CHAPTER – 9

PERMUTATIONS AND COMBINATIONS

Permutations and Combinations is one of the important areas in many exams because of two reasons. The first is that solving questions in this area is a measure of students' reasoning ability. Secondly, solving problems in areas like Probability requires thorough knowledge of Permutations and Combinations.

Before discussing Permutations and Combinations, let us look at what is called as the "fundamental rule"

“If one operation can be performed in ‘m' ways and (when, it has been performed in any one of these ways), a second operation then can be performed in ‘n' ways, the number of ways of performing the two operations will be m x n".

This can be extended to any number of operations.

If there are three cities A, B and C such that there are 3 roads connecting A and B and 4 roads connecting B and C, then the number of ways one can travel from A to C is 3 x 4, i.e., 12.

This is a very important principle and we will be using it extensively in Permutations and Combinations. Because we use it very extensively, we do not explicitly state every time that the result is obtained by the fundamental rule but directly write down the result.

PERMUTATIONS

Each of the arrangements which can be made by taking some or all of a number of items is called a Permutation. Permutation implies "arrangement" or that "order of the items” is important.

The permutations of three items a, b and c taken two at a time are ab, ba, ac, ca, cb and bc. Since the order in which the items are taken is important, ab and ba are counted as two different permutations. The words "permutation" and "arrangement" are synonymous and can be used interchangeably.

The number of permutations of n things taking r at time is denoted by ${}^nP_r$ (and read as "nPr")

COMBINATIONS

Each of the groups or selections which can be made by taking some or all of a number of items is called a Combination. In combinations, the order in which the items are taken is not considered as long as the specific things are included.

The combination of three items a, b and c taken two at a time are ab, bc and ca. Here, ab and ba are not considered separately because the order in which a and b are taken is not important but it is only required that a combination including a and b is what is to be counted. The words "combination" and "selection" are synonymous.

The number of combinations of n things taking r at time is denoted by ${}^nC_r$ (and read as "nCr")

When a problem is read, it should first be clear to you as to whether it is a permutation or combination that is being discussed. Some times the problem specifically states whether it is the number of permutations (or arrangements) or the number of combinations (or selections) that you should find out. The questions can be as follows:

For permutations, "Find the number of permutations that can be made ....." OR "Find the number of arrangements that can be made....." OR "Find the number of ways in which you can arrange....."

For combinations,
"Find the number of combinations that can be made ....."
OR "Find the number of selections that can be made....."
OR "Find the number of ways in which you can select....."

Some times, the problem may not explicitly state whether what you have to find out is a permutation or a combination but the nature of what is to be found out will decide whether it is the number of permutations or the number of combinations that you have to find out. Let us look at the following two examples to clarify this.

"How many four digit numbers can be formed using the digits 1, 2, 3 and 4 using each digit once?"
Here, since we are talking of numbers, the order of the digits matters and hence what we have to find out is permutations.

"Out of a group of five friends that I have, I have to invite two for dinner. In how many different ways can I do this?"

Here, if the five friends are A, B, C, D and E, whether the two friends that I call for dinner on a particular day are A and B **or** B and A, it does not make any difference, i.e., here the order of the "items" does not play any role and hence it is the number of combinations that we have to find out.

Now we will find out the number of permutations and combinations that can be made from a group of given items.

Initially, we impose two constraints (conditions) while looking at the number of permutations. They are

- all the n items are distinct or dissimilar (or no two items are of the same type)
- each item is used at most once (i.e., no item is repeated in any arrangement)

Number of linear permutations of 'n' dissimilar items taken 'r' at a time without repetition (${}^nP_r$)

Consider r boxes each of which can hold one item. When all the r boxes are filled, what we have is an arrangement of r items taken from the given n items. So, each time we fill up the r boxes with items taken from the given n items, we have an arrangement of r items taken from the given n items without repetition. Hence the number of ways in which we can fill up the r boxes by taking things from the given n things is equal to the number of permutations of n things taking r at a time.

Boxes ....

1 2 3 4 r

The first box can be filled in n ways (because any one of the n items can be used to fill this box). Having filled the first box, to fill the second box we now have only (n – 1) items; any one of these items can be used to fill the second box and hence the second box can be filled in (n – 1) ways; similarly, the third box in (n – 2) ways and so on the r^{th} box can be filled in {n – (r – 1)} ways, i.e. [n – r + 1] ways. Hence, from the Fundamental Rule, all the r boxes together can be filled up in
$n \cdot (n-1) \cdot (n-2)..................(n-r+1)$ ways.

So, ${}^nP_r = n \cdot (n-1) \cdot (n-2).............(n-r+1)$

This can be simplified by multiplying and dividing the right hand side by (n – r) (n – r – 1).........3.2.1 giving us
${}^nP_r = n(n-1)(n-2) .... [n-(r-1)]$

$$= \frac{(n-1)(n-2).....[n-(r-1)].(n-r).......3.2.1}{(n-r).....................3.2.1}$$

$$= \frac{n!}{(n-r)!}$$

The number of permutations of n distinct items taking r items at a time is

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

If we take n items at a time, then we get ${}^nP_n$. From a discussion similar to that we had for filling the r boxes above, we can find that ${}^nP_n$ is equal to n!

The first box can be filled in n ways, the second one in (n – 1) ways, the third one in (n – 2) ways and so on, then the n^{th} box in 1 way; hence, all the n boxes can be filled in n(n – 1) (n – 2) ....3.2.1 ways, i.e., n! ways. Hence,

$$\boxed{{}^nP_n = n!}$$

But if we substitute r = n in the formula for ${}^nP_r$, then we get ${}^nP_n = \frac{n!}{0!}$; since we already found that ${}^nP_n = n!$, we can conclude that 0! = 1

Number of combinations of n dissimilar things taken r at a time

Let the number of combinations ${}^nC_r$ be x. Consider one of these x combinations. Since this is a combination, the order of the r items is not important. If we now impose the condition that order is required for these r items, we can get r! arrangements from this one combination. So each combination can give rise to r! permutations. x combinations will thus give rise to x . r! permutations. But since these are all permutations of n things taken r at a time, this must be equal to ${}^nP_r$. So,

$$x.r! = {}^nP_r = \frac{n!}{(n-r)!} \Rightarrow {}^nC_r = \frac{n!}{r!.(n-r)!}$$

The number of combinations of n dissimilar things taken all at a time is 1.

Out of n things lying on a table, if we select r things and remove them from the table, we are left with (n-r) things on the table - that is, whenever r things are selected out of n things, we automatically have another selection of the (n – r) things. Hence, the number of ways of making combinations taking r out of n things is the same as selecting (n – r) things out of n given things, i.e.,

$$\mathbf{{}^nC_r = {}^nC_{n-r}}$$

When we looked at ${}^nP_r$, we imposed two constraints which we will now release one by one and see how to find out the number of permutations.

