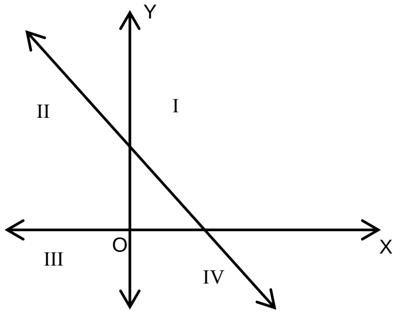
## Chapter – 6
## (Co-ordinate Geometry)

### Concept Review Questions

**Solutions for questions 1 to 30:**

**1.** As can be seen in the figure above, if a line with slope –1 passes through the I quadrant, it can't pass through the III quadrant




Choice (B)

**2.** Two non-parallel lines lying in the same plane have to intersect.
$\therefore$ The least distance is 0. Ans : (0)

**3.** As seen in solution I above, a line can pass through at most 3 quadrants. Ans : (3)

**4.** II[nd] quadrant. Choice (B)

**5.** If a line passes through the origin, both intercepts are 0. Choice (C)

**6.** The distance of $(x_1, y_1)$ from $ax + by + c = 0$ is $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$\therefore$ The distance of (0, 0) from $mx – y + c = 0$ is $\left|\dfrac{c}{\sqrt{1+m^2}}\right|$.
Choice (A)

**7.** Distance from (0, 0) to (3, 4) is $\sqrt{3^2 + 4^2}$ = 5 units.
Ans : (5)

**8.** The required distance is $\left|\dfrac{c}{\sqrt{a^2+b^2}}\right|$ Choice (D)

**9.** Let $A = (x_1,y_1)$ and $B = (–3x_1,–3y_1)$
The origin (0, 0) divides AB in the ratio say m : n ( say).
$\therefore 0 = \dfrac{m(-3x_1) + n(x_1)}{m+n}$
$\therefore -3m + n = 0 \Rightarrow n = 3m$
$\therefore m : n = m : 3m = 1:3$
As m : n is positive, this is internal division.

**Alternative Solution:**
The slope of the line joining $(x_1, y_1)$ and $(–3x_1, –3y_1)$ is $\dfrac{y_1}{x_1}$.

$\therefore$ Its equation is $y = \dfrac{y_1}{x_1}x$.

The origin lies on the line.

Distance between the origin and $(x_1, y_1) = \sqrt{x^2{}_1 + y^2{}_1}$.

Distance between the origin and $(–3x_1, –3y_1) = 3\sqrt{x_1^2 + y_1^2}$

The origin divides the line segment joining $(x_1, y_1)$ and $(–3x_1, –3y_1)$ in the ratio 1 : 3 internally. ($\because$ The origin lies on the line segment). Choice (B)

**10.** The points that are at unit distance from (0, 0) lie on a circle of radius 1 and centre at the origin. There are infinitely many such points on this circle. Choice (D)

**11.** The side of the square is distance between (0, 0) and (0, $\alpha$) i.e. $\alpha$ then its diagonal is $\sqrt{2}\,\alpha$. Choice (C)

**12.** The largest chord of a circle is its diameter. If the diameter is 2, then the radius is 1 and the area is $\pi(1^2) = \pi$
Choice (A)

**13.** $(x_1, y_1)$ and $(–x_1, y_1)$ are two opposite vertices of the square.
$\therefore$ The midpoint of the line segment joining these vertices is the common midpoint of the diagonals i.e., point of intersection of the diagonals is $(0, y_1)$ ($\because$ Midpoint of the line segment joining $(x_1, y_1)$ and $(x_2, y_2)$ is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.
The diagonal joining the given opposite vertices lies on the x-axis.
$\therefore$ The other diagonal would lie on the y-axis ($\because$ The diagonals of the square are perpendicular). ------(1)
Distance from the intersection point of the diagonals to each of the given vertices is $|x_1|$ ------- (2)
From (1) and (2), the other two vertices are $(0, y_1 + x_1)$ and $(0, y_1 – x_1)$ . Choice (B)

**14.** Since AB + BC = AC, A, B, C are collinear. Hence, the required equation is the same as the equation of the line joining A and B. (i.e) y = mx + c Choice (D)

**15.** Let the coordinates of the fourth point D be $(x_4, y_4)$.
The 3 possible values of $x_4$ are
$x_1 + x_2 – x_3$,
$x_1 – x_2 + x_3$
$–x_1 + x_2 + x_3$.
Corresponding $y_4$
$y_1 + y_2 – y_3$
$y_1 – y_2 + y_3$
$–y_1 + y_2 + y_3$. Ans : (3)

**16.** The area of $\Delta$ABC is zero means A, B, C, are collinear. If A and B are known points, we can say that C lies on line AB.
Choice (C)

**17.** The distance between two parallel lines,
$y = mx + c_1$ and $y = mx + c_2$ is $\left|\dfrac{c_1 - c_2}{\sqrt{1+m^2}}\right| = 1$

Given $\left|\dfrac{c_1 - c_2}{\sqrt{1+m^2}}\right| = 1$

$\therefore |c_1 - c_2| = \left|\sqrt{1+m^2}\right|$

i.e., $c_1 – c_2 = \left|\sqrt{1+m^2}\right|$ or $c_1 – c_2 = -\left|\sqrt{1+m^2}\right|$

$\Rightarrow c_2 = c_1 - \left|\sqrt{1+m^2}\right|$ or $c_2 = c_1 + \left|\sqrt{1+m^2}\right|$

$\therefore$ If the equation of one line is y = mx + c, that of the other is
$y = mx + c - \left|\sqrt{1+m^2}\right|$ or

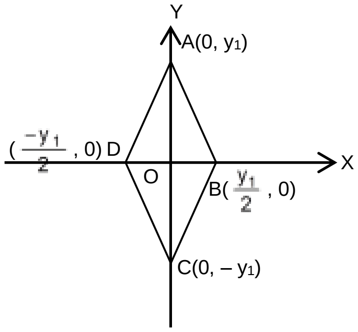
$y = mx + c + \left|\sqrt{1+m^2}\right|$ Choice (D)

**18.** The coordinates of the point p(x, y) that divides $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m : n are given by

$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$ Choice (B)

**19.** Given $\theta = 135°$
Slope = tan135° = –1 ⇒ –1 + 1 = 0 Ans : (0)

**20.** The vertices A and C of the rhombus ABCD are given

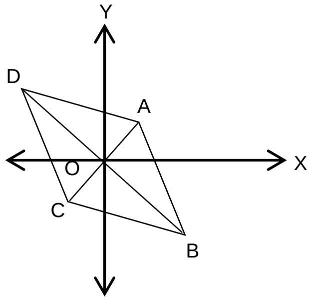



∴The midpoint of AC, i.e., (0, 0) is the point where BD intersects AC. Further BD ⊥ AC.
∴BD lies on the x-axis

As BD = (1/2) AC, B = $\left(\frac{y_1}{2}, 0\right)$ and D = $\left(\frac{-y_1}{2}, 0\right)$

Both (A) and (B) are vertices. Choice (C)

**21.** The rhombus is shown in the figure below

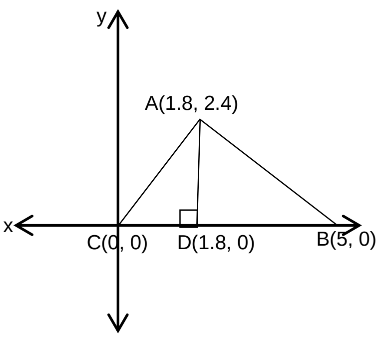



If the equation of AB is y = mx + c, that of CD is y = mx – c because the slope of CD is the same as that of AB, while its y-intercept is –c. Choice (A)

**22.** The points in the choices A, B and C lie on the line.
Choice (D)

**23.** Given equation is x – y = 0 i.e., y = x
comparing with y = mx + c
We have, m = 1
But m = tanθ ∴ tanθ = 1 ⇒ θ = 45°
∴ The required angle is 45° Ans : (45)

**24.** The given points are plotted in the figure below.




We suspect that ΔABC is right angled. We have to verify.
DC = 1.8, AD = 2.4 ∴ AC = 3
AD = 2.4, DB = 3.2 ∴ AB = 4
and we can see that BC = 5.
∴ Our suspicion is well-founded, ΔABC is right-angled at A.
∴ The orthocenter is A (1.8, 2.4), the vertex of the right angle. Choice (C)

**25.** The diagonals of a rhombus are perpendicular.
∴The product of their slopes is –1 (In case the diagonals are along the coordinate axes, the slopes are 0 and ∝ and the product is indeterminate).
∴ The required product is either –1 or indeterminate.
Choice (D)

**26.** The line is parallel to y = 2x.
∴ The line has the same slope as that of y = 2x
The slope of any line in the form y = mx + c is m. Slope of the considered line = That of y = 2x, which is 2.
Equation of the line is $2 = \frac{y-4}{x-3}$, i.e. y = 2x – 2.
Choice (D)

**27.** The required line (ray m) is perpendicular to the given line (say $\ell$) y = 3x + 1
∴ The product of the slopes of $\ell$ and m is –1.
The slope of $\ell$ is 3. ∴ The slope of m has to be $\frac{-1}{3}$
Equation of m is $\frac{-1}{3} = \frac{y-1}{x-1}$ i.e. 3y = –x + 4.
Choice (C)

**28.** The circle is centered at the origin and pass through (3, 4).

∴ Radius of the circle = distance between the origin and (3, 4) $= \sqrt{(3-0)^2 + (4-0)^2} = 5$.
Circumference of the circle = 2π (Radius) = 10 π.
Choice (B)

**29.** The centroid of the triangle whose vertices are $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$ is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

The centroid of the given triangle is

$$\left(\frac{0+5+0}{3}, \frac{0+0+12}{3}\right) \text{ i.e., } \left(\frac{5}{3}, 4\right)$$ Choice (C)

**30.** The midpoint of the line segment whose end points are $(x_1, y_1)$, $(x_2, y_2)$, is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Mid point of the line segment whose ends are (1, 6) and (9, 12) is $\left(\frac{1+9}{2}, \frac{6+12}{2}\right)$ i.e., (5, 9)

Choice (A)

## Exercise – 6(a)

**Solutions for questions 1 to 35:**

**1.** The centre of the circle $(x - g)^2 + (y - f)^2 = r^2$ is (g, f)
∴The centre of the circle $(x - 6)^2 + (y - 3)^2 = 25$ is (6, 3)
Distance between the points (3, 7), (6, 3) is

$\sqrt{(3-6)^2 + (7-3)^2} = \sqrt{9+16} = 5$ units Ans : (5)

**2.** We know that the ratio in which the y-axis divides the line

joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is $-x_1 : x_2$. Here the points are (4, 3), (–6, 2).
∴ The required ratio is –4 : –6 = 2 : 3 Choice (C)

**3.** Let the given points be A(3, 5), B(5, 9), C(10, k). If A, B and C are collinear, then the slope of AB = slope of BC.

∴ Slope of AB = $\frac{9-5}{5-3}=\frac{4}{2}=2$

Slope of BC = $\frac{k-9}{10-5}=\frac{k-9}{5}$

∴ $\frac{k-9}{5}=2$ ⇒ k = 10 + 9 = 19 Choice (C)

**4.** In a triangle, the centroid divides the segment joining the orthocentre, 'O' and the circumcentre, S in the ratio 2 : 1 internally.
Given O(4, 5), G(3, 3).
Let S= (x, y)
The centroid G is

$\left(\frac{2x+4}{2+1}, \frac{2x+5}{2+1}\right)=(3,3)$

2x + 4 = 9; 2y + 5 = 9
2x = 5 2y = 4
x = $\frac{5}{2}$ y = 2

∴ Circumcentre $\left(\frac{5}{2}, 2\right)$ Choice (A)

**5.** If $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ are the three consecutive vertices of a parallelogram, then the fourth vertex is $(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$
The given vertices are (2, 6), (–4, 2) and (8, –4).
∴ The fourth vertex is (2 + 8 –(–4), 6 – 4– 2) = (14, 0)
Choice (A)

**6.** We know that, the slope of the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is $\frac{y_2-y_1}{x_2-x_1}$.
Given, points are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

⇒ slope = $\frac{2at_2-2at_1}{at_2^2-at_1^2}$ = $\frac{2a(t_2-t_1)}{a(t_2+t_1)(t_2-t_1)}$ = $\frac{2}{t_2+t_1}$
Choice (C)

**7.** We know that, the equation of the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is given by $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$.
∴ The equation of the line joining the points (5, 6) and (4, 3) is

y – 6 = $\frac{3-6}{4-5}(x-5)$
y – 6 = 3(x – 5)
3x – 15 – y + 6 = 0
3x – y – 9 = 0 Choice (D)

**8.** The given line is $\sqrt{3}x$ – y + 9 = 0.
Slope of the line is $\sqrt{3}$ .
∴ The angle made by the line with x-axis is 60º.
Hence, the angle made by the line with y-axis is 90 – 60 = 30º. Ans : (30)

**9.** If $m_1$ and $m_2$ are the slopes of two lines and θ is the acute angle between the lines, then tanθ = $\left|\frac{m_1-m_2}{1+m_1m_2}\right|$.
Given lines are 2x + 3y + 7 = 0 ------- (1)
and x – 5y + 3 = 0 ------- (2)

Slope of lines (1) = $m_1 = -\frac{2}{3}$

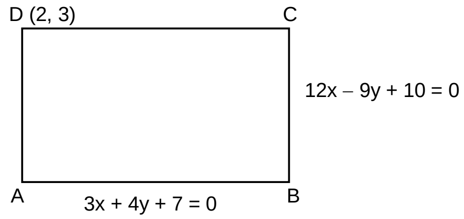
Slope of lines (2) = $m_2 = \frac{1}{5}$

tanθ = $\left|\frac{-\frac{2}{3}-\frac{1}{5}}{1+\left(-\frac{2}{3}\right)\frac{1}{5}}\right|$

tanθ = $\frac{\frac{13}{15}}{\frac{13}{15}}$

tanθ = 1 ⇒ θ = 45º Ans : (45)

**10.** The given lines 3x + 4y + 7 = 0 and 12x – 9y + 10 = 0 represent the adjacent sides AB and BC of a rec0tangle ABCD. Since the point (2, 3) does not lie on the two given lines. So, (2, 3) represents the vertex 'D'.




Length/breadth of the rectangle is the perpendicular distance from (2, 3) to the line 12x – 9y + 10 = 0

i.e. $\left|\frac{12(2)-9(3)+10}{\sqrt{12^2+(-9)^2}}\right|=\frac{7}{15}$ units.

Breadth of the rectangle is the perpendicular distance from (2, 3) to the line 3x + 4y + 7 = 0

i.e., $\left|\frac{2(3)+4(3)+7}{\sqrt{3^2+4^2}}\right|=\frac{25}{5}=5$ units

∴ Area of the rectangle = length × breadth

= 5 × $\frac{7}{15}$ = $\frac{7}{3}$ sq units. Choice (C)

**11.** The given lines are
2x + 3y + 7 = 0 -------- (1)
4x + 9y + 12 = 0 -------- (2)
3x – 2y + 9 = 0 -------- (3)
Clearly equation (1) and equation (3) are perpendicular to each other.
Hence, the points of intersection of the lines (1) and (2) and (1) and (3) are the end points of the hypotenuse.

∴ Solving (1) and (2) we get x = $-\frac{9}{2}$ , y = $\frac{2}{3}$

∴ $\left(-\frac{9}{2}, \frac{2}{3}\right)$ is one end point of the hypotenuse.

Choice (C)

**12.** Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$

Given, a + b = 7 ⇒ b = 7 – a
Since the above line passes through (4, –5)

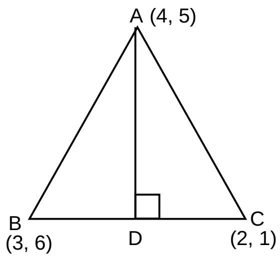
$\frac{4}{a}+\frac{-5}{7-a}=1$

28 – 4a – 5a = a(7 – a)
$a^2$ – 16a + 28 = 0
∴ a = 14 or 2 ⇒ b = –7 or 5

Equation of the line can be $\frac{x}{2}+\frac{y}{5}=1$

5x + 2y = 10. Choice (D)

**13.** Given, A(4, 5) B(3, 6) and C(2, 1) are the vertices of the triangle ABC.




Let AD be the altitude, then AD $\perp$ BC.
Slope of AD $\times$ Slope of BC $= -1$
Slope of AD $\times 5 = -1$
Slope of AD $= -\frac{1}{5}$.
Equation of AD having slope $= -\frac{1}{5}$ and passing through the point A(4, 5) is
$y - y_1 = m(x - x_1)$
i.e. $y - 5 = -\frac{1}{5}(x - 4)$
$5y - 25 = -x + 4$
$x + 5y - 29 = 0$ Choice (D)

**14.** If two lines are perpendicular to each other, then, $m_1 \times m_2 = -1$, where $m_1$ & $m_2$ are the slopes of the two lines.
The given lines are $ax + 3y + 7 = 0$, $4x + 9y + 15 = 0$.
The slopes of the lines are $-\frac{a}{3}, -\frac{4}{9}$ respectively.
$\therefore\ -\frac{a}{3} \times -\frac{4}{9} = -1$
$4a = -27 \Rightarrow a = -\frac{27}{4}$ Choice (A)

**15.** Given, lines are $4x + 5y - 23 = 0$, $x + 3y - 11 = 0$
Solving these equations we get, $x = 2$, $y = 3$
Since the line $x + ky + 3k + 2 = 0$ passes through the point (2, 3),
$2 + 3k + 3k + 2 = 0$
$6k = -4$
$k = -\frac{4}{6} = -\frac{2}{3}$ Choice (D)

**16.** The given line is $3x + 4y + 5 + k(x - 3y + 2) = 0$
i.e. $(3 + k)x + (4 - 3k)y + 5 + 2k = 0$
If a line is parallel to x-axis, then the coefficient of x must be zero.
$\therefore\ 3 + k = 0 \Rightarrow k = -3 \Rightarrow k^2 = 9$ Ans : (9)

**17.** Given lines are $8x + 5y = 48$ and $y = kx + 6$
At the point of intersection of the lines P(say) $5y = 48 - 8x = 5(kx + 6)$
At p, $48 - 8x = 5kx + 30$
$18 = x(5k + 8) \Rightarrow \frac{18}{5k + 8} = x$
The coordinates of P are integers $\therefore 5k + 8$ is a factor of 18. This factor may be positive or negative.
$5k + 8 = \pm1, \pm2, \pm3, \pm6, \pm9, \pm18$
$5k = -7$, or $-9$, $-6$ or $-10$, $-5$ or $-1$, $-2$ or $-14$, 1 or $-17$, 10 or $-26$
k is an integer when $5k = -10$, $-5$ or 10
k has three integer values. Also, if both x and k are integers, then $y = kx + 6$ has to be an integer. Ans : (3)

**18.** On solving the equation $x^2 - 5x - 6 = 0$, we get the roots 6, $-1$.
$\therefore$ The equation of the line can be
$y = 6x - 1$ (when slope = 6 and y – intercept is $-1$) or $y = -x + 6$ (when slope = $-1$ and y – intercept is 6).
Choice (B)

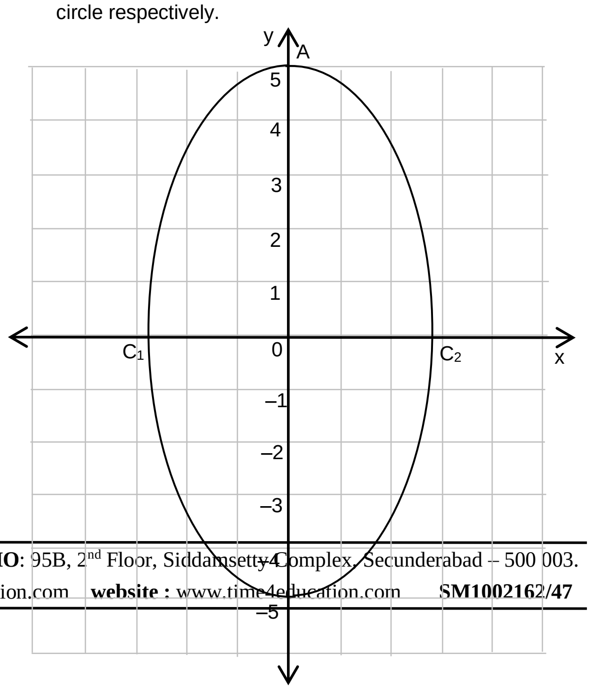
**19.** Two of the tangents to the circle are $x + y = 7$ and $x + y = \frac{-13}{2}$. These are parallel tangents. The diameter of the circle is the distance between the tangents.
Distance between the tangents is $\frac{\left|-7 - \left(\frac{13}{2}\right)\right|}{\sqrt{2}}$ i.e. $\frac{27}{2\sqrt{2}}$
Circumference of the circle $= \frac{27}{2\sqrt{2}}\pi$. Choice (B)

**20.** We know that, the distance between the parallel lines $ax + by + c_1 = 0$, $ax + by + c_2 = 0$ is given by $\left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$.
Given,
$5x + 12y + 24 = 0$ -------- (1)
$10x + 24y + 49 = 0$ -------- (2)
(2) is equivalent to $5x + 12y + \frac{49}{2} = 0$
$\therefore$ Distance $= \left|\frac{\frac{49}{2} - 24}{\sqrt{5^2 + 12^2}}\right|$
$= \frac{1}{2 \times 13} = \frac{1}{26}$ units. Choice (A)

**21.** The area of the triangle formed by the line $ax + by + c = 0$ with the coordinate axes is $\frac{c^2}{2|ab|}$.
The area of the triangle formed by the line $4x - 5y + 20 = 0$ with the coordinate axes is $\frac{20^2}{2|4(-5)|}$ = 10 sq units.
Ans : (10)

**22.** Centroid of the triangle $= \left(\frac{-4 + 7 + 5}{3}, \frac{0 + 0 + a}{3}\right) = \left(\frac{8}{3}, \frac{a}{3}\right)$
$\left(\frac{8}{3}, \frac{a}{3}\right) = \left(\frac{8}{3}, \frac{5}{3}\right)$ $a = 5$
The triangle has a base of 11 and a height of a i.e. 5
The area of the triangle $= \frac{1}{2}(11)(5) = \frac{55}{2}$. Choice (C)

**23.** Let $C_1$ and $C_2$ be the centres of the first circle and second circle respectively.



Let A and B be the points of intersection of the circles
The common radius of the two circles is $C_1C_2$.
$C_1C_2 = C_1A = C_2A$. $\therefore$ Triangle $AC_1C_2$ is equilateral,
Triangle $AC_1O$ is a 30 – 60 – 90 triangle.

