Exercise – 11
(Mensuration)

Directions for questions 1 to 50: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If the sides of a triangle are 26 cm, 24 cm and 10 cm, what is its area (in cm^2)?

2. What is the area of an equilateral triangle of side 16 cm?

(A) $64\sqrt{3}$ cm^2 (B) $128\sqrt{3}$ cm^2

(C) $9.6\sqrt{3}$ cm^2 (D) $32\sqrt{3}$ cm^2

3. The perimeter of a triangle is 28 cm and the inradius of the triangle is 2.5 cm. What is the area of the triangle?

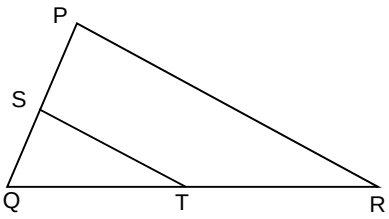
(A) 25 cm^2 (B) 35 cm^2 (C) 49 cm^2 (D) 70 cm^2

4. An equilateral triangle is inscribed in a circle of radius 12 cm. What is the area of the triangle?

(A) $48\sqrt{3}$ cm^2 (B) $72\sqrt{3}$ cm^2

(C) $98\sqrt{3}$ cm^2 (D) $108\sqrt{3}$ cm^2

5.



In the figure given above, S and T are the mid points of PQ and QR respectively. If the area of the triangle PQR is 48 cm^2, what is the area of the triangle SQT (in cm^2)?

6. If ABC is a right angled triangle right angled at B, where AB = 3 cm and BC = 4 cm, what is the area of the circle inscribed in the triangle?

(A) π cm^2 (B) 2π cm^2

(C) $\pi/2$ cm^2 (D) 6π cm^2

7. The area of a square is equal to five times the area of a rectangle of dimensions 125 cm $\times$ 64 cm. What is the perimeter of the square (in cm)?

8. The lengths of the parallel sides of a trapezium are x cm and y cm and the area of the trapezium is $\frac{1}{2}(x^2 - y^2)cm^2$. What is the distance between the parallel sides? (in cm)

(A) x (B) y

(C) x + y (D) x – y

9. A rectangle of length 24 cm and breadth 18 cm is inscribed in a circle. What is the area of the circle?

(A) 900π cm^2 (B) 800π cm^2

(C) 300π cm^2 (D) 225π cm^2

10. The length and breadth of a rectangular cardboard are 40 cm and 35 cm respectively. If the largest possible square is cut out from this, what is the area of the rectangle unused (in cm^2)?

11. Find the area of the following.

(a) A rectangle with length 20 cm and breadth 16 cm.

(A) 320 cm^2 (B) 180 cm^2

(C) 160 cm^2 (D) 144 cm^2

(b) A square, the product of whose diagonals is 50 cm^2.

(A) 30 cm^2 (B) 40 cm^2

(C) 20 cm^2 (D) 25 cm^2

(c) A rhombus whose diagonals are 15 cm and 20 cm.

(A) 150 cm^2 (B) 180 cm^2

(C) 210 cm^2 (D) 240 cm^2

(d) A parallelogram with base 24 cm and height 16 cm.

(A) 262 cm^2 (B) 384 cm^2

(C) 192 cm^2 (D) 131 cm^2

12. A wire is bent to form a rectangle of length 16 cm and breadth 6 cm. If it is bent to form a circle, what is its area?

(A) 154 cm^2 (B) 231 cm^2

(C) 168 cm^2 (D) 158 cm^2

13. A square is inscribed in a circle of radius 20 cm. What is the area of the square?

(A) 400 cm^2 (B) 800 cm^2

(C) 1200 cm^2 (D) 1600 cm^2

14. A wire in the form of a circle of radius 3.5 m is bent in the form of a rectangle, whose length and breadth are in the ratio of 6 : 5. What is the area of the rectangle (in cm^2)?

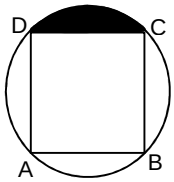
15. In the following figure ABCD is a square inscribed in a circle of radius 10 cm. Find the area of the shaded region.

(A) $(25\pi - 12.5)$ cm^2

(B) $(50\pi - 25)$ cm^2

(C) $(100\pi - 25)$ cm^2

(D) $(25\pi - 50)$ cm^2



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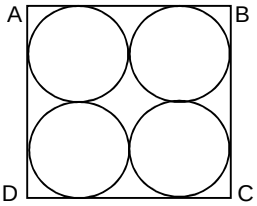


In the figure above, ABCD is a square of side 14 cm. Four equal circles are drawn inside the square, touching the sides of the square as shown. What part of the square is not covered by the circles?

(A) 11/14 (B) 3/14 (C) 1/7 (D) 2/7

17. What is the area of a regular hexagon whose side measures 4 cm?

(A) $12\sqrt{3}$ cm^2 (B) $18\sqrt{3}$ cm^2

(C) $24\sqrt{3}$ cm^2 (D) $36\sqrt{3}$ cm^2

18. The radius of a sphere and that of a cylinder are equal. The ratio of the radius and the height of the cylinder is 3 : 4. What is the ratio of their volumes?
(A) 9 : 16 (B) 27 : 64 (C) 1 : 1 (D) 1 : 2

19. What is the total surface area of a cube with side 8 cm (in cm^2)?

20. Two cubes of sides 8 cm each are kept adjacent to each other. What is the total surface area of the cuboid formed?
(A) 756 cm^2 (B) 840 cm^2
(C) 1210 cm^2 (D) 640 cm^2

21. What is the volume of a rectangular box of dimensions 20 cm $\times$ 8 cm $\times$ 15 cm (in cm^3)?

22. The volumes of two cones are in the ratio of 1 : 10 and the radii of the cones are in the ratio of 1 : 2. What is the ratio of their vertical heights?
(A) 2 : 5 (B) 1 : 5 (C) 3 : 5 (D) 4 : 5

23. There is a rectangular garden of dimensions 100 meters $\times$ 80 meters. Two paths are laid running midway in the garden, one parallel to the length and the other parallel to the breadth. What is the total area of the paths, if the width of the path is 2 meters?
(A) 356 m^2 (B) 360 m^2
(C) 344 m^2 (D) 336 m^2

24. The radius of a wheel is 28 cm. What is the distance covered by the wheel in making 500 revolutions (in m)?

25. A metallic sphere of radius 12 cm is melted and recasted into a cylinder, whose radius is 16 cm. What is the height of the cylinder? (in cm)

26. The area of the base of a right circular cone is 154 cm^2 and its vertical height is 24 cm. What is its curved surface area?
(A) 816 cm^2 (B) 924 cm^2
(C) 550 cm^2 (D) 660 cm^2

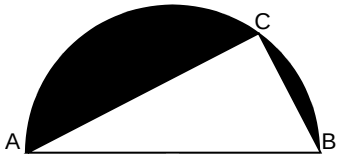
27. A total of twenty cubic metres of aluminium was used to make a sheet of area 1600 sq.m. Find the thickness of the sheet (in cm).

28. The base radii of two cylinders are 30 cm and 42 cm. A rope can make a maximum of 200 rounds around the second cylinder. Find the maximum number of rounds it can make around the first cylinder.

29. A cow is tied to one corner of a closed shed located in a field. The dimensions of the shed are 15 m $\times$ 12 m and the length of the rope that is used to tie the cow is 10 m. What is the area in which the cow can graze?
(A) 25π m^2 (B) 50π m^2
(C) 75π m^2 (D) 100π m^2

30. The radius of a garden that is in the shape of a circle is 35 m. There is a path of uniform width of 7 m all around and outside the garden. What is the area of the path (in m^2)?

31. In the following figure ABC is a triangle inscribed in the semi circle. What is the area of the shaded region if AB = 26 cm and BC = 10 cm?



(A) 1/2(169π – 88) cm^2 (B) 1/2(676π – 196) cm^2
(C) 1/2(169π – 240) cm^2 (D) 1/2(676π – 212) cm^2

32. The side of a cubical box is 12 cm. What is its volume?
(A) 1728 cm^3 (B) 1320 cm^3
(C) 1560 cm^3 (D) 1440 cm^3

33. A trapezium has parallel sides of 20 cm and 10 cm, and the distance between them is 15 cm. Find its area in cm^2.

34. In a rhombus ABCD, AC = 8 cm and BD = 6 cm. What is the perimeter of the rhombus (in cm)?

35. The diameter of the base of a cylinder is 14 cm and its height is 12 cm. What is the volume of the cylinder?
(A) 1600 cm^3 (B) 1200 cm^3
(C) 1848 cm^3 (D) 2400 cm^3

36. A sphere of the maximum possible volume is carved out of a cubical wooden block of edge 42 cm. What is the volume of the sphere carved (in cm^3)?

37. A picture measures 80 cm $\times$ 50 cm. It has a frame of uniform width of 10 cm all around and outside it. What is the area of the frame (in cm^2)?

38. The ratio of the curved surface area and the base area of a cylinder is 4 : 3. Find the ratio of its radius and its height.
(A) 3 : 2 (B) 2 : 3 (C) 3 : 4 (D) 4 : 3

39. C and E denote the areas of a circle and an equilateral triangle having the same perimeter. Which of the following can be concluded?
(A) 2C > E > C (B) E = C
(C) E $\geq$ 2C (D) E < C

40. S_1 and S_2 are two squares. C is a circle such that C is inscribed in S_2 and S_1 is inscribed in C. Find the ratio of the areas of S_1 and S_2.
(A) 1 : 2 (B) 2 : 3
(C) 1 : $\sqrt{3}$ (D) 1 : $\sqrt{2}$

41. A field in the shape of a triangle has a base of 25 meters and an altitude (corresponding to the base) of 20 meters. If the cost of tilling the field is ₹15 per square meter, what is the total cost incurred (in ₹)?
[ ]

42. A circle is inscribed in an equilateral triangle of side 18 cm. What is the area of the incircle?
(A) 9π cm^2 (B) 18π cm^2
(C) 27π cm^2 (D) 81π cm^2

43. The perimeters of two equilateral triangles are in the ratio of 4 : 1. What is the ratio of their areas?
(A) 2 : 1 (B) 4 : 1
(C) 16 : 1 (D) 8 : 1

44. What is the area of the triangle, given its circumradius is 5 cm and the product of the sides of the triangle (in cm^3) is 1500?
(A) 50 cm^2 (B) 75 cm^2
(C) 102 cm^2 (D) 125 cm^2

45. The sector of a circle has a radius of 21 cm and a central angle of 135°. Find its perimeter. (in cm)
[ ]

46. The volume of a right circular cone is 196π cm^3. If the area of its base is 154 cm^2, what is the vertical height of the cone (in cm)?
[ ]

47. When a cubical metallic piece of edge 3 cm is dropped in a cylindrical glass of water, the water column rises by 3 cm. What is the radius of the base of the glass?
(A) $2/\sqrt{\pi}$ cm (B) $3/\sqrt{\pi}$ cm
(C) $6/\sqrt{\pi}$ cm (D) $9/\sqrt{\pi}$ cm

48. A cow is tied to a corner of a closed shed of dimensions 20 ft $\times$ 12 ft. If the length of the rope is 14 feet, what is the area that the cow can cover?
(A) 148π sq. feet (B) 236π sq. feet
(C) 147π sq. feet (D) 235π sq. feet

49. What is the area of a regular octagon (in cm^2) of side 12 cm?
(A) $288\cot\frac{45^\circ}{2}$ (B) $288\tan\frac{45^\circ}{2}$
(C) $144\cot\frac{45^\circ}{2}$ (D) $144\tan\frac{45^\circ}{2}$

50. A sector of a circle of radius 15 cm and central angle 288° is folded to form a cone, by joining its edges. What is the volume of the cone thus formed?
(A) 135π cm^3 (B) 270π cm^3
(C) 432π cm^3 (D) 512π cm^3

Exercise – 12
(Coordinate Geometry)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The distance between the points (2, 3) and (–4, 5) is _____.
(A) $2\sqrt{2}$ (B) $2\sqrt{10}$ (C) $2\sqrt{17}$ (D) $\sqrt{10}$

2. If (4, 5) is one end of the diameter of a circle with radius 5 units and centre on x-axis, then the x-coordinate of the centre of the circle is [ ].

3. The point on x-axis, equidistant from (1, 4) and (2, 3) is _____.
(A) (–1, 0) (B) (0, –1) (C) (–2, 0) (D) (0, –2)

4. If (3, 1) and (2, 4) are the points of trisection of the line segment joining two points A and B, then the distance between A and B is _____.
(A) $4\sqrt{10}$ (B) $\sqrt{10}$ (C) $2\sqrt{10}$ (D) $3\sqrt{10}$

5. The ratio in which (2, –3) divides the line joining (–2, 5) and (0, 1) is _____.
(A) 2 : 1 internally (B) 2 : 1 externally
(C) 1 : 2 internally (D) 1 : 2 externally

6. The area of triangle formed by (3, 2), (–1, –2) and (–1, 4) in square units is [ ].

7. If the midpoints of the sides of a triangle are (–6, –4), (–5/2, –1/2) and (–9/2, 3/2), then the area of the triangle in square units is [ ].

