CHAPTER – 5

MENSURATION

AREAS OF PLANE FIGURES

Mensuration is the branch of geometry that deals with the measurement of length, area and volume. We have looked at properties of plane figures till now. Here, in addition to areas of plane figures, we will also look at surface areas and volumes of "solids". Solids are objects, which have three dimensions (plane figures have only two dimensions).

Let us briefly look at the formulae for areas of various plane figures and surface areas and volumes of various solids.

TRIANGLES

The area of a triangle is represented by the symbol Δ. For any triangle, the lengths of the three sides are represented by a, b and c and the angles opposite these sides are represented by A, B and C respectively.

(i) For any triangle in general,

(a) when the lengths of three sides a, b, c are given,

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ **,** where

$s = \frac{a+b+c}{2}$

(This is called Hero's formula.)

(b) when base (b) and altitude (height) to that base are given,
Area = ½ (base)(altitude) = ½ bh

(c) **Area = ½ absinC = ½ bcsinA = ½ casinB**

(d) **Area =** $\frac{abc}{4R}$ where R is the circumradius of the triangle.

(e) **Area = rs** where r is the inradius of the triangle and s, the semi-perimeter.

(ii) For a right angled triangle,

Area = (½)(Product of the sides containing the right angle)

(iii) For an equilateral triangle

Area = $\frac{\sqrt{3}a^2}{4}$, where "a" is the side of the triangle

The height of an equilateral triangle $= \frac{\sqrt{3}a}{2}$

(iv) For an isosceles triangle

Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$ where "a" is length of each of the two equal sides and b is the third side

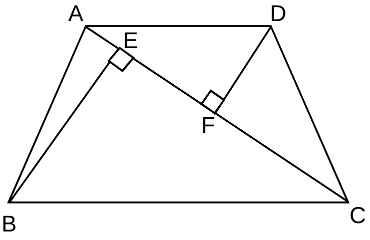
QUADRILATERALS

(i) For any quadrilateral

Area of the quadrilateral = (½)(One diagonal)(Sum of the offsets drawn to that diagonal)

Hence, for the quadrilateral ABCD shown in Fig. 5.01, area of quadrilateral ABCD = ½ (AC)(BE + DF)

Fig. 5.01

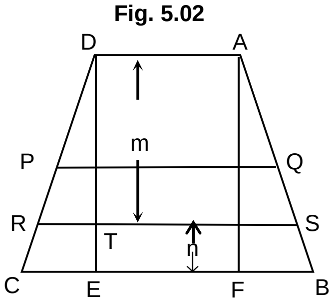


(ii) For a cyclic quadrilateral in which the lengths of the four sides are a, b, c and d,

Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semi-perimeter, i.e., s = (a + b + c + d)/2

(iii) For a trapezium
Area of a trapezium = ½(Sum of parallel sides)(Distance between them)
= (½)(AD + BC)(AF) (refer to Fig. 5.02)

Fig. 5.02



(iv) For a parallelogram
  (a) **Area = Base(Height)**
  (b) **Area = Product of two adjacent sides (Sine of included angle)**

(v) For a rhombus
Area = ½ (Product of the diagonals)
Perimeter = 4(Side of the rhombus)

(vi) For a rectangle
Area = Length(Breadth)

Perimeter = 2(ℓ + b), where ℓ and b are the length and the breadth of the rectangle respectively

(vii) For a square
  (a) **Area = Side^2**
  (b) **Area = ½(Diagonal^2)**

[where the diagonal = $\sqrt{2}$ (side)
Perimeter = 4(Side)

(viii) For a polygon
  (a) **Area of a regular polygon = ½(Perimeter)(Perpendicular distance from the centre of the polygon to any side)**
  (Note that the centre of a regular polygon is equidistant from all its sides)

(b) For a polygon which is not regular, the area has to be found out by dividing the polygon into suitable number of quadrilaterals and triangles and adding up the areas of all such figures present in the polygon.

CIRCLE

(i) **Area of the circle** = πr^2 where r is the radius of the circle
Circumference = $2\pi r$

(ii) Sector of a circle

Length of arc = $\frac{\theta}{360^\circ}(2\pi r)$

Area = $\frac{\theta}{360}(\pi r^2)$**,** where θ is the angle of the sector in degrees and r is the radius of the circle.

Area = (1/2)lr; ℓ is length of arc and r is radius.

(iii) Ring : Ring is the space enclosed by two concentric circles.

Area = $\pi R^2 - \pi r^2 = \pi(R + r)(R - r)$, where R is the radius of the outer circle and r is the radius of the inner circle.

AREAS AND VOLUMES OF SOLIDS

Solids are three-dimensional objects which, in addition to area, have volume also. For solids, two different types of areas are defined

(a) Lateral surface area or curved surface area and
(b) Total surface area

As the name itself indicates, lateral surface area is the area of the LATERAL surfaces of the solid. Total surface area includes the areas of the top and the bottom surfaces also of the solid. Hence, Total surface area = Lateral surface area + Area of the top face + Area of the bottom face

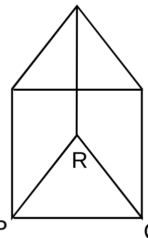
In solids (like cylinder, cone, sphere) where the lateral surface is curved, the lateral surface area is usually referred to as the "curved surface area."

For any solid, whose faces are regular polygons, there is a definite relationship between the number of vertices, the number of sides and the number of edges of the solid. This relationship is given by "Euler's Rule".

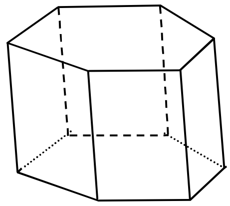
Number of faces + Number of vertices
= Number of edges + 2 (Euler's Rule)

PRISM

A right prism is a solid whose top and bottom faces (bottom face is called base) are parallel to each other and are identical polygons (of any number of sides) that are parallel. The faces joining the top and bottom faces are rectangles and are called lateral faces. There are as many lateral faces as there are sides in the base. The distance between the base and the top is called height or length of the right prism.



Triangular prism


Hexagonal prism

In a right prism, if a perpendicular is drawn from the centre of the top face, it passes through the centre of the base.

For any prism,

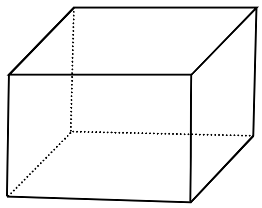
Lateral Surface Area = Perimeter of base (Height of the prism)

Total Surface Area = Lateral Surface Area + 2 × Area of base

Volume = Area of base x Height of the prism

CUBOID OR RECTANGULAR SOLID

A right prism whose base is a rectangle is called a rectangular solid or cuboid. If ℓ and b are respectively the length and breadth of the base and h, the height of the prism, then

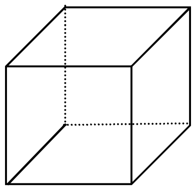

Cuboid

Volume = ℓ bh
Lateral Surface Area = 2(ℓ + b)h
Total Surface Area = 2(ℓ + b)h + 2ℓb
= 2(ℓb + ℓh + bh)

Longest diagonal of the cuboid = $\sqrt{\ell^2 + b^2 + h^2}$

CUBE

A right prism whose base is a square and height is equal to the side of the base is called a cube.

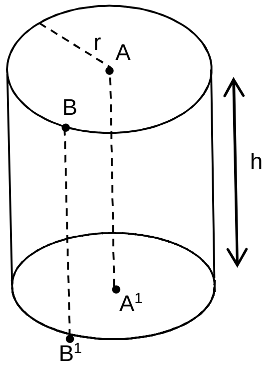

Cube

Volume = a^3, where a is the edge of the cube
Lateral Surface Area = $4a^2$
Total Surface Area = $6a^2$

The longest diagonal of the cube (i.e., the line joining one vertex on the top face to the diagonally opposite vertex on the bottom face) is called the diagonal of the cube. The length of the diagonal of the cube is $a\sqrt{3}$.

CYLINDER

A cylinder is equivalent to a right prism whose base is a circle. A cylinder has a single curved surface as its lateral faces. If r is the radius of the base and h is the height of the cylinder,



Volume = $\pi r^2 h$
Curved Surface Area = $2\pi rh$
Total Surface Area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

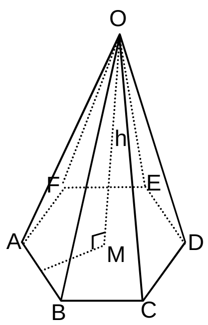
A hollow cylinder has a cross-section of a ring.
Volume of the material contained in a hollow cylindrical shell = $\pi (R^2 - r^2)h$ where R is the outer radius, r is the inner radius and h, the height.