$\therefore AC_1 = \frac{2}{\sqrt{3}}$ (OA) $= \frac{10}{3}\sqrt{3}$

The area of each circle $= \pi(AC_1)^2 = \pi\left(\frac{100}{3}\right)$.

Choice (C)

**24.** $81y^2 - x^2 + 14x = c$
$\Rightarrow 81y^2 - (x-7)^2 + 49 = c$
$\Rightarrow 81y^2 - (x-7)^2 = c - 49$. This represents a pair of straight lines.
$\therefore$ c – 49 must be 0 $\therefore$ c = 49. Choice (B)

**25.** The points (0, 6) and (0, 17) are the ends of a diagonal of a square. $\therefore$The length of this diagonal is 11. Let the side of the square be a.
$\sqrt{2}$ a = 11
The diagonals of the square bisect each other.
$\therefore$The midpoint of the diagonal is the same.
The midpoint of the diagonal whose ends are (0, 6) and (0, 17) is $\left(\frac{0+0}{2}, \frac{6+17}{2}\right)$ i.e. (0, 11.5). $\therefore$The midpoint of the other diagonal is also (0, 11.5)
One of the ends of the other diagonal of the square is $\frac{\sqrt{2}a}{2}$ away from (0, 11.5) i.e. 5.5 away from (0, 11.5) ----- (1)
The diagonals of the square must be perpendicular to each other. The diagonal whose ends are (0, 6) and (0, 17) lies on the y -axis. $\therefore$ the other diagonal must be parallel to the x-axis --------- (2)
From (1) and (2) the two end points of the other diagonal must be (5.5, 11.5) and (–5.5, 11.5). Choice (D)

**26.** (p – q, p +q) and (p + q, p – q) are either adjacent vertices or alternate vertices.
If (p – q, p + q) and (p + q, p –q) are adjacent vertices (6p – q, 6p + q) must be vertex opposite to either (p – q, p +q) or (p + q, p – q).
Let the fourth vertex be (a, b).
Common midpoint of the two diagonals.

$= \left(\frac{6p-q+p-q}{2}, \frac{6p+q+p+q}{2}\right)$ or

$\left(\frac{6p-q+p+q}{2}, \frac{6p+q+p-q}{2}\right)$

$\left(\frac{p+q+a}{2}, \frac{p-q+b}{2}\right) = \left(\frac{6p-q+p-q}{2}, \frac{6p+q+p+q}{2}\right)$

Or

$\left(\frac{p-q+a}{2}, \frac{p+q+b}{2}\right) = \left(\frac{6p-q+p+q}{2}, \frac{6p+q+p-q}{2}\right)$

(a, b) = (6p – 3q, 6p + 3q) or (6p + q, 6p – q)

If (p – q, p + q) and (p + q, p – q) are alternate vertices, |6p – q, 6p + q| must be opposite to the fourth vertex. Let the fourth vertex be (c, d)

$\left(\frac{p-q+p+q}{2}, \frac{p+q+p-q}{2}\right)$

$= \left(\frac{6p-q+c}{2}, \frac{6p+q+d}{2}\right)$

$\therefore$ (c, d) = (q – 4p, –4p – q).
The fourth vertex can be (6p – 3q, 6p + 3q) (6p + q, 6p – q) or (q – 4p, –4p – q).

**Alternative solution:**
Three of the vertices are
A = (p–q, p+q)
B = (p+q, p–q)
C = (6p–q, 6p+q)
Let the fourth vertex be D(x,y)
If A and C opposite vertices, (7p–2q, 7p+2q) = (p+q+x, p–q+y) i.e. (x,y) = (6p –3q, 6p+3q)
If A and B are opposite vertices,
(2p, 2p) = (6p –q+x, 6p+q+y)
i.e. (x,y) = (6p+q,6p–q)
$\therefore$The coordinates of the point D can be any of the ones in options A, B or C. Choice (D)

**27.** Area of a triangle formed by joining the midpoints of the sides of a triangle T is one-fourth the area of T

Using the formula $\frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$, the area of the triangle whose vertices are (3, 5), (5, 8), and (7, 5) is $\frac{1}{2}$ $\begin{vmatrix} -2 & -3 \\ -2 & 3 \end{vmatrix}$ i.e. $\frac{1}{2}$ (|–6 –6|) i.e. 6.

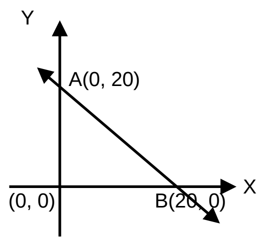
Area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (3, 5), (5, 8), and (7, 5) is $\frac{1}{4}$ (6) i.e. 1.5. Ans : (1.5)

**28.** The given line is x (p + 5q) + 4y (p + q) = 5p - 1q i.e.
p(x + 4y) + q(5x + 4y) = 5p + q
The given line passes through a certain point for all real values of p and q. This is possible, only if x + 4y = 5 and 5x + 4y = 1.
If x + 4y = 5 and 5x + 4y = 1, x = -1 and y = 1.5
The point through which the line passes for all real values of p and q is (–1, 1.5). Choice (A)

**29.** The line cuts the x-axis at (–2, 0) and y-axis at (0, –3)
$\therefore$ Equation of the line is $\frac{x}{-2} + \frac{y}{-3} = 1$
$\Rightarrow 3x + 2y + 6 = 0$ Choice (C)

**30.** The inclination of the line is $\theta = 60^\circ$
$\therefore$ Slope = tan60º = $\sqrt{3}$
$\therefore$ Y intercept is = –1
$\therefore$ Equation of the line is y – (–1) = $\sqrt{3}$ (x – 0)
y + 1 = $\sqrt{3}$ x Choice (A)

**31.** The triangle formed by the given vertices is shown in the figure.




The triangle meets the coordinate axes at A(20, 0) and B(0, 20).
$\therefore$ Equation of line AB is
$\frac{x}{20} + \frac{y}{20} = 1$
x + y = 20

Now, we find the intercepts which satisfies the following conditions.

$x + y < 20$

$x > 0$ ; $y > 0$

When x = 1 the number of corresponding values are 18. Similarly when x = 2, the number of corresponding values are 17.

$\therefore$ The total number of ordered pairs of x and y which satisfies the above conditions are

$18 + 17 + \ldots\ldots + 1 = \frac{19 \times 18}{2} = 171$ Ans : (171)

**32.** Let L be the line,

Slope of the line $m = \tan 60^o = \sqrt{3}$.

Equation of the line having slope $\sqrt{3}$ and passing through the point (5, 3) is $y - 3 = \sqrt{3}\,(x - 5)$

$\sqrt{3}\,x - y - 5\sqrt{3} + 3 = 0$

Since, this cuts the y-axis at Q, x = 0

$\therefore -y - 5\sqrt{3} + 3 = 0$

$\Rightarrow y = 3 - 5\sqrt{3}$,

$\therefore$ The point $Q(0, 3 - 5\sqrt{3})$.

$PQ = \sqrt{(5-0)^2 + \left(3 - \left(3 - 5\sqrt{3}\right)\right)^2}$

$= \sqrt{25 + 75} = \sqrt{100} = 10$ units Ans : (10)

**33.** The given four points form a square.

$\therefore$ The required equation is the equation of the diagonal passing through the points (6, 6) and (– 1, 3).

$y - 6 = \frac{-3}{-7}(x - 6)$

$7y - 42 = 3x - 18$

$3x - 7y + 24 = 0$ Choice (D)

**34.** Translation equations are:

$X = x - \alpha$, $Y = y - \beta$

Given: $(x, y) = (2, 3)$, $(\alpha, \beta) = (-4, 5)$

$X = 6$, $Y = -2$

$\therefore$ The required point is (6, – 2) Choice (B)

**35.** Given: $(\alpha, \beta) = (1, 1)$ and $f(X, Y) = 2X^2 - 3XY - Y^2 - 5 = 0$

The original equation is $f(x - \propto, y - \beta) = f(x - 1, y - 1) = 0$

$2(x - 1)^2 - 3(x - 1)(y - 1) - (y - 1)^2 - 5 = 0$

$2x^2 - 3xy - y^2 - x + 5y - 7 = 0$ Choice (D)

## Exercise – 6(b)

**Solutions for questions 1 to 35:**

**1.** We have, slope of the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is,

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Given, $(x_1, y_1) = (4, p)$
$(x_2, y_2) = (p, 5)$ and
$m = -2$

$\therefore \frac{5 - p}{p - 4} = -2$

$\Rightarrow 5 - p = -2(p - 4) \Rightarrow 5 - p = -2p + 8$

$\Rightarrow p = 3$ Ans : (3)

**2.** We know that,

Equation of a line passing through $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Here, $A(x_1, y_1) = (1, p)$
$B(x_2, y_2) = (p, 1)$

$\therefore y - p = \frac{(1 - p)}{(p - 1)}(x - 1)$

$\Rightarrow y - p = \frac{1 - p}{-(1 - p)}(x - 1)$

$\Rightarrow y - p = -(x - 1)$

Hence, equation of the required line is $x + y - p - 1 = 0$ Choice (B)

**3.** Equation of line passing through (0, 0) and having slope 3 is

$y - 0 = 3(x - 0)$

$\Rightarrow y = 3x \Rightarrow 3x - y = 0$ Choice (B)

**4.** Intercepts of the line $x - y + 7 = 0$ i.e $\frac{x}{-7} + \frac{y}{7} = 1$ are – 7 and 7.

Intercepts of the line $2x + 3y = 6$ i.e. $\frac{x}{3} + \frac{y}{2} = 1$ are 3 and 2.

Intercepts of the line $x + y - 10 = 0$ i.e. $\frac{x}{10} + \frac{y}{10} = 1$ are 10 and 10.

Intercept of the line $2x - 3y = 6$ i.e. $\frac{x}{3} + \frac{y}{-2} = 1$ are 3 and – 2.

Clearly the line $x + y - 10 = 0$ has equal intercepts

x – intercept = 10

y – intercept = 10 Choice (C)

**5.** Given lines are,

$x + y - 8 = 0$ ---------- (1)

$3x - 2y + 1 = 0$ ---------- (2)

$x - y = 0$ ---------- (3)

The vertex opposite to the hypotenuse is the vertex containing the right angle.

Also, the vertex containing the right angle is the point of intersection of the perpendicular sides (1) and (3) as the product of the slopes of these lines is – 1

Now, solving (1) and (3), we get

x = 4 and y = 4

Hence, the required vertex is (4, 4) Choice (C)

**6.** Given quadratic equation is $x^2 + 7x + 12 = 0$

$\Rightarrow (x + 3)(x + 4) = 0$

$\Rightarrow$ Roots are –3, –4

$\therefore$ The ordered pair (x, y) = (–3, –4)

Clearly it satisfies $2x - 5y = 14$ Choice (C)

**7.** Given lines are $5x + 3y = 2$ and $x - 2y = 3$

Since, the given lines are intersecting lines, (as their slopes are not equal) the shortest distance between them will be zero. Ans : (0)

**8.** Given line is, $5x + 6y = 30$

$\Rightarrow \frac{5x}{30} + \frac{6y}{30} = 1$

$\Rightarrow \frac{x}{6} + \frac{y}{5} = 1$

Comparing with $\frac{x}{a} + \frac{y}{b} = 1$, we get

a = 6 and b = 5

We have, Area of the triangle

$= \frac{1}{2}|ab| = \frac{1}{2}|6 \times 5| = \frac{1}{2} \times 30 = 15$ sq.units Choice (A)

**9.** Given lines are $3x - ky + 6 = 0 \rightarrow (1)$
$2x + 3y + 7 = 0 \rightarrow (2)$

Since (1) and (2) are parallel, their slopes must be equal.

$\therefore \frac{3}{k} = \frac{-2}{3}$

$\Rightarrow -2k = 9 \Rightarrow k = \frac{-9}{2}$ Choice (A)

**10.** Given lines are

$\sqrt{k}x - 3y + 10 = 0 \rightarrow (1)$

$6x + ky + 25 = 0 \rightarrow (2)$

Since the angle between the lines (1) and (2) is 90°, the product of the two slopes = –1

i.e., $m_1 m_2 = -1$

$\Rightarrow \frac{\sqrt{k}}{3} \times \frac{-6}{k} = -1$

$\Rightarrow 2\sqrt{k} = k$

$\Rightarrow 2\sqrt{k} - k = 0$

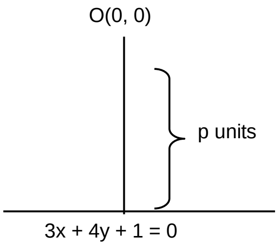
$\Rightarrow \sqrt{k}(2 - \sqrt{k}) = 0$

$\Rightarrow \sqrt{k} = 0$ or $2 - \sqrt{k} = 0$

$\Rightarrow \sqrt{k} = 0$ or $\sqrt{k} = 2$

$\Rightarrow k = 0$ or $k = 4$ Choice (D)

**11.**




Let 'p' be the perpendicular distance from (0, 0) to the line 3x + 4y + 1 = 0

Then, p = $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|3(0) + 4(0) + 1|}{\sqrt{3^2 + 4^2}}$

$\Rightarrow p = \frac{1}{5}$ units Ans: (0.2)

**12.** Slope of the line joining (0, 0) and (p, q) is $\frac{q - 0}{p - 0}$

Slope of the line joining (0, 0) and (p, q), the slope of the line joining (p, q) and (–p, –q), the slope of the line joining (–p, –q) and (pq, $q^2$) are all the same.

$\therefore$ The given points are collinear. Choice (D)

**13.** In the figure, the angle made by line $l_1$, with the positive direction of x = axis = 45°

$\therefore$ m = tan45° = 1

Also, the required line is passing through the origin

$\therefore$ Equation of the line is, $y - y_1 = m(x - x_1)$

i.e., y – 0 = 1(x – 0)

i.e., x – y = 0 Choice (C)

**14.** We know that, the centroid of the triangle formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Given,

$(x_1, y_1) = (7, 0)$

$(x_2, y_2) = (5, 1)$

$(x_3, y_3) = (3, 5)$

$\therefore$ Centroid, G = $\left(\frac{7+5+3}{3}, \frac{0+1+5}{3}\right) = \left(\frac{15}{3}, \frac{6}{3}\right) = (5, 2)$

Now, Distance between the points

(–3, –4) and (5, 2) = $\sqrt{(5+3)^2 + (2+4)^2}$

= $\sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100}$ = 10 units Ans : (10)

**15.** Let A(2, –3), B(0, 0) and C(3, 2) be the given vertices of triangle ABC

Then,

AB = $\sqrt{4+9} = \sqrt{13}$

BC = $\sqrt{9+4} = \sqrt{13}$

CA = $\sqrt{1+25} = \sqrt{26}$

$\Rightarrow AB^2 + BC^2 = \left(\sqrt{13}\right)^2 + \left(\sqrt{13}\right)^2 = \left(\sqrt{26}\right)^2 = CA^2$

$\therefore AB^2 + BC^2 = CA^2$

$\angle B = 90°$

Also, AB = BC

$\therefore$ $\Delta$ABC is a right angled isosceles triangle. Choice (B)

**16.** If the circumcentre, centroid and orthocentre are S, G and O respectively

$\frac{SG}{GO} = \frac{1}{2}$

$O = (x_1, y_1)$

G = (0, 0)

$S = (x_2, y_2)$ say

$\frac{x_2}{0 - x_1} = \frac{1}{2} \Rightarrow x_2 = \frac{-x_1}{2}$

Similarly $y_2 = \frac{-y_1}{2}$ Choice (D)

**17.** We know that, x-axis divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$

in the ratio $-y_1 : y_2$

Given, $A(x_1, y_1) = (-3, 2)$

$B(x_2, y_2) = (4, 6)$

$\therefore$ Required ratio = –2 : 6 = –1 : 3

Hence, the ratio in which x-axis divides the line joining the points (–3, 2) and B(4, 6) is 1 : 3 externally. Choice (A)

**18.** The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are said to be collinear if,

Slope of AB = slope of BC

Given, $A(x_1, y_1) = (p + 1, 1)$

$B(x_2, y_2) = (2p + 1, 3)$

$C(x_3, y_3) = (2p + 2, 2p)$

We have,

Slope, m = $\frac{y_2 - y_1}{x_2 - x_1}$

Slope of AB = Slope of BC

$\Rightarrow \frac{3-1}{(2p+1)-(p+1)} = \frac{2p-3}{(2p+2)-(2p+1)}$

$\Rightarrow \frac{2}{p} = \frac{2p-3}{1} \Rightarrow p(2p - 3) = 2$

$\Rightarrow 2p^2 - 3p - 2 = 0$

$\Rightarrow (2p + 1)(p - 2) = 0$

$\Rightarrow p = 2, \frac{-1}{2}$ Choice (C)

**19.** Equation of the line passing through the points (1, 3) and (5, –5) is $y - 3 = \frac{-5-3}{5-1}(x - 1)$

$\Rightarrow y - 3 = \frac{-8}{4}(x - 1)$

$\Rightarrow y - 3 = -2x + 2$

$\Rightarrow 2x + y - 5 = 0$

Of the given choices, only the point (4, –3) satisfies this equation. Choice (C)

**20.** Let D(x, y) be the fourth vertex.

Let A (4, 1), B(7, 4) and C(13, –2) be the given vertices.

Since ABCD is a rectangle,

The mid point of $\overline{BD}$ is same as the mid point of $\overline{AC}$.

