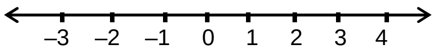
CHAPTER – 6

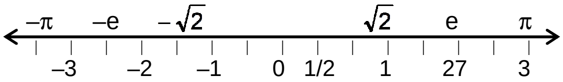
CO-ORDINATE GEOMETRY

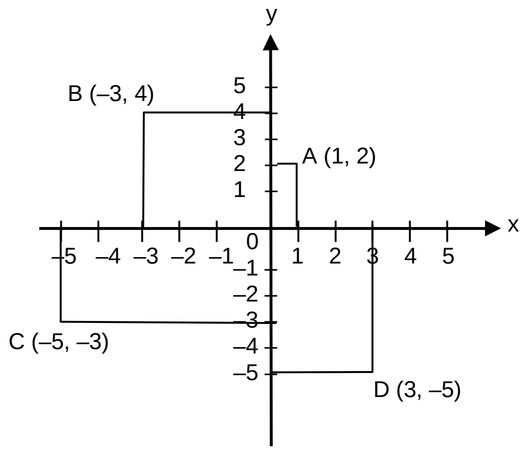
Real numbers can be represented geometrically on a horizontal line. We begin by selecting an arbitrary point O, called the origin and associate it with real number 0. By convention, we take all positive real numbers to the right of 0 and negative real numbers to the left of 0.



Integer line

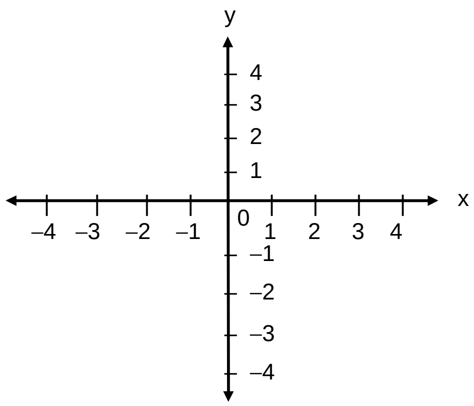
Primarily, we plot the integers. On subdivision of these segments, it is possible to locate rational and irrational numbers.





Rectangular coordinates:

Consider two lines, one vertical and the other horizontal. Let the horizontal line be named as “x-axis” and the vertical line the “y-axis”.



This time, we take the point of intersection of the axes as origin (O). Once again on x-axis we follow the convention of associating positive real numbers to the right of O and negative real numbers to the left of O. On the y-axis, positive real numbers are associated above O and negative real numbers below O.

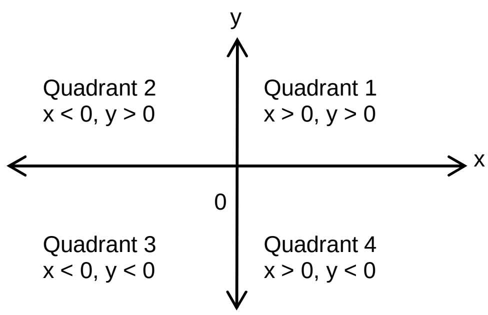
Ordered Pair:

Any point P in the plane formed by the x-axis and y-axis can be located by using an ordered pair of real numbers.

Let x denote the signed distance of P from the y-axis (by signed distance we mean if P is to the right of y-axis, then x > 0 and if P is to the left of y-axis, then x < 0); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y) is now the coordinates of P. This gives us the information to locate the point P. The points A, B, C, D located on the figure can be appreciated by the reader.

x-coordinate and y-coordinate:

If (x, y) are the coordinates of a point P, then x is called the x-coordinate of P and y is called the y-coordinate of P. For instance, the coordinates of origin are (0, 0). The x-coordinate of any point on the y-axis is 0, the y-coordinate of any point on the x-axis is 0. The coordinate system described here is also termed as cartesian coordinate system. The plane is divided into 4 sections termed as quadrants.



Examples:

The point (7, –2) lies in 4th quadrant.
The point (–3, –4) lies in 3rd quadrant.
The point (–8, 10) lies in 2nd quadrant.
The point (7, 7) lies in 1st quadrant.

Some basic rules and formulae are given below, which have to be remembered. Each formula is followed by one or more examples, which clearly explain its application.

1. Distance formula:

(i) The distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(ii) The distance between the origin (0, 0) and the point (x, y) is $\sqrt{x^2 + y^2}$

Examples:

6.01. Find the distance between the points (3, 4) and (–2, 3).

Sol: Distance = $\sqrt{(-2-3)^2+(3-4)^2}$

$= \sqrt{25+1} = \sqrt{26}$ units.

6.02. What is the distance between the points (0, 0) and (24, 7)?

Sol: Distance = $\sqrt{24^2+7^2} = \sqrt{576+49}$

$= \sqrt{625}$ = 25 units.

6.03. Prove that the points (1, –1), (–1, 4), (4, 6) are the vertices of an isosceles right-angled triangle.

Sol: Let A = (1, –1), B (–1, 4) and C = (4, 6).
Using the distance formula we find AB, BC and CA.

AB = $\sqrt{(-1-1)^2+(4+1)^2}$

$= \sqrt{4+25} = \sqrt{29}$ units

$\Rightarrow AB^2 = 29$.

BC = $\sqrt{(4+1)^2+(6-4)^2}$

$= \sqrt{25+4} = \sqrt{29}$ units

$\Rightarrow BC^2 = 29$.

CA = $\sqrt{(4-1)^2+(6+1)^2}$

$= \sqrt{9+49} = \sqrt{58}$ units

$\Rightarrow CA^2 = 58$

$AB^2 + BC^2 = 29 + 29 = 58$

Hence, $AB^2 + BC^2 = CA^2$ and AB = BC

$\therefore$ The given points form an isosceles right-angled triangle.

2. Area of triangle:

(i) The area of the triangle formed by the vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is equal to the value of the determinant

$$\frac{1}{2}\begin{vmatrix} x_1-x_2 & y_1-y_2 \\ x_2-x_3 & y_2-y_3 \end{vmatrix}$$; i.e.

$1/2|(x_1 - x_2)(y_2 - y_3) - (y_1 - y_2)(x_2 - x_3)|$.

Alternately, the area can be found using

$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$= \frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)\right|$$

(ii) The area of the triangle formed by the vertices (0, 0) (x_1, y_1) and (x_2, y_2) is $1/2\ |x_1y_2 - x_2y_1|$.

6.04. Find the area of the triangle formed by joining the points (1, 4), (–2, 5) and (4, –3).

Sol: Given $(x_1, y_1) = (1, 4)$;
$(x_2, y_2) = (-2, 5)$ and $(x_3, y_3) = (4, -3)$

Area = $\frac{1}{2}\begin{vmatrix} 1-(-2) & 4-5 \\ -2-4 & 5-(-3) \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 3 & -1 \\ -6 & 8 \end{vmatrix} = \frac{1}{2}[3 \times 8 - (-1) \times (-6)]$

$= \frac{1}{2}[24 - 6] = \frac{1}{2} \times (18) = 9$ sq. units.

6.05. Find the area of the triangle formed by joining the points (0, 0), (3, 0) and (3, 5).

Sol: Given $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (3, 5)$

Area = $\frac{1}{2}|x_1y_2 - x_2y_1|$

$= \frac{1}{2}|3 \times 5 - 3 \times 0| = \frac{1}{2}|15| = 7{\cdot}5$ sq. units.

6.06. Find the area of the triangle formed by joining the points (–3, –4), (–2, 5) and (–1, 14).

Sol: Area = $\frac{1}{2}\begin{vmatrix} -3-(-2) & (-4)-5 \\ -2-(-1) & 5-14 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -1 & -9 \\ -1 & -9 \end{vmatrix} = \frac{1}{2}[(-1)(-9) - (-1)(-9)]$

$= \frac{1}{2}[9 - 9] = 0$.

