CHAPTER – 5

FUNCTIONS

SETS:

A set is a well defined collection of objects. The objects of the set are called its elements. Sets are usually denoted by capital letters and the elements of the set are denoted by lower case letters. If an element x belongs to set A, it is denoted by $x \in A$. If x is not an element of A, it is denoted by $x \notin A$.

A set, in general is represented in two forms:

1. ***Roster Form:*** In this form a set is described by actually listing out the elements. For example, the set of all even natural numbers less than 12 is represented by {2, 4, 6, 8, 10}

2. ***Set Builder Form:*** In this form a set is described by a characterising property. For example, the set of all even natural numbers less than 12 is represented by $\{x \in N \mid x < 12$ and x is even$\}$. The symbol | is read as "such that."

Some Definitions:

Null Set: A set is said to be a null set if it has no elements. It is also called an empty set or a void set and is denoted by ϕ.

Examples:

(i) $\{x \mid x$ is an integer, $1 < x < 2\}$
(ii) $\{x \mid x$ is a real number and $x^2 < 0\}$

Finite and Infinite sets: A set 'A' is said to be **finite** if it is either an empty set or contains a finite number of elements. Otherwise, it is said to be **infinite**.

Examples:

(i) The set of vowels in the English alphabet is finite.
(ii) The set of natural numbers is infinite.

Cardinality of a Set: The number of distinct elements in a set is called the cardinality (or order) of the set. If a finite set A has n distinct elements, the cardinality of the set is n and is denoted by O(A) or n(A). The cardinality of the empty set is 0.

Example: The cardinality of A = {x, y, z, t} is 4.

Singleton Set: A set consisting of a single element is called a singleton set, i.e. a set whose cardinality is 1 is a singleton set.

Examples: (i) {3} (ii) {a}

Equal Sets*:* Two sets A and B are said to be equal if they have the same elements, i.e. if every element of A is an element of B and every element of B is an element of A.

Subsets and Supersets:

Let A and B be two sets. If every element of A belongs to B, then A is said to be a subset of B. This is written as $A \subseteq B$.

$\therefore A \not\subseteq B$ means A is not a subset of B. If A is a subset of B and there is at least one element in B that is not there in A, A is said to be a proper subset of B. This is written as $A \subset B$.

$\therefore A \not\subset B$ means A is a not a proper subset of B.
Note: $A \subset B \Rightarrow A \subseteq B$. But the converse is not true.

If A is a subset of B we say that B contains A or B is a superset of A. This is written as $B \supseteq A$

Note:

(1) Every set is a subset of itself.
(2) The empty set is a subset of every set.
(3) If A is a finite set of cardinality n, then the total number of subsets of A is 2^n.

Power Set: If A is any set, then the set of all subsets of A is called the power set of A and is denoted by P(A), i.e. $P(A) = \{S \mid S \subseteq A\}$

Example: If A = {1, 2, 3}, then P(A) = {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

Note:

(1) $A, \phi \in P(A)$.
(2) If A is a finite set having n elements, then the cardinality of P(A) is 2^n.

Universal Set*:* The set that contains all the elements in a given context is called the Universal Set. It is denoted by μ or U.

Examples:

(i) In Plane Geometry, the set of all points in the plane is the universal set.
(ii) In the context of divisibility tests, the set of natural numbers is the universal set.

BASIC SET OPERATIONS

Union of Sets: If A and B are two sets, the union of A and B is the set of all those elements which belong to either A or to B or to both A and B. This is denoted by $A \cup B$.
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Note:

(1) If $A \subseteq B$, then $A \cup B = B$
(2) $A \cup \phi = A$
(3) $A \cup \mu = \mu$

Intersection of Sets:

Let A and B be two sets. The intersection of A and B is the set of all those elements that belong to both A and B. It is denoted by $A \cap B$.
$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Note:

(1) $A \subseteq B$, then $A \cap B = A$.
(2) $A \cap \phi = \phi$.
(3) $A \cap \mu = A$.

Disjoint Sets:

Two sets A and B are said to be disjoint if they have no element in common.
$\therefore$ If A and B are disjoint, then $A \cap B = \phi$.

Difference of Sets:

The difference of two sets is the set of all elements which are there in one set but not in the other.
Let A and B be two sets. A – B is the set of all those elements of A which do not belong to B.

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$
Similarly, $B - A = \{ x / x \in B \text{ and } x \notin A\}$.

Example: $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 8, 10\}$
$A - B = \{1, 2\}$
$B - A = \{8, 10\}$

Complement of a Set:

The complement of set A is the set of all those elements that do not belong to set A, i.e. the complement of a set A is the difference of the universal set and set A and is denoted by A' or A^c.

Example: If μ is the set of natural numbers, A is the set of even natural numbers, then A' is the set of odd natural numbers.

Symmetric Difference of Two Sets:

Let A and B be two sets. The symmetric difference of the sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$.

Example: $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ then
$A \Delta B = \{1, 2, 5, 6\}$

Some Results:

(1) $A \cup A = A$; $A \cap A = A$
(2) $A \cup B = B \cup A$
(3) $A \cap B = B \cap A$
(4) $A \cup (B \cup C) = (A \cup B) \cup C$
(5) $A \cap (B \cap C) = (A \cap B) \cap C$
(6) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(7) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(8) $C - (C - A) = C \cap A$
(9) $C - (A \cup B) = (C - A) \cap (C - B)$
(10) $C - (A \cap B) = (C - A) \cup (C - B)$
(11) $(A^c)^c = A$
(12) $(A \cup B)^c = A^c \cap B^c$
(13) $(A \cap B)^c = A^c \cup B^c$

Cartesian Product of Two Sets:

Let A and B be any two sets. Then the Cartesian Product of A and B is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. The product is denoted by $A \times B$
$A \times B = \{(a, b) \mid a \in A, b \in B\}$

Example: $A = \{1, 2, 3\}$ $B = \{a, b\}$ then
$A \times B = \{(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)\}$
$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

Note:

(1) If A and B are two sets such that O(A) = m and O(B) = n, then the number of ordered pairs in A x B is mn
(2) $A \times B \neq B \times A$
(3) $n(A \times B) = n(B \times A)$

RELATIONS

Any subset of $A \times B$ is a relation from A to B. A relation pairs up elements of A with elements in B. If a from A is paired with b from B in a relation R, then we write $(a, b) \in R$ or aRb.
A subset of $A \times A$ is called a relation in the set A.

Example: $A = \{1, 2, 3\}$, $B = \{a, b\}$ then $R = \{(1, a) (3, b)\}$ is a possible relation.

