CHAPTER – 7

INDICES - SURDS

Indices – Surds

If a number 'a' is added three times to itself, then we write it as 3a. Instead of adding, if we multiply 'a' three times with itself, we write it as a^3.

We say that 'a' is expressed as an exponent. Here, 'a' is called the 'base' and 3 is called the 'power' or 'index' or 'exponent'.

Similarly, 'a' can be expressed to any exponent 'n' and accordingly written as a^n. This is read as "a to the power n" or "a raised to the power n."

$a^n = a \times a \times a \times a \times$ .... n times

For example,
$2^3 = 2 \times 2 \times 2 = 8$ and $3^4 = 3 \times 3 \times 3 \times 3 = 81$

While the example taken is for a positive integer value of n, the powers can also be negative integers or positive or negative fractions. In the sections that follow, we will also see how to deal with numbers where the powers are fractions or negative integers.

If a number raised to a certain power is inside brackets and quantity is then raised to a power again, {i.e., a number of the type $(a^m)^n$ - read as "a raised to the power m whole raised to the power n" or "a raised to power m whole to the power n"}, then the number inside the brackets is evaluated first and then this number is raised to the power which is outside the brackets.

For example, to evaluate $(2^3)^2$, we first find out the value of the number inside the bracket (2^3) as 8 and now raise this to the power 2. This gives 8^2 which is equal to 64. Thus $(2^3)^2$ is equal to 64.

If we have powers in the manner of "steps", then such a number is evaluated by starting at the topmost of the "steps" and coming down one "step" in each operation.

For example, 2^{4^3} is evaluated by starting at the topmost level '3'. Thus we first calculate 4^3 as equal to 64. Since 2 is raised to the power 4^3, we now have 2^{64}.

Similarly, 2^{3^2} is equal to "2 raised to the power 3^2" or "2 raised to the power 9" or 2^9 which is equal to 512.

There are certain basic rules/formulae for dealing with numbers having powers. These are called Laws of Indices. The important ones are listed below but you are not required to learn the proof for any of these formulae/rules. The students have to know these rules and be able to apply any of them in solving problems. Most of the problems in indices will require one or more of these formulae. These formulae should be internalised by the students to the extent that after some practice, application of these rules should come naturally and the student should not feel that he is applying some specific formula.

Table of Rules/Laws of Indices

	Rule/Law	Example
(1)	$a^m \times a^n = a^{m+n}$	$5^2 \times 5^7 = 5^9$
(2)	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^5}{7^3} = 7^2 = 49$
(3)	$(a^m)^n = a^{mn}$	$(4^2)^3 = 4^6$
(4)	$a^{-m} = \frac{1}{a^m}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$
(5)	$\sqrt[m]{a} = a^{1/m}$	$\sqrt[3]{64} = 64^{1/3} = 4$
(6)	$(ab)^m = a^m \cdot b^m$	$(2 \times 3)^4 = 2^4 \cdot 3^4$
(7)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$
(8)	$a^0 = 1$ (where $a \neq 0$)	$3^0 = 1$
(9)	$a^1 = a$	$4^1 = 4$

These rules/laws will help you in solving a number of problems. In addition to the above, the student should also remember the following rules:

<u>Rule 1</u>: When the bases of two EQUAL numbers are equal, then their powers also will be equal. (If the bases are neither zero nor ± 1.)
For example : If $2^n = 2^3$, then it means n = 3

<u>Rule 2</u>: When the powers of two equal numbers are equal (and not equal to zero), two cases arise:
(i) if the power is an odd number, then the bases are equal. For example, if $a^3 = 4^3$ then a = 4.
(ii) if the powers are even numbers, then the bases are numerically equal but can have different signs. For example, if $a^4 = 3^4$ then a = +3 or –3.

The problems associated with indices are normally of THREE types:

<u>Simplification</u>
Here, the problem involves terms with different bases and powers which have to be simplified using the rules/formulae discussed in the table above.

<u>Solving for the value of an unknown</u>

Here, the problem will have an equation where an unknown (like x or y) will appear in the base or in the power and using Rule 1 and Rule 2 discussed above, values of unknown are to be determined.

<u>Comparison of numbers</u>
Here two or more quantities will be given – each being a number raised to a certain power. These numbers have to be compared in magnitude – either to find the largest or smallest of the quantities or to arrange the given quantities in ascending or descending order.

The following examples will make clear the different types of problems that you may be asked.

Examples

7.01. Simplify: $\left(\frac{256}{576}\right)^{1/4} \times \left(\frac{64}{27}\right)^{-1/3} \times \left(\frac{216}{8}\right)^{-1}$

Sol: As $256 = 2^8$; $576 = 24^2$

$64 = 2^6$; $27 = 3^3$; $216 = 6^3$ and $8 = 2^3$,

$$\left(\frac{256}{576}\right)^{1/4} \times \left(\frac{64}{27}\right)^{-1/3} \times \left(\frac{216}{8}\right)^{-1}$$

$$= \left(\frac{2^8}{24^2}\right)^{1/4} \times \left(\frac{3^3}{2^6}\right)^{1/3} \times \frac{8}{216}$$

$$= \frac{2^2}{\sqrt{24}} \times \frac{3}{4} \times \frac{1}{27} = \frac{1}{2\sqrt{6}\ x\ 9} = \frac{1}{18\sqrt{6}}$$

7.02. Simplify the following

$$\left(\frac{l^4 m^6}{n^8}\right)^3 \times \left(\frac{m^8 n^4}{l^{-6}}\right)^{-2} \times \left(\frac{n^6 l^6}{m^4}\right)^2$$

