CHAPTER – 8

OPERATOR BASED QUESTIONS

Questions based on operators or definition based, questions have been given in various entrance exams including CAT.

The basic idea deals with giving a set of operators / definitions followed by questions which require the use of these operators and definitions.

Examples:

Directions for questions 8.01 to 8.03:

Consider the following operators:
$a \alpha b = a^3 + b^3$;
$a \cup b = a^3 - b^3$;
$a \beta b = (a^2 - b^2)/(a^2 + b^2)$;
$a \subset b = (a^2 + b^2)/(a^2 - b^2)$;
$a \rightarrow b = ab$.

8.01. What is the value of $(2 \alpha 1) \beta (2 \cup 1)$?

Sol: $2 \alpha 1 = 2^3 + 1^3 = 8 + 1 = 9$;
$2 \cup 1 = 2^3 - 1^3 = 8 - 1 = 7$
$\therefore (2 \alpha 1) \beta (2 \cup 1) = \dfrac{9^2 - 7^2}{9^2 + 7^2}$
$= \dfrac{32}{130} = \dfrac{16}{65}$.

8.02. What is the value of $(37 \beta 4) \rightarrow (37 \subset 4)$

Sol: Since $a \beta b = \dfrac{a^2 - b^2}{a^2 + b^2}$ and $a \subset b = \dfrac{a^2 + b^2}{a^2 - b^2}$.

The given expression is of the form

$= (a \beta b) \rightarrow (a \subset b) = \left(\dfrac{a^2 - b^2}{a^2 + b^2}\right) \rightarrow \left(\dfrac{a^2 + b^2}{a^2 - b^2}\right)$

$= \left(\dfrac{a^2 - b^2}{a^2 + b^2}\right)\left(\dfrac{a^2 + b^2}{a^2 - b^2}\right) = 1$, for all values of a and b, where $a^2 + b^2, a^2 - b^2 \neq 0$.

8.03. Find the value of the expression:
$((a \alpha b) \alpha (a \cup b)) \rightarrow ((a \alpha b) \cup (a \cup b))$

Sol: Considering the first term:
$(a \alpha b) \alpha (a \cup b) = (a^3 + b^3) \alpha (a^3 - b^3)$
$= (a^3 + b^3)^3 + (a^3 - b^3)^3 = 2a^9 + 6a^3 b^6$
$= 2a^3(a^6 + 3b^6)$

Second term is $(a \alpha b) \cup (a \cup b)$
$= (a^3 + b^3) \cup (a^3 - b^3) = (a^3 + b^3)^3 - (a^3 - b^3)^3$
$= 6a^6 b^3 + 2b^9 = 2b^3 (3a^6 + b^6)$
$\therefore ((a \alpha b) \alpha (a \cup b) \rightarrow (a \alpha b) \cup (a \cup b))$
$= 4a^3 b^3 (a^6 + 3b^6)(b^6 + 3a^6)$

Directions for questions 8.04 to 8.08:

Consider the following definitions
$a(x, y) = |(x^2 + y^3) - (x^3 + y^2)|$
$b(x, y) = (x^3 - y^2) + (y^3 - x^2)$
$c(x, y) =$ L.C.M. $(a(x, y), b(x, y))$
$d(x, y) =$ H.C.F. $(a(x, y) b(x, y))$

8.04. Find the value of a(4, 7).

Sol: $a(4, 7) = |(4^2 + 7^3) - (4^3 + 7^2)|$
$= |359 - 113| = 359 - 113 = 246$.

8.05. Find the value of b(3, 6).

Sol: $b(3, 6) = (3^3 - 6^2) + (6^3 - 3^2)$
$= -9 + 207 = 198$

8.06. Find the value of c (3, 5).

Sol: c(3, 5) = L.C.M. (a(3, 5), b(3, 5))
$a(3, 5) = |(3^2 + 5^3) - (3^3 + 5^2)|$
$= |134 - 52| = 82$
$b(3, 5) = (3^3 - 5^2) + (5^3 - 3^2) = 2 + 116 = 118$
$\therefore c(3, 5)$ = L.C.M. (82, 118) = 4838.