Domain and Range of a Relation:

The set of all first coordinates of the ordered pairs of a relation R is called the Domain of R and the set of all second coordinates of the ordered pairs of R is called the range of R, i.e. if R is a relation from A to B then Domain of R = $\{x/(x, y) \in R$ for some y$\}$ and Range of R = $\{y/ (x, y) \in R$ for some x$\}$. The domain and range of R are denoted as Dom R and Range R.

Inverse of a Relation:

Let A and B be two sets and let R be a relation from A to B. Then the inverse of R denoted by R^{-1} is a relation from B to A and is defined by
$R^{-1} = \{(b, a) \mid (a, b) \in R\}$
We note that Domain of R = Range of R^{-1} and Range of R = Domain of R^{-1}

Examples:

5.01. The sets A and B are given as $A = \{a, b, c, d\}$ and $B = \{a, d, i\}$ Find $A \cup B$, $A \cap B$, $A - B$, $B - A$, $A \Delta B$, $A \times B$ and $B \times A$.

Sol.
(i) $A \cup B$ is the set of all elements which are in set A OR set B or in both
$\therefore A \cup B = \{a, b, c, d, i\}$
(ii) $A \cap B$ is the set of elements which are in both set A AND set B (common elements)
$\therefore A \cap B = \{a, d\}$.
(iii) A – B is the set of elements that are present in set A but NOT in set B
$\therefore A - B = \{b, c\}$.
(iv) B – A is the set of elements that are in set B but NOT in set A
$\therefore B - A = \{i\}$.
(v) $A \Delta B = (A \cup B) - (A \cap B) = \{a, b, c, d, i\} - \{a, d\} = \{b, c, i\}$.
(vi) $A \times B$ = set of all ordered pairs with first element from set A and second element from set B
$A \times B = \{(a, a), (a, d), (a, i), (b, a), (b, d), (b, i), (c, a), (c,d), (c, i), (d, a), (d, d), (d, i)\}$.
(vii) $B \times A$ = Set of ordered pairs with first element from set B and second element from set A
$\therefore B \times A = \{(a, a), (a, b), (a, c), (a, d), (d, a), (d, b), (d, c), (d, d), (i, a), (i, b), (i, c), (i, d)\}$.

5.02. The sets A and B are given as A = {a, b, c, d, e} and B = {a, c, e f} and μ = {a, b, c, d, e, f, g, h} (μ is the universal set) then verify the following
(i) $(A \cup B)^c = A^c \cap B^c$
(ii) $(A \cap B)^c = A^C \cup B^C$

Sol. $A \cup B$ = {a, b, c, d, e, f} $\therefore (A \cup B)^c$ = {g, h} - (1)
A^c = {f, g, h}, and B^c = {b, d, g, h}
$\therefore A^c \cap B^c$ = {g, h} ----------- (2)
From (1) and (2) $(A \cup B)^c = A^c \cap B^C$.
(ii) $A \cap B$ = {a, c, e} $\therefore (A \cap B)^c$ = {c, d, f, g, h} ---- (1)
$A^c \cup B^c$ = {b, d, f, g, h} ----------- (2)
From (1) and (2), $(A \cap B)^c = A^c \cup B^c$.

5.03. If A = {a, b, c, d, e}, then find the number of subsets of A that contain ‘a’ but not ‘e’.

Sol. In the formation of any subset, each element of A either gets included or excluded. Thus each element has two ways to be dealt with. Now that a is included and e is excluded, each of the remaining elements b, c, d can be dealt with in 2 ways. $\therefore$ The number of subsets containing ‘a’ but not ‘e’ is 2^3 or 8.

5.04. If A = {a, d, f, h}, R_1 = {(a, f), (a, h), (f, h)} and R_2 = {(a, d), (d, f), (f, h), (a, h)}, then find R_2^{-1}, $R_1 \cup R_2$ and $R_1 \cap R_2$.

Sol. (i) R_2^{-1} = {(d, a), (f, d), (h, f), (h, a)} (Reverse the order).
(ii) $R_1 \cup R_2$ = {(a, f), (a, h), (f, h), (a, d), (d, f)}.
(iii) $R_1 \cap R_2$ = {(a, h), (f, h)}.

Function:

A relation f, which associates to each element of a set A exactly one element of set B is called a function from A to B and is denoted as f: $A \to B$ (read as f maps A into B) If (a, b) $\in$ f then ‘b’ is called the image of 'a' under f and is written as b = f(a) and 'a' is called the pre-image of 'b'.

If f: $A \to B$ then A is called the domain of f and B is called the co-domain of f and the set R = {f(a)/a $\in$ A} is called the range of f. The range of f is also called the image of f and is denoted by Im(f) or f(A)

Note:
(1) Range $\subseteq$ Co-domain.
(2) $f \subseteq A \times B$.
(3) Every element of A has a unique f-image in B.
(4) Two or more elements of A can have the same f-image in B.
(5) There may be elements in B which are not f-images of any element of A.
(6) The number of functions from a set A containing m elements to another set B containing n elements is n^m.

One-One Function (Injection):

A function f: $A \to B$ is called as one-to-one function if distinct elements of A have distinct images in B.

i.e. f: $A \to B$ is one-one if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ equivalently $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Note:
(1) If n(A) = m, n(B) = n, then the number of one to one functions is ${}^nP_m$ $(n \geq m)$.
(2) A one-one function is possible from A to B if $n(A) \leq n(B)$.

Many-One Function:

A function which is not one-one is called many-one function.

Onto Function (Surjection):

A function f: $A \to B$ is called an onto function if every element of B is an image of at least one element of A i.e. f: $A \to B$ is onto, if for each $y \in B$, there exists $x \in A$ such that f(x) = y

Note:
(1) If f is onto, Range of f = co-domain of f.
(2) An onto function is possible from A to B if $n(A) \geq n(B)$.
(3) If n (A) = m and n(B) = 2, there are $2^m - 2$ onto functions from A to B.
(4) If n(A) = m, n(B) = n then number of onto functions are $n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m ....+…(n \leq m)$

Into Function:

A function which is not onto is called into function, i.e. f: $A \to B$ is into if Range of f is a proper subset of B.

Bijection:

If a function is both one-one and onto, then it is called a bijective function or bijection.

Note:
(1) A bijection from A to B is possible if n(A) = n(B).
(2) If ‘A’ is set with n elements, the number of bijections from A to A (or in A) is n!.

Constant Function:

A function f: $A \to B$ is said to be a constant function if f(x) = k, for all $x \in A$, where k is a fixed element of B.

Note: Range of f = {k}

Identity Function:

The function f : $A \to A$ defined by f(x) = x is called the identity function, denoted by I_A.

Note:
(1) For the identity function, Range = Domain. Symbolically, $I_A(x) = x$ for $x \in A$.
(2) Identity function is a bijection.