Number of arrangements of n items of which p are of one type, q are of a second type and the rest are distinct

When the items are all not distinct, then we **cannot** talk of a general formula for $\mathbf{{}^nP_r}$ for any r but we can talk of only ${}^nP_n$ (which is given below). If we want to find out ${}^nP_r$ for a specific value of r in a given problem, we have to work on a case to case basis (this has been explained in one of the solved examples).

The number of ways in which n things may be arranged taking them all at a time, when p of the things are exactly alike of one kind, q of them exactly alike of another kind, r of them exactly alike of a third kind, and the rest all distinct is

$$\frac{n!}{p!\,q!\,r!}.$$

Number of arrangements of n distinct items where each item can be used any number of times (i.e., repetition allowed)

You are advised to apply the basic reasoning given while deriving the formula for ${}^nP_r$ to arrive at this result also. The first box can be filled up in n ways; the second box can be filled in again inn ways (even though the first box is filled with one item, the same item can be used for filling the second box also because repetition is allowed); the third box can also be filled in n ways and so on ... the r^{th} box can be filled in n ways. Now all the r boxes together can be filled in {n.n.n.n ... r times} ways, i.e., $\mathbf{n^r}$ ways.

The number of permutations of n things, taken r at a time when each item may be repeated once, twice, ... up to r times in any arrangement is $\mathbf{n^r}$

What is important is not this formula by itself but the reasoning involved. So, even while solving problems of this type, you will be better off if you go from the basic reasoning and not just apply this formula.

Total number of combinations:

Out of n given things, the number of ways of selecting **one or more** things is where we can select 1 or 2 or 3 ... and so on n things at a time; hence the number of ways is ${}^nC_1 + {}^nC_2 + {}^nC_3 + ... {}^nC_n$

This is called "the total number of combinations" and is equal to $2^n - 1$ where n is the number of things.
The same can be reasoned out in the following manner also.

There are n items to select from. Let each of these be represented by a box.

	1	2	3	4		n
No. of ways of dealing with the boxes	2	2	2	2	....	2

The first box can be dealt with in two ways. In any combination that we consider, this box is **either** included **or** not included. These are the two ways of dealing with the first box. Similarly, the second box can be dealt with in two ways, the third one in two ways and so on, the n th box in two ways. By the Fundamental Rule, the number of ways of dealing with all the boxes together in $2 \cdot 2 \cdot 2 \cdot$ .... n times ways, i.e., in 2^n ways. But out of these, there is one combination where we "do not include the first box, do not include the second box, do not include the third box and so on, do not include the n th box." That means, no box is included. But this is not allowed because we have to select **one or more** of the items (i.e., at least one item). Hence this combination of no box being included is to be subtracted from the 2^n ways to give the result of

Number of ways of selecting one or more items from n given items is $2^n - 1$

Dividing given items into groups:

Dividing (p + q) items into two groups of p and q items respectively

Out of (p + q) items, if we select p items (which can be done in ${}^{p+q}C_p$ ways), then we will be left with q items, i.e., we have two groups of p and q items respectively. So, the number of ways of dividing (p + q) items into two groups of p and q items respectively is equal to ${}^{p+q}C_p$ which is equal to $\frac{(p+q)!}{p! \cdot q!}$

The number of ways of dividing (p + q) items into two groups of p and q items respectively is $\frac{(p+q)!}{p! \cdot q!}$

If p = q, i.e., if we have to divide the given items into two EQUAL groups, then two cases arise

(i) when the two groups have distinct identity and

(ii) when the two groups do not have distinct identity.

In the first case, we just have to substitute p = q in the above formula which then becomes

The number of ways of dividing 2p items into two equal groups of p each is $\frac{(2p)!}{(p!)^2}$ where the two groups have distinct identity.

In the second case, where the two groups do not have distinct identity, we have to divide the above result by 2!, i.e., it then becomes

The number of ways of dividing 2p items into two equal groups of p each is $\frac{(2p)!}{2!(p!)^2}$ where the two groups do not have distinct identity.

Dividing (p + q + r) items into three groups consisting of p, q and r items respectively

The number of ways in which (p + q + r) things can be divided into three groups containing p, q and r things respectively is $\frac{(p+q+r)!}{p!q!r!}$

If p = q = r, i.e., if we have to divide the given items into three EQUAL groups, then we have two cases where the three groups are distinct and where the groups are not distinct.

When the three groups are distinct, the number of ways is $\frac{(3p)!}{(p!)^3}$

When the three groups are not distinct, then the number of ways is $\frac{(3p)!}{3!\,(p!)^3}$

Circular Permutations:

When n distinct things are arranged in a straight line taking all the n items, we get n! permutations. However, if these n items are arranged in a circular manner, then the number of arrangements will not be n! but it will be less than that. This is because in a straight line manner, if we have an arrangement ABCDE and if we move every item one place to the right (in cyclic order), the new arrangement that we get EABCD is not the same as ABCDE and this also is counted in the n! permutations that we talked of. However, if we have an arrangement ABCDE in a circular fashion, by shifting every item by one place in the clockwise direction, we still get the same arrangement ABCDE. So, if we now take n! as the number of permutations, we will be counting the same arrangement more than once.

The number of arrangements in circular fashion can be found out by first fixing the position of one item. Then the remaining (n – 1) items can be arranged in (n – 1)! ways. Now even if we move these (n – 1) items by one place in the clockwise direction, then the arrangement that we get will not be the same as the initial arrangement because one item is fixed and it does not move.

Hence, the number of ways in which n distinct things can be arranged in a circular arrangement is (n – 1)!

If we take the case of five persons A, B, C, D and E sitting around a table, then the two arrangements ABCDE (in clockwise direction) and AEDCB (the same order but in anticlockwise direction) will be different and distinct. Here we say that the clockwise and anticlockwise arrangements are different. However, if we consider the circular arrangement of a necklace made of five precious stones A, B, C, D and E, the two arrangements talked of above will be the same because we take one arrangement and turn the necklace around (front to back), then we get the other arrangement. Here, we say that there is no difference between the clockwise and anticlockwise arrangements. In this case the number of arrangements will be half of what it is in the case where the clockwise and anticlockwise arrangements are different.

> The number of **circular arrangements** of **n distinct items** is
> **(n – 1)!** if there is **DIFFERENCE** between clockwise and anticlockwise arrangements and
> **(n – 1)!/2** if there is **NO DIFFERENCE** between clockwise and anticlockwise arrangements

Sum of all numbers formed from given digits:

If n distinct digits are used to make all the possible n-digit numbers, we get n! numbers. We now want to find out the sum if all these n! numbers are added. Let us take an example and understand how it is to be done and then look it as a formula.