8.07. Find the value of d(4, 5).

Sol: d(4, 5) = H.C.F. (a (4, 5), b(4, 5))
$a(4, 5) = |(4^2 + 5^3) - (4^3 + 5^2)|$
$= |141 - 89| = 52$
$b(4,5) = (4^3 - 5^2) + (5^3 - 4^2)$
$= 39 + 109 = 148$.
$\therefore$ HCF (a(4, 5), b(4, 5)) = H.C.F. (52, 148) = 4.

8.08. Find the value of c(1, 2) – d(2, 1).

Sol: c(1, 2) = L.C.M. (a (1, 2), b(1, 2))
$a(1, 2) = |(1^2 + 2^3) - (1^3 + 2^2)| = 4$
$b(1, 2) = (1^3 - 2^2) + (2^3 - 1^2) = 4$
$\therefore c(1, 2)$ = L.C.M. (4, 4) = 4
d(2, 1) = H.C.F. (a (2, 1), b(2, 1))
$a(2, 1) = |(2^2 + 1^3) - (2^3 + 1^2)| = 4$
$b(2, 1) = (2^3 - 1^2) + (1^3 - 2^2) = 4$
$\therefore d(2, 1)$ = H.C.F. (4, 4) = 4
$\therefore c(1, 2) - d(2, 1) = 4 - 4 = 0$.

Concept Review Questions

Directions for questions 1 to 15: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If $a \Delta b = a^2 + b^2 - ab$, then $1 \Delta 2 =$ ☐ .

2. If x – y = sum of x and y, then 3 – 2 = ☐ .

3. If $a \Delta b = a - b$ and $a \nabla b = a + b$, then which of the following is always non-negative?
(A) $(a \nabla b) - (a \Delta b)$ (B) $(a \nabla b)^2 - (a \Delta b)^2$
(C) $(a \Delta b) + (a \nabla b)$ (D) $(a \Delta b)^2 + (a \nabla b)^2$

4. For two real numbers 'a' and 'b', $a \downarrow b = a - b$, $a \uparrow b = a + b$, $a \rightarrow b = a \times b$ and $a \leftarrow b = \frac{a}{b}$. Which of the following is an integer?
(A) $\frac{1}{2} \downarrow \frac{1}{4}$ (B) $(2 \leftarrow 3) \uparrow (4 \leftarrow 3)$
(C) $(2 \rightarrow 3) \leftarrow 4$ (D) None of these

5. For two positive real numbers 'a' and 'b',
let $a \$ b = \log_{10}(ab)$

$a \% b = \left(\frac{1}{2}\right)^{ab}$

$a \Delta b = a - b$
$a \setminus b = a + b - ab$

Which of the following is always positive?
(A) a $ b (B) $a \Delta b$ (C) a % b (D) $a \setminus b$

Directions for questions 6 to 8: These questions are based on the following data.

For two real numbers a and b, let $a \odot b = a + b + ab$.

6. If $a \neq -1$, find the number e such that $a \odot e = a$.
(A) 0 (B) 1
(C) –1 (D) None of these

7. Find the number a such that $a \odot 2 = 0$.
(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{-3}{2}$ (D) $\frac{-2}{3}$

8. Find a such that $a \odot 1 = a$.
(A) 1 (B) –1 (C) 0 (D) 2

Directions for questions 9 to 12: These questions are based on the following data.

For real numbers a, b, c and d an operator $\oplus$ is defined as follows:

$\oplus$	a	b	c	d
a	b	d	a	c
b	d	c	b	a
c	a	b	c	d
d	c	a	d	b

i.e., $a \oplus a = b$, $a \oplus b = d$, $a \oplus c = a$ and so on.
Further, for x = a, b, c and d,
$x^2 = x \oplus x$, $x^3 = x^2 \oplus x$, etc.

9. $(a \oplus b) \oplus (d \oplus c) =$ _____.
(A) a (B) c (C) b (D) d

10. The least positive integer n such that $d^n = c$ is _____.
(A) 1 (B) 2 (C) 3 (D) 4

11. $c^{100} \oplus a^{100} =$ _____.
(A) a (B) b (C) c (D) d

12. $a \oplus b \oplus (c \oplus d) =$ _____.
(A) a (B) b
(C) c (D) Cannot be determined

Directions for questions 13 to 15: These questions are based on the following data.

Let L (a, b) = L.C.M (a, b)
and H (a, b) = H.C.F (a, b)

13. H [L(4,18), L(12,18)] = ☐ .

14. H [H[H[H(H(64,32), 16), 8], 4], 2] = ☐ .

15. L[L[L[L(L(1, 3), 6), 12], 24], 48] = ☐ .

Exercise – 8(a)

Directions for questions 1 to 25: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Directions for questions 1 to 3: These questions are based on the following data.

