + b = 0 \quad \rightarrow \quad (1)$
$(bx-3y+1 = 0)a \Rightarrow abx - 3ay + a = 0 \quad \rightarrow \quad (2)$

Solving (1) and (2)
$y = \frac{a-b}{3a-2b}$ and $x = \frac{-1}{-2b+3a}$
substitute x and y in equation 3
$c\left[\frac{-1}{-2b+3a}\right] - 4\left[\frac{a-b}{-2b+3a}\right] + 1 = 0$
$-c - 4a + 4b + 3a - 2b = 0$
$-a + 2b = c$
$c + a = 2b$ Ans : (2)

35. As (1, –2) and (–1, 4) lie on $y = mx + c$;
We have, $m + c = -2$ and $-m + c = 4$
Solving above equations, we get m = –3, c = 1
$\therefore$ Ordered pair (c, m) = (1, –3) Choice (A)

36. Given, $x = a + e^t$ and $y = b + e^{-t}$
$\Rightarrow x - a = e^t$ and $y - b = e^{-t}$
$\Rightarrow (x - a)(y - b) = e^t \cdot e^{-t}$
$\Rightarrow xy - bx - ay + ab = 1$
i.e., $xy - bx - ay + ab - 1 = 0$ Choice (C)

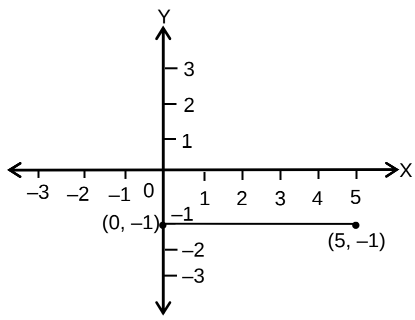
37.



According to the given conditions, the circle must pass through the point (0, 0), (3, 0) and (0, 4).
Only Choice (C) satisfies all the three points.
Hence, Choice (C) is the right option. Choice (C)

38. Required point is the image of B(–1, 2) w.r.t the line $x + 2y + 1 = 0$. We know Image (h, k) of point (x_1, y_1).
with respect to $lx + my + n = 0$ is given by
$\frac{h - x_1}{\ell} = \frac{k - y_1}{m} = \frac{-2(lx_1 + my_1 + n)}{\ell^2 + m^2}$
$\frac{h + 1}{1} = \frac{k - 2}{2} = \frac{-2(-\ell + 4 + 2)}{5}$
$\Rightarrow h + 1 = -2$ and $k - 2 = -4$
$\Rightarrow h = -3; k = -2$
$\therefore$ (h, k) = (–3, –2) Choice (C)

39.



Foot of the perpendicular from the point (5, –1) to y-axis is (0, –1) Choice (B)

40. The equation of the line passing through P(3, 4) and making an angle 60° with x-axis is $y - 4 = \tan 60°(x - 3)$
$y - 4 = \sqrt{3}(x - 3)$
$\sqrt{3x} - y + 4 - 3\sqrt{3} = 0$

When y = 0 $\Rightarrow$ x = $\dfrac{3\sqrt{3}-4}{\sqrt{3}}$

$\therefore$ Q $\left(3-\dfrac{4}{\sqrt{3}},\ 0\right)$

PQ = $\sqrt{\left(3-\left(3-\dfrac{4}{\sqrt{3}}\right)\right)^2+(4-0)^2} = \sqrt{\dfrac{16}{3}+16} = \dfrac{8}{\sqrt{3}}$

$a/\sqrt{3} = \dfrac{8}{\sqrt{3}} \Rightarrow a = 8.$ Ans: (8)

Exercise – 13
(Trigonometry)

Solutions for questions 1 to 40:

1. $\dfrac{4}{5}\pi^\circ = \dfrac{4}{5}\times 180 = 36\times 4 = 144^\circ$ Ans : (144)

2. $108^\circ = 108\times\dfrac{\pi}{180} = \dfrac{3\pi^c}{5}$ Choice (B)

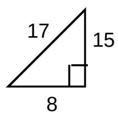
3. sin (60 – π/6) = sin (60° – 30°)
= sin 30° = 1/2 Ans : (0.5)

4. tan (A + B) = $\sqrt{3}$, cos A = $1/\sqrt{2}$
Least A + B = 60°, Least A = 45°
Least B = 60° – 45° = 15° Ans : (15)

5. tan (A + B) = $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$
$\Rightarrow$ Least A + B = 75°
cos A = ½ $\Rightarrow$ Least A = 60°
Least B = 75 – 60 = 15° = $\dfrac{\pi}{12}$ Choice (A)

6. sin θ = $\dfrac{15}{17}$

tanθ = $\dfrac{15}{8}$



Ans : (1.875)

7. $\dfrac{360}{60}$ = 6° in 1 min
$\therefore$ for 25 min - ?
25 × 6 = 150° Choice (D)

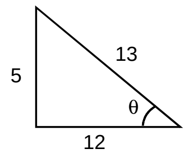
8. $3(\sin^2 30^\circ + \cos^2 120^\circ) - 4(\sin^6 45^\circ + \cos^6 135^\circ)$
= $3((½)^2 + (–½)^2)$
$-4\left((1/\sqrt{2})^6 + (-1/\sqrt{2})^6\right)$
= $3(½) - 4\times\dfrac{1}{4} = \dfrac{3}{2} - 1 = \dfrac{1}{2}$ Ans : (0.5)

9. $3\sin^2 A = \sin 30^\circ + \cos^2 45^\circ$
$3\sin^2 A$ = ½ + ½
sin A = $1/\sqrt{3}$
cosec A = $\sqrt{3}$
$\text{cosec}^2 A = 3$ Ans : (3)

10. L = rθ = 1100
θ = $\dfrac{1100}{2L}$
θ = 50° = 5π/18 Choice (A)

11. $\sec\dfrac{8\pi}{4} = \sec(2\pi) = 1$ Ans : (1)

12. Since tanθ = 5/12, we get the right angled triangle as follows:



As θ is in third quadrant, sinθ = –5/13
Now cot θ + sin θ = $\dfrac{12}{5} + \left(\dfrac{-5}{13}\right) = \dfrac{156-25}{65} = \dfrac{131}{65}$
Choice (B)

13. cosA = $\dfrac{4}{5}$; cotB = $\dfrac{12}{5}$
sinA = $\dfrac{3}{5}$; cosB = $\dfrac{-12}{13}$
$25\sin^2 A + 13\cos B$
= $25\left(\dfrac{9}{25}\right) + 13\left(\dfrac{-12}{13}\right)$ = 9 – 12 = –3. Choice (B)

14. $\cos^2\theta + 5\sin^2\theta = 4$
$\Rightarrow 1 + 5\tan^2\theta = 4\sec^2\theta$
$\Rightarrow 1 + 5\tan^2\theta = 4 + 4\tan^2\theta$
$\Rightarrow \tan^2\theta = 3$ Ans : (3)

15. $\dfrac{1-\cos\theta+1+\cos\theta}{1-\cos^2\theta} = \dfrac{2}{\sin^2\theta}$
= $2\text{cosec}^2\theta$. Choice (C)

16. $\sqrt{\dfrac{1-\cos x}{1+\cos x}} = \sqrt{\dfrac{(1-\cos x)^2}{(1+\cos x)(1-\cos x)}} = \dfrac{1-\cos x}{\sqrt{\sin^2 x}}$
= $\dfrac{1-\cos x}{\sin x}$ = cosecx – cotx. Choice (B)

17. cos1°· cos2°· cos3°....... cos179° = 0 [$\because$ cos90° = 0].
Ans : (0)

18. tan 89° = tan (90 – 1) = cot1°.
We start pairing the extreme terms log tan 1° + log tan 89°
= log tan 1 + log cot 1
= log tan1°. cot 1° = log 1 = 0
Like wise every other pair would vanish except log tan 45.
log tan 45° = log 1 = 0.
Thus the expression equals zero. Choice (B)

19. $12(\sin^4 x + \cos^4 x) - 8(\sin^6 x + \cos^6 x)$
= $12[(\sin^2 x)^2 + (\cos^2 x)^2] - 8[(\sin^2 x)^3 + (\cos^2 x)^3]$
= $12[(1)^2 - 2\sin^2 x\,.\,\cos^2 x] - 8[(1)^3 - 3\sin^2 x\,.\,\cos^2 x(1)]$
= $12 - 24\sin^2 x\,.\,\cos^2 x - 8 + 24\sin^2 x\,.\,\cos^2 x = 4$

Alternate:
Put x = 0 $\Rightarrow$ 12 (0 + 1) – 8 (0 + 1)
$\Rightarrow$ 12 – 8 = 4. Choice (A)

20. We know that, $\sec^2\theta - \tan^2\theta = 1$
$\Rightarrow$ (secθ + tanθ) (secθ – tanθ) = 1
$\Rightarrow$ secθ – tanθ = $\dfrac{1}{\sec\theta+\tan\theta} = \dfrac{1}{-5}$
–(secθ – tanθ) = $1/5$ = 0.2 Ans : (0.2)

21. For 0 ≤ θ ≤ 180°,
cosθ lies in the interval [–1, 1] and sinθ in the interval [0, 1]. Choice (D)

22. Let the angle be θ. Then the complement of θ = 90° – θ and supplement of θ = 180° – θ.
Given, 90° – θ = $\dfrac{1}{3}$(180° – θ)
$\Rightarrow$ 270° – 3θ = 180° – θ $\Rightarrow$ 2θ = 90° $\Rightarrow$ θ = 45°
Hence, θ = 45°. Ans : (45)

23. cos30 cosx + sin30 sinx = cos (30 – x) or cos (x – 30).
Now cos (x – 30) =1/2.
$\Rightarrow$ x – 30 = 60 or x = 90° or $\pi/2$ Choice (D)

24. $k\left(\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}\right) = \cos^2\theta.$

$\Rightarrow k\left(\frac{(1+\sin\theta)/\cos\theta}{(1-\sin\theta)/\cos\theta}\right) = \cos^2\theta$

$\Rightarrow k\left(\frac{1+\sin\theta}{1-\sin\theta}\right) = (1-\sin^2\theta) \Rightarrow k = (1-\sin\theta)^2$

Choice (A)

25. $x^2 = (2k\cos A \sin 2A - k\sin A)^2$
$= 4k^2\cos^2 A \sin^2 2A + k^2\sin^2 A - 4k^2 \sin A \cos A \sin 2A$
$y^2 = 4k^2\cos^2 A\cos^2 2A + k^2\cos^2 A - 4k^2\cos^2 A\cos 2A$
$\Rightarrow x^2 + y^2 = 4k^2\cos^2 A(\sin^2 2A + \cos^2 2A) + k^2(\sin^2 A + \cos^2 A)$
$-4k^2\sin A\cos A(2\sin A\cos A) - 4k^2\cos^2 A\cos^2 2A$
$=4k^2\cos^2 A(1) + k^2(1) - 4k^2\cos^2 A(2\sin^2 A + \cos^2 2A)$
$= 4k^2\cos^2 A + k^2 - 4k^2\cos^2 A(2\sin^2 A + 1 - 2\sin^2 A)$
$= 4k^2\cos^2 A + k^2 - 4k^2\cos^2 A = k^2$
Hence, $x^2 + y^2 = k^2$.

Alternate method:
Put A = 45°
x = 2kcosAsin2A – ksinA
$x = \sqrt{2}k - \frac{k}{\sqrt{2}}$

$y = 2k\cos A\cos 2A - k\cos A = 0 - \frac{k}{\sqrt{2}}$

$\therefore x^2 + y^2 = \left(\sqrt{2}k - \frac{k}{\sqrt{2}}\right)^2 + \left(\frac{-k}{\sqrt{2}}\right)^2 = 2k^2 + \frac{k^2}{2} - 2k^2 + \frac{k^2}{2}$
$x^2 + y^2 = k^2$ Choice (C)

26. We know that sinAcosB + cosAsinB = sin (A + B) and
cosAcosB – sinAsinB = cos(A+B)
$\therefore(\sin A\cos B + \cos A\sin B)^2 + (\cos A\cos B - \sin A\sin B)^2$
$= ((\sin(A + B))^2 + (\cos(A + B))^2$
$\sin^2(A + B) + \cos^2(A + B) = 1$ Ans : (1)

27. We know that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

So, $\frac{\tan 48° + \tan 12°}{1 - \tan 48°\tan 12°} = \tan(48° + 12°)$

$= \tan 60° = \sqrt{3}$
$\tan^2 60 = 3$ Ans : (3)

28. We know that cos(A – B) = cosAcosB+ sinAsinB
Put A = 45° and B = 30° we have
cos(45° – 30°) = cos45°cos30° + sin45°sin30°

$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}.$ Choice (B)

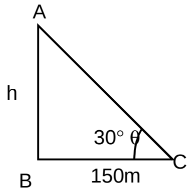
29. Minimum value $= -\sqrt{(8)^2 + (15)^2} = -17.$

Choice (B)

30. $\frac{\cos 75° - \sin 75°}{\cos 75° + \sin 75°} = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}} - \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}}}$

$= \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$ Choice (C)

31.

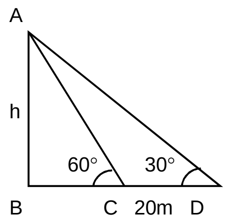


AB represents the height of the pole and C the point of observation.
Given BC = 150m
tan 30° = AB/BC = h/150

$\Rightarrow h = 150\left(\frac{1}{\sqrt{3}}\right) = 50\sqrt{3}m$

$\therefore$ Height of flag pole = $50\sqrt{3}m$ Choice (B)

32.



Let AB be the height of building.
From ΔABC, tan60° = AB/BC

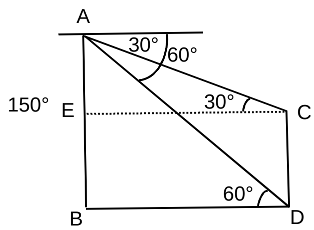
$\Rightarrow BC = \frac{h}{\sqrt{3}} \to (1)$

from ΔABD, tan 30° = AB/BD = AB/(BC + CD)
$\Rightarrow BC + 20 = AB(\sqrt{3})$
$\Rightarrow BC = \sqrt{3}h - 20 \to (2)$

From (1) and (2) we have $\frac{h}{\sqrt{3}} = \sqrt{3}h - 20$

$\Rightarrow h = 3h - 20\sqrt{3} \Rightarrow 2h = 20\sqrt{3}$
$\Rightarrow h = 10\sqrt{3}$ Choice (D)

33.



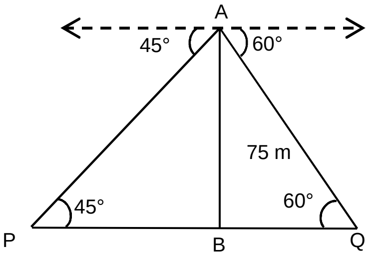
Let AB be the tower and CD be the building
Given AB = 150m
In Δ AEC, tan 30° = AE/CE
$\Rightarrow CE = AE\sqrt{3}$ and
In Δ ABD, tan 60° = AB/BD
$BD = AB\left(\frac{1}{\sqrt{3}}\right)$
But CE = BD and AB = AE + EB
$\Rightarrow AE\sqrt{3} = AB\left(\frac{1}{\sqrt{3}}\right)$
$\Rightarrow$ 3AE = AB = 150
$\Rightarrow$ AE = 50m

$\therefore$ CD = BE = 150 – 50 = 100m Ans : (100)

34. (i) $5\pi/18 = (5\pi/18) \times (180°/\pi)$ [since $1^c = 180°/\pi$]
= 50° Ans : (50)

(ii) $7\pi/5 = (7\pi/5) \times (180/\pi)$ = 252°
Ans : (252)

35.



In ΔPBA, $\tan 45° = \dfrac{AB}{PB}$

$\Rightarrow 1 = \dfrac{75}{PB}$

$\Rightarrow$ PB = 75 m

In ΔQBA, $\tan 60° = \dfrac{AB}{QB}$

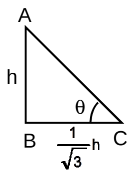
$\Rightarrow \sqrt{3} = \dfrac{75}{QB} \quad \Rightarrow QB = \dfrac{75}{\sqrt{3}} = \dfrac{75\sqrt{3}}{3}$

Hence, the distance PQ = PB + QB = $75 + \dfrac{75\sqrt{3}}{3}$

$= 75\left(\dfrac{3+\sqrt{3}}{3}\right)$m $= 25(3 + \sqrt{3})$ m. Choice (A)

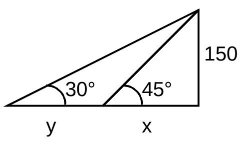
36. $\tan\theta = \dfrac{h}{\frac{1}{\sqrt{3}}h} = \sqrt{3}$

θ = 60°.



Ans : (60)

37.



$\tan 45° = \dfrac{150}{X}$

x = 150

$\tan 30° = \dfrac{150}{x+y}$

$x + y = 150\sqrt{3}$

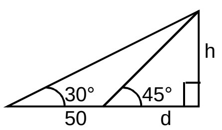
$y = 150\sqrt{3} - 150 = (\sqrt{3} - 1)\,150$

= 0.7 × 150 = 110 m less Choice (A)

38. $\tan 45° = \dfrac{h}{d}$

h = d

$\tan 30° = \dfrac{h}{50+d}$



$\sqrt{3}$ h = 50 + d

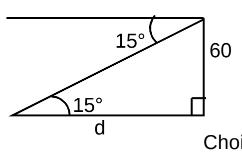
$\sqrt{3}$ d = 50 + d

$h = d = \dfrac{50}{\sqrt{3}-1} = 25\left(\sqrt{3}+1\right) = 25(\sqrt{3} + 1)$ m

Choice (D)

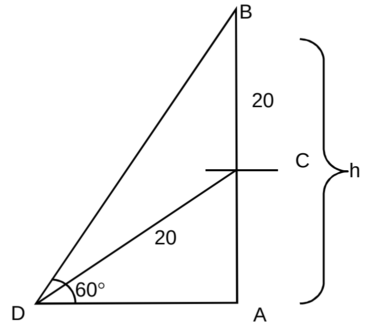
39. $\tan 15° = \dfrac{60}{d}$

$\dfrac{1}{2+\sqrt{3}} = \dfrac{60}{d}$



$d = 60\,(2 + \sqrt{3})$ Choice (C)

40. Let AB be the flag whose height be 'h' feet.



Since ∠ADB = 60°, ∠DAB = 90°. So ∠ABD = 30°.
In ΔDCB, DC = CB,
$\therefore$ ∠CBD = ∠CDB = 30°
$\Rightarrow$ ∠ADC = 60°– 30°= 30°

$\sin 30° = \dfrac{AC}{DC}$

$\dfrac{1}{2} = \dfrac{AC}{20} \Rightarrow$ AC = 10 feet.

