Solutions

Exercise – 1

Solutions for questions 1 and 2:

Let the number of erasers, pencils and pens be x, y and z respectively.
$x + y + z = 38$ and $z > y > x$ and $x, y, z \geq 11$.
$\therefore x = 11$, and $y + z = 27$

There are two cases possible,
$\Rightarrow y = 12, z = 15$ or $y = 13, z = 14$
Let, $x = 12$, then $y + z = 26$

which is not possible because, if $y > x$, and $y = 13$, then $z = 13$ which would make $y = z$ which cannot be considered as $z > y$. Any other value of $y > 13$ would make $y > z$ which also is not acceptable.
$\therefore$ Two sets of values are possible.
$\therefore$ $x = 11, y = 12, z = 15$ and $x = 11, y = 13, z = 14$

1. 11 Choice (A)

2. If the number of pencils cannot be divided equally among the four brothers, then the number of pencils should not be a multiple of 12.
$\therefore$ The number of pencils = 13
$\therefore$ The number of pens = 14 Choice (D)

Solutions for questions 3 and 4:

If a person has x with him and the loser gives him 2/3 x, his amount would be $x + \frac{2}{3}x = \frac{5}{3}x$

Now if the amount with a person at the end of a round (i.e., after receiving $\frac{2}{3}^{rd}$ of what he had) is N, then the amount with him before that round would have been $\frac{3}{5}N$

Since, $\frac{3}{5}N + \frac{2}{3}\left(\frac{3}{5}N\right) = N$, for $\frac{3}{5}N$ to be an integer, N must be divisible by 5. Similarly if a person has 'y' with him and the loser gives him $\frac{1}{2}$ y his total amount would become $y + \frac{1}{2}y = \frac{3}{2}y$

Now if the amount with a person at the end of a round is 'M', (i.e. after receiving $\frac{1}{2}$ of what he had) then the amount with him before that round was $\frac{2}{3}N$

$$\frac{2}{3}N + \frac{1}{2}\left(\frac{2}{3}N\right) = N$$

Now for (2/3)M to be an integer, 'M' must be divisible by 3. Since there are two rounds where the loser gives 2/3 and two rounds where the loser gives 1/2 in order to have integral values, the value 'P' must be divisible by both 5^2 i.e., 25 and 3^2 i.e., 9.
P must be a multiple of $(5)^2 (3)^2$ i.e., 225.
Let P = 225k
Proceeding in the reverse direction we can get the following table

	Anil	Bimal	Charan	Deepak
	396k	252k $\boxed{168k}$	144k $\boxed{96k}$	108k $\boxed{72k}$
After Round I	60k $\boxed{40k}$	420k	240k $\boxed{160k}$	180k $\boxed{120k}$
After Round II	100k $\boxed{50k}$	100k $\boxed{50k}$	400k	300k $\boxed{150k}$
After Round III	150k $\boxed{75k}$	150k $\boxed{75k}$	150k $\boxed{75k}$	450k
After Round IV	225k	225k	225k	225k

The amounts in the boxes are those received from the loser.

3. We will get the least value of P, when K = 1. Therefore, the least value of P is ₹225. Choice (C)

4. From the above table, we can see that the amount with none of them shows such a trend. For each person, there is 1 decrease and 3 increases. They can't alternate. Choice (D)

Solutions for questions 5 and 6:

Let, cost = C and, number of diamonds be n
Given that, $C \propto n^2 \Rightarrow C = kn^2$ where k is the constant of proportionality $1,60,000 = k \times (8)^2 \Rightarrow k = 2,500$

5. The cost of the necklace with 10 diamonds (in rupees)
$= 2,500 (10)^2 = 2,50,000$ Choice (C)

6. Percentage decrease in value (or in the amount received by Seth Heeramal)
$$= \frac{2,500[10^2 - (5^2 + 3^2 + 2^2)}{2,500 \times 10^2} \times 100\% = 62\%$$
Choice (C)

Solutions for questions 7 and 8:

7. We want the net score of Smitha to be 60 with maximum possible number of incorrect answers. Given that Smitha answered all the questions and got exactly 40 correct answers in level C. So, the remaining 40 questions of level C are incorrect and the net score in level C is $(1 \times 40) - (1/2 - 40) = 20$. Since we want the maximum number of incorrect answers, the incorrect answers in level B must be made maximum possible. Say all the 60 questions of level B are wrong. Then the net score becomes $20 - (1 \times 60) = -40$. But the required net score is 60. So from level A, a net score of $60 - (-40) = 100$ must be scored. Let a be the number of incorrect answers in level A then the number of correct answers is $(60 - a)$.
$\Rightarrow 3(60 - a) - 2a = 100 \Rightarrow 5a = 80 \Rightarrow a = 16$
Hence the total (maximum possible) number of incorrect answers = 40 (in level C) + 60 (in level B) + 16 (in level A) = 116 Choice (B)

8. Rohit has attempted at least one question of each type. Therefore he has scored at least 3 from level B and C. If he attempts 39 questions correctly of A level, he gets 39 (3) = 117.
$\therefore$ Rohit has to attempt a minimum of 41 questions to get 120 marks. Choice (B)

Solutions for questions 9 to 14:

9. Let the number of Re.1 and ₹2 coins be x and y.
$\therefore$ Then the number of ₹5 coins is $500 - x - y$.
As the total amount is ₹960,
we have $x (1) + y (2) + (500 - x - y) 5 = 960$
$\Rightarrow 4x + 3y = 1540$ ……(1)
Also, $(500 - x - y) (1) + y (2) + x (5) = 1440$.
$\Rightarrow 4x + y = 940$ …… (2)
From (1) and (2), we have $2y = 600 \Rightarrow y = 300$.
Ans: 300

10. Let the number of apples with Sheela be x.
$\frac{3}{4}x + \frac{3}{4} = x$
$\frac{1}{4}x = \frac{3}{4} \Rightarrow x = 3$ Choice (A)

11. Let the topper get 7x chocolates
$\therefore$ The second ranker got $\frac{5}{7}(7x) = 5x$ chocolates and the third ranker $\frac{3}{5}(5x) = 3x$ chocolates

Now, $7x - 3x = 60 \Rightarrow x = 15$
$\therefore$ The topper got 7 (15) = 105 chocolates
The second ranker got 5 (15) = 75 chocolates and the third ranker got 3 (15) = 45 chocolates.
The total number of chocolates is 105 + 75 + 45 = 225
Ans: 225

12. The sum of the thousands and hundreds digit (H) is 15 and the sum of the tens and units digit (u) is 15, i.e., the number is (15 – H) H (15 – U) U. As 15 – H + U = 2H, U is a multiple of 3 and 15 – H is a digit, i.e., U = 6 or 9 and 15 – U = 9 or 6. It can't be 7. Choice (C)

13. Let the fraction be a/b.
Given $\frac{2a}{b+4} = \frac{5}{6} \Rightarrow 12a - 5b = 20$ …….. (1)
Also $\frac{a-2}{b+1} = \frac{1}{3} \Rightarrow 3a - b = 7$ ……. (2)
Solving (1) and (2), we get a = 5 and b = 8.

Alternate method:
$\frac{2a}{b+4} = \frac{5}{6}$ and $\frac{a-2}{b+1} = \frac{1}{3}$
Hence b + 1 must be a multiple of 3.
From the choices when b = 3 or 9, b + 1 is not a multiple of 3. Hence b = 8 or 5
a = 5/12 (b + 4). Hence b + 4 must be a multiple of 12 for a to be an integer. Only when b = 8, b + 4 is a multiple of 4.
Ans: 8

14. Let the number of cats be x.
$\therefore$ Number of rats = x – 1.
Also, (x – 1) – 1 = x/2
$\Rightarrow x - 2 = x/2 \Rightarrow x = 4$ Choice (B)

Solutions for questions 15 and 16:

The table below shows the share of A, B, C and D in the four months.

	A	B	C	D
First month	$\frac{2}{14} \times 4{,}200$ = 600	$\frac{3}{14} \times 4{,}200$ = 900	$\frac{4}{14} \times 4{,}200$ = 1,200	$\frac{5}{14} \times 4{,}200$ = 1,500
Second month	$\frac{3}{14} \times 8{,}400$ = 1,800	$\frac{4}{14} \times 8{,}400$ = 2,400	$\frac{5}{14} \times 8{,}400$ = 3,000	$\frac{2}{14} \times 8{,}400$ = 1,200
Third month	$\frac{4}{14} \times 12{,}600$ = 3,600	$\frac{5}{14} \times 12{,}600$ = 4,500	$\frac{3}{14} \times 12{,}600$ = 2,700	$\frac{2}{14} \times 12{,}600$ = 1,800
Fourth month	$\frac{4}{14} \times 6{,}300$ = 1,800	$\frac{3}{14} \times 6{,}300$ = 1,350	$\frac{5}{14} \times 6{,}300$ = 2,250	$\frac{2}{14} \times 6{,}300$ = 900
Total	7,800	9,150	9,150	5,400

15. Average monthly earning of C = ₹ $\frac{9{,}150}{4}$ = ₹2,287.50
Ans: 2287.50

16. B should have earned (3,500 (4) – 9,150) = 4,850 more, to make his average monthly earning ₹3500. Ans: 4850

Solutions for questions 17 to 19:

Let Sonali's age be x years
$\therefore$ Sagar's age is 2x years.
Let Monali's age be y years
$\therefore y + x = 2(2x) \Rightarrow y = 3x$
Let Surya's age be z years
$\therefore z + 2x = 2(x + 3x) \Rightarrow z = 6x$
Prithvi's age = 21 years
Now, $21 + 6x + 2x = 5(x + 3x) \Rightarrow 12x = 21 \Rightarrow x = 1^3/_4$
$\therefore$ Sonali's age = $1^3/_4$ years = 1 year 9 months
Sagar's age = 2 ($1^3/_4$) = $3^1/_2$ years
Monali's age = 3 (13/4) = $5^1/_4$ years
= 5 years and 3 months
Surya's age = 6 ($1^3/_4$) = $10^1/_2$ years = 10 years 6 months

17. Sonali is the youngest. Choice (C)

18. {5 years 3 months – 1 year 9 months = 3½ years}
Choice (B)

19. Let Surya be twice as old as sagar in 't' years.
$10^1/_2 + t = 2(3^1/_2 + t) \Rightarrow 10^1/_2 + t = 7 + 2t \Rightarrow t = 3^1/_2$
Choice (C)

Solutions for questions 20 and 21:

Let the number of men be x and the number of children be y.
Given that $x : 7 = 7 : y \Rightarrow xy = 49$
There are two possibilities x = 1, y = 49 or x = 7, y = 7 but since the maximum number of visitors were the children, the number of children is 49.
$\therefore$ Total number of visitors = 1 + 7 + 49 = 57

20. 49 children visited the shop. Choice (B)

21. Required ratio = 19 : (57 – 19) = 1 : 2 Choice (C)

Solutions for questions 22 to 25:

22. On the LHS, the first two terms have to be inserted in brackets and on the RHS, the second and third terms have to be inserted in brackets.
Thus, $\frac{(6a-12b)+5c}{(6a-12b)-5c} = \frac{6a+(12b-5c)}{6a-(12b-5c)}$
$\Rightarrow \frac{6a-12b}{5c} = \frac{6a}{12b-5c}$
$\Rightarrow (a - 2b)(12b - 5c) = 5ac$
$\Rightarrow 12ab - 5ac - 24b^2 + 10bc = 5ac$
$\Rightarrow 12ab + 10bc = 10ac + 24b^2$
$\Rightarrow 6ab + 5bc = 5ac + 12b^2$. Choice (C)

23. $\frac{a+b}{b+c} = \frac{c+d}{d+e} = \frac{e+f}{f+a}$ = k (say) ---------- (1)
(1) $\Rightarrow$ k = $\frac{a+b+c+d+e+f}{b+c+d+e+f+a}$ = 1 ---------- (A) or
a + b + c + d + e + f = 0 ----------- (B)
k = 1 $\Rightarrow$ a = c = e we see that if a = c = e, then the two equations in (1) are satisfied $\therefore$ b, d, f can have arbitrary values.
Consider the other given condition
$c^2 - c(a + d) + ad \neq 0$
$\Rightarrow (c - a)(c - d) \neq 0$. In particular $a \neq c$
From the above argument, (1) $\Rightarrow$ A or B and A $\Rightarrow$ a = c.
As $a \neq c$, it follows that B has to be true
$\therefore \frac{a+b+c}{d+e+f} = -1$(as $d + e + f \neq 0$). Choice (C)

24. $\frac{b+c}{c+d} = \frac{d+e}{e+f} = \frac{f+a}{a+b}$ = k (say) ------------ (1)
$\therefore$ k = $\frac{b+c+d+e+f+a}{c+d+e+f+a+b}$ = 1 ----------- (A)
or a + b + c + d + e + f = 0 ------------ (B)
As $a + b + c \neq -(d + e + f)$, (A) follows
$\frac{b+c}{c+d} = 1 \Rightarrow b = d$ and $\frac{f+a}{a+b} = 1 \Rightarrow f = b$
$\therefore$ b = d = f. These values satisfy (1) (Therefore, a, c, e can have arbitrary values)
$\frac{b}{d} + \frac{d}{f} + \frac{f}{b}$ = 3 or b = d = f = 0. Choice (D)

25. Tax = F + K (Salary – 50,000)
6,200 = F + K (60,000 – 50,000)
6,200 = F + 10,000K …….. (1)
7,700 = F + K (75,000 – 50,000)
7,700 = F + 25,000K ……. (2)
From (1) and (2), F = 5,200 and K = 1/10
Now T = 8,200

$8,200 = 5,200 + \frac{1}{10}$ (Salary – 50,000)

Annual Salary = Wr. 80,000 Choice (A)

Solutions for question 26:

26.

Dates	13	14	15	16	17	18	19	20	21
Money	100	90	180	170	340	330	660	650	1,300

Dates	22	23	24	25
Money	1,290	2,580	2,570	5,140

∴ I would have received 5140 – 3860
i.e., ₹1,280 more.

Choice (C)

Solutions for questions 27 to 36:

27. Let x be the value of the magic number on the first day. The following table shows the values of the magic numbers from day 1 to day 10.

Day	1	2	3	4	5	6	7	8	9	10
Value of the magic numbers	x	3x	$\frac{3x}{2}$	$\frac{5x}{2}$	$\frac{15x}{2}$	$\frac{15x}{4}$	$\frac{25x}{4}$	$\frac{75x}{4}$	$\frac{75x}{4}$	$\frac{125x}{8}$

Given that, $\frac{125x}{8} = 375 \Rightarrow x = 24$ Ans: 24

28. Let the initial quantity of wheat the shopkeeper had be i kg.
Quantity of wheat the shopkeeper sold to his

- first customer = $\left(\frac{i}{2}-\frac{1}{2}\right)$ kg

- second customer = $\frac{1}{2}\left(i-\left(\frac{i}{2}-\frac{1}{2}\right)\right)-\frac{1}{2}$

$= \frac{1}{2}\left(\frac{i}{2}+\frac{1}{2}\right)-\frac{1}{2}=\left(\frac{i}{4}-\frac{1}{4}\right)$ kg and so on

The table below shows all the quantities considered, in terms of i.

Customer	Sold	Remaining
1	$\frac{i}{2}-\frac{1}{2}$	$\frac{i}{2}+\frac{1}{2}=\frac{i}{2}+\left(1-\frac{1}{2}\right)$
2	$\frac{i}{4}-\frac{1}{4}$	$\frac{i}{4}+\frac{3}{4}=\frac{i}{2^2}+\left(1-\frac{1}{2^2}\right)$
3	$\frac{i}{8}-\frac{1}{8}$	$\frac{i}{8}+\frac{7}{8}=\frac{i}{2}+\left(1-\frac{1}{2^3}\right)$
4	$\frac{i}{16}-\frac{1}{16}$	$\frac{i}{16}+\frac{15}{16}=\frac{i}{2^4}+\left(1-\frac{1}{2^4}\right)$

It can be seen that the remaining quantity after selling to the nth customer is $\frac{i}{2^n}+\left(1-\frac{1}{2^n}\right)$. If we notice this for n=1 and n = 2, we can work out the remaining quantity faster for n=3 and 4.

$\frac{i}{16}+\frac{15}{16} = 3$

i = 33
∴ i lies between 30 and 35 Choice (C)

29. Prof. Singh has been working for 8 years and Prof. Mathur for 4 years. (2 years back Prof. Singh had 6 years experience while Prof. Mathur had 2 years experience.)

Ans: 8

30. Let Steve's age be x years and that of Mark's be y years.
(x + 2) = 2(x – 2)
⇒ x + 2 = 2x – 4 ⇒ x = 6
∴ Steve's present age is 6 + 27 = 33 years.
Also, (y + 3) = 3(y – 3)
⇒ y + 3 = 3y – 9 ⇒ y = 6
∴ Mark's present age is 6 + 27 = 33 years
∴ Both of them are of the same age.

Choice (B)

31. Let the distance covered be S.
S = A + B
$A \propto t \Rightarrow A = K_1t$
$B \propto t^2 \Rightarrow B = K_2t^2$
So, $S = K_1 t + K_2t^2$
When t = 5s, S = 100 m
$100 = K_1 \times 5 + K_2 \times 5^2$
$100 = 5K_1 + 25K_2$
$20 = K_1 + 5K_2$ ……….. (1)
when t = 6s, S = 138 m
$138 = K_1 \times 6 + K_2 \times 6^2$
$138 = 6K_1 + 36K_2$
$23 = K_1 + 6K_2$ …… (2)
Solving (1) and (2)
$K_1 = 5, K_2 = 3$
So, $S = 5t + 3t^2$
t = 20
$S = 5 \times 20 + 3 \times 20^2 = 1,300$ m Choice (B)

32. The number of diamonds with the thief before giving to

the third watchman = $(14 + 6)\ \frac{3}{2} = 30$

the second watchman = $(30 + 6)\ \frac{3}{2} = 54$

the first watchman = $(54 + 6)\ \frac{3}{2} = 90$ Ans: 90

33. Let the number of questions the students got wrong be x. So the number of questions, (for which his answers were) correct = 48 – x
he number of questions not attempted = 60 – 48 = 12.

Net score is $(48 – x) – \frac{x}{2} – 3 = 33$

1.5x = 12 ⇒ x = 8. Ans: 8

34. The ages of A, B, C at the 4 different times mentioned are tabulated below

	P – 3	P	P + 1	P + 2
A	2a			2a + 5
B	3a	3a + 3		
C			4a + 4	4a + 5

$\frac{2a+5}{4a+5} = \frac{3}{5} \Rightarrow 10a + 25 = 12a + 15 \Rightarrow a = 5$

Their present ages are (2a + 3) + (3a + 3) + (4a + 3)
= 9a + 9
= 9(5) + 9 = 54. Choice (A)

35. Let h and t be the hundreds and tens digit of the three-digit number respectively.
Then the units digit is (t + 1)
Given that, h + t + (t + 1) = 15
$\Rightarrow$ h + 2t = 14 ……… (1) $\Rightarrow$ h is even.
It can't be 5. Choice (A)

36. Let Mr. Sundaram's age be x years and that of Ajit y years.
$\frac{x-5}{4} = y - 5$
$\Rightarrow$ x = 4y – 15 …….. (1)
also, $\frac{x+15}{2} = y + 15 \Rightarrow x = 2y + 15$ ……. (2)
From (1) and (2);
4y – 15 = 2y + 15 $\Rightarrow$ y = 15
$\therefore$ x = 2 x 15 + 15 = 45
Choice (B)

Solutions for questions 37 and 38:

The intended ratio is $\frac{1}{2}:\frac{1}{3}:\frac{1}{4} = 6 : 4 : 3$

$\therefore$ Sachin's share = $\frac{6}{13}$ (117) = 54

Rahul's share = $\frac{4}{13}$ (117) = 36

Saurav's share = $\frac{3}{13}$ (117) = 27

The actual ratio is 2 : 3 : 4

$\therefore$ Sachin's share = $\frac{2}{9}$ (117) = 26

Rahul's share = $\frac{3}{9}$ (117) = 39

Saurav's share = $\frac{4}{9}$ (117) = 52

37. Saurav got 52 – 27 = 25 sweets more than the intended number. Ans: 25

38. Let the total number be x.
Saurva's share is $\frac{3x}{13}$. Rahul's actual share is 39.
$\therefore \frac{3}{13}$ (x) = 39 $\Rightarrow$ x = 169 Ans: 169

Solutions for questions 39 and 40:

Let the requirement for the entire village per day be x kilo litres.
(Note: 1 kilo litre = 1000 litres)
Volume of the tank = 50x
Also, volume of the tank = 40 (x + 20) = 40x + 800
$\therefore$ 50x = 40x + 8,000
$\Rightarrow$ x = 80 kilo litres

39. The total requirements of the village per day is 80 kilo litres.
Choice (D)

40. Let the number of days be n.
50 (80) = n (80 + 5) $\Rightarrow n = \frac{50(800)}{85} = 47^{1}/_{17}$
The tank would be emptied on the 48th day.
Choice (D)

Solutions for questions 41 to 43:

41. Let t (in hrs) be the time required to carry n passengers through a distance of S km.
Given that t $\propto$ ($\sqrt{n}$). S
$t_1 = k\sqrt{n_1}$. (S_1)
$t_2 = k\sqrt{n_2}$. (S_2)
$\frac{t_1}{t_2} = \left(\sqrt{\frac{n_1}{n_2}}\right)\frac{S_1}{S_2}$
Given, $\frac{t_1}{t_2} = \frac{2}{1}, \frac{S_1}{S_2} = \frac{80}{30} = \frac{8}{3}$
So, $\frac{2}{1} = \left(\sqrt{\frac{n_1}{n_2}}\right)\left(\frac{8}{3}\right)$
$\therefore \sqrt{\frac{n_1}{n_2}} = \frac{3}{4} \Rightarrow \frac{n_1}{n_2} = \frac{9}{16}$
Given that n_2 = 64.
$\therefore n_1 = \frac{9}{16}$ (64) = 36 Ans: 36

42. Let the amount of ice-cream be A.
A $\propto$ r^2h, A = kr^2h
$\therefore$ 77 = k (3.5) (3.5) (6) ……… (1)
154 = k (r^2) (3) …….. (2)
Dividing (2) by (1) $\frac{154}{77} = \frac{r^3(3)}{(3.5)(3.5)(6)}$
= $\frac{(2)(3.5)(3.5)(6)}{3} = r^2$
r = 2 (3.5) = 7 Choice (C)

43. Let the drop in temperature from the previous day be t°c and the rainfall be r mm
t $\propto$ $\sqrt{r} \Rightarrow t = k\sqrt{r}$. When r = r_1 = 25, t = t_1 = (42 – 32).
$t_1 = k\sqrt{r_1} \Rightarrow 10 = k\sqrt{25}$
$\Rightarrow$ k = 2 and $t_2 = k\sqrt{r_2} = 2\sqrt{36} = 12$
$\therefore$ Temperature in May drops by 12°C from 44°C to 32°C on the previous day. Ans: 32

Solutions for questions 44 and 45:

Let the number of pieces actually left be 10x + y. The software mistakes it as 10y + x.
Now, (10x + y) – (10y + x) = 63
$\Rightarrow$ 9 (x – y) = 63 $\Rightarrow$ x – y = 7

Since x and y are single digit numbers, there are two possibilities for 'xy', 92 and 81.

As the price per piece is an integer, its reverse (the incorrect price) as well as the incorrect closing stock are factors of 936 = (18) (52). As 29 is not a factor, the incorrect closing stock is 18 (the correct closing stock is 81) and the incorrect price is 52 (the correct price is 25)

44. The number of items actually left at the end of the day is 81.
Choice (B)

45. The number of items sold was shown as 63 more than it was. Let the actual number of items sold be x.
$\therefore$ (x + 63) is the incorrect number of items sold. Correct price is ₹25 while incorrect price is 52.
$\therefore$ Incorrect value of the items sold = (x + 63) (52)
Correct value of items sold = x (25)
Difference = 52x + 63 (52) – 25x which cannot be determined since x is not known. Choice (D)

Solutions for questions 46 to 48:

46. Coins left after Amar gave 23 coins to his wife, Akbar donated 37 coins and Anthony sold 42 coins.
= 2,602 – [23 + 37 + 42] = 2,500
Now, the coins with them are in the ratio 11 : 9 : 5
$\therefore$ Amar has $\frac{11}{25}$ (2,500) = 1,100 coins
Initially Amar had 1,100 + 23 = 1,123 coins.
Ans: 1123

47. If A gives ₹15 to B, they would have equal amounts.
$\therefore$ A has ₹30 more than B. If C takes half of B's money, he would have 5 times as much as B.
$\therefore$ If B has 2x, C has 4x. The ratio of the money with A and B is 5 : 2, i.e., A has 3 parts more than B (which is ₹30). Each part is ₹10.
$\therefore$ A has ₹50, B has ₹20 and C has ₹40. They have ₹110 in all. Choice (B)

48. Five years ago the sum of their ages was
140 − (4 × 5) = 120 years
Let, Inder's age = 2k
Sunder's age = 3k
Mrs.Rathore's age = 7k and Mr.Rathore's age = 8k
Now, 2k + 3k + 7k + 8k = 120 $\Rightarrow$ k = 6
Inder's age five years ago was 2 (6) = 12 years
$\therefore$ His present age is 12 + 5 = 17 and his mother's present age is 42 + 5 = 47.
$\therefore$ After 30 years Inder will be as old as his mother is today.
Ans: 30

Solutions for questions 49 and 50:

Let the number of daughters in the Nanda family be x and that of sons be y.
5(y – 1) = x ………. (1)
and x – 1 = 2y ……. (2)
Solving (1) and (2), we get;
x = 5, y = 2
Let the number of daughters in the Parekh family be a and that of the brothers be b.
b – 1 = a ………(3)
2(a – 1) = b …….. (4)
Solving (3) and (4), we get; a = 3, b = 4

49. Mr. Parekh had 3 + 4 = 7 children. Choice (A)

50. Required ratio = (2 + 4) : (5 + 3) = 3 : 4 Choice (B)

Exercise – 2

Solutions for questions 1 to 20:

1. Let the price at which Rohan purchased the flat be 400x.
Profit percentage = x%
$\therefore$ $400x + \left(\frac{x}{100}\right)(400x) = 43824$
$\Rightarrow 400x + 4x^2 – 43,824 = 0$
$\Rightarrow x^2 + 100x – 10,956 = 0$
$= x^2 + 166x – 66x – 10,956 = 0$
$\Rightarrow (x – 66)(x + 166) = 0 \Rightarrow x = 66$
$\therefore$ CP = 400 (66) = 26400
$\therefore$ Rohan purchased the item for ₹26,400. Ans: 26400

2. Let the number of days required be n.
On the n^{th} day the level of pollution (as a percentage) would become $10\,(0.6)^n$
This should be less than 0.02.
$\Rightarrow 10\,(0.6)^n < 0.02$
Taking log on both the sides
$\Rightarrow$ log10 + nlog0.6 < log2 – log(1/100)
$\Rightarrow 1 + n\log 6 – n\log 10 < \log 2 – \log\left(\frac{1}{100}\right)$
$\Rightarrow$ n (log 6 – 1) < 0.3010 – 2 – 1
$\Rightarrow$ n (log2 + log3 – 1) < –2.699
$\Rightarrow n(–0.2219) < –2.699 \Rightarrow n > \frac{2.699}{0.2219} \Rightarrow n > 12.15$
$\therefore$ The pollution would come back to the acceptable level on the 13^{th} day. Ans: 13

3. Let the cost of the article be ₹x. Anand sells it at 1.1x. Srinath buys it at 0.9x and sells it at (1.1x + 25) making 50% profit
$\therefore$ 1.5 (0.9x) = 1.1x + 25
0.25x = 25 $\Rightarrow$ x = 100 Choice (A)

4. Let the face value be ₹100
Market price = ₹80
Interest = 15% of the face value = ₹15
$\therefore$ Return on investment = $\frac{15}{80}$ (100%) = 18.75%
This will be the same for any investment.
Choice (C)

5.

	For	Against
case (i)	(900 – x)	(x)
case (ii)	(800 – 1.5x)	(1.5x)

given
$(900 – 2x)\left(1+\frac{300}{100}\right) = 3x – 800$
$\therefore 4400 = 11x \Rightarrow x = 400$
$\therefore 1.5x = 600$ Ans: 600

6. The sales be ₹X since he earns a commission of ₹400 per month, his annual commission = ₹4800
4800 = 0.08 (x – 1,00,000)
x = 1,60,000 Choice (C)

7. Let Ram buy a dozen each
$\therefore$ cost = ₹7 + ₹6 = ₹13
selling price for two dozens = 7.50X2 = ₹18
So, profit on 2 dozens is ₹2
$\therefore$to get a Profit of ₹80 = 80 dozens need to be sold.
Ans: 80

8. Since the profit decreases by ₹14 when the discount Is increased from 5% to 10%
14 = 5% of the marked price
$\Rightarrow$ marked price = ₹280
since mark up = 40% of the cost price = 280/1.4 = 200
If a discount of 20% is given on the marked price
Selling price = 0.8 × 280 = ₹224. Profit = ₹24
Ans: 24

9. Let the height, length and breadth of the first room be h, l and b. The area of the 4 walls is 2h (l + b).The area of the 4 walls of the second room is $2\left(\frac{5h}{4}\right)\left(\frac{6l}{5}+\frac{3b}{2}\right)$
We have $\frac{2\left(\frac{5h}{4}\right)\left(\frac{6l}{5}+\frac{3b}{2}\right)}{2h(l+b)} = \frac{3300}{2000} = \frac{33}{20}$
$\Rightarrow \frac{6l}{5}+\frac{3b}{2} = \left(\frac{4}{5}\right)\left(\frac{33}{20}\right)(l+b) = \frac{33}{25}(l+b)$
$\Rightarrow b\left(\frac{3}{2}-\frac{33}{25}\right) = l\left(\frac{33}{25}-\frac{6}{5}\right) \Rightarrow b\left(\frac{9}{50}\right) = \frac{3l}{25} \Rightarrow \frac{l}{b} = \frac{3}{2}$.
Choice (C)

10. Percentage profit = $\frac{100-800}{800} \times 100 = 25\%$
Percentage error = $\frac{100-800}{800} \times 100 = 20\%$
$\therefore$ Percentage profit difference = 5
Ans: 5

11. Cost is to be calculated for painting an area given by 2 (l + b) h + lb
= 2(x + 2x) 3x + x.2x {$\because$ l : b : h = 2 : 1 : 3} = 20 x^2
Cost for painting a similar room is known.
The area of the second room is 2 (1.1l + 0.9b) 0.9h + (1.1l) (0.9b) = 2 (0.9x + (1.1) 2x) 0.9 (3x) + (0.9x) (1.1) (2x) = 18.72x^2
For painting an area of 18.72x, the cost = ₹1872
$\therefore$ For painting an area of 20x^2, the cost = ₹2000
Choice (B)

12. Let the list price be ₹100. Akai gives a commission of ₹25 to its dealer
So, cost = ₹75
To get a profit of 20%, S.P. for the dealer = ₹75 (1.2) = ₹90.
So, he can offer a discount of 10% to his customers.
Since the list price is ₹25,000 the dealer can give a discount = 10% of 25,000 = ₹2,500. Choice (D)

13. Selling price = ₹12
Profit = 20%
$\therefore$ Cost = ₹10
Excise duty = ₹5, other costs = ₹5
New Excise duty = ₹ $5\left(1-\frac{20}{100}\right)$ = ₹4
$\therefore$ New cost = ₹9
profit = 20%
$\therefore$ New selling price = 1.2 × 9 = ₹10.80
So the manufacturer should reduce the Selling price by ₹1.20 Ans: 1.20

14. The man invests ₹19,000 in 5% stock at 95.
$\therefore$ number of shares to be bought = $\frac{19000}{95}$ = 200
annual income = ₹(200)(5) = ₹1000
After he invests in 6% stock at 80, annual income increases by ₹200
Now, the annual income = ₹1200
$\therefore$ The number of shares = $\frac{1200}{6}$ = 200
Investment = ₹200 (80)
Since this amount is obtained by selling 200 shares of the 5% stock, he must have sold the initial stock at ₹80.
Choice (C)

15. Initial price = ₹10
Consumption = 50 kg
$\therefore$ Total expenditure = ₹500
New price = ₹14
Let the consumption be x kg
New expenditure = ₹560
x = $\frac{560}{14}$ = 40 kg Ans: 40

16. Every 900ml of milk he sells as one litre.
So, 18 litres will be sold as 20 litres.
The C.P. for 18 litres = 10 (18) = 180
selling price = 12 (20) = 240
$\therefore$ profit = ₹60
Profit percentage = $\frac{60}{180}$ (100%) = $33\frac{1}{3}$%.
Choice (B)

17. The option that gives the maximum effective (i.e., the least selling price) discount is the best for me. The effective discount percent doesn't depend on the list price. So, a convinient figure of ₹100 can be used.
Option I.
Final selling price = 100 × 0.85 × 0.85 = 72.25
Option II
Final selling price = 100 × 0.9 × 0.9 × 0.9 = 72.9
Option III
Final selling price = 100 × 0.8 × 0.95 × 0.95 = 72.2
So option III is the best for me Choice (C)

18. Ratio of these capitals = 74,000 : 1,11,000 : 1,48,000
= 2 : 3 : 4
From the annual profit of ₹62,000, Rajesh is first paid a monthly salary of ₹1,000.
Total annual salary = ₹12,000
Remaining salary = ₹50,000
Then Vinod gets a commission of 10% = ₹5,000
Profit remaining = ₹45,000
This is divided in the ratio 2 : 3 : 4
Vinod gets 2/9 (45,000) = 10,000
Vinod's share of total profit = 10,000 + 5,000 = 15,000
From the options, 25% of 62,000 = 15,500
19% of 62,000 = 620

$\therefore$ 24% of 62,000 = 14,880
and 26% of 62,000 = 16,120
15,000 is approximately 24% of 62,000.
(Note: If the options give values like 24.1, 24.2, we need to work out what is 0.1% of 62,000.) Choice (C)

19. Let the total profit be ₹P, then Gaurav's share
= $\frac{1}{7}$ P + $\frac{3}{9}\left(\frac{6}{7}\right)$ P = 90,000
$\frac{3}{7}$ (P) = 90,000 $\Rightarrow$ P = 2,10,000
Ans: 210000

20. Let the cost price be ₹C
0.95C + 56.25 = 1.1C
0.15C = 56.25 $\Rightarrow$ C = 375
If the selling price = ₹450
Profit = 20% Ans: 20

Solutions for questions 21 and 22:

Sales man	Number of pieces sold	Fixed income	Commission/Penalty	Total income (in ₹)
A	6,000	5,000	$\frac{10}{100}$ (2,000) 5 = 1,000	6,000
B	4,000	5,000	0	5,000
C	2,000	5,000	$\frac{-20}{100}$ (2,000) 5 = –2,000	3,000
D	5,000	5,000	$\frac{10}{100}$ (1,000) 5 = 500	5,500
E	7,000	5,000	$\frac{10}{100}$ (3,000) 5 = 1,500	6,500
				26,000

21. Average income of the five salesmen
= $\frac{6,000 + 5,000 + 3,000 + 5,500 + 6,500}{5}$ = ₹5,200
Choice (B)

22. For A, B or D the incomes do not change. Let us now look at the incomes of C and E. The number of pieces sold by C is shown as 4,000 and hence his income is ₹5,000. For E, the number of pieces sold shown will be 5,000 and hence his income is ₹5,500.
$\therefore$ Total income of C and E, now
= 5,000 + 5,500 = 10,500 which is ₹1,000 more than the previous amount for C & E which was 3000 + 6500 = 9500.
Choice (C)

Solutions for questions 23 and 24:

Let us make the table for the next month:

Salesman	Number of pieces sold	Fixed income	Variable income	Total income
A	7,000	2,000	$\left(\frac{5}{100}+\frac{10}{100}+\frac{15}{100}+\frac{20}{100}+\frac{25}{100}\right)1{,}000(5)$ = 3,750	5,750
B	5,000	2,000	$\left(\frac{5}{100}+\frac{10}{100}+\frac{15}{100}\right)1{,}000(5)=1{,}500$	3,500
C	3,000	2,000	$\frac{5}{100}\ (1{,}000)\,(5)=250$	2,250
D	4,000	2,000	$\left(\frac{5}{100}+\frac{10}{100}\right)1{,}000(5)=750$	2,750
E	5,000	2,000	$\left(\frac{5}{100}+\frac{10}{100}+\frac{15}{100}\right)1{,}000(5)=1{,}500$	3,500

23. Under the first scheme, A's income would have been = 5,000 + $\frac{10}{100}$ (3,000) (5) = 6,500
$\therefore$ Under the second scheme, it is 5,750
It would be less by 6,500 − 5,750 = 750
So, he would get ₹750 less in May. Choice (B)

24. Average earning in the month of May= $\frac{17{,}750}{5}$ = 3,550
which is less than that in April by 5,200 − 3,550 = 1,650
Choice (B)

Solutions for questions 25 and 26:

Let Ram attempt x questions. He gets 10% of x wrong
$\therefore$ (0.9x) – (0.1x) = 0.8x is the net score
Given that 0.8x = 0.64T where T is the total marks
So, x = 80% of total = 0.8T.
Similarly let Rajat attempt y questions. He gets 0.2y questions wrong.
$\therefore$ (0.8y)– (0.2y) = 0.54T $\Rightarrow$ 0.6y = 0.54T
y = 90% of T = 0.9T
Now, 0.64T = P + 22 ….. (1) where P is the pass mark
0.54T = P + 12 ……. (2)
0.1T = 10 $\Rightarrow$ T = 100
$\therefore$ x = 80, y = 90. P = 42

25. Rajat attempted 90% of the questions Ans: 90

26. Pass mark is 42. Ans: 42

Solutions for questions 27 to 38:

27. Number of males = (11/18) (7200) = 4400
Number of married males = $\frac{40}{100}$ (4400) = 1760
which is equal to the number of married females.
Number of females = 7200 – 4400= 2800
So, percentage of married females = $\frac{1760}{2800}$ (100%)= $62^{6}/_{7}$%
Choice (C)

28.

	A	B	C
Q_1	4	1	15
Q_2	2	2	15
Q_3	1	4	15
Q_4	1/2	8	15
Entire year	15/2	15	60

So, the ratio of capitals is 1 : 2 : 8
B's share = 2/11 (Total) = 22,000
Total = ₹1,21,000 Ans: 121000

29. Let the marked price be ₹100. Profit = ₹30
$\therefore$ The cost price = ₹70
But discount of 20% has been given, so the selling price = ₹80
$\therefore$ Profit % = $\frac{80-70}{70}\times 100$ = $14^{2}/_{7}$% Choice (A)

30. Total number of shares purchased = 46,000/92 = 500
He sold ₹25,000 worth
Stock = 250 shares at ₹95 each
So he will get 250 (95) = ₹23,750
200 shares at 90, so he will get
200 (90) = ₹18,000
and the remaining 50 shares at no profit, no loss i.e., at ₹92.
$\therefore$ he will get 50 (92) = 4,600
Since he retained the shares for one dividend date, he will get one dividend = 500 × 5 = 2,500
Total realization = 23,750 + 18,000 + 4,600 + 2,500 = 48,850
Profit = 48850 – 46000 = 2,850 Choice (B)

31. The ratio of the capitals of Praveen, Ramu and Shashi = 62,000 : 93,000 : 1,24,000 = 2 : 3 : 4. Since 10% of the profit was paid as a commission for managing the business, the remaining 90% is the net profit. Profit share of the profit = $\frac{2}{9}\left(\frac{90}{100}\right)12000$ = 2400 i.e., the remaining ₹500 was paid for managing the business.
Since ₹500 out of a total annual commission of ₹1200 was paid to Praveen, he managed the business for $\frac{500}{1200}$ (12) i.e., for 5 months. Ans: 5

32. Let her buy a number given by LCM of 12, 8 and 10 = 120 of each variety. Cost of the 1st variety = ₹10 (at 12 a rupee).
Cost of the 2nd variety = ₹15 (at 8 a rupee)
Total cost = ₹25
Selling price = ₹24 (at 10 a rupee)
$\therefore$ loss = ₹1
loss percent = (1/25) (100%) = 4% Choice (A)

33. Total number of students passed
n = 0.7 (6,00,000) = 4,20,000
The number of males who appeared
= 0.6 (6,00,000) = 3,60,000
The number of males who passed
= $\frac{3}{4}$ (3,60,000) = 2,70,000
$\therefore$ The number of females

= 6,00,000 – 3,60,000 = 2,40,000
The number of females who passed
= 4,20,000 – 2,70,000 = 1,50,000

Pass percentage for females = $\frac{1,50,000}{2,40,000}$ (100%) = 62.5%

Choice (C)

34. The initial and largest values of x, y, z and a, b are tabulated below.

x	y	z	2xyz	a	b	3ab	V
$\frac{5x}{4}$	$\frac{6y}{5}$	$\frac{9z}{10}$	1.8 (2xyz)	$\frac{6a}{5}$	$\frac{3b}{2}$	1.8 (3ab)	1.8V

Since 2xyz and 3ab, each increase by 80%, V increases by 80%. Choice (B)

35. Venkat buys 12 dozen at ₹10 a dozen
$\therefore$ cost = ₹120
He spent 10% on transport costs
$\therefore$ total cost = ₹132
profit = 20%
So, overall selling price = ₹132 (1.2) = ₹158.4
This has to be realized from the sale of 10 dozen bananas as 2 dozen have got spoilt.
$\therefore$ selling price per dozen = ₹15.84 Ans: 15.84

36. Let the list price be ₹x the cost price for Ramanand = 0.7x
Selling price = 1.6x
Profit = 0.9x = 81 $\therefore$ x = 90 Ans: 90

37. The percentage increase for 11 bats and 11 balls is same as that for 1 bat and 1 ball.
Old value = 2500 + 100 = 2600
New value = 2750 + 120 = 2870

Percentage increase = $\frac{2870 - 2600}{2800}(100)$

= $\frac{270}{26}$ % = 10.38% $\approx$ 10% Choice (D)

38. Let the number of employees be 90
Women Men
30 60
15 women are married and 5 of them have children
45 men are married and 30 men have children.
Totally 35 employees have children and 55 do not have children.