Since the area = 0, the points do not form a triangle but form a straight line.

Note:
The area of a quadrilateral formed by the points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) describing the consecutive vertices is given by $\frac{1}{2}\begin{vmatrix} x_1-x_3 & y_1-y_3 \\ x_2-x_4 & y_2-y_4 \end{vmatrix}$.

Alternately, the area can be found using $\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$ i.e.,

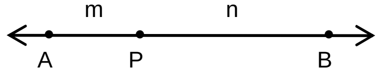
$$\frac{1}{2}\left|(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)\right|$$

3. Section formulae:

(i) Internal Division:

If A (x_1, y_1) and B (x_2, y_2) are two points given, then the coordinates of a point P, which divides the line joining AB internally in the ratio m : n is given by

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$



Note: The point P is between A and B for internal division.

6.07. Find the co-ordinates of the point P which divides the line joining the points A (2, 4) and B (3, –3) in the ratio 3 : 4 internally.

Sol: From the section formula,

$P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Here m = 3, n = 4; $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (3, –3)$

The co-ordinates of P

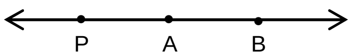
$= \left(\frac{3\times3+4\times2}{3+4}, \frac{3\times-3+4\times4}{3+4}\right)$

$= \left(\frac{17}{7}, \frac{7}{7}\right) = \left(\frac{17}{7}, 1\right)$

(ii) External Division:

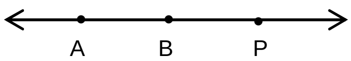
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points then the coordinates of a point P, which divides the line joining AB in the ratio m : n externally are given by

$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

Fig 1. 

(if m < n)

OR

Fig 2. 

(if m > n)

Note: The point P is beyond A and B for external division. It can be either beyond A (Fig 1) or beyond B (fig 2).

6.08. Find the co-ordinates of the point P which divides the line joining the points (3, –5) and (4, 6) in the ratio 2 : 1 externally.

Sol: $P = \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

Here $(x_1, y_1) = (3, –5)$; $(x_2, y_2) = (4, 6)$, m = 2 and n = 1.

$P = \left(\frac{2(4)-1(3)}{2-1}, \frac{2(6)-1(-5)}{2-1}\right) = (5, 17)$

Note: The midpoint of the line segment joining two points is a special case of section formula, when the ratio is 1 : 1.

6.09. Find the centre of the circle which has (5, 12) and (3, –8) as the extremities of its diameter.

Sol: The centre of any circle C is the midpoint of its diameter.

$\therefore C = \left(\frac{5+3}{2}, \frac{12-8}{2}\right) = (4, 2).$

Centroid: If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle, then the centroid of the triangle ABC is given by $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Note: The centroid of a triangle is the point of concurrence of the medians of a triangle.

6.10. Find the centroid of the triangle formed by the vertices A(–1, 3), B(5, –2) and C(5, –4).

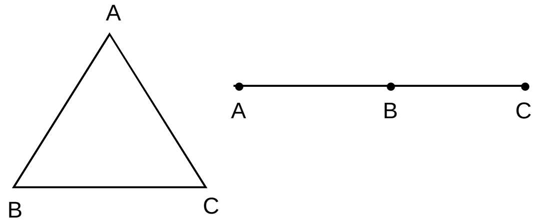
Sol: G = Centroid of the triangle

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Here A (–1, 3) = (x_1, y_1), B (5, –2) = (x_2, y_2), C (5, –4) = (x_3, y_3).

$G = \left(\frac{-1+5+5}{3}, \frac{3-2-4}{3}\right) = (3, -1)$

Collinearity: Given three distinct points in a plane $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, there are two possibilities. They may form a triangle or a straight line.



In case the three points A, B and C form a straight line, we say that they are "collinear".

Any of the following conditions are enough to show collinearity of the given three points.

1. AB + BC = CA or AC + CB = AB or AB + AC = BC.
2. The area of the triangle formed by A, B and C equals zero.

6.11. Show that the points A(3, –4), B(7, –10), C(5, –7) are collinear.

Sol: $AC = \sqrt{(5-3)^2+(-7+4)^2} = \sqrt{4+9} = \sqrt{13}$

$BC = \sqrt{(5-7)^2+(-7+10)^2} = \sqrt{4+9} = \sqrt{13}$

$AB = \sqrt{(7-3)^2+(-10+4)^2}$

$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

$\Rightarrow$ AC + BC = AB

$\therefore$A, B, C are collinear (OR)

Applying the area of a triangle formula,

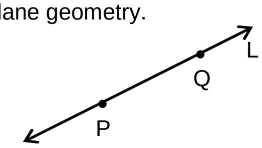
$\frac{1}{2}\begin{vmatrix} 3-7 & -4+10 \\ 7-5 & -10+7 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -4 & 6 \\ 2 & -3 \end{vmatrix}$

$= \frac{1}{2}$ [(–4) (–3) – (B) (6)] = 0

$\therefore$ A, B and C are collinear.

The Straight Line:

We now deal with a case when a specified relationship (equation) between x and y is given for various points P(x, y). One such relationship is the linear equation. The graph of it is called a straight line. We know that there is one and only one line containing two distinct points P and Q from plane geometry.

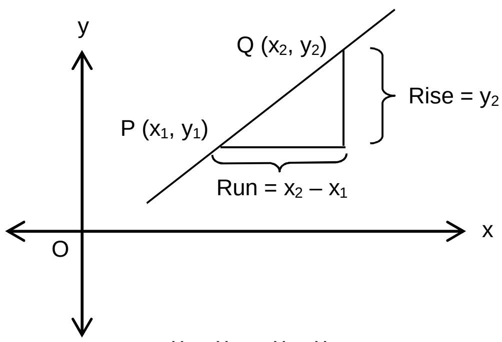


If P, Q are each represented by ordered pairs of real numbers, the following definition can be given:

Slope of a line: The slope of a line is a number that describes both the direction and steepness of a line.
Let P and Q be two distinct points with coordinates (x_1, y_1) and (x_2, y_2) respectively. The slope m of the line L containing P and Q is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ if } x_1 \neq x_2.$$

If $x_1 = x_2$, the slope m of line L is undefined (since this results in division by 0) and L is a vertical line.
For a non vertical line, slope = $\frac{\text{Change in } y}{\text{Change in } x}$ or "rise over run"



Note: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$
So the slope is same whether changes are computed from P to Q or Q to P.

Alternately, the slope of a line is the tangent value of the angle (θ) made by the line with the positive direction of the x-axis in the anticlockwise direction.
$m = \tan\theta$.

6.12. Compute the slope of the line passing through the points (–1, 4) and (3, –1).

Sol: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{3+1} = \frac{-5}{4}$.

6.13. The lines L_1, L_2, L_3 and L_4 contain the following pairs of points. Find their slopes and graph them.
L_1 : (3, –4); (–1, 2) L_2 : (3, –4); (3, 3)
L_3 : (3, –4); (1, – 4) L_4 : (3, –4); (5, 2)

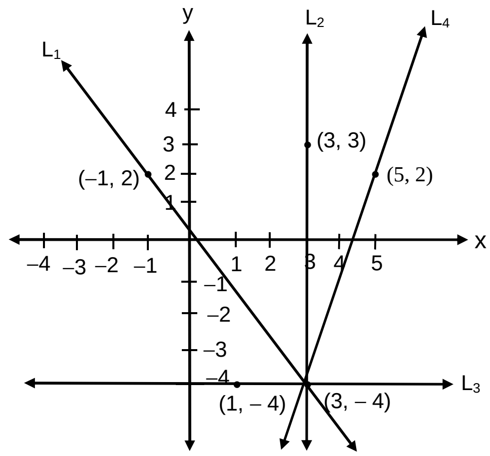
Sol: Let m_1, m_2, m_3, m_4 denote the slopes of the lines L_1, L_2, L_3 and L_4 respectively. Then

$m_1 = \frac{2-(-4)}{-1-3} = \frac{6}{-4} = \frac{-3}{2}$

$m_2 = \frac{3-(-4)}{3-3} = \frac{7}{0}$

$m_3 = \frac{-4+4}{1-3} = 0$

$m_4 = \frac{2+4}{5-3} = 3$



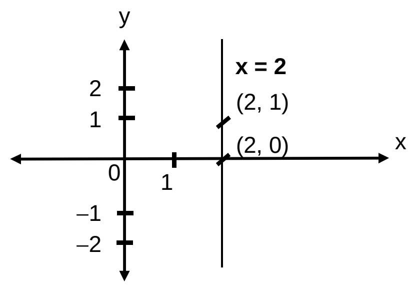
m_1 is negative, the line L_1 "slants downwards" or decreasing trend; m_2 is not defined, the line L_2 is vertical; m_3 is 0, the line L_3 is horizontal; m_4 is positive, the line L_4 "slants upward" or increasing trend.