$\therefore$ AB = AC + CB = 10 + 20 = 30 feet. Ans : (30)

Exercise – 14
(Operator Based Questions)

Solutions for questions 1 and 2:

1. A (B (1, – 1), D (2, 1))
= A (0, 1) = 1 Ans : (1)

2. We have, C (x, y). D (x, y) – B (x, y)
= (x + y) (x – y) – ($x^2 - y^2$)
= ($x^2 - y^2$) – ($x^2 - y^2$) = 0 Choice (C)

Solutions for questions 3 to 5:

3. When a = 0 and b = 2,
f (a, b) = $(0 + 2)^2$ = 4
g (a, b) = 0 + 2 = 2
h (a, b) = $\sqrt{0+2}$ = 2
Here, f (a, b) ≥ g (a, b) ≥ h (a, b)

When a = $\dfrac{1}{8}$ and b = $\dfrac{1}{8}$,

f (a, b) = $\left(\dfrac{1}{8}+\dfrac{1}{8}\right)^2 = \dfrac{1}{16}$

g (a, b) = $\dfrac{1}{8} + \dfrac{1}{8} = \dfrac{1}{4}$

and h (a, b) = $\sqrt{\dfrac{1}{8}+\dfrac{1}{8}} = \dfrac{1}{2}$

Here, f (a, b) ≤ g (a, b) ≤ h (a, b)
$\therefore$ None of the given choices is necessarily true.
Choice (D)

4. When a and b are non–negative integers,
$(a + b)^2 \geq a + b \geq \sqrt{a+b}$
So, f (a, b) ≥ g (a, b) ≥ h (a, b)
Choice C has to be rejected.
Consider a = 0, b = 1

f(a, b) = 1, g(a, b) = 1 and h (a, b) = 1 Choice (B)

5. None of the given choices is true. Choice (D)

Solutions for question 6:

6. Given (p, q) $\downarrow$ (r, s) = (ps – qr, ps + qr)
$\therefore$ (p, q) $\downarrow$ (q, p) = (p^2–q^2, p^2+q^2). . . . ..(A)
(2, 3) $\downarrow$ (1, –2) = (– 4 – 3, – 4+3) = (– 7, –1)
(1, 2) $\downarrow$ (3, 4) = (4 – 6, 4 + 6) = (– 2, 10)
(x, y) = (–7, –1) $\downarrow$ (–2, 10) = (– 70 – 2, – 70 + 2)
= (– 72, – 68) $\therefore$ x = –72, y = – 68
($x^2 – y^2$, x y) $\downarrow$ (xy, $x^2 – y^2$) = [$(x^2 – y^2)^2 – x^2y^2$, $(x^2 –y^2)^2 + x^2y^2$]
$x^2 –y^2$ = 4(140) = 560 and xy = 4896.
$\therefore$ The required ordered pair is [$560^2 – 4896^2$, $560^2 + 4896^2$]
Choice (C)

Solutions for questions 7 to 9:

7. $4 \uparrow 6 = \frac{3x4x6}{2} = 36$
$36 \to 9 = 2 (36) + 3 (9) = 99$
$9 \downarrow 12 = \frac{4(99)}{12} = 33$
$33 \leftarrow 5 = 4 (33) – 5 (5) = 107.$ Ans : (107)

8. Considering option (B), we get
$3 \to 7 = 2(3) + 3(7) = 27$
$((3 \to 7) \downarrow 9) \uparrow 5 = (27 \downarrow 9) \uparrow 5$
$= \left(\frac{4(27)}{9}\right) \uparrow 5 = \frac{3(12)(5)}{2} = 90$ Choice (B)

9. Considering option (C), we get
$((a \to b) \uparrow b) \leftarrow ab) \downarrow b$
$((2a + 3b) \uparrow b) = \frac{3(2a+3b)b}{2}$
$\left(\frac{6ab+9b^2}{2}\right) \leftarrow ab = \frac{4(6ab+9b^2)}{2} – 5ab = 7ab + 18b^2$
$(7ab + 18b^2) \downarrow b = \frac{4(7ab+18b^2)}{b} = 28a + 72b$
Choice (C)

Solutions for questions 10 and 11:

10. Given (x, y) $\uparrow$ (z, w) = ((x z – y w), (x w + y z))
$\therefore$ (x, y) $\uparrow$ (y, x) = (0, $x^2 + y^2$)
$\therefore$ (a_1, b_1) $\uparrow$ (b_1, a_1) = (0, $a_1^2 + b_1^2$)
and (a_2, b_2) $\uparrow$ (b_2, a_2) = ($a_2 b_2 – a_2 b_2$, $a_2^2 + b_2^2$) = (0, $a_2^2 + b_2^2$)
(p, q) = (0, $a_1^2 + b_1^2$) $\uparrow$ (0, $a_2^2 + b_2^2$) = [0 – $(a_1^2 + b_1^2)(a_2^2 + b_2^2)$, 0]
$\therefore$ p = – $(a_1^2 + b_1^2)(a_2^2 + b_2^2)$ and q = 0
(q – p, pq) $\uparrow$ (pq, q – p) = [0,$(p–q)^2 + p^2q^2$] = (0, p^2)
[0, $(a_1^2 + b_1^2)^2 (a_2^2 + b_2^2)^2$] Choice (A)

11. Given (a, b) Δ (c, d) = (ac – bd, ad – bc)
$\therefore$ (a, b) Δ (b, a) = (0, $a^2 – b^2$)
$\therefore$ (3, 4) Δ (4, 3) = (0, –7)
and (1, 3) Δ (3, 1) = (0, – 8)
(x, y) = (0, –7) Δ (0, – 8) = (0 – 56, 0 – 0)
$\therefore$ x = –56 and y = 0
($x^2 + y^2$, $x^2 – y^2$) Δ ($x^2 – y^2$, $x^2 + y^2$) = [0, $(x^2 + y^2)^2 – (x^2 – y^2)^2$]
= (0, ($4x^2 y^2$)) = (0, 0) Choice (D)

Solutions for question 12:

12. Let the given expression be E.
Let A = (a $\downarrow$ b) = ($a^2 – b^2$)
Let B = (a $\uparrow$ b) = ($a^2 + b^2$)
A $\downarrow$ B = $(a^2 – b^2)^2 – (a^2 + b^2)^2 = –4a^2 b^2$
(a $\to$ b) = $a^2 b^2$
$\therefore$ E $= \frac{-4a^2b^2}{a^2b^2} = –4$

–K = – 4 $\Rightarrow$ K = 4 Ans : (4)

Solutions for questions 13 to 15:

13. $c^3 = c^2 = c \otimes c = a = a \otimes c = c$,
$c^4 = c^3 \otimes c = c^4 = c \otimes c = a$
b^2 =c
$b^3 = c \otimes b = d$
$\therefore c^4 \otimes b^3 = a \otimes d = d$ Choice (A)

14. $a^3 = (a \otimes a) \otimes a = a \otimes a = a$
$b^3 = (b \otimes b) \otimes b = c \otimes b = d$
$a^3 \oplus b^3 = a \oplus d = d$
$c^3 = c^2 \otimes c = (c \otimes c) \otimes c = a \otimes c = c$
$d^3 = (d \otimes d) \otimes d = c \otimes d = b$
$c^3 \oplus d^3 = c \oplus b = a$
$3b = 2b \oplus b = (b \oplus b) \oplus b = d \oplus b = c$
$4c = 3c \oplus c = (2c \oplus c) \oplus c = ((c \oplus c) \oplus c) \oplus c$
$= (d \oplus c) \oplus c = b \oplus c = a$
$\therefore 3b \oplus 4c = c \oplus a = c$
$[(a^3 \oplus b^3) \otimes (c^3 \oplus d^3)] \oplus (3b + 4c)$
$= [d \otimes a] \oplus c = d \oplus c = b$ Choice (C)

15. $b \oplus a = b$
$(b \oplus a) \otimes c = b \otimes c = d$
$[(b \oplus a) \otimes c] \oplus d = d \oplus d = a$
$d \oplus c = b$
$(d \oplus c) \otimes b = b \otimes b = c$
$[(d \oplus c) \otimes b] \oplus a = c \oplus a = c$
$\{[(b \oplus a) \otimes c] \otimes d\} \oplus [((d \oplus c) \otimes b) \oplus a]$
$a \otimes c = c$ Choice (D)

Exercise – 15
(Statistics)

Solutions for questions 1 to 20:

1. The AM of 5, 10, 12, 18 and 20 is
$\frac{5+10+12+18+20}{5} = \frac{65}{5} = 13$ Ans : (13)

2. The combined mean is $\frac{n_1x_1 + n_2x_2}{n_1 + n_2}$
n_1 = 10, n_2 = 12, x_1 = 12; x_2 = 10
Combined mean = $\frac{10 \times 12 + 12 \times 10}{12 + 10} = \frac{240}{22}$
$= \frac{120}{11}$ Choice (B)

3. Here, n_1 = 60; x_1 = 1200
n_2 = 40; x_2 = ?
$\Rightarrow$ Combined Mean = $\frac{n_1x_1 + n_2x_2}{n_1 + n_2}$
$\Rightarrow 1000 = \frac{60 \times 1200 + 40x_2}{100}$
$\Rightarrow 40x_2 = 100000 – 72000$
$\Rightarrow x_2 = 700$ Ans : (700)

4. Arithmetic mean of first n natural numbers is $\frac{n+1}{2}$.
Given,
$\frac{n+1}{2} = 8 \Rightarrow n = 15$ Ans : (15)

5. The sum of n terms of an AP whose first term is a, common difference d is
$S_n = \frac{n}{2}[2a + (n – 1)d]$

AM = $\frac{S_n}{n} = \frac{1}{2}$ [2a + (n – 1)d] Choice (D)

6. Since sum of correct observations is equal to sum of wrong observations, there is no change in the mean.
Ans : (39)

7. GM (3, 5, 15, 45, 75) = $\sqrt[5]{3 \times 5 \times 15 \times 45 \times 75}$
= $\sqrt[5]{15 \times 15 \times 15 \times 15 \times 15}$ = $\sqrt[5]{15^5}$ = 15 Ans : (15)

8. Let the number be b.
$\therefore \sqrt{48 \times b}$ = 12 $\Rightarrow$ b = $\frac{12 \times 12}{48}$ = 3 Ans : (3)

9. Mean of squares of first 'n' natural numbers
= $\frac{(n+1)(2n+1)}{6}$ = $\frac{(10+1)(2(10)+1)}{6}$
= $\frac{(11)(21)}{6}$ = $\frac{77}{2}$ = 38.5 Choice (B)

10. HM (a, b, c, d) is
= $\frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$
HM (2, 4, 6, 8) is
= $\frac{4}{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}}$ = $\frac{4}{\frac{12+6+4+3}{24}}$
= $\frac{96}{25}$ = 3.84 Ans : (3.84)

11. We know that, $(GM)^2$ = (AM) (HM)
$(8)^2$ =10 (HM)
HM = $\frac{64}{10}$ = 6.4 Ans : (6.4)

12. The median of 12, 13, 18, 25, 30 is 18. Ans : (18)

13. The first 12 primes are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37
$\therefore$ Median = $\frac{13+17}{2}$ = 15 Ans : (15)

14. mode = 3 median– 2 mean
$\Rightarrow$ 12 = 3 median – 2 $\times$ 3
$\Rightarrow$ 18 = 3 median $\Rightarrow$ median = 6. Ans : (6)

15. Range = Max. value – Min. value.
= 93 – 13 = 80. Ans : (80)

16. The ascending order of the observation is
12, 15, 18, 19, 20, 23, 25
$Q_1 = \left(\frac{7+1}{4}\right)^{th}$ i.e., 2nd observation = 15.
$Q_3 = \left[3\frac{(7+1)}{4}\right]^{th}$ i.e., 6th observation = 23.
$\therefore$ Q.D. = $\frac{Q_3 - Q_1}{2} = \frac{23-15}{2} = \frac{8}{2}$ = 4. Ans : (4)

17. The ascending order is 10, 13, 18, 21, 24, 36, 50, 63, 75, 84, 90
$Q_1 = \left(\frac{11+1}{4}\right)^{th}$ i.e., 3rd observation = 18.
$Q_3 = \left[3\frac{(11+1)}{4}\right]^{th}$ i.e , 9th observation = 75
Q.D. = $\frac{Q_3 - Q_1}{2} = \frac{75-18}{2}$ = $\frac{57}{2}$ = 28.5
Ans : (28.5)

18. Mean of (113, 117, 120, 122, 128) is
$\frac{113+117+120+122+128}{5}$ = 120.
Mean deviation = $\frac{\sum|x_i - A|}{n}$
= $\frac{|113-120|+|117-120|+|120-120|+|122-120|+|128-120|}{5}$
= $\frac{7+3+2+8}{5}$ = 4. Ans : (4)

19. Given S.D.(x_1, x_2, …. x_n) = 7.8
$\therefore$ S.D. (x_1 + 3, x_2 + 3, …. x_n + 3) = 7.8 Ans : (7.8)

20. Given, S.D. = 4; $\frac{\sum x_i}{n}$ = 50
n = 100, $\sum x_i^2$ = ?
S.D. = $\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$
$\Rightarrow 4 = \sqrt{\frac{\sum x_i^2}{100} - (50)^2}$
16 = $\frac{\sum x_i^2}{100}$ – 2500
2516 = $\frac{\sum x_i^2}{100}$
$\therefore \sum x_i^2$ = 251600 Ans : (251600)

Exercise – 16
(Special Equations)

Solutions for questions 1 to 15:

1. Remainder of $\left(\frac{4a}{7}\right)$ = 2
when a = 4, remainder is 2 Ans : (4)

2. Remainder of $\left(\frac{3x}{5}\right)$ = 3
One possible value of x is 1. This value and the remaining values of x form an arithmetic progression with common difference 5. The set of possible values of x is {1, 6, 11, 16, …}.
Choice (A)

3. Given 5x + 3y = 34 $\Rightarrow$ y = $\frac{34-5x}{3}$ $\rightarrow$ (1)
We notice that x = 2 $\Rightarrow$ y = 8
The other possible values of x are 5, 8, 11, …
But from (1), x $\le$ 6
$\therefore$The possible values of x and y are (2, 8), (5, 3)
$\therefore$Number of positive integral solutions is 2. Ans : (2)

4. Given 3x + 4y = 29 $\Rightarrow$y = $\frac{29-3x}{4}$
The following are the possible solutions of this equation
x : 3, 7, 11, …
y : 5, 2, –1, …
When x = 7, y = 2, i.e. 0 < y < x
Choice (D)

5. Given 14x + 5y = 57 $\Rightarrow$ y = $\frac{57-14x}{5}$
Solutions of the above are x : …. 8, 3, –2, …..

y : ….. –11, 3, 17, ….

$\therefore xy > 0 \Rightarrow (x, y) = (3, 3)$ Choice (B)

6. Given $12x - 5y = 19 \Rightarrow y = \frac{12x - 19}{5}$

Both options A, B satisfy this equation. Choice (D)

7. Given $7a - 3b = 20 \Rightarrow b = \frac{7a - 20}{3}$

Possible values of a, b that satisfy this equation are listed below.

a : 2, 5, 8, –1, –4, …

b : –2, 5, 12, –9, –16, …

Possible values of a + b are 0, 10, 20, –10, –20, …

Choice (D)

8. Remainder $\left(\frac{7x}{9}\right) = 7$

The values of x that satisfy the above equation are 1, 10, 19, 28, … which is an arithmetic progression with common difference 9. Ans : (9)

9. Let the number of toys purchased be t_1 and t_2 respectively.

$11t_1 + 17t_2 = 123$

Remainder $\left(\frac{17t_2}{11}\right)$ = Remainder $\left(\frac{123}{11}\right)$

$\Rightarrow R\left(\frac{6t_2}{11}\right) = 2$

$t_2 = 4$ satisfies the above. Correspondingly, $t_1 = 5$ (If $t_2 = 4 + 11$, $t_1 = 5 - 17$, which is negative)

$\therefore$The total number of toys Mr Raghu purchased = 5 + 4 = 9

Choice (C)

10. Let the number of oranges and apples, Deepika purchased be x and y respectively (i.e. x + y = n)

$\therefore 6x + 14y = 200$ or $3x + 7y = 100$

Dividing both sides by 3 we have Remainder of $\left(\frac{7y}{3}\right)$

= Remainder$\left(\frac{100}{3}\right) \Rightarrow R\left(\frac{y}{3}\right) = 1$

Possible values of y and x are listed below

y : 1 4 7 10 13

x : 31 24 17 10 3

of all x + y = 13 + 3 = 16 is the least value

$\therefore$The minimum value of n is 16 Ans : (16)

11. Let the number of flower pots and sparklers purchased be x and y respectively

$\therefore 12x + 8y = 96 \quad y = \frac{24 - 3x}{2}$

Possible values of x and y are x : 2 4 6 8

y : 9 6 3 0

As Tinku wants at least one of each, the maximum number of flower pots she can purchase is 6. Choice (D)

12. Let the number of pens and pencils Radha purchases be x and y respectively

$\therefore 12x + 5y = 97 \quad y = \frac{97 - 12x}{5}$

Possible values of x, y are x : 1 6

y : 17 5

$\therefore$ She can purchase two combinations. Ans : (2)

13. Total amount spent = 74.

Amount spent for 2 of each item = 2 (6 + 9 +7) = 44

Amount left = 74 – 44 = 30

With ₹30, Dhanush can order 4 kachoris. But as he spent exactly ₹74 (and hence exactly ₹30 on the items over and above 2 of each kind), he must have ordered 3 kachoris and 1 cutlet.

$\therefore$The maximum number of kachoris he could have ordered = 2 + 3 = 5 Choice (C)

14. Let the number of ₹5, ₹2 and ₹1 coins in the bag be x, y and z respectively. Also x = y + z

x + y + z = 26 — (1) and 5x + 2y + z = 57 — (2)

Eliminating z from (1) and (2), we have 5x + 2y + 26 – x – y = 57

$\Rightarrow 4x + y = 31 \quad \Rightarrow y = 31 - 4x$

Possible values of x, y , z are x: 2 3 4 5 6 7

y: 23 19 15 11 7 3

z: 1 4 7 10 13 16

Only when x = 7, y = 3 and z = 16 is x + y < z. In all other cases, x + y ≥ z.

$\therefore$The number of ₹2 coins in the bag y = 3. Ans : (3)

15. As per the given conditions

N = 6x + 5 and N = 5y + 2

$\Rightarrow 6x + 5 = 5y + 2$

$6x - 5y = -3. \Rightarrow y = \frac{6x + 3}{5}$

When x = 2, 7 or 12, N is 15, 47, 77 correspondingly.

As 20 < N < 50, N = 47. Ans : (47)

Exercise – 17
(Quadratic Equations)

Solutions for questions 1 to 40:

1. (a) $x^2 - 8x + 15 = 0$

$x^2 - 5x - 3x + 15 = 0$

$x(x - 5) - 3(x - 5) = 0$

$(x - 5)(x - 3) = 0$

x = 3, 5 Choice (A)

(b) $x^2 + 4x + 3 = 0$

$x^2 + 3x + x + 3 = 0$

$x(x + 3) + 1(x + 3) = 0$

$(x + 3)(x + 1) = 0$

x + 3 = 0 or x + 1 = 0

x = –3 or – 1 Choice (D)

(c) $x^2 - 5x - 6 = 0$

$x^2 - 6x + x - 6 = 0$

$x(x - 6) + 1(x - 6) = 0$

$(x - 6)(x + 1) = 0$

x = 6 or – 1 Choice (B)

(d) $x^2 + x - 20 = 0$

$x^2 + 5x - 4x - 20 = 0$

$x(x + 5) - 4(x + 5) = 0$

$(x + 5)(x - 4) = 0$

x = – 5 or 4. Choice (C)

2. (a) $8x^2 - 17x + 2 = 0$

$8x^2 - 16x - x + 2 = 0$

$8x(x - 20) - 1(x - 2) = 0$

$(8x - 1)(x - 2) = 0$

8x – 1 = 0 or x = 2

$x = \frac{1}{8}$ or 2 Choice (A)

(b) $7x^2 + 51x + 14 = 0$

$7x^2 + 49x + 2x + 14 = 0$

$7x(x + 7) + 2(x + 7) = 0$

$(x + 7)(7x + 2) = 0$

x + 7 = 0 or 7x+ 2 = 0

$x = -7$ or $-\frac{2}{7}$ Choice (A)

(c) $-6x^2 + 29x - 20 = 0$

$-6x^2 + 24x + 5x - 20 = 0$

$-6x(x - 4) + 5(x - 4) = 0$

$(-6x + 5)(x - 4) = 0$

–6x + 5 = 0 or x – 4 = 0

$x = \frac{5}{6}$ or 4 Choice (A)

3. (a) $2x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

From the general form of an equation, sum of the roots = $\frac{7}{4}$ Product of roots = $\frac{3}{4}$. Choice (A)

(b) Sum of the roots = $-\frac{(-36)}{6\sqrt{3}} = 2\sqrt{3}$

Product of the roots = $= \frac{12\sqrt{3}}{6\sqrt{3}} = 2$ Choice (A)

(c) Sum of the roots

$= \frac{-(a^2 - b^2)}{a-b} = -(a+b)$

Product of the roots

$= \frac{\frac{1}{a} - \frac{1}{b}}{a-b} = -\frac{1}{ab}$ Choice (D)

4. For a quadratic equation $ax^2 + bx + c = 0$

If $b^2 - 4ac > 0$ but not a perfect square, the roots are real and unequal and irrational.

If $b^2 - 4ac > 0$ and a perfect square, the roots are real and unequal and rational.

If $b^2 - 4ac = 0$, the roots are rational and equal.

If $b^2 - 4ac < 0$, the roots are complex.