To find the sum of all the four digit numbers formed using the digits 2, 3, 4 and 5 without repetition:

We can form a total of 4! or 24 numbers. When we add all these numbers, let us look at the contribution of the digit 2 to the sum.

When 2 occurs in the thousands place in a particular number, its contribution to the total will be 2000. The number of numbers that can be formed with 2 in the thousands place is 3!, i.e., 6 numbers. Hence, when 2 is in the thousands place, its contribution to the sum is 3! x 2000.

Similarly, when 2 occurs in the hundreds place in a particular number, its contribution to the total will be 200 and since there are 3! numbers with 2 in the hundreds place, the contribution 2 makes to the sum when it comes in the hundreds place is $3! \times 200$.

Similarly, when 2 occurs in the tens and units place respectively, its contribution to the sum is $3! \times 20$ and $3! \times 2$ respectively. Thus the total contribution of 2 to the sum is $3! \times 2000 + 3! \times 200 + 3! \times 20 + 3! \times 2$, i.e., $3! \times 2222$. This takes care of the digit 2 completely.

In a similar manner, the contribution of 3, 4 and 5 to the sum will respectively be $3! \times 3333$, $3! \times 4444$ and $3! \times 5555$ respectively.

The sum can now be obtained by adding the contributions of these four digits. Hence the sum of the numbers formed by using the four digits is $3! \times (2222 + + 3333 + 4444 + 5555)$, i.e.,
$3! \times (2 + 3 + 4 + 5) \times 1111$

We can now generalize the above as

> If all the possible n-digit numbers using n distinct digits are formed, the sum of all the numbers so formed is equal to **(n-1)!** $\times$ **{sum of the n digits}** $\times$ **{11111 ....} n times**

Rank of a word:

Finding the rank of a given word is basically finding out the position of the word when all possible words have been formed using all the letters of this word exactly once and arranged in alphabetical order as in the case of dictionary. Let us understand this by taking an example

Let us look at the word “POINT”. The letters involved here, when taken in alphabetical order are I, N, O, P, T.

To arrive at the word “POINT”, initially we have to go through the words that begin with I, then all those that begin with N, those that begin with O which are 4! in each case. Then we have words that begin with PI, PN which are 3! in each case. Then we arrive at the word POINT.

There are $3 \times 4! + 2 \times 3! = 84$ words that precede the word POINT i.e., POINT is the 85th word. Hence rank of ‘POINT’ is 85.

The number of diagonals in an n-sided regular polygon

An n-sided regular polygon has n vertices. Joining any two vertices we get a line of the polygon which are $^nC_2$ in number. Of these $^nC_2$ lines, n of them are sides. Hence diagonals are

$$^nC_2 - n = \frac{n(n-3)}{2}$$

Number of integral solution of the equation
$x_1 + x_2 + \ldots\ldots + x_n = s$

Consider the equation

$x_1 + x_2 + x_3 = 10$
If we consider all possible integral solutions of this equation, there are infinitely many. But the number of positive (or non-negative) integral solutions is finite.

We would like the number of positive integral solutions of this equation, i.e., values of (x_1, x_2, x_3) such that each $x_i > 0$.

We imagine 10 identical objects arranged on a line. There are 9 gaps between these 10 objects. If we choose any two of these gaps, we are effectively splitting the 10 identical objects into 3 parts of distinct identity. Conversely, every split of these 10 objects corresponds to a selection of 2 gaps out of the 9 gaps.

Therefore, the number of positive integral solutions is $^9C_2$. In general, if $x_1 + x_2 + \ldots\ldots + x_n = s$ where $s \geq n$, the number of positive integral solutions is $^{s-1}C_{n-1}$.

If we need the number of non negative integral solutions, we proceed as follows. Let $a_1, a_2, \ldots\ldots$ be a non-negative integral solution. Than $a_1 + 1, a_2 + 1, \ldots\ldots a_n + 1$ is a positive integral solution of the equation $x_1 + x_2 + \ldots. + x_n = s + n$. Therefore, the number of non-negative integral solutions of the given equation is equal to the number of positive integral solutions of $x_1 + x_2 + \ldots\ldots + x_n = s + n$, which is $^{s+n-1}C_{n-1}$.

For $x_1 + x_2 + x_3 + \text{---------} + x_n = s$ where $s \geq 0$, the number of **positive integral solutions** (when $s \geq n$) is $^{s-1}C_{n-1}$ and the number of **non-negative integral solutions** is $^{n+s-1}C_{n-1}$

Some additional points:

1. Suppose there are n letters and n corresponding addressed envelopes. The numbers of ways of placing these letters into the envelopes such that no letter is placed in its corresponding envelope is often referred as derangements. The number of derangements of n objects is given by

$$D(n) = n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+....+(-1)^n\frac{1}{n!}\right]$$

For example, when n = 3, the number of derangements is

$D(3) = 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right]=2$ and when n = 4,

$D(4) = 4!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right]=9$

2. The total number of ways in which a selection can be made by taking some or all out of p + q + r + ... things where p are alike of one kind, q alike of a second kind, r alike of a third kind and so on is
$[\{(p + 1)(q + 1)(r + 1) ....\} – 1]$.

3. $^{n+1}C_r = {}^nC_r + {}^nC_{r-1}$ and $^nP_r = r.{}^{n-1}P_{r-1} + {}^{n-1}P_r$

Examples

9.01. Out of 8 persons in a group, find the number of ways of selecting 3 persons and also the number of ways of arranging these 3 selected persons in a row.

Sol. The number of ways of selecting 3 persons from 8 persons is

$$^8C_3 = \frac{8!}{3!\ 5!} = 56 \quad \left[{}^nC_r = \frac{n!}{(n-r)!\ r!}\right]$$

The number of ways of arranging 3 persons taken from 8 persons is

$$^8P_3 = \frac{8!}{5!} = 336 \quad \left[{}^nP_r = \frac{n!}{(n-r)\,!}\right]$$

Note:

$^8C_3$ can be calculated as $\frac{8\times7\times6}{3\times2\times1}$ while $^8P_3$ can be calculated as $8\times7\times6$.

9.02. There are 6 distinct letters of English alphabet and 4 distinct digits. All possible 6 character alpha – numero codes are generated using any 4 letters of the alphabet and any 2 available digits. If in any given code, the characters are all distinct, then what is the maximum number of such codes that can be generated?

Sol. We are looking for all possible codes that can be generated using 4 of the 6 letters of the given alphabet and 2 of the 4 given digits. Initially we need to look at the total number of such choices that can be made which are $^6C_4 \times {}^4C_2$. Having selected the 6 characters, they can be arranged among themselves in 6! ways. Hence the total number of codes that can be generated is $^6C_4 \times {}^4C_2 \times 6! = 64800$

9.03. In a cricket tournament, each participating team plays once against every other team and in all 36 matches are played. Find the number of teams that participated in the tournament.