$a \sim b$ is LCM of a^3 and b^3
$a \% b$ is HCF of a^3 and b^3
$a \$ b$ is $(a + b)^2 - (a - b)^2$
$a \sigma b$ is $(a + b)^2 + (a - b)^2$
$a \Delta b$ is $a^2 - b^2$

1. Which of the following is/are true for two positive numbers a and b?
(A) $(a \sim b)$ is always less than $(a \% b)$
(B) $a \$ b > 0$
(C) $a \Delta b + a \$ b > 0$
(D) Both (A) and (B)

2. Which of the following is true?
(A) $(a \sim b)(a \% b)$ is divisible by a^2 but not b^2
(B) $(a \sim b)(a \% b)$ is divisible by both a^2 and b^2
(C) $(a \sim b)(a \% b)$ is divisible by b^2 but not a^2
(D) $(a \sim b)(a \% b)$ is divisible by neither a^2 nor b^2

3. If a = 9 and b = 6, which of the following is true?
(A) $(a \% b)(a \sim b) - (a \Delta b)(a \sigma b) = 1,46,934$
(B) $\sqrt{(a \$ b)(a \sigma b)} = b$
(C) $a \Delta b > a \sigma b$
(D) $a \$ b = a \sigma b$

Directions for questions 4 to 6: These questions are based on the following data.

$f(x, y) = 2^{x+y}$, $g(x, y) = 2^{x-y}$

$p(x, y) = \log_2 xy$, $q(x, y) = \log_2\left(\frac{x}{y}\right)$

4. The value of $q(f(x, -x), g(x, x))$ is ☐.

5. The value of $p(f(3, 4), g(4, 5))$ is ☐.

6. $\frac{f[p(5,6), q(6,5)]}{g[p(4,5), q(5,6)]} =$ ☐.

Directions for questions 7 and 8: These questions are based on the following data.

$C(x, y) = (x + y)^3$,
$D(x, y) = (x - y)^3$
$A(x, y) = (x + y)^3 + (x - y)^3$
$S(x, y) = (x + y)^3 - (x - y)^3$

7. When x = 10; y = 5 the value of $\frac{C(x,y) - D(x,y)}{A(x,y) + S(x,y)}$ is
(A) $\frac{13}{27}$ (B) $\frac{27}{13}$ (C) $\frac{39}{27}$ (D) $\frac{26}{59}$

8. If x and y are positive real numbers, which of the following is/are true?
(A) $D(x, y) > 0$ (B) $C(x, y) > 0$
(C) $A(x, y) > S(x, y)$ (D) Both (A) and (B)

Directions for questions 9 and 10: These questions are based on the following data.

In the expression $ax^2 + by^2 + 2gx + 2fy + c$; two quantities Δ and ∇ are defined as follows
$\Delta = \sqrt{g^2 - ac}$; $\nabla = \sqrt{f^2 - bc}$ and $\Delta^2 = \Delta.\Delta$; $\nabla^2 = \nabla.\nabla$

9. For the expression $x^2 + y^2 + 2x + 7y + 9$, which of the following is/are true?
(A) $\Delta^2 = 0$ (B) $\nabla^2 < 0$
(C) $\Delta^2 < \nabla^2$ (D) Both (B) and (C)

10. For the expression given in the previous question, which of the following is true?
(A) $\Delta^2 + \nabla^2$ is positive (B) $\nabla^2 - \Delta^2$ is negative
(C) Δ is not a real number (D) $\Delta = \nabla$

Directions for questions 11 and 12: These questions are based on the following data.

a and b are two non-zero real numbers and $*$, $\oplus$ are defined as $a * b = \frac{ab}{3}$; $a \oplus b = a + b - ab$

11. If $a * b = a \oplus b$ then which of the following is true?
(A) $\frac{1}{a} + \frac{1}{b} = 2$ (B) $\frac{1}{a} + \frac{1}{b} = \frac{4}{3}$
(C) $\frac{a+b}{ab} = 5$ (D) $\frac{ab}{a+b} = \frac{4}{3}$