8. If the points (a, 4), (2, 2) and (5, 5) are collinear, then a = [ ].

9. If (0, 0), (3, 3) and (k, 0) form a right angled isosceles triangle, then k could be _____.
(A) 4 (B) 2 (C) –3 (D) 6

10. If (3, 3), (–1, –1) are two vertices of an equilateral triangle, then the third vertex can be _____.
(A) (2, 2) (B) (–5, –5)
(C) (–2, –2) (D) $\left(-2\sqrt{3}+1, 2\sqrt{3}+1\right)$

11. The centroid of the triangle whose midpoints of sides are (3, 6), (5, 4) and (4, 2) is _____.
(A) (4, 4) (B) (12, 12)
(C) (4/3, 4/3) (D) None of these

12. If the medians of a triangle meet at (1, 1) and two of its vertices are (2, 5) and (4, 6), then the third vertex is _____.
(A) (–3, –8) (B) (3, 8)
(C) (0, –3) (D) (3, 5)

13. If A(–1, 7) and D(3, 5) are the ends of a median drawn from A, of a triangle ABC, then the centroid is _____.
(A) (1/2, 19/3) (B) (5/3, 17/3)
(C) (1, 19) (D) (2/3, 19/3)

14. The orthocentre of the triangle formed by (1, 2), (1, 3) and (3, 2) is _____.
(A) (1, 3) (B) (1, 2) (C) (2, 3) (D) (5/3, 7/3)

15. The vertices of a triangle are (0, 0), (0, 6) and (6, 0). The distance between the orthocentre and the circumcentre is _____.
(A) $5\sqrt{2}$ (B) $2\sqrt{2}$ (C) $3\sqrt{2}$ (D) $\sqrt{2}$

16. If (5, 6), (7, 8) and (4, 10) are the three vertices of a parallelogram taken in order, then the fourth vertex is _____.
(A) (1, 1) (B) (0, 0) (C) (2, 2) (D) (2, 8)

17. The value of k for which the lines 3x + 4y – 12 = 0 and kx + y + 1 = 0 do not intersect is [ ].

18. The value of k for which the lines x – y = 0 and 7x + ky – 7 = 0 are perpendicular is [ ].

19. The line parallel to 4x + 3y + 7 = 0 and passing through (2, 1) is _____.
(A) 4x + 3y + 11 = 0 (B) 4x + 3y + 8 = 0
(C) 4x + 3y – 8 = 0 (D) 4x + 3y – 11 = 0

20. The line through (2, 1) and perpendicular to 2x + 5y – 11 = 0 is _____.
(A) 5x – 2y – 11 = 0 (B) 5x – 2y + 11 = 0
(C) 5x – 2y + 8 = 0 (D) 5x – 2y – 8 = 0

21. The equation of the line passing through the point (1, –2) and perpendicular to the line segment joining the points (2, 3) and (–4, 1) is _____.
(A) 2x – y – 4 = 0 (B) 3x + y – 1 = 0
(C) 3x + y – 3 = 0 (D) 3x – y – 5 = 0

22. The line through (–5, 8) and parallel to x-axis is y = c. The value of c is [ ].

23. The point of intersection of 4x + 6y = 10 and 9x + 5y = 14 is _____.
(A) (–2, 3) (B) (–1/9, 3) (C) (1, 1) (D) (2, 1/3)

24. The line x + y + a = 0 makes equal intercepts on the axes and passes through (1, 2). Then the sum of the intercepts is [ ].

25. The equation of the line making equal intercepts on the coordinate axes and passing through $\left(-2, \frac{1}{2}\right)$ is _____.
(A) 2(x + y) + 3 = 0 (B) 3(x + y) + 2 = 0
(C) 2(x + y) – 3 = 0 (D) 2(x + y) = 5

26. The area of the triangle formed by the lines x = 0, y = 0 and 7x + 4y – 28 = 0 in square units is [ ].

27. The status of the points (1, 2) and (4, 5) with respect to the line 3x + 2y – 12 = 0 is _____.
(A) both lie on the same side of the line
(B) both lie on the opposite side of the line
(C) lie on the line
(D) None of these

28. If the intercepts of a line are the roots of the equation $x^2 – 3x + 2 = 0$, then the line is _____.
(A) x/1 + y/2 = 1 (B) x/2 + y/1 = 1
(C) Either (A) or (B) (D) None of these

29. The distance of the line 4x + 3y – 9 = 0 from the origin is [ ].

30. The distance between the lines 4x + 3y – 5 = 0 and 4x + 3y – 11 = 0 is [ ].

31. If (1, 2) and (3, 4) are diagonally opposite vertices of a square, then the area of the square is (in square units) [ ].

32. If the lines x + y + 3 = 0, x – y – 1 = 0 and ax + y + 4 = 0 are concurrent, then a = [ ].

33. The line concurrent with the lines 3x + y = 2, x + 4y + 3 = 0 and perpendicular to x + 2y + 4 = 0 is
(A) 2x – y + 3 = 0 (B) 2x + y + 3 = 0
(C) 2x – y – 3 = 0 (D) 2x + y – 3 = 0

34. If the lines ax – 2y + 1 = 0, bx – 3y + 1 = 0 and cx – 4y + 1 = 0 are passing through the same point, and $\frac{a+c}{b}$ = k, then the value of k is [ ].

35. If the line y = mx + c passes through (1, –2) and (–1, 4) then the ordered pair (c, m) is _____.
(A) (1, –3) (B) (–3, 1) (C) (1, 3) (D) (–3, –1)

36. If $x = a + e^t$ and $y = b + e^{-t}$, where t is a parameter, then the locus of (x, y) is _____.
(A) xy – ax – by + ab – 1 = 0
(B) xy – bx – ay + ab + 1 = 0
(C) xy – bx – ay + ab – 1 = 0
(D) xy – bx – ay + ab = 0

37. Which of the following represents the equation of a circle passing through the origin and making intercepts 3 and 4 on positive x–axis and positive y-axis respectively?
(A) $x^2 + y^2 – 3x – 4y – 5 = 0$
(B) $x^2 + y^2 + 3x + 4y = 0$
(C) $x^2 + y^2 – 3x – 4y = 0$
(D) $x^2 + y^2 – 6x – 8y = 0$

38. If the equation of perpendicular bisector of AB is x + 2y + 2 = 0 and B = (–1, 2), then A is _____.
(A) (–3, 2) (B) (3, –2) (C) (–3, –2) (D) (3, 2)

39. Find the foot of the perpendicular from the point (5, –1) to y-axis.
(A) (0, 5) (B) (0, –1) (C) (0, 1) (D) (0, –5)

40. If a line drawn through P(3, 4) makes an angle of 60° with the x–axis at Q, then the distance between the points P and Q is $a/\sqrt{3}$. The value of a is [ ].

Exercise – 13
(Trigonometry)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The value of $4\pi/5$ in sexagesimal measure is $\square$.

2. The value of 108° in circular (radian) measure is _____.
(A) $5\pi/3$ (B) $3\pi/5$ (C) $\pi/5$ (D) $6\pi/7$

3. $\sin 60° \cos(\pi/6) - \cos 60° \sin(\pi/6) = \square$.

4. If $\tan(A + B) = \sqrt{3}$ and $\cos A = \frac{1}{\sqrt{2}}$, the least value of B in degrees is $\square$.

5. $\tan(A + B) = \frac{\sqrt{3}+1}{\sqrt{3}-1}$; $\cos A = \frac{1}{2}$. The least value of B in circular measure is _____.
(A) $\pi/12$ (B) $3\pi/5$ (C) $2\pi/5$ (D) $\pi/5$

6. If θ is acute and $\sin\theta = \frac{15}{17}$, then $\tan\theta = \square$.

7. The minutes hand of a clock is 11 cm long. The angle it covers in 25 minutes is _____.
(A) 120° (B) 135° (C) 200° (D) 150°

8. $3(\sin^2 30° + \cos^2 120°) - 4(\sin^6 45° + \cos^6 135°) = \square$.

9. If $3\sin^2 A = \sin 30° + \cos^2 45°$, then $\operatorname{cosec}^2 A = \square$.

10. The radius of a circle is 22 cm. Angle subtended by its arc of length 1100 cm is _____.
(A) $5\pi/18$ (B) $5\pi/9$ (C) $\pi/3$ (D) $4\pi/3$

11. The value of $\sec\frac{8\pi}{4}$ is $\square$.

12. If $180° < \theta < 270°$ and $\tan\theta = \frac{5}{12}$, then $\cot\theta + \sin\theta$ = _____.
(A) $\frac{181}{65}$ (B) $\frac{131}{65}$ (C) $\frac{125}{65}$ (D) $\frac{108}{65}$

13. If $\cos A = \frac{4}{5}$, $\cot B = \frac{12}{5}$, $0° < A < 90°$ and $180° < B < 270°$, then the value of $25\sin^2 A + 13\cos B$ is _____.
(A) 21 (B) –3 (C) 1 (D) –1

14. If $\cos^2\theta + 5\sin^2\theta = 4$, then $\tan^2\theta = \square$.

15. $\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} =$ _____.
(A) $2\tan^2\theta$ (B) $2\sin^2\theta$
(C) $2\operatorname{cosec}^2\theta$ (D) $2\sec^2\theta$

16. $\sqrt{\frac{1-\cos x}{1+\cos x}} =$ _____.
(A) $\operatorname{cosec} x + \cot x$ (B) $\operatorname{cosec} x - \cot x$
(C) 1 (D) $\operatorname{cosec}^2 x + \cot^2 x$

17. $\cos 1° \cdot \cos 2° \cdot \cos 3° \ldots\ldots \cos 179° = \square$.

18. $\log_{10}\tan 1° + \log_{10}\tan 2° + \log_{10}\tan 3° \ldots\ldots + \log_{10}\tan 89° =$ _____.
(A) 1 (B) 0 (C) –1 (D) 2

19. $12(\sin^4 x + \cos^4 x) - 8(\sin^6 x + \cos^6 x) =$ _____.
(A) 4 (B) –4 (C) 2 (D) –2

20. If $\sec\theta + \tan\theta = -5$, then $-\sec\theta + \tan\theta = \square$.

21. If $0 \le \theta \le 180°$, then which of the following is necessarily true?
(A) $0 \le \cos\theta \le 1$ (B) $-1 \le \cos\theta < 0$
(C) $-1 \le \sin\theta \le 1$ (D) $-1 \le \cos\theta \le 1$

22. An angle θ in sexagesimal measure is such that its complement is equal to one-third of its supplement. Then θ is equal to $\square$ degrees.

23. Which of the following values of x satisfies $\cos 30°.\cos x + \sin 30°.\sin x = \frac{1}{2}$?
(A) 45° (B) 60° (C) 0° (D) 90°

24. If $k(\sec\theta + \tan\theta) = (\sec\theta - \tan\theta)\cos^2\theta$, then k = _____.
(A) $(1 - \sin\theta)^2$ (B) $(1 + \cos\theta)^2$
(C) $(\cos\theta + \sin\theta)^2$ (D) None of these

25. If $x = 2k\cos A \sin 2A - k\sin A$ and $y = 2k\cos A\cos 2A - k\cos A$, then $x^2 + y^2 =$ _____.
(A) k (B) $-k^2$ (C) k^2 (D) –k

26. The value of $(\sin A\cos B + \cos A\sin B)^2 + (\cos A\cos B - \sin A\sin B)^2$ is _____.
(A) $2\sin A\cos B$ (B) $2\cos A\sin B$
(C) 1 (D) 0

27. The value of $\left(\frac{\tan 48° + \tan 12°}{1 - \tan 48°\tan 12°}\right)^2$ is $\square$.

28. The value of $\cos 15°$ is _____.
(A) $\frac{\sqrt{3}+1}{4\sqrt{2}}$ (B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (D) $\frac{\sqrt{3}-1}{4\sqrt{2}}$

29. If f(x) = 8cosx – 15 sinx, then the minimum value of f(x) is _____.
(A) 17 (B) –17 (C) 16 (D) –16

30. $\frac{\cos 75^\circ - \sin 75^\circ}{\cos 75^\circ + \sin 75^\circ}$ = _____.
(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{\sqrt{3}}$ (D) 0

31. The top of a flag pole makes an angle of 30° at a point on the ground. If the distance between the point of observation and the foot of the pole is 150m, find the height of the flag pole. (in m)
(A) 50 (B) $50\sqrt{3}$
(C) $100\sqrt{3}$ (D) $50/\sqrt{3}$

32. A man wishes to find the height of the building which stands on a horizontal plane. At a point on the plane, he finds the angle of elevation of the top of the building to be 60° and on moving 20m along the same line away from the building, he finds the angle of elevation to be 30°. Find the height of the building.
(A) 30 m (B) $15\sqrt{3}$ m
(C) $20\sqrt{3}$m (D) $10\sqrt{3}$ m

33. From the top of a tower 150 m high, the angles of depression of the top and bottom of a building are observed to be 30° and 60° respectively. Find the height of the building (in m).
[ ]

34. The angles given below are in radians. Give their corresponding values in degrees.
(i) 5π/18 = [ ].
(ii) 7π/5 = [ ].

35. From the top of a tower of height 75 m, the angles of depression of two points P and Q on the ground lying on either side of the tower are observed to be 45° and 60°. Find the distance between P and Q.
(A) $25\left(3+\sqrt{3}\right)$m (B) $25\left(3-\sqrt{3}\right)$ m
(C) $25\left(\sqrt{3}+1\right)$m (D) $25\left(\sqrt{3}-1\right)$m

36. What is the angle of elevation of the sun (in degrees) when the length of the shadow of a pole is $\frac{1}{\sqrt{3}}$ times the height of the pole?
[ ]

37. A tower is 150m high. Its shadow when the sun's altitude is 45°, as compared to its shadow when the sun's altitude is 30°, is nearly _____.
(A) 110 m less (B) 100 m more
(C) same (D) 150 m more

38. Find the height of a chimney, when it is found that on walking 50m towards it, in a horizontal line through its base, the angle of elevation of its top changes from 30°to 45°.
(A) $25(\sqrt{3} - 1)$ m (B) $25\sqrt{3}$ m
(C) $50(\sqrt{3} + 1)$ m (D) $25(\sqrt{3} + 1)$ m

39. From the top of a light house 60m high, with its base at the sea level, the angle of depression of a boat is 15°. Find the distance of the boat from the foot of the light house.
(A) $60(2 - \sqrt{3})$ m (B) 120 m
(C) $60(2 + \sqrt{3})$ m (D) $120(2 + \sqrt{3})$ m

40. A ladder 20 feet long reaches a point 20 feet below the top of a flag. The angle of elevation of the top of the flag at the foot of the ladder is 60°, the heig ht of the flag in ft is [ ].