$\therefore$55/90 of the employees do not have children. i.e., $\frac{11}{18}$ of the employees don't have children. Ans: 11

Solutions for questions 39 and 40:

39. Cost = ₹8
Selling price = ₹10

Tax = $\frac{12.5}{100}$ (8) = ₹1

So the total cost = ₹9

Profit = 10 – 9 = ₹1. Now Tax = $\frac{12.5}{100}$ (10)
= ₹1.25
$\therefore$ The total cost = ₹9.25
Selling price = ₹9.50
Profit = ₹0.25. Percentage decrease in profit = 75%.
Choice (D)

40. Let the earlier sales be 10 pens
Government revenue at the old tax regime
= 10 $\times$ 1= ₹10. Now, sales have increased by 10%, to 11 pens and the tax per pen = ₹1.25
$\therefore$Total government revenue = 1.25 $\times$ 11 = ₹13.75
Increase = 37.5%
Choice (B)

Exercise – 3

Solutions for questions 1 to 17:

1. Normally, Raju's father reaches the school by 3:30 p.m. and Raju and his father are back at a certain time. On the given day, they are back 24 minutes earlier, the father saves 12 minutes on either direction; so, the father met Raju at 3:18 p.m. In 48 minutes before meeting his father, the son travels $\left(\frac{48}{60}\right)(6)$ km. If speed of father is f,

$\left(\frac{48}{60}\right)(6) = (f)\left(\frac{12}{60}\right) \Rightarrow$ f = 24 km/hr Choice (C)

2. A, B and C meet for the first time after

LCM $\left(\frac{1260}{5-3}, \frac{1260}{7-5}\right)$ = 630 seconds

A travels a distance of 630 x 3 = 1890
= 1(1260) + 630
So, they meet for the first time at a point, which is diametrically opposite the starting point.
A, B and C meet for the first time at the starting point.

after LCM $\frac{1260}{3}, \frac{1260}{5}, \frac{1260}{7}$ = 1260 seconds.

So, they meet at two distinct points on the track. The starting point and its diametically opposite point. Ans: 2

3. Let the actual speed of the boat in still water be ku and the speed of the current be u.
Let the distance covered be d.
Time taken for the round trip

= $\frac{d}{(k+1)u} + \frac{d}{(k-1)u} = \frac{2k}{k^2-1}\frac{d}{u}$.

If the speed of the boat in still water doubles, the time taken would be $\frac{2(2k)}{(2k)^2-1}\frac{d}{u} = \frac{4k}{4k^2-1}\left(\frac{d}{u}\right)$

We have $\frac{2k}{k^2-1}\frac{d}{u} = 8$ ----------- (1)

And $\frac{4k}{4k^2-1}\frac{d}{u} = \frac{16}{5}$ ------------ (2)

(1) $\div$ (2) $\Rightarrow \frac{4k^2-1}{k^2-1} = 5$

$\Rightarrow 4k^2 - 1 = 5k^2 - 5 \Rightarrow k^2 = 4$
$\therefore$ k = 2 (k = –2 would mean, the speed of the boat in still water is –2u. the minus sign would signify the opposite direction, if some direction is assumed as positive. The magnitude is anyway 2)
$\therefore$The speed of the boat in still water is twice the speed of the current. Choice (C)

4. The boy travels half the distance at 10 km/hr. The average speed for the other part = (15 + 20)/2 = 17.5 km/hr. Therefore, the average for the entire trip is $\frac{2(10)(17.5)}{10+17.5}$ km/hr = $\frac{350}{27.5}$ km/hr = $\frac{140}{11}$ km/hr = $12\frac{8}{11}$ km/hr. Choice (B)

5. As A beats B by 1 minute and B beats C by 30 seconds, A beats C by 90 seconds. Say A takes t seconds, C takes t + 90 seconds and B takes (t + 60) seconds to run 1 kilometre. A beats C by 600 m.

$\frac{1000}{1000-600} = \frac{5}{2} = \frac{t+90}{t} \Rightarrow 5t = 2t + 180$

$\Rightarrow 3t = 180 \Rightarrow t = 60$

Ratio of speeds of A and B = $\frac{t+60}{t} = \frac{120}{60} = \frac{2}{1}$

Choice (D)

6. In the first 10 min,

Harry travels = $\frac{10}{60}$ x 60 = 10 km

Second 10 min he travels = 9 km.

3^{rd} 10 min = 8 km,

4^{th} 10 min = 7 km

5^{th} 10 min = 6 km

So, in 50 minutes = 40 km

So, he travelled 40 km in 50 minutes. To travel the remaining 2 km at 30 km/hr, he would take 4 minutes. i.e., a total of 54 minutes. Ans: 54

7. Let the man open the parachute exactly t seconds after he jumps. Then he falls $5t^2$ m before he opens the parachute and $2(40 – t)^2 - 3(40 - t)$ m after he opens the parachute. But total distance is 2300 m.

$\therefore 5t^2 + 2(1600 + t^2 - 80t) - 3(40 - t) = 2300$

$\Rightarrow 7t^2 - 160t + 3200 - 120 + 3t = 2300$

$\Rightarrow 7t^2 - 157t + 3080 = 2300$

$\Rightarrow 7t^2 - 157t + 780 = 0$

$\Rightarrow (t – 15)(7t – 52) = 0$

$\Rightarrow t = 15$ or $t = 52/7$.

$\therefore$The height at which he should open the parachute is $2{,}300 – 5t^2 = 1{,}175$ m or $2300 – 5(52/7)2 \approx 2024$ m. Among the choices we have only the former.

Ans: 1175

8. Let the escalator have n steps. Let the man walk 'm' steps per second and the escalator moves at one step per second.

Then $\frac{n}{(m-1)} = 108 \Rightarrow (m – 1) = \frac{n}{108}$ ------------ (1)

$\frac{n}{(m+1)} = 36 \Rightarrow m + 1 = \frac{n}{36}$ ------------ (2)

From (1) + (2)

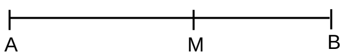
$\Rightarrow 2m = \frac{n}{36} + \frac{n}{108} = \frac{4n}{108} \Rightarrow \frac{n}{m} = 54$

$\therefore$when he walks on the stationary escalator, he would take 54 seconds. Choice (A)

9. The speeds of P and Q are in the ratio 5 : 4. They start from diametrically opposite points and both run clockwise. They start, P has to cover $\frac{1}{2}$ round more than Q to catch up and an additional $\frac{1}{2}$ round to be diametrically opposite for the first time. The second time they are diametrically opposite, P has to cover exactly 2 rounds more than Q. This happens when P covers 10 rounds and Q covers 8 rounds. The time taken would be 10(12) min (or 8(15) min).

Choice (B)

10. Let V_1 be the speed of the first person and V_2 be the speed of the second person. Let t be the time taken by them to meet. Let M be the meeting point



Time taken by them to cover AM is t hrs and 6.25 hrs respectively and time taken by them to cover MB is 2.25 hrs and t hrs respectively.

Now $\frac{V_1}{V_2} = \frac{6.25}{t} = \frac{t}{2.25}$

$\Rightarrow t^2 = 6.25 \times 2.25 \Rightarrow t = 3.75 \Rightarrow V_1 : V_2 = 5 : 3$

Since $V_1 – V_2 = 8$,

$V_1 = 20$ km/h

and $V_2 = 12$ km/h

Since the second man has taken 4 hours more than the first man,

$t_1 = 6$ hours

$t_2 = 10$ hours

d = 20 x 6 = 120 km Ans: 120

11. Given that boat P can do 8 m/sec in still water and 6 m/sec upstream. Therefore, the speed of the current is 2 m/sec. Speed of Q downstream is 5 m/s.

By the time the two passengers on P decide to get off, their boat had already travelled for 20 min i.e., covered (20) (60) (10) = 12,000 m and the boat Q had travelled for 15 min and covered 15 (60) (5) = 4500 m.

$\therefore$The raft is 7500 m ahead of boat Q.

The boat Q gains on the raft at 3 m/sec.

To reach the raft, time taken = $\frac{7500}{3}$ sec = 2500 sec

= $\frac{2500}{60} = 41^2/_3$ min $\approx$ 42 min. Choice (A)

12. Let L_m be the length of the track.

Let V_a and V_b be their speeds of A and B respectively.

$\frac{L}{V_a + V_b} = 40$ and $\frac{L}{V_a - V_b} = 160$

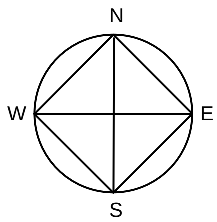
$\Rightarrow V_a = \frac{5L}{320}$ and $V_b = \frac{3L}{320}$

They meet at the starting point after LCM

$\left(\frac{L}{5L/320}, \frac{L}{3L/320}\right)$

= LCM $\left(64, \frac{320}{3}\right)$ = 320 seconds Ans: 320

13.



Since $\overline{NW} = \overline{WS} = \overline{SE} = \overline{EN}$ by the time the slower shire goes from W to S, the faster shire would have travelled five times the distance $\overline{WS}$ i.e. he would arrive at S exactly, but after having completed one full round. Similarly they meet at E, N, W, S ....

Therefore the 19^{th} time they would meet at the North gate.

Choice (A)

14. Since Amul can beat Cadbury by 500m in a race of 2,000m, their speeds are in the ratio of 4 : 3. So, Amul overtakes Cadbury after travelling 2,000m, and thereafter after every 3,000m. So, in a 12 km race Amul overtakes Cadbury at 2km, 5km, 8km and 11km i.e. 4 times. Ans: 4

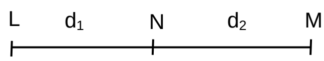
15. The distance covered (in km) in first 2 hrs (starting with the usual rate) is (2) (2 + 4) = 12. But the next 2 hrs, the speed of the current doubles and the distance travelled in those 2 hrs = (2) (4 + 4) = 16. So every 4hrs distance travelled = 28 km. As 130 = (28) (4) + 18. So, in 16 hrs he covers 112 km. Now 12 out of the 18 km will be covered at the usual rate of flow in 2 hrs and the remaining 6 km will take 3/4 hr with the river flowing at 4 km/hr.

$\therefore$Total time = 4 (4) + 2 + (3/4) = $18\frac{3}{4}$ hrs Choice (D)

16. Ratio of their times is 11 : 9

By going at $3/4^{th}$ of usual speed, Harinder takes 10 minutes more time to travel the same distance. If he travels at $3/4^{th}$ of the usual speed, he takes 4/3 rd of his usual time i.e. one-third of his usual time is 10 minutes. So, his usual time is 30 minutes. If he travels at $5/4^{th}$ of this usual speed, he takes $4/5^{th}$ of his usual time. i.e., $\frac{4}{5} \times 30 = 24$ minutes

So, he reaches 6 minutes early. Ans: 6

17. 

Let L be the anthill, N the point from where Anne returned and M the point from where Amy returned. If v was their initial speed, then Anne returned with speed $\frac{v}{2}$ and Amy returned with speed $\frac{3v}{4}$.

Total time for Anne's trip = $\frac{d_1}{v} + \frac{d_1}{(v/2)} = \frac{3d_1}{v}$

Total time for Amy's trip

$= \frac{d_1 + d_2}{v} + \frac{d_1 + d_2}{3v/4} = \frac{7(d_1 + d_2)}{3v}$

But $\frac{3d_1}{v} = \frac{7(d_1 + d_2)}{3v}$ or $(d_1 + d_2) = \frac{9}{7} d_1$

$\Rightarrow d_1 + d_2 = 1\frac{2}{7} d_1$ or d_2 is $\frac{2}{7}$ (100)% of d = 28.57% of d,

Amy's trip is 29% longer than Anne's trip. Choice (A)

Solutions for questions 18 to 20:

18. The dial of a clock on planet OZ will have 18 hours instead of 12 hrs and hence the hour hand travels $\frac{360}{18}$ = 20° per hr. and $\frac{2°}{9}$ per min and minute hand travels 360° per hr and 4° per min.

But since one hr = 90 min

Minutes hand travels $\frac{360}{90}$ = 4° per min now to calculate the angle at 16:50 a.m.

First find angle at 16:00 a.m. it will be $\frac{16}{18}$ x 360°= 320°

The relative speed of the minute and the hour hands is

$4 - \frac{2}{9} = \frac{34°}{9}$ degrees per min in 50 min angle reduces by 50 x $\frac{34}{9} = 188^8/_9{}^{\circ}$

$\therefore$ The final angle = $320° - 188^8/_9{}^{\circ} = 131^1/_9{}^{\circ}$ Choice (B)

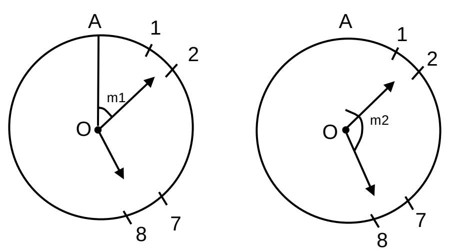
19. At 14'O clock the angle is $\frac{14}{18}$ (360) = 280°

if this has to become 60° then the minute hand covers either 280 + 60 = 340 or 280 − 60 = 220 degrees relative to hour hand.

$\therefore \frac{340}{(34/9)} = \frac{340 \times 9}{34}$ =90 min or

$\frac{220}{(34/9)} = \frac{220 \times 9}{34} = 58^4/_{17}$ min. Choice (C)

20. Say, Ziba left home at m_1 minutes past 7 a.m. and returned at m_2 min past 1 p.m.



The angle made by the hours hand with OA when Ziba left is equal to the angle made by the minutes hand with OA when Ziba returned, i.e., $7(20) + (2/9)m_1 = 4m_2$ -- (1)

Similarly $20 + m_2 \times \frac{20}{90} = 4m_1$ ----------- (2)

solving (1) and (2) for m_1 and m_2

Time of leaving is 7:07 a.m. approx

Arriving is 1 : 35 p.m. approx

$\therefore$ Time out = 12 hrs 28 min. Choice (C)

Solutions for questions 21 to 28:

21. Time taken by all the three to meet for the first time in minutes= $LCM\left(\frac{1600}{50-40}, \frac{1600}{50+30}\right)$ = LCM (160, 20) = 160

Hence for the third meeting time taken will be 3 (160) min = 8 hours. Choice (B)

22.

Ravi	Rahul	Ramu
9000	8100	
	9000	8500

By the time Rahul covers 8,100m Ramu must have to cover

$\frac{8,100}{9,000} \times 8,500 = 7,650$

i.e., by the time Ravi covered 9,000m, Ramu could cover 7,650m. i.e. in a race of 9,000m Ravi beat Ramu by 1,350m.

So, in a race of 2,000m, Ravi would beat Ramu by $\frac{1,350}{9,000} \times$ 2,000 = 300 m Ans: 300

23. To complete running at least once between any two corners of the tracks they have to run at least one extra length or breadth of the track. Since the breadth is shorter it will be one that will be run twice. The length, breadth and diagonal of Samanth's field are 80 m, 60 m and 100 m while that of Saurav's field are 40 m, 30 m and 50 m. Hence the distance Samanth has to cover is 2(80 + 60) + 2 (100) + 60 = 540 m while Saurav will have to cover

2(30 + 40) + 2(50) + 30 = 270 m. Time taken by Saurav = $\frac{270}{135}$ s = 200. If Samanth has to beat Saurav by 20s then he has to take 180s to finish 540 m.

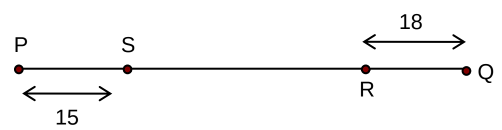
His speed must be at least $\frac{540}{180}$ = 3 m/s. Choice (A)

24. The minute hand is 165° with respect to the hour hand. After M minutes again the minute hand is 165° with respect to the hour hand. The angle covered by the minute hand in M minutes is 5.5M (relative to the hour hand).

$\Rightarrow$ 165° + 5.5M + 165 = 360.

The minimum value of M is

$\frac{360-(165+165)}{5.5} = \frac{30}{5.5} = 5\frac{5}{11}$ min Choice (C)

25. 

Given that the person starting from P is faster than the other person. Since his speed is more than twice the speed of the other person, before the slower person reaches P, the faster person will reach Q, turns back and meets the slower person (Say at S).

Let PQ = d

In the entire process, the ratio of their speeds is constant.

So, $\frac{d-18}{18} = \frac{2d-15}{d-15}$

$d^2 - 69 + 540 = 0 \Rightarrow d = 9$ or 60

Since d ≠ 9,

d has to be 60 km. Ans: 60

26. Let the usual time taken be t min

Then first, if Anil left, say, t_1 minutes late and by travelling at 4/3 of his usual speed and reached t_1 min early then

$\frac{t - 2t_1}{t} = \frac{3}{4} \Rightarrow t = 8t_1$ or $t_1 = t/8$

[Since he travelled at $4/3^{rd}$ of his speed, he would take $3/4^{th}$ of the time]

Now if the next day. Anil left say $t_1$min early and reached say $t_2$min (say) late then

$\frac{t + (t_1 + t_2)}{t} = \frac{11}{8} \Rightarrow 8(t_1 + t_2) = 3t \Rightarrow t_1 + t_2 = \frac{3t}{8}$

or since $t_1 = \frac{t}{8}$ we get $t_2 = \frac{2t}{8}$ or $2t_1$

$\therefore$ Anil reached the school twice as many minutes late.

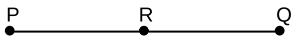
Choice (B)

27. Since the hare overtakes the mongoose 2 times, per round the hare's speed is 3 times that of the mongoose and Similarly the mongoose's speed is 5 times that of the tortoise.

$\therefore$ The hare's speed is 15 times that of the tortoise.

Choice (C)

28. Since Amar overtakes Banti, Amar is faster. Let Amar overtake Banti at R.



Let Banti take t hours to travel from P to R. Then, Amar takes (t – 2) hours to travel from P to R.

So, $\frac{t - 2}{t} = \frac{2}{3} \Rightarrow t = 6$ hours

Let RQ = d. Then $\frac{d + 9}{d - 9} = \frac{3}{2} \Rightarrow d = 45$km

So, RQ = 45. Hence we cannot find PQ

Choice (D)

Solutions for question 29:

29. Let the coordinate of X be O and that of Y be L. Let the coordinates of the cars A, B, C be a, b, c respectively.

These coordinates are functions of time, given by different expressions in different time intervals as shown below. The symbol t represents the time (in hours) elapsed after 7:00 am.

	7 to 8	8 to 9	After 9
a	50 t	50 t	50 t
b	0	40 (t – 1)	40 (t – 1)
c	L	L	L – 30 (t – 2)

Let the event of AC being equal to AB be denoted by E. Depending on in which time interval E occurs, we would write down different equations to determine the time.

1. that E occurs at T where 7 am < T ≤ 8 am

   $L - 50t = 50t - 0 \Rightarrow t = \frac{L}{100}$ (A) (if $\frac{L}{100} \le 1$, this is the required time)

2. 8 < T ≤ 9 am

   $L - 50t = 50t - 40(t - 1)$

   $\Rightarrow t = \frac{L - 40}{60}$ (B)

   If $1 \le \frac{L - 40}{60} \le 2$, this is the required time

   i.e. if $100 \le L \le 160$, the required time would be given by $\frac{L - 40}{60}$ (Note that for L = 100, both A and B given the same value of 1)

3. 9 am < T

   $L - 30(t - 2) - 50t = 50t - 40(t - 1)$

   $\Rightarrow L + 60 - 40 = 90t \Rightarrow t = \frac{L + 20}{90}$ … (C)

   If $2 \le \frac{L + 20}{90}$, i.e. if $L \ge 160$, the required time is given by $\frac{L + 20}{90}$ (Note: that for L = 160, both B and C give the same value of 2).

   As L = 200, we have to use (C). $t = \frac{200 + 20}{90} = \frac{22}{9}$

   $= 2\frac{4}{9}$ This represents the time 9:26:40 am.

Choice (A)

Solutions for questions 30 to 40:

30.

Let AB = L

Let the first crossing occur at T_1 and the second at T_2. Amrish and Biswas start from A and B respectively at T = 0.

From T = 0 to T = T_1, Amrish covers 17 Biswas covers L – 17.

From T = T_1 to T = T_2, Amrish covers $L - 17 - 19\frac{5}{6}$

and Biswas covers $17 + L - 19\frac{5}{6}$

$\therefore \frac{17}{L - 36\frac{5}{6}} = \frac{L - 17}{L - 2\frac{5}{6}} \Rightarrow \frac{6(17)}{6L - 221} = \frac{6(L - 17)}{6L - 17}$

$\Rightarrow 36(17)L - 6(17^2) = 36L^2 - 6L(323) + 221\,(6)\,(17)$

$\Rightarrow 102L - 17^2 = 6L^2 - 323L + 221\,(17)$

$\Rightarrow 6L^2 - 425L + 17^2\,(14) = 0$

$\Rightarrow [2L - 7(17)]\,[3L - 2(17)] = 0$

$\Rightarrow L = 59.5$ or $\frac{34}{3}$

As L > 17, L has to be 59.5. Ans: 59.5

31. Let the speeds of P, Q and the initial speed of R be p, q, r respectively (in km/hr). From 10:00 am to 11:00 am, P covers p towards (say) the east of the starting point. R begins at 11:00 and catches up with P in $\frac{p}{r - p}$ hours

$\therefore \frac{p}{r - p} = \frac{1}{2}$------------- (1)

R then reverses his direction and increases his speed by $61\frac{1}{9}$% are catches up with Q who is going west in 1 hour.

$\left(61\frac{1}{9}\% = \frac{550}{9}\% = \frac{550}{900} = \frac{11}{18}\right)$

$\therefore \frac{1.5q + 0.5r}{\frac{29}{18}r - q} = 1$ – – – – – – – (2)

$(1) \Rightarrow p = \frac{r}{3} = \frac{3r}{9}$

$(2) \Rightarrow 1.5q + 0.5r = \frac{29}{18}r - q$

$\Rightarrow 2.5q = \frac{20r}{18} = \frac{10r}{9} \Rightarrow q = \frac{4r}{9}$

$\therefore \frac{p}{q} = 3 : 4.$ Choice (C)

32. Let the speed of the first and the second trains be 4x km/hr and 5x km/hr. The lengths of the first and second trains are

$(4x+6)\left(\frac{5}{18}\right)50$ and $(5x)\left(\frac{5}{18}\right)30$

Given that $2(5x)(30) = (4x + 6)\,50$

$\Rightarrow 100x = 300 \Rightarrow x = 3$

Length of the second train = $(5x)\left(\frac{5}{18}\right)(30)$ m = 125 m

Ans: 125

33. As they are travelling in the same direction, we have $\frac{600}{a=b}=60$

$\Rightarrow$ a = b = 10 m/s

(where a and b are the respective speeds of Anubhav and Rakesh).

Also given L.C.M of (600/a, 600/b) = 60 seconds,

As the difference of speeds is10 m/s,

the possible speeds could be

(1) (10m/s, 20m/s)

(2) (20 m/s, 30 m/s)

(3) (30 m/s, 40 m/s)

(4) (40 m/s, 50 m/s)

(5) (50 m/s, 60 m/s)

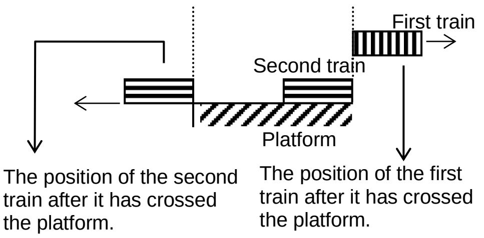
In the given choices, as 20 m/s, 30 m/s 40 m/s, are given, speed of the faster person cannot be 10 m/s, because the other person is also running. Choice (A)

34. After the boat meets the trunk at 2:00 pm, the boat travels for 2 hr upstream and 2hr downstream.

Therefore it reaches the trunk.

But it also reaches P (exactly at 4:00 pm). Therefore the trunk also reaches P at 4:00 pm. Choice (D)

35. At the instant the second train enters the platform completely, it also crosses the first train. Hence, the second train, now, has yet to cover exactly the length of the platform by the time the first train crosses the platform (refer the figure given)



$\therefore$ Time taken = $\frac{\text{Length of the platform}}{\text{Speed of the second train}}$

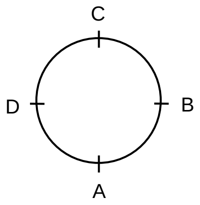
= $\frac{1125}{(75 \times 5/18)}$ = 54 seconds Ans: 54

36. Let the speeds of the boat in still water and the speed of the stream be 2x km/hr and x km/hr.

$\frac{18}{2x-x} = \frac{18}{2x+x} + 3 \Rightarrow \frac{18}{x} = \frac{18}{3x} + 3 \Rightarrow \frac{12}{x} = 3$

$\Rightarrow$ x = 4 km/hr Choice (B)

37.



Let Ajit and Ajay start at A and run towards B.

If Ajit runs at a speed v then Ajay's speed is 5v. Now Ajay overtakes Ajit four times by the time Ajay does one round (at B, C and D and at A, the starting point)

Now, at B, C, D Ajit's tokens double and they will halve at A. Ajit starts with 1 token. In the first round, Ajit's tokens become 2 at B, 4 at C, 8 at D and 4 at A.

Similarly, after another full round, Ajit has 16 tokens. Say the race lasts for n rounds. Ajit will have $(4)^n$ tokens totally of which $(4^n - 1)$ are what Ajay gave.

$\therefore$ If Ajay has N tokens initially N – $(4^n - 1)$ are left with him, which is 246.

N = 246 + $(4^n - 1)$ trying n = 1, 2, 3 etc we get n = 4, N = 501 fits the choices. Ans: 501

38. Let the speeds of P and Q be x km/hr and y km/hr. In two hours, they travel 2x km and 2y km respectively.

After that Q takes $\frac{2x}{y}$ hr to reach A and P takes $\frac{2y}{x}$ hr to reach B. We have

$\frac{2x}{y} - \frac{2y}{x} = 3$

Let $\frac{x}{y}$ be a. Then $2a - \frac{2}{a} = 3$

$2a^2 - 3a - 2 = 0$

$2a^2 - 4a + a - 2 = 0$

$(2a + 1)(a - 2) = 0$

$\Rightarrow a = 2$ or $-\frac{1}{2}$. As a > 0, a = 2

Hence x = 2y = 4 Choice (C)

39. Let x be the hours the two cars travelled with original speeds. Then the distance to be travelled by the two cars in km is (420 – 60x) and (420 – 90x) respectively after x hours.

Given that

$\frac{420 - 60x}{90} = \frac{420 - 90x}{60} + 1$

$\Rightarrow 840 - 120x = 1440 - 270x$

$\Rightarrow x = \frac{600}{150}$ = 4 hours

$\therefore$ The speeds got reversed after 4 hours. Ans: 4

40. Charan takes 1.6 times his normal time.

$\therefore$ The extra back and forward motion was o.6 times the total time (as distance). He went back by 0.3 (PQ). Thus, he covered (0.5 + 0.3) PQ in the time that Dinesh covered (0.2)PQ. Charan is thus 4 times as fast as Dinesh.

To cover $\frac{4}{5}$(PQ), Dinesh normally takes time t (say)

To cover the same distance at Charan's speed, he takes t/4.

$\therefore t - \frac{t}{4} = \frac{3t}{4} = 20 \text{ min} = \frac{1}{3}\text{hr} \Rightarrow t = \frac{4}{9}\text{hr}.$

To cover PQ, Dinesh would take $\frac{5t}{4} = \frac{5}{4}\left(\frac{4}{9}\right)$ hr= $\frac{5}{9}$ hr.

His speed is $\frac{PQ}{5/9} = \frac{2.5}{5/9}$ km/hr = 4.5 km/hr. Ans: 4.5

Exercise – 4

Solutions for questions 1 and 2:

Let the first tap fill the cistern in t minutes. Then the second tap alone will fill the cistern in 3/2t

($\because$ capacity is 2/3 times). The total rate of the first two taps is $\frac{5}{3}$ times the rate of the first tap. Therefore, the first two taps can fill the cistern in 0.6t minutes.

The first three taps will take 0.6 (0.6t) = $(0.6)^2$t

[Imagine the first two taps as a single tap that takes t_1 minutes then the next tap takes $3t_1/2$ min. and hence together the tap takes $3/5t_1$ = (3/5) (3/5) (t min)]

1. Given $(0.6)^9$ (t) = 1 $\Rightarrow$ t = $\left(\frac{5}{3}\right)^9$

$\therefore$ Time taken by 6 taps

$t_6 = \left(\frac{3}{5}\right)^5 t = 1 \times \left(\frac{5}{3}\right)^4 = \frac{625}{81}$

Time taken by 5 taps $t_5 = \left(\frac{3}{5}\right)^4 t = \frac{3125}{243}$

If the 6^{th} tap alone takes t_6' minutes

Then $\frac{1}{t_5} + \frac{1}{t_6'} = \frac{1}{t_6}$

$\Rightarrow \frac{1}{t_6'} = \frac{1}{t_6} - \frac{1}{t_5} = \frac{81}{625} - \frac{243}{3125} = \frac{162}{3125}$

$t_6' = 19\frac{47}{162}$

Choice (D)

2. Given the total time taken by 10 taps is 1 minute
$\Rightarrow (0.6)^9 \times t = 1$
Now the time taken by 7 taps would be $(0.6)^6 \times t$

$= \frac{0.6^9 \times t}{(0.6)^3} = 1 \times \frac{5}{3} \times \frac{5}{3} \times \frac{5}{3} = \frac{125}{27} = 4^{17}/_{27}$ min

Choice (A)

Solutions for questions 3 and 4:

3. The time that Anoop took to read and answer each of the four passages is tabulated below (where t is the time taken to answer a question)

Passage	1	2	3	4	Total
Read	12t	12t	12t	12t	48t
Answer	5t	8t	8t	6t	27t
					75t
Total	17t	20t	20t	18t	150t

The percentage of time that he spent for reading the first passage is

$\frac{17t}{75t}(100\%) = 17\left(\frac{4}{3}\right)\% = \frac{68}{3}\%$ = 22.7%   Choice (C)

4. If he has to cut down the total time by 20%, i.e., decrease it to 60t, he can spend only 33t on the four passages, or 8.25t per passage.

He has to increase his reading speed to $\frac{1200}{825} = \frac{16}{11}$ times the original speed or he has to increase his speed by $\frac{5}{11}$ of his original speed i.e $\frac{500}{11}\%$ = 45.45%.

Choice (D)

Solutions for questions 5 and 6:

5. Portion of work completed by A on the first day = 1/12
Portion of work completed by A on the second day = 1/24
Portion of work completed by A on the third day = 1/48
$\therefore$ Portion of work completed on the first three days

$= \frac{1}{12} + \frac{1}{24} + \frac{1}{48} = \frac{4+2+1}{48} = \frac{7}{48}$

Portion of work completed in the first 18 (6 X 3) days = $\frac{6 \times 7}{48} = \frac{7}{8}$

On the 19^{th} day A would work with full efficiency. He does $1/12^{th}$ of the work. Portion of work done on the 19^{th} day = 1/12.
$\therefore$ The remaining work now = 1/8 – 1/12 = 1/24
which would be completed at the end of the 20^{th} day (with half efficiency)   Choice (B)

6. Portion of work completed in the first three days = 7/48
Remaining work = 41/48
$\therefore$ Time taken to complete the remaining 41/48 of the work = 11 – 3 = 8

$\therefore$ Portion of work done by A (in all) = $\frac{7}{48} + \frac{18}{12} = \frac{39}{48}$

and, hence portion of work done by B = $1 - \frac{39}{48} = \frac{9}{48}$

$\therefore$ Ratio of A's share to that of B's share = 39 : 9

$\therefore$ B's share in their earning = ₹ $\frac{9}{48}$ (7200) = ₹1350.

Ans: 1350

Solutions for questions 7 and 8:

7. Tap A fills a 60,000 lt tank in 20 hrs, i.e., it supplies water at 3000 lt/ hr.
Tap B fills the tank in 15 hrs, i.e., it supplies water at 4000 lt/hr.
Tap C fills the tank in 12 hrs, i.e., it supplies water at 5000 lt/hr.
Together, they supply water at the rate of 12,000 lt/hr.
Ans: 12000

8. The requirement of each of the 100 houses is 900 lt per day. For all the houses, it is 90,000 lt/day, which is $1\frac{1}{2}$ times the capacity of the tank. To fill the tank, the least time is $\frac{60,000}{12,000}$ hrs = 5 hrs. We suppose that all the pipes are opened at t = 0. It is not necessary to wait until the tank is full, before pipe D is opened D can be opened at t = 4-40. At t = 5, 60,000 lt of water is supplied D has to be closed at this time.
A, B, C take 2hr 30 min more to fill up 30,000 lt, i.e, upto t = 7.30, D can be opened again at 7.20, so that by 7.30 the tank is again empty and all the houses receive their quota of water.
Therefore the least time needed to supply all the houses with the daily requirement is 7-30 hr or 450 min.   Ans: 450

Solutions for questions 9 and 10:

9. Let the time taken to build one side of the square be t days, then when the first mason works alone he takes t + 12 days and when the second mason works alone he takes (t + 39 – 12) = t + 27 days
Now we can write the eqn.

$\frac{1}{t+12} + \frac{1}{t+27} = \frac{1}{t}$ .... (1)

Solving the above eqn. (1) we get t = 18 days
i.e., first mason takes t + 12 = 30 days
second mason takes t + 27 = 45 days
(when working alone)
If both work together it takes 't' = 18 days per side. To finish 2 sides (remaining) they will take 36 days.   Choice (C)

10. The two masons finished the other two sides of the square by working together. They worked for another 18 (2) = 36 days. Now they should share the amount in the ratio in which they did the work
First mason worked for 30 + 36 days
2^{nd} mason for 45 + 36

$\therefore$ Ratio = $\frac{(30+36)}{30} : \frac{(45+36)}{45} = \frac{66}{30} : \frac{81}{45} = \frac{11}{5} : \frac{9}{5}$ =11 : 9

$\therefore$ the second mason would get

₹9 x $\frac{200000}{(11+9)}$ = ₹90000   Choice (D)

Solutions for questions 11 to 13:

Time taken by A = 12 hours/ switch board
Time taken by B = 12/1.2 = 10 hours/ switch board
Time taken by C = 12/1.5 = 8 hours/ switch board
Time taken by D = 12/2 = 6 hours/switch board

11. To complete 90 switch boards C would take
90 (8) = 720 hours Choice (A)

12. Portion of work done by A, B, C and D in 1 hour
$= \frac{1}{12}+\frac{1}{10}+\frac{1}{8}+\frac{1}{6}=\frac{19}{40}$
Portion of work done by A and B in the next 2 hours.
$= 2\left(\frac{1}{12}+\frac{1}{10}\right)=\frac{11}{30}$
Portion of work remaining = $1-\left(\frac{19}{40}+\frac{11}{30}\right)=\frac{19}{120}$
Time taken by A to complete $19/120^{th}$ of the job
$= \frac{19}{120}$ (12) = 1.9 hours Choice (D)

13. Number of switch boards made by A in 1 day (12 hours) = 1
Number of switch boards made by B in 1 day = 12/10 = 1.2
Number of switch boards made by C in 1 day = 12/8 = 1.5
Number of switch boards made by D in 1 day = 12/6 = 2
$\therefore$The total number of pieces made by A, B, C and D in the first 4 days = 1 + 1.2 + 1.5 + 2 = 5.7
$\therefore$ The number of days required to make 114 switch boards
$= \frac{114}{5.7}$ (4) = 80 days Choice (C)

Solutions for questions 14 and 15:

14. The given table shows the depth of the well dug and also the depth at the beginning of each day.

Day	Depth upto which the well is dug	Actual depth at the end of the day
1	10	10 – 0.2 (10) = 8
2	18	18 – 0.2 (18) = 14.4
3	24.4	24.4 – 0.2 (24.4) = 19.52
4	29.52	29.52 – 0.2 (29.52) = 23.516

$\therefore$Digging of the well would be completed on the 5^{th} day.
Choice (B)

15. The depth of the well at the beginning of the fourth day or at the end of the third day = 19.52 m Choice (B)

Solutions for questions 16 to 20:

16. Efficiency during the first shift = 80%
80% of total efficiency = 1/60
Efficiency = $\frac{100}{80}\times\frac{1}{60}=\frac{1}{48}$
Work done in the three shifts in one day
$= \frac{1}{60}+\frac{70}{100}\times\frac{1}{48}+\frac{50}{100}\times\frac{1}{48}$
$= \frac{1}{60}+\frac{7}{480}+\frac{1}{96}=\frac{1}{24}$
The work can be completed in 24 days. 36 less days when compared to 60 days. Ans: 36

17. Only 200 men are working in the first shift. 80% of the work done by 200 men is done in the first shift = 160 men working at 100% efficiency
People working in the second shift
= 200 men + 150 women
= 200 m + 75 m = 275 m
So, 275 men work at 70% efficiency = 192 men work at 100% efficiency. 300 women and 200 men work in the third shift at 50% efficiency = 350 men work at 50% efficiency = 175 men at 100% efficiency in the second shift, optimum work is done.
Choice (B)

18. Since P is the fastest of the three, let P work on everyday, Q and R on the alternate days.
The work done on the consecutive days will be
$= \frac{1}{15}+\frac{1}{20};\frac{1}{15}+\frac{1}{30};\frac{1}{15}+\frac{1}{20}\ .\ .\ .\ .$ so on.
In two days the work done = $\frac{2}{15}+\frac{1}{20}+\frac{1}{30}=\frac{13}{60}$
In 4 such time periods i.e., 8 days 52/60 of the work is done. Remaining work = 8/60. On the 9^{th} day P and Q do 7/60 of the remaining work = 1/60 which is done by P and R in $1/6^{th}$ of the 10^{th} day. So, the work takes $9^1/_6$ days.
Choice (B)

19. Let B take x days.
$\therefore$A took (x + 12) days.
To make 2 models, they would take 16 days.
$\therefore$ To make 1 model, they would take 8 days.
$\frac{1}{x}+\frac{1}{x+12}=\frac{1}{8}\Rightarrow\frac{2x+12}{x^2+12x}=\frac{1}{8}$
$\Rightarrow x^2 - 4x - 96 = 0 \Rightarrow (x - 12)(x + 8) = 0$
$\therefore$ x =12.
$\therefore$ Time taken by A = 12 + 12 = 24 days. Ans: 24

20. Let A, B and C complete the work in x days.
Work done in one day = $\frac{1}{x}$
Work done in 9 days = $\frac{9}{x}$
Let the work done on the last day be $\frac{1}{y}$, $\frac{9}{x}+\frac{1}{y}=1$
A completes $\frac{1}{y}$ work in $\frac{1}{2}$ days.
One work in $\frac{y}{2}$ days.
Similarly, B and C complete one work in $\frac{3y}{4}$ and y days.
Work done by them in one day
$= \frac{2}{y}+\frac{4}{3y}+\frac{1}{y}=\frac{1}{x}=\frac{13}{3y}+\frac{1}{x}$
$y = \frac{13x}{3}$
$\frac{9}{x}+\frac{3}{13x}=1$
$\frac{120}{13x}=1, x=\frac{120}{13}$ = $9^3/_{13}$ days Choice (B)

Solutions for questions 21 and 22:

21. A can dig one well in 10 days. But in the difficult terrain his efficiency falls by 10% every day
$\therefore$On the first day A does $1/10^{th}$ of the work
on the second day he does
$\frac{1}{10}\left(1-\frac{10}{100}\right)=\frac{1}{10}\times(0.9)^{th}$ of the work
on the third day he does
$\frac{1}{10}$ (0.9) $*$ (0.9) the of the work and so on.
$\therefore$the total work done after n days will be

$\frac{1}{10}$ + $\frac{1}{10}$ $\times$ (0.9) + $\frac{1}{10}$ $(0.9)^2$ + ... + $\frac{1}{10}$ $(0.9)^{n-1}$

$\therefore$ If the work is done in n days

$\Rightarrow \frac{[1 - (0.9)^n]}{[1 - 0.9]} = 1$

$\Rightarrow 1 - (0.9)^n = 1 \Rightarrow (0.9)^n = 0 \Rightarrow n \to \infty$

That is A will never be able to finish the work.

Choice (D)

22. Similar to the above solution

If B takes n days to finish the work, then

$\frac{1}{5} + \frac{1}{5}$ (0.9) + $\frac{1}{5}$ $(0.9)^2$ +...... + $\frac{1}{5}$ $(0.9)^{n-1}$ = 1

$\frac{\frac{1}{5}\left[1-(0.9)^n\right]}{[1-\ 0.9]} = 1 \Rightarrow 1 - (0.9)^n = 0.5$

or all we need to see is for what least possible integral value of 'n'

will $1 - (0.9)^n \geq 0.5 \Rightarrow 1 - (0.9)^n \leq 0.5$

smallest value of n possible is

$(0.9)^6 = 0.531$

$(0.9)^7 = 0.478$

$\therefore n = 7$

i.e., on the 7^{th} day the work will be done. Choice (B)

Solutions for questions 23 and 24:

23. Let x be the number of days taken on the first moat, on which all three men are working together

$\therefore$ The second moat took (25/6) days on which only (say) the first man worked.

The 3^{rd} moat took (5/2)x + 10 days, on which only the second man worked

And the 4^{th} moat took (5/2)x days, on which only the third man worked.