Inverse Function:

If f: $A \to B$ be a bijective function then the function f^{-1}: $B \to A$, where f^{-1} = {(b, a)/(a, b) $\in$ f} is called the inverse of the function f.

Note:
(1) If f: $A \to B$ is bijective, $f^{-1} : B \to A$ is also bijective
(2) If f (a) = b, then a = f^{-1} (b)

Composition of functions:

If f: A → B and g: B → C are two functions, then gof is a function from A to C such that gof(a) = g(f(a)) for every a ∈ A and is called the composition of f and g, read as 'g composition f'.

Note:

(1) If f: A → B and g : B → C are two functions then,
  (a) f and g are injective ⇒ gof is injective.
  (b) f and g are surjective ⇒ gof is surjective.
  (c) f and g are bijective ⇒ gof is bijective.
(2) If f: A → A and is injective. If A is finite then f is bijective. But if A is infinite this is not true.
(3) If f: A → B, g: B → C are two bijective functions then $(gof)^{-1} = f^{-1}og^{-1}$.
(4) If f: A → B is bijective then $f^{-1}of = I_A$ and $fof^{-1} = I_B$.
(5) If f: A → B and g: B → A are two functions such that gof = I_A and fog = I_B then g = f^{-1}.
(6) If h: A → B, g: B → C and f: C → D be any three functions then fo(goh) = (fog)oh.
(7) $(fof^{-1})(x) = x$ or $fof^{-1} = I$.
(8) If gof is one-one then 'f' must be one-one but 'g' need not be.
(9) If gof is surjection then 'g' must be onto but 'f' needs not be.
(10) If gof is bijection then 'f' must be one-one and g' must be onto.

Real Function:

If A is a non-empty subset of R, then a function f: A → R is called a real function.

Operations on Real-Valued Functions:

If f: D_1 → R and g: D_2 → R and D = $D_1 \cap D_2$ then we can define

(a) f ± g : D → R such that for all x ∈ D, (f ± g)(x) = f(x) ± g(x)
(b) f·g : D → R such that for all x ∈ D, (fg)(x) = f(x) · g(x)
(c) f/g: D → R such that for all x ∈ D, f/g(x) = f(x)/g(x) provided g(x) ≠ 0
(d) for some constant c, (cf) : D_1 → R such that for all x ∈ D_1, (cf)(x) = c.f(x).

Note: Domain of f ± g, fg, f/g, is $D_1 \cap D_2$ where D_1 is the domain of f and D_2 is domain of g.

Some Real Functions:

(1) Explicit and Implicit Functions:

If the relation between the variables is of the form y = f(x) then y is an explicit function of x. Similarly x is an explicit function of y if x = f(y).

Examples:
(i) $x = y^2 + 3$;
(ii) $y = \sin x + x^3 + 2$

A function which is not explicit is called an implicit function and it is of the form f(x, y) = 0,

Example: $x^2 + e^{xy} \log y = \sin x$; $x^2 + y^2 = a^2$

(2) Even and Odd Functions:

A function f(x) is said to be an even function if f(–x) = f(x) for all x in its domain.

Examples: (i) $x^2 + 5$ (ii) cosx

A function f is said to be odd if f(–x) = – f(x) for every x in its domain.

Examples: (i) x^3 (ii) sinx

Note: Only certain functions are either even or odd. The others are neither even nor odd. This property (of being even or odd) is called parity of functions which has to be carefully distinguished from the property of parity of numbers. If f, g are two functions, the following table shows how the parity of f ± g, fg, f/g gof and fog depends on the parity of f and g. (e denotes even and d denotes odd)

f	g	f+ g	f – g	fg	f/g	gof	fog
e	e	e	e	e	e	e	e
e	d	–	–	d	d	e	e
d	e	–	–	d	d	e	e
d	d	d	d	e	e	d	d

The student should construct the corresponding table for x ± y, xy, x/y where x and y are integers and carefully note the points at which the two tables differ.

(3) Polynomial Function:

A function of the form $f(x) = a_0x^n + a_1x^{n-2} + a_2x^{n-2} + ... + a_n$ where $a_0, a_1, a_2 .... a_n$ are real numbers, $a_0 \neq 0$ and n ∈ N is called polynomial function of degree n.

Note: The domain of a polynomial function is R.

(4) Modulus Function:

The function defined by

$$f(x) = |x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Example:

|7| = 7; |– 7| = 7
Here Domain of f = R, range of f = set of all non-negative real numbers

(5) Greatest Integer Function (Step Function):

f(x) = [x] = the greatest integer less than or equal to x.

Example :
[7·5] = 7, [8 · 5] = 8, [–7·1] = –8
Domain of f = R, Range of f = Z (set of all integers).

(6) Signum Function:

The function f defined by

$$sg(x) = \begin{cases} \dfrac{|x|}{x} \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$$

is called signum function.
Domain of f = R, Range = {–1, 0, 1}.

(7) Exponential function:

The function that associates every real number x to e^x is called the exponential function i.e., $f(x) = e^x$ Domain of f = R, Range = set of all positive real numbers, R^+.

(8) Logarithmic Functions:

The function that associates every positive real number x to log x is called logarithmic function. Domain of f = set of all positive real numbers (R^+); Range of f = R.

(9) Square Root Function:

A function f(x) defined by $f(x) = \sqrt{x}$, $x \in R^+$, is called the square root function.
Domain = Range = $[0, \infty)$ = set of all non-negative real numbers.

(10) Trigonometric Function:

We give the domain and range of various trigonometric functions.

Function	Domain	Range
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sin x	R	$[-1, 1]$
cos x	R	$[-1, 1]$
tan x	$R-\{(2n+1)\,\pi/2 \mid n\in Z\}$	R
cot x	$R - \{n\pi \mid n\in Z\}$	R
sec x	$R-\{(2n+1)\pi/2 \mid n\in Z\}$	$(-\infty, -1] \cup [1, \infty)$
cosec x	$R-\{n\pi \mid n \in Z\}$	$(-\infty,-1] \cup [1, \infty)$

Periodic Function:

A function f(x) is said to be periodic if there exists a non-negative real number T such that $f(x + T) = f(x)\ \forall\ x \in R$. If T is the smallest positive real number such that $f(x + T) = f(x)$ for all $x \in R$, then T is called the fundamental period of f(x). Any integral multiple of T is also a period. But when we speak of the period, without further qualification, we refer to the fundamental period.

Note:

(1) If f(x) is a periodic function with period T, then the function f(ax + b), where a > 0 is periodic with period T/a. The period of both sin x and cos x is 2π and the period of tanx is π

(2) The constant function is a periodic function without a fundamental period.