Exercise – 14
(Operator – based Questions)

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Directions for questions 1 and 2: For real numbers x and y, let

$A(x, y) = x^2 + y^2$
$B(x, y) = x^2 - y^2$
C (x, y) = x + y and
D (x, y) = x – y

1. The value of A (B (1, –1), D (2, 1)) is [ ].

2. Which of the following equals zero?
(A) $[C(x, y)]^2 - A(x, y)$
(B) $[D(x, y)]^2 - B(x, y) + 2xy$
(C) C (x, y). D(x, y) – B(x, y)
(D) None of these

Directions for questions 3 to 5: These questions are based on the following information. For real numbers a and b,

$f(a, b) = (a + b)^2$
g (a, b) = a + b and
$h(a, b) = \sqrt{a+b}$

3. If a and b are non-negative real numbers, then which of the following is necessarily true?
(A) $f(a, b) \le g(a, b) \le h(a, b)$
(B) $f(a, b) \ge g(a, b) \ge h(a, b)$
(C) $f(a, b) > g(a, b) > h(a,b)$
(D) None of these

4. If a and b are non-negative integers, then which of the following is necessarily true?
(A) $f(a, b) \le g(a, b) \le h(a, b)$
(B) $f(a, b) \ge g(a, b) \ge h(a, b)$
(C) $f(a, b) > g(a, b) > h(a, b)$
(D) None of these

5. If $a + b \ge 0$, then which of the following is necessarily true?
(A) $f(a, b) \ge g(a, b) \ge h(a, b)$
(B) $f(a, b) \le g(a, b) \le h(a, b)$
(C) $f(a, b) > g(a, b) > h(a, b)$
(D) None of these

Directions for question 6: Select the correct alternative from the given choices.

6. If $(p, q) \downarrow (r, s) = ((ps - qr), (ps + qr))$ and $(x, y) = [(2, 3) \downarrow (1, -2)] \downarrow [(1, 2) \downarrow (3, 4)]$, then $(x^2 - y^2, xy) \downarrow (xy, x^2 - y^2) =$
(A) (0, 0)
(B) $(560^2 + 4896^2, 0)$
(C) $((560^2 - 4896^2), (560^2 + 4896^2))$
(D) None of these

Directions for questions 7 to 9: These questions are based on the following data:

$a \uparrow b = \frac{3ab}{2}$

$a \downarrow b = \frac{4a}{b}$

$a \rightarrow b = 2a + 3b$

$a \leftarrow b = 4a - 5b$

7. The value of $(((4 \uparrow 6) \rightarrow 9) \downarrow 12) \leftarrow 5$ is ☐ .

8. Which of the following is an integer?
(A) $((6 \downarrow 8) \rightarrow 5) \uparrow 3$
(B) $((3 \rightarrow 7) \downarrow 9) \uparrow 5$
(C) $((4 \uparrow 7) \downarrow 5) \rightarrow 3$
(D) $((5 \uparrow 7) \rightarrow 6) \downarrow 7$

9. Which of the following is always true?
(A) $(((p \uparrow q) \rightarrow q) \downarrow pq) \leftarrow q = 3p - 7q$
(B) $(((c \leftarrow d) \uparrow d) \rightarrow cd) \downarrow d = 6c + 5d$
(C) $(((a \rightarrow b) \uparrow b) \leftarrow ab) \downarrow b = 28a + 72b$
(D) $(((r \downarrow s) \rightarrow s) \uparrow rs) \leftarrow s = \frac{3r - 7s}{5}$

Directions for questions 10 and 11: Select the correct alternative from the given choices.

10. If $(x, y) \uparrow (z, w) = (xz - yw, xw + yz)$ and $(p, q) = [(a_1, b_1) \uparrow (b_1, a_1)] \uparrow [(a_2, b_2) \uparrow (b_2, a_2)]$, then $(q - p, pq) \uparrow (pq, q - p) =$ _____
(A) $[(0, (a_1^2 + b_1^2)^2(a_2^2 + b_2^2)^2]$
(B) $[(0, (a_1^2 - b_1^2)^2 (a_2^2 - b_2^2)^2]$
(C) $[(a_1^2 + b_1^2)^2, (a_2^2 + b_2^2)^2]$
(D) (0, 0)

11. If $(a, b) \Delta (c, d) = (ac - bd, ad - bc)$ and $(x, y) = [(3, 4) \Delta (4, 3)] \Delta [(1, 3) \Delta (3, 1)]$ then $(x^2 + y^2, x^2 - y^2) \Delta (x^2 - y^2, x^2 + y^2) =$ _____
(A) $(56^2, 56^2)$ (B) (56, 56)
(C) (–56, –56) (D) (0, 0)

Direction for question 12: These questions are based on the following information.

Let $a \$ b$ = HCF (a, b), $a \downarrow b = a^2 - b^2$, $a \uparrow b = a^2 + b^2$, $a \rightarrow b = a^2 b^2$ and $a \leftarrow b = a^2/b^2$.

12. $[(a \downarrow b) \downarrow (a \uparrow b)] \leftarrow (a \rightarrow b) = -k$ then k = ☐

Directions for questions 13 to 15: Two operators $\oplus$ and $\otimes$ are defined in the following tables as follows.

$\oplus$	a	b	c	d
a	a	b	c	d
b	b	d	a	c
c	c	a	d	b
d	d	c	b	a

$\otimes$	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

and $a^2 = a \otimes a$, $a^3 = a^2 \otimes a$....., $2a = a \oplus a$, $3a = 2a \oplus a$, ……

13. $c^4 \otimes b^3$ = _____.
(A) d (B) a (C) c (D) b

14. $[(a^3 \oplus b^3) \otimes (c^3 \oplus d^3)] \oplus (3b \oplus 4c)$ = _____.
(A) a (B) c (C) b (D) d

15. $[(b \oplus a) \otimes c) \oplus d] \otimes [(d \oplus c) \otimes b) \oplus a]$ = _____.
(A) a (B) b (C) d (D) c

Exercise – 15
(Statistics)

Directions for questions 1 to 20: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The arithmetic mean of 5, 10, 12, 18, 20 is ☐ .

2. If the mean of 10 observations is 12 and that of another 12 observations is 10, then the mean of the total observations is _____.
(A) 10 (B) 120/11 (C) 31/25 (D) 27/20

3. If the average wage of 100 workers is ₹1,000 and the average wage of 60 of them is ₹1,200, then the average wage of remaining workers is (in rupees) ☐ .

4. If the arithmetic mean of first n natural numbers is 8, then n = ☐ .

5. The first term and common difference of an arithmetic progression are a and d respectively. The arithmetic mean of first n terms of the progression is _____.
(A) $\frac{a + (n - 1)d}{n}$ (B) $\frac{n}{2}$ [2a + (n – 1)d]

(C) $a + (n - 1)d$ (D) $\frac{1}{2}[2a + (n - 1)d]$

6. The mean of 100 observations was found to be 39. Three observations were incorrectly copied as 13, 19, 26 instead of 18, 12, 28 respectively. The correct value of mean is ☐ .

7. The geometric mean of 3, 5, 15, 45, 75 is ☐ .

8. If the geometric mean of two numbers is 12, one of the numbers being 48, the other number is ☐ .

9. The mean of squares of first 10 natural numbers is _____.
(A) 5.5 (B) 38.5 (C) 35.8 (D) 53.8

10. The harmonic mean (correct to the two decimal places) of the numbers 2, 4, 6, 8 is ☐ .

11. The AM of two numbers is 10 and their GM is 8. Then their HM is ☐ .

12. The median of 12, 13, 18, 25, 30 is ☐ .

13. The median of first 12 prime numbers is ☐ .

14. If mode of a distribution is 12 and mean is 3, then the median is ☐ .

15. The range of 13, 38, 51, 56, 70, 72, 63, 65, 93 is ☐ .

16. The quartile deviation of 12, 18, 20, 15, 19, 23, 25 is ☐ .

17. The quartile deviation of 10, 13, 18, 21, 24, 36, 50, 84, 63, 90, 75 is ☐ .

18. The mean deviation of 113, 117, 120, 122 and 128 is ☐ .

19. If the standard deviation of $x_1, x_2, x_3 \text{ -- } x_n$ is 7.8, then the standard deviation of $x_1 + 3, x_2 + 3, \ldots\ldots, x_n + 3$ is ☐ .

20. If the mean of 100 observations is 50 and their standard deviation is 4, then the sum of squares of the observations is ☐ .

Exercise – 16
(Special Equations)

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If the remainder of $\left(\frac{4a}{7}\right) = 2$, then the least possible positive integer value of a is ☐ .

2. If $x > 0$ and the remainder of $\left(\frac{3x}{5}\right) = 3$, then which of the following set could be the values of x?
(A) {1, 6, 11, 16, ……} (B) {1, 3, 6, 9, …..}
(C) {1, 6, 9, 12, …} (D) None of these

3. The number of positive integral solutions of $5x + 3y = 34$ is ☐ .

4. Which of the following is the value of x that satisfies the equation $3x + 4y = 29$ and the condition $0 < y < x$?
(A) 5 (B) 3 (C) 6 (D) 7

5. The value of (x, y) if $14x + 5y = 57$ and $xy > 0$ is _____.
(A) (2, 2) (B) (3, 3) (C) (4, 4) (D) (5, 5)

6. Which of the following is a possible solution or are possible solutions for the equation $12x – 5y = 19$?
(A) (7, 13) (B) (12, 25)
(C) Neither (A) nor (B) (D) Both (A) and (B)

7. Which of the following is/are a possible value of $a + b$ for which $7a – 3b = 20$?
(A) 0
(B) 20
(C) –20
(D) All the previous choices

8. The values of x that satisfy the condition that remainder of $\left(\frac{7x}{9}\right) = 7$, form an arithmetic progression with common difference ☐ .

9. Mr Raghu purchased toys of two types – T_1 and T_2 –costing ₹11 and ₹17 respectively. Raghu spent an amount of ₹123 for purchasing them. Find the maximum number of toys he could have purchased if he purchased at least one of each type.
(A) 7 (B) 5
(C) 9 (D) 10

10. Deepika purchased a total of n oranges and apples costing ₹6 and ₹14 each respectively. If she spent an amount of ₹200, then the least possible value of

n is .

11. Tinku wants to purchase some flower pots and sparklers. Each flower pot costs ₹12 and each sparkler costs ₹8. If Tinku has ₹96 and she wants to buy at least one of each, find the maximum number of flower pots she can buy.
(A) 4 (B) 8 (C) 2 (D) 6

12. Radha went to a stationery shop to purchase pens and pencils each costing ₹12 and ₹5 respectively. She wants to spend an amount of ₹97. Find the number of combinations of pens and pencils she can buy.

13. Dhanush wants to offer samosas, cutlets and kachoris to his friends, each of which costs ₹6, ₹9 and ₹7 respectively. Dhanush orders at least two of each item and spends exactly ₹74. Find the maximum number of kachoris he could have ordered.
(A) 3 (B) 4 (C) 5 (D) 6

14. A bag contains 26 coins in denominations of ₹5, ₹2 and ₹1. The money in the bag amounts to ₹57. The number of one rupee coins is more than the total number of other coins. The number of ₹2 coins in the bag is .

15. A two digit number N, (20 < N < 50) is such that when divided by 6, it leaves a remainder 5 and when divided by 5, it leaves a remainder 2. The number is .

Exercise – 17
(Quadratic Equations)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the roots of the following quadratic equations.
(a) $x^2 - 8x + 15 = 0$
(A) 3, 5 (B) 3, –5 (C) –3, 5 (D) –3, –5

(b) $x^2 + 4x + 3 = 0$
(A) –3, 1 (B) 3, –1 (C) 3, 1 (D) –3, –1

(c) $x^2 - 5x - 6 = 0$
(A) –6, –1 (B) 6, –1 (C) 6, 1 (D) –6, 1

(d) $x^2 + x - 20 = 0$
(A) 5, –4 (B) –5, –4 (C) –5, 4 (D) 4, 5

2. Find the roots of the following quadratic equations.
(a) $8x^2 - 17x + 2 = 0$
(A) $2, \frac{1}{8}$ (B) $-\frac{1}{8}, -2$
(C) $-\frac{1}{8}, 2$ (D) $\frac{1}{8}, -2$

(b) $7x^2 + 51x + 14 = 0$
(A) $-7, \frac{-2}{7}$ (B) $-7, \frac{2}{7}$ (C) $7, \frac{-2}{7}$ (D) $7, \frac{2}{7}$

(c) $-6x^2 + 29x - 20 = 0$
(A) $4, \frac{5}{6}$ (B) $-4, \frac{5}{6}$ (C) $4, \frac{-5}{6}$ (D) $-4, \frac{-5}{6}$

3. Find the sum and the product of the roots of the following equations.
(a) $2x^2 - \frac{7}{2}x + 2 = \frac{1}{2}$
(A) $\frac{7}{4}, \frac{3}{4}$ (B) $\frac{7}{2}, \frac{3}{4}$
(C) $\frac{7}{2}, 2$ (D) $-\frac{7}{2}, 2$

(b) $6\sqrt{3}\,x^2 - 36x + 12\sqrt{3} = 0$
(A) $2\sqrt{3}, 2$ (B) $-2\sqrt{3}, 2$
(C) 12, 2 (D) –12, 2

(c) $(a - b)\,x^2 + (a^2 - b^2)\,x + \frac{1}{a} - \frac{1}{b} = 0$
(A) $a + b, \frac{1}{ab}$ (B) $-(a + b), \frac{1}{ab}$
(C) $a + b, \frac{-1}{ab}$ (D) $-(a + b), \frac{-1}{ab}$