$\Rightarrow \left(\frac{7+x}{2}, \frac{4+y}{2}\right) = \left(\frac{4+13}{2}, \frac{1-2}{2}\right)$

$\left[\because \text{midpont} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$

$\Rightarrow \frac{7+x}{2} = \frac{17}{2}, \Rightarrow \frac{4+y}{2} = \frac{-1}{2}$

$\Rightarrow x = 10, y = -5$

Hence the fourth vertex is (10, –5) Choice (A)

**21.** Clearly for the lines 9x + y = 3 and 4x + y + 2 = 0, the product of the intercepts is 1
Also, the point (1, –6) satisfies both these equations.
Hence, the required line is 9x + y = 3 or 4x + y + 2 = 0
Choice (D)

**22.** Slope – intercept form of a line is y = mx + c
where, m = slope
c = y – intercept

$\frac{-c}{m}$ = x – intercept

m = tanθ = tan60° = $\sqrt{3}$

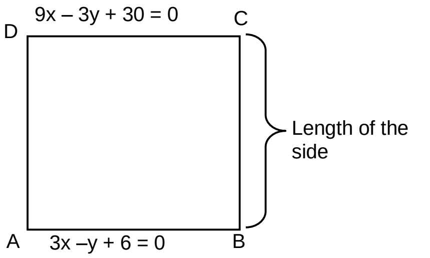
Also, $\frac{-c}{\sqrt{3}} = 3 \Rightarrow c = -3\sqrt{3}$

∴ Required equation of the line is

$y = \sqrt{3}x - 3\sqrt{3}$ i.e., $\sqrt{3}x - y - 3\sqrt{3} = 0$ Choice (D)

**23.** Three lines are said to be concurrent, if the point of intersection of any two lines lies on the third line.
Given lines are
3x – y = 2 → (1)
2x + y = 3 → (2) and
5x – ay = 3 → (3)
Solving (1) and (2), we get (x, y) = (1, 1)
Substituting (1, 1) in (3), we have
5 (1) – a(1) = 3
⇒ 5 – a = 3
⇒ a = 5 – 3 = 2. Ans: (2)

**24.**




Given lines are
3x – y + 6 = 0 → (1) and 9x – 3y + 30 = 0 → (2)
(2) can be written as 3x – y + 10 = 0 → (3)
Length of the side = Distance (d) between the lines (1) and (3)

$$d = \left|\frac{c_2 - c_1}{\sqrt{a^2+b^2}}\right| = \frac{|10-6|}{\sqrt{9+1}} = \frac{4}{\sqrt{10}}$$

∴ Perimeter of the square = 4 × side

$$= 4 \times \frac{4}{\sqrt{10}} = \frac{16}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{16\sqrt{10}}{10} = \frac{8\sqrt{10}}{5} \text{ units}$$

Choice (D)

**25.** Let ABCD be the square one of whose vertices is A (3, 10). Since (3, 10) is not on the line 5x – y + 12 = 0, the equation represents diagonal BD. Let AC be the required diagonal. Since AC is perpendicular to BD, the
slope of AC × slope of BD = – 1
slope of AC × 5 = –1.

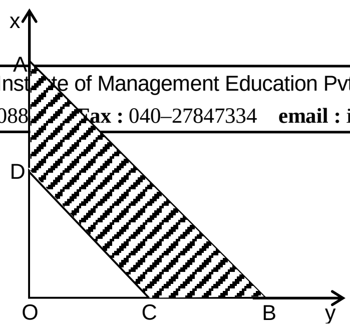
slope of AC = $\frac{-1}{5}$

The equation of AC passing through the point A(3,10) and having slope $\frac{-1}{5}$ is

$y - 10 = \frac{-1}{5}(x - 3) \Rightarrow 5y - 50 = -x + 3$

x + 5y – 53 = 0 Choice (A)

**26.**




Let 5x + 12y =13 intersect the y-axis and x-axis at A and B.
Let the line 5x + 4y = 3 intersect the x-axis and y-axis at C and D.
The area of shaded region = The area of Δ OAB – The area of ΔOCD

$$= \left(\frac{1}{2}\right)\left(\frac{13}{12}\right)\left(\frac{13}{5}\right) - \frac{1}{2}\left(\frac{3}{5}\right)\left(\frac{3}{4}\right)$$

$$= \frac{169-27}{120} = \frac{142}{120} = \frac{71}{60}$$ Choice (A)

**27.** If $m_1$ and $m_2$ are the slopes of two lines, then the acute angle between the lines is given by

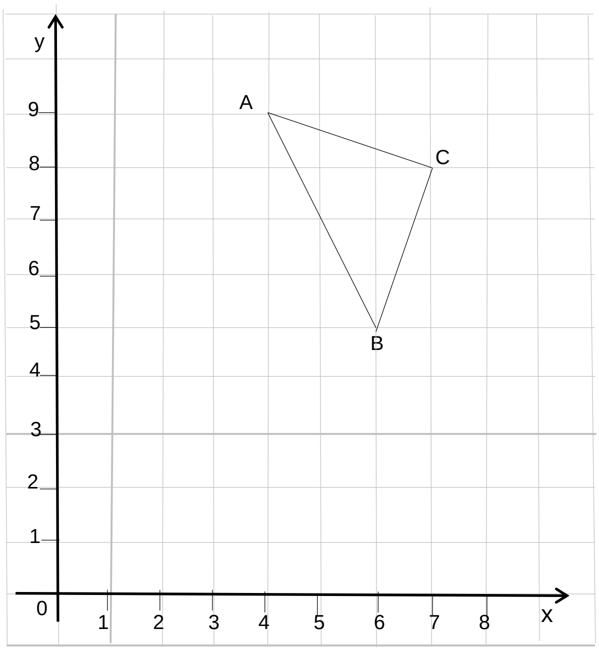
$\tan\theta = \frac{|m_1 - m_2|}{1 + m_1m_2}$

Given, $m_1 = \frac{1}{5}$ and $m_2 = \frac{-1}{7}$

$$\therefore \tan\theta = \frac{\left|\frac{1}{5}+\frac{1}{7}\right|}{1-\left(\frac{1}{5}\right)\left(\frac{1}{7}\right)} = \frac{\left|\frac{7+5}{35}\right|}{\frac{35-1}{35}} = \frac{12}{34} = \frac{6}{17}$$

$\therefore \theta = \tan^{-1}\left(\frac{6}{17}\right)$ Choice (C)

**28.**




We note that CA = CB = $\sqrt{10}$ , i.e. Δ CAB is isosceles.
The angle bisector of ∠C is perpendicular to AB. Slope of AB is –2.

∴ The slope of the line which bisects ∠ACB is $\frac{1}{2}$.

Choice (D)

**29.** The equation of a straight line passing through the points (3,6) and (–3, 9) is $\frac{y-6}{x-3} = \frac{9-6}{-3-3}$

$= \frac{y-6}{x-3} = \frac{-1}{2}$

$2y – 12 = –x + 3$

$= 1x + 2y – 15 = 0$

Let the line $x + 2y –15 = 0$

Intersect the x-axis at A, y-axis at B

$\therefore$ A = (15, 0) and B = $\left(0, \frac{15}{2}\right)$

$\therefore$ Length of intercept between the axes = distance between AB

$= \sqrt{(15)^2 + \frac{(15)^2}{(2)^2}} = 15\sqrt{\frac{5}{4}}$

$= \frac{15}{2}\sqrt{5}$ Choice (D)

**30.** The given lines are

$3x + 4ky + 6 = 0$

$kx – 3y + 9 = 0$

Solving these two equations,

We get, $\left(\frac{36k+18}{-(9+4k^2)}, \frac{6k-27}{-(9+4k^2)}\right)$

Since, the point is in the second quadrant, x < 0, y > 0

$\therefore \frac{36k+18}{-(9+4k^2)} < 0; \frac{6k-27}{-(9+4k^2)} > 0$

$36k + 18 > 0$ and $6k – 27 < 0$

$k > -\frac{1}{2}$ $k < \frac{9}{2}$

$\therefore -\frac{1}{2} < k < \frac{9}{2}$

$\therefore$ The integral values that k can take are 0, 1, 2, 3, 4.

Hence, 5 integral values of k satisfy the equation.

Ans: (5)

**31.** Given line is $3x + 4y – 9 = 0 \rightarrow (1)$

Any line parallel to equation (1) has the form $3x + 4y + k = 0$.

Let the other line be $3x + 4y + k_1 = 0 \rightarrow (2)$

Distance between the lines (1) and (2) is $\frac{3}{10}$ units (given)

$\therefore \frac{|k+9|}{\sqrt{3^2+4^2}} = \frac{3}{10}$

$\Rightarrow \frac{|k+9|}{5} = \frac{3}{10} \Rightarrow |k+9| = \frac{3}{2}$

$k + 9 = \frac{3}{2}$ or $k + 9 = \frac{-3}{2}$

$\Rightarrow k = \frac{3}{2} - 9$ or $k = \frac{-3}{2} - 9$

$k = \frac{-15}{2}$ or $k = \frac{-21}{2}$

$\therefore$ Equation of the required line is $3x + 4y - \frac{15}{2} = 0$ or

$3x + 4y - \frac{21}{2} = 0$

i.e., $6x + 8y – 15 = 0$ or $6x + 8y – 21 = 0$

Choice (D)

**32.** Here, x – intercept (a) = 3

y – intercept (b) = –2

$\therefore$ Equation of the required line is

$\frac{x}{a} + \frac{y}{b} = 1$

i.e., $\frac{x}{3} + \frac{y}{-2} = 1 \Rightarrow 2x – 3y – 6 = 0$ Choice (A)

**33.** Let the coordinates of a point P be (x, y) in a system R. Let the origin of R be translated to ($\alpha$ $\beta$). The coordinates of P in the new system, R′, say (x, y) are obtained from

$x = X + \alpha, y = Y + \beta$

Given: (X, Y) = (– 2, – 1), ($\alpha$, $\beta$) = (– 3, 1)

$\therefore$ The required coordinates are (x, y) = (– 5, 0)

Choice (B)

**34.** Let the coordinates of a point P in R and R′ be (x, y) and (X, Y) respectively. If the axes of R are rotated through $\theta$ (considered positive in the anticlockwise direction), then

$X = x \cos\theta + y \sin\theta$ and

$Y = – x \sin\theta + y \cos\theta$

Given: (x, y) = $(-2\sqrt{2}, 5\sqrt{2})$ and $\theta = -45°$

($\because$ rotation is clockwise, So '$\theta$' is negative)

$\therefore$ (X, Y) = (– 7, 3). Choice (A)

**35.** Given: $(\alpha, \beta) = (-1, 1)$ and $f(x, y) = 2x^2 – xy + y^2 – 4x + 7y – 5 = 0$

The transformed equation is $f(X + \alpha, Y + \beta) = f(X – 1), (Y+1) = 0$

$2(X – 1)^2 – (X – 1)(Y + 1) + (Y + 1)^2 – 4(X –1) + 7(Y + 1) – 5 = 0$

$2X^2 – XY + Y^2 – 9X + 10Y + 10 = 0$

Choice (A)

**Solutions for questions 36 to 40:**

**36.** From statement I, the equation of the line AB is $4x + 3y = 12$.

When y = 0, x = 3

When x = 0, y = 4

So, OA = 3 and OB = 4

$\therefore$The area of $\Delta$AOB = 1/2 x OA x OB

= 1/2 x 3 × 4 = 6 sq. units

So, statement I alone is sufficient.

From statement II, the midpoint of the line segment AB is $\left(\frac{3}{2}, 2\right)$

Let A(a, 0) and B (0, b)

$\left(\frac{a+0}{2}, \frac{0+b}{2}\right) = \left(\frac{3}{2}, 2\right)$

$\Rightarrow$ a = 3 and b = 4

$\therefore$The area of $\Delta$AOB = 1/2 |a b|

= 1/2 x 3 x 4 = 6 sq. units.

So statement II alone is sufficient.

Hence, either of the statements alone is sufficient to answer the question.

Choice (B)

**37.** As OA = OB = OC, the shaded region is a semi-circle.

From statement I alone, AB = 10

$\Rightarrow$ OA = OB = 5

i.e. the radius of the circle is 5 units.

$\therefore$The area of the shaded portion is

$\frac{\pi r^2}{2} = \frac{\pi(5)^2}{2} = \frac{25\pi}{2}$ sq. units.

So statement I alone is sufficient.

From statement II alone, the area of $\Delta$ABC is 25 sq. units.

$\Rightarrow$ 1/2 × AB × OC = 25

$\Rightarrow \frac{1}{2}$ x (OA + OB) x OC = 25

$\Rightarrow$ (OA + OA) x OA = 50 ($\because$ OA = OB = OC)

$\Rightarrow 2 \times OA^2 = 50$ $\Rightarrow$ OA = 5

i.e. the radius of the circle is 5 units.

$\therefore$ The area of the semi circle is $\frac{\pi r^2}{2}$

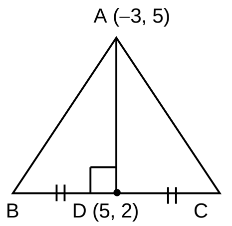
$= \frac{\pi(5)^2}{2} = \frac{25\pi}{2}$ sq. units.

So statement II alone is sufficient.

Hence, either of the statements alone is sufficient to answer the question. Choice (B)

**38.** Let C be the centre of the circle.
From statement I, A (4, 0) $\Rightarrow$ CA = CB = 4
So, the centre of the circle is (4,4) and its radius is 4 units.
$\therefore$ the equation of the circle is $(x-4)^2 + (y-4)^2 = 16$
So, statement I alone is sufficient.
From statement II, B (0, 4) $\Rightarrow$ CB = CA.
So, the centre of the circle is (4, 4) and its radius is 4 units.
So, the equation of the circle is $(x-4)^2 + (y-4)^2 = 16$.
$\therefore$Statement II alone is sufficient. Hence, either of the statements alone is sufficient to answer the question.
Choice (B)

**39.** Clearly, either of the statements alone is not sufficient to answer the question. Combining both the statements, $\Delta$ABC is an equilateral triangle and the vertex is A (–3, 5), the mid point of BC is D (5, 2).




So, the height of the $\Delta$ABC corresponding to the base BC is AD = $\sqrt{64+9} = \sqrt{73}$ .

$\Rightarrow$ The side a of the $\Delta$ABC = $\frac{2}{\sqrt{3}}$ AD

[$\because$ h = $\sqrt{3}/2$ a]

a = $\frac{2}{\sqrt{3}}\sqrt{73}$ units

$\therefore$The area of $\Delta$ABC = $\frac{\sqrt{3}}{4}$ a$^2$ = $\frac{\sqrt{3}}{4}$ x $\frac{4}{3}$ x 73 = $\frac{73}{\sqrt{3}}$ sq. units

Hence, both I and II together are sufficient to answer the question. Choice (C)

**40.** Clearly, either of the statements alone is not sufficient to answer the question. Combining both I and II, the equation of the line '$\ell$' is, x = 3. The line x = 3 meets the x-axis at the point (3, 0). Hence, both I and II together are sufficient to answer the question. Choice (C)

## Chapter – 7
## (Trigonometry)

### Concept Review Questions

**Solutions for questions 1 to 30:**

**1.** (i) $\frac{6\pi^c}{5} = \frac{6}{5}\times 180 = 216°$ Choice (B)

(ii) $72° = 72\times\frac{\pi}{180} = \frac{2\pi^c}{5}$ . Choice (A)

**2.** (i) sin(270° – A) = –cos A Choice (C)

(ii) sin (750°) = sin [2(360°) + 30°]
= sin30° = $\frac{1}{2}$
Choice (C)

(iii) sin25° + cos115°
= sin25° + cos (90° + 25°)
= sin25° – sin25° = 0 Choice (A)

**3.** sin$\theta$ is positive
$\Rightarrow$ $\theta$ belongs to I quadrant or II quadrant
cos$\theta$ is negative
$\Rightarrow$ $\theta$ belongs to II quadrant or III quadrant
Hence, $\theta$ belongs to II quadrant
Choice (B)

**4.** 8, 15 and 17 are Pythagorean numbers. So cosec$\theta$ = $\frac{17}{8}$ ;

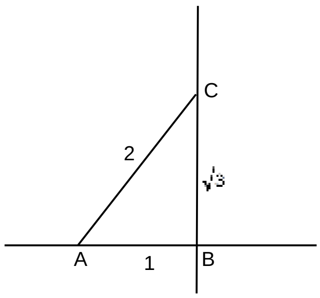
sin$\theta$ = $\frac{8}{17}$ ; cos$\theta$ = $\frac{15}{17}$

cot$\theta$ = $\frac{\cos\theta}{\sin\theta} = \frac{15}{8}$ . Choice (C)

**5.** sin$\theta$ cosec$\theta$ = sin$\theta$ $\frac{1}{\sin\theta}$ = 1 Choice (B)

**6.** Greatest side. The same can be proved using sine rule.
Choice (B)

**7.**




Let $\Delta$ABC be the given triangle and AB = 1 unit,
BC = $\sqrt{3}$ units and AC = 2 units.
Since, $(1)^2 + \left(\sqrt{3}\right)^2 = 4 = (2)^2$ i.e., AB$^2$ + BC$^2$ = AC$^2$,
$\Delta$ABC is a right-angled triangle.
$\therefore$ sin A = $\frac{BC}{AC} = \frac{\sqrt{3}}{2}$
$\Rightarrow$ A = 60°
$\therefore$ C = 90° – 60° = 30°
Hence, the angles of the triangle are 30°, 60° and 90°.
Choice (B)

**8.** sin$\theta$ = cos$\theta$ $\Rightarrow$ $\theta$ = 45° ($\because$ $\theta$ is acute)
Hence, the curves y = sin$\theta$ and y = cos$\theta$ meet at $\theta = \frac{\pi}{4}$
Choice (C)

**9.** $\sec^4\theta + \tan^4\theta - 2\sec^2\theta\tan^2\theta$
$= (\sec^2\theta)^2 + (\tan^2\theta)^2 - 2\sec^2\theta\tan^2\theta$
$= (\sec^2\theta - \tan^2\theta)^2 = (1)^2 = 1$ Ans : (1)

**10.** $\text{cosec}^4\theta + \cot^4\theta - 2\,\text{cosec}^2\theta\cot^2\theta = (\text{cosec}^2\theta - \cot^2\theta)^2$
$= 1^2 = 1$ Ans : (1)

**11.** $\sin\frac{\pi}{6}\cos\left(\frac{\pi}{3}\right) + \cos\frac{\pi}{6}\cdot\sin\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)$
$(\because \sin A\cos B + \cos A\sin B = \sin(A+B))$
$= \sin\frac{\pi}{2} = 1.$ Choice (A)

**12.** Given tan$\alpha$ = cot$\beta$ = 1
$\Rightarrow$ tan $\alpha$ = 1; cot$\beta$ = 1
$\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{4}$

$2\alpha + \beta = 2\left(\frac{\pi}{4}\right) + \frac{\pi}{4}$

$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$ Choice (A)

**13.** $\cos\theta$ is positive $\Rightarrow \theta$ belongs to I quadrant or IV quadrant.

As $\theta$ is not acute $\theta \in Q_4$. $\theta = \frac{7\pi}{4}$ ($\because \cos\theta = \frac{1}{\sqrt{2}}$)

Hence, $\tan\theta$ is negative.

$\therefore \tan\theta = \tan\frac{7\pi}{4} = -1$ Choice (B)

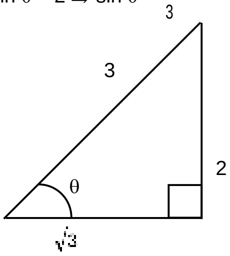
**14.** We know that, $\sin^2\theta + \cos^2\theta = 1$
$\therefore \sin^2 45° + \cos^2 45° = 1$ Ans : (1)

**15.** $\sin 30° + \sqrt{3}\tan 60° - \sec 0°$

$= \frac{1}{2} + \sqrt{3}\sqrt{3} - 1 = \frac{1}{2} + 2 = \frac{5}{2}$ Choice (D)

**16.** For $0 < \theta < \frac{\pi}{4}$, $\sin\theta < \cos\theta$ Choice (C)

**17.** Given, $3\sin\theta = 2 \Rightarrow \sin\theta = \frac{2}{3}$




Also $\theta$ is in II quadrant.