5.05. Which of the following relations is a function on A = {1, 3, 5, 7}?

(A) $f_1 = \{(1, 3), (1, 5), (1, 7)\}$
(B) $f_2 = \{(1, 3), (3, 5), (5, 7)\}$
(C) $f_3 = \{(1, 1), (3, 3), (5, 5)\}$
(D) $f_4 = \{(1, 3), (3, 5), (5, 7), (7, 1)\}$

Sol. f_4 is a function on A f_1 is not a function on A as 1 has more than one image. Further 3, 5, 7 have no images f_2, f_3 are not functions on A as 7 has no image

5.06. Compute the inverse of $f(x) = \dfrac{2x+1}{x-5}$; $(x \neq 5)$, $x\in R$ $x \neq 2$

Sol. Let y = f(x) (given y in terms of x)

$y = \dfrac{2x+1}{x-5} \Rightarrow xy - 5y = 2x + 1$

$\Rightarrow x(y - 2) = 5y + 1$

$\Rightarrow x = \dfrac{5y+1}{y-2}$ (expressing x in terms of y)

$\therefore f^{-1}(y) = \dfrac{5y+1}{y-2} \Rightarrow f^{-1}(x) = \dfrac{5x+1}{x-2}$ (put y = x)

5.07. If f(x) = 2 x + 3, g(x) = log cosx, $0 < x < \dfrac{\pi}{2}$, then find fog (x) and gof (x).

Sol. (fog)(x) = f[g(x)] = f(log cos x) = 2 (log cos x) + 3
(gof) (x) = g[f(x)] = g(2 x + 3) = log cos (2 x + 3)
NOTE: fog $\neq$ gof

5.08. If $f(x) = e^{2x} + 3x^2$, $g(x) = \dfrac{f(x)+f(-x)}{2}$ and $h(x) = \dfrac{f(x)-f(-x)}{2}$, show that

(i) g(x) is even and (ii) h(x) is odd

Sol. (i) If g(x) = g(–x), then g(x) is said to be an even function.

$g(x) = \dfrac{f(x)+f(-x)}{2}$ = while

$g(-x) = \dfrac{f(-x)+f(x)}{2}$

As g(x) = g (–x),
g(x) is even

(ii) If h (–x) = – h(x), then h(x) is said to be an odd function.

$h(x) = \dfrac{f(x)-f(-x)}{2}$, while $h(-x)\ \dfrac{f(-x)-f(x)}{2}$

As h (–x) = –h(x), h (x) is odd.

Note: (1) For any real function f(x), $\dfrac{f(x)+f(-x)}{2}$ is even and $\dfrac{f(x)-f(-x)}{2}$ is odd function.

(2) Every function can be expressed as a sum of an even and an odd function

$f(x) = \dfrac{f(x)+f(-x)}{2} + \dfrac{f(x)-f(x)}{2}$

5.09. If $f(x) = \dfrac{4x-5}{6}$ for all $x \in R$, then show that f is bijection.

Sol. We first show that f is one - one

$f(x_1) = f(x_2) \Rightarrow \dfrac{4x_1-5}{6} = \dfrac{4x_2-5}{6}$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one one.
Then we show that f is onto

$y = \dfrac{4x-5}{6} \Rightarrow x = \dfrac{6y+5}{4}$

Consider $f\left(\dfrac{6y+5}{4}\right) = \dfrac{4\left(\dfrac{6y+5}{4}\right)-5}{6} = y$

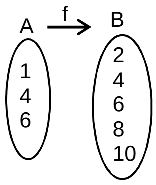
So in order to get any real number y as an f image, the pre image should be $\frac{6y+5}{4}$

Thus every real number gets included in the range and hence f is onto.

$\therefore$ As is one - one and onto, it is a bijection.

5.10. If A = {1, 4, 6}, B = {2, 4, 6, 8, 10}. How many one - one functions are possible from A to B?

Sol.



f: A $\to$ B is any one–one function

Now 1 of A can be assigned to any one of the five elements of B.

Then 4 can be assigned to any one of the four remaining elements of B. Similarly 6 can be assigned to any three of the remaining elements.

Thus 5(4)(3), i.e. ${}^5P_3$ one-one functions are possible from A to B.

5.11. A = {1, 3, 5, 7, 9, 11, 13} and B = {2, 4}. How many onto functions are possible from A to B?

Sol. The total number of functions from A to B is 2^7.

The constant functions, f(x) = 2 and f(x) = 4 are not onto.

$\therefore$ The number of onto functions is $2^7 - 2 = 126$.

5.12. If $f(x) = x + \frac{1}{x^2}$ $x \neq 0$; $x \in R$, then compute

$f\left(\frac{1}{x}\right)$, $(fof)\left(\frac{1}{x}\right)$.

Sol. Given $f(x) = x + \frac{1}{x^2}$

$f\left(\frac{1}{x}\right) = \frac{1}{x} + \frac{1}{\left(\frac{1}{x}\right)^2} = x^2 + \frac{1}{x}$

(fof)

$\left(\frac{1}{x}\right) = f\left[f\left(\frac{1}{x}\right)\right] = f\left(x^2 + \frac{1}{x}\right) = \left(x^2 + \frac{1}{x}\right) + \frac{1}{\left(x^2 + \frac{1}{x}\right)^2}$

5.13. If f(x) = sin log(x) where x > 0, $x \in R$, then find $f\left(\frac{x}{y}\right)$ and $f\left(\frac{y}{x}\right)$.

Also show that $f\left(\frac{y}{x}\right) = -f\left(\frac{x}{y}\right)$.

Sol. $f\left(\frac{x}{y}\right)$ = sin log$\left(\frac{x}{y}\right)$ = sin(log x – log y)

$f\left(\frac{y}{x}\right)$ = sin log$\left(\frac{y}{x}\right)$ = sin (log y – log x)

= sin(– (log x – log y)) = – sin (log x – log y) = $-f\left(\frac{x}{y}\right)$

$\therefore f\left(\frac{y}{x}\right) = -f\left(\frac{x}{y}\right)$

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Which of the following represents the set A = {2, 4, 6, 8, 10,....20}?
(A) A = {x/x $\in$ N, x $\leq$ 20}
(B) A = {x/x = 2y – 1, y $\in$ N}
(C) A = {x/x = 2 n, n $\in$ N and n $\leq$ 10}
(D) A = {x/x = 2 y, y $\in$ W, y $\leq$ 10}

2. P = {x$\in$N/x is a perfect square less than 50}. Which of the following sets represents P?
(A) P = {1, 4, 9, 16, 25, 49}
(B) P = {1, 4, 9, 25, 49}
(C) P = {4, 9, 16, 36, 49}
(D) P = {1, 4, 9, 16, 25, 36, 49}

3. Which of the following is/are a null set(s)?
I. {x/x $\neq$ x, x$\in$ R}
II. {x/x = 2y, y $\in$ N, x is an odd natural number}
III. {x/x = 2y – 1, x is an even natural number x, y$\in$ N}
(A) I, II (B) II, III
(C) I, III (D) I, II, III

4. The cardinal number of the set formed by the letters of the word "MISSISSIPPI" is _______.
(A) 4. (B) 5. (C) 11. (D) 6.