12. $(((3 * 5) \oplus 7) * 9) \oplus 4 =$
(A) 211 (B) –69 (C) –211 (D) 63

13. If $\$(x, y) = HCF(x, y)$, $\Delta(x, y) = AM(x, y)$, $\nabla(x, y) = LCM(x, y)$, $\sigma(x, y) =$ quotient when x is divided by y, then find the value of $\sigma(\Delta(\nabla(\$(\Delta(240,180),70),50),90),10)$.
☐

14. AM(a, b, c, d) is the arithmetic mean of a, b, c, d AMS(a, b, c, d) is the arithmetic mean of squares of a, b, c, d. AMC(a, b, c, d) is the arithmetic mean of cubes of a, b, c, d. Which of the following is true for all a, b, c, d > 0?
(A) AM(a, b, c, d) > AMS(a, b, c, d)
(B) AMS(a, b, c, d) > AMC(a^3, b^3, c^3, d^3)
(C) AMC(a, b, c, d) > AM(a, b, c, d)
(D) None of these

Directions for questions 15 to 17: These questions are based on the following data.

$f(x, y) = \frac{a^x + a^{-y}}{2}$, $g(x, y) = \frac{a^x - a^{-y}}{2}$

$p(x, y) = \log_a\left(\frac{x+y}{x-y}\right)$,

$q(x, y) = \log_a\left(\frac{x-y}{x+y}\right)$

15. The value of q(f(x, x), g(x,–x)) is _____.

(A) $\frac{a^x - a^{-x}}{a^x + a^{-x}}$ (B) $\frac{a^x + a^{-x}}{a^x - a^{-x}}$

(C) 1 (D) 0

16. $\frac{f(p(x,y),q(x,y))}{g(q(x,-y),p(x,y))}$ = _____.

(A) $\frac{(x+y)^2}{2xy}$ (B) $\frac{x+y}{2xy}$

(C) $\frac{2xy}{x^2-y^2}$ (D) $\frac{2xy}{(x+y)^2}$

17. The value of q(p(f(6, 4),g(6, 4)), 4) = _____.

(A) $\log_a 21$ (B) $\log_a\left(\frac{3}{7}\right)$

(C) $\log_a\frac{7}{3}$ (D) $\log_a 343$

18. If (x, y) ∗ (z, w) = (xw + yz, xz – yw) and (p, q) = [(a₁, b₁) ∗ (b₁, a₁)] ∗ [(a₂, b₂) ∗ (b₂, a₂)] then (p + q, pq) ∗ (pq, p + q) = _____.

(A) $[(a_1^2 + b_1^2)(a_2^2 + b_2^2), 0)]$

(B) $(a_1^2 b_1^2 + a_2^2 b_2^2, 0)$

(C) $[(a_1^2 + b_1^2)^2(a_2^2 + b_2^2)^2, 0]$

(D) $[(a_1^2 - b_1^2)(a_2^2 - b_2^2), 0)]$

Directions for questions 19 and 20: These questions are based on the following data.

Let $x \oplus y = x^2 + y^2$, $x \circledcirc y = x^4 - x^2y^2 + y^4$ and $x \Delta y = x^6 + y^6$

19. $\frac{(x \oplus y)(x \circledcirc y)}{x \Delta y}$ = _____.

(A) $x^6 + y^6$ (B) $x^3 + y^3$ (C) 1 (D) x + y

20. $\frac{x \Delta y}{x \circledcirc y}$ = _____.

(A) x + y (B) $x^2 + y^2$ (C) $x^3 + y^3$ (D) 1

Directions for questions 21 and 22: These questions are based on the following data.

Let $f(x, y) = a^{x+y}$, $g(x, y) = a^{x-y}$, $h(x, y) = f(x, y) \times g(x, y)$, $I(x, y) = \frac{f(x,y)}{g(x,y)}$

21. $\frac{h(x,y)}{I(x,y)}$ =

(A) $(g(x, y))^2$ (B) $(f(x, y))^2$

(C) x + y (D) x – y

22. h(x, y) × I(x, y) =

(A) 1 (B) f(x, y)

(C) g(x, y) (D) $(f(x, y))^2$

Directions for questions 23 to 25: These questions are based on the following data.

