CHAPTER – 7

TRIGONOMETRY

Trigonometry is a branch of mathematics that studies relationships involving lengths and angles of triangles. Trigonometry literally means the measuring (of angles and sides) of triangles.

Measurement of angles:

The angle between two lines can be measured in various systems. We consider two of these systems,
(a) Sexagesimal System and
(b) Circular System.

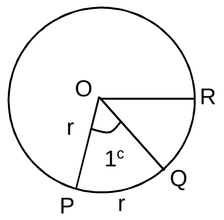
(a) Sexagesimal System:

A right angle is taken as the unit of measurement.
$^1/_{90}{}^{th}$ of a right angle is termed as a degree (1^o).
$^1/_{60}{}^{th}$ of a degree is termed as a minute (1').
$^1/_{60}{}^{th}$ of a minute is termed as a second (1'').

Basic Conversions:

60'' = 1'.
60' = 1°.
and 90° = 1 right angle.

(b) Circular System:



A radian (1^c) is taken as the basic unit of this measure.
A radian (1^c) is defined as the angle subtended at the centre of a circle by an arc whose length (ℓ) equals the radius (r).
i.e., $\ell = r$
$\therefore$ If arc PQ = r,
then $\angle POQ$ = 1 radian.

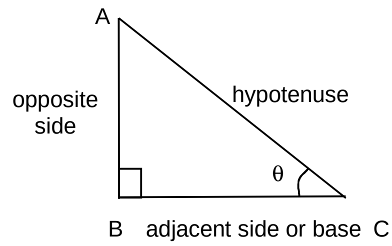
Note:

(i) Radian is a real constant.
(ii) The circular and sexagesimal measures are related by
π^c or π radians = 180°.
2π radians = 360°.
(iii) 1 revolution = 2π radians.

Important Conversions:

Sexagesimal Measure	Circular Measure
30°	$\pi/6$
45°	$\pi/4$
60°	$\pi/3$
90°	$\pi/2$
270°	$3\pi/2$

Given a right-angled triangle ABC, and $\angle ACB = \theta$ the trigonometric ratios are defined as follows:



$\sin\theta = \dfrac{opp}{hyp} = \dfrac{AB}{AC}$

$\cos\theta = \dfrac{adj}{hyp} = \dfrac{BC}{AC}$

$\tan\theta = \dfrac{opp}{adj} = \dfrac{AB}{BC}$

$\cot\theta = \dfrac{adj}{opp} = \dfrac{BC}{AB}$

$\sec\theta = \dfrac{hyp}{adj} = \dfrac{AC}{BC}$

$\text{cosec}\theta = \dfrac{hyp}{opp} = \dfrac{AC}{AB}$

From the ratios, we can easily observe the following relations:

(a) Reciprocal Relations:

$\text{cosec}\theta = \dfrac{1}{\sin\theta}$; $\sin\theta = \dfrac{1}{\text{cos ec}\,\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$; $\cos\theta = \dfrac{1}{\sec\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$

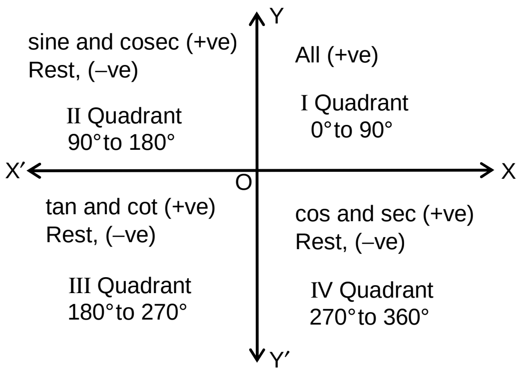
(b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

(c) $\sin^2\theta + \cos^2\theta = 1$
$\text{cosec}^2\theta - \cot^2\theta = 1$
$\sec^2\theta - \tan^2\theta = 1$

Signs of Trigonometric Ratios:

(i) If θ lies in the first quadrant ($0 < \theta < \pi/2$), all the trigonometric ratios are positive.
(ii) If θ lies in the second quadrant ($\pi/2 < \theta < \pi$), only sin θ and cosec θ are positive and the rest of the ratios are negative.
(iii) In the third quadrant ($\pi < \theta < 3\pi/2$), only tan θ and cot θ are positive and the rest of the ratios are negative.
(iv) In the fourth quadrant ($3\pi/2 < \theta < 2\pi$), only cos θ and sec θ are positive and the rest of the ratios are negative.