Sol: $$\frac{l^{4x3}\ m^{6x3}}{n^{8x3}} \times \frac{m^{8x-2}\ n^{4x-2}}{l^{-6x-2}} \times \frac{n^{6x2}\ l^{6x2}}{m^{4x2}}$$

$$= l^{12-12+12}\, m^{18-16-8} n^{-8+12-24} = \frac{l^{12}}{m^6 . n^{20}}$$

7.03. Solve the given equation for the value of x $\sqrt[3]{\left(\frac{5}{4}\right)^{x+2}} = \frac{4096}{15625}$

Sol: The given equation can be written as

$$\left(\frac{5}{4}\right)^{\frac{x+2}{3}} = \left(\frac{15625}{4096}\right)^{-1}$$

$$\Rightarrow \left(\frac{5}{4}\right)^{\frac{x+2}{3}} = \left(\frac{5^6}{4^6}\right)^{-1} \Rightarrow \left(\frac{5}{4}\right)^{\frac{x+2}{3}} = \left(\frac{5}{4}\right)^{-6}$$

$$\Rightarrow \frac{x+2}{3} = -6 \Rightarrow x + 2 = -18$$

$$\therefore x = -20$$

SURDS

Any number of the form p/q, where p and q are integers and q ≠ 0 is called a rational number. Any real number which is not a rational number is an irrational number. Amongst irrational numbers, of particular interest to us are SURDS. Amongst surds, we will specifically be looking at 'quadratic surds' – surds of the type $a + \sqrt{b}$ and $a + \sqrt{b} + \sqrt{c}$, where the terms involve only square roots and not any higher roots. We do not need to go very deep into the area of surds - what is required is a basic understanding of some of the operations on surds.

If there is a surd of the form $(a + \sqrt{b})$, then a surd of the form $\pm(a - \sqrt{b})$ is called the conjugate of the surd $(a + \sqrt{b})$. The product of a surd and its conjugate will always be a rational number.

RATIONALISATION OF A SURD

When there is a surd of the form $\frac{1}{a+\sqrt{b}}$, it is difficult to perform arithmetic operations on it. Hence, the denominator is converted into a rational number thereby facilitating ease of handling the surd. This process of converting the denominator into a rational number without changing the value of the surd is called rationalisation.

To convert the denominator of a surd into a rational number, multiply the denominator and the numerator simultaneously with the conjugate of the surd in the denominator so that the denominator gets converted to a rational number without changing the value of the fraction. That is, if there is a surd of the type $a + \sqrt{b}$ in the denominator, then both the numerator and the denominator have to be multiplied with a surd of the form $a - \sqrt{b}$ or a surd of the type $(-a + \sqrt{b})$ to convert the denominator into a rational number.

If there is a surd of the form $(a + \sqrt{b} + \sqrt{c})$ in the denominator, then the process of multiplying the denominator with its conjugate surd has to be carried out TWICE to rationalise the denominator.

SQUARE ROOT OF A SURD

If there exists a square root of a surd of the type $a + \sqrt{b}$, then it will be of the form $\sqrt{x} + \sqrt{y}$. We can equate the square of $\sqrt{x} + \sqrt{y}$ to $a + \sqrt{b}$ and thus solve for x and y. Here, one point should be noted – when there is an equation with rational and irrational terms, the rational part on the left hand side is equal to the rational part on the right hand side and, the irrational part on the left hand side is equal to the irrational part on the right hand side of the equation.

However, for the problems which are expected in the entrance exams, there is no need of solving for the square root in such an elaborate manner. We will look at finding the square root of the surd in a much simpler manner. Here, first the given surd is written in the form of $(\sqrt{x} + \sqrt{y})^2$ or $(\sqrt{x} - \sqrt{y})^2$. Then the square root of the surd will be $(\sqrt{x} + \sqrt{y})$ or $(\sqrt{x} - \sqrt{y})$ respectively.

COMPARISON OF SURDS

Sometimes we will need to compare two or more surds either to identify the largest one or to arrange the given surds in ascending/descending order. The surds given in such cases will be such that they will be close to each other and hence we will not be able to identify the largest one by taking the approximate square root of each of the terms. In such a case, the surds can both be squared and the common rational part be subtracted. At this stage, normally one will be able to make out the order of the surds. If even at this stage, it is not possible to identify the larger of the two, then the numbers should be squared once more.

Examples

7.04. If $(32768)^{x-2} = (32)^x$, then find the value of x.

Sol: The given equation can be rewritten as
$(2^{15})^{x-2} = (2^5)^x$
$\Rightarrow 15x - 30 = 5x \Rightarrow 10x = 30$
$x = 3$

7.05. If $\left(\frac{1}{16\times 81}\right)^{3-\frac{x}{2}} = (48)^{2x-12}$ then find the value of x

Sol: The given equation can be rewritten as
$\left(3^{-4}\times 2^{-4}\right)^{3-\frac{x}{2}} = (2^4)^{2x-12}\times 3^{2x-12}$
$\Rightarrow 3^{2x-12}\times 2^{2x-12} = 2^{8x-48}\times 3^{2x-12}$
$\Rightarrow 2^{2x-12} = 2^{8x-48}$
$\Rightarrow 2x - 12 = 8x - 48 \Rightarrow x = 6$

7.06. Arrange the following number in ascending order $(125)^{10}$, $(625)^9$ and $(25)^{16}$

Sol: Each of the given numbers can be expressed with 5 as the base.
Hence the given numbers can be written as $(5^3)^{10}$, $(5^4)^9$ and $(5^2)^{16}$
$\Rightarrow 5^{30}$, 5^{36} and 5^{32}
As the bases are equal, the values can be compared on the basis of the powers.
Hence $5^{30} < 5^{32} < 5^{36}$
i.e., $(125)^{10} < (25)^{16} < (625)^9$

7.07. Arrange the following numbers in descending order $(144)^3$, $(256)^2$ and $(36)^6$.

Sol: Each of the given bases can be expressed in exponential form.
Hence the given numbers are
12^6, 16^4 and 6^{12} i.e., $(12^{1/2})^{12}$, $(16^{1/3})^{12}$ and $(6)^{12}$
All these numbers have the same power. Hence, they can be compared on the basis of the bases of the numbers.
As $12^{1/2}$ lies between 3 and 4 and $16^{1/3}$ lies between 2 and 3, $16^{1/3} < 12^{1/2} < 6$.
Thus $6^{12} > 12^6 > 16^4$
$\Rightarrow 36^6 > 144^3 > 256^2$ is the descending order.