5. The cardinal number of the set of digits in the 10 digit number 5 5 …… 5 is $\boxed{\quad\quad}$.

6. The number of elements of the set {2, {2, 3}, 3, {1, 2, 3}}is _______.
(A) 3. (B) 7.
(C) 4. (D) None of these

7. Which of the following sets is/are subset(s) of {{1, 2}, 2, 3, 4}?
(A) {1, 2} (B) {2, 3}
(C) {1, 2, 3} (D) All of the above

8. A = {1, 2, 3, 4}, B = {x/x $\in$ N, x $\leq$ 4}. Which of the following is/ are true?
I. A $\subseteq$ B
II. B $\subseteq$ A
III. A = B
(A) Only I (B) Only II (C) Only III (D) I, II, III

9. The minimum number of subsets that are possible for any non–empty set is _______.
(A) 1 (B) 2
(C) 0 (D) None of these

10. Which of the following statements is/are true?
1. The null set is a subset of every set.
2. Every set is a subset of itself.
3. There is no set with exactly one subset.

(A) Only 1 (B) Only 2
(C) 1 and 2 (D) 1, 2 and 3

11. If A = {1, 2, 3, 4, 5, 6}, then the number of subsets of A is $\boxed{}$

12. The number of proper subsets of the set A = {a, b, c, d} is _______.
(A) 16. (B) 15. (C) 32. (D) 31.

13. If A = {1, 3, 4, 5} and B = {1, 2, 4, 6}, then $A \cup B$ = _______.
(A) {1, 2, 4, 5, 6}. (B) {1, 2, 3, 4, 5, 6}.
(C) {1, 3, 4, 6}. (D) {1, 2, 3, 5, 6}.

14. If A = {2, 3, 5, 7} and B = {1, 2, 3, 4}, then $A \cap B$ = _______.
(A) {2, 3}. (B) {1, 2, 3}.
(C) {2, 3, 4}. (D) {1, 2, 3, 4, 5, 7}.

15. Which of the following can be the number of proper subsets of a given non-empty set?
(A) 24 (B) 31 (C) 20 (D) 120

16. If A = {a, b, c, d, e, f, g} and B = {a, c, f, g, h}, then A – B = _______.
(A) {a, c, g, f}. (B) {b, c, f, g}.
(C) {b, d, e}. (D) {b, d, f, g}.

17. If A = {1, 2, 3, 4, 5, 6, 7} and B = {2, 4, 6, 8, 10}, then B – A = _______.
(A) {8, 10}. (B) {1, 3, 5, 7}.
(C) {2, 4, 6}. (D) {2, 4, 6, 8}.

18. If μ = {1, 2, 3, 4,.....,10}, A = {1, 3, 5, 7}, then A^c = _______.
(A) {1, 3, 5, 7}. (B) {2, 4, 6, 8, 10}.
(C) {2, 4, 6, 8, 9, 10}. (D) {1, 2, 3, 4, 5, 6}.

19. If $A \subseteq B$, then $A \cup B$ = _______.
(A) A. (B) B.
(C) $A \cap B$. (D) None of these

20. If $A \subseteq B$, then $A \cap B$ = _______.
(A) A. (B) B.
(C) $A \cup B$. (D) None of these

21. Set A has 5 elements and set B has 8 elements. Then the maximum number of elements that $A \cup B$ can have is $\boxed{}$.

22. Set A has 6 elements and set B has 9 elements. Then the minimum number of elements $A \cup B$ can have is _______.
(A) 6. (B) 9. (C) 15. (D) 3.

23. Set A has 4 elements and set B has 7 elements. Then the maximum number of elements that $A \cap B$ can have is $\boxed{}$.

24. If μ is the universal set and A is any non-empty set, then $A \cup A'$ = _______.
(A) μ. (B) A. (C) A'. (D) ϕ.

25. If μ is the universal set and A is any non-empty set, then $A \cap A'$ = _______.
(A) μ. (B) A. (C) ϕ. (D) A'.

26. If n(A) =3 and n(B) = 4, then $n(A \times B)$ equals ______.
(A) 3. (B) 4. (C) 12. (D) 7.

27. Which of the following is $A \times B$, where A and B are any two non empty sets?
(A) $\{(x, y)/, x \in B, y \in A\}$ (B) $\{(x, y)/x, y \in A \cap B\}$
(C) $\{(x, y)/x \in A, y \in B\}$ (D) $\{(y, x)/ y \in B, x \in A\}$

28. The maximum number of elements in any relation from set A to set B, where A and B are non-empty finite sets is _______.
(A) n(A). (B) n(B).
(C) $n(A \times B)$. (D) n(A) + n(B).

29. Which of the following is a relation from set A to set B, where A = {1, 2, 3} and B = {4, 5}?
(A) {(1, 4), (2, 4), (3, 5)}
(B) {(1, 4), (2, 4), (4, 5)}
(C) {(1, 5), (4, 5), (3, 4)}
(D) {(1, 4), (4, 2), (3, 4), (5, 4)}

30. A relation R = {(1, 2), (2, 3), (3, 4), (4, 1)}. R^{-1}, the inverse relation of R = _______.
(A) {(1, 4), (3, 4), (3, 2), (2, 1)}
(B) {(2, 1), (3, 2), (4, 3), (1, 4)}
(C) {(4, 1), (4, 3), (2, 3), (2, 1)}
(D) {(4, 1), (4, 3), (2, 1), (2, 3), (3, 4)}

Exercise – 5(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The number of elements in the power set of A is denoted by n(P(A)). Find n(P(A)), where A = {a, b, c, d, e}.
(A) 25 (B) 16 (C) 32 (D) 31

2. If the number of subsets of a set A that contain a but not b is 16, then the number of elements in set A is $\boxed{}$.

3. The number of functions that can be defined from set A to set B is 625 and n(A) = 4. Then n(B) = _______.
(A) 4. (B) 5. (C) 3. (D) 10.

4. The number of one-one functions that can be defined from set A to set B where n(B) = 16 is 3360. Then n(A) = $\boxed{}$.

5. The number of onto functions possible from set A to set B when n(A) = 6 and n(B) = 4 is _______.
(A) 1560. (B) 1260. (C) 1460. (D) 1660.