The signs of the trigonometric ratios can be remembered with the help of the diagram given below.



In the above graph angles in different quadrants and the signs of the trigonometric ratios are given. As a mnemonic, this can be remembered as All Silver Tea Cups

Angles greater than 90°:

To get the value of various ratios for angles greater than 90°, let us look at the absolute value and the sign of the ratio separately.

(i) Sign of the ratio:

Depending on which quadrant the angle falls in, the sign of the result of required ratio should be determined as per the "All Silver Tea Cups" rule above.

(ii) The absolute value of the ratio is to be arrived at as per the following table:

Ratio	when n is even	when n is odd
$\sin(n\pi/2 \pm \theta)$	$\sin\theta$	$\cos\theta$
$\cos(n\pi/2 \pm \theta)$	$\cos\theta$	$\sin\theta$
$\tan(n\pi/2 \pm \theta)$	$\tan\theta$	$\cot\theta$

Values of Trigonometric functions of some standard angles

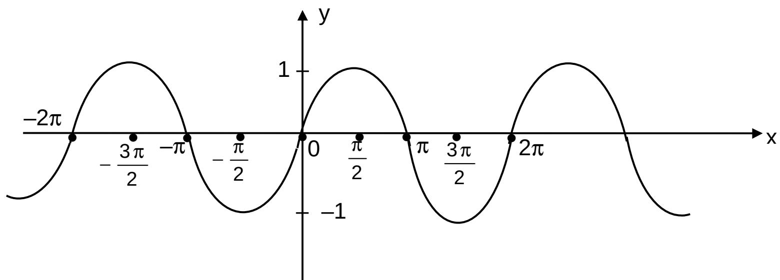
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	–1	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	–1	0	1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞	0

Maximum and Minimum values of Trigonometric Ratios:

(i) The sine and cosine of an angle can never be less than –1 and cannot be greater than +1.
(ii) The secant and cosecant of an angle will not have any value between –1 and +1.
(iii) The tangent and co-tangent of an angle can take any real value.
(iv) The minimum value of asinx + bcosx + c is $c-\sqrt{a^2+b^2}$
(v) The maximum value of asinx + bcosx + c is $c+\sqrt{a^2+b^2}$

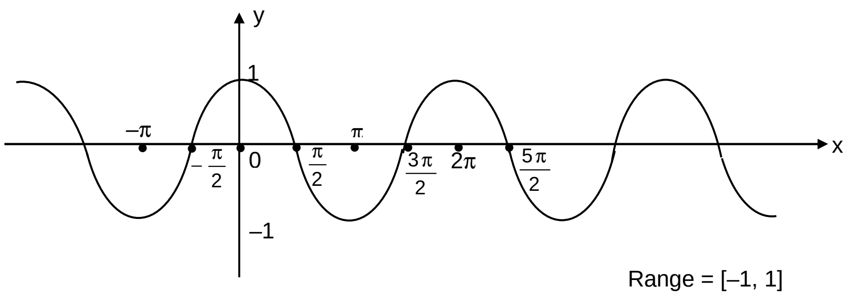
Graphs of sine, cosine and tangent functions

sine function: y = sin x



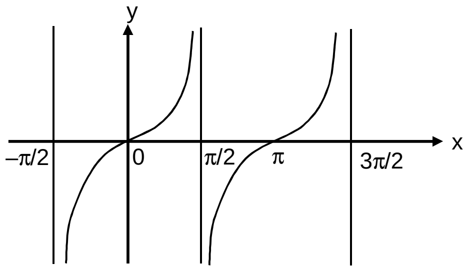
Range = [–1, 1]

cosine function: y = cos x



Range = [–1, 1]

tangent function: y = tan x



Range = R or (–∞, ∞)

Some important trigonometric identities to remember:

1. (a) $\sin(-\theta) = -\sin\theta$
   (b) $\cos(-\theta) = \cos\theta$
   (c) $\tan(-\theta) = -\tan\theta$

"T-ratios of compound angles"

Compound Angle: An angle obtained by the sum or difference of two or more angles is called a compound angle.