$a \rightarrow b = a \times 3b$

$a \leftarrow b = \frac{2a}{b}$

$a \uparrow b = 3a + b$

$a \downarrow b = 2a - 3b$

23. Which of the following is a perfect cube?

(A) $[(9 \uparrow 7) \rightarrow 4] \downarrow 29$ (B) $[(9 \rightarrow 7) \downarrow 4] \uparrow 29$

(C) $[(9 \downarrow 7) \uparrow 4] \rightarrow 29$ (D) $[(9 \uparrow 7) \downarrow 4] \rightarrow 29$

24. Which of the following is a multiple of 13?

(A) $((15 \uparrow 6) \rightarrow 9) \leftarrow 2$ (B) $((15 \rightarrow 6) \uparrow 9) \leftarrow 2$

(C) $((15 \downarrow 6) \leftarrow 9) \rightarrow 2$ (D) $((15 \leftarrow 6) \downarrow 9) \uparrow 2$

25. The value of the expression $\sqrt[3]{30 \uparrow 35} - \sqrt{14 \downarrow 4}$ is

(A) 6 (B) 5 (C) 1 (D) –1

Exercise – 8(b)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

Directions for questions 1 to 3: These questions are based on the following data:

a ~ b is HCF of a^2, b^2

a% b is LCM of a^2, b^2

$a \Delta b$ is $(a + b)^2 - 4ab$

$a \nabla b$ is $(a - b)^2 + 4ab$

1. The value of the expression $(((4 \sim 5) \Delta 6) \nabla 3)$ = ☐.

2. If a = 5, b = 6, then which of the following is true?

(A) a % b = a Δ b (B) a ~ b = a Δ b

(C) a % b = a ∇ b – a Δ b (D) a ∇ b = a Δ b + 1

3. For distinct integers a and b, which of the following is/are always false?

(A) a ~ b > a % b

(B) a Δ b > a ∇ b

(C) a % b < a ∇ b

(D) Both (A) and (B)

4. To simplify the algebraic expression, we follow the order BOADSM where B stands for Bracket, O stands for of, A stands for addition, D stands for division, S stands for subtraction, and M stands for Multiplication. Using the above rule what is the value of $(3 \times 9) + 28 \div 7 \times 24 - 10$?

(A) 113 (B) $\frac{1250}{7}$ (C) 110 (D) $\frac{55}{158}$

Moderate

Directions for questions 5 to 7: These questions are based on the following data:
For real numbers a, b
$\& (a, b) = a^2 - b^3$
$\$ (a, b) = a^3 - b^2$
$\sigma (a, b) = a^3 + b^3$
$\phi (a, b) = a^2 + b^2$

5. The value of $\frac{\&(3,6)+\sigma(3,6)}{\$(3,6)-\phi(6,3)}$ is ____.

(A) –3 (B) 3 (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

6. Which of the following is/are true for all values of a, b?
(A) $\&(a, b) + \sigma(a, b) > 0$
(B) $\$(a, b) + \phi(a, b) < 0$
(C) $\&(a, b) - \phi(a, b) = \$(a, b) - \sigma(a, b)$
(D) Both (A) and (B)

7. The value of $\&(\$(\sigma(\phi(0, 1), -1), 2), -2)$ is [ ].

Directions for questions 8 to 10: These questions are based on the following data:

$a \uparrow b = \frac{3ab}{2}$

$a \downarrow b = \frac{4a}{b}$

$a \rightarrow b = 2a + 3b$

$a \leftarrow b = 4a - 5b$

8. Which of the following is an integer?
(A) $((3 \downarrow 7) \rightarrow 9) \uparrow 5$ (B) $((3 \rightarrow 7) \downarrow 9) \uparrow 5$
(C) $((3 \uparrow 7) \downarrow 9) \rightarrow 5$ (D) $((3 \uparrow 7) \rightarrow 9) \downarrow 5$

9. Which of the following is always true?
(A) $(((a \uparrow b) \rightarrow b) \downarrow ab) \leftarrow b = 3a - 7b$
(B) $(((a \leftarrow b) \uparrow b) \rightarrow ab) \downarrow b = 6a + 5b$
(C) $(((a \rightarrow b) \uparrow b) \leftarrow ab) \downarrow b = 28a + 72b$
(D) $(((a \downarrow b) \rightarrow b) \uparrow ab) \leftarrow b = \frac{3a-7b}{5}$