Sol. Let us assume that ‘n’ teams participated in the tournament. As exactly 2 teams are required to play a match, the total number of matches that are played in the tournament are $^nC_2$.

Given that $^nC_2 = 36$ i.e., $\frac{n(n-1)}{2} = 36$

We get $n(n – 1) = 9 \times 8$ which gives n = 9.
Hence 9 teams participated in the tournament.

Directions for examples 9.04 to 9.09:

The letters of the word ADROIT are permuted in all possible ways to form 6! different words. In each of the examples 9.04 to 9.10 certain conditions are given. We will find out the number of Permutations subject to these conditions.

9.04. How many of these words begin with T?

Sol. The word ADROIT has got 6 letters. As we want to consider words that begin with T, we fix T in the first position. Now the other 5 positions can be filled with the remaining 5 letters in 5! = 120 ways. Hence, 120 words begin with T.

9.05. How many words begin with A, but do not end in T?

Sol. The first position can be filled in only one way (must be A). As the words cannot end in T, the last position can be filled with any of the other 4 letters i.e., D, R, O, I. Having taken care of the first and the last positions, the other 4 positions can be filled with the remaining 4 letters in 4! ways.
Hence total possibilities are $4 \times 4! = 96$

9.06. How many words can be formed which either begin with T or end in A?

Sol. There are 5! words that begin with T and 5! words that end in A, while 4! words begin with T and end in A. Hence of the 5! + 5! words that either begin with T or end in A, we exclude the 4! words, which begin with T and also end in A. Hence the required number of words = $5! + 5! – 4! = 2 \times 5! – 4!$ = 240 – 24 = 216.

9.07. How many words can be formed which neither begin with T nor end in A?

Sol. We will consider the following two cases

Case1: Words that begin with A.
Since A is in the first place, it takes care of both the conditions given (that T should not be in the first place and A should not be in the last place). Now, there are 5! words that begin with A.
Case 2: Words that do not begin with A. Here we have to ensure that T does not come in the first place. So the first place can be filled up in 4 ways (using any letter except A and T). Out of the remaining 5 letters A cannot go into the last place. Hence the last place can be filled in 4 ways. The other 4 places (2^{nd} to 5^{th}) can now be filled in 4! ways. Hence, the number of words are $4 \times 4 \times 4!$. Combining both the cases, the total number of words are $5! + 16 \times 4! = 504$

Alternative method:

From example 9.06 we know that 216 words either begin with T or end in A. If we exclude these from the total number of words which are 6! = 720, then we get the words that neither begin with T nor end in A. Hence, 720 – 216 = 504, words neither begin with T nor end in A.

9.08. How many words can be formed so that the vowels occupy the even places?

Sol. Out of a total of 6 places, there are 3 even places and the word ADROIT has 3 vowels. The 3 vowels can be arranged in the 3 even places in 3! ways and the consonants which are 3 can be arranged in the remaining 3 places in 3! ways. Hence, the total number of words in which vowels occupy even places is $3! \times 3! = 36$.

9.09. How many words can be formed such that the vowels are always together?

Sol. As the vowels have to be together, the 3 vowels can be treated as one unit. Now the 3 consonants and this unit, can be arranged in 4! ways. Further the three vowels can be arranged among themselves in 3! ways. Hence, the total number of words in which the vowels are together is $4! \times 3! = 144$.

9.10. If the letters of the word 'NUMBERS' are permuted in all possible ways, then in how many of these permutations, are the vowels never together i.e., vowels are separated?

Sol. The word NUMBERS has 7 distinct letters of which 2 are vowels. The word can be permuted in 7! ways, of which $6! \times 2!$ permutations have the vowels together. Hence $7! – 2!\,6! = 7.6! – 2.6! = 5.6! = 3600$ permutations will not have the vowels together.
(Indirect counting is done here).

9.11. In how many ways can the letters of the word MANIFOLD be arranged so that the vowels are separated?

Sol. The word MANIFOLD has 8 distinct letters of which 3 are vowels and 5 are consonants. The vowels have to be separated is same as saying no two vowels are together. Hence between any two vowels there is at least one consonant present which acts as a separator. We first arrange the 5 consonants in 5! ways and then the 3 vowels can be arranged in the 6 possible places as shown below
('C' represents a consonant and - represents a possible position for vowels)

-- C -- C -- C -- C -- C --
The vowels can be arranged in $^6P_3$ ways. Hence the total number of words is $5! \times {}^6P_3 = 14400$.

Note:
Problems 9.10 and 9.11 both require the arrangements where vowels are separated. But the approach is different. Though the approach taken in 9.11 is applicable for 9.10, the reverse is not true, as removing the arrangements where vowels are together from total possibilities in the word MANIFOLD will only give cases where all the 3 vowels are not together. It will not take care of the vowels being separated. Hence when there are more than 2 items to be separated, we should take the approach as discussed in 9.11. This is the most general method for this type of problems.

Directions for examples 9.12 to 9.17:

Anusha has a collection of 7 C.D's. with her viz., Ben Hur, The Sound of Music, Titanic, Small Wonders, Stuart Little, Chariots of Fire and Toy Story. Nikhil wanted to borrow 4 of these C.D's to watch during vacation. Anusha has no reservation against lending any of the C.D's.

9.12. In how many ways can Nikhil borrow the C.D's?

Sol. Nikhil can borrow any 4 of the 7 C.D's i.e, he can select any 4 of the 7 C.D's in $^7C_4 = 35$ ways.

9.13. In how many ways can Nikhil borrow the C.D's, if he wants to include Stuart Little in his selection?

Sol. Since 'Stuart Little' is to be included in the selection, Nikhil can choose any 3 of the remaining 6 C.D's in $^6C_3 = 20$ ways.

9.14. If Nikhil wants to exclude Titanic, then in how many ways can the selection be made?

Sol. If Titanic has to be excluded, then Nikhil has to select his 4 C.D's from the remaining 6 C.D's which can be done in $^6C_4 = 15$ ways.

9.15. in how many ways can Nikhil borrow the C.D's, if he is particular that his selection should include both Small Wonders and Toy Story if at all one of them is included?

Sol. Here we need to consider two cases –
Case(i): Both Small Wonders and Toy Story are included.
Case(ii): Neither Small Wonders nor Toy Story is included.
In case(i), Nikhil needs to select 2 more C.D's which can be done in $^5C_2$ ways.
In case (ii), Nikhil needs to select 4 CD's from the remaining 5 C.D's which can be done in $^5C_4$ ways. Hence, the required number of ways $= {}^5C_2 + {}^5C_4 = 15$.