1. Addition and Subtraction formulae:

(i) $\sin(A + B) = \sin A\cos B + \cos A\sin B$
(ii) $\sin(A - B) = \sin A\cos B - \cos A\sin B$
(iii) $\cos(A + B) = \cos A\cos B - \sin A\sin B$
(iv) $\cos(A - B) = \cos A\cos B + \sin A\sin B$
(v) $\tan(A + B) = \dfrac{\tan A+\tan B}{1-\tan A\tan B}$
(vi) $\tan(A - B) = \dfrac{\tan A-\tan B}{1+\tan A\tan B}$
(vii) $\cot(A + B) = \dfrac{\cot A\cot B-1}{\cot A+\cot B}$
(viii) $\cot(A - B) = \dfrac{\cot A\cot B+1}{\cot B-\cot A}$
(ix) $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$
$= \cos^2 B - \cos^2 A$
(x) $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$
(xi) $\tan(A + B + C) = \dfrac{\sum\tan A-\pi\tan A}{1-\sum\tan A\tan B}$

2. Double angle formulae:

(i) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$
(iii) From (ii) above,
$\cos\theta = \sqrt{\dfrac{1+\cos 2\theta}{2}}$; $\sin\theta = \sqrt{\dfrac{1-\cos 2\theta}{2}}$
(iv) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
(v) $\cot 2\theta = \dfrac{\cot^2\theta-1}{2\cot\theta}$

3. Half - angle formulae:

Replacing θ by θ/2 in the formulae given in (2), we get the following formulae.

(i) $\sin\theta = 2\sin\theta/2\cos\theta/2$
$= \dfrac{2\tan\theta/2}{1+\tan^2\theta/2}$
(ii) $\cos\theta = \cos^2\theta/2 - \sin^2\theta/2$
$= 2\cos^2\theta/2 - 1$
$= 1 - 2\sin^2\theta/2 = \dfrac{1-\tan^2\theta/2}{1+\tan^2\theta/2}$
(iii) $\tan\theta = \dfrac{2\tan\theta/2}{1-\tan^2\theta/2}$
(iv) $\cot\theta = \dfrac{\cot^2\theta/2-1}{2\cot\theta/2}$

4. Triple angle formulae:

(i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
(ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
(iii) $\tan 3\theta = \dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$
(iv) $\cot 3\theta = \dfrac{\cot^3\theta-3\cot\theta}{3\cot^2\theta-1} = \dfrac{3\cot\theta-\cot^3\theta}{1-3\cot^2\theta}$

5. Formulae for changing the Product into Sum or Difference:

$2\sin A\cos B = \sin(A + B) + \sin(A - B)$
$2\cos A\sin B = \sin(A + B) - \sin(A - B)$
$2\cos A\cos B = \cos(A + B) + \cos(A - B)$
$2\sin A\sin B = \cos(A - B) - \cos(A + B)$

6. Formulae for changing the Sum or Difference into Products:

$\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

$\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{C-D}{2}\right)$

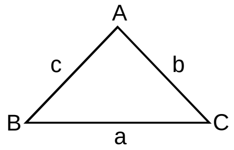
$\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right)\cos\left(\dfrac{C-D}{2}\right)$

$\cos C - \cos D = 2\sin\left(\dfrac{C+D}{2}\right)\sin\left(\dfrac{D-C}{2}\right)$

Other standard results

(i) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$
(R is the circumradius)

(ii)



Area of a triangle
ABC, (Δ) = (1/2) ab sinC
(where C is the angle included between the sides of the lengths a and b)
Alternatively Δ = 1/2 bc sinA and 1/2 ca sinB also give the same result.