PYRAMID

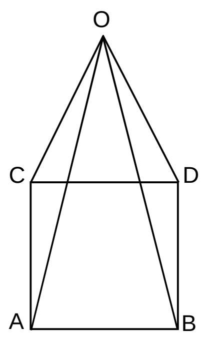
A solid whose base is a polygon and whose faces are triangles is called a pyramid. The triangular faces meet at a common point called vertex. The perpendicular from the vertex to the base is called the height of the pyramid.

A pyramid whose base is a regular polygon and the foot of the perpendicular from the vertex to the base coincides with the centre of the base, is called a right pyramid.

The length of the perpendicular from the vertex to any side of the base (note that this side will be the base of one of the triangular lateral faces of the prism) is called the slant height of the prism.



Hexagonal pyramid



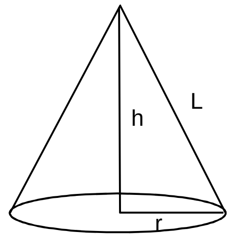
Square pyramid

Volume of a pyramid = ($^1/_3$)(Area of base) (Height)
Lateral Surface area = ($^1/_2$) (Perimeter of the base) (Slant height)

Total Surface Area = Lateral Surface Area + Area of the base.

CONE

Fig. 5.03



A cone is equivalent to a right pyramid whose base is a circle. The lateral surface of a cone does not consist of triangles like in a right pyramid but is a single curved surface.

If r is the radius of the base of the cone, h is height of the cone and l is the slant height of the cone, then we have the relationship (Fig. 5.03)

$l^2 = r^2 + h^2$

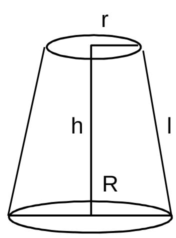
Volume = $^1/_3\, \pi r^2 h$
Curved Surface Area = πrl
Total Surface Area = $\pi rl + \pi r^2 = \pi r(l + r)$

A cone can be formed by folding a sector of a circle and joining together its straight edges. If the radius of the sector is R and the angle of the sector is θ°, then we have the following relationships between the length of the arc and area of the sector on the one hand and base perimeter of the cone and curved surface area of the cone on the other hand.

Radius of the sector = Slant height of the cone
i.e., R = l -------- (1)
Length of the arc of the sector = Circumference of the base of the cone
i.e., $\frac{\theta}{360}(2\pi R) = 2\pi r \Rightarrow\ r = \frac{\theta R}{360}$ -------- (2)
and Area of the sector = Curved surface area
[$\therefore$ curved surface area = Area of sector = $\frac{\theta}{360}\pi R^2 = \pi rl$
(from (1).(2)]

CONE FRUSTUM

Fig. 5.04



If a cone is cut into two parts by a plane parallel to the base, the portion that contains the base is called the frustum of a cone.

If r is the top radius ; R, the radius of the base; h the height and l the slant height of a frustum of a cone (Fig. 5.04), then,
Lateral Surface Area of the cone = $\pi l(R + r)$

Total Surface Area = π (R^2 + r^2 + Rl + rl)
Volume = 1/3 π h (R^2 + Rr + r^2)
l^2 = $(R – r)^2$ + h^2

If H is the height of the complete cone from which the frustum is cut, then from similar triangles, we can write the following relationship.

$$\frac{r}{R} = \frac{H-h}{H}$$

A bucket that is normally used in a house is a good example of the frustum of a cone. The bucket is actually the inverted form of the frustum that is shown in the figure above.

FRUSTUM OF A PYRAMID

The part of a pyramid left after cutting of a portion at the top by a plane parallel to the base is called a frustum of a pyramid.

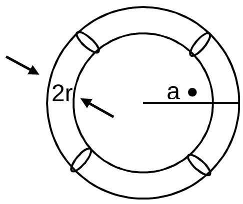
If A_1 is the area of the base; A_2 the area of the top and h, the height of the frustum,

Volume of frustum = $^1/_3 \times h \times \left(A_1 + A_2 + \sqrt{A_1A_2}\right)$

Lateral Surface Area = ½ x (Sum of perimeters of base and top) x Slant height
Total Surface Area = Lateral Surface Area + A_1 + A_2

TORUS

Fig. 5.05



A torus is a three-dimensional figure produced by the revolution of a circle about an axis lying in its plane but not intersecting the circle. The shape of the rubber tube in a bicycle (when it is inflated fully) is an example of a torus. If r is the radius of the circle that rotates and a is the distance between the centre of the circle and the axis of revolution,

Surface Area of the torus = $4\pi^2ra$
Volume of the torus = $2\pi^2r^2a$

A torus is also referred to as a solid ring. (Fig. 5.05)

SPHERE

Any point on the surface of a sphere is equidistant from the centre of the sphere. This distance is the radius of the sphere.

Surface Area of a sphere = $4\pi r^2$
Volume of a sphere = $(4/3)\pi r^3$

The curved surface area of a hemisphere is equal to half the surface area of a sphere, i.e., $2\pi r^2$

The following examples cover various properties / theorems discussed in Geometry as well as areas and volumes discussed in Mensuration. You should learn all the properties of triangles, quadrilaterals and circles as well as areas/volumes of plane figures and solids thoroughly before starting with the worked out examples and the exercise that follows the worked out examples.

Note: We shall be using symbols in two different ways. In some problems we shall use symbols to represent quantities (like length, area, volume). In other problems we shall use symbols to represent only a number.

Examples:

5.01. The base of a right-angled triangle is 7 cm and its area is 84 cm^2. Find its hypotenuse.

Sol: Given base = 7 cm.

Area of the triangle = $\frac{1}{2}$bh, (where b is the base and h is the height)

Given, $\frac{1}{2}$bh = 84 cm^2,

h = $\frac{2(84)}{7}\ \frac{cm^2}{cm} = 24\,cm$

In a right angled triangle, the sum of the squares of perpendicular sides = square of hypotenuse,
$\therefore$ Length of the hypotenuse
= $\sqrt{7^2+24^2}\,cm = 25\,cm$.

5.02. The sides of a triangle are 7 cm, 8 cm and 9 cm. Find its area.

Sol: Area of a triangle whose sides are given as a, b and c is $\sqrt{s(s-a)(s-b)(s-c)}$,

where s = $\frac{a+b+c}{2}$.

As a = 7 cm; b = 8 cm, c = 9 cm $\Rightarrow$ s = 12 cm.

Hence area = $\sqrt{12(5)(4)(3)\ cm^4}$

= $12\sqrt{5}$ cm^2.

5.03. If the area of an equilateral triangle is $64\sqrt{3}$ cm^2, then find its side.

Sol: Area of an equilateral triangle of side a = $\frac{\sqrt{3}}{4}a^2$.

$\therefore \frac{\sqrt{3}}{4}a^2 = 64\sqrt{3}\ cm^2$
$\Rightarrow a^2 = 256\ cm^2$
$\Rightarrow$ a = 16 cm
Hence, side is 16 cm.

5.04. Find the number of revolutions made by a wheel to cover a distance of 352 m, if its diameter is 40 cm.

Sol: Number of revolutions

= $\frac{\text{Distance travelled}}{\text{Circumference of the wheel}}$

= $\frac{(352 \times 100)\,cm}{2\left(\frac{22}{7}\right)\left(\frac{40}{2}\right)cm}$ = 280.

5.05. A piece of wire is bent in the form of a circle of radius 42 cm. An equal length of wire is bent in the form of a square. Find the difference in the areas of the square and the circle.

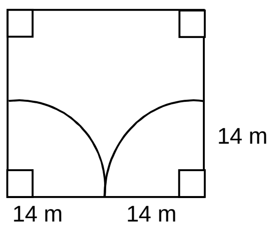
Sol: Perimeter of the circular wire
$= (2)\,(22/7)(42)\text{ cm} = 264\text{ cm}.$
If an equal length of wire is bent and made a square, perimeter of the square is equal to that of the circle.
Perimeter of the square
= 4(side) = 264 cm.
∴ The side of the square = 66 cm.
Area of the square = 66^2 cm^2 = 4356 cm^2.
Area of the circle = $\pi\,(42)^2$ cm^2
$= (22/7)(42)(42)\text{ cm}^2 = 5544\text{ cm}^2.$
Difference in the areas of the square and the circle = 1188 cm^2.

5.06. If the circumference of one circle is 5/2 times that of the other, how many times the area of the smaller circle is the bigger circle?

Sol: Let the radius of smaller circle = r
As the circumference of the bigger circle is (5/2) times that of the smaller circle, radius of the bigger circle will be (5r/2).
Area of the smaller circle = πr^2.
Area of the bigger circle $= \pi\left(\frac{5r}{2}\right)^2 = \frac{25}{4}\pi r^2.$
Therefore, ratio of the area of the bigger circle to that of the smaller circle
$$= \frac{\frac{25}{4}\pi r^2}{\pi r^2} = \frac{25}{4}$$

5.07. Two goats are each tied to two adjacent corners of a square plot of side 28 m with ropes 14 m long. Find the area of the plot over which the goats cannot graze.