4. Identify the nature of roots of the following equations.
(a) $9x^2 - 10x + 3 = 0$
(A) Real and equal (B) Rational and unequal
(C) Complex (D) Irrational

(b) $x^2 - 2x - 3 = 0$
(A) Real and equal (B) Rational and unequal
(C) Complex (D) Irrational

(c) $25x^2 + 60x + 36 = 0$
(A) Real and equal (B) Rational and unequal
(C) Complex (D) Irrational

5. The quadratic equation whose roots are 5 and –2 is _____.
(A) $x^2 - 3x - 10 = 0$ (B) $x^2 + 3x - 10 = 0$
(C) $x^2 - 3x + 10 = 0$ (D) $x^2 - 3x + 10 = 0$

6. Find the quadratic equation whose roots are
(a) $-5 \pm \sqrt{3}$
(A) $x^2 - 100x + 22 = 0$ (B) $x^2 + 10x + 22 = 0$
(C) $x^2 - 28x + 22 = 0$ (D) $x^2 + 28x + 22 = 0$

(b) $\sqrt{p} \pm \sqrt{q}$
(A) $x^2 + 2\sqrt{p}\,x + p - q = 0$
(B) $x^2 - 2\sqrt{p}\,x + p - q = 0$
(C) $x^2 - 2\sqrt{p}\,x + p + q = 0$

(D) $x^2 + 2\sqrt{p}\,x + p + q = 0$

7. In the quadratic equation $16x^2 - qx + 81 = 0$, one root is the cube of the other. Find the value of q.
(A) ±78 (B) 39 (C) –39 (D) 56

8. If $x^2 - px + 9 = 0$ has 3 as one root, then the value of p is $\square$.

9. Find the possible values of x of the equation $\sqrt{3-2x} + \sqrt{7+2x} = 4$.
(A) –3, 1 (B) 3, –1 (C) 3, –2 (D) 3, 2

10. The roots of the equation
$(k - m + \ell)x^2 + 2kx + (k + m - \ell) = 0$ are
(A) $-1, \dfrac{2k}{k-m+\ell}$ (B) $-1, \dfrac{-k+m-\ell}{k-m+\ell}$
(C) $-1, \dfrac{-2k}{k-m+\ell}$ (D) $-1, -\left(\dfrac{k+m-\ell}{k-m+\ell}\right)$

11. If x is an integer and $\dfrac{6}{x+2} + \dfrac{8}{x+3} = \dfrac{10}{x+4} + \dfrac{4}{x+1}$, the value of x equals to $\square$.

12. If m and n are the roots of the quadratic equation $x^2 + 2x + 3 = 0$, find the value of the following.

(a) $\dfrac{1}{m} + \dfrac{1}{n}$
(A) –2/3 (B) 1/2 (C) –1/2 (D) –2

(b) $m^2 + n^2$
(A) –5 (B) –4 (C) –3 (D) –2

(c) $m^3 + n^3$
(A) 10 (B) 8 (C) 16 (D) 24

(d) $\dfrac{m}{n} - \dfrac{1}{m^2} + \dfrac{n}{m} - \dfrac{1}{n^2}$
(A) –4/9 (B) –1 (C) 4/9 (D) –2

13. The discriminant of the quadratic equation $(a - b)x^2 + 2ax + (a + b) = 0$, where a and b are real numbers is _____.
(A) non-positive (B) non-negative
(C) zero (D) None of these

14. If $b^2 - 4ac$ has a value of 49 where a, b and c are rational numbers satisfying $ax^2 + bx + c = 0$, the roots are
(A) irrational. (B) rational and equal.
(C) complex. (D) rational and unequal

15. If the square of the sum of the roots of a quadratic equation $ax^2 + bx + c = 0$ where a, b, c are rational, equals to 4 times the product of the roots, then the roots are _____.
(A) rational and equal (B) rational and unequal
(C) irrational (D) complex

16. If a quadratic equation $ax^2 + bx + c = 0$ has the sum of the roots equal to the product of the roots, which of the following holds true?
(A) b = c (B) b = –c
(C) a = b (D) a = –c

17. A natural number when added to 6 times its reciprocal equals $\dfrac{151}{5}$. Find the number $\square$.

18. A natural number is tripled and then increased by 2. The result when squared equals 32 times the original number. Find the number $\square$.

19. Ashok bought some pencils for ₹180. He sold all but 2 pencils for ₹160, making a profit of ₹2 per pencil. Find the number of pencils he bought $\square$.

20. Sum of squares of two consecutive natural numbers is 841. Find the least among the two numbers $\square$.

21. A positive number was subtracted from its reciprocal. If the result is $\dfrac{-48}{7}$, then find the positive number $\square$.

22. The minimum value of the quadratic expression $3x^2 - 7x + 6$ is _____.
(A) $-\dfrac{23}{6}$ (B) $\dfrac{23}{12}$ (C) $-\dfrac{31}{6}$ (D) $\dfrac{7}{6}$

23. Find the range of x in the following inequations.
(a) $x^2 + 19x + 70 < 0$
(A) x < – 14 or x > – 5 (B) x > – 14
(C) –14 < x < – 5 (D) x < – 5

(b) $x^2 - 15x + 50 > 0$
(A) x < 5 or x > 10 (B) –10 < x < –5
(C) 5 < x < 10 (D) x < –10 or x > –5

(c) $x^2 - 27x - 90 < 0$
(A) –3 < x < 30 (B) 3 < x < 30
(C) –30 < x < 3 (D) –30 < x < –3

(d) $x^2 - 60x + 900 > 0$
(A) x > 30
(B) x < 30
(C) Any real value except x = 30
(D) None of these

24. The maximum value of the quadratic expression $-x^2 + 10x + 3$ occurs at x = $\square$.

25. Find the maximum or minimum value of

(a) $3x^2 - 7x + 4$
(A) Minimum of $\dfrac{1}{12}$ (B) Maximum of $\dfrac{1}{12}$
(C) Minimum of $-\dfrac{1}{12}$ (D) Maximum of $-\dfrac{1}{12}$

(b) $-8x^2 + 10x + 3$
(A) Minimum of $-\dfrac{49}{8}$ (B) Maximum of $-\dfrac{49}{8}$

(C) Minimum of $\frac{49}{8}$ (D) Maximum of $\frac{49}{8}$

26. Solve for x: $x^4 - 33x^2 + 216 = 0$

(A) $\pm 3, \pm 2\sqrt{6}$ (B) $\pm 3, \pm 3\sqrt{6}$

(C) $\pm 4, \pm 4\sqrt{6}$ (D) $\pm 4, \pm 3\sqrt{6}$

27. The sum of the squares of two consecutive positive integers exceeds their product by 91. Find the integers.

(A) 9, 10 (B) 10, 11 (C) 11, 12 (D) 12, 13

28. A man could buy a certain number of notebooks for ₹300. If each notebook costed ₹5 more, he could have bought 10 notebooks less for the same amount. Find the price of each notebook (in ₹) ☐ .

29. One of the roots of the equation $x^2 - kx + 1 = 0$ is the cube of the other. Find k, given that both the roots of the equation are positive.

☐

30. The roots of the quadratic equation $ax^2 + bx + c = 0$ are in the ratio of 3 : 4, which of the following holds true?

(A) $49ac = 12b^2$ (B) $12ac = 49b^2$

(C) $25ac = 12b^2$ (D) $12ac = 25b^2$

31. One root of $64x^2 + ax + 1 = 0$ is the square root of the other. Find the value of a.

(A) 68 (B) –20 (C) –85 (D) 85

32. Find the remainder when $x^4 - 3x^3 + 2x^2 - 3x + 7$ is divided by x + 1.

☐

33. If the expression $x^4 - 3x^3 + kx^2 + 7$ is exactly divisible by (3x – 1), then the value of k is _____.

(A) $\frac{550}{9}$ (B) $-\frac{559}{9}$ (C) $\frac{557}{9}$ (D) $\frac{558}{9}$

34. If the expression $2x^3 - 7x^2 + 5x - 3$ leaves a remainder 5k – 2 when divided by x + 1, then find the value of – k.

☐

35. Find the remainder when $x^6 - 7x^3 + 8$ is divided by $x^3 - 2$.

(A) –2 (B) 2 (C) 7 (D) 1

36. If (2x – 1) is a factor of $2x^2 + px - 2$, then the other factor is _____.

(A) x + 1 (B) x + 2

(C) x – 2 (D) 2x + 1

37. What should be subtracted from $2x^3 + 4x^2 - 7x + 4$ to make it exactly divisible by x – 1?

☐

38. If $ax^2 + bx + c$ is exactly divisible by (2x – 3), then which of the following is true?

(A) 9a – 6b – 4c = 0 (B) 6a + 9b + 4c = 0

(C) 9a + 6b + 4c = 0 (D) 4a + 9b + 6c = 0

39. For what values of a and b is $x^4 + 4x^3 + ax^2 - bx + 3$ a multiple of $x^2 - 1$?

(A) a = 1, b = 7 (B) a = –3, b = –5

(C) a = 3, b = –5 (D) a = –4, b = 4

40. A polynomial f(x) leaves remainders of 6 and 3 when divided by (x – 4) and (x – 1) respectively. Find the remainder when the polynomial f(x) is divided by $x^2 - 5x + 4$.

(A) x + 2 (B) x – 2

(C) 3 + x (D) 2 – x

Exercise – 18
(Inequations and Modulus)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If 3x + 14 > 5x + 24, then _____.

(A) x > 5 (B) x < 5 (C) x > –5 (D) x < –5

2. If 7x – 12 > 5x – 11, then _____.

(A) x > 1/2 (B) x < 1/2 (C) x > –1/2 (D) x < –1/2

3. If 5x – 7 ≥ 3 and 3x – 4 ≥ 8, then _____.

(A) x > 2 (B) x ≥ 4 (C) x ≤ –2 (D) x ≤ 4

4. If 3x – 8 ≤ 5x – 2 and 6x – 4 ≤ 4x + 1, then _____.

(A) $x \in [5/2, 3]$ (B) $x \in (-3, 5/2)$

(C) $x \in [-3, 5/2]$ (D) $x \in (5/2, 3)$

5. Solve for real x: $6x^2 + x - 12 \geq 0$.

(A) R (B) R – (–3/2, 4/3)

(C) R – (–2/3, 3/4) (D) (–3/2, 4/3)

6. If $x^2 - x - 6 < 0$, then which of the following always holds true?

(A) $x \in (2, 3)$ (B) $x \in (-3, 2)$

(C) $x \in (-2, 3)$ (D) $x \in (-3, -2)$

7. If $x^2 - 4x + 10 > -4$, then which of the following represent the solution set for x?

(A) { } (B) R (C) (1, ∞) (D) (–∞, 1]

8. If $x^2 + 8x + 20 < 0$, then the solution set for the inequality is _____.

(A) { } (B) R

(C) (–4, 4) (D) R – [–4, 4]

9. If $\frac{x^2 - 2x - 8}{3x^2 + 7x + 2} < 0$, then the solution set for the inequality is _____.

(A) R (B) R – [–1/3, 4]

(C) (–1/3, 4) (D) None of these

10. If $\frac{2x^2 - 3x - 2}{x^2 - 4} \leq 0$, then the solution set for the inequality is _____.

(A) (–2, 2) (B) (–1/2, 2)

(C) (–2, –1/2] (D) (–1/2, 2]

11. If $\frac{x-3}{x+3} > 3$ with $x \neq -3$, then which of the following always holds true?
(A) $x \in (-6, -3)$ (B) $x \in (-3, 3)$
(C) $x \in R - [-3, 3]$ (D) $x \in [-6, -2]$

12. The values of x that do not satisfy $\frac{x+4}{x-1} \geq 0$ where $x \neq 1$ are _____.
(A) (– 4, ∞) (B) (1, ∞)
(C) (– 4, 1) (D) (–1, 4)

13. Solve: $|x + 2| = 5$.
(A) {3, 7} (B) {–3, –7}
(C) {–7, 3} (D) {–3, 7}

14. Solve : $|x^2 – 9| = 0$.
(A) [–3, 3] (B) (–3, 3)
(C) {–3, 3} (D) None of these

15. If $|x + 5| < 7$, then solution set for the inequality is _____.
(A) (–12, 2) (B) (–5,2)
(C) (–12, –5) (D) (–12, –5) ∪ (–5, 2)

16. If $|3x + 4| < 5$, then the solution set for the inequality is _____.
(A) (–3,1/3) (B) (–3, –4/3) ∪ (–4/3, 1/3)
(C) (–4/3, 1/3) (D) None of these

17. Solve: $|2x + 3| > 2$.
(A) (–5/2, –1/2)
(B) (–∞, –5/2)
(C) (–∞, –5/2) ∪ (–1/2, ∞)
(D) (–1, ∞)

18. If $|x + 3| > – 5$, then the solution set of x is _____.
(A) { } (B) R
(C) (–∞, –8) ∪ (2, ∞) (D) None of these

19. The maximum value of the function $f(x) = 15 – |x – 7|$ is ☐.

20. The minimum value of $f(x) = 12 + |x + 5|$ is ☐.

21. If $A = \{x : x |x – 6| = 7\}$ and $x \in N$, then A = _____.
(A) { } (B) {1, 7}
(C) {1} (D) {7}

22. The number of distinct solutions of the equation f(x) = 0, where $f(x) = |x + 2| + |x – 2|$; $x \in R$ is ☐.

23. The number of distinct solutions of the equation $x^2 – 9|x| + 18 = 0$ is ☐.

24. The number of distinct solutions of the equation $|x – |4x + 1|| = 7$ is ☐.

25. $|3 – 5x| > 5 – x$, $x > 0$, $\Rightarrow x >$ _____.
(A) 3/4 (B) 5/3
(C) 2/7 (D) 4/3