7.08. Simplify $\frac{1}{4+\sqrt{2}} - \frac{1}{4-\sqrt{2}}$

Sol: $\frac{1}{4+\sqrt{2}} - \frac{1}{4-\sqrt{2}}$
$= \frac{(4-\sqrt{2})-(4+\sqrt{2})}{(4+\sqrt{2})\ (4-\sqrt{2})} = \frac{-2\sqrt{2}}{14} = \frac{-\sqrt{2}}{7}$

7.09. Rationalize the denominator of the surd $\frac{1}{4-\sqrt{13}}$.

Sol: Since the denominator of the surd is $4-\sqrt{13}$, to rationalize, we multiply both the numerator and the denominator with $4+\sqrt{13}$
$\frac{1}{4-\sqrt{13}} \text{ x } \frac{4+\sqrt{13}}{4+\sqrt{13}} = \frac{4+\sqrt{13}}{16-13} = \frac{4+\sqrt{13}}{3}$

7.10. Rationalize the denominator of the surd $\frac{1}{4+\sqrt{6}-\sqrt{10}}$.

Sol: Here first take $4+\sqrt{6}$ as one term and $\sqrt{10}$ as the second term and carry out the rationalization by multiplying the numerator and denominator with $\left(4+\sqrt{6}-\sqrt{10}\right)$
$\frac{1}{(4+\sqrt{6}-\sqrt{10})}$
$= \frac{(4+\sqrt{6}+\sqrt{10})}{(4+\sqrt{6}-\sqrt{10})(4+\sqrt{6}-\sqrt{10})}$
$= \frac{(4+\sqrt{6}+\sqrt{10})}{(4+\sqrt{6})^2-(\sqrt{10})^2}$
$= \frac{4+\sqrt{6}+\sqrt{10}}{16+6+8\sqrt{6}-10} = \frac{4+\sqrt{6}+\sqrt{10}}{12+8\sqrt{6}}$
As the denominator has still irrational part, it should be rationalized one more time by multiplying the numerator and denominator with its conjugate surd.
$= \frac{(4+\sqrt{6}+\sqrt{10})\ (3-2\sqrt{6})}{4(3+2\sqrt{6})\ \ (3-2\sqrt{6})}$
$= \frac{12+3\sqrt{6}+3\sqrt{10}-8\sqrt{6}-12-2\sqrt{60}}{4(9-24)}$
$= \frac{-5\sqrt{6}+3\sqrt{10}-2\sqrt{60}}{-60} = \frac{5\sqrt{6}-3\sqrt{10}+4\sqrt{15}}{60}$

7.11. Find the positive square root of the surd $32+4\sqrt{15}$.

Sol: The given surd is to be written as $\left(a+\sqrt{b}\right)^2$, as the irrational part is positive.
(if the irrational part is negative, we would have written it as $\left(a-\sqrt{b}\right)^2$
In the expansion of $\left(a+\sqrt{b}\right)^2$, we get the terms a^2, b and $2a\sqrt{b}$.
Since the coefficient of irrational term is 2, we will keep the co-efficient of irrational term of the given surd as 2.
Consider the term $4\sqrt{15}$. Here the coefficient of the irrational term is 4, we will retain only 2 and take the remaining 2 under the square root. Hence $4\sqrt{15}$ will become $2\sqrt{60}$. Thus the given surd is $32 + 2\sqrt{60}$
Now, resolve 60 into a pair of factors such that their sum is 32. The pair of factors 30 and 2 satisfies the condition.
$32 + 2\sqrt{60} = (30 + 2) + 2\sqrt{30\text{x}2}$
Hence, the positive square root
$= \sqrt{30}+\sqrt{2} = \sqrt{2}\left(\sqrt{15}+1\right)$.
(Please note that although the square root can be with a positive or a negative sign, when written in the form $\sqrt{32+2\sqrt{60}}$, positive root is implied.)

7.12. Find which of the following two surds is greater: $\sqrt{6}+\sqrt{26}$ and $\sqrt{3}+\sqrt{31}$.

Sol: If we try to take approximate values of both the surds we find that both are more than 7; and we will not able to judge which surd is greater. The comparison can be done by squaring the surds and then comparing them. Now we get

$(\sqrt{6}+\sqrt{26})^2 = 32+2\sqrt{156}$; $(\sqrt{3}+\sqrt{31})^2$

$= 34 + 2\sqrt{93}$

The square of the second surd can be written as $32 + 2 + 2\sqrt{93}$. Since 32 is the common rational part for both the surds, we need to compare $2\sqrt{156}$ and $2 + 2\sqrt{93}$ only.

We know that $\sqrt{156}$ lies between 12 and 13 (since $12^2 = 144$ and $13^2 = 169$)

$\therefore 2\sqrt{156}$ lies between 24 and 26.

Also $\sqrt{93}$ lies between 9 and 10, so $2 + 2\sqrt{93}$ lies between 2 + 2(9 to 10) i.e., 20 to 22.

As 24 to 26 > 20 to 22, $32 + 2\sqrt{156} > 34 + 2\sqrt{93}$

i.e., $\sqrt{6}+\sqrt{26} > \sqrt{3}+\sqrt{31}$.

Concept Review Questions

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If $4^{2x} = 256$, find x.