Hence, $\tan\theta = \frac{-2}{\sqrt{5}}$ Choice (A)

**18.** Given, $\text{cosec}\,\theta = -\sqrt{2}$ and $\tan\theta = -1$
$\Rightarrow \theta$ is in IV quadrant and $\theta = 360° - 45° = 315°$
$\therefore \cos\theta = \cos 315° = \cos(360° - 45°) = \cos 45° = \frac{1}{\sqrt{2}}$
Choice (B)

**19.** We know that,

for $0 \le \theta < \frac{\pi}{4}$, $\sin\theta < \cos\theta$

and for $\frac{\pi}{4} < \theta \le \frac{\pi}{2}$, $\sin\theta > \cos\theta$ Choice (A)

**20.** Options A, B and C can't be true ($\because \sin\theta$ and $\cos\theta$ lie in the range [–1, 1]). Choice (D) follows. Choice (D)

**21.** Since $-\pi \le x < \pi$,
$\sin x = 0 \Rightarrow x = -\pi$ or 0.
$\therefore$ The curve $y = \sin x$ meets the x-axis in two points viz $(-\pi, 0)$ and $(0, 0)$. Choice (B)

**22.** At any point that the graph of the function $y = \cos x$ meets the x – axis, $y = 0$ i.e., $\cos x = 0$.

When $-\pi \le x \le \pi$, $\cos x = 0$ at $x = \frac{\pi}{2}$ or $-\frac{\pi}{2}$

There are two points. Ans : (2)

**23.** $\sec^2\theta - \tan^2\theta = 1$ for all $\theta$. Choice (A)

**24.** $\text{cosec}^2\theta - \cot^2\theta = 1$ for all $\theta$. Ans : (1)

**25.** $\sec\theta = \frac{13}{5} \therefore \cos\theta = \frac{5}{13}$
$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \left(\frac{5}{13}\right)^2 = \frac{144}{169}$

As $\theta$ is acute, $\sin\theta$, $\cos\theta$, $\tan\theta$ are all positive.

$\sin\theta = +\sqrt{\frac{144}{169}} = \frac{12}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$.

Alternately, $\sec^2\theta - \tan^2\theta = 1$ for all $\theta$

$\tan^2\theta = \left(\frac{13}{5}\right)^2 - 1 = \frac{144}{25} = \left(\frac{12}{5}\right)^2$.

As $\theta$ is acute, $\tan\theta$ is positive

$\tan\theta = +\sqrt{\left(\frac{12}{5}\right)^2} = \frac{12}{5}$. Choice (A)

**26.** $\tan\theta = \frac{4}{5}$ i.e., $\frac{\sin\theta}{\cos\theta} = \frac{4}{5}$

$\frac{\sin\theta}{\cos\theta}$ is positive. $\therefore \sin\theta$ and $\cos\theta$ have the same sign.
If both $\sin\theta$ and $\cos\theta$ are positive, $\theta$ lies in $Q_1$.
If both $\sin\theta$ and $\cos\theta$ are negative, $\theta$ lies in $Q_3$.
$\theta$ lies in $Q_1$ or $Q_3$
Alternately, since $\tan\theta$ is positive, $\theta$ lies in $Q_1$ or $Q_3$.
Choice (D)

**27.** The range of $\sin\theta$ is [–1, 1].
$\therefore$ The range of $2\sin\theta$ is [–2, 2] Choice (D)

**28.** $\cot\theta + \tan\theta = 2$

$\frac{1}{\tan\theta} + \tan\theta = 2$

$\tan^2\theta - 2\tan\theta + 1 = 0$
$(\tan\theta - 1)^2 = 0$ i.e., $\tan\theta = 1$
$\sec^2\theta = 1 + \tan^2\theta = 1 + 1^2 = 2$
$\sec\theta = \pm\sqrt{2}$

$\cos\theta = \frac{1}{\sec\theta} = \pm\frac{1}{\sqrt{2}}$ Choice (C)

**29.** $\cot\theta$ is negative.
$\tan\theta$ is also negative ($\tan\theta = \frac{1}{\cot\theta}$)($\therefore \theta$ lies in $Q_2$ or $Q_4$.)
$\sin\theta$ is positive. ($\therefore \theta$ lies in $Q_1$ or $Q_2$.)
$\therefore \theta$ lies in $Q_2$. Choice (B)

**30.** $\sin^2\theta + \cos^2\theta = 1$ for all $\theta$. Ans : (1)

**Exercise – 7(a)**

**Solutions for questions 1 to 30:**

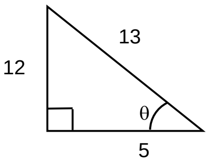
**1.** $\pi = 180°$

$\Rightarrow 225° = \frac{225° \times \pi}{180°} = \frac{5\pi^c}{4}$ Choice (C)

**2.** $\cos 28° + \cos 65° + \cos 115° + \cos 240° + \cos 208° + \cos 300°$
$\Rightarrow \cos 28° + \cos 65° - \cos 65° - \cos 60° - \cos 28° + \cos 60° = 0$. Ans : (0)

**3.** $\frac{3\pi}{2}$ radius = $\frac{3}{2}$ (180°) = 270° A minute hand covers 360° in 60 minutes
So, the time which it sweeps after covering 270°
= $\frac{270}{360} \times 60$ = 45 min
$\therefore$ The clock shows 12:45 p.m. now. Choice (C)

**4.** 13 sinθ = 12




$\Rightarrow$ sinθ = $\frac{12}{13}$ ; cosθ = $\frac{5}{13}$ ;
secθ = $\frac{13}{5}$ ;
cosec θ = $\frac{13}{12}$ ; cotθ = $\frac{5}{12}$
and tanθ = $\frac{12}{5}$

$$\therefore \frac{\text{cosec}\theta + \cot\theta}{\tan\theta + \sec\theta} = \frac{\frac{13}{12} + \frac{5}{12}}{\frac{12}{5} + \frac{13}{5}} = \frac{18}{25} \times \frac{5}{12} = \frac{3}{10}$$

Choice (B)

**5.** 1 hour = 12 revolutions
60 minutes = 12 × $2\pi^c$
1 minute = $\frac{12 \times 2\pi}{60} = \frac{2\pi^c}{5}$
20 minutes = $\frac{2\pi}{5}(20) = 8\pi$. Choice (A)

**6.** $3\cos^2A = \frac{1}{2} + \left(\frac{1}{\sqrt{2}}\right)^2$
$3\cos^2A = \frac{1}{2} + \frac{1}{2}$
$\cos^2A = \frac{1}{3}$
$\sec^2A = 3$. Ans : (3)

**7.** $3\left(\frac{1}{2} + \frac{1}{2}\right) - 3\left(\frac{1}{2} + \frac{1}{4}\right) = 3 - 3 \cdot \frac{3}{4}$
$= 3\left[1 - \frac{3}{4}\right] = 3 \cdot \frac{1}{4} = \frac{3}{4}$. Choice (C)

**8.** (1 + tanθ + secθ) (1 + secθ – tanθ) – 2 secθ
= [(1 + secθ) + tanθ] [(1 + secθ) – tanθ] – 2 secθ
= $(1 + \sec\theta)^2 - \tan^2\theta - 2\sec\theta$
= $1 + \sec^2\theta + 2\sec\theta - \tan^2\theta - 2\sec\theta$
= $1 + (\sec^2\theta - \tan^2\theta)$
= 1 + 1 = 2 Ans : (2)

**9.** $\cos\theta + \cos^2\theta = 1 \Rightarrow \cos\theta = 1 - \cos^2\theta = \sin^2\theta$ and
$\cos^2\theta = (\sin^2\theta)^2 = \sin^4\theta$
$\therefore \sin^2\theta + \sin^4\theta = \cos\theta + \cos^2\theta = 1$. Ans : (1)

**10.** In the figure given, ABCD is a cyclic quadrilateral
$\Rightarrow$ A + C = 180° = B + D
$\Rightarrow$ tanA = tan (180°– C) = –tanC
$\therefore$ tanB = tan (180° – D) = –tanD
$\Rightarrow$ tanA + tanB = –(tanC + tanD)
$\Rightarrow$ tanA + tanB + tanC = –tanD
Given, tanA + tanB + tanC = 5 $\Rightarrow$ –tanD = 5
$\therefore$ tanD = –5 Choice (D)

**11.** Given, α +β = 180°
$\Rightarrow$ secα = sec (180° – β) = –secβ
$\Rightarrow$ secα + secβ = 0
secα and secβ are the roots of the equation $ax^2 + bx + c = 0$
$\Rightarrow$ secα + secβ = $\frac{-b}{a} = 0$ $\therefore$ b = 0 Choice (C)

**12.** (i) Given, secθ + tanθ = m
$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = m$
Squaring on both the sides, we get
$\Rightarrow \frac{(1+\sin\theta)^2}{\cos^2\theta} = m^2$
$\Rightarrow \frac{(1+\sin\theta)^2}{(1-\sin^2\theta)} = m^2$
$\Rightarrow \frac{1+\sin\theta}{1-\sin\theta} = m^2 \Rightarrow \sin\theta = \frac{m^2-1}{m^2+1}$
$\therefore \cos\theta = \sqrt{1-\left(\frac{m^2-1}{m^2+1}\right)^2} = \frac{2m}{m^2+1}$

**Alternative solution:**
Given, secθ + tanθ = m ---------- (1)
$\Rightarrow$ secθ – tanθ = $\frac{1}{m}$ --------(2)
Adding (1) and (2), we get
2secθ = m + $\frac{1}{m}$ $\Rightarrow$ secθ = $\frac{m^2 + 1}{2m}$
$\therefore$ cosθ = $\frac{2m}{m^2 + 1}$ Choice (D)

(ii) Given, cosecθ – cotθ = p
$\therefore$ cosecθ + cotθ = $\frac{1}{\text{cosec}\,\theta - \cot\theta} = \frac{1}{p}$
Choice (D)

**13.** Given, a = 2, b = $3\sqrt{3}$ and c = 7
cosC = $\frac{a^2 + b^2 - c^2}{2ab}$
= $\frac{4 + 27 - 49}{2 \times 2 \times 3\sqrt{3}}$
= $\frac{-18}{12\sqrt{3}} = \frac{-\sqrt{3}}{2}$.
$\therefore \angle$C = 150°. Ans : (150°)

**14.** Given, x = secθ; and y = tanθ
$\therefore \sqrt{\frac{x-1}{x+1}} - \sqrt{\frac{x+1}{x-1}} = \frac{(x-1)-(x+1)}{\sqrt{x^2-1}}$
$= \frac{-2}{\sqrt{x^2-1}} = \frac{-2}{\tan\theta} = \frac{-2}{y}$ Choice (C)

**15.** sinθ + cosθ = $\sqrt{2}$
It is possible only when θ = 45°
$\tan^n\theta + \cot^n\theta = \tan^n45 + \cot^n45 = 1 + 1 = 2$ Ans : (2)

**16.** Given: sin 12° sin 48° sin 54°
$= \frac{\sin12°\sin(60°-12°)\sin(60°+12°)\sin54°}{\sin72°}$
(We know that sinA . sin(60° + A) sin(60° – A)
$= \frac{1}{4}\sin3A$)
$= \left(\frac{1}{4}\right)\frac{\sin3(12°)\sin54°}{\sin72°}$
$= \left(\frac{1}{8}\right)\frac{2\sin36°\cos36°}{\sin72°} = \frac{\sin72°}{8.\sin72} = \frac{1}{8}$ Choice (B)

**17.** If A + B = 45° then cot (A + B) = cot 45

$\frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$

$\Rightarrow$ cot A cot B – 1 = cot B + cot A

$\Rightarrow$ – 1 = cot A + cot B – cot A cot B

–2 = cot A (1 – cot B) – 1 (1 – cot B)

–2 = (cot A – 1) (1 – cot B)

Or (1 – cot A) (1 – cot B) = 2

$\therefore$ (1 – cot 4°) (1 – cot 41°) (1 – cot 5°) (1 – cot 40°) - - - - - - - - - - (1 – cot 22) (1 – cot 23) = $2^P$

i.e. $2^{19} = 2^P \Rightarrow P = 19$ Ans : (19)

**18.** $\sqrt{2-\sqrt{2-\sqrt{2-2\cos 2\theta}}} = \sqrt{2-\sqrt{2-\sqrt{2(2\sin^2\theta)}}}$

$= \sqrt{2-\sqrt{2-2\sin\theta}}$ Choice (C)

**19.** (i) $\cos^2\theta + \sin^4\theta = \sin^4\theta + 1 - \sin^2\theta$

$= \left\{(\sin^2\theta)^2 - 2.\sin^2\theta \times \frac{1}{2} + \left(\frac{1}{2}\right)^2\right\} + 1 - \left(\frac{1}{2}\right)^2$

$= \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

When $\sin^2\theta = 0$ the expression is maximum

Maximum value = $\left(\frac{-1}{2}\right)^2 + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = 1$

When $\sin^2\theta = \frac{1}{2}$ the expression is minimum.

Minimum value = $0 + \frac{3}{4} = \frac{3}{4}$

$\therefore$ The range = $\left[\frac{3}{4}, 1\right]$ Choice (A)

(ii) $\sin^2 x - \cos 2x = \frac{1-\cos 2x}{2} - \cos 2x$

$= \frac{1-3\cos 2x}{2}$

The above expression will take the maximum value if cos2x is – 1 $\frac{1-3\cos 2x}{2} = \frac{1+3}{2} = 2$ Choice (B)

(iii) Min = $-\sqrt{(3)^2 + (-4)^2} = -5$

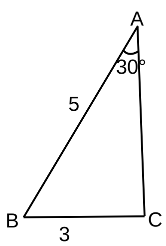
Choice (A)

**20.** We know that,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \frac{3}{\sin 30°} = \frac{5}{\sin C}$

$\Rightarrow \sin C = \frac{5\left(\frac{1}{2}\right)}{3} = \frac{5}{6}$




Choice (D)

**21.** We know that,

$a^2 = b^2 + c^2 - 2bc \cos A$

$\Rightarrow a^2 = 4^2 + 3^2 - 2 \times 4 \times 3 \times \cos 60$

$\Rightarrow a^2 = 13 \therefore a = \sqrt{13}$ units

Choice (C)

**22.** (i) We note that this graph is above x-axis. So it takes the form y = $|f(x)|$ . It resembles sinx as it passes through the origin. The function equals zero at x = $\frac{\pi}{3}$

$\Rightarrow$ The function resembles sin3x.

Also the maximum value of y is 2.

$\therefore$ The function y = $2|\sin 3x|$ is most appropriate.

Choice (D)

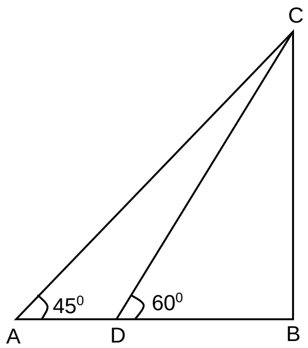
(ii) The graph represents that of x = cosy.

But x = 0 at y = $\frac{\pi}{4}$ and x = 1 at y = $\frac{\pi}{2}$

$\therefore$ The function x = –cos 2y best describes the graph.

Choice (B)

**23.**




Let the height of the lighthouse be h.

Distance covered by the man in 5 minutes = 25 m.

AD = 25 m

AB – DB = 25 m

$\frac{BC}{\tan 45^0} - \frac{BC}{\tan 60^0} = 25$ m

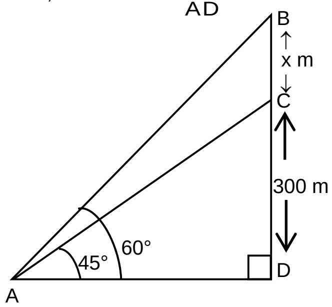
i.e., $\frac{h}{1} - \frac{h}{\sqrt{3}} = 25 \Rightarrow h(\sqrt{3}-1) = 25\sqrt{3}$

$\Rightarrow h = \frac{25\sqrt{3}}{\sqrt{3}-1} = \frac{25\sqrt{3}(\sqrt{3}+1)}{2} = \frac{25(3+\sqrt{3})}{2}$.

Choice (A)

**24.** Let the height of the flag post be x metres

In $\Delta$ACD, tan45° = 1 = $\frac{300}{AD} \Rightarrow$ AD = 300 m



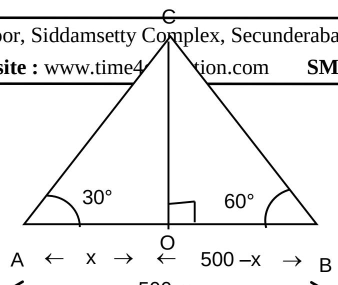

In $\Delta$DAB, tan60° = $\sqrt{3} = \frac{(300+x)}{300}$

$\Rightarrow x = 300\ (\sqrt{3}-1)$ m = 219.6 m

$\therefore$ The height of the flag post is 219.6 m Ans : (219.6)

**25.** Let the pole be at a distance of x m from the point A.





In $\Delta$AOC, $\tan 30° = \frac{1}{\sqrt{3}} = \frac{OC}{x}$

In $\Delta$COB,

$\tan 60° = \sqrt{3} = \frac{OC}{(500 - x)}$

$\Rightarrow \frac{x}{\sqrt{3}} = (500 - x)\sqrt{3}$

$\Rightarrow x = 1500 - 3x$

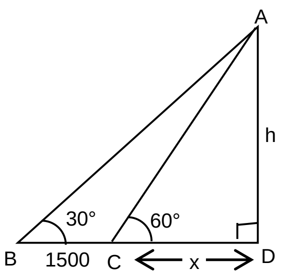
$\Rightarrow 4x = 1500$

$\Rightarrow x = 375$ m

$\therefore$ The pole is at a distance of 375 m from the point A

Ans: (375)

**26.** Let the height of the tower be h metres. In $\Delta$ADC,




$\tan 60° = \frac{AD}{CD}$

$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

In $\Delta$ADB, $\tan 30° = \frac{AD}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{1500 + x}$

$1500 + x = h\sqrt{3}$

$= \sqrt{3}\,x.\sqrt{3}$

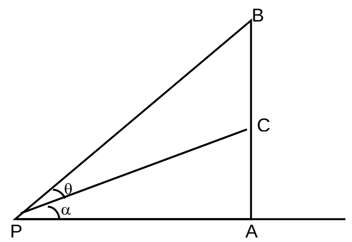
$1500 + x = 3x \Rightarrow 1500 = 2x$

$750 = x$

$\therefore$ The height of the tower is $\sqrt{3}x = 750\sqrt{3}$ m

Choice (B)

**27.**




AC = CB

In $\Delta$PAC, $\tan\alpha = \frac{AC}{AP} = \frac{AB}{2AP} = \frac{1}{2n}$

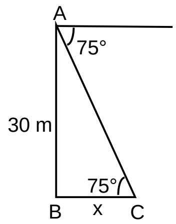
In $\Delta$PAB, $\tan(\alpha + \theta) = \frac{AB}{AP} = \frac{1}{n}$

$\Rightarrow \frac{\tan\alpha + \tan\theta}{1 - \tan\alpha.\tan\theta} = \frac{1}{n} \Rightarrow \frac{\frac{1}{2n} + \tan\theta}{1 - \frac{1}{2n}\tan\theta} = \frac{1}{n}$

$\Rightarrow \frac{1}{2n} + \tan\theta = \frac{1}{n} - \frac{1}{2n^2}\tan\theta \Rightarrow \frac{1}{2n} = \left(1 + \frac{1}{2n^2}\right)\tan\theta$

$\therefore \cot\theta = \frac{2n^2 + 1}{n}$ Choice (A)

**28.**




$\tan 75° = \frac{30}{x}$

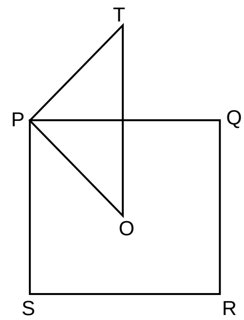
$\Rightarrow x = 30\cot 75° = 30\left(2 - \sqrt{3}\right)$

$= 30\,(2 - 1.732) = 30\,(0{\cdot}268)$

$x = 8.04$ m

Ans: (8.04)

**29.**




Let the side of the square be a. Let OT be the flagstaff

$4a = 240 \Rightarrow a = 60.$

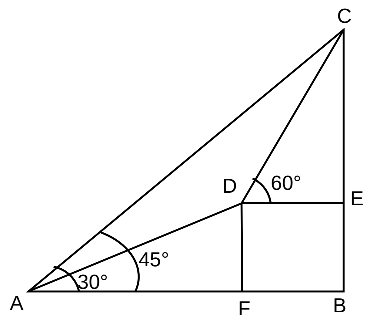
$OP = \frac{\text{Diagonal of PQRS}}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}} = \frac{60}{\sqrt{2}}$

$\text{Tan}\angle OPT = \frac{OT}{OP}$

$\text{Tan } 60° = \frac{OT\sqrt{2}}{60} \Rightarrow OT = \frac{60}{\sqrt{2}}\sqrt{3} = 30\sqrt{6}$

Choice (B)

**30.**




Let D be the point up to which the man goes.