Equations of lines

Vertical lines:

The equation of a vertical line passing through a point (a, 0) is given by the equation x = a where a is a given real number.

Example:



Non-vertical lines: Let L be a non-vertical line with slope m containing (x_1, y_1). For any other point (x, y) on L, we have

$m = \frac{y - y_1}{x - x_1}$ or $y - y_1 = m(x - x_1)$

Point-Slope Form: The equation of a non-vertical line of slope m and passing through the point (x_1, y_1) is $y - y_1 = m(x - x_1)$

6.14. Find the equation of the line which passes through (1, –1) and has slope of 2.

Sol: $y - y_1 = m(x - x_1)$
Here m = 2, (x_1, y_1) = (1, –1)
y + 1 = 2 (x – 1)
y + 1 = 2x – 2
⇒ y = 2x – 3.

Two-point Form: The equation of a non-vertical line passing through $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

6.15. Find the equation of the line passing through the points (2, 1) and (4, 5).

Sol: Here $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (4, 5)$.

$y - 1 = \frac{5-1}{4-2}(x - 2)$

$\Rightarrow y - 1 = 2(x - 2) \quad \Rightarrow y = 2x - 3.$

General Form: The equation of a line L is in the general form when it is written as $ax + by + c = 0$ where a, b and c are real numbers with either $a \neq 0$ or $b \neq 0$.

6.16. Find the equation of the line joining the points (5, 3) and (2, 1) in the general form.

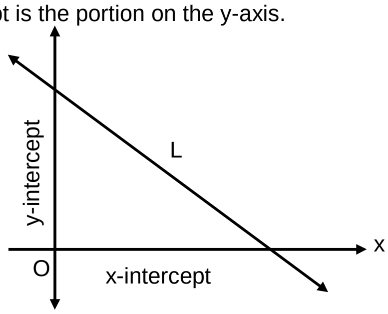
Sol: Using the two-point form, we get

$y - 3 = \frac{1-3}{2-5}(x - 5)$

$\Rightarrow$ $y - 3 = 2/3\,(x - 5)$
$\Rightarrow$ $3y - 9 = 2x - 10$
$\Rightarrow$ $2x - 3y - 1 = 0$ $(ax + by + c = 0)$
Here $a = 2$, $b = -3$ and $c = -1$.

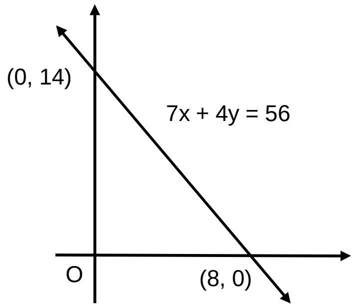
Note:
(A) In algebra $ax + by + c = 0$ is termed as a first-degree equation in x and y.
(B) If $a = 0$ and $b \neq 0$, then L will be a horizontal line.
(C) If $b = 0$ and $a \neq 0$, then L will be a vertical line.
(D) If $c = 0$, then L passes through the origin.

Intercepts: The portions cut off by a line on the coordinate axes are called intercepts.
The x-intercept is the portion on the x-axis and the y-intercept is the portion on the y-axis.



6.17. Find the x and y intercepts of the line $7x + 4y - 56 = 0$.

Sol: The given line cuts the x-axis when y = 0
$\Rightarrow 7x = 56$
$\Rightarrow x = 8$
$\therefore$ x-intercept = 8
The given line cuts the y-axis when x = 0
$\Rightarrow 4y = 56$
$\Rightarrow y = 14$
$\therefore$ y-intercept = 14.



Intercept Form: $\frac{x}{a} + \frac{y}{b} = 1$, where x-intercept is 'a' and y-intercept is 'b'.

6.18. Write the intercept form of the line whose general form is $3x + 4y - 12 = 0$.

Sol: Putting x = 0 in the equation, we get y = 3 and y = 0 in the equation, we get x = 4

$\therefore$ The intercept form of the line is $\frac{x}{4} + \frac{y}{3} = 1$.

Slope Intercept Form: The equation of a line with slope m and y-intercept b is $y = mx + b$

Note:
(A) When the equation is written in this form, the coefficient of x is the slope and the constant term gives the y-intercept of the line.
(B) y is explicitly written in terms of x. So this form is also termed as the explicit form of the line.

6.19. Find the slope and the y-intercept of the line $y = 2x - 5$.

Sol: The given line is of the form $y = mx + b$ where $m = 2$, $b = -5$.
$\therefore$ slope (m) = 2
y-intercept (b) = –5

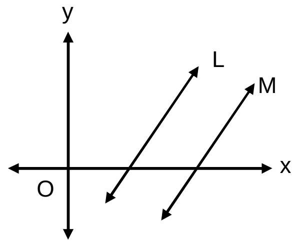
The following table summarises the various forms of equations of straight lines.

	You are Given	You Use	Equation
1.	Point (x_1, y_1) and slope m	Point - slope form	$y - y_1 = m(x - x_1)$
2.	Two points (x_1, y_1), (x_2, y_2)	If $x_1 = x_2$, use vertical line equation	$x = x_1$
		If $x_1 \neq x_2$, Two Point Form	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
3.	x and y intercepts a and b	Intercept-Form	$x/a + y/b = 1$
4.	Slope m, y-intercept b	Slope - Intercept Form	$y = mx + b$

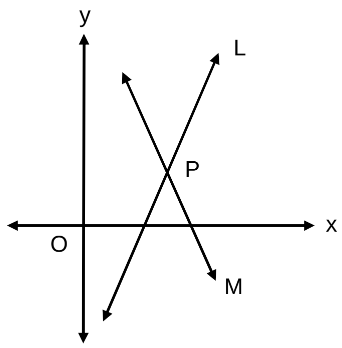
Parallel and intersecting lines:

Let L and M be two lines. Exactly one of the following three relationships must hold for two lines L and M.

1. All the points on L are the same as the points on M. (Identical lines)
2. L and M have no points in common. (Parallel lines)
3. L and M have exactly one point in common. (intersecting lines)



Parallel lines



Intersecting lines

Note: To find the coordinates of the point of intersection of the lines
$L : a_1x + b_1y + c_1 = 0$
$M : a_2x + b_2y + c_2 = 0$
We solve the two equations to get the point of intersection as

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$$

6.20. Find the point of intersection of the lines $3x - 2y - 12 = 0$ and $4x + y - 5 = 0$ and also find the number of regions into which the xy plane is divided by these lines.

Sol: $L_1 = 3x - 2y - 12 = 0$ and $L_2 = 4x + y - 5 = 0$
On solving we get $x = 2$ and $y = -3$.
So $(2, -3)$ is the point of intersection.
As the lines intersect, the plane gets divided into 4 infinite regions.