$\boxed{}$

2. Find the conjugate of the surd $\sqrt{13} - 2$.

(A) $\sqrt{13} + 2$ (B) $\sqrt{13} - 2$

(C) $-(2+\sqrt{13})$ (D) $2 - \sqrt{3}$

3. Simplify : $\dfrac{3}{\sqrt{5}+\sqrt{2}} + \dfrac{1}{3+\sqrt{8}}$.

(A) $3+\sqrt{5}+3\sqrt{2}$ (B) $3+\sqrt{5}-3\sqrt{2}$

(C) $3-\sqrt{5}-3\sqrt{2}$ (D) $-3+\sqrt{5}+3\sqrt{2}$

4. Find the positive square root of $\sqrt{108}-\sqrt{75}$.

(A) $\sqrt{6}$ (B) 3 (C) $\sqrt{3}$ (D) $\sqrt[4]{3}$

5. Find the positive square root of $\sqrt{128} - \sqrt{56}$.

(A) $\sqrt[4]{2}\left(\sqrt{7}-1\right)$ (B) $\sqrt[4]{2}\left(\sqrt{7}+1\right)$

(C) $\sqrt{2}(\sqrt{7}-1)$ (D) $\sqrt{2}(\sqrt{7}+1)$

6. Find the positive square root of $14 + 6\sqrt{5}$.

(A) $3+\sqrt{5}$ (B) $3-\sqrt{5}$

(C) $3+2\sqrt{5}$ (D) $3-2\sqrt{5}$

7. Simplify : $x^{\frac{b+c-a}{a+b+c}} \cdot x^{\frac{a+c-b}{a+b+c}} \cdot x^{\frac{a+b-c}{a+b+c}}$, where a, b and c are positive numbers.

(A) 1 (B) x (C) x^2 (D) x^3

8. $\dfrac{3}{5^{\frac{2}{3}}+10^{\frac{1}{3}}+2^{\frac{2}{3}}}$ = _______.

(A) $5^{\frac{2}{3}}+2^{\frac{2}{3}}$ (B) $5^{\frac{2}{3}}-2^{\frac{1}{3}}$

(C) $5^{\frac{1}{3}}-2^{\frac{1}{3}}$ (D) $5^{\frac{1}{3}}+2^{\frac{1}{3}}$

9. Find the rationalizing factor of $4^{\frac{1}{3}}+3^{\frac{1}{3}}$.

(A) $4^{\frac{2}{3}}-2^{\frac{1}{3}}$ (B) $4^{\frac{2}{3}}+2^{\frac{1}{3}}$

(C) $4^{\frac{2}{3}}+12^{\frac{1}{3}}+3^{\frac{2}{3}}$ (D) $4^{\frac{2}{3}}-12^{\frac{1}{3}}+3^{\frac{2}{3}}$

10. Which of the following is greater?

(a) $\dfrac{2}{\sqrt{27}+\sqrt{11}}$ (b) $\dfrac{1}{\sqrt{11}+\sqrt{3}}$

(A) a (B) b (C) Both are equal

11. Which is greater : 2^{3^3} or 3^{3^2} ?

(A) 2^{3^3} (B) 3^{3^2} (C) Both are equal

12. Find the rationalizing factor of $12 + 4\sqrt{7}$.

(A) $(3-\sqrt{21})$ (B) $4(3+\sqrt{7})$

(C) $4(3-\sqrt{7})$ (D) $\sqrt{3}-\sqrt{7}$

13. Find x if $2^{x^x} = 16$.

$\boxed{}$

14. If $2^p3^q = 432$, and p and q are integers, find p + q.

$\boxed{}$

15. If $2^{\frac{1}{2}}\,2^{\frac{3}{2}}\,2^{\frac{5}{2}}\,2^{\frac{7}{2}}\,2^{\frac{9}{2}}\,2^{\frac{11}{2}} = 8^x$, find x.

(A) 6 (B) 4

(C) 8 (D) 10

Exercise – 7(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Simplify : $\left((64)^{-1/6} \times (81)^{3/4} \times (64)^{2/3}\right)^{1/3}$.

$\boxed{}$

2. Simplify: $\left(\dfrac{x^4\ y^{-5}\ z^2}{x^{-3}\ y\ z^4}\right)^{11} \div \left(\dfrac{x^2\ y\ z^3}{x^{-8}\ y^2\ z^5}\right)^7$.

(A) $\dfrac{x^7}{y^{59}\ z^8}$ (B) $x^7\ y^{59}\ z^8$

(C) $\dfrac{y^{59}}{x^7\ z^8}$ (D) $\dfrac{x^7\ y^{59}}{z^8}$

3. If $(1.761)^x = (0.1761)^y = 10^z$, then find the relationship between x, y and z.

(A) $\dfrac{1}{z} = \dfrac{1}{x} - \dfrac{1}{y}$ (B) $\dfrac{1}{z} = \dfrac{1}{x} + \dfrac{1}{y}$

(C) $\dfrac{1}{y} = \dfrac{1}{z} - \dfrac{1}{x}$ (D) $\dfrac{1}{y} = \dfrac{1}{x} + \dfrac{1}{z}$

4. If $x = 4 + 4^{1/3} + 4^{2/3}$, then find the value of $x^3 - 12x^2 + 36x + 8$.

(A) 24 (B) 32 (C) 36 (D) 44

5. Solve for x : $(2\sqrt{3})^{3x-4} = 20736$

$\boxed{}$

6. Solve for x and y : $x^y = y^x$ and $y = x^2$ where $x > 1$.

(A) x = 0, y = 0 (B) x = 0, y = 2

(C) x = 1, y = 2 (D) x = 2, y = 4

7. Solve for x and y : $2^x - 3^y = 23$, $2^{x-4} + 3^{2y-3} = 5$.

(A) x = 4, y = 3 (B) x = 3, y = 4

(C) x = 2, y = 5 (D) x = 5, y = 2

8. If x and y are integers and $2^x \cdot 3^{2y} = 144$, then what is the value of (x + y)?

$\boxed{}$

9. If p + q + r = 0, then find the value of

$\sqrt[qr]{a^{p^2}} \cdot \sqrt[rp]{a^{q^2}} \cdot \sqrt[pq]{a^{r^2}}$.