(a) As $b^2 - 4ac = -8$ which is < 0, the roots are complex. Choice (C)

(b) As $b^2 - 4ac > 0$ and a perfect square, the roots are rational and unequal. Choice (B)

(c) As $b^2 - 4ac = 0$, the roots are real and equal. Choice (A)

5. As the roots are 5 and –2, sum of the roots is 3. Product of the roots is –10. Hence the quadratic equation is $x^2 - 3x - 10 = 0$. Choice (A)

6. Any quadratic equation is of the form

x^2 – (sum of the roots) x + (product of the roots) = 0.

Hence if the roots are known the sum of the roots and the product of the roots can be computed and then the equation can be formed.

(a) Sum of the roots = –10

Product of the roots = 22

Hence the equation is $x^2 + 10x + 22 = 0$ Choice (B)

(b) Sum of the roots = $\sqrt{p}$.

Product of the roots = p – q

Hence the equation is $x^2 - 2\sqrt{p}\;x + p - q = 0$

Choice (B)

7. Let the roots be α and α^3

Product of the roots = $\alpha^4 = \frac{81}{16} \Rightarrow \alpha = \pm\frac{3}{2}$

$\frac{q}{16} = \alpha + \alpha^3 = \pm\frac{3}{2} \pm \frac{27}{8}$

$\Rightarrow q = \pm 78$ Choice (A)

8. Let the other roots be r

Product of roots = 3r = 9 $\Rightarrow$ r = 3

sum of roots = –(–p)= p = 3 + r

As r = 3, p = 6. Ans : (6)

9. Squaring both sides,

$3 - 2x + 7 + 2x + 2\sqrt{(3-2x)(7+2x)} = 16$

$\Rightarrow \sqrt{21 - 8x - 4x^2} = 3$

Squaring on both sides,

$21 - 8x - 4x^2 = 9$

$\Rightarrow 4(x^2 + 2x - 3) = 0$

$\Rightarrow 4(x^2 + 3x - x - 3) = 0$

$\Rightarrow 4(x(x + 3) - 1(x + 3)) = 0$

$\Rightarrow 4((x - 1)(x + 3) = 0$

$\Rightarrow x - 1 = 0$ or $x + 3 = 0$

$\Rightarrow x = 1$ or $x = -3$.

both these values satisfy the original equation.

Choice (A)

10. When x = –1, the L. H. S of the quadratic equation becomes 0.

Hence x = –1 is one root.

Let the other root be y

Sum of the roots $= \frac{-2k}{k-m+l}$

$-1 + y = \frac{-2k}{k-m+l}$

$y = \frac{-2k}{k-m+l} = \frac{-k-m+l}{k-m+l} = -\left(\frac{k+m-l}{k-m+l}\right)$

–1 and $-\left(\frac{k+m-l}{k-m+l}\right)$ Choice (D)

11. $\frac{6(x+3)+8(x+2)}{(x+2)(x+3)} = \frac{10(x+1)+4(x+4)}{(x+4)(x+1)}$

$\frac{14x+34}{x^2+5x+26} = \frac{14x+26}{x^2+5x+4}$

$(x^2+5x)14x + (x^2+5x)34 + 4(14x+34)$

$= (x^2+5x)(14x) + (x^2+5x)26 + 6(14x+26)$

$\Rightarrow 8x^2 + 12x - 20 = 0$

$\Rightarrow 8x^2 + 20x - 8x - 20 = 0$

$\Rightarrow x(8x+20) - 1(8x+20) = 0$

$(8x+20)(x-1) = 0$

8x + 20 = 0 (or) x – 1 = 0

$x = \frac{-20}{8}$ or 1

Going by the choices, x = 1

Note: By directly substituting the choices in the given equation L.H.S = R.H.S

Only when x = 1. Ans : (1)

12. Given $x^2 + 2x + 3 = 0$

Since m, n are the roots of the equation,

m + n = –2; mn = 3

(a) $\frac{1}{m} + \frac{1}{n} = \frac{n+m}{mn}$ = –2/3 Choice (A)

(b) $m^2 + n^2 = (m+n)^2 - 2mn$ = 4 – 6 = –2 Choice (D)

(c) $m^3 + n^3 = (m+n)^3 - 3mn(m+n)$

–8 – 3(3) (–2) = 10 Choice (A)

(d) $\frac{m}{n} - \frac{1}{m^2} + \frac{n}{m} - \frac{1}{n^2}$

$= \frac{m}{n} + \frac{n}{m} - \left(\frac{1}{m^2} + \frac{1}{n^2}\right)$

$= \frac{m^2+n^2}{mn} - \frac{n^2+m^2}{m^2n^2} =$

From above results, we get $m^2 + n^2 = -2$

Hence the required value is

$= \frac{-2}{3} + \frac{2}{9} = \frac{-6+2}{9} = \frac{-4}{9}$. Choice (A)

13. The discriminant of the equation is

$(2a)^2 - 4(a - b)(a + b)$

$= 4a^2 - 4(a^2 - b^2) = 4b^2$ which is 0 when b is 0 and positive otherwise. Choice (B)

14. As $b^2 - 4ac$ is positive and a perfect square, the roots are rational and unequal. Choice (D)

15. $\left(-\frac{b}{a}\right)^2 = 4\left(\frac{c}{a}\right)$

$b^2 = 4ac$

Hence $b^2 - 4ac = 0$

Hence the roots are rational and equal. Choice (A)

16. Sum of the roots $= -\frac{b}{a}$

Product of the roots $= \frac{c}{a}$

$-\frac{b}{a} = \frac{c}{a}$

$\Rightarrow$ cross multiplying, $-ab = ac$

$a(c + b) = 0$

$a = 0$ or $c + b = 0$, i.e., $b = -c$

As a cannot be 0, for any quadratic equation,

$\therefore$ $b = -c$. Choice (B)

17. Let the number be x.

$x + \frac{6}{x} = \frac{151}{5}$

Multiplying both sides by 5, we get

$5x^2 + 30 = 151x$

$\Rightarrow 5x^2 - 151x + 30 = 0$

$\Rightarrow 5x^2 - 150x - x + 30 = 0$

$\Rightarrow 5x(x - 30) - 1(x - 30) = 0$

$\Rightarrow (5x - 1)(x - 30) = 0$

$x = \frac{1}{5}$ or 30

As the number is a natural number x can only take a value of 30. Ans : (30)

18. Let the natural number be x

$(3x + 2)^2 = 32x$

$9x^2 + 12x + 4 = 32x$

$9x^2 - 20x + 4 = 0$

$9x^2 - 18x - 2x + 4 = 0$

$9x(x - 2) - 2(x - 2) = 0$

$(9x - 2)(x - 2) = 0$

$9x - 2 = 0$ or $x - 2 = 0$

$\Rightarrow x = \frac{2}{9}$ or 2

As x is a natural number x = 2 Ans : (2)

19. Let the number of pencils bought by Ashok be x and cost of each pencil be y

$\therefore$ $xy = 180$

$(x - 2)(y + 2) = 160$

$xy - 2y + 2x - 4 = 160$

$180 - 2\left(\frac{180}{x}\right) + 2x - 164 = 0$

Multiplying both sides by –x and simplifying,

$x^2 + 8x - 180 = 0$

$\Rightarrow x^2 + 18x - 10x - 180 = 0$

$\Rightarrow x(x + 18) - 10(x + 18) = 0$

$\Rightarrow (x - 10)(x + 18) = 0$

$\Rightarrow x - 10 = 0$ or $x + 18 = 0$

$\Rightarrow x = 10$ or -18

As $x > 0$, $x = 10$. Ans : (10)

20. Let the consecutive numbers be x and x + 1.

$x^2 + (x + 1)^2 = 841$

$\Rightarrow x^2 + x^2 + 2x + 1 - 841 = 0$

$\Rightarrow 2(x^2 + x - 420) = 0$

$x^2 + 21x - 20x - 420 = 0$

$x(x + 21) - 20(x + 21) = 0$

$x - 20 = 0$ or $x + 21 = 0$

$x = 20$ or $x = -21$

As x is a natural number $x > 0$. Hence the smaller natural number is 20. Ans : (20)

21. Let the positive number be x

$\frac{1}{x} - x = \frac{-48}{7}$

Multiplying both sides by 7x and simplifying

$7x^2 - 48x - 7 = 0$

$7x^2 - 49x + x - 7 = 0$

$7x(x - 7) + 1(x - 7) = 0$

$(x - 7)(7x + 1) = 0$

$x - 7 = 0$ or $7x + 1 = 0$

$\Rightarrow X = 7$ or $x = \frac{-1}{7}$

$\therefore$ The required number is 7. Ans : (7)

22. The minimum value of a quadratic expression $ax^2 + bx + c$ is $\frac{4ac - b^2}{4a}$

As a = 3 and b = –7, c = 6 the minimum value is

$\frac{4(3)(6) - (-7)^2}{4(3)} = \frac{72 - 49}{12} = \frac{23}{12}$. Choice (B)

23. (a) $x^2 + 19x + 70 < 0$

$x^2 + 14x + 5x + 70 < 0$

$x(x + 14) + 5(x + 14) < 0$

$(x + 5)(x + 14) < 0$

We have two possibilities

(i) $x + 5 > 0$ and $x + 14 < 0$

i.e., $x < 5$ and $x < -14$

No value of x satisfies both these inequalities

(ii) $x + 5 < 0$ and $x + 14 > 0$

i.e., x _ 5 and $x > -14$

Hence $-14 < x < -5$

Thus $-14 < x < -5$ Choice (C)

(b) $x^2 - 15x + 50 > 0$

$x^2 - 10x - 5x + 50 > 0$

$x(x - 10) - 5(x - 10) > 0$

$(x - 10)(x - 5) > 0$

We have two possibilities

(i) $x - 10 > 0$ and $x - 5 > 0$

i.e., $x > 10$ and $x > 5$

Hence $x > 10$

(ii) $x - 10 < 0$ and $x - 5 < 0$

i.e., $x < 10$ and $x < 5$

Hence $x < 5$ or $x > 10$. Choice (A)

(c) $x^2 - 27x - 90 < 0$

$x^2 - 30x + 3x - 90 < 0$

$x(x - 30) + 3(x - 30) < 0$

$(x - 30)(x + 3) < 0$

We have two possibilities

(i) $x - 30 > 0$ and $x + 3 < 0$

i.e. $x > 30$ and $x < -3$

No value of x satisfies both these inequalities.

(ii) $x - 30 < 0$ and $x + 3 > 0$

i.e., $x < 30$ and $x > -3$

Hence $-3 < x < 30$ Choice (A)

(d) $x^2 - 60x + 900 > 0$

$x^2 - 30x - 30x + 900 > 0$

$x(x - 30) - 30(x - 30) > 0$

$(x - 30)^2 > 0$

This is true for any real value of x, except x = 30. Choice (C)

24. The minimum/maximum value of a quadratic expression $ax^2 + bx + c$ is occurs when $x = -b/2a$

As a = –1 and b = 10, the maximum value occurs at $x = \frac{-10}{2(-1)} = 5$ Ans : (5)

25. The maximum/minimum value of the quadratic expression $ax^2 + bx + c$ is given by

$\frac{4ac - b^2}{4a}$ The expression has a minimum value when a > b and a maximum value when a > 0

(a) As a > 0, the expression has minimum value which is given by

$$\frac{4(3)(4)-(-7)^2}{4(3)} = -\frac{1}{12}$$ Choice (C)

(b) As a < 0, the expression has maximum value which is given by

$$\frac{4(-8)(3)-(10)^2}{4(-8)} = \frac{49}{8}$$ Choice (D)

26. Let $x^2 = a$
$(x^2)^2 - 33x^2 + 216 = 0$
$a^2 - 33a + 216 = 0$
$a^2 - 24a - 9a + 216 = 0$
$a(a - 24) - 9(a - 24) = 0$
$(a - 24)(a - 9) = 0$
$a - 24 = 0$ or $a - 9 = 0$
a = 24 or 9
Hence $x = \sqrt{24}$ or $\sqrt{9}$
$\therefore x = \pm 2\sqrt{6}$ or ± 3 Choice (A)

27. Let the two consecutive positive integers be x and x + 1 respectively $x^2 + (x + 1)^2 - x(x + 1) = 91$
$x^2 + x - 90 = 0$
$(x + 10)(x - 9) = 0$
x = –10 or 9
As x is positive x = 9
Hence the two consecutive positive integers are 9 and 10 respectively. Choice (A)

28. Let the price of each note book be ₹x.
Let the number of note books which can be brought for ₹300 each at a price of Rs. x be y. Hence xy = 300

$\Rightarrow y = \frac{300}{x}$

$(x+5)(y-10) = 300$

$\Rightarrow xy + 5y - 10x - 50 = xy$

$\Rightarrow 5\left(\frac{300}{x}\right) - 10x - 50 = 0 \Rightarrow -150 + x^2 + 5x = 0$

Multiplying both sides by $\frac{-1}{10x}$

$\Rightarrow x^2 + 15x - 10x - 150 = 0$
$x(x + 15) - 10(x + 15) = 0$
$(x + 15)(x - 10) = 0$
x = –15 or 10
As x > 0, x = 10 Ans : (10)

29. Let the roots of the equation be α and α^3. The products of the roots $= \alpha^4 = 1 \Rightarrow \alpha = \pm\sqrt[4]{1}$
$\alpha = \pm 1 \Rightarrow \alpha = 1$ ($\because \alpha > 0$)
Sum of the roots = k
$\therefore k = \alpha + \alpha^3 = 1 + 1 = 2$ Ans : (2)

30. Let the roots of the quadratic equation be 3α and 4α respectively.

Sum of the roots $= -\frac{b}{a} = 7\alpha$

$\Rightarrow$ Squaring both sides $\frac{b^2}{a^2} = 49\alpha^2$ —— (1)

Product of the roots $= \frac{c}{a} = 12\alpha^2$ —— (2)

Dividing (1) by (2)

$\frac{b^2}{ac} = \frac{49}{12} \Rightarrow 49ac = 12b^2$ Choice (A)

31. Let the roots of the quadratic equation be p and p^2

Product of roots $= p^3 = \frac{1}{64} \Rightarrow p = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$

$p^2 = \frac{1}{16}$

sum of roots = p + p^2

$= \frac{1}{4} + \frac{1}{16} = \frac{5}{16} = -\frac{a}{64}$

$\Rightarrow a = -20$ Choice (B)

32. Let $f(x) = x^4 - 3x^3 + 2x^2 - 3x + 7$
The remainder when f(x) is divided by x + 1 is f(–1)
$\Rightarrow f(-1) = 1 + 3 + 2 + 3 + 7 = 16$ Ans : (16)

33. Given $f(x) = x^4 - 3x^3 + kx^2 + 7$ is exactly divisible by 3x – 1.

$\Rightarrow f\left(\frac{1}{3}\right) = 0 \Rightarrow \left(\frac{1}{3}\right)^4 - 3\left(\frac{1}{3}\right)^3 + k\left(\frac{1}{3}\right)^2 + 7 = 0$

$\Rightarrow k = \frac{-559}{9}$ Choice (B)

34. Let $f(x) = 2x^3 - 7x^2 + 5x - 3$
When f(x) is divided by x + 1, remainder is f(–1).
$\therefore f(-1) = 2(-1)^3 - 7(-1)^2 + 5(-1) - 3 = -17$
The remainder is 5k – 2.
$\therefore 5k - 2 = -17$
$\Rightarrow k = -3 \Rightarrow -k = 3$ Ans : (3)

35. Let $x^3 = a$.
$\Rightarrow f(x^3) = (x^3)^2 - 7x^3 + 8$. If $f(a) = a^2 - 7a + 8$ is divided by a – 2, then remainder is f(2).
$\Rightarrow (2)^2 - 7(2) + 8 = -2$ Choice (A)

36. Given 2x – 1 is a factor of $f(x) = 2x^2 + px - 2$.

$\Rightarrow f\left(\frac{1}{2}\right) = 0 \Rightarrow f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 + p\left(\frac{1}{2}\right) - 2 = 0$

$\Rightarrow 1 + p - 4 = 0 \Rightarrow p = 3$
$\therefore f(x) = 2x^2 + 3x - 2$. Hence other factor is x + 2
Choice (B)

37. If f(x) is exactly divisible by x – 1, sum of coefficients must be zero.
Here sum of coefficients is 2 + 4 – 7 + 4 = 3
$\therefore$ If 3 is subtracted or –3 is added the given expression is exactly divisible by x – 1. Ans : (3)

38. If f(x) is exactly divisible by 2x – 3, then $f\left(\frac{3}{2}\right) = 0$

$\Rightarrow a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) + c = 0 \Rightarrow 9a + 6b + 4c = 0$ Choice (C)

39. Given $x^2 - 1$ is a factor of $x^4 + 4x^3 + ax^2 - bx + 3$
$\Rightarrow f(1) = 0$
$\Rightarrow 1 + 4 + a - b + 3 = 0$
$\Rightarrow a - b = -8$ and ------- (1)
f(–1) = 0
$\Rightarrow 1 - 4 + a + b + 3 = 0$
$\Rightarrow a + b = 0$ ------ (2)
Solving (1) and (2) we get a = – 4 and b = 4.
Choice (D)

40. $f(x) = Q(x).(x^2 - 5x + 4) + ax + b$
given f(4) = 6, and f(1) = 3
6 = 4a + b . . . (1)
a + b = 3 . . . (2)
solving (1) and (2), we get
a = 1 and b = 2
$\therefore$ required remainder is x + 2 Choice (A)

Exercise – 18
(Inequations and Modulus)

Solutions for questions 1 to 40:

1. Given that $3x + 14 > 5x + 24$.
$\Rightarrow 3x - 5x > 24 - 14 \Rightarrow -2x > 10 \Rightarrow x < -5$
Choice (D)

(**Note:** When an inequality is multiplied by a negative number, the inequality is reversed)

2. Given that $7x - 12 > 5x - 11$
$\Rightarrow 7x - 5x > -11 + 12$
$\Rightarrow 2x > 1 \Rightarrow x > 1/2$ Choice (A)

3. Given that $5x - 7 \geq 3$ and $3x - 4 \geq 8$.
$\Rightarrow 5x \geq 10$ and $3x - 4 \geq 8$
$\Rightarrow x \geq 2$ and $x \geq 4$
Solution of the given inequalities is the common segment of $x \geq 2$ and $x \geq 4$ and this is given by $x \geq 4$.
Choice (B)

4. $3x - 8 \leq 5x - 2 \Rightarrow 3x - 5x \leq -2 + 8$
$\Rightarrow -2x \leq 6 \Rightarrow x \geq -3$ ---- (1)
and $6x - 4 \leq 4x + 1 \Rightarrow 6x - 4x \leq 1 + 4$
$\Rightarrow 2x \leq 5 \Rightarrow x \leq 5/2$ ---- (2)
The region that satisfies both (1) and (2) i.e., $x \geq -3$ and $x \leq 5/2$ is $x \in \left[-3, \frac{5}{2}\right]$ Choice (C)

5. $6x^2 + x - 12 \geq 0 \Rightarrow (2x + 3)(3x - 4) \geq 0$
This is of the form $(x - \alpha)(x - \beta) \geq 0$ $(\alpha < \beta)$, whose solution is $x \in R - (\alpha, \beta)$
$\therefore$ solution is $x \in R - \left(\frac{-3}{2}, \frac{4}{3}\right)$ Choice (B)

6. $x^2 - x - 6 < 0 \Rightarrow (x - 3)(x + 2) < 0$, which is of the form $(x - \alpha)(x - \beta) < 0$ whose solution is given by $x \in (\alpha, \beta)$
$\therefore$ Required solution is $x \in (-2, 3)$ Choice (C)

7. $x^2 - 4x + 10 > 4 \Rightarrow (x - 2)^2 + 10 > 0$, as $(x - 2)^2 \geq 0$
$\Rightarrow (x - 2)^2 + 10 > 0$ holds for all real numbers $(x - 2)^2 + 10 \geq 10 > 0$.
$\therefore$ solution set is R . Choice (B)

8. $x^2 + 8x + 20 < 0$
$\Rightarrow (x + 4)^2 + 4 < 0$
But $(x + 4)^2 + 4 \geq 4 \; \forall \; x \in R \; \therefore \exists$ no real number that satisfies the given inequality.
$\Rightarrow$ solution set is { } or null set. Choice (A)