9.16. How many selections can be made if Nikhil does not want to include Small Wonders and Toy Story together?

Sol. The total number of selections that can be made are ${}^7C_4$ = 35. Of these, we want to exclude the selections where Toy Story and Small Wonders are together, which are ${}^5C_2$ = 10 in number. Hence, the required number of ways are 35 – 10 = 25.

9.17. In how many of the selections would exactly one among Ben Hur or Chariots of Fire is included?

Sol. The given problem can be broken into two cases (i) Ben Hur is included but not Chariots of Fire (ii) Chariots of Fire is included but not Ben Hur.
Case (i) When Ben Hur is included with Chariots of Fire being excluded, the other three C.D's can be borrowed from the remaining 5 CD's in ${}^5C_3$ =10 ways.
Similarly in case (ii) there are 10 ways of selecting. Hence, the total number of ways are 20.

9.18. Consider the word PRECIPITATION. Find the number of ways in which
(i) a selection
(ii) an arrangement
of 4 letters can be made from the letters of this word.

Sol. The word PRECIPITATION has 13 letters I, I, I, P, P, T, T, E, R, C, A, O, N of 9 different sorts.
In taking 4 letters, the following are the possibilities to be considered.
(a) all 4 distinct.
(b) 3 alike, 1 distinct.
(c) 2 alike of one kind, 2 alike of other kind.
(d) 2 alike, 2 other distinct.

Selections

(a) 4 distinct letters can be selected from 9 distinct letters (I, P, T, E, R, C, A, O, N) in ${}^9C_4$ = 126 ways.

(b) As 3 letters have to be alike, the only possibility is selecting all the I's. Now the 4th letter can be selected from any of the remaining 8 distinct letters in ${}^8C_1$ = 8 ways.

(c) Two pairs of two alike letters can be selected from I's, Q's and T's in ${}^3C_2$ = 3 ways.

(d) The two alike letters can be selected in ${}^3C_1$ = 3 ways and the two distinct letters can now be selected from the 8 distinct letters in ${}^8C_2$ = 28 ways. Hence required number of ways are $3 \times 28 = 84$
Hence, the total number of selections = 126 + 8 + 3 + 84 = 221.

Arrangements

For arrangements, we find the arrangements for each of the above selections and add them up.

(a) As the 4 letters are distinct, there are 4! arrangements for each selection. Hence required arrangements are $126 \times 4! = 3024$

(b) Since 3 of the 4 letters are alike, there are $\frac{4!}{3!}$ arrangements for each of the selection. Hence required number of arrangements are $8 \times \frac{4!}{3!} = 32$.

(c) The required number of arrangements here are $3 \times \frac{4!}{2!2!} = 18$

(d) The required number of arrangements are $84 \times \frac{4!}{2!}$ = 1008.
Total number of arrangements are 3024 + 32 + 18 + 1008 = 4082.

9.19 If all possible four-digit numbers are formed using the digits 5, 6, 7, 8 without repetition and arranged in ascending order of magnitude then find the position of the number 7685.

Sol. To find the position of the number 7685, we first count all four-digit numbers that precede 7685 when arranged in ascending order of magnitude.
The number of four-digit numbers that
begin with 5 is 3!.
begin with 6 is 3!.
begin with 75 is 2!.
begin with 765 is 1.
The next number in the order is 7685. Hence 3! + 3! + 2! + 1 = 15 numbers precede 7685. Hence, the position of 7685 is 16.

9.20 A committee of 7 is to be formed from 6 ladies and 8 gentlemen. In how many ways can it be done when the committee consists of
(i) exactly 4 ladies.
(ii) at least 4 ladies.

Sol. (i) The committee must have 4 ladies and 3 gentleman. The selection of 4 ladies and 3 gentlemen from 6 ladies and 8 gentlemen can be done in ${}^6C_4 \times {}^8C_3 = 15 \times 56 = 840$ ways.
(ii) Here the committee can consist of 4 or 5 or 6 ladies and accordingly 3 or 2 or 1 gentleman respectively. This can be done in ${}^6C_4 \times {}^8C_3 + {}^6C_5 \times {}^8C_2 + {}^6C_6 \times {}^8C_1$
= 840 + 168 + 8 = 1016 ways.

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the value of $^{12}C_2$.
(A) 55 (B) 66 (C) 132 (D) 110

2. Find the value of $^{10}C_7$.
(A) 60 (B) 360 (C) 120 (D) 720

3. If $^nC_{12} = {}^nC_{18}$, then find $^nC_3$.
(A) 4060 (B) 240 (C) 30 (D) 1140

4. Find the value of $^8P_3$.
(A) 166 (B) 336 (C) 672 (D) 536

5. If $^nP_4 = {}^nP_5$, then n = $\boxed{}$

6. In how many ways can three marbles be drawn simultaneously from a box containing 12 different marbles?
(A) 110 (B) 220 (C) 660 (D) 240

7. In how many ways can 4 girls be seated on 10 different chairs in a row?
$\boxed{}$

8. Relation between ^nC_r and ^nP_r is _______.
(A) $^nP_r = {}^nC_r$ (B) $^nP_r = {}^nC_{n-r}$
(C) $^nP_r = r!{}^nC_r$ (D) $^nC_r = r!{}^nP_r$

9. The value of $^{240}C_{239}$ = $\boxed{}$

10. $^8C_7 + {}^8C_6$ = _______.
(A) $^9C_6$ (B) $^8C_7$ (C) $^9C_7$ (D) 8

11. If $^nC_7 = {}^nC_3$, then n = _______.
(A) 10 (B) 4 (C) 7 (D) 3

12. If $^nP_r = 5040$ and $^nC_r = 210$, then find the value of r.
(A) 3 (B) 6 (C) 4 (D) 5

13. Number of ways of arranging 3 persons in 8 distinct chairs is _______.
(A) $^8P_3$ (B) $^8C_3$ (C) 8! (D) 3!

14. Number of ways of selecting 5 out of 7 persons is _______
(A) $^7P_5$ (B) $^7C_5$ (C) 7! (D) 5!

15. The number of different words that can be formed using all the letters of the word SPECIAL that begin with S and end with L is _______.
(A) 5! (B) 7! (C) 6! (D) 4!

16. Number of odd numbers that can be formed using the digits 2, 4, 6, 8 is $\boxed{}$.

17. Number of even numbers that can be formed using all the digits 1, 3, 5, 6, 7 is _______.
(A) 5! (B) 5 (C) 4! (D) 0

18. The number of ways of forming a garland using 21 different flowers is _______.
(A) 20! (B) 21! (C) 21 (D) $\frac{20!}{2}$

19. Find the number of ways of arranging all the letters of the word ARRANGE.
(A) 7! (B) $\frac{7!}{(2!)^2}$ (C) $\frac{7!}{2!}$ (D) 4!7!