26. Solve for x : $\frac{1}{x - 2} < 0$.
(A) (–∞, 4) (B) (2, ∞)
(C) (–∞, 2) (D) (4, ∞)

27. Which of the following is true?
(A) $|x| + |y| \leq |x + y|$ (B) $|x – y| \leq |x| – |y|$
(C) $|x + y| \leq |x| + |y|$ (D) $|xy| > |x| . |y|$

28. The range of x which does not satisfy the inequation $3x^2 – 7x – 6 > 0$ is _____.
(A) (–2/3, 3) (B) (–2/3, 0)
(C) [–2/3, 3] (D) R

29. Solve for x: $x^2 – 5x + 6 < 0$
(A) (2, 3) (B) (3, ∞)
(C) R (D) (–∞, 2)

30. If $(x – 3)(x – 4)(x + 2)^2 > 0$, then the solution is _____.
(A) (–∞, –2) (B) (–2, 4)
(C) (4, ∞) (D) R – [3, 4] ∪ {–2}

31. Solve for x: $\frac{x + 4}{x - 2} > 3$
(A) (2, 7) (B) (2, 5)
(C) (0, ∞) (D) (–∞, 2)

32. If $\frac{3x^2 – x – 9}{x^2 – 2x – 3} < 1$, then the solution set of x is _____.
(A) (–2, –1) (B) (3/2, 3)
(C) (–2, –1) ∪ (3/2, 3) (D) None of these

33. Solve for x: $\frac{x^2 – 4x + 3}{2x^2 – 3x – 2} < 0$
(A) (–1/2, 1) ∪ (2, 3) (B) (2, 7)
(C) (–1/2, 3) (D) (3, 9)

34. Solve for x : $|x – 3| > 4$
(A) (–∞, –1) ∪ (7, ∞) (B) (–1, 7)
(C) (6, ∞) (D) (–∞, ∞)

35. If $21x + 71 \geq 17x + 95$ and $3x – 13 \leq 2x – 10$ then _____.
(A) $x \in [-2, 2]$ (B) $x \in [3, 4]$
(C) $x \in (-\infty, -3]$ (D) None of these

36. Solve : $|2x – 3| < 1$.
(A) (–1, 3/2) (B) (0, 1)
(C) (5, 7) (D) (1, 2)

37. Solve : $|x + 7| = 5$.
(A) {12, 2} (B) {2, –12}
(C) {–2, 12} (D) {–2, –12}

38. If $|x + 7| > –8$, then the solution set is _____.
(A) (–15, 15) (B) (0, ∞)
(C) ϕ (D) R

39. If $|x + 2| < -3$, then the solution set is _____.
(A) $(0, \infty)$ (B) $(-5, 5)$
(C) R (D) None of these

40. Solve: $| x + 4 | < 12$.
(A) $0 < x < 8$ (B) $-16 < x < 0$
(C) $-16 < x < 8$ (D) $8 < x < 16$

Exercise – 19
(Sequences and Series)

Directions for questions 1 to 50: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The sixteenth term of 1, 5, 9, 13, 17, . . is ☐.

2. The nth term of the arithmetic progression 11, 16, 21, 26, … is 66. then n = ☐ .

3. Find the eleventh term of the series 4, –2, –8, –14, ……. .
(A) –56 (B) –62 (C) –68 (D) –50

4. The sum of first n terms of the arithmetic progression 21, 28, 35, 42, …….is 175. Find n.
(A) 7 (B) 8 (C) 5 (D) 9

5. The 41st term of the series $8, 10\frac{1}{2}, 13, 15\frac{1}{2}, \ldots$ is ☐ .

6. Which term of the arithmetic progression 5, 9, 13, … is 105?
(A) 26 (B) 27 (C) 28 (D) 29

7. In an arithmetic progression, the fifth term is 11 and its 15th term is 31, the 25th term of the arithmetic progression is ☐ .

8. Seven times the seventh term of arithmetic progression equals to sixteen times its sixteenth term, then the 23rd term of the arithmetic progression is ☐ .

9. The sum of the terms of the series 49, 42, 35, …, 7 is ☐ .

10. The seventeenth term of an arithmetic progression is 33. Find the sum of the first 33 terms of the arithmetic progression.
(A) 1069 (B) 1089
(C) 1056 (D) Cannot be determined

11. If the sum of the first n terms of an arithmetic progression is $3n^2 + 6n$, then the nth term is _____.
(A) 6n + 3 (B) 12n – 6
(C) 12n + 3 (D) 6n + 6

12. Three numbers in an arithmetic progression have a sum of 39 and a product of 2145. Find the numbers.
(A) 13, 15, 17 (B) 11, 13, 15
(C) 15, 17, 19 (D) 14, 15, 16

13. The sum of three terms of an arithmetic progression is 30. The sum of the squares of the terms is 318, find the three terms.
(A) 6, 10, 14 (B) 7, 10, 13
(C) 8, 10, 12 (D) 9, 10, 11

14. In an exam, A scored as many marks less than B, as B scored less than C. If the total marks scored by the three are 240, then the marks scored by B is _____.
(A) 70 (B) 80 (C) 75 (D) 85

15. The sum of 15 terms of an arithmetic progression whose first and last terms are 20 and 100 respectively is ☐ .

16. Five terms in an arithmetic progression have a sum of 40. The extreme terms have a product of 48. Write the terms in increasing order.
(A) 4, 6, 8, 10, 12 (B) 6, 7, 8, 9, 10
(C) 6, 8, 10, 12, 14 (D) 8, 10, 12, 14, 16

17. The sum of all two-digit numbers which leave remainder of 2, when divided by 5 is ☐ .

18. The arithmetic mean of the first n natural numbers is 34. Find n.
(A) 68 (B) 67 (C) 69 (D) 70

19. Find the first four terms of a geometric progression whose n^{th} term is $3(-5)^n$.
(A) 15, –75, 375, –1875
(B) –15, –75, –375, –1875
(C) 15, 75, 375, 1875
(D) –15, 75, –375, 1875

20. The sixth term of the series $\frac{1}{8}, -\frac{1}{4}, \frac{1}{2}, -1, \ldots$ is _____.
(A) –4 (B) 4 (C) –8 (D) 8

21. The fifth and nineth terms of a geometric progression are 3 and 1/27 respectively, find its eleventh term.
(A) $\frac{1}{729}$ (B) $\frac{1}{243}$ (C) $\frac{1}{81}$ (D) $\frac{1}{2187}$

22. The number of terms of the series 2, 6, 18, … that make a sum of 2186 is ☐ .

23. If the sum and product of three numbers which are in geometric progression are respectively 63 and 1728, then find the numbers.
(A) 3, 12, 48 (B) 8, 16, 32
(C) 4, 12, 36 (D) 8, 16, 24

24. Which term of the geometric progression 2, $2\sqrt{2}$, 4 … is 256?

(A) 14^{th} (B) 15^{th} (C) 16^{th} (D) 17^{th}

25. Find the sum to infinity of a geometric progression whose first term is 5 and fifth term is $\frac{16}{125}$.

(A) $\frac{35}{3}$ (B) $\frac{40}{3}$

(C) $\frac{25}{3}$ (or) $\frac{25}{7}$ (D) $\frac{50}{3}$

26. The common ratio of a geometric progression whose first term is 5, last term is 1536 and sum of its terms 3067 is $\boxed{}$.

27. The geometric mean of 3, 9, 27, 81 and 243 is $\boxed{}$.

28. Each term in an infinite geometric progression is equal to one third of the sum of all its following terms. Find the common ratio if all terms are positive.
(A) 1/2 (B) 2/5 (C) 3/4 (D) 2/3

29. The series formed by the reciprocals of the terms of a geometric progression is a/an _____.
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) None of these

30. Ajay, Bala and Charan are three friends. The amount with Ajay is thrice the amount with Bala. The amount with Bala is thrice the amount with Charan. If the total amount with them is ₹260, find the amounts with Ajay, Bala and Charan respectively.
(A) 180, 60, 20 (B) 90, 30, 10
(C) 135, 45, 15 (D) 160, 80, 20

31. The harmonic mean and the geometric mean of two numbers are $\frac{243}{41}$ and 27 respectively, find the numbers.
(A) 3, 81 (B) 3, 243 (C) 9, 243 (D) 27, 27

32. The arithmetic mean and geometric mean of the roots of a quadratic equation are a and b respectively. Which of the following represents the quadratic equation?
(A) $x^2 + 2ax + b^2 = 0$ (B) $x^2 - 2ax - b^2 = 0$
(C) $x^2 + 2ax - b^2 = 0$ (D) $x^2 - 2ax + b^2 = 0$

33. If p, q and r are in geometric progression then qr, pr and pq will be in _____.
(A) arithmetic progression
(B) geometric progression
(C) harmonic progression
(D) None of these

34. P, Q and R are in arithmetic progression. X, Y and Z are in geometric progression.

$X^{R-Q}.Y^{P-R}.Z^{Q-P} = \boxed{}$.

35. $\frac{1}{3.6} + \frac{1}{6.9} + \frac{1}{9.12} + ...\infty$ equals to
(A) 1/6 (B) 1/9 (C) 1/3 (D) 1/12

36. The least value of n for which $1 + 2 + 2^2 + 2^3 + ... + 2^n > 1400$ is $\boxed{}$.

37. If the sum of an arithmetic progression of first 16 terms is equal to the sum of its first 18 terms, then the sum of the first 34 terms of the arithmetic progression is $\boxed{}$.

38. If $x_p = (\sqrt{5})^p$ where p is any natural number and $\sum_{p=1}^{n} x_p = 155 + 31\sqrt{5}$, then n = $\boxed{}$.

39. The sum of the terms of the series 1, $\frac{1}{3}$, $\frac{1}{9}$, ...∞ is _____.
(A) 3/2 (B) 5/2 (C) 7/2 (D) 9/2

40. In an arithmetic progression $3t_1 = t_{10} - t_8$. The ratio of the first term and the common difference of the arithmetic progression is _____.
(A) 4/3 (B) 2/3 (C) 3/4 (D) 3/2

41. The fourth, tenth and last terms of an arithmetic progression are $\frac{5}{2}$, 7 and $\frac{23}{2}$ respectively. Find the respective values of seventh term and number of terms in the arithmetic progression.
(A) $\frac{17}{4}$, 15 (B) $\frac{19}{4}$, 15 (C) $\frac{17}{4}$, 17 (D) $\frac{19}{4}$, 16

42. If the sum of the fifth, sixth and the seventh terms of a geometric progression is one fourth of the sum of the ninth, tenth and the eleventh terms, find the ratio of the sum of the second and third terms to the sum of the eighth and the ninth terms, given the common ratio is positive.
(A) 1 : 4 (B) 1 : 2 (C) 1 : 8 (D) 1 : 16

43. The sum of all the two-digit numbers which when divided by 6 leave 3 as a remainder is $\boxed{}$.

44. Four numbers in arithmetic progression have a sum of 32. If the product of the extremes is 8 less than the product of the other two numbers, find the four numbers.
(A) 5, 7, 9, 11 (B) 7, 9, 11, 13
(C) 9, 11, 13, 15 (D) 6, 8, 10, 12

45. In an arithmetic progression the first term is 8, the last term is 23 and the sum of its terms is 248, then the common difference is $\boxed{}$.

46. If t_n denotes the nth term of an arithmetic progression, whose first term is a and common difference is d, then which of the following choices is/are true?
(A) $t_n = t_{n-1} + t_{n-2}$
(B) $t_{n+1} = a + nd$
(C) $t_4 + t_8 = t_2 + t_{10}$
(D) More than one of the above

47. In a geometric progression the first term is 4 and the last term is 972. The sum of all the terms is 1456. Find the number of terms and the common ratio.
(A) 6, 3 (B) 4, 3 (C) 8, 2 (D) 7, 4

48. The geometric progression has a first term as 2 and sum of the first 4 terms is 2222. Find the common ratio.
(A) 10 (B) 20
(C) 100 (D) None of these

49. In a network system, each person has to join three persons under him and such that the chain should continue. A person would get ₹1 as commission per person below him in his group. If exactly one person earns ₹39, the number of persons who did not earn any commission is ☐ .

50. In a geometric progression, the first term and the common ratio are both equal to 2. The fourth term is ☐ .

Exercise – 20
(Functions)

Directions for questions 1 to 55: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Which of the following represents the roster form of the set A = {x/x is an odd natural number less than 15}?
(A) {1, 3, 5, 7, 9, 11, 13, 15}
(B) (1, 3, 5, 7, 9, 11, 13)
(C) {0, 1, 3, 5, 7, 9, 11, 13}
(D) {1, 3, 5, 7, 9, 11, 13}

2. Which of the following represents the set builder form of A = {1, 2, 3, 4, …..12}?
(A) {x / x is a whole number ≤ 12}
(B) {x / x is a natural number}
(C) {x / x is a natural number ≤ 12}
(D) {x / x is a natural number < 12}

3. Which of the following is/are a subset of {2, 4, 6, 8}?
(A) { } (B) {2, 8}
(C) Both (A) and (B) (D) None of these

4. If A = {2, 3, 5, 7} and B = {2, 4, 6, 8}, then which of the following is true for the sets A and B?
(A) A and B are equal.
(B) A and B are disjoint.
(C) A and B are equivalent.
(D) None of these

5. If A denotes the set of all factors of 10 and B denotes the set of all factors of 12, then A ∩ B is _____.
(A) {1, 3, 4} (B) {1, 2} (C) {1, 3} (D) {1, 2, 5}

6. If A = {2, 4, 5, 7, 9}, B = {1, 3, 5, 7, 8} and C = {1, 2, 5, 8}, then (A ∪ B) ∩ C = _____.
(A) C (B) A ∩ B (C) B (D) A

7. If A and B are any two sets, then (B – A) ∪ A =
(A) A (B) A ∪ B (C) B (D) A ∩ B

8. If A = {0, 2, 4, 6, 9} and B = {1, 3, 5, 7}, then A Δ B = _____.
(A) A ∩ B (B) A (C) A ∪ B (D) B

9. Which of the following laws gives the relation A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)?
(A) De Morgan's law (B) Associative law
(C) Distributive law (D) Commutative law

10. Which of the following cannot be the cardinal number of power set of any finite set?
(A) 1056 (B) 256 (C) 1 (D) 64

11. If the letters of the word "EXAMINATION" are written in the form of a set, then the number of elements in the set would be _____.
(A) 7 (B) 8 (C) 11 (D) 10

12. If Z_p is the set of all factors of p for p = 1, 2, 3 …. and N is the set of all composite numbers, then $\bigcap_{n\in N} Z_n$ = _____.
(A) ϕ (B) {0} (C) {1} (D) $\not\exists$

13. If $D_n = \{ x : 0 < x < 2n\}$, then $D_5 \cap D_6$ = _____.
(A) D_{10} (B) D_{30} (C) D_6 (D) D_5

14. If $D_n = \left\{x : 0 \le x < \frac{1}{3n}\right\}$, then $\bigcap_{n=1}^{\infty} D_n$ = _____.
(A) R (B) {0} (C) ϕ (D) $\left\{\frac{1}{3}\right\}$

15. If A and B are two sets containing 4 and 6 elements respectively, then the minimum number of elements in A ∪ B = ☐ .

16. If A = {p ∈ N; p is a prime and $p = \frac{7n^2 + 3n + 3}{n}$ for some n ∈ N}, then the number of elements in the set A is ☐ .