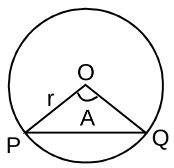
(iii) Cosine Rule:

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

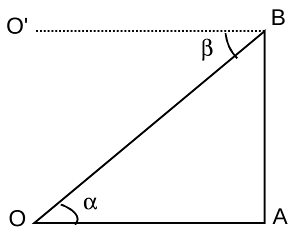
(iv)



O is the centre and PQ is a chord.

Chord PQ = $2r\sin\frac{A}{2}$, r = radius of the circle.

Heights and Distances



Let AB be a vertical line drawn from B to A to meet the horizontal line drawn from O (which is at a lower level than B) and let BO' be the horizontal line drawn through B (i.e., BO' is parallel to OA)

Then ∠AOB is called the angle of elevation of point B as seen from point O and ∠O' BO is the angle of depression of point O as seen from point B. It can be readily seen that $\alpha = \beta$ i.e.

Angle of Elevation = Angle of Depression (since AO is parallel to BO').

Using the trigonometric ratios on angles of elevation and depression, we can find out heights and distances as seen in the examples given in the later part of this chapter.

Examples:

7.01. Convert $\frac{7\pi}{12}$ radians into degrees.

Sol: π radians = 180°

$\therefore \frac{7\pi}{12} = \frac{7\pi}{12} \cdot \frac{180°}{\pi} = 105°$

7.02. If an athlete runs 1800 m around a circular path of circumference 250 m, then find the angle covered by the athlete in radians.

Sol: The athlete makes $\frac{1800}{250} = 7\frac{1}{5}$ rounds of the circular path.
1 round = 2π radians

$7\frac{1}{5}$ rounds = $7\frac{1}{5} \times 2\pi$ radians

= $\frac{36}{5} \times 2\pi$ radians = $\frac{72\pi}{5}$ radians.

7.03. If the angles of a triangle are in arithmetic progression with a common difference of 20° then express the angles in radian measure.

Sol: Let A, B, C denote the angles of the triangle.
So, A + B + C = 180°
The angles being in A.P., with a common difference of 20°, A = B – 20° and C = B + 20°
∴ B – 20° + B + B + 20° = 180°
⇒ 3B = 180°
⇒ B = 60°, A = 40° and C = 80°
∴ Angles in circular measure are $\frac{2\pi}{9}, \frac{\pi}{3}, \frac{4\pi}{9}$.

7.04. Find the angle covered by a minute hand in 54 minutes.

Sol: The angle covered by a minute hand in 60 minutes = 360°

∴ Angle covered in 54 minutes = $\frac{360° \times 54}{60}$

= 324° = $\frac{9\pi}{5}$ radians.

7.05. If $\tan^2\theta - 3 = 0$ and θ is in the III Quadrant, then find the values of $\sin\theta$ and $\sec\theta$.

Sol: $\tan^2\theta - 3 = 0$
$\Rightarrow \tan^2\theta = 3$
$\Rightarrow \tan\theta = \pm\sqrt{3}$

Since θ lies in the III quadrant $\tan\theta$ is positive, whereas, $\sin\theta$ and $\sec\theta$ are negative.

$\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{4} = -2$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{1}{4}} = -\frac{\sqrt{3}}{2}$.

7.06. Show that $\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}$ = 2cosecθ.

Sol: $\frac{\sin\theta}{1-\cos\theta}+\frac{\sin\theta}{1+\cos\theta}$

$=\frac{\sin\theta[1+\cos\theta+1-\cos\theta]}{(1-\cos^2\theta)}$

$=\frac{2\sin\theta}{\sin^2\theta}$ = 2cosecθ

[Using the identity $\cos^2\theta+\sin^2\theta=1$]

7.07. Prove that (cosecθ – cotθ) (cosecθ + cotθ) – sin2θ + $(\sin\theta+\cos\theta)^2 = 2$.

Sol: (cosecθ – cotθ) (cosecθ + cotθ) – sin2θ + $(\sin\theta+\cos\theta)^2$

= $(\text{cosec}^2\theta-\cot^2\theta)-\sin2\theta+\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=2$.