As all the three men worked on the first moat, the work that they did in 1 day is

$\frac{6}{25x} + \frac{2}{5x+20} + \frac{2}{5x} = \frac{1}{x} \Rightarrow \frac{2}{5x+20} = \frac{9}{25x}$

$\Rightarrow 50x = 45x + 180 \Rightarrow x = 36$

$\therefore$ The second moat took $\left(\frac{25}{6}\right)x = \left(\frac{25}{6}\right)(36)$ =150 days

$\therefore$ The third and fourth moats took 100 days and 90 days respectively.

$\therefore$ total time for all the wells to be complete

= 36 + maximum of [150, 100, 90]

= 36 + 150 = 186 days Ans: 186

24. The ratio in which the amount should be split is

$\frac{1}{150}(36 + 150) : \frac{1}{100}(36 + 100) : \frac{1}{90}(36 + 90)$

$= \frac{31}{25} : \frac{34}{25} : \frac{35}{25}$ = 31 : 34 : 35

If amount = 10,000

Then the shares are 3100, 3400, and 3500

$\therefore$ The difference = ₹(3500 – 3100) = ₹400 Ans: 400

Solutions for questions 25 and 26:

25. Since only $2/5^{th}$ (40%) of the work was done till the 5^{th} day, at that rate, the 50 workers would have taken, (5) (5/2) = 12.5 days to complete the work.

Also, to complete the entire work 20 workers would normally have taken 10 $\left(\frac{50}{20}\right)$ = 25 days

$\therefore$ Portion of work done by the 70 workers (50 + 20) everyday

$= \frac{1}{12.5} + \frac{1}{25} = \frac{3}{25}$

Time taken to complete the remaining 60% work

$= \frac{25}{3} \times \left(\frac{60}{100}\right) = 5$

$\therefore$ Total time taken = 10 days. Choice (C)

26. Portion of work done by the 50 workers

$= 10 \times \frac{1}{12.5} = \frac{4}{5}$ and

$\therefore$ Portion of work done by the 20 workers = $\frac{1.4}{5}$ = 1/5

Amount distributed among the workers in rupees

$= 75000 - \frac{20}{100}(75000) = 60000$

$\therefore$ Share of the 20 workers = $\frac{1}{5}(60000) = 12000$

$\therefore$ Share of each worker = $\frac{12000}{20} = 600$

$\therefore$ Average daily earnings = $\frac{600}{5} = 120$ Choice (D)

Solutions for questions 27 and 28:

27. Let the time taken to fill the tank when both pipes are filling it be t minutes. Then the time taken for both pipes together to empty the tank will also be t minutes. It is given that in t minutes only pipe A fills $3/5^{th}$ of the tank. Hence, in t minutes pipe B can fill $2/5^{th}$ of the tank. Now it also follows that pipe A alone can empty $3/5^{th}$ of the tank in 't' minutes and pipe B alone would empty $2/5^{th}$ of the tank in 't' minutes (when they are used to empty the tank individually). If pipe A is used to fill and pipe B is used to empty for t/2 minutes, then the tank will be (1/2) (3/5 – 2/5) = 1/10 = 10% full. Therefore the tank will be 90% empty. Ans: 90

28. The rates and the times taken by pipes A and B are tabulated below:

	A	B	AB	A + B
Rate $\left(\frac{LCM}{T}\right)$	4	3	5	7
Time (min)	18	24	14.4	
T	3	4	2.4	LCM = 12

If the tank capacity is taken as 72 units, the rates of A, B, AB are 4, 3, 5 respectively.

$\therefore$ The net flow rate (NFR) when both pipes are open is 5 units while the sum of the rates is 7. The NFR is $\frac{200}{7}$ % less than the sum of the individual rates. Choice (C)

Solutions for questions 29 and 30:

29. If the four outlet pipes are opened simultaneously, they can empty the top $\frac{1}{4}^{th}$ part in $\frac{1}{4}$ hour or 15 minutes.

Now, only 3 taps work. Three taps can empty the next $\frac{1}{4}^{th}$ part in one hour.

$\frac{1}{4}^{th}$ part of the tank in $\frac{1}{4} \times \frac{4}{3} \times 60$ = 20 minutes.

Now only two taps work. They can empty half part in one hour.

They can empty the $\frac{1}{4}^{th}$ part in $\frac{1}{2}$ an hour or 30 minutes.

Now the last tap is left alone to empty $\frac{1}{4}^{th}$ part.

It takes one hour. The total time taken to empty

= 15 + 20 + 30 + 60 = 125 minutes. Ans: 125

30. In the first 15 minutes $\frac{1}{4}^{th}$ of the tank is emptied. In the next 20 minutes $\frac{1}{4}^{th}$ of the tank is emptied.

Totally in 35 minutes = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ of the tank is emptied.

Now, 25 minutes are left. In 30 minutes, the third $\frac{1}{4}^{th}$ part can be emptied.

In the remaining 25 minutes, the part that is emptied.

$= \frac{25}{30} \times \frac{1}{4} = \frac{5}{24}$

Total part of the tank emptied = $\frac{1}{4} + \frac{1}{4} + \frac{5}{24} = \frac{17}{24}$.

The inlet pipe can fill the tank in 60 minutes.

It can fill $\frac{17}{24}$ in $\frac{17}{24} \times 60 = \frac{85}{2}$ = 42½ minutes

The tank will be filled in 42½ more minutes. Ans: 42.5

Solutions for questions 31 and 32:

Let the marks scored in Test V, Test VII, Test XI and Test XII be x + 2, y, x and y + 12 respectively.
Now, (x + 2) + y + x + (y + 12) = 1,020 − 674 = 346
$\Rightarrow$ x + y = 166. Also, x − y = 18
$\therefore$x = 92, y = 74

31. Score in Test V = x + 2 = 92 + 2 = 94. Choice (A)

32. To increase his average by 1.
Satish should have got (12 x 1) i.e.; 12 marks more than that he got presently.
His present score in test XII is 86.
$\therefore$ His score would have been (86 + 12) = 98
Choice (B)

Solutions for questions 33 to 41:

33. Let the number of innings be n.

$$\frac{6{,}000+90}{n+5} = \frac{6{,}000}{n} - 2$$

$2n^2 + 100n - 30{,}000 = 0$
$n^2 + 50n - 15{,}000 = 0$
$\Rightarrow$ n = 100 or –150
As n can't be negative n = 100.
$\therefore$The total number of innings he played = 100 + 5 = 105
Ans: 105

34. At each step the concentrations of the milk decreases to 2/3 of what it is at the beginning. Thus after the first 4 steps, the concentration is respectively $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}$.
Therefore, if Doodhimal carries out the process for 4 times, the concentration drops to $\frac{16}{81}$ as below 20%.

Ans: 4

35. The quantities of milk and water in A and B at the 3 different stages are tabulated below

A		B	
Milk	Water	Milk	Water
N	0	0	N
N – X	0	X	N
$(N - X) + \frac{X^2}{N+X}$	$\frac{X(N)}{N+X}$	$\frac{XN}{N+X}$	$\frac{N^2}{N+X}$

The concentration of milk in A = $\frac{N}{N+X}$

The concentration of water in A = $\frac{X}{N+X}$

The difference = $\frac{N-X}{N+X} = \frac{3}{4}$ (given)

$\therefore$ 4N – 4X = 3N + 3X or N = 7X
The concentration of milk in B = The concentration of water in A = $\frac{X}{N+X} = \frac{X}{8X} = \frac{1}{8}$ = 12.5%. Choice (A)

36. The effective rate of interest earned for the total investment

$\frac{2{,}750}{25{,}000} \times 100\% = 11\%$

12   8

11

3   :   1

$\therefore$ Amount invested in the 12% bond

= ₹ $\frac{3}{4}$ (25,000) = ₹18,750 Choice (C)

37. Total number of mistakes = 2 (1,007) = 2,014
$\therefore$ The number of mistakes in the last 1007 − 612 or 395 pages = 2,014 − 434 = 1,580
$\therefore$ Average number of mistakes per page for the last 395.
pages = $\frac{1{,}580}{365} = 4$ Ans: 4

38. Price per kg of the mixture

$= \frac{(1 \times 74) + (2 \times 68) + (4 \times 63)}{1+2+4} = 66$

C.P of the final mixture = $\frac{84}{1.2}$ = 70

using the rule of alligation;

Mixture T_1

66   74

70

4   4

1 : 1

$\therefore$ He mixed 4 kg of T_1 Choice (A)

39. Let the number of senior level employees be X.

$\frac{125 \times 5{,}500 + 14{,}000X}{125 + X}$ = 8,687.5

$\Rightarrow$ 6,87,500 + 14,000X = 10,85,937.5 + 8,687.5X
$\Rightarrow$ 5,312.5X = 3,98,437.5
$\Rightarrow$ X = 75

$\therefore$The total number of employees = $\frac{(125+75)}{0.8}$ = 250
Ans: 250

40. Total percentage depreciation
= (8/2) [2 (20) + 7 (–2.5)] = 90
$\therefore$ The cost of the machine after 8 years, in rupees,

= 2,5,0000 – $\frac{90}{100}$ (25,000)

i.e., $\frac{10}{100}$ (25,000) = 25,000 Ans: 25000

41. Total marks obtained by the top 10 students = 10 (67) = 670
Total marks obtained by the last 11 (from 10th to the 20th) students = 11 (64) = 704
If we add these two, Rohit's score is getting added twice.
Now, total marks scored by the 20 students = 20 (65) = 1,300
$\therefore$Rohit's score = 670 + 704 − 1,300 = 74
Choice (C)

Solutions for questions 42 and 43:

Let A = x – (1)
∴ B = x + 5 – (2)
and let, C = y
D = y + 11
Also, x = (y + 11) − 23 ⇒ y + 11 = x + 23
⇒ D = x + 23 – (3)
and y = x + 23 − 11= x + 12
⇒ C = x + 12 – (4),
Now from (1), (2), (3) and (4)
$\frac{x+x+5+x+12+x+23}{4}$ = 22
⇒ x + 10 = 22 ⇒ x = 12

42. A = 12 Ans: 12

43. C = y = x + 12 = 12 + 12 = 24 Ans: 24

Solutions for questions 44 and 45:

Let, the amounts with A, B, C and D be ₹x, ₹y, ₹z, and ₹w respectively.
x + y + z + w = 1200 ------------ (1)
Also, 2x + 2y + z + w = 1500 ⇒ x + y = 300 ----------- (2)
It is also given that 2z + 1.4w = 2 (690)
⇒ 2z + 1.4w = 1380 ----------------- (3)
But, 2 x (1) – (2)
⇒ z + w = 900 ----------------- (4)
Solving (3) and (4), we get; z = 200 and w = 700

44. C had ₹z = ₹200. Choice (C)

45. x + y = 200 ------------------ (1)
x + 50 = y − 50 ⇒ y − x = 100 --------------- (2)
Solving (1) and (2), we get; x = 100 Choice (B)

Solutions for questions 46 to 50:

46. Let the weight of the acid solution A be 9x kg
Then, the weight of the acid in solution A = 4x kg
The weight of the acid in 25 kg of solution B
= $\frac{2}{5}(25)$ kg = 10 kg
32 kg of acid is mixed with 9x kg of A and 25 kg of B. The total weight of the acid
= 4x + 10 + 32. This has to be equal to $66\frac{2}{3}$ % of
(9x + 25 + 32)
4x + 42 = $\frac{200}{3}\times\left(\frac{1}{100}\right)(9x+57)$
⇒ 2x+ 21 = 3x + 19 ⇒ x = 2
∴ Required weight of acid solution A
= 9x kg = 9(2) kg = 18 kg Choice (C)

47. After one execution of process P (two symmetrical) operations the amount of alcohol in drum A and the amount of water in drum W will be the same. Since at the end of any execution the total content of each drum = total quantity of the alcohol in the drums = total quantity of water in the drums = 1000 litres.
Even after 3 executions of P the amount of alcohol in drum A and the amount of water in drum W will be the same. After this, 2 litres of the contents of drum A are mixed with contents of drum W. So the amount of alcohol in drum A will be less than the amount of water in drum W (∵ The amount of alcohol in A has decreased while that of water in W has increased).
a < w. Choice (B)

48. The concentration of the resulting solution's will be more than
= $\frac{(0.45)(8)+(0.3)(7)}{15}(100\%)$ = 38%
∴C > 38 Choice (D)

49. Let the ages of the six people be x_1, x_2, x_3, x_4, x_5, x_6 in ascending order
$x_1 + x_2 + x_3 + x_4 + x_5$ = 29 (5) ………….(1)
$x_1 + x_2 + x_3 + x_4 + x_6$ = 39 (5) …………(2) and so on
Finally,
$x_2 + x_3 + x_4 + x_5 + x_6$ = 41 (5) …………(6)
(6) – (1) ⇒ $X_6 – X_1$= 5 (41 – 29) = 60 years Ans: 60

50. Let the amount of solution at first be 9K (2K of A and 7K of B) 18 litres will contain 4 of A and 14 of B. From the given condition,
$\frac{2K-4+18}{7K-14}=\frac{12}{17}$ ⇒ K = $\frac{19}{5}$
∴ 9 K = $\frac{171}{5}$
So, there is $\frac{171}{5}$ litres of the solution. Choice (A)

Exercise – 5

Solutions for questions 1 and 2:

Let the total number of students be x.
Number of students who went to the temple = x/7.
Number of students who went to the museum
= $\sqrt{x-x/7}=\sqrt{\frac{6x}{7}}$

Number of students who went to the fair = $\frac{1}{5}\left(x-\left(\frac{x}{7}+\sqrt{\frac{6x}{7}}\right)\right)$

As the total number of students who went to the temple or the museum is twice the number who went to the fair
$\frac{x}{7}+\sqrt{\frac{6x}{7}}=\frac{2}{5}\left(x-\frac{x}{7}-\sqrt{\frac{6x}{7}}\right)$

⇒ $\frac{x}{7}+\sqrt{\frac{6x}{7}}=\frac{2}{5}\left(\frac{6x}{7}-\sqrt{\frac{6x}{7}}\right)$

⇒ $\frac{7}{5}\sqrt{\frac{6x}{7}}=\frac{x}{5}$ ⇒ $\sqrt{\frac{6x}{7}}=\frac{x}{7}$

Squaring both sides ⇒ $\frac{6x}{7}=\frac{x^2}{7\times 7}$ ⇒ x = 7 x 6 = 42 (since x ≠ 0)
∴Total number of students = 42
Number of students who went to the temple = $\frac{42}{7}$ = 6
Number of students who went to the museum = $\sqrt{42-6}$ = 6
Number of students who went to the fair = $\frac{1}{5}$[42 – (6 + 6)] = 6
Number of students who went to the science exhibition
= $\frac{1}{3}$(42 – 18) = 8

1. ∴ Number of students who went for the movie
= 42 – (6 + 6 + 6 + 8) = 16. Choice (C)

2. Total money paid
= (5 x 6) + (4 x 6) + (2 x 8) + (25 x 16) = ₹470/-
Choice (D)

Solutions for questions 3 to 65:

3. Let the age of the son be x years.
∴ Mrs. Mathur's age = x^2 years
Now, x^2 + 4 = 4 (x + 4)
⇒ x^2 – 4x – 12 = 0 ⇒ (x – 6) (x + 2) = 0
∴ x = 6 (since x ≠ – 2)
∴ Mrs. Mathur's present age = 36 years
Her son would be of her age
36 – 6 i.e., 30 years hence
∴ Mrs. Mathur's age 30 years
hence is 36 + 30 = 66 years. Choice (C)

4. Let the number of monkeys be x.

$$\frac{x}{5}+8\sqrt{x-\frac{x}{5}} + 20 = x$$

$$n_1\, 8\sqrt{\frac{4x}{5}} = \frac{4x}{5} - 20$$

Squaring both sides, we get;

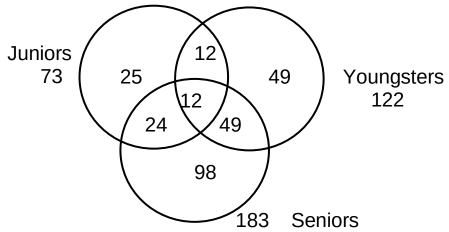
$\frac{256x}{5} = \frac{16x^2}{25} - 32x + 400 \Rightarrow 16x^2 - 2080x + 10000 = 0$

$\Rightarrow x^2 - 130x + 625 = 0 \Rightarrow (x - 5)(x - 125) = 0$

But, $x \neq 5$ (since $x > 20$)

$\therefore x = 125.$ Ans: 125

5. All the members were present on 15th August 1987.



From 15th Aug 1987 to 14th August 1988, there were 366 days. (including February 29th, 1988.)

Number of times the juniors visited the club = $1 + \frac{365}{5} = 74$

Number of times the youngsters visited the club

$= 1 + \frac{365}{3} = 122$

Number of times the seniors visited the club = $1 + \frac{365}{2} = 183$

Number of times juniors and youngsters visited the club

$= 1 + \frac{365}{15} = 25$

Number of times youngsters and seniors visited the club

$= 1 + \frac{365}{6} = 61$

Number of times the seniors and the juniors visited the club

$= 1 + \frac{365}{10} = 37$

Number of times all the three groups visited the club

$= 1 + \frac{365}{30} = 13.$

$\therefore$ The total number of days one or more of the groups visited the club = 74 + 122 + 183 – 25 – 61 – 37 + 13 = 269

$\therefore$ The number of days when none of the groups visited the club = 366 – 269 = 97.

Alternate method:

All the members came to the club on Aug 15, 87 (day 0) counting Aug 16 as day 1, Aug 14, 88 is day 365. (1988 was a leap year.)

In the first 30 days (LCM of 2, 3, 5), there were no visitors on days 1, 7, 11, 13, 17, 19, 23 and 29. (i.e., on 8 days)

In the first 360 days, there were no visitors on 12 (8) = 96 days. On day 361, again there were no visitors.

$\therefore$ The number of days on which none of the groups came to the club is 97. Ans: 97

6. We know that $5^2 = 4^2 + 3^2$

$45 = 9 \times 5 \Rightarrow 45^2 = (9 \times 5)^2$

$\therefore (9 \times 5)^2 = (9 \times 4)^2 + (9 \times 3)^2 \Rightarrow 45^2 = 36^2 + 27^2$

This is the only way of expressing 45^2 as a sum of two non-zero perfect squares.

Let us try to express $45^2 = 2025 = 3^4 \times 5^2$ as a difference of two non-zero perfect squares. 45^2 (i.e., can be written as a product of two numbers in $\frac{(4+1)(2+1)+1}{2}$ = 8 ways which includes 45×45.

Let $A^2 - B^2 = (A + B)(A - B) = 45^2$

Since both A and B are positive, (A + B) has to be greater than (A – B). In each way of expression of 45^2 as a product of two factors, take the larger factor as (A + B) and the smaller as (A – B).

For example, $(A + B)(A - B) = 5 \times 405$

So, take A + B = 405 and A – B = 5

$\Rightarrow$ A = 205 and B = 200

(A + B) and (A – B) both are taken as 45, B = 0. This violates the condition that A and B are non-zero perfect squares.

Therefore, 45^2 can be expressed as the difference of two perfect squares in (8 – 1) = 7 ways and as a sum of two perfect squares in only one way.

So, total of 8 ways.

Choice (D)

7. $\frac{n(n+1)}{2}$ is the sum of first n natural numbers, so choice (A) is true.

$\left\{\frac{n(n+1)}{2}\right\}^2$ is the sum of the cubes of first n natural numbers. So, Choice (B) is true.

n(n + 1) is the product of 2 successive numbers.

$\therefore$ one of them has to be even

$\therefore \{n(n + 1)\}^2$ is always even. Choice (C) is true.

Choice (D) is false because for n = 237, $\left\{\frac{n(n+1)}{2}\right\}^2$ is divisible by 237.

Choice (D)

8. This is the 2nd model of HCF problems. When we make exact measurements the volume of the can is dividing the volumes of three jars and leaving the same remainders.

Let the volume of the can be x and the equal remainder be y

x = HCF [(177 – 57), (177 – 129)]

= HCF [120, 48] = 24 Ans: 24

9. Since, the number of chocolates involved in the entire process are to be minimised, let the number of chocolates received by Diana (or each of her 5 sisters) also be the minimum possible i.e., one.

Number of chocolates received by Celina = (6 x 1) + 3 = 9

So, the number of chocolates received by Bipasha

= (9 x 5) + 2 = 47

$\therefore$ The number of chocolates Aishwarya initially had

= (4 x 47) + 3 = 191.

Choice (A)

10. A number when divided by D leaves a remainder of r. The possible remainders when the same number is divided by n D are r, (D + r), (2D + r), (3D + r), ….., [(n – 1) D + r].

Let the number be N. Let the remainder left when N is divided by D be represented by Rem $\left(\frac{N}{D}\right)$.

Given that Rem $\left(\frac{N}{D}\right)$ = 7 ---- (1)

and Rem $\left(\frac{N}{3D}\right)$ = 20 ----- (2)

Rem $\left(\frac{N}{3D}\right)$ could be either (D + 7) or (2D + 7)

Since $2D + 7 \neq 20$ when D is a natural number, D + 7 = 20 $\Rightarrow$ D = 13. $\therefore$ 3D = 39.

Rem $\left(\frac{2N}{3D}\right) = 2 \times 20 = 40$ (from (2))

Since 40 > 3D, Rem $\left(\frac{2N}{3D}\right)$ = Rem $\left(\frac{40}{3D}\right)$ = Rem $\left(\frac{40}{39}\right)$ = 1

Choice (A)

11. Let N be the number.

Given that Rem $\left(\frac{N}{D}\right)$ = 9 --- (1)

and Rem $\left(\frac{N}{3D}\right)$ = 35 ---- (2)

$\Rightarrow$ Either D + 9 = 35 or 2D + 9 = 35.
$\Rightarrow$ D = 13 or 26 $\Rightarrow$ 3D = 39 or 78

Rem $\left(\frac{2N}{39}\right)$ = Rem $\left(\frac{2\times 35}{39}\right)$ = 31

Rem $\left(\frac{2N}{78}\right)$ = 2 × 35 = 70

$\therefore$ Rem $\left(\frac{2N}{3D}\right)$ = 31 or 70. Choice (D)

12. Let the marks of Garry and Robin be a and b respectively.
ab = 10(a + b) $\Rightarrow$ b(a – 10) = 10a
$\Rightarrow$ b = 10a / (a – 10)
When, a = 11, b = 110.
When, a = 12, b = 60.
When, a = 13, b = 130/3.
When a = 14, b = 35 and when a = 15, b = 30.
Since 35 lies in the interval (32, 40)
Garry's Score = 14. Ans: 14

13. Distributing the sweets among 11 players is same as dividing by 11, distributing the sweets among 11 players, 3 extras and 1 coach is the same as dividing by 15.
This problem is equivalent to first model of LCM

divisors	11	15
remainders	2	2

The number of sweets is given by
LCM (11, 15) × K + 2 = 165K + 2
$\therefore$ Minimum number of sweets = 167. Choice (A)

14. Suppose there are two numbers, the operation has to be done once. Suppose there are three numbers a, b, c
operation (a, b) $\to$ find HCF operation {HCF, C}
So, operation has to be done twice. Suppose there are four numbers a, b, c, d
operation (a, b) $\to$ HCF 1
operation (c, d) $\to$ HCF 2
operation {HCF1, HCF2} $\to$ HCF. So, operation has to be done thrice. Hence, if there are n numbers
operation has to be done (n – 1) times.
Since there are 50 numbers, it has to be done 49 times.
Choice (C)

15. Let N be the numbers.

Given that Rem $\left(\frac{N}{D}\right)$ = 5 ------ (1)

and Rem $\left(\frac{N}{4D}\right)$ = 41 ------- (2)

$\Rightarrow$ D + 5 = 41 or 2D + 5 = 41 or 3D + 5 = 41
$\Rightarrow$ 4D = 144 or 72 or 48

Consider Rem $\left(\frac{11N}{4D}\right)$ when 4D = 144;

Rem $\left(\frac{11N}{144}\right) = \text{Rem}\left(\frac{11\times 41}{144}\right)$ = Rem $\left(\frac{451}{144}\right)$

= 451 – 3 × 144 = 19
when 4D = 72

Rem $\left(\frac{11N}{72}\right)$ = Rem $\left(\frac{11\times 41}{48}\right)$ = Rem $\left(\frac{451}{72}\right)$

= 451 – 6 × 72 = 19
when 4D = 48

Rem $\left(\frac{11N}{48}\right)$ = Rem $\left(\frac{11\times 41}{48}\right)$ = Rem $\left(\frac{451}{48}\right)$

= 451 – 9 × 48 = 19

$\therefore$ Though we have three possible values of D, the value of Rem $\left(\frac{11N}{4D}\right)$ in each case is same i.e., 19. Choice (C)

16. Subtracting one from each number in S, we have
0, 1, 2, ……..360.
In this sequence, we are selecting all the multiples of 2 for set-A, multiples of 3 for set-B and multiples of 5 for set-D. Set C is a subset of A.
As 360 = 8 (9) (5) = $2^3 3^2 5$, the 'left-overs' in the sequence are the numbers that are relatively prime to 360.
The sum of all these numbers is

S = $\left(\frac{360}{2}\right)\phi(360)$, where ϕ (360) is the number of numbers less than 360 and relatively prime to 360.

ϕ (360) = 360 $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 360\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)$ = 96 and s = 180 (96) and the sum of all the numbers in y is 180 (96) + 96 = 17,376 (as each number in y is one more than the corresponding number that is left in the sequence.)
Choice (A)

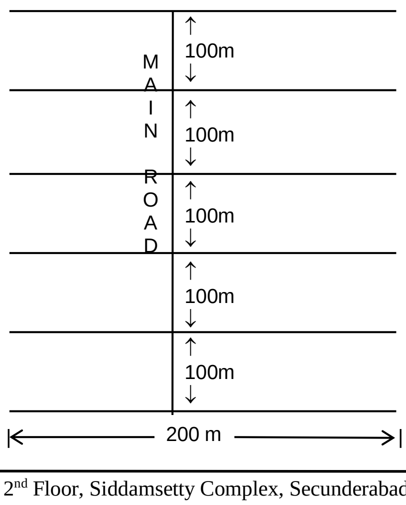
17. Any number ending in 9 raised to an even power ends in 1. 17 raised to a multiple of 4 ends in 1 as the last digit is same as that of 7 raised to a multiple of 4.
$\therefore$ The resulting number ends in 1. Ans: 1

18. Let the volume of the measuring can be x litres respectively. Since the process has to take the least number of measurements, x is the HCF of (3,248 – 7) and (4,175 – 8)
HCF (3241, 4167)
$\therefore$ x = 463 Choice (A)

19. This question can be best done from the answer choices.
Choice (A): 40, 80
40 = $2^3 \times 5^1$; i.e., 8 factors with 2 prime factors
80 = $2^4 \times 5^1$; 10 factors with 2 prime factors
Incorrect choices 80 has only 2 factors more
Choice (B): 20, 40
20 = $2^2 \times 5$; i.e., 6 factors with 2 prime factors
40 = $2^3 \times 5^1$; i.e., 8 factors with 2 prime factors
Incorrect choice
Choice (C): 30, 60
30 = 2 × 3 × 5; i.e., 8 factors with 3 prime factors
60 = $2^2 \times 3 \times 5$; 12 factors with 3 prime factors.
Since 60 has 4 more factors than 30, this is the correct choice. Choice (C)

20. Only three digits 0, 1 and 8 look like digits when viewed in the mirror. Since the reference is to a year in the nineteenth century it has to be 18 followed by either 01, 10 a 08, 80 or 18, 81. Since the year is multiplied by 4.5 and we get an integral value, therefore 01 and 81 are ruled out. Of all the other given numbers it is possible only for 1,818, since 1818 × 4.5 = 8,181. Ans: 1818

21.



Six Avenues each separated by 100m will be there for each avenue, number of trees = $\left(\frac{200}{10}+1\right)$ x 2 = 42

So, for 6 avenues, there are 42 x 6 or 252 trees.

Ans: 252

22. Let the volumes of the two glasses be x and y ml
x + y = 250. LCM of x, y = 300
So, x and y are 100 ml and 150 ml. Choice (D)

23. The side of the square tile must be the HCF of the two dimensions = HCF (24 ft, 36 ft) = 12ft
$\therefore$3 pairs of tiles along the length (or 2 sets of 3 tiles along the breadth) i.e., 6 tiles are required. Ans: 6

24. For both the wheels the red mark will touch the ground after a distance equal to the respective circumferences.

Circumference of the front wheel = $2\left(\frac{22}{7}\right)(14)$ = 88 cm

Circumference of the rear wheel = $2 \times \frac{22}{7} \times 21$ = 132 cm

Both red marks will touch the ground simultaneously after the LCM (88, 132) = 264 cm. Choice (D)

25. When the stick is divided into parts of 7cm each, it is the same as being divided by 7.
$\therefore$ This problem is equivalent to

7	9	divisors
2	3	remainders

9k + 3 – 2 is divisible by 7
9k + 1 or 2k + 1 is divisible by 7
k = 3, 10, 17, 24, ………
$\therefore$ The minimum possible number is 9(3) + 3 = 30
$\therefore$ Length of the required piece = 30 – 8 (3) = 6 cm.

Ans: 6

26. Let the number of trees be x. If he plants in rows of 6, 8, 10 or 12; 5 trees are left unplanted.
This is the first model of LCM problems

Divisor	6	8	10	12
Remainder	5	5	5	5

$\therefore$ x = LCM (6, 8, 10, 12) + 5 = 120k + 5
but when planted in rows of 13 no trees are left unplanted.
$\therefore$ x is a multiple of 13. If we take k = 1, 120k leaves a remainder of 3, and takes successive values of k, i.e., k = 2, 3, 4, 5, 6, 7, …… this remainder increases by 3 at each step, i.e., the remainders are 6, 9, 12, 2, 5, 8 i.e., for k = 7, 120k + 5 is divisible by 13, and 120 (7) + 5 = 845 is the minimum number that satisfies all the conditions. Choice (C)

27. This problem is the second model LCM problem

10	9	8	7	6	divisors
9	8	7	6	5	remainders

The least number is the LCM of (10, 9, 8, 7, 6) – 1 = 2,519

Ans: 2519

28. Let ₹x crore be sent to reserves.
Then ₹2,483 – x crores are to be distributed among 10 crore, 15 crore or 20 crore shareholders, leaving a remainder of 3 crores in each case.
This is a first model of LCM problem.
So, 2,483 – x = 60K + 3
The smallest value of x = 20 crores, as the largest value of (60K + 3) is 2,463. Choice (C)

29. All the shooters will hit their targets together after a time given by LCM of 5, 6, 7 and 8 seconds
= 840 seconds or 14 minutes.
They next hit the target together at 10:14 a.m.

Choice (A)

30. If the six digit number is divisible by 7, 11 and 13, it would be divisible by 7 × 11 × 13 = 1001.
$\therefore$The only six-digit number whose first three digits are 267 and which is a multiple of 1001 is 267267. Hence its units digit is 7. Choice (B)

31. All the classes start at 9:00 a.m. the primary section has classes at 9:00, 9:30, 10:00 etc. and at intervals of 30 minutes each. The middle section has classes at 9:00, 9:45, 10:30 etc at intervals of 45 minutes each and the secondary section has classes at 9:00, 10:00, 11:00 etc at intervals of 60 minutes each. All of them will end a class after a time given by the LCM of 30, 45 and 60 minutes = 180 minutes. So, lunch break can be given after 180 m i.e., at 12:00 p.m. Choice (C)

32. 2160 = 16(135) = 16(27)(5) = 243351
$\therefore$ The sum of all the factors of 2160
$= (2^5 - 1)\frac{(3^4-1)}{2}\frac{(5^2-1)}{4}$ = (31) (40) (6)
and the sum of all the odd factors of 2160
$= \frac{(3^4-1)}{2}\frac{(5^2-1)}{4}$ = (40) (6)
$\therefore$ The sum of all the even factors of 2160 = (30) (40) (6) = 7200.

Ans: 7200

33. In each room equal number of students must sit, and take the same exam therefore this number must be a common factor of 60, 72 and 96. Since the number of rooms must be minimum, the common factor must be the highest
= HCF (60, 72, 96) = 12
$\therefore$5 rooms for the first subject, 6 for the second and 8 for the third i.e., a total of 19 rooms are needed.

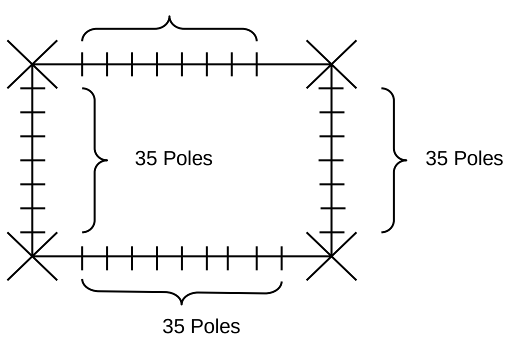
Ans: 19

34. Any odd number can be expressed as a difference of two perfect squares and any number which is a multiple of 4 can also be expressed as a difference of two perfect squares. Divide the numbers of the series into groups of four.
1, 4, 7, 10
13, 16, 19, 22
25, 28, 31, 34 and so on.
First and third numbers of the each group are odd numbers and the second number is a multiple of 4. So, three in each group can be expressed as a difference of two perfect squares. In the first 300 numbers of the series there are 75 such groups.
$\therefore$ The number of such numbers = 3 × 75 = 225.

Choice (B)

35. For the minimum number of tiles the dimensions of the rectangle have to be maximum.
Let the dimension of each tile be 3k and 2k.
So, the number of tiles required to cover the floor is $\frac{72}{3k} \times \frac{72}{2k} = \frac{24}{k} \times \frac{36}{k}$
$\therefore$ For minimum number of tiles k = HCF (24, 36) = 12
Then the number of tiles = 2 x 3 = 6. For maximum number of tiles k = 1
Then the number of tiles = 24 x 36 = 864
$\therefore$ Difference = 858 Ans: 858

36.



Total number of poles = (35 × 4) + 4 (at the corners) = 144

Choice (C)

37. Let the number of chocolates with Mohit, Rohit, Lohit be m, r, l respectively.
The date is tabulated below.

m		r		l	
m	+	r			= 6 k
		r	+	l	= 7k
M			+	l	= 8k

$\therefore 2$ (m + r + l = 21k $\Rightarrow$ m + r + l = 10.5
$\Rightarrow$ m = 3.5k, r = 2.5k and l = 4.5k $\Rightarrow$ m : r : l = 7 : 5 : 9.
As m + r + l = 42, r = 10, l = 18.
$\therefore$ r = 10 Choice (B)

38. The total number of eggs were such that when he counted 3 eggs at a time 1 was left and when he counted 4 at time 1 was left. This is LCM I model problem

3	4	divisors
1	1	remainders

Total number = 12K + 1
When counted 5 at a time, no eggs are left. So 12K + 1 is a multiple of 5 $\Rightarrow$ k = 2 and the required total number = 25. Since 10 eggs are unbroken, 15 eggs were broken
Ans: 15

39. We consider the choices.
Choice A: 1, 4, 10, 16. We can't weigh 2 pounds (and possibly some other weights), i.e., we can't express 2 as the sum or difference of the given numbers.
Choice B: 1, 2, 3, 25. We can't weigh any weight from 7 to 18 pounds.
Choice C: 2, 3, 4, 22. We can't weigh 10, 11, 12 pounds.
Choice D: 1, 3, 9, 18. We can weigh any weight from 1 to 31 by taking the sum or difference of these four weights.
Choice (D)

40. Let a, v, p and s be thier scores respectively
$\frac{a}{b} = 2v = p - 2 = s + 2 = k$
$\Rightarrow a = 2k,\ v = \frac{k}{2},\ p = k + 2$ and $s = k - 2$
$2k + \frac{k}{2} + k + 2 + k - 2 = 45 \Rightarrow k = 10$
$\therefore$ Prakash scored = k + 2 = 10 + 2 = 12. Ans: 12

41. The number of times Jayesh has to press the keys is as follows:
1 – digit number 1 to 9 = 9
9×1 = 9 times
2 – digit numbers 10 to 99 = 90
90×2 = 180 times
3 – digit numbers 100 to 300 = 201
201×3 = 603 times
Total number of times he has to press the keyboard
= 603 + 180 + 9 = 792 Ans: 792

42. Consider Rem $\left(\frac{3^{1000}}{730}\right)$ = Rem $\left[\frac{3^4(3^6)^{166}}{730}\right]$
= Rem $\left(\frac{3^4}{730}\right) \times$ Rem$\left(\frac{729^{166}}{729-(-1)}\right)$ = 34 $\times$ $(-1)^{166}$ = 81
Since 73 is a factor of 730.
Rem $\left(\frac{3^{1000}}{73}\right)$ = Rem$\left[\frac{\text{Rem}(3^{1000}/730)}{73}\right]$
= Rem $\left(\frac{81}{73}\right)$ = 8. Choice (B)

43. Since one of the numbers is 7, the number would be greater than 7^3 i.e., 343. If it starts with 3; then $3^3 + 7^3 = 370$, which means that the number can be 371, as $3^3 + 7^3 + 1^3 = 371$.
Choice (B)

44. The only perfect square which satisfies the given conditions is 12321 which is $(111)^2$. Choice (B)

45. If a number is divisible by 36, it would be divisible by 9 and by 4. If the number is divisible by 4, the last two digits must be divisible by 4. So, x can take 0, 2, 4, 6 or 8. If the number is divisible by 9, the sum of the digits (2x + 11) must be divisible by 9. i.e., x can take only 8. Choice (C)

46. Let the age of Richard's son be x.
Given that $\sqrt{x+6} = x - 6$
$\Rightarrow x + 6 = x^2 - 12x + 36$
$\Rightarrow x^2 - 13x + 30 = 0$
$\Rightarrow x = 3$ or 10
Since (x – 6) > 0, x = 10. Ans: 10

47. The six-digit number which occurs in the cyclic form is 1,42,857. Hence, the last digit is 7. Ans: 7

48. N = 54 …… 5454. Let the sum of all the alternate digits starting with the units digit (the digit on the extreme right) be U and let the sum of all the other digits be T. Let the digit in the 12th place (from the left) be a. Now, U = 27(4) + a and T = 28(5).
Rem N/11 = Rem (U – T)/11
= Rem $\frac{27(4)+a-27(5)-5}{11}$ = Rem $\frac{27(-1)+a-5}{11}$
= Rem $\frac{-32+a}{11}$ If a = –1 or 10, then – 32 + a is divisible by 11. But as $0 \le a < 10$, there is no value of a, which satisfies this condition.