17. Let Z denote the set of integers, then $\{x \in Z : |x - 3| < 4\} \cap \{x \in Z : |x - 4| < 5\}$=
(A) {–1, 0, 1, 2, 3, 4}
(B) {0, 1, 2, 3, 4}
(C) {0, 1, 2, 3, 4, 5, 6}
(D) {–1, 0, 1, 2, 3, 5, 6, 7, 8, 9}

18. If in a group of people, m persons can speak Malayalam, r persons can speak Kannada and c persons can speak both Malayalam and Kannada, then the number of persons who can speak either Malayalam or Kannada is _____.
(A) m + c – r (B) m + r – c
(C) c + r – m (D) m + r + c

19. If A and B are two sets and X′ denotes the complement of the set X, then the dual of A' ∩ (A' ∪ B) = A' is _____.
(A) A ∩ (A ∪ B) = A (B) A ∪ (A ∩ B) = A
(C) A' ∪ (A' ∩ B) = A' (D) (A ∪ B) ∩ A' = A

20. If n(X) denotes the number of elements in the set X and if n(A) = 12, n(B) = 18 and n(A ∪ B) = 25, then n(A ∩ B) = ☐ .

Directions for questions 21 to 24: These questions are based on the following data.

If A = {1, 2, 3, 4, 6, 12}, then how many of the subsets of A

21. contain 3 and 4 is ☐.

22. contain 6 but not 12 is ☐.

23. contain exactly 4 elements is ☐.

24. have atmost one element is ☐.

25. If a set A has 5 elements and set B has 3 elements, then the number of relations possible from A to B is _____.
(A) 25 (B) 2^{15} (C) 2^{25} (D) 5^{25}

26. If A = {2, 3, 5, 7}, then the maximum number of elements in a relation defined on A is ☐.

27. If A = {1, 2, 3} and B = {4, 5}, then which of the following set of ordered pairs can represent a function from A to B?
(A) {(1, 4), (2, 5), (1, 5)}
(B) {(1, 4), (2, 5)}
(C) {(1, 4), (2, 4), (3, 5)}
(D) {(1, 4), (1, 5), (2, 4), (3, 5)}

28. Which of the following set of ordered pairs does not represent a function on A = {1, 2, –2, 3}?
(A) {(1, 1), (2, 1), (–2, 2)}
(B) {(1, 2), (2, –2), (3, 2), (–2, 3)}
(C) {(1, 3), (2, 3), (–2, 3), (3, 3)}
(D) {(1, 2), (2, –2), (–2, 3), (3, 1)}

29. The domain of the function {(2, 2), (3, 2), (4, 1), (5, 3)} is _____.
(A) {2, 3, 4} (B) {4, 3, 5, 2}
(C) {5, 4, 2} (D) {3, 4, 5}

30. The range of the function {(1, 2), (2, 3), (3, 4), (4, 1)} is _____.
(A) {1, 2, 3} (B) {2, 3, 4}
(C) {1, 3, 4} (D) {1, 2, 3, 4}

31. If f (x, y) = $2x^2 - y$, then f(3, f(2, 1)) = ☐.

32. If f = {(1, 2), (2, 3), (3, 4), (4, 1)}, then fof = _____.
(A) {(1, 4), (2, 4), (3, 1)}
(B) {(1, 3), (2, 4), (3, 1)}
(C) {(2, 4), (3, 1), (4, 2), (2, 1)}
(D) {(1, 3), (2, 4), (3, 1), (4, 2)}

33. If f(x)= 3x – 2 and g(x) = 2x + 5, then fog(x) = _____.
(A) 6x + 13 (B) 6x – 13
(C) 6x – 1 (D) 6x + 1

34. If f(x) = x^2 + 3 and g(x) = x + 5, then f(g(x)) = _____.
(A) x^2 + 8 (B) x^2 + 10x + 28
(C) $x^3 + 5x^2 + 3x + 15$ (D) x^3 + 15

35. If f(x) = $ax^2 + bx + a$, then f(1/x) = _____.
(A) xf(x) (B) x^2f(x) (C) $\frac{1}{x^2}$ f(x) (D) f(x^2)

36. Which of the following function is bijective on R?
(A) f(x) = $2x^2 - 1$ (B) f(x) = $1 - x^4$
(C) f(x) = |3x – 2| (D) f(x) = 2x + 1

37. If f(2x – 3) = $4x^2 - 12x + 4$, then f(3x – 2) = _____.
(A) $9x^2 + 12x + 1$ (B) $9x^2 - 12x - 1$
(C) $x^2 - 12x - 1$ (D) $8x^2 - 12x - 1$

38. If f(x) = 2x + 1 and g(x) = x^2 – 3, then which of the following is true $\forall$ x $\in$ R?
(A) f and g are equal (B) fog(x)= gof(x)
(C) (f + g)x = f(x) + g(x) (D) Both (B) and (C)

39. If A = {1, 2, 3, 4} and B is a non empty finite set and f: A $\rightarrow$ B is an onto function, then the number of elements in B is _____.
(A) atleast 4 (B) atmost 4
(C) exactly 4 (D) atleast 5

40. If a set A has 3 elements and another set B has 4 elements, then the number of one to one functions that can be defined from A to B is _____.
(A) 0 (B) 12 (C) 24 (D) 3

41. If A and B have p and q elements respectively and f : A $\rightarrow$ B is one-one, then which of the following is true?
(A) p = q (B) p $\geq$ q
(C) p $\leq$ q (D) None of these

42. If A = {1, 2, 3, 4} and B = {a_1, a_2}, then the number of onto functions from A to B is ☐.

43. If f(x) = $3x^2 + 4\cos x + 2\sin^2 x$, then f(x) is _____.
(A) even
(B) odd
(C) neither even nor odd
(D) None of these

44. If f(x) = $\frac{3x-5}{2x+1}$ for x $\neq$ –1/2, then f^{-1}(x) = _____.
(A) $\frac{3x+5}{2x-1}$ (B) $\frac{3x-5}{2x+1}$
(C) $\frac{x+5}{3-2x}$ (D) $\frac{x+5}{2x-3}$

45. If f : R $\rightarrow$ R is defined by f(x) = 5x + 7, then f^{-1}({2, 22}) = _____.
(A) {–1, 10} (B) {–1, 3}
(C) {1, 3} (D) {–1, –3}

46. The domain of f(x) = $\frac{5}{|5-x|}$ is _____.
(A) (5, ∞) (B) (–5, ∞) (C) R – {–5} (D) R – {5}

47. If f(x) = $\sqrt{2x-3}$, then the domain of f is _____.
(A) [3/2, ∞) (B) (3/2, ∞) (C) [0, ∞) (D) (–∞,∞)

48. The domain of f(x) = log(x – 2) + $\frac{1}{\sqrt{x^2-4}}$ is _____.
(A) [–2, 2] (B) (–2, 2)
(C) (2, ∞) (D) (–∞, –2) $\cup$ (2, ∞)

49. If W is a set of all whole numbers and f(n) = $(-1)^n$ + 1 $\forall$ n $\in$ W, then the range of f is _____.

(A) {0, 1, 2} (B) {0, 2} (C) W (D) {0, 1}

50. If f : N → N is defined as f(x) = 3x – 2, then which of the following is true about the range of f?
(A) Range = {1, 3, 5, 7, …}
(B) Range = N
(C) Range = {1, 4, 7, 10, …}
(D) Range = {5, 8, 11, …}

51. The range of f(x) = $\frac{2x-1}{|2x-1|}$ where x ≠ 1/2 is _____.
(A) {–1, 0, 1} (B) {1}
(C) {–1} (D) {–1, 1}

52. If g : R → R and h : R → R are defined as g(x) = x – [x] and h(x) = [x], where [x] is the greatest integer less than or equal to x, then the range of h(g(x)) is _____.
(A) ϕ (B) {0, –1} (C) {0} (D) Z

53. If f(x) = 3(x^2 + $\frac{1}{x^2}$) + 7(x + $\frac{1}{x}$) + 8, then {β ∈ R : f(β) = 0} = _____.
(A) {–2, $\frac{1}{3}$} (B) {–1, 1}
(C) {–2, $\frac{-1}{3}$} (D) {–1}

54. The range of the function f(x) = $\frac{|x|}{x}$ x ≠ 0 is _____.
(A) {–1, 0, 1}
(B) {–1, 1}
(C) {1}
(D) R

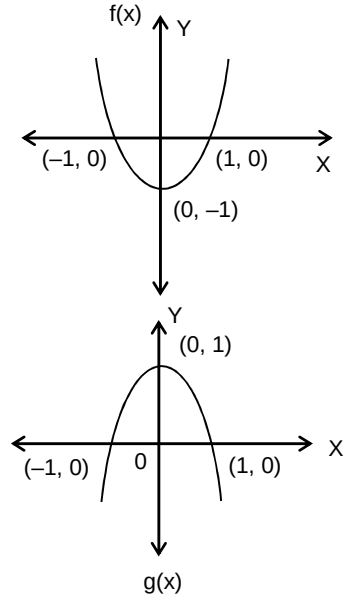
55. If A = {1, 2, 3, 4, 5}; B = {H, T}, then the number of onto functions from A to B is [ ] .

Exercise – 21
(Graphs)

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Directions for question 1: In the following question the graphs of f(x) and g(x) are given. Select the choice which describes the relation between the given graphs.

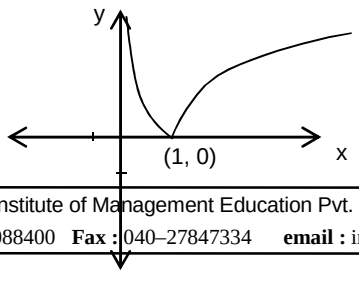
1.



(A) f(x) = g(x)
(B) f(x) = g(–x), for x > 0
(C) f(x) = g(–x) for x < 0
(D) f(x) = –g(–x)

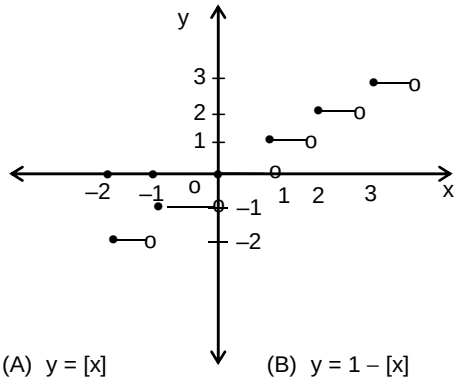
Directions for questions 2 to 4: In these questions a graph is given. Select the choice which best describes the given graph.

2.



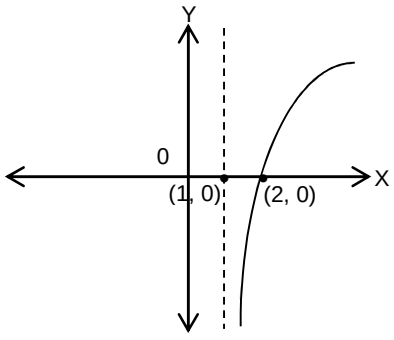
(A) y = logx (B) y = –logx
(C) y = |logx| (D) y = –|logx|

3.



(A) y = [x] (B) y = 1 – [x]
(C) y = 1 + [x] (D) y = [2x]

4.



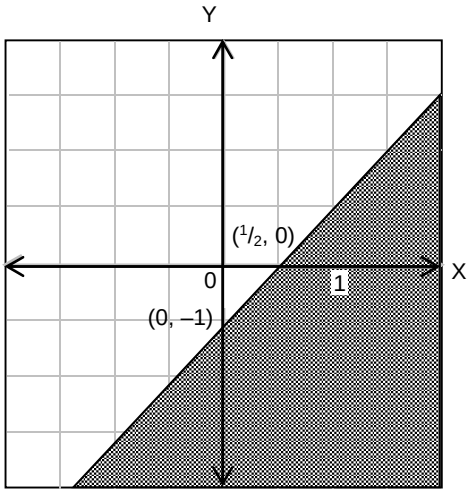
(A) y = $\log_e$x (B) y = $\log_e$(x – 1)
(C) y = $\log_e\left(\frac{x}{2}\right)$ (D) y = $\log_e$2x

Directions for questions 5 to 15: Select the correct alternative from the given choices.

