

CHAPTER – 11

CLOCKS

The hour hand and the minute hand of a clock move in relation to each other continuously and at any given point of time, they make an angle between 0° and 180° with each other.

If the time shown by the clock is known, the angle between the hands can be calculated. Similarly, if the angle between two hands is known, the time shown by the clock can be found out.

When we say angle between the hands, we normally refer to the acute/obtuse angles (upto 180°) between the two hands and not the reflex angle ($> 180^\circ$).

For solving the problems on clocks, the following points will be helpful.

- Minute hand covers 360° in 1 hour, i.e., in 60 minutes. Hence, MINUTE HAND COVERS 6° PER MINUTE.
- Hour hand covers 360° in 12 hours. Hence, hour hand covers 30° per hour. Hence, HOUR HAND COVERS $1/2^\circ$ PER MINUTE.

The following additional points also should be remembered. In a period of 12 hours, the hands make an angle of

- 0° with each other (i.e., they coincide with each other), 11 times.
- 180° with each other (i.e., they lie on the same straight line but pointing in two directions), 11 times.
- Any angle other than 0° and 180° occur with each other, 22 times.

Note: We can also solve the problems on clocks using the method of "Relative Velocity".

In 1 minute, Minute Hand covers 6° and Hour hand covers $1/2^\circ$.

Therefore, Relative Velocity = $6 - 1/2 = 5\frac{1}{2}^\circ$ per minute. Alternately, in 1 hour, the minute hand covers 60 minute divisions whereas the hour hand covers 5 minute divisions.

\therefore Relative Speed = $60 - 5 = 55$ minutes per hour. However, adopting the approach of actual angles covered is by far the simplest and does not create any confusion.

Points to Note:

- Any angle other than 0° and 180° is made 22 times in a period of 12 hours.
- In a period of 12 hours, there are 11 coincidences of the two hands, when the two hands are in a straight line and facing and opposite directions.
- The time gap between any two coincidences is $12/11$ hours or $65\frac{5}{11}$ minutes.
- If the hands of a clock (which do not show the correct time) coincide every p minutes, then
If $p > 65\frac{5}{11}$, then the watch is going slow or losing time.
If $p < 65\frac{5}{11}$, then the watch is going fast or gaining time.
To calculate the angle ' θ ' between the hands of a clock, we use the following formula (where m = minutes and h = hours)

$$(i) \quad \theta = \frac{11}{2}m - 30h \quad \left(\text{when } \frac{11}{2}m > 30h \right)$$

$$(ii) \quad \theta = 30h - \frac{11}{2}m \quad \left(\text{when } 30h > \frac{11}{2}m \right)$$

Worked Out Examples:

11.01. What is the angle between the minute hand and the hour hand of a clock at 3 hours 40 minutes?

- (A) 20° (B) 70°
(C) 90° (D) 130°

Sol. The angle between the hands can be calculated by $\theta = \left| \frac{11}{2}m - 30h \right|$, where m is minutes and h is hours. Here, $m = 40$ and $h = 3$

$$\therefore \theta = \left| \frac{11}{2} \times 40 - 30 \times 3 \right| = |220 - 90| = 130^\circ$$

The angle between the two hands is 130° .

Choice (D)

11.02. Find the time between 2 and 3 O'clock at which the minute hand and the hour hand

- (i) make an angle of 60° with each other.
(ii) overlap.
(iii) are perpendicular to each other.
(iv) are on the same straight line but are facing opposite directions.

Sol. (i) In the formula $\theta = \left| \frac{11}{2}m - 30h \right|$,

$$\theta = 60^\circ \text{ and } h = 2$$

$$\therefore 60 = \frac{11}{2}m - 30 \times 2$$

$$\frac{11}{2}m = 120$$

$$m = \frac{240}{11} = 21\frac{9}{11} \text{ min past 2}$$

or

$$60 = 30 \times 2 - \frac{11}{2}m$$

$$\therefore \frac{11}{2}m = 0$$

$$m = 0$$

Therefore, the angle between the hour hand and the minute hand is 60°

at 2 O'clock and at $21\frac{9}{11}$ minutes past 2 O'clock.

