Solutions for SM1002108

Chapter – 1
(Numbers – I)

Concept Review Questions

Solutions for questions 1 to 75:

1. $3^6 = 729$ Ans : (729)

2. $2^{10} = 1024$
$2^{15} = (2^{10})(2^5) = (1024)(32) = 32768$. Ans : (32768)

3. The sum of an even number of odd numbers is always even. Choice (A)

4. The product of 2 or more even numbers is always even. Choice (A)

5. If all the numbers are even, the sum is even. If one of them is even and the others are odd, the sum is odd.
$\therefore$ We cannot say. Choice (C)

6. If all of them are odd, the product is odd. If one of them is even, the product is even.
$\therefore$ we cannot say. Choice (C)

7. If at least one of them is 2, the product is even, otherwise the product is odd.
$\therefore$ we cannot say. Choice (C)

8. 35 is odd
If the sum of an odd number of prime numbers is even, then one of them is always 2. Ans : (2)

9. $13013 = 13 (1001) = 13 (13) (11) (7)$
$= 13^2 \times (11) \times (7)$
$\therefore$ 13013 has 3 distinct prime factors. Ans : (3)

10. Let $x = 0 \cdot 2\overline{77}$
$10x = 2 \cdot \overline{77} \rightarrow$ (1)
$100x = 27 \cdot \overline{77} \rightarrow$ (2)
Subtracting (1) from (2), we get $90x = 25$
$\therefore \; x = \frac{5}{18}$ Choice (A)

11. Let $x = 0.4\overline{56}$
$1000x = 456 \cdot \overline{56}$ -------- (1)
$10x = 4.56$ -------- (2)
Subtracting (2) from (1), we get
$990x = 452$
$x = \frac{226}{495}$. Choice (D)

12. Let $x = 0 \cdot 12\overline{3}$
$100x = 12 \cdot \overline{3}$ -------- (1)
$1000x = 123 \cdot \overline{3}$ -------- (2)
Subtracting (1) from (2), we get
$900x = 111$
$x = \frac{37}{300}$. Choice (C)

13. 231 is not prime.
$\therefore$ 229 and 231 cannot be twin primes. Choice (D)

14. 437 = (19) (23)
323 = (19) (17)
567 = 7 (81)
651 = (21) (31)
241 is prime. Choice (D)

15. A number divisible by 11 must have the sum of its odd digits and the sum of its even digits equal, or else their difference should be a multiple of 11. Only choice (C) satisfies this condition. Choice (C)

16. Sum of the digits of 7654321A = 28 + A, so it must be divisible by 9. As $0 \le A \le 9$, $28 \le 28 + A \le 37$. Only when 28 + A = 36 is the number divisible by 9.
$\therefore$ A = 8. Ans : (8)

17. Sum of digits of 24687x = 27 + x
This is divisible by 9 when x = 0 or 9.
$\therefore$ value of x cannot be determined uniquely. Choice (D)

18. The number formed by the last 5 digits of PQRSTU6736 must be divisible by 32. When U = 1, this condition is satisfied. When U = 2, this condition is not satisfied.
$\therefore$ We can't say. Choice (C)

19. The number can be written as 10000 (PQRST) + 9875
As 10000 is divisible by 625
10000 (PQRST) is divisible by 625, while 9875 is not divisible by 625.
$\therefore$ The number is not divisible by 625.
Note: For a number to be divisible by 625, the number formed by its last 4 digits must be divisible by 625. Choice (B)

20. Take any number. Find the sum of its digits and subtract the sum from the number. The result is always divisible by 9. Choice (B)

21. Let the other number be x
(LCM) (HCF) = product of the numbers
$(264)(2) = (22)(x) \Rightarrow x = 24$ Ans : (24)

22. If the LCM of two or more numbers equals their product, the numbers must be co-prime, hence the HCF of any two numbers would be 1. In the given problem,
LCM (x, y, z) = x.y.z
$\therefore$ HCF (y, z) = 1. Choice (A)

23. HCF (2, 3, 5) = 1 and LCM (2, 3, 5) = 2 (3) (5)
But HCF (2, 3, 6) = 1 and LCM (2, 3, 6) $\ne$ 2 (3) (6)
$\therefore$ We can't say. Choice (C)

24. $(3^8) \times (6^4) = (3^8) \times (2)^4 \times (3)^4 = (3^{12}) \times (2^4)$
Number of factors of $(3^8)(6^4) = (12 + 1)(4 + 1) = 65$. Ans : (65)

25. $(3^3)(7^7)(21^5) = (3^3)(7^7)(7 \times 3)^5 = (3^8) \times (7^{12})$
The index of each prime factor is even.
$(3^3)(7^7)(21^5)$ is a perfect square. Choice (A)

26. As the number has an odd number of factors, when the number is expressed as a product of powers of prime factors, each index is even. If each index is divisible by 6, then the number is a perfect cube. Otherwise it is not a perfect cube.
We can't say. Choice (C)

27. A perfect square has an odd number of factors
$\therefore$ the number is not a perfect square. Choice (B)

28. The square of a number consisting of n ones ($1 \le n \le 9$) equals 1234….(n) (n – 1) (n – 2) …..1.
In the given problem, n = 5
$\therefore 11111^2 = 123454321$ Ans : (123454321)

29. $(5^8)(7^{10})$ is a perfect square
Number of ways of expressing it as a product of two distinct natural numbers= $\frac{(8+1)(10+1)-1}{2} = 49$ Choice (C)

30. $(3^6)(7^3)$ can be written as a product of 2 distinct natural numbers in $\frac{(6+1)(3+1)}{2}$ or 14 ways Ans : (14)

31. $(2^6)(3^{10})$ can be written as a product of 2 co-primes as $(1)[(2^6)(3^{10})]$ or $(2^6)(3^{10})$ i.e., in 2 ways.
Alternately, number of ways = 2 raised to the power of number of distinct prime factors minus 1 = $2^{2-1} = 2$. Choice (A)

32. $(2^3)(3^4)(5^6)(7^8)$ can be written as a product of 2 co-primes in 2^{4-1}, i.e., 8 ways. Ans : (8)

33. Sum of the factors of $(2^4)(3^3) = \frac{2^5-1}{2-1}\cdot\frac{3^4-1}{3-1} = 1240$.
Choice (B)

34. Sum of the factors of 437 or (19) (23)
$= \frac{19^2-1}{19-1}\cdot\frac{23^2-1}{23-1} = 480$. Ans : (480)

35. A perfect number is half the sum of its factors.
Choice (B)

36. There are 2^{13} odd numbers less than 2^{14}.
$\therefore$ There are 2^{13} numbers co-prime to it. Choice (D)

37. $N = 3^{p+2r}2^q$
Number of numbers less than N and co-prime to it
$= N\left(1-\frac{1}{3}\right)\left(1-\frac{1}{2}\right) = \frac{N}{3}$ Choice (B)

38. $289 = 17^2$
Number of numbers co-prime to 289 and less than it = 288 – (Number of numbers having a common factor with 289) = 288 – 16 = 272
(The only numbers less than 289 and not co-prime to 289 are 1×17, $2 \times 17 \times$……… 16×17.) Ans : (272)

39. $48 = 2^4 3$
Sum of the co-primes of 48 less than
$(48) = \frac{48}{2} \times 48\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right) = 24 \times 48 \times \frac{1}{3} = 384$
Ans : (384)

40. $\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2 = 4^2$ (given)
$\therefore\ x^2+\frac{1}{x^2} = 14$ Choice (D)

41. $\left(y-\frac{1}{y}\right)^2 = y^2+\frac{1}{y^2}-2 = 3^2$ (given)
$\therefore\ y^2+\frac{1}{y^2} = 11$ Ans : (11)

42. $x^4 - 3x^2 + 1 = (x^2)^2 - 2x^2 + 1 - x^2$
$= (x^2 - 1)^2 - x^2 = (x^2 - x - 1)(x^2 + x - 1)$
$\therefore\ x^2 - x - 1$ and $x^2 + x - 1$ are both factors of $x^4 - 3x^2 + 1$.
Choice (D)

43. $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$
$= 12^3 - 3(12)(18) = 1080$. Ans : (1080)

44. If $p^3 + q^3 + r^3 = 3pqr$,
$P^3 + q^3 + r^3 - 3pqr = 0$
$(p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp) = 0$
$(p + q + r) = 0$ or $\frac{1}{2}[(p - q)^2 + (q - r)^2 + (r - p)^2] = 0$
i.e., $p + q + r = 0$ or $p = q = r$. Choice (C)

45. $p^3 + q^3 + r^3 - 3pqr$
$= (p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp)$
Replacing q by – q on both sides of the equation above,
$p^3 - q^3 + r^3 + 3pqr$
$= (p - q + r)(p^2 + q^2 + r^2 + pq + qr - rp)$
$\therefore\ \frac{p^3+r^3-q^3+3pqr}{p^2+q^2+r^2+pq+qr-rp} = p-q+r$ Choice (B)

46. If $p + q + r = 0$, $p^3 + q^3 + r^3 = 3pqr$. Choice (A)

47. $480 = 40 \times 12$
$360 = 40 \times 9$
$320 = 40 \times 8$
LCM (480, 360, 320) = 40 LCM (12, 9, 8)
= (40) (72) = 2880. Ans : (2880)

48. $63 = 21 \times 3$
$84 = 21 \times 4$
$147 = 21 \times 7$
HCF (63, 84, 147) = 21. Ans : (21)

49. $\text{LCM}\left(\frac{5}{6},\frac{9}{10},\frac{8}{9}\right) = \frac{\text{LCM}(5,9,8)}{\text{HCF}(6,10,9)} = \frac{360}{1} = 360.$
Ans : (360)

50. $\text{HCF}\left(\frac{7}{12},\frac{21}{5},\frac{14}{18}\right) = \text{HCF}\left(\frac{7}{12},\frac{21}{5},\frac{7}{9}\right) = \frac{\text{HCF}(7,21,7)}{\text{LCM}(12,5,9)}$
$= \frac{7}{180}$ Choice (B)

51. Yes, both expressions are equal to HCF (p, q, r, s)
Choice (A)

52. Yes, both expressions are equal to LCM (p, q, r, s).
Choice (A)

53. The given expression is in the form of $\frac{a^3-b^3}{a^2+b^2+ab}$, which is always equal to $a - b$.
Here, $a = 10{\cdot}59$ and $b = 4{\cdot}78$.
$\therefore$ The expression is $a - b = 5.81$. Ans : (5.81)

54. Dividing 256 successively by 2, we get

2 | 256
2 | 128
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2 | 2
  1

$\therefore$ The number of twos in 256! is 1 + 2 + .4 + ... + 128
$= 2^8 - 1 = 255$. Ans : (255)

55. The remainder when a number is divided by 2^n is equal to the remainder when the 'tail' (the number formed by the last n digits of the given number) is divided by 2^n. Here n = 3 and the 'tail' is 677. Therefore, the remainder is 5. Therefore, the least number to be added such that the sum is divisible by 8 is 3. Ans : (3)

56. The product of any 6 consecutive natural numbers is always divisible by 6! i.e., 720. Choice (D)

57. The product of 10 consecutive even natural numbers is always divisible by $2^{10} \times 10!$ As each number is even, the product is divisible by 2^{10}. The other factors are 10 consecutive numbers. Their product is divisible by10!
$\therefore$ The product is always divisible by $2^{10} \times 10!$
Choice (C)

58. The index of each prime factor must be even. If we multiply the number by (5) (7) i.e., 35, the resulting indices are all even. Ans : (35)

59. If we divide the number by (5) (7), the quotient is a perfect square. This is the least number by which we have to divide.
Ans : (35)

60. The index of each prime factor must become divisible by 3 upon division. The least number which achieves this objective is (2) (3^2) i.e., 18. Choice (D)

61. Least perfect cube greater than 395 is 512. 117 should be added to 395 to obtain 512. Ans : (117)

62. 484 is the greatest perfect square below 500.
$\therefore$ 16 is the least natural number to be subtracted from 500.
Ans : (16)

63. Let the number be N. Let the quotient obtained, when the number is divided by 54, be q.
N = 54q + 31
When N is divided by 27, the quotient is 2q + 1 and the remainder is 4. Ans : (4)

64. All such numbers are of the form kLCM(7, 8) + 3. The least natural number of this kind occurs when k = 0. This number is 3. Choice (A)

65. Let the least natural number be N. Let the number divided by 24 and 18 result in quotients of q_1 and q_2 respectively.
$N = 24q_1 + 18 = 18q_2 + 12$ we get
$N + 6 = 24 (q_1 + 1) = 18 (q_2 + 1)$
$\therefore$ N + 6 is the least number divisible by 24 and 18 i.e., by LCM (24, 18) = 72. $\Rightarrow$ N = 66. Ans : (66)

66. Numbers which leave a remainder of 3 when divided by 5 are 3, 8, 13, 18, 23, 28, . . .
Numbers which leave a remainder of 5 when divided by 6 are 5, 11, 17, 23, 29, . . .
Therefore numbers of the form kLCM(5, 6) + 23 satisfy both the conditions. Putting k = 0, gives the least natural number.
Ans : (23)

67. The length (in cm) of the side of the smallest square must be divisible by 7 as well as 5.
$\therefore$ It must be L.C.M (7, 5) or 35 cm.
Its area = $35^2\,cm^2 = 1225\,cm^2$. Choice (A)

68. Required number = HCF (107 – 17, 78 – 18) = 30.
Ans : (30)

69. Let the remainder in each case be r.
Let the largest number be N and the quotients when N divides 34, 58 and 94 be q_1, q_2 and q_3 respectively
$34 = Nq_1 + r \rightarrow (1)$
$58 = Nq_2 + r \rightarrow (2)$
$94 = Nq_3 + r \rightarrow (3)$
Subtracting (1) from (2), we get $24 = N (q_2 - q_1) \rightarrow (4)$
Subtracting (1) from (3), we get $60 = N (q_3 - q_1) \rightarrow (5)$
From (4) and (5), N divides 24 and 60.
$\therefore$ N = HCF (24, 60) = 12. Choice (C)

70. The successive division is shown below

Number/ Quotient	N q_1 q_2
Divisor	5 6 7
Remainder	3 4 5

The least number = $((5 \times 6) + 4)\,5 + 3 = 173$
Ans : (173)

71. Number of three digit natural numbers divisible by 8, 12 and 15 = Number of three digit natural numbers divisible by LCM (8, 12, 15), i.e., 120.
There are 8-three digit natural numbers divisible by 120, 120(1), 120(2), . . . 120(8). Ans : (8)

72. The number of digits in the product must be at least the number of digits in (10^6) (10^9) (10^{11}) and less than the number of digits in (10^7) (10^{10}) (10^{12}).
$\therefore$ The number has at least 27 digits and less than 30 digits.
Choice (D)

73. Suppose a number x has m digits
i.e., $10^{m-1} \le x < 10^m$
$\therefore 10^{2m-2} \le x^2 < 10^{2m}$
i.e., x^2 has 2m or 2m – 1 digits. Conversely, if a number has 2m – 1 or 2m digits, its square root has m digits. Therefore, if a number has 13 digits, its square root has 7 digits. Ans : (7)

74. $(2PQR)^4$ must be at least $(2000)^4$ and less than $(3000)^4$
$(2000)^4$ as well as $(3000)^4$ have 14 digits.
$\therefore (2PQR)^4$ has 14 digits. Choice (B)

75. Suppose a number x has m digits
$10^{m-1} \le x < 10^m$
$\therefore 10^{3m-3} \le x^3 < 10^{3m}$
i.e., x^3 has 3m – 2, 3m – 1 or 3m digits, so if a number has 28, 29 or 30 digits, its cube root has 10 digits.
Choice (A)

Exercise – 1(a)

Solutions for questions 1 to 40:

1. Let the number be N.
Let N = DK + 13, where K is the quotient
3N = 3DK + 39
As the remainder of 3N divided by D is 2, 37 must go into the quotient in the form $\frac{37}{D}$.
$\therefore 3N = D\left(3k + \frac{37}{D}\right) + 2$
As $\frac{37}{D}$ must be a natural number, D can be 37.
But as D exceeds the remainder when N is divided by D, D can be 37 only. Ans : (1)

2. Any natural number having an even number of factors is not a perfect square. Any natural number that is not a perfect square can be written as a product of two factors where one of the factors lies between 1 and its -square root and the other factor lies between its square root and itself.
$\therefore$ In the given problem, both (a) and (b) are true.
Choice (A)

3. The remainder of X divided by 16 is equal to the remainder when the number formed by the by the last 4 digit of X is divided by 16. We tabulate below the numbers, the number of numbers, the number of digits and the total number of digits in X

Numbers	Number of Numbers	Number of Digits	Total number of Digits
1 – 9	9	9	19
10 – 54	45	90	99
55	Parts of 1 number	1	100

We see that the number formed by the last 4 digits of X is 3545 (The 3 from 53, then 54 and the first 5 from 55) Rem (3545/16) = 9 Ans : (9)

4. Let P be 100a + 10b + c
Q = 100c + 10b + a
Q – P = 99 (c – a)
As Q – P is divisible by 5.
c – a is divisible by 5.
As Q > P, c – a is positive
c – a = 5, Hence (a, c) can be (1, 6), (2, 7), (3, 8) or (4, 9) or P can be 1b6,2b7,3b8 or 4b9 where b can be any digit.
$\therefore 106 \le P \le 499$
Only choice (B) accommodates all the values of P.
Choice (B)

5. (i) $18^3 = (2 \times 3^2)^3 = 3^6\, 2^3$. The number of factors is 7(4) = 28
The product of all these factors is $(18^3)^{14} = 18^{42}$.
Choice (B)

(ii) 12! = (2.4.6.8.10.12) (3.5.7.9.11)
$= (2^{10} . 3^2 . 5)$ (3.5 7.9.11)
$= 2^{10}\, 3^5\, 5^2\, 7\, (11)$
The number of factors of 12! is 11(6) (3) (2) (2) = 792
The product of all these factors is $(12!)^{\frac{792}{2}}$.
$= (12!)^{396}$. Choice (B)

6. If a number N = $a^p.b^q$. c^r …… where a, b, c are prime numbers and p, q, r …… are integers, then, the number of different ways, in which N can be written as product of two co-primes factors, is 2^{n-1} where 'n' is the number of different prime numbers used in resolving N into prime factors.
Here, N = 11025 = 25 × 441
$= 5^2 \times 21^2 = 3^2 \times 5^2 \times 7^2$
Three different prime numbers i.e., 3, 5 and 7 are used in the resolution into prime factors. Hence, n = 3
$\therefore 2^{n-1} = 2^{3-1} = 4$
i.e., 11025 can be written as product of a pair of co-prime factors in 4 different ways. Ans : (4)

7. Up to 1400, there are 200 multiples of 7. There are 280 multiples of 5. There are 40 multiples of both 5 and 7 (i.e. of LCM (5, 7) i.e. 35).
∴ Up to 1400, there are 100 odd numbers divisible by 7, 140 odd numbers divisible by 5, and 20 odd number divisible by both 5 and 7.
There are 140 + 100 – 20 i.e. 220 odd numbers which are divisible by either 5 or 7.
∴ The remaining 700 – 220 i.e. 480 odd numbers are divisible by neither 5 nor 7. Ans : (480)

8. $840 = 2^3 \times 3 \times 5 \times 7$
Number of co-primes to 840 and less than it
$= 840\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{7}\right)$
$= 840\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right) = 192$
Sum of the co-primes to 840 and less than it
$= \frac{(192)\,(840)}{2} = 80640$ Choice (B)

9. LCM of 15, 20, 25 and 30 is 300.
15 = 3(5)
$20 = 2^2(5)$
$25 = 5^2$
30 = 2(3)(5)
$\therefore$ LCM $= 2^2(3)(5^2) = 300$
∴The buses would start together again after 300 minutes i.e. after 5 hours from 8:00 a.m. i.e., 1:00 p.m. Choice (C)

10. LCM of 10, 16 and 20 is 80. The number of sweets should be of the form 80k + 1, where k = 1, 2, ….. It must also be divisible by 23. Also it is less than 200
80k + 1 < 200
$k < \frac{199}{80}$ i.e., $2\frac{39}{80}$
k = 1 or 2.
Only when k = 2, 80k + 1 is divisible by 23. The number of sweets with me is 161. Ans : (161)

11. LCM of 7, 11 and 21 is 231.
The number would be in the form 231k + 5.
10164 is the smallest five digit multiple of 231.
∴The required number is 10164 + 5 = 10169.
Choice (B)

12. Let the number be N.
$N = 3k_1 + 2 = 5k_2 + 4 = 7k_3 + 6 = 9k_4 + 8$
$N + 1 = 3(k_1 + 1) = 5(k_2 + 1) = 7(k_3 + 1) = 9(k_4 + 1)$
N + 1 = KLCM(3, 5, 7, 9)
N = KLCM(3, 5, 7, 9) –1.
As N is the least number, N = LCM (3, 5, 7, 9) –1
= 314. Ans : (314)

13. Let the number be 8k + 3, where k = 0, 1, 2, 3, 4, ……
When (8k + 3) is divided by 7, the remainder is 1.
∴ (8k + 3 – 1), i.e. (8k + 2) is divisible by 7.
The smallest value of k for which 8k + 2 is divisible by 7 is 5.
∴ The smallest such number is 8 × 5 + 3 = 43.
The general form of the required number is 56p + 43
(56 is the LCM of 8 and 7).
The largest five digit multiple of 56 is 99960.
∴ The required number is 99960 + 43 – 56 = 99947.
Choice (D)

14. The required number would divide 565 – 5, 847 – 7 and 1551 – 11 i.e. 560, 840 and 1540.
Therefore, we have to find the HCF of 560, 840 and 1540.
$560 = 2^4(5)(7)$
$840 = 2^3(3)(5)(7)$
$1540 = 2^2(5)(7)(11)$
∴ HCF of the three numbers $= 2^2\,(5)(7) = 140$
Ans : (140)

15. Required time = LCM $\left(3+7\frac{5}{6}, 3+1\frac{1}{3}, 3+5\frac{2}{3}\right)$
= LCM $\left(\frac{65}{6}, \frac{13}{3}, \frac{26}{3}\right)$
$= \frac{130}{3} = 43\frac{1}{3}$ seconds. Choice (D)

16. Let the numbers be 11x and 11y, where x and y are relative primes and x ≤ y.
LCM of 11x and 11y is 11 xy.
11xy = 1001 ⇒ xy = 91
∴ x = 1 and y = 91
or x = 7 and y = 13
∴ The numbers could be 11, 1001 or 77, 143.
Since the sum of the two numbers is 220, the required number is 77. Ans : (77)