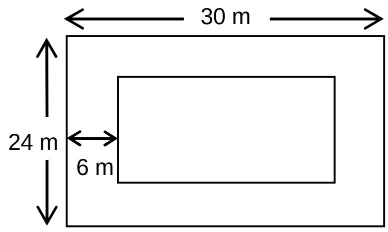
Sol:



Area of the square = (28)(28) m^2 = 784 m^2.
Area over which the goats can graze
$= 2\left(\frac{90°}{360°}\right)\left(\frac{22}{7}\right)(14)(14)\ \text{m}^2 = 308\ \text{m}^2$
Area over which the goats cannot graze
= (784 – 308) m^2
= 476 m^2.

5.08. A path 6 metres wide was laid all around and inside a rectangular plot of length 30 metres and breadth 24 metres. Find the area of the rectangular plot, not covered by the path.

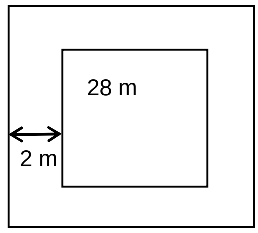
Sol:



Length of the inner rectangular plot
= [30 – 2(6)] m = 18 m.
Breadth of the inner rectangular plot
= [24 – 2(6)] m = 12 m.
Area of the rectangular plot not covered by the path = Area of the inner rectangle = (18)(12) m^2
= 216 m^2.

5.09. A path of uniform width of two metres runs around and outside a square plot of side 28 metres. If the path is to be covered with tiles at the rate of ₹15 per square metre, then find the total cost of the work.

Sol: Let the area of the path = A



Area of the outer square plot
= [28 + 2 (2)]^2 m^2 = 1024 m^2.
Area of the inner square plot = 28^2 m^2
= 784 m^2
Total cost of the work in rupees
= (1024 – 784) (15) = 240(15) = 3600.

5.10. The angle subtended by an arc at the centre of a circle is 70°. If the circumference of the circle is 132 cm, then find the area of the sector formed.

Sol: Let the radius of the circle be r cm.
Given $2\pi r = 132 \Rightarrow r = 21$.
Area of the sector
$= \frac{70°}{360°}\left(\frac{22}{7}\right)(21^2)\text{ cm}^2 = 269.5\ \text{cm}^2.$

5.11. Two circles of radii 9 cm and 4 cm touch each other externally. Find the length of their direct common tangent.

Sol: As the circles touch each other externally, distance between centers = sum of the radii
= 13 cm.
Length of the direct common tangent (L)
$$= \sqrt{\begin{pmatrix}\text{Distance between}\\ \text{the centres}\end{pmatrix}^2 - \begin{pmatrix}\text{Difference in the}\\ \text{radii of the two circles}\end{pmatrix}^2}$$
The difference in the radii is 5 cm.
$\therefore\ L = \sqrt{(13^2 - 5^2)\text{cm}^2} = 12\text{ cm}$

5.12. If the base of a parallelogram is 9 cm and the height of the parallelogram is 6 cm, find the area of the parallelogram.

Sol: The area of the parallelogram
= base $\times$ height = (9)(6) cm^2 = 54 cm^2.

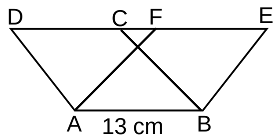
5.13. ABCD is a parallelogram. Are the areas of ΔABC and ΔADB equal?

Sol: ABCD is a parallelogram. Therefore, AB is parallel to CD. ΔABC and ΔADB lie on the same base AB. They also lie between the same parallels AB and DC. Hence their heights are equal. Therefore, ΔABC and ΔADB are equal in area.

5.14. In a rhombus ABCD, AC = 24 cm and BD = 20 cm. Find the area of the rhombus.

Sol: Area of the rhombus
$= (1/2)(\text{Pr oduct of the diagonals })$.
$= (1/2)(AC)(BD) = (1/2)(24)(20)$ cm^2 = 240 cm^2

5.15. In the figure given below, if the area of parallelogram ABCD is 208 cm^2, then find the height of the parallelogram ABEF.



Sol: Parallelograms ABCD and ABEF lie on the same base AB and between the same parallels AB and DE. Hence they are equal in area. Therefore, area of ABEF = 208 cm^2,

height = Area/base = $\dfrac{208\ cm^2}{13\ cm}$ = 16 cm

5.16. A rectangle has its length 10 cm more than twice the side of a square and breadth 12 cm less than the side of the square. If the rectangle has the same perimeter as that of the square, find the area of the square.

Sol: Let the side of the square be S cm, Length of the rectangle is (2S + 10) cm and breadth is (S – 12) cm
= S – 12.
2 (2S + 10 + S – 12) = 4S.
6S – 4 = 4S $\Rightarrow$ S = 2
Area of the square = 2^2 cm^2 = 4 cm^2.

5.17. Find the lateral surface area and volume of a cuboid whose length is 12 cm, breadth is 10 cm and height is 8 cm.

Sol: Lateral surface area of a cuboid
= 2h(ℓ + b) = 2 (8) (12 + 10) cm^2= 352 cm^2.

Volume of a cuboid = ℓ bh
= 12(10)(8) = 960 cm^3

5.18. The cross section of a canal is a trapezium of 5 m width at the top, 3 m at the bottom and the depth is 2 m. Find the quantity of earth dug out in digging a 150 m long canal.

Sol: Area of cross section of the canal = Area of trapezium = $\dfrac{h}{2}$(a + b) = $\dfrac{2}{2}$ (5 + 3) m^2 = 8 m^2
Volume of the earth dug out = area of cross section(length) = 8 (150) m^3 = 1200 m^3

5.19. A wall of dimensions 12 m $\times$ 8 m $\times$ 36 m is to be constructed with bricks of dimensions 10 cm $\times$ 9 cm $\times$ 8 cm. Find the number of bricks required to construct the wall.

Sol: Volume of the wall = 1200(36)(800) cm^3
Volume of brick = 10(9)(8) cm^3
Number of bricks
$= \dfrac{1200(3600)(800)}{10(9)(8)} \dfrac{cm^3}{cm^3} = 48{,}00{,}000$

5.20. The cost of flooring a room 20 m long at ₹7 per square metre is ₹2520. Find the breadth of the room.

Sol: Area of the room = $\dfrac{₹\ 2520}{₹7/m^2} = 360\ m^2$.

Breadth of the room = $\dfrac{360}{20}\dfrac{m^2}{m} = 18$ m.

5.21. A cylindrical vessel of diameter 48 cm has water to a height of 10 cm. A metal cube of 14 cm edge is immersed in it. Calculate the approximate height to which the water level rises.

Sol: Let the increase in the water level be h. Volume of water that rises = $\pi r^2 h$. Since the volume of water that rises is equal to the volume of the cube,
$(22/7)(24)(24)(h)\ cm^2 = 14(14)(14)\ cm^3$

h = $\dfrac{14\,(14)(14)}{(^{22}/_{7})(24)(24)}\ \dfrac{cm^3}{cm^2} \simeq$ 1.516 cm.

5.22. The external radius of a steel pipe is 1.6 cm and its thickness is 1 cm. If 1 cm^3 of steel weighs 20 gms, then find the weight of the steel pipe of length 28 cm.

Sol: External radius, R = 1.6 cm.
Internal radius, r = (1.6 – 1) cm = 0.6 cm.
Volume of the steel pipe of length 28 cm
= $\dfrac{22}{7}\left(1.6^2 - 0.6^2\right)$ (28) cm^3.
Weight of the steel pipe
= $\left(\dfrac{22}{7}\right)\left(1.6^2 - 0.6^2\right)(28)(20)\ cm^3\ \dfrac{gm}{cm^3}$
= 3872 gm = 3.872 kg.

5.23. A conical cup is completely filled with ice cream and topped with a hemispherical scoop at its open end. Find the volume of ice cream, if the radius of the base of the cone is 7 cm and the vertical height of the cone is 12 cm.

Sol: Volume of ice cream inside the cone

= volume of the cone = $\frac{1}{3}$ πr^2 h

= $\frac{1}{3}\left(\frac{22}{7}\right)(7)(7)(12)$ cm^3 = 616 cm^3

Volume of the hemispherical scoop of ice cream

= $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\left(\frac{22}{7}\right)(7)(7)(7)\text{ cm}^3 = \frac{2156}{3}\text{ cm}^3$

Total volume of ice cream

= $\left(616 + \frac{2156}{3}\right)\text{cm}^3 = \frac{4004}{3}\text{ cm}^3$

5.24. If the radius of a right cylinder is increased by 20% and its height is decreased by 10%, find the percentage increase in its volume.

Sol: Original volume = πr^2h.