(ii) When the two hands overlap, the angle between them is 0° .

$$\theta = \left| \frac{11}{2}m - 30h \right|$$

$$\therefore \theta = 0^\circ \text{ and } h = 2$$

$$\frac{11}{2}m = 30 \times 2$$

$$m = \frac{120}{11} = 10\frac{10}{11} \text{ min past 2.}$$

- (iii) When two hands are perpendicular,
 $\theta = 90^\circ$ and $h = 2$

$$\therefore \theta = \left(\frac{11}{2}m - 30h \right) \text{ or } \left(30h - \frac{11}{2}m \right)$$

$$90 = \frac{11}{2}m - 30 \times 2$$

$$\frac{11}{2}m = 150$$

$$m = \frac{300}{11} = 27 \frac{3}{11} \text{ minutes past 2.}$$

or

$$90 = 30 \times 2 - \frac{11}{2}m$$

$$\frac{11}{2}m = -30$$

As m cannot be negative, this case is not possible.

So, the hands are perpendicular to each other only once i.e. at $27 \frac{3}{11}$ minutes past 2 O'clock.

- (iv) When two hands are pointing opposite directions and are on a straight line the angle between them would be 180° . i.e.
 $\theta = 180^\circ$ and $h = 2$.

$$180^\circ = \frac{11}{2}m - 30h$$

$$\frac{11}{2}m = 180 + 60 = 240$$

$$m = \frac{480}{11} = 43 \frac{7}{11}$$

So, at $43 \frac{7}{11}$ minutes past 2 O'clock the hands will be at 180° .

Exercise – 11

Directions for questions 1 to 25: Select the correct alternative from the given choices.

- How many degrees does an hour-hand move in 10 minutes?
 (A) 10° (B) 20° (C) 15° (D) 5°
- How many degrees will the minute-hand move, in the same time, in which the hour-hand moves 10° ?
 (A) 40° (B) 80° (C) 90° (D) 120°
- How many degrees will the minute-hand move, in the same time in which the second hand moves 300° ?
 (A) 6° (B) 5° (C) 4° (D) 10°
- A boy observes the reflection of a wall clock in a mirror. The time observed by the boy in the mirror is 4 hours 20 minutes. What is the actual time shown on the clock?
 (A) 7 hours 15 minutes (B) 7 hours 50 minutes
 (C) 7 hours 40 minutes (D) 7 hours 35 minutes
- If the time in a clock is 7 hours 15 minutes, then what time does it show on the mirror?
 (A) 4 hours 50 minutes (B) 4 hours 40 minutes
 (C) 4 hours 35 minutes (D) 4 hours 45 minutes
- At what angle are the hands of a clock inclined at 4 hours 20 minutes?
 (A) 5° (B) 10° (C) 20° (D) 25°
- The time on the watch is 4:30. If the minute hand points towards the south, the hour hand will point towards
 (A) South-east (B) East
 (C) West (D) North-west
- What is the angle between the two hands of a clock, when the clock shows 3 hours 25 minutes?
 (A) $45\frac{1}{2}^\circ$ (B) 46° (C) $46\frac{1}{2}^\circ$ (D) $47\frac{1}{2}^\circ$
- What is the angle between the two hands of a clock, when the time is 2 hours 35 minutes?
 (A) $122\frac{1}{2}^\circ$ (B) $142\frac{1}{2}^\circ$
 (C) $132\frac{1}{2}^\circ$ (D) $116\frac{1}{2}^\circ$
- At what time between 6 and 7 O'clock, are the hands of a clock together?
 (A) 6 hours $32\frac{8}{11}$ minutes
 (B) 6 hours $33\frac{9}{11}$ minutes
 (C) 6 hours $34\frac{5}{11}$ minutes
 (D) 6 hours $29\frac{7}{11}$ minutes
- At what time between 3 and 4 O'clock are the hands of a clock in the opposite direction?
 (A) 3 hours $48\frac{6}{11}$ minutes
 (B) 3 hours $53\frac{2}{11}$ minutes
 (C) 3 hours $50\frac{4}{11}$ minutes
 (D) 3 hours $49\frac{1}{11}$ minutes
- At what angle are the hands of a clock inclined at 20 minutes past 7?
 (A) 80° (B) 90°
 (C) 100° (D) 120°
- The angle between the two hands of a clock is 70° , when the hour hand is between 7 and 8. What time does the watch show?
 (A) 7 hours $50\frac{10}{11}$ minutes
 (B) 7 hours $25\frac{5}{11}$ minutes
 (C) 7 hours $42\frac{8}{11}$ minutes
 (D) Both (A) and (B)
- What time does the clock show when the hour hand is between 3 and 4 and the angle between the two hands of the clock is 50° ?
 (A) $8\frac{5}{11}$ min past 3
 (B) $25\frac{5}{11}$ min past 3
 (C) $24\frac{6}{11}$ min past 3
 (D) Both (A) and (B)
- At what time between 5 and 6 O'clock, will the hands of a clock be at an angle of 62° ?
 (A) 5 hours $17\frac{2}{11}$ minutes
 (B) 5 hours $38\frac{6}{11}$ minutes
 (C) 5 hours 16 minutes
 (D) Both (B) and (C)