(A) 1 (B) a^3

(C) a^{pqr} (D) None of these

10. Simplify : $\dfrac{6}{2\sqrt{3}+\sqrt{6}} - \dfrac{1}{\sqrt{3}-\sqrt{2}} + \dfrac{4}{\sqrt{6}-\sqrt{2}}$.

(A) 1 (B) $\sqrt{3}$

(C) $2\sqrt{6}$ (D) $\sqrt{2}-\sqrt{3}+\sqrt{6}$

11. Write the following surds in ascending order:

$\sqrt{5}$, $\sqrt[4]{7}$, $\sqrt[3]{11}$.

(A) $\sqrt[4]{7}, \sqrt[3]{11}, \sqrt{5}$ (B) $\sqrt[4]{7}, \sqrt{5}, \sqrt[3]{11}$

(C) $\sqrt[3]{11}, \sqrt[4]{7}, \sqrt{5}$ (D) $\sqrt{5}, \sqrt[3]{11}, \sqrt[4]{7}$

12. Find the ascending order of $3^{\frac{1}{2}}, 4^{\frac{1}{3}}$ and $5^{\frac{1}{4}}$.

(A) $5^{\frac{1}{4}}, 3^{\frac{1}{2}}, 4^{\frac{1}{3}}$ (B) $4^{\frac{1}{3}}, 3^{\frac{1}{2}}, 5^{\frac{1}{4}}$

(C) $3^{\frac{1}{2}}, 4^{\frac{1}{3}}, 5^{\frac{1}{4}}$ (D) $5^{\frac{1}{4}}, 4^{\frac{1}{3}}, 3^{\frac{1}{2}}$

13. Given that $5^a = 7^b = 35^c$. Find the relation between a, b and c.

(A) $\dfrac{2}{a}+\dfrac{2}{b}=\dfrac{3}{c}$ (B) $\dfrac{1}{a}+\dfrac{2}{b}=\dfrac{1}{c}$

(C) $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{2}{c}$ (D) $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{c}$

14. Find the value of

$$\frac{\sqrt{9+\sqrt{24}+\sqrt{48}+\sqrt{32}}+\sqrt{9-\sqrt{24}+\sqrt{48}-\sqrt{32}}}{\sqrt{9+\sqrt{24}+\sqrt{48}+\sqrt{32}}-\sqrt{9-\sqrt{24}+\sqrt{48}-\sqrt{32}}}.$$

(A) $\sqrt{\dfrac{3}{2}}+2$ (B) $\dfrac{\sqrt{3}}{2}+2$

(C) $\sqrt{\dfrac{3}{2}}+\dfrac{1}{\sqrt{2}}$ (D) $\sqrt{\dfrac{3}{2}}+\sqrt{2}$

15. Simplify : $\dfrac{\sqrt{3-2\sqrt{2}}}{\sqrt{2}+1} + \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{5-\sqrt{24}}}$

(A) $8+2\sqrt{2}-2\sqrt{6}$ (B) $8-2\sqrt{2}-2\sqrt{6}$

(C) $8-2\sqrt{2}+2\sqrt{6}$ (D) $8+2\sqrt{2}+2\sqrt{6}$

16. If $x = \dfrac{1}{2}$, then find the value of

$$\sqrt{\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}}.$$

(A) $\dfrac{1}{\sqrt{2}}\left(2+\sqrt{6}\right)$ (B) $\dfrac{1}{2}\left(\sqrt{2}+\sqrt{6}\right)$

(C) $\sqrt{2}+1$ (D) $\sqrt{3}+1$

17. If $\dfrac{12}{3+\sqrt{5}-2\sqrt{2}} = x + a\sqrt{2} + b\sqrt{5} + c\sqrt{10}$ and x, a, b and c are rational, then find the value of x + a + b + c.

$\boxed{}$

18. If $x = \sqrt{\dfrac{\sqrt{5}+1}{\sqrt{5}-1}}$, then find the value of $7x^2 - 7x$.

$\boxed{}$

19. If $\sqrt{x}+\sqrt{2-x}=\sqrt{y}+\sqrt{2-y}=\sqrt{z}+\sqrt{2-z}$, and x, y and z lie between 0 and 2, including 0 and 2, then which of the following is true?
(A) x = y, y = z and z = x
(B) x + y = 2, y + z = 2 and z + x = 2
(C) Either (A) or (B)
(D) None of these

20. If $x = \dfrac{\left(\sqrt{\left(\sqrt{3}+\sqrt{2}\right)}+\sqrt{\left(\sqrt{3}-\sqrt{2}\right)}\right)^2}{\sqrt{3}-1}$, then find $\sqrt{x}$.
(A) $\left(\sqrt{3}+1\right)$ (B) $\left(\sqrt{3}-1\right)$
(C) $\left(2+\sqrt{3}\right)$ (D) $\left(2-\sqrt{3}\right)$

21. If $\dfrac{\left(x-\sqrt{3}\right)\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}-\sqrt{8}\right)}$ = 1, then find the value of x.
(A) $5-2\sqrt{3}+\sqrt{6}$ (B) $5+2\sqrt{6}+\sqrt{3}$
(C) $5+\sqrt{3}-2\sqrt{6}$ (D) $5-\sqrt{3}+2\sqrt{6}$

22. If $x = \dfrac{3+\sqrt{20}-\sqrt{45}}{7+\sqrt{320}-\sqrt{125}}$, then find the positive square root of x.
(A) $\sqrt{10}+\sqrt{2}$ (B) $\sqrt{10}-\sqrt{2}$
(C) $\sqrt{5}+2$ (D) $\sqrt{5}-2$

23. One of the values of
$\sqrt{x^2+2y\sqrt{x^2-y^2}}+\sqrt{x^2-2y\sqrt{x^2-y^2}}$ is ______.
(A) $y^2 + x^2$ (B) $x^2 - y^2$
(C) 2y (D) 2x

24. If $\sqrt{\left(4+\sqrt{15}\right)^3}+\sqrt{\left(4-\sqrt{15}\right)^3}=a\sqrt{5}$, then find a.
(A) 5 (B) $7\sqrt{2}$ (C) 7 (D) $10\sqrt{2}$

25. Simplify :
$\left(2^{1/2}\cdot4^{3/4}\cdot8^{5/6}\cdot16^{7/8}\cdot32^{9/10}\cdot64^{11/12}\cdot128^{13/14}\cdot256^{15/16}\right)^{5/16}$.