9. $\dfrac{x^2 - 2x - 8}{3x^2 + 7x + 2} < 0$

$\Rightarrow \dfrac{(x + 2)(x - 4)}{(x + 2)(3x + 1)} < 0 \Rightarrow \dfrac{x - 4}{3x + 1} < 0$; $(x \neq -2)$

$\Rightarrow \dfrac{(x - 4)(3x + 1)}{(3x + 1)^2} < 0 \Rightarrow (x - 4)(3x + 1) < 0$

$\Rightarrow x \in (-1/3, 4)$ Choice (C)

10. Given $\dfrac{2x^2 - 3x - 2}{x^2 - 4} \leq 0$

$\Rightarrow \dfrac{(2x + 1)(x - 2)}{(x + 2)(x - 2)} \leq 0$

$\Rightarrow \dfrac{2x + 1}{x + 2} \leq 0$

$\Rightarrow \dfrac{(2x + 1)(x + 2)}{(x + 2)^2} \leq 0$

$\Rightarrow (2x + 1)(x + 2) \leq 0$
$\therefore$ solution set is $x \in (-2, -1/2]$. Choice (C)

11. $\dfrac{(x - 3)}{(x + 3)} \times \dfrac{(x + 3)}{(x + 3)} > 3 \Rightarrow x^2 - 9 > 3(x + 3)^2$
$\Rightarrow -2x^2 - 18x - 36 > 0 \Rightarrow x^2 + 9x + 18 < 0$
$\Rightarrow (x + 3)(x + 6) < 0 \Rightarrow x \in (-6, -3)$
$\therefore$ solution set is $(-6, -3)$ Choice (A)

12. $\dfrac{x + 4}{x - 1} \geq 0$

$\Rightarrow \dfrac{(x - 1)(x + 4)}{(x - 1)^2} \geq 0$

$\Rightarrow (x - 1)(x + 4) \geq 0$ (with $x \neq 1$) $\Rightarrow x \leq -4$ or $x > 1$
As we need the values that do not satisfy the inequality, we get $-4 < x < 1$. Choice (C)

13. Given $|x + 2| = 5$,
$\Rightarrow x + 2 = 5$ or $x + 2 = -5 \Rightarrow x = 3$ or $x = -7$
$\therefore x \in \{-7, 3\}$ Choice (C)

14. $|x^2 - 9| = 0$
$\Rightarrow (x + 3)(x - 3) = 0$
$\Rightarrow x = 3$ or -3
$\therefore$ solution set is $\{-3, 3\}$ Choice (C)

15. Given $|x + 5| < 7$, $-7 < x + 5 < 7$
$\Rightarrow -12 < x < 2$
$\therefore$solution is $(-12, 2)$ Choice (A)

16. Given $|3x + 4| < 5$.
$\Rightarrow -5 < 3x + 4 < 5$
$\Rightarrow -9 < 3x < 1$
$\Rightarrow -3 < x < \dfrac{1}{3}$

$\therefore$solution is $(-3, \frac{1}{3})$. Choice (A)

17. We know that $|x| > a$
$\Rightarrow x < -a$ or $x > a$
$|2x + 3| > 2 \Rightarrow 2x + 3 < -2$ or $2x + 3 > 2$
$x < -5/2$ or $x > -1/2$
$\Rightarrow (-\infty, -5/2) \cup (-1/2, \infty)$. Choice (C)

18. Given $|x + 3| > -5$
Clearly $|x + 3|$ is a positive value for all $x \in R$ which is always greater than any negative value i.e. -5
$\therefore$ solution set is R Choice (B)

19. $f(x) = 15 - |x - 7|$, is maximum when $|x - 7|$ is minimum. The minimum value of $|x - 7| = 0$.
$\therefore$ Maximum value of f(x) is 15. Ans : (15)

20. The minimum value of $f(x) = 12 + |x + 5|$ is attained when $|x + 5|$ is minimum. The minimum value of $|x + 5| = 0$.
$\therefore$The minimum value of f(x) is $12 + 0 = 12$ Ans : (12)

21. $x|x - 6| = 7$
If $x \geq 6$, $|x - 6| = x - 6 \therefore x|x - 6| = x(x - 6) = 7$.
$x^2 - 6x - 7 = 0$.
$\therefore x = 7$ or -1.
but $x > 6 \Rightarrow x = 7$
$x < 6$ we get non natural number

Alternate method:
Proceeding from the choices only $x = 7$ satisfies $x|x - 6| = 7$
Choice (D)

22. $|x + 2| + |x - 2|$ is a positive quantity and so it cannot be zero. Hence, no solution is possible for $f(x) = 0$. Ans : (0)

23. Case (i): When $x \geq 0$, $|x| = x$.
$\therefore x^2 - 9x + 18 = 0$
$\Rightarrow (x - 3)(x - 6) = 0 \Rightarrow x = 6$ or $x = 3$
Case (ii): When $x < 0$, $|x| = -x$
$\therefore x^2 + 9x + 18 = 0$
$\Rightarrow (x + 3)(x + 6) = 0$
$\Rightarrow x = -6$ or $x = -3$
Thus there are 4 distinct solutions of the given equation.

Alternate method:
$x^2 = |x|^2$
$\therefore$ The given equation becomes $|x|^2 - 9|x| + 18 = 0$
$|x| = 6$ or 3.

$\therefore$ x = ± 6 or ± 3.
$\therefore$ x has 4 distinct solutions. Ans : (4)

24. Case (i): Let $x \geq -\frac{1}{4}$.
Then, |4x + 1| = 4x + 1
So, $\left| x - |4x + 1| \right| = 7$
$\Rightarrow$ |x – 4x – 1| = 7
$\Rightarrow$ |3x + 1| = 7 $\Rightarrow$ x = 2 or $x = \frac{-8}{3}$

Since $x \geq \frac{-1}{4}$ x = 2 is the only possibility.

Case (ii): Let $x < -\frac{1}{4}$
Then, |4x + 1| = – (4x + 1)
So, $\left| x - |4x + 1| \right| = 7$
$\Rightarrow$ |x + 4x + 1| = 7
$\Rightarrow$ |5x + 1| = 7 $\Rightarrow$ $x = \frac{6}{5}$ or $x = \frac{-8}{5}$

Since $x < \frac{-1}{4}$ So, $x = \frac{-8}{5}$ is the only possibility.
Thus we get two solutions for this equation. Ans: (2)

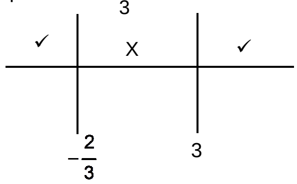
25. Given |3 – 5x| > 5 – x, if $x < \frac{3}{5}$, |3 – 5x| = 3 – 5x

$\therefore$ 3 – 5x > 5 – x
–2 > 4x
$x < -\frac{1}{2}$
Given x > 0 there is no solution
x > 3/5 |3 – 5x| = 5x – 3
Inequation becomes 5x – 3 > 5 – x
6x > 8
$x > \frac{8}{6} \Rightarrow x > \frac{4}{3}$ Choice (D)

26. $\frac{1}{x-2} < 0 \Rightarrow x - 2 < 0 \Rightarrow x < 2.$ Choice (C)

27. |x + y| ≤ |x| + |y| is true (standard result). Choice (C)

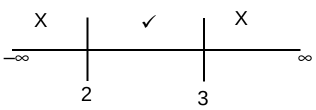
28. $3x^2 - 7x - 6 > 0$
$\Rightarrow$ (3x + 2) (x – 3) > 0
The critical points are $-\frac{2}{3}$ and 3.



When x = 0, the inequation is not satisfied.
$\therefore$ The solution is $x < -\frac{2}{3}$ or x > 3.

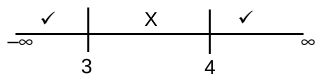
$\Rightarrow x \notin \left[-\frac{2}{3}, 3\right]$ Choice (C)

29. Given $x^2 - 5x + 6 < 0$
$\Rightarrow$ (x – 2) (x – 3) < 0
The critical points are 2 and 3.



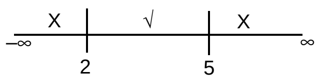
$\therefore$ Solution is (2, 3) Choice (A)

30. $(x - 3)(x - 4)(x + 2)^2 > 0$
Since, $(x + 2)^2$ must be positive, (x – 3) (x – 4) > 0.
Critical points are 3 and 4.



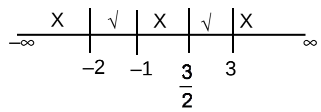
$\therefore$ Solution set (–∞, 3) ∪ (4, ∞)
But, at x = –2, the inequation is not valid.
$\therefore$ x ∉ [3, 4] and x ≠ –2. Choice (D)

31. $\frac{x+4}{x-2} > 3 \Rightarrow \frac{x+4}{x-2} - 3 > 0$
$\Rightarrow \frac{x+4-3x+6}{x-2} > 0 \quad \Rightarrow \frac{10-2x}{x-2} > 0$
$\Rightarrow \frac{2(5-x)(x-2)}{(x-2)^2} > 0$
(5 – x) (x – 2) > 0 $\Rightarrow$ (x – 2) (x – 5) < 0
Critical points are 5 and 2.



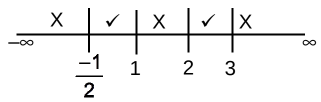
When x = 0, the inequation is not true.
The solution is (2, 5). Choice (B)

32. $\frac{3x^2 - x - 9}{x^2 - 2x - 3} < 1$
$\Rightarrow \frac{3x^2 - x - 9 - x^2 + 2x + 3}{x^2 - 2x - 3} < 0$
$\frac{2x^2 + x - 6}{x^2 - 2x - 3} < 0$
$\frac{(2x-3)(x+2)}{(x-3)(x+1)} < 0$



The solution is (–2, –1) ∪ $(\frac{3}{2}, 3)$ Choice (C)

33. $\frac{x^2 - 4x + 3}{2x^2 - 3x - 2} < 0$
$\Rightarrow \frac{(x-3)(x-1)}{(x-2)(2x+1)} < 0$
$\Rightarrow$ (x – 3) (x – 1) (x – 2) (2x + 1) < 0
The critical points are 1, 2, 3, $-\frac{1}{2}$



$\therefore$ Solution is $\left(-\frac{1}{2}, 1\right) \cup (2, 3)$ Choice (A)

34. If |x| > a
$\Rightarrow$ x < –a or x > a
|x – 3| > 4
$\Rightarrow$ x – 3 > 4 or x – 3 < –4
x > 7 or x < –1
$\therefore$Solution is x < –1 or x > 7 Choice (A)

35. 21x + 71 ≥ 17x + 95
4x ≥ 24;
x ≥ 6 …….. (1)
3x – 13 ≤ 2x – 10
x ≤ 3 …….. (2)

From (1) and (2), the common solution is an empty set. Choice (D)

36. $|2x - 3| < 1$
$\Rightarrow -1 < 2x - 3 < 1 \Rightarrow 1 < x < 2$ Choice (D)

37. $|x + 7| = 5$
$x + 7 = \pm 5$
$x = -2$ or -12
$\therefore$The solution set is $\{-2, -12\}$ Choice (D)

38. Since, $|x + 7|$ is always positive,
$|x + 7| > -8$ for all values of x.
$x \in R$. Choice (D)

39. Since, $|(x + 2)|$ is positive, so for any value of x,
$|(x + 2)|$ will never be less than -3.
Solution set is empty set. Choice (D)

40. We know that,
$|x| < a$
$\Rightarrow -a < x < a$
$|x + 4| < 12$
$\Rightarrow -12 < x + 4 < 12 \Rightarrow -16 < x < 8$. Choice (C)

Exercise – 19
(Sequences and Series)

Solutions for questions 1 to 50:

1. The series is an A.P with a first term 1 and common difference 4,
16^{th} term of the A.P = 1 + 15 (4) = 61. Ans : (61)

2. The A.P has a first term of 11 and a common difference of 5. The nth term of the A. P is 66.
i.e., 11 + (n – 1)5 = 66
n = 12. Ans : (12)

3. The series is an A.P with first term as 4 and common difference –6 the 11^{th} term of the A.P
= 4 + [(11 – 1) (– 6)] = –56. Choice (A)

4. Let the number of terms of the A. P to be added be n. The A. P has a first term of 21 and common difference of 7.
Sum of first n terms of the A. P = $= \frac{n}{2}$ [2 (21) + (n – 1)7]
$\Rightarrow \frac{n}{2}$ [35 + 7n] = 175 $\Rightarrow$ 7n [5 + n] = 350
Substituting each of the choices for n in the above equation only n = 5, satisfies it. Choice (C)

5. The series is an arithmetic progression with first term 8, and common difference $2\frac{1}{2}$. So a =8, d= $2\frac{1}{2}$
$t_n = a+(n-1)d$
$t_{41} = 8 + (40 \times 2^1/_2) = 108$ Ans : (108)

6. Let the nth term of the A.P 5, 9, 13, … be 105.
Here a = 5, d = 4
$t_n = 5 + (n - 1)\ 4$
$\Rightarrow$ 1 + 4n = 105
4n = 104
n = 26 Choice (A)

7. Let the first term and the common difference of the A.P be a and d respectively.
$t_5 = a + 4d = 11$ —— (1)
$t_{15} = a + 14d = 31$ —— (2)
Subtracting equation (1) from equation (2), we have
10d = 20 $\Rightarrow$ d = 2
25^{th} term of the A.P = a + 24d
= a + 4d + 20d = 11 + 20 (2) = 51 Ans : (51)

8. Let the first term and the common difference of an A.P be a and d respectively. The 7^{th} and 16^{th} terms are a + 6d and a + 15d respectively.
7(a + 6d) = 16(a + 15d) $\Rightarrow$ 7a + 42d = 16a + 240d
$\Rightarrow$ 9 (a + 22d) = 0
$\therefore$ The 23^{rd} term a + 22d is 0. Ans : (0)

9. The series has a first term as 49, common difference –7 and the last term is 7.
Let the number of terms in the series be n.
$\therefore$ 7 = 49 – 7 (n – 1)
42 = 7 (n – 1)
6 = n – 1
7 = n
Sum to n terms of the series = $\frac{n}{2}$ (First term + last term)
= $\frac{7}{2}(49+7) = 196$ Ans : (196)

10. Let the first term and common difference of the A.P be a and d respectively.
17^{th} term of A.P = a + 16d = 33.
Sum of the first 33 terms of the A.P = $\frac{33}{2}[2a+32d]$
$= 33\,[a + 16\,d] = 33\,[33] = 1089$ Choice (B)

11. nth term of the A.P = Sum of the first n terms of the A.P – Sum of the first (n – 1) terms of the A.P = $3n^2 + 6n - [3(n-1)^2 + 6(n-1)] = 6n + 3$ Choice (A)

12. Let the three numbers in A.P be a – d, a and a + d
a – d + a + a + d = 39
a = 13
Product of the numbers = (a – d) (a) (a + d)
$\Rightarrow$ a (a – d) (a + d) = 2145 $\Rightarrow$ a $(a^2 - d^2)$ = 2145
$d^2 = 4$
d = ± 2
If d = 2, the numbers are 11, 13 and 15, If d = –2 the same numbers are obtained in the reverse order. Choice (B)

13. Let the three terms in A.P be a – d, a, a + d.
a – d + a + a + d = 30
3a = 30 $\Rightarrow$ a = 10
Sum of the squares of the terms = $(a - d)^2 + a^2 + (a + d)^2$
i.e., $3a^2 + 2d^2 = 318$
$2d^2 = 318 - 3a^2 = 18$
$d^2 = 9 \Rightarrow d = \pm 3$.
If d = 3, the terms are 7, 10, 13.
If d = –3, the same terms are obtained in the reverse order. Choice (B)

14. Let us say A scored x marks less than B and B scored x marks less than C. If B got y marks, sum of the marks of A, B and C = y – x + y + y + x
$\Rightarrow$ 3y = 240 $\Rightarrow$ y = 80 Choice (B)

15. Sum of 15 terms of the A.P = $\frac{15}{2}$ (first term + last term)
$= \frac{15}{2}[20+100] = 900$ Ans : (900)

16. Let the five terms in A.P be
a – 2d, a – d, a, a + d and a + 2d respectively.
a – 2d + a – d + a + a + d + a + 2d = 40
5a = 40 $\Rightarrow$ a = 8
Product of the extreme terms = (a – 2d) (a + 2d) = $a^2 - 4d^2$
$\therefore 64 - 4d^2 = 48$
$4d^2 = 16 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$
If d = 2, the terms are 4, 6, 8, 10 and 12. If d = –2, the same terms are obtained in reverse order. Choice (A)

17. The least two–digit number which leaves a remainder of 2 when divided by 5 is 12. The greatest two–digit number which leaves a remainder of 2 when divided by 5 is 97. Hence the last term t_n is 97.
$\therefore$ 97 = 12 + (n – 1) 5

$85 = (n - 1) 5$
$\Rightarrow n - 1 = 17 \Rightarrow n = 18.$
Sum of all two–digit numbers which leaves a remainder of 2 when divided by $5 = \frac{15}{2}[12 + 97] = 981$ Ans : (981)

18. The arithmetic mean of the first n natural numbers is $\frac{n+1}{2}$

$\therefore \frac{n+1}{2} = 34 \Rightarrow n = 67$ Choice (B)

19. When n= 1, 2, 3 and 4, the first four terms are obtained as $3(-5)^1$, $3(-5)^2$ $3(-5)^3$ and $3(-5)^4$ i.e –15, 75, –375 and 1875 respectively. Choice (D)

20. The series is a G.P with first term as $\frac{1}{8}$ and common ratio equal to –2. The sixth term of the series equals $\frac{1}{8}(-2)^5$

$= \frac{-32}{8} = -4.$ Choice (A)

21. Let the first term of the G.P be a and the common ratio be r
$ar^4 = 3$ —— (1)
$ar^8 = \frac{1}{27}$ —— (2)
Dividing (2) by (1) we have $r^4 = \frac{1}{81}$
$r = \pm\frac{1}{3}$
substituting r in (1) we get a = 243
11^{th} term of the G.P = $ar^{10} = 243\left(\pm\frac{1}{3}\right)^{10}$

$= \frac{3^5}{(3^5)^2} = \frac{1}{3^5} = \frac{1}{243}$ Choice (B)

22. Let the number of terms of the series 2, 6, 18, …which result in a sum of 2186 be n.
The series is a G.P with first term 2 and common ratio 3. Sum of the terms of a G.P having n terms with first term a and common ratio r is given by $\frac{a(r^n - 1)}{r - 1}$
As a = 2 and r = 3,
Sum of the terms of the series = $\frac{2(3^n - 1)}{3 - 1} = 2186$
$3^n = 2187 = 3^7$
Equating powers of 3 both sides, n = 7. Ans : (7)

23. Let the numbers in G. P be $\frac{a}{r}$, a and ar. The product of the numbers is $a^3 = 1728. \Rightarrow a = 12$
Hence the numbers are $\frac{12}{r}$, 12 and 12r

$\frac{12}{r} + 12 + 12r = 63$

$\frac{12}{r} - 51 + 12r = 0$

Multiplying both sides by $\frac{r}{3}$,

$4r^2 - 17r + 4 = 0$
$4r^2 - 16r - r + 4 = 0$
$4r(r - 4) - 1(r - 4) = 0$
$(r - 4)(4r - 1) = 0$
$r - 4 = 0$ or $4r - 1 = 0$
$r = 4$ or $\frac{1}{4}$

If r = 4, the numbers are 3, 12 and 48. If $r = \frac{1}{4}$, the same numbers are obtained in reverse order.
Choice (A)

24. Let the n^{th} term of the G. P be 256 n^{th} term of the G.P = $ar^{n-1} = 2\left(\sqrt{2}\right)^{n-1} = 256$
$\left(\sqrt{2}\right)^{n-1} = 128$, $\left(\sqrt{2}\right)^{n-1} = 2^7 = \left(\sqrt{2}\right)^{14}$
Equating powers of $\sqrt{2}$ both sides, n – 1 = 14
$\Rightarrow n = 15.$ Choice (B)

25. Let the first term of the G.P be a and the common ratio be r.

Fifth term = $ar^4 = \frac{16}{125} \Rightarrow r = \sqrt[4]{\frac{16}{125a}}$

As a = 5 $\Rightarrow r = \pm\sqrt[4]{\frac{16}{625}} = \pm\frac{2}{5}$

The sum to infinity of the G.P is $\frac{a}{1-r} = \frac{25}{3}$ if $r = \frac{2}{3}$ and

$= \frac{25}{7}$ if $r = \frac{-2}{5}$ Choice (C)

26. Let the common ratio be r. Sum of terms of a G.P is given by $\frac{r\ (\text{last term}) - (\text{first term})}{r - 1}$
As first term = 5 and last term = 1536.
Sum of terms of
$G.P = \frac{r[1536] - 5}{r - 1} = 3067$
$1536r - 5 = 3067(r - 1) \Rightarrow r = 2$ Ans : (2)

27. The geometric mean of the n terms $a_1, a_2, \ldots, a_n$ is $\sqrt[n]{a_1 \times a_2 \times a_3 \times \ldots\ldots \times a_n}$
As n = 5, the geometric mean of the 5 terms $= \sqrt[5]{(3)(9)(27)(81)(243)} = 27.$ Ans : (27)

28. Let the first term of the G. P be a and the common ratio be r.
Sum to infinity of the G. P = $\frac{a}{1-r}$
The first term would be one third of the sum of all the following terms to it. Hence $a = \frac{1}{3}(\frac{a}{1-r} - a)$

$4a = \frac{a}{1-r}$

$\Rightarrow (1 - r) 4 = 1 \Rightarrow 4r = 3 \Rightarrow r = \frac{3}{4}$ Choice (C)

29. Let the G.P be a, ar, ar^2, …
The reciprocals of the terms of the G.P are $\frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, \ldots$
The above series is also in G.P with common ratio $\frac{1}{r}$.
Choice (B)

30. Let the amount with Charan be ₹x, amount with Bala would be ₹3x.
Amount with Ajay would be ₹3 (3x) = ₹9x
9x + 3x + x = 260
13x = 260
x = 20
3x = 60
9x = 180
Amounts with Ajay, Bala and Charan are ₹180, ₹60 and ₹20 respectively. Choice (A)

31. Let the two numbers be a and b.

Harmonic mean (a, b) = $\frac{2ab}{a+b} = \frac{243}{41}$ —— (1)

Geometric mean (a, b) = $\sqrt{ab}$ = 27

$\Rightarrow$ ab = 729 —— (2)
Going by the choices and verifying which choice satisfies both equations above, we find only Choice (B) satisfying both equations (1) and (2). Choice (B)

32. Let the roots of the quadratic equation be α and β respectively. Arithmetic mean (α, β) = a.

i.e., $\frac{\alpha+\beta}{2} = a \Rightarrow \alpha + \beta = 2a$

Geometric mean (α, β)

$\sqrt{\alpha\beta} = b$

$\alpha\beta = b^2$
The quadratic equation whose roots are α and β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
i.e., $x^2 - 2ax + b^2 = 0$ Choice (D)

33. As p, q and r are in G.P $\Rightarrow q^2 = pr$

$\frac{p}{q} = \frac{q}{r} \Rightarrow \frac{pr}{qr} = \frac{pq}{pr}$

i.e., qr, pr and pq are in G.P. Choice (B)

34. Let the common difference of the arithmetic progression be d. Let the common ratio of the geometric progression be r.