[$\because$ $\text{cosec}^2-\cot^2\theta=1$; $\sin^2\theta+\cos^2\theta=1$ and $2\sin\theta\cos\theta=\sin2\theta$]

7.08. If 6, 7, 8 represent the lengths of the sides of a triangle, then find the cosine of the greatest angle in the triangle.

Sol: Let a = 6, b = 7 and c = 8

The greatest angle of a triangle will be opposite to the greatest side of the triangle.

Here c is the greatest side

$\therefore$ $\angle C$ is the greatest angle.

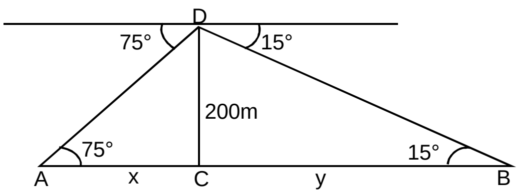
$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{36+49-64}{2(6)(7)}$

$=\frac{21}{2(6)(7)}=\frac{1}{4}$

7.09. From the top of a tower of height 200 m, the angles of depression of two points on opposite sides of the tower are observed to be 15° and 75°. Find the distance between the two points.

(Hint: tan15° = $2-\sqrt{3}$ and tan 75° = $2+\sqrt{3}$)

Sol:



Let CD be the height of the tower and A and B be two points on the ground. From ΔACD,

$\tan75° = \frac{200}{x}$

$\Rightarrow x=\frac{200}{2+\sqrt{3}}$ $\Rightarrow x=200\left(2-\sqrt{3}\right)$m

From ΔBCD, $\tan15° = \frac{200}{y}$

$\Rightarrow y=\frac{200}{2-\sqrt{3}}$

$\Rightarrow y = 200\left(2+\sqrt{3}\right)$m

Distance AB is x + y = $200\left[2-\sqrt{3}+2+\sqrt{3}\right]$ = 800 m.

7.10. Find $\sin^8\theta+\cos^8\theta$, when $\theta=\frac{\pi}{3}$.

Sol: $\sin^8\theta+\cos^8\theta=\sin^8\frac{\pi}{3}+\cos^8\frac{\pi}{3}$

$=\left(\frac{\sqrt{3}}{2}\right)^8+\left(\frac{1}{2}\right)^8=\frac{1}{2^8}[3^4+1]$

$=\frac{82}{256}=\frac{41}{128}$.

7.11. Find the value of cos (–1200°).

Sol: We know that cos (– θ) = cos θ.

$\Rightarrow$ cos (–1200°) = cos 1200°

= cos (3 $\times$ 360° + 120°)

= cos 120°

= cos (180° – 60°) = – cos 60° = –1/2

7.12. Find the value of $\cos^2 15°-\cos^2 75°$.

Sol: $\cos^2 15-\cos^2 75$

= sin (75° + 15°) sin (75° – 15°) = sin 90°.

$\sin 60°=\frac{\sqrt{3}}{2}$.

7.13. Find the value of $\frac{\sqrt{3}}{\sin20°}-\frac{1}{\cos20°}$.

Sol: $\frac{\sqrt{3}\cos20°-\sin20°}{\sin20°\cos20°}=\frac{2\left[\frac{\sqrt{3}}{2}\cos20°-\frac{1}{2}\sin20°\right]}{\frac{1}{2}\left(2\sin20°\cos20°\right)}$

$=\frac{4(\sin60°\cos20°-\cos60°\sin20°)}{\sin40°}$

$=4\left(\frac{\sin40°}{\sin40°}\right)=4$

7.14. If α and β are acute angles such that $\sin\alpha=\frac{4}{5}$ and $\cos\beta=\frac{5}{13}$, then find the value of $\sin\frac{\alpha-\beta}{2}$.