Alternative Method:
Let the digit in the 12th place be x
N = 54545454545x5454…….54 = (5454 …….5x) 10^{44} + 54 …..22 times
54 ……. 22 times is divisible by 11
$\therefore$ For N to be divisible by 11,
5454 ……… 5x must be divisible by 11. This is not possible for any value of x. Choice (D)

49. The divisors and corresponding remainders are tabulated below

Divisors	7	8	9	7	72
Remainders	3	1	2	3	65

Numbers of the form 8p + 1 and 9q + 2 are of the form 72n – 7 (LCM model 2). If such a number is also of the form 7n + 3, 72n – 7 – 3 should be a multiple of 7, i.e., 2n – 3 should be a multiple of 7. $\therefore$ n = 5, 12, 19, …. The First number would be 353.
The General form of number is 7(72) N + 353 where N is anywhole number. The only numbers in this form less than 1500 are those for which N is 0, 1 or 2.
N = 0, 1, 2, these numbers are
less than 1500.
$\therefore$There are 3 such numbers. Ans: 3

50. 720 =(2^4) (3^2)(5)240 = 2^4(3)(5)
120 = 2^3 (3) (5)
As 720, 240 and 120 all have 2, 3 and 5 as their only prime factors coprime to any of these is also co-prime to each of the other two
$\therefore$ Number of positive integers less than 240 and coprime to 720
$= 240\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 240\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = 64$
No. of positive integers less than 120 and coprime to 720
$= 120\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 120\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = 32$
Required no. of positive integers = 64 – 32 = 32
Choice (B)

51. The last two digits of the first 4 powers of 7 are 07, 49, 43, 01. These add up to 00.
$\therefore 7^1 + 7^2 + \ldots\ldots 7^{12}$ ends in 00 and $7^0 + 7^1 + \ldots\ldots + 7^{12}$ ends in 01. Choice (D)

52. Given that N when expressed in base 12, has 7 in its units place. Hence N = 12k + 7.
Now, when 12k + 7 is converted to base 5, the units place depends on the remainder when 12k + 7 is divided by 5. A unique value cannot be determined as for k = 1, the remainder is 4 whereas for k = 2, the remainder is 1.
Choice (D)

53. Let us find the natural numbers which correspond to a 4-digit number in base 5 system.
The smallest such number being $(1000)_5 = 125$ and the largest such number is $(4444)_5 = 624$
(i.e., 625 – 1). Similarly the smallest and the largest three digit numbers in base 6 system are
$(100)_6 = 36$ and $(555)_6 = 215$. From the above observations, we can say that all numbers from 125 to 215 satisfy both the conditions.
Hence, 215 – 125 + 1 = 91 such numbers exist.
Choice (B)

54. Given in base n, $32 \times 45 = 2133$; if the numbers 32 and 45 were in base 10, we would have got the product as 1440. Since 2133 > 1440, (both considered as numbers in base ten), n < 10. Also to use 5 as a digit, $n \geq 6$.
$\therefore$ n = 6, 7, 8 or 9. Consider the units place multiplication $2 \times 5 = 10$. Since the digit in the units place of the result is 3, the base has to be 7.
We convert $(424)_{10}$ to base 7 as follows,

7	424		
7	60	–	4
7	8	–	4
	1	–	1

$\therefore (424)_{10} = (1144)_7$ Ans: 1144

55. Let $y = (72345)_8 - (46436)_8$

```
      8 8 8
      -------------
      7 2 3 4 5
(–)   4 6 4 3 6
      ------------
      2 3 7 0 7
      ------------
```

Note: While subtracting a larger number from a smaller number we borrow a number whose value is equal to the base from the digit on the left. Choice (B)

56. Consider the number $(1331)_n = n^3 + 3n^2 + 3n + 1 = (n + 1)^3$
As $(n + 1)^3$ is a perfect cube for all $n \in N$, 1331 is the required number. Choice (A)

57. As $3^8 < 6796 < 3^9$, and $3^8 = (100000000)_3$, the smallest nine-digit number.
and $3^9 = (1000000000)_3$, the smallest ten-digit number any number between 3^8 and 3^9 will be a 9 digit number in base 3.
Ans: 9

58. We try to find some 5 digit numbers ABCDE and PQRST such that ABC + 3DE = 269 ------ (1) and PQR + 3ST = 329 --------- (2)
As $\text{Rem}\frac{269}{3}$ = 2, we select ABC such that $\text{Rem}\frac{ABC}{3}$ = s2
Let ABC = 101. $\therefore$ DE = $\frac{269-101}{3} = \frac{168}{3} = 56$
Let PQR = 200 $\therefore$ ST = $\frac{329-200}{3} = 43$
$\therefore$ ABCDE = 10156
PQRST = 20043
Sum = 30199
On checking with the divisors in the options, we find that only 23 divides 30199. This is enough to go for B. One proof is given below
ABC + 3DE + 30 = 299 ----------- (3)
PQR + 3ST – 30 = 299 ---------- (4)
ABC + 3(DE + 10) = 299 i.e. ABC + 3FE = 299
(where D + 1 = F) -------------- (5)s
PQR + 3(ST – 10) = 299 i.e. PQR + 3UT = 299
(where S – 1 = U) ------------ (6)
(We are assuming $D \leq 8$ and $S \geq 2$. The cases D = 9 and S = 1 or 0 can be worked out separately)
$\therefore$ 300ABC + 3FE = 299 (ABC + 1) -------------- (7) and
300 PQR + 3UT = 299 (PQR + 1) ----------- (8)
(Adding 299 ABC on both sides of (5)
Add 299 PQR on both sides of (6)
As 3 does not divide 299, ABCFE and PQRUT are multiples of 299 and hence ABCFE + PQRUT is a multiple of 299.
As D = F – 1 and S = U + 1, ABCDE + PQRST is also a multiple of 299 and hence of 13 and 23. Choice (C)

59. $13^{196} = 199q + 126$
$13^3\ 13^{193} = 199q + 126$ -------- (1)
The divisor that we are interested in is 199
Now $13^2 = 169 = –30$
$13^3 = 13\ (–30) = –390 \equiv –191 \equiv 8$
(1) can be written as $8(13^{193}) \equiv 126$ ----------- (2)
If we divided (2) by 8, we will get the desired value. But the RHS has to be recast so that dividing by 8 gives an integer. The recasting has to be done by adding a suitable number of 199's
Now $\text{Rem}\frac{126}{8}$ = 6 Adding one 199 is equivalent to subtracting 1 (when the divisor is 8, because 199 = 200 -1)
Therefore, we need to add six 199's
(2) $\Rightarrow 8(13^{193}) \equiv 126 + 6\ (199) = 126 + 1200 – 6$
$\Rightarrow 13^{193} \equiv 15 + 150 = 165$. Choice (D)

60. By Fermat's theorem, $\text{Rem}\frac{5^{78}}{79} = 1$
Let $5^{77} = 79p + r$
$\therefore 5^{78} = 79q + 5r$ (where q = 5p) $= 79q_1 + 1$
i.e. 5r = 79a + 1 or r = 15a + $\frac{4a+1}{5}$
Setting a = 1, we get r = 15 + 1 = 16
If we set a = 6, we get r = 15(6) + 5 = 95, which is 79 + 16
In general, if a = 5k + 1, r = 15(5k + 1) + $\frac{20k+5}{5}$
= 75k + 15 + 4k + 1 = 79k + 16. Choice (A)

61. $\text{Rem}\frac{7^{100}}{2399} = \text{Rem}\frac{\left(7^4\right)^{25}}{2399}$
$= \text{Rem}\frac{2401^{25}}{2399} = \text{Rem}\frac{2^{25}}{2399}$
$= \text{Rem}\frac{\left(2^{11}\right)\left(2^{11}\right)8}{2399} = \text{Rem}\frac{(2048)(2048)(8)}{2399}$
$= \text{Rem}\frac{(-351)(-351)8}{2399} = \text{Rem}\frac{(351)(351)(8)}{2399}$
$= \text{Rem}\frac{27^2\,13^2 8}{2399} = \text{Rem}\frac{(729)8(169)}{2399}$
$= \text{Rem}\frac{(5832)(169)}{2399}$
$= \text{Rem}\frac{(1034)(13)(13)}{2399} = \text{Rem}\frac{(13442)(13)}{2399}$
$= \text{Rem}\frac{(1447)(13)}{2399} = \text{Rem}\frac{18811}{2399}$
= 2018. Ans: 2018

62. $\text{Rem}\frac{3^{100}}{241} = \text{Rem}\frac{\left(3^5\right)^{20}}{241} = \text{Rem}\frac{(243)^{20}}{241}$
$= \text{Rem}\frac{2^{20}}{241} = \text{Rem}\frac{\left(2^8\right)^2 16}{241}$

$= \text{Rem}\dfrac{(256)^2\ (16)}{241} = \text{Rem}\dfrac{(15^2)16}{241}$

$= \text{Rem}\dfrac{(225)(16)}{241} = \text{Rem}\dfrac{(-16)(16)}{241}$

$= \text{Rem}\dfrac{-256}{241} = -15 = 226.$ Choice (B)

63. Let the partially specified prime number be 100a − 1. The other prime factor has to be 10b + 1
$\therefore$ (100a − 1) (10b + 1) = 60499
$\Rightarrow$ 1000ab − 10b + 100a = 60500
We see that b has to be a multiple of 10, say b = 10c
$\therefore$ (100a – 1) (100c + 1)
= 60499 [This possibility is also indicated by the options. The sum of the factors would be 100 (a + c)]
$\Rightarrow$ 10,000ac + 100 (a – c) = 60500
$\Rightarrow$ 100ac + (a – c) = 605 ------------- (1)
Now, the product ac could be 1, 2, 3, 4, 5 or 6.
By trial, a = 1, c = 6 satisfy (1). We see that the product of 100(1) + 1 and 100 (6) – 1 is 60499. The sum of these factors is 700. Choice (C)

64. The 6 – digit number N = ababab = ab (10101) = ab(3) (3367) = ab(3) (7) (481) = 3(7) (13) (37) ab = (111) (91) ab. Therefore, all such numbers are multiples of 111. Now if a = 0, we would get a 5-digit number. Therefore, a can have 9 values and for each such value, b can have 10 values. Therefore, there are 90 numbers of the required form.
Choice (D)

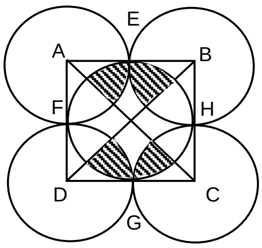
65. The 6 – digit number N = abcabc = abc (1001) = 7(11) (13) abc, while 301 = 7(43). If N has to be a multiple of 301, a bc has to be a multiple of 43. Now 43(23) = 989. There are 23 multiples of 43 having at the most 3 digits. But if abc = 043 or 086, abcabc would be a 5 digit number. Therefore there are 21 six-digit numbers of the form abcabc, which are multiples of 301. Ans: 21

Exercise – 6

Solutions for questions 1 to 22:

1. In ΔABC, AB = AC. $\therefore \angle$ABC = $\angle$ACB = 30°
In ΔABD, AB = AD.
$\therefore \angle$ABD = $\angle$ADB = x° say
In ΔDBC, the sum of the angles is
x° + (x° + 30°) + 30°
= 180° $\Rightarrow$ x° = 60°. Choice (A)

2.



Consider the circle with centre A. Join the line EF. The shaded area in that circle is equal to, (because of symmetry), the difference of the areas of sector AFE and the triangle AFE.
$\therefore$Shaded area = (sector AFE) – (Triangle AFE)

$= \frac{1}{4}\cdot \pi r^2 - \frac{1}{2}r^2 = \dfrac{\pi \times 3^2}{4} - \dfrac{3^2}{2}$

(because r = AE = $\frac{1}{2}$AB = $\frac{1}{2}\times 6 = 3$)

$= \frac{9}{4}(\pi - 2)$.

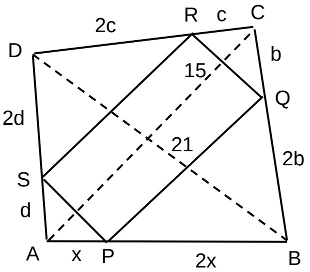
The required total shaded area is 4 times the shaded area shown in the circle with centre A.

Hence, the required area = $4 \times \frac{9}{4}(\pi - 2)$

= $9(\pi - 2)$.

Choice (B)

3.



Consider ΔABC
The line segment PQ divides both BC & BA in the ratio 2 : 1
$\therefore$ PQ is parallel to AC ($\because$ Any line which divides the two sides of a triangle in ratios which are equal is always parallel to the third side of a triangle)
$\therefore$ ΔPQB is similar to ΔACB

$\dfrac{PB}{AB} = \dfrac{QB}{CB} = \dfrac{PQ}{AC} = \dfrac{2}{3}$

$\Rightarrow$ PQ = $\dfrac{2}{3}\times AC = 10\,\text{cm}$

Similarly from Δle DAC we get

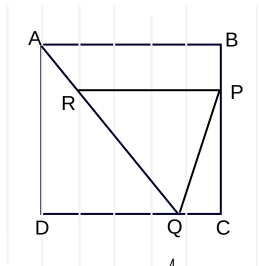
SR = $\dfrac{2}{3}\times AC = 10\,\text{cm}$

In ΔBCD $\dfrac{CR}{CD} = \dfrac{CQ}{CB} = \dfrac{RQ}{DB} = \dfrac{1}{3} \Rightarrow$ RQ = $\dfrac{21}{3} = 7\,\text{cm}$

Similarly from ΔABD. We get PS = $\dfrac{21}{3} = 7\,\text{cm}$.

Choice (A)

4.



Join AP. The area of PQR is $\frac{4}{5}$ of the area of PQA

($\because$RQ is $\frac{4}{5}$ of AQ)

Ar of ABP = $\frac{1}{3}$ of ABC = ($\because$BP = $\frac{1}{3}$ BC) = $\frac{1}{6}$(ABCD)

Ar of CQP = $\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)$ of BCD

($\because$CP = $\frac{2}{3}$ of CB, CQ = $\frac{1}{4}$ of CD)

= $\dfrac{1}{12}$ of ABCD

Ar of ADQ = $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$ABCD = $\frac{3}{8}$ of ABCD.

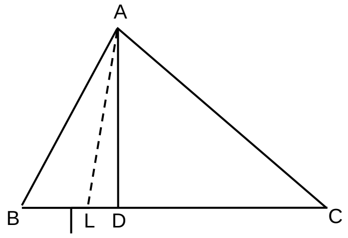
The LCM of 6, 12, 8 is 24. Let the area of ABCD be 24.
Ar of ABP = 4, Ar of CQP = 2, Ar of ADQ = 9.
$\therefore$ Ar of APQ = 24 − (4 + 2 + 9) = 9

and Ar of PQR = $\frac{4}{5}$ (9) = 7.2.

The required ratio is $\dfrac{7.2}{24} = \dfrac{72}{240} = \dfrac{3}{10}$

Choice (A)

5.



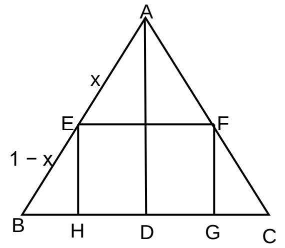
$$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ADC} = \frac{\frac{1}{2} \times BD \times AL}{\frac{1}{2} \times CD \times AL}$$

$= \frac{BD}{CD} = \frac{AB}{AC} = \frac{1}{3}$ ($\because$ AD is the angle bisector of $\angle BAC$)

$\Rightarrow$ Area of $\Delta ADC = 3 \times 30 = 90$

Area of $\Delta ABC = 90 + 30 = 120$ sqcm. Ans: 120

6. Consider the variable rectangle, one of whose sides is EF.



Let $\frac{AE}{AB} = x$. $\therefore \frac{EB}{AB} = 1 - x$

$\therefore$ EF = x BC

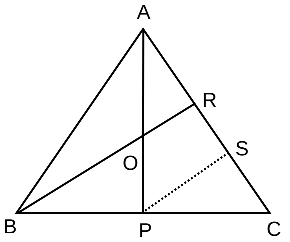
and EH = (1 − x) AD

EF (EH) = x (1 − x) BC (AD).

The maximum value of this occurs when $x = \frac{1}{2}$ and this maximum value is $\frac{1}{4}$(BC) (AD) = $\frac{10AD}{4} = 10\sqrt{3}$

Given $\therefore$ AD = $4\sqrt{3}$ Choice (A)

7.



From P draw the line segment PS parallel to BQ

$\because \frac{BP}{PC} = 1 \Rightarrow \frac{CS}{RS} = 1 \Rightarrow CS = RS$ --- (1)

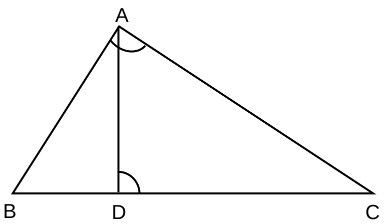
From ΔAPS

$\because \frac{AQ}{QP} = 1 \Rightarrow \frac{AR}{RS} = 1 \Rightarrow AR = RS$ --- (2)

From (1) & (2)

$CS = RS = AR = \frac{AC}{3} \Rightarrow AR = \frac{45}{3} = 15\,\text{cm}$. Ans: 15

8.



In triangles ABC and DAC, $\angle BAC = \angle ADC$

$\angle C = \angle C$

$\therefore$ ΔABC is similar to ΔADC

$$\frac{AB}{DA} = \frac{CA}{CD} = \frac{CB}{CA}$$

$\Rightarrow CA^2 = (CB)(CD)$

$\Rightarrow$ (CB)(CD) = 36

For BD to be maximum

CB should be the maximum possible length

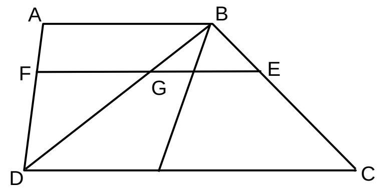
$\therefore$CD should be the minimum possible length

(CB)(CD) = (18)(2) = 36

$\therefore$The maximum value of BD = (18 − 2) cm = 16 cm

Choice (B)

9.



Triangle DFG is similar to triangle DAB (AAA)

[$\therefore \angle 1 = \angle 2$ (Corresponding angles) and

$\angle FDG = \angle ADB$]

$\frac{FG}{AB} = \frac{DF}{DA}$ ---- (1)

$\therefore$ AB, EF, CD are parallel to each other

$\Rightarrow \frac{DF}{FA} = \frac{CE}{EB} = \frac{4}{3} \Rightarrow \frac{DF}{DA} = \frac{4}{7}$ ---- (2)

From (1) and (2),

$\frac{FG}{AB} = \frac{4}{7} \Rightarrow FG = \frac{4}{7}(AB)$ -- (a)

In Δ BEG and BCD we have

$$\left.\begin{aligned} &\angle BEG = \angle BCD \\ &\angle B = \angle B \end{aligned}\right\} \quad \text{AAA}$$

$\Rightarrow \Delta BEG \sim \Delta BCD \Rightarrow \frac{BE}{BC} = \frac{EG}{CD}$

$\Rightarrow \frac{BE}{BE + EC} = \frac{EG}{CD} \Rightarrow \frac{EG}{CD} = \frac{3}{3+4} = \frac{3}{7}$

$EG = \frac{3}{7}(CD) = \frac{6}{7}(AB)$ --- (b)

From (a) and (b)

$EF = EG + GE = \frac{10}{7}(AB)$

$\Rightarrow$ EF : AB = 10 : 7

Alternate method:

As FE moves from AB to DC, its length increases from one times AB to 2 times AB. When it covers $\frac{3}{7}$ parts of the distance between the AB and CD, its length is AB + $\frac{3}{7}$ (DC – AB) = AB [1 + $\frac{3}{7}$(2 – 1)] = $\frac{10}{7}$ AB.

$\therefore$FE : AB = $\frac{10}{7}$: 1 = 10 : 7

Choice (D)

10. Let the length of my plot be x m.

$\therefore$ Breadth = (x – 44) m.

$\therefore$ Area = x (x – 44) = ($x^2 - 44x$) sq.m.

The length of Mr.Tarapore's plot

= (x + 16) m. and breadth = 16 mt

Now, $x^2 - 44x = (x + 16)\,16$

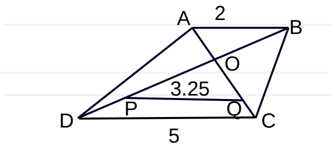
$\Rightarrow x^2 - 60x - 256 = 0 \Rightarrow (x - 64)(x + 4) = 0$

$\therefore x = 64$ (since $x \neq -4$)

$\therefore$The length of Mr. Tarapore's plot = 64 + 16 = 80 m.

Ans: 80

11.



Let the diagonals BD and AC intersect at O. $\Delta AOB \sim \Delta COD$ and the corresponding parts are in the ratio 2 : 5. If we draw a line perpendicular to CD (and AB as well as PQ) passing through O and let it intersect AB at R, PQ at S and CD at T, then OR : OT = 2 : 5 and OS : OT = 3.25 : 5 s = 13 : 20

These ratios are tabulated below:

RO	OS	ST	OT
2			5
8	13	7	20

$\therefore$ BP : PD = RS : ST = (8 + 13) : 7 = 21 : 7 = 3 : 1

As BD = 6, PB = $\frac{3}{4}$(BD) = $\frac{3}{4}$(6) = 4.5. Choice (C)

12. Let $\angle ADB = x°$

$\Rightarrow \angle ADC = 180 - x°$

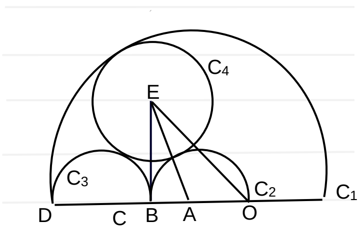
$\angle ODE = \frac{x°}{2}$

$\angle ODF = \frac{180° - x°}{2} = 90° - \frac{x°}{2}$

$\angle EDF = \angle ODE + \angle ODE = \frac{x°}{2} + 90° - \frac{x°}{2} = 90°$

$\angle EFD = 180° - (35° + 90°) = 55°$. Choice (D)

13.



Let the center and radius of C_4 be E and r respectively.

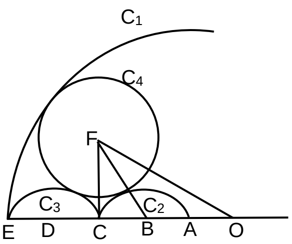
In Δ EBA, AB = 5, EA = 5 + r. $\therefore EB^2 = r^2 + 10r$

In Δ EBO, $EB^2 = r^2 + 10r$, $BO^2 = 10^2$ and $EO^2 = (20 - r)^2$

$\therefore r^2 + 10r + 100 = 400 - 40r + r^2$

$\Rightarrow 50r = 300 \Rightarrow r = 6$. Ans: 6

14. Let the centre of C_4 be F and the radius be r.



In Δ BCF, BC = 3, BF = r + 3. $\therefore FC^2 = r^2 + 6r$ (applying $FC^2 + BC^2 = BF^2$)

In ΔOCF, OC = 9, OF = 15 − r.

$\therefore 81 + (r^2 + 6r) = r^2 - 30r + 225 \Rightarrow r = 4$ Choice (B)

15. $\angle BDA = \angle ADC = 90°$

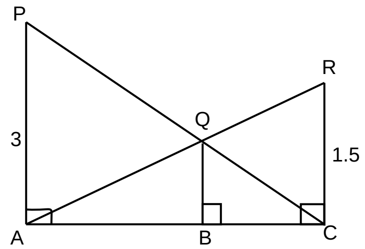
$\angle BAD = \angle ACD$ (Given)

$\therefore \Delta BAD \approx \Delta ACD$

$\frac{BD}{DA} = \frac{AD}{DC} \Rightarrow (BD)(DC) = AD^2 = 35$

BD could be 1, 5, 7 or any other number. Choice (D)

16.



$\frac{QB}{PA} = \frac{BC}{AC}$ $[\because$ QB is parallel to PA$]$

$\frac{QB}{RC} = \frac{AB}{AC}$ $(\because$ QB is parallel to RC$)$

$\frac{QB}{PA} + \frac{QB}{RC} = \frac{BC + AB}{AC}$

$\frac{QB}{PA} + \frac{QB}{RC} = 1$

$\Rightarrow \frac{1}{PA} + \frac{1}{RC} = \frac{1}{QB} \Rightarrow QB = \frac{(PA)(RC)}{PA + RC}$

$= \frac{3(1.5)}{4.5}$ cm = 1 cm Ans: 1

17. As BO bisects $\angle ABC$ and DO bisects $\angle ADC$,

AB/BC = AO/OC and AO/OC = DA/DC = 1/4.

As ABCD is a quadrilateral,

DC < DA + AB + BC

$\Rightarrow 12 < 3 + 5x \Rightarrow 1.8 < x$

and BC < BA + AD + DC $\Rightarrow 4x < x + 15 \Rightarrow x < 5$.

i.e., 1.8 < x < 5 or 7.2 < 4x < 20.

Only 12.8 is a possible value of BC. Choice (C)

18. Given that the volume of air that an exhaust fan can evacuate per unit time is proportional to the square of its speed in rev. per min $\Rightarrow v \propto n^2$

given it takes 20 min. to evacuate 2 m x 3 m x 4 m

i.e., v_1 = 24 cu.m.

If the same fan is running 5 times its speed it can evacuate 5^2 times v_1.

i.e., 25 x 24 = 600 cu.m. in 20 min

But in 10 min it will evacuate only 300 cu.m.

$\therefore$The total volume of air in the room should be 300 cu.m.

Total volume of the room = 6 x 8 x 12.5 = 600 cu.m.

$\therefore$ Volume of the boxes = 600 – 300 = 300 cu.m.

but the volume of each box = 1/2 x 1/2 x 1 = 1/4 cu.m.

$\therefore$ One needs 300 x 4 = 1200 such boxes.

Ans: 1200

19. Let the width be xm.

(48 x 40) – (48 – 2x) (40 – 2x) = 336

$\Rightarrow 1920 - [1920 - 176x + 4x^2] = 336$

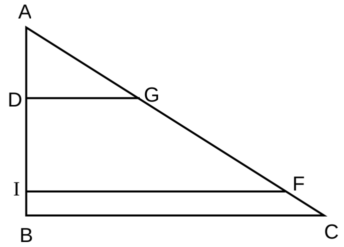
$\Rightarrow 4x^2 - 176x + 336 = 0$

$\Rightarrow x^2 - 44x + 84 = 0 \Rightarrow (x - 2)(x - 42) = 0$

$\Rightarrow$ x = 2 (since x cannot be 42)

Choice (A)

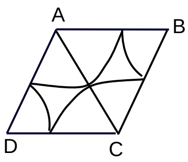
20.



The median DG in Δ ADF is parallel to BC. Draw a line through F parallel to BC to intersect AB at I.

As AG = GF, AD = DI. As AD : DB = 4 : 5, AD : DI : IB = 4 : 4 : 1 and consequently AG : GF : FC = 4 : 4 : 1 or CF : FA = 1 : 8 Choice (D)

21.

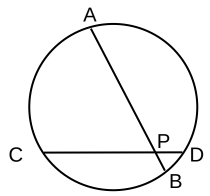


Area of figure = Area of rhombus − Area of full circle

$= \frac{\sqrt{3}}{2}(2^2) - \pi(1^2)\ cm^2 = (2\sqrt{3} - \pi)\ cm^2$

Choice (D)

22. $\angle APC = 60°$



$\therefore \angle DAP + \angle ADP = 60°$

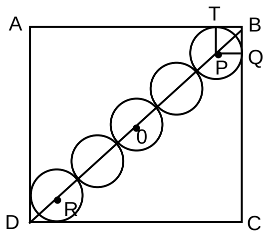
or $\angle DBP + \angle ADC = 60°$

$\therefore$ AC+ DB = 120° and CB+ DA = 240°. As CB= 110°, DA = 130°.

Ans: 130

Solutions for question 23:

23.



Let P and R be the centres of the two circles in the corners.
Then PR = 8r, where r is the radius of each circle.
Let PQ and PT be perpendiculars to BC and BA respectively.
$\therefore$ Q and T are points of contact of tangents BQ and BT.

PQBT is a square. Hence, PA = $\sqrt{2}$ r.
Hence diagonal DB of the square is:

DR + RP + PB = $\sqrt{2}$ r + 8r + $\sqrt{2}$ r = r $(8 + 2\sqrt{2})$.

But diagonal DB = $\sqrt{2}$ (side of square) = $\sqrt{2}$ a.

$\therefore \sqrt{2}\,a = r(8 + 2\sqrt{2})$

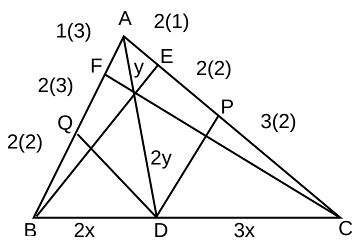
$\therefore r : a = \sqrt{2} : 8 + 2\sqrt{2} = 1 : (4\sqrt{2} + 2)$

$= 1 : 2(2\sqrt{2} + 1)$

Choice (C)

Solutions for questions 24 to 60:

24.



Through D, we can draw lines parallel to BE intersecting AC at P and another line parallel to CF intersecting AB at Q.
In ΔCEB, CP/PE = 3/2
In ΔADP, PE/AE = 2/1

$\therefore$ AE : EP : PC = 2 : 4 : 6 or
AE : EC = 2 : 10 = 1: 5
In ΔBCF, BQ/QF = 2/3
In ΔAQD, QF/AF = 2/11
$\therefore$ AF : FQ : QB = 3 : 6 : 4 or
AF : BF = 3 : 10
Let AE = b. Therefore EC = 5b
Let AF = 3C. Therefore BF = 10c

$$\frac{AF^2 \cdot AE + BF^2 \cdot EC}{AF^2 \cdot EC + BF^2 \cdot AE} = \frac{9c^2b + 500c^2b}{45c^2b + 100c^2b} = \frac{509}{145}$$

Choice (B)

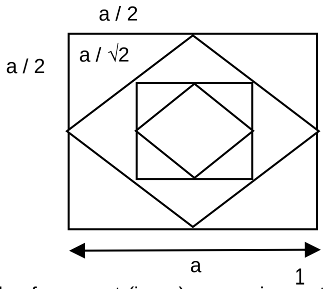
25. In the right triangle with sides a, b, c (a < b < c),
2a + 7c = 9b.
i.e., 7(c – b) = 2(b – a) or (b – a) : (c – b) = 7 : 2
Checking with the common Pythagoras triplets, we see that a : b : c could be 8 : 15 : 17.

As a = 12 = $\frac{3}{2}$(8), c = $\frac{3}{2}$(17) = 25.5

Note: If we don't notice immediately that a : b : c = 8 : 15 : 17, we would proceed as follows.
b – a = 7p and c – b = 2p.
Let a = kp. Therefore, b = (k + 7)p and c = (k + 9)p.
$\therefore k^2 + (k + 7)^2 = (k + 9)^2$
$\Rightarrow 2k^2 + 14k + 49 = k^2 + 18k + 81$
$\Rightarrow k^2 - 4k - 32 = 0$
$\Rightarrow (k - 8)(k + 4) = 0$
As k can't be –4, k = 8 and a = 8p, b = 15p, c = 17p.

Ans: 25.5

26. The pattern that Manoj was doodling is



The side of any next (inner) square is $\frac{1}{\sqrt{2}}$ times that of the immediately outer square.

$\therefore$ (1) the first sum : of all the squares

$= a^2 + \frac{a^2}{2} + \frac{a^2}{4} + \frac{a^2}{8} = \frac{a^2}{1 - 1/2} = 2a^2$

(2) The second sum
The difference between the areas of 1st and 2nd square

$= a^2 - \frac{a^2}{2} = \frac{a^2}{2}$

similarly between 2nd and 3rd = $\frac{a^2}{2} - \frac{a^2}{4} = \frac{a^2}{4}$

$\therefore$ sum = $\frac{a^2}{2} + \frac{a^2}{4} + \frac{a^2}{8} = a^2$

(3) The third sum : the sum of alternative squares starting from the second from outside.

$= \frac{a^2}{2} + \frac{a^2}{8} + \frac{a^2}{32}$

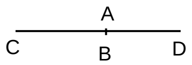
$= \frac{\frac{a^2}{2}}{1 - \frac{1}{4}} = \frac{a^2}{2} \times \frac{4}{3} = \frac{2a^2}{3}$

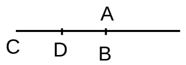
$\therefore$ The ratio will be $2a^2 : a^2 : \frac{2a^2}{3}$ = 6 : 3 : 2

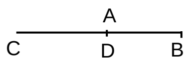
Choice (B)

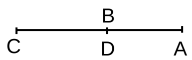
27. Of the 4 points, C lies at one extreme. There could be 2, 3 or 4 distinct points. The various possibilities are listed below.

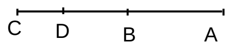
(I)

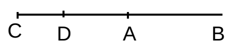

(IIA)


(IIB)


(IIC)


(IID)


(IIIA)


(IIIB)


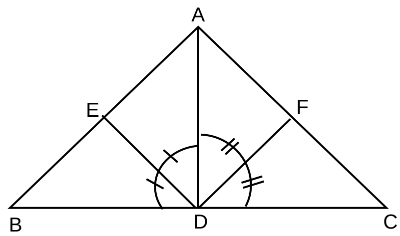
We see that if there are only 2 distinct points, there is only one possibility.
If there are 3 distinct points, there are 4 possibilities (IIA to IID)
If there are 4 distinct points, there are 2 possibilities (IIIA to IIIB)
Consider the options.
(A) A = B. We need to consider I, IIA, IIB. False.
(B) A = D. Consider I, IIC False.
(C) B =D. Consider I, IID. False.
(D) A ≠ B. Consider IIC, IID, IIIA, IIIB. True.
Choice (D)

28. $\angle AOD = \angle COB$ (vertically opposite angles)
$\angle DAO = \angle BCO$ (Alternate interior angles)
$\therefore \Delta\, AOD \sim \Delta COB$
$\therefore \dfrac{OD}{OB} = \dfrac{AO}{CO} = \dfrac{1}{3}$
$\Rightarrow$ OB = 3(OD) = 3(1.2) cm = 3.6 cm.
Ans: 3.6

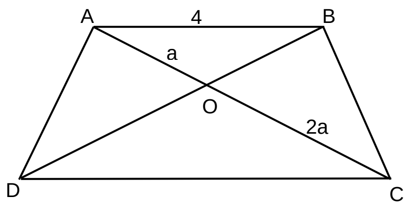
29. Area of the base circle = 22 X 8 = 176 sq.m.
Let, the radius of the base circle be r.
$\dfrac{22}{7} \times r^2 = 176 \Rightarrow r^2 = 56$
Also, volume of the tent $= \frac{1}{3}\pi r^2 h = 22 \times 40$
$\Rightarrow \frac{1}{3} \times \dfrac{22}{7} \times 56 \times h = 22 \times 40$
$\Rightarrow h = \dfrac{40 \times 3 \times 7}{56} = 15$ m　　Choice (A)

30. Area of $\Delta XBY = \frac{1}{2}(XB)(BY) \sin B = \frac{1}{2}(3BA)(4BC).\sin B$
$= (12)\left[\dfrac{1}{2}(BA)(BC) . \sin B\right] = 12 . \Delta ABC$
Area of ΔBXY is 1100% greater than the area of ΔABC.
Ans: 1100

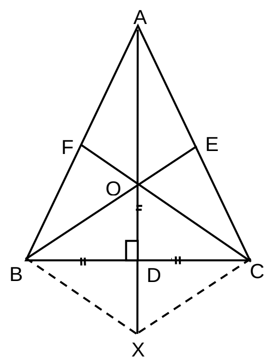
31. The speed required is minimum when the path taken is the shortest possible path.
The shortest possible path for fifi to reach the diagonally opposite corner will be the diagonal of the room itself.
The length of the diagonal of the cuboid of dimensions 3 x 4 x 12 mt. will be
$\sqrt{3^2 + 4^2 + 12^2} = 13$ m.
Hence fifi needs to cover 13 m in a max. of 20 sec.
i.e., at least = 65 cm/sec.　　Ans: 65

32.


$\because$ AD is the median BD = DC　　--- (1)
$\because$ DE bisects the angle ADB
$\dfrac{AE}{EB} = \dfrac{AD}{BD}$　　--- (2)
$\because$ DF bisects the angle ADC
$\dfrac{AD}{DC} = \dfrac{AF}{FC}$　　--- (3)
from (1), (2) & (3)
$\dfrac{AF}{FC} = \dfrac{AE}{EB} = \dfrac{1}{8}$
As AF = 4 cm,
FC = 32 cm $\Rightarrow$ AC = 36 cm.　　Choice (B)

33.


Let the parallel sides be AB and CD
Given that AB = 4 cm
Triangle AOB is similar to triangle COD.
$\therefore \dfrac{CD}{AB} = \dfrac{CO}{AO} = 2$
$\Rightarrow$ CD = 2(AB) = 8 cm.　　Ans: 8

34.


Let OD = x, AO = 4x, AD = 5x, OX = 2x
Join BX and CX. In quadrilateral BOCX, the diagonals bisect each other at right angles
$\therefore$BOCX is a rhombus
i.e., BX is parallel to OC and CX is parallel to OB --- (A)
Consider Δ'le ABX

$\because$ BX is parallel to OF

$\therefore \frac{AX}{OX} = \frac{AB}{FB}; \Rightarrow \frac{4x + 2x}{2x} = \frac{AB}{3}$

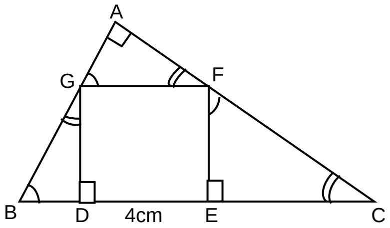
$\Rightarrow$ AB = 9 Choice (C)

35. Let the radii of the first, second and the third packs be 2r, 3r and 4r. Rate of increase of temperature (T)

$= K \frac{(\text{Lateral surface area})}{\text{volume}} = K \frac{(2\pi RH)}{\pi R^2 H} = \frac{2K}{R}$

$\therefore$ Smaller the radius greater the rate at which the temperature increases. So, the first pack must be enclosed in the flask. Choice (A)

36.



ΔGBD ~ ΔAGF (AAA property)

ΔAGF ~ ΔFEC (AAA property)

$\frac{BD}{EF} = \frac{BG}{FC} = \frac{DG}{EC} \Rightarrow \frac{BD}{EF} = \frac{DG}{EC}$

$\Rightarrow BD \cdot EC = DG \cdot EF$

$\Rightarrow BD \cdot EC = DE^2$ ($\because$ DEFG is a square)

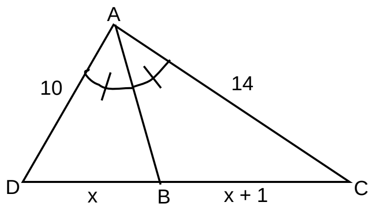
$BD \cdot EC = 16$

Given that BD > 1, EC > 1, BD $\neq$ EC

$\Rightarrow BD \cdot EC = 8 \cdot 2$ or $2 \cdot 8$

In either case their sum is 10. Ans: 10

37.



Let, BD = x. Then, CB = x + 1

$\frac{x}{x+1} = \frac{AD}{AC} = \frac{10}{14}$

$7x = 5x + 5 \Rightarrow x + 1 = 3.5$

$\therefore$ BC = 3.5 Choice (A)

38. In ΔBMC and ΔEDM

$\angle EMD = \angle CMB$

MC = MD

$\angle MCB = \angle MDE$ ($\because$ AE is parallel to BC)

$\Rightarrow \Delta$ BMC $\cong$ Δ EDM (ASA congruence)

BC = ED

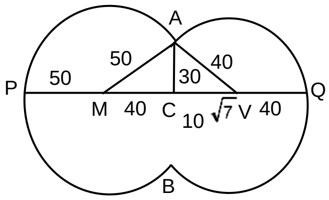
In ΔAEL and ΔCBL, $\angle A = \angle C$ and $\angle E = \angle B$

$\therefore \Delta AEL \sim \Delta CBL$

$\frac{EA}{BC} = \frac{EL}{BL} \Rightarrow \frac{EL}{BL} = \frac{ED + DA}{BC} = 2$

EL = 2BL = 6 cm Ans: 6

39.



The longest distance will be PQ, which is the perpendicular bisector of AB. PQ

= PM + MC + CN + NQ

In right-angled triangle ACM, AM = 50, AC = 30, ($\because$ C is the midpoint of AB)

$\therefore$ MC = 40

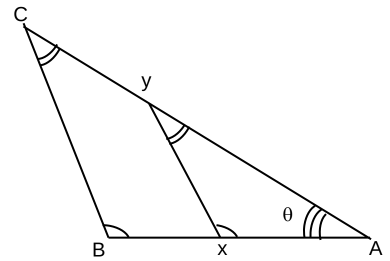
In the right-angled triangle CAN, AN = 40, AC = 30.

$\therefore$ CN = $10\sqrt{7}$

$\therefore$ PQ = $(50 + 40 + 10\sqrt{7} + 40)$ m

= $10(13 + \sqrt{7})$ m Choice (C)

40.



$\frac{\text{Area of } \Delta AXY}{\text{Area of } \Delta ABC} = \frac{1}{2}$

$\Rightarrow \frac{\frac{1}{2}(AX)(AY)\sin\theta}{\frac{1}{2}(AB)(AC)\sin\theta} = \frac{1}{2}$

$\Rightarrow \frac{(AX)(AY)}{(AB)(AC)} = \frac{1}{2}$ --- (1)

$\because \Delta AXY \sim \Delta ABC$ (AAA property)

$\frac{XY}{BC} = \frac{AY}{AC} = \frac{AX}{AB}$ --- (2)

from (1) & (2)

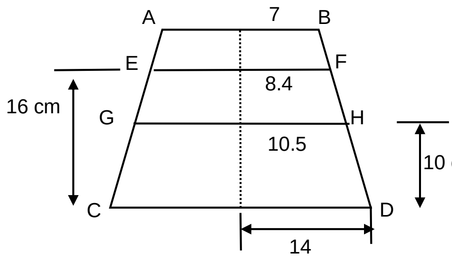
$\frac{AX}{AB} = \frac{1}{\sqrt{2}}$

$\frac{AB - BX}{AB} = \frac{1}{\sqrt{2}} \Rightarrow 1 - \frac{BX}{AB} = \frac{1}{\sqrt{2}} \Rightarrow \frac{BX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$

Choice (B)

41. The volume of water in the bucket when the height is half would be $\frac{1}{3}\pi \times \frac{h}{2}\pi \times (R^2 + Rr + r^2)$

R = 14 cm; r = 7 cm, h = 20 cm



Initial height = 20/2 = 10 cm

Radius at this height = 10.5 cm

Base radius = 14 cm. When height = 80% i.e., 16 cm, then radius = 14 – 0.8 x (14 – 7) = 8.4 cm

$\therefore$ Required volume is the volume of the frustum of dimensions R = 10.5 cm, r = 8.4 cm and h = 16 – 10 = 6 cm

$\therefore$ Number of pebbles required = $\frac{\frac{1}{3}\pi h(R^2 + Rr + r^2)}{\frac{4}{3}\pi(1)^3}$

= 404 pebbles. Ans: 404

42. $\Delta CDA \sim \Delta CEB$ and the corresponding sides are in the ratio 13 : 14 i.e., CD : DA : AC = CE : EB : BC

$CD = 5 \Rightarrow CE = \frac{14}{13}(5) = \frac{70}{13}$

$\Delta CDE \sim \Delta CAB$

(As $\angle AEB = \angle BDA = 90°$, quadrilateral ABDE is cyclic.
$\angle ABD$ and $\angle AED$ are supplementary; $\angle AED$ and $\angle CED$ are supplementary
$\therefore$ $\angle CED$ of ΔCDE = $\angle CBA$ of ΔCAB)
$\therefore$ CD : DE : EC = CA : AB : BC.
It's given that DE : AB = 5 : 13.

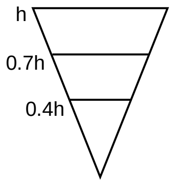
$CD = 5 \Rightarrow CA = \left(\frac{13}{5}\right)(5) = 13$

As EABD is a cyclic quadrilateral,
CE . CA = CD. CB

$\Rightarrow \left(\frac{70}{13}\right)(13) = 5\,(CB) \Rightarrow CB = 14.$

$\therefore$ BD = CB – CD = 14 – 5 = 9. Choice (B)

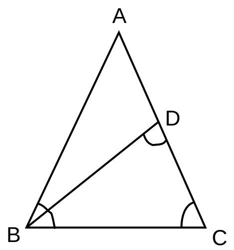
43. The data is tabulated below



Level	Vol. of water left	Time (s)
100%	V	O
70%	0.343V	3942
40%	0.064V	

Let the total volume of the cone be 1000 cc.
For 657 cc (1000 – 343) to leak out, 3942 s = 65.7 min are needed
$\therefore$For 279 cc (343 – 64) to leak out, 27.9 min are needed.
Ans: 27.9

44.