5. The reflection of the point (–1, 4) in the origin is _____.
(A) (–1, 4) (B) (1, 4)
(C) (–4, 1) (D) (1, –4)

6. The reflection of the point (2, –3) in y = 2 is _____.
(A) (2, 7) (B) (2, 2)
(C) (7, –3) (D) (4, –3)

7.



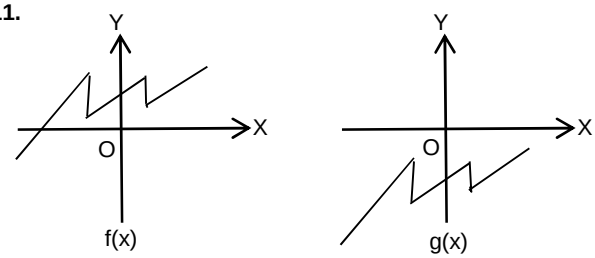
Which of the following best describes the graph given above?
(A) $y \le 2x + 1$
(B) $2y \le x - 1$
(C) $y \ge 2x + 1$
(D) $y \le 2x - 1$

8. Let G be the graph of y = f(x). If a is a positive number, the graph of y = f (x + a) is obtained by _____.
(A) shifting G horizontally towards right by a units
(B) shifting G horizontally towards left by a units
(C) shifting G vertically up by a units
(D) shifting G vertically down by a units.

9. Let G be the graph of y = f(x). If c is a positive number, the graph of y = f(x) – c is obtained by _____.
(A) shifting G horizontally towards left by c units
(B) shifting G horizontally towards right by c units
(C) shifting G vertically up by c units.
(D) shifting G vertically down by c units.

10. The area of the region (in sq units) described by the relations $4x + 3y \le 12$, $2x - y + 4 \ge 0$ and $y \ge 0$ is ☐.

11.



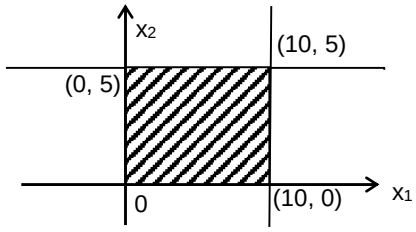
Which of the following is true?
(A) g(x) = f(x – a) (B) g(x) = f(x) – c
(C) g(x) = f(x + c) (D) None of these

12. If y = f(x) represents the curve c, then y = f (–x) represents a curve obtained by
(A) reflecting c in the x – axis
(B) reflecting c about origin
(C) reflecting c in the y – axis
(D) None of these

13. y = f(x) represents the curve c, then x = f(y) represents a curve obtained by
(A) reflecting c in x – axis
(B) reflecting c about origin
(C) reflecting c in the line y = x
(D) reflecting c in the y – axis

14. The number of points in which the graph $4x^2 - 12x + 9 = y$ meets x – axis is _____.
(A) 0 (B) 1 (C) 2 (D) infinite

15.



Which of the following constraints best represent the graph in first quadrant?
(A) $x_1 + x_2 \le 10$ and $x_1 - x_2 \le 5$
(B) $x_1 \le 10$ and $x_2 \le 5$
(C) $x_1 \le 5$ and $x_2 \le 10$
(D) $x_1 \ge 10$ and $x_2 \ge 5$

Exercise – 22
(Indices and Surds)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Simplify the following:

$$\left(\frac{169}{121}\right)^{\frac{-3}{2}} \times \frac{27}{2} \times \left(\frac{13}{22}\right)^{-1}$$

(A) $13^{-1}\,11^4\,3^3$ (B) $13^{-2}\,11^4\,3^4$
(C) $13^{-3}\,11^4\,3^4$ (D) $13^{-4}\,11^4\,3^3$

2. Simplify the following:

$(3^{a+1}\,9^{a+2}\,27^a) \div (3^{a-1}\,9^a\,27^{a+1}) =$ ☐.

3. Simplify the following:

$$\frac{3^{-3} \times 9^{5/2}}{27^{2/3} \times 3^{-4}} =$$ ☐.

4. Simplify the following:

$27^{\frac{2}{3}} + 81^{\frac{1}{4}} - 16^{\frac{1}{2}} =$ ☐.

5. Simplify the following:

$$2\sqrt{\frac{5}{2}} + 5\sqrt{\frac{2}{5}} + \sqrt{10} + \sqrt{1000}$$

(A) $15\sqrt{10}$ (B) $13\sqrt{10}$

(C) $14\sqrt{10}$ (D) $16\sqrt{10}$

6. Simplify the following:
$1-\{1 + (a^2 - 1)^{-1}\}^{-1}$.

(A) $\frac{1}{a^2}$ (B) a^2 (C) $\frac{-1}{a^2}$ (D) $-a^2$

7. Simplify: $y^{p(q-r)} \cdot y^{q(r-p)} \cdot y^{r(p-q)}$.

☐

8. Simplify: $\frac{y^{4b-a} \cdot y^{4c-b} \cdot y^{4a-c}}{\left(y^a \cdot y^b \cdot y^c\right)^3}$.

☐

9. If a + b = –c, find the value of $\left(x^{\frac{a^4}{a^2bc}}\right)\left(x^{\frac{b^4}{ab^2c}}\right)\left(x^{\frac{c^4}{abc^2}}\right)$.

(A) x^3 (B) x^2 (C) x^0 (D) x

10. If $\frac{\left(a+\frac{1}{b}\right)^c\left(a-\frac{1}{b}\right)^d}{\left(b+\frac{1}{a}\right)^c\left(b-\frac{1}{a}\right)^d} = \left(\frac{a}{b}\right)^e$ where a, b, c, d and e are all real numbers, find e.

(A) c + d (B) c – d (C) d – c (D) c + 2d

11. If $3^p = x$, $3^q = y$ and $3^r = z$, find the value of 3^{q-p-r}.

(A) $\frac{y}{xz}$ (B) $\frac{x}{yz}$ (C) $\frac{z}{xy}$ (D) $\frac{xy}{z}$

12. If $3^{4m+1} = 3^{7m-5}$, solve for m.

☐

13. If $2^{4m+2} = 4^{6m-4}$, then m = ☐.

14. Solve for y if $2^{3y+3} = 2^{3y+1} + 48$.

☐

15. Solve for m if $3 \cdot (\sqrt{3})^{m+4} = (\sqrt{3})^{2m+7}$.

(A) 0 (B) –1 (C) 2 (D) 3

16. Solve for m if $6^{8m} = 1296 \times 6^{3(2m+4)}$.

☐

17. If $\left(\sqrt[3]{\frac{7}{4}}\right)^{y-1} = \frac{64}{343}$, what is the value of y?

(A) –7 (B) –8 (C) 7 (D) 8

18. If x is an integer, find the minimum value of x such that $0.00001154111 \times 10^x$ exceeds 1000.

☐

19. Solve for y if $2^{y^{\sqrt{2}^2}} = 512$.

(A) ±3 (B) ±2 (C) ±4 (D) ±6

20. If $y = 7 + 4\sqrt{3}$, then $\frac{y+1}{\sqrt{y}} =$ ☐.

21. $\left(\frac{\sqrt[6]{pq} - \sqrt[3]{q}}{\sqrt[3]{p} - \sqrt[6]{pq}}\right)^{-6} =$ _____.

(A) $\frac{p}{q}$ (B) $\frac{-p}{q}$ (C) $\frac{q}{p}$ (D) $\frac{-q}{p}$

22. $\frac{12}{\sqrt{15} - \sqrt{11}}$ is equal to _____.

(A) $3\sqrt{15} + 3\sqrt{11}$ (B) $3\sqrt{15} - 3\sqrt{11}$

(C) $2\left(\sqrt{15} - \sqrt{11}\right)$ (D) None of these

23. $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} =$ ☐.

24. If $y = 12 + 8\sqrt{2}$, then $\sqrt{y} - \frac{4}{\sqrt{y}}$ is ☐.

25. Find the square root of $6 + 2\sqrt{5}$.

(A) $\sqrt{5} + 1$ (B) $\sqrt{\frac{5}{2}} + \sqrt{2}$

(C) $\frac{\sqrt{5}}{2} + 2$ (D) $\sqrt{\frac{5}{3}} + \sqrt{3}$

26. Find $\sqrt{\sqrt{245} + \sqrt{240}}$.

(A) $5^{\frac{1}{4}}\left(2 - \sqrt{2}\right)$ (B) $5^{\frac{1}{4}}\left(2 + \sqrt{3}\right)$

(C) $5^{\frac{1}{4}}\left(\sqrt{3} - \sqrt{2}\right)$ (D) $5^{\frac{1}{4}}\left(\sqrt{3} + \sqrt{2}\right)$

27. If two surds have an arithmetic mean of $\frac{5}{2}\sqrt{2} + 3$ and one of the surds is $3\left(\sqrt{2} + 1\right)$, find the square root of the other.

(A) $\sqrt{2} + 1$ (B) $2\sqrt{2} - 1$

(C) $2\sqrt{2} - 2$ (D) $\sqrt{2} + 2$

28. In each of the following sets of numbers, identify the greatest number.

(i) 4^{50}, 2^{100}, 16^{25}

(A) 4^{50} (B) 2^{100}

(C) 16^{25} (D) All are equal

(ii) 4^{50}, 5^{40}, 10^{20}, 100^2

(A) 4^{50} (B) 5^{40}

(C) 10^{20} (D) 100^2

29. Which of the following is the greatest?

(A) 3^{3^3} (B) 333^3 (C) 33^{33} (D) 3^{333}

30. Which among the following is the greatest?

(A) $6^{\frac{1}{6}}$ (B) $4^{\frac{5}{12}}$ (C) $3^{\frac{1}{8}}$ (D) $9^{\frac{1}{4}}$

31. Compare the following surds and determine the greatest of them.

$\sqrt{2}+\sqrt{13}, \sqrt{4}+\sqrt{11}, \sqrt{34}+\sqrt{6}$ and $\sqrt{17}+\sqrt{12}$

(A) $\sqrt{2}+\sqrt{13}$ (B) $\sqrt{4}+\sqrt{11}$

(C) $\sqrt{34}+\sqrt{6}$ (D) $\sqrt{17}+\sqrt{12}$

32. $\frac{1}{\sqrt{9}+\sqrt{10}}+\frac{1}{\sqrt{10}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{12}}+.....$ up to 91 terms is _____.

(A) 7 (B) 8 (C) 6 (D) 9

33. Which of the following is the greatest?

(A) 2222^{22} (B) 222^{222}

(C) 22^{2222} (D) 2^{22222}

34. If $\beta \neq 0$, then $\frac{1}{1+\beta^3+\beta^6}+\frac{1}{1+\beta^3+\beta^{-3}}+\frac{1}{1+\beta^{-3}+\beta^{-6}}$ = _____.

(A) $1+\beta^3+\beta^6$ (B) β

(C) 1 (D) $1+\beta^3-\beta^6$

35. $\sqrt{5\sqrt{5\sqrt{5\sqrt{5.....}}}}$ ... 8 times = _____.

(A) $5^{\frac{1}{8}}$ (B) $5^{\frac{15}{16}}$

(C) $5^{\frac{255}{256}}$ (D) None of these

36. Which of the following is true?

(A) $8^{\frac{1}{6}}<7^{\frac{1}{6}}<6^{\frac{1}{6}}<5^{\frac{5}{24}}$ (B) $5^{\frac{5}{24}}<6^{\frac{1}{6}}<7^{\frac{1}{6}}<8^{\frac{1}{6}}$

(C) $6^{\frac{1}{6}}<5^{\frac{5}{24}}<7^{\frac{1}{6}}<8^{\frac{1}{6}}$ (D) $6^{\frac{1}{6}}<7^{\frac{1}{6}}<5^{\frac{5}{24}}<8^{\frac{1}{6}}$

37. $\sqrt[6]{\sqrt[5]{\sqrt[4]{\sqrt[3]{x}}}}$ = ?

(A) $x^{\frac{1}{180}}$ (B) $x^{\frac{1}{360}}$

(C) $x^{\frac{1}{270}}$ (D) None of these

38. $\sqrt{400\sqrt{400\sqrt{400.....\infty}}}$ = ☐ .

39. x, a, b and c are real numbers.

$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}=$

☐

40. If $a=\frac{9}{9+6\sqrt{2}}$, find the value of $a^2 + 18a$.

(A) 300 (B) 305

(C) $300+216\sqrt{2}$ (D) $315-216\sqrt{2}$

Exercise – 23
(Logarithms)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the value of the following.

(a) $\log_{125} 625$

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$ (C) $\frac{5}{4}$ (D) $\frac{6}{5}$

(b) $\log_{2^8} 512$

(A) $\frac{9}{8}$ (B) $\frac{8}{9}$ (C) 2 (D) $\frac{1}{2}$

2. $\log_{81}\left(\frac{1}{81}\right)$

(A) 1 (B) –1 (C) 0 (D) 2

3. Simplify the following logarithm.

log125 + log8 – log10.

☐

4. Simplify the following logarithm.

log2 + log50 + 1.

☐

5. Simplify the following logarithm.

$5+\log\left(\frac{1}{1000}\right)$

☐

6. Solve the following for x.

logx + log3 = log15

☐

7. Solve the following for x.

log (x + 2) + log (x – 2) = log5.

☐

8. Solve the following for x.

$\frac{\log 2401}{\log 343}=\log x$

(A) 1 (B) 10

(C) 100 (D) $\sqrt[3]{10000}$

9. $\log_5 40 + \log_5 150 - \log_5 48 - 2 =$ ☐.