26. If 3^{2x+2} = 6561 and 2^{3y-1} = 2048, then find the value of $\dfrac{2x+3y}{y-x}$.

27. If $11^{2/p} - 11^{1/p} + 1 = 73$ and $11^{2/q} - 11^{1/q} + 1 = 91$, $11^{1/p-1/q}$ = ______.
(A) $\dfrac{8}{9}$ (B) $\dfrac{9}{10}$ (C) $\dfrac{10}{11}$ (D) $\dfrac{7}{8}$

28. If $\sqrt{11\sqrt{11\sqrt{11......\infty}}}$ = 14641^x, then find x.
(A) 11 (B) 4
(C) 1/4 (D) None of these

29. Arrange the given surds in ascending order:
$\sqrt{7}+\sqrt{17}, \sqrt{8}+\sqrt{14}, \sqrt{10}+\sqrt{14}$ and $\sqrt{3}+\sqrt{22}$.
(A) $\sqrt{8}+\sqrt{14}, \sqrt{7}+\sqrt{17}, \sqrt{10}+\sqrt{14}, \sqrt{3}+\sqrt{72}$
(B) $\sqrt{8}+\sqrt{14}, \sqrt{3}+\sqrt{22}, \sqrt{10}+\sqrt{14}, \sqrt{7}+\sqrt{17}$
(C) $\sqrt{8}+\sqrt{14}, \sqrt{3}+\sqrt{22}, \sqrt{7}+\sqrt{17}, \sqrt{10}+\sqrt{14}$
(D) $\sqrt{3}+\sqrt{22}, \sqrt{8}+\sqrt{14}, \sqrt{7}+\sqrt{17}, \sqrt{10}+\sqrt{14}$

30. Find the value of
$\dfrac{1}{\sqrt{2}+1}+\dfrac{1}{\sqrt{3}+\sqrt{2}}+\dfrac{1}{\sqrt{4}+\sqrt{3}}+..........+\dfrac{1}{\sqrt{100}+\sqrt{99}}$

31. Solve for x if $\left(\sqrt[4]{\dfrac{2}{7}}\right)^{(3x+1)}=\left(\sqrt[3]{\dfrac{49}{4}}\right)^{\frac{5x-37}{2}}$.

32. If $\dfrac{x-y}{\sqrt{x}+\sqrt{y}}+\dfrac{x-y}{\sqrt{x}-\sqrt{y}}$ = 4, then find the value of x.

33. Find the square root of $\dfrac{3}{2}(x+4)+\sqrt{2x^2+19x+35}$
(A) $\dfrac{1}{2}\left(\sqrt{x+7}+\sqrt{2x+5}\right)$
(B) $\dfrac{1}{2}\left(\sqrt{x-7}+\sqrt{2x+5}\right)$
(C) $\dfrac{1}{\sqrt{2}}\left(\sqrt{x+7}+\sqrt{2x+5}\right)$
(D) $\dfrac{1}{\sqrt{2}}\left(\sqrt{x+7}+\sqrt{2x-5}\right)$

34. If $5^x + 4^y = 141$ and $5^{2x-5} + 4^{y+1} = 69$, then find the value of x – y.

35. If xyz = 1, then
$\dfrac{1}{1+y^2+x^{-2}}+\dfrac{1}{1+z^2+y^{-2}}+\dfrac{1}{1+x^2+z^{-2}}$ = ______.
(A) 1
(B) 3
(C) $(x + y + z)^2$
(D) $x^{-2} + y^{-2} + z^{-2}$

Exercise – 7(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. Simplify : $\left(8a^6 \div 64a^{12}\right)^{-2/3} \times \left(216a^9 \div 512a^{-3}\right)^{2/3}$.

(A) $\frac{3}{2}a^6$ (B) $\frac{3}{2}a^{12}$ (C) $\frac{9}{4}a^6$ (D) $\frac{9}{4}a^{12}$

2. Simplify: $\frac{2^{m+2}\ 6^{m-2}\ 10^{2m}\ 15^{m-1}\ 25^{m-n}}{4^{2m}\ 3^{2m-3}\ 5^{5m-2n+5}\ 5^{-8}}$.

(A) 1 (B) $2^m\ 3^{m-1}\ 5$
(C) $2\ 3^m\ 5^{2m-2n}$ (D) 25

3. If x is a positive integer, the greatest integer by which $5^x + 5^{x+1} + 5^{x+2}$ would be divisible is _______.
(A) 31 (B) 155
(C) 225 (D) None of these

4. Simplify : $\sqrt{\sqrt[3]{\sqrt[4]{\sqrt[6]{25^{72}}}}}$.

$\boxed{}$

Moderate

5. If $2^x = 7^y = 14^z$, then find the value of z in terms of x and y.

(A) $\frac{x+y}{x-y}$ (B) $\frac{xy}{x+y}$
(C) x + y + xy (D) xy – (x + y)

6. If $\frac{5^x}{5^y}$ = 125, and $\frac{2^x}{4^y}$ = 2, then find the value of $\left(\frac{3^{2x}}{27^y} + 4\right)$.

$\boxed{}$

7. Find the value of $\frac{1+4x}{1+\sqrt{1+4x}} + \frac{1+4x}{1-\sqrt{1+4x}}$, when x = 1/4.

$\boxed{}$

8. If $x = \frac{1}{2}\left(\sqrt{5} + \frac{1}{\sqrt{5}}\right)$, then find the value of $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$.