17. The procedure is as follows

```
248) 480 (1
     248
   ---------
     232) 248 (1
          232
         --------
          16) 232 (14
              224
             --------
               8) 16 (2
                  16
                 -----
                  0
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∴ The numbers are 248 and 480. Choice (B)

18. Given function is $n(n^2 + 20)$, n being an even number.
Let n = 2k, when k is any positive integer
Hence, $n(n^2 + 20) = 2k(4k^2 + 20)$
$= 8k\,(k^2 + 5) = 8k\,[(k^2 – 1) + 6]$
$= 8k\,(k^2 – 1) + 48k$
= 8 (k – 1) k (k + 1) + 48 k
= 8[multiple of 6] + 48 k
= 4.8 L + 48k, where L is an integer.
= 48 x an integer.
i.e., $x\,(x^2 + 20)$ is always a multiple of 48, as long as n is even.
Hence, 48 is the HCF of all numbers represented by
$n\,(n^2 + 20)$, n being even. Choice (D)

19. Let the number be N. Let the quotient when the number is divided by 6 be q_1,
$N = 6q_1 + 4$.
Let the quotient be q_2 when q_1 is divided by 7.

$q_1 = 7q_2 + 5$
$N = 6(7q_2 + 5) + 4$
$= 42q_2 + 34$.
When N is divided by 21, the quotient is $2q_2 + 1$ and the remainder is 13. Ans : (13)

20. The smallest number is {(4)(4) + 3}3 + 2 = 59
The general form of the number would be 60k + 59.
60 = 3(4)(5)
The greatest 4 digit multiple of 60 is 9960.
Since 9960 + 59 = 10019 becomes a five-digit number, 10019 − 60 = 9959 is the required number.
Choice (D)

21. 20 = 4(5)
The index of the greatest power (IGP) of 5, that can divide 200! is 40 + 8 + 1 = 49
The IGP of 2 that can divide 200! is 100 + 50 + 25 + 12 + 6 + 3 + 1 = 197
∴ The IGP of 4 that can divide 200! is 98.
∴ The IGP of 20 that can divide 200! is 49. (The lower of the two values) Ans : (49)

22. The number of zeros at the end of 175! is same as the greatest power of 5 in 175!. 175! has 35 fives, 7 twenty-fives and 1 one twenty five.
∴ Index of the greatest power of 5 in 175! is (35 + 7 + 1) = 43. Ans : (43)

23. $N = a^2 - b^2 = (a + b)(a - b)$ [a > b]
For a and b to be natural numbers, a + b and a − b must be of the same parity.
∴ We need to identify N for which the number of ways of expressing N as a product of 2 factors of the same parity is the least.

Choice (A)
$187 = 1 \times 187 = 11 \times 17$
∴ k(187) = 2

Choice (B)
$120 = 2 \times 60 = 4 \times 30 = 6 \times 20 = 10 \times 12$
∴ k(120) = 4

Choice (C)
k(110) = 0

Choice (D)
$105 = 3 \times 35 = 5 \times 21 = 7 \times 15$
∴ k(105) = 3
∴k(110) which is equal to 0, is the least. Choice (C)

24. $1125 = (5)(225) = 5^3 3^2$
Number of factors of 1125 = (3 + 1) (2 + 1) = 12
$1800 = 8(225) = 2^3(3^2)(5^2)$.
Number of factors of 1800 = (3 + 1) (2 + 1) (2 + 1) = 36
Number of common factors of 1125 and 1800 = Number of factors of HCF (1125, 1800) i.e. of 225 (i.e. $3^2 5^2$)
= (2 + 1) (2 + 1) = 9.
Number of factors of 1125 which are not factors of 1800
= 12 – 9 = 3.
Number of factors of 1800 which are not factors of 1125
= 36 – 9 = 27.
Number of factors of only one of 1125 and 1800 = 3 + 27 = 30. Ans : (30)

25. Let X = 33333333 = 3(11111111) = 3(11110000 + 1111)
= 3(1111)(10000 + 1)
= 3(1111)(10001).
We can recognize 10,001 as the difference of two squares i.e. 10, 001 = 11025 – 1024 = $105^2 - 32^2$ = 137 (73).
∴ X = 3(11) (101) (137) (73)

Sum of all the factors of $p_1^a\ p_2^b\ p_3^c$ ………where p_1, p_2, p_3…..are primes and a, b, c…….are whole numbers is

$$\left(\frac{p_1^{a+1}-1}{p_1-1}\right)\left(\frac{p_2^{b+1}-1}{p_2-1}\right)\left(\frac{p_3^{c+1}-1}{p_3-1}\right)\ldots.$$

Sum of all the factors of 33333333 =

$$\left(\frac{3^2-1}{3-1}\right)\left(\frac{11^2-1}{11-1}\right)\left(\frac{101^2-1}{101-1}\right)\left(\frac{137^2-1}{137-1}\right)\left(\frac{73^2-1}{73-1}\right)$$

(3 + 1) (11 + 1) (101 + 1) (137 + 1) (73 + 1)
= 4(12) (102) (138) (74)
= 49997952. Choice (D)

26. The given function $n(n^2 - 4)(n^4 - 10n^2 + 9)$ can be written as:
$n(n + 2)(n - 2)(n^2 - 9)(n^2 - 1)$
= n (n + 2) (n – 2) (n + 3) (n – 3) (n + 1) (n – 1)
= (n – 3) (n – 2) (n – 1) n (n + 1) (n + 2) (n + 3) ;
i.e. the function is the product of 7 consecutive positive integers. (As n > 3, n – 3 > 0); and hence, the function is divisible by 7!
7! = 1(2)(3)(4)(5)(6)(7)
The given options are
126 = 3(6)(7)
72 = 3(4)(6)
52 = 4(13)
and 144 = 2(3)(4)(6)
Except 52, all others are factors of 7!
Hence 52 is the only number that is not a factor.
Choice (C)

27. $A = 3(2n_1 - 1)$ and $B = 5(2n_2 - 1)$
$\frac{5A - 3B}{15} = \frac{(30n_1 - 15) - (30n_2 - 15)}{15} = 2(n_1 - n_2)$
$\frac{5A - 3B}{30} = n_1 - n_2$
$5A - B = 30n_1 - 15 - 10n_2 + 5 = 10(3n_1 - n_2 - 1)$
$\Rightarrow \frac{5A - B}{10} = 3n_1 - n_2 - 1$ while $\frac{5A - B}{20} = \frac{3n_1 - n_2 - 1}{2}$
which may not be an integer. Choice (D)

28. $72000 = 8(9)(2^3 5^3) = 2^6(3^2)(5^3)$
We should multiply with $2^4(3^3)(5^2)$
i.e. 10800 so that the product is a perfect 5^{th} power.
Choice (A)

29. 36 + 37 + 38 + 39 + 40 = 190
$1085 + 190 = 1275 = \frac{50(51)}{2}$
So the least of the numbers on the intact houses can be 36.
26 + 27 + 28 + 29 + 30 = 140
$1085 + 140 = 1225 = \frac{49(50)}{2}$
So it can be 26 also.

Alternate method:

The sum of the numbers on all the houses is $\frac{N(N+1)}{2}$.
Let the numbers on the houses that remained intact be M – 2, M – 1, M, M + 1, M + 2
The sum of these 5 numbers is 5M.
∴ The sum of the numbers on the destroyed houses is $\frac{N(N+1)}{2} - 5M$.

∴ $\frac{N(N+1)}{2} - 5M = 1085$

$\Rightarrow N(N+1) = 10M + 2170 = 10(M - 2) + 2190$.

The values of M – 2 (the least of the numbers on the intact houses suggested in the options and the corresponding values of N(N + 1) and N (for those values of M – 2 which produce an integral value of N) are tabulated below.

M – 2	N(N+ 1)	N
25	2440	-
26	2450	49
36	2550	50

∴ M – 2 can be 26 or 36. Choice (D)

30. The factors of N, which are perfect cubes will be of the form $= 2^a (3^b)(5^c)$, where a can be 0, 3, 9.,…… 24, b can be 0, 3, 9, … 15, and c can be 0, 3, … 15. The number of factors which are perfect cubes is
= 9(6)(6) = 324. Choice (A)

31. $X = \left\{\frac{7}{128}, \frac{7}{64}, \frac{7}{32}, ... 7(512)\right\}$.

Let $A = \{2^{-7}, 2^{-6}, 2^{-5}, ... 2^9\}$ and

let $B = \{-7, -6, ... 9\}$

The 7+1+9 (viz 17) elements of B can be arranged as shown.

– 7	–6	–5	–4	–3	–2	–1	0	1
9	8	7	6	5	4	3	2	

From each of the 9 columns, we can select only one element. If we select both, the numbers in the (say) first column, the corresponding numbers in A would be 2^{-7} and 2^9 and the corresponding elements in X would be $7(2^{-7})$, $7(2^9)$. They would have a product of 4(49), i.e. 196.

$\therefore$ The subset Y can have at the most 9 elements.

Ans : (9)

32. If the sum of 3 numbers x, y and z is constant (in this case 9m + 10), $x^2 + y^2 + z^2$ (say s) will have its minimum value when x, y and z are as close to each other as possible, i.e. x = 3m + 3, y = 3m + 3 and z = 3m + 4 Therefore, the minimum value of S is
$(3m + 3)^2 + (3m + 3)^2 + (3m + 4)^2$ or $27m^2 + 60m + 34$.

Choice (C)

33. The following results are useful in all such problems.
If the index of the greatest power, (IGP), of p in A is m and the IGP of p in B is n, then
(1) the IGP of p in AB is m + n.
(2) if $m \neq n$, the IGP of p in A + B is the smaller of m and n.
If m = n, the IGP of p in A + B could be M or more than m.
Let S = 64! + 65! + 66! + 67! + …..+120!
The IGPs (Index of Greatest power) of 2 in successive terms are 63, 63, 64, 64, 66, 66, …..
We have to express S as $S_1 + S_2$.
Here, we have to club the first four forms.
S_1 = 64! [1 + 65 + 65(66) + 65(66)(67)], the expression in the bracket is a multiple of 2 but not of 4.
The IGP of 2 in 64! is 63.
The IGP of 2 in the bracket is 1.
$\therefore$ The IGP of 2 in S_1 is 64 which the IGP of 1 in S_2 is 66.
$\therefore$ The IGP of 2 in S is 64. Ans : (64)

34. p = q + 2 = r + 4. $\therefore$ p = q + 2, q = r + 2.
Each prime number greater than 3 is of the form 6k + 1.
If r is of the form 6k + 1, than q is of the form 6k + 3 and then q is divisible by 3. Also q is prime. $\therefore$ Only possible value of q is 3. But then r(= 1 and hence) will not be prime.
If r is of the form 6k –1, p is of the form 6k + 3. Only possible value of p is 3. But then q(= 1 and hence) will not be prime.
r is neither of the form 6k + 1 or 6k – 1.$\therefore$ r is not a prime number greater than 3.
r = 2 or 3.
If r = 2, then q = 4 which is not prime.
$\therefore$ r = 3 and q = 5 and p = 7.
Only one combination exists for p, q, r.

Alternative Solution:
r, q, p is an increasing AP with common difference 2.
For any 3 terms in an AP, with common difference which is not a multiple of 3, one of the numbers is a multiple of 3, another leaves a remainder of 1 and the third leaves a remainder of 2 (when divided by 3). The only way in which all 3 can be prime is when the multiple of 3 is 3 itself.
i.e. (r, q, p) = (3, 5, 7). We can also consider (– 7, –5, –3).
As primes are considered to be positive, we have only one combination. Ans : (1)

35. The successive expressions have been relabeled as shown below.
F(10, F(9, F(8, F(7, F(6, i)))) = 1
A B C D
$\therefore$ F(10, A) = 1 $\Rightarrow$ 0 < A $\leq$ 10
$\therefore$ 0 < F(9, B) $\leq$ 10 $\Rightarrow$ 0 < B $\leq$ 10(9)
$\therefore$ 0 < F(8, C) $\leq$ 10(9) $\Rightarrow$ 0 < C $\leq$ 10(9)(8)
$\therefore$ 0 < F(7, D) $\leq$ 10(9)(8) $\Rightarrow$ 0 < D $\leq$ 10(9)(8)(7)
$\therefore$ 0 < F(6, i) $\leq$ 10(9)(8)(7) $\Rightarrow$ 0 < i $\leq$ 10(9)(8)(7)(6)
$\therefore$ i can have all values greater than 0 and up to 30, 240.
(i) is false.
(ii) is false. Choice (D)

36. P_1 = {1, 2, 3, 4, 5, 6}
P_2 = {2, 3, 4, 5, 6, 7}
P_3 = {3, 4, 5, 6, 7, 8}
:
:
:

P_8 = {8, 9, 10, 11, 12, 13}
The sets P_1 and P_2 do not contain 8 or its higher multiple.
The sets P_3 to P_8 contain a multiple of 8.
$\therefore$ In the first 8 sets, 6 contain a multiple of 8.
Similarly, it can be shown that for each collection of 8 successive sets, 6 sets contain a multiple of 8 while the other 2 don't.
$\therefore$ The total number of sets which contain a multiple of 8 = $\frac{88}{8}(6) = 66$. Ans : (66)

37. $y = \frac{x\sqrt{x}\left(\sqrt{x}+3\right)+27\left(\sqrt{x}+3\right)}{\left(\sqrt{x}+3\right)^2}$

$= \frac{x\sqrt{x}+27}{\sqrt{x}+3} = \frac{\left(\sqrt{x}\right)^3+3^3}{\sqrt{x}+3} = x - 3\sqrt{x} + 9$

$= x - 2\left(\frac{3}{2}\right)\sqrt{x} + 2.25 + 6.75 = \left(\sqrt{x} - 1.5\right)^2 + 6.75$

It can be seen that y is an increasing function for all non negative values of x
$\therefore$ when x = 25 and x = 49, the minimum and the maximum values of y for x in the given range are obtained as 19 and 37 respectively.
$\therefore$ y satisfies 19 $\leq$ y $\leq$ 38 Choice (B)

38. We should take the smallest 5 digit number and the greatest 4 digit number for the difference to be the least.
The required difference = 12345 − 9876 = 2469
Ans : (2469)

39. We can write 540 as 5 (2) (6) (9) or (5) (4) (3) (9) since the digits are all distinct
So number of four digit numbers
= 4! + 4! = 48. Ans : (48)

40. If p is any prime number, (p – 1)! is not divisible by p.
1 $\leq$ P $\leq$ 40
Each prime value of P satisfies the given condition.
There are 12 such values.
When P is 1, (P – 1)! = 1 (which is divisible by P).
The only composite number satisfying the condition is 4.
There are 13 values of P satisfying the condition.
Choice (D)

Exercise – 1(b)

Solutions for questions 1 to 60:

1. p is a prime number greater than 3, hence it can be represented by either (6k + 1) or (6k – 1), k being a positive, integer.
Hence $p^2 - 1 = [(6k + 1)^2 - 1]$ or $[(6k - 1)^2 - 1]$
Consider $(6k + 1)^2 - 1$:
This is equal to [(6k + 1) + 1] [(6k + 1) – 1]
= (6k + 2) (6k) = 12k (3k + 1) $\rightarrow$ (1)

when 'k' is odd, 3k + 1 is even, hence k (3k + 1) is even; and when k is even. k (3k + 1) is even;
$\Rightarrow$ k (3k + 1) is always divisible by 24 $\rightarrow$ (2)
when p = 6k – 1
$p^2 - 1 = 12k(3k - 1)$ and this is also divisible by 24, for all values of k.
Hence, for all prime numbers which are greater than 3, $(p^2 - 1)$ is always divisible by 24.
Hence, 6 and 2 are also factors of $(p^2 - 1)$. Choice (D)

2. When 478185 is divided by 19, the remainder is 12.
Ans : (12)

3. A zero at the end of a product comes from the product of 2 and 5. None of the prime numbers except 2 is even. None of the prime numbers except 5 is divisible by 5.
$\therefore$ The product ends with 1 zero. Ans : (1)

4. Let the least number to be added to 1648 so that a remainder of 10 is left when the resulting number is divided by 14 or 21 be 'x'.
$\therefore$1648 + x = k(L.C.M (14, 21)) + 10.
= 42k + 10.
$\Rightarrow$ 1638 + x = 42k. ----------- (1)
As 'x' is a natural number, x>0.
$\therefore$42k > 1638.
$k > \frac{1638}{42}$
$\Rightarrow$ k > 39.
So, the least value of k is 40.
Substituting k = 40 in (1),
x = 42 (40) – 1638 = 1680 – 1638 = 42
$\therefore$x = 42.
$\therefore$ Required number is 42. Choice (B)

5. $N^3 - N = N(N^2 - 1) = N(N - 1)(N + 1)$
= (N – 1) N (N + 1) = Product of 3 consecutive integers; which is divisible by 6.i.e. $(N^3 - N)$ is divisible by 6, when N > 1. Therefore, the remainder is zero.
Hence, the product of the two remainders is zero.
Ans : (0)

6. Let x = 10. 04343. . . .
$\therefore$ 10x = 100.4343 --------- (1)
1000x = 10043.4343 --------- (2)
(2) – (1) $\Rightarrow$ 990x = 9943
$\therefore x = \frac{9943}{990}$ Choice (D)

7. Let the number be
1000x + 100y + 10z + u
Let another four digit number formed by permuting its digits be 1000y + 100z + 10u + x
The difference between these two is 999x – 900y – 90z – 9u
i.e. 9(111x – 100y – 10z – u) which is always a multiple of 9.
Let us consider another four digit number with the same digits, 1000z + 100u + 10x + y.
The difference is 990x + 99y – 900z – 99u
i.e. 9(110x + 11y – 110z – 11u) which again would be a multiple of 9. Choice (D)

8. $n^7 - n = n(n^6 - 1) = n(n^3 - 1)(n^3 + 1)$
$= n(n - 1)(n^2 + n + 1)(n + 1)(n^2 - n + 1)$
When n = 1, $n^7 - n = 0$ is divisible by all numbers.
When n = 2, $n^2 + n + 1 = 7$
$\Rightarrow n^7 - n$ is divisible by 7
When n = 3, $n^2 - n + 1 = 7 \Rightarrow n^7 - n$ is divisible by 7.
Similarly for n = 4, 5, 6, and 7, $n^7 - n$ is divisible by 7.
From n = 8 onwards the same pattern repeats.
$\therefore n^7 - n$ is always divisible by 7. Choice (C)

9. As a < (a + b) and b < (a + b), (a + b)! is divisible by both a! and b!.
$$\frac{(a+b)!}{a!\,b!} = \frac{(1)(2)\ldots(a)(a+1)\ldots(a+b)}{(1)(2)\ldots(a)(1)(2)\ldots(b)}$$
$$= \frac{(a+1)(a+2)\ldots\ldots(a+b)}{(1)(2)\ldots(b)} = \text{an integer}$$
The product of b consecutive natural numbers is always divisible by b! Choice (D)

10. $1 \le k \le 40$
$\Rightarrow 5 \le 5k \le 200$
$\Rightarrow 6 \le 5k + 1 \le 201$
$\therefore$ All the prime numbers between 6 and 201, which are of form 5k + 1 are 11, 31, 41, 61, 71, 101, 131, 151, 181 and 191 i.e., 10 in all. Ans : (10)

11. Required divisor = 238 + 342 – 156 = 424. Choice (C)

12. As each digit of S is even, the first two digits must be 2 each. Their sum is 4. The other two digits, being even, must have an even sum. As the sum of the digits of S is divisible by 3, the sum of the last two digits can be 2 or 8 or 14. (The last two digits are even, each at most must be 8.
$\therefore$ Their sum at most can be 16).
If the sum of the last two digits is 2, the third digit must be 2 and the fourth digit must be 0 or vice versa.
x has 2 possibilities. If the sum of the last two digits is 8, (third digit, fourth digit can be (0, 8), (2, 6), (4, 4), (6, 2) or (8, 0)
$\therefore$ x has five possibilities.
If the sum of the last two digits is 14, the third digit can be 6 and the fourth digit can be 8 or vice versa.
$\therefore$ x has 2 possibilities.
$\therefore$ x has a total of 9 possibilities. Ans : (9)

13. (i) $900 = 2^2 \times 3^2 \times 5^2$
Number of factors (divisors) = (2 + 1) (2 + 1) (2 + 1)
$= 3 \times 3 \times 3 = 27.$
Number of divisors excluding '1' and itself = 27 – 2 = 25
Choice (A)

(ii) Number of ways in which 1500 can be expressed as a product of two of its factors
$= \frac{\text{Number of factors of 1500}}{2}$
$1500 = 3 \times 5^3 \times 2^2$
Number of factors of 1500 = (1 + 1) (3 + 1) (2 + 1) = 24.
Number of ways in which 1500 can be expressed as a product of two of its factors = $\frac{24}{2}$ = 12.
Choice (B)

14. Given number = 784
When resolved into primes factors,
$784 = 4 \times 196 = 2^2 \times 14^2 = 2^4 \times 7^2$
The number of ways in which N can be expressed as product of a pair of different factors is
$\frac{1}{2}\left[(p+1)(q+1)\ldots\ldots-1\right]$ where N = $a^p.b^q$. c^r ……, a, b, c being prime factors of N and p, q, r are whole numbers.
Hence, the answer to the question is
$\frac{1}{2}\left[(4+1)(2+1)-1\right] = \frac{1}{2}(14) = 7$ Ans : (7)

15. $4!\,5! = 24(120) = 2^3(3)\,2^3(3)(5) = 2^6\,3^2\,5^1$
The number of factors is 7(3)(2) = 42
The product of all these factors is $[24(120)]^{21} = (2880)^{21}$
Choice (C)

16. $8^9 9^8 = 2^{27}3^{16}$. The number of factors is 28(17) = 476
The product of all these factors is $(2^{27}3^{16})^{238} = 2^{6426} \times 3^{3808}$.
Choice (D)

17. $8640 = 2^6 \times 3^3 \times 5$
Number of co-primes to it and less than it
$= 8640\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 8640\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = 2304$
Ans : (2304)

18. 3 m 78 cm = 378 cm
4 m 80 cm = 480 cm
$378 = 2 \times 3^3 \times 7$
$480 = 2^5 \times 3 \times 5$
HCF of 378 and 480 = $2 \times 3 = 6$
$\therefore$ Each side of the square tile = 6 cm.
$\therefore$ Minimum number of tiles required = $\frac{378 \times 480}{6 \times 6}$ = 5040
Choice (B)

19. $\frac{900!}{450!}$ = (900) (899) (898)…..(451).
The index of the greatest power (IGP) of 11 that divides $\frac{900!}{450!}$ is obtained as below.

900	81	7
11	11	

450	40	3
11	11	

IGP of 11 in 900! is 81 + 7 i.e. 88 and the IGP of 11 in 450! is 40 + 3 i.e. 43.
IGP of 11 in $\frac{900!}{450!}$ = 88 – 43 = 45. Ans : (45)

20. Divisors are : 5, 7 and 11.
Remainders are : 4, 6, and 10.
Complements are : 1, 1, and 1.
Hence, (LCM of 5, 7 and 11)K–1 is the general form of the selection.
$\Rightarrow$ (5 x 7 x 11 x k) – 1 or (385 K – 1) is the number that satisfies the above condition, K being a position integer.
The required number shell be a multiple of 17; let it be 17Q, when Q is a positive integer.
Hence, 385 K – 1 = 17Q.
$\Rightarrow$ Remainder of $\frac{385K - 1}{17}$ is zero
$\Rightarrow$ Remainder of 22K + $\frac{11K - 1}{17}$ is zero
$\Rightarrow$ Remainder of $\frac{11K - 1}{17}$ is zero
$\Rightarrow$ 11K – 1 is a multiple of 17.
By trial and error, it can be seen that when
K is 14, 11K – 1 = 154 – 1
= 153 which is a multiple of 17.
And K = 14 is the last value of K that satisfies the above equation.
Hence, the least value of the number is
$385K - 1 = 17\left(22K + \frac{11K - 1}{17}\right)$
$= 17\left((22 \times 14) + \left(\frac{11 \times 14 - 1}{17}\right)\right)$
= 17 (308 + 9) = 17 × 317
Hence, 317 the multiple of 17 is the least value of the number required.
Hence the answer is 317. Choice (B)

21. Let the number be 31k + 7, where k = 0 1, 2, ……
If (31k + 7) is divided by 25, the remainder is 6.
$\therefore$ 31k + 7 – 6 i.e. 31k +1 is divisible by 25.
The smallest value of 'k' for which (31k + 1) is divisible by 25 is k = 4
$\therefore$ The required number is 31 × 4 + 7 = 131 Ans : (131)

22. The greatest value of the divisor is given by
HCF [(6155, – 5), (4935, – 15)]
= HCF (6150, 4920) = 1230 Ans : (1230)

23. Weight of each part
= HCF$\left(5\frac{1}{4}\text{lb}, 7\frac{3}{4}\text{lb}, 8\frac{1}{5}\text{lb}\right)$
= HCF$\left(\frac{21}{4}\text{lb}, \frac{31}{4}\text{lb}, \frac{41}{5}\text{lb}\right) = \frac{1}{20}\text{lb}$
Number of guests = Number of pieces
$= \frac{\frac{21}{4}}{\frac{1}{20}} + \frac{\frac{31}{4}}{\frac{1}{20}} + \frac{\frac{41}{5}}{\frac{1}{20}} = 424$ Ans : (424)

24. Complete remainder is the smallest number which when successively divided by 7, 11 and 5, the respective remainders are 5, 1 and 1.
i.e. {(1 × 11) + 1} × 7 + 5 = 89 Ans: (89)

25. Let the quotients obtained when the number is successively divided by 4, 5 and 6 be denoted by K_1, K_2 and K_3 respectively.
Let the number be denoted by N.
$N = 4K_1 + 3$
$K_1 = 5K_2 + 4$
$K_2 = 6K_3 + 5$
$N = 4(5K_2 + 4) + 3$
$= 20K_2 + 19$
$= 20(6K_3 + 5) + 19$
$= 120K_3 + 119$
When $K_3 = 0$, the smallest value of N is obtained as 119.
When $K_3 = 1$, the second smallest value of N is obtained as 239.

(i) Hundreds digit of 119 is 1. Choice (A)

(ii) Hundreds digit of 239 is 2. Choice (A)

26. A number whose units digit equals the units digit of its square must end with 0, 1, 5 or 6.
In the given problem, AB ends with a digit which is the same as the units digit of its square.
$\therefore$ B must be 0, 1, 5 or 6.
The square of any number ending with 0 must end with 2 zeroes. If B = 0, $(AB)^2$ cannot be CCB (it can be CBB)
$\therefore$ B $\neq$ 0
$\therefore$ B is 1 or 5 or 6.
As CCB is a three-digit number, its square root i.e. AB must be from 10 to 31.
If B = 1, AB = 11 or 21 or 31.
$(AB)^2$ will be of the form CCB only when AB = 21.
If B = 5, AB = 15 or 25
$(AB)^2$ will be of the form CCB only when AB = 15
If B = 6, AB = 16 or 26
$(AB)^2$ can never be of the form CCB.
$\therefore$ The sum of all the possible values of CCB
$= 21^2 + 15^2 = 666$ Ans : (666)

27. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
$= (11)^3 - 3 \times 7 \times 11$
= 1331 – 231 = 1100 Ans : (1100)

28. In the sum of the first 15828 prime numbers there is one even number and 15827 odd numbers.
$\therefore$ The sum is odd. Choice (D)

29. The sum of first 10 natural numbers is 55. When 10 consecutive natural numbers are added the sum will be of the form (10k + 55) where, k is a natural number.
Among the given options only 785 is of the above form.
Choice (A)

30. $Y_1 = (-1)Y_0 = y$
$Y_2 = (-1)^2Y_1 = y$
$Y_3 = (-1)^3Y_2 = -y$
$Y_4 = (-1)^4Y_3 = -y$
The cycle repeats after every 4 terms.
(A) $\therefore$ Y_n is positive when n is even and is not divisible by 4.
(B) Y_n is not positive for all odd values of n(for n = 3, 7, etc.)
(C) is false, because (A) is true.
Choice (A)

31. $\dfrac{1}{6^2-1}=\dfrac{1}{(5)(7)}=\dfrac{1}{2}\left[\dfrac{7-5}{(5)(7)}\right]$

$=\dfrac{1}{2}\left[\dfrac{1}{5}-\dfrac{1}{7}\right]$

Similarly $\dfrac{1}{8^2-1}=\dfrac{1}{2}\left[\dfrac{1}{7}-\dfrac{1}{9}\right]$ and so on

Finally, $\dfrac{1}{16^2-1}=\dfrac{1}{2}\left[\dfrac{1}{15}-\dfrac{1}{17}\right]$

Value of the given expression.

$=\dfrac{20}{3}\left[\dfrac{1}{2}\left[\dfrac{1}{5}-\dfrac{1}{7}+\dfrac{1}{7}-\dfrac{1}{9}+\dfrac{1}{9}-\dfrac{1}{11}+.......+\dfrac{1}{15}-\dfrac{1}{17}\right]\right]$

$=\dfrac{20}{3}\left(\dfrac{1}{2}\right)\left(\dfrac{1}{5}-\dfrac{1}{17}\right)=\dfrac{20}{3}\dfrac{1}{2}\dfrac{12}{(5)(17)}=\dfrac{8}{17}$ Choice (A)

32. 1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4 …… are all triangular numbers.

$\therefore$ Any triangular number would be of the form

$\dfrac{n(n+1)}{2}$ (i.e. 1 + 2 + 3 + 4 + …… + n)

where n is a natural number.

Now, let = $\dfrac{n(n+1)}{2}$ = 903

$\Rightarrow n^2 + n - 1806 = 0 \Rightarrow n^2 + 43n - 42n + 1806 = 0$

$\Rightarrow n(n + 43) - 42(n + 43) = 0$

$\Rightarrow (n - 42)(n + 43) = 0 \quad \Rightarrow n = 42$ or -43

But n = – 43 is not possible.

$\therefore$ n = 42

For the other numbers given, we find that on solving the equations the values that n takes are not natural numbers. Hence, 903 is a triangular number. Choice (D)

33. Let S_N denote the sum of the first N natural numbers.

$S_{36} = 666$, $S_{37} = 703$

$\therefore$ N = 36. The number which the student had added twice by mistake = 700 – 666 = 34.

The sum of its digits = 7. Ans : (7)

34. Since the number is a three-digit number, 7, 8, 9 are ruled out as 7! = 5040 which is a four digit number 8! and 9! are more than that. 6! is also ruled out, as 6! = 720, which would then require 7! which is not possible. Therefore, the three numbers x, y and z can be chosen from 0, 1, 2, 3, 4 and 5. Since it is a three-digit number, 5 has to be one of them, and since at most one 5 is allowed the number has to be below 5! + 4! + 3! = 150 and how it is obvious that 1 is in the hundreds place and 5 has to be present (but not in the tens place). Hence, the number is of the form 1Y5. Y may be 0, 2, 3 or 4. Clearly 145 = 1! + 4! + 5!.

$\therefore$ 541 – 145 = 396 Ans : (396)

35. (i) $x^2 - 4x + 1 = 0$

Dividing both sides by x;