Q – P = R – Q = d and $\frac{Y}{X} = \frac{Z}{Y} = r$

$\left(\frac{Y}{r}\right)^d . Y^{P-Q+Q-R}(Yr)^d = \frac{Y^d}{r^d} . Y^{-d+(-d)} . Y^d . r^d$

= $Y^{d-2d+d} = Y^0 = 1$. Ans : (1)

35. The sum of the first n terms of $\frac{1}{3.6} + \frac{1}{6.9} + \frac{1}{9.12} + ...\infty$

$\frac{1}{3.6} + \frac{1}{6.9} + \frac{1}{9.12} + ... \frac{1}{3n(3n+3)}$

$= \frac{1}{3}\left[\frac{6-3}{3.6} + \frac{9-6}{6.9} + \frac{12-9}{9.12} + ... + \frac{3(n+1)-3n}{3n.3(n+1)}\right]$

$= \frac{1}{3}\left[\frac{1}{3} - \frac{1}{6} + \frac{1}{6} - \frac{1}{9} + \frac{1}{9} - \frac{1}{12} + ... \frac{1}{3n} - \frac{1}{3(n+1)}\right]$

$= \frac{1}{3}\left[\frac{1}{3} - \frac{1}{3(n+1)}\right]$

As $n \to \infty$ $\frac{1}{n+1} \to 0$ and, we get the required sum as

$\frac{1}{3}\left[\frac{1}{3} - 0\right] = \frac{1}{9}$ Choice (B)

36. The value of sum of the terms of the G. P 1, 2, 2^2, 2^3… 2^n
$= \frac{1(2^{n+1}-1)}{2-1} = 2^{n+1} - 1$
If $2^{n+1} - 1 > 1400$,
$2^{n+1} > 1401$
$2^{10} = 1024$
10 is the least value of n satisfying $2^{n+1} > 1401$. Ans : (10)

37. Let the first term of the A.P be a and the common difference be d.
Sum of the first 16 terms and the sum of the first 18 terms are $\frac{16}{2}[2a+15d]$ and $\frac{18}{2}[2a+17d]$ respectively

$\frac{16}{2}[2a+15d] = \frac{18}{2}[2a+17d]$

0 = 2a + 33d

Sum of the first 34 terms of the arithmetic progression
$= \frac{34}{2}[2a+33d] = \frac{34}{2}[0] = 0$ Ans : (0)

38. $X_p = \sqrt{5}.(\sqrt{5})^{p-1} = \sqrt{5}\,x_{p-1}$.

$\therefore \frac{x_p}{x_{p-1}}$ = a constant which is $\sqrt{5}$.

$\therefore$ The terms of x_p are in G.P whose common ratio is $\sqrt{5}$.

Its first term = $x_1 = \sqrt{5}$.

$\therefore \sum_{p=1}^{n} x_p = \frac{\sqrt{5}\left(\sqrt{5}^n - 1\right)}{\sqrt{5}-1}$ = 155 + 31 $\sqrt{5}$

$\sqrt{5}^n - 1 = \frac{155+31\sqrt{5}}{\sqrt{5}}\left(\sqrt{5}-1\right)$

$= \frac{31\sqrt{5}\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)}{\sqrt{5}}$ = 31(5 – 1) = 124

$\sqrt{5}^n = 125$

$5^{\frac{n}{2}} = 5^3$

Equating the powers of n both sides, $\frac{n}{2} = 3$

$\therefore$ n = 6. Ans : (6)

39. The series is a G.P with first term 1 and common ratio $\frac{1}{3}$

Sum of the terms of the series = $\frac{a}{1-r}$

$= \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$ Choice (A)

40. Let the first term and common difference of the A.P be a and d respectively.
$t_1 = a$
$t_{10} = a + 9d$
$t_8 = a + 7d$
$3t_1 = 3a$
$t_{10} - t_8 = 2d$
As $3t_1 = t_{10} - t_8$,
3a = 2d
Hence $\frac{a}{d} = \frac{2}{3}$ Choice (B)

41. Let the first term and common difference of an A.P be be a and d respectively the fourth and the tenth terms of the A. P are a + 3d are a + 9d respectively

$a + 3d = \frac{5}{2} ----(1)$

$a + 9d = 7 ----(2)$

Subtracting (1) from (2)

$6d = \frac{9}{2} \Rightarrow d = \frac{3}{4}$ and $a = \frac{1}{4}$

Seventh term = a + 6d = a + 3d + 3d

$= \frac{5}{2} + 3\left(\frac{3}{4}\right) = \frac{19}{4}$.

Last term = $\frac{23}{2}$

$\Rightarrow \frac{1}{4} + (n-1)\frac{3}{4} = \frac{23}{2}$

$\Rightarrow \frac{1}{4} + \frac{3n}{4} - \frac{3}{4} = \frac{23}{2}$

$\Rightarrow$ n = 16. Choice (D)

42. The sum of the 5th, 6th and 7th terms = $ar^4 + ar^5 + ar^6$

This is equal to $\frac{ar^8 + ar^9 + ar^{10}}{4}$

$\Rightarrow r^4 = 4 \Rightarrow r = \pm\sqrt{2} \Rightarrow r = \sqrt{2}$

Now, $ar + ar^2$ and $ar^7 + ar^8$ have a ratio of $1 : r^6$.

1 : 8 Choice (C)

43. The least two–digit number which when divided by 6 leaves a remainder of 3 is 15.

The largest two–digit number when divided by 6 leaves a remainder of 3 is 99.

If a total of n numbers leave a remainder of 3 when divided by 6, 99 = 15 + (n – 1) 6 (since the two digit numbers are in A. P with first term as 15 and common difference as 6)

n = 15

Sum of all the two digit numbers

$= \frac{15}{2}[15 + 99] = 15[57] = 855$ Ans: (855)

44. Let the four number in A. P be a – 3d, a – d, a + d and a+ 3d.

a – 3d + a – d + a + d + a + 3d = 32

4a = 32 $\Rightarrow$ a = 8

(a – 3d) (a + 3d) = (a – d) (a + d) – 8

$a^2 - 9d^2 = a^2 - d^2 - 8$

$d^2 = 1$

$d = \pm 1$

If d = 1, the four numbers are 5, 7, 9 and 11. if d = –1, the same four numbers are obtained in the reverse order.

Choice (A)

45. Let the number of terms in the A.P be n

sum of the n terms = $\frac{n}{2}$ (first term + last term)

i.e., $\frac{n}{2}[8 + 23] = 248$

n = 16

The last term = 23 = 8 + (n – 1)d

Substituting n = 16 in the equation above, common difference = 1. Ans : (1)

46. Given the first term of A.P. is a and the common difference be d.

Choice (A)

$t_n = a + (n - 1)d$

$t_{n-1} = a + (n - 2)d$

$t_{n-2} = a + (n - 3)d$

$t_{n-1} + t_{n-2} = 2a + (2n - 5)d$, which need not be equal to t_n

$t_{n+1} = a + (n + 1 - 1)d = a + nd$.

$\therefore$ Choice (B) is true

Choice (C)

$t_4 = a + 3d$

$t_8 = a + 7d$

$t_2 = a + d$

$t_{10} = a + 9d$

As $t_4 + t_8 = t_2 + t_{10}$, Choice (C) is true, as both choices (B) and (C) are true, Choice (D) follows.

Choice (D)

47. Let the number of terms of G. P be n, and the common ratio be r.

Sum of n terms of the G. P

$\frac{r\,[\text{last term of G.P}] - [\text{First term of G.P}]}{r - 1}$

$\Rightarrow \frac{r[972] - 4}{r - 1} = 1456$

972r – 4 = 1456 (r – 1)

r = 3

$972 = 4\,(3)^{n-1}$

$243 = 3^5 = 3^{n-1}$

equating powers of 3, both sides we have 5 = n – 1

$\therefore$ n = 6. Choice (A)

48. Let the common ratio be r

The first four terms have a sum of $\frac{2(r^4 - 1)}{r - 1} = 2222$

Substituting the choices for r in the above equation, only r = 10 satisfies it. Choice (A)

49. The person who earns Rs.40 must be having a total of 40 persons under him. The person must have three persons directly under him, each of the three persons must have three persons, directly under him, i.e. a total of 9 persons under them and so on. Hence total number of persons under the person earning ₹39 = 3 + 9 + 27= 40.

Hence there are three levels of members under him.

Persons who do not earn any commission are the number of persons in the bottom most level of hierarchy i.e., 3^{n-1}= 27.

Ans : (27)

50. Fourth term = $ar^3 = 2\,(2)^3 = 16$.

Ans : (16)

Exercise – 20
(Functions)

Solutions for questions 1 to 20:

1. Given, A = {x/x is an odd natural number less than 15}

Roster form of the set A is {1, 3, 5, 7, 9, 11, 13}

Choice (D)

2. Given, A = {1, 2, 3, 4, ….. 12}

Set builder form of A is {x/x is a natural number $\leq$ 12}

Choice (C)

3. Let A = {2, 4, 6, 8}

{ } and {2, 8} are the subsets of A

[since empty set is a subset of every set] Choice (C)

4. Given A = {2, 3, 5, 7} and B = {2, 4, 6, 8}

$\therefore$ n(A) = 4 and n(B) = 4

$\therefore$ A and B are equivalent Choice (C)

5. Given, A = {1, 2, 5, 10}

B = {1, 2, 3, 4, 6, 12}

$\Rightarrow A \cap B = \{1, 2\}$ Choice (B)

6. Given, A = {2, 4, 5, 7, 9}, B = {1, 3, 5, 7, 8} and C = {1, 2, 5, 8}

$\therefore (A \cup B) = \{1, 2, 3, 4, 5, 7, 8, 9\}$

$\Rightarrow (A \cup B) \cap C = \{1, 2, 5, 8\} = C$ Choice (A)

7. For any two sets A and B, $(B - A) \cup A = A \cup B$.

Choice (B)

8. Given, A = {0, 2, 4, 6, 9} and B = {1, 3, 5, 7}

We know that, $A \Delta B = (A \cup B) - (A \cap B)$

Since $A \cap B = \phi$, $A \Delta B = A \cup B$. Choice (C)

9. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is distributive law.

Choice (C)

10. If a set has ‘n’ elements then the cardinal number of its power set is 2^n.

Options (B), (C) and (D) can be expressed as 2^8, 2^0 and 2^6 respectively, but option (A) cannot be expressed in the form 2^n.

$\therefore$ cardinal number of power set of any finite set cannot be 1056.

Choice (A)

11. Given word is ‘EXAMINATION’

This word can be written in the form of a set as A = {E, X, A, M, I, N, T, O}

$\therefore$ number of elements in the set is 8. Choice (B)

12. Given,

Z_p = {factors of p}

Z_1 = {1}

$Z_2 = \{1, 2\}$
$Z_3 = \{1, 3\}$
$Z_4 = \{1, 2, 4\}$
$Z_5 = \{1, 5\}$
$Z_6 = \{1, 2, 6\}$
And N = { 4, 6, 8, 9, 10, 12, 14…..}
$Z_n = Z_4 \bigcap_{n \in N} Z_n = Z_4 \cap Z_6 \cap Z_8 ...... \cap Z_n = \{1\}$ Choice (C)

13. Given $D_n = \{x : 0 < x < 2n\}$
$D_5 = \{x : 0 < x < 10\}$
$D_6 = \{x : 0 < x < 12\}$
$\therefore D_5 \cap D_6 = D_5$ Since $D_5 \subseteq D_6$ Choice (D)

14. Given $D_n = \left\{x : 0 \le x < \frac{1}{3n}\right\}$

$\therefore D_1 = \left\{x : 0 \le x < \frac{1}{3}\right\}$

$D_2 = \left\{x : 0 \le n < \frac{1}{6}\right\}$

$D_1 \cap D_2 = D_2$ ($\because D_2 \subseteq D_1$)

$\therefore$ Similarly $D_1 \cap D_2 \cap D_3 \ldots . \cap D_n = D_n$.
When $n \to \infty$, D_n must be a singleton set i.e. = {0}

$\therefore \bigcap_{n=1}^{\infty} D_n = \{0\}$ Choice (B)

15. Given, n(A) = 4 and n(B) = 6
Minimum number of elements in $A \cup B$ will be 6
i.e., when A is a subset of B. Ans : (6)

16. $p = \frac{7n^2 + 3n + 3}{n}$ = 7n + 3 + 3/n
For p to be a natural number, n = 1 or n = 3. Accordingly p = 13 or p =25. As p is also a prime p = 13 is the only possible value.
So, n (A) = 1. Ans : (1)

17. $|x - 3| < 4$
$\Rightarrow -1 < x < 7$
As x is an integer, x = 0, 1, 2, 3, 4, 5, 6
$|x - 3| < 5$
$\Rightarrow -1 < x < 9$. As x is an integer, x = 0, 1, 2, 3, 4, 5, 6, 7, 8
On taking the intersection, we get {0, 1, 2, 3, 4, 5, 6}.
Choice (C)

18. We know, that, $n (M \cup K) = n (M) + n(K) - n (M \cap K)$
$\Rightarrow n (M \cup K) = m + r - c$. Choice (B)

19. Dual is obtained by replacing $\cup$ with $\cap$ and $\cap$ with $\cup$. Accordingly, we get $A' \cup (A' \cap B) = A'$. Choice (C)

20. $n (A \cup B) = n (A) + n (B) - n (A \cap B)$ (Standard Result)
So, $n (A \cap B)$
$= n (A) + n (B) - n (A \cup B)$
= 18 + 12 – 25 = 5 Ans : (5)

Solutions for questions 21 to 24:

21. Given, A = {1, 2, 3, 4, 6, 12}
The number of subsets which contain 3 and 4 is 2^4 i.e., 16.
Ans: (16)

22. The number of subsets which contain 6 but not 12 is 2^4 i.e., 16
Ans : (16)

23. The number of subsets which contain exactly 4 elements is ${}^6C_4$ i.e., 15 . Ans : (15)

24. The number of subsets which have atmost one element is ${}^6C_0 + {}^6C_1$ i.e., 1 + 6 = 7 Ans : (7)

Solutions for questions 25 to 55:

25. Given, n(A) = 5 and n(B) = 3. Then the number of relations from A to B is $2^{(5)(3)} = 2^{15}$ Choice (B)

26. Given, A = {2, 3, 5, 7}
$\Rightarrow$ n(A) = 4
$\therefore$ maximum number of elements in any relation is $4 \times 4 = 16$.
Ans : (16)

27. In options (A) and (D), multiple images exist for 1. So these are not functions.
In option (B), 3 is not having any image. So it is not a function. Choice (C)

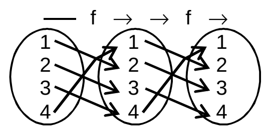
28. In option (A), the number 3 does not have any image. So it is not a function. Choice (A)

29. Domain of the given function is set of all first coordinates. Hence it is {2, 3, 4, 5}. Choice (B)

30. Range is set of second coordinates of the ordered pairs i.e., {1, 2, 3, 4}. Choice (D)

31. Given $f(x, y) = 2x^2 - y$
$f(2, 1) = 2(2)^2 - 1 = 7$
$\therefore f(3, f(2, 1)) = f(3, 7) = 2(3)^2 - 7 = 11$.
Ans : (11)

32.