10. A perfect square among the following is ____.
(A) $((((4 \downarrow 5) \rightarrow 7) \uparrow 9) \leftarrow 8)$
(B) $((((4 \uparrow 5) \downarrow 7) \rightarrow 9) \leftarrow 8)$
(C) $((((4 \leftarrow 5) \rightarrow 7) \downarrow 9) \uparrow 8)$
(D) $((((4 \rightarrow 5) \leftarrow 7) \downarrow 9) \uparrow 8)$

Directions for questions 11 to 13: These questions are based on the following data:

$f(x, y) = \lceil 2x \rceil + \lceil 2y \rceil + \lceil x + y \rceil$
$g(x, y) = \lceil 3x \rceil + \lceil 3y \rceil$
$h(x, y) = \lfloor 3x \rfloor + \lfloor 3y \rfloor$
where $\lceil x \rceil$ denotes the least integral value greater than or equal to x.
$\lfloor x \rfloor$ denotes the greatest integral value less than or equal to x.

11. For real numbers x and y which of the following is/are true?
(A) $f(x, y) = g(x, y)$ (B) $g(x, y) \geq h(x, y)$
(C) $f(x, y) < h(x, y)$ (D) Both (A) and (B)

12. Under which of the following conditions is g(x, y) = h(x, y)?
(A) Neither x nor y is an integer.
(B) x is an integer but y is not an integer.
(C) x is not an integer but y is an integer.
(D) None of these

13. The value of h(g(f(3.5, 7.9), 8.2), 7) = [ ].

Directions for questions 14 to 16: These questions are based on the following data:

Two operations $*$ and $\oplus$ are defined in a set of {a, b, c, d} as follows:

$*$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

$\oplus$	a	b	c	d
a	b	c	d	a
b	c	d	a	b
c	d	a	b	c
d	a	b	c	d

$a^2 = a * a$; $a^3 = a * a * a$ etc
$2a = a \oplus a$; $3a = a \oplus 2a$ etc

14. $(((a * b) \oplus c) * d) \oplus 3b =$
(A) $d \oplus c$ (B) $a \oplus b$ (C) $b \oplus d$ (D) $c \oplus d$

15. If $b^n = b$, the minimum value of n is [ ].

16. $(((a^{10} \oplus 3b) * 5c) \oplus d^5) =$
(A) $a * b$ (B) $b * b$ (C) $c * d$ (D) $a * d$

17. If $(a, b) \otimes (c, d) = (ab + cd, ab - cd)$ and
$(x, y) = [(p_1, q_1) \otimes (q_1, p_1)] \otimes [(p_2, q_2) \otimes (q_2, p_2)]$ then
$(x + y, xy) \otimes (xy, x + y) =$
(A) (1, 1)
(B) (0, 0)
(C) $(2p_1q_1, 2p_2q_2)$
(D) $(p_1^2 + q_1^2, p_1^2 - q_1^2)$

Difficult / Very Difficult

Directions for questions 18 to 20: These questions are based on the following data.

For positive real numbers x, y, z,

f(x, y, z) = min(max(x, y), max(y, z), max (z, x))
g(x, y, z) = max(min(x, y), min(y, z), min (z, x))
h(x, y, z) = max(min(x, y), min(y, z), max(z, x))
k(x, y, z) = min(min(x, y), min(y, z), max(z, x))
j(x, y, z) = min(min(x, y), min(y, z), min(z, x))
i(x, y, z) = max(max(x, y), max(y, z), max(z, x))

18. Which of the following is greater than 1?
(A) $\frac{f(x,y,z)-g(x,y,z)}{h(x,y,z)+j(x,y,z)}$ (B) $\frac{f(x,y,z)+k(x,y,z)}{g(x,y,z)+i(x,y,z)}$
(C) $\frac{h(x,y,z)-g(x,y,z)}{k(x,y,z)-i(x,y,z)}$ (D) $\frac{i(x,y,z)-k(x,y,z)}{h(x,y,z)-g(x,y,z)}$

19. Which of the following is negative?

(A) $\frac{-k(x,y,z)+f(x,y,z)}{h(x,y,z)}$ (B) $\frac{h(x,y,z)+g(x,y,z)}{k(x,y,z)+j(x,y,z)}$

(C) $\frac{k(x,y,z)+g(x,y,z)}{j(x,y,z)-f(x,y,z)}$ (D) $\frac{g(x,y,z)-i(x,y,z)}{j(x,y,z)-f(x,y,z)}$

20. Which of the following expressions is undefined?

(A) $\frac{f(x,y,z)-h(x,y,z)}{-k(x,y,z)-j(x,y,z)}$

(B) $\frac{g(x,y,z)}{k(x,y,z)-i(x,y,z)}$

(C) $\frac{h(x,y,z)+j(x,y,z)}{j(x,y,z)-k(x,y,z)}$

(D) $\frac{g(x,y,z)-f(x,y,z)}{h(x,y,z)-j(x,y,z)}$

Directions for questions 21 to 23: These questions are based on the following information.