$x - 4 + \dfrac{1}{x} = 0$

$\Rightarrow x + \dfrac{1}{x} = 4$

Now, $x^2 + \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)^2 - 2$

$= (4)^2 - 2 = 14$

Now, $x^4 + \dfrac{1}{x^4} = \left(x^2 + \dfrac{1}{x^2}\right)^2 - 2$

$= (14)^2 - 2 = 196 - 2 = 194$ Ans : (194)

(ii) $x^4 - \dfrac{1}{x^4} = \left(x^2 + \dfrac{1}{x^2}\right)\left(x^2 - \dfrac{1}{x^2}\right)$

$= \left(x^2 + \dfrac{1}{x^2}\right)\left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x}\right)$

Now, $\left(x - \dfrac{1}{x}\right) = \pm\sqrt{\left(x + \dfrac{1}{x}\right)^2 - 4}$

$= \sqrt{4^2 - 4} = \sqrt{12} = 2\sqrt{3}$

$x^2 + \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)^2 - 2 = (4)^2 - 2 = 14$

$\therefore x^4 - \dfrac{1}{x^4} = 14 \times 4 \times 2\sqrt{3} = 112\sqrt{3}$ Choice (B)

36. Let the 600-digit number be N. The first positive integer occurs once in N, the second occurs twice, the third thrice and so on.

The number has 600 digits.

$\therefore$ The last (positive) integer occurring in N is a two-digit number or part of a two-digit number.

Let us work out how many digits we would get if we go up to 25.

Total number of digits in the number up to 99…… 9 (nine times) = 1 + 2 + …9 = 45.

Total number of digits in the number from 1010…..10 (ten times) to 2525…….25 (twenty five times) = 2(10 + 11 + ….25) = 560.

Total number of digits would be 605. We need to leave out the last 5 digits.

The number = (1223334444…) (2525…)2.

There are twenty two 25's in the second bracket above.

The last four digits of the number are 5252.

$\therefore$ Remainder of the number divided by 16 is equal to that of 5252 divided by 16. i.e., 4. Choice (A)

37. $Y^2 - 8Y = (X^2 - 2X)^2 - 8(X^2 - 2X)$

$= (X^2 - 2X)(X^2 - 2X - 8) = X(X - 2)(X - 4)(X + 2)$

$= (X - 4)(X - 2)X(X + 2)$

i.e. The product of 4 consecutive even integers.

The product of any 4 consecutive integers is divisible by 24. When each of X – 4, X – 2, X and X + 2 is expressed as 2(an integer), $Y^2 - 8Y$ = 16 (Product of 4 consecutive integers).

$\therefore Y^2 - 8Y$ is divisible by (16)(24) = 384 Choice (C)

38. $324 = (18)^2 = 2^2\,3^4$

The sum of all the factors which are multiples of 3

$= (2^0 + 2^1 + 2^2)(3^1 + 3^2 + 3^3 + 3^4)$

= 7(120) = 840. Ans : (840)

39. X = {8, 14, 20, …….368,374}

8 = 2 + 6 (1) and 374 = 2 + 6 (62)

Therefore, there are 62 elements in X. If we form pairs like (8,374), (14, 368), there will be 31 pairs. From each pair, if we can choose only one number, the sum of no two numbers selected will be 382. Thus, we can choose at the most 31 numbers so that the given condition is satisfied.

Ans : (31)

40. 838695 is divisible by 5. Sum of its digits is 42; it is divisible by 3.

$\Rightarrow$ 838695 is divisible by 15.

838695 = 15 (55913)

55913 is divisible by 11.

55913 = 11(5083) = 11(5100 – 17) = 11 (17) (300 – 1) = (11) (17) (299) = 11(17) (13) (23)

838695 = (11) (13) (15) (17) (23)

The five two-digit numbers are 11, 13, 15, 17, and 23.

Their sum is 79. Choice (A)

41. For N to be divisible by 8, N must be divisible by 4.

The last two digits of N must be 56, 76, 96 or 68 (last two digits of N cannot be 88 since N has distinct digits).

If the last two digits of N are 56, the last three digits of N must be 656 or 856 (for N to be divisible by 8).

If the last two digits of N are 76, the last three digits of N must be 576, or 776 or 976.

If the last two digits of N are 96, the last three digits of N are 696 or 896.

If the last two digits of N are 68, the last three digits of N are 568, 768, or 968.

N has distinct digits. $\therefore$ The last three digits of N can be 856, 576, 976, 896, 568, 768 or 968.
For any value that the last three digits can take, the thousands digit of N has two possible values.
N has (2) (7) i.e. 14 values satisfying the given conditions.
Ans : (14)

42. Let x = 3p + 6q – 9r, y = 3p – 6q + 9r, z = –3p + 6q + 9r.
$E = x^3 + y^3 + z^3 - 3xyz$ where x + y + z = 3p + 6q + 9r
= 3(p + 2q + 3r) = 0.
As x + y + z = 0, $x^3 + y^3 + z^3 = 3xyz$.
$\therefore$ E = 0
E is at least zero as well as at most zero. Choice (C)

43. $T_n = n$
$T_{n+1} = n + 1$
$T_n + T_{n+1} = 2n + 1$, which is always odd.
$T_n + T_{n+1}$ is an odd perfect square.
$n \le 150$
$\therefore 2n + 1 \le 301$
$T_n + T_{n+1}$ is an odd perfect square not excluding 301.
$T_n + T_{n+1}$ can be 9, 25, 49, 81, 121, 169, 225 or 289.
(It cannot be 1 as 1 = 0 + 1 but 0 is not a natural number).
$T_n + T_{n+1}$ has eight values. Ans : (8)

44. The number is divisible by 9 and 4. The sum of the digits of 7 5 4 3 2 9 9 p 6 = 45 + p.
As the number is divisible by 9, 45 + p must be divisible by 9.
$\therefore$ p = 0 or 9.
As the number is divisible by 4, p6 must be divisible by 9. This condition is satisfied only if p = 9, $\therefore$ p = 9. Ans : (9)

45. L.C.M of fractions = $\frac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$

Hence, L.C.M of $\frac{3}{14}, \frac{6}{35}$ and $\frac{16}{21}$ is

$\frac{\text{L.C.M of 3}}{\text{H.C.F of 14}}, \frac{6}{35}$ and $\frac{16}{21} = \frac{48}{7} \rightarrow (1)$

H.C.F of fractions = $\frac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$

Hence, H.C.F of $\frac{3}{14}$, $\frac{6}{35}$ and $\frac{16}{21}$ is

$\frac{\text{H.C.F of 3}}{\text{L.C.M of 14}}, \frac{6}{35}$ and $\frac{16}{21} = \frac{1}{120} \rightarrow (2)$

From (1) and (2),
$\frac{\text{L.C.M of fraction}}{\text{H.C.F of fraction}} = \frac{48}{7} \Big/ \frac{1}{210}$ = 1440;
$\Rightarrow$ 1440 times the H.C.F = L.C.M
Hence answer is 1440. Choice (A)

46. We have to find how many three digit numbers from 100 to 500 are divisible by 7 but not by 11.
The least multiple of 7 exceeding 99 = 7(15)
The largest multiple of 7 less than 500 = 7(71)
The number of numbers divisible by 7 from 100 to 500
= The number of numbers from 7(15) to 7(71), i.e. number of numbers from 15 to 71, i.e. 57.
The number of numbers divisible by 7 but not 11 = the number of numbers divisible by 7 – (The number of numbers divisible by both 7 and 11, i.e. 77)
There are 5 numbers from 100 to 500 divisible by 77[77(2), 77(3), 77(4), 77(5), 77(6)]
$\therefore$ 52 numbers satisfy the given conditions. Ans : (52)

47. Let N be abcd.
N exceeds dcba by M, where M is a multiple of 45
M = abcd – dcba = 45k
(1000a + 100b + 10c + d) – (1000d + 100c + 10b + a) = 45k
999 (a – d) + 90(b – c) = 45k
90(b – c) is divisible by 45. $\therefore$ 999 (a – d) must also be divisible by 45 i.e. by both 9 and 5
a – d must be divisible by 5
$\therefore$ (a, d) can be (5, 0). Choice (D)

48. P = 1 + 2 + 6 + 24 + 120 + 720 + 5040 + 40320 + 9! + …..+ 60!
= 46233 + 9! + …..+60!
Each of 9!, 10!, ……60! is divisible by 3. Also, 46233 is divisible by 3. P must be divisible by 3.
If P is a perfect cube, it is a perfect cube divisible by 3.
Any perfect cube divisible by 3 must be divisible by 3^3 i.e. 27
46233 is not divisible by 27. Each of 9!, 10!......60! is divisible by 27.
9! + 10! + …. + 60! is divisible by 27.
P is not divisible by 27. $\therefore$ P is divisible by 3 but not by 27.
$\therefore$ P is not a perfect cube.
A:P = 46233 + 9! + …….+60!
Each of 9!, 10!,…….,60! ends with 0. $\therefore$ P ends with 3.
No positive integer ending with 3 is a perfect square.
P is not a perfect square.
P is neither a perfect square nor a perfect cube.
Choice (D)

49. The typist numbers 9 single digit pages (1 to 9), 90 two digit pages (10 to 99), 900 three digit pages (100 to 999), 201 four digit pages (1000 to 1200).
The typist would have pressed
–9 times to number all the 9 single digit pages,
–180 times to number all the 90 two digit pages,
–2700 times to number all the 900 three digit pages,
–804 times to number all the 201 four digit pages.
Number of times that the typist pressed the number keys
= 9 + 180 + 2700 + 804 = 3693 . Ans : (3693)

50. Let the four consecutive even natural numbers be 2a, 2a + 2, 2a + 4, 2a + 6.
P = 2a (2a + 2) (2a + 4) (2a + 6) = 16a (a + 1) (a + 2) (a + 3)
Q = P + 16 = 16[a(a + 1) (a + 2)(a + 3) + 1] = $16[(a^2 + 3a)(a^2 + 3a + 2) + 1]$
= $16 (a^2 + 3a + 1)^2 = 16[a(a + 3) + 1]^2$.
One of a and a + 3 must be even. $\therefore$ a (a + 3) must be even.
$\therefore$ a (a + 3) + 1 must be odd. $\therefore$ $[a (a + 3) + 1]^2$ must be odd
i.e. $\frac{Q}{16}$ is odd.
Also it is a perfect square. (1) is true
Q is an odd multiple of 16. $\therefore$ Q is not divisible by 32.
Q is divisible by both 8 and 16. Ans : (3)

51. The product of all the factors of a positive integer N is $N^{\phi(N)/2}$, where $\phi(N)$ is the number of factors of N.
Product of all the factors of N is N^2.
$\therefore \frac{\phi(N)}{2} = 2$
$\phi(N)$ = 4 i.e. N has 4 factors.
Any positive integer having 4 factors is either the cube of a prime number or the product of two distinct primes.
Suppose N is the cube of a prime number p.
The factors of N are 1, p, p^2, p^3.
Sum of all the factors of N excluding N = $1 + p + p^2$
$1 + p + p^2 = 57 \Rightarrow p^2 + p - 56 = 0 \Rightarrow p = 7 (\because p > 0)$
Suppose N is the product of the primes p_1 and p_2.
Let $p_1 < p_2$.
The factors of N are 1, p_1, p_2, p_1p_2.
$1 + p_1 + p_2 = 57 \Rightarrow p_1 + p_2 = 56$
(p_1, p_2) = (3, 53), (13, 43) or (19, 37)
N has four values. Ans : (4)

52. As the remainders obtained are the same, N must divide 39276 – 38304 = 972.
972 = 1 (972) = 2 (486) = 3 (324) = 4 (243) = (6) (162) = (9) (108).
$\therefore$ N can be 972 or 486 or 324 or 243 or 162 or 108.
$\therefore$ N has 6 possibilities. Choice (C)

53. The divisor would be a factor of (64484 – 62767) i.e. 1717 which is 17 × 101. Hence, the required three-digit number is 101. Choice (A)

54. Let P and Q be denoted by 10a + b and 10c + d respectively.
$\Rightarrow$ (10a + b) (10c + d) = (10b + a) (10d + c)
100ac + 10ad + 10bc + bd
= 100bd + 10ad + 10bc + ac $\Rightarrow$ ac = bd

Let us assume a < c. (one of the digits has to be smaller)
As ac is not prime, we have the following possibilities.

(i) If a = 1, c = 4 or 6 or 8 or 9 then
ac = 4 or 6 or 8 or 9.
As a, b, c and d are distinct, ac = 4 and ac = 9 are not possible.
If ac = 6, b = 2 and d = 3 or vice versa.
If ac = 8, b = 2 and d = 4 or vice versa.
$\therefore$(P, Q) could be (12, 63), (13, 62), (12, 84), (14, 82) or (63, 12), (62, 13) (84, 12), (82, 14)

(ii) If a = 2, c = 3 or 4
$\therefore$ ac = 6 or 8
If ac = 6, b = 1 and d = 6 or vice versa.
If ac = 8, b = 1 and d = 8 or vice versa.
$\therefore$ (P, Q) could be (21, 36), (26,31), (21,48), (28, 41) or (36, 21), (31, 26), (48, 21), (41, 28)

(iii) If a = 3, ac will not be a single digit number.
$\therefore$ a $\geq$ 3 is not possible.
$\therefore$ We have a total of 16 possibilities. Choice (B)

55. HCF of $3^p\ 5^{q+4}\ 7^7\ 11^5$ and $3^{q+5}\ 5^{p+4}\ 7^x 11^x$ is $3^x\ 5^x\ 7^x\ 11^x$. We can say x ≤ 5 ------- (1)
Consider the indices of 3 and 5. If between p and q + 5, q + 5 is the one which is not greater, then between p + 4 and q + 4, q + 4 would be the one which is definitely smaller. As the HCF = $3^x\ 5^x\ 7^x\ 11^x$, q + 5 = x and q + 4 = x.
This is not possible.
$\therefore$ Between p and q + 5, p is the one which is not greater, i.e., p = x. Between q + 4 and p + 4, p + 4 cannot be the index of 5 in the HCF.
($\because$ that index is x and x = p).
$\therefore$ q + 4 = x. As q ≥ 1, it follows that x = q + 4 ≥ 5 ------- (2)
(1), (2) $\Rightarrow$ x = 5
$\therefore$ p = 5. ($\because$ x = min (5, x)).$\Rightarrow$ p, q ≥ 1.
$\therefore$ p + 4, q + 4 ≥ 5.
x = min(p + 4, q + 4) $\Rightarrow$ x ≥ 5 – (2)
From (1) and (2), x = 5.
Also x = min(p, q + 5) and q + 5 ≥ 6.
$\therefore$ Only possibility is p = 5. Ans : (5)

56. Fermat's last theorem states that there are no solutions in positive integers for the equation $a^d + b^d = c^d$ for d ≥ 3.
Therefore, d has to be 2. There are many solutions for a, b and c. eg $3^2 + 4^2 + = 5^2$, $5^2 + 12^2 = 13^2$, $8^2 + 15^2 = 17^2$ etc.
But we see that the minimum possible value of a or b is 3 [for c, the minimum value is 5]
$\therefore$ d is less than the minimum of a and b. Choice (A)

57. (p # q) # (r # s) = 1
$\Rightarrow \dfrac{\text{HCF}(p\#q, r\#s)}{\text{LCM}(p\#q, r\#s)} = 1$
$\Rightarrow$ LCM (p # q, r # s) = HCF (p # q, r # s) $\Rightarrow$ p # q = r # s
Choice (D)

58. We have to compare $\lceil x\rceil + \lceil y\rceil + \lceil x + y\rceil$ with $\lceil 2x\rceil + \lceil 2y\rceil$
If the fractional part of x is less than 1/2, $\lceil 2x\rceil = 2\lceil x\rceil + 1$
If the fractional part of x is equal to or greater than 1/2 $\lceil 2x\rceil = 2\lceil x\rceil + 2$.
Let a be the greatest integer less than or equal to x and b be the greatest integer less than or equal to y.
Let f denote a fraction less than 1/2 and g a proper fraction equal to or greater than 1/2. We tabulate the possibilities for x, y, A = $\lceil 2x\rceil + \lceil 2y\rceil$ and B = $\lceil x\rceil + \lceil y\rceil + \lceil x + y\rceil$

A	B	$\lceil 2x\rceil + \lceil 2y\rceil$	$\lceil x\rceil + \lceil y\rceil + \lceil x + y\rceil$
a + f	b + f	(2a + 1) + (2b + 1)	(a + 1) + (b + 1) + (a + b + 1)
a + f	b + g	(2a + 1) + (2b + 2)	(a + 1) + (b + 1) + (a + b + 1) or (a + b + 2)
a + g	b + f	(2a + 2) + (2b + 1)	(a + 1) + (b + 1) + (a + b + 1) or (a + b + 2)
a + g	b + g	(2a + 2) + (2b + 2)	(a + 1) + (b + 1) + (a + b + 2)

We see that B can be greater than A or equal to but not less than A. Choice (B)

59. Let the four two-digit numbers be ab, ab + 1, ab + 2, ab + 3.
S = ab + (ab + 1) + (ab + 2) + (ab + 3) = 4ab + 6.
S = 10(q) + 0 = 10q.
S is divisible by 10. $\therefore$ 4ab + 6 ends with 0.
$\therefore$ 4ab ends with 4. $\therefore$ ab ends with 1 or 6
S = 10q = 4ab + 6
10q = 4(10a + b) + 6 = 4(10a + 1) + 6 or 4(10a + 6) + 6
10q = 40a + 10 or 40a + 30
q = 4a + 1 or 4a + 3.
q is a perfect square. If b is 1, then 4a + 1 is a perfect square.
If b is 6, then 4a + 3 is a perfect square.
ab is two digit number $\therefore$ a ≥ 1
1 ≤ a ≤ 9. $\therefore$ 5 ≤ 4a + 1 ≤ 37 and 7 ≤ 4a + 3 ≤ 39.
The perfect square values of 4a + 1 are 9 and 25.
No perfect square value exists for 4a + 3.
b cannot be 6. $\therefore$ b must be 1.
ab has two possibilities.
$\therefore$ Two combinations exist.

<u>Alternative Solution:</u>
The sum of 4 consecutive numbers is 10 times a square.
As the sum is 10 times square, it has to be 10, 40, 90, 160, 250 or 360. (As all the 4 numbers are two-digit numbers, the sum has to be less than 400). The sum of 4 consecutive numbers a, a + 1, a + 2, a + 3 is of the form 4a + 6. It cannot be 40, 160 or 360.
$\therefore$ 4a + 6 = 10, 90 or 250 and a = 1, 21 or 61.
As the numbers have to be two-digited, there are only two possibilities.
21 + 22+ 23 + 24 = 90 and 61 + 62 + 63 + 64 = 250.
Ans : (2)

60. $45^2 = 2025$ and $46^2 = 2116$.
There are 45 perfect squares up to 2050.
$12^3 = 1728$ and $13^2 = 2197$.
There are 12 perfect cubes up to 2050 (Of these 1^3, 4^3 and 9^3 are squares).
Number of natural numbers up to 2050 which are either perfect squares or perfect cubes = 45 + 12 – 3 = 54.
Number of natural numbers up to 2050 which are neither perfect squares nor perfect cubes = 2050 – 54 = 1996.
Choice (C)

Solutions for questions 61 to 75:

61. It is given that x, y, z are three successive prime numbers.
From statement I
x – y = 6, y – z = 4
$\therefore$ x – z = 10
(x, y, z) could be (29, 23, 19) or (53, 47, 43).
Statement I alone is not sufficient.
From statement II, there is a limit given which is x < 60.
Independently this statement is not sufficient.
If we combine both statements, we get two possibilities i.e., 19, 23, 29 and 43, 47, 53. Hence data is insufficient.
Choice (D)

62. From statement I,
When k = $2^2 \times 3^1 \times 5^1$ or $2^1 \times 3^2 \times 5^1$ or $2^1 \times 3^1 \times 5^2$ the number of factors of k is 12.
So we can't determine k uniquely.
From statement II,
When k = $2^2 \times 5^1 \times 3^1$ or few of its multiples then k is a multiple of 4. Again, we do not get a unique value of k.
Statement II alone is not sufficient.
Using both statements, k = $2^2 \times 5^1 \times 3^1$ Choice (C)

63. From statement I
The given expression is
$$7\frac{1}{2} + k\left(\frac{1}{3} + \frac{\ell}{6}\right) \Rightarrow \therefore\ 7\frac{1}{2} + k\left(\frac{1}{2}\right)$$
$\therefore$ This sum is an integer only when k is odd. So statement I alone is sufficient

From statement II $7\frac{1}{2}+\ell\left(\frac{5}{3}+\frac{1}{6}\right) = 7\frac{1}{2}+\ell\left[\frac{11}{6}\right]$

$\therefore$ It is an integer only when ℓ is a multiple of 3. But we do not know whether ℓ is a multiple of 3 or not. So we can't answer the question. Choice (A)

64. From statement I, a = b.
$\therefore$ a + b = 2b and ab = b^2
$b^2 < 2b$ if b = 1 and $b^2 = 2b$ if b = 2
otherwise $b^2 > 2b$
So statement I alone is insufficient
From statement II, a = 1
$\therefore$ a + b = b + 1 and ab = b
As, b + 1 > b is definitely true, a + b > ab.
Hence statement II alone is sufficient. Choice (A)

65. From statement I, N is a product of two different single digit numbers and N > 70. Hence N is (9)(8) = 72.
Hence statement I alone is sufficient
From statement II, we can say that the two-digit number greater than 70 which is a product of 3 distinct primes is 78 [as (2)(3)(13) = 78]. Hence statement II alone is sufficient.
Choice (B)

66. From statement I,
The units digit of x and x^2 is the same. So the units digit of x is 0, 1, 5, or 6. So x could be 10, 11, 15, 16, 20, 21, 25, 26, 30, or 31.
Statement I alone is not sufficient. From statement II, x^2 can be 121, 484 or 676. So x is either 11 or 26.
Statement II alone is also not sufficient.
Using both statements also x can be 11 or 26. So we can't answer the question. Choice (D)

67. The sum of (2a – b) and (2a + 5b – 4c) is 4a + 4b – 4c.
From statement I, c < 0, while a, b > 0. We can conclude that 4a + 4b – 4c is positive. But, this is not sufficient.
From statement II, if c > 0, the numbers a, b, c are a, a + 1, a + 2 and 4(a + b –c) is 4(a – 1), which may or may not be divisible by 3.
If c < 0, the numbers a, b and c are a, a + 1, –a –2 and 4(a + b –c) = 4[a+(a + 1) + (a + 2)] which is always divisible by 3.
$\therefore$ Statement II alone is not sufficient, but statements I and II, taken together are sufficient to say a, b, |c| are successive numbers. Choice (C)

68. Is N the HCF of two numbers X, Y.
From statement I, if N is the HCF it is definitely factor of x, y. But there may be more common factors of x, y.
$\therefore$ I alone is insufficient.
From statement II, let N = 5, x = 30 and y = 20.
$\therefore$ x – y = 2N = 10 and the HCF is 10 which is 2N. If N = 5, x = 25 and y = 15, x – y = 2N and HCF(x, y) = N
Hence II alone is insufficient. Even if I & II are used together, we cannot answer the question. Choice (D)

69. $K\times 0\cdot\overline{ab} = K\times\frac{ab}{99}$

$\therefore$ The product of K and $0\cdot\overline{ab}$ is an integer if K is a multiple of both 9 and 11.
Both Statements I and II are required to answer the question. Choice (C)

70. From statement I, the possibilities for (x y, z) are (28, 3, 31), (42, 5, 47). So, the value of x is not unique. So statement I alone is insufficient.
From statement II. LCM of y and z is 527. Here 527 can be expressed as 17(31), both of which are prime. So x is 31 – 17 = 14.
$\therefore$ statement II alone is sufficient. Choice (A)

71. From statement I x and y are integers. If xy is odd, then both x and y are odd. Hence statement I alone is sufficient.
From statement II, x + y is odd so either x or y is odd and the other is even. So we cannot answer the question. Hence statement II alone is insufficient. Choice (A)

72. The given number is 810A4B6C. We need to find A + B + C.
From statement I, as the number is a multiple of 5 and 8, C = 0 and B is odd.

As the number is a multiple of 9, the possible values of A and B are as listed below.

8	1	0	A	4	B	6	C
–	–	–	7	–	1	–	0
–	–	–	5	–	3	–	0
–	–	–	3	–	5	–	0
–	–	–	1	–	7	–	0
–	–	–	8	–	9	–	0

From statement II, both A and B are non prime. There are many possibilities for A, B and C.
Combining both statements, (A, B, C) = (8, 9, 0)
$\therefore$ A + B + C = 17 Choice (C)

73. X is a natural number greater than 189.
From statement I, the number has only five multiples less than 1000.
i.e., it is less than 200
$\therefore$ X $\in$ {190, 191,…….199}
$\therefore$ This statement alone does not determine X.
From statement II, the number is odd and does not end in 5.
$\therefore$ X $\in$ {191, 193, 197, 199 -----------}
$\therefore$ this statement alone is also not sufficient.
Combining statements I and II,
there are 4 possible values of X which are {191, 193, 197, 199} All these numbers are prime, so X is definitely a prime number. Choice (C)

74. Let the three integers be a, b and c.
Given, abc = 40 $\Rightarrow$ At least one of a, b, c is even.
Statement I: a + b + c is odd. Possibilities: 1 odd or 3 odd. As at least one is even, exactly 1 is odd.
Statement II: ab + bc + ca is odd. Possibilities: 2(of a, b, c) are odd or all (of a, b, c) are odd. As at least 1(a, b, c) is even, exactly 2 are odd.
We can answer the question from either statement.
Choice (B)

75. From statement I, N is the smallest number that leaves a remainder of 4 when divided by 12, 13 or 14. Such numbers are of the form (LCM of 12, 13, 14)
k + 4 = 1092k + 4 and the smallest such number is 1096.
Statement I alone is sufficient
From statement II, N is the smallest number of the form kLCM(16, 17) – 10 or 272k – 10, which is 262
So N = 262
Again this alone is sufficient. Hence each statement alone, is sufficient to answer the question. Choice (B)

Chapter – 2
(Numbers – II)

Concept Review Questions

Solutions for questions 1 to 20:

1. $a^n + b^n$ is divisible by a + b when n is odd. Since 103 is odd, $11^{103} + 14^{103}$ is divisible by 11 + 14, i.e., 25. Choice (C)

2. $38^{2n} - 11^{2n} = (38^2)^n - (11^2)^n = (1444)^n - (121)^n$. This is always divisible by 1444 – 121 = 1323. The greatest number which divides it among the choices is 1323. Choice (D)

3. $3^{200} = 3^{4\times(50)}$ As the index of the power of 3 is divisible by 4, 3^{200} has the same units' digit as 3^4 i.e. 1.
4^{500} has an even index.
Its units' digit is 6.
$\therefore$ Units' digit of $(3^{200})(4^{500})$ is 6. Ans : (6)

4. $3^{3n} - 1 = (3^3)^n - 1 = 27^n - 1^n$
If N is a natural number, $a^N - b^N$ is divisible by a – b.
$27^n - 1^n$ is divisible by 27 – 1 i.e. 26.
The given statement is true for all values of n.
Choice (C)

5. $2^{5n} + 1 = (2^5)^n + 1 = 32^n + 1^n$
If N is a natural number, $a^N + b^N$ is divisible by a + b when N is odd.
$32^n + 1^n$ is divisible by 32 + 1 i.e. 33.
The given statement is true for odd values of n.
Choice (B)

6. Units digit of $(13687)^{3265}$ is the same as units digit of $7^{3265} = 7^{4\,(816)+1}$
$\therefore$ Units digit of 7^{3265} is the same as that of 7^1, i.e. 7.
Ans : (7)

7. The remainder when any number is divided by 25 is the remainder when the number formed by the last two digits of that number (i.e., 69) is divided by 25 which is 19.
Ans : (19)

8. The remainder, when any number is divided by 9, is the remainder when the sum of its digits is divided by 9. In the given problem, the sum of the digits of the number = 37. Remainder, when it is divided by 9, is 1. Ans : (1)

9. The remainder of a^{b-1} divided by b when a and b are co-primes is 1. In the given problem, 18 and 19 are co-primes
$\therefore$ the remainder is 1. Choice (A)

10. Any 10 consecutive natural numbers have a multiple of among them 5.
$\therefore$The product ends with a 5. Ans : (5)

11. The largest power of 5 in 14! is 2.
$\therefore$ 14! ends with 2 zeros. The tens digit is 0. Ans : (0)

12. When 10000 is divided by 19, the remainder is 6.
$\therefore$ 10000 – 19 = 9981 is the largest 4 digit number which leaves a remainder of 6 when divided by 19.
Choice (D)

13. The remainder, when 1000 is divided by 36, is 28.
$\therefore$ 1000 – 28 = 972 is the largest 3-digit number divisible by 36.
$\therefore$ 972 + 36 = 1008 is the least 4-digit number divisible by 36.
$\therefore$ The least 4-digit number which leaves a remainder of 10 = 1008 + 10 = 1018. Ans : (1018)

14. The successive division is shown below

Number / Quotient	192	27	13	3
Division	7	2	4	
Remainder	3	1	1	

The last remainder is 1. Ans : (1)

15. pqr86 is an even number, but not a multiple of 4. It cannot be a square. Choice (B)

16. Any perfect square ending with a 5 must have a tens digit of 2. As the tens digit of 2a4b75 is 7, it is not a perfect square.