Angle between two lines:

If m_1 and m_2 are the slopes of two lines, the angle 'θ' between them is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

Note:

(1) Condition for parallel lines:
$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ or $m_1 = m_2$

(2) Condition for perpendicular lines:
$a_1 a_2 + b_1 b_2 = 0$ or $m_1 m_2 = -1$.

6.21. Show that the lines $2x - 3y - 4 = 0$ and $-8x + 12y - 6 = 0$ are parallel.

Sol: $a_1 = 2, a_2 = -8, b_1 = -3, b_2 = 12$

$$\frac{a_1}{a_2} = \frac{2}{-8} = \frac{-1}{4};\ \frac{b_1}{b_2} = \frac{-3}{12} = \frac{-1}{4}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\therefore$ The given lines are parallel.

Note: It can be concluded that the lines are parallel, by observing $m_1 = m_2 = 2/3$.

6.22. Show that the lines $3x + 5y - 9 = 0$ and $10x - 6y + 7 = 0$ are perpendicular.

Sol: $a_1 = 3, b_1 = 5, a_2 = 10, b_2 = -6$
$a_1a_2 + b_1b_2 = (3)(10) + (5)(-6) = 30 - 30 = 0$
$\therefore$ The given lines are perpendicular.

Note: The equation of a line through a point $P(x_1, y_1)$ and
(i) parallel to $ax + by + c = 0$ is given by $a(x - x_1) + b(y - y_1) = 0$.
(ii) perpendicular to $ax + by + c = 0$ is given by $b(x - x_1) - a(y - y_1) = 0$.

6.23. Find the equation of the line through $(3, -3)$ and
(i) parallel to $3x + y + 1 = 0$
(ii) perpendicular to $2x + 5y - 2 = 0$

Sol: (i) $3(x - 3) + 1(y + 3) = 0$
$\Rightarrow 3x - 9 + y + 3 = 0$
$\Rightarrow 3x + y - 6 = 0$
(ii) $5(x - 3) - 2(y + 3) = 0$
$\Rightarrow 5x - 15 - 2y - 6 = 0$
$\Rightarrow 5x - 2y - 21 = 0$.

Some formulae to remember:

(1) The general form of the equation of a straight line is $ax + by + c = 0$. Here, the y-intercept is $-c/b$, the x-intercept is $-c/a$ and the slope is $-a/b$.

(2) If $ax + by + c = 0$ is the equation of a line, the perpendicular distance from a point (x_1, y_1) to this line is given by: $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

(3) The distance between two parallel straight lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

(4) The equation of a circle centred at (h, k) with radius 'r' units is $(x - h)^2 + (y - k)^2 = r^2$.

(5) The equation of a circle centred at the origin with radius 'r' units is $x^2 + y^2 = r^2$.

Some more worked examples:

6.24. Find the equation of the line whose x and y intercepts are $2\frac{1}{3}$ and –3 respectively.

Sol: $\frac{x}{a}+\frac{y}{b} = 1$

$\Rightarrow \quad \frac{x}{7/3}+\frac{y}{-3} = 1$

$\Rightarrow \quad 9x – 7y = 21$

$\Rightarrow \quad 9x – 7y – 21 = 0.$

6.25. Find the equation of the line passing through (2, –3) and parallel to the x-axis.

Sol: Equation of a line parallel to the x-axis is of the form y = k.
Substituting (2, –3) we get –3 = k
$\therefore$ Equation of the required line is y = –3.

6.26. Find the perpendicular distance of the point (2, 2) from the line 4x + 3y – 4 = 0.

Sol: The length of the perpendicular from the point $P(x_1, y_1)$ to the line ax + by + c = 0 is given by

$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

So d = $\left|\frac{4(2)+3(2)-4}{\sqrt{4^2+3^2}}\right| = \left|\frac{8+6-4}{5}\right|$

$=\left|\frac{10}{5}\right|$ = 2 units.

6.27. Find the equation of the circle with centre as (3, 2) and radius 4 units.

Sol: Let $P(x_1, y_1)$ be any point on the circle.
The distance between P and the center of the circle equals the radius.
Accordingly, $\sqrt{(x_1-3)^2+(y_1-2)^2} = 4$
On squaring, we get $(x_1 – 3)^2 + (y_1 – 2)^2 = 16$
$x_1^2 + y_1^2 – 6x_1 – 4y_1 – 3 = 0$
As (x_1, y_1) was arbitrarily chosen, the equation of the circle is
$x^2 + y^2 – 6x – 4y – 3 = 0$

Note: The general form of the circle is
$x^2 + y^2 + 2gx + 2fy + c = 0$ with $g^2 + f^2 – c \geq 0$.

6.28. A line drawn through P (2, –3) making an angle of 45° with the x-axis cuts the x-axis at Q. Find PQ.

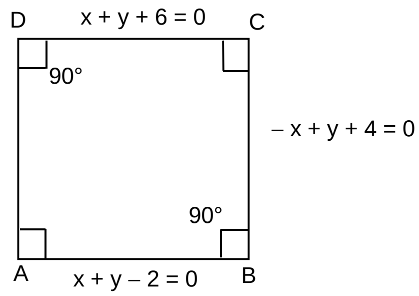
Sol: Equation of the line is y + 3 = 1 (x – 2)
$\Rightarrow$ x – y – 5 = 0.
This line cuts the x-axis, for y = 0
$\Rightarrow$ x = 5
$\therefore$ Q (5, 0)
PQ = $\sqrt{(5-2)^2+(0+3)^2}$
$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units.

6.29. Find the value of k, if
(x + y – 1) – k (3x – 7y + 12) = 0 is parallel to the y-axis.

Sol: x (1 – 3k) + y (1 + 7k) – (1 + 12k) = 0. – (A)
Slope = $\frac{-(1-3k)}{1+7k} = \frac{3k-1}{1+7k}$
Since (A) is parallel to the y-axis, slope = ∞
$\Rightarrow$ 1 + 7k = 0
$\Rightarrow k = \frac{-1}{7}$.

6.30. The lines x + y – 2 = 0, x + y + 6 = 0 and –x + y + 4 = 0 form three sides of a square. Find the equation of the fourth side.

Sol:



Let the fourth line by –x + y + k = 0
Since ABCD is a square, the distance between the lines AB and CD = the distance between the lines AD and BC.

$\frac{|6+2|}{\sqrt{1^2+1^2}} = \frac{|4-k|}{\sqrt{(-1)^2+1^2}}$

$\Rightarrow \quad \frac{|8|}{\sqrt{2}} = \frac{|4-k|}{\sqrt{2}}$

$\Rightarrow \quad 4 – k = \pm 8.$

$\Rightarrow \quad k = –4 \text{ or } k = 12$

$\therefore$ Equation of the fourth side is –x + y – 4 = 0 or –x + y + 12 = 0.

Change of Axes:

Sometimes, to be able to express an equation in a simpler form, it may be required to change the original co-ordinate axes, either by way of shifting the origin or by changing the direction of the axes.

Translation of Axes:

In this case, the origin is shifted to a new point, keeping the direction of the axes intact. The new axes are parallel to the original axes.
Suppose the origin is shifted to (h, k). If the original coordinates of a point P are (x, y) and (X, Y) denote the coordinates of P with reference to the new axes, then we have x = X + h and y = Y + k.

Rotation of Axes:

In this process, the origin is kept intact and the axes are rotated about the origin, through a required angle.

If θ is the angle of rotation and (x, y) are the coordinates of a point P with reference to the original axes and (X, Y) with reference to the new axes, then the relation between them is given below:

$x = X\cos\theta - Y\sin\theta$	$y = X\sin\theta + Y\cos\theta$
$X = x\cos\theta + y\sin\theta$	$Y = -x\sin\theta + y\cos\theta$

These equations are called transformation equations.