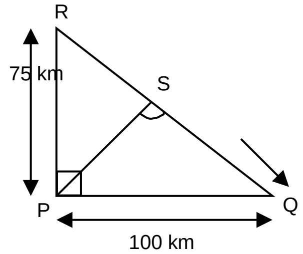


Given that $BC^2 = (AC)(CD) = (AB)(CD)$

$\Rightarrow \frac{BC}{CD} = \frac{AB}{BC}$ and $\angle BCD$ of ΔBCD = $\angle ABC$ of ΔABC

$\Rightarrow \Delta ABC \simeq \Delta BDC$ (SAS property)
$\Rightarrow \angle BDC = \angle ACB$ (corresponding angles)
$\Rightarrow$ BD = BC = 4·5 cm. Choice (B)

45.

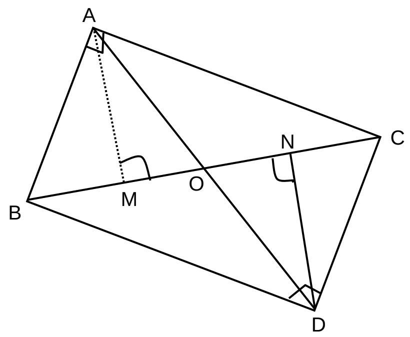


$QR = \sqrt{75^2 + 100^2} = 125$ km

Now, $\frac{1}{2} \times 75 \times 100 = \frac{1}{2} \times 125 \times SP$

$\Rightarrow SP = \frac{75 \times 100}{125} = 60$ km Ans: 60

46.



Draw AM and DN, both perpendicular to BC
Consider ΔAMO and ΔDNO
$\angle O = \angle O$
$\angle AMO = \angle DNO = 90°$

$\therefore \Delta AMO \sim \Delta DMO \;\Rightarrow\; \frac{AM}{DN} = \frac{AO}{DO} = \frac{3}{2}$ (given)

$\Rightarrow \frac{\text{Area of ABC}}{\text{Area of DBC}} = \frac{\frac{1}{2}(BC)(AM)}{\frac{1}{2}(BC)(DN)} = \frac{AO}{DO} = \frac{3}{2}$. Choice (C)

47. The data is tabulated below.

	Width (cm)	Height (cm)	Length (cm)
Carton	51	52	35
Cake	5	3	4

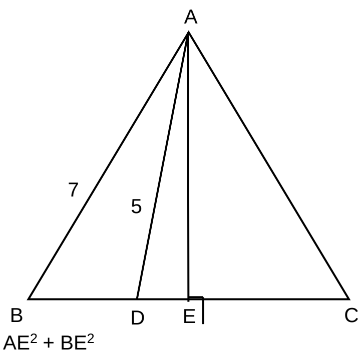
We see that the width, height and length of the carton are multiples of the height, length, and width of the cake respectively. Therefore, we can pack the cakes in the cartons with zero wastage of space. The minimum number of cartons would correspond to this packing. The number of cakes that can be placed this way is $\left(\frac{51}{3}\right)\left(\frac{52}{4}\right)\left(\frac{35}{5}\right)$

= (17) (13) (7) = 1547.

To pack 24000 cakes, we need at least $\frac{24000}{1547}$

= 15.51 or 16 cartons.
Apporva is actually left with 2240 cakes unpacked after she uses 16 cartons, i.e., she has packed 24,000 – 2240 or 21,760 cakes in 16 cartons or 1360 cakes per carton. For the remaining 2240 cakes she needs $\frac{2240}{1360}$ = 1.65 or 2 more cartons. Ans: 2

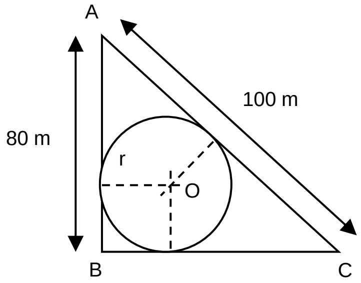
48.



$AB^2 = AE^2 + BE^2$

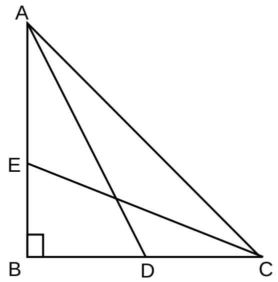
$AD^2 = AE^2 + DE^2 \Rightarrow AB^2 - AD^2 = BE^2 - DE^2$
$= (BE + DE)(BE - DE) = (CE + DE)(BE - DE)$
(In an isosceles triangle the altitude on to the base bisects the base; hence BE = CE) = (CD)(BD)
$AB^2 - AD^2 = 49 - 25 = 24 = 1(24) = 2(12) = 3(8) = 4(6)$
Therefore, these are the possibilities for BD and CD which follow from the condition that they are an integral number of centimeters. Therefore BD + CD could be 25, 14, 11 or 10.
But BD + CD = BC < AB + AC = 14. Therefore the only values that are possible are 11 and 10. Only 11 is there among the choices. Choice (B)

49.



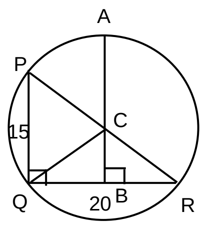
BC = $\sqrt{100^2 - 80^2}$ = 60 m
Let O be the center of the pool and r its radius.
Ar . ΔABC = rs
where s = $\frac{80 + 60 + 100}{2}$ m = 120 m
$\Rightarrow$ r = $\frac{1/2 \times 80 \times 60}{120}$ m = 20 m
$\therefore$ Area = $\pi (20)^2$ sq.m. = 400π sq.m. Choice (A)

50.



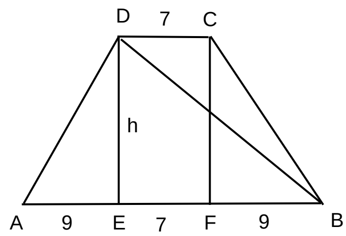
AC = 20
$AD^2 = AB^2 + BD^2 = AB^2 + \frac{BC^2}{4}$
($\because$BD = BC/2)
$CE^2 = EB^2 + BC^2 = \frac{AB^2}{4} + BC^2$ ($\because$EB = AB/2)
$\Rightarrow AD^2 + CE^2 = \frac{5}{4}\left[AB^2 + AC^2\right] = \frac{5}{4}AC^2$
$= \frac{5}{4}(400)$ cm^2 = 500 cm^2. Choice (A)

51. Since $\angle$Q = 90°, PR is the diameter of the circle.
Also as PQ = 3(5) cm and QR = 4(5) cm, it follows that PR = 5(5) cm = 25 cm
Radius of the circle = $\frac{PR}{2} = \frac{25}{2}$cm = 12.5 cm



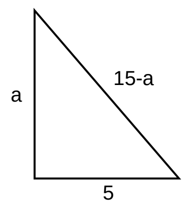
Of all the perpendiculars that can be drawn to the chord QR, from different positions of A on the circle, the perpendicular passing through the center C will have the greatest length. This greatest length AB = AC + BC = (12.5 + 7.5) cm.
[$\because$ BC = 1/2) PQ] = 20 cm] Choice (B)

52.



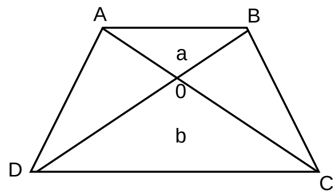
$AD^2 = 9^2 + h^2$
$BD^2 = 16^2 + h^2$
$AB^2 = AD^2 + BD^2 = 16^2 + 9^2 + 2h^2 = 25^2$
$\Rightarrow 2h^2 = 25^2 - 16^2 - 9^2 \Rightarrow h^2 = 144 \Rightarrow h = 12$
Area of trapezium = $\frac{1}{2}(12)(7 + 25) = 192$ Ans: 192

53. $a^2 + 25 = (15 - a)^2$
$\Rightarrow a^2 + 25 = 225 - 30a + a^2$
$\Rightarrow a = \frac{20}{3}$
Area = $\frac{1}{2}(5)\left(\frac{20}{3}\right) = \frac{50}{3}$
difference in areas = $25 - \frac{50}{3} = \frac{25}{3}$



Choice (B)

54.



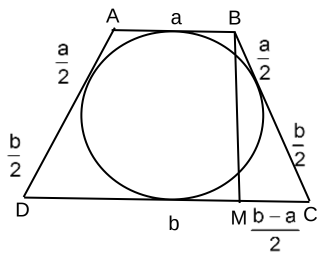
$\frac{OD}{OB} = \sqrt{\frac{a}{b}}$

Area of OAD = Area of OBC= $a\sqrt{\frac{b}{a}} = \sqrt{ab}$

Area of trapezium = $a + b + 2\sqrt{ab} = \left(\sqrt{a} + \sqrt{b}\right)^2$

Note: If the dimensions of the sides of the trapezium are in cm, the area will be in sq cm. But in choice A the units are in cm.
$\therefore$ Choice B is ruled out.
If the trapezium is a square of side 2, its area will be 4.
Then, a = b = $\frac{1}{4}(4) = 1$.
Only choice C would equal 4.
choice C follows . Choice (C)

55.



Diameter of circle = BM = $\sqrt{\left(\frac{a+b}{2}\right)^2 - \left(\frac{b-a}{2}\right)^2}$

= $\sqrt{ab}$

And it is given

$\frac{a+b}{2}$ = 13 cm $\Rightarrow$ a + b =26 cm------------ (1) and

$$\frac{\frac{h}{2}\left(a+\frac{a+b}{2}\right)}{\frac{h}{2}\left(b+\frac{a+b}{2}\right)} = \frac{7}{19} \Rightarrow \frac{3a+b}{a+3b} = \frac{7}{19}$$

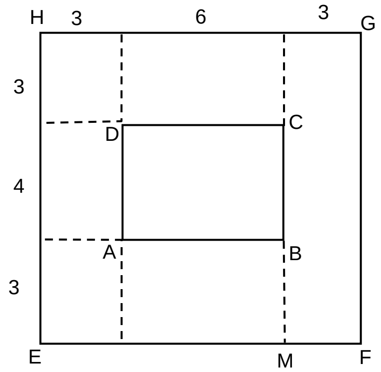
$\Rightarrow$ 25a = b ------------ (2)

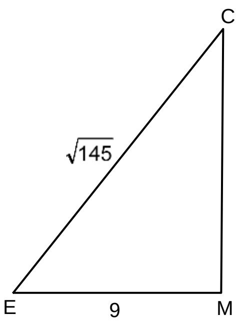
Solving (1) and (2) we get

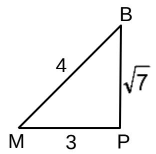
a = 1 cm and b = 25 cm

diameter of the circle = $\sqrt{1(25)}$ cm = 5 cm. Ans: 5

56.





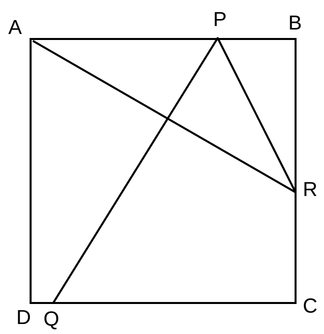


ABCD is the top face and EFGH is the base. The ant has to go from E to C. We can fold the front face EFBA along BA so that the face is in the same plane as ABCD. The horizontal distance from B to M is 3. The height of B above the base (BP) is $\sqrt{7}$ (given). Therefore, the slant height BM is 4.

$EC^2 = EM^2 + MC^2 = (3 + 6)^2 + (4 + 4)^2 = 81 + 64 = 145$

$\therefore$ EC = $\sqrt{145}$. Choice (C)

57.



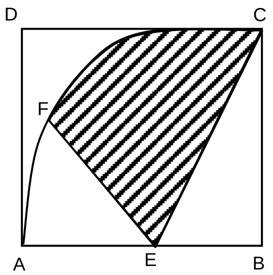
Let AP = 13 $\therefore$ PB = 5. (Given AP : PB = 13 : 5) Also PR = 13

($\because$ PQ is the perpendicular bisector of AR)

$\therefore$ BR = 12 and RC = 6. ($\because$ BR + RC = AB = 18)

tan $\angle$RDC = RC/CD = 6/18 = 1/3. Choice (C)

58.



Let AB = a and BE = h

Area of the shaded region = sector FBC + ΔFBE – ΔCBE

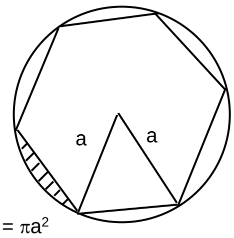
= $\frac{1}{6}\pi a^2 + \frac{1}{2}$(a) (h) sin $\angle$FBE – $\frac{1}{2}$ ah

So $\frac{\pi a^2}{6} - \frac{1}{4}$ ah = $\frac{\pi}{8}a^2$ ($\because$ $\angle$FBE = 30°) $\Rightarrow$ h = $\frac{\pi a}{6}$

But a = 18 cm $\Rightarrow$ h = 3π cm

So AE = AB – BE = (18 – 3π) cm. Choice (B)

59.



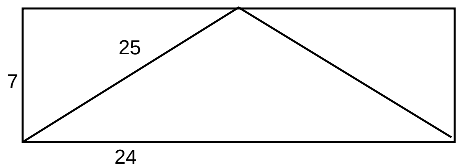
Area of circle = πa^2

Area of hexagon= $\frac{\sqrt{3}}{4}a^2 \times (6)$

Area of the shaded region

$$= \frac{\pi a^2 - \frac{3\sqrt{3}}{2}a^2}{6} = \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)a^2.$$ Choice (A)

60.



Let the total surface area of the cylinder be S.

S = $2\pi rh + 2\pi r^2$

The total area of the part that is left, say T, includes the lateral surface area of the cylinder, the area of the top and the curved surface area which is exposed after the cone is cut. i.e T = $2\pi rh + \pi r^2 + \pi rl$

T – S = $\pi rl - \pi r^2 = \pi r(l - r) = 24\pi$. Ans: 24

Exercise – 7

Solutions for questions 1 to 43:

1. AB = $\sqrt{12}$, BC = $\sqrt{12}$, CA = $2\sqrt{3} = \sqrt{12}$

Δ ABC is an equilateral triangle.

$\therefore$ its circumcentre coincides with the centroid.

$\therefore$ Circumcentre $\equiv \left(\frac{\sqrt{3}+0+-\sqrt{3}}{3}, \frac{0+3+0}{3}\right) = (0, 1)$

Choice (A)

2. Intersection of diagonals of a parallelogram is at its midpoint.
Let D (x, y) be its 4th vertex.

The midpoint of diagonal BD = $\left(\frac{2+x}{2}, \frac{4+y}{2}\right)$

The midpoint of diagonal AC = $\left(\frac{1+5}{2}, \frac{5+1}{2}\right) \equiv (3, 3)$

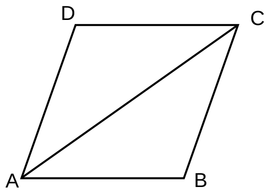
$\therefore \frac{2+x}{2} = 3 \therefore$ x = 4, and $\frac{4+y}{2} = 3$

$\therefore$ y = 2

D $\equiv$ (4, 2) Choice (D)

3. Translation equations are X = x – 7 & Y = y + 3
$\therefore$ New co-ordinates of (1, 1) are (1 – 7, 1 + 3) = (– 6, 4)
Choice (C)

4. The diagonal AC divides the rhombus into two equal parts.



$\therefore$ Area of ABCD = 2 Area of Δ ABC

$= 2\times\frac{1}{2}\begin{vmatrix}5-2 & 2-(-2)\\ 7-3 & 3-0\end{vmatrix} = \begin{vmatrix}3 & 4\\ 4 & 3\end{vmatrix}$ = 7 sq.units

Ans: 7

5. Side of the equilateral triangle is $8\sqrt{2}$.
The given points lie on the line y = x.
$\therefore$ The third vertex lies on the line y = -x.
$\therefore$ Its coordinates have the form (a, –a)

$\left(\sqrt{2}a\right)^2 = \left(4\sqrt{6}\right)^2$

a = $\pm 4\sqrt{3}$

$\therefore$ The third vertex can be $\left(-4\sqrt{3}, 4\sqrt{3}\right)$

$\therefore$ Third vertex can be $\left(-4\sqrt{3}, 4\sqrt{3}\right)$ Choice (D)

6. Let ABC be an isosceles right angled triangle right angled at A. Then BC is hypotenuse.
Let equal sides have lengths of a each. Let D be midpoint of BC.

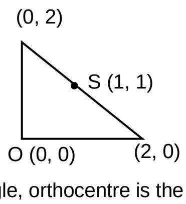
Then from geometry, BC = $\sqrt{2}a$, $AD = \frac{a}{\sqrt{2}}$

Now, $\frac{a}{\sqrt{2}} = \sqrt{(0-3)^2 + (0-3)^2} = 3\sqrt{2}$

$\therefore$ a = 6 units.

$\therefore$ Area = $\frac{1}{2}\times a^2 = 18$ square units. Ans: 18

7.



For a right triangle, orthocentre is the vertex containing right angle and circumcenter is the midpoint of hypotenuse.
$\therefore$ Orthocentre = (0, 0)
Circumcentre = (1, 1)
$\therefore$ distance = $\sqrt{1^2+1^2} = \sqrt{2}$ Choice (C)

8. x + 2ay + a = 0 $\rightarrow$ (1)
x + 3by + b = 0 $\rightarrow$ (2)
x + 4cy + c = 0 $\rightarrow$ (3)
Since the three lines are concurrent, the point of intersection of (1) and (2) must be lie on (3).
The point of intersection of (1) and (2) is

$\left(\frac{-ab}{3b-2a}, \frac{a-b}{3b-2a}\right)$

Substitute this point in (3)

$\frac{-ab}{3b-2a} + \frac{4c(a-b)}{3b-2a} + c = 0$

–ab + 4ac – 4bc + 3bc – 2ac = 0

–ab + 2ac – bc = 0 $\Rightarrow$ 2ac = ab + bc $\Rightarrow \frac{2ac}{a+c} = b$

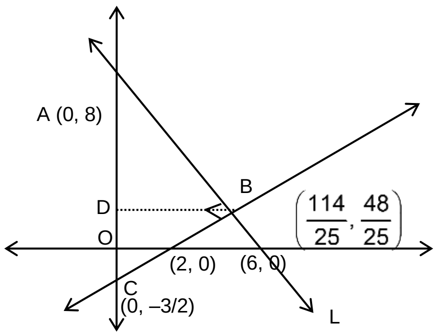
Hence a, b, c are in H.P. Choice (C)

9. Point of intersection of 5x – y = 9 and x + 6y = 8 is (2, 1).
Equation of the line through (2, 1) and (3, 4) is 3x – y – 5 = 0
Choice (C)

10. Equation of the line through (3, 4) and (6, 0) is

$\equiv \frac{y-4}{x-3} = \frac{-4}{3}$

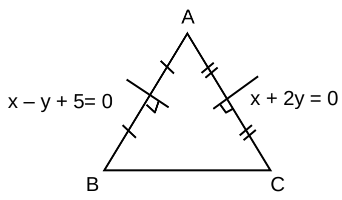
$\Rightarrow$ 4x + 3y = 24.
Equation of the line perpendicular to 4x + 3y = 24 and passing through (2, 0) is 3x – 4y = 6
Plotting the lines on a graph, we see that the point of intersection of the lines is $\left(\frac{114}{25}, \frac{48}{25}\right)$



$\therefore$ Area of Δ ABC = $\frac{1}{2} AC \times BD$

$= \frac{1}{2}\times 9.5\times\frac{114}{25} = 21.66$ = 21 sq. units Ans: 21

11.



B is the image of A (1, –2) in x – y + 5 = 0 and
C is the image of A (1, –2) in x + 2y = 0

$\frac{h-1}{1} = \frac{k+2}{-1} = \frac{-2(1+2+5)}{2}$

$\Rightarrow$ h = – 8 + 1 = – 7
k = 8 – 2 = 6
(h, k) = (–7, 6) = B

$\frac{h-1}{1} = \frac{k+2}{2} = \frac{-2(1-4)}{1+4} = \frac{6}{5}$

$\Rightarrow$ h = $\frac{6}{5}+1 = \frac{11}{5}$, k = $\frac{12}{5}-2 = \frac{2}{5}$

(h, k) = $\left(\frac{11}{5}, \frac{2}{5}\right)$ = C

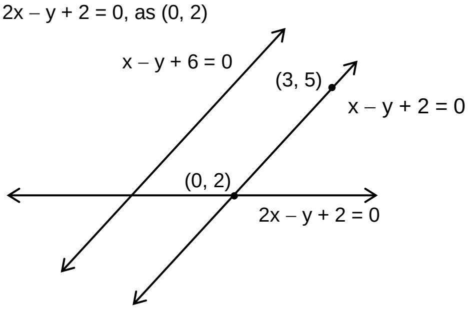
Equation of BC is $y - 6 = \dfrac{\frac{2}{5} - 6}{\frac{11}{5} + 7}$ (x + 7)

i.e., $y - 6 = \dfrac{-28}{46}$ (x + 7)

23 (y – 6) = – 14 (x + 7)
14x + 23y – 138 + 98 = 0
14x + 23y – 40 = 0

Choice (A)

12. The line passing through (3, 5) and parallel to l : x – y + 6 = 0 is given by m : x – y + 2 = 0.
Now, we find the point of intersection of x – y + 2 = 0 and 2x – y + 2 = 0, as (0, 2)



The distance between (0, 2) and (3, 5) gives the required distance.

$\therefore$ The required distance = $\sqrt{3^2 + 3^2} = 3\sqrt{2}$ units.

Choice (C)

13. Clearly the slope of the line through A (1, 2) is tan 45 = 1
$\therefore$ the equation of the line is y – 2 = x – 1
i.e., x – y + 1 = 0 $\to$ (i)
Given line 3x + 4y = 12 $\to$ (ii)
Solving (i) and (ii)

$$\frac{x}{12-4} = \frac{y}{3+12} = \frac{1}{4+3}$$

$\therefore x = \dfrac{8}{7}$, $y = \dfrac{15}{7}$

$\therefore P = \left(\dfrac{8}{7}, \dfrac{15}{7}\right)$

$$AP = \sqrt{\left(\frac{8}{7}-1\right)^2 + \left(\frac{15}{7}-2\right)^2} = \sqrt{\frac{1}{49}+\frac{1}{49}} = \frac{\sqrt{2}}{7}$$

Choice (D)

14. Let the slope of a line through (2, 3) and making an angle of 60° with x + y = 2 be m

Then $\tan 60 = \left|\dfrac{m+1}{1-m}\right| = \dfrac{1+m}{1-m} = \pm\sqrt{3}$

Case (i): $\dfrac{1+m}{1-m} = -\sqrt{3}$

$1 + m = \sqrt{3}\,(1 - m)$

$\Rightarrow = 1 + m = \sqrt{3} - \sqrt{3}\,m$

$\Rightarrow = (1 + \sqrt{3})m = (\sqrt{3} - 1)$

$m = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

Or

$m = \dfrac{3+1-2\sqrt{3}}{2} = 2 - \sqrt{3}$

Case (ii) if $\dfrac{1+m}{1-m} = -\sqrt{3}$,

$(1 + m) = -\sqrt{3}\,(1 - m)$

$1 + m = -\sqrt{3} + \sqrt{3}\,m$

$m(1 - \sqrt{3}) = -(1 + \sqrt{3})$

$m = \dfrac{1+\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{3+1+2\sqrt{3}}{2} = 2 + \sqrt{3}$

The equation of the lines passing through the point (2, 3) and having slopes $2-\sqrt{3}$, $2+\sqrt{3}$ are

(i): $y - 3 = (2 - \sqrt{3})(x - 2)$

$(2 - \sqrt{3})x - y + 2\sqrt{3} - 1 = 0$

(ii): $y - 3 = (2 + \sqrt{3})(x - 2)$

$(2 + \sqrt{3})x - y - 1 - 2\sqrt{3} = 0$

Choice (C)

15. Any point on x + y = 4 is in the form (t, 4 – t)

$$\frac{|4t + 3(4-t) - 10|}{\sqrt{16+9}}$$

$= 1 = |t + 2| = 5$

$\Rightarrow t = \pm 5 - 2 \Rightarrow t = 3, -7$

$\therefore$ Required. points are (3, 1), (–7, 11)

Choice (C)

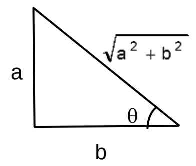
16. **(a)** A, B, C and D are angles of a cyclic quadrilateral
A + C = 180° and B + D = 180°
$\Rightarrow$ C = 180° - A and D = 180° – B
cos A + cos B + cos (180 – A) + cos (180 – B)
= cos A + cos B – cos A – cos B = 0

Ans: 0

(b) cos 28° + cos 65° + cos 115° + cos 240° + cos 208° + cos 300°
$\Rightarrow$ cos 28° + cos 65° – cos 65° – cos 60° – cos 28° + cos 60°
= 0.

Ans: 0

17.



Given: $\cot\theta = \dfrac{b}{a}$ and $\theta \notin Q_1$

$\Rightarrow \theta \in Q_3$

$\therefore \sin\theta = \dfrac{-a}{\sqrt{a^2+b^2}}$, $\cos\theta = \dfrac{-b}{\sqrt{a^2+b^2}}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \dfrac{\dfrac{-a^2+b^2}{\sqrt{a^2+b^2}}}{\dfrac{-(a^2+b^2)}{\left(\sqrt{a^2+b^2}\right)}}$$

$$= \frac{a^2-b^2}{a^2+b^2}$$

Alternate method:

$$\cot\theta = \frac{b}{a}, \Rightarrow \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a - \frac{b}{a}}{a + \frac{b}{a}} = \frac{a^2-b^2}{a^2+b^2}$$

Choice (D)

18. $\left(\frac{\cot A}{\operatorname{cosec}A+1}+\frac{\cot A}{\operatorname{cosec}A-1}\right)\frac{1}{\sec A}$

$= \left(\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}+1}+\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}-1}\right)\frac{1}{\sec A}$

$= \left(\frac{\cos A}{1+\sin A}+\frac{\cos A}{1-\sin A}\right)\frac{1}{\sec A}$

$= \cos A\left(\frac{2}{\cos^2 A}\right)\cos A = 2$ Ans: 2

19. $x = a\cot\theta + b\operatorname{cosec}\theta \rightarrow (1)$
$y = a\cot\theta - b\operatorname{cosec}\theta \rightarrow (2)$
Solving (1) and (2) for $\cot\theta$ and $\operatorname{cosec}\theta$

$\cot\theta = \frac{(x+y)}{2a}$ and $\operatorname{cosec}\theta = \frac{(x-y)}{2b}$

Since $\operatorname{cosec}^2\theta - \cot^2\theta = 1$,

$\frac{(x-y)^2}{4b^2} - \frac{(x+y)^2}{4a^2} = 1$ Choice (D)

20. $\sin\theta + \cos\theta = \sqrt{2}$
It is possible only when $\theta = 45°$
$\tan^n\theta + \cot^n\theta = \tan^n 45 + \cot^n 45 = 1 + 1 = 2$
Ans: 2

21. Given A + B = 45° ⇒ B = 45° – A
(1 – cot A) (1 + cot B) = (1 – cot A) (1 + cot (45 – A)

$= (1 - \cot A)\left[1+\frac{1+\cot A}{\cot A - 1}\right]$

$= \frac{1-\cot A}{-(1-\cot A)}$ [cot A – 1 +1 + cot A] = – 2 cot A

Choice (D)

22. Given: sin 12° sin 48° sin 54°

$\frac{\sin 12°\sin(60°-12°)\sin(60°+12°)\sin 54°}{\sin 72°}$

$= \left(\frac{1}{4}\right)\frac{\sin 3(12°)\sin 54°}{\sin 72°}$

$= \left(\frac{1}{8}\right)\frac{2\sin 36°\cos 36°}{\sin 72°}$

$= \frac{\sin 72°}{8.\sin 72} = \frac{1}{8}$ Choice (B)

23. tan 9° + tan (90° – 9) – (tan 27° + tan (90° – 27°))
⇒ tan 9° + cot 9° – (tan 27°+ cot 27°)

$\Rightarrow \frac{\sin 9°}{\cos 9°}+\frac{\cos 9°}{\sin 9°}-\left(\frac{\sin 27°}{\cos 27°}+\frac{\cos 27°}{\sin 27°}\right)$

$\Rightarrow \frac{2(\sin^2 9°+\cos^2 9°)}{2\sin 9°\cos 9°}-\frac{2(\sin^2 27°+\cos^2 27°)}{2\sin 27°\cos 27°}$

$\Rightarrow \frac{2}{\sin 18°}-\frac{2}{\sin 54°} = 2\left[\frac{1}{\sin 18}-\frac{1}{\cos 36}\right]$

$\Rightarrow 2\left[\frac{4}{\sqrt{5}-1}-\frac{4}{\sqrt{5}+1}\right] = 8\left[\frac{\sqrt{5}+1-\sqrt{5}+1}{4}\right] = 4$

Ans: 4

24. $\cos^2\theta + \sin^4\theta = \sin^4\theta + 1 - \sin^2\theta$

$= \left\{(\sin^2\theta)^2 - 2.\sin^2\theta\times\frac{1}{2}+\left(\frac{1}{2}\right)^2\right\} + 1 - \left(\frac{1}{2}\right)^2$

$= \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

When $\sin^2\theta = 0$, the given expression is maximum

Maximum value $= \left(\frac{-1}{2}\right)^2 + \frac{3}{4} = \frac{1}{4}+\frac{3}{4} = 1$

Minimum value $= 0 + \frac{3}{4} = \frac{3}{4}$

∴The range $= \left[\frac{3}{4}, 1\right]$ Choice (A)

25. $1 + 8\sin^2x^2\cos^2x^2 = 1 + 2(2\sin x^2\cos x^2)^2$
$= 1 + 2\sin^2 2x^2$
$= 1 + (1 - \cos 4x^2) = 2 - \cos 4x^2$
Minimum value is $c - \sqrt{a^2+b^2}$ here, c = 2, a = 1, b = 0
∴ The required minimum value is 2 – 1 = 1.
Ans: 1

26. $\sin^2(\theta - 45°) + \sin^2(\theta + 15°) - \sin^2(\theta - 15°)$
$= \sin^2(\theta - 45°) + (\sin 2\theta.\ \sin 30°)$
$(\sin^2 A - \sin^2 B) = \sin(A + B).\sin(A - B)$

$= \frac{1-\cos(90°-2\theta)}{2}+\frac{1}{2}\sin 2\theta$

∵ $\cos\theta$ is an even function.

$= \frac{1}{2}-\frac{1}{2}\sin 2\theta+\frac{1}{2}\sin 2\theta = \frac{1}{2}$ Choice (B)

27. We know that AM ≥ GM

$\Rightarrow \frac{4\tan^2 x + 9\cot^2 x}{2} \geq \sqrt{4\tan^2 x.9\cot^2 x}$

∴ $4\tan^2 x + 9\cot^2 x \geq 12$
∴ The minimum value of the function is 12. Ans: 12

28. Given: A, B, C are in A.P and A + C = 2B
We know that A + B + C = 180°
2B + B = 180°
B = 60°

Given: $\frac{b}{c} = \frac{\sqrt{6}}{\sqrt{3}-1} = \frac{\sin B}{\sin C} = \frac{\sqrt{6}}{\sqrt{3}-1}$

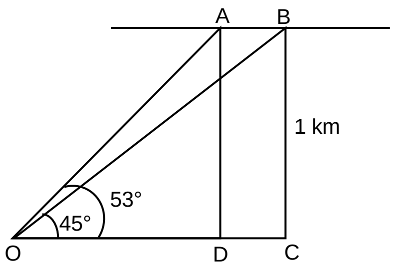
$\sin C = \frac{(\sqrt{3}-1)\sin B}{\sqrt{6}} = \frac{\sqrt{3}-1}{\sqrt{6}}\sin 60°$

$= \frac{\sqrt{3}-1}{\sqrt{6}}.\frac{\sqrt{3}}{2}$

$\sin C = \frac{\sqrt{3}-1}{2\sqrt{2}}$

∴ C = 15°
∴ A = 105°. Choice (D)

29.



Let A be the position of the aeroplane at the given instant of time.
Let v km/sec be the speed of the aeroplane.

Then DC = AB = 12v km
$\angle BOC = 45°$,
OC = BC = 1km
$\Rightarrow$ OD + DC = 1
OD = 1 − 12v

Now, $\tan 53° = \dfrac{AD}{OD} = \dfrac{1}{1-12v}$

$\Rightarrow \tan(90° - 37°) = \dfrac{1}{1-12v}$

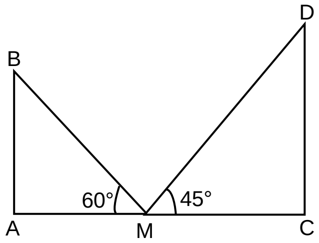
$\Rightarrow \dfrac{1}{\tan 37°} = \dfrac{1}{1-12v}$

Given: $\sin 37° = 0{\cdot}6 = \dfrac{3}{5}$ $\therefore$ $\tan 37° = \dfrac{3}{4}$

$\Rightarrow \dfrac{4}{3} = \dfrac{1}{1-12v} \Rightarrow 4 - 48v = 3$

$\therefore\ v = \dfrac{1}{48}\text{km/sec}$. Choice (B)

30.



Let M be the midpoint of AC and AB = $112\sqrt{3}$ m

In ΔMAB, $\tan 60° = \dfrac{AB}{AM}$ $\therefore$ $AM = \dfrac{112\sqrt{3}}{\sqrt{3}} = 112$ m

Since $\angle DMC = 45°$, MC = CD = 112 m. ($\because$ MC = AM)
Ans: 112

31. The number of mappings from A $\to$ B when n(A) = p and n(B) = q is $\{n(B)\}^{n(A)} = q^p$
Here p = 3 and q = 5 $\therefore q^p = 5^3$ Choice (A)

32. We know that, the number of one-one functions possible from set A to set B when n(A) = m and n(B) = n where n > m is ^nP_m. Here, n = 5 and m = 4
$\therefore$ The number of one-one functions possible is
$^5P_4 = 5! = 120$ Choice (C)

33. We know that, the number of onto functions from A $\to$ B when n(B) < n(A), where n(B) = q and n(A) = p are
$q^p - \{^qC_1(q-1)^p - {}^qC_2(q-2)^p + {}^qC_3(q-3)^p \ldots\}$
Here, p = 4, and q = 3,
$\therefore$ The required number
$= 3^4 - (^3C_1(3-1)^4 - {}^3C_2(3-2)^4) = 81$
$- (3 \times 2^4 - 3(1)^4) = 81 - 45 = 36$
Ans: 36

34. The number of bijections from set A to set A is (n(A))!
Given: n(A) = 4
$\therefore$ The number of bijections = 4! = 24
Ans: 24

35. If $f(p).f\left(\dfrac{1}{p}\right) = f(p) + \left(\dfrac{1}{p}\right)$, $f(p) = 1 \pm p^n$

Here, given $f(5) = -124 = 1 - 5^3 \Rightarrow f(x) = 1 - x^n$
$\therefore f(9) = -9^3 + 1 = -728$. Ans: −728

36. Given: f(x+ y) = f(x) + f(y)
$\sum\limits_{i=1}^{10} f(i) = f(1) + f(2) + f(3) + f(4) + \ldots + f(10)$

Given, f(1) = 2
f(2) = f(1 + 1) = f(1) + f(1) = 2 + 2 = 2.2= 4
f(3) = f(1 + 2) = f(1) + f(2) = 2 + 4 = 2.3 = 6
.
.
.
f(10) = f(1 + 9) = f(1) + f(9) = 2 + 18 = 2.10

$\therefore \sum\limits_{i=1}^{10} f(i) = 2(1 + 2 + 3 + \ldots + 10)$

$= \dfrac{2(10)(10+1)}{2} = 110$

Ans: 110

37. Let $x = \dfrac{1}{100}$. $f\left(\dfrac{1}{100}\right) + f\left(2 - \dfrac{1}{100}\right) = 4$

$\Rightarrow f\left(\dfrac{1}{100}\right) + f\left(\dfrac{199}{100}\right) = 4$

Let $x = \dfrac{2}{100}$. $f\left(\dfrac{2}{100}\right) + f\left(2 - \dfrac{2}{100}\right) = 4$

$\Rightarrow \left(\dfrac{2}{100}\right) + f\left(\dfrac{198}{100}\right) = 4$

.
.
.

Let $x = \dfrac{99}{100}$. $f\left(\dfrac{99}{100}\right) + f\left(2 - \dfrac{99}{100}\right) = 4$

$\Rightarrow f\left(\dfrac{99}{100}\right) + f\left(\dfrac{101}{100}\right) = 4$

Let $x = \dfrac{100}{100} \Rightarrow f\left(\dfrac{100}{100}\right) + f\left(2 - \dfrac{100}{100}\right) = 4$

$\Rightarrow f\left(\dfrac{100}{100}\right) + f\left(\dfrac{100}{100}\right) = 4 \Rightarrow f\left(\dfrac{100}{100}\right) = 2$

$\therefore$ The value of $f\left(\dfrac{1}{100}\right) + f\left(\dfrac{2}{100}\right) + \ldots\ldots\ldots + f\left(\dfrac{199}{100}\right)$

$= 4 \times 99 + 2 = 396 + 2 = 398$.
Ans: 398

38. f(− 2.5) + f(− 1.5) + f(− 3.5) = (− 2.5 − 1) − 1.5 + (− 3.5 − 1) = − 9.5. Ans: −9.5

39. Given: $f(x) = 2^x + 2^{-x}$ and $g(x) = 2^x - 2^{-x}$
$f(x).g(y) + f(y)\,g(x) = (2^x + 2^{-x})(2^y - 2^{-y}) + (2^y + 2^{-y})(2^x - 2^{-x})$
$\Rightarrow 2^{x+y} - 2^{x-y} + 2^{-x+y} - 2^{-x-y} + 2^{x+y} - 2^{-x+y} - 2^{-x-y} + 2^{-y+x}$
$\Rightarrow 2(2^{x+y} - 2^{-(x+y)}) = 2g(x + y)$ Choice (C)

40. The domain of $\dfrac{1}{3x-4}$ is $x \neq \dfrac{4}{3}$ and

the domain of $\sqrt{2x-1}$ is $x \geq \dfrac{1}{2}$

$\therefore$ The domain is $x \geq \dfrac{1}{2}$, $x \neq \dfrac{4}{3}$ Choice (B)

41. $\dfrac{5-|x|}{7-|x|} \geq 0$,

$\therefore$ 5 – |x| and 7 – |x| $\geq$ 0 or 5 – |x| $\leq$ 0 and 7 – |x| $\leq$ 0
5 – |x| and 7 – |x| are positive in the interval (–5, 5) and both are negative in the intervals (– ∞, –7) and in (7, ∞).
Also 5 – |x| can be 0 but not 7 – |x|
$\therefore$ The domain is [– 5, 5]$\cup$(– ∞, – 7) $\cup$ (7, ∞)
Choice (B)

42. Let $\frac{x}{\sqrt{x^2+1}} = y \Rightarrow y^2(x^2+1) = x^2$ or $x^2(y^2-1)+y^2 = 0$
For the roots to be real, $b^2 - 4ac \geq 0$ must hold true
i.e. $-4(y^2-1)y^2 \geq 0$. But y cannot be 1 or – 1.
$y^2(y^2-1) \leq 0$
$\therefore y^2 - 1 \leq 0$ …. $\therefore -1 \leq y \leq 1$
$\therefore$ The range is (– 1, 1) Choice (A)

43. f(x) = max (4x + 3, 5 – 6x)
Let 4x + 3 > 5 – 6x
10x > 2.
x > 1/5.
$\therefore$ When $x > \frac{1}{5}$, f(x) = 4x + 3
$\therefore$ When $x > \frac{1}{5}$, $f(x) > \frac{19}{5}$.
Let 4x + 3 < 5 – 6x
10x < 2.
x < 1/5.
For $x < \frac{1}{5}$, $f(x) > \frac{19}{5}$
$\therefore$ The minimum value of f(x) is $\frac{19}{5}$ and this occurs when 4x + 3 = 5 – 6x Choice (A)

Solutions for questions 44 to 46:

44. Given f (1) = 4 and f (x + y) = f (x) f (y) for all real values of x and y.
$\Rightarrow f(2) = f(1+1) = f(1)\,f(1) = 4^2$
$\therefore f(4) = f(2+2) = f(2)\,f(2) = (4^2)(4^2) = 4^4$
$\therefore f^6(4) = f(4)\,f(4)$ ……… 6 times
$= 4^4 \times 4^4 \times$ ….. 6 times $= 4^{24} = 2^{48}$ Choice (C)

45. Given g (2) = 3 and f (3) = 2.
f (g (8)) = f [g (2 × 2 × 2)] = f [g (2) + g (2) + g (2)]
= f (3 + 3 + 3)
= f (3) f (3) f (3) = 2 × 2 × 2 = 8 Ans: 8

46. Given $f^n(x) = f(x)\,f(x)\,f(x)$ …… n times
$\Rightarrow f^n(x) = f(x + x + x +$ …. n times$) = f(nx)$.
Consider, g (1) = g (1 × 1) = g (1) + g (1)
$\Rightarrow$ g (1) = 2 g (1)
$\Rightarrow$ g (1) = 0
Consider, f (0) = f (0 + 0) = f (0) · f (0)
$\Rightarrow$ f (0) = f (0) · f (0)
$\therefore$ f (0) = 0 or 1.
If f (0) = 0,
f (5) = f (5 + 0) = f (5) · f (0) = f (5) × 0 = 0
$\Rightarrow$ f (5) = 0, but given that for $x \neq 0$, $f(x) \neq 0$.
$\therefore$ f (0) = 1.
g(4) = g(2×2) = g(2) + g(2) = 3 + 3 = 6
f(g(4)) = f(6) = f(3+3) = f(3).f(3) = 4·4 = 16 true
Choice (D)

Solutions for question 47:

47. Radius of the circle = $\sqrt{3^2+4^2} = 5$ units
Area of the shaded region
$= \frac{1}{4}$ (Area of circle) – (Area of Δ OPQ + Area of Δ OPR)
$= \frac{1}{4}(25\pi) - \left(\frac{1}{2}\times5\times4+\frac{1}{2}\times5\times3\right)$
$= \frac{25\pi}{4} - \frac{35}{2} = \frac{25\pi-70}{4}$
$= \frac{25\times3.14-70}{4} = \frac{78.5-70}{4} = \frac{8.5}{4} = \frac{17}{8}$ sq. units.
Choice (C)

Solutions for questions 48 to 52:

To stretch the graph of f(x) vertically, define g(x) = k f(x) where k > 1.