Increase in radius = $\frac{20}{100}r = 0\cdot 2r$

Decrease in height = $\frac{10}{100}h = 0.1h$.

New volume = π (r + 0·2r)^2 (h – 0·1h)

= π (1.2r)^2 (0.9h) = $\pi \times 1{\cdot}44\ r^2 \times 0{\cdot}9h$

= 1·296 πr^2h.

Percentage increase in the volume of the cylinder

= $\frac{1\cdot 296\pi r^2h - \pi r^2h}{\pi r^2h}(100\%)$ = 29.6%.

5.25. A swimming pool 150 m long and 50 m wide is 1 m deep at the shallow end and 6 m deep at the deep end. Find the volume of the pool.

Sol: Area of cross section perpendicular to the width

= (150)(1/2) (1 + 6) m^2 = 525 m^2

Volume = Area of cross section $\times$ Width

= 525 (50) m^3= 26,250 m^3

5.26. A metallic solid cylinder of 14 cm diameter and 32 cm height is cast into 77 solid cubes of equal size. What is the edge of each cube thus formed?

Sol: Volume of the cylinder

= $\frac{22}{7}\left(\frac{14}{2}\right)^2$ (32) cm^3 = 4298 cm^3 ----------- (1)

If the edge of each cube is a, total volume of the cubes is 77a^3 ----------- (2)

77a^3 = 4928 cm^3

$\Rightarrow$ a = $\sqrt[3]{64}$ cm = 4 cm .

Hence edge of the cube formed is 4 cm.

5.27. A cone has its height equal to a third of its diameter. If the height measures p cm, find its volume, in terms of p.

Sol: Volume of the cone = $\frac{1}{3}\pi r^2h$ cm^3

h = $\frac{2r}{3} = p \Rightarrow r = \frac{3}{2}p$

(Here, h, r, p represent only numbers.)

Volume of the cone

= $\frac{1}{3}\pi\left(\frac{3}{2}p\right)^2 p\text{ cm}^3 = \frac{3}{4}\pi p^3\text{ cm}^3$

5.28. A metallic cone of diameter 48 cm and height 18 cm is melted into identical spheres each of radius 2 cm. How many such spheres can be made?

Sol: Volume of each sphere = $\frac{4}{3}\pi r^3$ cm^3

= $\frac{4\pi}{3}$ (2^3) cm^3 -------- (1)

Volume of the cone

= $\frac{1}{3}\pi\left(\frac{48}{2}\right)^2(18)$ cm^3 -------- (2)

Dividing (2) by (1), number of spheres

= $\frac{24(24)(18)}{4(8)}$ = 324

5.29. A solid is in the form of a cylinder surmounted by a cone. The diameter of the cone is 28 cm. The height of the cylindrical part is 16 cm and that of the conical part is10 cm. Find the volume of the solid.

Sol: Volume of the solid

= Volume of the cylinder + volume of the cone

= $\frac{22}{7}(14)^2(16) + \frac{1}{3}\left(\frac{22}{7}\right)(14)^2(10)$ cm^3

= $\frac{22}{7}(14^2)\left(16+\frac{10}{3}\right)$ cm^3

= $\frac{22}{7}(14^2)\left(\frac{58}{3}\right)$ cm^3 =11,909 cm^3 (rounded off to the nearest unit)

5.30. The area of the base of a right circular cone is 154 cm^2 and its height is 24 cm. Find its volume and curved surface area.

Sol: Area of the base = πr^2 = 154 cm^2

$\Rightarrow$ r^2 = $\frac{7}{22}$ (154) cm^2 = 49 cm^2.

Slant height = ℓ = $\sqrt{r^2+h^2}$

= $\sqrt{7^2+24^2}$ cm

= $\sqrt{49+576}$ cm = 25 cm.

Curved surface area = πrl

= $\frac{22}{7}$(7)(25) cm^2= 550 cm^2.

Volume of the cone = $\frac{1}{3}$ (πr^2h)

= $\frac{1}{3}$(154)(24) cm^3= 1232 cm^3

5.31. Find the volume of the largest sphere which can be cut from a cube having an edge of 7 cm length.

Sol: For the largest sphere, the diameter is equal to the edge of the cube. Hence radius r of this sphere = $\frac{7}{2}$ cm.

Volume of this sphere

$= \frac{4}{3}\left(\frac{22}{7}\right)\left(\frac{7}{2}\right)^3 \text{cm}^3 = \frac{539}{3}\text{cm}^3$.

5.32. What is the maximum number of spherical balls of radius 4 cm that can be packed in a box of size 40 cm $\times$ 24 cm $\times$ 8 cm?

Sol: Diameter of the spherical ball = 8 cm.
Number of spherical balls that can be adjusted along the length = 40/8 = 5; along the breadth = 24/8 = 3; and along the height = 8 cm/8 = 1.
There will be only one layer.
Total number of spherical balls = 5(3)(1) = 15.

5.33. The area of the floor of a conical tent is 616 ft^2. If its height is $2\sqrt{15}$ ft, then find the area of the canvas required for the tent.

Sol: Area of the base = πr^2 = 616 ft^2

$r^2 = \frac{616(7)}{22} = \text{ft}^2$

$r = \sqrt{28(7)\ \text{ft}^2} = 14$ ft.

Slant height (l) = $\sqrt{14^2 + \left(2\sqrt{15}\right)^2}$ ft

$\sqrt{196+60}$ ft $= 16$ ft.

Canvas required = $\pi rl = \frac{22}{7}(14)(16)$ = 704 ft^2.

5.34. A roller is 8 m long and has a diameter of 0.7 m. It takes exactly 1000 rotations of the roller to level a road. If the cost of using the roller is ₹2 per square metre, then find the total cost of levelling the road.

Sol: Radius of the roller = $0.7/2$ m $= 0.35$ m
Length of the roller = 8 m.
Curved surface area of the roller = $2\pi rh$
= 2(22/7)(0.35)(8) m^2 = 17.6 m^2.
In 1 rotation the roller covers 17.6 m^2.
In 1000 rotations, the roller covers 17600 m^2.

Hence the cost = $(17600)(2)\ \text{m}^2\ \frac{\text{Rs}}{\text{m}^2}$

= ₹35,200

5.35. A cylinder has its height equal to its diameter. If the diameter measures p cm, find its volume in terms of p.

Sol: Volume of the cylinder = $\pi r^2 h$
As diameter is p cm, height is also p cm.

Volume = $\pi \left(\frac{p}{2}\right)^2$ p cm^3 $= \pi\frac{p^3}{4}$ cm^3.

Concept Review Questions

Directions for questions 1 to 40: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The lengths of two sides of a triangle are 6 cm and 8 cm. The angle between them is 30°. Find the area of the triangle (in cm^2).

2. Find the area of the triangle (in cm^2), whose sides are 13 cm, 13 cm and 10 cm.

3. A triangle has a semi perimeter of s and an inradius of r. Find the area of the triangle.

(A) rs (B) $\frac{rs}{2}$ (C) 2rs (D) $\frac{rs}{4}$

4. A triangle has sides a, b and c and a circumradius of R. Find its area.

(A) $\frac{abc}{R}$ (B) $\frac{abc}{2R}$ (C) $\frac{abc}{4R}$ (D) $\frac{abc}{8R}$

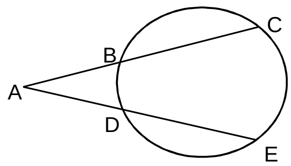
5. Find the area of an equilateral triangle (in cm^2), whose side is 6 cm.

(A) $6\sqrt{3}$ (B) $12\sqrt{3}$ (C) $8\sqrt{3}$ (D) $9\sqrt{3}$

6. In triangle PQR, PQ = 3 cm, PR = 4 cm and QR = 6 cm. Find the length of the median which can be drawn from P to QR. (in cm).

(A) $\sqrt{2.5}$ (B) $\sqrt{1.5}$ (C) $\sqrt{3.5}$ (D) $\sqrt{4.5}$

7.



In the figure, AB = 2 cm, BC = 22 cm and DE = 8 cm. Find AD (in cm).

8. A circle has a diameter of 14 cm. Find the length of the tangent (in cm) which can be drawn to it from a point outside the circle which is at a distance of 25 cm from the centre.of the circle.

9. Two adjacent sides of a parallelogram have lengths of 6 cm and 10 cm. The angle between them is 30°. Find the area of the parallelogram (in cm^2).

10. Find the area enclosed by a ring if the radii of the inner circle and the outer circle are 6 cm and 8 cm respectively (in cm^2). (Take $\pi = 22/7$)

(A) 66 (B) 88 (C) 77 (D) 99

11. Find the area (in cm^2) of a rhombus whose diagonals are 6 cm and 8 cm.

12. Find the area of a trapezium (in cm^2) whose shorter and longer parallel sides have lengths 12 cm and 18 cm respectively and are separated by 15 cm.