Note:

(1) If the axes are rotated at an angle θ in the anticlockwise direction, θ is considered positive.
(2) Sometimes, we may have to translate and rotate or rotate and translate. The order is unimportant.

6.31. (a) Find the coordinates of the point (3, 5) when the origin is translated to (1, 3).
(b) The origin is translated to (5, −6) and the point P is transformed to (1, 3). Find the original coordinates of P.

Sol: (a) Here x = X + h, y = Y + k
Given (h, k) = (1, 3) and (x, y) = (3, 5)
X = x − h = 3 − 1 = 2
Y = y − k = 5 − 3 = 2
∴ (X, Y) = (2, 2)

(b) Here (h, k) = (5, −6) and (X, Y) = (1, 3)
x = X + h = 1 + 5 = 6
y = Y + k = 3 − 6 = −3
(x, y) = (6, −3)

6.32. (a) If the axes are rotated through an angle 30°, then find the coordinates of the point (3, 5) in the new system.
(b) When the axes are rotated through an angle of 45°, the coordinates of a point P in the new system are $(5\sqrt{3}, 6\sqrt{3})$. Find the coordinates of P in the original system.

Sol: (a) Here $X = x\cos\theta + y\sin\theta$, $Y = -x\sin\theta + y\cos\theta$
Where θ = 30° and (x, y) = (3, 5)

$X = 3.\frac{\sqrt{3}}{2} + 5.\frac{1}{2} = \frac{5+3\sqrt{3}}{2}$

$Y = -3.\frac{1}{2} + 5.\frac{\sqrt{3}}{2} = \frac{-3+5\sqrt{3}}{2}$

The point in the new system is

$\left(\frac{5+3\sqrt{3}}{2}, \frac{5\sqrt{3}-3}{2}\right)$

(b) Here $x = X\cos\theta - Y\sin\theta$, $y = X\sin\theta + Y\cos\theta$
Where θ = 45° and (X, Y) = $\left(5\sqrt{3}, 6\sqrt{3}\right)$

$x = 5\sqrt{3}.\frac{1}{\sqrt{2}} - 6\sqrt{3}.\frac{1}{\sqrt{2}}$

$= \frac{5\sqrt{3} - 6\sqrt{3}}{\sqrt{2}} = -\sqrt{\frac{3}{2}}$

$y = 5\sqrt{3}.\frac{1}{\sqrt{2}} + 6\sqrt{3}.\frac{1}{\sqrt{2}} = \frac{11\sqrt{3}}{\sqrt{2}} = 11\sqrt{\frac{3}{2}}$

Coordinates of P are $\left(-\sqrt{\frac{3}{2}}, 11\sqrt{\frac{3}{2}}\right)$

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. A straight line with a slope of –1 passes through the Ist quadrant. The quadrant through which it can't pass through is _____.
(A) IInd (B) IIIrd
(C) IVth (D) None of these

2. What is the least distance (in units) between two non-parallel lines lying in the same plane, if the distance is measured along a direction which is perpendicular to one of the lines?

$\boxed{}$

3. What is the maximum number of quadrants that a straight line can pass through?

$\boxed{}$

4. The point (–4, 3) lies in _______ quadrant
(A) Ist (B) IInd (C) IIIrd (D) IVth

5. What are the intercepts of a line passing through the origin?
(A) one unit each (B) two units each
(C) each zero (D) none of these

6. What is the distance of the straight line y = mx + c from the origin?
(A) $\left|\frac{c}{\sqrt{1+m^2}}\right|$ (B) $\left|\frac{c}{m}\right|$
(C) $\left|\frac{c}{1+m}\right|$ (D) $\left|\frac{c}{1+m^2}\right|$

7. Find the distance from the origin to (3, 4).

$\boxed{}$

8. Distance from origin to the line ax + by + c = 0 is
(A) $\left|\frac{c}{a^2+b^2}\right|$ (B) $\left|\frac{c^2}{a^2+b^2}\right|$
(C) $\left|\frac{c^2}{\sqrt{a^2+b^2}}\right|$ (D) $\left|\frac{c}{\sqrt{a^2+b^2}}\right|$

9. What is the ratio in which the origin divides the line segment joining the points (x_1, y_1) and $(-3x_1, -3y_1)$?
(A) 1 : 3 externally (B) 1 : 3 internally
(C) 1 : 2 internally (D) 2 : 1 internally

10. How many points are there which are at unit distance from the origin?
(A) 2 (B) 4 (C) 1 (D) infinite

11. In a square, two adjacent vertices are (0, 0) and (0, α) What is the length of the diagonal of the square?
(A) $\sqrt{\alpha}$ (B) $2\sqrt{\alpha}$ (C) $\sqrt{2}\alpha$ (D) α^2

12. What is the area of the circle whose largest chord is 2 units? (in sq units)
(A) π (B) 4π
(C) 2π (D) None of these

13. If (x_1, y_1) and $(-x_1, y_1)$ are two opposite vertices of a square, the other two vertices are _____.
(A) $(y_1 + x_1, 0)$ $(y_1 - x_1, 0)$
(B) $(0, y_1 + x_1)$ $(0, y_1 - x_1)$
(C) $(-x_1, -y_1)$, $(x_1, -y_1)$
(D) None of these

14. If the equation of the line joining A and B is y = mx + c, what is the equation of the line joining B and C, if AB + BC = AC?
(A) y = mx – c (B) y = –mx + c
(C) y = –mx – c (D) y = mx + c

15. ABCD is a parallelogram If A = (x_1, y_1), B = (x_2, y_2) and C = (x_3, y_3), how many possible positions of D exist?

$\boxed{}$

16. The area of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is zero. What can we say about the three points?
(A) They are collinear
(B) One of the point divides the join of the other two in some ratio
(C) Both (A) and (B)
(D) Neither (A) nor (B)

17. If two parallel lines are 1 unit apart and one of the lines is y = mx + c, what is the equation of the other line?
(A) $y = mx - c + \sqrt{1+m^2}$
(B) $y = mx + c + \sqrt{1+m^2}$
(C) $y = mx + c - \sqrt{1+m^2}$
(D) Either (B) or (C)

18. Which is the point that divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio of m : n internally?
(A) $\left(\frac{mx_1+nx_2}{m+n}, \frac{my_1+ny_2}{m+n}\right)$
(B) $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
(C) $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$
(D) None of these

19. The slope of the line whose inclination is 135° is m. $m + 1 = \boxed{}$.

20. $(0, y_1)$ and $(0, -y_1)$ are the end points of a diagonal of a rhombus. The other diagonal is half the first diagonal. Which of the following is/are vertices of the rhombus?

(A) $(\frac{y_1}{2}, 0)$ (B) $(\frac{-y_1}{2}, 0)$
(C) Both (A) and (B) (D) Neither (A) nor (B)

21. In a rhombus ABCD, the diagonals intersect at the origin. If the equation of AB is y = mx + c, what is the equation of CD?
(A) y = mx – c (B) y = –mx + c
(C) y = mx + c (D) y = –mx – c

22. Which of the following points lie on the line 2x – y + 7=0?
(A) (–2, 3) (B) (0, 7)
(C) (1, 9) (D) All the above

23. The angle made (in degrees) by the line x – y = 0 with the positive direction of x – axis is $\boxed{}$.

24. If (1.8, 2.4) (5, 0) and (0, 0) are the vertices of a triangle, what are the coordinates of its orthocenter?
(A) (0, 0) (B) (5, 0)
(C) (1.8, 2.4) (D) None of these

25. What is the product of the slopes of the diagonals of a rhombus?
(A) –1
(B) zero
(C) indeterminate
(D) Either (A) or (C)

26. The equation of a line parallel to y = 2x and passing through (3, 4) is ______.
(A) y = 2x +2 (B) $y = \frac{1}{2}x + \frac{5}{2}$
(C) $y = \frac{-1}{2}x + \frac{11}{2}$ (D) y = 2x – 2

27. The equation of a line perpendicular to y = 3x + 1 and passing through (1, 1) is _____.
(A) 3y = –x + 12 (B) y = –3x + 4
(C) 3y = -x + 4 (D) 3y = x + 2

28. A circle is centered at the origin and passes through (3, 4). Find the circumference of the circle.
(A) 5 π (B) 10 π (C) 2.5 π (D) 20 π

29. The centroid of the triangle whose vertices are (0, 0), (5, 0) and (0, 12) is _____.
(A) $\left(\frac{5}{2},6\right)$ (B) $\left(\frac{5}{4},3\right)$
(C) $\left(\frac{5}{3},4\right)$ (D) $\left(\frac{5}{6},2\right)$

30. The midpoint of the line segment whose ends are (1, 6) and (9, 12) is _____.
(A) (5, 9) (B) (7, 9)
(C) (5, 8) (D) (6, 8)

Exercise – 6(a)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. The distance between (3, 7) and the centre of the circle $(x - 6)^2 + (y - 3)^2 = 25$ (in units) is $\boxed{}$.