6. Of all the functions that can be defined from set A to set B, where n(A) = 4 and n(B) = 6, the number of functions which are not one-one is _______.
(A) 900. (B) 1296. (C) 936. (D) 1200.

7. The number of bijections that can be defined from set A to set B is 120. The number of elements in set B is $\boxed{}$

8. If the function h: R $\to$ B defined by $h(x) = \dfrac{x^2}{3x^2+1}$ is an onto function, then B = _______.

(A) $\left[0, \dfrac{1}{5}\right]$ (B) $\left[0, \dfrac{1}{3}\right)$ (C) $\left[0, \dfrac{1}{5}\right)$ (D) $\left[0, \dfrac{1}{3}\right]$

9. f: N $\to$ N is defined as follows:

$f(n) = \dfrac{n}{2}$ if n is even

$= \dfrac{n+1}{2}$ if n is odd

f(n) is _______.

(A) one – one but not onto
(B) onto but not one – one
(C) both one – one and onto
(D) neither one – one nor onto

10. A set X contains 2, 3, 4 and some other elements. The difference between the number of proper subsets of X and the number of subsets that contain 2, 3, 4 is 111. The cardinal number of set X is $\boxed{}$.

11. If f(x) = 2x + 1 and g(x) = x + 3, then which of the following is true?

(A) (f + g)x = f(x) + g(x) (B) fog(x) = gof(x)
(C) (fg)(x) = g[f(x)] (D) (fg)(x) = f[g(x)]

12. If f(x) = 3x – 1; x$\in$R and g(x) = 2x + 3; x$\in$R, then (gof) (–1) = $\boxed{}$.

13. If $f(x) = \dfrac{3x+5}{4x-3}$, $x \neq \dfrac{3}{4}$; x$\in$R, then fofofofof(4) = _______.

(A) 4 (B) $\dfrac{17}{13}$ (C) –4 (D) $\dfrac{-17}{13}$

14. If $f(x) = \dfrac{2x-3}{5x+2}$, $x \neq \dfrac{-2}{5}$ and x$\in$R, then $f^{-1}(x)$ is ______.

(A) $\dfrac{3-2x}{2-5x}$ (B) $\dfrac{5x+2}{2x-3}$

(C) $\dfrac{2x-3}{5x+2}$ (D) $\dfrac{3+2x}{2-5x}$

15. If $f(x) = (k - x^{1/n})^n$, where n $\in$ N and x > 0, then (fof)(2) = _______.

(A) –2 (B) 2 (C) $\dfrac{1}{2}$ (D) 2^n

16. If f(x) = px + q, where p and q are constants and $f(f(f(x))) = 64x + 105$, then find 3p – 2q.

$\boxed{}$

17. The domain of the real function

$f(x) = \sqrt{2x+3} + \log(4 - x^2) + \dfrac{1}{\sqrt{9-x^2}}$ is ______.

(A) $\left[\dfrac{-3}{2}, 2\right)$ (B) (–3, 3)
(C) (–2, 2) (D) (–3, –2) $\cup$ (2, 3)

18. The domain of the real function

$f(x) = \sqrt{1-2x} + \cos^{-1}\left(\dfrac{2x-1}{3}\right)$ is _______.

(A) (0, 1/2] (B) (–∞, –1/2]
(C) [–1, –1/2] (D) [–1, 1/2]

19. If $f\left(x - \dfrac{1}{x}\right) = x^3 - \dfrac{1}{x^3}$, x $\neq$ 0 x$\in$R, then $f\left(x + \dfrac{1}{x}\right)$ is _______.

(A) $x^3 + \dfrac{1}{x^3}$ (B) $x^3 + \dfrac{1}{x^3} + 6x + \dfrac{6}{x}$

(C) $x^3 - \dfrac{1}{x^3} - 6x - \dfrac{6}{x}$ (D) $x^3 + \dfrac{1}{x^3} - 6x - \dfrac{6}{x}$

20. A real function f(x) is defined as $f(x) = \dfrac{5}{\sqrt[6]{x-[x]}}$; where [x] is the greatest integer less than or equal to x. The domain of f(x) is _______.

(A) R – {0} (B) R – Z
(C) Z^+ (D) $R - Z^-$

21. $f(x) = \sqrt{\dfrac{(x+1)(x-3)}{(x-1)(x+2)}}$, x $\neq$ 1, x $\neq$ –2 and x$\in$R. The domain of f(x) is _______.

(A) (–2, 3)
(B) (–∞, –2) $\cup$ (3, ∞)
(C) (–∞, –2) $\cup$ [–1, ∞)
(D) (–∞, –2) $\cup$ [–1, 1) $\cup$ [3, ∞)

22. A real function f(x) is defined as $f(x) = \dfrac{1}{3+2\cos x}$. The domain of f(x) is _______.

(A) R (B) Z
(C) W (D) N

23. If g(x + y) = g(x) g(y) for all real values of x and y and g(4) = 2401, then g(2) = $\boxed{}$.

24. Let h(x) be a function satisfying $\dfrac{h(a)}{h(b)} = h\left(\dfrac{a}{b}\right)$ for all real values of a and b. If $h(6) = \dfrac{1}{12}$, then find $h\left(\dfrac{1}{6}\right)$.

$\boxed{}$

25. If f (x + y) = f (x) + f (y), ∀ x, y $\in$ R and f (3) = 18 then find the value of $f\left(\dfrac{1}{3}\right)$.

$\boxed{}$

26. If $f(x) = \frac{4^x}{4^x + 2}$, then find $f\left(\frac{1}{2}\right) + f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$.

(A) $\frac{3}{2}$ (B) 2

(C) 4 (D) None of these

27. H(x) is a function such that H(1) = 2460 and $H(1) + H(2) + H(3) + ... + H(N) = \frac{2N[H(N)]}{N-1}$, where N is any natural number greater than 3. Find H(4).

(A) 2952 (B) 3640 (C) 2840 (D) 4280

28. If h(x) is a real function such that h(0) = 1, h(1) = 2 and h(x + 1) = 3h(x) – 2h(x –1) for all x ≥ 1, then h(6) is equal to $\boxed{}$.

29. If $g(x) = 3x^2 + 8$; $x \in R$ and $h(x) = 3x^2 + 4x - 56$; $x \in R$, The value of x such that g(x – 2) = h(x + 2) is

$\boxed{}$

30.

x	1	2	3	4
I(x)	3	4	1	2

The table above defines I(x) for x = 1, 2, 3, 4. For x > 4, I(x)= I(I (x – 1)). The value of I(750) is _______.