Choice (B)

17. 1076 is not a perfect square while 5776 is a perfect square.
Choice (C)

18. $PQ1 = (10R \pm 1)^2$ or $(10R \pm 9)^2$ where R is a single digit number.
$PQ1 = 100R^2 \pm 20R + 1$ or $100R^2 \pm 180R + 81$
Tens digit of L.H.S is Q.
Tens digit of R.H.S is even.
$\therefore$ Q must be even. Choice (B)

19. The only three-digit perfect squares in the form P6Q are 169 or 361 or 961.
$\therefore$ P has to be 1 or 3, or 9, i.e. odd. Choice (B)

20. The only three-digit perfect square in the form A9B is 196.
$\therefore$ A + B = 7. Choice (A)

Exercise – 2(a)

Solutions for questions 1 to 25:

1. Given number is $2^{48} \times 7^{40} \times 4^{48}$
2^n ends with 2, 4, 8 or 6.
$\therefore$ 2^{48} ends with 6, since 48 is multiple of '4'.
7^n ends with 7, 9, 3 or 1.
$\therefore$ 7^{40} ends with 1, since 40 is multiple of 4.
4^n ends with 4 or 6.
4^{48} ends with 6, since 48 is multiple of 2.
$\therefore$ The units digit of
$2^{48} \times 7^{40} \times 4^{48}$ is 6.
($\because$ 6 × 1 × 6 ends with 6) Ans : (6)

2. The units digit of
7^{4k} is 1
7^{4k+1} is 7
7^{4k+2} is 9 and 7^{4k+3} is 3
192567 is of the form 4k + 3
$\therefore$ 57867^{192567} would end with a 3.
Also, the units digit of
2^{4k} is 6
2^{4k+1} is 2
2^{4k+2} is 4 and 2^{4k+3} is 8
876 is of the form 4k.
$\therefore$ 1452^{876} would end with a 6.
$\therefore$ The units digit would be 13 – 6
i.e. 7 (since 3 < 6, 6 is deducted from 13). Ans : (7)

3. Let $f(x) = 5x^3 - 2x^2 - ax - b = 0$
Since (x – 1) and (x + 1) are factors of f(x),
f(1) = 0
$\Rightarrow$ 5 – 2 – a – b = 0
$\Rightarrow$ a + b = 3 $\rightarrow$ (1)
and f(–1) = 0
$\Rightarrow$ – 5 – 2 + a – b = 0
$\Rightarrow$ a – b = 7 $\rightarrow$ (2)
Solving (1) and (2), we get
a = 5 Choice (B)

4. $2^{360} = (2^3)^{120}$
$7 = 2^3 - 1$
Remainder when $(2^3)^{120}$ is divided by $2^3 - 1$ is $1^{120} = 1$.
(By remainder theorem). Ans : (1)

5. When a^n is divided by a prime number p and n is a multiple of p – 1 then the remainder would always be equal to 1.
Here, 50 = (11 – 1) × 5

$$\text{Rem}\left(\frac{3^{50}}{11}\right) = 1$$

Alternate method:

<u>powers of 3</u>	<u>Remainder</u>
3	3
9	9
27	5
81	4
243	1
729	3

729(3^6), 3^7, 3^8, 3^9, 3^{10} leave the same remainders as those of 3, 3^2, 3^3, 3^4, 3^5 respectively.

$$\text{Rem}\left(\frac{3^x}{11}\right) = \text{Rem}\left(\frac{3^{x-5}}{11}\right)$$

$$\text{Rem}\left(\frac{3^{50}}{11}\right) = \text{Rem}\left(\frac{3^5}{11}\right) = 1$$ Ans : (1)

6. $21^3 + 24^3$ is divisible by 21 + 24 i.e., 45
$22^3 + 23^3$ is divisible by 22 + 23 i.e., 45
$\therefore$ $Y = 21^3 + 22^3 + 23^3 + 24^3$ is divisible by 45.

Moreover, Y is even (there are two odd numbers in the expression)

$\therefore$ Y is divisible by 90 or Rem Y/90 = 0 Ans : (0)

7. M = 49, 51, 49, 51, 49, 51, …. 49, 51 (600 digits or 300 groups of 2 digits each. Of these 300 groups, there are 150 groups of 49 and 150 groups of 51).

L – S = (51) (150) – (49) (150) = 2(150) = 300

P = Rem $\left(\frac{300}{101}\right)$ = 98. Choice (C)

8. Let N = 676, 767,... (900 digits or 300 groups of 3 digits each)

M = (676 + 767) 150 = 216, 450

P = 216 + 450 = 666

$\therefore$ Rem $\left(\frac{N}{999}\right) = 666$. Ans : (666)

9. Rem $\frac{347^{347}}{100} = \text{Rem}\frac{47^{340+7}}{100} = \text{Rem}\frac{47^{7}}{100}$

47^2 = 2309

09^2 = 81

47^6 = (81)(09) = 29

$\therefore$ 47^7 = (47)(29) = 63 Choice (C)

10. Rem 78^{1234} = Rem $78^{1220+14}$ = Rem 78^{14}

Rem 78^2 = 84, Rem 84^2 = 56, Rem 56^2 = 36

Rem 78^{14} = Rem $(78^8)(78^4)(78^2)$ = Rem (36) (56) (84)

= Rem (16) (84) = 44 Ans : (44)

11. Rem $\frac{326^{972}}{100} = \text{Rem}\frac{26^{12}}{100}$. All powers of 26 (except the first) end in 76. Choice (D)

12. Let $(1 + x + 2x^2)^{100} = a_0 + a_1 x + a_2 x^2 + ... + a_{200} x^{200}$

Setting x = 1, we get $a_0 + a_1 + ... + a_{200} = 4^{100} = 2^{200}$

Choice (A)

13. We need Rem 2^{275}/137. The divisor d is prime and the index of the power (275) is of the form nd + 1 [275 = 2(137) + 1]. By Fermat's theorem, 2^{136} leaves a remainder of 1

$2^{275} = 2^{3a}\, 2^{272}$ $\therefore$ Rem $2^{275} = 8\,(1)^2 = 8$. Choice (D)

14. Rem 32^{180}/149 = Rem 2^{900}/149. By Fermat's Theorem, Rem 2^{148}/149 = 1

$\therefore$ Rem $2^{6(148)}$ = Rem 2^{888} = 1

Rem 2^{900} = Rem 2^{888} Rem 2^{12} = Rem 2^{12} = Rem 4096

(In all such questions, i.e Rem a^n/P, we have to be careful and treat the index (n) and the power a^n separately. We divided 2^{900} by 149 but 900 by 148.

Rem $\left(\frac{2^{r+148k}}{149}\right)$ = Rem $\left(\frac{2^r}{149}\right)$

For the index, the periodicity is p – 1. (and not p).

$\therefore$ We divided 900 by 148 (and not 149) when we wrote with the power, we have to divide it by p (and not p – 1)

Rem $\frac{4096}{149}$ = Rem 149 (27) + 73 = 73

$\therefore$ Rem $\left(\frac{32^{180}}{149}\right) = 73$. Ans : (73)

15. We want Rem 24^{1202}/1446 Now 1446 = 2(723) = 2(3)(241)

Rem $\frac{24}{2}$ = 0 $\Rightarrow$ Rem 24^{1202}/2 = 0

Rem $\frac{24}{3}$ = 0 $\Rightarrow$ Rem 24^{1202}/3 = 0

Rem 24^{1202}/241 = Rem $24^{1200}(24^2)$/241

= Rem $(24^{240})^5\,(24^2)$/241 = 1(Rem 576/241) = 94

This is LCM model 3

$\therefore$ 6 x = 241 y + 94

y = 6 z + 2

Let N = 24^{1202}. N leaves a remainder of 0 when divided by 6 and 94 when divided by 241. Let us say all such numbers leave a remainder of r when divided by 6(241) = 1446. We obtain r as shown below.

Let X = 6 x = 241 y + 94

$\Rightarrow$ y = 6 z + 2

[As 94 leaves a remainder of 4 when divided by 6, 241y (and hence y) leaves a remainder of 2]

Least z = 0. $\therefore$ Least y = 2

x = $\frac{241(2)+94}{6} = \frac{482+94}{6} = \frac{576}{6}$ = 96. $\therefore$ Least x = 6.96

The number x is of the form LCM (6,241) k + 6.96 = 6(241) k + 576 (This family of numbers represents an AP whose common difference is 1446).

N is one of the numbers of this AP

$\therefore$ r = Rem $\frac{N}{1446}$ = 576 Choice (B)

16. By Wilson's theorem, (P – 1)! is of the form K_1 P – 1

An immediate corollary of this is that, (P – 2)! is of the form K_2P + 1

$\therefore$ 95! is of the form 97 K + 1. The required remainder is 1

Ans : (1)

17. By Wilson's theorem, (P – 1)! is of the form KP – 1

$\therefore$ 46! = 47 K – 1

$\Rightarrow$ 46! (47) (48) (49) (50) = (47 K – 1) (47) (48) (49) (50)

Let N = $\frac{50!}{47}$ = (47 K – 1) (48) (49) (50)

Consider the remainder when N is divided by 47

Rem $\frac{N}{47} \equiv (-1)(1)(2)(3) = -6 \equiv 41$

$\therefore$ Rem $\frac{50!}{47^2} \equiv$ (41) (47) = 1927

(Rem $\frac{KN}{KD} = K\,\text{Rem}\frac{N}{D}$) Choice (D)

18. N = 1! + 2! + 3! + …. 100!

168 = 7(24)

$\text{Rem}\left(\frac{N}{24}\right) = \text{Rem}\left(\frac{1+2+6}{24}\right) = 9$

$\text{Rem}\left(\frac{N}{7}\right) = \text{Rem}\left(\frac{1+2+6+24+120+720}{7}\right) = 5$

$\therefore$ N = 7p + 5 = 24q + 9. N and all the other numbers X which have there two properties (Rem $\frac{X}{24}$ = 9 and Rem $\frac{X}{7}$ = 5) can be obtained as follows.

7p = 24q + 4 ------ (1)

$\Rightarrow$ 3q = 7q_1 + 3 (Consider the remainders of the different terms when they are divided by the smaller of the two coefficients in (1), viz 7. The LHS is a multiple of 7. $\therefore$ The RHS has to be a multiple. As the second term on the RHS leaves a remainder of 4, the first term (i.e 24q or equivalently 3q) has to leave a remainder of 3. $\therefore$ q_1 is a multiple of 3)

The values of q_1, q, p, 7p + 5 and 24q + 9 are tabulated below.

q_1	q	p	7p + 5	24q + 9
0	1	4	33	33
3	8	28	201	201
6	15	52	369	369

We see that any such number X has the form 168k + 33.

$\therefore$ N is also of the form 168k + 33.

Alternate method:

168 = 7(24).

$\therefore$For n $\geq$ 7, n! is divisible by 168.

Rem $\frac{N}{168}$ depends only on the first 6 terms of N, Rem $\left(\frac{1!+2!+3!+4!+5!+6!}{168}\right)$

= Rem $\left(\frac{1\ +\ 2\ +\ 6\ +\ 24\ +\ 120\ +\ 720}{168}\right)$

= Rem $\frac{873}{168}$ = 33. Ans: (33)

19. N = 127127 ………….. 1271 (a total of 202 digits)
N = 1 + 1270 + 1270000 + ……
= 1+ 1270 (1 + 10^3 + 10^6 + …….. 10^{198})
The given divisor is 143. Instead of that we can consider a convenient multiple of 143, viz, 143(7) = 1001.

Rem $\frac{N}{1001}$ = $\text{Rem}\left(\frac{1+1270(1-1+1.....+1)}{1001}\right)$ = 1271

$\text{Rem}\left(\frac{N}{143}\right)$ = $\text{Rem}\left(\frac{1271}{143}\right)$ = 127. Choice (C)

20. $\text{Rem}\left(\frac{1111}{19}\right)$ = 9.

Successive powers of 1111 (or Rem $\frac{1111}{19}$ = 9) leave the following remainders 9, 5, 7, 6, 16, 11, 4, 17 and 1 when divided by 19. After that (viz after the 9^{th} power), the pattern repeats. As 2222 = 9(246) + 8, $\text{Rem}\left(\frac{1111^{2222}}{19}\right)$ = 17, viz the 8^{th} number in the sequence above. Choice (D)

21. x = $\frac{(60-58)\left(60^{98}+60^{97}\,58+60^{96}\,58^2+.......+58^{98}\right)}{60^{98}+58^{98}}$

= 2 × (1 + some positive number)
∴ x > 2 Choice (D)

22. $2^{14N+7} + 3^{10N+5} - 7 = (2^7)^{2N+1} + (3^5)^{2N+1} - 7$
$= 128^{2N+1} + 243^{2N+1} - 7$
2N + 1 is odd.
$a^n + b^n$ is always divisible by a + b when n is odd.
As 2N + 1 is odd, $128^{2N+1} + 243^{2N+1}$ is divisible by 128 + 243 i.e. 371.
$2^{14N+7} + 3^{10N+5} - 7 = 371k - 7$
Remainder of $2^{14N+7} + 3^{10N+5} - 7$ divided by 371 is –7.
The equivalent positive remainder is 364. Choice (C)

23. I : X = $10^{57} - 450 = 10(10^{56} - 45)$
$10^{56} = (11-1)^{56} = (11-1)(11-1)$ …………….. 56 times
$= 11k + (-1)^{56} = 11k + 1$
$10^{56} - 45 = 11k + 1 - 45 = 11(k-4)$
X = 10 (11 (k – 4)) ⇒ X is divisible by 11.
I is true.
II: X = (7 + 3) (7 + 3) ………….. 57 times = 7k + 3^{57}
$3^{57} = (3^3)^{19} = 27^{19} = (28-1)^{19} = 28k_1 + (-1)^{19} = 28k_1 - 1$
$10^{57} = 7k + 28k_1 - 1 = 7k_2 - 1$
$X = 7k_2 - 1 - (7(63) + 9) = 7(k_2 - 63) - 10$

Rem $\left(\frac{X}{7}\right)$ = –10.

The equivalent positive remainder is 4.
II is true.

Alternate Solution:

Rem $\frac{10^n}{11}$ = 10 if n is odd, and 1 if n is even.

Rem $\frac{450}{11}$ = 10.

∴Rem $\frac{10^{57}-450}{11}$ = 0.

The remainders of successive powers of 10, when divided by 7, show the following pattern : 3, 2, 6, 4, 5, 1; 3, 2, 6, 4, 5, 1; . . . .
i.e they form a cyclic pattern with cycle length 6.
∴ Rem $\frac{10^{57}}{7}$ = Rem $\frac{10^3}{7}$ = 6 and Rem $\frac{450}{7}$ = 2.

∴Rem $\frac{X}{7}$ = 6 – 2 =4 ⇒ Both I, II are true. Choice (C)

24. Difference of 7^P and P^3 is divisible by 10 i.e. 7^P and P^3 have the same units digit.
7^P is odd, ∴ P^3 is odd ⇒ P is odd.
P = $4k_1$ + 1 or $4k_2$ + 3 where k_1 and k_2 are whole numbers.
If P = $4k_1$ + 1, 7^P has a units digit of 7.
∴ P^3 should have a units digit of 7. This is possible only if the units digit of P is 3.
If P = $4k_1$ + 1, P = 13, 33, 53, 73 or 93.
If P = $4k_2$ + 3, 7^P has a units digit as that of 7^3 i.e. 3.
∴ P^3 also has a units digit of 3. This is possible only if the units digit of P is 7.
If P = $4k_2$ + 3, P = 7, 27, 47, 67 or 87.
P has 10 values. Ans: (10)

25. $256^{15} = 16^{30} = (17-1)^{30} = 17^{30} + {}^{30}C_1\,(17)^{29}\,(-1) + {}^{30}C_2(17)^{28}(-1)^2 + \ldots\ldots\ldots\ldots + {}^{30}C_{29}\ (17)\ (-1)^{29} + {}^{30}C_{30}(-1)^{30}$
Except for the last two terms which are –30(17) and1, all the terms in the expansion are divisible by 17^2.
$256^{15} = 17^2k + 30\ (17)\ (-1) + 1 = 17^2k - 509$
$= 17^2(k-2) + 2(289) - 509 = 17^2k_1 + 69$

∴ Rem $\frac{289(256^{15})}{17^4}$ = 17^2 Rem $\frac{(256)^{15}}{17^2}$ = 289(69) = 19941. Ans: (19941)

Exercise – 2(b)

Solutions for questions 1 to 35:

1. Given,
$7^{33} \times 14^{31} \times 6^{30}$.
Units digit of 7^{33} = 7
Units digit of 14^{31} = units digit of 4^{31} = 4
Units digit of 6^{30} = 6.
Units digit of $7^{33} \times 14^{31} \times 6^{30}$ is the same as the units digit of (7 × 4 × 6)
Required units digit = 8 Ans : (8)

2. Given,
3^{44} + 131 × 56 + 34 × 46
Units digit of 3^{44} = 1
Units digit of 131 × 56 = 6
Units digit of 34 × 46 = 4
Units digit of (3^{44} + 131 × 56 + 34 × 46) is the same as the units digit of (1 + 6 + 4)
Required units digit = 1 Ans : (1)

3. The right most non-zero digit of 70^{5340} is the right most digit of 7^{5340}. The right most digit of 7^{5340} is 1.
(As 7^{5340} is divisible by 4, 7^{5340} has the same units digit as that of 7^4).
∴ Required digit is 1. Choice (D)

4. The last digit in 4^{2k} is 6 and that in 4^{2k+1} is 4.
782 is of the form 2k.
∴ The last digit of 424^{782} is 6.
The last digit of 9^{2k} is 1 and that in 9^{2k+1} is 9.
∴ The last digit of 179^{137} is 9.
(Since, 137 is of the form 2k+1).
∴ The last digit of $424^{782} + 179^{137}$ is 6 + 9 i.e., 5.
Choice (C)

5. Rem $\frac{1576^{689}}{100} = \text{Rem}\frac{76^9}{100}$ = 76. All powers of 76 end in 76. Choice (D)

6. $\frac{a^n}{a+1}$ leaves the remainder 1 when n is even
∴ Required remainder is 1. Ans : (1)

7. $(19^{23} + 17^{23})$ is divisible by 19 + 17 viz 36 because 23 is an odd number.
$\therefore$ The units digit of the remainder when $(19^{23} + 17^{23})$ is divided by 36 is zero. Ans : (0)

8. $(2^4)^{19}$ is divided by $2^4 - 1$.
Let $x = 2^4$, $f(x) = x^{19}$. When f(x) is divided by x – 1, the remainder is f(1)
$f(1) = (1)^{19} = 1$ Ans : (1)

9. $(2^6)^{11}$ is divisible by $2^6 - 1$
Using a similar method as in the above solution the remainder is $(-1)^{11} = -1$.
A negative remainder is not possible. It is converted to a positive remainder by adding the divisor. Hence the actual remainder is –1 + 65 = 64. Choice (D)

10. Rem (2582/3) = 2 (or – 1)
$\therefore$ Rem $(2582^{801}/3)$ = 2 (or – 1)
Rem (2579/3) = 2
$\therefore$ Rem $(2579^{401}/3) = 2$
$\therefore$ Rem $[(2582^{801} - 2579^{401})/3] = 0$
Ans : (0)

11. $7^{83} = 7^3 (7^{80}) = 7^3(7^4)^{20} = 7^3 (2400 + 1)^{20}$
$= 7^3(2400k + 1) = (7^3)(2400k) + 343$ where k is a natural number.
When this is divided by 20, the remainder is 3.
Choice (C)

12. To find the remainder of any number N (say) when divided by 99..9 (n nines), we group the digits of N such that there are n per group starting from the right and add up all the groups. Then we can apply this process as often as needed until we get a value less than the divisor. This value is the remainder. n = 56, 78, 56, 78,....56, 78 (1000 digits or 500 groups of 2 digits each. Of these 500 groups, there are 250 groups of 56 and 250 groups of 78).
Let Rem $\left(\frac{N}{99}\right)$ = M.
M = (56 + 78)250 = (134) (250) = 3,35,00
Since M > 99, we repeat the process until we get the remainder.
$\therefore$ Rem $\left(\frac{N}{99}\right)$ = 3 + 35 + 00 = 38. Choice (C)

13. To find the remainder of any number N (say) when divided by $10^n + 1$, we group the digits of N such that there are N digits per group starting from the right. We then add up all the alternative groups starting with the last group. Let the sum we obtain be called L. We add up all the alternative groups starting from the second last group. Let the sum we obtain be called S. Then we apply this process as often as needed until we get a value less than the divisor. This value is the remainder.
Let N = 468 468 ... 468 (333 digits
= 1000 M + 468 (where M = 468468.......330 digits)
M = 468,468, ... 468 (330 digits or 110 groups of 3 digits each of these 110 groups, L as well as S is the sum of 55 groups).
P = (468) (55) – (468) (55) = 0
$\therefore$ Rem $\frac{M}{1001} = 0$
Rem $\frac{N}{1001} = 468$ Ans : (468)

14. Rem $\frac{793^{1008}}{100} = \text{Rem}\frac{93^{1000+8}}{100} = \text{Rem}\frac{93^8}{100}$
Also, Rem $\frac{793^{1008}}{25}$ can be obtained from Rem $\frac{793^{1008}}{100}$
$93^2 \equiv 49$, $49^2 \equiv 01$, $01^2 \equiv 01$
$\therefore$ Rem $\left(\frac{93^8}{25}\right) = \text{Rem}\left(\frac{01}{25}\right) = 1$
($\because 93^8$ has the form 100k + 01 = 25 (4k) + 01).
Note: $\equiv$ means equivalent remainder.
Choice (B)

15. Rem $\frac{114^{210}}{25}$ can be obtained from Rem $\frac{114^{210}}{100}$
Rem $\frac{114^{210}}{100} = \text{Rem}\frac{14^{10}}{100}$
$14^2 \equiv 96$, $96^2 \equiv 16$, $16^2 \equiv 56$
$\therefore 14^8 \equiv 56$
$14^{10} \equiv 14^8 (14^2) \equiv 56(96) \equiv 76$
$\therefore$ Rem $\left(\frac{14^{10}}{25}\right) = \text{Rem}\left(\frac{76}{25}\right) = 1$ ($\because 14^{10}$ has the form 100k + 76, i.e. 25 (4k) + 76).
$\therefore$ Rem $\left(\frac{114^{210}}{25}\right) = 1$

Alternate method:
14^{10} = Tenth power of an even number.
The tenth power of any even number ends with 24 or 76. Also 14^{10} ends with 6.
$\therefore$ Rem $\left(\frac{114^{210}}{25}\right) = 1$. Choice (C)

16. Rem $\frac{784^{489}}{100} = \text{Rem}\frac{84^9}{100}$
$84^2 \equiv 56$, $56^2 \equiv 36$, $36^2 \equiv 96$.
$\therefore 84^8 \equiv 96 \Rightarrow 84^9 \equiv (96)(84) \equiv 64$.

Alternate method:
The tenth power of any even number must end with 24 or 76.
Also 84^{10} ends with 6. $\therefore 84^{10}$ ends with 76.
84^9 will end with 4. Let us say it ends with a4.
(a4) (84) = 76
Tens digit of L.H.S = units digits of (a.4 + 4.8 + tens digits of 4.4) = that of 4a + 3 $\therefore$ 4a + 3 ends with 7
$\therefore$ 4a ends with 4
$\therefore$ a ends with 1 or 6.
$\therefore$ a4 = 14 or 64.
But 84^9 must be divisible by 4.
$\therefore$ a4 cannot be 14. $\therefore$ It must be 64. Ans : (64)

17. Rem $\frac{1532^{786}}{100} = \text{Rem}\frac{32^6}{100}$
$32^2 \equiv 24$, $24^2 \equiv 76$ $\therefore 32^4 \equiv 76$
$32^6 = 32^4 (32^2)$
$\therefore$ Rem $32^6 \equiv (76)(24) \equiv 24$ Choice (B)

18. Rem $\frac{71}{72} = -1$, Rem $\frac{73}{72} = 1$
$\therefore$ Rem $71^{72} + 73^{72} = (-1)^{72} + 1^{72} = 2$ Ans : (2)

19. Rem $\frac{91}{31} = -2$ and Rem $\frac{95}{31} = 2$
$\therefore$ Rem $(91^{150} + 95^{150})$ = Rem $(-2)^{150}$ + Rem 2^{150} = 2 Rem 2^{150}
= 2 Rem $(2^5)^{30}$ = 2 (1) = 2.
Ans : (2)

20. Let $(2 + 3x)^{75} = a_0 + a_1 x + \ldots\ldots + a_{75} x^{75}$
Setting x = 1, we get $a_0 + a_1 + \ldots\ldots + a_{75} = 5^{75}$
Choice (C)

21. We want Rem $6^{722}/73$
722 = 72 (10) + 2.
Rem $6^{72}/73 = 1$
$\therefore$ Rem $6^{720}/73 = 1$ ($\because$ Fermat's theorem)
$\therefore$ Rem $6^{722}/73$ = Rem $6^2/73 = 36$
Choice (B)

22. $81^{225} = 3^{900}$ the divisor is the prime number 179.
900 = 178 (5) + 10

Let R = Rem $\frac{81^{225}}{179} = \text{Rem}\frac{3^{900}}{179} = \text{Rem}\frac{(3^{178})^5(3^{10})}{179}$

= Rem $\frac{3^{10}}{179}$

Rem $\frac{3^5}{179} = \text{Rem}\frac{243}{179}$ = 64

R = Rem $\frac{64^2}{179} = \text{Rem}\frac{4096}{179}$ = 158 $\therefore$ R = 158

Ans : (158)

23. By Wilson's Theorem (P − 1)! is of the form K_1 P − 1
An immediate corollary of this is that, (P − 2)! is of the form K_2 P + 1
$\therefore$ 101! is of the form 103 K + 1. The required remainder is 1.
Choice (B)

24. 100 = 25(4)

$\text{Rem}\left(\frac{7^{349}}{4}\right) \equiv (-1)^{349} \equiv -1 \equiv 3$

$\text{Rem}\left(\frac{7^{349}}{25}\right) = \text{Rem}\left(\frac{(49)^{174} \times 7}{25}\right)$

$\equiv \text{Rem}\left(\frac{49^{174}}{25}\right) \text{Rem}\left(\frac{7}{25}\right) \equiv (-1)^{174}\, 7 = 7$

$\therefore 7^{349}$ is of the form 4x + 3 and 25y + 7. Numbers of the form mx + a and ny + b (where (m, n) = 1) are of the form mnz + c where c is obtained as shown below.
4x + 3 = 25y + 7 $\Rightarrow$ 4x = 25y + 4 $\Rightarrow$ y = $4y_1$
Values of y_1, y, x and 4x + 3, 25y + 7 are tabulated below.

y_1	y	x	4x + 3	25y + 7
0	0	1	7	7
1	4	26	107	107
2	8	51	207	207

$\therefore$ N is of the form 100k + 7

Alternate method:
Successive powers of any positive integer n, when divided by any divisor d leave remainders that are in a cyclic pattern. When d = 100, the remainders are simply the last two digits of the powers. When n = 7 and d = 100, the pattern is very short. There are only 4 possible values for the last two digits of the powers of 7 which are 07, 49, 43 and 01.

$\therefore$ Rem $\frac{7^{349}}{100}$ = Rem $\frac{7^{4(87)+1}}{100}$ = Rem $\frac{7}{100}$ = 7.

Choice (B)

25. N = $12^1 + 12^2 + 12^3 + \ldots.\ 12^{100}$

$\text{Rem}\left(\frac{N}{7}\right) = \text{Rem}\left(\frac{5+5^2+\ldots..5^{100}}{7}\right)$

The remainders when power of 5 are divided by 7 are listed below
$5^1 - 5$
$5^2 - 4$
$5^3 - 6$
$5^4 - 2$
$5^5 - 3$
$5^6 - 1$

$5^7 - 5$
The remainders show a cyclic pattern and the cycle length is 6.
Each of the first 16 complete cycles contributes 0. There are 4 powers in the next (the 17^{th}) cycle

$\therefore$ Rem $\frac{N}{7}$ = Rem $\frac{5+4+6+2}{7}$ = 3. Ans : (3)

26. 1742 is divisible by 13.
Required remainder = Remainder of (1742 + 8) (1742 + 10) (1742 + 12) divided by 13 = Remainder of (8) (10) (12) divided by 13 = 11. Ans : (11)

27. Rem $\frac{(3333333333)}{16}$ = Rem $\left(\frac{3333}{16}\right)$ = 5 ---------(1)

Rem $\left(\frac{3^{144}}{16}\right)$ = Rem $\left(\frac{81^{36}}{16}\right)$ = Rem $\left(\frac{(16k+1)^{36}}{16}\right)$

$(16k + 1)^{36}$ = (16k + 1) (16k + 1),………………… 36 times = 16M + 1^{36} = 16M + 1

Rem $\left(\frac{(16k+1)^{36}}{16}\right)$ = 1 i.e. Rem $\left(\frac{3^{144}}{16}\right)$ = 1 --------------(2)

$\therefore$ Required remainder = 5 + 1 = 6. Choice (D)

28. Rem $\frac{31^{3300}}{32} = (-1)^{3300} = 1$

Rem $\left(\frac{3332}{32}\right)$ = 4

Rem $\left(\frac{31^{3300} - 3332}{32}\right)$ = 1 – 4 = –3

Equivalent positive remainder is 32 –3, viz 29.
Choice (C)

29. Every natural number is of the form 3k or 3k + 1 or 3k + 2 where k is a whole number.
$(3k)^2$, when divided by 9, will leave a remainder of 0.
$(3k + 1)^2 = 9k^2 + 6k + 1$
k can be of the form 3k, or $3k_1$ + 1 or $3k_1$ + 2 where k_1 is a whole number.
$\therefore$ 6k + 1 can be $18k_1$ + 1 or $18k_1$ + 7 or $18k_1$ + 13.
$\therefore$ $(3k + 1)^2$ when divided by 9 leaves a remainder of 1 or 7 or 4.
Similarly, it can be shown that $(3k + 2)^2$ leaves a remainder of 1 or 7 or 4.
$\therefore$ The square of a natural number leaves a remainder of 0 or 1 or 4 or 7. The sum of all the possible remainders is 12.
Ans : (12)

30. We want Rem 10^{1283}/514. Now 514 = 2(257)
Rem 10/2 = 0 $\Rightarrow$ Rem 10^{1283}/2 = 0

Rem $\frac{10^{1283}}{257} = \text{Rem}\frac{10^{5(256)+3}}{257}$ = Rem $\frac{10^3}{257}$

($\because$ Rem $\frac{10^{256}}{257} = 1$) = 229

Let N = 10^{1283}. N leaves a remainder of 0 when divided by 2 and 229 when divided by 257. Let us say all such numbers leave a remainder of r when divided by 2(257) or 514. We obtain r as shown below.
Let X = 2 x = 257 y + 229 ($\therefore$ y has to be odd)
y = 1 $\Rightarrow$ 2 x = (257 + 229) = 486
In general, X = 2(257) K + 486. (This family of numbers represents an AP whose common difference is 514). N is one of the numbers of this AP. The required remainder is 486. $\therefore$ r = 486. Choice (D)

31. By Wilson's theorem, (P – 1)! is of the form Kp – 1.
$\therefore$ 22! = 23 K – 1
$\Rightarrow$ 22! (23) (24) (25) = (23 K – 1) (23) (24) (25).

Let N = $\frac{25!}{23}$ = (23 K – 1) (24) (25)

Rem $\left(\frac{N}{23}\right) \equiv -1$ (1) (2) = – 2 $\equiv$ 21

$\therefore$ Rem $\left(\frac{N}{23^2}\right) \equiv$ (21) (23) = 483 $\left(\text{Rem}\frac{KN}{KD} = K\ \text{Rem}\frac{N}{D}\right)$.

Choice (B)

32. $N = 1^4 + 2^4 + 3^4 \ldots + 100^4$ (say)

$$\text{Rem}\left(\frac{N}{7}\right)$$

$$= \text{Rem}\left(\frac{1^4+2^4+3^4+\ldots 100^4}{7}\right)$$

$$= \text{Rem}\left[\frac{(1^4+\ldots+7^4)+\ldots+(92^4+\ldots+98^4)+99^4+100^4}{7}\right]$$

$$= \text{Rem}\left[\frac{14(1+2+4+4+2+1+0)+1+2}{7}\right] = 3. \quad \text{Ans : (3)}$$

33. Let A = 111111 = 111(1001) = 111(7)(11)(13)
$\therefore$ A is divisible by 13.
N = 111…..1(a total of 363 digits)
= 1000 M + 111 where M = 111…..1 (a total of 360 digits)
$\therefore \text{Rem}\,\frac{N}{13} = \text{Rem}\,\frac{111}{13} = 7.$ ($\because$ M is a multiple of 13)

Choice (C)

34. $X = 40k_1 + 1$ and $Y = 40k_2 + 2$
I: $3^X = 3^{40k_1+1} = 3^{4(10k_1)+1}$
Units digits of 3^N has a cycle of 4.
$\therefore$ Units digit of 3^X is that of 3^1 i.e. 3.
$3X = 3(40k_1 + 1) = 120k_1 + 3$
$\therefore$ Units digit of 3X is 3
3^X and 3X have the same units digits.
$\therefore 3^X - 3X$ is divisible by 10.
I is true.

II: $7^Y = 7^{40k_2+2} = 7^{4(10k_2)+2}$
Units digit of 7^N has a cycle of 4.
$\therefore$ Units digit of 7^Y is same as that of 7^2 i.e. 9.
$7^Y + 7(Y + 1) = 10k + 9 + 7(40k_2 + 3) = 10k + 280k_2 + 30$
$7^Y + 7(Y + 1)$ is divisible by 10.
II is true.

Alternate Solution:
The units digit of successive powers of 3 or 7 show a cyclic pattern and the cycle length is 4.