2. The ratio in which y-axis divides the line joining the points (4, 3) and (–6, 2) is ______.
(A) 2 : 3 externally (B) 3 : 2 internally
(C) 2 : 3 internally (D) 3 : 2 externally

3. If the points (3, 5), (5, 9) and (10, k) are collinear, then k = _____.
(A) $\frac{1}{19}$ (B) 1 (C) 19 (D) –1

4. If the orthocenter and the centroid of a triangle are (4, 5) and (3, 3) respectively, then the circumcentre of the triangle is _____.
(A) $\left(\frac{5}{2}, 2\right)$ (B) (5, 4) (C) (1, 6) (D) (6, 1)

5. If (2, 6), (–4, 2) and (8, –4) are three consecutive vertices of a parallelogram, then the fourth vertex is
(A) (14, 0) (B) (0, –4)
(C) (–14, 0) (D) (–10, 12)

6. The slope of the line joining the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ is _____.
(A) $\frac{1}{2(t_1+t_2)}$ (B) $\frac{1}{t_2+t_1}$
(C) $\frac{2}{t_2+t_1}$ (D) $\frac{1}{2a(t_1+t_2)}$

7. The equation of the line passing through the points (5, 6) and (4, 3) is _____.
(A) $3x - y + 9 = 0$ (B) $2x + 3y - 17 = 0$
(C) $x + 3y - 24 = 0$ (D) $3x - y - 9 = 0$

8. The acute angle made (in degrees) by the line $\sqrt{3}\,x - y + 9 = 0$ with y–axis is $\boxed{}$.

9. The acute angle (in degrees) between the lines $2x + 3y + 7 = 0$ and $x - 5y + 3 = 0$ is $\boxed{}$.

10. If $3x + 4y + 7 = 0$ and $12x - 9y + 10 = 0$ are two adjacent sides of a rectangle and one of its vertices is (2, 3), then the area of the rectangle in sq. units is
(A) 5 (B) 2 (C) $\frac{7}{3}$ (D) $\frac{7}{5}$

11. If the lines $2x + 3y + 7 = 0$, $4x + 9y + 12 = 0$ and $3x - 2y + 9 = 0$ form a right angled triangle, then one end of the hypotenuse is _____.
(A) $\left(-\frac{13}{5}, -\frac{3}{5}\right)$ (B) $\left(\frac{9}{2}, -\frac{2}{3}\right)$
(C) $\left(-\frac{9}{2}, \frac{2}{3}\right)$ (D) (0, –3)

12. A line passes through the point (4, –5). The sum of the intercepts is 7. Its equation can be _____.
(A) $x - 2y + 14 = 0$ (B) $2x - y - 14 = 0$
(C) $2x + 5y - 10 = 0$ (D) $5x + 2y - 10 = 0$

13. If A(4, 5), B(3, 6), C(2, 1) are the vertices of a triangle, then the equation of the altitude through A is _____.
(A) $3x - 7y + 23 = 0$ (B) $5x - y - 15 = 0$
(C) $x - 5y + 4 = 0$ (D) $x + 5y - 29 = 0$

14. If the lines $ax + 3y + 7 = 0$ and $4x + 9y + 15 = 0$ are perpendicular to each other, then the value of a is _____.
(A) $-\frac{27}{4}$ (B) $-\frac{3}{2}$ (C) $\frac{4}{27}$ (D) $\frac{2}{3}$

15. If the line $x + ky + 3k + 2 = 0$ passes through the point of intersection of the lines $4x + 5y - 23 = 0$ and $x + 3y - 11 = 0$, then k = _____.
(A) $\frac{2}{3}$ (B) $-\frac{3}{2}$ (C) $\frac{3}{2}$ (D) $-\frac{2}{3}$

16. If the line $3x + 4y + 5 + k(x - 3y + 2) = 0$ is parallel to the x-axis, then the value of k^2 is $\boxed{}$.

17. The point of intersection of $8x + 5y = 48$ and $y = kx + 6$ has integral coordinates. The number of integral values of k is $\boxed{}$.

18. If the roots of the equation $x^2 - 5x - 6 = 0$ represents the slope and y-intercept of a line, then the equation of the line can be _____.
(A) $x + y + 6 = 0$ (B) $6x - y - 1 = 0$
(C) $6x + y + 1 = 0$ (D) $x + y + 1 = 0$

19. Two of the tangents to a circle are $x + y - 7 = 0$ and $2x + 2y + 13 = 0$. Find the circumference of the circle.
(A) $\frac{27}{\sqrt{2}}\pi$ (B) $\frac{27}{2\sqrt{2}}\pi$
(C) $27\sqrt{2}\,\pi$ (D) $\frac{27}{4\sqrt{2}}\pi$

20. The distance between the parallel lines $5x + 12y + 24 = 0$ and $10x + 24y + 49 = 0$ is _____.
(A) $\frac{1}{26}$ (B) $\frac{25}{13}$
(C) $\frac{1}{13}$ (D) $\frac{2}{13}$

21. The area of the triangle (in sq.units) formed by the line $4x - 5y + 20 = 0$ with the coordinate axes is $\boxed{}$.

22. The vertices of a triangle are (–4, 0), (7, 0) and (5, a). If the centroid of the triangle is $\left(\frac{8}{3}, \frac{5}{3}\right)$, the area of the triangle is ______.

(A) $\frac{65}{2}$ (B) $\frac{75}{2}$ (C) $\frac{55}{2}$ (D) $\frac{45}{2}$

23. Two identical circles intersect at (0, 5) and (0, –5). Each circle passes through the centre of the other circle. Find the area of either circle.

(A) $\frac{50\pi}{3}$ (B) $\frac{200\pi}{3}$ (C) $\frac{100\pi}{3}$ (D) $\frac{400\pi}{3}$

24. If $81y^2 - x^2 + 14x = c$ represents a pair of straight lines, the value of c is ______.

(A) 64 (B) 49 (C) 81 (D) 36

25. The points (0, 6) and (0, 17) are the ends of a diagonal of a square. One of the ends of the other diagonal of the square is ______.

(A) (4.5, 11.5) (B) (–7.5, 11.5)
(C) (–6.5, 11.5) (D) (–5.5, 11.5)

26. If three of the vertices of a parallelogram are (p – q, p + q), (p + q, p – q), and (6p – q, 6p + q), the fourth vertex of the parallelogram can be ______.

(A) (6p – 3q, 6p + 3q)
(B) (6p + q, 6p – q)
(C) (q – 4p, –4p –q)
(D) Any of the previous choices

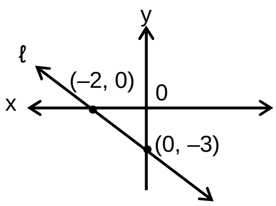
27. The area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are (3, 5), (5, 8) and (7, 5) is $\boxed{}$.