Let F be the point vertically below D which is in line with A, the initial position of the man.

The man covers AD.

Let AD = x.

AB = AF + FB = x cos 30° + DE = x cos 30° + CE cot 60° =

$\frac{x\sqrt{3}}{2} + \frac{CE}{\sqrt{3}} =$

$$\frac{x\sqrt{3}}{2} + \frac{(CB - EB)}{\sqrt{3}} = \frac{x\sqrt{3}}{2} + \frac{(40 - DF)}{\sqrt{3}}$$

$$= \frac{x\sqrt{3}}{2} + \frac{40 - x\sin 30°}{\sqrt{3}}$$

Also, AB = BC tan 45°, BC = 40.

$$\therefore BC = \frac{x\sqrt{3}}{2} + \frac{40 - \frac{x}{2}}{\sqrt{3}} = 40$$

$3x + 80 - x = 80\sqrt{3}$

$2x = 80(\sqrt{3} - 1)$

$x = 40(\sqrt{3} - 1)$ Choice (C)

**Exercise – 7(b)**

**Solutions for questions 1 to 45:**

**1.** We know that, in 1 revolution the wheel makes an angle of $2\pi$. Given that in 1 minute (60 sec), the wheel makes 180 revolutions.
$\therefore$ In 5 sec the wheel makes 15 revolutions.
$\therefore$ The angle made by the wheel. $= 15 \cdot 2\pi = 30\pi$.
Choice (B)

**2.** Given, $\theta = 30°$
$\cos2\theta \cdot \text{cosec}3\theta - \sec2\theta \tan\theta$
$= \cos60° \text{ cosec}90° - \sec60° \tan30°$
$= \frac{1}{2}(1) - 2\left(\frac{1}{\sqrt{3}}\right) = \frac{\sqrt{3}-4}{2\sqrt{3}}$. Choice (C)

**3.** Given, $\Delta ABC$ is a right-angled isosceles triangle,
$\therefore$ If one of the angle is 90°, then the other two angles will be 45° each.
$\therefore \sin A + \sin B + \sin C = \sin 90° + \sin 45° + \sin 45°$
$= 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 1 + \sqrt{2}$. Choice (A)

**4.** $1 + 8\sin^2x^2\cos^2x^2 = 1 + 2(2\sin x^2\cos x^2)^2$
$= 1 + 2\sin^2 2x^2$
$= 1 + (1 - \cos4x^2) = 2 - \cos4x^2$
Minimum value is $c - \sqrt{a^2 + b^2}$ here, c = 2, a = 1, b = 0
$\therefore$ The required minimum value is 2 – 1 = 1. Ans : (1)

**5.** Given, $\tan\theta = \frac{\sin 2\theta}{1 + \cos 2\theta}$

substitute $\theta = 22\frac{1}{2}^{\circ}$

Then, $\tan 22\frac{1}{2}^{\circ} = \frac{\sin 45°}{1 + \cos 45°}$

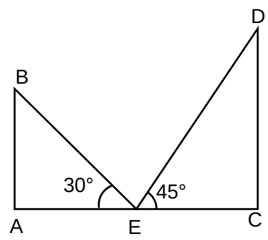
$$= \frac{\left(\frac{1}{\sqrt{2}}\right)}{1 + \left(\frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2}+1} = \frac{(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \sqrt{2} - 1$$

Hence, $\tan 22\frac{1}{2}^{\circ} = \sqrt{2} - 1$. Choice (B)

**6.** cos20° cos40° cos80°
We know that $\cos A \cos(60° - A)\cos(60° + A) = \frac{1}{4}\cos3A$.
cos20° cos(60 – 20°) cos(60° + 20°)
$= \frac{1}{4}\cos(3 \times 20) = \frac{1}{4}\cos 60° = \frac{1}{8}$. Choice (D)

**7.** Let us assume that $\Delta ABC$ is right angled at A
$\Rightarrow$ cosecA = cosec 90° = 1
$\Rightarrow$ log cosec 90° = log1 = 0.
$\therefore$ (log cosecA) (log cosecB) (log cosecC) = 0 Ans : (0)

**8.** Let AB and CD be the towers.
Given, AC = 600m
Let AB = $h_1$ and CD = $h_2$
In $\Delta EAB$,




$\tan30° = \frac{AB}{AE}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h_1}{AE}$

$AE = h_1\sqrt{3}$
In $\Delta ECD$,
$\tan45° = \frac{CD}{EC}$

$1 = \frac{h_2}{EC}$ EC = $h_2$.

$\frac{AE}{EC} = \frac{h_1\sqrt{3}}{h_2} = \frac{1}{1}$

$\frac{h_1}{h_2} = \frac{1}{\sqrt{3}}$

$\therefore h_1 : h_2 = 1 : \sqrt{3}$ Choice (A)

**9.** Given, $\tan\theta + \cot\theta = 3$
$\sec^2\theta + \text{cosec}^2\theta = 1 + \tan^2\theta + 1 + \cot^2\theta$
$= 2 + \tan^2\theta + \cot^2\theta = (\tan\theta + \cot\theta)^2 = (3)^2 = 9$.
Ans : (9)

**10.** Given, $\cos\theta + \sec\theta + \cos^2\theta + \sec^2\theta = 0$
$\Rightarrow \cos\theta + \sec\theta + (\cos\theta + \sec\theta)^2 - 2\cos\theta\sec\theta = 0$.
$(\because a^2 + b^2 = (a + b)^2 - 2ab)$
$\Rightarrow (\cos\theta + \sec\theta) + (\cos\theta + \sec\theta)^2 - 2 = 0$
Let, $\cos\theta + \sec\theta = x$
$\Rightarrow x + x^2 - 2 = 0$
$\Rightarrow x = -2$ or $x = 1$
$\therefore \cos\theta + \sec\theta = -2$ (or) $\cos\theta + \sec\theta = 1$
But, $\cos\theta + \sec\theta \geq 2$ (or) $\leq -2$
Hence, $\cos\theta + \sec\theta = -2$
$\Rightarrow \cos\theta = \sec\theta = -1$
$\Rightarrow \theta = \pi$
$\therefore \tan\theta = \tan\pi = 0$. Ans : (0)

**11.** Given, $\text{cosec}\theta$ and $\cot\theta$ are the roots of $cx^2 + bx + a = 0$.
$\therefore \text{cosec}\theta + \cot\theta = \frac{-b}{c}$

$\Rightarrow \text{cosec}\theta - \cot\theta = \frac{-c}{b}$.

$\text{cosec}\theta \cdot \cot\theta = \frac{a}{c}$

$(a + b)^2 - (a - b)^2 = 4ab$
$(\text{cosec}\theta + \cot\theta)^2 - (\text{cosec}\theta - \cot\theta)^2 = 4\,\text{cosec}\theta \cdot \cot\theta$

$\frac{b^2}{c^2} - \frac{c^2}{b^2} = \frac{4a}{c}$

$\Rightarrow \frac{b^4 - c^4}{b^2c^2} = \frac{4a}{c}$

$\Rightarrow b^4 - c^4 = 4ab^2c$
$\Rightarrow b^4 = 4ab^2c + c^4$. Choice (A)

**12.** Given, $\tan\theta = \sqrt{\frac{1 - \cos2\theta}{1 + \cos2\theta}}$

Put $\theta = 67\frac{1}{2}^{o}$

$\tan 67\frac{1}{2}^{o} = \sqrt{\frac{1-\cos 135^{\circ}}{1+\cos 135^{\circ}}} = \sqrt{\frac{1-\left(\frac{-1}{\sqrt{2}}\right)}{1+\left(\frac{-1}{\sqrt{2}}\right)}}$

$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \sqrt{3+2\sqrt{2}} = \sqrt{2}+1$ Choice (D)

**13.** Given, $1 + \sec\theta + \tan\theta = p$
$\sec\theta + \tan\theta = p - 1$ -------→ (1)
$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p-1}$ ----→ (2)
Adding (1) & (2), we get

$2\sec\theta = \frac{(p-1)^2+1}{p-1}$

$\Rightarrow \sec\theta = \frac{p^2-2p+2}{2(p-1)}$

$\Rightarrow \cos\theta = \frac{2(p-1)}{p^2-2p+2}$. Choice (B)

**14.** Given, $3\tan\theta - 4 = 0$, $3\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$

$\sin\theta = \frac{4}{5}$, $\mathrm{cosec}\theta = \frac{5}{4}$,

$\cot\theta = \frac{3}{4}$, $\sec\theta = \frac{5}{3}$.

$\therefore \frac{3\sec\theta + 2\mathrm{cosec}\theta}{\cot\theta - 5\sin\theta}$

$= \frac{3\left(\frac{5}{3}\right)+2\left(\frac{5}{4}\right)}{\frac{3}{4}-5\left(\frac{4}{5}\right)}$

$= \frac{5+\frac{5}{2}}{\frac{3}{4}-4} = \frac{\frac{15}{2}}{\frac{-13}{4}} = \frac{-30}{13}$. Choice (C)

**15.** $\left[\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1} + \frac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}\right]\sin x$

$= \left[\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x}\right]\sin x$

$= \frac{\sin^2 x + \cos^2 x + 1 + 2\cos x}{[1+\cos x]}$

$= \frac{2[1+\cos x]}{[1+\cos x]} = 2.$

Ans : (2)

**16.** Given, $\frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x} = k - \sin x\cos x$

$\Rightarrow \frac{\sin^2 x}{1+\frac{\cos x}{\sin x}} + \frac{\cos^2 x}{1+\frac{\sin x}{\cos x}} = k - \sin x\cos x$

$\Rightarrow \frac{\sin^3 x}{\sin x + \cos x} + \frac{\cos^3 x}{\cos x + \sin x} = k - \sin x\cos x$

$\Rightarrow \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = k - \sin x\cos x$

$\Rightarrow \frac{(\sin x + \cos x)(\sin^2 x - \sin x\cos x + \cos^2 x)}{(\sin x + \cos x)}$
$= k - \sin x\cos x$

[$\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]
$\Rightarrow \sin^2x + \cos^2x - \sin x\cos x = k - \sin x\cos x$
$\Rightarrow 1 - \sin x\cos x = k - \sin x\cos x \Rightarrow k = 1$

Ans : (1)

**17.** $\left[\frac{\frac{\cos x}{\sin x}}{\frac{1+\sin x}{\sin x}} + \frac{\frac{1+\sin x}{\sin x}}{\frac{\cos x}{\sin x}}\right]\cos x = \left[\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}\right]\cos x$

$= \left[\frac{\cos^2 x + 1 + \sin^2 x + 2\sin x}{(1+\sin x)\cos x}\right]\cos x$

$= \frac{2+2\sin x}{1+\sin x} = 2$ Ans : (2)

**18.** $\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}$

$= \frac{1-\sin x}{\cos x}$

$= \sec x - \tan x$ Choice (A)

**19.** $\sin A = 1/3$
$\tan B = 3/4$
$9\cos^2A + 20\sec B$

$= 9.\frac{8}{9} + 20.(-5/4)$

$= 8 - 25 = -17$ Ans : (–17)

**20.** $\sin\theta + \frac{1}{\sin\theta} = 2 \Rightarrow \sin\theta = 1.$

So $\sin^4\theta + \cos^4\theta = 1 + 0 = 1$ Choice (D)

**21.** We know that 17° + 28° = 45°
$\tan(17° + 28°) = \tan 45°$
$\frac{\tan 17° + \tan 28°}{1-\tan 17°\tan 28°} = 1 \Rightarrow \tan 17° + \tan 28° = 1 - \tan 17°\tan 28°$
$\tan 17° + \tan 28° + \tan 17°.\tan 28° = 1$ Ans : (1)

**22.** $\frac{\sin\theta}{\sqrt{2}} + \frac{\cos\theta}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$
$\sin(\theta + 45) = 1$
$\theta + 45 = 90$
$\theta = 45°$
$\sin^2\theta - \cos^2\theta = (1/\sqrt{2})^2 - (1/\sqrt{2})^2 = 0$ Ans : (0)

**23.** $(\sin^2 x)^3 + (\cos^2 x)^3 + 3\sin^2 x\cos^2 x .1$
$= (\sin^2x)^3 + (\cos^2x)^3 + 3\sin^2x\cos^2x(\sin^2 x + \cos^2x)$
$= (\sin^2x + \cos^2x)^3 = 1^3 = 1$ Ans : (1)

**24.** $\frac{\cos x}{\frac{\cos x - \sin x}{\cos x}} + \frac{\sin x}{\frac{\sin x - \cos x}{\sin x}}$

$= \frac{\cos^2 x - \sin^2 x}{(-\sin x + \cos x)}$

$= \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x - \sin x)}$

$= \sin x + \cos x$ Choice (A)

**25.** Let a = 8, b = 10 and $\angle C = 45°$

Area of $\Delta ABC = \frac{1}{2}ab\sin C$

$= \frac{1}{2}\cdot 8\cdot 10\cdot \sin 45°$

$= \frac{40}{\sqrt{2}}$ sq units.

Choice (A)

**26.** The given expression is $\sqrt{29}$ sin x + $\sqrt{7}$ cos x + 4.
The maximum value and minimum value are
$4 \pm \sqrt{29+7} = 4 \pm 6 \Rightarrow 10$ and – 2
$\therefore$ The range of the given function is
$\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{10}, \infty\right)$

Choice (D)

**27.** We know that AM $\geq$ GM
$\Rightarrow \frac{4\tan^2 x + 9\cot^2 x}{2} \geq \sqrt{4\tan^2 x . 9\cot^2 x}$
$\therefore$ $4\tan^2 x + 9\cot^2 x \geq 12$
$\therefore$ The minimum value of the function is 12.

Ans : (12)

**28.** We know AM (a, b) $\geq$ GM (a, b)
AM $\left(81^{\cos^2 x}, 81^{\sin^2 x}\right) \geq$ GM $\left(81^{\cos^2 x}, 81^{\sin^2 x}\right)$
$\frac{81^{\cos^2 x} + 81^{\sin^2 x}}{2} \geq \sqrt{81^{\cos^2 x} \times 81^{\sin^2 x}}$

$81^{\cos^2 x} + 81^{\sin^2 x} \geq 2\sqrt{81^{\cos^2 x + \sin^2 x}} = 18.$

$\therefore$ The minimum value of $81^{\cos^2 x} + 81^{\sin^2 x}$ is 18. It occurs when $\cos^2 x = \sin^2 x = 1/2$.

Ans : (18)

**29.** $\frac{\sin 2A}{1+\cos 2A} \frac{(1-\cos 2A)}{\sin A} \cos A$

$= \frac{(2\sin A \cos A)}{(2\cos^2 A)} . \frac{(2\sin^2 A).\cos A}{\sin A}$

$= 2\sin^2 A$

Choice (D)

**30.** $a\cos\alpha + b\sin\alpha = c$
$\Rightarrow a\cos\alpha = c - b\sin\alpha$
$\Rightarrow a^2\cos^2\alpha = c^2 + b^2\sin^2\alpha - 2bc\sin\alpha$
$\Rightarrow a^2 - a^2\sin^2\alpha = c^2 + b^2\sin^2\alpha - 2bc\sin\alpha$
$\Rightarrow (b^2 + a^2)\sin^2\alpha - 2bc\sin\alpha + c^2 - a^2 = 0$
If $\sin\theta_1$ and $\sin\theta_2$ are the roots of this equation, then
The sum of the roots = $\sin\theta_1 + \sin\theta_2 = \frac{2bc}{a^2+b^2}$

Choice (A)

**31.** $\log_3(\sin^2\theta + \cos^2\theta) = \log_3 1 = 0$. Similarly,
$\log_5(\sin^2\theta + \cos^2\theta) = 0$
$\therefore$ The required value is 0.

Ans : (0)

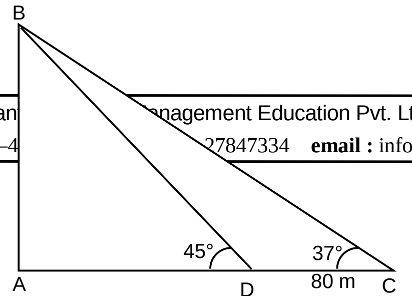
**32.** When $x = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$
y is zero.
when $x = 0, \frac{\pi}{2}, \pi, \ldots\ldots$
y is one.
In given choices,
only $y = |\cos 2x|$ satisfies both conditions.

Choice (D)

**33.** When $y = \frac{\pi}{2}$, $x = -1$.
This condition is satisfied only by $x = -\sin y$.

Choice (C)

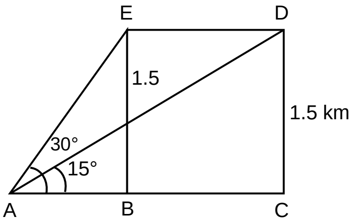
**34.**




Let AB be the lighthouse and C and D denote the positions of the two ships.
CD = 80 m
Since, $\angle ADB = 45°$, AB = AD
$\therefore$ AC = AD + 80 = AB + 80
Now, $\tan 37° = \frac{AB}{AC}$
$\Rightarrow \frac{3}{4} = \frac{AB}{AB+80}$ $(\because \sin 37° = 0{\cdot}6 = \frac{3}{5})$
$\Rightarrow 3AB + 3 \times 80 = 4AB$
$\therefore$ AB = 240 m.

Ans : (240)

**35.** Let E be the initial position of the aeroplane at an altitude of 1.5 km and after 20 sec, let the position be at D.

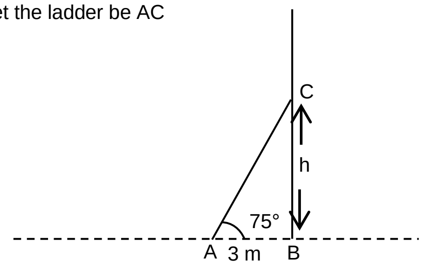



In $\Delta$ABE,
$\tan 30° = \frac{BE}{AB} = \frac{1}{\sqrt{3}} = \frac{1.5}{AB}$
$\Rightarrow AB = 1.5\sqrt{3}$
In $\Delta$ACD,
$\tan 15° = \frac{CD}{AC}$
$\Rightarrow 2 - \sqrt{3} = \frac{1.5}{AC}$
$AC = \frac{1.5}{2-\sqrt{3}} = 1.5(2+\sqrt{3})$ km
Now ED = BC = AC − AB
$= 1.5(2+\sqrt{3}) - 1.5\sqrt{3} = 3$ km
$\therefore$ The distance travelled by the aeroplane in 20 sec is 3 km

$\therefore$ Speed $= \frac{\text{distance}}{\text{time}} = \frac{3}{\frac{20}{60\times 60}}$ kmph
$= \frac{3\times 60\times 60}{20} = 540$ kmph.

Ans : (540)

**36.** Let the ladder be AC

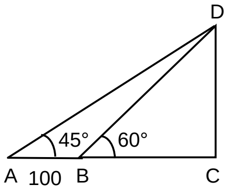



$\tan 75° = \frac{h}{3} \Rightarrow 2 + \sqrt{3} = \frac{h}{3}$

$3\left(2+\sqrt{3}\right)$ = h

$\therefore$ The tip of the ladder is at a height of 3 $(\sqrt{3}+2)$ m from the ground. Choice (D)

**37.** $\tan 60° = \frac{y}{x}$ $\quad \sqrt{3} = \frac{y}{x}$




$x = \frac{y}{\sqrt{3}} \rightarrow \quad (1)$

$\tan 45° = \frac{y}{100+x}$

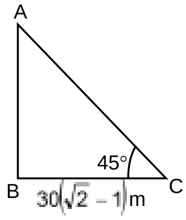
$100 + x = y \Rightarrow 100+\frac{y}{\sqrt{3}} = y$ (from (1))

$100 = y-\frac{y}{\sqrt{3}} \Rightarrow y\left[1-\frac{1}{\sqrt{3}}\right] = 100$

$y = \frac{100\sqrt{3}}{\sqrt{3}-1}\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = \frac{100\sqrt{3}(\sqrt{3}+1)}{2}$

$y = 50[3 + \sqrt{3}\,]$ m. Choice (B)

**38.**




From the figure, AB + AC represents the height of the pole and AC is the broken part of the pole.