To compress the graph of f(x) vertically, define $g(x) = \frac{1}{k}f(x)$ where k > 1.

To stretch the graph of f(x) horizontally, define $g(x) = f\left(\frac{x}{k}\right)$ where k > 1.

To compress the graph of f(x) horizontally, define g(x) = f(kx), where k > 1.

48. The graph g(x) can be obtained by stretching the graph f(x) vertically without changing it horizontally.
f(0) = 1 whereas g(0) = 2
$\therefore$ g(x) = 2f(x) Choice (C)

49. The graph g(x) can be obtained by reflecting the negative points of f(x) in the x – axis.
$\therefore$ g (x) = |f (x)| Choice (B)

50. The graph g(x) can be obtained by stretching the graph of f(x), horizontally and compressing it vertically. Therefore, $\frac{1}{2}f\left(\frac{x}{2}\right) = g(x)$ or $\left(\text{setting}\frac{x}{2} = y\right)$; f(y) = 2g(2y)
$\therefore$ f(x) = 2g(2x). Choice (D)

51. The graph g (x) is a reflection of f(x) in the y axis.
$\therefore$ f (x) = g (– x). Choice (A)

52. From the graph of f (x) it can be observed that
f(x) = – f (– x) $\Rightarrow$ f (x) + f (– x) = 0.
Since g (x) = 0, f (x) + f (– x) = 0 Choice (B)

Solutions for question 53:

53. Since x = 1.5, $i(x) = (1.5)^2 - 4 < 0$,
hence $h(i(x)) = 1 - 2\,(i(x))^5$, as i(x) < 0, h(x) > 0.
$\Rightarrow g(h(x)) = \frac{|h(x)|}{h(x)} = 1$, as h(x) > 0.
$\therefore f(g(x)) = f(1) = 1^2 + 3 + 6 = 10$
Choice (B)

Solutions for questions 54 to 60:

54. Let us consider a point in the interior of the square, say (1.5, 1.5). For this point, $\lfloor x\rfloor + \lfloor y\rfloor = 1 + 1 = 2$, $\lceil x\rceil + \lceil y\rceil = 2 + 2 = 4$, $\lfloor x\rfloor + \lceil y\rceil = 1 + 2 = 3$ and $\lceil x\rceil + \lfloor y\rfloor = 2 + 1 = 3$ $\therefore$ All the four options can be considered. The differences occur only on the boundaries. For the given graph, the left and lower sides of each square are included. If we have the function $\lfloor x\rfloor$, the left boundary would be included, while the right one would be left out. (For $\lceil x\rceil$, it is vice versa.) If we have the function $\lfloor y\rfloor$, the lower side of each square would be included while the upper side would be left out. For $\lceil y\rceil$, it is vice versa.) Therefore, only choice (A) represents the given graph.
Note: The points (0, 3); (1, 2); (2, 1) and (3, 0) are excluded.
Choice (A)

55. The graph shows an ascending series of rectangles.
$\therefore$We should expect minus signs, if all the terms are on one side. (mx – y = 0 when m > 0 represents an ascending line while mx + y = 0 (m > 0) represents a descending line). The length is along x-axis and it is 2 units. We should expect the term $\frac{x}{2}$ (rather than say 2x). Also, the left boundaries are included. (We should expect $\lfloor x/2\rfloor$ rather than $\lceil x/2\rceil$).
The breadth is along the y-axis and it is 1 unit. The upper boundaries are included. We should expect $\lceil y\rceil$ rather than $\lfloor y\rfloor$.

$\therefore$ We select choice (D).
Note: The points (–4, –2); (–2, –1); (0, 0); (2, 1) and (4, 2) are included. Choice (D)

56. In the given domain, $-2 \le x \le -1$ and $1 \le x \le 2$, four unit squares are included and for each of them the right and bottom edges are included.
Let us first consider the square in the first quadrant (say S_1) we may as well drop the mod function. As the right edge is included, we need $\lceil x \rceil$ (rather than $\lfloor x \rfloor$). As the bottom edge is included, we need $\lfloor y \rfloor$ (rather than $\lceil y \rceil$). To describe just this square (in terms of the floor/ceil functions), we can write down $\lceil x \rceil + \lfloor y \rfloor = 3$ ($\therefore$ We can consider only choices (B) and (C).
We need to consider the points (both internal and on the boundary, if necessary) in the other three squares (which we can denote as S_2, S_3, S_4 lying in Q_2, Q_3, Q_4, respectively) and see which of these two choices are satisfied by all of them.

Point	$\lceil x \rceil$	$\lfloor y \rfloor$	$\lvert\lceil x \rceil\rvert + \lvert\lfloor y \rfloor\rvert$	$\lvert x \rvert$	$\lvert y \rvert$	$\lceil \lvert x \rvert \rceil + \lfloor \lvert y \rvert \rfloor$
(–1.5, 2.5)	– 1	2	1 + 2 = 3	1.5	2.5	2 + 2 = 4
(–1.5, –1.5)	– 1	– 2	1 + 2 = 3	1.5	1.5	2 + 1 = 3
(1.5, –0.5)	2	– 1	2 + 1 = 3	1.5	0.5	2 + 0 = 2

We see that it is not necessary now to consider the boundary points. Only choice (B) covers all these internal points.
We can confirm that for each of the squares S_2, S_3, S_4, the boundaries are covered by choice (B).
Note: All the corners, where the dotted line meets the dark line are excluded. Choice (B)

57. There are only two squares, one in Q_1 (say S_1), the other in Q_2 (say S_2). For S_1, the right and upper edges are included. The mod of the x coordinates have the same range, viz., $2 < \lvert x \rvert \le 4$ for both S_1 and S_2 and y coordinates have the same range, viz, $1 < y \le 3$ for both S_1 and S_2. We should expect mod signs with the x-coordinates and no mod signs with the y-coordinates. (If the mod sign appears with the y-coordinates, there should be two more squares in Q_3 and Q_4, respectively) (We should reject B and even D). As the right edge is included, we should expect $\lceil x \rceil$ (rather than $\lfloor x \rfloor$) and therefore we should reject A also. We can verify that choice (C) describes the given squares.
Note: All the corners, where the dotted line meets the dark line are excluded. Choice (C)

58. There are 2 series of squares: $\lceil x \rceil + \lceil y \rceil = 3$ and $\lceil x \rceil = \lceil y \rceil$, i.e., $(a + b - 3)(a - b) = 0 \Rightarrow a^2 - b^2 = 3(a - b)$
$\Rightarrow a^2 - 3a = b^2 - 3b$. Where the two series cross each other, a bigger (2 $\times$ 2) square is formed.
Note: All the corners, where the dotted line meets the dark line are excluded. Choice (D)

59. There are 2 series of squares: $\lfloor x \rfloor + \lceil y \rceil = 0$ and
$\lfloor x \rfloor - \lceil y \rceil = -2$, i.e., $(p + q)(p - q + 2) = 0$
$p^2 - q^2 + 2(p + q) = 0$
$p^2 + 2p = q^2 - 2q$
For the square, which is common to the series, two of the vertices are included.
Note: All the corners, where the dotted line meets the dark line are excluded. Choice (B)

60. The following relation can be used to simplify the options (k is any integer, positive, negative or 0)
(1) $\lfloor x + k \rfloor = \lfloor x \rfloor + k$,
(2) $\lceil -x \rceil = -\lfloor x \rfloor$
(for both integral and non-integral values of x)
$\therefore$ The four choices are simplified below
(1) $\lfloor x + 1 \rfloor + \lfloor y + 2 \rfloor = 6 \Leftrightarrow \lfloor x \rfloor + \lfloor y \rfloor = 3$
(2) $\lfloor x \rfloor + \lfloor y \rfloor = 2$
(3) $\lfloor x \rfloor + \lfloor y \rfloor = 2$
(4) $\lfloor x + 2 \rfloor + \lfloor y + 1 \rfloor = 5 \Leftrightarrow \lfloor x \rfloor + \lfloor y \rfloor = 2$
Only (A) does not represent the graph.
Choice (A)

Exercise – 8

Solutions for questions 1 and 2:

1. f {g [(3, 4), (12, 16)], h [(3, 4), (12, 16)]}
$= \sqrt{\dfrac{3^2 + 4^2}{12^2 + 16^2}} = \sqrt{\dfrac{25}{400}} = \sqrt{\dfrac{1}{16}} = \dfrac{1}{4}$. Choice (C)

2. f {i [(3, 4), (12, 6)], j [(3, 4), (12, 16)]}.
$= \sqrt{(3^2 + 4^2) \times (12^2 + 16^2)}$
$= \sqrt{25 \times 400} = 100$ Choice (B)

Solutions for questions 3 and 4:

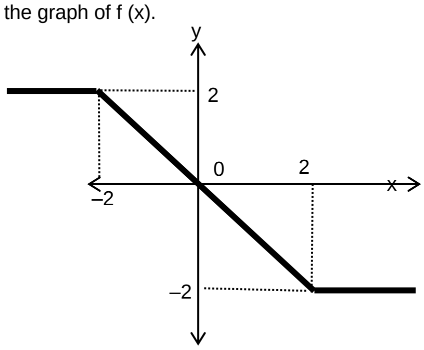
Given f (x) = $\dfrac{\lvert x - 2 \rvert - \lvert x + 2 \rvert}{2}$

$\Rightarrow$ f (x) = $\dfrac{(x-2) - (x+2)}{2} = -2;\ x \ge 2$;

f (x) = $\dfrac{-(x-2) - (x+2)}{2} = -x;\ -2 \le x \le 2$ and

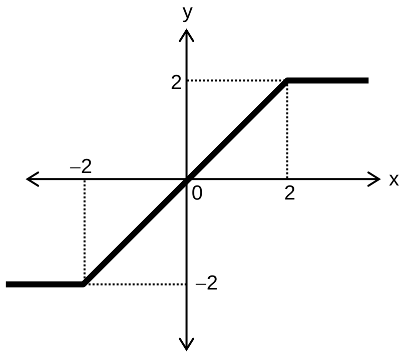
f (x) = $\dfrac{-(x-2) + (x+2)}{2} = 2;\ x \le -2$

We plot the graph of f (x).



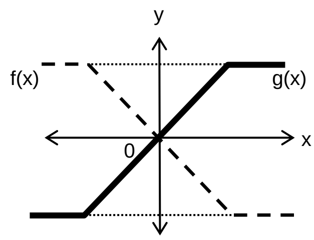
f(x); i (x)

The graph of g(x) can be obtained by reflecting f(x) in the x axis, the graph h (x) can be obtained by reflecting f(x) in the y-axis and the graph i(x) can be obtained by double reflecting f(x) in the x-axis followed by a reflection in the y-axis or vice versa.



g(x); h(x)

3.



The region of the graph, above the x-axis represents F (x).
$\therefore$ F (x) = |f (x)| Choice (B)

4. From the graphs it is clear that f (x) and g (x) coincide at only one point. Choice (A)

Solutions for questions 5 to 12:

5. Given $(x * y) * (y * z) = 1$
If $p * q = 1$, then LCM (p, q) = HCF (p, q) or equivalently, p = q.
$\therefore x * y = y * z \quad \to \quad (1)$
If p is a prime number that occurs m times in x and n times in y (m or n could be any non negative integers), then it occurs
(i) min (m, n) times in HCF (x, y)
(ii) max (m, n) times in LCM (x, y)
(iii) $|m - n|$ times in $x * y \quad \to \quad (2)$
$\therefore$ Let p be a prime number that occurs a times in x, b times in y and c times in z. From (1), (2)
$|a - b| = |b - c|$
$\therefore$ If a < b, then $|a - b| = b - a$ and $b - c = b - a$ or $-(b - c) = b - a$, i.e., c = a or c = 2b – a.
z need not be equal to x (As far as y is concerned, it is not related to x or z at all)
Also, x, y, z need not be in G.P. Choice (D)

6. $(x + y, x - y) * (x - y, x + y)$
$= [(x + y)^2 - (x - y)^2, (x^2 - y^2) - (x^2 - y^2)] = (4xy, 0)$ ……(1)
$(1, 3) * (2, 4) = (4 - 6, 3 - 8) = (-2, -5)$
$(5, 6) * (7, 8) = (40 - 42, 30 - 56) = (-2, -26)$
$\therefore (x, y) = (-2, -5) * (-2, -26) = (52 - 10, 10 - 52)$
$= (42, -42)$
$\therefore (x, y) = (42, -42)$ and $(x + y, x - y) * (x - y, x + y)$
$= (4xy, 0) = (-84^2, 0)$ Choice (B)

7. $a \odot b = \dfrac{1}{(a+1)(b+1)} - \dfrac{1}{ab}$

$\therefore a \odot (a + 1) = \dfrac{1}{(a+1)(a+2)} - \dfrac{1}{a(a+1)}$

$= \dfrac{1}{a+1} - \dfrac{1}{a+2} - \dfrac{1}{a} + \dfrac{1}{a+1}$

$= \dfrac{-1}{a} + \dfrac{2}{a+1} - \dfrac{1}{a+2}$

$\therefore 1 \odot 2 = \dfrac{-1}{1} + \dfrac{2}{2} - \dfrac{1}{3} \quad \to \quad (1)$

$2 \odot 3 = \dfrac{-1}{2} + \dfrac{2}{3} - \dfrac{1}{4} \quad \to \quad (2)$

$3 \odot 4 = \dfrac{-1}{3} + \dfrac{2}{4} - \dfrac{1}{5} \quad \to \quad (3)$

\- - - - - - - - - -

$7 \odot 8 = \dfrac{-1}{7} + \dfrac{2}{8} - \dfrac{1}{9} \quad \to \quad (7)$

$8 \odot 9 = \dfrac{-1}{8} + \dfrac{2}{9} - \dfrac{1}{10} \quad \to \quad (8)$

$9 \odot 10 = \dfrac{-1}{9} + \dfrac{2}{10} - \dfrac{1}{11} \quad \to \quad (9)$

When these 9 equations are added, only the first two terms from (1), the first from (2), the third from (8) and the second and third terms from (9) remain.

$\therefore$ The given expression is $\left(-1+1-\dfrac{1}{2}\right) + \left(\dfrac{-1}{10} + \dfrac{2}{10} - \dfrac{1}{11}\right)$

$= \dfrac{-1}{2} + \dfrac{1}{110} = \dfrac{-54}{110} = \dfrac{-27}{55}$

Choice (B)

8. Without loss of generality, let
$a \le b \le c \le d$
f(a, b, c, d) = min (c, d, d, d) = c
g(a, b, c, d) = max (c, d, d, d) = d
h(a, b, c, d) = max (c, a , a, d) = d
i(a, b, c, d) = min (c, d, d, b) = b
$\therefore$ g(a, b, c, d) = h(a, b, c, d)
$\therefore$ Option c is not defined. Choice (C)

9. Given 3x + 13y = 85, if integral solutions exists for this equation then the minimum positive value of y satisfying the equation is such that $1 \le y \le 3$, similarly minimum possible positive value of x is such that $1 \le x \le 13$.
Since, y has a lesser range, by trial and error we find the minimum positive value of y. This comes out to be 1.
For y = 1, x = 24.
$\therefore$Possible values of x are in the form 13k + 24; $k \in Z$
(13 is coefficient of y).
for $-50 \le x \le 50$
$-50 \le 13k + 24 \le 50$
$-76 \le 13k \le 26$
$-5{\cdot}8 \le k \le 2 \Rightarrow k = -5, -4, -3, -2, -1, 0, 1, 2$
$\therefore$ 8 such solutions exist. Ans: 8

10. Let x, y and z be the cost of one pencil, one sharpener and one pen respectively.
$\therefore$ 3x + 6y + 9z = 129 ---- (1)
and 7x + 11y + 15z = 232 ---- (2)
Coefficients x, y, z are in an A.P. in both equations, with common differences of 3 and 4 respectively.
$4 \times$ equation 1 : 12x + 24y + 36z = 516
$3 \times$ equation 2 : 21x + 33y + 45z = 696

$-9x - 9y - 9z = -180$

$\Rightarrow$ x + y + z = 20. Choice (A)

11. Let x, y and z be the number of oranges, apples and bananas respectively
2x + 5y + 6z = 55 -- (1)
x + y + z = 12 -- (2)
$(1) - 2 \times (2)$
$\Rightarrow$ 3y + 4z = 31-- (3)
Dividing by 3 (least coefficient) and then separating the integer and fraction parts we have:

$\dfrac{z-1}{3} = 10 - y - z = k$

$\dfrac{z-1}{3} = k \Rightarrow z = 3k + 1, k \ge 0$ for z to be a positive integer.

Substituting z = 3k + 1, is (3), we have
3y = 31 – 12k – y or y = 9 – 4k
Since y is a positive integer k < 3.
$\therefore 0 \le k < 3$, i.e., k = 0, 1, 2
for k = 0; y = 9, z = 1, x = 2
k = 1; y = 5, z = 4, x = 3
k = 2; y = 1, z = 7, x = 4
$\therefore$ John could have bought 9 apples. Ans: 9

12. Let x and y be the number of 25 paise and 10 paise coins respectively.
$\therefore$ 25x + 10y = 495
5x + 2y = 99 -- (1)
Dividing by 2 (least coefficient) and then separating the integer and fraction parts we have,

$\dfrac{x-1}{2}$ = k(integer part)

x = 2k + 1, substituting this in (1)
we have 2y = 94 – 10k
or y = 47 – 5k
as y is a positive integer k < 10
$\Rightarrow$ k = 0, 1, 2, ..., 9
Hence, 10 such combinations exist.
Choice (C)

Solutions for questions 13 and 14:

Let the number of Mathematics, Physics and Chemistry books bought be x, y and z respectively.
$\therefore$ x + y + z = 21-- (1)
7x + 4y + 19z = 168 -- (2)

(2) – 4 × (1): ⇒ 3x + 15z = 84
⇒ x + 5z = 28
The possible values of x and z and the corresponding values of y have been tabulated below.

x	y	z
18	1	2
13	5	3
8	9	4
3	13	5

13. Four such combinations exist. Ans: 4

14. Since x, y, z < 10; x = 8, y = 9, z = 4 and y – x = 1.
Ans: 1

Solutions for questions 15 to 20:

15. Let the number of 4's and 6's scored be x and y respectively.
∴ 4x + 6y = 200
or 2x + 3y = 50
As 3y = 50 – 2x, 3y is an even number, i.e., y is an even number. The possible values of y are 0, 2, 4, 6, 8,…, 16.
∴ 17 such combinations exist. Ans: 17

16. $\frac{1}{x} + \frac{1}{y} = \frac{1}{18}$

$\frac{y+x}{xy} = \frac{1}{18} \Rightarrow 18x + 18y = xy$
xy – 18x – 18y = 0
xy – 18x – 18y + 324 = 324
(y – 18) (x – 18) = 4 (81) → (1)
(y – 18) (x – 18) = 2^2 (3^4)
The number of factors of the RHS is 15
∴ (y – 18) (x – 18) can be expressed in 15 ways.
∴ The number of ordered pairs of positive integers (x, y) is 15.
Note: Although x and y are positive integers, (y – 18) and (x – 18) can be negative. But those negative values do not result in positive integer values of x and y.
∴ They can be ignored. Had (1) been (y – 19) (x – 20) = (–18, –18) would give a possible (y, x) as (1, 2).
Choice (B)

17. $\frac{1}{x} + \frac{3}{y} = \frac{1}{29} \Rightarrow (x - 29)(y - 87) = 87(29) = 3(29)^2$

If we add 87 (29) to both sides, we can factorise the LHS.
The number of positive factors of $3(29)^2$ is 6. We also have 6 negative factors, so total number of factors of $3(29)^2$ is 12.
But x > 0; x – 29 > – 29
The possible values of x – 29 can be −3, −1, 1, 3, 29, 3(29), $(29)^2$, $3(29)^2$ i.e., 8 values
y has the 8 corresponding values.
∴ Total number of solutions is 8. Choice (C)

18. $\frac{13}{x} - \frac{5}{y} = \frac{1}{8}$

$\frac{13y-5x}{xy} = \frac{1}{8}$
104y – 40x = xy
xy + 40x – 104y = 0
x(y + 40) – 104y – 40.104 = −40.104
x(y + 40) – 104(y + 40) = −40(104)
(x – 104) (y + 40) = − 40(104)
= −8.5.8.13 = − 2^6.5.13
The number of positive factors of $2^6$5.13 is (7) (2) (2)
i.e. 28. We also have 28 negative factors.
∴ Total number of factors is 56.
∴ The total number of ordered pairs of integers is 56.
Ans: 56

19. $x^2 - y^2 = 60$
(x + y) (x – y) = 60 = (4) (3) (5) = 2^2 (3) (5)
The total number of factors is 3(2) (2) i.e 12
since x, y are integers
(x + y) and (x – y) must be both even numbers.
∴ The possible values of (x + y) are 2, 6, 10, 30, –2, –6, –10, –30. Correspondingly, x –y would be 30, 10, 6, 2, –30, –10, –6, –2. Hence, the number of possible integral solutions is 8. Choice (D)

20. $x^2 - y^2 = 385 \Rightarrow (x - y)(x + y) = (5)(7)(11)$
The number of positive factors of 5(7) (11) is 8. We also have 8 negative factors.
∴ Total number of factors is 16. There are 16 possible values of x + y and correspondingly 16 values of (x, y)
Ans: 16

Solutions for questions 21 to 50:

21. $\frac{1}{5.10} + \frac{1}{10.15} + \frac{1}{15.20}$ …….∞ terms

$= \left(\frac{1}{5} - \frac{1}{10}\right)\frac{1}{5} + \left(\frac{1}{10} - \frac{1}{5}\right)\frac{1}{5} + \left(\frac{1}{15} - \frac{1}{20}\right)\frac{1}{5} + \infty$

$= \frac{1}{5}\left\{\frac{1}{5} - \frac{1}{10} + \frac{1}{10} - \frac{1}{15} + \frac{1}{15} - \frac{1}{20} + \frac{1}{20} \ldots\ldots \infty\right\}$

$= \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$ Choice (D)

22. S = 1 + (1 + b)r + (1 + b + b²)r² + (1 + b + b² + b³)r³ + . . .
multiplying both sides by r
rS = r + (1 + b)r² + (1 + b + b²)r³ + (1 + b + b² + b³)r⁴ + . . .
Subtracting the second equation from the first equation, we get (1 – r)S = 1 + br + b²r² + . . . The R.H.S is a G.P. with first term as 1 and common ratio as br.
The sum of this series on RHS = a/(1 – br)
i.e., (1 – r)S = 1/(1 – br); So, S = 1/{(1 – r) (1 – br)}
Choice (C)

23. $S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3}$ . . . .

1/5 . S = 1/5 + 4/5² + 7/5³ + 10/5⁴ + ...
(Multiplying both sides by 1/5)
Subtracting the second equation from the first equation,
(1 – 1/5)S = 1 + 3/5 + 3/5² + 3/5³ + . . .
= 1 + 3{(1/5)/1 – 1/5} = 1 + 3(1/4) = 1 + 3/4 = 7/4
S = 7/4 x 5/4 = 35/16 Choice (A)

24. S_n =

$\frac{1^3}{2} + \frac{1^3+2^3}{2+4} + \frac{1^3+2^3+3^3}{2+4+6} + \ldots\ldots + \frac{1^3+2^3+\ldots.n^3}{2+4+6+\ldots.+2n}$

$\therefore t_n = \frac{1^3+2^3+\ldots\ldots..+n^3}{2+4+6+\ldots\ldots+2n}$

$= \frac{\Sigma n^3}{2(1+2+\ldots\ldots+n)} = \frac{\Sigma n^3}{2\Sigma n}$

$= \frac{1}{2}\frac{n^2(n+1)^2}{4}\frac{n(n+1)}{2} = \frac{n(n+1)}{4}$

∴ $S_n = \Sigma t_n$

$= \Sigma\frac{n(n+1)}{4} = \frac{1}{4}[\Sigma n^2 + \Sigma n]$

$= \frac{1}{4}\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)(n+2)}{12}$ Choice (C)

25. Given $7^{1+x} + 7^{1-x}$, $\frac{p}{2}$ and $10^x + 10^{-x}$ are in A.P

$\therefore p = 7^{1+x} + 7^{1-x} + 10^x + 10^{-x}$

$= 7 \cdot 7^x + \frac{7}{7^x} + 10^x + \frac{1}{10^x}$

$= 7\left(7^x + \frac{1}{7^x}\right) + \left(10^x + \frac{1}{10^x}\right)$

We know that the sum of any positive number and its reciprocal is always greater than or equal to 2

$\therefore$ p ≥ 14 + 2 ($\because 7^x$ and 10^x are positive).

p ≥ 16

$\therefore$ The range of p is [16, ∞) Choice (B)

26. Given $a_1 + a_{10} + a_{15} + a_{25} + a_{30} + a_{39} = 270$

$(a_1 + a_{39}) + (a_{10} + a_{30}) + (a_{15} + a_{25}) = 270$

We know that, in an finite arithmetic progression the sum of the terms equidistant from the beginning and end is equal to the sum of first and last term

$\therefore a_1 + a_{39} = a_{10} + a_{30} = a_{15} + a_{25}$

$\therefore 3(a_1 + a_{39}) = 270$

$a_1 + a_{39} = 90 \rightarrow$ (1)

Now $a_1 + a_{19} + a_{21} + a_{39} = (a_1 + a_{39}) + (a_{19} + a_{21})$

$= 2(a_1 + a_{39})$

= 2 (90) = 180 Ans: 180

27. 0.7 + 0.77 + 0.777 + ……..

= 7 (0.1 + 0.11 + 0.111 + ………)

$= \frac{7}{9}$ (0.9 + 0.99 + 0.999 + …….. n terms)

$= \frac{7}{9}\left(1-\frac{1}{10}+1-\frac{1}{10^2}+1-\frac{1}{10^3}+\ldots\ldots.+n \text{ terms}\right)$

$= \frac{7}{9}\left(1+1+1+..n\text{terms} - \left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+..n\text{terms}\right)\right)$

$= \frac{7}{9}\left(n - \left(\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right)\right)$

$= \frac{7}{9}\left(n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right) = \frac{7}{81}\left(9n + \frac{1}{10^n} - 1\right)$

Choice (B)

28. Let

$S = 1 + 5x + 9x^2 + 13x^3 + \ldots\ldots\ldots.$

$Sx = x + 5x^2 + 9x^3 + 13x^4 + \ldots\ldots\ldots.$

$(S - Sx) = 1 + 4x + 4x^2 + 4x^3 + \ldots\ldots\ldots.$

$= 1 + 4x(1 + x + x^2 + \ldots\ldots\ldots)$

$S(1 - x) = 1 + \frac{4x}{1-x}$

$S(1 - x) = \frac{1-x+4x}{1-x}$

$S = \frac{1+3x}{(1-x)^2}$

Alternate Solution:

Given $1 + 5x + 9x^2 + 13x^3 + \ldots\ldots.$

The n^{th} term of 1, 5, 9, 13,…….

1 + (n – 1) (4) = 4n – 3

$\therefore$ The n^{th} term of the series $T_n = (4n - 3)x^{n-1}$

$S_n = \Sigma T_n = \Sigma[4nx^{n-1} - 3x^{n-1}]$

$= 4[1 + 2x + 3x^2 + 4x^3 + \ldots..] - 3[1 + x + x^2 + \ldots\ldots]$

$= 4\frac{1}{(1-x)^2} - \frac{3}{1-x}$

$S = \frac{4-3(1-x)}{(1-x)^2} = \frac{1+3x}{(1-x)^2}$ Choice (B)

29. Given series can be written as

$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \ldots\ldots\ldots$

$\therefore$ The n^{th} term $T_n = \frac{(2n+1)}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

Putting n = 1, 2, 3 ……n and adding, we get

$S_n = 1 - \frac{1}{(n+1)^2} = \frac{n^2+2n}{n^2+2n+1}$

Choice (D)

30. Interest on ₹1,80,000= $\frac{10}{100} \times$ 1,80,000 = 18,000

$\therefore$ First instalment = 10,000 + 18,000 = ₹28,000

Amount left = ₹1,70,000

Interest of ₹1,70,000 = $\frac{10}{100} \times$ 1,70,000 = 17,000

$\therefore$Second instalment = 10,000 + 17,000 = ₹27,000

Total amount paid is in 18 months.

$= \frac{18}{2}$ {2 x 28,000 + 17 (–1,000)} = ₹3,51,000

$\therefore$ Additional amount paid = 3,51,000 – 1,80,000

= ₹1,71,000 Ans: 1,71,000

31. Let the first instalment be a and the increment be d.

(20/2) {2a + 19d} = (3/4) 60,000

$\Rightarrow$ 20a + 190d = 45,000

$\Rightarrow$ 2a + 19d = 4,500 ------- (1)

Also, (25/2) [2a + 24d] = 60,000

$\Rightarrow$ 2a + 24d = 4,800 ----- (2)

(2) – (1) $\Rightarrow$ 5d = 300

d = 60

$\therefore$ a = $\frac{4{,}500 - 19 \times 60}{2} = \frac{3{,}360}{2}$ = 1,680 Choice (D)

32. Total distance travelled by the ball =

25 + 2 {3/4 × 25 + $(3/4)^2$ × 25 + $(3/4)^3$ × 25 + …to ∞}

= 25 + 2 x $\frac{(3/4)\ 25}{1-(3/4)}$ = 25 + 150 = 175 m Ans: 175

33. Let the time taken be n minutes.

$\frac{n}{2}$ [2 x 220 + (n – 1) (–10)] = 2,500

$5n^2 - 225n + 2{,}500 = 0$

$\Rightarrow n^2 - 45n + 500 = 0$

$\Rightarrow$ (n – 20) (n – 25) = 0

$\Rightarrow$ n = 20 or 25

Since, 20 < 25, he would have finished counting in 20 minutes. Choice (A)

34. Here, a = 500; d = 50

Let the loan be cleared in n months

$\frac{n}{2}$ {2 x 500 + (n – 1) 50} = 25,000

$\Rightarrow 50n^2 + 950n - 50{,}000 = O$

$\Rightarrow n^2 + 19n - 1{,}000 = 0$

Solving this equation, we find that n ≈ 23.5

$\therefore$ n = 24

$\therefore$ The amount paid in 23 months

$= \frac{23}{2}$ 2 x 500 + 22 x 50 = ₹24,150

$\therefore$ last instalment = 25,000 – 24,150 = ₹850

Ans: 850

35. The distances to be run form an A.P. where

a = 100, d = 25, n =10

Total distance run = 2 x $\frac{10}{2}\{200 + 9 \text{ x } 25\}$ m = 4,250 m

Choice (D)

36. The maximum distance that I can cover would tend to $\frac{10}{1-1/2}$ = 20 km, and hence, I can never be 25 km away from the starting points. Choice (D)

37. If Common difference $\quad$ age of Molly

= 1	14 – 2 x 1 = 12
= 2	14 – 2 x 2 = 10
= 3	14 – 2 x 3 = 8

We cannot take c.d as 4, since then the age of Lolly would be 14 – 4 x 4 which is a negative value.
So, only II is acceptable. Choice (A)

38. The distance that the ball falls through in successive seconds is in A.P. with a common difference of 10 m. The distance that it falls through in the 20th second (in m) is 6 + 19 (10) = 196. Ans: 196

39. Let the number of wickets be a + d, a and a – d.
(a + d) + a + (a – d) = 21 $\Rightarrow$ a = 7
Now, 7 + d + 15, 7 + 2 and 7 – d + 1
i.e. 22 + d, 9 and 8 – d are in G.P.
$(22 + d)(8 – d) = 9^2$
$\Rightarrow 176 – 14d – d^2 = 81 \Rightarrow d^2 + 14d – 95 = 0$
$\Rightarrow (d + 19)(d – 5) = 0$
$\therefore$ d = 5
(Since if d = –19; 7 + (–19) = –12 which cannot be the number of wickets taken)
$\therefore$ The number of wickets taken by Kumble = 7 + 5 = 12
Ans: 12

40. Let the first instalment be a
Then, $\frac{24}{2}$ {2a + 23 (100)} = 32,400
$\Rightarrow$ 24a + 27,600 = 32,400
$\Rightarrow$ 24a = 4,800 $\Rightarrow$ a = 200 Ans: 200

41. Let there be n friends
$a + ar^{n–1} = 66$
$\Rightarrow ar^{n–1} = 66 – a$ – (1)
and, $ar.\,ar^{n–2} = 128 \Rightarrow a^2r^{n–1} = 128$
$\Rightarrow a[66 – a] = 128$
$\Rightarrow a^2 – 66a + 128 = 0$
$\Rightarrow (a – 64)(a – 2) = 0$
a = 2 or 64.
As the amounts are in an increasing G.P., a = 2.
The sum of the amounts with all the friends is 126.
i.e., $2\,\frac{r^n-1}{r-1} = 126 \Rightarrow \frac{r^n-1}{r+1} = 63$ ….. (2)
Also, from (1);
$2(r^{n-1}) = 66 – 2 \Rightarrow r^{n-1} = 32 \Rightarrow r^n = 32r$
From (2);
$\frac{32r-1}{r-1} = 63$
$\Rightarrow 32r – 1 = 63r – 63$
$\Rightarrow 31r = 62 \Rightarrow r = 2$
Now, $2^{n–1} = 32 = 2^5$
$\Rightarrow n – 1 = 5 \Rightarrow n = 6$ Choice (A)

42. The numbers in the 25th bracket are {577, 576, ……, 625}
The sum of all these numbers n $\left(\frac{577+625}{2}\right)$ (49)
= (601) (49) = 29,449. Ans: 29449

43. Total savings in rupees = $\frac{24}{2}$ [2 (120) + 23 (5)] = 4,260
Ans: 4260

44. Given $T_1 = 19$ and $T_n = 5\,T_{n-1} – 16n$ ($\therefore T_2 = 63, T_3 = 267$, ….) From the options, we guess that T_n could be $a(5^n) + bn + c$…(1) OR $d(5^{n-1}) + e(n – 1) + f$ …… (2)
Consider (1)
$T_1 = 5a + b + c = 19$ ……. (3)
$T_2 = 25a + 2b + c = 63$ ….. (4)
$T_3 = 125a + 3b + c = 267$….. (5)
(4) – (3): 20a + b = 44 …… (6)
(5) – (4) : 100a + b = 204 .. (7)
(7) – (6): 80a = 160 $\therefore$ a = 2, b = 4, c = 5
i.e., $T_n = 2(5^n) + 4n + 5$
$T_1 = 2(5^1) + 4(1) + 5 = 19$
$T_2 = 2(5^2) + 4(2) + 5 = 63$
and $T_{30} = 2(5^{30}) + 4(30) + 5 = 2(5^{30}) + 125$
Note: We could have assumed (2).
As $2(5^n) + 4n + 5 \equiv d(5^{n-1}) + e(n – 1) + f$ ……(A) it follows that d = 10, e = 4 and f = 9
Instead of choice (B), if we were given $10(5^{29}) + 4(29) + 9$, this form would be more directly useful.
(As A is an identity, the exponential, linear and constant terms on the LHS are equal to the corresponding terms on the RHS).

Alternate method:
$T_1 = 19 = 5(2) + 9$
$T_2 = 63 = 5^2(2) + 13$
$T_3 = 267 = 5^3(2) + 17$
We see that the nth term of T_n is $5^n(2) + 5 + 4n$.
$\therefore T_{30}$ is $2(5^{30}) + 125$. Choice (B)

45. Consider
$(1 + x)^2 x^0 + (1 + x) x + (1 + x)^0 x^2$
$= 1 + 2x + x^2 + x + x^2 + x^2$
$= 1+3x + 3x^2 = (1 + x)^3 – x^3$
Similarly $(1 + x)^3.\,x^0 + (1 + x)^2 x + (1+ x)^1 (x)^2 + (1 + x)^\circ x^3$
$= (1 + x)^4 – x^4$
$\therefore (1 + x)^{350} x^\circ + (1 + x)^{349} x + (1 + x)^{348} x^2 + (1 + x)^{347} x^3 + …… + (1 + x)^0 x^{350} = (1 + x)^{351} – x^{351}$.
The coefficients of x^{150} and x^{151} in the above expansion are
${}^{351}C_{150}$ and ${}^{351}C_{151}$
$\therefore$ sum of the coefficients =
${}^{351}C_{150} + {}^{351}C_{151} = {}^{352}C_{151}$

Alternative solution:
$(1 + x)^n x^m$ can be represented as a series of terms. In each term there are two factors $(1 + x)^r$ and x^s and the coefficients of all terms (unlike in the binomial theorem) are all 1. We can let the indices of 1 + x decrease and those of x increase. Thus
$(1 + x)^n x^m = (1 + x)^{n-1} x^m (1 + x)$
$= (1 + x)^{n–1}x^m + (1 + x)^{n–1}x^{m+1}$
and so on.
Thus, $(1 + x)^{351} = (1 + x)^{350} (1 + x) = (1 + x)^{350} + (1 + x)^{350}x$ ….(1)
$(1 + x)^{350} x = (1 + x)^{349}x(1 + x) = (1 + x)^{349}x + (1 + x)^{349} x^2$ ….(2)
- - - - - - -
$(1 + x) x^{350} = (1 + x)^0x^{350} + x^{351}$ …………. (351)
Adding these 351 equations, we get
$(1 + x)^{351} = (1 + x)^{350} + (1 + x)^{349} x + …. + (1 + x)^0 x^{350} + x^{351}$
$\therefore$ The given series is equal to $(1 + x)^{351} – x^{351}$
$T_{150} = {}^{351}C_{150}\,x^{150}$ and $T_{151} = {}^{351}C_{151}\,x^{151}$
The sum of these coefficients (i.e., ${}^{351}C_{150} + {}^{351}C_{151}$) is ${}^{352}C_{151}$ Choice (C)

46. Consider I
$1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + ……. - \frac{1}{250}$
$= \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+…..+\frac{1}{250}\right)$
$-\,2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+…..+\frac{1}{250}\right)$
$= 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+…..+\frac{1}{250} - \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+…..+\frac{1}{125}\right)$
$= \frac{1}{126}+\frac{1}{127}+\frac{1}{128}+…..+\frac{1}{250}$
Consider II
$(188)\left[\frac{1}{(126)(250)}+\frac{1}{(127)(249)}+\frac{1}{(128)(248)}+…..+\frac{1}{(250)(126)}\right]$

$\frac{1}{2}\left[\frac{376}{(126)(250)}+\frac{376}{(127)(249)}+......+\frac{376}{(250)(126)}\right]$

$\frac{1}{2}\left[\frac{126+250}{(126)(250)}+\frac{127+249}{(127)(249)}+......+\frac{250+126}{(250)(126)}\right]$

$= \frac{1}{2}\left[\frac{1}{250}+\frac{1}{126}+\frac{1}{249}+\frac{1}{127}+\frac{1}{248}+\frac{1}{128}.....+\frac{1}{126}+\frac{1}{250}\right]$

$= \frac{2}{2}\left[\frac{1}{126}+\frac{1}{127}+\frac{1}{128}+\frac{1}{129}+\frac{1}{130}+......+\frac{1}{250}\right]$

$= \frac{1}{126}+\frac{1}{127}+\frac{1}{128}+......+\frac{1}{250}$

$\therefore$ Both I and II are true Choice (B)

47. $\frac{1}{(2)(4)}+\frac{1}{(4)(6)}+\frac{1}{(6)(8)}+......+\frac{1}{(28)(30)}$

$= \frac{1}{4}\left[\frac{1}{(1)(2)}+\frac{1}{(2)(3)}+\frac{1}{(3)(4)}+......\frac{1}{(14)(15)}\right]$

$= \frac{1}{4}\left[1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+.......\frac{1}{14}-\frac{1}{15}\right]$

$= \frac{1}{4}\left[1-\frac{1}{15}\right] = \frac{14}{4(15)} = \frac{7}{30}$ Choice (C)

48. $3(4)^2 + 5(5^2) + 7(6^2) + 9(7^2) + ...$
$\therefore$ the nth term in the above series is $t_n = (2n + 1)(n + 3)^2$
$\therefore t_n = (2n+1)(n^2 + 6n + 9)$
$= 2n^3 + 13n^2 + 24n + 9$
$S_n = \Sigma t_n = 2\Sigma n^3 + 13\,\Sigma n^2 + 24\Sigma n + 9\Sigma 1$

$= \frac{n^2(n+1)^2}{2} + 13\,\frac{n(n+1)(2n+1)}{6} + 12n(n+1) + 9n$

Put n = 20.

$\therefore S_{20} = \frac{(400)(441)}{2} + 13\frac{(20)(21)(41)}{6} + 12(20)(21) + 9(20)$

$= 88200 + 37310 + 5040 + 180 = 130730.$

Choice (B)

Note: We can consider the nth term (i.e., t_n) to be $(2n - 5)n^2$. We would then be dealing with only two summations.