28. The point through which the line x (p + 5q) + 4y (p + q) = 5p + q passes for all real values of p and q is a _______.

(A) (–1, 1.5) (B) (–1, 0.5)
(C) (–2, 1.5) (D) (–2, 2.5)

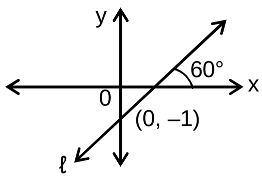
29. The equation of the line ℓ is _____.?

(A) 2x + 3y – 6 = 0
(B) 2x + 3y + 6 = 0
(C) 3x + 2y + 6 = 0
(D) 3x – 2y – 6 = 0



30. The equation of the line ℓ is _____.

(A) $\sqrt{3}x - y - 1 = 0$
(B) $\sqrt{3}y - x - 1 = 0$
(C) $\sqrt{3}x + y + 1 = 0$
(D) $\sqrt{3}y + x + 1 = 0$



31. In the xy-plane, the vertices of a triangle are (0, 20), (20, 0), (0, 0). The number of points with integer coordinates inside the triangle (excluding all the points on the boundary) is $\boxed{}$.

32. If a line passing through P(5, 3) makes an angle of 60º with the x-axis and cuts the y-axis at Q, then the length of PQ (in units) is $\boxed{}$.

33. (– 1, 3) (1, 8) (6, 6) and (4, 1) are the consecutive vertices of a quadrilateral. Find the equation of a line passing through (6, 6) which divides the quadrilateral into two equal areas.

(A) 7x – 5y + 12 = 0
(B) 5x – 7y + 12 = 0
(C) 3x – 7y – 24 = 0
(D) 3x – 7y + 24 = 0

34. The co-ordinates of the point (2, 3) in the new system when the origin is translated to the point (– 4, 5) are _____.

(A) (6, 2) (B) (6, – 2)
(C) (2, 6) (D) (– 2, 6)

35. The transformed equation of a curve when the origin is translated to (1, 1) is $2x^2 - 3xy - y^2 = 5$. Its equation with respect to the original axes is

(A) $2x^2 - 3xy - y^2 + x + 5y + 7 = 0$
(B) $2x^2 - 3xy + y^2 - x - 5y + 7 = 0$
(C) $2x^2 - 3xy - y^2 - x - 5y - 7 = 0$
(D) $2x^2 - 3xy - y^2 - x + 5y - 7 = 0$

Exercise – 6(b)

Directions for questions 1 to 35: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. If the slope of the line joining the points (4, p) and (p, 5) is –2, then the value of p is $\boxed{}$.

2. The equation of a line passing through the points (1, p) and (p, 1), where p ≠ 1, is _____.

(A) 2x + y – p – 2 = 0 (B) x + y – p – 1 = 0
(C) x + y + p = 0 (D) x + y = p

3. Which of the following lines passes through the origin and has a slope of 3?

(A) x – 3y = 0 (B) 3x – y = 0
(C) x – y = 0 (D) x – 3y + 5 = 0

4. Among the following, the line that makes equal intercepts on the coordinate axis is
(A) $x - y + 7 = 0$ (B) $2x + 3y = 6$
(C) $x + y - 10 = 0$ (D) $2x - 3y = 6$

Moderate

5. If the sides of a right-angled triangle lie on the lines $x + y - 8 = 0$, $3x - 2y + 1 = 0$ and $x - y = 0$, then the vertex opposite to the hypotenuse is _____.
(A) (3, 5) (B) (0, 0)
(C) (4, 4) (D) (–1, –1)

6. Let the values of x that satisfy the quadratic equation $x^2 + 7x + 12 = 0$ form an ordered pair (a, b) such that a > b. Which of the following relations does the point (a, b) satisfy?
(A) $a - 3b = -6$ (B) $2a - 3b = 10$
(C) $2a - 5b = 14$ (D) $a - b = 8$

7. The shortest distance (in units) between the lines $5x + 3y = 2$ and $x - 2y = 3$ is $\boxed{\quad}$.

8. The area of the triangle formed by the line $5x + 6y = 30$ with the coordinate axes is _____.
(A) 15 sq.units (B) 30 sq.units
(C) 25 sq.units (D) 27.5 sq.units

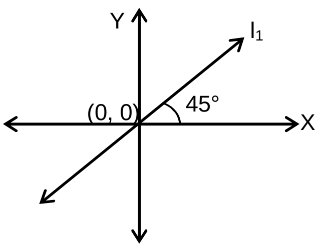
9. If the lines $3x - ky + 6 = 0$ and $2x + 3y + 7 = 0$ are parallel, then k =
(A) $\frac{-9}{2}$ (B) $\frac{-7}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$

10. If the angle between the lines $\sqrt{k}x - 3y + 10 = 0$ and $6x + ky + 25 = 0$ is 90°, then k = _____.
(A) 0 (B) 4
(C) 2 (D) Either (A) or (B)

11. The perpendicular distance (in units) of the origin from the line $3x + 4y + 1 = 0$ is $\boxed{\quad}$.

12. The points (0, 0), (p, q), (–p, –q) and (pq, q^2) are ______.
(A) vertices of a rectangle
(B) vertices of a rhombus
(C) Both A and B
(D) Neither A nor B

13. Which of the equations represent the following graph?
(A) $2x - 3y - 6 = 0$
(B) $3x - 2y - 6 = 0$
(C) $x - y = 0$
(D) $2x + y = 0$



14. The distance (in units) between the point (–3, –4) and the centroid of the triangle whose vertices are (7, 0), (5, 1) and (3, 5) is $\boxed{\quad}$.

15. The points (2, –3), (0, 0) and (3, 2) form a / an _____.
(A) scalene triangle.
(B) right angled isosceles triangle.
(C) equilateral triangle.
(D) straight line.

16. The orthocenter of a triangle is (x_1, y_1) and the centroid is origin. The circumcentre is ______.
(A) $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$ (B) $\left(\frac{-x_1}{2}, \frac{y_1}{2}\right)$
(C) $\left(\frac{x_1}{2}, \frac{-y_1}{2}\right)$ (D) $\left(\frac{-x_1}{2}, \frac{-y_1}{2}\right)$

17. The join of (–3, 2) and (4, 6) is cut by x – axis in the ratio _____.
(A) 1 : 3 externally (B) 1 : 2 externally
(C) 2 : 3 internally (D) 3 : 2 internally

18. The points (p + 1, 1), (2p + 1, 3) and (2p + 2, 2p) lie on the same straight line only if _____.
(A) p = –1 or 2 (B) p = 2 or 1
(C) p = 2 or $\frac{-1}{2}$ (D) p = $\frac{1}{2}$ or 2

19. Two points (1, 3) and (5, –5) are joined by a straight line. Which of the following points lies on this line?
(A) (1, –3) (B) (1, 2)
(C) (4, –3) (D) (5, 2)

20. The fourth vertex of the rectangle whose other vertices are (4, 1), (7, 4) and (13, –2) taken in that order, is _____.
(A) (10, –5) (B) (8, 3)
(C) (8, –3) (D) (–10, –5)

21. The equation of a line passing through the point (1, –6) and product of whose intercepts on the axes is 1, is _____.
(A) $3x + y = -3$ (B) $9x + y = 3$
(C) $4x + y + 2 = 0$ (D) Either (B) or (C)

22. A line makes an angle of 60° with the positive direction of the x-axis. If its x-intercept is 3, its equation is _____.
(A) $x + \sqrt{3}y + \sqrt{3} = 0$ (B) $x - y + \sqrt{3} = 0$
(C) $\sqrt{3}x + y + 3\sqrt{3} = 0$ (D) $\sqrt{3}x - y - 3\sqrt{3} = 0$