(A) 180 (B) 195 (C) 210 (D) 225

13. ABCD is a quadrilateral. AC = 12 cm. The perpendicular distances from B and D to AC are 4 cm and 6 cm respectively. Find the area (in cm^2) of the quadrilateral ABCD.

(A) 30 (B) 45 (C) 60 (D) 90

14. Find the perimeter (in cm) of a semi circle of radius 14 cm.

15. A circle has a radius of 7 cm with centre O. AOB represents a sector where $\angle AOB = 90^o$. Find the area (in cm^2) of the sector AOB.

16. Find the area (in cm^2) of a cyclic quadrilateral whose sides are 8 cm, 10 cm, 12 cm and 16 cm.

(A) $\sqrt{15015}$ (B) $\sqrt{30030}$

(C) $\sqrt{10010}$ (D) None of these

17. The circumference of a circle and the perimeter of a rectangle are in the ratio π : 2. Find the ratio of their areas if the radius of the circle equals one of the dimensions of the rectangle.

(A) π : 1 (B) π : 2 (C) π : 3 (D) π : 4

18. Three circles of radii 2 cm, 4 cm and 6 cm are tangent to one another. What is the area of the triangle formed by joining the centers of the three circles (in cm^2)?

19. The ratio of the areas of a square and an equilateral triangle is $3\sqrt{3}:4$. If s and t are the perimeters of the square and the triangle, then what is the relation between s and t?

(A) s > t (B) s = t

(C) s < t (D) Cannot be determined

20. The height of a rectangular prism is 8 cm. The length and the breadth of the base are 4 cm and 2 cm respectively. Find the lateral surface area of the prism (in cm^2).

21. Find the total surface area of a prism having a height of 10 cm and whose base is a square of side 6 cm (in cm^2).

$\boxed{}$

22. The volume of a prism (in cm^3) whose height is 8 cm and whose base is an equilateral triangle of side 4 cm is $x\sqrt{3}$. Find x.

$\boxed{}$

23. The length, breadth and height of a cuboid are 12 cm, 10 cm and 9 cm respectively. Find the volume of the cuboid (in cm^3).

$\boxed{}$

24. Find the longest diagonal of a cuboid having the length, breadth and height as l, b and h respectively.

(A) $\sqrt{l^2 + b^2 + h^2}$ (B) $2\sqrt{l^2 + b^2 + h^2}$

(C) $\frac{\sqrt{l^2 + b^2 + h^2}}{2}$ (D) None of these

25. Find the lateral surface area of a cuboid having length, breadth and height as l, b and h respectively.

(A) $2h(\ell + b)$ (B) $h(\ell + b)$

(C) $3h(\ell + b)$ (D) $4h(\ell + b)$

26. The breadth and the height of a cuboid are 4 cm and 2 cm respectively. If the total surface area of the cuboid is 88 cm^2, find the length of the cuboid (in cm).

$\boxed{}$

27. Find the total surface area of a solid hemisphere having a radius of r.

(A) πr^2 (B) $2\pi r^2$ (C) $3\pi r^2$ (D) $4\pi r^2$

28. Find the volume of a sphere (in cm^3) having a radius of 6 cm.

(A) 216π (B) 240π (C) 288π (D) 264π

29. A right cone of maximum possible volume is carved out of a solid cylinder. What is the ratio of the volume of the remaining part of the cylinder to the volume of the original cylinder?

(A) 1/2 (B) 1/6 (C) 1/3 (D) 2/3

30. A prism and a pyramid have equal bases and equal heights. Find the ratio of the volumes of the prism and the pyramid.

$\boxed{}$

31. Find the total surface area (in cm^2) of a pyramid having a slant height of 8 cm and a square base of side 4 cm.

$\boxed{}$

32. Find the total surface area (in cm^2) of a cone having a slant height of 10 cm and a radius of 6 cm.

(A) 72π (B) 83π (C) 96π (D) 108π

33. Find the lateral surface area of the frustum of a cone (in cm^2) having top radius, radius of the base and slant height equal to 8 cm, 10 cm and 9 cm respectively.

(A) 81π (B) 108π (C) 162π (D) 144π

34. A frustum of a square pyramid has a top of side 6 cm and a base of side 10 cm. Its slant height is 8 cm. Find its total surface area (in cm^2).

$\boxed{}$

35. Find the slant height of a cone (in cm), whose radius is 5 cm and height is 12 cm.

$\boxed{}$

36. A small cone is cut off from a bigger cone. Find the volume of the frustum if the top radius, the base radius and the height are r, R and h respectively.

(A) $\frac{1}{3}\pi h(R^2 - Rr + r^2)$ (B) $\frac{1}{3}\pi h(R^2 + Rr + r^2)$

(C) $\frac{1}{3}\pi h(R^2 + r^2)$ (D) None of these

37. In question 36, if the height of the frustum is two thirds the height of the bigger cone, find the ratio of the curved surface area of the smaller cone to that of the bigger cone.

(A) 1 : 8 (B) 2 : 7 (C) 1 : 9 (D) 1 : 3

38. In question 36, if the height of the frustum is two thirds the height of the bigger cone, find the ratio of the slant height of the frustum to that of the bigger cone.

(A) 1 : 2 (B) 3 : 4 (C) 5 : 6 (D) 2 : 3

39. A cube of side one meter length is cut into small cubes of side 10 cm each. How many such small cubes can be obtained?

$\boxed{}$

40. Approximately, what is the length (in cm) of the longest pencil that can be placed in a pencil box of dimensions 15 cm × 20 cm × 25 cm?

$\boxed{}$

Exercise – 5(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. In a triangle, if the length of each side is subtracted from the sum of the lengths of the other two sides, the results are 10 cm, 20 cm and 30 cm. Find the area of the triangle (in cm^2).

2. The perimeter of a triangular field is 58 m. If two of its sides are 16 m and 22 m, then the shortest altitude of the sides is approximately _____.
(A) 12.0 m (B) 12.5 m (C) 14.0 m (D) 15.0 m

3. What is the greatest possible area (in cm) of a right - angled triangle whose hypotenuse is 4 cm?

4. A circle is inscribed in an equilateral triangle of side 24 cm. Another equilateral triangle is inscribed in the circle. Find the area of the inscribed triangle.
(A) 25 cm^2 (B) $24\sqrt{3}$ cm^2
(C) $36\sqrt{3}$ cm^2 (D) 72 cm^2

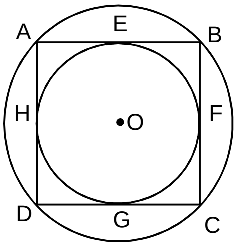
5. What is the circumradius of the triangle whose sides are 67.2 cm, 89.6 cm and 112 cm?
(A) 48.4 cm (B) 50.2 cm
(C) 56 cm (D) Cannot be determined

6. Two of the medians of a triangle are perpendicular. If the lengths of these medians are 12 and 9, the area of the triangle is ☐ .

7. The area of a rhombus is 21 cm^2. If the perimeter of the rhombus is 40 cm, then find the sum of its diagonals (in cm).

8. The centres of two circles with equal radii are A and B. The circles intersect at C and D. ACBD is a square. The radius of each circle is 2 cm. Find the area common to the two circles (in cm^2).
(A) $2(\pi - 1)$ (B) $2(\pi - 2)$ (C) $4(\pi - 1)$ (D) $4(\pi - 2)$

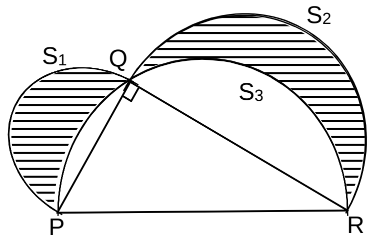
9. In the given figure, there are two concentric circles with centre O. ABCD is a square inscribed in the outer circle and circumscribing the inner circle E, F, G and H are the points of contact of AB, BC, CD and DA respectively with the inner circle. Find the ratio of the areas of the outer circle and the quadrilateral EFGH.



(A) $\pi : 1$ (B) $\pi : 2$ (C) $2\pi : 3$ (D) $3\pi : 4$

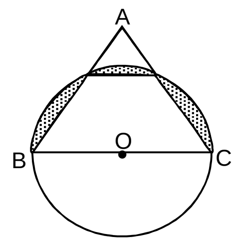
10. A right-angled isosceles triangle has a perimeter of $8\sqrt{2} + 8$. The largest possible quadrant that can be cut out from the triangle is cut out from it. Find the area of the remaining region. (Take $\pi = 22/7$).
(A) $\frac{12}{7}$ (B) $\frac{24}{7}$ (C) $\frac{18}{7}$ (D) $\frac{36}{7}$

11.