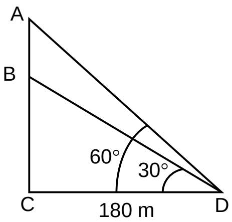
tan 45° = AB/BC = AB/ $30(\sqrt{2}-1)$

$\Rightarrow$ AB = 30 m and cos 45° = BC/AC

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{30\left(\sqrt{2}-1\right)}{AC} \Rightarrow AC = 30\sqrt{2}(\sqrt{2}-1)$

$\therefore$ Height of the pole = AB + AC = 30 m Ans : (30)

**39.** Let BC represent the tower, AB the flag post and D be the point of observation.




Given CD = 180m

From triangle BCD, tan30° = BC/CD

$\Rightarrow$ BC

$= 180\left(\frac{1}{\sqrt{3}}\right) = 60\sqrt{3}m$

from $\Delta$ ACD, tan 60° = AC/CD

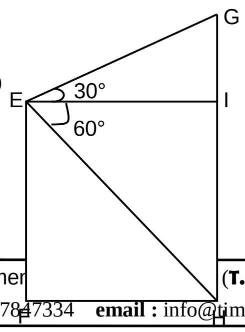
= (AB + BC) / CD

$\Rightarrow$ AB + BC = $180\left(\sqrt{3}\right)$

$\Rightarrow$ AB = $180\sqrt{3}-60\sqrt{3}$

$= 60\sqrt{3}(3-1) = 120\sqrt{3}$ m Choice (A)

**40.** GH = 120




GI + IH = 120

EI tan 30° + EI tan 60° = 120

$\frac{EI}{\sqrt{3}} + EI\sqrt{3} = 120$

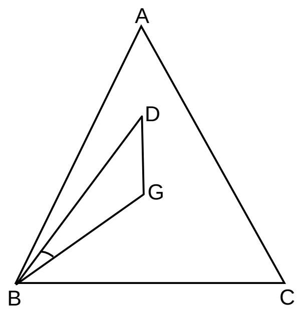
$\Rightarrow$ EI = $30\sqrt{3}$

EF = IH = EI tan $60^0$

$= 30\sqrt{3}\left(\sqrt{3}\right) = 90$

Ans : (90)

**41.**




Let G be the centroid and DG be the flagstaff.

Tan $\angle$ DBG = $\frac{DG}{BG}$ i.e., TAN 30° = $\frac{24}{BG}$

BG = $24\sqrt{3}$

BG = Distance between vertex B of the equilateral triangle AND and the centroid G

The distance between any vertex of an equilateral triangle and the centroid of the triangle is $\frac{a}{\sqrt{3}}$

$BG = \frac{a}{\sqrt{3}} = 24\sqrt{3}$ ,

a = 72. Ans : (72)

**42.** $6((\sin^2 x)^3 + (\cos^2 x)^3) - 9((\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x)$

$= 6((\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x\,(\sin^2 x + \cos^2 x)) - 9(1 - 2\sin^2 x\,\cos^2 x)$

$= 6 - 18\sin^2 x\,\cos^2 x - 9 + 18\sin^2 x\cos^2 x = -3$

**Alternate Solution:**

Put x = 90°,

6 (1 + 0) – 9 (1 + 0) = –3 Ans : (–3)

**43.** $\sin^2(120° + \theta) + \sin^2(120° - \theta)$

$= 1 - [\cos^2(120° + \theta) - \sin^2(120° - \theta)]$

$= 1 - \{\cos 240 \cos 2\theta\}$ ($\because \cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)$)

$= 1 + \frac{1}{2}\cos 2\theta$

The required range = $1 \pm \frac{1}{2}$ ie $\left[\frac{1}{2},\frac{3}{2}\right]$ Choice (C)

**44.** $\frac{\cos(90°-70°)+\sin 50°}{\sin 20°+\cos(90°-40°)} = \frac{\sin 70°+\sin 50°}{\sin 20°+\sin 40°}$

$= \frac{2\sin 60°\cos 10°}{2\sin 30°\cos 10°}$

$\left(\because \sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}\right)$

$= \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$ Choice (B)

**45.** $\sin^2(\theta - 45°) + \sin^2(\theta + 15°) - \sin^2(\theta - 15°)$

$= \sin^2(\theta - 45)° + (\sin(\theta + 15°) + \sin(\theta - 15°))\,(\sin(\theta + 15°) - \sin(\theta - 15°))$

$= \sin^2(\theta - 45°) + (\sin 2\theta.\ \sin 30°)$

$= \frac{1-\cos(90°-2\theta)}{2} + \frac{1}{2}\sin 2\theta$

$= \frac{1}{2} - \frac{1}{2}\sin 2\theta + \frac{1}{2}\sin 2\theta = \frac{1}{2}$ Choice (B)

## Chapter – 8
## (Operator Based Questions)

### Concept Review Questions

**Solutions for questions 1 to 5:**

**1.** $1 \Delta 2 = (1)^2 + (2)^2 - 1 (2) = 3$ Ans : (3)

**2.** 3 – 2 = sum of 3 and 2 = 5 Ans : (5)

**3.** Choice (D) is the sum of two squares. It has to be non-negative. Choice (D)

**4.** $(2 \leftarrow 3) \uparrow (4 \leftarrow 3) = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$
$\therefore$ Choice (B) gives an integer value. Choice (B)

**5.** $\left(\frac{1}{2}\right)^{ab}$ is positive for any two real numbers a and b.
Choice (C)

**Solutions for questions 6 to 8:**

**6.** $a \odot e = a$
$\Rightarrow a + e + ae = a$
$\Rightarrow e (1 + a) = 0$
$\Rightarrow e = 0$ ($\because a \neq -1$) Choice (A)

**7.** $a \odot 2 = 0$
$\Rightarrow a + 2 + 2a = 0$
$\Rightarrow 3a = -2$
$\Rightarrow a = \frac{-2}{3}$ Choice (D)

**8.** $a \odot 1 = a$
$\Rightarrow a + 1 + a = a$
$\Rightarrow a + 1 = 0 \Rightarrow a = -1$ Choice (B)

**Solutions for questions 9 to 12:**

**9.** $(a \oplus b) \oplus (d \oplus c)$
$= d \oplus d = b$ Choice (C)

**10.** $d^2 = d \oplus d = b$
$d^3 = b \oplus d = a$
$d^4 = a \oplus d = c$
$\therefore n = 4$ Choice (D)

**11.** $c^2 = c \oplus c = c$
$\therefore c^{100} = c$
$a = a$
$a^2 = a \oplus a = b$
$a^3 = a^2 \oplus a = b \oplus a = d$
$a^4 = a^3 \oplus a = d \oplus a = c$
Similarly $a^5, a^6, a^7, a^8$ are a, b, d, c respectively.
$\therefore a^{100} = c$
$\therefore c^{100} \oplus a^{100} = c \oplus c = c$ Choice (C)

**12.** $a \oplus b \oplus (c \oplus d) = a \oplus b \oplus d$
$= (a \oplus b) \oplus d = d \oplus d = b$
or $a \oplus (b \oplus d) = a \oplus a = b$ Choice (B)

**Solutions for questions 13 to 15:**

**13.** H [L(4,18), L(12,18)] = H (36, 36) = 36 Ans : (36)

**14.** H {H[H[H(H(64, 32), 16), 8], 4], 2}
= H {H[H[64, 8], 4], 2} = 2 Ans : (2)

**15.** L {L[L[L(L(1, 3), 6), 12], 24], 48}
= L {L[L[6, 12], 24], 48} = 48 Ans : (48)

### Exercise – 8(a)

**Solutions for questions 1 to 3:**

**1.** We know that LCM of two distinct numbers is always greater than HCF of the numbers.
$\therefore$ Choice (A) is false
$a \$ b = (a + b)^2 - (a - b)^2 = 4ab$
Since a and b are positive $a \$ b > 0$
$\therefore$ Choice (B) is true
Consider (C)
$a \Delta b + a \$ b = a^2 - b^2 + (a + b)^2 - (a - b)^2$
$= a^2 - b^2 + 4ab$
When a = 10 and b = 1 then $a^2 - b^2 + 4ab > 0$. When a = 1 and b = 10 then $a^2 - b^2 + 4ab < 0$. Choice (B)

**2.** Let p and q be positive numbers, L, G be their respective LCM and HCF.
$L \times G = p \times q$
$\therefore$ Since $a \sim b$ and a % b are the LCM and HCF of the numbers $a^3$ and $b^3$ respectively $(a \sim b)$ (a % b) must be divisible by $a^3$ and $b^3$.
Hence $(a \sim b)$ (a % b) is also divisible by $a^2$ and $b^2$
Choice (B)

**3.** Given a = 9; b = 6

Choice (A)
a % b = HCF of $a^3$ and $b^3$
= HCF of $9^3$ and $6^3 = 3^3 = 27$
$a \sim b$ = LCM of $a^3$ and $b^3$
= LCM of $9^3$ and $6^3 = 18^3$
$a \Delta b = a^2 - b^2 = 9^2 - 6^2 = 45$
$a \sigma b = (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
$= 2(9^2 + 6^2)$
= 2 (117) = 234
$(a \% b)(a \sim b) - (a \Delta b)(a \sigma b)$
$27 (18^3) - 45 (234)$ = 1, 46, 934

Choice (B)
$a \$ b = (a + b)^2 - (a - b)^2 = 4ab$
9 \$ 6 = 4 (9) (6) = 216
$9 \sigma 6 = 234$
$\sqrt{(a\$b)(a\sigma b)} = \sqrt{216(234)}$ = 6 is false

Choice (C)
$a \Delta b = a^2 - b^2$ and $a \sigma b = 2 (a^2 + b^2)$
$a^2 - b^2 > 2 (a^2 + b^2)$ is false. Choice (A)

**Solutions for questions 4 to 6:**

**4.** Given $f(x, y) = 2^{x+y}$, $g(x, y) = 2^{x-y}$
$p(x, y) = \log_2 xy$, $q(x, y) = \log_2 \left(\frac{x}{y}\right)$
$f(x, -x) = 2^{x-x} = 2^0 = 1$
$g(x, x) = 2^{x-x} = 2^0 = 1$
$q(f(x, -x), g(x, x)) = q(1, 1)$
$= \log 2 \left(\frac{1}{1}\right) = 0$ Ans : (0)

**5.** $f(3, 4) = 2^{3+4} = 2^7$
$g(4, 5) = 2^{4-5} = 2^{-1} = 1/2$
$p(f(3, 4), g(4, 5)) = p(2^7, 1/2)$
$= \log_2 2^7 . \frac{1}{2} = \log_2 2^6 = 6$ Ans : (6)

**6.** $p(5, 6) = \log_2 5 (6) = \log_2 30$
$q(6, 5) = \log_2 \left(\frac{6}{5}\right)$
and p (5, 6) + q 6,5)

$= \log_2 (30) + \log \frac{6}{5} = \log_2 36$

$\therefore$ f[g(5, 6), q(6, 5)]
$= 2^{\log_2 36} = 36$
$p(4, 5) = \log_2 (4)(5) = \log_2 20$
$q(5, 6) = \log_2 5/6$
$\therefore$ g[p(4, 5), q(5, 6)]
$\therefore$ p (4,5) – q (5, 6) = $\log_2 20 - \log_2 5/6$

$= \log_2 \frac{20(6)}{5} = \log_2 24 = 2^{\log_2 24} = 24$

$\therefore \frac{f(p(5,6), q(6,5))}{g(p(4,5), q(5,6))} = \frac{36}{24} = \frac{3}{2}$ Ans : (1.5)

**Solutions for questions 7 and 8:**

**7.** $C(10, 5) = (10 + 5)^3 = 15^3$
$D(10, 5) = (10 - 5)^3 = 5^3$
$A(10, 5) = (10 + 5)^3 + (10 - 5)^3 = 15^3 + 5^3$
$S(10, 5) = (10 + 5)^3 - (10 - 5)^3 = 15^3 - 5^3$

$\therefore$ Given $\frac{C(10,5) - D(10,5)}{A(10,5) + S(10,5)} = \frac{15^3 - 5^3}{15^3 + 5^3 + 15^3 - 5^3}$

$= \frac{3250}{2x3375} = \frac{13}{27}$ Choice (A)

**8.** When x < y; $(x-y)^3 < 0$
So D(x, y) < 0 when x < y
Option (A) is not always true.
Similarly x < y; A(x, y) < S(x, y)
Option (C) is not always true.
Given x and y are positive real numbers
x + y > 0
$\therefore$ C(x, y) > 0 Choice (B)

**Solutions for questions 9 and 10:**

**9.** Given $x^2 + y^2 + 2x + 7y + 9$

g = 1; f = $\frac{7}{2}$; c = 9

$\Delta = \sqrt{g^2 - ac} = \sqrt{1 - 1x9} = \sqrt{-8}$

$\nabla = \sqrt{f^2 - bc} = \sqrt{\left(\frac{7}{2}\right)^2 - 9} = \sqrt{\frac{49}{4} - 9} = \sqrt{\frac{13}{4}}$

Clearly $\Delta^2 < 0$; $\nabla^2 > 0$
$\therefore \Delta^2 < \nabla^2$
Choice (C)

**10.** Option A, $\Delta^2 + \nabla^2 = -8 + \frac{13}{4} = -\frac{19}{4} < 0$

Option B, $\nabla^2 - \Delta^2 = \frac{13}{4} - (-8) = \frac{45}{4} > 0$

Option C, $\Delta = \sqrt{-8}$ is not a real number

Option D, $\Delta = \sqrt{-8}$ $\nabla = \sqrt{1314}$

$\therefore \Delta \neq \nabla$ Choice (C)

**Solutions for questions 11 and 12:**

**11.** Given $a * b = a \oplus b$

$\frac{ab}{3} = a + b - ab$

$\frac{4ab}{3} = a + b \Rightarrow \frac{4}{3} = \frac{1}{b} + \frac{1}{a}$

Choice (B)

**12.** $3 * 5 = \frac{(3)(5)}{3} = 5$

$((3 * 5) \oplus 7) = (5 \oplus 7) = 5 + 7 - 35 = -23$

$(-23 * 9) = \frac{-23\ (9)}{3} = -69$

$-69 \oplus 4 = -69 + 4 + 69\ (4) = 211$ Choice (A)

**Solutions for questions 13 and 14:**

**13.** Given \$(x, y) = HCF (x, y), $\Delta$(x, y) = AM (x, y),
$\nabla$ (x, y) = LCM (x, y), $\sigma$(x, y) = quotient when x is divided by y

$\Delta(240, 180) = AM(240, 180) = \frac{240 + 180}{2} = 210$

\$($\Delta$(240,180), 70) = \$(210, 70) = HCF(210, 70) = 70
$\nabla$(70, 50) = LCM of (70, 50) = 350
$\Delta$(350, 90) = AM(350, 90)

$= \frac{350 + 90}{2} = 220$

$\sigma$(220, 10) = Quotient when 220 is divided by 10 = 22
Ans : (22)

**14.** When 0 < a, b, c, d < 1
AM > AMS > AMC
When a, b, c, d $\geq$ 1
AM $\leq$ AMS $\leq$ AMC
Option (A) is false when 1 < a, b, c, d.
Option (B) is false when 1 < a, b, c, d.
Option (C) is false when 0 < a, b, c, d < 1. Choice (D)

**Solutions for questions 15 to 17:**

**15.** $f(x, x) = \frac{a^x + a^{-x}}{2}$

$g(x, -x) = \frac{a^x - a^{-(-x)}}{2} = 0$

$\therefore q(f(x, x), g(x,-x)) = q\left(\frac{a^x + a^{-x}}{2}, 0\right)$

$= \log_a \left(\frac{a^x + a^{-x} - 0}{a^x + a^{-x} + 0}\right) = \log_a 1 = 0$ Choice (D)

**16.** $f(p(x, y), q(x, y)) = \frac{a^{\log_a \frac{x+y}{x-y}} + a^{-\left(\log_a \frac{x-y}{x+y}\right)}}{2}$

$= \frac{\frac{x+y}{x-y} + \frac{x+y}{x-y}}{2} = \frac{(x+y)}{(x-y)} \rightarrow (1)$

$g(q(x, -y), p(x, y) = \frac{a^{\log_a \frac{x+y}{x-y}} - a^{-\log_a\left(\frac{x+y}{x-y}\right)}}{2}$

$= \frac{\frac{x+y}{x-y} - \frac{x-y}{x+y}}{2} = \frac{(x+y)^2 - (x-y)^2}{2(x^2 - y^2)} = \frac{2xy}{x^2 - y^2}$

$\therefore \frac{f(p(x,y), q(x,y))}{g(q(x-y), p(x,y))} = \frac{\frac{(x+y)}{(x-y)}}{\frac{2xy}{x^2 - y^2}}$

$= \frac{(x+y)}{(x-y)} \frac{(x^2 - y^2)}{2xy} = \frac{(x+y)^2}{2xy}$ Choice (A)

**17.** $f(6, 4) = \frac{a^6 + a^{-4}}{2}$

$g(6, 4) = \frac{a^6 - a^{-4}}{2}$

$p(f(6, 4), g(6, 4) = \log_a \left(\frac{\frac{a^6 + a^{-4} + a^6 - a^{-4}}{2}}{\frac{a^6 + a^{-4} - (a^6 - a^{-4})}{2}}\right)$

$$= \log_a\left(\frac{a^6}{a^{-4}}\right) = \log_a a^{10} = 10$$

$$q(10, 4) = \log_a\left(\frac{10-4}{10+4}\right) = \log_a \frac{6}{14} = \log_a\left(\frac{3}{7}\right)$$

Choice (B)

**Solution for question 18:**

**18.** Given $(x, y) * (z, w) = (xw + yz, xz - yw)$
$\therefore (x, y) * (y, x) = (x^2 + y^2, 0)$ … (1)
$\therefore (a_1, b_1) * (b_1, a_1) = (a_1^2 + b_1^2, 0)$
and $(a_2, b_2) * (b_2, a_2) = (a_2^2 + b_2^2, 0)$
$(p, q) = (a_1^2 + b_1^2, 0) * (a_2^2 + b_2^2, 0)$
$= [(0 + 0, (a_1^2 + b_1^2)(a_2^2 + b_2^2) - 0)]$
$= [(0, (a_1^2 + b_1^2)(a_2^2 + b_2^2)]$
$\therefore$ p = 0 and $q = (a_1^2 + b_1^2)(a_2^2 + b_2^2)$
We need the value of $(p + q, pq) * (pq, p + q)$ which is $[(p + q)^2 + p^2q^2, 0]$ (from 1) $= [q^2, 0]$. In all the options, the second element is 0. We have to work out the first element
$q^2 = (a_1^2+b_1^2)^2 (a_2^2+b_2^2)^2$

Choice (C)

**Solutions for questions 19 and 20:**

**19.** $x \oplus y = x^2 + y^2$
$x \odiv y = x^4 - x^2y^2 + y^4$
$(x \oplus y)(x \odiv y) = (x^2 + y^2)(x^4 - x^2y^2 + y^4) = x^6 + y^6$
$x \Delta y = x^6 + y^6$

$$\therefore \frac{(x \oplus y)(x \odiv y)}{x \Delta y} = \frac{x^6 + y^6}{x^6 + y^6} = 1$$ Choice (C)

**20.** $x \Delta y = x^6 + y^6$ and $x \odiv y = x^4 - x^2y^2 + y^4$

$$\frac{x \Delta y}{x^4 - x^2y^2 + y^4} = \frac{x^6 + y^6}{x^4 - x^2y^2 + y^4} = x^2 + y^2$$

$$= \frac{(x^2 + y^2)(x^4 - x^2y^2 + y^4)}{x^4 - x^2y^2 + y^4} = x^2 + y^2 = x \oplus y$$

Choice (B)