20. How many arrangements can be made by using all the letters of the word COMPLEX?
$\boxed{}$

21. Neha has 10 different chocolates. In how many ways can she give them to two of her friends?
(A) 10! (B) 2^{10} (C) 10 (D) 9

22. A set has 8 elements. How many non-empty subsets are there for the set?
$\boxed{}$

23. In how many ways can 6 letters be posted into 4 letterboxes?
(A) 6^4 (B) $^6P_4$ (C) 4^6 (D) $^6C_4$

24. The number of diagonals in a regular decagon is
$\boxed{}$

25. In how many ways can 6 persons be seated around a circular table?
$\boxed{}$

26. In a library there are 12 research scholars. In how many ways can we select 4 of them?
(A) 400 (B) 495
(C) 320 (D) 240

27. In how many ways can we select two vertices in an octagon?
$\boxed{}$

28. Find the number of ways in which 11 delegates can be arranged around a table for a meeting.
$\boxed{}$

29. Find the number of diagonals in a regular duodecagon.
$\boxed{}$

30. Number of ways of arranging 5 persons in a row is _______.
(A) 5 (B) 10 (C) 5! (D) 20

Exercise – 9(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. Find the number of ways of arranging all the letters of the word THUNDER so that the vowels appear in the odd places.

$\boxed{}$

2. The number of words that can be formed using all the letters of the word LIMPET so that the vowels are always together is $\boxed{}$.

3. In how many ways can the letters of the word COUPLE be arranged so that no two vowels are together?
(A) 6! – 4! 3! (B) 3! 3!
(C) 3! 4! (D) (2) $(3!)^2$

4. Twelve varieties of ice-cream are available at an ice-cream parlour. The number of ways of selecting
(i) at least one of them is _______.
(A) 4096 (B) 4095
(C) 12! (D) 12! – 1

(ii) at least 2 of them is _______.
(A) 4084 (B) 13
(C) 66 (D) 4083

5. How many 5-digit numbers can be formed using the digits 0, 2, 3, 4, 5, 8 and 9, if repetition of the digits is not allowed?

$\boxed{}$

6. How many 5-digit numbers that are divisible by 4 can be formed using the digits 0 to 6, if no digit is to occur more than once in each number?

$\boxed{}$

7. In how many ways can 10 post-cards be posted into 6 letter boxes?
(A) 10^6 (B) 6^{10}
(C) ${}^{10}C_6\,(10)$ (D) ${}^{10}P_6$

8. In how many ways can the crew of a ten-oared boat be arranged, when of the ten persons available two can row only on the bow side and two of the others can row only on the stroke side?
(A) $\frac{(6!)(5!)^2}{(3!)^2}$ (B) $\frac{5.6!}{(3!)^2}$
(C) $\frac{6!(5!)^2}{3!}$ (D) $\frac{6!(5!)}{(3!)}$

9. A cinema hall has a capacity of 100 seats – 40 seats in the balcony and 60 seats on the ground floor. In how many ways can 100 ticket holders be accommodated, if 10 of them want to be in the balcony and 15 others refuse to be in the balcony?
(A) $\frac{75!}{45!}$ (B) 75! 40! 60!
(C) $\frac{75!}{30!}$ (D) $\frac{75!}{45!\,30!}$

10. Find the number of ways of arranging the letters of the word INSTALLATION in all possible ways so that the Ns come together?
(A) 11!2! (B) $\frac{11!}{(2!)^4}$ (C) $\frac{12!}{(2!)^5}$ (D) $\frac{11!}{(2!)^3}$

11. In how many ways can 20 distinct books be divided equally
(i) among 4 girls?
(A) $\frac{20!}{(4!)^5}$ (B) $\frac{20!}{(5!)^4\,4!}$
(C) $\frac{20!}{4!\,5!}$ (D) $\frac{20!}{(5!)^4}$

(ii) into 4 parcels?
(A) $\frac{20!}{(4!)^5}$ (B) $\frac{20!}{(5!)^4\,4!}$
(C) $\frac{20!}{4!\,5!}$ (D) $\frac{20!}{(5!)^4}$

12. In how many ways can 6 girls and 6 boys sit around a circular table so that no two boys sit together?
(A) $(5!)^2$ (B) $(6!)^2$ (C) 5! 6! (D) 11!

13. How many ten-digit numbers can be formed using all the digits of 1324642119 such that the even digits appear only in odd places?
(A) $(5!)^2$ (B) $\frac{(5!)^2}{3!}$
(C) 5! 3! 2! (D) $\frac{(5!)^2}{(2!)^2\,3!}$

14. Consider the word ANTIBIOTIC.
(i) In how many ways can 4 letters be selected from the word?
(A) 68 (B) 63 (C) 66 (D) 72

(ii) How many arrangements can be made by taking 4 letters from the word?
(A) 1728 (B) 1230
(C) 1444 (D) 1634

15. How many numbers with at most 4 digits can be formed using the digits 3, 4, 5, and 6, when repetition of digits is allowed?

$\boxed{}$

16. A regular polygon has the number of diagonals equal to one and a half times the number of sides. The polygon is a/an _______.
(A) pentagon (B) heptagon
(C) octagon (D) hexagon

17. The letters of the word TINSEL are permuted in all possible ways and the words formed are arranged as in a dictionary. The rank of the word LISTEN is

$\boxed{}$

18. Find the sum of all numbers that can be formed using all the digits 1, 2, 5, 8, and 9 without repetition.
(A) 5555500 (B) 666600
(C) 4444400 (D) 6666600

19. Let S be the set of all 5-digit numbers with distinct digits that can be formed using the digits 1, 2, 4, 5 and 8 such that exactly two odd positions are occupied by the even digits. Find the sum of the digits in the rightmost position of all the numbers in S.

20. Among all convex nonagons, what is the maximum number of points of intersection of the diagonals inside the nonagon?

21. In a regular polygon with 15 sides, find the number of triangles that can be formed with the vertices of the polygon, such that none of the sides of the triangles are taken from the sides of the polygon.