23. Three lines $3x - y = 2$, $5x - ay = 3$ and $2x + y = 3$ are concurrent. a = $\boxed{\quad}$.

24. If the two opposite sides of a square are represented by the lines $3x - y + 6 = 0$ and $9x - 3y + 30 = 0$, then its perimeter in units is _____.
(A) $\frac{8}{5}$ (B) $\frac{4}{\sqrt{10}}$ (C) $\frac{8}{\sqrt{10}}$ (D) $\frac{8\sqrt{10}}{5}$

25. If (3,10) is a vertex and $5x - y + 12 = 0$ is the equation of a diagonal of a square, then find the equation of the other diagonal of the square.
(A) $x + 5y - 53 = 0$ (B) $x + 5y - 17 = 0$
(C) $x + 5y - 18 = 0$ (D) $x + 5y + 53 = 0$

26. Find the area bounded by the lines 5x + 12y = 13, 5x + 4y = 3 and coordinate axes.

(A) $\frac{71}{60}$ (B) 71 (C) $\frac{71}{40}$ (D) $\frac{63}{40}$

27. If $\frac{1}{5}$ and $\frac{-1}{7}$ are the slopes of two lines, then the acute angle between the lines is _____.

(A) $\tan^{-1}\left(\frac{7}{13}\right)$ (B) $\tan^{-1}\left(\frac{6}{11}\right)$

(C) $\tan^{-1}\left(\frac{6}{17}\right)$ (D) $\tan^{-1}\left(\frac{8}{13}\right)$

28. If A (4, 9), B (6, 5) and C(7, 8) are the vertices of the triangle ABC, the slope of the line which bisects $\angle ACB$ is _________.

(A) $\frac{-1}{2}$ (B) 2 (C) –2 (D) $\frac{1}{2}$

29. The length of the intercept made by the line which passes through the points (3, 6) and (–3, 9) between the coordinate axes is _____

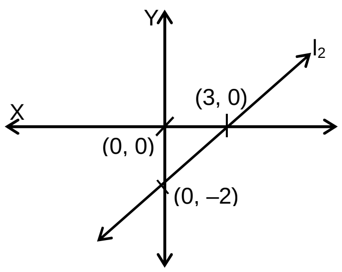
(A) $8\sqrt{5}$ (B) $\frac{7\sqrt{5}}{2}$ (C) $15\sqrt{5}$ (D) $\frac{15}{2}\sqrt{5}$

30. How many integral values of k are possible, if the lines 3x + 4ky + 6 = 0, and kx – 3y + 9 = 0 intersect in the 2nd quadrant?

31. The distance between two parallel lines is $\frac{3}{10}$ units. If one of the parallel lines is 3x + 4y = 9, then the other line could be _____.

(A) 6x + 8y = 21
(B) 9x + 12y = 15
(C) 6x + 8y = 15
(D) Either (A) or (C)

32. Which of the equations represent the following graph?



(A) 2x – 3y – 6 = 0
(B) 3x – 2y – 6 = 0
(C) x – y = 0
(D) None of these

Difficult / Very Difficult

33. The coordinates of the point A when the origin is translated to (–3, 1) are (–2, –1). The original coordinates of the point A are _____.

(A) (1, 0) (B) (– 5, 0)
(C) (5, 0) (D) (0, 5)

34. The axes in the coordinate system R are rotated through 45° in the clockwise direction to obtain the coordinate system R′. If the coordinates of a point P in R are $\left(-2\sqrt{2}, 5\sqrt{2}\right)$, what are its coordinates in R′?

(A) (– 7, 3) (B) (7, 3)
(C) (7, – 3) (D) (– 7, – 3)

35. The transformed equation of $2x^2 - xy + y^2 - 4x + 7y - 5 = 0$ when the origin is translated to (–1, 1) is _____.

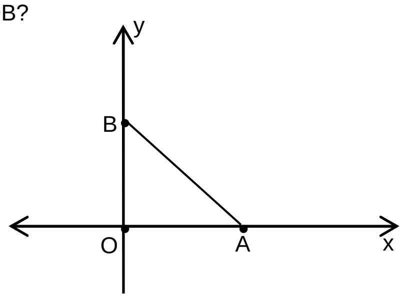
(A) $2X^2 - XY + Y^2 - 9X + 10Y + 10 = 0$
(B) $X^2 - XY + Y^2 - 9X + 10Y + 10 = 0$
(C) $2X^2 + XY + Y^2 + 9X + 10Y + 10 = 0$
(D) $2X^2 - XY - Y^2 - 9X - 10Y + 10 = 0$

Data Sufficiency

Directions for questions 36 to 40: Each question is followed by two statements, I and II. Answer each question using the following instructions:

Choice (A) if the question can be answered by using one of the statements alone, but cannot be answered by using the other statement alone.
Choice (B) if the question can be answered by using either statement alone.
Choice (C) if the question can be answered by using both statements together, but cannot be answered by using either statement alone.
Choice (D) if the question cannot be answered even by using both the statements together.”

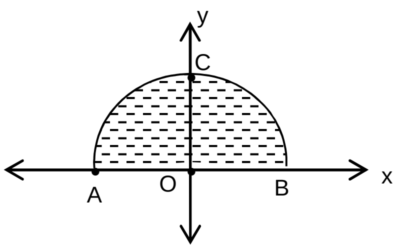
36. In the figure given below, what is the area of triangle AOB?



I. The equation of the line AB is 4x + 3y = 12.

II. The midpoint of the line segment AB is $\left(\frac{3}{2}, 2\right)$

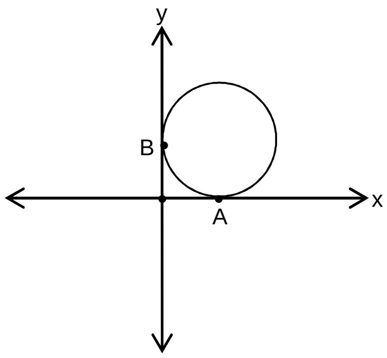
37. In the figure given below OA = OB = OC. What is the area of the shaded region?



I. The length of the line segment AB is 10 units.

II. The area of ΔABC is 25 sq. units.

38.



The circle given in the above figure touches the coordinate axes at the points A and B respectively.

What is the equation of the circle?

I. The coordinates of the point A are (4, 0)

II. The coordinates of the point B are (0, 4)

39. What is the area of triangle ABC?

I. ABC is an equilateral triangle.

II. The coordinates of the vertex A and the midpoint of the side BC are respectively (–3, 5) and (5, 2).

40. At which point does the line 'ℓ' intersect the x-axis?

I. The line ℓ is parallel to y-axis.

II. The line ℓ passes through the point (3, –4).

Key

Concept Review Questions

1. B	9. B	17. D	25. D
2. 0	10. D	18. B	26. D
3. 3	11. C	19. 0	27. C
4. B	12. A	20. C	28. B
5. C	13. B	21. A	29. C
6. A	14. D	22. D	30 A
7. 5	15. 3	23. 45	
8. D	16. C	24. C	

.

Exercise – 6(a)

1. 5	8. 30	15. D	22. C	29. C
2. C	9. 45	16. 9	23. C	30. A
3. C	10. C	17. 3	24. B	31. 171
4. A	11. C	18. B	25. D	32. 10
5. A	12. D	19. B	26. D	33. D
6. C	13. D	20. A	27. 1.5	34. B
7. D	14. A	21. 10	28. A	35. D

Exercise – 6(b)

1. 3	9. A	17. A	25. A	33. B
2. B	10. D	18. C	26. A	34. A
3. B	11. 0.2	19. C	27. C	35. A
4. C	12. D	20. A	28. D	36. B
5. C	13. C	21. D	29. D	37. B
6. C	14. 10	22. D	30. 5	38. B
7. 0	15. B	23. 2	31. D	39. C
8. A	16. D	24. D	32. A	40. C