For 3 and 7, the patterns are shown below.

Units digit of n	3	7
Units digit of n^2	9	9
Units digit of n^3	7	3
Units digit of n^4	1	1

As X = 40P + 1, the units digit of 3^X is 3 while 3X =120P+3.
$\therefore 3^x - 3X$ is divisible by 10.
As Y = 40Q+2, the units digit of 7^y is 9 while
$7(Y + 1) = 7(40Q + 3) = 280Q + 21$. $\therefore 7^Y + 7(Y + 1)$ is divisible by 10.
Both I, II are true. Choice (C)

35. N = 120121122 ………………… 165
Let M = 120 + 121 + 122 + …………… + 165

$$\text{Rem}\left(\frac{N}{9}\right) = \text{Rem}\left(\frac{M}{9}\right)$$

$$M = \frac{(165)(166)}{2} - \frac{(119)(120)}{2} = (165)(83) - (119)(60)$$

= [9(18) + 3] [(9 (9) + 2)] – [9 (13) + 2] [9 (6) + 6]

$\text{Rem}\left(\frac{M}{9}\right) = -6$. The equivalent positive remainder is 3.

$\text{Rem}\left(\frac{N}{9}\right) = 3.$

Alternate Solution:
Let N = 120121 ------------ 165
and M = 120+121+ --------- +165
M is the sum of the 46 consecutive integers. We can ignore the first 45 of these (their sum would be a multiple of 9 and think of only 165. As Rem $\frac{165}{9} = 3$, it follows that Rem $\frac{N}{9} = 3$.
Note: The sum of n consecutive numbers is a multiple of n, provided n is odd. If n is even, say n = 2m, then the sum of n consecutive numbers leaves a remainder of m, when divided by n. Choice (A)

Chapter – 3
(Number Systems)
Concept Review Questions

Solutions for questions 1 to 15:

1. We have 502 = 256 + 128 + 64 + 32 + 16 + 4 + 2
$= 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^2 + 2^1$
$\therefore (502)_{10} = (111110110)_2$ Choice (D)

2. $(1000001)_2 = 1 \times 2^6 + 1 \times 1 = 65$ Ans : (65)

3.
```
7 | 532
7 | 76 – 0
7 | 10 – 6
7 | 1 – 3
  | 0 – 1
```
$\therefore (532)_{10} = (1360)_7$ Ans : (1360)

4.
```
12 | 1463
12 | 121 – 11 (B)
12 | 10 – 1
   | 0 – 10 (A)
```
$\therefore (1463)_{10} = (A1B)_{12}$ Choice (C)

5. The largest 3-digit septenary number is 666.
Ans : (666)

6.
```
8 | 239
8 | 29 - 7
  | 3 - 5
```
$\therefore (239)_{10} = (357)_8$ Choice (D)

7. $(AEB)_{16} = A \times 16^2 + E \times 16 + BX1$
$= 10 \times 256 + 14 \times 16 + 11 \times 1 = (2795)_{10}$ Choice (B)

8. Let N be the number and $(a_k\, a_{k-1} \ldots a_4\, a_3\, 000)_2$
$= a_k.\, 2^k + a_{k-1}.\, 2^{k-1}$
$+ \ldots\ldots\ldots + a_4.2^4 + a_3.2^3$
$+ 0.2^2 + 0.2^1 + 0.2^0$
$= a_k.2^k + a_{k-1}.2^{k-1} + \ldots$
$+ a_4.2^4 + a_3.2^3$
Clearly N is divisible by 8. Among the choices only 48 is divisible by 8. Choice (D)

9.
```
16 | 1734
16 | 108 – 6
16 |   6 – C
   |   0 – 6
```
$\therefore (1734)_{10} = (6C6)_{16}$. Choice (D)

10. We have
$(243)_6 = 2 \times 6^2 + 4 \times 6 + 3 \times 1 = (99)_{10}$
$(201)_7 = 2 \times 7^2 + 0 \times 7 + 1 \times 1 = (99)_{10}$
and $(143)_8 = 1 \times 8^2 + 4 \times 8^1 + 3 \times 1 = (99)_{10}$
Choice (D)

11. To express a number in base B, the digits we use are 0, 1, 2,…B – 1.
To express a number in binary, the digits we use are 0 and 1 Choice (C)

12. In any number system in which the base is at least 11, the numerical value of A is 10. In the duodecimal system, the base is 12. The numerical value of A is 10. Ans : (10)

13. The binary representation of any multiple of 16 ends with 0000. Choice (D)

14. The decimal equivalent of the binary number 1. 011 is
$1(2^0) + 0(2^{-1}) + 1(2^{-2}) + 1(2^{-3})$
$= 1 + 0 + \frac{1}{4} + \frac{1}{8} = 1\frac{3}{8} = 1.375$ Choice (D)

15. $(224)_5 = 5^2(2) + 5(2) + 5^0(4) = 64$. Its cube root is 4.
$(4)_{10} = (11)_3$. Ans : (11)

Exercise – 3(a)

Solutions for questions 1 to 8:

1. We have $(374)_8 = 3 \times 8^2 + 7 \times 8^1 + 4 \times 1 = (252)_{1\phi}$
Now, $252 = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2$
$\therefore$ $(252)_{10} = (11111100)_2$ Choice (C)

2. $(386)_{12} = (534)_{10}$
$(177)_{12} = (235)_{10}$
$\therefore$ $(386)_{12} - (177)_{12} = (299)_{10}$

Now, 12 | 299
12 | 24 – B
  | 2 – 0

$\therefore (299)_{10} = (20B)_{12}$ Choice (B)

3. We have $(418)_{10} = (110100010)_2$
$\therefore$ The minimum number of bits required is 9 Ans : (9)

4. We have $(A)_{16} = (10)_{10}$
$(110101111)_2 = 1 + 2 + 4 + 8 + 32 + 128 + 256$
$= 15 + 32 + 128 + 256$
$= 47 + 128 + 256 = 384 + 47$
$= (431)_{10}$
When $(431)_{10}$ is divided by 10, then the remainder is 1.
Choice (B)

5. We have
$57 = 32 + 16 + 8 + 1 = 2^5 + 2^4 + 2^3 + 2^0$
$\therefore$ $(57)_{10} = (111001)_2$
Also,
$0.140625 \times 2 = 0{\cdot}28125$
$0.28125 \times 2 = 0{\cdot}5625$
$0.5625 \times 2 = 1{\cdot}125$
$0.125 \times 2 = 0{\cdot}25$
$0.25 \times 2 = 0{\cdot}50$
$0.5 \times 2 = 1{\cdot}0$
$\therefore$ $(57{\cdot}143251)_{10}$ is equal to $(111001.\ 001001)_2$
Choice (B)

6. We have
$(13.24)_5 = 1 \times 5 + 3 \times 5^\circ + \frac{2}{5} + \frac{4}{25}$
$= 8 + \frac{14}{25} = 8.56.$ Ans : (8.56)

7. Let $n = (a_k\, a_{k-1}\, a_{k-2} \ldots\ldots\ldots\, a_1\, a_0)_{16}$
Then, $n = a_k\, 16^k + a_{k-1}\, 16^{k-1} + \ldots\ldots + a_1\, 16^1 + a_0\, 16^0$
Concatenating n with '0', we have
$(a_k\, a_{k-1} \ldots\ldots\ldots\, a_1\, a_0\, 0)_{16}$
$= a_k\, 16^{k+1} + a_{k-1}\, 16^k \ldots\ldots\ldots + a_0\, 16 + 0$
$= 16\, (a_k.16^k + a_{k-1}\, 16^{k-1} \ldots\ldots\ldots\ldots\ldots + a_0)$
$= 16n$ Choice (A)

8. Let a, b, c be 7-digit, 8-digit and 9-digit numbers respectively in base m.
i.e., $m^6 \le a < m^7$
$m^7 \le b < m^8$
$m^8 \le c < m^9$
Let x, y be a 5-digit number and a 6-digit number respectively in base n i.e., $n^4 \le x < n^5$
$n^5 \le y < n^6$
$\therefore$ The range m^6 to m^9 is a subset of the range n^4 to n^6.
We tabulate these four values for the given options (all in base 10)

		A		B		
m	n	m^6	m^4	n^4	n^6	$A \subset B$
2	3	64	512	81	729	False
3	6	729	19, 683	1296	–	False
3	7	729	–	2401	–	False
4	8	2^{12}	2^{18}	2^{12}	2^{18}	True

Choice (D)

Solutions for questions 9 and 10:

9. We have, 400 = 256 + 128 + 16
$\therefore$ The minimum number of times he needs to use the machine is 3. Ans : (3)

10. Instead of one 256, he can use the quantity 128 twice, this implies, 400 = 128 + 128 + 128 + 16
$\therefore$ The number of times he has to use the machine
Now is 4. Ans : (4)

Solutions for questions 11 to 25:

11. Given, $(11{\cdot}5)_n = (1001{\cdot}101)$
$\Rightarrow \left(n + 1 + \frac{5}{n}\right) = 9 + \frac{1}{2} + \frac{1}{8}$
$\Rightarrow \frac{(n^2 + n + 5)}{n} = \frac{77}{8}$
$8n^2 + 8n + 40 = 77n$
$8n^2 - 69n + 40 = 0$
$8n^2 - 64n - 5n + 40 = 0$
$8n\,(n - 8) - 5\,(n - 8)$
$(8n - 5)\,(n - 8) = 0$
$\Rightarrow n = 8.$ Ans : (8)

12. We have $(25)_{12} = (29)_{10}$
$\therefore$ $(25)_{12}\ \$\ (17)_{10} = (29)_{10}\ \$ (17)_{10}$
$= 5\,(29) + 2\,(17) + 2)_{10}$
$= (145 + 34 + 2) = (181)_{10}$ Choice (B)

13. $(51)_k = (5k + 1)_{10}$
$(50)_{k+2} = (5k + 10)_{10}$
Given,
GCD is $(9)_{10}$ and LCM is $(180)_{10}$.
We have, the product of 2 numbers is equal to product of their LCM and GCD.
$\therefore (5k + 1)\,(5k + 10) = 9 \times 180 \Rightarrow k = 7.$ Ans : (7)

14. We have, $(305)_{13} = 3 \times 169 + 5 = (512)_{10}$
$(305)_7 = (152)_{10}$
So, out of the given three options, only $(512)_{10}$, i.e., $(305)_{13}$ is a perfect cube. Choice (A)

15. We have, $(33)_7 = (24)_{10}$ and
$(28)_9 = (26)_{10}$
Arithmetic mean $= (25)_{10}$
$(IC)_{13} = (13 + 12) = (25)_{10}$
$\therefore$ The required radix is 13 Ans : (13)

16. Let the base of the system be n. All numbers which appear without a base are in base ten. The sum of the roots is 13. $\therefore$ a = 13. The product of the roots is 40. $\therefore$ $(44)_n = 4n + 4 = 40$ or n = 9. Hence, the base or radix of the number system is 9. Ans : (9)

17. We have, $(34)_7 = (25)_{10}$
$(31)_8 = (25)_{10}$
$\therefore$ $(34)_7 \times (31)_8 = (625)_{10} = (441)_{12}$ Choice (A)

18. LCM (3, 4, 5, 7) = 420
$\therefore$ 420 – 2 = 418 is the required positive integer
Ans : (418)

19. $(314)_5 = 3\,(5^2) + 1\,(5^1) + 4\,(5^0)$
$= 75 + 5 + 4 = (84)_{10}$
$(412)_6 = 4\,(6^2) + 1\,(6) + 2\,(6^0)$
$= (144 + 6 + 2)_{10} = (152)_{10}$
$\therefore (314)_5\ @\ (412)_6 = (84)_{10}\ @\ (152)_{10}$

= 5(84) – 2(152) + 60
= 420 – 304 + 60 = 176

$$\begin{array}{r|l} 7 & 176 \\ \hline 7 & 25 - 1 \\ \hline & 3 - 4 \end{array}$$

$\therefore (176)_{10} = (341)_7$ Choice (D)

20. $(23516)_8 = (010\ 011\ 101\ 001\ 110)_2$
$= (0010\ 0111\ 0100\ 1110)_2$
$= (274E)_{16}$ Choice (C)

21. $(21)_8 = 2\ (8^1) + 1\ (8^0) = (17)_{10}$
$(23)_5 = 2\ (5^1) + 3\ (5^0) = 10 + 3 = (13)_{10}$
$\therefore$ f[$(23)_{10}$, $(21)_8$, $(23)_5$]
= f[$(23)_{10}$, $(17)_{10}$, $(13)_{10}$]
= [3(23) + 2(17) – 13]
= 69 + 34 - 13
$= (90)_{10}$

$$\begin{array}{r|l} 2 & 90 \\ \hline 2 & 45 - 0 \\ \hline 2 & 22 - 1 \\ \hline 2 & 11 - 0 \\ \hline 2 & 5 - 1 \\ \hline 2 & 2 - 1 \\ \hline & 1 - 0 \end{array}$$

$\therefore (90)_{10} = (1011010)_2$
The choices are of the form 8a + 1, 2b, 6c + 5 and 14d +11
We have to consider only choice B. Choice (B)

22. $(26)_7 = 2\ (7) + 6\ (7^0) = 14 + 6 = (20)_{10}$
$(104)_6 = 1\ (6^2) + 0\ (6^1) + 4 = (40)_{10}$
$(88)_9 = 8\ (9) + 8\ (9^0)$
$= 72 + 8 = (80)_{10}$
$(40)^2 = 1600 = 80\ (20)$
(i.e.) the numbers satisfy the condition $b^2 = ac$
Hence they are in G.P Choice (B)

23. $(2000)_8 = 2 \times 5^3 = 64 \times 16 = (1024)_{10} = (32)^2$
$\therefore$The square root of $(2000)_8$ is $(32)_{10}$. Ans : (32)

24. We have, $(325)_8 = (213)_{10}$
$(213)^2 = (45369)_{10}$
$= (130471)_8$ Choice (A)

25. Given $(1002)_n = (345)_{10}$
$\Rightarrow 2 + n^3 = 345$
$\Rightarrow n^3 = 343$
$\Rightarrow n = 7$ Ans : (7)

Exercise – 3(b)

Solutions for questions 1 to 25:

1. $(3AC)_{13} = C\ (13^0) + A\ (13^1) + 3\ (13^2)$
= 12 (A) + 10 (13)+ 3 (169)
= 12 + 130 + 507
$= (649)_{10}$ Ans : (649)

2. $(100\ \ 101\ \ 011)_2 = [(100)_2\ (101)_2\ (011)]_8 = (453)_8$
Ans : (453)

3. $[1110011101]_2 = [(0011)_2\ (1001)_2\ (1101)_2]_{16}$
(append two zeros at left)
$= (39D)_{16}$ Choice (C)

4. A number abc in base n is
c $an^2 + bn^2 + c$
If 0 is appended to the right most digit of the number then the number becomes
$an^3 + bn^2 + cn$
= n (old number)
So the new number is n times the old number.
Choice (C)

5.

$$\begin{array}{r|l} 12 & 123456 \\ \hline 12 & 10288 - 0 \\ \hline 12 & 857 - 4 \\ \hline 12 & 71 - 5 \\ \hline & 5 - B \end{array}$$

$\therefore (123456)_{10} = (5B540)_{12}$ Choice (B)

6. $(101101)_2 = 1\ (2^5) + 0\ (2^4) + 1(2^3) + 1\ (2^2) + 0\ (2^1) + 1\ (2^0)$
= 32 + 0 + 8 + 4 + 1
$= (45)_{10}$
$(201)_8 = 2\ (8^2) + 0\ (8^1) + 1\ (8^0)$
= 128 + 0 + 1
$= (129)_{10}$
$(453)_{10} = (453)_{10}$
$\therefore (101101)_2 + (201)_8 + (453)_{10}$
$= (45 + 129 + 453)_{10}$
$= (627)_{10}$

$$\begin{array}{r|l} 9 & 627 \\ \hline 9 & 69 - 6 \\ \hline & 7 - 6 \end{array}$$

$\therefore (627)_{10} = (766)_9$ Choice (B)

7. $(231)_{16} = 2\ (16^2) + 3\ (16^1) + 1\ (16^0)$
= 2 (256) + 48 + 1
$= 512 + 49 = (561)_{10}$
$(231)_8 = 2\ (8^2) + 3\ (8) + 1\ (8^0)$
= 2 (64) + 24 + 1
= 128 + 25
$= (153)_{10}$
$\therefore (231)_{16} – (231)_8 = (561 – 153)_{10} = (408)_{10}$

$$\begin{array}{r|l} 11 & 408 \\ \hline 11 & 37 - 1 \\ \hline & 3 - 4 \end{array}$$

$\therefore (408)_{10} = (341)_{11}$ Choice (D)

8.

$$\begin{array}{r} 110110 \\ -10001 \\ \hline 100101 \\ \hline \end{array}$$

$(100101)_2 = 32 + 4 + 1 = 37$

When any number in any base is divided by the base, it leave a remainder which is equal to the units digit. For example, choice (A) i.e $(112)_4$ is of the form 4a + 2
Similarly the other choices are of the form 7b + 5, 5c + 4 and 4d + 1.
$\therefore$ We have to consider only choice D. $(211)_4$
= 2(16) + 1(4) + 1 = 37. Choice (D)

9.

0.7265625 x 2 = 1.4531250	1
0.4531250 x 2 = 0.906250	0
0.90625 x 2 = 1.81250	1
0.8125 x2 = 1.6250	1
0.625 x 2 = 1.250	1
0.25 x 2 = 0.5	0
0.5 x 2 = 1.0	1

$\therefore (0.7265625)_{10} = (0.1011101)_2$ Choice (C)

10. $(110101.11011)_2$
$= 1(2^5) + 1(2^4) + 0(2^3) + 1(2^2) + 0(2) + 1 + 1(2^{-1}) + 1(2^{-2}) + 0(2^{-3}) + 1(2^{-4}) + 1(2^{-5})$
= 32 + 16 + 4 + 1 + 0.5 + 0.25 + 0.0625 + 0.03125
$= (53.84375)_{10}$ Ans : (53.84375)

11. $(281)_{10} < 2^n$
$(281)_{10} = 256 + 25$
$= 2^8 + 25$
$\therefore (281)_{10} < 2^n$
$\Rightarrow (2^8 + 25) \le 2^n$
$\Rightarrow n \ge 9$
$\therefore$Number of bits required = 9 bits.

It can be noted that 256 = $(100000000)_2$ and $(111111111)_2$ = 511 is the largest 9 bit number. So any number that lies between 256 and 511 would require a minimum of 9 bits to represent it in binary. Ans : (9)

12. The remainder when $(abcde)_{10}$ is divided by 9 is equal to the remainder when a + b + c + d + e is divided by 9. in general, the remainder when $(abcde)_{n+1}$ is divided by n is equal to the remainder when (a + b + c + d + e) is divided by n.
$\therefore$ The required remainder is 3. Choice (C)

13. $(1331)_8 = 1\ (8^3) + 3\ (8^2) + 3\ (8) + 1\ (8^0)$
$= 512 + 192 + 24 + 1 = (729)_{10}$
$\sqrt{(729)_{10}} = (27)_{10}$
The choices are 12 + 6, 42 + 3, 26 + 1, 32 + 3.
Choice (C)

14. $(132)_4 = 1 \times 4^2 + 3 \times 4 + 2 \times 4^0$
= 16 + 12 + 2
$= (30)_{10}$
$(30)^2_{10} = (900)_{10}$
The choices are of the form 7a + 2, 4b, 7c + 4, 4d + 2.
We need to consider only choices 2 and 3
$(10230)_4 = 256 + 2(16) + 3(4)$ and
$(2424)_7 = 2(343) + 4(49) + 2(7) + 4$
= 686 + 196 + 14 + 4 = 900. Choice (C)

15. $(10111001)_2 = 1(2^7) + 0(2^6) + 1(2^5) + 1(2^4) + 1(2^3) + 0(2^2) + 0(2^1) + 1(2^0)$
$= 2^7 + 2^5 + 2^4 + 2^3 + 1$
$= (185)_{10}$
$(11110)_2 = 1(2^4) + 1(2^3) + 1(2^2) + 1(2) + 0(2^0)$
$= 16 + 8 + 4 + 2 = (30)_{10}$
Remainder when $(185)_{10}$ is divided by $(30)_{10}$ is $(5)_{10}$
Choice (C)

16.
$$\begin{array}{rl} & \underline{\ 11\ \ \ } \\ & (215)\ 8 \\ + & \underline{(476)\ 8} \\ & \underline{(713)\ 8} \end{array}$$

$\therefore (215)_8 + (476)_8 = (713)_8$

Note: that the above addition is carried on in the base-8 system. Choice (B)

17. $(112)_3 = 1(3^2) + 1(3) + 2(3^0) = 9 + 3 + 2 = (14)_{10}$
$(115)_5 = 1(5^2) + 1(5) + 1(5^0) = 25 + 5 + 1 = (31)_{10}$
Product of the numbers $= (14)_{10}\ (31)_{10} = (434)_{10}$

$$\begin{array}{r|l} 12 & 434 \\ \hline 12 & 36 - 2 \\ \hline & 3 - 0 \end{array}$$

$\therefore (434)_{10} = (302)_{12}$ Ans : (302)

18. Let the scale of the number be n
$\Rightarrow 1654 = n^3 + 6n^2 + 5n + 4n^0 = n^3 + 6n^2 + 5n + 4 \rightarrow (1)$
From the choices substituting n = 7 in (1)
we get 676 which is a perfect square. Choice (B)

19. $(39)_{11} = 3 \times 11 + 9 \times 11^0 = 33 + 9 = (42)_{10}$
$(62)_9 = 6 \times 9 + 2 = 54 + 2 = (56)_{10}$
$\therefore$Arithmetic mean of $(39)_{11}$ and $(62)_9$
$= \left(\frac{42+56}{2}\right)_{10} = \left(\frac{98}{2}\right)_{10} = (49)_{10}$
Given that $(49)_{10} = (94)_n$
$\Rightarrow 49 = 9n + 4 \Rightarrow n = 5$
$(32)_4 + (21)_5 = (14)_{10} + (11)_{10} = \mathbf{(25)_{10} = (100)_5}$
Ans : (100)

20. Given, $(125)_k = (68)_{10}$
$\Rightarrow k^2 + 2k + 5 = 68 \Rightarrow k^2 + 2k - 63 = 0$
$\Rightarrow (k + 9)\ (k - 7) = 0$
$\Rightarrow k = -9, 7$
But, k cannot be negative.
$\therefore k = 7$ Ans : (7)

21. We have, $(62)_8 = (50)_{10}$
$(144)_8 = 4 + 32 + 64 = (100)_{10}$
and $(226)_8 = 6 + 16 + 128 = (150)_{10}$
$\therefore$ $(62)_8$ and $(144)_8$ and $(226)_{10}$ are clearly in arithmetic progression. Choice (A)

22. $(310)_4 = 3\ (4^2) + 1\ (4^1) + 0\ (4^0)$
$= 48 + 4 + 0 = (52)_{10}$
$(110)_4 = 1\ (4^2) + 1\ (4^1) + 0\ (4^0) = (20)_{10}$

$$\begin{array}{r|l} 4 & 52,\ 20 \\ \hline & 13,\ 5 \end{array}$$

$\therefore$ L.C.M of $(310)_4,(110)_4 = 13\ (20) = (260)_{10}$
The first 3 choices are of the form 5a + 1, 6b + 2 and 4c + 1
We need to consider only choice B i.e. $(1112)_6 = 216 + 36 + 6 + 2 = 260$. Choice (B)

23. We have, $(A)_{16} = 10$
$(11)_2 = 3$
$(13)_8 = 11$
f (x, y, z) = (x + 2y) (2y + 2) (z + x)
$= (16) \times (17) \times (21) = (5712)_{10}$ Ans : (5712)

24. $(346)_n = (1211)_5$
$\Rightarrow 3n^2 + 4n + 6 = 125 + 50 + 6$
$\Rightarrow 3n^2 + 4n - 175 = 0$
$\Rightarrow 3n^2 + 25n - 21n - 175 = 0$
$\Rightarrow n(3n + 25) - 7\ (3n+25) = 0$
$\Rightarrow (n - 7)\ (3n + 25) = 0$
$\Rightarrow n = 7$ or $\frac{-25}{3}$
As radix cannot be a fraction, n = 7

$$\begin{array}{r|l} 7 & 235 \\ \hline 7 & 33 - 4 \\ \hline & 4 - 5 \end{array}$$

Ans : (454)

25. As 2 and 9 are single digit numbers, their values are 2 and 9 respectively in all bases which are 10 or more. Their product is 18 which is represented as $(15)_n$ i.e.
$18 = 5 + n \Rightarrow n = 13$.
$(543)_6 = 5(36) + 4(6) + 3 = 207$

$$\begin{array}{r|l} 13 & 207 \\ \hline 13 & 15 - 12 \\ \hline & 1 - 2 \end{array}$$

$(543)_6 = (207)_{10} = (12C)_{13}$ Choice (B)

Chapter – 4
(Geometry)

Concept Review Questions

Solutions for questions 1 to 35:

1. $6^2 + 8^2 = 10^2$
$\therefore$ the triangle is right angle.
$\therefore$ Circumradius $= \frac{\text{Hypotenuse}}{2} = 5\text{cm}$ Ans : (5)

2. $12^2 + 16^2 = 20^2$
$\therefore$ The triangle is a right-angled triangle.
$\therefore$ The point of intersection of the perpendicular sides is the orthocentre.
$\therefore$ Sum of the distances from the orthocentre to the vertices of the triangle = (0 + 12 + 16) cm = 28 cm. Ans : (28)

3. The inradius of a triangle is always less than $\frac{1}{2}$(smallest altitude in the triangle). In the given problem, as the smallest altitude = 18 cm, the inradius has to be less than 9 cm.
Choice (C)

4. The larger part and the smaller part are in the ratio of 2 : 1. Choice (C)

5. Inradius = $\frac{18}{2\sqrt{3}}$ cm = $3\sqrt{3}$ cm. Choice (A)

6. Circumradius = $\frac{18}{\sqrt{3}}$ cm = $6\sqrt{3}$ cm. Choice (C)

7. Area of triangle GFB = $\frac{\text{Area of } \Delta ABC}{6} = \frac{24}{6}$ cm² = 4 cm².

Ans : (4)

8. $\frac{AB}{AC} = \frac{BD}{DC}$

BD = $\frac{AB}{AC}(DC) = \frac{10}{12}(8)\frac{cm^2}{cm} = 6\frac{2}{3}$ cm. Choice (D)

9. $\angle QIR = 90° + \frac{1}{2}\ \angle QPR = 90° + \frac{50°}{2} = 115°$ Ans : (115)

10. Only in an obtuse angled triangle,
$AB^2 + AC^2 < BC^2 \Rightarrow \angle BAC > 90°$
i.e. $x > 90°$ Choice (C)

11. $8^2 > 6^2 + 4^2$
$\therefore$ The triangle is obtuse angled.
$\therefore$ Its circumcentre lies outside the triangle. Choice (C)

12. Incentre. Choice (A)

13. As ST is parallel to QR, $\frac{PS}{SQ} = \frac{PT}{TR}$

PT = $\frac{PS}{SQ}(TR) = \frac{8}{4}(6)\frac{cm^2}{cm} = 12$ cm Ans : (12)

14. As PQRS is a cyclic quadrilateral,
$\angle R = 180° - \angle P = 130°$
$\angle S = 180° - \angle Q = 110°$ Choice (D)

15. QS = $\sqrt{(PS)(SR)} = \sqrt{(32)(18)}$ cm $= 24$ cm Ans : (24)

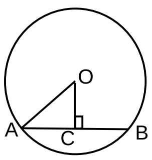
16. EF = $\left[\frac{2}{5}(24) + \frac{3}{5}(12)\right]$ cm = 16.8 cm Ans : (16.8)

17. As ST is parallel to QR,
$\frac{PS}{PQ} = \frac{ST}{QR}$

QR = $\frac{PQ}{PS}(ST) = \frac{16}{4}(3)\frac{cm^2}{cm} = 12$ cm Choice (D)

18. Number of diagonals = $\frac{(10)(10-3)}{2} = 35$ Ans : (35)

19. Let the centre of the circle be O and the chord be AB.
Let OC be the perpendicular line drawn from O to AB,
$OC^2 + AC^2 = OA^2$
$AC^2 = OA^2 - OC^2 = (15^2 - 9^2)cm^2$
$\Rightarrow$ AC = 12 cm
AB = 2AC = 24 cm Ans : (24)



20. $\angle POQ = 2\angle PRQ$

$\angle PRQ = \frac{\angle POR}{2} = 50°$ Choice (C)

21. As Z is a point on the circumference and XY is the diameter of the circle, $\angle XZY = 90°$

$\therefore XY^2 = XZ^2 + ZY^2$
YZ = $\sqrt{XY^2 - ZX^2} = \sqrt{26^2 - 24^2}$ cm $= 10$ cm.

Ans : (10)

22. Only an isosceles trapezium is necessarily a cyclic quadrilateral. Ans : (1)

23. $\angle BAC = \angle BDC = 50°$
(Angles in the same segment are equal).
In triangle ABC, $\angle ABC + \angle BAC + \angle BCA = 180°$
$\angle ABC = 180° - (\angle BAC + \angle BCA)$
$= 180° - (50° + 45°) = 85°$

Ans : (85)

24. Area = $\frac{3\sqrt{3}}{2}(4)^2 = 24\sqrt{3}$ cm². Choice (A)

25. Let R and H represent the radius and the height respectively of the original cone.
Let r and h represent the radius and the height respectively of the frustum.
Then it follows that
$\frac{r}{R} = \frac{H-h}{H}$

h = $\frac{2}{3}H$ $\therefore$ r = $\frac{1}{3}R$

$\therefore$ The height of the smaller cone = $\frac{H}{3}$

As the smaller cone and the original cone are similar, the radius of the smaller cone will be $\frac{R}{3}$.

$\therefore$ Required ratio = $\frac{1}{3}\pi\left(\frac{R}{3}\right)^2 \frac{H}{3} : \frac{1}{3}\pi R^2 H = 1 : 27$

Choice (B)

26. Reflex $\angle$ POR = 300°
$2\angle PQR$ = Reflex $\angle POR$
$\therefore \angle PQR = 150°$ Ans : (150)

27. As XZ is the diameter of the circle,
$\angle XYZ = 90°$.
In triangle XYZ, $\angle YXZ + \angle XYZ + \angle YZX = 180°$
$\angle YXZ = 180° - (\angle XYZ + \angle YZX)$
$= 180° - (90° + 35°) = 55°$ Ans : (55)

28. As AC and BC are tangents to the circles,
$\angle OAC = \angle OBC = 90°$
In quadrilateral OABC, $\angle OAC + \angle ACB + \angle OBC + \angle AOB = 360°$
$\angle AOB = 360° - (\angle OAC + \angle ACB + \angle OBC)$
$= 360° - (90° + 50° + 90°) = 130°$ Choice (A)

29. As RS || TU, $\angle XZT = \angle XNR \rightarrow$ (1)
(Corresponding angles are equal).

As VW || XY, $\angle VMR = \angle XNR \rightarrow$ (2)
(Corresponding angles are equal)

As PQ || RS, $\angle VOP = \angle VMR \rightarrow$ (3)
(Corresponding angles are equal)

From (1), (2) and (3), $\angle VOP = \angle XZT = 130°$
Choice (B)

30. The quadrilateral formed by joining the midpoints of another quadrilateral is always a parallelogram.
Its area is always half the area of the outer quadrilateral.
In the given problem, the quadrilateral formed is a parallelogram of area $\frac{200}{2}$ cm² i.e., 100 cm².

Choice (A)

31. To two non-intersecting and non-enclosing circles, two direct common tangents and two transverse common tangents can be drawn. Choice (D)

32. To two circles which touch each other externally, two direct common tangents and one transverse common tangent can be drawn. Ans : (3)

33. To two circles which intersect each other, two direct common tangents can be drawn. No transverse common tangent can be drawn. Ans : (2)

34. A triangle which has its circumcentre on one its sides must have its circumcentre as the midpoint of its longest side. Such a triangle must be right angled. Choice (C)

35. The incentre, the centroid and the circumcentre coincide, since the triangle is equilateral.
$\therefore$ The required area is 0. Ans : (0)

Exercise – 4(a)

Solutions for questions 1 to 35:

1. (i) $\angle 1 = \angle 3$ (vertically opposite angles)
$\angle 5 = \angle 7$ (vertically opposite angles)
$\angle 2 = \angle 4$ (vertically opposite angles)
$\angle 6 = \angle 8$ (vertically opposite angles)
$\angle 4 = \angle 6$ and $\angle 3 = \angle 5$ (alternate angles)
$\angle 1 + \angle 8 = 180°$ (sum of the exterior angles on the same side of the transversal) ----- (1)
$\angle 3 - \angle 8 = 90°$ given
$\angle 1 - \angle 8 = 90°$ --- (2) $\{\because \angle 1 = \angle 3\}$
$\Rightarrow$ solving equations (1) and (2),
we get $\angle 1 = 135°$ and $\angle 8 = 45°$
$\therefore \angle 1 = \angle 3 = \angle 5 = \angle 7 = 135°$
and $\angle 2 = \angle 4 = \angle 6 = \angle 8 = 45°$ Choice (C)

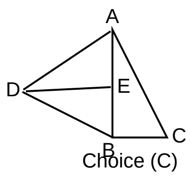
(ii) The perpendicular distances between (l and m) and (m and n) are in the ratio 3 : 4, so $\frac{AC}{BC} = \frac{3}{4}$.

$\therefore AB = \frac{7}{3}(12)$ cm = 28 cm $\{\because AC = 12 \text{ cm}\}$

Ans : (28)

2. Since PUSR is a parallelogram
$\angle UPR = 50°$
$\therefore \angle TPU = 180° - \angle QPU = 180° - (\angle QPR + \angle UPR)$
$= 180° - (60° + 50°) = 70°$ Ans : (70)

3. $AB = \sqrt{AC^2 - BC^2} = \sqrt{(41)^2 - (9)^2}$

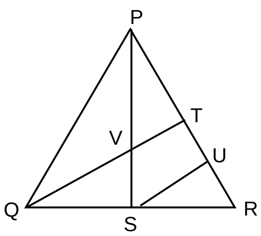


$\sqrt{1681 - 81} = \sqrt{1600} = 40$

$\therefore DE = \frac{\sqrt{3}}{2} \times (40) = 20\sqrt{3}$

Choice (C)

4. Let U be a point on PR such that SU is parallel to QT.



As QT || SU,
$\frac{PT}{TU} = \frac{PV}{VS} = \frac{4}{3}$ (given) $\rightarrow$ (1)

and $\frac{TU}{UR} = \frac{QS}{SR} = \frac{4}{3}$ (given) $\rightarrow$ (2)

From (1), $TU = \frac{3}{4} PT = 6$ cm

From (2), $UR = \frac{3}{4} TU = 4.5$ cm

$\therefore PR = PT + TU + UR = 18.5$ cm Ans : (18.5)

5. $\frac{n(n-3)}{2} = 20$, where n is the number of sides.

$\Rightarrow$ n = 8. Interior angle $= \frac{(2n-4)}{n}(90°) = \frac{12}{8}(90)$
$= 135°$ Choice (D)

6. Let, number of sides = n
Now, $\frac{n-2}{n}(180) = 144$
$\Rightarrow 180n - 360 = 144n \Rightarrow 36n = 360$
$\therefore n = 10$ Ans : (10)

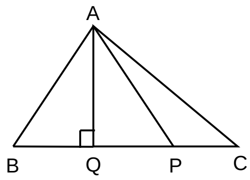
7. Circumradius $= \frac{36}{\sqrt{3}}$ cm $= 12\sqrt{3}$ cm. Choice (C)

8. Let the sides be a cm, b cm and c cm
Let a < b < c. $\therefore$ a + b < 2c and a + b + c < 3c.
As a + b + c = 20 it follows that $c > \frac{20}{3}$
$\therefore$ c can be 7, 8 or 9 .
The possible values are tabulated below