$\therefore$ fof = {(1, 3), (2, 4), (3, 1), (4, 2)} Choice (D)

33. fog(x) = f[g(x)] = f[2x + 5] = 3(2x + 5) – 2
= 6x + 13. Choice (A)

34. f(g(x)) = f(x + 5)
$= (x + 5)^2 + 3$
$= x^2 + 10x + 28$ Choice (B)

35. $f(x) = ax^2 + bx + a$
$f(1/x) = \frac{a}{x^2} + \frac{b}{x} + a$

$= \frac{1}{x^2} (a + bx + ax^2)$

$= \frac{1}{x^2} (ax^2 + bx + a) = \frac{1}{x^2} f(x)$ Choice (C)

36. The functions containing even powers of x i.e., x^2, x^4 are not bijectives and also having modulus are not bijective.
$\therefore$ option (4) i.e, f(x) = 2x + 1 is a bijective function.
Choice (D)

37. Given $f(2x - 3) = 4x^2 - 12 + 4 \Rightarrow f(2x - 3) = (2x - 3)^2 - 5$
$\therefore f(x) = x^2 - 5 \Rightarrow f(3x - 2) = (3x - 2)^2 - 5$
$= 9x^2 - 12x - 1$. Choice (B)

38. We know (f + g)(x) = f(x) + g(x).
$\therefore$ option (C) is satisfied by the given functions.
Choice (C)

39. A has 4 elements and given f: $A \to B$ is an onto function.
$\therefore$ The number of elements B can have is atmost 4.
Choice (B)

40. Number of one-one functions possible are ${}^{n(B)}P_{n(A)}$.
Here n(B) = 4 and n(A) = 3. ${}^4P_3 = \frac{4!}{1!} = 24$.
Choice (C)

41. If f: A $\to$ B, is one-one, then n(A) $\le$ n(B).
Hence p $\le$ q. Choice (C)

42. If n(A) = m and n(B) = 2 then (2^m – 2) onto functions are possible from A to B. Here m = 4.
Thus $2^4 - 2 = 14$. Ans : (14)

43. Given $f(x) = 3x^2 + 4\cos x + 2\sin^2 x$
$f(-x) = 3(-x)^2 + 4\cos(-x) + 2\sin^2(-x)$
$= 3x^2 + 4\cos x + 2\sin^2 x = f(x)$
$\therefore$ f (x) is even. Choice (A)

44. Let $y = \dfrac{3x-5}{2x+1}$
$\Rightarrow 2xy + y = 3x - 5$
$\Rightarrow x(2y-3) = -(y+5)$
$\Rightarrow x = \dfrac{-(y+5)}{2y-3}$ or $x = \dfrac{y+5}{3-2y}$
$\therefore f^{-1}(x) = \dfrac{x+5}{3-2x}$.
Choice (C)

45. y = f(x) = 5x + 7
$\Rightarrow x = \dfrac{y-7}{5}$ $\Rightarrow f^{-1}(x) = \dfrac{x-7}{5}$
Now $f^{-1}(2) = \dfrac{2-7}{5} = -1$
and $f^{-1}(22) = \dfrac{22-7}{5} = 3$
Thus $f^{-1}(\{2, 22\}) = \{-1, 3\}$ Choice (B)

46. The denominator $5 - x \ne 0 \Rightarrow x \ne 5$.
Thus domain is R – {5}. Choice (D)

47. f(x) is real when $2x - 3 \ge 0 \Rightarrow x \ge 3/2$
$\therefore$ domain of f(x) is $[3/2, \infty)$. Choice (A)

48. Domain of log(x – 2) is x – 2 > 0
$\Rightarrow x \ge 2$ and domain of $\dfrac{1}{\sqrt{x^2-4}}$ is $x^2 - 4 > 0$
$\Rightarrow (x+2)(x-2) > 0$
$\Rightarrow x > 2$ or $x < -2$
$\therefore$ domain of the given function is $\{x \ge 2\} \cap \{x > 2 \text{ or } x < -2\}$
$\therefore$domain is x > 2 or $(2, \infty)$. Choice (C)

49. $(-1)^n = +1$ or -1 for $n \in W$. So $(-1)^n + 1 = +2$ or 0 for $n \in W$. Thus f(n) = +2 or 0 for $n \in W$.
So Range = {0, 2}. Choice (B)

50. f(x) = 3x – 2 for x = {1, 2, 3, …}
f(1) = 3 – 2 = 1
f(2) = 6 – 2 = 4
f(3) = 9 – 2 = 7 and so on.
$\therefore$ Range = {1, 4, 7, …} Choice (C)

51. When $x < \dfrac{1}{2}$ then $|2x - 1| < 0 \Rightarrow \dfrac{2x-1}{|2x-1|} = -1$ and when
$x > \dfrac{1}{2}$ then $|2x - 1| > 0$
$\Rightarrow \dfrac{2x-1}{|2x-1|} = 1$
$\therefore$ Range of $\dfrac{(2x-1)}{|2x+1|}$ is {–1, 1}.
Choice (D)

52. By definition of [x], we have [x] < [x] + 1
Accordingly $0 \le x - [x] < 1$
Now [x – [x]] = 0
h(g(x)) = h(x – [x])
= [x – [x]] = 0
Hence the range is {0}. Choice (C)

53. Given $f(x) = 3(x^2 + \dfrac{1}{x^2}) + 7(x + \dfrac{1}{x}) + 8$
$f(x) = \left\{3\left(x + \dfrac{1}{x}\right)^2 - 2\right\} + 7\left(x + \dfrac{1}{x}\right) + 8$
$f(\beta) = 3\left\{\left(\beta + \dfrac{1}{\beta}\right)^2 - 2\right\} + 7\left(\beta + \dfrac{1}{\beta}\right) + 8$
Let $\beta + \dfrac{1}{\beta} = m$
$= 3m^2 - 6 + 7m + 8$
$= 3m^2 + 7m + 2$
$= (3m + 1)(m + 2)$
$\therefore f(\beta) = 0$
$(3m + 1)(m + 2) = 0$
$m = -2$ or $m = \dfrac{-1}{3}$
$\beta + \dfrac{1}{\beta} = -2$
$\Rightarrow \beta = -1$
When $\beta + \dfrac{1}{\beta} = \dfrac{-1}{3}$ β is not a real number.
Choice (D)

54. Given, $f(x) = \dfrac{|x|}{x}$
When $x \ge 0$, |x| = x.
$\therefore f(x) = \dfrac{x}{x} = 1$
When x < 0, |x| = –x.
$\therefore f(x) = -\dfrac{x}{x} = -1$
$\therefore$ Range = {–1, 1} Choice (B)

55. If n(A) = m, n(B) = 2, then the number of onto functions defined from A to B is (2^m – 2)
Here, n(A) = 5, n(B) = 2 = 32 – 2 = 30. Ans : (30)

Exercise – 21
(Graphs)

Solutions for questions 1 to 4:

1. We can notice that the graph g(x) can be obtained from that of f(x) by reflecting it in the in y-axis and then in the x-axis
$\therefore$ It is a double reflection
i.e., f(x) = –g (–x) Choice (D)

2. The graph represents the function $y = |\log x|$
Choice (C)

3. The figure given is a graph of discontinuous function.
According to the figure,
$f(x) = 0 \ \forall\ x \in [0, 1)$
$f(x) = 1 \forall\ x \in [1, 2)$ and so on.
Clearly, this represents the function f(x) = [x].
Choice (A)

4. Both options (B) and (C) are satisfied by the point (2, 0)
But, the graph is not defined for $x \le 1$.
So, $y = \log_e(x - 1)$ is the required relation. Choice (B)

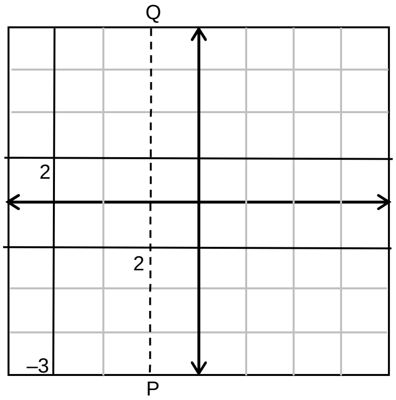
Solutions for questions 5 to 15:

5. The reflection of the point (x, y) in the origin is $(-x_1, -y_1)$

∴ The reflection of (–1, 4) in the origin is (1, –4).

Choice (D)

6.



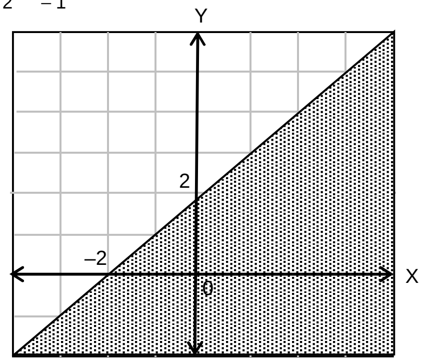
P (2, –3) is the given point. Let Q be the image of P in the line y = 2.Q is at a distance of 5 units from the line y = 2 as shown above.

∴ The coordinates of Q are (2, 2 + 5) = (2, 7)

Choice (A)

7. The given line meets the x and the y– axis at (1/2, 0) and (0, –1) respectively.
The equation of the line is

$\frac{x}{1/2} - \frac{y}{-1} = 1 \Rightarrow 2x - y = 1$ or $y = 2x - 1$

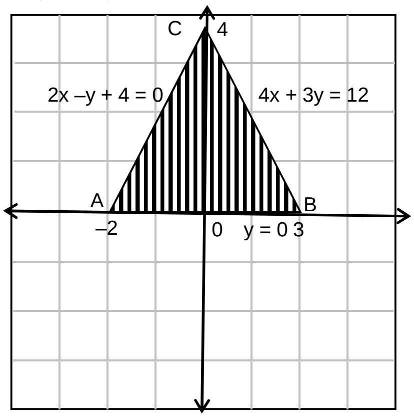


As the shaded region is not the origin side, the origin is not a solution Hence the relation that represents the graph is $y \le 2x - 1$. Choice (D)

8. y = f (x + a) is the graph obtained by shifting the graph of y = f(x) horizontally towards left by a units (standard result). Choice (B)

9. y = f(x) – c is the graph obtained by shifting the graph of y = f(x) vertically down by c units (standard result) Choice (D)

10. The graph of 4x + 3y = 12 is the line BC and that of $4x + 3y \le 12$ (1) is the region BOC. The graph of 2x –y= –4 in the line CA and that of $2x - y \ge -4$ (2) is the region AOC. The graph of y = 0 in the x axis and that of $y \ge 0$ (3) in the upper half plane. The region represented by (1), (2), (3) is the triangular region ABC.



Its area $= \frac{1}{2}(AB)(OC) = \frac{1}{2}(5)(4) = 10.$ Ans : (10)

11. The graph of g(x) is obtained by shifting the graph of f(x) vertically down by c units.
$\therefore g(x) = f(x) - c \quad (c > 0)$ Choice (B)

12. The graph of y = f(–x) is obtained by reflecting the graph of y = f(x) in y – axis. Choice (C)

13. If y = f(x) represents c, then x = f(y) represents d, which is obtained by reflecting c in the line y = x. Choice (C)

14. $4x^2 - 12x + 9 = y$
On x–axis y = 0
$\Rightarrow 4x^2 - 12x + 9 = 0 \Rightarrow (2x - 3)^2 = 0$

$x = \frac{3}{2}$

$\therefore$ The graph intersects x-axis at on one point
Choice (B)

15. The feasible region is bounded by the lines $x_1 = 10$ and $x_2 = 5$. Also (0, 0) is in feasible region. Accordingly $x_1 \le 10$ and $x_2 \le 5$. Choice (B)

Exercise – 22
(Indices and Surds)

Solutions for questions 1 to 40:

1. $\left(\frac{169}{121}\right)^{\frac{-3}{2}} \times \frac{27}{2} \times \left(\frac{13}{22}\right)^{-1}$

$\left(\frac{13^2}{11^2}\right)^{\frac{-3}{2}} \times \frac{27}{2} \times \frac{22}{13} = \frac{13^{-3}}{11^{-3}} \times \frac{27}{2} \times \frac{22}{13}$

$= 13^{-4}\, 11^4\, 3^3$. Choice (D)

2. $\frac{3^{a+1} 9^{a+2} 27^a}{3^{a-1} 9^a 27^{a+1}} =$

$\frac{3^{a+1} 3^{2(a+2)} (3^3)^a}{3^{a-1} (3^2)^a (3^3)^{a+1}} = \frac{3^{6a+5}}{3^{6a+2}} = 3^3 = 27$ Ans : (27)

3. $\frac{3^{-3} \times 9^{\frac{5}{2}}}{27^{\frac{2}{3}} \times 3^{-4}} = \frac{3^{-3} \times (3^2)^{\frac{5}{2}}}{(3^3)^{\frac{2}{3}} \times 3^{-4}}$

$= \frac{3^{-3} \times 3^5}{3^2 \times 3^{-4}}$

$= \frac{3^{-3+5}}{3^{2-4}} = \frac{3^2}{3^{-2}} = 3^2 \times 3^2 = 81$ Ans : (81)

4. $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

$81^{\frac{1}{4}} = \sqrt[4]{81} = 3.$

$16^{\frac{1}{2}} = \sqrt{16} = 4.$

Required value = 9 + 3 – 4 = 8. Ans : (8)

5. $2\sqrt{\frac{5}{2}} = \sqrt{2}.\sqrt{5} = \sqrt{10}$

$5\sqrt{\frac{2}{5}} = \sqrt{5}.\sqrt{2} = \sqrt{10}$

$\sqrt{1000} = \sqrt{100.10} = 10\sqrt{10}$

Required value = $\sqrt{10} + \sqrt{10} + \sqrt{10} + 10\sqrt{13} = 13\sqrt{10}$
Choice (B)

6. $1 - \{1 + (a^2 - 1)^{-1}\}^{-1}$

$= 1 - \left\{1 + \frac{1}{a^2 - 1}\right\}^{-1}$

$= 1 - \left\{\frac{a^2}{a^2 - 1}\right\}^{-1} = 1 - \frac{a^2 - 1}{a^2} = \frac{1}{a^2}.$ Choice (A)

7. $y^{p\,(q-r)} \cdot y^{q\,(r-p)} \cdot y^{r\,(p-q)}$
$= y^{pq-pr} \cdot y^{qr-qp} \cdot y^{rp-rq}$
$= y^{pq-pr+qr-qp+rp-rq}$
$= y^0 = 1.$ Ans : (1)

8. $\frac{y^{4b-a} \cdot y^{4c-b} \cdot y^{4a-c}}{\left(y^a \cdot y^b \cdot y^c\right)}$

$$= \frac{y^{4b-a} \cdot y^{4c-b} \cdot y^{4a-c}}{\left(y^{a+b+c}\right)^3}$$

$$= \frac{y^{4b-a+4c-b+4a-c}}{y^{3(a+b+c)}}$$

$$= \frac{y^{3(a+b+c)}}{y^{3(a+b+c)}} = 1.$$

Ans : (1)

9. $\left(x^{a^4}\right)^{\frac{1}{a^2bc}} \cdot \left(x^{b^4}\right)^{\frac{1}{ab^2c}} \cdot \left(x^{c^4}\right)^{\frac{1}{abc^2}}$

$= x^{\frac{a^3}{abc}+\frac{b^3}{abc}+\frac{c^3}{abc}} = x^{\frac{a^3+b^3+c^3}{abc}}$

As a + b = – c, a + b + c = 0

When a + b + c = 0, $a^3 + b^3 + c^3 = 3abc$.

Hence the value of the required expression is x^3.

Choice (A)

10. $\dfrac{\left(a+\frac{1}{b}\right)^c \left(a-\frac{1}{b}\right)^d}{\left(b+\frac{1}{a}\right)^c \left(b-\frac{1}{a}\right)^d} = \dfrac{\left(\frac{ab+1}{b}\right)^c \left(\frac{ab-1}{b}\right)^d}{\left(\frac{ba+1}{a}\right)^c \left(\frac{ba-1}{a}\right)^d} = \left(\frac{a}{b}\right)^c \times \left(\frac{a}{b}\right)^d$

$= \left(\frac{a}{b}\right)^{c+d} = \left(\frac{a}{b}\right)^e$

Equating powers of $\frac{a}{b}$ on both sides, e = c + d.

Choice (A)

11. $3^{q-p-r} = \dfrac{3^q}{(3^p)(3^r)} = \dfrac{y}{xz}$

Choice (A)

12. $3^{4m+1} = 3^{7m-5}$ equating powers of 3 on both sides, 4m + 1 = 7m – 5

3m = 6 ⇒ m = 2.

Ans : (2)

13. $2^{4m+2} = \left(2^2\right)^{6m-4}$

$2^{4m+2} = 2^{12m-8}$

Equating powers of 2 both sides,

4m + 2 = 12m – 8

10 = 8m ⇒ m = 5/4 = 1.25

Ans : (1.25)

14. $2^{3y+3} = 2^{3y+1} + 48$

$2^3 (2^{3y}) = 2^1 (2^{3y}) + 48$

$6 (2^{3y}) = 48$

$2^{3y} = 2^3$

equating powers of 2 both sides,

3y = 3

y = 1.

Ans : (1)

15. $3\left(\sqrt{3}\right)^{m+4} = 3\left(3^{\frac{1}{2}}\right)^{m+4} = 3^{1+\frac{m+4}{2}}$

$\sqrt{3}^{2m+7} = \left(3^{\frac{1}{2}}\right)^{2m+7} = 3^{\frac{2m+7}{2}}$

As $3(\sqrt{3})^{m+4} = \sqrt{3}^{2m+7}$

$3^{1+\frac{m+4}{2}} = 3^{\frac{2m+7}{2}}$

equating powers of 3 on both sides, m = –1.

Choice (B)

16. $6^{8m} = 6^4 \times 6^{3(2m+4)}$

$= 6^{6m+16}$

equating powers of 6 on both sides, 8m = 6m + 16

m = 8.

Ans : (8)

17. $\left(\sqrt[3]{\frac{7}{4}}\right)^{y-1} = \dfrac{64}{343}$

$\left(\left(\frac{7}{4}\right)^{\frac{1}{3}}\right)^{y-1} = \left(\left(\frac{7}{4}\right)^3\right)^{-1}$

$\left(\frac{7}{4}\right)^{\frac{y-1}{3}} = \left(\frac{7}{4}\right)^{-3}$, Equating powers of $\frac{7}{4}$ on both sides,

$\frac{y-1}{3} = -3$

y = –8.

Choice (B)

18. Considering from the left if the decimal point is shifted by 8 places to the right, the number becomes 1154.111 Therefore $0{\cdot}00001154111 \times 10^x$ exceeds 1000 when x has a minimum value of 8.

Ans : (8)

19. $2^{y^{\sqrt{2}^2}} = 512$

$2^{y^2} = 2^9$ $\left(\because \sqrt{2}^2 = (2^{1/2})^2 = 2\right)$

equating powers of 2 on both sides,

$y^2 = 9 \Rightarrow y = \pm 3$

Choice (A)

20. $\dfrac{y-1}{\sqrt{y}} = \sqrt{y} - \dfrac{1}{\sqrt{y}}$

$y = 4 + 3 + 2\sqrt{4}.\sqrt{3} = \left(2+\sqrt{3}\right)^2$

$\sqrt{y} = 2 + \sqrt{3}$

$\dfrac{1}{\sqrt{y}} = \dfrac{2-\sqrt{3}}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} = 2 - \sqrt{3}$

$\sqrt{y} + \dfrac{1}{\sqrt{y}} = 4$

Ans : (4)

21. $\left(\dfrac{\sqrt[6]{pq} - \sqrt[3]{q}}{\sqrt[3]{p} - \sqrt[6]{pq}}\right)^{-6} = \left(\dfrac{\sqrt[6]{q}\left(\sqrt[6]{p} - \sqrt[6]{q}\right)}{\sqrt[6]{p}\left(\sqrt[6]{p} - \sqrt[6]{q}\right)}\right)^{-6} = \left(\sqrt[6]{\dfrac{q}{p}}\right)^{-6} = \left(\left(\dfrac{q}{p}\right)^{\frac{1}{6}}\right)^{-6}$

$\left(\dfrac{q}{p}\right)^{\frac{1}{6} \cdot -6} = \dfrac{p}{q}.$

Choice (A)

22. $\dfrac{12}{\sqrt{15} - \sqrt{11}}$ becomes $\dfrac{12(\sqrt{15}+\sqrt{11})}{(\sqrt{15}-\sqrt{11})(\sqrt{15}+\sqrt{11})}$

on multiplying both numerator and denominator by $\left(\sqrt{15}+\sqrt{11}\right)$

$\dfrac{12(\sqrt{15}+\sqrt{11})}{(\sqrt{15}-\sqrt{11})(\sqrt{15}-\sqrt{11})} = \dfrac{12(\sqrt{15}+\sqrt{11})}{(\sqrt{15})^2 - (\sqrt{11})^2}$

$= 3\sqrt{15} + 3\sqrt{11}$.

Choice (A)

23. $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} =$

$\dfrac{(\sqrt{7}+\sqrt{5})^2 + (\sqrt{7}-\sqrt{5})^2}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} =$

$= \frac{(\sqrt{7}+\sqrt{5})^2 + (\sqrt{7}-\sqrt{5})^2}{2} = 12$

Ans : (12)

24. $\sqrt{y} - \frac{4}{\sqrt{y}} = \frac{y-4}{\sqrt{y}} = \frac{8+8\sqrt{2}}{\sqrt{y}}$

$\sqrt{y} = \sqrt{4(3+2\sqrt{2})} = 2\sqrt{(3+2\sqrt{2})}$

$\sqrt{3+2\sqrt{2}} = (\sqrt{2}+1)$ as shown in solution 1.