(A) –1 (B) 1 (C) –2 (D) 2

9. If $\frac{\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}} + \sqrt{3-2\sqrt{2}}} = \sqrt{\frac{x}{32}}$, then find the value of x.
(A) 2 (B) 4 (C) 8 (D) 16

10. Simplify : $\frac{10\sqrt{2}}{\sqrt{18} - \sqrt{3+\sqrt{5}}} + \sqrt{30 - 10\sqrt{5}}$.

$\boxed{}$

11. If $32^a = 25b$ and $2^a = 50b$, then find the value of a.
(A) –1 (B) –1/2 (C) –1/4 (D) 2

12. Simplify : $\frac{2+\sqrt{2}+\sqrt{6}}{\sqrt{3+\sqrt{2}+\sqrt{3}+\sqrt{6}}}$.

(A) 1 (B) 2 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

13. Simplify : $\frac{1}{\sqrt{\sqrt{4+2\sqrt{3}} + \sqrt{3}\left(3+2\sqrt{3}\right)}}$.

(A) $2-\sqrt{3}$ (B) $2+\sqrt{3}$
(C) $3-\sqrt{2}$ (D) $3+\sqrt{2}$

14. If $\sqrt{2}$ = 1.414 and $\sqrt{3}$ = 1.732, find the value of $\frac{\left(1-\sqrt{2}\right)}{3+2\sqrt{2}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$.
(A) 1.076 (B) 0.673
(C) 0.198 (D) None of these

15. $\left(24-16\sqrt{2}\right)^{\frac{1}{6}} \left(32+16\sqrt{2}\right)^{\frac{1}{3}}$ = _______.

(A) 2 (B) 4 (C) $(2)^{\frac{1}{3}}$ (D) $(4)^{\frac{1}{3}}$

16. $\left[(512)^{\frac{2}{9}} \times (2187)^{\frac{3}{7}} \times (3375)^{\frac{1}{3}} \times 5\right]^{\frac{1}{2}}$ = _______.
(A) 30 (B) 90
(C) $30\sqrt{5}$ (D) $30\sqrt{3}$

17. Simplify : $\frac{2^a.2^{a+2}.3^{a+2}.2^{6a}.3^{4a+4b}}{3^{a-b}.3^{2a+b-2}.2^{4a+2b-4}.2^6.3^2}$.

(A) $\frac{(4)^{2a-b}}{(9)^{(a+2b+1)}}$ (B) $\frac{4}{9}$

(C) $\frac{2^{4a-2b}}{9^{a+b}}$ (D) $4^{2a-b} \times 9^{a+2b+1}$

18. If $(2.56)^a = (0.00256)^b = 10^c$, then find the relationship between a, b and c.
(A) $\frac{2}{c} = \frac{1}{a} - \frac{1}{b}$ (B) $\frac{3}{c} = \frac{1}{b} - \frac{1}{a}$

(C) $\frac{1}{a} = \frac{3}{c} + \frac{1}{b}$ (D) $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

19. Write the following terms in descending order: $\sqrt[3]{4}, \sqrt[4]{7}$ and $\sqrt[6]{17}$.

(A) $\sqrt[3]{4}, \sqrt[6]{17}$ and $\sqrt[4]{7}$ (B) $\sqrt[6]{17}, \sqrt[3]{4}$ and $\sqrt[4]{7}$
(C) $\sqrt[4]{7}, \sqrt[6]{17}$ and $\sqrt[3]{4}$ (D) $\sqrt[4]{7}, \sqrt[3]{4}$ and $\sqrt[6]{17}$

20. If $\frac{8}{2+2\sqrt{2}+2\sqrt{3}} = x + y\sqrt{2} + z\sqrt{6}$, then find the value of x + y + z.

$\boxed{}$

21. Simplify : $\frac{\sqrt{10}}{\sqrt{2-\sqrt{3}}} + \sqrt{20-10\sqrt{3}}$.

(A) $2\sqrt{5}$ (B) 0
(C) $2\sqrt{15}$ (D) $\sqrt{15}$

22. If x = $\sqrt[3]{4+\sqrt{8}} + \sqrt[3]{4-\sqrt{8}}$, then $x^3 - 6x =$ $\boxed{}$

23. Simplify : $\frac{\left(17+2\sqrt{72}\right)^{\frac{3}{2}} + \left(17-2\sqrt{72}\right)^{\frac{3}{2}}}{\left(6+4\sqrt{2}\right)^{\frac{3}{2}} + \left(6-4\sqrt{2}\right)^{\frac{3}{2}}}$.

(A) $\frac{99}{20}$ (B) $\frac{5}{4}$ (C) $\frac{35}{23}$ (D) $35\sqrt{8}$

24. **(a)** Find the ascending order of 2^{300}, 3^{200} and 6^{100}.
(A) $6^{100}, 3^{200}, 2^{300}$ (B) $6^{100}, 2^{300}, 3^{200}$
(C) $2^{300}, 3^{200}, 6^{100}$ (D) $3^{200}, 2^{300}, 6^{100}$

(b) The smallest of $5^{1/6}$, $3^{1/4}$, $4^{1/3}$, $2^{1/2}$ is _______.
(A) $5^{1/6}$ (B) $3^{1/4}$
(C) $4^{1/3}$ (D) $2^{1/2}$

25. If x = 3 +2$\sqrt{2}$, then the value of $x^2 - \frac{1}{x^2}$ is _______.

(A) $24\sqrt{2}$ (B) $6\sqrt{6}$
(C) $18\sqrt{3}$ (D) 34

26. If p is a rational number, $\sqrt{q}$, $\sqrt{r}$, and $\sqrt{s}$ are surds, and $\sqrt{p+\sqrt{q}+\sqrt{r}+\sqrt{s}} = \sqrt{a}+\sqrt{b}+\sqrt{c}$, then a + b + c = _______.
(A) p (B) q
(C) pqrs (D) p + q + r + s

27. $\sqrt{15+2\sqrt{35}-2\sqrt{15}-2\sqrt{21}}$ = _______.