Sol: Consider $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$=\frac{3}{5}\cdot\frac{5}{13}+\frac{4}{5}\cdot\frac{12}{13}$ $\Rightarrow\cos(\alpha-\beta)=\frac{63}{65}$

We know that $\sin\left(\frac{\alpha-\beta}{2}\right)$

$=\sqrt{\frac{1-\cos(\alpha-\beta)}{2}}=\sqrt{\frac{1-\frac{63}{65}}{2}}$

$=\sqrt{\frac{2}{2(65)}}=\frac{1}{\sqrt{65}}$

Concept Review Questions

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. (i) The value of $\frac{6\pi}{5}$ in sexagesimal measure is _____.

(A) 144° (B) 216° (C) 240° (D) 120°

(ii) The value of 72° in circular measure is _____ .

(A) $\frac{2\pi}{5}$ (B) $\frac{4\pi}{5}$ (C) $\frac{3\pi}{5}$ (D) $\frac{6\pi}{5}$

2. (i) sin (270° - A) = ______.

(A) cos A (B) sin A (C) –cos A (D) –sin A

(ii) sin 750° = ______.

(A) 1 (B) –1 (C) 1/2 (D) –1/2

(iii) sin 25° + cos 115° = _______.

(A) 0 (B) 1 (C) –1 (D) $\frac{1}{\sqrt{2}}$

3. Given that sinθ is positive and cosθ is negative, then θ is in _____ quadrant.

(A) 1st (B) 2nd (C) 3rd (D) 4th

4. If θ is acute and cosecθ = $\frac{17}{8}$, then cotθ is _____.

(A) $\frac{8}{15}$ (B) $\frac{15}{17}$ (C) $\frac{15}{8}$ (D) $\frac{17}{15}$

5. sinθ · cosecθ = _____.

(A) 0 (B) 1 (C) tanθ (D) cotθ

6. The greatest angle in a triangle is opposite to the _____.

(A) smallest side (B) greatest side

(C) either (A) or (B) (D) Neither (A) nor (B)

7. If the lengths of the sides of a triangle are 1, $\sqrt{3}$ and 2, then the angles opposite to these sides are ______ respectively.

(A) 45°, 60°, 75° (B) 30°, 60°, 90°

(C) 30°, 45°, 105° (D) None of these

8. If 'θ' is an acute angle, and sin θ = cosθ, then θ = _______.

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

9. $\sec^4\theta + \tan^4\theta - 2\sec^2\theta\tan^2\theta$ = $\boxed{}$.

10. $\mathrm{cosec}^4\theta + \cot^4\theta - 2\,\mathrm{cosec}^2\theta\cot^2\theta$ = $\boxed{}$.

11. sin30° cos60° + cos30°.sin60° = _____.

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\sqrt{3}$

12. If tanα = cotβ = 1 and α, β are both acute angles, then find the value of 2α + β.

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

13. If the value of cosθ = $\frac{1}{\sqrt{2}}$ and θ is not acute, then the value of tanθ = _____.

(A) 1 (B) –1 (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{-1}{\sqrt{3}}$

14. $\sin^2 45° + \cos^2 45°$ = $\boxed{}$.

15. sin 30° + $\sqrt{3}$ tan 60° – sec 0° = _____.

(A) 0 (B) 1 (C) 2 (D) 5/2

16. If x = sinθ and y = cosθ and 0° < θ < 45°, then _____.

(A) x = y (B) x > y

(C) x < y (D) x ≥ y

17. If 3sinθ = 2 and θ is in II quadrant, then tan θ = ______.

(A) $\frac{-2}{\sqrt{5}}$ (B) $\frac{2}{\sqrt{5}}$ (C) $\frac{2}{5}$ (D) $\frac{-2}{5}$

18. If cosecθ = $-\sqrt{2}$ and tanθ = –1, then cosθ = ______.

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $-\sqrt{2}$ (D) $\frac{-1}{\sqrt{2}}$

19. Which of the following statements is true if cosθ < sinθ?

(A) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$ (B) $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$

(C) $0 < \theta < \frac{\pi}{3}$ (D) $0 < \theta < \frac{\pi}{4}$

20. Which of the following can be true?

(A) sinθ = π (B) cosθ = –π

(C) secθ = $\frac{1}{2}$ (D) cosecθ = –3

21. At how many points does the graph of the function y = sinx meet the x – axis for –π ≤ x < π?

(A) 1 (B) 2 (C) 3 (D) 0

22. At how many points does the graph of the function y = cos x meet the x – axis for – π ≤ x ≤ π?

$\boxed{}$

23. $\sec^2 60° - \tan^2 60°$ = _____.