Hence $\sqrt{y} = 2(\sqrt{2}+1)$

$\sqrt{y} - \frac{4}{\sqrt{y}} = \frac{8(1+\sqrt{2})}{2(\sqrt{2}+1)} = 4$ Ans : (4)

25. Let the square root of $6+2\sqrt{5}$ be $\sqrt{a}+\sqrt{b}$

$\sqrt{6+2\sqrt{5}} = \sqrt{a}+\sqrt{b}$

squaring both sides, $6+2\sqrt{5} = a+b+2\sqrt{ab}$

equating the rational and irrational parts on both sides,

$a + b = 6 \quad \rightarrow \quad (1)$

$ab = 5$

$a-b = \sqrt{(a+b)^2 - 4ab}$

As a + b = 6 and ab = 5,

$a-b = \sqrt{6^2 - 4(5)} = 4 \quad \rightarrow \quad (2)$

as a > b

Adding (1) and (2),

$2a = 10 \Rightarrow a = 5$

$\Rightarrow b = 6 - a = 1.$

The square root is $\sqrt{5}+1$ Choice (A)

26. Let $\sqrt{\sqrt{245}+\sqrt{240}}$ be $\sqrt{a}+\sqrt{b}$ where a > b

$\sqrt{245}+\sqrt{240} = \sqrt{5}(\sqrt{149}+\sqrt{48}) = \sqrt{5}(7+\sqrt{48})$

$\sqrt{\sqrt{245}+\sqrt{240}} = \sqrt{\sqrt{5}(7+\sqrt{48})}$

Let $\sqrt{7+\sqrt{48}}$ be $\sqrt{c}+\sqrt{d}$

where c > d

squaring both sides,

$7+\sqrt{48} = 7+2\sqrt{12} = c+d+2\sqrt{cd}$

Equating the rational terms and equating the irrational terms both sides,

$c + d = 7 \quad \rightarrow \quad (1)$

$cd = 12$

$c-d = \sqrt{(c+d)^2 - 4cd} = 1$ as c > d $\rightarrow$ (2)

Adding (1) and (2),

$2c = 8$

$c = 4$

$\Rightarrow d = 7 - c = 3.$

Hence $\sqrt{\sqrt{a}+\sqrt{b}} = 5^{\frac{1}{4}}(2+\sqrt{3}).$ Choice (B)

27. Let the two surds be p and q

Arithmetic mean of the two surds = $\frac{p+q}{2} = \frac{5}{2}\sqrt{2}+3$

p + q = $5\sqrt{2}+6$. As $p = 3(\sqrt{2}+1)$ (given)

q = $5\sqrt{2}+6-p = 2\sqrt{2}+3$

$= 2 + 1 + 2\sqrt{2} = (\sqrt{2}+\sqrt{1})^2$

$\sqrt{q} = \sqrt{2}+1$ Choice (A)

28. (i) $4^{50} = (2^2)^{50} = 2^{100}$

$16^{25} = (2^4)^{25} = 2^{100}$

Hence 4^{50}, 2^{100} and 16^{25} are all equal.

Choice (D)

(ii) $4^{50} = (4^5)^{10} = 1024^{10}$

$5^{40} = (5^4)^{10} = 625^{10}$

$10^{20} = (10^2)^{10} = 100^{10}$

$100^2 < 100^{10} < 625^{10} < 1024^{10}$

Hence 4^{50} is the greatest. Choice (A)

29. $3^{3^{3^3}} = 3^{3^9}$

In Choice (B), the base 333 lies between 243 = (3^5) and 729 = (3^6)

Hence 333^3 lies between $(3^5)^3$ and $(3^6)^3$.

In Choice (C), the base 33 lies between 27 = (3^3) and 81 = (3^4).

Hence 33^{33} lies between $(3^3)^{33}$ and $(3^4)^{33}$ i.e. between 3^{99} and 3^{132}. Hence $3^{3^{3^3}}$ is the greatest. Choice (A)

30. Since the smallest common multiple of 6, 12, 8 and 4 is 24, arrange the terms such that each term is raised to power of 1/24.

$6^{\frac{1}{6}} = (6^4)^{\frac{1}{24}} = (1296)^{\frac{1}{24}}$

$4^{\frac{5}{12}} = (4^{10})^{\frac{1}{24}} = [1024]^{\frac{1}{24}}$

$3^{\frac{1}{8}} = (3^3)^{\frac{1}{24}} = 27^{\frac{1}{24}}$

$9^{\frac{1}{4}} = (3^6)^{\frac{1}{24}} = (729)^{\frac{1}{24}}$

Hence the greatest among the four is $4^{5/12}$.

Choice (B)

31. Squaring the surds $\sqrt{2}+\sqrt{13}$, $\sqrt{4}+\sqrt{11}$, $\sqrt{34}+\sqrt{6}$ and $\sqrt{17}+\sqrt{12}$, we get

$15+2\sqrt{26}$, $15+2\sqrt{44}$, $40+2\sqrt{204}$ and $29+2\sqrt{204}$ respectively.

$40+2\sqrt{204} > 29+2\sqrt{204} > 15+2\sqrt{44} > 15+2\sqrt{26}$

Therefore $\sqrt{40+2\sqrt{204}}$ or $\sqrt{34}+\sqrt{6}$ is the greatest.

Choice (C)

32. $\frac{1}{\sqrt{9}+\sqrt{10}} = \frac{\sqrt{9}-\sqrt{10}}{(\sqrt{9}-\sqrt{10})(\sqrt{9}-\sqrt{10})}$ (multiplying both its numerator and denominator by $\sqrt{9}-\sqrt{10}$)

Hence $\frac{1}{\sqrt{9}+\sqrt{10}} = \frac{\sqrt{9}-\sqrt{10}}{(\sqrt{9})^2-(\sqrt{10})^2} = \frac{\sqrt{9}-\sqrt{10}}{-1}$

By multiplying numerator and denominator of each term by the conjugate of their denominators, the given expression becomes

$\frac{\sqrt{9}-\sqrt{10}}{-1} + \frac{\sqrt{10}-\sqrt{11}}{-1} + \frac{\sqrt{11}-\sqrt{12}}{-1} + ... \frac{\sqrt{99}-\sqrt{100}}{-1}$

$= \frac{\sqrt{9}-\sqrt{100}}{-1} = 7.$

Choice (A)

33. $2^{11} = 2048$ and $2^{12} = 4096$.

$2^{11} < 2222 < 2^{12}$

$\therefore (2^{11})^{22} < 2222^{22} < (2^{12})^{22}$ i.e. $2^{242} < 2222^{22} < 2^{264}$

$2^7 = 128$ and $2^8 = 256$

$\therefore 2^7 < 222 < 2^8$.

$\therefore (2^7)^{222} < 222^{222} < (2^8)^{222}$ i.e. $2^{1554} < 222^{222} < 2^{1776}$

$2^4 = 16$ and $2^5 = 32$.
$2^4 < 22 < 2^5$.
$\therefore (2^4)^{2222} < 22^{2222} < (2^5)^{2222}$ i.e., $2^{8888} < 22^{2222} < 2^{11110}$.
It can be seen that the greatest term is $2^{22222.}$
Choice (D)

34. $\dfrac{1}{1+\beta^3+\beta^{-3}} = \dfrac{\beta^3}{\beta^3\left(1+\beta^3+\beta^{-3}\right)} = \dfrac{\beta^3}{1+\beta^3+\beta^6}$.

$\dfrac{1}{1+\beta^{-3}+\beta^{-6}} = \dfrac{\beta^6}{\beta^6\left(1+\beta^{-3}+\beta^{-6}\right)} = \dfrac{\beta^6}{1+\beta^3+\beta^6}$.

Required sum = $\dfrac{1+\beta^3+\beta^6}{1+\beta^3+\beta^6} = 1$. Choice (C)

35. $\sqrt{5\sqrt{5\sqrt{5\sqrt{5\ldots\ldots\ldots\ldots}}}}$ 8times$= 5^{\frac12}\cdot 5^{\frac14}\cdot 5^{\frac18}\ldots\ldots 5^{\frac{1}{256}}$

$= 5^{\frac12+\frac14+\frac18+\ldots\ldots\ldots+\frac{1}{256}}$

The power of 5 is a geometric progression with first term being $\dfrac12$ having 8 terms.

Sum of these terms = $\dfrac{\frac12\left(1-\left(\frac12\right)^8\right)}{1-\frac12}$

$= 1-\left(\dfrac12\right)^8 = \dfrac{255}{256}$

Alternate method:

$\sqrt{5} = 5^{1-\frac12}$

$\sqrt{5\sqrt{5}} = 5^{1-\left(\frac12\right)^2}$

$\sqrt{5\sqrt{5\sqrt{5}}} = 5^{\frac12}5^{\frac14}5^{\frac18}$

$= 5^{\frac12+\frac14+\frac18} = 5^{\frac78} = 5^{1-\left(\frac12\right)^3}$

Hence from the above patterns, the value of

$\sqrt{5\sqrt{5\sqrt{5\sqrt{5\ldots\ldots.8\text{times}}}}}$

$= 5^{1-\left(\frac12\right)^8} = 5^{\frac{255}{256}}$ Choice (C)

36. As $8 > 7 > 6, 8^{\frac16} > 7^{\frac16} > 6^{\frac16}$

$5^{\frac{5}{24}} = \left(5^5\right)^{1/24} = (3125)^{1/24}$

$7^{\frac16} = 7^{\frac{4}{24}} = \left(7^4\right)^{\frac{1}{24}}$

$= (2401)^{1/24}$

$8^{\frac16} = \left(8^4\right)^{1/24} = (4096)^{1/24}$,

$7^{\frac16} < 5^{\frac{5}{24}} < 8^{\frac16}$

Hence $6^{\frac16} < 7^{\frac16} < 5^{\frac{5}{24}} < 8^{\frac16}$ Choice (D)

37. $\sqrt[6]{\sqrt[5]{\sqrt[4]{\sqrt[3]{x}}}} = \left(\left(\left(x^{\frac13}\right)^{\frac14}\right)^{\frac15}\right)^{\frac16} = \left(\left(x^{\frac13\times\frac14}\right)^{\frac15}\right)^{\frac16}$

$= \left(x^{\frac13\times\frac14\times\frac15}\right)^{\frac16} = x^{\frac13\times\frac14\times\frac15\times\frac16} = x^{\frac{1}{360}}$. Choice (B)

38. Let $\sqrt{400\sqrt{400\sqrt{400\ldots\ldots\infty}}} = x$
Hence $\sqrt{400x} = x$
Squaring both sides
$400x = x^2$
$x(x-400) = 0$
$\Rightarrow x = 0$ or $x - 400 = 0$ i.e., $x = 400$
As x cannot be 0, x = 400 Ans : (400)

39. Multiplying both the numerator and the denominator of $\dfrac{1}{1+x^{b-a}+x^{c-a}}$ by x^a, it becomes $\dfrac{x^a}{x^a+x^b+x^c}$. Similarly multiplying both the numerator and the denominator of $\dfrac{1}{1+x^{a-b}+x^{c-b}}$, by x^b, it becomes $\dfrac{x^b}{x^b+x^a+x^c}$. Multiplying both the numerator and the denominator of $\dfrac{1}{1+x^{a-c}+x^{b-c}}$ by x^c, it becomes $\dfrac{x^c}{x^c+x^a+x^b}$. The value of the required expression is $\dfrac{x^a}{x^a+x^b+x^c}+\dfrac{x^b}{x^b+x^a+x^c}+\dfrac{x^c}{x^c+x^a+x^b}$

$= \dfrac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$ Ans : (1)

40. $a = \dfrac{9}{9+6\sqrt2}$
By multiplying both numerator and denominator of a by $9-6\sqrt2$,

$a = \dfrac{9\left(9-6\sqrt2\right)}{\left(9+6\sqrt2\right)\left(9-6\sqrt2\right)} = \dfrac{9\left(9-6\sqrt2\right)}{9^2-\left(6\sqrt2\right)^2}$

$a = 9-6\sqrt2$

$a^2 = 153-108\sqrt2$

$18a = 162 - 108\sqrt2$.

Hence $a^2 + 18a = 315 - 216\sqrt2$. Choice (D)

Exercise – 23
(Logarithms)

Solutions for questions 1 to 25:

1. (a) $\log_{125}625 = \log_{5^3}5^4$

$= \dfrac43 \left[\text{As } \log_{a^n}a^m = \dfrac{m}{n}\right]$ Choice (B)

(b) $\log_{2^8}512 = \log_{2^8}2^9 = \dfrac98$ Choice (A)

2. $\log_{81}\left(\dfrac{1}{81}\right) = \log_{81}81^{-1} = -1$ Choice (B)

3. log125 + log8 – log10

$= \log\left(\frac{125 \times 8}{10}\right) = \log 100$

As the base is not mentioned, the value of the given expression is $\log_{10}100 = \log_{10}10^2 = 2$.

Ans : (2)

4. $\log 2 + \log 50 + 1$
$= \log (2 \times 50) + 1$
$= \log 100 + 1$
As the base is not mentioned, the value of the given expression is $\log_{10} 100 + 1 = 3$. Ans : (3)

5. $5 + \log\left(\frac{1}{1000}\right)$
$= 5 + \log 10^{-3}$
As the base is not mentioned, the value of the given expression becomes $5 + \log_{10}10^{-3}$
$= 5 - 3 = 2$ Ans : (2)

6. $\log x + \log 3 = \log 15$
$\log (3x) = \log 15$
Equating arguments of logarithms both sides,
$\Rightarrow 3x = 15 \Rightarrow x = 5$ Ans : (5)

7. $\log (x + 2) + \log (x - 2) = \log 5$
$\log[(x + 2)(x - 2)] = \log 5$
Comparing arguments of logarithms both sides,
$x^2 - 4 = 5$
$x^2 = 9$
$x = \sqrt{9} = \pm 3$
Logarithms are defined only for positive values. Hence in the given equation, $x - 2$ must be positive.
Hence $x = 3$. Ans : (3)

8. $\frac{\log 2401}{\log 343} = \frac{\log 7^4}{\log 7^3} = \frac{4 \log 7}{3 \log 7}$

$= \frac{4}{3} = \log_{10} x$ (since the base is not mentioned)

$x = 10^{\frac{4}{3}} = \sqrt[3]{10000}$ Choice (D)

9. $\log_5 40 + \log_5 150 - \log_5 48 - 2$
$= \log_5\left[\frac{(40)(150)}{48}\right] - 2 = \log_5\left(5^3\right) - 2 = 1$. Ans : (1)

10. $0.9\log_5 25 + 0.09\log_5 25 + 0.009\log_5 25 + \ldots\ldots\infty$
$= [\log_5 25]\ (0.9 + 0.09 + 0.009 + \ldots\ldots.)$
$0.9 + 0.09 = 0.99$
$0.9 + 0.09 + 0.009 = 0.999$
$0.9 + 0.09 + 0.009 + 0.0009 = 0.9999$
Hence, it can be seen that the more terms of the expression in brackets are taken, the sum increases to 1. When the number of terms taken and added becomes infinite, we expect their sum to be 1.
Hence the value of the required expression is $[\log_5 25]\ (1) = 2$ Ans : (2)

11. $\log 4 + \log 8 - \log\left(\frac{32}{1000}\right) + \log_{32} 1024$

$= \log(4 \times 8) - \log\left(\frac{32}{1000}\right) + \log_{32} 32^2$

$= \log 32 - \log\left(\frac{32}{10^3}\right) + 2 = \log\left(\frac{32}{\frac{32}{10^3}}\right) + 2 = \log 10^3 + 2$

As the base is not mentioned it can be taken to be 10. Hence the value of the given expression
$= \log_{10}10^3 + 2 = 5$. Ans : (5)

12. $3\log_{100}2 - \log_{100}4 + 3\log_{100}5 - \log_{100}250$

$= \log_{100}\left(\frac{2^3}{4} \times \frac{5^3}{250}\right)$

$= \log_{100} 1 = 0$, since logarithm of 1 to any base is 0.
Ans : (0)

13. $\frac{1}{1 + \log_{ab} c} = \frac{1}{\log_{ab} ab + \log_{ab} c} = \frac{1}{\log_{ab} abc} = \log_{abc}ab$

$\frac{1}{1 + \log_{ac} b} = \frac{1}{\log_{ac} ac + \log_{ac} b} = \frac{1}{\log_{ac} abc} = \log_{abc}ac$

$\frac{1}{1 + \log_{bc} a} + \frac{1}{\log_{bc} bc + \log_{bc} a} = \frac{1}{\log_{bc} abc} = \log_{abc}bc$

Hence the value of the required expression $= \log_{abc}ab + \log_{abc}ac + \log_{abc}bc = \log_{abc}[(ab)\ (ac)\ (bc)]$
$= \log_{abc} (abc)^2 = 2$. Choice (C)

14. $\frac{\log_9 81}{\log_{11} 1331} = \frac{\log_9 9^2}{\log_{11} 11^3} = \frac{2}{3}$. Choice (A)

15. $49^{2\log_7 \sqrt{2}} = 7^{4\log_7 \sqrt{2}} = 7^{\log_7 \left(\sqrt{2}\right)^4} = 7^{\log_7 4}$

$= 4$ $\left[\because a^{\log_a N} = N\right]$ Choice (B)

16. $\log_9\left(\frac{1}{729}\right) = \log_9 729^{-1} = \log_9 9^{-3} = -3$ Choice (A)

17. $\frac{\log_5 64}{\log_5 4} = \log_3 x \Rightarrow \frac{\log_5 4^3}{\log_5 4} = \log_3 x$
$\Rightarrow 3 = \log_3 x \quad \Rightarrow x = 3^3 = 27$ Choice (B)

18. $\log\left[\frac{14x + 114}{4x + 1}\right] = \log 10$
Equating arguments of logarithms both sides,
$14x + 114 = 10\ (4x + 1)$
$14x + 114 = 40x + 10$
$104 = 26x$
$4 = x$. Ans : (4)

19. $(3x + 8) \log 4 = (7x + 9) \log 4^2$
$(3x + 8) \log 4 = 2\ (7x + 9) \log 4$
Cancelling $\log 4$ on both sides, $3x + 8 = 14x + 18$
$\Rightarrow -10 = \frac{1}{x} \Rightarrow x = \frac{-10}{11}$ Choice (A)

20. $\log y = \log 160 - \log 40 = \log 4$
$\Rightarrow y = 4$ Ans : (4)

21. $3\log y + \log 32 = \log 256$
$\log y^3 + \log 32 = \log 256$
$\log\left(32y^3\right) = \log 256$
Equating arguments of logarithms both sides,
$32y^3 = 256$
$y^3 = 8$.
$y = \sqrt[3]{8} = 2$. Choice (B)

22. $\log_6 (\log_3 x) = 7^0 = 1$
$\Rightarrow \log_3{}^x = 6^1 = 6$
$\Rightarrow x = 3^6 = 729$. Choice (A)

23. $\log_5 (\log_6 (\log_7 49^3)) = \log_5 (\log_6 (\log_7 (7^2)^3))$
$= \log_5 (\log_6 (\log_7 7^6))$
$= \log_5 (\log_6(6)) = \log_5 (1) = 0$, since logarithm of 1 to any base is 0. Choice (A)

24. $6^{3 - \log_6 18 + 2 - \log_6 3}$

$= 6^{3 - \log_6 6 - \log_6 18 + \log_3 3^2}$

$= 6^{\log_6 6^3 - \log_6 18 + \log_6 3^2}$

$= 6^{\log_6 \left[\left(6^3 \times 3^2\right)/18\right]}$

$= \dfrac{6^3 \times 3^2}{18} = 6^2 \times 3 = 108$ Choice (B)

25. $\log_5 x = \log_5 25 + \log_5 y^2$
$\Rightarrow \log_5 x = \log_5 (25y^2)$
$\Rightarrow x = 25y^2$
$x - y^2 = 25y^2 - y^2 = 24y^2$ Choice (C)

Exercise – 24
(Permutations and Combinations)

Solutions for questions 1 to 40:

1. One can travel from city A to city B in 3 ways and from city B to city C in 5 ways. Since the two operations are independent, by fundamental theorem, number of distinct routes from A to C are 3 x 5 = 15 Ans : (15)

2. The boy can select one trouser in nine ways.
The boy can select one shirt in 12 ways.
$\therefore$The number of ways in which he can select one trouser and one shirt is $9 \times 12 = 108$ ways. Choice (D)

3. The number of letters in the given word is four.
The number of three letter words that can be formed using these four letters is ${}^4P_3 = 4 \times 3 \times 2 = 24$. Choice (D)

4. Total number of letters = 8
Using these letters the number of 8 letter words formed is ${}^8P_8 = 8!$. Choice (B)

5. There are five letters in the given word.
Consider 5 blanks . . . .
The first blank and last blank must be filled with N and A and the remaining three blanks can be filled with the remaining 3 letters in 3! ways.
$\therefore$ The number of words = 3! = 6. Choice (B)