10. Simplify:
$0.9 \log_5 25 + 0.09 \log_5 25 + 0.009 \log_5 25 + \ldots. \infty$.
☐

11. The value of $\log 4 + \log 8 - \log\left(\frac{32}{1000}\right) + \log_{32} 1024$
= ☐.

12. The value of $3\log_{100}2 - \log_{100}4 + 3\log_{100}5 - \log_{100}250$ is ☐.

13. The value of $\frac{1}{1+\log_{ab} c} + \frac{1}{1+\log_{ac} b} + \frac{1}{1+\log_{bc} a}$
equals _____.
(A) 1 (B) 0 (C) 2 (D) log abc

14. $\frac{\log_9 81}{\log_{11} 1331}$ = _____.
(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $\frac{4}{3}$

15. The value of $49^{2\log_7 \sqrt{2}}$ is _____.
(A) 7 (B) 4 (C) $\frac{1}{7}$ (D) $\frac{1}{4}$

16. Find the value of $\log_9\left(\frac{1}{729}\right)$.
(A) –3 (B) –2 (C) –4 (D) –5

17. If $\frac{\log_5 64}{\log_5 4} = \log_3 x$, then find x.
(A) 9 (B) 27
(C) 3 (D) None of these

18. If log (14x + 114) – log (4x + 1) = log 10, then x = ☐.

19. If (3x + 8) log 4 = (7x + 9) log 16, then find x.
(A) $\frac{-10}{11}$ (B) $\frac{-15}{11}$ (C) $\frac{-20}{11}$ (D) $\frac{-25}{11}$

20. If logy + log40 = log160, then the value of y = ☐.

21. If 3logy + log 32 = log 256, then find y
(A) 1 (B) 2 (C) 3 (D) 4

22. If $\log_7 [\log_6 (\log_3 x)] = 0$, then find x.
(A) 729 (B) 27 (C) 81 (D) 243

23. $\log_5 (\log_6 (\log_7 49^3))$ = _____.
(A) 0 (B) 1 (C) 2 (D) 3

24. Find the value of $6^{3 - \log_6 18 + 2\log_6 3}$.
(A) 36 (B) 108 (C) 18 (D) 216

25. Find the value of $x - y^2$ in terms of y, if $\log_5 x = \log_5 25 + 2\log_5 y$.
(A) $25y^2$ (B) $49y^2$ (C) $24y^2$ (D) y^2

Exercise – 24
(Permutations and Combinations)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If there are 3 routes from a city A to city B and 5 routes from city B to city C, then the number of distinct routes form A to C through B are ☐.

2. A boy has nine trousers and 12 shirts. In how many different ways can he select a trouser and a shirt?
(A) 21 (B) 12 (C) 9 (D) 108

3. How many three letter words can be formed using the letters of the word TIME?
(A) 12 (B) 20 (C) 16 (D) 24

4. Using all the letters of the word "THURSDAY", how many different words can be formed?
(A) 8 (B) 8! (C) 7! (D) 7

5. Using all the letters of the word "NOKIA", how many words can be formed, which begin with N and end with A?
(A) 3 (B) 6 (C) 24 (D) 120

6. The number of arrangements that can be made with the letters of the word MEADOWS so that the vowels occupy the even places is .

7. The number of arrangements using all the letters of the word 'JACKSON' such that the word ends with K is ☐.

8. Using all the letters of the word PIGMENT, words that begin with P but do not end with T have to be formed. How many such words are there?
(A) 120 (B) 480 (C) 600 (D) 500

9. There are 20 points on a plane of which only 6 of them are collinear. Except for the triplets selected exclusively from these 6 points, no other set of 3 points are collinear. Find the number of lines that can be formed by joining these points.
(A) 175 (B) 176 (C) 190 (D) 185

10. In the above problem find the number of triangles that can be formed by joining those points.
(A) 1020 (B) 980 (C) 1000 (D) 1120

11. Find the number of ways of seating 7 persons in a row if a particular person occupies the middle position.
(A) 720 (B) 600 (C) 540 (D) 760

12. Find the number of different words that can be formed using all the letters of the word 'SPECIAL' if the words start with a vowel and do not end with L.
(A) 1200 (B) 1800 (C) 1500 (D) 1600

13. How many 3 - digit numbers with distinct digits can be formed using the digits 0, 1, 2, 4, 5 such that they are divisible by 5 and are greater than 300?
(A) 9 (B) 6 (C) 8 (D) 10

14. The number of ways in which 3 letters can be posted in 5 letter boxes is ☐.

15. At a party, each person greeted each of the other persons once. The total number of greetings was 132, then the number of persons who attended the party is ☐.

16. In how many ways can a committee of 6 be formed from 8 boys and 6 girls such that the committee consists of at least 3 boys and at least one girl.
(A) 2506 (B) 2410 (C) 2200 (D) 2650

17. Find the number of ways of distributing 4 prizes to 6 persons when each person is eligible for more than one prize.
(A) 6^4 (B) 4^6 (C) 216 (D) 1028

18. There are 18 stations between Hyderabad and Bangalore. How many second class tickets have to be printed, so that a passenger can travel from any station to any other station?
(A) 200 (B) 380 (C) 95 (D) 100

19. The number of permutations of the letters of the word 'MESMERISE' is _____.
(A) $\frac{9!}{(2!)^2(3!)}$ (B) $\frac{9!}{(2!)^3 3!}$
(C) $\frac{9!}{(2!)^2(3!)^2}$ (D) $\frac{5!}{(2!)^2 3!}$

20. The number of arrangements of the different words, not necessarily meaningful, that can be formed using all the letters of the word MATHEMATICS is _____.
(A) 11! (B) $\frac{11!}{2!}$ (C) $\frac{11!}{(2!)^2}$ (D) $\frac{11!}{(2!)^3}$

21. How many multiples of 6 can be formed using all the digits 1, 2, 3, 4 and 5?
☐

22. The number of ways 4 teachers and a principal can be seated in 5 chairs such that the principal always sits in the middle is _____.
(A) 5! (B) 4! (C) $3! \times 2!$ (D) 3!

23. How many 7 digit numbers can be formed using the digits 0, 1, 1, 2, 2, 3, 3?
(A) 198 (B) 200 (C) 240 (D) 540

24. In how many ways can 6 letters be posted when 4 mail boxes are available for posting the letters?
(A) ${}^6C_4$ (B) 4^6 (C) ${}^6P_4$ (D) 6^4

25. In how many ways can 12 prizes be distributed among 3 boys?
(A) 3^3 (B) 12^3 (C) 3^{12} (D) 3^{15}

26. A letter lock consists of three rings each marked with six different letters. The number of distinct unsuccessful attempts to open the lock is at the most ☐.

27. The number of ways in which six boys and six girls can be seated in a row for a photograph so that no two girls sit together is ☐.

28. The number of four digit numbers that can be formed using the digits {1, 3, 4, 5, 7, 9}? (repetition of digits is not allowed) is ☐.

29. The number of four digit even numbers that can be formed using the digits {2, 3, 5, 1, 7, 9} (repetition of digits is not allowed) is ☐.

30. The number of ways six members can be selected from a group of ten members is ☐.

31. A bag contains nine yellow balls, three white balls and four red balls. In how many ways can two balls be drawn from the bag?
(A) ${}^9C_2$ (B) ${}^3C_2$ (C) ${}^{16}C_2$ (D) ${}^{12}C_2$

32. If 12 archers participate in a shooting competition, in how many ways can they finish the competition for the first four positions?
(A) ${}^{12}C_4$ (B) ${}^{12}P_4$ (C) 12^4 (D) 4^{12}

33. In how many ways can three consonants and two vowels be selected from the letters of the word "TRIANGLE"?
(A) 25 (B) 30 (C) 40 (D) 20

34. In a class there are 20 boys and 25 girls. The number of ways in which a boy and a girl be selected is ☐.

35. In how many ways can a committee consisting of three men and four women be formed from a group of six men and seven women?
(A) ${}^6C_4\ {}^7C_3$ (B) ${}^6C_3\ {}^7C_5$
(C) ${}^6C_3\ {}^7C_4$ (D) ${}^7C_5\ {}^6C_4$

36. A question paper consists of five problems, each problem having three internal choices. The number of ways a candidate can attempt one or more problems is ☐.

37. Six points are marked on a straight line and five points are marked on another line which is parallel to the first line. How many straight lines, including the first two, can be formed with these points?
(A) 32 (B) 33 (C) 55 (D) 30

38. The number of ways in which five boys and three girls can be made to sit in a row such that all boys sit together is ☐.

39. The number of sequences in which 7 players can throw a ball, so that the youngest player may not be the last is ☐.

40. The number of ways of arranging the letters of the word "MATERIAL" such that all the vowels are always together is ☐.

Exercise – 25
(Probability)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The probability of getting no head when five fair coins are tossed, is ☐.

2. The probability of getting at least three heads when six coins are tossed, is ☐.

3. In rolling an unbiased dice, the probability of getting an even number or a number less than 4 is _____.
(A) 2/3 (B) 2/5 (C) 1/3 (D) 5/6

Directions for questions 4 and 5: These questions are based on the data given below.

Three six faced dice are rolled together. The probability that

4. no two dice show the same number on them is _____.
(A) 7/12 (B) 5/9 (C) 1/36 (D) 5/12

5. exactly two dice show the same number on them is _____.
(A) 5/9 (B) 5/12 (C) 1/36 (D) 7/12

6. If two dice are thrown together, the probability of getting an even number on one dice and an odd number on the other dice is ☐.

7. If a card is picked at random from a well shuffled pack of cards, what is the probability that it is neither a king nor a spade?
(A) 4/13 (B) 11/13 (C) 2/13 (D) 9/13

8. If two cards are drawn at random from a well shuffled pack, the probability that both are queens is _____.
(A) 3/221 (B) 1/221 (C) 1/17 (D) 4/17

9. A number is chosen at random from the set, A = {2, 4, 6, 8, 10, 12, 14, 16}. The probability that it is a multiple of 4 is ☐.

10. A number is selected at random from the first fifty natural numbers. The probability that it is a composite number is ☐.

11. A natural number is chosen at random from 1 to 100. The probability that it is a prime number is ☐.

12. If a number is chosen at random from the set {1, 2, 3, ……, 100}, then the probability that the chosen number is a perfect cube is ☐.

13. If two distinct numbers are picked at random from the set A where A = {2, 3, 5, 7, 11, 13}, then what is the probability that the sum of the numbers picked is even?
(A) 2/3 (B) 1/3 (C) 1/4 (D) 1

14. A bag contains five red balls, three black balls and a white ball. If three balls are drawn from the bag, the probability that the three balls are of different colours is _____.
(A) 23/28 (B) 5/28
(C) 3/28 (D) None of these

15. If six people sit around a circular table, the probability that two specified persons always sit side by side is ☐.

16. If ten students are to be seated in a row, then the probability that two particular students never sit together is ☐.

17. Out of seven fruits in a basket, two are rotten. If two fruits are drawn at random from the basket, the probability of both being rotten is _____.
(A) 1/21 (B) 10/21 (C) 20/21 (D) 11/21

18. The probability that a non-leap year chosen at random has 52 Mondays and 52 Tuesdays is _____.
(A) 2/7 (B) 4/7 (C) 5/7 (D) 1/7

19. A two-digit number is formed by using the digits 0, 2, 3, 5, 8 without repetition. If a two digit number is selected at random out of these numbers, then the probability that it is a multiple of '2' is ☐.

20. The probability of getting at least one 4 when two unbiased dice are thrown together is _____.
(A) 5/18 (B) 11/36 (C) 1/3 (D) 13/36

21. If two balls are drawn at random from a bag containing four green balls and three red balls, the odds against the two balls being green is _____.
(A) 3 : 5 (B) 2 : 5 (C) 5 : 2 (D) 5 : 3

Directions for questions 22 to 26: These questions are based on the following information.

A box contains twelve bulbs of which two are defective.

If three bulbs are chosen at random, find the probability that

22. all the bulbs are good.
(A) 9/22 (B) 5/11 (C) 6/11 (D) 7/22

23. two bulbs are good and one is defective.
(A) 9/22 (B) 1/22 (C) 1/5 (D) 7/11

24. one bulb is good and the other two are defective.
(A) 7/22 (B) 1/22 (C) 1/11 (D) 2/5

25. at least one bulb is defective.
(A) 9/11 (B) 8/11 (C) 7/22 (D) 5/11

26. at least one bulb is good is $\square$.

27. The probability that a student is not a singer is 3/5. The probability that out of six students two are singers is _____.
(A) $\frac{422}{3125}$ (B) $\frac{972}{3125}$
(C) $\frac{1}{3}$ (D) $\frac{2}{5}$

28. X and Y are independent events. The probability that both X and Y occur is 1/8 and the probability that neither of these occur is 3/8. The probability of occurrence of X can be _____.
(A) 2/3 (B) 1/4
(C) 1/3 (D) 3/4

29. A person gets as many rupees as the number shows up when an unbiased 6 faced dice is rolled. If two dice are rolled, the probability that the person gets ₹9 is _____.
(A) 5/36 (B) 1/9
(C) 1/6 (D) 1/12

30. An unbiased coin is tossed. A person gets ₹30 if the coin shows head, and he loses ₹15 if the coin shows tail. If three unbiased coins are tossed, the probability that the person gets ₹45 is _____.
(A) 3/8 (B) 1/2
(C) 1/10 (D) 1/25

31. Let E be the set of all integers with 6 in their unit place. The probability that a number chosen from the set {10, 11, 12, . . . 59} is a member of E, is $\square$.

32. If A and B are two mutually exclusive events and P(A) = 1/6 and P(B) = 1/3, then $P(A \cup B) = \square$.

33. If A and B are two independent events such that P(A) = 2/3 and $P(\overline{B}) = 1/5$, then find $P(\overline{A} \cup \overline{B})$.
(A) 8/15 (B) 4/15
(C) 7/15 (D) 11/15

34. If A and B are two independent events such that $P(A \cup B) = 0.75$ and P(B) = 0.25, then P(A) = _____.
(A) 2/3
(B) 1/3
(C) 1/2
(D) None of these

35. A committee of three is to be chosen from a group of three men and four women. The probability that the committee would contain only women is _____.
(A) 4/35 (B) 3/35
(C) 31/35 (D) 34/35