In the given figure, $\angle PQR$ is a right angle. The diameters of semicircles S_1, S_2, S_3 are PQ, QR, PR respectively. The area of the shaded region is 30. Find the area of the triangle.
(A) 15 (B) 30
(C) 45 (D) Cannot be determined

12.

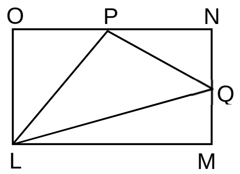


In the given figure, ABC is an equilateral triangle of side 2 cm and the radius of the circle with centre O is 1 cm. Find the area of the shaded region.
(A) $\left(\frac{\pi}{2} - \frac{\sqrt{3}}{4}\right)$cm^2 (B) $\left(\frac{\pi}{2} - \frac{3\sqrt{3}}{4}\right)$cm^2
(C) $\left(\frac{\pi}{2} - \frac{5\sqrt{3}}{4}\right)$cm^2 (D) $\left(\frac{\pi}{2} + \frac{3\sqrt{3}}{4}\right)$cm^2

13. There is a fenced rectangular plot in a large field. The dimensions of the plot are 40 m $\times$14 m. A cow is tethered outside the plot at one corner by a rope 21 metres long. Find the total area of the field, over which it can graze (in m^2).

14. A circle has a radius of 1 cm. It has 8 sectors. The second sector is adjacent to the first, the third to the second and so on with the eighth adjacent to the seventh. The sum of the lengths of all the arcs of the sectors is $(1/10)^{th}$ of the circumference of C. Moreover, the arc length of the kth sector is twice that of the $(k - 1)^{th}$ sector, where $2 \le k \le 8$. Find the central angle of the 1st sector.
(A) $\frac{\pi}{2555}$ (B) $\frac{\pi}{1275}$ (C) $\frac{\pi}{1270}$ (D) $\frac{\pi}{635}$

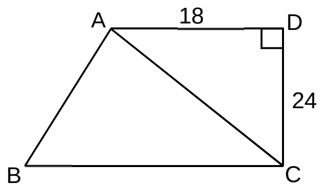
15. In the given figure, LMNO is a rectangle. P and Q are the midpoints of $\overline{ON}$ and $\overline{MN}$ respectively. If l and b are the length and the breadth of the rectangle respectively, find the area of the triangle PQL.



(A) $\frac{\ell b}{8}$ (B) $\frac{7\ell b}{8}$ (C) $\frac{5\ell b}{8}$ (D) $\frac{3\ell b}{8}$

16. Find the area of a regular pentagon (in cm^2), whose side is 20 cm (Take $\cot 36° = 1.376$).

17. The given figure (not to scale) shows a plot ABCD. The sides AD and DC measure 18 m and 24 m respectively. The perimeter of triangle ABC is 80 m and AB = BC. Find the area of the plot (in m^2).



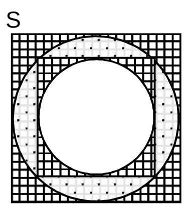
18. ABCDEF is a regular hexagon. X is a point such that AX = BX = 2 AB. Find the ratio of the areas of ABX and ABCDEF.

(A) $\frac{\sqrt{3}}{5}$ (B) $\frac{\sqrt{5}}{7}$ (C) $\frac{\sqrt{5}}{6}$ (D) $\frac{\sqrt{3}}{4}$

19. In triangle ABC, D, E, F are points on BC, CA, AB respectively, such that BD : DC = 1 : 2, CE : EA = 1 : 2 and AF : FB = 1 : 2. Find the ratio of the areas of ΔDEF and ΔABC.

(A) 2 : 5 (B) 1 : 4
(C) 1 : 3 (D) 2 : 7

20.



In the given figure, a circle is inscribed in the square S. A smaller square is inscribed in this circle and a smaller circle is inscribed in the smaller square. Find the ratio of the areas of the regions marked by lines and marked by dots.

(A) $\frac{4(4-\pi)}{3(\pi-2)}$ (B) $\frac{3(4-\pi)}{4(\pi-2)}$

(C) $\frac{3(\pi-3)}{2(\pi-2)}$ (D) $\frac{3(4-\pi)}{2(\pi-2)}$

21. A square sheet has a side of 30 cm. From each of its corners, a square of side y cm is cut. The remaining sheet is folded to form an open cuboid. Find the value of y which maximizes the volume of the cuboid formed.

(A) 5 (B) 6 (C) 3 (D) 10

22. A drum in the shape of a frustum of a cone with radii 24 ft and 15 ft, and height 5 ft is full of water. The drum is emptied into a rectangular tank of base dimensions 99 ft $\times$ 43 ft. Find the rise in the water level in the tank.

(A) $\frac{4}{7}$ ft (B) $1\frac{1}{4}$ ft

(C) $1\frac{3}{4}$ ft (D) None of these

23. The interior of a building is in the form of a cylinder of diameter 7 m, surmounted by a cone whose vertical angle is a right angle. The height of the building at the centre is 14 m. Find the volume enclosed by the building.

(A) $404\frac{1}{4}$ m^3 (B) $449\frac{1}{6}$ m^3

(C) $488\frac{1}{7}$ m^3 (D) None of these

24. There is a rectangular plot of dimensions 50 m $\times$ 30 m. Two trenches of width 5 m and depth 1.5 m each are dug, one along the entire length and the other along the entire breadth. The earth dug out is spread uniformly on the remaining part of the plot. What is the increase in the height of the remaining part of the plot?

(A) $\frac{6}{11}$ m (B) $\frac{5}{12}$ m (C) $\frac{1}{2}$ m (D) $\frac{1}{3}$ m

25. The largest possible right circular cylinder is cut out from a wooden cube of edge 7 cm. Find the volume of the cube left over after cutting the cylinder (in cm^3).

26. There are two identical cubes A and B. A sphere C is inscribed in A. B is cut into a number of identical small cubes. A sphere is inscribed in each of the smaller cubes. Find the ratio of the volume of C and the total volume of the spheres in the small cubes.

27. A rectangular reservoir is 80 m long and 35 m wide. At what speed must water flow into it through a pipe of cross-section of area 625 cm² so that water rises by 2 m in 10 hours?

(A) $8\frac{24}{25}$ kmph (B) $8\frac{12}{25}$ kmph

(C) $12\frac{24}{25}$ kmph (D) $9\frac{12}{24}$ kmph

28. Bricks of dimensions 10 cm $\times$ 8 cm $\times$ 4 cm have to be used to construct a platform of dimensions 15 m $\times$ 10 m $\times$ 8 m. 10% of the volume of the platform will be occupied by mortar. If 100 bricks cost ₹400, find the cost of the bricks required for constructing the platform in crores of rupees.

(A) 0.80 (B) 1.35
(C) 1.80 (D) None of these

29. The sides of a triangle are 8, 15, 17. The triangle is rotated about its longest side. Find the volume of the figure generated.
(A) $\frac{4800\pi}{17}$ (B) $\frac{2400\pi}{17}$ (C) $\frac{3600\pi}{17}$ (D) $\frac{5400\pi}{17}$

30. A metallic sphere of radius 30 cm was recast into identical small spheres of radius 20 mm each. What is the number of small spheres formed?

31. A lawn mower is in the shape of a cylinder. The radius of crosssection is 14 cm and the length is 1 m. What is the area covered by the lawn mower in making 200 revolutions? (in m^2)

32. The length, the breadth and the height of a cuboid are in the ratio 5 : 4 : 3. The longest rod that can be placed in the cuboid is $10\sqrt{2}$ cm. What is the volume of the cuboid (in cm^3)?

33. Find the volume of the largest right circular cone that can be cut out of a cube of side 12 cm.
(A) $452\frac{4}{7}$ cm^3 (B) $462\frac{6}{7}$ cm^3
(C) $482\frac{2}{7}$ cm^3 (D) $505\frac{5}{7}$ cm^3

34. The total surface area of a cuboid is 214 cm^2. The areas of two of its faces are 42 cm^2 and 35 cm^2. Find the greatest dimension of the cuboid (in cm).

35. A box has dimensions of 30 m $\times$ 20 m $\times$ 15 m. Find the maximum number of boxes with each having dimensions of 6 m $\times$ 5 m $\times$ 3 m, which can be placed in it.

Exercise – 5(b)

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. The areas of a circle, a square and an equilateral triangle are equal. If the circumference of the circle is C, the perimeters of the square and the triangle are S and T respectively, then which of the following holds true?
(A) S > C > T (B) C > S > T
(C) C > T > S (D) None of these

2. A circle is placed in a rectangle such that it touches both the lengths of the rectangle. If the length of the rectangle is two times the diameter of the circle, then find the ratio of the area of the rectangle and the area of the circle. (Take π = 22/7)
(A) 14 : 11 (B) 22 : 7 (C) 44 : 21 (D) 28 : 11

3. What is the length of the pendulum (in cm) which swings through an angle of 60° in describing an arc of 44 cm in length?