**Solutions for questions 21and 22:**

**21.** $h(x, y) = f(x, y) \times g(x, y) = a^{x+y} \times a^{x-y} = a^{2x}$

$$I(x, y) = \frac{f(x,y)}{g(x,y)} = \frac{a^{x+y}}{a^{x-y}} = a^{2y}$$

$$\frac{h(x,y)}{I(x,y)} = \frac{a^{2x}}{a^{2y}} = a^{2(x-y)} = (a^{x-y})^2 = (g(x, y))^2$$

Choice (A)

**22.** $h(x, y) \times I(x, y) = a^{2x} \times a^{2y} = a^{2(x+y)} = (a^{x+y})^2 = (f(x, y))^2$.
Choice (D)

**Solutions for questions 23 to 25:**

**23.** $((9 \uparrow 7) \to 4) \downarrow 29 = (34 \to 4 \downarrow 29)$
$= (34 \times 12) \downarrow 29$
$= (34 \times 24) - 3 \times 29$
$= 729$
which is a perfect cube.
The other 3 expression are not perfect cubes.
$((9 \to 7) \downarrow 4) \uparrow 29$
$= (189 \downarrow 4) \uparrow 29$
$= 366 \uparrow 29$
$= 3 \times 366 + 29 = 1127$
$((9 \downarrow 7) \uparrow 4) \to 29 = -435$
$((9 \uparrow 7) \downarrow 4) \to 29 = 4872$ Choice (A)

**24.** Consider
$(15 \uparrow 6 \to 9) \leftarrow 2 = 1377$
$(15 \to 6) \uparrow 9) \leftarrow 2 = 819$
$(15 \downarrow 6) \leftarrow 9) \to 2 = 16$
$(15 \leftarrow 6) \downarrow 9) \uparrow 2 = -49$
among these only 819 is divisible by 13. Choice (B)

**25.** $\sqrt[3]{30 \uparrow 35} - \sqrt{14 \downarrow 4}$
$= \sqrt[3]{90 + 35} - \sqrt{28 - 12}$
$= \sqrt[3]{125} - \sqrt{16} = 5 - 4 = 1$ Choice (C)

**Exercise – 8(b)**

**Solutions for questions 1 to 3:**

**1.** $4 \sim 5$ = HCF of $4^2$ and $5^2 = 1$
$(4 \sim 5) \Delta 6 = 1 \Delta 6 = (1 + 6)^2 - 4(1)(6) = 25$
$((4 \sim 5) \Delta 6) \nabla 3 = 25 \nabla 3$
$= (25 - 3)^2 + 4(25)3 = 784$ Ans : (784)

**2.** $a \% b$ = LCM of $a^2$ and $b^2$
= LCM of $5^2$ and $6^2 = 900$
$a \Delta b = (a + b)^2 - 4ab = (5 + 6)^2 - 4(5)6 = 1$
$a \sim b$ = HCF of $a^2$ and $b^2$ = HCF of $5^2$ and $6^2 = 1$
$a \nabla b = (a - b)^2 + 4ab = (5 - 6)^2 + 4(5)(6) = 121$
$\therefore$ From the above $a \Delta b = a \sim b$ Choice (B)

**3.** a and b are distinct integers.
$a \Delta b = (a + b)^2 - 4ab = (a - b)^2$
$a \nabla b = (a - b)^2 + 4ab = (a + b)^2$
When $a > 0$; $b < 0$ $a \Delta b > a \nabla b$
$\therefore$ Choice (B) is not false.
$a \% b$ = LCM of $a^2$, $b^2$
$a \nabla b = (a + b)^2$
Let a = 1; b = 2
$\therefore a \% b = 4$; $a \nabla b = 9$
$a \% b < a \nabla b$ Choice (C) is also not false.
Since a and b are distinct integers HCF of $a^2$, $b^2$ is always less than LCM of $a^2$, $b^2$
$a \sim b < a \% b$
Choice (A) is false. Choice (A)

**Solution for question 4:**

**4.** $(3 \times 9) + 28 \div 7 \times 24 - 10$
$= (27 + 28) \div 7 \times 24 - 10 = 55 \div 7 \times 24 - 10$
$= \frac{55}{7} \times 24 - 10 = \frac{55}{7} \times 14 = 110$ Choice (C)

**Solutions for questions 5 to 7:**

$\& (a, b) = a^2 - b^3$
$\$ (a, b) = a^3 - b^2$
$\sigma (a, b) = a^3 + b^3$
$\phi (a, b) = a^2 + b^2$

**5.** $\& (3, 6) = 3^2 - 6^3 = -207$
$\sigma (3, 6) = 6^3 + 3^3 = 243$
$\$ (3, 6) = 3^3 - 6^2 = -9$
$\phi (6,3) = 6^2 + 3^2 = 45$

$$\frac{\&(3,6) + \sigma(3,6)}{\$(3,6) - \phi(6,3)} = \frac{-207 + 243}{-9 - 45} = \frac{36}{-54} = \frac{-2}{3}$$ Choice (D)

**6.** $\&(a, b) + \sigma(a, b)$
$= a^2 - b^3 + a^3 + b^3$
$= a^3 + a^2$
When a is negative integer $a^3 + a^2 \le 0$
$\therefore$ Option (A) is not true
$\$(a, b) + \phi(a, b)$
$a^3 - b^2 + a^2 + b^2 = a^3 + a^2$
Then a is positive $a^3 + a^2 > 0$
Option (B) is not true
$(a, b) - \phi(a, b) = a^2 - b^3 - (a^2 + b^2) = -b^3 - b^2 \to (1)$
$(a, b) - \sigma(a, b) = a^3 - b^2 - (a^3 + b^3) = -b^2 - b^3 \to (2)$

$\therefore$ (1) = (2).
Option (C) is always true. Choice (C)

**7.** $\phi(0, 1) = 0^2 + 1^2 = 1$
$\sigma(1, -1) = 1^3 + (-1)^3 = 0$
$\$(0, 2) = 0 - 2^2 = -4$
$\&(-4, -2) = 16 + 8 = 24$ Ans : (24)

**Solutions for questions 8 to 10:**

**8.** Considering option (B), we get
$3 \to 7 = 2(3) + 3(7) = 27$
$((3 \to 7) \downarrow 9) \uparrow 5 = (27 \downarrow 9) \uparrow 5$
$= \left(\frac{4(27)}{9}\right) \uparrow 5 = \frac{3(12)(5)}{2} = 90$ Choice (B)

**9.** Considering option (C), we get
$((a \to b) \uparrow b) \leftarrow ab) \downarrow b$
$((2a + 3b) \uparrow b) = \frac{3(2a+3b)b}{2}$
$\left(\frac{6ab+9b^2}{2}\right) \leftarrow ab = \frac{4(6ab+9b^2)}{2} - 5ab = 7ab + 18b^2$
$(7ab + 18b^2) \downarrow b = \frac{4(7ab+18b^2)}{b} = 28a + 72b$
Choice (C)

**10.** Considering option (A), we get
$((4 \downarrow 5) \to 7) \uparrow 9) \leftarrow 8) = \frac{7198}{5}$
$((4 \uparrow 5) \downarrow 7) \to 9) \leftarrow 8) = \frac{1436}{7}$
$((4 \leftarrow 5) \to 7) \downarrow 9) \uparrow 8) = 16$
$((4 \to 5) \leftarrow 7) \downarrow 9) \uparrow 8) = 304$ Choice (C)

**Solutions for questions 11 to 13:**

**11.** When, x = 2.5; y = 1.5
f(x, y) = 12, g(x, y) = 13
$\therefore$ f(x, y) < g(x, y)
Option (A) is false.
When, x = 2.5; y = 1.5
f(x, y) = 12
h(x, y) = 11
f(x, y) > h(x, y)
Option (C) is false.
When x and y are integers g(x, y) = h (x, y)
When x and y are not integers
g(x, y) > h(x, y)
$\therefore$ Option (B) is always true. Choice (B)

**12.** When $x = \frac{1}{3}$ and $y = \frac{1}{3}$
then g (x, y) = h(x, y)
So for option (A), the statement is not true.
When x = 2 and $y = \frac{1}{3}$
then g(x, y) = h(x, y)
So for option (B), the statement is not true.
When $x = \frac{1}{3}$ and y = 2
then g (x, y) = h (x, y)
So for option (C), the statement is not true.
Choice (D)

**13.** $f(3.5, 7.9) = 7 + 16 + 12 = 35$
$g(35, 8.2) = 105 + 25 = 130$
$h(130, 7) = 390 + 21 = 411$ Ans : (411)

**Solutions for questions 14 to 16:**

**14.** $((a * b) \oplus c) * d) \oplus 3b$
$((b \oplus c) * d) \oplus (b \oplus b \oplus b)$
$= (a * d) \oplus (d \oplus b)$
$= d \oplus b = b$
Evaluating the options, we see that
Choice (A) = c, Choice (B) = c, Choice (C) = b, Choice (D) = c
$\therefore$ The given expression is equal to the expression in choice C. Choice (C)

**15.** $b^3 = (b * b) * b = a * b = b$
$\therefore$ The minimum value of n = 3 Ans : (3)

**16.** From the table, $a^{10} = a$;
5c = c, 3b = b, $d^5 = d$
$(((a^{10} \oplus 3b) * 5c) \oplus d^5 = ((a \oplus b) * c) \oplus d)$
$= (c * c) \oplus d = a \oplus d = a$
Evaluating the options, we see that
Choice (A) = b, Choice (B) = a, Choice (C) = b, Choice (D) = d
$\therefore$ The given expression is equal to the expression in choice (B). Choice (B)

**Solutions for question 17:**

**17.** Given $(a, b) \otimes (c, d) = (ab + cd, ab - cd)$
$\therefore (a, b) \otimes (b, a) = (2ab, 0)$
$\therefore (p_1, q_1) \otimes (q_1, p_1) = (2\ p_1q_1, 0)$
and $(p_2, q_2) \otimes (q_2, p_2) = (2\ p_2q_2, 0)$
$\therefore (x, y) = (2p_1\ q_1, 0) \otimes (2p_2q_2, 0) = (0, 0)$
$(x + y, xy) \otimes (xy, x + y) = 2(x + y)\ xy, 0 = (0, 0)$
Choice (B)

**Solutions for questions 18 to 20:**

Without loss of generality let x < y < z.
f(x, y, z) = min(y, z, z) = y
g(x, y, z) = max(x, y, x) = y
h(x, y, z) = max(x, y, z) = z
k(x, y, z) = min(x, y, z) = x
j(x, y, z) = min(x, y, x) = x
i(x, y, z) = max(y, z, z) = z
As x < y < z,
k = j < f = g < h = i

**18.** (i) $\frac{f(x,y,z) - g(x,y,z)}{h(x,y,z) + j(x,y,z)} = \frac{y - y}{z + x} < 1$
(ii) $\frac{f(x,y,z) + k(x,y,z)}{g(x,y,z) + i(x,y,z)} = \frac{y + x}{y + z} < 1$
(iii) $\frac{h(x,y,z) - g(x,y,z)}{k(x,y,z) - i(x,y,z)} = \frac{z - y}{x - z} < 1$
(iv) $\frac{i - k}{h - g} = \frac{z - x}{z - y} > 1.$ Choice (D)

**19.** (i) $\frac{-k(x,y,z) + f(x,y,z)}{h(x,y,z)} = \frac{-x + y}{z} \nless 0$
(ii) $\frac{h(x,y,z) + g(x,y,z)}{k(x,y,z) + j(x,y,z)} = \frac{z + y}{x + x} \nless 0$
(iii) $\frac{k(x,y,z) + g(x,y,z)}{j(x,y,z) - f(x,y,z)} = \frac{x + y}{x - y} < 0$
(iv) $\frac{g - i}{j - f} = \frac{y - z}{x - y} \nless 0$ Choice (C)

**20.** If the function is undefined, then the denominator will be = 0. Among the options only option C equals 0.
Choice (C)

**Solutions for questions 21 to 23:**

**21.** $a \$ b = a^{a^3 - b^3} + b.$
$a * b = (a + b)^{a^3 - b^3}$
$\therefore 2 \$ 1 = 2^{2^3 - 1} + 1^{2^3 - 1} = 2^7 + 1$
and $2 * 1 = (2 + 1)^{2^3 - 1} = 3^7$

$(2 \$ 1) - (2 * 1) = 2^7 + 1 - 3^7$ Choice (B)

**22.** Consider option (B)

$a \wedge b > b^{a^3 - b^3}$

a = 1; b = 2 we get $2^{1^3 - 2^3} = 2^{-7} = \frac{1}{2^7} < 1$

$\therefore$ $a \wedge b > 1$ is false

Consider option (C)

Given $a * b = a \$ b$

Let a = b

$a * b = (a + b)^{a^3 - b^3} = (2a)^{a^3 - a^3} = 1$

$a \$ b = a^{a^3 - b^3} + b^{a^3 - b^3}$

$= a^{a^3 - a^3} + a^{a^3 - a^3} = 1 + 1 = 2$

$a * b \neq a \$ b$

Consider option (A)

$a \vee b = a^{a^3 - b^3}$

$a \wedge b = b^{a^3 - b^3}$

$\frac{a \vee b}{a \wedge b} = \frac{a^{a^3 - b^3}}{b^{a^3 - b^3}} = \left(\frac{a}{b}\right)^{a^3 - b^3}$

If a < b, $a^3 - b^3 \leq 0$ and $= \left(\frac{a}{b}\right)^{a^3 - b^3} \geq 1$

Similarly if a > b, $\left(\frac{a}{b}\right)^{a^3 - b^3} \geq 1$

In both cases $\left(\frac{a}{b}\right)^{a^3 - b^3} \geq 1$

$\therefore$ $\frac{a \vee b}{a \wedge b} > 1 \Rightarrow a \vee b \geq a \wedge b$ Choice (A)

**23.** Given a = 1, b = 2

$a * b = (a + b)^{a^3 - b^3}$

$= (1 + 2)^{1 - 2^3} = 3^{-7} = \frac{1}{3^7} > 0$

which is true.

$a \vee b = a^{a^3 - b^3} = 1^{1-8} = 1$

It is also true.

$a \wedge b = b^{a^3 - b^3} = 2^{1-8} = \frac{1}{2^7}$

$a \$ b = a^{a^3 - b^3} + b^{a^3 - b^3} = 1 + \frac{1}{2^7} \Rightarrow a \$ b = 1 + a \wedge b$

$\therefore$ Choice (C) is not true. Choice (C)

**Solutions for questions 24 to 26:**

**24.** (18 \$ 24) = HCF(18, 24) = 6

$(8 \downarrow 7) = 8^2 - 7^2 = 15$

$(18 \$ 24) \rightarrow (8 \downarrow 7) = 6^2 (15^2)$

$(6 \uparrow 8) = 6^2 + 8^2 = 100$

$\sqrt{\frac{(18 \$ 24) \rightarrow (8 \downarrow 7)}{6 \uparrow 8}} = \sqrt{\frac{6^2 \times 15^2}{100}} = 9$ Ans : (9)

**25.** Let $A = (41 \downarrow 40) = 41^2 - 40^2 = 81$

Let $B = (9 \uparrow 27) = 9^2 + 27^2 = 9^2 (10)$

$A \$ B = HCF [81, 9^2 (10)] = 81 \Rightarrow 81 \leftarrow 9 = 81^2/9^2$

$\therefore$ Given expression $= 81 \leftarrow 81 = 1$ Ans : (1)

**26.** Let the given expression be E.

Let $A = (a \uparrow b) = (a^2 + b^2)$

Let $B = (a \downarrow b) = (a^2 - b^2)$

$A \downarrow B = (a^2 + b^2)^2 - (a^2 - b^2)^2 = 4a^2 b^2$

$(a \rightarrow b) = a^2 b^2$

$= \frac{4a^2 b^2}{a^2 b^2} = 4$ Ans : (4)

**Solutions for questions 27 to 30:**

**27.** Let $A = (8 \cup 12) = \frac{8 + 12}{8 - 12} = \frac{20}{-4} = -5$

Let $B = (15 \ominus 1) = 15 + 1 - 15(1) = 1$

$A \cap B = -5 \cap 1 = \frac{2(-5)(1)}{-5 + 1} = \frac{5}{2}$ Choice (A)

**28.** Let $A = 21 \oplus 7 = 21 - 7 + 21(7) = 161$

Let $B = 12 \ominus 8 = 12 + 8 - 12(8) = -76$

$A \cup B = (161 \cup -76) = \frac{161 - 76}{161 - (-76)} = \frac{85}{237}$

Choice (B)

**29.** $(5 \nabla 8) = (5 \cup 8)(5 \cap 8)$

$= \left(\frac{5+8}{5-8}\right)\left(\frac{2 \times 5 \times 8}{5+8}\right) = -\frac{80}{3}$

$(6 \Delta 3) = (6 \cup 3) - (6 \cap 3)$

$= \frac{6+3}{6-3} - \frac{2 \times 6 \times 3}{6+3} = -1$

$(5 \nabla 8) \cup (6 \Delta 3) = \left(-\frac{80}{3}\right) \cup (-1) = \frac{-\frac{80}{3} - 1}{-\frac{80}{3} + 1} = \frac{83}{77}$

Choice (C)

**30.** Let $A = 6 \oplus 4 = 6 - 4 + 6(4) = 26$

Let $B = 6 \ominus 4 = 6 + 4 - 6(4) = -14$

$A \Delta B = (26) \Delta (-14)$

$= (26 \cup -14) - (26 \cap -14)$

$= \left(\frac{26 - 14}{26 + 14}\right) - \frac{2(26) - 14}{26 - 14}$

$= \frac{12}{40} + \frac{182}{3} = \frac{1829}{30}$

Let $C = 8 \cup 7 = \frac{8 + 7}{8 - 7} = 15$

$\therefore$ $(A \Delta B) \cap C$

$= \left(\frac{1829}{30}\right) \cap (15) = \frac{2\left(\frac{1829}{30}\right)(15)}{\frac{1829}{30} + 15}$

$= \frac{54870}{2279}$ Choice (A)

## Chapter – 9
## (Statistics)

### Concept Review Questions

**Solutions for questions 1 to 20:**

**1.** The mid value of the class 45 – 65 is $\frac{45 + 65}{2} = \frac{110}{2} = 55$

Ans : (55)

**2.** The size of the class 12 – 22 is 22 – 12 = 10. Ans : (10)

**3.** A.M (3a, 3b, 3c) = $\frac{3a + 3b + 3c}{3} = \frac{3(a + b + c)}{3} = a + b + c$

Choice (B)

**4.** For a symmetric distribution, mean = median = mode.

Choice (C)

**5.** Mode = 3 median – 2 mean. Choice (C)

**6.** 1 occurs more frequently in the given data. So, the mode is 1. Ans : (1)

**7.** Since no observation occurs more than once, the mode is ill defined. Choice (D)

**8.** G.M. (a, b, c) = $\sqrt[3]{abc}$ Choice (C)

**9.** G. M (5, 7, 5, 9) = $\sqrt[3]{5 \times 75 \times 9} = \sqrt[3]{5^3 \times 3^3} = 5 \times 3 = 15$ Ans : (15)

**10.** Given, A = $\frac{a+b}{2}$

G = $\sqrt{ab}$

and H = $\frac{2ab}{a+b}$

$\therefore$ A.H. = $\left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right)$ = ab = G$^2$ Choice (D)

**11.** Range = maximum value – minimum value = 82 – 8 = 74 Ans : (74)

**12.** Range = maximum value – minimum value

$\therefore$ 15 = 101 – minimum value

So, minimum value = 101 – 15 = 86 Ans : (86)

**13.** $Q_2$ is equal to the median. Choice (B)

**14.** We know that, A.M. = $\frac{\text{Sum of the observations}}{\text{Number of the observations}}$

So, 12 = $\frac{\text{sum}}{15}$

$\therefore$ The sum of the observations = 12 $\times$ 15 = 180 Choice (A)

**15.** We know that, on adding a constant value to each of the given observation, the standard deviation remains unchanged.