a	b	c
5	7	8
3	8	9
4	7	9
5	6	7

Choice (B)

9. $\angle PAB = \frac{1}{2} \angle BAC$
$\angle PAQ = \frac{1}{2} \angle BAC - \angle BAQ$
$= \frac{1}{2} \angle BAC - (90° - \angle ABC)$



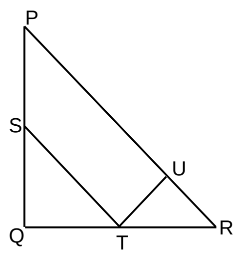
$= \frac{\angle BAC + 2\angle ABC - 180°}{2}$

$= \frac{\angle BAC + 2\angle ABC - (\angle ABC + \angle BAC + \angle BCA)}{2}$

$= \frac{\angle ABC - \angle BCA}{2} = \frac{80° - 40°}{2} = 20°$ Choice (C)

10. The three medians divide the triangle into six triangles of equal areas.
Quadrilateral CQAG consists of two such triangles.
The required ratio is 2 : 6 or 1 : 3 Choice (B)

11.



In ΔPQR,
$\angle PQR + \angle QPR + \angle QRP = 180$
$\angle PQR + \angle QRP = 180° - \angle QPR = 144 \rightarrow$ (1)
As QS = QT, $\angle QST = \angle STQ$ $\rightarrow$ (2)

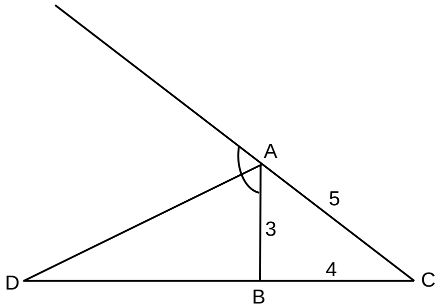
As RU = RT, $\angle RTU = \angle RUT$ $\rightarrow$ (3)
In ΔQST and ΔRUT,
$\angle SQT + \angle STQ + \angle QST = 180^\circ$
$\angle URT + \angle RTU + \angle RUT = 180^\circ$
Adding the two equations above, we get
$\angle STQ + \angle RTU = \dfrac{360^\circ - 144^\circ}{2} = 108^\circ$
(From (1), (2) and (3))
$\angle STU + \angle STQ + \angle RTU = 180^\circ$
(QR is a straight line)
$\angle STU = 180^\circ - 108^\circ = 72^\circ$ Choice (C)

12. (i) As per angle bisector theorem, $\dfrac{AB}{AC} = \dfrac{BD}{DC}$

$\Rightarrow DC = \dfrac{AC}{AB} BD = \dfrac{16}{12}(3)\dfrac{cm^2}{cm} = 4$ cm

$\therefore BC = BD + DC = (3 + 4)$ cm = 7 cm Choice (D)

(ii)



$AB = \sqrt{AC^2 - BC^2} = \sqrt{5^2 - 3^2}$ cm = 4 cm

Now, $\dfrac{AC}{AB} = \dfrac{DC}{DB}$

Let DB = x cm. $\therefore \dfrac{5}{3} = \dfrac{x+4}{x} \Rightarrow 5x = 3x + 12 \Rightarrow x = 6$

Choice (D)

13. Given that
D, E and F are midpoints of BC, CA and AB
and P, Q and R are midpoints of EF, FD and DE
we know that, Area of ΔABC = 4 Area of ΔDEF
But area of ABC = 64 sq. units
4 Area of ΔDEF = 64
Area of $\Delta DEF = \dfrac{64}{4}$
Area of ΔDEF = 16 sq. units.
Area of ΔDEF = 4 Area of ΔPQR
4 Area of ΔPQR = 16
Area of $\Delta PQR = \dfrac{16}{4} = 4$
Area of ΔPQR = 4 sq. units. Ans : (4)

14. In ΔPQR and ΔPRS,
$\angle QPR = \angle RPS$
$\angle QRP = \angle RSP$
As two pairs of corresponding angles of ΔPQR, ΔPRS are equal, the third pair of angles must also be equal.
$\therefore$ ΔPQR and ΔPSR are similar.

Raito of corresponding sides of ΔPQR and ΔPSR
$= \dfrac{QR}{SR} = \dfrac{PQ}{PR} = \dfrac{PR}{PS}$
$PR^2 = (PS)(PQ) = (32)(18)\ cm^2$

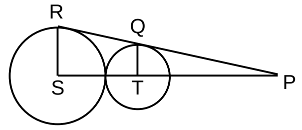
$PR = \sqrt{(32)(18)}\,cm = 24\ cm$

Ratio of perimeters of ΔPQR and $\Delta PRS = \dfrac{PQ}{PR} = 3 : 4$.

Ratio of perimeters of ΔPQR and ΔPRS = 3 : 4.

Choice (D)

15.



Given that, RS : QT = 5 : 3. Clearly ΔPQT and ΔPRS are similar triangles. $\therefore \dfrac{PT}{PS} = \dfrac{QT}{RS} \Rightarrow \dfrac{12}{(12 + ST)} = \dfrac{3}{5}$

$\Rightarrow$ ST = 8 cm. $\therefore$ RS = 5 cm and QT = 3 cm.
Now, the length of the common tangent RQ

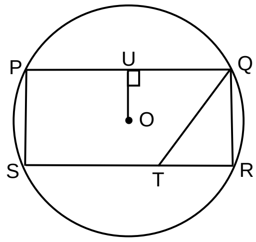
$= \sqrt{(ST)^2 - (RS - QT)^2} = \sqrt{64 - 4}$.

$RQ = 2\sqrt{15}$ cm. $\Rightarrow QP = 3\sqrt{15}$ cm.
(i) Clearly RQTS is a trapezium.
$\therefore$ Its area $= \dfrac{1}{2}(2\sqrt{15})(5 + 3) = 8\sqrt{15}\ cm^2$.

(ii) Area of $\Delta PQT = \dfrac{1}{2}\left(3\sqrt{15}\right)(3) = \dfrac{9\sqrt{15}}{2}\ cm^2$.

Choice (B)

16.



Let U be the point on PQ such that $OU \perp PQ$
Consider ΔOUQ and ΔTRQ
$\angle OUQ = \angle TRQ = 90^\circ$ and $\angle URO = \angle RQT$
As two pairs of angles of ΔUQO and ΔRQT are equal, the third pair of angles must also be equal.
$\therefore$ ΔUQO is similar to ΔRQT

$\therefore \dfrac{QR}{TR} = \dfrac{QU}{OU} = k$ (say) $\rightarrow$ (1)

Area of the rectangle PQRS = (PQ) (QR) = (2UQ) $\times$ (2OU) = $4k\,(OU)^2$
($\because$ As O is the centre of the circle, PQ = 2UQ and QR = 2OU).
Area of the circle $= \pi\,(OU^2\,(1 + k^2))$

Given $\dfrac{\text{Area of rec tan gle}}{\text{Area of circle}} = \dfrac{2\sqrt{5}}{3\pi}$

$\therefore \dfrac{4k\,(OU)^2}{\pi\left(k^2 + 1\right)(OU)^2} = \dfrac{2\sqrt{5}}{3\pi} \Rightarrow 12k = 2\sqrt{5}\,k^2 + 2\sqrt{5}$

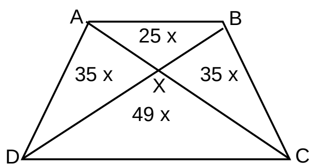
$\Rightarrow \sqrt{5}\,k^2 - 6k + \sqrt{5} = 0 \Rightarrow (\sqrt{5}\,k - 1)(k - \sqrt{5}) = 0$

$\Rightarrow k = \sqrt{5}$ or ${}^{1}\!/\!{}_{\sqrt{5}}$

As PQ > PS, UQ > OU and k = QU/OU > 1.
$\therefore k = \sqrt{5}$ Choice (C)

17. $\Delta\, AXB \sim \Delta\, DXC$

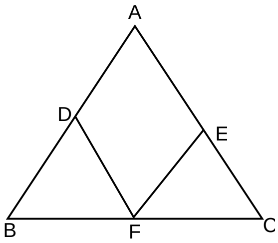
$\dfrac{BX}{XD} = \sqrt{\dfrac{25}{49}} = \dfrac{5}{7}$



Area of Δ A D X
$= \dfrac{7}{5}(25x) = 35x$

Ratio of the areas of Δ AXD and trapezium ABCD
$= \dfrac{35x}{35x + 25x + 35x + 49x} = \dfrac{35}{144}$ Choice (A)

18.



DF || AC and EF || AB.

Since DF || AC, triangles DBF and ABC are similar ($\because \angle B$ is common to the two triangles. Corresponding angles are equal)

Since EF || AB, triangles EFC and ABC are similar.

Ratio of the areas of DBF and ABC is $\left(\frac{BF}{BC}\right)^2$. Ratio of the areas of EFC and ABC is $\left(\frac{CF}{CB}\right)^2$.

$\left(\frac{CF}{CB}\right)^2 = \frac{64}{400} = \frac{4}{25} \therefore \frac{CF}{CB} = \frac{2}{5} \therefore \frac{BF}{BC} = \frac{3}{5}$.

$\therefore$ Ratio of the areas of DBF and ABC is $\frac{9}{25}$.

$\therefore$ Area of DBF is 144.

Area of ADFE = 400 – (144 + 64) = 192 Ans : (192)

19. In triangles PAB and PQR, $\angle PAB = \angle PQR$ and $\angle PBA = \angle PRQ$ (corresponding angles) and $\angle QAR$ is common.

Since the ratio of areas is 1 : 4, ratio of corresponding sides is $\sqrt{1} : \sqrt{4}$ = 1 : 2

$\therefore$ Perimeter of PAB will be half of the perimeter of triangle PQR i.e., 12 cm.

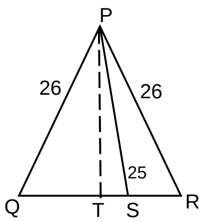
Ans : (12)

20. Let $\overline{PT} \perp \overline{QR}$. All lengths are in cm.

(i) In ΔPQT,
$(PT)^2 = 676 - (QT)^2 \rightarrow$ (1)

(ii) In ΔPTS,
$(PT)^2 = 625 - (TS)^2 \rightarrow$ (2)

(iii) In ΔPTR,
$(PT)^2 = 676 - (TS + 3)^2 \rightarrow$ (3)



From (2) and (3),

$625 - (TS)^2 = 676 - (TS + 3)^2$

$625 = 676 - 9 - 6(TS) \Rightarrow TS = 7$.

$\Rightarrow TS + SR = 10$ cm. $\Rightarrow TR = 10$.

But QT = TR = 10 ($\because$ ABC is an isosceles triangle)

$\therefore$ QS = QT + TS = 17. Choice (B)

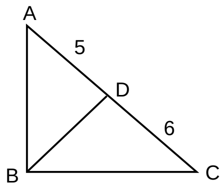
21. As BD $\perp$ AC,

$BD^2 = AD.\ DC$

$\Rightarrow BD = \sqrt{30}$.

$BC^2 = \left[\sqrt{30}^2 + 6^2\right] = 66$

$AB^2 = \left(\sqrt{30}\right)^2 + 5^2 = 55$ Choice (D)



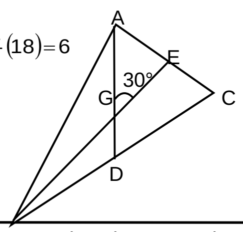
22. AG = $\frac{2}{3}(12) = 8$ and GE = $\frac{1}{3}(18) = 6$

Area of AGE

$= \frac{1}{2}(8)(6) \sin 30° = 12$

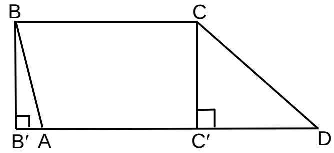
Area of triangle

= 12(6) = 72



Ans : (72)

23.



Given AB = 8, BC = 10

AD = 16, CD = 12

Let B'A = x, AC' = 10 – x B'D = 16 + x

BB' = y

$x^2 + y^2 = 64 \rightarrow$ (1)

$(6 + x)^2 + y^2 = 144 \rightarrow$ (2)

$BD^2 + AC^2 = (16 + x)^2 + 2y^2 + (10 - x)^2$

$= 2x^2 + 2y^2 + 12x + 356 = x^2 + y^2 + (6 + x)^2 + y^2 + 320$

= 64 + 144 + 320 = 528 Ans : (528)

24. In a parallelogram, the sum of the adjacent angles is 180°.

$\Rightarrow x + 20 + x - 40 = 180° \Rightarrow x = 100°$

Opposite angles are equal.

x + 20 = y + 10

$100 + 20 = y + 10 \Rightarrow y = 110°$ Ans : (110)

25. $AC^2 + BD^2 = 2(AB^2 + BC^2)$.

$\therefore 2(AB^2 + BC^2) = 10^2 + 12^2$ sq. units

and $AB^2 + BC^2 = 122$ sq.units. Choice (C)

26. (i) $\angle CDX = \angle ABC = 130°$
(exterior angle of a cyclic quadrilateral is equal to the opposite interior angle)
Since ABCE is a cyclic quadrilateral,
$\angle ABC + \angle AEC = 180°$
$130° + \angle AEC = 180° \Rightarrow \angle AEC = 50°$ Ans : (50)

(ii) $\angle SOR = 180° - (\angle SOR + \angle ORS) = 180° - 2(20)°$
= 140° ($\because$ OR = OS)
$\therefore \angle SQR = \frac{1}{2} \angle SOR = 70°$
$\angle PTS = \angle SQR = 70°$ (Exterior angle of a cyclic quadrilateral equals the interior angle at the opposite vertex). Ans : (70)

27. $\angle PTR = \angle PRT = \frac{180° - 60°}{2} = 60°$

$\therefore \angle TSR = \angle PRT = 60°$ (alternate segment theorem)

Choice (B)

28. Let the radius of each circle be denoted by r cm.

Perimeter of XYZ

= XC + CD + DY + YF + FE + EZ + ZB + BA + AX

= (XC + CD + CD + YD – CD) + (YF + FE + FE + EZ – FE) + (ZB + BA + BA + AX – AB)

= (2r – CD) + (2r – FE) + (2r – AB)

= [6(30) – (6 + 18 + 12)] cm = 144 cm. Ans : (144)

29. The two parallel chords could be on either side of the centre of the circle or on the same side of the centre of the circle.

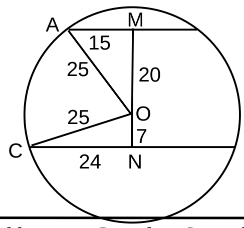
Case I: (See in Fig A)

The two parallel chords lie on either side of the centre of the circle.

MN = MO + ON

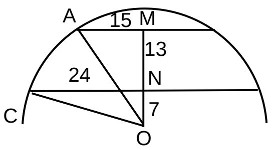
MO = 20 ($\because$ AM = 15, OA = 25)

Fig: A



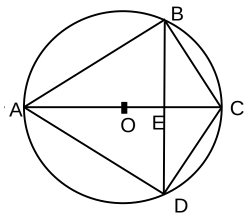
ON = 7 (∵ CN = 24, OC = 25)
∴ MN = 27
Case II: See Fig B

Fig: B



MN = MO – NO = 20 – 7 = 13 Choice (B)

30.

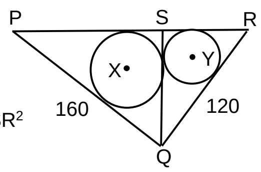


If an equilateral triangle is inscribed in a circle, the centre of the circle is the incentre (also the orthocentre, centroid and circumcentre) of the triangle. Hence, AC bisects ∠A or ∠BAO = 30° As ∠ABC is an angle in a semicircle, ∠ABC = 90°.
∴ Angles in ΔABC (and similarly ΔADC) are 30°, 60° and 90°, and the ratio of sides BC : AB : AC = 1 : $\sqrt{3}$: 2

∴ The required ratio = $\dfrac{AB+BC+CD+DA}{AC}$

= $\dfrac{\sqrt{3}+1+1+\sqrt{3}}{2}=1+\sqrt{3}$ Choice (C)

31. In ΔPQR, $PR^2 = PQ^2 + QR^2$



PR = $\sqrt{160^2+120^2}=200$
Let PS = x
SR = 200 – x
$SQ^2 = PQ^2 - PS^2 = QR^2 - SR^2$
$= 160^2 - x^2$
$= 120^2 - (200 - x)^2$
x = 128
SQ = 96, SR = 72
Perimeter of ΔPSQ = 384
If the inradius of ΔPQS is a,

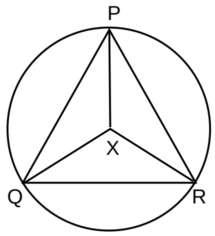
a = $\dfrac{\frac{1}{2}(128)(96)}{\frac{384}{2}}=32$

If the inradius of ΔRSQ is b,

b = $\dfrac{\frac{1}{2}(72)(96)}{\frac{288}{2}}=24$

$XY^2 = (a + b)^2 - (a - b)^2$
= 3136 + 64 = 3200 Ans : (3200)

32.

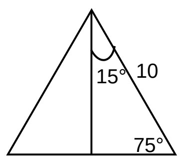


X is a point inside ΔPQR which is equidistant from P, Q, R.
∴ X must be the circumcentre of PQR.
XP = XQ (= XR) = Circumradius of PQR
∴∠PQX = ∠QPX = 35°(given)
∠PXQ = 180° – (∠XPQ + ∠XQP) = 180° – 2(35)° = 110°
Similarly, as ∠PRX = 25°, it follows that ∠PXR = 130°
∠QXR = 360° – (∠PXQ + ∠PXR) = 360° – 240° = 120°
∴ ∠PXQ < ∠QXR < ∠PXR
Since PQ, QR and RP are chords of the circle circumscribing PQR, PQ < QR < PR (∵ lesser the angle a chord subtends at centre, shorter will be the length of it)

Alternatively:
In two isosceles triangles, say ABC, DEF. if AB = AC = DE = DF, if ∠A < ∠D, then BC < EF. Triangles PXQ, QXR, PXR are isosceles. The equal sides of each of these are all equal to the circumradius of PQR
The order of PQ, QR, PR is the same as the order of ∠PXQ, ∠QXR, ∠PXR
PQ < QR < PR
Perimeter of PQR is 24 cm
Both I and II follow (Perimeter > 3PQ and perimeter < 3PR).
Choice (C)

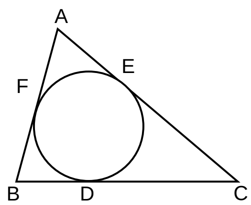
33. In a cyclic quadrilateral, if a, b, c and d are the lengths of the four consecutive sides and the diagonals are d_1 and d_2, then $d_1 d_2 = ac + bd$ (Ptolemy's theorem)
⇒ $d_2 = \dfrac{(10)(12)+(14)(15)}{15} = 22$ Ans : (22)

34. Side of the polygon = 2(10) cos 75°



= 2(10) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$
= 5($\sqrt{6}-\sqrt{2}$)
Perimeter of polygon
= 60($\sqrt{6}-\sqrt{2}$) . Choice (B)

35. In the given triangle

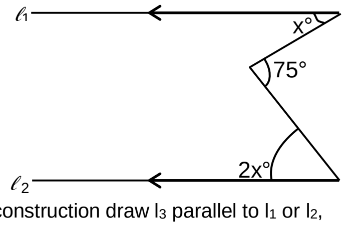


DC = EC = K_1 (say)
AE = AF = K_2 (say)
BF = BD = K_3 (say)
∴ $2(K_1 + K_2 + K_3)$
= AB + BC + CA
⇒ $K_1 + K_2 + K_3 = 9$. ….. (1)
$K_2 + K_3$ = AF + BF = AB = 5 ….. (2)
(1) – (2) : K_1 = 9 – 5 = 4
∴ DC = 4 cm Ans : (4)

Exercise – 4(b)

Solutions for questions 1 to 23:

1.



By construction draw l_3 parallel to l_1 or l_2,
a = x and b = 2x (alternate angles)
a + b = 75° but a +b = x + 2x =3x
∴3x = 75° ⇒ x = 25° Ans : (25)

2. ∠PQY + ∠PYQ = ∠QPA (The exterior angle of a triangle is equal to the sum of the interior angles opposite to it)
∠PQY = 120° – 40° = 80°
∴∠ZQR = 180° – 80° = 100°
∴∠BZX = ∠RQZ + ∠ZQR

= 100° + 25° = 125° Ans : (125)

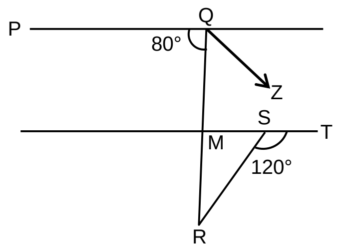
3. $\angle A + \angle B + \angle C = 180°$
$\angle B - \angle A = \angle C - \angle B$
$\Rightarrow 2\angle B = \angle A + \angle C$
$\therefore \angle A + \angle B + \angle C = 3\angle B$
$\therefore \angle B = 60°$ and hence
$\angle A + \angle C = 120°$
$\angle A : \angle C = 3 : 5$
$\therefore\ \angle A = 45°$ and $\angle C = 75°$
$\therefore \angle C - \angle A = 30°$. Choice (C)

4. Let the side of the rhombus be a cm.
Let the longer and the shorter diagonals be ℓ cm and s cm respectively.
$a + \frac{l+s}{2} = 60$
$\Rightarrow 2a + l + s = 120$
$2a + l = 100$
$\therefore\ s = 20$. Ans : (20)

5. As AB is a chord and $OM \perp AB$
AM = MB
$PM^2 - AM^2 = (PM + AM)(PM - AM)$
= (PM + MB) (PA) = (PB)(PA)
$= 8(3) = cm^2 = 24\ cm^2$.
Choice (C)

6. $\angle CRS + \angle ESR = 180°$
($\because$ sum of interior angles on the same side of the transversal)
$\angle ESR = 180 - 50 = 130°$ {$\because \angle CRS = 50°$}
$\angle ESR = \angle STA = 130°$
($\because$ corresponding angles)
Ans : (130)

7.



Join S and U
$\angle RSU = 180° - 120° = 60°$
$\angle QUT = \angle PQR = 80°$
$\therefore \angle RUS = 180° - 80° = 100°$
$\therefore \angle SRU = 180° - (60° + 100°) = 20°$
$\therefore \angle RQZ = 2 \times 20° = 40°$ Choice (C)

8. If X is any point inside the triangle ABC, then AX + BX > AB, BX + CX > BC and CX + AX > AC
By adding the three inequalities, we get
2(AX + BX + CX) > AB + BC + CA
$\Rightarrow (AX) + (BX) + (CX) > \frac{P}{2}$ Choice (D)

9.

In a triangle, the sum of two of the sides is greater than the third side. Also the third side is greater than the difference between the first two sides.
(21 – 12) < AC < (21 + 12)
9 < AC < 33
(21 + 12 + 9) cm < perimeter < (21 + 12 + 33) cm
$\Rightarrow$ 42 cm < p < 66 cm Choice (D)

10. $PQ = QT = \frac{QR}{2}$

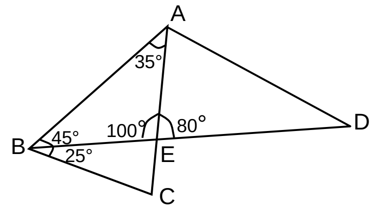
As $\angle QTR = 90°$ and $QT = \frac{1}{2}QR$, $\angle RQT = 60°$.

$\angle QPT = \angle QTP = \frac{180 - \angle PQT}{2}$

$= \frac{180° - (90° + 60°)}{2} = 15°$ Choice (A)

11. $\angle CAD = \angle CDA = 20°$ (Since AC = CD)
$\therefore \angle BAD = 25° + 20° = 45°$
$\therefore \angle BCD = 360° - (40° + 45° + 20°) = 255°$
$\therefore \angle BCT = 360° - (255° + 35°)$
$= 360° - 290° = 70°$ Ans: (70)

12.



Given that, $\angle EBC = 25°$
$\angle BAC = 35°$ and $\angle AED = 80°$
(i) $\angle AED + \angle AEB = 180°$
(linear pair) $\Rightarrow \angle AEB = 100°$

(ii) In ΔABE,
$\angle ABE = 180° - (100 + 35°) = 45°$.
$\Rightarrow \angle ABC$
$= \angle ABE + \angle EBC = 70°$.

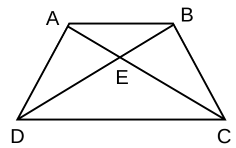
(iii) In ΔAED,
$\angle EAD + \angle ADE = 180° - 80° = 100°$.
$\therefore$ [$\angle ABC + \angle EAD + \angle ADE$] = 170°. Ans: (170)

13. Given BC = 5 cm and BD : DC = 2 : 3
$\therefore$ BD = 2 cm and DC = 3 cm.
As $AD \perp BC$, AD = BD tan60° = $2\sqrt{3}$ cm.

$AC^2 = AD^2 + DC^2 = \sqrt{\left[(2\sqrt{3})^2 + 3^2\right]}\,cm^2 = 21\ cm^2$

$\therefore$ AC = $\sqrt{21}$ cm. Choice (D)

14.



$\Delta AEB \sim \Delta DEC$ (AA Similarity)

$\frac{\text{area of }(\Delta AEB)}{\text{area of }(\Delta DEC)} = \frac{AB^2}{DC^2}$

$\therefore \frac{64\ cm^2}{\text{area of }(\Delta DEC)} = \frac{4}{9}$ $\therefore$ Area of (ΔDEC) = $\frac{9}{4}(64)\ cm^2$
$= 144\ cm^2$ Ans : (144)

15. Required length = (1/2) (BE) = (1/2) $\sqrt{8^2 + 12^2}$
$= 2\sqrt{13}$ Choice (A)

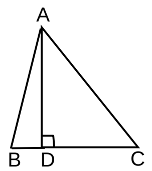
16. Let, FB = x cm
$\therefore$ CF = (15 – x)cm
EF/AB = CF/BC
$\Rightarrow$ EF = $\frac{15-x}{15}$ (30) cm = (30 – 2x) cm
Also, EF/CD = x/15
$\Rightarrow$ EF = $\frac{x}{15}$ (45) = 3x
$\therefore$ 30 – 2x = 3x $\Rightarrow$ x = 6
$\therefore$ EF = 3(6) = 18 cm Ans : (18)

17. $a^2 + b^2 + c^2 = 50$ ------- (1)
$d^2 + e^2 + f^2 = 50$ ------- (2)
ad + be + cf = 50 ------- (3)
Adding (1) and (2) and subtracting (2) (3) from the result, we get
$(a - d)^2 + (b - e)^2 + (c - f)^2 = 0$
$\therefore$ a = d, b = e and c = f
$\therefore$ The two triangles are congruent.
$\therefore$ They will have the same perimeter and the same area.
Choice (C)

18. The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides, (as perimeters).
$\therefore (32/40)^2$ = area of (ΔABC)/100 cm^2
$\Rightarrow$ area of ΔABC = $\frac{16}{25}$ (100) cm^2 = 64 cm^2 Ans : (64)

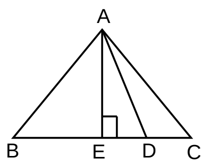
19. As per Apollonious theorem,
$PQ^2 + PR^2 = 2(PS^2 + QS^2)$
$\therefore QS^2 = \frac{PQ^2 + PR^2}{2} - PS^2 = \left[\frac{121+169}{2} - 64\right] cm^2$
= 81 cm^2
$\Rightarrow$ QS = 9 cm Choice (B)

20. $AD^2 = AB^2 - BD^2$



$= AB^2 - \frac{BC^2}{16}$
(Since BD = ¼ BC)
Similarly, $AD^2 = AC^2 - CD^2$
$= AC^2 - \frac{9BC^2}{16}$
$\therefore AB^2 - \frac{BC^2}{16} = AC^2 - \frac{9BC^2}{16}$
$\Rightarrow BC^2 = 2(AC^2 - AB^2) = 2(21^2 - 9^2)\ cm^2 = 720\ cm^2$
$\therefore$ BC = 12 $\sqrt{5}$ cm
Choice (A)

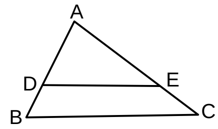
21. $AB^2 = AE^2 + BE^2$



$AD^2 = AE^2 + ED^2$
$\therefore AB^2 - AD^2 = BE^2 - ED^2$
= (BE – ED) (BE + ED)
= (CE – ED) (BE + ED) (Since BE = CE) = CD (BD)
$\therefore$ BD = $\frac{65^2 - 63^2}{8} \frac{cm^2}{cm}$
= 32 cm Ans : (32)

22. Let BD = x cm
$\therefore 7^2 - (3 + x)^2 = 5^2 - x^2$
$\Rightarrow 49 - (9 + x^2 + 6x) = 25 - x^2$
$\Rightarrow$ 15 = 6x $\Rightarrow$ x = 2.5 cm
$\therefore$ AD = $\sqrt{5^2 - (2.5)^2}$ cm
= 2.5 $\sqrt{3}$ cm
Choice (D)

23. $\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{1}{2}$



[Since ar(ΔABC) = ar(ΔADE) + ar(trap.DECB) = 2ar(ΔADE)]
$\Rightarrow \frac{AD^2}{AB^2} = \frac{1}{2} \Rightarrow$ AD = AB/ $\sqrt{2}$
= (12/ $\sqrt{2}$) cm = 6 $\sqrt{2}$ cm
$\therefore$ BD = AB – AD = (12 – 6 $\sqrt{2}$) cm Choice (B)

Solutions for questions 24 and 25:

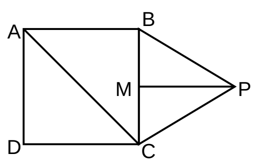
24. CK = CH – HK = AJ – IB = AJ – $\frac{AJ}{2} = \frac{AJ}{2}$
KD = HG = $\frac{AJ}{3} \Rightarrow$ tan$\angle$CDK = $\frac{CK}{DK} = \frac{3}{2}$
$\angle$CDK = $\tan^{-1}\left(\frac{3}{2}\right)$ Choice (D)

25. Area of ABIJ = $\frac{1}{2}$ (AJ + BI) (JI)
$= \frac{1}{2}\left(AJ + \frac{AJ}{2}\right)\frac{AJ}{3} = \frac{AJ^2}{4}$
Area of EFHC = $\frac{1}{2}$ (CH + EF) HF
= $\frac{1}{2}$ (AJ + AJ) (HG + GF)
= $\frac{1}{2}$ (2AJ) $\left(\frac{AJ}{3} + \frac{AJ}{3}\right) = \frac{2AJ^2}{3}$
Required ratio = 3 : 8 Choice (C)

Solutions for questions 26 to 28:

26. Given $\angle$AOB = 90°
$\therefore$ AO = $\sqrt{AB^2 - OB^2} = \sqrt{8^2 - 6^2}$ cm = $2\sqrt{7}$cm
In triangle AOD, as $\angle$ADO = 45°, and $\angle$AOD = 90°
$\angle$DAO = 45°
$\therefore$ AO = OD = $2\sqrt{7}$cm
$\therefore$ AD = $\sqrt{\left(2\sqrt{7}\right)^2}$ = $2\sqrt{14}$ cm. Choice (B)

27.



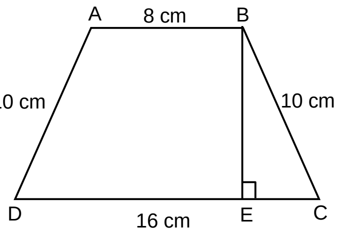
ABCD is a square and ΔBCP is an equilateral triangle.
And $\overline{PM} \parallel \overline{AB}$.

$\therefore$ $\overline{PM}$ is an altitude of the ΔBCP.

Given that, AC = $22\sqrt{2}$ cm.
$\therefore$ BC = 22cm. $\therefore$ PM = $\frac{\sqrt{3}(BC)}{2}$ = $11\sqrt{3}$ cm.

$\therefore$ Required ratio, PM : AC = $11\sqrt{3}$: $22\sqrt{2}$
= $\sqrt{3}$: $2\sqrt{2}$. Choice (B)

28.



Given that, AB = 8 cm,
BD = AC = 10 cm
CD = 16 cm and $\overline{AB} \parallel \overline{CD}$.

$\therefore$ ABCD is an isosceles trapezium. If $\overline{BE} \perp \overline{CD}$, then ED = 4 cm. Since ABCD is an isosceles trapezium.

$\therefore$ BE = $\sqrt{(BD)^2 - (ED)^2}$ = $\sqrt{100 - 16}$ cm = $\sqrt{84}$ cm.

Now, BC = $\sqrt{(BE)^2 + (CE)^2}$ = $\sqrt{84 + 144}$ cm = $\sqrt{228}$ cm

= $2\sqrt{57}$ cm $\Rightarrow$ BC = AD = $2\sqrt{57}$ cm. ($\because$ diagonals are equal in an isosceles trapezium).

$\Rightarrow$ BC + AD = $4\sqrt{57}$ cm. Choice (A)

Solutions for questions 29 and 30:

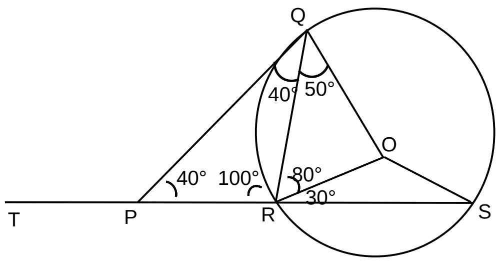
29. Given OR = 5 cm and OP = 3 cm
As $\angle$OPR = 90°, PR = 4 cm.
$\Rightarrow$ AR = (8 + 4) = 12 cm.
In a circle when equal chords intersect, then the line segments from the point to the circumference of the circle which are adjacent to the angle made by the point with the centre are equal.
As PR = PB, (In ΔOPR and ΔOQR, OR = OP, $\angle$P = $\angle$Q, OP = OQ) it follows that AR = CR
CR = AR = 12 cm. Ans : (12)

30. As OP$\perp$ AR, OR $\perp$ RC, and AR = RC,
OP = OQ, PR = RQ
$\therefore$ QC = AP = 8 cm Ans : (8)

Solutions for questions 31 to 35:

31. The perimeter of the triangle PQR
= PQ + PR + QR = PQ + PR + QM + RM
= PQ + PR + QB + RC = PB + PC = (8 + 8) = 16 cm.
Ans : (16)

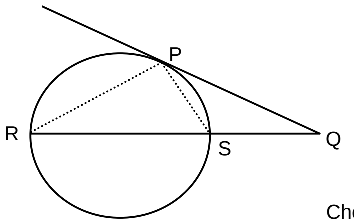
32.



$\angle$QPR = 180 – $\angle$QPT = 40º
As QR = PR, $\angle$QPR = $\angle$RQP = 40°
$\angle$OQP = 90 ($\because$ PQ is tangent to the circle)
$\therefore$ $\angle$OQR = $\angle$OQP – $\angle$RQP = 90° – 40° = 50°
In ΔOQR, OQ = OR
$\therefore$ $\angle$ORQ = $\angle$OQR = 50°
$\angle$QRS = $\angle$QPR + $\angle$PQR = 80º
$\angle$ORS = $\angle$QRS – $\angle$ORQ = 80° – 50° = 30°
In ΔORS, OR = OS
$\therefore$ $\angle$OSR = $\angle$ORS = 30° and
$\angle$ROS = 180º – ($\angle$ORS + $\angle$ROS) = 120°
Ans : (120)

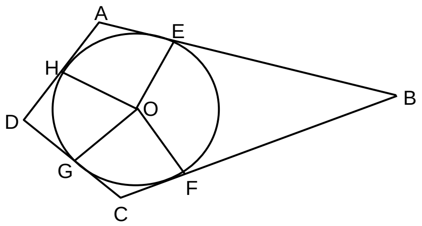
33. Let AD = x cm.
AB = AC = (x + 4) cm.
In triangle ABD, $AB^2 = AD^2 + BD^2$.
$\Rightarrow (x + 4)^2 = x^2 + 5^2 \Rightarrow x^2 + 8x + 16 = x^2 + 25$.
$\Rightarrow x = \frac{9}{8} = 1.125$ Ans : (1.125)

34. Join RP and PS
$\angle RPS = 90°$ ($\because$ Angle in a semi circle).
$\therefore \angle RPQ > 90°$.
Triangle RPQ is an obtuse angled triangle.
$\therefore$ RQ > RP and RQ > PQ.



Choice (A)

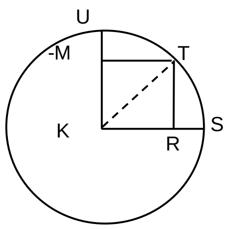
35.



Let 'O" be the centre of the circle.
Let E, F, G, H be the respective points of contact of AB, BC, CD, DA with the circle.
$\therefore$ BE = BF, AE = AH, DH = DG and CG = CF
Let EB = EF = x cm.
Given AB = 10 cm, AD = 3 cm and CD = 4 cm.
$\therefore$ AE = AH = (10 – x) cm.
DH = DG = 3 – (10 – x)
= (x – 7) cm.
$\Rightarrow$ CG = CF = (1 – x) cm.
$\therefore$ BC = BF + FC = 11 cm. Choice (A)

Solutions for questions 36 and 37:

36. K is the centre of the circle.
Let radius of the circle be r
$\therefore$ The diagonal of the square = KT = r
$\therefore$ The side of the square
= KR = $\frac{r}{\sqrt{2}}$.
$\therefore$ Ratio of the perimeter of the square and circumference of the circle = $4\left(\frac{r}{\sqrt{2}}\right) : 2\pi r$



$= \frac{2}{\sqrt{2}} : \frac{22}{7} = 7 : 11\sqrt{2}$ Choice (B)

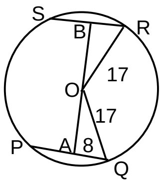
37. Ratio of the area of square and the area of the circle

$= \left(\frac{r}{\sqrt{2}}\right)^2 : \pi r^2 = \frac{1}{2} : \pi = 7 : 44.$

Choice (C)

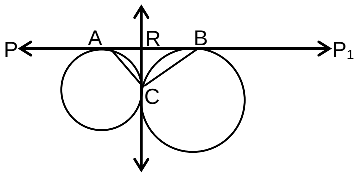
Solutions for questions 38 to 46:

38. OA = $\sqrt{17^2 - 8^2}$ cm = 15 cm
$\therefore$ OB = AB – 15 cm = (22 – 15)cm = 7 cm
$\therefore$ SR = 2BR = $2\sqrt{17^2 - 7^2}$ cm
= $2\sqrt{240}$ cm = $8\sqrt{15}$ cm



Choice (C)

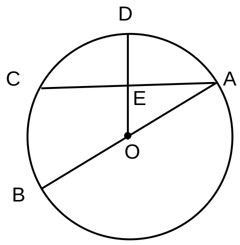
39.



RC is a common tangent to both the circles.
Now, AR = RC and BR = RC.
Let $\angle CAB = x$ and, $\angle CBA = y$
$\therefore \angle ACR = x$ and $\angle BCR = y$
Now, $\angle ACB + \angle CAB + \angle CBA = 180°$
$\Rightarrow (x + y) + x + y = 180° \Rightarrow x + y = 90°$
$\therefore \angle ACB = 90°$ Choice (B)

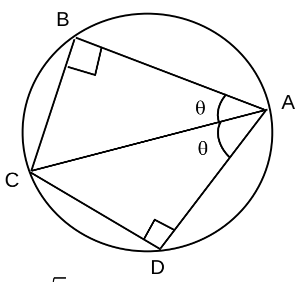
40. Let the number of angles which are x° be a.
Let the number of sides be N.
Sum of all the angles = 180°(N – 2) = 360° $\therefore$ N = 4
ax° + (4 – a) (180° – x°)
= Sum of all the exterior angles = a(180° – x°) + (4 – a)x
(2x – 180) (2a – 4) = 0
As x ≠ 90, a = 2. Ans : (2)

41.



The perpendicular drawn from the centre of a circle to any chord bisects the chord.
So CE = EA = 12 cm
As OEA is a right angled triangle, OE = $\frac{7}{2}$ cm
DE = OD – OE = $\left(\frac{25}{2} - \frac{7}{2}\right)$ cm = 9 cm Ans: (9)

42.



$\cos\theta = \frac{AB}{AC} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$
$\angle BCD = 360° – 180° – 60° = 120°$. Choice (B)

43. In a triangle ABC let D, E and F be the midpoints of BC, CA and AB respectively. Then

$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2} \quad \rightarrow \quad (1)$

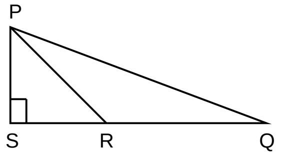
$BC^2 + AC^2 = 2CF^2 + \frac{AB^2}{2} \quad \rightarrow \quad (2)$

$AB^2 + BC^2 = 2BE^2 + \frac{AC^2}{2} \quad \rightarrow \quad (3)$

Adding (1), (2), (3) we get $3(AB^2 + BC^2 + CA^2)$
$= 4(AD^2 + BE^2 + CF^2)$
In any triangle, three times the sum of the squares of its sides is equal to four times the sum of the squares of its medians.

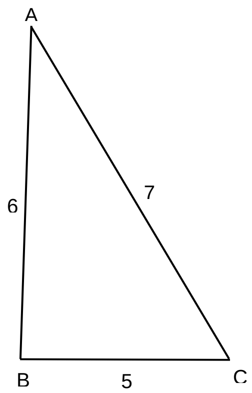
$\therefore$ Required sum = $\frac{4}{3}(36)$ cm^2 = 48 cm^2 Ans : (48)