(A) 4 (B) 2 (C) 1 (D) 3

31. Let f(x) = min (3 – 2x, 3x + 8) ; $x \in R$. Find the maximum possible value of f(x).

$\boxed{}$

32. If $2f(x) + 3f\left(\frac{1}{x}\right) = 3x + 2$, then find f(3).

(A) –13/5 (B) 5/13 (C) –5/13 (D) 13/5

33. If h(x, y, p, q) = x q – y p and h(z + 1, z, z + 2, 7) = h(5, 4, 3, 1), then the possible values of z are ______.

(A) –2, 7 (B) 2, –7 (C) 3, 4 (D) –5, 1

34. Let f(x) = 3, if x is rational

= –3, if x is irrational

Find the value of the expression

$f(\sqrt{3}) + f(\sqrt{4}) + |f(\sqrt{5})| + \sqrt{f(3)} + \sqrt{|f(6)|} + |f(\sqrt{7})|$.

(A) $2(\sqrt{3}+1)$ (B) $2(\sqrt{3}+\sqrt{2})$

(C) $2(\sqrt{3}-\sqrt{2})$ (D) $2(3+\sqrt{3})$

35. $f(x) = 11 - x^2 + p$, $g(x) = 6 + x^2 + q$, p > 0, q > 0. If f(4) g(2) < 0 then the common range of p and q is _______.

(A) (5, ∞) (B) (0, 5) (C) (3, 10) (D) (5, 10)

Exercise – 5(b)

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. The number of one-one functions from set A = {1, 2, 3, 4, 5, 6} to the set B = {4, 5, 6, 7, 8} is _______.

(A) 6. (B) 5. (C) $^6P_5$. (D) 0.

2. A = {1, 3, 5, 7, 9, 11}. The number of subsets of A that contain 1, 3 but not 11 is $\boxed{}$.

3. The number of functions that can be defined from set A to set B when n(A) = 4 and n(B) = 3 is $\boxed{}$.

4. The number of one-one functions possible from set A to set B, where n(A) = 4 and n(B) = 7 is _______.

(A) 600. (B) 540.

(C) 820. (D) 840.

5.

x	1	2	3	4
h(x)	2	3	1	4

The table above defines h(x) for x = 1, 2, 3, 4. For x > 4, h(x) = h(h(x – 1)). The value of h(796) is $\boxed{}$.

Moderate

6. The number of onto functions possible from set A to set B where n(B) = 2 is 2046. Then n(A) = _______.

(A) 10. (B) 11. (C) 9. (D) 12.

7. The number of into functions that can be defined from set A to set B, where n(A) = 6 and n(B) = 5 is _______.

(A) 13500. (B) 13626.

(C) 13800. (D) 13825.

8. The number of bijections that can be defined from set A to itself when n(A) = 6 is $\boxed{}$.

9. A set P contains a, c, e, f and some other elements. The sum of the number of proper subsets of P and the number of subsets of p that contain a, c, f but not e is 33. n(P) = $\boxed{}$.

10. If f(x) = 2 x – 1, $x \in R$ and g(x) = 3 x + 4, $x \in R$ then $(gof)^{-1}(0)$ = _______.

(A) $\frac{1}{6}$. (B) $-\frac{1}{6}$. (C) $\frac{-7}{6}$. (D) $\frac{7}{6}$.

11. The domain of the real function $f(x) = \frac{2x}{\sqrt{x^2 - 2x - 15}}$ is _______.

(A) [–3, 5]. (B) [–3, ∞).

(C) (–∞, –3) ∪ (5, ∞). (D) (–5, 3).

12. If $f(2x + 1) = 4x^2 + 4x + 7$ $x \in R$, then $f(1 - 2x)$ = _______.
(A) $4x^2 - 4x - 7$. (B) $4x^2 + 4x - 7$.
(C) $4x^2 - 4x + 7$. (D) $2x^2 - 2x + 7$.

13. Find the domain of the function
$f(x) = \dfrac{1}{\log|x-1|} + \dfrac{1}{x+1}$.
(A) R – {–1, 0, 2} (B) R – {–1, 0, 1}
(C) R – {–1, 0, 1, 2} (D) R – {0, 1, 2}

14. Find the range of the function f(x) = x – [x], where [x] is the greatest integer less than or equal to x.
(A) [0, 1] (B) (0, 1]
(C) [0, 1) (D) (0, 1)

15. Domain of the function $f(x) = \dfrac{2}{1-[x]}$ is _______.
(A) $(-\infty, 1) \cup [2, \infty)$ (B) $(-\infty, -1] \cup [2, \infty)$
(C) $(-\infty, -1) \cup (2, \infty)$ (D) $[-4, -1] \cup [2, 4]$

16. Domain of the function $f(x) = \sqrt{x+5} + \dfrac{1}{\sqrt{|x|-3}}$ is _______.
(A) $[-5, \infty)$
(B) $(-5, -3] \cup (3, \infty)$
(C) $[-5, -3) \cup (3, \infty)$
(D) $(-5, -3] \cup [3, \infty)$

17. Range of the function y = 2 + |sinx| is _______.
(A) [2, 4] (B) [2, 3]
(C) $(2, \infty)$ (D) $[4, \infty)$

18. The domain of the real function
$f(x) = \sqrt{x^2 - 4} + \log|x - 5|$ is _______.
(A) $(-\infty, -2)$.
(B) $[5, \infty)$.
(C) $(-\infty, -2] \cup [2, 5)$.
(D) $(-\infty, -2] \cup [2, 5) \cup (5, \infty)$.

19. The range of the real function
f(x) = 8 cos x – 15 sin x + 20 is _______.
(A) (8, 15). (B) (8, 20).
(C) [3, 37]. (D) [–15, 20].

20. The range of the function $f(x) = \dfrac{1}{4 - \sin 2x}$; $x \in R$ is _______.
(A) $\left(\frac{1}{4}, \frac{1}{2}\right)$. (B) $\left(\frac{-1}{2}, \frac{1}{4}\right)$.
(C) $\left[\frac{-1}{4}, \frac{1}{2}\right]$. (D) $\left[\frac{1}{5}, \frac{1}{3}\right]$.

21. If $f(x) = \sqrt{x(x+1)(x+2)(x-3)}$ is a real valued function, find the domain of f(x).
(A) $(-\infty, -2] \cup [3, \infty)$.
(B) $(-\infty, 3]$.
(C) $(-\infty, -2] \cup [-1, 0] \cup [3, \infty)$.
(D) $[-1, \infty)$.

22. If $f(x) = K^x$, then which of the following is not true?
(A) f(x) f(–x) = 1.
(B) $f(x + 3) f(x - 3) = (f(x))^2$.
(C) f(x + 3) – 3 f(x + 2) + 3 f(x + 2) f(–1) – f(x) f(0) $= (K - 1)^3 f(x)$.
(D) All of the previous choices are true

23. $f_1(x) = 5x^2 + 7$; $x \in R$, and $f_2(x) = 3x^2 - 3x + 12$; $x \in R$. The positive value of x such that $f_1(x) = f_2(x - 1)$ Is $\boxed{}$.