16. A watch, which gains uniformly, was observed to be 4 minutes slow at 6 a.m. on a Monday. On the subsequent Thursday at 7 p.m. it was noticed that the watch was 6 minutes fast. When did the watch show the correct time?
 (A) 5 p.m. Tuesday
 (B) 4 p.m. Tuesday
 (C) 6 p.m. Tuesday
 (D) 3 p.m. Tuesday
17. The minute-hand of a clock overtakes the hour-hand at intervals of 66 minutes of the correct time. How much in a day does the clock gain or lose?
 (A) $10^{113/121}$ minutes
 (B) $11^{115/121}$ minutes
 (C) $11^{109/121}$ minutes
 (D) $10^{104/121}$ minutes
18. The minute-hand of a clock overtakes the hour-hand at intervals of 62 minutes of a correct time. How much in a day does the clock gain or lose?
 (A) $80^{80/341}$ minutes
 (B) $85^{90/311}$ minutes
 (C) $80^{70/341}$ minutes
 (D) $80^{60/341}$ minutes
19. A watch, which gains uniformly, was observed to be 5 minutes slow at 12 noon on Monday. It was noticed 10 minutes fast at 6 p.m. on the next day. When did the watch show the correct time?
 (A) 9 p.m., the same day.
 (B) 9 hours 30 minutes p.m., the same day.
 (C) 10 hours 30 minutes p.m., the same day.
 (D) 10 p.m., the same day
20. A watch showed 5 minutes past 3 O'clock on Sunday evening when the correct time was 3 O'clock. It loses uniformly and was observed to be 10 minutes slow on the subsequent Tuesday at 9 p.m. When did the watch show the correct time?
 (A) 8:00 a.m. Monday
 (B) 10:00 a.m. Monday
 (C) 7:00 a.m. Monday
 (D) 9:00 a.m. Monday
21. A clock is set to show the correct time at 8 a.m. The clock gains 10 minutes in a day. What will be the approximate time, when the watch indicates 4 p.m. the next day?
 (A) 3 hours 36 minutes
 (B) 3 hours 47 minutes
 (C) 3 hours 50 minutes
 (D) 3 hours 54 minutes
22. A clock is set to show the correct time at 10 a.m. The clock loses 12 min in a day. What will be the appropriate time when the watch indicates 5 p.m. on the next day?
 (A) 4:35 p.m. (B) 4:55 p.m.
 (C) 4:45 p.m. (D) 4:50 p.m.
23. A watch, which loses uniformly was observed to be 12 minutes fast at 4 a.m. on 6th of a month. It showed 20 minutes less than the correct time at 6 p.m. on the 10th of the same month. When did the watch show the correct time?
 (A) 9:15 p.m. on the 7th.
 (B) 9:50 a.m. on the 8th.
 (C) 9:35 p.m. on the 9th.
 (D) 9:20 p.m. on 7th.
24. There are two clocks on a wall, both set to show the correct time at 8 a.m. One clock loses two minutes in an hour while the other gains one minute in one hour. By how many minutes do the two clocks differ at 12 noon on the same day?
 (A) 6 minutes (B) 9 minutes
 (C) 8 minutes (D) 12 minutes
25. There are two clocks on a wall, both set to show the correct time at 12 noon. Both the clocks gain 1 minute and 2 minutes respectively in an hour. If the clock which gains 1 minute in one hour shows the time as 8 minutes past 8 p.m. on the same day, then what time does the other watch show?
 (A) 8 hours 4 minutes
 (B) 8 hours 8 minutes
 (C) 8 hours 16 minutes
 (D) 7 hours 52 minutes

Key

Exercise – 11

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| 1. D | 6. B | 11. D | 16. B | 21. B |
| 2. D | 7. A | 12. C | 17. C | 22. C |
| 3. B | 8. D | 13. D | 18. A | 23. A |
| 4. C | 9. C | 14. B | 19. D | 24. D |
| 5. D | 10. A | 15. D | 20. D | 25. C |