44.



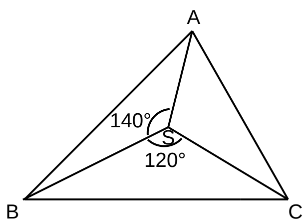
$PQ^2 = PS^2 + SQ^2 = PS^2 + (2QR)^2$ [$\because$ QR = RS]
$= PS^2 + 4QR^2$
Also, $PR^2 = PS^2 + SR^2 = PS^2 + QR^2$
$\therefore PQ^2 - PR^2 = 3QR^2$
$\therefore PR^2 = PQ^2 - 3QR^2 = [20^2 - 3(8)^2]$ cm^2 = 208 cm^2
$\therefore PR = 4\sqrt{13}$ cm Choice (B)

45. Let D be the foot of the perpendicular drawn from A to BC.



Let BD = x cm.
$\therefore 6^2 - x^2 = 7^2 - (5 - x)^2$
$\Rightarrow 36 - x^2 = 49 - (25 + x^2 - 10x) \Rightarrow 10x = 12$
x = 1.2 cm
E is the mid point of BC $\therefore$ BE = 2.5 cm
$\therefore$ DE = 1.3 cm Ans : (1.3)

46.



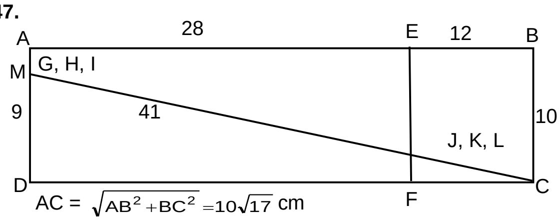
The point S is the circumcentre of the triangle.

$\angle CSA = 360° - (140° + 120°) = 100°$.

$\angle ABC = \frac{\angle CSA}{2} = 50°$. Ans: (50)

Solutions for questions 47 and 48:

47.



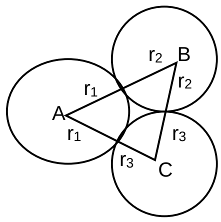
AC = $\sqrt{AB^2 + BC^2} = 10\sqrt{17}$ cm

This is very close to the distance (41) between the 6 pairs of points.
$\therefore$ Each of the three points in rectangle AEFD must lie close to one of the vertices A or D and each of the three points in the other rectangle must lie close to the opposite vertex. i.e., C or B respectively.
$\therefore$ Choice (A) must be true. Choice (A)

48. From the above solution the points may lie on MA which is 1 cm in length. All the points may lie on MA.
So, the distance between any pair of points among G, H, I is at most 1 cm.
From the choices GI can be 0.5 cm. Choice (A)

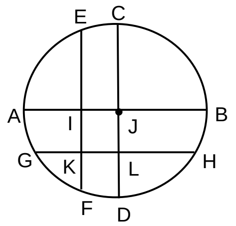
Solutions for questions 49 to 55:

49. $r_1 + r_2 = 15 \quad \rightarrow \quad (1)$
$r_2 + r_3 = 16 \quad \rightarrow \quad (2)$
$r_3 + r_1 = 17 \quad \rightarrow \quad (3)$
Adding (1), (2) and (3), we get;
$2(r_1 + r_2 + r_3) = 48$
$\Rightarrow r_1 + r_2 + r_3 = 24$
$\therefore$ The radius of the largest circle is r_3.
$r_3 = 24 - (r_1 + r_2)$
= (24 – 15) cm = 9 cm



Ans : (9)

50.

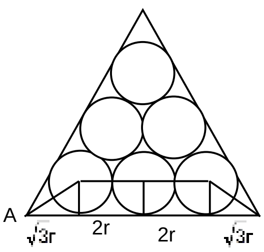


AI = (1/4) AB = (1/4)(8) = 2
IJ = AJ – AI = 2
Similarly, DL = LJ = 2
Consider the right-angled triangle GJL. GJ = 4, JL = 2

$\therefore GL = \sqrt{4^2 - 2^2} = 2\sqrt{3}$

$\therefore GK = GL - KL = 2\sqrt{3} - 2 = 2(\sqrt{3} - 1)$ Choice (A)

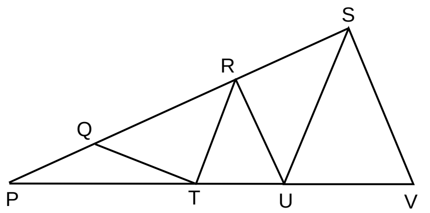
51.



$4r + 2\sqrt{3}r = a$

$r = \frac{a}{2(2-\sqrt{3})} = \frac{a(2-\sqrt{3})}{2}$ Choice (B)

52.

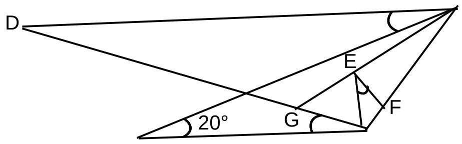


Let $\angle P = x$. In ΔPUQ, the external angle at Q is 2x
($\because \angle P = \angle QUP = x$)
In ΔPSU, the external angle at U, i.e., $\angle SUV = 3x$
In ΔPSV, the external angle at V, i.e., $\angle VSY = 4x$
In ΔPRT, the external angle at T, i.e., $\angle RTX = 2x$
In ΔPRV, the external angle at R, i.e., $\angle VRY$
$= \angle VSR = 3x$
$\angle VSR + \angle VSY = 180°$
$\Rightarrow 3x + 4x = 180° \Rightarrow x = 25\frac{5°}{7}$ Choice (C)

53. We know that the area of a quadrilateral is
$= \frac{1}{2} d_1 d_2 \sin\theta$
$= \frac{1}{2}(10)(14)\sin 60° = 70\left(\frac{\sqrt{3}}{2}\right) = 35\sqrt{3}$ sq.units.
$[\because \sin(180° - \theta) = \sin\theta]$ Choice (C)

54. In a cyclic quadrilateral the product of the diagonals is equal to the sum of the products of the opposite sides.
$\therefore$ (AC) (BD) = (AB) (CD) + (BC) (AD).
$\therefore$(AC) (BD) = 6(4) + 5(3) = 39 sq.units.
Choice (C)

55.



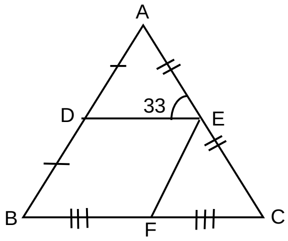
For the sake of clarity, the circle has not been shown.
Let CE intersect BD at G
$\angle BEG = \angle BCE + \angle EBC$ ($\because$ Exterior angle)
$= \frac{1}{2}(\angle BCA + \angle DBC)$
$\angle BEF$ ($\therefore \angle ABD$) is the complement of this angle. Hence $2\angle ABD$ (or $\angle ABD + \angle ACD$) is the supplement of ($\angle DBC + \angle BCA$). But together these angles make up the two interior angles made by line CD and BA on the same side of the transversal BC. As these angles are supplementary, CD is parallel to AB.
$\therefore \angle ACD = \angle CAB = 20°$
Ans : (20)

Solutions for questions 56 to 65:

56. As AD and BE are two of the medians, G must be the centroid of the triangle.
$\therefore$ AG : GD = 2 : 1and BG : GE = 2 : 1.
Statement I alone is not sufficient as it gives no lengths.
Statement II alone is not sufficient, as it gives the information about only two sides
From I and II, we have, AB = 10cm, BC = 20 cm.
AG : GE = 2: 1,
$\therefore$ GD = GE and BG = AG.
$\Rightarrow$ AD = BE.
In a triangle, if two medians are equal, then the sides as to which these medians are drawn must be equal.
$\therefore$ BC = AC
$\therefore$AC = 20 cm.
As we know the three sides of the triangle, we can find the area of the triangle.
$\therefore$ We can answer the question, using both the statements.
Choice (C)

57. Given, $\angle BAC = 90°$
$\therefore$ BC is the diameter of the circumcircle of the triangle
From statement I, BC = 21cm
Hence AM would be the circumradius.
$\therefore$AM = 1/2BC
$\therefore$ Statement I alone is sufficient.
From statement II, AD alone cannot yield the length of AM.
Statement II alone is not sufficient.
Choice (A)

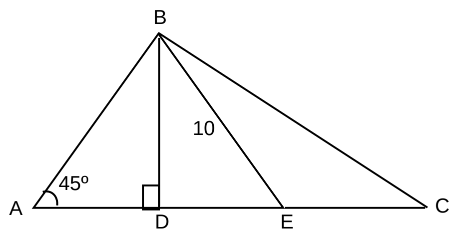
58.



As D, E, F are the mid points of AB, AC and BC respectively, the triangles AED, ECF, DFB and FDE are congruent.
$\therefore \angle FCE = \angle AED = 33°$.
From statement I, $\angle FEC = 78°$.
$\therefore \angle BFE = \angle FEC + \angle ECF$
= 78 + 33 = 111°
So, $\angle BFE = 111°$
Hence statement I alone is sufficient .
From statement II $\angle BAC = 78°$.
$\therefore \angle FEC = \angle BAC = 78°$.
$\therefore$ We can find $\angle BFE$.
Hence statement II alone is sufficient. Choice (B)

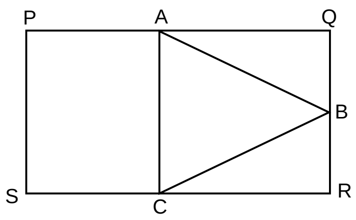
59. Statement I alone is not sufficient, as it gives no lengths.
Statement II alone is not sufficient, as we don't know the information about the other side or any of the angles of the triangle.
From I and II, we have
$\angle ABC = 60°$ and AB = 10 cm, AC = 10 cm.
$\therefore \angle BCA = \angle ABC = 60°$
$\therefore \angle BAC = 180° - (60° + 60°) = 60°$
$\therefore$ BC = 10 cm
We know all the sides, therefore, we can find the area.
Choice (C)

60.



From statement I, $\angle BCA = 45°$. So triangle ABC is a right angled triangle, but we do not know the lengths of AC or BC, so statement I is not sufficient.
From statement II, we have $\angle BEA = 60°$ As triangle BDE is a right angled triangle, BD = BE sin(60°)
$BD = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$ cm.
$AB = \frac{BD}{\text{Sin } 45°}$ [$\because \Delta$ ABD is a right triangles].
$\therefore AB = 5\sqrt{6}$ cm. Hence statement II alone is sufficient.
Choice (A)

61.



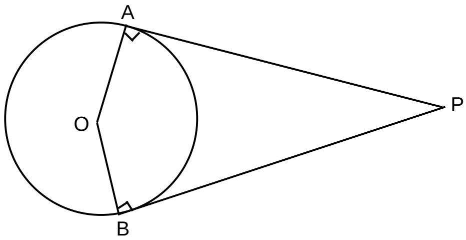
From statement I, we do not know the information about the point C, so we can't answer the question. Statement I alone is not sufficient.
From statement II, by knowing AC is parallel to PS we can't answer the question as we don't know the information about B. Statement II alone is not sufficient.
Using both the statements,

we have AC = PS and $\frac{2}{3}$PQ = AQ

$$\therefore \ \frac{\text{area of the triangle ABC}}{\text{area of the rectangle PQRS}} = \frac{\frac{1}{2} \times AC \times AQ}{PS \times PQ} = \frac{1}{3}$$

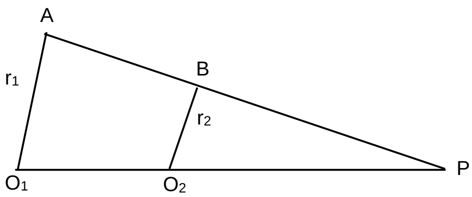
Using both the statements we can answer the question.
Choice (C)

62.



From the statement I, by knowing the radius of circle we can't find the length of PO, because to find PO, we need OA as well as AP.
Statement I alone is insufficient.
From statement II, we have circumradius of triangle AOP as 14 cm. As $\angle$OAP = 90°, the triangle AOP is a right angled triangle As OP is the hypotenuse it must be twice the circumradius.
Statement II alone is sufficient. Choice (A)

63.

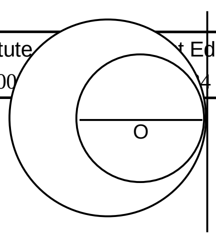


From statement I, we have,
r_1 = 15 cm and r_2 = 9 cm.
Also, $O_1 O_2 = r_1 + r_2$ = 24 cm.
The two triangles, AO_1P and BO_2P are similar.

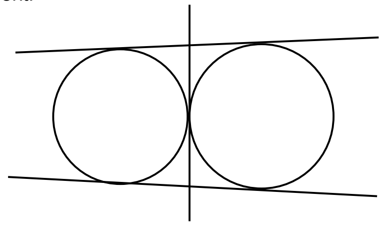
$$\therefore \ \frac{AO_1}{BO_2} = \frac{O_1P}{O_2P}. \Rightarrow \frac{15}{9} = \frac{O_2P + O_1O_2}{O_2P}$$

$\Rightarrow 5O_2P = 3O_2P + 3 \times 24 \Rightarrow O_2P$ = 36 cm.
As $\angle O_2BP = \angle O_2AP$ = 90°
BP = $\sqrt{O_2P^2 - O_2B^2}$
AP = $\sqrt{O_1P^2 - O_2A^2}$
As we know
O_2P, O_1P, O_2B, O_1A, we can find AP and BP, and thereby AB.
Statement I alone is sufficient.
Statement II alone is not sufficient, as it does not have any length. Choice (A)

64.

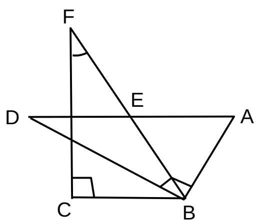


From statement I, the two circles have only one common tangent therefore, one circle lies inside the other, which means the distance between the centres is equal to the difference between the radii. So statement I alone is sufficient.



From statement II, when two circles having only three common tangents the circles touch each other externally. Therefore, the distance between centres is equal to the sum of the radii. Statement II alone is also sufficient.
Choice (B)

65.



As $\angle$ABD = $\angle$BCF = 90°, AD and BF are diameters of the circle. $\therefore$ E must be the centre of the circle.
$\therefore$ $\angle$CEB = 2$\angle$CFB and $\angle$BEA = 2$\angle$BDA.
[$\because$Angle at centre is twice the angle subtended by the arc at any point on the circumference of the circle].
From statement I, $\angle$BDA = 30°. $\therefore$ $\angle$BEA = 60°.
$\therefore$ Statement I alone is sufficient.
From statement II, $\angle$CFB = 30°.
$\therefore$ $\angle$CEB = 60°
But we cannot find $\angle$BEA.
$\therefore$ Statement II alone is not sufficient.
Choice (A)

Chapter – 5
(Mensuration)

Concept Review Questions

Solutions for questions 1 to 36:

1. Area of the triangle = $\frac{1}{2}$ (6) (8) sin30° = 12 cm^2.
Ans : (12)

2. Area of an isosceles triangle of base b cm and equal sides a cm each = $\frac{b}{4}\sqrt{4a^2 - b^2}$.
In the given problem, b = 10 and a = 13.
Area of the triangle = $\frac{10}{4}\sqrt{4(13)^2 - 10^2}$ = 60 cm^2. Ans : (60)

3. Area of the triangle = rs. Choice (A)

4. Area of the triangle = $\frac{abc}{4R}$. Choice (C)

5. Area of the triangle = $\frac{\sqrt{3}}{4}(6)^2$ cm^2 = $9\sqrt{3}$ cm^2.

Choice (D)

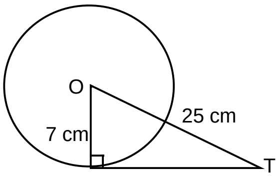
6. Let PS be the median from P to QR.
By Apollonius theorem, $2(PS^2 + QS^2) = PQ^2 + PR^2$

$PS^2 = \frac{3^2+4^2}{2} - \left(\frac{6}{2}\right)^2$ cm^2 = 3.5 cm$^2 \Rightarrow$ PS = $\sqrt{3.5}$ cm.

Choice (C)

7. AC and AE are secants. Let AD = X cm
(AB) (AC) = (AD) (AE)
(2) (2 + 22) = (X) (X + 8) $\Rightarrow$ X = 4 $\therefore$ AD = 4 cm Ans : (4)

8.



Let O be the centre of the circle and T be the point from which the tangent is drawn.
Length of the tangent = $\sqrt{OT^2 - (\text{Radius})^2}$ = $\sqrt{25^2 - 7^2}$ cm
= 24 cm Ans : (24)

9. Area = (6) (10) sin30° cm^2 = 30 cm^2. Ans : (30)

10. Area enclosed by a ring whose inner circle radius is r and outer circle is R is given by $\pi(R^2 - r^2)$.
In the given problem, R = 8 cm and r = 6 cm
$\therefore$ Required area = $\pi(8^2 - 6^2)$ cm^2 = 28 π cm^2 = 88 cm^2
Choice (B)

11. Area = $\frac{1}{2}$(6) (8) cm^2 = 24 cm^2. Ans : (24)

12. Area = $\frac{1}{2}$(12 + 18) (15) cm^2= 225 cm^2. Choice (D)

13. Area of ABCD = Area of triangle ABC + Area of triangle ACD
= $\left[\frac{1}{2}(4)(12) + \frac{1}{22}(6)(12)\right]$ cm^2 = 60 cm^2. Choice (C)

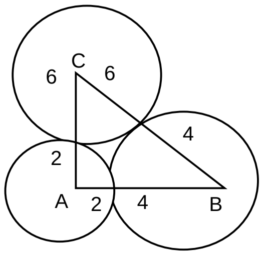
14. Perimeter = [π(14) + 2(14)] cm ≈ $\left[\frac{22}{7}(14) + 2(14)\right]$ cm
= 72 cm. Ans : (72)

15. Area of the sector AOB = $\frac{90}{360}\pi(7)^2$ cm^2 = 38.5 cm^2
Ans : (38.5)

16. Semi Perimeter 23 cm.
Area = $\sqrt{(23-8)(23-10)(23-12)(23-16)}$ cm^2
= $\sqrt{(15)(13)(11)(7)}$ cm^2 = $\sqrt{15015}$ cm^2. Choice (A)

17. Let the radius of the circle be r cm.
Let the length and the breadth of the rectangle be ℓ cm and b cm respectively.
$\frac{\pi}{2}[2(\ell + b)] = 2\pi r \Rightarrow \ell + b = 2r$
If ℓ = r, b = r
If b = r, ℓ = r
In either case, ℓ = b = r
Required ratio = $\pi r^2 : \ell b = \pi : 1$ Choice (A)

18.



Let A, B and C be the centres of the circles.
AB = 6 cm, BC = 10 cm and CA = 8 cm
Semi perimeter(s) of the circle $= \frac{6+8+10}{2}$ cm $= 12$ cm
Area of the triangle $= \sqrt{12(12-8)(12-6)(12-10)}$ cm^2
24 cm^2.

Note: We observe that the above triangle is a right triangle.
$\therefore$ Area = 1/2(6)(8) cm^2 = 24 cm^2. Ans : (24)

19. Let the area of the square be $3\sqrt{3}$ k and that of the triangle be 4k
S(S) = $3\sqrt{3}$ k $\Rightarrow$ S = $(3\sqrt{3}\,k)^{1/2}$
$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 4k \Rightarrow a = \left(\frac{16k}{\sqrt{3}}\right)^{1/2}$
Perimeter of the square = 4 $(3\sqrt{3}k)^{1/2}$
Perimeter of the triangle $= 3\,\frac{(4\sqrt{k})}{(\sqrt{3})^{1/2}}$
The required ratio is $= 4(3\sqrt{3}k)^{1/2} : 3\frac{(4\sqrt{k})}{(\sqrt{3})^{1/2}}$
= 1 : 1 Choice (B)

20. Lateral Surface Area of the Prism = (Perimeter of the base) (Height) = 2 (4 + 2) (8) = 96 cm^2. Ans : (96)

21. Lateral Surface Area = (4) (6) (10) = 240 cm^2.
Total Surface Area = Lateral Surface Area + 2(Base Area) = 240 + 2(6)^2 = 312 cm^2. Ans : (312)

22. Volume of the Prism = (Area of the base) (Height of the Prism) = $\frac{\sqrt{3}}{4}(4)^2(8) = 32\sqrt{3}$ cubic cm. Ans : (32)

23. Volume = (12) (10) (9) = 1080 cubic cm. Ans : (1080)

24. Longest diagonal = $\sqrt{l^2 + b^2 + h^2}$ Choice (A)

25. Lateral Surface Area = 2h (l + b) Choice (A)

26. Let the length of the cuboid be l cm.
4l + 2l + (4) (2) = 44 $\Rightarrow$ l = 6 Ans : (6)

27. Total Surface Area = $2\pi r^2 + \pi r^2 = 3\pi r^2$. Choice (C)

28. Volume = $\frac{4}{3}\pi(6)^3$ = 288π cubic cm. Choice (C)

29. Let the radius and the height of the cylinder be r cm and h cm respectively.
Volume of the cylinder = $\pi r^2 h$ cubic cm.
Volume of the cone = $\frac{1}{3}\pi r^2 h$ cubic cm

$\therefore \frac{2}{3}$rd of the cylinder is remaining. Choice (D)

30. Volume of the prism = (Area of the base) (Height)

Volume of the pyramid = $\frac{1}{3}$(Area of the base) (Height)

$\therefore$ Ratio of the volumes of the prism and the pyramid = 3 : 1. Ans : (3)

31. Total Surface Area of the Pyramid

$= \left[\frac{1}{2}(\text{Perimeter of the base})(\text{Slant height})\right]$

$+ \text{ Area of the base} = \frac{1}{2}(4)(4)(8) + 4^2 = 80\ cm^2$. Ans : (80)

32. Total Surface Area = $\pi(6)\ (10 + 6) = 96\pi\ cm^2$.
Choice (C)

33. Lateral Surface Area of the Frustum of a cone having top radius of r cm, radius of the base of R cm and slant height of ℓ cm = $\pi l(R + r)\ cm^2$.
In the given problem, r = 8, R = 10 and l = 9
Lateral Surface Area = $\pi(9)\ (18) = 162\pi\ cm^2$.
Choice (C)

34. Total Surface Area = Lateral Surface Area + Top Area + Base Area = $\frac{1}{2}$(Sum of the perimeters of the base and the top) slant height + Top Area + Base Area

$= \frac{1}{2}\ [4(6) + 4(10)8] + 6^2 + 10^2 = 392\ cm^2$. Ans : (392)

35. Slant height = $\sqrt{5^2 + 12^2}$ = 13 cm Ans : (13)

36. Volume = $\frac{1}{3}\pi h\ (R^2 + Rr + r^2)$ Choice (B)

Solutions for questions 37 and 38:

R is the base radius of the frustum, also of the cone from which the frustum is obtained.
Let H represent the height of this cone.
Then it follows that

$\frac{r}{R} = \frac{H - h}{H}$

As h = $\frac{2}{3}$H , it follows that r = $\frac{1}{3}$R

$\therefore$The height of the smaller cone = $\frac{H}{3}$

As the smaller cone and the original cone are identical, the radius of the smaller cone will be $\frac{R}{3}$.

37. Ratio of the curved surface area = (Ratio of the radii) (Ratio of the slant heights) = $\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9}$ .Choice (C)

38. As ratio of the slant heights of the smaller and the original cone = 1 : 3, the ratio of the slant heights of the frustum and the original cone = 2 : 3. Choice (D)

Solutions for questions 39 to 40:

39. Along one edge, the number of small cubes that can be cut $= \frac{100}{10} = 10$

Along each edge 10 cubes can be cut. (Along length, breadth and height). Total number of small cubes that can be cut = $10 \times 10 \times 10 = 1000$ Ans : (1000)

40. The longest diagonal of the cuboid = $\sqrt{l^2 + b^2 + h^2}$

$= \sqrt{15^2 + 20^2 + 25^2}$ cm = $\sqrt{1250}$ cm ≈ 35 cm
Ans : (35)

Exercise – 5(a)

Solutions for questions 1 to 35:

1. Let the sides of the triangle be a cm, b cm and c cm
Let a + b – c = 10, b + c – a = 20 and c + a – b = 30.
Adding these equations, we get a + b + c = 60.

Area of the triangle = $\sqrt{s(s-a)\,(s-b)\,(s-c)}$

$= \sqrt{\frac{a+b+c}{2}\left(\frac{a+b+c-a}{2}\right)\left(\frac{a+b+c-b}{2}\right)\left(\frac{a+b+c+-c}{2}\right)}$

$= \sqrt{\frac{(a+b+c)\,(b+c-a)\,(a+c-b)\,(a+b-c)}{16}}$

$= \sqrt{\frac{(60)\,(10)\,(20)\,(30)}{16}}$ = 150 cm^2. Ans : (150)

2. Third side = 58 – (16 + 22) = 20 cm

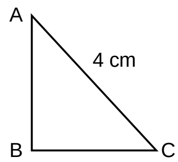
Now, S = $\frac{16 + 22 + 20}{2}$ = 29

$\therefore$ Area of the triangle

$= \sqrt{29(29-16)\,(29-22)\,(29-20)}$ = 154.11 cm^2

Let the length of the altitude be h cm
= ½ × 22 × h = 154.11
$\therefore$ h = 14.01 m Choice (C)

3.



Let AB = x + a and BC = x – a.
Area of the triangle ABC

$= \frac{(x+a)(x-a)}{2} = \frac{x^2 - a^2}{2}$.

But area of the triangle is maximum.
$\therefore$ a = 0. ⇒ AB = BC.
$\therefore$ AB = BC = $2\sqrt{2}$ cm.
$\therefore$ Maximum possible area of the triangle is 4 cm^2.
Ans : (4)

4. Radius of the inscribed circle = $\frac{24}{2\sqrt{3}}$

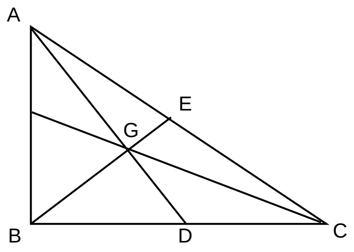
Let each side of the inscribed triangle be 'a' cm
Radius of the inscribed circle = circumradius of the inscribed triangle

$\frac{24}{2\sqrt{3}} = \frac{a}{\sqrt{3}} \Rightarrow a = 12$

$\therefore$ Required area = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\ (12)^2\ cm^2$

$= 36\sqrt{3}\ cm^2$ Choice (C)

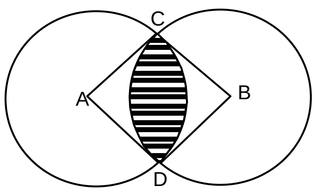
5. $67.2 = 3 \times 22.4$
$89.6 = 4 \times 22.4$
$112 = 5 \times 22.4$
$\therefore$ The given triangle is a right-angled triangle and hence the hypotenuse of 112 cm is the diameter of the circle.
$\therefore$ Radius of the circumcircle = $^1/_2(112)$ cm = 56 cm
Choice (C)

6.


Let AD and BE be the perpendicular medians.
The point of intersection of any two medians of a triangle is the same. AD, BE and the median through C intersect at G, the centroid. As G trisects AD and BE, BG = $\frac{2}{3}$(BE) = 6 and

GD = $\frac{1}{3}$ AD = 4.
The medians of a triangle divide the triangle into six triangles of equal area.
Area of ABC = 6 (Area of BGD)
= 6 ($\frac{1}{2}$(BG) (GD)) = 6 ($\frac{1}{2}$(6) (4)) = 72. Ans : (72)

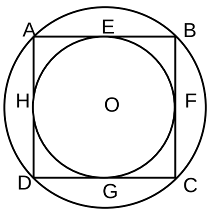
7. $\frac{1}{2} d_1 d_2 = 21$
$d_1 d_2 = 42$
Also, side of the rhombus = 40/4 = 10 cm
$\therefore \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = 10^2 \Rightarrow d_1^2 + d_2^2 = 400$
Now, $(d_1 + d_2)^2 = d_1^2 + d_2^2 + 2d_1d_2$
$= 400 + (2 \times 42) = 484 = (22)^2$
$\therefore d_1 + d_2 = 22$ Ans : (22)

8.


The shaded area in the above figure is the area common to the two circles.
Shaded Area = Area of sector ACD − Area of ΔACD + Area of sector BCD − Area of ΔBCD

$= 2\left[\frac{90}{360}\pi(2)^2 - \frac{1}{2}(2)(2)\right] = 2(\pi - 2)$ cm^2.

Choice (B)

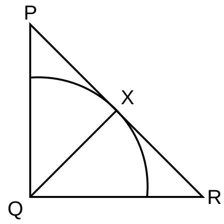
9.


Let OF = 1. $\therefore$ EF = $\sqrt{2}$ and area of square EFGH = 2

Radius of bigger circle OB = $\sqrt{2}$. $\therefore$ Area = 2π

Required ratio = $2\pi : 2 = \pi : 1$ Choice (A)

10. Let the equal sides of the triangle be a each.
Perimeter of the triangle = $2a + a\sqrt{2}$



$2a + a\sqrt{2} = 8\sqrt{2} + 8$
$\Rightarrow a = 4\sqrt{2}$
Let the radius of the largest possible quadrant cut out be r.
The hypotenuse of the triangle is a tangent to the quadrant at X.
$\therefore$ r = QX = Altitude drawn to the hypotenuse.

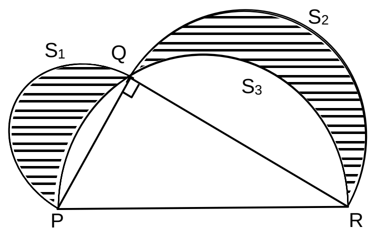
$r = \frac{a}{\sqrt{2}} = 4$

Area of the remaining region = Area of the triangle − Area of the quadrant

$= \frac{1}{2}a^2 - \frac{\pi r^2}{4} \approx \frac{1}{2}\left(4\sqrt{2}\right)^2 - \frac{22}{7}\frac{(4)^2}{(4)}(4)^2 = \frac{24}{7}$.

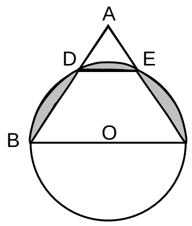
Choice (B)

11. Let the shaded areas in S_1 and S_2 be A_1 and A_2 respectively.



Let the shaded and unshaded regions in S_1 be a and b respectively.
Let the shaded and unshaded regions in S_2 be c and d respectively and let the area of ΔPQR be e.
As $PQ^2 + QR^2 = PR^2$ ($\because \angle Q = 90°$)
$S_1 + S_2 = S_3$ i.e. (a + b) + (c + d) = (b + e + d)
$\therefore$ a + c = e
As a + c = 30, e is also 30.

Choice (B)

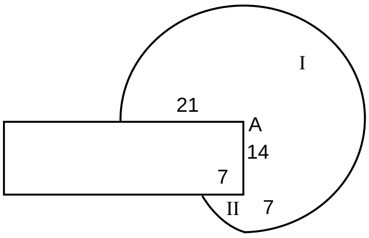
12.


As BC = 2 cm and the radius of the circle is 1 cm, BC is the diameter of the circle.
Let O be the centre of the circle and D and E be the respective points of intersection of AB and AC with the circle.
As OB = OD, and $\angle$ABO = 60°, ΔBDO is an equilateral triangle.
Similarly, ECO is an equilateral triangle.
As $\angle$BOC = 180°, $\angle$DOE = 60°.
The triangles BDO, DOE and EOC are congruent.
The area of the shaded region
= Area of semi-circle– 3(Area of each equilateral triangle)

$= \left[\frac{\pi}{2}(1)^2 - 3x\frac{\sqrt{3}}{4}(1)^2\right]$

$= \left(\frac{\pi}{2} - 3\frac{\sqrt{3}}{4}\right)$ sq.cm. Choice (B)

13. The cow can graze the shaded areas numbered I and II.



Let the cow be tied at A.

Area of the region I = $\frac{22}{7} \times 21 \times 21 \times \frac{270}{360} = 1039^1/_2$ sq.m

Area of the region II = $\frac{22}{7} \times 7 \times 7 \times \frac{90}{360} = 38^1/_2$ sq.m

$\therefore$ Total area that the cow can graze
$= 1039^1/_2 + 38^1/_2 = 1078$ sq.m

Ans : (1078)

14. Let the arc length of 1st sector be x cm. Arc lengths of 2nd, 3rd, 4th, 5th, 6th, 7th and 8th sectors are 2x cm, 4x cm, 8x cm, 16x cm, 32x cm, 64x cm and 128x cm respectively.
Sum of the arc lengths of the sectors = 255x cm

$\therefore 255x = \frac{2\pi(1)}{10} \Rightarrow x = \frac{\pi}{1275}$

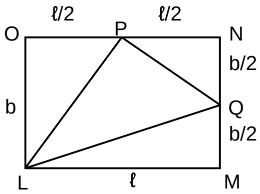
If the central angle of the 1st sector is θ,

$\frac{\theta}{2\pi}(2\pi(1)) = \frac{\pi}{1275}$

$\theta = \frac{\pi}{1275}$

Choice (B)

15.



(i) Area of the rectangle = ℓb.

(ii) Area of the triangle NPQ = $\frac{\ell b}{8}$

(iii) Area of the triangle LMQ = $\frac{\ell b}{4}$

$\therefore$ Area of the triangle PQL

$= \ell b - \left(\frac{\ell b}{8} + \frac{\ell b}{4} + \frac{\ell b}{4}\right) = \frac{3\ell b}{8}$. Choice (D)

16. Area of the pentagon $= 5 \times \frac{(20)^2}{4} \times \cot\left(\frac{180°}{5}\right)$

$= 500 \times 1.376 = 688$ cm^2 Ans : (688)

17. $AC^2 = AD^2 + CD^2$

$AC = \sqrt{18^2 + 24^2} = 30$ m

In ΔABC, AB = BC = $\frac{80-30}{2} = 25$ m

Let BE be the perpendicular from B to AC.
$BC^2 = BE^2 + EC^2$
As ΔABC is isosceles,

EC = $\frac{AC}{2}$ = 15 m

BE = $\sqrt{BC^2 - EC^2}$ = 20 m

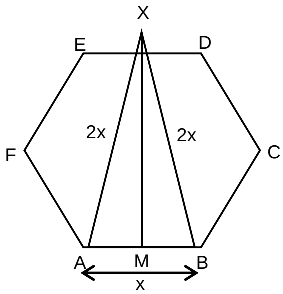
Area of ΔABC = $\frac{1}{2}$ (20) (30) = 300 sq.m.

Area of ΔADC = $\frac{1}{2}$ (18) (24) = 216 sq.m.

Area of the plot = Area of ΔABC + Area of ΔADC = 516 sq.m.

Ans : (516)

18.



Let M be the midpoint of AB and XM = h

$h^2 + \frac{x^2}{4} = 4x^2 \Rightarrow h^2 = \frac{15x^2}{4} \Rightarrow h = \frac{\sqrt{15}}{2}x$

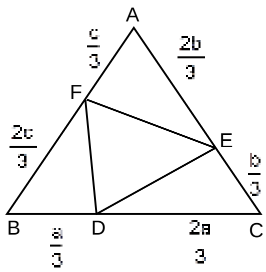
Area of Δ ABX = $\frac{1}{2}\frac{\sqrt{15}}{2}x^2 = \frac{\sqrt{15}}{4}x^2$

Area of ABCDEF = $6\left(\frac{\sqrt{3}}{4}\right)x^2 = \frac{3\sqrt{3}}{2}x^2$

Ratio of areas of ABX and ABCDEF = $\frac{\sqrt{15}}{4} \cdot \frac{2}{3\sqrt{3}} = \frac{\sqrt{5}}{6}$

Choice (C)

19. Let AB = c; BC = a, AC = b



Area of ABC = $\frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$

Area of CDE = $\frac{1}{2}\left(\frac{2a}{3}\right)\left(\frac{b}{3}\right)\sin C = \frac{2}{9}\Delta$

Area of AFE = $\frac{1}{2}\left(\frac{2b}{3}\right)\left(\frac{c}{3}\right)\sin A = \frac{2}{9}\Delta$

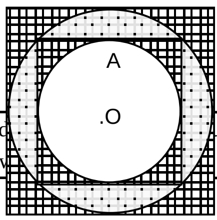
Similarly area of BDE = $\frac{2\Delta}{9}$

Area of DEF = $\Delta - 3\left(\frac{2\Delta}{9}\right) = \frac{\Delta}{3}$

The ratio of the areas of DEF and ABC = $\frac{\Delta/3}{\Delta}$ = 1 : 3

Choice (C)

20.



Let the radius of the smaller circle (say D) be r.
The side of the smaller square (say T) = r + r = 2r

The radius of the bigger circle (say C) = $\sqrt{2}$ r

The side of the bigger square (say S)= $2\sqrt{2}$ r
The area shaded by lines is (S – C) + (T – D).
The area shaded by dots is (C – T).

The required ratio = $\dfrac{4r^2 - \pi r^2 + 8r^2 - 2\pi r^2}{2\pi r^2 - 4r^2}$

= $\dfrac{12r^2 - 3\pi r^2}{2\left(\pi r^2 - 2r^2\right)}$ = $\dfrac{3(4-\pi)}{2(\pi-2)}$ Choice (D)

21. The length and breadth of the cuboid formed are both equal to (30 – 2y) cm each.