49. 75(5) + 74 (6) +73 (7) + …….+5(75)
$\therefore$ If we append 4 more terms at the beginning, the nth term of the resulting series is n(80 – n)

$\therefore \sum_{n=5}^{75} n(80-n) = \sum_{n=5}^{75}(80n-n^2)$

$= \sum_{n=1}^{75}(80n-n^2) - \sum_{n=1}^{4} n(80n-n^2)$

$\sum_{n=1}^{75}\left[80\frac{(n(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}\right]$

$-\sum_{n=1}^{4}\left[\frac{80n(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}\right]$

$= 40(75)(76) - \frac{(75)(76)(151)}{6} - [40(4)(5) - \frac{(4)(5)(9)}{6}]$

$= 228000 - 143450 - 800 + 30 = 83780$

Choice (D)

50. $T_n = \left(1+\frac{4}{n^2-4}\right)T_{n-1} = \left(\frac{n^2-4+4}{n^2-4}\right)T_{n-1}$

$= \left(\frac{n^2}{n^2-4}\right)T_{n-1} = \left(\frac{n}{n-2}.\frac{n}{n+2}\right)T_{n-1}$

$\therefore T_3 = \left(\frac{3}{1}\frac{3}{5}\right)T_2 = \left(\frac{3}{1}\right)\left(\frac{3}{5}\right)T_2$

$T_4 = \left(\frac{4}{2}\frac{4}{6}\right)T_3 = \left(\frac{3}{1}\frac{4}{2}\right)\left(\frac{3}{5}\frac{4}{6}\right)T_2$

$T_5 = \left(\frac{5}{3}\frac{5}{7}\right)T_4 = \left(\frac{3}{1}\frac{4}{2}\frac{5}{3}\right)\left(\frac{3}{5}\frac{4}{6}\frac{5}{7}\right)T_2$

Similarly,

$T_{20} = \left(\frac{3}{1}\frac{4}{2}\frac{5}{3}.....\frac{18}{16}\frac{19}{17}\frac{20}{18}\right)\left(\frac{3}{5}\frac{4}{6}\frac{5}{7}.....\frac{18}{20}\frac{19}{21}\frac{20}{22}\right)T_2$

$= \frac{3^2(4)^2 19^2(20)^2 T_2}{1(2)(3)(4)19(20)(21)(22)} = \frac{3(4)19(20)}{2(21)(22)}\left(\frac{3}{4}\right) = \frac{285}{77}$

Choice (A)

Exercise – 9

Solutions for questions 1 to 11:

1. $\alpha\beta^2 + \alpha^2\beta = \alpha\beta(\beta + \alpha)$
$= q(-p) = -pq$ = sum of the roots
$\alpha\beta^2.\alpha^2\beta = \alpha^3\beta^3 = (\alpha\beta)^3 = q^3$ = product of the roots.
The required equation is $x^2 - (-pq)x + q^3 = 0$
$= x^2 + pqx + q^3 = 0$

Choice (D)

2. Let $\frac{x^2+2x-11}{2(x-3)} = k$
$x^2 + 2x - 11 = 2kx - 6k$
$x^2 + (2 - 2k)x + (6k - 11) = 0$
Since x is real, $b^2 - 4ac \geq 0$
$\Rightarrow 4(1 - k)^2 - 4.1.(6k - 11) \geq 0$
$\Rightarrow 4 + 4k^2 - 8k - 24k + 44 \geq 0$
$4k^2 - 32k + 48 \geq 0$
$k^2 - 8k + 12 \geq 0 \Rightarrow (k - 6)(k - 2) \geq 0$
$\Rightarrow k - 6 \geq 0$ and $k - 2 \geq 0$ or $k - 6 \leq 0$ and $k - 2 \leq 0$
$\Rightarrow k \geq 6$ and $k \geq 2$ or $k \leq 6$ and $k \leq 2$
$\Rightarrow k \geq 6$ or $k \leq 2$
Therefore, $k \leq 2$ and $k \geq 6$. Hence, values k can take are $(-\infty, 2] \cup [6, \infty)$ Choice (D)

3. Let the roots be 2α and 3α.
Sum of the roots = $2\alpha + 3\alpha = -2b/3a$
$5\alpha = -2b/3a$
$\alpha = -2b/15a \rightarrow (1)$
Product of the roots = $2\alpha \times 3\alpha = c/3a$
$6\alpha^2 = c/3a$
$\alpha^2 = c/18a \rightarrow (2)$
As $\alpha = -2b/15a$,
$\alpha^2 = (-2b/15a)^2 = c/18a \Rightarrow 4b^2/225a^2 = c/18a$
$8b^2 = 25ac$ Choice (D)

4. It can be seen that x = 1 satisfies the equation.
Product of the roots is $\frac{\ell+m-k}{k+m-\ell}$.
$\therefore$ The roots are 1 and $\frac{\ell+m-k}{k+m-\ell}$

Choice (B)

5. Let the roots be α and β. Given $3\alpha = 2\beta$

$\frac{\alpha}{\beta} = \frac{2}{3}$ i.e., $\alpha = \frac{2}{3}\beta$

$\alpha + \beta = \frac{5\beta}{3}$ and $\alpha\beta = \frac{2}{3}\beta^2$, $\frac{3q}{p} = \frac{5\beta}{3}$ and $\frac{2r}{p} = \frac{2\beta^2}{3}$

$\therefore \frac{q}{p} = \frac{-5}{9}\beta$ and $\frac{r}{p} = \frac{1}{3}\beta^2$

$\frac{\left(\frac{q}{p}\right)^2}{\frac{r}{p}} = \frac{\left(\frac{-5}{9}\right)^2}{\frac{1}{3}} = \frac{25}{81}.3 = \frac{25}{27}$. Choice (D)

6. Each root is positive.
$\therefore$ Sum and product of roots are positive.
Also discriminant is non-negative.
$\therefore$ $4m + 1 > 0$, $4m + 2.25 > 0$ and $(4m + 1)^2 - 4m + 2.25 \geq 0$
i.e., $m > \frac{-1}{4}$, $m > \frac{-9}{4}$, $(2m + 1)(m - 1) \geq 0$
i.e., $m > \frac{-1}{4}$, $m \leq \frac{-1}{2}$ or $m \geq 1$.
$\therefore$ It effectively follows that $m \geq 1$. Choice (D)

7. Given: p and q are the roots of $x^2 - ax - 1 = 0$
$p + q = a$ and $pq = -1$. Further r, s are the roots of
$x^2 - bx - 1 = 0$; $r + s = b$ and $rs = -1$
Now $(r + p)(s + p)(r - q)(s - q) = (r + p)(s - q)(s + p)(r - q)$
$= [rs + ps - rq - pq][sr + pr - sq - pq]$
$= [-1 + ps - rq + 1][-1 + pr - sq + 1]$
$= [ps - rq][pr - sq]$
$= p^2rs - pqr^2 - pqs^2 + rsq^2$
$= -p^2 + r^2 + s^2 - q^2 = r^2 + s^2 - (p^2 + q^2)$
$= ((r + s)^2 - 2) - ((p + q)^2 - 2)$
$= ((-b)^2 - 2) - ((-a)^2 - 2) = b^2 - a^2$ Choice (D)

8. $3x^4 + 10x^3 - 42x^2 + 10x + 3 = 0$
On dividing both sides by x^2, we get
$3x^2 + 10x - 42 + \frac{10}{x} + \frac{3}{x^2} = 0$
$$3\left(x^2 + \frac{1}{x^2}\right) + 10\left(x + \frac{1}{x}\right) - 42 = 0$$
Let $y = x + \frac{1}{x}$
$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = y^2 - 2$$
$\therefore$ $3(y^2 - 2) + 10y - 42 = 0$
$3y^2 + 10y - 48 = 0$
$(3y - 8)(y + 6) = 0$
$y = \frac{8}{3}$ or -6
$\therefore$ $x + \frac{1}{x} = \frac{8}{3}$ or -6. Choice (A)

9. Let the expression be denoted by k
$$\frac{2\sqrt{2}by - (y^2 + b^2)}{2\sqrt{2}y - b} = k$$
$2\sqrt{2}yk - kb + y^2 + b^2 - 2\sqrt{2}by = 0$
$y^2 + 2\sqrt{2}(k - b)y - kb + b^2 = 0$
y is real $\Rightarrow$ Discriminant of the equation above must be ≥ 0
i.e., $\left(2\sqrt{2}(k - b)\right)^2 - 4(-kb + b^2) \geq 0$
$4(2k^2 + 2b^2 - 3bk - b^2) \geq 0$
$2k^2 - 3bk + b^2 \geq 0$
$(2k - b)(k - b) \geq 0$ i.e., $2\left(k - \frac{b}{2}\right)(k - b) \geq 0$
i.e., $\left(k - \frac{b}{2}\right)(k - b) \geq 0$
$k \leq \frac{b}{2}$ or $k \geq b$ i.e k lies in the interval $\left[-\infty, \frac{b}{2}\right] \cup [b, \infty]$.
($\because$ b > 0) Choice (A)

10. Let α be the root of $x^2 - 3x + 2k = 0$. 2α is the root of $x^2 - 7x + 12k = 0$
$\alpha^2 - 3\alpha + 2k = 0 \quad \rightarrow \quad (1)$
and $4\alpha^2 - 14\alpha + 12k = 0 \quad \rightarrow \quad (2)$
From (1) and (2),
On eliminating k we get $-2\alpha^2 + 4\alpha = 0$
$\alpha = 0$ or $\alpha = 2$
If $\alpha = 2$, from (1), $2^2 - 3(2) + 2k = 0$
$2k = 2 \Rightarrow k = 1$. Ans: 1

11. $px^2 + qx + r = 0$ ---- (1)
$qx^2 + rx + p = 0$ ---- (2)
Let α be the common root of (1) and (2)
$p\alpha^2 + q\alpha + r = 0$ ---- (1)
$q\alpha^2 + r\alpha + p = 0$ ----- (2)
On solving (1) and (2), we get
$\alpha^2 = \frac{pq - r^2}{pr - q^2}$ and $\alpha = \frac{qr - p^2}{pr - q^2}$
$$\left(\frac{qr - p^2}{pr - q^2}\right)^2 = \frac{pq - r^2}{pr - q^2}$$
$(qr - p^2)^2 = (pq - r^2)(pr - q^2)$
$\Rightarrow p^3 + q^3 + r^3 - 3pqr = 0$
$\Rightarrow p^3 + q^3 + r^3 = 3pqr$. Choice (D)

Solutions for questions 12 and 13:

Let the number of movies released be x.
$$\frac{1}{5}x + 10\sqrt{x - \frac{x}{5}} + \frac{1}{4} \times 10\sqrt{x - \frac{x}{5}} + 150 = x$$
$$\Rightarrow \frac{25}{2}\sqrt{\frac{4x}{5}} = \frac{4x}{5} - 150$$
Squaring both sides, $\frac{625}{4}\left(\frac{4x}{5}\right) = \frac{16x^2}{25} - 240x + 22500$
$\Rightarrow \frac{16x^2}{25} - 365x + 22500 = 0 \Rightarrow 16x^2 - 9125x + 562500 = 0$
$\therefore$ $x = 500$ (since $x \neq \frac{1125}{16}$ as x has to be an integer).

12. Total number of action films released
$10\sqrt{\frac{4 \times 500}{5}} = 200$
$\therefore$The number of action films which did good business
$= 1/20 \times 200 = 10$. Ans: 10

13. Number of art films = $1/4 \times 200 = 50$.
$\therefore$ Required percentage = $\frac{5}{50} \times 100 = 10\%$ Ans: 10

Solutions for questions 14 and 15:

Let there be 2x children in the group. Expenditure on 2x children is equal to the expenditure on x adults.
$\therefore$ Expenditure per day for an adult = $\frac{4,800}{14 + x}$
Total expenditure per day after the addition of the guests
$= \frac{(4,800 - 1,920)}{8} = 5,760$
$\therefore$ Expenditure per head = $\frac{5,760}{16 + x + y}$, where y is the number of guests in Mr. Singh's house.
Now, $y = \sqrt{2x - 8}$
$$\therefore \frac{4,800}{14 + x} = \frac{5,760}{16 + x + \sqrt{2x - 8}}$$
$\Rightarrow 5(16 + x + \sqrt{2x - 8}) = 6(14 + x)$
$\Rightarrow 80 + 5x + 5\sqrt{2x - 8} = 84 + 6x$
$\Rightarrow 5\sqrt{2x - 8} = x + 4$
Squaring both sides;

$25(2x - 8) = x^2 + 8x + 16$
$\Rightarrow x^2 - 42x + 216 = 0$
$\Rightarrow (x - 6)(x - 36) = 0$
$\Rightarrow x = 6$ or 36.
But since the number of children was less than that of the adults.
$\therefore x = 6$.
$\therefore$ Number of children = 12.
Number of adults = $16 + \sqrt{12 - 8} = 18$
Number of children = 12
Expenditure of 12 children = Expenditure of 6 adults.

14. Expenditure per day of each adult = $\frac{5760}{18 + 6}$ = ₹240.
Choice (D)

15. Number of guests at Mr. Singh's residence = $\sqrt{12 - 8} = 2$.
Choice (B)

Solutions for questions 16 to 50:

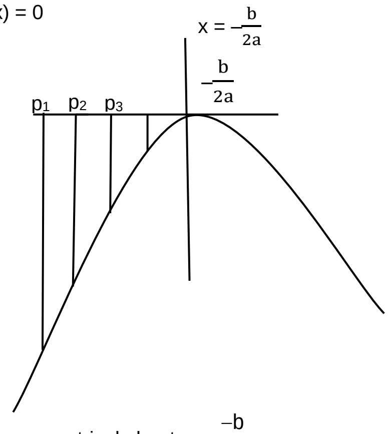
16. Let the total number of students be x.
$\frac{x}{4} + 4\sqrt{x} + 16 = x \Rightarrow 4\sqrt{x} = \frac{3x}{4} - 16$
$\Rightarrow 16\sqrt{x} = 3x - 64$
Squaring both sides,
$256x = 9x^2 - 384x + 4096$
$\Rightarrow 9x^2 - 640x + 4096 = 0$
$\Rightarrow (x - 64)(9x - 64) = 0$
$\Rightarrow x = 64$ since $x \neq \frac{64}{9}$ Ans: 64

17. Let there be 'n' rows in the first arrangement
$\therefore$ The number of balls = $1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$
According to the condition, $\frac{n(n+1)}{2} + 116 = (n - 4)^2$
$\Rightarrow n^2 + n + 232 = 2n^2 - 16n + 32$
$\Rightarrow n^2 - 17n - 200 = 0$
$\Rightarrow (n - 25)(n + 8) = 0$.
As n, the number of rows, can't be negative, n = 25.
$\therefore$ Initial number of balls = $\frac{25 \times (25 + 1)}{2} = 25 \times 13 = 325$
Ans: 325

18. Let the number of children who planned to go for picnic initially be x.
$\frac{500}{x} - \frac{500}{(x+5)} = 5 \Rightarrow x = 20$
$\therefore$ The number of children who went for picnic is 20 + 5 i.e., 25. Ans: 25

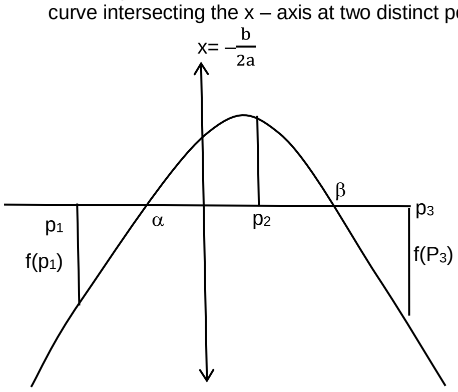
19. Let α, β, γ, δ be the distinct negative integral roots of the equation
$x^4 + px^3 + qx^2 + \gamma x + 105 = 0$ ------------ (1)
Now, $\alpha\beta\gamma\delta = 105$
$= 5(3)(7)(1)$
$= (-5)(-3)(-7)(-1)$
Hence, the roots have to be
$-7, -5, -3$ and -1.
Now,
$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\gamma$
$\Rightarrow (-7)(-5)(-3)+(-7)(-3)(-1)+(-5)(-3)(-1)(-7)(-5)(-1) = -r$
$\Rightarrow -105 - 21 - 15 - 35 = -r \Rightarrow r = 176$.
Ans: 176

20. Given,
$f(x) = ax^2 + bx + c, a < 0$ ------------ (1)
Since $a < 0$, the parabola represented, by (1), downwards is concave, as shown in the figure.
Further, the curve that touches the x – axis, since the roots of f(x) = 0 are equal and $\frac{-b}{2a}$ is the double root of the equation f(x) = 0



The curve is symmetrical about $x = \frac{-b}{2a}$
$\therefore p_1 < p_2 < p_3 < \frac{-b}{2a}$
$\Rightarrow f(p_1) < f(p_2) < f(p_3), < f\left(\frac{-b}{2a}\right) = 0$. Choice (B)

21. Given, S
$F(x) = ax^2 + bx + c, a < 0$
α, β are the distinct real roots of the equation f(x) = 0
$\therefore$ The parabola would be concave downwards, with the curve intersecting the x – axis at two distinct points.



Given,
$p_1 < \alpha < p_2 < \beta < p_3$
From the figure,
$f(p_1) < 0, f(\alpha) = 0, f(p_2) > 0, f(\beta) = 0, f(p_3) < 0$
$\therefore f(p_1) < f(p_2) > f(p_3)$.
Choice (D)

22. P is the greatest negative integer such that
$P^2 \geq 20P + 3500$,
i.e., $P^2 \geq 20P + 3500$
$\Rightarrow P^2 - 20P - 3500 \geq 0$
$\Rightarrow (P + 50)(P - 70) \geq 0$
$\Rightarrow P \leq -50$ or $P \geq 70$
i.e., the greatest negative integral value of P is –50.
Choice (C)

23. a, b and c are the roots of the equation
$x^3 + p_1x^2 + p_2x + p_3 = 0$ ------------ (1)
such that $a + b + c = -1$
$\Rightarrow -p_1 = -1 \Rightarrow p_1 = 1$
Since, coefficients p_1, p_2 and p_3 are in GP, co-efficients p_1, p_2 and p_3 would be in the form of 1, r, r^2 respectively.
Equation (1) reduces to
$x^3 + x^2 + rx + r^2 = 0$

Now,
$ab + bc + ca = r$ ------------ (2)
and $abc = -r^2$ ------------ (3)
From (2) and (3)
$(ab + bc + ca)^2 = r^2 = -abc$
$(ab + bc + ca)^2 = -abc.$ Choice (D)

24. The roots of the given equation $x^3 - kx^2 + 336x - 512 = 0$ are in geometric progression

Let the roots be $\frac{\alpha}{\beta}$, α, $\alpha\beta$

$\left(\frac{\alpha}{\beta}\right)(\alpha)(\alpha\beta) = 512$
$\alpha^3 = 512$
$\alpha = 8$
Since 8 is the root of the equation $x^3 - kx^2 + 336x - 512 = 0$,
$512 - 64k + 336 \times 8 - 512 = 0$
$64k = 336 \times 8 \Rightarrow k = 42$ Ans: 42

25. α, β and γ are the roots of the equation $x^3 - 8x^2 + 9x + 3 = 0$
$\alpha + \beta + \gamma = 8$
$\alpha\beta + \beta\gamma + \gamma\alpha = 9$
$\alpha\beta\gamma = -3$
$\Sigma \frac{1}{\alpha\beta} = \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$

$= \frac{8}{-3}$ Choice (C)

26. $x^3 + px^2 + qx + r = 0$
let the roots in A.P. be: $\alpha - \beta, \alpha, \alpha + \beta$
(common difference = β)
sum of roots = $3\alpha = -p$
product of roots = $\alpha(\alpha^2 - \beta^2) = -r$
sum of product of roots taken two at a time
$3\alpha^2 - \beta^2 = q \Rightarrow 2\alpha^2 + (\alpha^2 - \beta^2) = q$
$\Rightarrow 2(-p/3)^2 - r/\alpha = q \Rightarrow 2p^2/9 - r/(-p/3) = q$
$\Rightarrow 2p^2/9 + 3r/p = q \Rightarrow 2p^3 + 27r = 9pq$
$\Rightarrow 2p^3 = 9(pq - 3r)$ Choice (D)

27. $p^2 + q^2 + r^2 = (p + q + r)^2 - 2(pq + qr + pr)$
$p + q + r = 6$, $pq + qr + pr = 8 \Rightarrow p^2 + q^2 + r^2 = 20$
Ans: 20

28. For any $x \geq 1$, we have $2 \leq (1 + 1/x)^x < 2.8$

consider choice A: $\frac{255^{49}}{250^{50}} = \frac{1}{255}\left(\frac{255}{250}\right)^{50}$

$= \frac{1}{255}\left(1 + \frac{1}{50}\right)^{50} < \frac{2.8}{255} < 1$

$\therefore 255^{49} < 250^{50}$ is true
consider choice B:

$\frac{31^{29}}{30^{30}} = \frac{1}{31}\left(\frac{31}{30}\right)^{30} = \frac{1}{31}\left(1 + \frac{1}{30}\right)^{30} < \frac{2.8}{31} < 1$

$\therefore 31^{29} < 30^{30}$ is true
Consider choice C:

$\frac{240^{38}}{234^{39}} = \left(\frac{1}{240}\right)\left(\frac{240}{234}\right)^{39}$ $\frac{1}{240}\left(1 + \frac{1}{39}\right)^{39} < \frac{2.8}{240} < 1$

$\therefore 240^{38} > 234^{39}$ is false. Choice (C)

29. Since $x \geq 1$, $(x + 1)^x$ and $x^x + 1$ are positive.
now, $(x + 1)^x < x^x \cdot x$
or $(x + 1)^x/x^x < x \Rightarrow (1 + 1/x)^x < x$
we know that for $x \geq 1$; $2 \leq (1 + 1/x)^x < 2.8$
$\therefore$ for $x \geq 3$; $(1 + 1/x)^x < x$ Choice (C)

30. Given $x^2 + y^2 + z^2 = 4$
Now $(x + y + z)^2 \geq 0$
$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) \geq 0$
$\Rightarrow (xy + yz + zx) \geq -(x^2 + y^2 + z^2)/2$
or $xy + yz + zx \geq -2$ Choice (A)

31. Given $x = 10/(a + b)$ and $y = 10(a + b)$
$\therefore x + y = 10[1/(a + b) + (a + b)] \geq 10(2)$
As $k + \frac{1}{k} \geq 2$, for $k > 0$
$\therefore x + y \geq 20$ Choice (B)

32. Say $k = \frac{a^2(b+c) + b^2(c+a) + c^2(a+b)}{abc}$

$\Rightarrow k = \frac{a^2b + a^2c + b^2c + b^2a + c^2a + c^2b}{abc}$

Now $a^2b + a^2c + b^2c + b^2a + c^2a + c^2b \geq 6\sqrt[6]{a^6b^6c^6}$
(since A.M $\geq$ G.M)
$\Rightarrow a^2b + a^2c + b^2c + b^2a + c^2a + c^2b \geq 6abc$
$\Rightarrow k \geq 6abc/abc$ i.e., $k \geq 6$
Since a, b, c are distinct, $k > 6$. Choice (C)

33. Given $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$

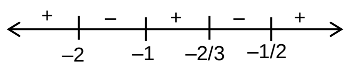
$\Rightarrow \frac{2x}{2x^2+5x+2} - \frac{1}{x+1} > 0$

$\Rightarrow \frac{2x(x+1) - (2x^2+5x+2)}{(2x^2+5x-2)(x+1)} > 0$

or $\frac{2x^2+2x-2x^2-5x-2}{(2x^2+5x+2)(x+1)} > 0$

$\Rightarrow \frac{(-3x-2)}{(2x^2+5x+2)(x+1)} > 0$

$\Rightarrow (x + 1)(3x + 2)(2x^2 + 5x + 2) < 0$
or $(x + 1)(3x + 2)(x + 2)(2x + 1) < 0$



$\therefore x \in (-2, -1) \cup (-2/3, -1/2)$ Choice (C)

34. Given $x^{\log_6 x} > 6$; $x > 0$
If $\log_6 x = y \Rightarrow 6^y = x$
$\therefore x^{\log_6 x} = (6^y)^y = 6^{y^2}$
$\Rightarrow 6^{y^2} > 6$ or $y^2 > 1 \Rightarrow y > 1$ or $y < -1$
$\Rightarrow x > 6$ or $x < 1/6$
$\therefore x \in (0, 1/6) \cup (6, \infty)$, as $x > 0$ Choice (A)

35. As we are looking at only negative values of x, we take $x < 0$
Case 1: $x \leq -4$:
$|x + 4| + |x - 7| < 13$
$-x - 4 + 7 - x < 13$; $-2x < 10 \Rightarrow x > -5$
As $x \leq -4$, $x = -4$
Case 2: $-4 < x < 0$
$|x + 4| + |x - 7| < 13$
$x + 4 + 7 - x < 13$
$\Rightarrow 11 < 13$, this being always true, the inequality is true for $-4 < x < 0$, hence $x = -1, -2, -3,$
$\therefore$ x can take four negative integral values
i.e. $-1, -2, -3, -4$. Ans: 4

36. For $x = 0$; $\frac{x^4}{1+x^8} = 0$
for $x \neq 0$; x^4 is positive and $1/x^4$ is positive.
As A.M $\geq$ G.M.

$\frac{x^4 + \frac{1}{x^4}}{2} = \sqrt{x^4 \times \frac{1}{x^4}} \Rightarrow \frac{x^8+1}{x^4} \geq 2$ or $\frac{x^4}{1+x^8} \leq \frac{1}{2}$
Choice (C)

37. $y = 25 - |x^2 - 2x + 4|$
consider, $x^2 - 2x + 4 = x^2 - 2x + 1 + 3 = (x - 1)^2 + 3$
$\therefore x^2 - 2x + 4 \ge 3$, as $(x - 1)^2 \ge 0$.
$\Rightarrow y = 25 - |x^2 - 2x + 4| \le 25 - 3$ or $y \le 22$
Ans: 22

38. We know that A.M (a, b, c, d) ≥ G.M (a, b, c, d)
$\frac{a+b+c+d}{4} \ge (abcd)^{1/4}$
$a + b + c + d \ge 4\ (abcd)^{1/4}$
$\therefore k = 4$
Ans: 4

39. We know that A.M ($p^4 q^4$, $q^4 r^4$) ≥ G.M ($p^4 q^4$, $q^4 r^4$)
$\frac{p^4q^4+q^4r^4}{2} \ge \left(p^4q^4q^4r^4\right)^{\frac{1}{2}}$
$p^4q^4 + q^4r^4 \ge 2\ p^2q^4r^2 \quad \rightarrow \quad (1)$
Similarly, $q^4r^4 + r^4 p^4 \ge 2\ p^2q^2r^4 \quad \rightarrow \quad (2)$ and
$r^4p^4 + p^4q^4 \ge 2p^4q^2\ r^2 \quad \rightarrow \quad (3)$
Adding (1), (2) and (3), we get
$2\ (p^4\ q^4 + q^4\ r^4 + r^4\ p^4) \ge 2(p^2q^2\ r^2\ (p^2 + q^2 + r^2))$
$p^4\ q^4 + q^4\ r^4 + r^4\ p^4 \ge p^2q^2\ r^2\ (p^2 + q^2 + r^2)$
Choice (D)

40. Consider the expression $\left(\frac{3x}{2}\right)^2\left(\frac{4y}{3}\right)^3$
Now the sum of all the factors of the above expression is
$2\left(\frac{3x}{2}\right) + 3\left(\frac{4y}{3}\right) = 3x + 4y = 15$ (constant)
$\therefore$ The sum of the factors is constant.
$\left(\frac{3x}{2}\right)^2\left(\frac{4y}{3}\right)^3$ is maximum when $\frac{3x}{2} = \frac{4y}{3}$ i.e., when
$\frac{3x}{2} = \frac{4y}{3} = \frac{3x+4y}{2+3} = \frac{15}{5} = 3$
$\frac{3x}{2} = 3;\ \frac{4y}{3} = 3$
$x = 2,\ y = \frac{9}{4}$
The maximum value of $x^2y^3 = 2^2\left(\frac{9}{4}\right)^3$.
Choice (A)

41. Let the positive numbers be $x_1, x_2 \ldots x_{100}$
We know that A.M ($x_1, x_2, x_3, \ldots x_{100}$) ≥ G.M ($x_1, x_2, x_3 \ldots x_{100}$)
$\frac{x_1+x_2+\ldots+x_{100}}{100} \ge \sqrt[100]{x_1\,x_2\ldots x_{100}}$
$x_1 + x_2 + \ldots + x_{100} \ge 100(1) \ge 100.$
$\therefore$ The sum of the positive numbers is never less than 100.
Choice (D)

42. Given: $3^{|x+2|} - 3^x = (|3^x - 1| + 2)$
If $x < -2$, then $|x + 2| = -(x + 2)$ and $|3^x - 1|$
$= -(3^x - 1)$
$\therefore$The given equation can be written as
$3^{-(x+2)} - 3^x = -3^x + 1 + 2$
$3^{-(x+2)} = 3$
$-x - 2 = 1$
$\Rightarrow x = -3$
Ans: –3

43. We know that A.M (a, b, c, d) ≥ H.M (a, b, c, d)
$\frac{(a+b+c+d)}{4} \ge \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$
$(a + b + c + d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \ge 16$
$(a + b + c + d)\ \frac{(bcd + acd + abd + abc)}{abcd} \ge 16$
$(a + b + c + d)\ (bcd + acd + abd + abc) \ge 16abcd$
$\therefore k = 16$
$k + 3 = 16 + 3 = 19.$
Ans: 19

44. We know that A.M. of two or more positive numbers is greater than or equal to their G.M.
$\therefore p^2 + q^2 \ge 2pq,\ p^2 + r^2 \ge 2pr,\ q^2 + r^2 \ge 2qr$
Adding these, we get $2(p^2 + q^2 + r^2) \ge 2(pq + pr + qr)$
$\therefore pq + pr + qr \le 7.$
Ans: 7

45. Given a, b, c are the sides of the triangle.
The sum of two sides is greater than the third side
i.e., $a + b > c$; $b + c > a$; $c + a > b$
let $p = a + b - c$, $q = a + c - b$ and $r = b + c - a$
$a + b + c = p + q + r$
We know that A .M (p, q, r) ≥ G.M (p, q, r)
$\therefore \frac{p+q+r}{3} \ge (pqr)^{\frac{1}{3}}$
$(p + q + r)^3 \ge 27\ pqr$
$(a + b + c)^3 \ge 27\ (a + b - c)\ (a + c - b)\ (b + c - a)$
$\frac{(a+b+c)^3}{(a+b-c)(a+c-b)(b+c-a)} \ge 27$
$\therefore$ The minimum value is 27.
Ans: 27

46. Let $m = p + q$ and $n = r + s$
$m + n = p + q + r + s = 10$
$mn = (p + q)\ (r + s)$
We know that when the sum of two positive quantities is constant the product of these quantities is maximum when they are equal.
$\therefore$ mn is maximum when $m = n = 5$.
$\therefore$ Maximum mn = 25
$\therefore 0 < x \le 25.$
Choice (C)

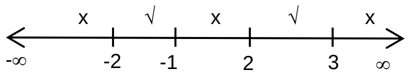
47. $(p + 2)\ (q + 2) = pq + 2p + 2q + 4$
$= 2\ (p + q) + 5$
We know that, A.M (p, q) ≥ G.M (p, q)
$\frac{p+q}{2} \ge (pq)^{\frac{1}{2}}$
$p + q \ge 2$
$2\ (p + q) \ge 4$
$2\ (p + q) + 5 \ge 4 + 5$
$2\ (p + q) + 5 \ge 9$
$\therefore (p + 2)\ (q + 2) \ge 9.$
$\therefore$ the minimum value = 9.
Ans: 9

48. We know that A.M. $(p_1 + p_1^2 + p_1^3) \ge$ G.M $(p_1 + p_1^2 + p_1^3)$
$= \frac{p_1 + p_1^2 + p_1^3}{3} \ge \sqrt[3]{p_1\ p_1^2\ p_1^3}$
$p_1 + p_1^2 + p_1^3 \ge 3\left(p_1\,p_1^2\,p_1^3\right)^3$
$p_1 + p_1^2 + p_1^3 \ge 3\ p_1^2$
Similarly $p_2 + p_2^2 + p_2^3 \ge 3\ p_2^2$
.
.
.
$p_n + p_n^2 + p_n^3 \ge 3\ p_n^2$
$(p_1 + p_1^2 + p_1^3)\ (p_2 + p_2^2 + p_2^3)\ \ldots\ (p_n + p_n^2 + p_n^3) \ge 3^n(p_1p_2 \ldots p_n)^2$
Choice (B)

49. B: p = 0.3, q = 0.2 satisfies B.
C: p = 0.3, q = –0.2 satisfies C.
Only B and C are possible.
Choice (D)

50. $\dfrac{x^2-5x+6}{x^2+3x+2}=\dfrac{(x-2)(x-3)}{(x+1)(x+2)} < 0$

$\dfrac{(x-2)\ (x-3)\ (x+2)\ (x+1)}{(x+1)^2\ (x+2)^2} < 0$



When x = 0 the inequation is not true
When $\in$ (–2, – 1) or [2, 3] the inequation is true.
Solution set is (–2, –1) $\cup$ [2, 3].
Choice (D)

Exercise – 10

Solutions for questions 1 to 40:

1. $2^{\frac{1}{p}} = 3^{\frac{1}{q}} = 576^{\frac{1}{r}}$. Let each of these be = k.
Then 2 = k^p, 3 = k^q and 576 = k^r.
576 = 9.64 = $3^2.2^6$
$\therefore k^r = (k^q)^2.(k^p)^6 = k^{2q}.k^{6p} = k^{6p+2q}$
Equating the powers of k both sides, r = 6p + 2q.
$\therefore \dfrac{r}{3p+q} = 2$ Ans: 2

2. $A^{\frac{1}{B}} = B^{\frac{1}{C}} = C^{\frac{1}{A}}$. Let each of these be = K.
Then A = K^B, B = K^C, and C = K^A.
$\therefore$A.B.C. = $K^{B+C+A} = K^{A+B+C}$
$\therefore K = (A.B.C)^{\frac{1}{A+B+C}}$ Choice (D)

3. Let $\sqrt[3]{12\sqrt[3]{12\sqrt[3]{12 - - - - - - - - -\infty}}}$ = y -------- (1)
$\sqrt[3]{12y} = y$
Cubing both sides, 12y = y^3
$\therefore y = \sqrt{12} = 12^{\frac{1}{2}}$ ($\because$ y is positive).
$\therefore 12^{\frac{1}{2}} = 144^x = (12^2)^x \quad 12^{\frac{1}{2}} = 12^{2x}$
Equating the powers of 12 on both sides,
$\dfrac{1}{2} = 2x \Rightarrow \dfrac{1}{4} = x$ Choice (B)

4. Let the value of the expression a a a …… be b.
$\therefore a^b = b \Rightarrow \log a = \dfrac{\log b}{b}$. As a = $e^{\frac{1}{e}}$
$\dfrac{1}{e} = \dfrac{\log b}{b}$
By inspection, b = e satisfies this equation.
Choice (B)

5. $5^{1/6} = 5^{2/12} = 25^{1/12}$
$3^{1/4} = 3^{3/12} = 27^{1/12}$
$4^{1/3} = 4^{4/12} = 256^{1/12}$
$2^{1/2} = 2^{6/12} = 64^{1/12}$
$\therefore$ Smallest is $5^{1/6}$ Choice (A)

6. Given $(3^y)^2 - 2.3^y3^2 = 243$
$(3^y)^2 - 18(3^y) = 243$
Adding 81 on both sides,
$(3^y)^2 - 2.9.3^y + 81 = 324$
$(3^y - 9)^2 = 324$
$3^y - 9 = \pm18$
3^y cannot be negative
$\therefore 3^y = 27 \quad \therefore y = 3$ Ans: 3

7. $2^{300} = (2^3)^{100} = 8^{100}$
$3^{200} = (3^2)^{100} = 9^{100}$
6 < 8 < 9.
$\therefore\ 6^{100} < 8^{100} < 9^{100}$ i.e. $6^{100} < 2^{300} < 3^{200}$ Choice (B)

8. $\dfrac{x^{2a}}{x^{2a}+x^{a+b}+x^{a+c}} = \dfrac{(x^a)^2}{(x^a)^2+(x^a)(x^b)+(x^a)(x^c)}$
$= \dfrac{x^a}{x^a+x^b+x^c}$
Similarly, $\dfrac{x^{2b}}{x^{2b}+x^{a+b}+x^{b+c}} = \dfrac{x^b}{x^a+x^b+x^c}$
and $\dfrac{x^{2c}}{x^{a+c}+x^{b+c}+x^{2c}} = \dfrac{x^c}{x^a+x^b+x^c}$
Required value
$= \dfrac{x^a}{x^a+x^b+x^c} + \dfrac{x^b}{x^a+x^b+x^c} + \dfrac{x^c}{x^a+x^b+x^c} = 1$
Ans: 1

9. $\left(\dfrac{2^{b^2}}{2^{ac}}\right)^a = \dfrac{2^{b^2a}}{2^{a^2c}} = 2^{b^2a-a^2c}$

Similarly $\left(\dfrac{2^{c^2}}{2^{ab}}\right)^b = 2^{c^2b-b^2a}$ and $\left(\dfrac{2^{a^2}}{2^{bc}}\right)^c = 2^{a^2c-c^2b}$

Required value = $2^{b^2a-a^2c+c^2b-b^2a+a^2c-c^2b} = 2^0 = 1$
Ans: 1

10. Given $5^a = 3^b = 225^{cs}$
Let $5^a = 3^b = 225^c$ = k. k must be positive.
$5^a = k \Rightarrow 5 = k^{1/a}$
$3^b = k \Rightarrow 3 = k^{1/b}$
And $225^c = k \Rightarrow (15^2)^c = k \Rightarrow 15 = k^{1/2c} \Rightarrow 3$
Consider 15 = $k^{1/2c}$
$3 \times 5 = k^{1/2c}$
$k^{1/b} \times k^{1/a} = k^{1/2c} \Rightarrow k^{1/b+1/a} = k^{1/2c}$
$\dfrac{1}{a}+\dfrac{1}{b} = \dfrac{1}{2c}$
or $\dfrac{a+b}{ab} = \dfrac{1}{2c}$
$2c = \dfrac{ab}{a+b} \Rightarrow c = \dfrac{ab}{2(a+b)}$ Choice (C)

11. Let 5^x = a.
$\Rightarrow a^2 - 16a - 225 = 0$
$a^2 - 25a + 9a - 225 = 0 \Rightarrow (a-25)(a+9) = 0$
a = 25 $\because$ a cannot be negative
$\Rightarrow 5^x = 5^2 \Rightarrow x = 2$ Ans: 2

12. Let $4^x = (0.008)^y = 10^z = k$
$4^x = k \Rightarrow 2^{2x} = k \Rightarrow 2 = k^{1/2x}$ ———— (1)
$(0.08)^y = k \Rightarrow (0.2)^{3y} = k \Rightarrow 5^{-3y} = k$
$\therefore\ 5 = k^{-1/3y}$ ———— (2)
$10^z = k \Rightarrow (2\times5)^z = k$
From (1) and (2)
$\left(k^{1/2x}k^{-1/3y}\right) = k^{1/z}$
$k^{1/2x}k_{-1/3y} = k^{1/z}$
$\Rightarrow \dfrac{1}{z} = \dfrac{1}{2x} - \dfrac{1}{3y} \Rightarrow \dfrac{1}{3y} + \dfrac{1}{z} = \dfrac{1}{2x}$ Choice (D)

13. Given $\left[(625)^{4x}\right]^{x}\left[(125)^{8y}\right]^{x}\left[(3125)(25)^{2}\right]^{y^2}$

$\Rightarrow (5^4)^{4x\times x}.(5^3)^{8y\times x}.(5^5.5^{2\times 2})^{y2} \Rightarrow 5^{16x^2}.5^{24xy}.5^{9y^2}$

$\Rightarrow 5^{16x^2+24xy+9y^2} = 5^{(4x+3y)^2}$ Choice (B)

14. $\log_b a(1+a) = 0$
$a(1+a) = b^0$
$a + a^2 = 1 \Rightarrow a^2 - 1 = -a$ Choice (C)

15. Let $k = x^{\log y - \log z}.y^{\log z - \log x}.z^{\log x - \log y}$.
Taking logarithms on both sides, we have
$\log k = (\log y - \log z)\log x + (\log z - \log x)\log y + (\log x - \log y)$
$\log z = 0$
Therefore, k = 1. Choice (B)

16. Given $a = b^2 = c^4 = d^6 = e^8$

$\therefore b = a^{\frac{1}{2}}$, $c = a^{\frac{1}{4}}$, $d = a^{\frac{1}{6}}$ and $e = a^{\frac{1}{8}}$

$\log_a abcde = \log_a a^1.a^{\frac{1}{2}}.a^{\frac{1}{4}}.a^{\frac{1}{6}}.a^{\frac{1}{8}}$

$= \log_a a^{\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}}$

$\log_a a^{\frac{24+12+6+4+3}{24}} = \log_a a^{\frac{49}{24}} = \frac{49}{24}$

Choice (B)

17. Let $x = 2^{142}$
$\therefore \log N = 142 \log 2 = 142\,(0.30103) = 42.74562$
From the values of log2, and log3 the values of log4, log5, log6 can be obtained. These are listed below.
log2 = 0.30103
log3 = 0.47712
log4 = 0.60206
log5 = 0.69897
log6 = 0.77815
$N = 10^{42.74567} = 10^{42}(10^{0.74567})$
As log5 < 0.74567 < log6, the first digit of $10^{0.74567}$ (as of N)
Ans: 5

18. $x = (1125)^{225} \Rightarrow \log x = 225 \log(1125)$
$\log x = 225 \log(9000/8)$
$\log x = 225 (\log 9000 - \log 8)$
$\log x = 225 [\log(3^2 \times 10^3) - \log(2^3)]$
$\log x = 225 [\log 3^2 + \log 10^3 - 3 \log 2]$
$\log x = 225 [2 \log 3 + 3 - 3 \log 2]$
$\log x = 225 [0.9542 + 3 - 0.9030]$
$\log x = 225 [3.9542 - 0.903] = 225(3.0512) = 686.52$
Hence, 687 digits Ans: 687

19. $\log_6 x < 1 \Rightarrow x < 6$ and $x > 1$
$\therefore 1 < x < 6$ Choice (A)

20. $\log_5 x > (0.9)^\circ \Rightarrow \log_5 x > 1$
$x > 5 \therefore x \in (5, \infty)$ Choice (B)

21. Let $\frac{\log a}{4} = \frac{\log b}{3} = \frac{\log c}{5} = k$
$\log a = 4k$; $\log b = 3k$; $\log c = 5k$
$a = 10^{4k}$; $b = 10^{3k}$; $c = 10^{5k}$
$a^2 = 10^{8k} = 10^{3k+5k} = bc$ Choice (D)

22. let $x = (2/3)^{1000} \Rightarrow \log x = 1000 [\log 2 - \log 3]$
$= 1000 [0.3010 - 0.4771] = 1000 [\bar{1}.1761]$
$= -176.1 = -177.9000$
Therefore, there will be 176 zeroes after the decimal point.
Choice (C)

23. $\log_n 2 + \log_n 3 + \ldots\ldots \log_n 100 = \log_n 100!$
Choice (C)

24. Given $\log_{15}\left(x^2-25\right) \le \log_{15}^{(4x+35)}$
$\Rightarrow x^2 - 25 \le 4x + 35$
$(\because \log_a^x \le \log_a^y \Rightarrow x \le y$ where $a > 1)$
$x^2 - 25 - 4x - 35 \le 0$
$x^2 - 4x - 60 \le 0$
$x^2 - 10x + 6x - 60 \le 0$
$x(x - 10) + 6(x + 10) \le 0$
$(x - 10)(x + 6) \le 0$
$\therefore x \in [-6, 10]$
but when $x \in [-5, 5]$, $\log(x^2 - 25)$ is not defined
$\therefore x \in [-6, -5) \cup (5, 10]$
Choice (D)

25. $\log_{\frac{1}{2}} x^2 - 6x + 3 \ge \log_{\frac{1}{2}} \frac{1}{4}$

$x^2 - 6x + 3 \le \frac{1}{4}$ $(\because \log_a x > \log_a y \Rightarrow x < y$ when $0 < a < 1)$

$x^2 - 6x + 3 - \frac{1}{4} \le 0$

$\Rightarrow 4x^2 - 24x + 11 \le 0$
$4x^2 - 2x - 22x + 11 \le 0$
$2x(2x - 1) - 11(2x - 1) \le 0$
$(2x - 1)(2x - 11) \le 0$

$x \in \left[\frac{1}{2}, \frac{11}{2}\right] \rightarrow (1)$

Since log is defined only for positive numbers, $x^2 - 6x + 3 > 0$
$x > 3 + \sqrt{6}$ or $x < 3 - \sqrt{6} \rightarrow (2)$

From (1) and (2) $\left[\frac{1}{2}, 3-\sqrt{6}\right) \cup \left(3+\sqrt{6}, \frac{11}{2}\right]$

Choice (A)

26. $p = \log_e \frac{(fg)^2}{e^3}$, $q = \log_f \frac{(eg)^2}{f^3}$ and $r = \log_g \frac{(ef)^2}{g^3}$

$e^p = \frac{(fg)^2}{e^3}$, $f^q = \frac{(eg)^2}{f^3}$ and $g^r = \frac{(ef)^2}{g^3}$

$e^p.f^q.g^r = \frac{(e^2f^2g^2)^2}{(efg)^3} = \frac{(efg)^4}{(efg)^3} = efg$

Equating the powers of p, q and r both sides, p = q = r = 1.