(A) $\sqrt{7}-\sqrt{5}-\sqrt{3}$ (B) $\sqrt{7}-\sqrt{5}+\sqrt{3}$
(C) $\sqrt{7}+\sqrt{5}-\sqrt{3}$ (D) $\sqrt{5}+\sqrt{3}-\sqrt{7}$

28. If $(0.abc)^p = (a.bc)^q = (ab.c)^r = abc$, then which of the following is true?
(A) p + 1 = q + r (B) p + q + r = 1
(C) $\frac{1}{p}+\frac{1}{q}+\frac{1}{r} = 1$ (D) $\frac{1}{p}+1 = \frac{1}{q}+\frac{1}{r}$

29. If $\left(6+\sqrt{35}\right)^{x/2} + \left(6-\sqrt{35}\right)^{x/2}$ = 12, the number of solutions for x is ________.
(A) 0 (B) 2 (C) 1 (D) 4

Difficult / Very Difficult

30. If x = $6+\sqrt{7}$, then find the value of $x^3 - 18x^2 + 101x - 132$.
(A) 42 (B) 68 (C) 77 (D) 84

31. If x = $2+\sqrt{3}+\sqrt{5}$, then find the value of $x^4 - 8x^3 + 8x^2 + 32x$.
(A) 36 (B) 44 (C) 48 (D) 70

32. Simplify : $\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}} + \frac{2-\sqrt{3}}{\sqrt{2}+\sqrt{2-\sqrt{3}}}$.

(A) 1 (B) $\frac{1}{\sqrt{2}}$
(C) $\sqrt{2}$ (D) $\frac{\sqrt{2}\left(3\sqrt{3}-1\right)}{\sqrt{3}\left(\sqrt{3}+1\right)}$

33. Simplify : $\frac{\sqrt{8}+\sqrt{6}-\sqrt{10}}{\sqrt{6+2\sqrt{3}-2\sqrt{5}-\sqrt{15}}}$.

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

34. If x = $\frac{13}{\sqrt{19-8\sqrt{3}}}$, then the value of $x^3 - 12x^2 + 45x + 50 =$ $\boxed{}$

35. The value of $\frac{13}{\sqrt{19-8\sqrt{3}}} - \frac{23}{\sqrt{31+12\sqrt{3}}} - \frac{24}{\sqrt{48+24\sqrt{3}}}$ is ______.

(A) $12 - 4\sqrt{3}$ (B) 0
(C) $8+2\sqrt{3}$ (D) 8

Data Sufficiency

Directions for questions 36 to 40: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.
Mark (B) if the question can be answered using either statement alone.
Mark (C) if the question can be answered using I and II together but not using I or II alone
Mark (D) if the question cannot be answered even using I and II together.

36. If a, b, c are all positive integers and a is a factor of c, then is x an integer?

I. $x = \left(\frac{a^3\ b^6\ c^9}{a^4\ b^8\ c^{12}}\right)^9 \div \left(\frac{a^2\ b^4\ c^6}{a^3\ b^7\ c^5}\right)^8$

II. $x = \left(\frac{a^5\ b^8\ c^{10}}{a^2\ b^6\ c^4}\right)^5 \div \left(\frac{a\ b^2\ c^2}{a^5\ b^7\ c^5}\right)^9$

37. If x, y are positive integers, what is the value of x + y?

I. $(4)^{x+2} + (729)^{y-4} = 1025$

II. $3^{2x}\, 2^{y} = 324$

38. If ab ≠ 0, what is the value of $\dfrac{(a^3)^2\, b^3}{a^2}$?

I. $(ab)^3 = \dfrac{3}{4a}$

II. $b = \dfrac{1}{2a^{\frac{4}{3}}}$

39. It is given that A, B are surds and $8(5+\sqrt{18})AB = (6+2\sqrt{3})$. What is the value of A?

I. $28AB = 15 - 3\sqrt{6} - 9\sqrt{2} + 5\sqrt{3}$

II. $\dfrac{1}{A} + \dfrac{1}{B} = 7 + 3\sqrt{2} - \dfrac{2}{\sqrt{3}}$

40. If x is a positive integer and x < 100, then is $\sqrt[x]{\sqrt[4]{\sqrt[3]{\sqrt{512^{216}}}}}$ an integer?

I. x is a multiple of 10

II. $x = 3^n$ where n is an integer.

Key
Concept Review Questions

1. 2	4. D	7. B	10. A	13. 2
2. C	5. A	8. C	11. A	14. 7
3. B	6. A	9. D	12. C	15. A

Exercise – 7(a)

1. 6	6. D	11. A	16. B	21. C	26. 18	31. 5
2. A	7. D	12. D	17. 2	22. D	27. B	32. 4
3. A	8. 5	13. D	18. 7	23. C	28. C	33. C
4. D	9. B	14. D	19. C	24. B	29. D	34. 1
5. 4	10. B	15. C	20. A	25. 1024	30. 9	35. A

Exercise – 7(b)

1. D	10. 10	19. C	27. C	36. A
2. D	11. C	20. 2	28. D	37. B
3. B	12. B	21. C	29. B	38. B
4. 5	13. A	22. 8	30. A	39. D
5. B	14. C	23. A	31. B	40. B
6. 85	15. B	24. (a) B	32. D	
7. –4	16. B	(b) A	33. C	
8. D	17. D	25. A	34. 102	
9. D	18. C	26. A	35. B	