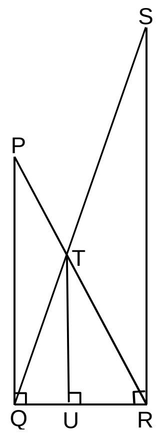
4. A water tank of dimensions 33 m $\times$ 10 m $\times$ 20 m is full of water. The tank is emptied through a sluice of cross section of 220 cm^2 in 5 hours. Find the speed of the water flow (in km/hr).

Moderate

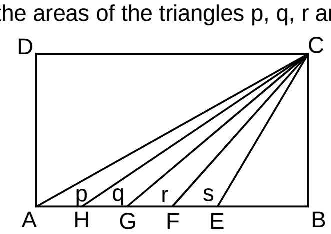
5. Find the two perpendicular sides of a right-angled triangle whose hypotenuse is 65 cm and whose perimeter is 144 cm.
(A) 16 cm, 63 cm (B) 29 cm, 60 cm
(C) 25 cm, 45 cm (D) None of these

6. The perimeter of an isosceles triangle is 52 cm and each of its equal sides is 16 cm. Find the length of the altitude to one of the equal sides.
(A) $5\left(\sqrt{29}\right)$cm (B) $2\left(\sqrt{39}\right)$cm
(C) $7.5\left(\sqrt{29}\right)$cm (D) $2.5\left(\sqrt{39}\right)$cm

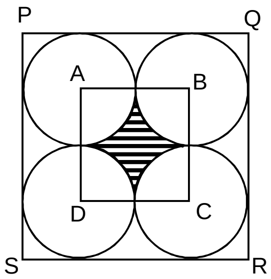
7. In the given figure, PQ = 12 cm, QR = 6 cm and RS = 18 cm. Find the area (in cm^2) of the triangle TQR.



8. In the figure below, (not to scale)
AH : HG : GF : FE : EB = 1 : 2 : 3 : 4 : 5.
Find the ratio of the area of the rectangle ABCD and the sum of the areas of the triangles p, q, r and s.

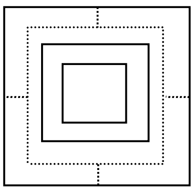


9. The figure below shows a square PQRS with 4 circles placed in it. For each circle, the circumference (in cm) is equal to the area (in cm^2). Find the area of the shaded region (in cm^2)



(A) $16 - 2\pi$ (B) $16 - 4\pi$
(C) $16 - 3\pi$ (D) $16 - \pi$

10. The figure below shows concentric squares. The perimeter of the innermost square is 16 cm. The dotted line indicates an indefinite number of squares. The perimeter of the nth square (counting the innermost as the first) is 8 cm more than the perimeter of the (n – 1)th square for all n > 1. Find the area between the 9th and the 10th squares (in cm^2).

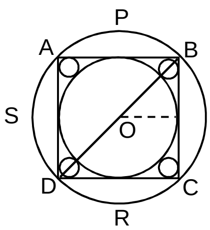


11. Find the radius (in cm) of the circle circumscribing a pentagon of side 10 cm. (Take cosec 36° = 1.7).

12. The minutes hand of a clock is 17.5 cm long. Find the area swept by it between 10:20 a.m. and 10:45 a.m.

(A) $401\frac{1}{24}$ cm^2 (B) $444\frac{5}{24}$ cm^2
(C) $482\frac{7}{24}$ cm^2 (D) None of these

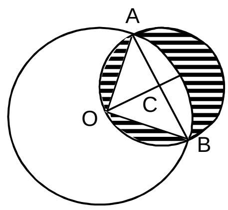
13.



In the given figure, ABCD is a square of side 5 units. The ratio of the areas of its circumcircle, its incircle and the sum of the areas of the four small circles is _____.

(A) $2 : 1 : (3 - 2\sqrt{2})$ (B) $2 : 1 : (3 - 2\sqrt{2})^2$
(C) $2 : 1 : 4(3 - 2\sqrt{2})$ (D) $2 : 1 : 4(3 - 2\sqrt{2})^2$

14.



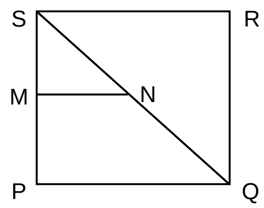
In the given figure, O and C are the centres of the bigger circle and the smaller circle respectively. AB = 10 cm and CO = 5 cm. Find the area (in cm^2) of the shaded region.

(A) $\frac{25}{8}\pi$ (B) $\frac{25}{4}\pi$ (C) $\frac{25}{2}\pi$ (D) $\frac{75}{4}\pi$

15. A, B, C and D are the four vertices of a square. P, Q and R are the midpoints of AB, BC and AD respectively. S is the midpoint of CD and T is the midpoint of RS. Which of the following is the ratio of the areas of ΔQTS and ΔRDT?

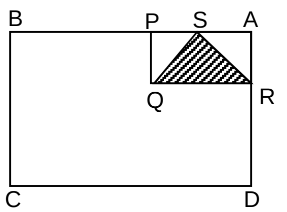
(A) $\frac{3}{5}$ (B) $\frac{1}{2}$ (C) $\frac{4}{5}$ (D) $\frac{2}{1}$

16. In the figure below, PQRS is a square iron sheet. M is the midpoint of $\overline{PS}$ and N is the point of intersection of $\overline{PR}$ and $\overline{SQ}$. If the quadrilateral PMNQ is removed from the sheet, then find the percentage of area of the sheet removed.



17. If the length and the breadth of a rectangle are increased by 3 cm, its area increases by 72 cm^2. If the length is increased by 1 cm, its area increases by 9 cm^2. Find the length of the rectangle (in cm).

18. In the following figure, ABCD is a rectangle, whose perimeter is 68 cm. The length of the rectangle is 6 cm more than its breadth. APQR is a rectangle such that AP : AB = 1 : 4 and AR : AD = 2 : 7. S is any point between A and P and ∠PQR = 90°. Find the area of ΔQRS.



(A) 10 cm^2 (B) $10\sqrt{3}$ cm^2
(C) $12\sqrt{3}$ cm^2 (D) Cannot be determined

19. The perimeter of a square is equal to that of a regular hexagon. Find the ratio of their areas.

(A) $2 : 3\sqrt{3}$ (B) $3\sqrt{3} : 2$

(C) $2 : \sqrt{3}$ (D) $\sqrt{3} : 2$

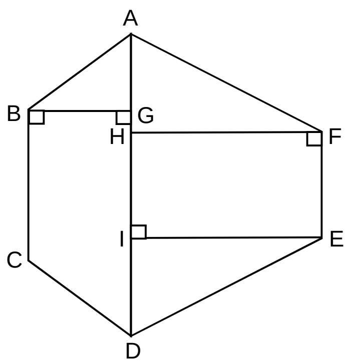
20. What is the difference in the areas of the regular hexagon circumscribing a circle of radius 10 cm and the regular hexagon inscribed in the circle?

(A) 50 cm^2 (B) $50\sqrt{3}$ cm^2

(C) $100\sqrt{3}$ cm^2 (D) $200\sqrt{3}$ cm^2

21. A square shed of side 7 m is in the middle of a huge grass field. A horse is tethered at one of the corners outside the shed with a rope of length 14 m. It can graze only outside the shed. What is the area of the field over which the horse can graze (in m^2)?

22.



In the figure above, BC = 10 cm, EF = 8 cm, ID = 7 cm, AH = 6 cm, HF = 12 cm, BG = 8 cm. Find the area of the figure (in cm^2).

23. R is a rectangular lawn of dimensions 50 m × 20 m. It has two 4-metre wide paths in the middle – one along the length and the other along the breadth. Find the total area of the two paths (in m^2).

(A) 256 (B) 196 (C) 264 (D) 280

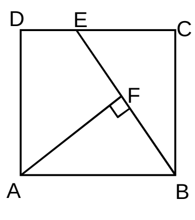
24. OABC is a rhombus. The vertices A, B and C lie on the circumference of a circle whose centre is at O. Find the area of the circle if the area of the rhombus is $8\sqrt{3}$ cm^2.

(A) 10π cm^2 (B) 12π cm^2

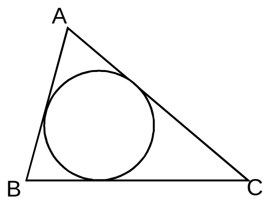
(C) 14π cm^2 (D) 16π cm^2

25. The perimeter of the sector of a circle of radius 21 cm is 64 cm. Find the area of the sector (in cm^2).

26. In the given figure, ABCD is a square BE = 36 and AF = 25. Find the area of the square.



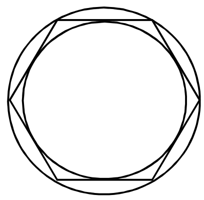
27.