$\therefore$Required value $= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

Choice (B)

27. $\log\left(\sqrt{6} - \sqrt{3} - 2 + \sqrt{2}\right) + \log\left(\sqrt{6} + \sqrt{3} + 2 + \sqrt{2}\right)$

$= \log[(\sqrt{6}+\sqrt{2}) - (\sqrt{3}+\sqrt{2})][(\sqrt{6}+\sqrt{2}) + (\sqrt{3}+\sqrt{2})]$

$= \log\left[\left(\sqrt{6}+\sqrt{2}\right)^2 - \left(\sqrt{3}+2\right)^2\right]$

$= \log\left(6 + 2 + 4\sqrt{3} - 3 - 4 - 4\sqrt{3}\right) = \log 1 = 0.$

Choice (D)

28. The first inequality states that the sum of a number and its reciprocal is less than or equal to 2.
The second states that the number is positive.
$\therefore$ The reciprocal is also positive. But the sum of a positive number and its reciprocal is greater than or equal to 2.
$\therefore$ This sum is exactly equal to 2, i.e., both the number and the reciprocal are 1.

$\log_{|p-4|}|p-3| = 1 \Rightarrow |p-3| = |p-4|$

$\Rightarrow p - 4 = \frac{-1}{2}$ and $p - 3 = \frac{1}{2}$ i.e., $p = 3\frac{1}{2} = \frac{7}{2}$.

Choice (C)

29. Let logx = a, log y = b, logz = c and log w = d

$\Rightarrow$ x = e^a, y = e^b, z = e^c, and w = e^d

Now,

$$x^{\log y(\log z-\log w)}+y^{\log z(\log w-\log x)}+z^{\log w(\log x-\log y)}+w^{\log x(\log y-\log z)}$$

$$=e^{ab(c-d)}+e^{bc(d-a)}+e^{cd(a-b)}+e^{da(b-c)}$$

$$=e^{abc-abd}+e^{bcd-abc}+e^{acd-bcd}+e^{abd-acd}$$

Now, $=e^{abc-abd}+e^{bcd-abc}+e^{acd-bcd}+e^{abd-acd}$

$$\geq 4\sqrt[4]{e^{abc-abd}e^{bcd-abc}e^{acd-bcd}e^{abd-acd}}$$

= 4. Ans: 4

30. $x=\dfrac{2}{3+\sqrt{7}}$

By rationalizing the denominator = $\dfrac{2\left(3-\sqrt{7}\right)}{\left(3+\sqrt{7}\right)\left(3-\sqrt{7}\right)}$

$$\frac{2\left(3-\sqrt{7}\right)}{(3)^2-\left(\sqrt{7}\right)^2}=\frac{2\left(3-\sqrt{7}\right)}{9-7}=3-\sqrt{7}$$

$x^2 - 6x + 2 = (3-\sqrt{7})^2 - 6(3-\sqrt{7}) + 2$

$= 9 - 6\sqrt{7} + 7 - 18 + 6\sqrt{7} + 2$

$= 18 - 18 - 6\sqrt{7} + 6\sqrt{7} = 0$

Ans: 0

31. $\sqrt{\sqrt{192}-\sqrt{180}}=\sqrt{\sqrt{3}\left(\sqrt{64}-\sqrt{60}\right)}=3^{1/4}\sqrt{8-2\sqrt{15}}$

$=3^{1/4}\sqrt{5+3-2\sqrt{5\times 3}}=3^{1/4}\left(\sqrt{5}-\sqrt{3}\right)$

$\sqrt{5\sqrt{3}}-\sqrt{3\sqrt{3}}$ Choice (A)

32. $x=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$=\dfrac{3+1-2\sqrt{3}}{3-1}$

$x = 2-\sqrt{3}$

$x^4 = (2-\sqrt{3})^4 = (4 + 3 - 4\sqrt{3})^2 = (7 - 4\sqrt{3})^2$

$= 49 + 48 - 56\sqrt{3} = 97 - 56\sqrt{3}$

Similarly $\dfrac{1}{x}=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=\dfrac{\left(\sqrt{3}+1\right)^2}{2}=2+\sqrt{3}$

$\dfrac{1}{x^4}=(2+\sqrt{3})^4=97+56\sqrt{3}$

$x^4+\dfrac{1}{x^4}=97-56\sqrt{3}+97+56\sqrt{3}=194$

Choice (C)

33. $\sqrt{\dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}}=\sqrt{\dfrac{(3\sqrt{2}+2\sqrt{3})\times(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})\times(3\sqrt{2}+2\sqrt{3})}}$

$=\sqrt{\dfrac{(3\sqrt{2}+2\sqrt{3})^2}{(3\sqrt{2})^2-(2\sqrt{3})^2}}=\dfrac{3\sqrt{2}+2\sqrt{3}}{\sqrt{18-12}}$

$=\dfrac{3\sqrt{2}+2\sqrt{3}}{\sqrt{6}}=\sqrt{3}+\sqrt{2}$

Choice (C)

34. $\sqrt[4]{49-20\sqrt{6}}=\sqrt[4]{49-2\sqrt{600}}$

$=\sqrt[4]{25+24-2\sqrt{25.24}}$

$=\sqrt[4]{(5-2\sqrt{6})^2}=\sqrt{5-2\sqrt{6}}$

$=\sqrt{(\sqrt{3}-\sqrt{2})^2}$

$=\sqrt{3}-\sqrt{2}$ Choice (D)

35. Let $(9\sqrt{3}+11\sqrt{2})^{1/3}=\sqrt{x}+\sqrt{y}$

Cubing on both sides we get

$9\sqrt{3}+11\sqrt{2}=x\sqrt{x}+y\sqrt{y}+3x\sqrt{y}+3y\sqrt{x}$

$=\sqrt{x}\,(x+3y)+\sqrt{y}\,(3x+y)$

Let x + 3y = 9 $\rightarrow$ (1)

3x + y = 11 $\rightarrow$ (2)

Solving (1) and (2),

we get x = 3, y = 2

$\sqrt[3]{9\sqrt{3}+11\sqrt{2}}=\sqrt{3}+\sqrt{2}=\sqrt{a}+\sqrt{b}$

Given that a > b

$\therefore$ a = 3 and b = 2

Choice (D)

36. Let $\left(8+3\sqrt{7}\right)^{x^2-3}=a$ then

$\left(8-3\sqrt{7}\right)^{x^2-3}=\dfrac{1}{a}$

$\Rightarrow a+\dfrac{1}{a}=16\Rightarrow a^2-16a+1=0$

$\Rightarrow a=\dfrac{16\pm\sqrt{256-4}}{2}=8\pm3\sqrt{7}$

$\left(8+3\sqrt{7}\right)^{x^2-3}=\left(8\pm3\sqrt{7}\right)=\left(8+3\sqrt{7}\right)^{\pm1}$

$x^2-3=\pm1\Rightarrow x=\pm2,\pm\sqrt{2}$

Choice (D)

37. $\dfrac{\left(\sqrt{5}+\sqrt{3}\right)^2+\left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$

$\dfrac{2(5+3)}{5-3}$ $\{(a+b)^2+(a-b)^2=2(a^2+b^2)\}=\dfrac{2\times8}{2}=8$

Ans: 8

38. $P=\sqrt{2}+\sqrt{24}$, $Q=\sqrt{3}+\sqrt{16}$, $R=\sqrt{6}+\sqrt{8}$ and

$S=\sqrt{12}+2$

Each of P, Q, R and S has the form $\sqrt{x}+\sqrt{y}$ where x, y is 48 i.e., a constant.

$(\sqrt{x}+\sqrt{y})^2=x+y+2\sqrt{xy}$

$=x+y+2\sqrt{48}$

Values of x + y for P, Q, R and S are 26, 19, 14 and 16 respectively.

$\therefore$R has the least x + y.

$\therefore R^2 < S^2, Q^2, P^2$.

$\therefore$R < S, Q, P i.e R is the least of P, Q, R and S.

Choice (C)

39. $E^2=26+2\sqrt{69}$, $F^2=25+2\sqrt{114}$,

$G^2=26+2\sqrt{105}$ and $H^2=32.5+2\sqrt{114}$.

$E^2 < G^2$ and $F^2 < H^2$. Also $E^2, G^2 < H^2$ ($\because$ 26 < 32.5 and 69 < 105 < 114).

$\therefore$E, G < H.

Only Choice (A) satisfies this condition

Choice (A)

40. Let $(5\sqrt{2}+7)^{\frac{1}{3}}$ = a and $(5\sqrt{2}-7)^{\frac{1}{3}}$ = b

$5\sqrt{2} + 7 = a^3$ and $5\sqrt{2} - 7 = b^3$

$ab = ((5\sqrt{2}+7)(5\sqrt{2}-7))^{\frac{1}{3}} = ((5\sqrt{2})^2 - 7^2)^{\frac{1}{3}}$

$(50 - 49)^{\frac{1}{3}} = 1$

$a^3 + b^3 = 10\sqrt{2}$.

$10\sqrt{2} = (a + b)^3 - 3ab\,(a + b)$

$\therefore 10\sqrt{2} = (a + b)^3 - 3(a + b)$

If a + b is rational, R.H.S would be rational. But L.H.S. is irrational.

$\therefore$ a + b cannot be rational.

$\therefore$ a + b must be irrational. Choice (C)

Exercise – 11

Solutions for questions 1 and 2:

1. In order to reach the cell Q from the cell P, the ant will have to take 4 horizontal steps and 3 vertical steps. Now, this implies that ant will have to take in all 7 steps of which 4 are horizontal (H) and 3 vertical (V), i.e., HHHHVVV in any possible order. Now HHHHVVV can be arranged in 7!/4!3! = 35 ways.

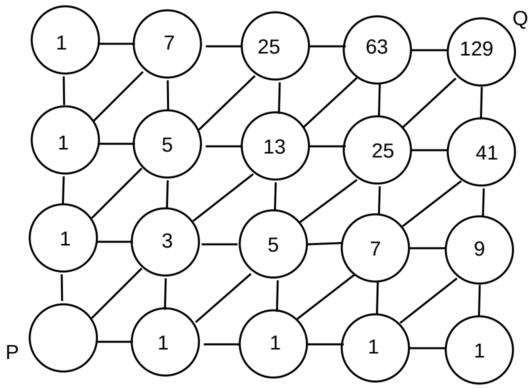
$\therefore$The number of such routes is 35. Choice (A)

2. Let H indicate each horizontal step, V a vertical step and D a diagonal step.
Initially we note that with each diagonal step the number of horizontal and vertical steps will come down by 1 each.
Now the different combinations, that can lead an ant from P to Q are.

1. HHHHVVV: Number of routes = 7!/4!3! = 35
2. HHHVVD: Number of routes = 6!/3!2! = 60
3. HHVDD: Number of routes = 5!/2!2! = 30
4. HDDD: Number of routes = 4!/3! = 4

$\therefore$ Total number of routes = 35 + 60 + 30 + 4 = 129

Alternative method:
In the diagram below, we write down the number of paths to each cell from P. The number of paths to any cell is equal to the sum of the numbers appearing in the cells that lead to the given cell.



Choice (B)

Solutions for questions 3 to 50:

3. Let the number of chocolates received by Jim, Joe and Julian be x, y and z respectively. We note that y is not the number of the card selected by Joe.
As per the given conditions x + y + z = 20; $x \geq 0$, $y \geq 0$, $z \geq 0$.
The number of solutions to x + y + z + …….r terms = n, where x, y, z, …… ≥ 0 and $n \in N$ is given by ${}^{n+r-1}C_{r-1}$

$\therefore$ Number of ways = ${}^{20+3-1}C_{3-1} = {}^{22}C_2 = 231$

Alternate method:
If Jim selected 0, Joe could select any number from 0 to 20, i.e., there are 21 way of distributing the chocolates. If Jim selects 1, there are 20 ways and so on. The total number of ways of distributing the chocolates is 21 + 20 + 19 + …… + 1 = (21) $\times$ (11) = 231. Ans: 231

4. The Venn diagram has to describe 5 sets, say A, B, C, D, E. Now, a point in a Venn diagram may belong to none, one or more of the five sets.
Number of regions belonging to all 5 sets = ${}^5C_5$
Number of regions common to only 4 sets = ${}^5C_4$
Similarly, the number of regions common to 3, 2 and 1 sets is given by ${}^5C_3$, ${}^5C_2$ and ${}^5C_1$.
Also ${}^5C_0$ gives the number of regions belonging to none of the sets.
$\therefore$ Total number of regions = ${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 = 32$. Ans: 32

5. To determine a plane uniquely, we require a set of 3 points. Hence, the number of planes that are determined by 15 points is ${}^{15}C_3$, if no 4 of them are coplanar.
Now, since 5 of the points are coplanar, instead of getting ${}^5C_3$ or 10 planes, these 5 points determine only 1 plane. .
$\therefore$ Total number of such planes = ${}^{15}C_3 - {}^5C_3 + 1$
= 455 – 10 + 1 = 446 Choice (D)

6. From the first 20 natural numbers a combination is to be selected such that when arranged in ascending order, we get an A.P.
Now, the middle element of any such combination has to be greater than 1.
For 2 as the middle element, we have only one A.P i.e., (1, 2, 3).
For 3 as the middle element, we have two A.P's possible i.e., (2, 3, 4) and (1, 3, 5). It may be noted that the number of A.P.'s with 2 as middle term is same as those with 19 as middle term and those with 3 as middle term is same as those with 18 as middle term and so on. We list below the number of A.P.'s possible in each case. For the middle element; 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 and 19, the number of A.P.'s. is 1, 2, 3, 4, 5, 6, 7, 8, 9, 9, 8, 7, 6, 5, 4, 3, 2 and 1 respectively.
$\therefore$ Total number of such combinations = 90.
Choice (B)

7. In order to get a point of intersection we need to select two line segments, i.e., we need to select two points from the first line and two from the second.
$\therefore$ The number of points of intersection = ${}^5C_2 \times {}^4C_2$
= 10 $\times$ 6 = 60 Ans: 60

8. The various ways of distributing the pencils are listed below.

C	B	A
1	2	7
1	3	6
1	4	5
2	3	5

$\therefore$ Only 4 such ways exist.
Choice (A)

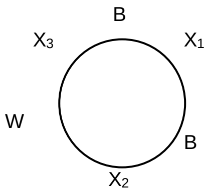
9. Number of 6 digit numbers that can be formed using 1, 2, 3, 4, 5 and 6 without repetition = 6! = 720.We are interested in finding out those numbers which are divisible by every factor of the number present in its units place. This implies that the number should be divisible by the number in the units place. Now any number having a 1, 2 and 5 in the units place is divisible by 1, 2 or 5 respectively, 3 $\times$ 5! i.e., 360 such numbers exist. Also since 1 + 2 + 3 + 4 + 5 + 6 = 21, is divisible by 3 any number with 3 in its units place is divisible by 3, 120 more such numbers exist. Numbers ending in 6 are also divisible by 6, as the number is divisible by 2 and also divisible by 3 as the sum of digits is 21. Thus, 120 more such numbers exist. Now if a number ending in 4 has to be divisible by 4, the last two digits should be 24 or 64. 48 such numbers exist.
$\therefore$ A total of 648 such numbers exist.
Ans: 648

10. Assuming that each child gets an odd number of chocolates, sum of 15 odd numbers should be odd. 50 being an even number, such a distribution is not possible. Choice (D)

11. The given word is 'SURJECTION'. Arranging the letters in alphabetical order we have CEIJNORSTU. This is the only possibility where in the letters are in alphabetical order. But since it does not satisfy the condition of vowels being in odd position, number of the arrangements of the required kind is 0. Choice (A)

12. Each line can intersect a circle at a maximum of 2 points. Hence a line can intersect 5 circles at a maximum of 10 points.
$\therefore$ 10 lines can intersect 5 circles at a maximum of 100 points. Ans: 100

13. Initially we use 2 black beads and one white bead to make a necklace, this can be done only in one way.
Now the two red beads can occupy the positions
X_2 and X_3 or X_1 and X_2
$\therefore$ Only two ways exist.



Note: The red beads occupying X_1 and X_2 or X_1 and X_3 positions will give the same arrangement.
Ans: 2

14. Arranging the letters of the word in the alphabetical order we get A, C, L, O.
The number of words starting with A are $\frac{7!}{3!\times 3!}$

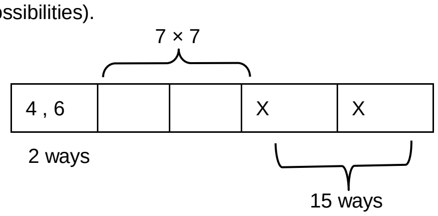
$= \frac{7\times6\times5\times4}{3\times2\times1} = 140$

The number of words starting with C are $\frac{7!}{3!\times 2!}$

$= \frac{7\times6\times5\times4}{2} = 420$

$\therefore$ The 561st word is LACCCOOO
Choice (C)

15. As the numbers lie between 40,000 and 70,000, the first digit could be 4 or 6.
Numbers whose last two digits are divisible by 4, could be: 00, 04, 40, 60, 08, 16, 36, 64, 48, 84, 76, 68, 44, 88 (15 possibilities).



The second and third digits can be selected in 7×7 ways.
$\therefore$The total required numbers are:
$= 2(7^2)(15) = 1470$ Ans: 1470

16. We need the number of derangements of the word ROBUST. This is

$= 6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)$

$= 720\left(\frac{360-120+30-6+1}{720}\right)$

$= 391 - 126 = 265.$
Choice (A)

17. Let the number of umbrellas hung in a row be n
We have,
${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 511$
$\Rightarrow 2^n - 1 = 511$
$\Rightarrow 2^n = 512 = 2^9 \Rightarrow n = 9$
The number of ways of selection four umbrellas such that no two are consecutive
${}^{n-r+1}C_r = {}^{9-4+1}C_4$
$= {}^{n-r+1}C_r = {}^{9-4+1}C_4$
$= {}^6C_4 = {}^6C_2 = \frac{6(5)}{2} = 15$.
Choice (D)

18. The number of ways in which 5 different coins can be placed into 3 different boxes is the number of onto functions that can be defined from A with 5 elements to B with 3 elements.
$= 3^5 - {}^3C_1(3-1)^5 + {}^3C_2(3-2)^5 - {}^3C_3(3-3)^5$
$= 243 - 3(32) + 3(1) - 0$
$= 243 - 96 + 3 = 150.$ Ans: 150

19. The number of ways in which 5 different coins can be placed into 3 different boxes is equal to the number of all possible mappings that can be defined from A with 5 elements to B with 3 elements. This is $= 3^5 = 243$.
Choice (B)

20. We need the number of positive integral solutions of $x_1 + x_2 + x_3 = 10$
where $x_1 \geq 2$, $x_2 \geq 2$, and $x_3 \geq 2$,
i.e., $x_1 - 2, \geq 0$ $x_2 - 2 \geq 0$ and $x_3 - 2 \geq 0$
let $X_1 = x_1 - 2$, $X_2 = x_2 - 2$, and $X_3 = x_3 - 2$,
So that $X_1 . X_2 . X_3 \geq 0$
and equation (1) yields
$X_1 + X_2 + X_3 = 4$ ---------- (2)
Now, the required number of ways is the
= number of nonnegative integral solutions of eq (2)
$= 4 + 3 - 1\ C_{3-1} = {}^6C_2 = 15.$
Ans: 15

21. We need the number of positive integral solutions of $x_1 + x_2 + x_3 = 20$……(1), where $3 \leq x \leq 8$ for $\ell = 1, 2, 3$. or of $y_1 + y_2 + y_3 = 14$………(2), where $1 \leq y_i \leq 6$ for $\ell = 1, 2, 3$. All the solutions of (2) can be easily listed as shown below.
$y_1 + y_2 + y_3 = 14$

2	6	6
3	5	6
	6	5
4	4	6
	5	5
	6	4
5	3	6
	4	5
	5	4
	6	3
6	2	6
	3	5
	4	4
	5	3
	6	2

Thus, there are 15 ways of placing 20 identical coins into 3 different boxes.

Alternate method:
The required number of ways (say N) is equal to the number of positive integral solutions of
$x_1 + x_2, + x_3 = 20$, where $3 \leq x_1, x_2, x_3 \leq 8$
$\therefore$ N = coefficient of x^{20} in $(x^3 + x^4 + \ldots. + x^8)^3$
= coefficient of x^{20} in $x^9(1 + x + \ldots + x^5)^3$

= coefficient of x^{11} in $\left(\frac{1-x^6}{1-x}\right)^3$

$(1 - x^6)^3 = (1 - 3x^6 + 3x^{12} - x^{18})$

$(1 - x)^{-3} =$

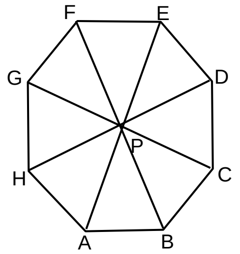
$(1 + 3x + \frac{3(4)}{2}x^2 + \ldots + \frac{3(4)(5)(6)(7)}{1(2)(3)(4)(5)}x^5 + \ldots + \frac{3(4)\ldots..(13)}{1(2)\ldots..(11)}$ $x^{11} + \ldots)$

$\therefore N = \frac{(-3)3(4)(5)(6)(7)}{1(2)(3)(4)(5)} + \frac{12(13)}{1(2)}$

$= -63 + 78 = 15.$

Choice (D)

22.



We have a total of 9 points (8 vertices and the centre P). The three points (two vertices and the centre P), on each of the diagonals AE, BF, CG and DH correspond to a straight line, and not a triangle.

∴ The total number of triangles is ${}^9C_3 - 4$. The number of triangles that do not include A or E is ${}^7C_3 - 3$

$\left({}^9C_3 - 4\right) - \left({}^7C_3 - 3\right) = {}^9C_3 - {}^7C_3 - 1$

$= \frac{9(8)(7)}{6} - \frac{7(6)(5)}{6} - 1 = 48.$ Choice (D)

23. The person who gets 2 objects can be selected in ${}^5C_1$ ways. The two objects to be given can be selected in ${}^6C_2$ ways. The other 4 objects can be given to the other 4 persons (one to each) in 24 ways. Therefore the required number of ways is ${}^5C_1{}^6C_2$ (24) = 5(15)(24) = 1800

Ans: 1800

24. At first 5 boys can be arranged in 5! ways as shown,

B	×	B	×	B	×	B	×	B
↓		↓		↓		↓		
x_1		x_2		x_3		x_4		

Let x_1, x_2, x_3 and x_4 be the number of girls in between successive boys

i.e., $x_1 + x_2 + x_3 + x_4 = 18$ ---- (1)

where $x_1 = 2n_1 - 1$, $x_2 = 2n_2 - 1$, $x_3 = 2n_3 - 1$ and x_4 $2n_4 - 1$ and $n_1, n_2, n_3, n_4 \in N$

Eq (1) reduces to $2(n_1 + n_2 + n_3 + n_4) = 18 + 4$

$\Rightarrow n_1 + n_2 + n_3 + n_4 = 11$

The number of positive solutions is ${}^{n-1}C_{4-1}$ or ${}^{10}C_3$

Number of ways in which the 18 girls and 5 boys can be arranged = 18! 5! ${}^{10}C_3$

= 18! (120) $\frac{10(9)(8)}{6}$ = 120^2 (18!)

∴ Required number of ways = $(120)^2$ 18! = (14400) (18!)

Choice (A)

25. We have $x_1\ x_2\ x_3\ x_4 = 600 = 2^3\ 3^1\ 5^2$ ----- (1)

The number of positive integral solutions of (1)

= ${}^6C_3\ {}^4C_3\ {}^5C_3$ = 20 (4) (10) = 800

Ans: 800

Note: The 3 identical 2's have to be divided into 4 distinct groups we need the number of nonnegative integral solutions of a + b + c + d = 3, which is equal to the number of positive integral solutions of $a_1 + b_1 + c_1 + d_1 = 7$, which is ${}^6C_3$. Similarly, the one 3 can be divided into 4 distinct groups in ${}^4C_3$ ways and the two 5's can be divided into 4 distinct groups in ${}^5C_3$ ways. The number of integral solutions of (1) is = 2^{4-1} (800) = 6400

26. The purse contains 50 paise and one-rupee coins in the ratio 1 : 2.

If some coins are selected at random from the box, the expected value of the ratio of the 50 paise coins and the 1 rupee coins is 1 : 2.

∴ If a coin is picked from the selection the probability of getting a one-rupee coin is 2/3. Choice (C)

27. Let P(A), P(B) and P(C) represent the probability of A, B and C attending the class respectively.

Given $P(A) + P(B) - 2P(A \cap B) = 7/10$

$P(B) + P(C) - 2P(B \cap C) = 4/10$

$P(C) + P(A) - P(A \cap C) = 7/10$

$P(A \cap B \cap C) = 9/100$

Now, P(at least one attending the class) = $P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{1}{2}[P(A) + P(B) - 2P(A \cap B) + P(B) + P(C) - 2P(B \cap C) + P(C) + P(A) - 2P(A \cap A)] + P(A \cap B \cap C)$

$P(A \cup B \cup C) = 1/2\ [7/10 + 4/10 + 7/10] + 9/100$

= 9/10 + 9/100 = 99/100 Choice (D)

28. Listed below are the combinations and their respective probabilities, in which the third draw can yield an apple.

Case 1: P(OOA) = 3/5 × 1/3 × 1

Case 2: P(AAA) = 2/5 × 3/6 × 4/7

Case 3: P(OAA) = 3/5 × 2/3 × 3/4

Case 4: P(AOA) = 2/5 × 3/6 × 3/4

∴ Required probability = 1/5 + 4/35 + 3/10 + 3/15

$= \frac{28 + 16 + 42 + 21}{140} = \frac{107}{140}$

Choice (A)

29. Since Anju should pick a ball numbered lesser than that picked by Anshu and in turn Anshu should pick a ball numbered lesser than Anitha, the number of ways is given by the combinations of three distinct numbers that we can select from the numbers 1 to 10.

∴ Favorable ways = ${}^{10}C_3$

Total number of ways = 10 × 10 × 10 = 1000

Required probability = ${}^{10}C_3$/1000 = 3/25

Choice (A)

30. The final card picked has to be queen or heart.

∴ The total number of ways in which the final card can be picked = (4 + 13) – 1 = 16

Number of favourable ways is 4, as we have to pick a queen.

∴ Required probability = 4/16 = 1/4

Choice (C)

31. Number of ways of arranging Ram and Shyam in any two of the ten positions available is ${}^{10}P_2$.

Now, if three students are required to be in between Ram and Shyam, the positions that Ram and Shyam can occupy are given by (1, 5) (2, 6) (3, 7) (4, 8) (5, 9) or (6, 10) in each of these positions they can arrange themselves in 2 ways.

∴ Required probability = 12/${}^{10}P_2$ = 12/10 × 2 = 2/15

Choice (C)

32. Without getting a 6 on any die the maximum sum that can be obtained is 15. Hence, we find the probability of getting the sum as 14 or 15. Given P(5) = 1/4 and P(1) = P(2) = P(3) = P(4) = P(6) = x.

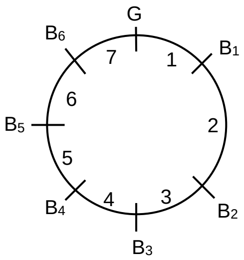
∴ 5x + 1/4 = 1 or x = 3/20

Combinations	Arrangements	Probability
(5, 5, 5)	1	1/4 × 1/4 × 1/4 = 1/64
(5, 5, 4)	3	3(1/4 × 1/4 × 3/20) = 9/320

∴ Required probability = 1/64 + 9/320 = 14/320 = 7/160

Choice (D)

33. Let the 6 boys and a girl be positioned as shown in the figure.



Now the second girl can occupy one of the seven places, numbered 1, 2, ……, 7.
To have at least 2 boys in between the 2 girls in both directions, the second girl can take the position 3, 4 or 5
$\therefore$ Required probability = 3/7

Choice (B)

34. Let p be the probabilty of success and q be the probability of a failure. p = 4/52 = 1/13 and q = 12/13
$\therefore$ The probability of A winning (when A starts the game)
= p + qqp + qqqqp + ……
= $1/13 + (12/13)^2\ 1/13 + (12/13)^{14}\ 1/13 + \ldots$

$$= \frac{\frac{1}{13}}{1-\frac{144}{169}} = \frac{13}{25}$$

$\therefore$ The probability of B winning = 12/25
$\Rightarrow$ Expectation of B = 12/25 $\times$ 400 = ₹192

Ans: 192

35. As per the description of the game, the winner is selected purely by chance and no person has an advantage over any other person i.e., each person has an equal probability of winning.
$\therefore$ Probability of A winning = 1/5

Choice (C)

36. Irrespective of when A, B and C deliver their speech with respect to the other speakers, we look at the order only in terms of A, B and C. The number of ways A, B and C can be arranged = 3! = 6. Out of these 6 ways there exists only one order which is favourable.
$\therefore$ Required probability = 1/6

Choice (B)

37. Total number of 6 digit numbers = 900000. The first digit can be 2, 4, 6 or 8. (4 possibilities). The third and fifth can be 0, 2, 6, 4 or 8 (5 possibilities each). The second, fourth and sixth digit can be 1, 3, 5, 7 or 9 (5 possibilities each). Total number of numbers of the required kind $4(5)^2\ (5)^3 = 4(5)^5$

$\therefore$The required probability = $\dfrac{4(5)^5}{9(2)^5\ (5)^5} = \dfrac{4}{288} = \dfrac{1}{72}$

Choice (C)

38. The favourable cases are HTHT or THTH
P(HTHT) = 1/2 $\times$ 1/2 $\times$ 1/2 $\times$ 1/2 = 1/16
P(THTH) = 1/2 $\times$ 1/2 $\times$ 1/2 $\times$ 1/2 = 1/16
$\therefore$ Required probability = 1/16 + 1/16 = 2/16 = 1/8

Choice (B)

39. Probability that no die shows a 6 = $(5/6)^4$
$\therefore$ probabiltiy that at least one die shows a 6
= $1 - (5/6)^4$

Choice (A)

40. Let x_1, x_2, x_3 and x_4 represent number of white, red, green and yellow coloured marbles contained in the selection of 12 marbles.
Number of ways of selecting 12 marbles is equal to the number of non-negative integral solutions of
$x_1 + x_2 + x_3 + x_4 = 12$
Total number of ways = ${}^{12+4-1}C_{4-1}$
$= {}^{15}C_3 = \dfrac{15(14)(13)}{3!}$
$= \dfrac{15(14)(13)}{6}$
= 5(7)(13)
The number of selections that contain at least one marble of each colour is equal to the number of positive integral solutions of $x_1 + x_2 + x_3 + x_4 = 12$
$= {}^{12-1}C_{4-1} = {}^{11}C_3 = \dfrac{11(10)(9)}{6} = 11(5)(3)$
Required probability = $\dfrac{11(5)(3)}{5(7)(13)} = \dfrac{33}{91}$

Choice (C)

41. Let x_1, x_2, x_3 and x_4 represent number of white, red green and yellow coloured marbles contained in the selection of 12 marbles
The number of ways of selecting 12 marbles is equal to the number of non–negative integral solutions of
$x_1 + x_2 + x_3 + x_4 = 12$
i.e., Total number of ways = ${}^{12+4-1}C_{4-1} = {}^{15}C_3$ = 5(7)(13)
The number of ways of selecting at least one of each colour and a distinct number of each colour is equal to the number of distinct positive integral solutions of $x_1 + x_2 + x_3 + x_4 = 12$
Without loss of generality, let $x_1 > x_2 > x_3 > x_4$
Let $x_2 = x_1 + \alpha$
$x_3 = x_2 + \beta$
$x_4 = x_3 + \gamma$ where α,β and γ are positive integers
Now, we have, $4x_1 + 3\alpha + 2\beta + \gamma = 12$
Where $x_1 = 1$, $\alpha = 1$, $1 \le \beta \le 2$, $1 \le \gamma \le 3$
number of ways
= 4! [coefficient of x^{12} in $x^4 . (x^3)\ (x^2 + x^4)\ (x + x^2 + x^3)$]
= 4! [coefficient of x^5 in $(x^2 + x^4)\ (x + x^2 + x^3)$]
= 4! [coefficient of x^5 in $(x^3 + x^4 + x^5 + x^5 + x^6 + x^7)$]
= 4! [coefficient of x^5 in $(x^3 + x^4 + 2x^5 + x^6 + x^7]$
= 4! (2) = 48
Required probability = $\dfrac{48}{5(7)(13)} = \dfrac{48}{455}$

Choice (A)

42. The data is chain in the diagram below

	A	B	C
Black	3	4	2
White	5	4	6

A prior Probability that a particular bag in selected is 1/3.The probability that white ball in selected is

$$\frac{1}{3}\left(\frac{5}{8}\right)+\frac{1}{3}\left(\frac{4}{8}\right)+\frac{1}{3}\left(\frac{6}{8}\right)=\frac{1}{3}\left(\frac{15}{8}\right)$$

The probability (a posterior) that the bag selected was C is

$$\frac{\frac{1}{3}\left(\frac{6}{8}\right)}{\frac{1}{3}\left(\frac{15}{8}\right)}=\left(\frac{6}{15}\right)=\frac{2}{5}$$

Choice (D)

43. The 8 numbers are

235	435
257	457
337	537
355	555

$P(A) = \frac{2}{8} = \frac{1}{4}$ $\quad P(A \cap B) = \frac{1}{8}$

$P(B) = \frac{4}{8} = \frac{1}{2}$ $\quad P(A \cap C) = \frac{1}{8}$

$P(C) = \frac{4}{8} = \frac{1}{2}$ $\quad P(B \cap C) = \frac{2}{8} = \frac{1}{4}$

As $P(A \cap B) = P(A)\ P(B)$, A,B are independent
As $P(A \cap C) = P(A)\ P(C)$, A,C are independent
As $P(B \cap C) = P(B)\ P(C)$, B,C are independent

Choice (C)

44.

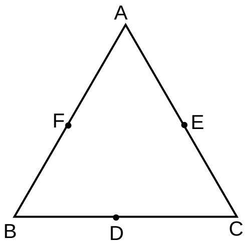


Fig.1

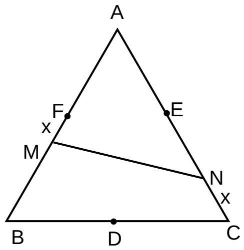


Fig.2

In one extreme position of the string MN, M would coincide with D, the midpoint of BC, while N would coincide with A. We can denote this as DBA. We can then slide the string along the perimeter of ABC. Three other positions of MN are BAE, FAC and ACD. This is the entire range of positions of string MN, as MN has to pass over A.

In any position, the segment MN (not string MN) would divide ABC into 2 parts – a smaller triangle and a quadrilateral. (In the 4 positions mentioned above both regions would be triangles) When M is on DB, A would be in the quadrilateral. When M is on BF, A would be in the triangle.

When M is on FA, (N would be on CD) and A would again be in the quadrilateral.

One of these positions (when M is on BF and N on EC) is shown in figure 2. Let the length of the string along AB (i.e., AM) be 1 + x. The length along AC (i.e., AN) is 2 – x (Also $0 \le x \le 1$). The area of the triangular part is (1/2)(1 + x)(2 –x)sin 60° = (1/2)(2 + x – x^2)sin 60°

while that of ΔABC is (1/2)(2)(2) sin 60°.

$\therefore$ The area of the quadrilateral is (1/2)[4 – (1 + x)(2 – x)] sin 60° = (1/2)(x^2 – x + 2) sin 60°. We can see immediately that the area of the triangular part is more

[$\because 2 + (x - x^2) > 2 - (x - x^2)$]

$\therefore$ If M lies on BF, A lies in the triangular part and the area of this part is more than half the area of the triangle. On the other hand, if M lies on DB or FA, A lies in the quadrilateral and the area of this part is less than half the area of the triangle.

$\therefore$ The required Probability

$= \dfrac{DB + FA}{DB + BF + FA} = \dfrac{2}{3}$

Choice (D)

45. The expected gain in the long run is

$$\frac{1}{3}(10) + \frac{1}{3}(20) + \frac{1}{3}\left[\frac{1}{3}(10) + \frac{1}{3}(20) + \frac{1}{3}(.....)\right]$$

Regrouping the coefficients of 10 and 20, we get

$$10\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ...\right) + 20\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ...\right)$$

= 10(1/2) + 20(1/2) = 15 Ans: 15

46. Given y = |x – 1| + |x – 2| + …. + |x –100|, represents the absolute deviation of the elements 1, 2, 3, 4, ……, 100 with respect to x.

We know that the sum of the absolute deviations is minimum about the median.

In this case the median can be any element such that $50 \le x \le 51$. Therefore, there are infinitely many values of x for which y has its minimum value. Choice (D)

47. Given that the sum of the deviations of the elements x_1, x_2, x_3, ……. x_i about 25 is – 2.

i.e., $(x_1 - 25) + (x_2 - 25) + (x_3 - 25) + …. + (x_{10} - 25)$

$= -2 \therefore x_1 + x_2 + x_3 + ….. + x_{10} = 248$.

Ans: 248

48. A.M. (1, 2, 3, 4, ……, n) = $\bar{x}$

Now, A.M. (2 × 1, 3 × 2, 4 × 3, 5 × 4, ..., (n + 1) n)

= A.M. (1^2 + 1, 2^2 + 2, 3^2 + 3, 4^2 + 4, …., n^2 + n)

$$= \frac{(1^2 + 2^2 + ...+ n^2) + (1 + 2 + 3 + ...+ n)}{n}$$

$$= \frac{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{n}$$

$$= \frac{(n+1)(2n+1)}{6} + \frac{(n+1)}{2}$$

$$= \frac{(2n+4)(n+1)}{6}$$

Choice (C)

49. Var ($5x_i$ + 7) = 225

SD ($5x_i$ + 7) = 15

SD ($5x_i$) = 15

SD (x_i) = 3

SD ($7x_i$ + 5) = 7 × SD (x_i) = 7 ×3 = 21

Choice (A)

50. Given SD = 2; $\bar{x}$ = 9, n = 15

$$SD = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$2 = \sqrt{\frac{\Sigma x_i^2}{15} - (9)^2}$$

$$4 = \frac{\Sigma x_i^2}{15} - 81$$

$$85 = \frac{\Sigma x_i^2}{15}$$

Σx_i^{2} = 85 × 15 = 1275

Ans: 1275