22. Meghana attempts a question paper consisting of 12 questions. Each question has 4 choices. If Meghana answers the questions randomly, then the number of ways in which she can attempt the entire paper is ________.
(A) 48 (B) $^{12}P_4$
(C) 4^{12} (D) 12^4

23. A question paper consists of 12 problems, each problem having an internal choice of 2 questions. In how many ways can a candidate attempt one or more problems?
(A) 12! (B) 2^{12}
(C) 13! (D) $3^{12} – 1$

24. There are 6 copies of a biography, 5 copies of an autobiography and 4 copies of a novel. The number of ways in which one or more books can be given away is

25. Twelve villages in a district are divided into 3 zones with four villages per zone. The telephone department of the district intends to connect the villages with telephone lines such that every two villages in the same zone are connected with three direct lines and every two villages belonging to different zones are connected with two direct lines. How many direct lines are required?
(A) 210 (B) 96
(C) 54 (D) 150

26. A certain number of students of a school have participated in the chess tournament of their Annual Sports Meet. It was found that in 105 games both the players were girls and in 300 games both the players were boys. The number of games in which one was a girl and the other was a boy is ________.
(A) 500 (B) 600
(C) 375 (D) 210

27. An advertisement board is to be designed with seven vertical stripes using some or all of the colours red, black, yellow and blue. In how many ways can the board be designed such that no two adjacent stripes have the same colour?
(A) 972 (B) 2916
(C) 729 (D) 2187

28. Let K be an integer such that the sum of the digits of K is 3 and $10^6 < K < 10^7$. How many values can 'K' have?

29. Six boxes numbered 1 to 6 are arranged in a row. Each is to be filled by either a blue or a green coloured ball such that no two adjacent boxes contain green coloured balls. In how many ways can the boxes be filled with the balls?

30. There are 12 points in a plane of which 4 are on a straight line and no three of the other points lie on a straight line.

(i) How many straight lines can be formed by joining these points?
(A) 60 (B) 61 (C) 59 (D) 66

(ii) How many triangles can be formed by joining these points?
(A) 220 (B) 216 (C) 224 (D) 66

31. How many non-negative integral solutions does the equation $x_1 + x_2 + x_3 + x_4 = 10$ have?
(A) 286 (B) 86
(C) 35 (D) 165

32. There is a grid of 6 uniformly spaced horizontal lines intersecting 6 uniformly spaced vertical lines. The distance between any two adjacent horizontal lines or any two adjacent vertical lines is 1 unit .An ant has to go from A(0,0) to B(5,5) .If it can move only along the lines, find the number of shortest paths it can take.

33. The number of positive integral solutions of the equation $x + y + z + t = 25$ is _______.
(A) 2024 (B) 2042
(C) 2204 (D) 2402

34. There is a 5×6 grid. Each of the 30 cells has a distinct label. In how many ways can I place 3 identical coins in 3 different cells (one in each cell) such that no two coins are in the same row or column?
(A) 1200 (B) 2400
(C) 3600 (D) 4060

35. There are 8 lines such that no two lines are parallel and no three lines are concurrent. Find the number of regions that are formed by these 8 lines.

Exercise – 9(b)

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. An urn contains 30 marbles of different colours. In how many ways can 10 marbles be selected from the urn?

(A) ${}^{30}P_{10}$ (B) ${}^{30}C_{10}$ (C) 30! (D) 10!

2. In how many ways can 6 prizes be distributed among 3 boys, if each boy is eligible to receive one or more prizes?

(A) 3^6 (B) 6^3 (C) ${}^{6}C_{3}$ (D) ${}^{6}P_{3}$

3. A group of friends reserve two circular tables in a restaurant for a dinner. One table has 4 chairs and the other has 6 chairs. In how many ways can the group seat themselves for the dinner?

(A) $\dfrac{10!}{4!6!}$ (B) $\dfrac{10!}{4!}$ (C) 10! (D) 3!5!

4. In how many ways can 15 different flowers be strung in the form of a garland?

(A) 15! (B) 14! (C) $\dfrac{15!}{2!}$ (D) $\dfrac{14!}{2!}$

5. A question paper consists of 15 'true or false' questions. In how many ways can a candidate answer the entire paper?

(A) 3^{15} (B) $3^{15} - 1$ (C) 2^{15} (D) $2^{15} - 1$

Moderate

6. In how many ways can the letters of the word SPECIAL be arranged in a row such that the vowels occupy only the odd places?

7. A man has 12 friends whom he wants to invite for lunch. In how many ways can he invite

(i) at least one of them?

(A) 4096 (B) 4095
(C) 2047 (D) 2048

(ii) at least 10 of them?

(A) 66 (B) 79 (C) 140 (D) 153

8. In how many ways can the letters of the word "VARIOUS" be arranged in a row so that the vowels and consonants appear alternately?

9. In how many ways can 4 letters be selected from the letters of the word ADDRESSEE?

10. How many arrangements can be made by taking 4 letters from the word in the previous question?

(A) 9! (B) 370
(C) 540 (D) 280

11. How many four-digit numbers, that are divisible by 3, can be formed, using the digits 0, 1, 2, 3 and 8 if no digit is to occur more than once in each number?

12. Find the number of ways of dividing 16 different books equally

(i) among 4 boys.

(A) $\dfrac{16!}{(4!)^4}$ (B) $\dfrac{16!}{(4!)^3}$

(C) $\dfrac{16!}{(4!)^5}$ (D) $(4!)^4$

(ii) into 4 parcels.

(A) $\dfrac{16!}{(4!)^4}$ (B) $\dfrac{16!}{(4!)^3}$

(C) $\dfrac{16!}{(4!)^5}$ (D) $(4!)^4$

13. If the letters of the word ARISE are arranged in a row in all possible ways and the arrangements are listed in alphabetical order as in a dictionary, then the rank of the word RAISE is

14. Find the sum of all possible 5-digit numbers that can be formed using the digits 1, 3, 4, 6 and 8 without repetition.

(A) 4880008 (B) 5040068
(C) 5866608 (D) 5866068

15. Among all convex polygons with n sides, the maximum number of points of intersection of the diagonals inside the polygon is 495. The value of n is

16. In a regular polygon with 10 sides, find the number of triangles that can be formed with the vertices of the polygon, such that the triangles formed have at least one side in common with the polygon.

17. From a group of 10 professors and 6 assistant professors, a management institute desires to send a delegation of 8 persons consisting of 5, professors and 3 assistant professors to the IIMs annual meet. If Prof. Balamurali, a science Professor refuses to be in the delegation if Assistant Prof. Sheshadri, an arts professor is included in the delegation, then in how many ways can the delegation be formed?

(A) ${}^{9}C_{4}\,{}^{4}C_{3}$ (B) ${}^{9}C_{5}\,{}^{4}C_{2}$

(C) ${}^{10}C_{5}\,{}^{6}C_{3} - {}^{9}C_{4}\,{}^{5}C_{2}$ (D) ${}^{9}C_{4}\,{}^{4}C_{3} + {}^{9}C_{5}\,{}^{4}C_{2}$

18. Sixteen villages in a district are divided into 4 zones with four villages per zone. The telephone department of the district intends to connect the villages with telephone lines such that every two villages in the same zone are connected with two direct lines and every two villages belonging to different zones are connected with one direct line. How many direct telephone lines are required?

19. A certain number of students of a school participated in the chess tournament of their Annual Sports Meet. Each player played 1 game against each of the other players. It was found that in 66 games both the players were girls, and in 240 games one was a girl and the other was a boy. The number of games in which both the players were boys is ______ .