In the figure above, AB = 5, BC = 6, CA = 7. Find the area of the circle.

(A) $\frac{4\pi}{3}$ (B) $\frac{8\pi}{3}$ (C) $\frac{8\pi}{9}$ (D) $\frac{16\pi}{9}$

28.



In the above figure, a circle circumscribes a regular hexagon and another circle of radius r is inscribed in the hexagon. Find the radius of the outer circle.

(A) 2r (B) $\frac{2r}{\sqrt{3}}$ (C) $\sqrt{3}r$ (D) $\sqrt{2r}$

29. The parallel sides of a trapezium of area 220 cm^2 are 20 cm and 24 cm. Find the length of the non-parallel sides if they are equal in length.

(A) $2\sqrt{26}$ cm (B) $3\sqrt{32}$ cm

(C) 5 cm (D) $4\sqrt{7}$ cm

30. The area of a rectangle ABCD is 768 m^2. AB : AD = 4 : 3. P divides AB internally in the ratio 3 : 1 while Q is a point on CD. The area of quadrilateral BCQP is 288 cm^2. Find CQ (in cm).

31. A 40-cm long cylindrical gold pipe is filled with bronze. The outer and the inner diameters of the gold pipe are 14 cm and 12 cm respectively. Find the weight of the pipe, if 1 cubic centimeter of gold weighs 21 gm and that of bronze weighs 28 gm (in kg).

32. A tank, in the shape of a frustum of a cone is full of water. The radii of the bases of the frustum are 6 m and 3 m and its height is 14 m. The tank was emptied into another tank, which is conical with a base radius 7 m. If the second tank is just full, find its height (in m).

33. The sum of the slant height and the base radius of a cone equals twice the difference of the base diameter and the slant height. Find the ratio of the base radius and the height of the cone.

(A) $1 : 2\sqrt{6}$ (B) $2\sqrt{6} : 1$

(C) $1 : \sqrt{6}$ (D) $\sqrt{6} : 1$

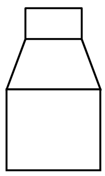
34. Three solid metallic cubes whose edges are in the ratio 3 : 4 : 5 are melted together and recast into one cube. If the diagonal of the cube formed is $12\sqrt{3}$ cm, then find the edge of the smallest of the initial cubes (in cm).

35. The sum of the radius of the base and the height of a solid cylinder is 21 cm. If the total surface area of the cylinder is 924 cm^2, then find the volume of the cylinder (in cm^3).
(A) 2156 (B) 4312
(C) Either 2156 or 4312 (D) None of these

36. An ink pen with a cylindrical barrel of diameter 1 cm and height 7 cm can write 2200 words. How many words can be written using 200 cm^3 of ink?

37. The cost of painting the entire outside surface of a closed cylinder at ₹4.25 per cm^2 is ₹1309. If the height of the cylinder is 3 times the radius, find its volume.
(A) 268.5 cm^3 (B) 302.25 cm^3
(C) 388.75 cm^3 (D) 404.25 cm^3

38. A bottle has the following cross-section.



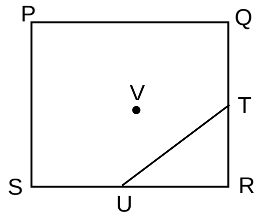
The radius of the bigger cylinder is 10 cm, and that of the smaller one is 5 cm. The height of the frustum of the cone is 5 cm, which is also equal to the height of each of the two cylinders. The volume of the bottle is ______ cm^3.
(A) $\frac{3265\pi}{3}$ (B) $\frac{3625\pi}{3}$ (C) $\frac{2365\pi}{3}$ (D) $\frac{2750\pi}{3}$

Difficult / Very Difficult

39. An equilateral triangle of side 12 cm is cut and a regular hexagon of the largest possible side is carved out of it. Find the area of the hexagon.
(A) $12\sqrt{3}$ cm^2
(B) $24\sqrt{3}$ cm^2
(C) $18\sqrt{3}$ cm^2
(D) Cannot be determined

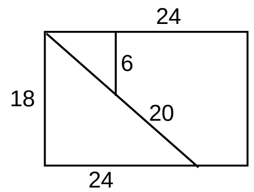
40. PQRSTU is a regular hexagon. Find the ratio of the areas of triangle QSU and the hexagon.
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$
(C) $\frac{1}{2}$ (D) $\frac{2}{5}$

41. In the figure below, PQRS is a square with T and U as the midpoints of QR and RS respectively. V is a point inside PQRS such that VT = VU and RV = 2PV Find the ratio of the areas of the triangles RUT and TVU.



(A) 1 : 2 (B) 2 : 3 (C) 1 : 1 (D) 3 : 5

42.

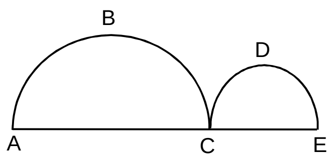


In the above figure, three pieces are placed as shown such that a rectangle is formed. Some of the dimensions of the sides of the pieces are as shown. If the pieces are rearranged to make a square, find the perimeter of the square.
(A) 96 (B) 72 (C) 108 (D) 60

43. A rectangular floor is fully covered with several square tiles of equal size. All the tiles on the floor's edges are blue and the tiles in its interior are green. The number of green tiles is twice the number of blue tiles. Which of the following represents a possible value of the number of tiles along the floor's length?
(A) 28 (B) 30 (C) 32 (D) 34

44. A cow is tied to the corner of a square shed of side 7 m with a rope of length 10.5 m. It cannot graze inside the shed but can graze outside it as far as the rope permits it. Find the total area over which it can graze (in m^2).

45.



In the figure above, AC = 10 cm, CE = 5 cm, and ABC, CDE are semicircles. Find the volume of the solid formed when the figure is rotated about AE (in cm^3).
(A) $375\,\pi$ (B) $\frac{375}{4}\pi$
(C) $\frac{125}{2}\pi$ (D) $\frac{375}{2}\pi$

Data Sufficiency

Directions for questions 46 to 50: Each question is followed by two statements, I and II. Indicate your responses based on the following directives:

Mark (A) if the question can be answered using one of the statements alone, but cannot be answered using the other statement alone.

Mark (B) if the question can be answered using either statement alone.

Mark (C) if the question can be answered using I and II together but not using I or II alone

Mark (D) if the question cannot be answered even using I and II together.

46. The circumference of the base of a cylinder is 44 cm. What is the total surface area of the cylinder?
I. The sum of the radius of the base and the height of the cylinder is 21 cm.
II. The curved surface area of the cylinder is 616 cm^2.

47. What is the area of a regular polygon?
I. The length of the longest diagonal is 20 cm.
II. One of the exterior angles of the polygon is 60°.

48. What is the area of the triangle ABC?
I. $\angle BAC = 30°$.
II. AC = 16 cm and BC = 12 cm.

49. What is the total surface area of a cuboid?
I. The length of body diagonal of the cuboid is 36 cm.
II. The sum of the length, breadth and the height of the cuboid is 38 cm.

50. The radius of the base of a cone is 3.5 cm. What is the volume of the cone?
I. The radius of the base subtends an angle of 60° at the vertex of the cone.
II. The ratio of the total surface area (in sq.cm.) to the volume (in cu.cm) is 1.5 : 1.

Key

Concept Review Questions

1. 12	9. 30	17. A	25. A	33. C
2. 60	10. B	18. 24	26. 6	34. 392
3. A	11. 24	19. B	27. C	35. 13
4. C	12. D	20. 96	28. C	36. B
5. D	13. C	21. 312	29. D	37. C
6. C	14. 72	22. 32	30. 3	38. D
7. 4	15. 38.5	23. 1080	31. 80	39. 1000
8. 24	16. A	24. A	32. C	40. 35

Exercise – 5(a)

1. 150	8. B	15. D	22. D	29. A
2. C	9. A	16. 688	23. B	30. 3375
3. 4	10. B	17. 516	24. C	31. 176
4. C	11. B	18. C	25. 73.5	32. 480
5. C	12. B	19. C	26. 1	33. A
6. 72	13. 1078	20. D	27. A	34. 7
7. 22	14. B	21. A	28. B	35. 100

Exercise – 5(b)

1. D	11. 8.5	21. 539	31. 161.04	41. D
2. D	12. A	22. 298	32. 18	42. A
3. 42	13. D	23. C	33. A	43. A
4. 60	14. C	24. D	34. 6	44. 279.125
5. A	15. D	25. 231	35. A	45. D
6. D	16. 37.5	26. 900	36. 80000	46. B
7. 21.6	17. 12	27. B	37. D	47. C
8. 3	18. A	28. B	38. D	48. C
9. B	19. D	29. A	39. B	49. C
10. 84	20. B	30. 16	40. C	50. B