24. Let g(x) = min (1 – 3x, 2x + 5); $x \in R$. Find the maximum possible value of g(x).
(A) $\frac{4}{5}$ (B) $\frac{-17}{5}$ (C) $\frac{17}{5}$ (D) $\frac{-4}{5}$

25. If f(a, b, c, d) = ab – cd and f(x – 1, 2x, x + 3, 2) = f(3, 9, 5, –3), then the number of positive integral values of x is $\boxed{}$.

26. If h(x) = 2x – 3, $x \in R$ and $g(x) = x^2 - 3$, $x \in R$, then $\dfrac{(hog)(2)}{(goh)(3)}$ = _______.
(A) $\frac{1}{6}$. (B) $\frac{-1}{6}$. (C) 6. (D) –6.

27. If $f(x) = 2x^2 - 1$; $x \in R$ and g(x) = 2 x + 3; $x \in R$, then (fog) (x) – (gof)(x) is a ______.
(A) linear function. (B) quadratic function.
(C) cubic function. (D) constant function.

28. Given f(x) + f(2 – x) = 6. The value of
$f\left(\frac{1}{150}\right) + f\left(\frac{2}{150}\right) + ... + f\left(\frac{299}{150}\right)$ is $\boxed{}$

29. If $f(x) = \dfrac{x}{\sqrt{1+x^2}}$, then find fofofofof(3).
(A) $\frac{3}{\sqrt{55}}$ (B) $\frac{3}{\sqrt{10}}$ (C) $\frac{3}{\sqrt{46}}$ (D) $\frac{1}{27}$

30. If f(x + 2) + f(x) = 0 $\forall x \in R$ and f(1) = 5, f(2) = 8, then find f(15) + f(16).
$\boxed{}$

31. Let set S = {x|x is a composite number}. For all natural numbers n > 2, A_n = {x|x is factor of n}. $\bigcap_{i \in S} A_i$ = _______.
(A) {1, 2} (B) {1} (C) S (D) {2, 3}

32. If n(A) = 4, n(B) = 7, then find the number of functions which can be defined from A to B, which are not one-one.
$\boxed{}$

33. $f(x) + 3f\left(\frac{1}{x}\right) = 3x + 1$. Find the value of f (3).
(A) $\frac{-1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{-1}{3}$ (D) –4

34. Which of the following functions is its own inverse?

(A) $f(x) = 9^{x(x-1)}$ (B) $g(x) = a^{\log x^2}$

(C) $l(x) = \frac{a-x}{a+x}$ (D) None of the above

35. If f (x) = p x + q, where p and q are constants and f [f[f(x)]] = 8x –56 then find the value of $f^{-1}(-6)$.

(A) 0 (B) 1 (C) –2 (D) 2

36. If $f(x) = x^{x-3}$ then find the value of 2f [2f(2f(2))].

37. The range of $f(x) = \frac{x-3}{|3x-9|}$, $x \neq 3$ is _______.

(A) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ (B) [–3, 3]

(C) (–∞, –3] ∪ [3, ∞) (D) $\left\{-\frac{1}{3}, \frac{1}{3}\right\}$

Difficult / Very Difficult

38. For all x ∈ R, h(x) = max (x + 2, x – 3), g(x) = min (3x – 1, 3x + 5) and f(x) = g(x) – h(x), then the set of values of x for which f(x) ≥ 9 (x∈R) is ______.

(A) $\left[\frac{-5}{2}, \frac{1}{2}\right]$. (B) (–2, 3).

(C) (–∞, 6]. (D) [6, ∞).

39. f(x) = |x – 2| + |x + 3| + |x – 1|. The range of the real function f(x) is _______.

(A) [–3, 2] (B) [–3, ∞)

(C) [5, ∞) (D) [3, ∞)

40. f(x) and g(x) are two real functions with domain A = {1, 2, 3, 4} defined by
f = {(1, 2), (2, 3), (3, 4), (4, 1)} and
g = {(2, 4), (3, 1), (1, 3) (4, 2)}.

Find the value of $\frac{f^2(2)+g(2)}{f(2)-g(2)}$.

41. If f (x) = min (x – 2, x + 3) g (x) = max (x – 4, x – 5) and h(x) = f(x) + g(x), then find the range of x such that h(x) ≤ 6.

(A) (0, 2) (B) (–∞, 1)

(C) (–∞, –5) (D) (–∞, 6]

42. For all x ∈ R, f(x) = min (x – 4, x + 2) and g(x) = max (x – 2, x + 3).If h(x) = f(x) + g(x), then the set of values of x for which h(x) ≤ 5 is _______.

(A) (–∞, 3] (B) [3, ∞)

(C) [–2, 4] (D) [–3, 4]

43. If f(xy) = f(x)f(y) ∀ x, y∈R and f(3) = 27 then find $\sum_{n=1}^{12} f(n)$

44. The function f(x) is defined such that f(2x) + f(3x) + f(x + 2) + f(3 – x) = x, for all real values of x. Find f (0).

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $-\frac{1}{2}$ (D) $-\frac{1}{4}$

45. $f(x) = 125x^3+\frac{1}{x^3}$. α and β are the roots of $5x + \frac{1}{x} = 3$. f(β) =

(A) 27 (B) 18 (C) –18 (D) –27

Key

Concept Review Questions

1. C	7. B	13. B	19. B	25. C
2. D	8. D	14. A	20. A	26. C
3. D	9. B	15. B	21. 13	27. C
4. A	10. C	16. C	22. B	28. C
5. 1	11. 64	17. A	23. 4	29. A
6. C	12. B	18. C	24. A	30. B

Exercise – 5(a)

1. C	6. C	11. A	16. 2	21. D	26. A	31. 5
2. 6	7. 5	12. 5	17. A	22. A	27. A	32. A
3. B	8. B	13. B	18. D	23. 49	28. 64	33. A
4. 3	9. B	14. D	19. B	24. 12	29. 2	34. D
5. A	10. 7	15. B	20. B	25. 2	30. B	35. B

Exercise – 5(b)

1. D	10. B	19. C	28. 897	37. D
2. 8	11. C	20. D	29. C	38. D
3. 81	12. C	21. C	30. –13	39. C
4. D	13. C	22. D	31. B	40. –13
5. 4	14. C	23. 1	32. 1561	41. D
6. B	15. A	24. C	33. A	42. A
7. D	16. C	25. 1	34. D	43. 6084
8. 720	17. B	26. B	35. B	44. D
9. 5	18. D	27. B	36. 1	45. C