6. The word MEADOWS has 7 letters of which 3 are vowels.
__ V __ V __ V __
As the vowels have to occupy even places, they can be arranged in the 3 even places in 3! i.e., 6 ways. While the consonants can be arranged among themselves in the remaining 4 places in 4! i.e., 24 ways. Hence the total ways are $24 \times 6 = 144$ Ans : (144)

7. Total number of arrangements such that the word ends with K is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$ ways Ans : (720)

8. "PIGMENT" has 7 letters. We have to fill seven blanks P_ _- __ ___ ___ ___ ___ _X. As the word has to begin with P fill the first blank with P and the word should not end with T.
$\therefore$ Last blank can be filled by any other letter than P and T. It can be done in 5 ways. The remaining 5 blanks can be filled with the remaining 5 letters in 5! ways.
$\therefore$ Total number of words is 5! × (5) = 600.
Choice (C)

9. We have to select two points to form a line.
$\therefore$ Total number of lines formed by joining the 20 points is ${}^{20}C_2$, Viz. 190. But of these, six points are collinear.
$\therefore$ ${}^6C_2$ .viz. 15 lines are not distinct. These six points form one line.
$\therefore$ Total number of lines formed = 190 – 15 + 1 = 176.
Choice (B)

10. By joining any three non-collinear points, a triangle is formed.
$\therefore$ The number of triangles formed = ${}^{20}C_3 - {}^6C_3 = 1120$
Choice (D)

11. A particular person has to occupy the middle place. The remaining 6 places can be filled with the remaining 6 persons in 6! viz, 720 ways. Choice (A)

12. 'SPECIAL' has 7 letters. Of these A, E, I are vowels.
$\therefore$ The first blank can be filled in 3 ways. The last letter cannot be L. $\therefore$ Last place can be filled in 5 ways. The remaining 5 blanks can be filled in 5!, viz 120 ways.
$\therefore$ The required number of words = 3(5!)(5) = 1800.
Choice (B)

13. As the, number is greater than 300, hundred's place can be filled in the following ways.
(1) $\dfrac{4}{1}\ \dfrac{}{(3)}\ \dfrac{0.5}{(2)} = 6$ or (2) $\dfrac{5}{1}\ \dfrac{}{(3)}\ \dfrac{0}{(1)} = 3$
$\therefore$ The required number of ways = 6 + 3 = 9
Choice (A)

14. To post each letter, there are 5 boxes.
$\therefore$ 3 letters can be posted in 5^3, viz 125 ways.
Ans : (125)

15. The number of greetings when 'n' persons are present is ${}^nP_2$ viz n(n – 1).
$\therefore$ n(n – 1) = 132 = 12 (11)
$\Rightarrow$ n = 12. Ans : (12)

16. The following are the possible ways of forming the committee.

No.of boys	No.of girls	Number of ways of selections
3	3	${}^8C_3 \times {}^6C_3 = 1120$
4	2	${}^8C_4 \times {}^6C_2 = 1050$
5	1	${}^5C_5 \times {}^6C_1 = 336$
		Total = 2506

$\therefore$ The number of ways of forming the committee = 2506.
Choice (A)

17. For each prize to be given, there are 6 persons who can receive it.
$\therefore$ The number of ways of giving away 4 prizes = 6^4
Choice (A)

18. The total number of stations is 20
From 20 stations we have to choose any two stations and the direction of travel (i.e., Hyderabad to Bangalore is different from Bangalore to Hyderabad) in ${}^{20}P_2$ ways.
$\therefore$ ${}^{20}P_2 = 20 \times 19 = 380$. Choice (B)

19. n items of which p are alike of one kind, q alike of the other, r alike of another kind and the remaining are distinct can be arranged in a row in $\dfrac{n!}{p!\,q!\,r!}$ ways.
The letter pattern 'MESMERISE' consists of 10 letters of which there are 2M's, 3E's, 2S's and 1I and 1R.
Number of arrangements = $\dfrac{9!}{(2!)^2\ 3!}$
Choice (A)

20. The number of letters in the word MATHEMATICS are 11. Out of which 2 M's, 2 A's, 2T's are alike
$\Rightarrow$ Total number of words formed = $\dfrac{11!}{2! \times 2! \times 2!} = \dfrac{11!}{(2!)^3}$
Choice (D)

21. As the sum of the digits is 15 the number formed will always be divisible by 3 for the number to be divisible by 6 it should also be divisible by 2
i.e., The number should end with even digit
4! numbers end with 2 and 4! numbers end with 4
$\therefore$ The required number of numbers = 4! + 4!
= 24 + 24 = 48 Ans : (48)

22. $\underset{4}{\_} \times \underset{3}{\_} \times \underset{1}{\overset{P}{\_}} \times \underset{2}{\_} \times \underset{1}{\_}$
The number of ways the 4 teachers can be seated in 4 chairs is 4! ways Choice (B)

23. Number of 7 digit numbers formed using the digits 0, 1, 1, 2, 2, 3, 3 is = $\frac{7!}{2!2!2!}$

X	X	X	X	X	X	X

Of which some of the numbers start with zero,
Now, numbers (7 digit) starting with zero are to be excluded

Number of 7 digit numbers starting with zero = $\frac{6!}{2!2!2!}$

Required number of numbers = $\frac{7!}{2!2!2!} - \frac{6!}{2!2!2!}$

= $\frac{6!}{(2!)^3}$ (7 – 1) = $\frac{720}{8}$ $\times$ 6 = 540 Choice (D)

24. Each letter can be dealt in 4 ways–post it into any of the 4 available boxes.
Hence 6 letters can be dealt in 4^6 ways. Choice (B)

25. Each prize can be given to any of the 3 boys in 3 ways
$\Rightarrow$ Total number of ways
= 3 $\times$ 3 $\times$ 3 $\times$ …… for 12 times = 3^{12} Choice (C)

26. Since each ring consists of six different letters, the total number of attempts possible with the three rings is = 6 $\times$ 6 $\times$ 6 = 216. Of these attempts, one of them is a successful attempt.
$\therefore$ Maximum number of unsuccessful attempts
= 216 – 1 = 215 Ans : (215)

27. We can initially arrange the six boys in 6! ways.
Having done this, now there are seven places and six girls to be arranged. This can be done in $^7P_6$ ways.
Hence required number of ways = 6! $\times$ $^7P_6$
= 3628800 Ans : (3628800)

28. The given digits are six.
$\therefore$ The number of four digit numbers that can be formed using six digits is $^6P_4 = 6 \times 5 \times 4 \times 3 = 360$ Ans : (360)

29. The given digits are 1, 2, 3, 5, 7, 9
A number is even when its units digit is even. Of the given digits, two is the only even digit.
Units place is filled with only '2' and the remaining three places can be filled in $^5P_3$ ways.
$\therefore$ Number of even numbers = $^5P_3$ = 60. Ans : (60)

30. Six members can be selected from ten members in $^{10}C_6 = {}^{10}C_4$ ways ($\therefore$ $^nc_r = {}^nc_{n-r}$). Ans : (210)

31. Total number of balls = 9 + 3 + 4 = 16
Two balls can be drawn from 16 balls in $^{16}C_2$ ways.
Choice (C)

32. 1st position can be attained by any person in 12 ways.
2nd position can be attained by any of the remaining persons in 11 ways.
3rd position can be attained by any of the remaining persons in 10 ways.
4th position can be attained by any of the remaining persons in 9 ways. $\therefore$Number of ways = 12 $\times$ 11 $\times$ 10 $\times$ 9 = $^{12}P_4$. Choice (B)

33. The word contains five consonants,
Three vowels, three consonants can be selected from five consonants in $^5C_3$ ways, two vowels can be selected from three vowels in $^3C_2$ ways.
$\therefore$ 3 consonants and 2 vowels can be selected in $^5C_2 \cdot {}^3C_2$ ways i.e., 10 $\times$ 3 = 30 ways. Choice (B)

34. We can select one boy from 20 boys in 20 ways
We select one girl from 25 girls in 25 ways
$\therefore$ we select a boy and girl in 20 $\times$ 25 ways
i.e., = 500 ways. Ans : (500)

35. The group contains six men and seven women
Three men can be selected from six men in $^6C_3$ ways.
Four women can be selected from seven women in $^7C_3$ ways
$\therefore$ total number of ways = ($^7C_4$) ($^6C_3$). Choice (C)

36. Given that, the question paper consists of five problems. For each problem, one or two or three or none of the choices can be attempted.
$\therefore$ Hence, the required number of ways = 4^5 – 1.
= 2^{10} – 1= 1024 – 1 = 1023 Ans : (1023)

37. We know that, the number of straight lines that can be formed by the 'n' points in which r points are collinear and no other set of three points, except those that can be selected out of these r points are collinear) is
$^nC_2 - {}^rC_2 + 1$.
$\therefore$ Hence, the required number of straight lines
= $^{11}C_2 - {}^6C_2 - {}^5C_2 + 1 + 1$
= 55 – 15 – 10 + 2 = 32 Choice (A)

38. Treat all boys as one unit. Now there are four students and they can be arranged in 4! ways Again the five boys can be arranged among themselves in 5! ways. $\therefore$ required number of arrangements 4! $\times$ 5! = 24 $\times$ 120 = 2880 Ans : (2880)

39. x Not younger_ _ _ _ _ _ $\uparrow$
The last ball can be thrown by any of the remaining 6 players. The first 6 players can throw the ball in $^6P_6$ ways.
$\therefore$ The required number of ways = 6 (6!) = 4320
Ans : (4320)

40. In the word, "MATERIAL" there are three vowels A, I, E.
If all the vowels are together, the arrangement is MTRL'AAEI'
Consider AAEI as one unit. The arrangement is as follows.

M T R L $\boxed{\text{A A E I}}$

The above 5 items can be arranged in 5! ways and AAEI can be arranged among themselves in $\frac{4!}{2!}$ ways.

$\therefore$ Number of required ways of arranging the above letters
= 5! $\times$ $\frac{4!}{2!}$ = $\frac{120 \times 24}{2}$ = 1440 ways Ans : (1440)

Exercise – 25
(Probability)

Solutions for questions 1 to 3:

1. The number of exhaustive events = 2^5 = 32.
Let E be the event of getting zero heads. Then n(E) = 1.
So P(E) = 1/32 = 0.03125. Ans : (0.03125)

2. The number of exhaustive outcomes is 2^6 = 64
Let E be the event of getting at least 3 heads
$\therefore$ Number of favourable cases are $^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$
= 20 + 15 + 6 + 1 = 42
P(E) = $\frac{42}{64} = \frac{21}{32}$ = 0.65625 Ans : (0.65625)

3. The favourable cases are 1, 2, 3, 4 and 6. These are 5 in number. Hence the required probability is $\frac{5}{6}$

Choice (D)

Solutions for questions 4 and 5:

4. No two dice show same number would mean all the three should show different numbers. The first can fall in any one of the 6 ways. The second die can show a different number in 5 ways. The third should show a number that is different from the first and the second. This can happen in 4 ways.
Thus, $6 \times 5 \times 4 = 120$ favourable cases.
The total number of cases is $6 \times 6 \times 6 = 216$.
Hence, the required probability = $\frac{120}{216} = \frac{5}{9}$. Choice (B)

5. Total number of cases is $6 \times 6 \times 6 = 216$.
Out of three any two dice show same number. This can be happen in ${}^3C_2$ ways. Let first two dice show the same number. This can be happen in 6 ways. The third die shows a different number. This can be happen in 5 ways
$\therefore$ total favourable cases = ${}^3C_2 . 6.5 = 90$
$\therefore$ Required probability = $\frac{90}{216}$ = 5/12 Choice (B)

Solutions for questions 6 to 21:

6. The number of exhaustive outcomes is 36.
Let E be the event of getting an even number on one die and an odd number on the other. Let $\overline{E}$ be the event of getting either both even or both odd then $P(\overline{E}) = 18/36 = 1/2$
$\therefore P(E) = 1 - 1/2 = 1/2 = 0.5$ Ans : (0.5)

7. The number of exhaustive outcomes = ${}^{52}C_1 = 52$
The number of favourable cases are 52 – 16 = 36
$\therefore$ Required probability = 36/52 = 9/13 Choice (D)

8. $n(S) = {}^{52}C_2$
There will be four queens and from this two are selected in ${}^4C_2$ ways. $n(E) = {}^4C_2$
$\therefore$ Probability = $\frac{{}^4C_2}{{}^{52}C_2} = \frac{1}{221}$ Choice (B)

9. A number can be chosen from the given set in 8 ways.
Let E be the event of choosing a number from the set which is a multiple of 4, then the number of favourable cases to the event E is 4.
$\therefore P(E) = 4/8 = 1/2 = 0.5$ Ans : (0.5)

10. The number of exhaustive events = ${}^{50}C_1 = 50$. We have 15 primes from 1 to 50.
Number of favourable cases are 34.
$\therefore$ Required probability = 34/50 = 17/25. Ans : (0.68)
NOTE: 1 is neither prime nor composite.

11. There are 25 primes from 1 to 100. Thus the probability of getting a prime is $\frac{25}{100} = \frac{1}{4}$ = 0.25 Ans : (0.25)

12. We have 1, 8, 27 and 64 as perfect cubes from 1 to 100. Thus, the probability of picking a perfect cube is $\frac{4}{100} = \frac{1}{25}$ = 0.04 Ans: (0.04)

13. The number of exhaustive cases = ${}^6C_2 = 15$.
The sum of two positive integers is even if both numbers are either odd or even. The number of favourable cases = ${}^5C_2 = 10$ ($\because$ Only one even number).
$\therefore$ Required probability = 10/15 = 2/3 Choice (A)

14. Three balls can be drawn from a bag containing 9 balls in ${}^9C_3$ ways.
Let E be the event of drawing different coloured balls.
$\therefore P(E) = \frac{{}^5C_1 \times {}^3C_1 \times {}^1C_1}{{}^9C_3} = \frac{15}{84} = \frac{5}{28}$ Choice (B)

15. Six people can sit around a circular table in (6 – 1)! = 5! ways.
Let E be the event of two specified persons sitting side by side.
Treat two persons as one unit. The total 5 members can be arranged around circular table in (5 – 1)! = 4! ways
The two persons can be arranged among themselves in 2! ways. n(E) = 2!4!
$\therefore$ P(E) = 2(4!)/5! = 2/5 = 0.4 Ans : (0.4)

16. Ten students can be seated in a row in 10! ways. Let E be the event that the students do not sit together.
Then $\overline{E}$ is the event of two students being seated together, then $P(\overline{E})$ = 2(9!)/10! = 2/10 = 1/5
$\therefore$ P(E) = 1 – 1/5 = 4/5 = 0.8 Ans: (0.8)

17. The number of exhaustive events = ${}^7C_2 = 21$.
Let E be event of the two fruits being rotten. The number of favourable cases are ${}^2C_2$ = 1 way.
$\therefore$ Required probability = 1/21. Choice (A)

18. A non leap year has 365 days i.e., 52 weeks and one odd day. This odd day can be any one of the 7 days in a week. Now other than Monday and Tuesday, there are 5 days in favour.
$\therefore$ Required probability = 5/7. Choice (C)

19. The number of exhaustive cases = $4 \times 4 = 16$.
The number of favourable cases to the given event are 10.
$\therefore$ Required probability = 10/16 = 5/8 = 0.625
Ans : (0.625)

20. When two dice are rolled the total outcomes = $6 \times 6 = 36$.
Let E be the event that atleast one dice shows 4.
$\overline{E}$ is the event no dice shows 4
$\therefore n(\overline{E}) = 5 \times 5 = 25$.
$P(\overline{E}) = \frac{25}{36}$.
$P(E) = 1 - P(\overline{E}) = 1 - \frac{25}{36} = \frac{11}{36}$. Choice (B)

21. The number of exhaustive events = ${}^7C_2 = 21$.
Let E be the event of the two balls drawn being green. The number of favourable cases = ${}^4C_2 = 6$.
$\therefore$ P(E) = 6/21 = 2/7
Hence the odds against the event E is $P(\overline{E})$: P(E) (1 – 2/7) : 2/7
= 5 : 2 Choice (C)

Solutions for questions 22 to 26:

As two bulbs are defective out of 12, there are 10 good bulbs. The total number of ways of choosing 3 bulbs out of 12 bulbs is ${}^{12}C_3 = \frac{12.11.10}{1.2.3}$ = 220 ways.

22. Three good bulbs can be chosen in ${}^{10}C_3$ = 120 ways
Hence the probability is $\frac{120}{220} = \frac{6}{11}$
Choice (C)

23. Two defective and one good can be chosen in ${}^{10}C_2 \times {}^2C_1 = 45 \times 2 = 90$ ways.
Hence the probability = $\frac{90}{220} = \frac{9}{22}$
Choice (A)

24. One good and two defective bulbs can be chosen in ${}^{10}C_1 \times {}^2C_2$ = 10 ways.

Hence the probability is $\frac{10}{220}$ = 1/22 Choice (B)

25. The possibilities are 1 defective and 2 good or 2 defective and 1 good. Number of ways = $({}^2C_1 \times {}^{10}C_2) + ({}^2C_2 \times {}^{10}C_1)$ = 90 + 10 = 100

Hence the probability = $\frac{100}{220} = \frac{5}{11}$

Alternate method

P(atleast one bulb is defective) = 1 – P(no bulb is defective)

$1 - \frac{120}{220} = 1 - \frac{6}{11} = \frac{5}{11}$ (indirect way)

Choice (D)

26. There can be at the most two defective bulbs when we choose 3 bulbs. Thus it is certain that we get at least one good bulb. Hence the probability is 1.

Ans : (1)

Solutions for questions 27 to 35:

27. Given, P(a student is not a singer) = $\frac{3}{5}$

So, P(a student is a singer) = $1 - \frac{3}{5} = \frac{2}{5}$.

Now out of 6, 2 have to be singers and remaining 4 are not singers.

So, the required probability is ${}^6C_4 \times \left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^4$

$= 15 \times \frac{4}{25} \times \frac{81}{625} = \frac{972}{3125}$. Choice (B)

28. Given P ($X \cap Y$) = 1/8 and P($\bar{X} \cap \bar{Y}$) = 3/8

As X and Y are independent,

P($X \cap Y$) = P(X). P(Y) and

P($\bar{X} \cap \bar{Y}$) = P($\bar{X}$). P($\bar{Y}$).

Let x = P(X) and y = P(Y).

Then, xy = 1/8 ……..(1) and (1–x)(1–y) = 3/8…….(2).

Solving (1) and (2) we get

$\Rightarrow x = \frac{1}{4}$ or $x = \frac{1}{2}$. Choice (B)

29. When the sum of the two dice is 9, then the person gets ₹9.

When two dice are rolled total out comes n(s) = 6 × 6 = 36.

The favourable out comes for getting sum 9 are

= (4, 5) (5, 4), (3, 6), (6, 3) = n(E) = 4

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$ Choice (B)

30. If two coins show head and third coin shows tail, then the person gets ₹45.

The total out comes = 2^3 = 8

favourable out comes = {HHT HTH, THH}

n(E) = 3.

P(E) = 3/8 Choice (A)

31. One number can be chosen from {10, 11, . . . 59} in 50 ways.

$\therefore$ n(S) = 50.

The favourable out comes = {16, 26, 36 46, 56}

i.e., = 5

$\therefore$ Required probability = $\frac{5}{50} = \frac{1}{10}$ = 0.1 Ans : (0.1)

32. Given A and B are mutually exclusive

$\Rightarrow P(A \cap B) = 0$.

$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6}$

$P(A \cup B) = \frac{1}{2} = 0.5$ Ans : (0.5)

33. Given A and B are two independent events

$\Rightarrow P(A \cap B) = P(A) . P(B)$

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

= 1 – P($A \cap B$) = 1 – P(A). P(B)

= 1 – 2/3. 4/5 = 7/15 Choice (C)

34. Given A and B are two independent events

$\Rightarrow P(A \cap B) = P(A) . P(B)$

Also given that P($A \cup B$) = 0.75, P(B) = 0.25.

We know that P($A \cup B$) = P(A) + P(B) – P($A \cap B$)

$\Rightarrow$ 0.75 = P(A) + 0.25 – P(A) (0.25)

$\Rightarrow$ 0.5 = 0.75P(A) $\Rightarrow$ P(A) = 2/3 Choice (A)

35. n(E) = ${}^4C_3$

n(S) = ${}^7C_3$

$\therefore P(E) = \frac{{}^4C_3}{{}^7C_3} = \frac{4}{35}$ Choice (A)