20. If all possible 5-digit numbers that can be formed using the digits 1, 2, 3, 4 and 5 without repetition, are arranged in the ascending order of magnitude, then the position of the number 21354 is _______.
(A) 26 (B) 32 (C) 25 (D) 50

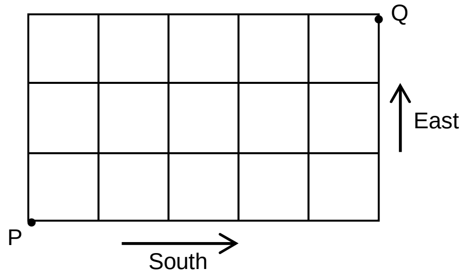
21. In an 'akhada', m wrestlers stand around a circle. Each possible pair of persons not standing next to each other play a match with in the middle of the circle for 1minute and 30 seconds. If the total time taken for all the matches is 30 minutes, then m is equal to

22. Let k be an integer such that the sum of the digits of k is 2 and $10^9 < k < 10^{10}$. How many values can k have?
(A) 10 (B) 7 (C) 9 (D) 45

23. Nine points are marked on a straight line and 10 points are marked on another line which is parallel to the first line. How many triangles can be formed with these points as vertices?

24. Ten points are plotted in a plane such that no three of them lie on a straight line. Four of these points are joined to each of the remaining six points and each of the remaining six points is joined to exactly five points. How many line segments are formed?
(A) 27 (B) 25 (C) 29 (D) 24

25. In the figure below, the lines represent the one-way roads allowing cars to travel only eastwards or southwards. In how many ways can a car travel from the point P to the point Q?



(A) 15 (B) 66 (C) 56 (D) 76

26. Using the first ten letters of the English alphabet how many strings of 4 letters can be formed such that each H is followed by A? (Repetition allowed)

27. How many positive integers less than 100,000 and divisible by 125 can be formed using the digits 0, 1, 2, 5 and 8, if repetition is allowed?
(A) 90 (B) 300 (C) 99 (D) 129

28. Two red pencils, three black pencils and two white pencils are to be arranged in a row such that:

(a) no two adjacent pencils are of the same colour and
(b) the pencils at the two ends of the row are of same colour.

In how many ways can the pencils be arranged?
(A) 12 (B) 8 (C) 9 (D) 10

29. Mangalam has forgotten his friend's 8-digit telephone number but remembers the following:

(a) The first 3 digits are either 270 or 279.
(b) The digit 0 occurs exactly three times and the digit 9 occurs exactly once.
(c) The number is an even number.

If Mangalam were to use a trial and error method to reach his friend, what is the maximum number of trials he has to make to be sure to succeed?

(A) 832 (B) 1664 (C) 1280 (D) 2000

30. There is an unlimited supply of identical red, blue and green coloured balls. In how many ways can 12 balls be selected from the supply?

31. How many numbers between 4000 and 6000 can be formed using the digits 1 to 6, when any digit can occur any number of times?
(A) 864 (B) 432 (C) 638 (D) 126

32. In how many ways can a group of 15 persons be arranged around two circular tables consisting of 7 and 8 chairs?

(A) $\frac{15!\,6!\,7!}{8!}$ (B) $\frac{15!\,6!}{8!}$

(C) $\frac{15!}{7!}$ (D) 15

33. (i) The number of positive integral solutions of the equation a + b + c = 15 is _______.
(A) 76 (B) 105 (C) 91 (D) 86

(ii) The number of non-negative integral solutions of the equation p + q + r + s = 30 is _______.
(A) 4598 (B) 5324
(C) 5546 (D) 5456

(iii) What is the number of non-negative integer solutions of A + B + C + D ≤ 15?
(A) $^{18}C_3$ (B) $^{19}C_3$
(C) $^{18}C_4$ (D) $^{19}C_4$

34. In how many ways can five letters be posted into 3 post boxes such that at least one letter is posted in each box?

$\boxed{}$

35. The number of positive integer solutions for the equation x + y + z = 20 where x ≥ 2, y ≥ 3, z ≥ 4 is $\boxed{}$.

36. There are 3 groups A, B and C –with 8, 6 and n persons respectively. Each person in a group shakes hands with every person in the other groups exactly once and no two persons within a group shake hands. The total number of handshakes among them is 104. Find the value of n.

$\boxed{}$

Difficult / Very Difficult

37. Fifteen lines are drawn in a plane such that four of them are parallel. What is the maximum number of regions into which the plane is divided?

(A) 121 (B) 116
(C) 115 (D) 114

38. Four atheletes Pravin, Visharath, Bhushan and Durandhar participate in 6 athletic events. There is only one prize for winning in each event and each of them won in at least one event. In how many ways could they have won the six prizes?

$\boxed{}$

39. There are 6 letters and corresponding 6 addressed envelops. If the letters are placed into the envelopes randomly (each letter is placed only in one envelope), in how many ways can exactly two letters be placed into their corresponding envelopes?

$\boxed{}$

40. From numbers 8, 9, 10, 11, …..25. Eight numbers are selected such that the greatest is 19. In how many ways can these 8 numbers be permuted?

(A) 11! (B) (330)8! (C) $\frac{(11)!}{3}$ (D) $\frac{(11)!}{6}$

Key

Concept Review Questions

1. B
2. C
3. A
4. B
5. 5
6. B
7. 5040
8. C
9. 240
10. C
11. A
12. C
13. A
14. B
15. A
16. 0
17. C
18. D
19. B
20. 5040
21. B
22. 255
23. C
24. 35
25. 120
26. B
27. 28
28. 3628800
29. 54
30. C

Exercise – 9(a)

1. 1440
2. 240
3. C
4. (i) B
   (ii) D
5. 2160
6. 624
7. B
8. A
9. D
10. B
11. (i) D
    (ii) B
12. C
13. D
14. (i) D
    (ii) B
15. 340
16. D
17. 281
18. D
19. 296
20. 126
21. 275
22. C
23. D
24. 209
25. D
26. C
27. B
28. 28
29. 21
30. (i) B
    (ii) B
31. A
32. 252
33. A
34. A
35. 37

Exercise – 9(b)

1. B
2. A
3. B
4. D
5. C
6. 576
7. (i) B
   (ii) B
8. 144
9. 30
10. B
11. 36
12. (i) A
    (ii) C
13. 76
14. C
15. 12
16. 70
17. C
18. 144
19. 190
20. A
21. 8
22. A
23. 765
24. A
25. C
26. 6805
27. C
28. D
29. B
30. 91
31. B
32. B
33. (i) C
    (ii) D
    (iii) D
34. 150
35. 78
36. 4
37. C
38. 1560
39. 135
40. B