Volume of the cuboid = (30 – 2y) (30 – 2y) y
= 2(15 – y) (15 – y) (2y)
The sum of 15 – y, 15 – y and 2y is constant. (i.e., 30)
Their product is maximum when 15 – y = 15 – y = 2y
i.e., y = 5
∴ the volume of the cuboid is maximum, when y = 5.
Choice (A)

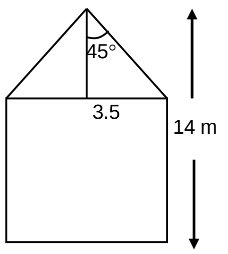
22. Volume of the drum = $\dfrac{\pi h}{3}$ ($R^2 + r^2 + Rr$)

= $\dfrac{22}{7}\left(\dfrac{5}{3}\right)$ ($24^2 + 15^2 + 24 \times 15$) = 42570/7 cu.ft

Let the rise in the water level be H ft.
99(43)H = 42570/7

∴ H = $\dfrac{42570}{7(99)(43)} = \dfrac{10}{7} = 1\dfrac{3}{7}$ Choice (D)

23.

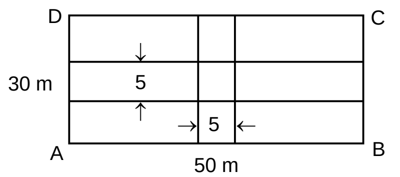


Height of the cone = Radius of the cone = 3.5 m
Height of the cylindrical portion = (14 – 3.5) m = 10.5 m
Volume enclosed by the building (in m^3)

= $\dfrac{22}{7}$ $(3.5)^2$ (10.5) + $\dfrac{1}{3}\left(\dfrac{22}{7}\right)$ $(3.5)^2$ (3.5)

= $449^1/_6$ Choice (B)

24.



(i) Area of the rectangular plot ABCD = 1500 m^2.
(ii) Area of the trenches = (30)(5) + (50)(5) – (5)(5) m^2.
= 375 m^2.
(iii) Area of the remaining portion = 1125 m^2.
(iv) Volume of the earth dug out = (375)(1.5) m^3 .

Rise in the level of the remaining plot

= $\dfrac{\text{Volume of the earth dug out}}{\text{Area of the remaining portion}} = \dfrac{562.5}{1125}$ m

= 0.5 m. Choice (C)

25. Volume of the cube = $(7)^3$ m^3 = 343 m^3.
Volume of the largest right circular cylinder = $\pi r^2 h$

= $\dfrac{22}{7}$ $(3.5)^2$ (7) cm^3 = 269.50 cm^3.

∴ The volume of the metal which is not used
= 343 – 269.5 cm^3 = 73.5 cm^3. Ans : (73.5)

26. Let the side of A as well as B be x cm.
Diameter of C = x cm

Volume of C = $\dfrac{4}{3}\pi\left(\dfrac{x}{2}\right)^3 = \dfrac{\pi x^3}{6}$ cm^3.

Let us say B is cut into n small cubes.

Volume of each cube = $\dfrac{x^3}{n}$ cm^3.

Diameter of the sphere in each cube = $\sqrt[3]{\dfrac{x^3}{n}}$ cm.

Volume of each of these spheres = $\dfrac{4}{3}\pi\left(\sqrt[n]{\dfrac{x^3}{n}}\right)^3$

= $\dfrac{\pi x^3}{6n}$ cm^3.

Total volume of these spheres = $\dfrac{\pi x^3}{6}$ cm^3.

∴Required ratio = 1 : 1. Ans : (1)

27. Let rate of flow per hour = x m/s
Volume of water flowing through the pipe per second
= x $\left(\dfrac{25}{100}\right)\left(\dfrac{25}{100}\right)\left(\dfrac{x}{16}\right)$ m^3

Volume of water flown in 10 hours = 80(35)(2) = 5600 m^3
∴ Volume of water flown into the tank per second

= $\dfrac{5600}{10(60)(60)}$ m^3 = $\dfrac{7}{45}$ m^3

∴ $\dfrac{x}{16} = \dfrac{7}{45} \Rightarrow x = \dfrac{7(16)}{45} = \dfrac{112}{45}$

The flow rate in $\dfrac{km}{hr}$ is $\dfrac{112}{45}\left(\dfrac{3600}{1000}\right) = 8\dfrac{24}{25}$
Choice (A)

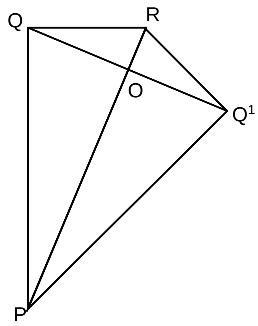
28. Number of bricks required = $\dfrac{0.9(1500)(1000)(800)}{10(8)(4)}$

= 3,375,000
Now cost of 3,375,000 bricks (in rupees)

= 3,375,000 $\left(\dfrac{400}{100}\right)$ = 13,500,000

∴ The cost of the bricks is ₹one crore, thirty five lakhs.
Choice (B)

29.



$8^2 + 15^2 = 17^2$.
$\therefore$ The triangle is right-angled.
By rotating the triangle about its hypotenuse, we get a double cone.
Volume of the figure generated = Volume of the double cone = Volume of the upper cone (U) +Volume of the lower cone (L)
Each of the two cones has a base diameter of QQ^1. O is the centre of the common circular base of either cone.
$\therefore$ $QO = Q^1O$ = Radius of the common circular base = r (say)
$QQ^1 \perp PR$

$U = \frac{1}{3}\pi(r^2)$ (RO) and $L = \frac{1}{3}\pi(r^2)$ (OP)

$U + L = \frac{1}{3}\pi r^2 (RO + OP) = \frac{1}{3}\pi r^2 (RP)$

r = Length of the altitude drawn from Q to PR.

Area of PQR = $\frac{1}{2}(8)(15) = \frac{1}{2}\,17r \Rightarrow r = \frac{120}{17}$.

Volume of the figure generated $= \frac{1}{3}\pi\left(\frac{120}{17}\right)^2 (17)$

$= \frac{4800\pi}{17}$. Choice (A)

30. Let n be the number of small spheres formed
$\therefore$ n (volume of small sphere) = volume of big sphere

$\Rightarrow n\left\{\frac{4}{3}\pi(2)(2)(2)\right\} = \frac{4}{3}\pi(30)(30)(30)$

$\Rightarrow n = 3375$ Ans : (3375)

31. The area covered by the lawn mower in one revolution is its curved surface area.
$\therefore$Area covered in 200 revolutions

$= 200(2\pi rh) = 200(2)\left(\frac{22}{7}\right)\left(\frac{14}{100}\right)(1)\ m^2 = 176\ m^2$

Ans : (176)

32. Let the length, the breadth and the height of the cuboid be 5x, 4x, and 3x respectively

Longest rod's length $= \sqrt{(5x)^2 + (4x)^2 + (3x)^2} = 10\sqrt{2}$ cm

$50x^2 = (10\sqrt{2})^2\ cm^2 \Rightarrow x = 2$ cm

Volume of the cuboid = (5x) (4x) (3x) = 480 cm^3
Ans : (480)

33. Radius of the cone = $^1/_2$ (12) cm = 6 cm
Height of the cone = 12 cm

$\therefore$ Volume $= \frac{1}{3}\left(\frac{22}{7}\right)(6)(6)(12)\ cm^3$

$= 452\frac{4}{7}\ cm^3$ Choice (A)

34. Let the length, breadth and the height of the cuboid be ℓ, b and h respectively (all in cm).
214 = 2(42 + 35 + bh)
$\Rightarrow$ bh = 30 ------- (1)
lb = 42 ------- (2)
lh = 35 ------- (3)
From (2) and (3)
b/h = 6/5 $\Rightarrow$ b = 6/5 h

Substituting in (1), we get $\frac{6}{5}h^2 = 30 \Rightarrow h = 5$

$\therefore b = \frac{6}{5}(5) = 6$

$\therefore l = \frac{42}{6} = 7$ Ans : (7)

35. Each dimension (in m) of the larger box is a multiple of the same dimension (in m) of each of the smaller boxes.

$\therefore$ Maximum number $= \frac{\text{volume of the larger box}}{\text{volume of each small box}}$

$\frac{30(20)(15)}{6(5)(3)} = 100$. Ans : (100)

Exercise – 5(b)

Solutions for questions 1 to 45:

1. Let the radius of the circle be r units. Let the sides of the square and the triangle be s units and a units respectively.

$\pi r^2 = s^2 = \frac{\sqrt{3}}{4}a^2$

$r = \frac{s}{\sqrt{\pi}}$ and $a = \frac{2s}{\sqrt[4]{3}}$

$C = 2\pi r = 2\sqrt{\pi}\ s$

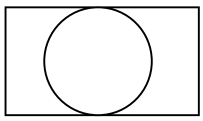
$S = 4s$

$T = 3a = \frac{6s}{\sqrt[4]{3}}$

As $\sqrt{\pi}$ is less than 2, $2\sqrt{\pi}$ s is less than 4s
$\therefore$ C < S.
Let us now compare S and T. Comparing S and T is equivalent to comparing their fourth powers.
$S^4 = 256$ s
$T^4 = 432$ s
$\therefore S^4 < T^4$
$\therefore$ S < T
$\therefore$ C < S < T

Alternate method:
Let us consider regular polygons with n sides.
If we take a fixed length P and take increasing values of n (3, 4, 5…) the area of the polygon keeps increasing with n. For a circle ($n = \infty$), this area is the maximum.
i.e., Area (Triangle) < Area (Square) < Area (Circle)
If we consider bigger triangle and squares so that their areas are equal to the area of the circle (of circumference P), the perimeters will naturally be greater.
$\therefore$ If we consider regular polygon with equal areas, the perimeter keeps decreasing with n and for a circle ($n = \infty$), this perimeter (i.e. circumference) is the least.
$\therefore$ T > S > C Choice (D)

2. Let the breath of the rectangle be 'b' units.
$\therefore$ Diameter of the circle = b
$\Rightarrow$ length of the rectangle = 2b.
$\therefore$ Ratio of the area of rectangle and the circle



$= (2b)(b) : \pi\left(\frac{b}{2}\right)^2 = 28 : 11.$ Choice (D)

3. The length of the pendulum is equal to the radius of the sector $= 2\pi r \times \frac{\theta}{360°} = \pi r = \frac{\theta}{180°}$

$\Rightarrow \frac{22}{7} \times r \times \frac{60}{180} = 44 \Rightarrow r = 42$ cm Ans : (42)

4. Volume of the water = $(33 \times 10 \times 20)$ cu.m.
Area of the cross section of the sluice = 220 cm^2.

$= \frac{220}{10000}$ sq.m.

$\therefore$ Length of the water column in 5 hr $= \frac{33 \times 10 \times 20}{\frac{220}{10000}} = 300$ km.

$\therefore$ Speed of the water flow $= \frac{300}{5}$ kmph = 60 kmph.

Ans : (60)

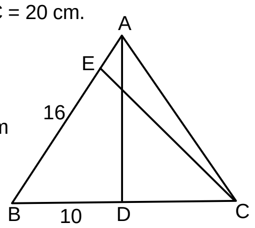
5. Let the other two sides be x cm and y cm
$x + y + 65 = 144 \Rightarrow x + y = 79$ ------ (1)
Also, $x^2 + y^2 = 65^2 = 4225$ ------ (2)
Now, $(x + y)^2 = 79^2 \Rightarrow (x^2 + y^2) + 2xy = 6241$
$\therefore$ $2xy = 6241 - 4225 = 2036$

$x - y = \sqrt{(x+y)^2 - 4xy}$

$= \sqrt{6241 - 2(2036)} = \sqrt{2209} = 47$ ------ (3)

Solving (1) and (3), we get
x = 63, y = 16 Choice (A)

6. Let AB = AC = 16 cm and BC = 20 cm.
$\therefore$ AD = 10 cm



$AD = \sqrt{(AB)^2 - (BD)^2}$

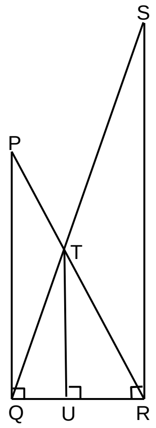
$= \sqrt{256 - 100}$ cm $= \sqrt{156}$ cm

(AB)(CE) = (AD)(BC)

$\therefore$ $CE = \dfrac{(AD)(BC)}{AB}$

$= \dfrac{20\left(\sqrt{56}\right)}{6} \; \dfrac{cm^2}{cm} = 2.5\sqrt{39}$ cm Choice (D)

7. Given that, PQ = 12 cm,
QR = 6 cm and SR = 18 cm.
Let QU = x cm and TU = y cm.
Clearly $\Delta PQR \sim \Delta TUR$



$\therefore$ $\dfrac{TU}{PQ} = \dfrac{UR}{QR}$

$\Rightarrow \dfrac{y}{12} = \dfrac{6 - x}{6}$ ------- (1)

Also $\Delta TUQ \sim \Delta SRQ$.

$\therefore$ $\dfrac{TU}{SR} = \dfrac{QU}{QR} \Rightarrow \dfrac{y}{18} = \dfrac{x}{6}$

$\Rightarrow y = 3x$ ---------- (2)

from (1) and (2) $\dfrac{3x}{12} = \dfrac{6 - x}{6}$

$\Rightarrow 3x = 12 - 2x$

$x = \dfrac{12}{5}$. $\Rightarrow TU = y = \dfrac{36}{5}$.

$\therefore$ Area of $\Delta TQR = \dfrac{1}{2}(6)\left(\dfrac{36}{5}\right)$ cm² = 21.6 cm². Ans : (21.6)

8. Let AH = x, HG = 2x, GF = 3x
FE = 4x. EB = 5x
Let BC = h
Sum of the areas of p, q, r and s

$= \dfrac{1}{2}$(sum of the bases of p, q, r and s) BC

$= \dfrac{1}{2}(x + 2x + 3x + 4x)h = 5xh$ (where h = BC)

Area of the rectangle ABCD = (x + 2x + 3x + 4x + 5x)h = 15xh.
Required ratio = 3

Alternate method:

Area of $\Delta ABC = \dfrac{1}{2}$(Area of rectangle ABCD)

$\dfrac{\text{Area of } \Delta BCE}{\text{Area of ABCD}} = \dfrac{\dfrac{(BC)(BE)}{2}}{(BC)(AB)}$

$= \dfrac{BE}{2AB} = \dfrac{5}{2(15)} = \dfrac{1}{6}$

(From the given ratio)
$\therefore$ Sum of the areas of p, q, r and s

$= \left(\dfrac{1}{2} - \dfrac{1}{6}\right)$ (Area of rectangle ABCD)

$= \dfrac{1}{3}$ (Area of rectangle ABCD)

$\therefore$ Required ratio = 3 Ans : (3)

9. Let the radius of each circle be r cm.
$2\pi r = \pi r^2 \Rightarrow r = 2$
Area of the shaded region = Area of square ABCD – Area of 4 sectors in it = $(16 - 4\pi)$ sq.units. Choice (B)

10. Side of the innermost square = 4 cm.
Side of the nth outer square will be 2 cm more than the side of (n – 1)th outer square to it.
$\therefore$ Side of the 9th outer square to it = 4 + 8(2) = 20 cm
Side of the 10th outer square to it = 4 + 9(2) = 22 cm.
Required area = $22^2 - 20^2$ or 84 cm². Ans : (84)

11. Let the circum radius be R and side of the polygon be a then

$R = \dfrac{a}{2}\text{cosec}\,\dfrac{\pi}{n} \Rightarrow R = \dfrac{10}{2}\text{cosec}\,\dfrac{\pi}{5}$

$= 5 \times \text{cosec}\,36 = 5 \times 1.7 = 8.5$ cm Ans : (8.5)

12. Angle covered by the minute hand in 25 minutes
$= 25 \times 6 = 150°$

$\therefore$ Required area $= \dfrac{22}{7}(17.5)^2\left(\dfrac{150}{360}\right)$

$= 401^{1}/_{24}$ cm² Choice (A)

13. Length of the diagonal of the square = $5\sqrt{2}$ units.

$\therefore$ Radius of the circumcircle $= \dfrac{5}{\sqrt{2}}$ units

Radius of the incircle $= \dfrac{5}{2}$ units

If radius of the smaller circle is a, then $\dfrac{5}{2} + a + \sqrt{2}a = \dfrac{5}{\sqrt{2}}$

$\Rightarrow a = \dfrac{\dfrac{5}{\sqrt{2}} - \dfrac{5}{2}}{\sqrt{2} + 1}$

$\Rightarrow a = \dfrac{5}{2}\left(3 - 2\sqrt{2}\right)$

$\therefore$The required ratio

$\pi\left(\dfrac{5}{\sqrt{2}}\right)^2 : \pi\left(\dfrac{5}{2}\right)^2 : 4\pi\left[\dfrac{5}{2}\left(3 - 2\sqrt{2}\right)\right]^2 = \dfrac{1}{2} : \dfrac{1}{4} : \left(3 - 2\sqrt{2}\right)^2$

$= 2 : 1 : 4\,(3 - 2\sqrt{2})^2$ Choice (D)

14. As AB = 10 cm and the radius of the smaller circle is 5 cm, AB is the diameter of the smaller circle.

$\therefore$ $OA = OB = \dfrac{10}{\sqrt{2}}$ cm.

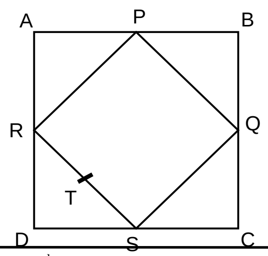
[$\because$ Δ AOB is a right angled isosceles triangle]
Area of the sector OAB

$= \dfrac{90°}{360°}\pi\left(5\sqrt{2}\right)^2 \text{ cm}^2 = \dfrac{25\pi}{2} \text{ cm}^2.$

Area of the shaded region

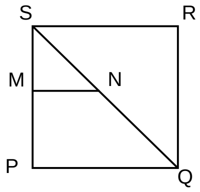
$= \left(25\pi - \dfrac{25\pi}{2}\right) \text{cm}^2 = \dfrac{25\pi}{2}$ cm² Choice (C)

15.



Let RT = 1. Area of ΔRTD = (1/2)(RT)(TD)
TS = 1 and SQ = 2, Area of ΔQTS = (1/2) (TS) (QS)
Required ratio = 2 : 1 Choice (D)

16.



Given that, PQRS is a square and M is the mid point of PS. Clearly N is the mid point of SQ, since N is the point of intersection of diagonals. $\Rightarrow \overline{MN} \parallel \overline{PQ}$.
Let the side of the square be 'a' units.
$\therefore$ PQ = a, PM = $\frac{a}{2}$ and MN = $\frac{a}{2}$.

(i) Area of the trapezium PMNQ = $\frac{1}{2}$(PM) (PQ + MN)

$= \frac{a}{4}\left(\frac{3a}{2}\right) = \frac{3a^2}{8}$.

(ii) Area of the square = a^2.

(iii) $\therefore$ Required percentage = $\frac{\left(\frac{3a^2}{8}\right)}{a^2} \times 100 = 37.5\%$.

Ans : (37.5)

17. Le the length and the breadth of the rectangle be ℓ cm and b cm respectively.
$(\ell + 3)(b + 3) = \ell b + 72 \rightarrow (1)$
$b(\ell + 1) = \ell b + 9$
b = 9
Substituting b in (1), $\ell = 12$ Ans : (12)

18. Let AB = ℓ, AD = b
$2(\ell + b) = 68$
$\Rightarrow \ell + b = 34$ --------- (1)
$\ell = b + 6$ --------- (2)
Solving (1) and (2), we get
l = 20, b = 14

Now AP = $\frac{1}{4}$(20) cm = 5 cm

AR = $\frac{2}{7}$(14) cm = 4 cm

$\therefore$ Area of rectangle APQR = 5(4) cm^2 = 20 cm^2
$\therefore$ Area of ΔQSR = ½ (Area of rectangle APQR)
$= \frac{1}{2}(20)\ cm^2 = 10\ cm^2$ Choice (A)

19. Let the side of the square be a units and that of the regular hexagon be x units
$4a = 6x \Rightarrow a = 3/2\ x$
Now, required ratio = $a^2 : \frac{3\sqrt{3}}{2}x^2$

$= \left(\frac{3}{2}x\right)^2 = \frac{3\sqrt{3}}{2}x^2 = 3 : 2\sqrt{3} = \sqrt{3} : 2$ Choice (D)

20. Side of the inscribed hexagon
= Radius of the circle = 10 cm
$\therefore$ Area of the inscribed hexagon = $\frac{3\sqrt{3}}{2}(10^2)$

$= \frac{3\sqrt{3}}{2}(100)\ cm^2 = 150\sqrt{3}\ cm^2$
Also, Radius of the circle = Distance between the parallel sides of the circumscribed hexagon = 2(10) = 20
$\Rightarrow \frac{\sqrt{3}}{2}$ a = 10, (a is the side of the circum hexagon)
$\therefore a = 20/\sqrt{3}$

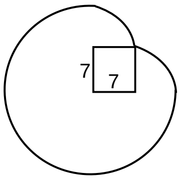
$\therefore$ Area of the circumscribed hexagon = $\frac{3\sqrt{3}}{2} \times \left(\frac{20}{\sqrt{3}}\right)^2$

$= \frac{3\sqrt{3}}{2}\left(\frac{400}{3}\right)$ cm^2 = 200 $\sqrt{3}$ cm^2

$\therefore$ Required difference = (200 $\sqrt{3}$ – 150 $\sqrt{3}$) cm^2

= 50 $\sqrt{3}$ cm^2 Choice (B)

21. Horse can cover 3 sectors of sector angles 270°, 90° and 90°.
(1) Area of the field of sector angle 270°



$= \frac{270°}{360°}\left(\frac{22}{7}\right)14^2 = 462$ m^2

(2) Combined area of the field of sector angles 90°

$= 2\left[\frac{90}{360}\frac{(22)}{7}(7)^2\right] = 77$ m^2.

$\therefore$ Required area = 462 + 77 = 539 m^2

Ans : (539)

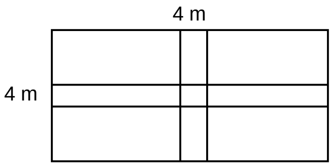
22. The figure is made up of trapeziums ABCD and ADEF
Area of the figure

$= \left\{\frac{1}{2}(BC + AD) \times BG\right\} + \left\{\frac{1}{2}(AD + EF)FH\right\}$

$= \left\{\frac{1}{2}(10 + 21)8\right\} + \left\{\frac{1}{2}(21 + 8)12\right\}$ cm^2

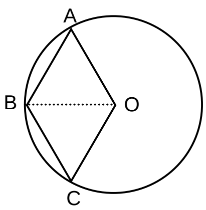
= 298 cm^2 Ans : (298)

23.



Total area = (4)(50) + (4)(20) –4(4) m^2
= 264 m^2 Choice (C)

24.



In the given figure, OA = AB = BC = OC = OB
Let the sides be ‘a’ units each.
Then, Area of the rhombus = 2 × (Area of the equilateral triangle OAB)

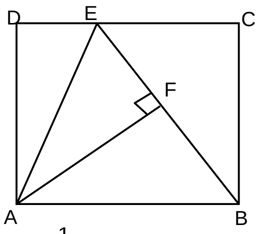
$= 2 \times \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3}a^2}{2}$

Now, $\frac{\sqrt{3}}{2}a^2 = 8\sqrt{3} \Rightarrow a = 4$

$\therefore$ Area of the circle = $\pi a^2 = 16\pi$ cm^2. Choice (D)

25. Perimeter of a sector (p) = l + 2r
$\therefore$ p = l + 2r = 64 $\Rightarrow$ l = 22 cm
$\therefore$ Area of the sector = ½ lr
= ½(22)(21) cm^2 = 11(21) cm^2 = 231 cm^2 Ans : (231)

26.

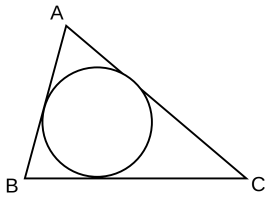


Area of Δ ABE = $\frac{1}{2}(BE)(AF)$

$= \frac{1}{2}(36)(25) = 450$

Area of the square ABCD = 2(450) = 900. Ans : (900)

27. AB = 5, BC = 6, CA = 7



$s = \frac{5+6+7}{2} = 9$

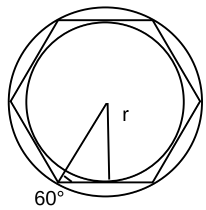
$\Delta = \sqrt{9(4)(3)(2)} = 6\sqrt{6}$

Inradius, $r = \frac{\Delta}{s} = \frac{6\sqrt{6}}{9}$

$= \frac{2\sqrt{6}}{3}$

Area of the circle = $\pi\left(\frac{4}{9}\right)6 = \frac{8\pi}{3}$ Choice (B)

28.

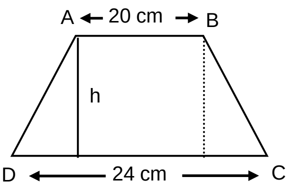


Let the radius of the smaller circle be 'r'.

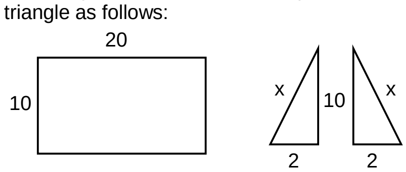
The radius of the bigger circle = $\frac{r}{\sin 60°} = \frac{2r}{\sqrt{3}}$

Choice (B)

29.

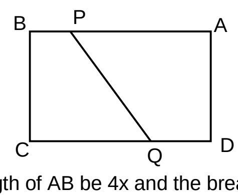


½ (20 + 24)h = 220 $\Rightarrow$ h = 10 cm
Now, trapezium ABCD can be split into a rectangle and a triangle as follows:



$x = \sqrt{100+4} = 2\sqrt{26}$ cm Choice (A)

30.



Let the length of AB be 4x and the breadth AD be 3x
Now 4x × 3x = 768 $\Rightarrow$ x = 8

$\therefore$ AB = CD = 4 × 8 = 32 cm
AD = BC = 3 × 8 = 24 cm
Now, PB = ¼ × 32 = 8 cm
Let CQ = y cm
½ (8 + y) × 24 = 288 ⇒ 8 + y = 24 ⇒ y = 16 cm
$\therefore$ CQ = 16 cm Ans : (16)

31. Volume of the gold in the pipe
$= \pi(7^2 - 6^2) \times 40 = 11440 = 1634^2/_7$ cu.cm
Weight of the gold = $\frac{11440}{7} \times 21$ = 34320 gm = 34.32 kg
Volume of bronze = $\pi \times 6^2 \times 40$
$= \frac{31680}{7} = 4525\frac{5}{7}$ cu.cm
$\therefore$ Weight of bronze = $\frac{31680}{7} \times 28$ = 126720 gm
= 126.72 kg
$\therefore$ Total weight = 34.32 + 126.72 kg = 161.04 kg
Ans : (161.04)

32. Volume of the water in the conical tank is equal to the volume of the water in the frustum.
$\Rightarrow \frac{1}{3}(\pi)(7)^2(h) = \frac{1}{3}\pi[6^2 + (6)(3) + 3^2]14.$
$\Rightarrow h = \frac{(36+18+9)(14)}{49}$ = 18 m. Ans : (18)

33. Let the radius and the slant height be r units and ℓ units respectively.
ℓ + r = 2 (Difference of 2r and ℓ)
If 2r = ℓ, ℓ + r = 0 which is not possible.
If 2r > ℓ, ℓ + r = 2(2r – ℓ)
ℓ = r which is not possible.
If 2r < ℓ, ℓ + r = 2(ℓ – 2r)
$\ell = 5r \Rightarrow \sqrt{r^2 + h^2} = 5r$
Squaring both sides, $h^2 = 24r^2 \Rightarrow h = 2\sqrt{6}r$ Choice (A)

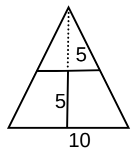
34. Let the edges of the cubes be 3x, 4x and 5x cm
Side of the new cube = $\frac{12\sqrt{3}}{\sqrt{3}}$ = 12 (Diagonal = $\sqrt{3}$ a)
$\therefore (3x)^3 + (4x)^3 + (5x)^3 = 12^3$
$\Rightarrow (27 + 64 + 125)\, x^3 = 1728 \Rightarrow x^3 = \frac{1728}{236} = 8 \therefore x = 2$
$\therefore$ Edge of the smallest cube will be 3 × 2 = 6 cm
Ans : (6)

35. Let radius = r cm and height = h cm
r + h = 21 and 2πr(r + h) = 924
$\therefore 2\left(\frac{22}{7}\right)(r)(21) = 924$
$\therefore r = \frac{924(7)}{2(22)(21)} = 7 \therefore h = 21 - 7 = 14$
$\therefore$ Volume = $\frac{22}{7}(7)(7)(14)$ cm³ = 2156 cm³ Choice (A)

36. Volume of the barrel = $\frac{22}{7}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(7) = 5^1/_2$ cu.cm
Number of words written using $5^1/_2$ cu.cm of ink is 2200.
$\therefore$ Number of words written with 200 ml (200 cu.cm)
i.e. 200 cu.cm = $\frac{2200}{11/2} \times 200$ = 80,000 Ans : (80000)

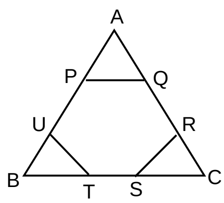
37. h = 3r
Total Surface Area = $2\pi r(r + h) = 8\pi r^2$
Cost of painting = $8\pi r^2 \times 4.25$
$\therefore 8\pi r^2 \times 4.25 = 1309$
$\therefore r^2 = \frac{1309 \times 7}{8 \times 22 \times 4.25} = 12.25$
$\therefore$ r = 3.5 cm $\therefore$ h = 3 × 3.5 = 10.5 cm
$\therefore$ Volume = $\frac{22}{7} \times (3.5)^2 \times 10.5$ = 404.25 cu.cm
Choice (D)

38. Volume of the frustum
$= \frac{\pi}{3}(10^2)(10) - \frac{\pi}{3}(5^2)(10) = \frac{875\pi}{3}$



Combined volume of the two cylinders
$= \pi\,(5^2)(5) + \pi\,(10^2)(5) = 625\pi$
Total volume = $\frac{875\pi}{3} + 625\pi$
$= \frac{2750\pi}{3}$ cubic cm Choice (D)

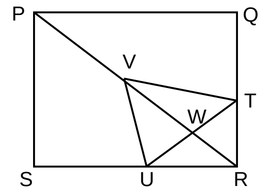
39.



In the given figure; PU = UT = PQ
But UT = BU and PQ = PA
$\therefore$ PA = PU = BU
$\therefore$ PU = 12/3 = 4 cm
$\therefore$ Area of hexagon PQRSTU = $\frac{3\sqrt{3}}{2} \times (4)^2$
$= 24\sqrt{3}$ cm² Choice (B)

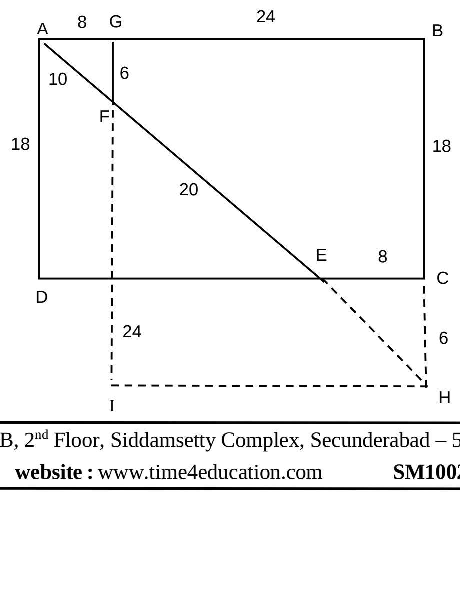
40. The area of the triangle formed by joining any three alternate vertices of a regular hexagon will always be half of that of the hexagon. As Q, S and U are alternate vertices of PQRSTU, required ratio is 2. Choice (C)

41.



As VT = VU, V lies on the diagonal PR.
As RV = 2PV, RV = (2/3) PR
RW = (1/4) PR $\therefore$ VW = $\left(\frac{2}{3} - \frac{1}{4}\right)PR = \frac{5}{12}PR$
$\frac{\text{Area of } \Delta \text{ RUT}}{\text{Area of } \Delta \text{ VUT}} = \frac{RW}{VW} = \frac{3/12}{5/12} = \frac{3}{5}$ Choice (D)

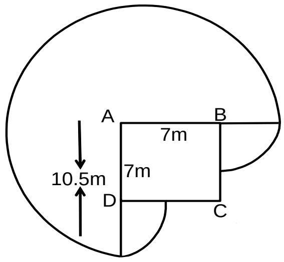
42. The given rectangle is shown in the figure below.



It can be re–arranged to form a square.
Δ ADE slides along AE, AE slides to FFI. Δ AGF can be cut out and placed on Δ ECH.
The perimeter of the square = 24 $\times$ 4 = 96 Choice (A)

43. Let us say there are x tiles along the length of the floor and y tiles along the breadth of the floor, excluding the tiles along the corners.
Number of blue tiles = 2x + 2y + 4
Number of green tiles = xy
xy = 2(2x + 2y + 4)
xy = 4x + 4y + 8
xy – 4x – 4y + 16 = 24
(x – 4) (y – 4) = 24
$x \geq y$
$\therefore$ x – 4 $\geq$ y – 4
Possibilities for (x – 4, y – 4) are (24, 1), (12, 2), (8, 3) and (6, 4).
$\therefore$ x is 28 or 16 or 12 or 10. Choice (A)

44.



Let us say the cow is tied at the corner denoted by A shown in the figure above. The area in which the cow can graze is

$= \frac{270°}{360°}\pi(10.5)^2 + \frac{2\,(90°)}{360}\pi(3.5)^2$

$= \frac{(\pi)}{2}(3.5)^2\left[\left(\frac{3}{2}\right)(9)+1\right] \approx \left(\frac{11}{7}\right)(2.5)(3.5)\left(\frac{29}{2}\right)$

= 279.125 Ans : (279.125)

45. As AC and CE are the diameters, the volume of the solid generated is the total volume of 2 spheres.
Volume of the solid generated

$= \frac{4}{3}\pi(5^3) + \frac{4}{3}\pi\left(\frac{5}{2}\right)^3$

$= \frac{4}{3}\pi\,(5^3)\left(1+\frac{1}{8}\right) = \frac{375\pi}{2}$ Choice (D)

Solutions for questions 46 to 50:

46. $2\pi r = 44 \Rightarrow r = 7$
Total surface area = $2\pi r\,(r + h)$
Statement I.
r + h = 21
Substituting r and r + h values in $2\pi r\,(r + h)$ we can find the total surface areas. Sufficient.
Statement II.
total surface area = curved surface area + $2\pi r^2$
As r = 7
Sufficient. Choice (B)

47. Statement I. We do not know the number of sides of the polygon, so we can't find the area of the polygon. Not sufficient.

Statement II. Each exterior angle is 60º, so the number of sides is $\frac{360}{60}$, viz 6.

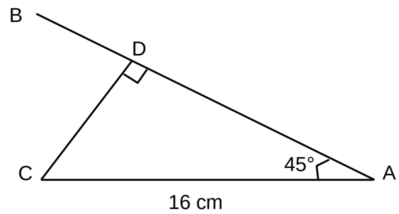
But we do not know the side of the polygon. Not sufficient.

Statements I, II. The length of longest diagonal in the hexagon is twice the length of side, so side of the hexagon is 10 cm area of the hexagon is $6\left(\frac{\sqrt{3}}{4}a^2\right)$.

Area of the hexagon = $6\left(\frac{\sqrt{3}}{4}\right)(10^2)$ cm^2.

Sufficient. Choice (C)

48. Statement I alone is not sufficient, as it gives no numerical data.
Statement II alone is not sufficient, as we don't know the information about the other side or any of the angles of the triangle.
From I and II, we have
$\angle$BAC = 45°, AC = 16 cm, BC = 12 cm.



Let D be the foot of the perpendicular from C to AB.
AB = AD + DB

$= CD + \sqrt{BC^2 - CD^2} = 8\sqrt{2} + \sqrt{12^2 - \left(8\sqrt{2}\right)^2}$

$= 8\sqrt{2+4}$

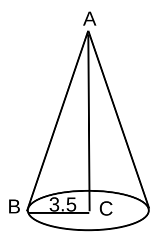
Area of ΔABC = $\frac{1}{2}$(AB)(CD)

CD = AC Sin 45° = $\frac{16}{\sqrt{2}} = 8\sqrt{2}$ cm ≃ 11.3 cm. Sufficient.

Choice (C)

49. Let the length, breadth, and the height of the cuboid be ℓ cm, b cm and h cm respectively.
We need to find $2(\ell b + \ell h + bh)$.
Statement I, $\ell^2 + b^2 + h^2 = 26^2$
From this we can't find the total surface area as we have one equation with three unknowns. Not sufficient.
Statement II, ℓ + b + h = 38
From this we can't find the total surface area as we have one equation with three unknowns. Not sufficient.
Statements I, II. $(\ell + b + h)^2 = 38^2$
$\Rightarrow \ell^2 + b^2 + h^2 + 2(\ell b + \ell h + bh) = 38^2 \Rightarrow 2\,(\ell b + \ell h + bh) = 38^2 - 26^2$. Sufficient. Choice (C)

50. Statement I. $\angle$BAC = 60°.
Hence the height of the cone
$= \frac{3.5}{\tan 60°}$ cm.
As we know the height and the radius of the base, we can find the volume of the cone. Sufficient.



Statement II. The ratio of the TSA (in cm^2) to the volume (in cm^3) is 1.5 : 1

Let ℓ cm and h cm be the slant height and the height of the cone respectively.
$\therefore \ell^2 - h^2 = (3.5)^2$ ……. (1)

Given, $\frac{\pi r\,(r + \ell)}{\frac{1}{3}\pi r^2 h} = \frac{3}{2}$

$2(r + \ell) = rh \Rightarrow 7 + 2\ell = 7/2\ h$ ……. (2)

Solving (1) and (2), we get ℓ and h.

Sufficient.

Choice (B)