$a * b = (a + b)^{a^3-b^3}$

$a \$ b = a^{a^3-b^3} + b^{a^3-b^3}$

$a \vee b = a^{a^3-b^3}$

$a \wedge b = b^{a^3-b^3}$

21. The value of $(a \$ b) - (a * b)$ when a = 2 and b = 1 is _____.

(A) $3^7 + 1 - 2^7$ (B) $2^7 - 3^7 + 1$

(C) $3^7 - 1 - 2^7$ (D) $2^7 - 3^7 - 1$

22. Which of the following is/are true for all values a, b > 0?

(A) $a \vee b \geq a \wedge b$ (B) $a \wedge b > 1$

(C) $a * b = a \$ b$ (D) Both (A) and (B)

23. Which of the following is/are false when a = 1, b = 2?

(A) $a * b > 0$ (B) $a \vee b = 1$

(C) $a \wedge b = 1 - a \$ b$ (D) Both (A) and (B)

Directions for questions 24 to 26: These questions are based on the following information.

Let $a \$ b$ = HCF (a, b), $a \downarrow b = a^2 - b^2$, $a \uparrow b = a^2 + b^2$, $a \rightarrow b = a^2 b^2$ and $a \leftarrow b = a^2/b^2$.

24. $\sqrt{\frac{(18 \$ 24) \rightarrow (8 \downarrow 7)}{6 \uparrow 8}}$ = ☐.

25. $[(41 \downarrow 40) \$ (9 \uparrow 27)] \leftarrow [81 \leftarrow 9]$ = ☐.

26. $\frac{(a \uparrow b) \downarrow (a \downarrow b)}{(a \rightarrow b)}$ = ☐.

Directions for questions 27 to 30: These questions are based on the following information.

$(a \cup b) = \frac{a+b}{a-b}$, $(a \cap b) = \frac{2ab}{a+b}$, $a \ominus b = a + b - ab$, $a \oplus b = a - b + ab$, $a \Delta b = (a \cup b) - (a \cap b)$, $a \nabla b = (a \cup b)(a \cap b)$

27. $(8 \cup 12) \cap (15 \ominus 1)$ = _____.

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$ (C) 2 (D) -1

28 $(21 \oplus 7) \cup (12 \ominus 8)$ = _____.

(A) $\frac{76}{161}$ (B) $\frac{85}{237}$ (C) $-\frac{85}{237}$ (D) $\frac{76}{161}$

29. $(5 \nabla 8) \cup (6 \Delta 3)$ = _____.

(A) 1 (B) $\frac{77}{83}$ (C) $\frac{83}{77}$ (D) –1

30. $[(6 \oplus 4) \Delta (6 \ominus 4)] \cap (8 \cup 7)$ = _____.

(A) $\frac{54870}{2279}$ (B) $\frac{18290}{2279}$ (C) $\frac{2279}{1829}$ (D) 1

Key

Concept Review Questions

1.	3	4.	B	7.	D	10.	D	13.	36
2.	5	5.	C	8.	B	11.	C	14.	2
3.	D	6.	A	9.	C	12.	B	15.	48

Exercise – 8(a)

1.	B	6.	1.5	11.	B	16.	A	21.	A
2.	B	7.	A	12.	A	17.	B	22.	D
3.	A	8.	B	13.	22	18.	C	23.	A
4.	0	9.	C	14.	D	19.	C	24.	B
5.	6	10.	C	15.	D	20.	B	25.	C

Exercise – 8(b)

1.	784	7.	24	13.	411	19.	C	25.	1
2.	B	8.	B	14.	C	20.	C	26.	4
3.	A	9.	C	15.	3	21.	B	27.	A
4.	C	10.	C	16.	B	22.	A	28.	B
5.	D	11.	B	17.	B	23.	C	29.	C
6.	C	12.	D	18.	D	24.	9	30.	A