$\therefore$ S.D (10, 20, 30, 40, 50)

= S.D (20, 30, 40, 50, 60) = S Choice (A)

**16.** We know that, Mean deviation (a, b) = $\frac{|a-b|}{2}$

$\therefore$ M.D (30, 40) = $\frac{|30-40|}{2}$ = 5 Ans : (5)

**17.** S.D ($x_1$ + c, $x_2$ + c, …., $x_3$ + c)

= S.D ($x_1$, $x_2$, ….., $x_3$) = S

$\therefore$ Variance ($x_1$ + c, $x_2$ + c, …., $x_n$ + c) = S$^2$ Choice (B)

**18.** Given, range ($x_1$, $x_2$, …., $x_n$) = R

$\therefore$ range ($x_1$ – 2, $x_2$ – 2, …., $x_n$ – 2) = R Choice (C)

**19.** A.M (($x_1$ + a, $x_2$ + a, …., $x_n$ + a)

= A.M ($x_1$, $x_2$, …., $x_n$) + a = A + a Choice (A)

**20.** Given, the A.M. of the first 'n' natural numbers is 8.

$\therefore \frac{n+1}{2} = 8 \Rightarrow n = 15$ Ans : (15)

### Exercise – 9(a)

**Solutions for questions 1 to 25:**

**1.** Arranging the given numbers in an increasing order, we get, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44 and 48 which is an arithmetic progression.

$\therefore$ The required arithmetic mean = $\frac{a+l}{2} = \frac{4+48}{2} = 26$

Choice (B)

**2.** Given, the first term and the common difference of an arithmetic progression are 3 and 4 respectively.

$\therefore$ 15$^{th}$ term = a + 14d = 3 + 14(4) = 59

So, the arithmetic mean of the first 15 terms is

$\frac{a+l}{2} = \frac{3+59}{2} = 31$ Ans : (31)

**3.** Given, the arithmetic mean of 17 observations is 20.

So, the sum of the 17 observations is 20 $\times$ 17 = 340

Now, the observations 13 and 27 are discarded from the set.

So, the new sum is 340 – (13 + 27) = 300

$\therefore$ The arithmetic mean of the new set of observations is $\frac{300}{15} = 20$. Ans : (20)

**4.** We know that, the sum of the cubes of the first 'n, natural numbers is $\frac{n^2(n+1)^2}{4}$

So, the required arithmetic mean is

$= \frac{\frac{n^2(n+1)^2}{4}}{n} = \frac{n(n+1)^2}{4}$ Choice (C)

**5.** A.M.($a_1 + b_1$, $a_2 + b_2$, ……, $a_n + b_n$)

$= \frac{(a_1+b_1)+(a_2+b_2)+..........+(a_n+b_n)}{n}$

$= \frac{(a_1+a_2+.......a_n)}{n} + \frac{(b_1+b_2+.......b_n)}{n}$ = A + B

Choice (B)

**6.** The multiples of 7 between 100 and 200 are 105, 112, 119, ….. , 196.

The above numbers are in arithmetic progression.,

$\therefore$ The required arithmetic mean is $\frac{a+l}{2} = \frac{105+196}{2}$

$= \frac{301}{2} = 150.5$ Ans : (150.5)

**7.** Given the arithmetic mean of $x_1$, $x_2$, ….. $x_n$ is A.

$\therefore \frac{x_1+x_2+........+x_n}{n} = A$

$\Rightarrow x_1 + x_2 + ……. + x_n = nA$ ………..(1)

Now, when $x_i$ is replaced by $x^1$, the new sum is $x_1 + x_2 + x_{i-1} + x_{i+1} ……. + x_n$.

= ($x_1 + x_2 + …… x_{i-1} + x_i + x_{i+1} + ……x_n + x^1 – x_i$)

= nA + $x^1 – x_i$

Hence, the arithmetic mean of the new series is $\frac{nA+x^1-x_i}{n}$. Choice (B)

**8.** Given, the arithmetic mean of a set of 15 observations is 25.

So, the sum of the 15 observations is 15 $\times$ 25 = 375.

But, four observations 4, 12, 19 and 35 were misread as 1, 3, 8 and 13 respectively.

So, the actual sum of the observations is 375 + (4 – 1) + (12 – 3) + (19 – 8) + (35 – 13) = 420

Hence, the correct mean is $\frac{420}{15} = 28$. Ans : (28)

**9.** Let the average of the remaining 40 observations be $\bar{x}$.

Then, 120 $\times$ 20 = 80 $\times$ 20 + 40$\bar{x}$.

$\Rightarrow 40\bar{x} = 2400 – 1600 \Rightarrow 40\bar{x} = 800$

$\therefore \bar{x} = 20$. Ans : (20)

**10.** We have, $45 = 3^2 \times 5$
$245 = 5 \times 7^2$, $21 = 3 \times 7$ and $525 = 3 \times 5^2 \times 7$
We know that, the G.M($x_1, x_2, \ldots\ldots x_n$)

$= (x_1 \times x_2 \times x_3 \ldots\ldots x_n)^{\frac{1}{n}}$

$\therefore$ The G.M. (45, 245, 21, 525)

$= (45 \times 245 \times 21 \times 525)^{\frac{1}{4}}$

$= (3^5 \times 5^4 \times 7^4)^{\frac{1}{4}} = 3 \times 5 \times 7 = 105.$ Choice (B)

**11.** Arranging the given numbers in increasing order, we get, 12.12344, 12.12345, 12.12346, 12.12346, 12.12349, 12.12355, 12.12382, 12, 12432, 12.13245, 12.15632 and 12.18932.
There are 11 values.
So, the median is the $6^{th}$ observation which is 12.12355.
Ans : (12.12355)

**12.** Median of the first 100 natural numbers is $\frac{50 + 51}{2} = 50.5$.
Ans : (50.5)

**13.** Arranging the given numbers other than 'x' in increasing order, we get, 3, 4, 5, 6, 8, 9, 11, 14, 15, 23, 25, 25, 29 and 39.
If $x \leq 11$, then the median is 11.
If $x \geq 14$, then the median is 14.
If $11 < x < 14$, then the median is x.
Hence, the range of the values of the median is [11, 14].
Choice (C)

**14.** If each observation is divided by 4, the median of the new set of observations will be one-fourth of the median of the original set of observations. Hence, the median of the new set of observations is $\frac{100}{4} = 25$. Choice (B)

**15.** 7 is the most often occurring observation in the given data.
So, the mode of the data is 7. Ans : (7)

**16.** On adding a constant value to each of the given observations, the range of the new set of observations remains unchanged.
Hence, the required range is 50. Ans : (50)

**17.** Arranging the given numbers in increasing order, we get, 1, 4, 8, 12, 14, 15, 19, 23, 25, 32, 35.

$\therefore$ $Q_1$ = size of $\left(\frac{n+1}{4}\right)^{th}$ observation.

= size of $\left(\frac{11+1}{4} = 3\right)^{rd}$ observation $= 8$

$\therefore$ $Q_3$ = size of $3\left(\frac{n+1}{4}\right)^{th}$ observation.

= size of $\left(3\left(\frac{11+1}{4}\right) = 9\right)^{th}$ observation $= 25$

Hence, Q.D. = $\frac{Q_3 - Q_1}{2} = \frac{25 - 8}{2} = 8.5$

Choice (C)

**18.** A.M.(1, 4, 12, 18, 13, 16, 25, 3, 5, 3) = $\frac{100}{10} = 10$.

$\therefore$ The mean deviation = $\frac{\sum|X_i - M|}{n}$

$= \frac{9 + 6 + 2 + 8 + 3 + 6 + 15 + 7 + 5 + 7}{10} =$

$\frac{68}{10} = 6.8$

Ans : (6.8)

**19.** The standard deviation of the series will be $3\sigma$.
(Standard result). Choice (D)

**20.** We know that when a constant term is subtracted from each of the given observations, the standard deviation remains unchanged.
$\therefore$ S.D.(101, 102, 103,………, 111)
= S.D(1, 2, 3, ………, 11)
(Subtracting 100 from each of the observations) = M
Choice (A)

**21.** Let $x_1, x_2, \ldots.., x_{11}$ be the 11 observations and $\bar{x}$ be their arithmetic mean

$\sqrt{\frac{\sum(x_i - \bar{x})^2}{n}} = \sqrt{\frac{275}{11}} = \sqrt{25} = 5$ Ans : (5)

**22.** The arithmetic mean of the first 13 natural numbers is $\frac{13+1}{2} = 7$.

$\therefore$ The standard deviation of the first 13 natural numbers is

$\sqrt{\frac{\sum(x_i - \bar{x})}{n}}$

$= \sqrt{\frac{6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2}{13}}$

$= \sqrt{\frac{182}{13}} = \sqrt{14}$ Choice (C)

**23.** We have, A.M. = $\frac{\sum f_i x_i}{\sum f}$

$= \frac{6 + 10 + 27}{2 + 5 + 3} = \frac{43}{10} = 4.3$ Ans : (4.3)

**24.** Let $t_1$, $t_2$ be the time taken by the man to cover the first 100 km and the second 100 km respectively.

Then, $V_1 = \frac{100}{t_1} \Rightarrow t_1 = \frac{100}{V_1}$

and $V_2 = \frac{100}{t_2} \Rightarrow t_2 = \frac{100}{V_2}$

Now, let V be the average speed of the motor cycle for the entire journey.

Then, $V = \frac{200}{t_1 + t_2}$

$\Rightarrow V = \frac{200}{\frac{100}{V_1} + \frac{100}{V_2}}$

$\therefore V = \frac{2}{\frac{1}{V_1} + \frac{1}{V_2}}$

i.e. V is the harmonic mean of $V_1$ and $V_2$. Choice (C)

**25.** Given $y_i$ = A.M.($x_i, x_{i+1}, x_{i+2}$) for $1 \leq i \leq n - 2$

So, $y_1 = \frac{x_1 + x_2 + x_3}{3}$

$\Rightarrow 3y_1 = x_1 + x_2 + x_3$

$y_{n-2} = \dfrac{x_{n-2} + x_{n-1} + x_n}{3}$
$\Rightarrow 3y_{n-2} = x_{n-2} + x_{n-1} + x_n$
Also, $y_{n-1}$ = A.M.($x_{n-1}$. $x_n$. $x_1$)
$\Rightarrow 3y_{n-1} = x_{n-1} + x_n + x_1$
and $y_n$ = A.M.($x_n$, $x_1$, $x_2$)
$\Rightarrow 3y_n = x_n + x_1 + x_2$
$\therefore$ $3y_1 + 3y_2 + \ldots\ldots + 3y_n = (x_1 + x_2 + x_3) + (x_2 + x_3 + x_4) + \ldots\ldots\ldots + (x_n + x_1 + x_2)$
$\Rightarrow 3(y_1 + y_2 + \ldots\ldots + y_n) = 3(x_1 + x_2 + \ldots.. + x_n)$
$\Rightarrow y_1 + y_2 + \ldots\ldots + y_n = x_1 + x_2 + \ldots\ldots. + x_n$
Hence, A.M.($y_1$, $y_2$, …..$y_n$) = A.M.($x_1$, $x_2$, … $x_n$) = M
Choice (B)

**Exercise – 9(b)**

**Solutions for questions 1 to 25:**

**1.** We know that, the A.M.($x_1$, $x_2$,…….. $x_n$)
$= \dfrac{x_1 + x_2 + \ldots\ldots. + x_n}{n}$
$\therefore$ The A.M. (2, 12, 8, 16, 17, 18, 23, 40)
$= \dfrac{2+12+8+16+17+18+23+40}{8} = \dfrac{136}{8} = 17$
Choice (B)

**2.** We know that, the arithmetic mean of first 'n' natural numbers is $\dfrac{n+1}{2}$.
$\therefore$ The arithmetic mean of the first 100 natural numbers is $\dfrac{100+1}{2} = 50.5$ Ans: (50.5)

**3.** Given, the A.M. of 22 observations is 25.
So, the sum of the 22 observations = 22 $\times$ 25 = 550
After discarding the observations 23 and 47, the new sum is 550 – (23 + 47) = 480
Hence, the required mean = $\dfrac{480}{20}$ = 24 Choice (A)

**4.** Given, the A.M.($x_1$, $x_2$, ……., $x_n$) = M
We know that, if the A.M. .($x_1$, $x_2$, ……., $x_n$) = A, then the A.M. ($ax_1$+ b, $ax_2$ + b……., $ax_n$+ b) = aA + b
$\therefore$ The A.M. $\left(\dfrac{2x_1-3}{5}, \dfrac{2x_2-3}{5}, \ldots\ldots\ldots. \dfrac{2x_n-3}{5}\right)$
$= \dfrac{2}{5}M - \dfrac{3}{5} = \dfrac{2M-3}{5}$ Choice (B)

**5.** Given, the arithmetic mean of a set of 10 observations is 30. So, the sum of the 10 observations is 10 $\times$ 30 = 300. But, the observations 18, 12 and 21 were misread as 38, 6 and 22. So, the actual sum of the observations
= 300 + (18 – 38) + (12 – 6) + (21 – 22) = 285
Hence, the actual mean is $\dfrac{285}{10} = 28.5$. Choice (B)

**6.** Given, the average wage of 40 employees is ₹2000 per month and the average wage of 60 employees is ₹3000 per month. So, the average wage of the 100 employees per month
$= \dfrac{n_1x_1 + n_2x_2}{n_1 + n_2} = \dfrac{40\times2000 + 60\times3000}{100}$
$= \dfrac{260000}{100} = ₹2600$ Ans : (2600)

**7.** Given, the 10th term is 48 and the common difference is 4.
So, the 11th term is 48 + 4 = 52
$\therefore$ The arithmetic mean of the 20 terms of A.P. = the average of the middle terms.
= The average of 10th and 11th terms = $\dfrac{48+52}{2} = 50$
Ans : (50)

**8.** Given,
The A.M. ($a_1$, $a_2$, …… $a_n$) = M and $a_1 < a_2 < \ldots\ldots. < a_n$
Also $b_i$ = max{$a_1$, $a_2$,….. $a_i$}
$\Rightarrow b_1 = a_1$, $b_2 = a_2$, $b_3 = a_3$,………., $b_n = a_n$
$\therefore$ A.M.($b_1$, $b_2$ …….. $b_n$) = the A.M.($a_1$, $a_2$, ….. $a_n$) = M
Choice (B)

**9.** Arranging the given values other than 'x' in increasing order, we have 5, 12, 14, 15, 29, 23.
If x $\le$ 14, then median is 14.
If x $\ge$ 15, then median is 15.
If 14 < x < 15, the median is x.
So, the range of the values of the median is [14, 15].
Choice (B)

**10.** Increasing order of the given numbers is
$\dfrac{1}{2}, \dfrac{2}{3}, 1, 2, \dfrac{13}{6}$ and $\dfrac{12}{5}$.
$\therefore$ The median is $\dfrac{1+2}{2} = 1.5$ Ans : (1.5)

**11.** If '2' is subtracted from each of the given set of observations, the median of the new set of observations reduces by 2.
Hence, the median of the new set of observations is 48.
Ans : (48)

**12.** Given, the median of the given set of numbers is 15.
$\Rightarrow \dfrac{x+y}{2} = 15 \Rightarrow x + y = 30$
If x = 15 and y = 15, then the mode of the given numbers is 8, 14 and 15.
If x = 14 and y = 16, then the mode of the given numbers is 14.
Hence, the mode of the given data cannot be determined uniquely. Choice (D)

**13.** We know that, the G.M.($x_1$, $x_2$, …….. $x_n$)
$\therefore$ The G.M.(1, 4, $4^2$, …… $4^{101)}$
$= \left(4^{1+2+\ldots\ldots\ldots+101}\right)^{\frac{1}{102}} = \left(4^{\frac{101\times102}{2}}\right)^{\frac{1}{102}} = (4)^{\frac{101}{2}} = 2^{101}$
Choice (A)

**14.** We have, 75 = 3 $\times$ $5^2$, 80 = $2^4$ $\times$ 5, 144 = $2^4$ $\times$ $3^2$, 225 = 3 x 5 x 3 x 5 and 20 = $2^2$ x 5
$\therefore$ The G.M.(75, 80, 225, 20, 144) = $(75.\ 80.\ 225.\ 20.\ 144)^{\frac{1}{5}}$
$= \left(2^{10}.3^5.5^5\right)^{\frac{1}{5}} = 2^2 \times 3 \times 5 = 60$ Ans : (60)

**15.** We know that, the sum of the first 'n' even natural numbers is n(n + 1).
Hence, the required arithmetic mean is $\dfrac{n(n+1)}{n} = n+1$
Choice (B)

**16.** We know that, the H.M. ($x_1$, $x_2$, …… $x_n$)
$= \dfrac{n}{\dfrac{1}{x_1} + \dfrac{1}{x_2} + \ldots\ldots\ldots. + \dfrac{1}{x_n}}$
$\therefore$ The H.M.(1, 2, 4, 7, 14, 28) = $\dfrac{n}{\dfrac{1}{x_1} + \dfrac{1}{x_2} + \ldots\ldots\ldots. + \dfrac{1}{x_n}}$

$= \dfrac{6}{\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}} = \dfrac{6}{\left(\frac{56}{28}\right)} = 3$ Ans : (3)

**17.** The first 20 prime numbers are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67 and 71.

$\therefore$ The median is $\dfrac{29+31}{2} = 30$ Ans : (30)

**18.** Let $x_1, x_2, \ldots\ldots x_{12}$ be the twelve numbers.
Then, $(x_1 - 9) + (x_2 - 9) + \ldots\ldots (x_{12} - 9) = 60$
Given,
$\Rightarrow x_1 + x_2 + \ldots\ldots\ldots x_{12} = 60 + 108 = 168$

Hence, the A.M.$(x_1. x_2 \ldots\ldots x_{12}) = \dfrac{168}{12} = 14$ Choice (A)

**19.** Since, each of the given observations is divided by 4, the range of the new observations is $\dfrac{100}{4} = 25$ . Ans : (25)

**20.** A.M. (12, 5, 9, 15, 31, 20, 4, 17, 22)

$= \dfrac{12+5+9+15+31+20+4+17+22}{9} = \dfrac{135}{9} = 15$

$\therefore$ The mean deviation = $\dfrac{\sum |x_i - M|}{n}$

$= \dfrac{3+10+6+0+16+5+11+2+7}{9} = \dfrac{60}{9} = \dfrac{20}{3}$

Choice (C)

**21.** We know that the standard deviation of a set of observations remains unchanged on adding or subtracting a constant from each of the observations.
$\therefore$ The S.D.$(x_1 - 2, x_2 - 2, \ldots\ldots\ldots\ldots x_n - 2) = \sigma$

Choice (A)

**22.** The arithmetic mean of the first 11 natural numbers is $\dfrac{11+1}{2} = 6.$

$\therefore$ The required variance = $\dfrac{\sum (x_i - \bar{x})^2}{n}$

$= \dfrac{(6-1)^2 + (6-2)^2 + (6-3)^2 + \ldots\ldots\ldots + (6-11)^2}{11}$

$= \dfrac{25+16+9+\ldots\ldots\ldots + 25}{11} = \dfrac{110}{11} = 10$ Ans : (10)

**23.** S.D. (7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 18)
= S.D (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11)
Now, A.M. (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11)

$= \dfrac{11}{11} = 1$ $\therefore$ S.D. (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11)

$= \sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n}}$ , where $\bar{x}$ is the arithmetic mean.

$= \sqrt{\dfrac{1+1+1+1+1+1+1+1+1+1+(10)^2}{11}} = \sqrt{\dfrac{110}{11}} = \sqrt{10}$

Choice (C)

**24.** We have, sin179º = sin1º, sin178º = sin2º, ……. , sin91º = sin89º
$\therefore$ The increasing order of the values sin1º . sin2º, sin3º, ….. sin89º, sin90º, ….. sin179º is sin1º . sin179º . sin2º, sin178º…. sin90º.

There are 179 observations. So, the middle observation is the 90th observation which is sin45º.

Hence, the median of the series is sin45º i.e. $\frac{1}{\sqrt{2}}$.

Choice (B)

**25.** Arranging the given numbers in increasing order, we get 2, 11, 12, 12, 17, 19, 23, 25, 32, 39, 52.

$\therefore Q_1$ = Size of $\left(\dfrac{n+1}{4}\right)^{th}$ observation.

= size of $\dfrac{11+1}{4}$ = 3rd observation = 12

$\therefore Q_3$ = Size of $3\left(\dfrac{n+1}{4}\right)^{th}$ observation.

= size of $3\left(\dfrac{11+1}{4}\right)$ = 9th observation = 32

Hence, Quartile Deviation = $\dfrac{Q_3 - Q_1}{2} = \dfrac{32-12}{2} = 10$

Ans : (10)