(A) 1 (B) –1 (C) 2 (D) –2

24. $\text{cosec}^2 120° - \cot^2 120° = \boxed{}$.

25. If θ is acute and sec θ = $\frac{13}{5}$, then tan θ can be _____.

(A) $\frac{12}{5}$ but not $-\frac{12}{5}$

(B) $-\frac{12}{5}$ but not $\frac{12}{5}$

(C) either $\frac{12}{5}$ or $-\frac{12}{5}$

(D) neither $\frac{12}{5}$ nor $-\frac{12}{5}$

26. If tan θ = $\frac{4}{5}$, then θ lies in _____.

(A) Q_2 (B) Q_3
(C) Q_1 (D) Q_1 or Q_3

27. The range of 2 sin θ is _____.

(A) [0, 1] (B) [–2, 1]
(C) [–1, 1] (D) [–2, 2]

28. If cot θ + tan θ =2, then cos θ = _____.

(A) $\pm\frac{1}{\sqrt{3}}$ (B) $\pm\frac{1}{2}$

(C) $\pm\frac{1}{\sqrt{2}}$ (D) $\pm\frac{\sqrt{3}}{2}$

29. If cot θ = $\frac{-3}{4}$ and sin θ = $\frac{4}{5}$, then θ lies in _____.

(A) Q_1 (B) Q_2
(C) Q_3 (D) Q_4

30. $\sin^2 240° + \cos^2 240° = \boxed{}$.

Exercise – 7(a)

Directions for questions 1 to 30: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

1. If the sexagesimal measure of an angle is 225°, the n the angle expressed in the circular measure is _____.
(A) $\frac{3\pi^c}{4}$ (B) $\frac{2\pi^c}{3}$ (C) $\frac{5\pi^c}{4}$ (D) $\frac{17\pi^c}{8}$

2. The value of cos 28°+ cos 65°+ cos 115°+ cos 208 °+ cos 240°+ cos 300° is $\boxed{}$.

3. A clock was set at noon. If the clock stops functioning after the minute hand has swept an angle of $\frac{3\pi}{2}$ radians, then the time which the clock shows now is _____.
(A) 12:30 p.m. (B) 12:35 p.m.
(C) 12:45 p.m. (D) 12:55 p.m.

4. If 13 sinθ = 12 and θ is acute, then $\frac{\text{cosec }\theta + \cot\theta}{\tan\theta + \sec\theta}$ = _____.
(A) $\frac{3}{8}$ (B) $\frac{3}{10}$ (C) $\frac{10}{3}$ (D) $\frac{8}{3}$

5. A wheel makes 12 revolutions per hour. The number of radians it turns through in 20 minutes is _____.
(A) 8π (B) 16π (C) 24π (D) 32π

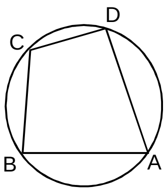
6. If $3\cos^2 A = \cos 60° + \sin^2 45°$, then $\sec^2 A$ = $\boxed{}$.

7. $3(\cos^2 45° + \sin^2 225°) - 3(\sin^2 225° + \cos^4 225°)$ = _____.
(A) 4/3 (B) 1/9 (C) 3/4 (D) 1/2

8. (1 + tanθ + secθ) (1 + secθ – tanθ) – 2secθ = $\boxed{}$.

9. If $\cos\theta + \cos^2\theta = 1$, then $\sin^2\theta + \sin^4\theta$ = $\boxed{}$.

10. In the figure shown below, if tan A + tan B + tan C = 5, then tan D = _____.
(A) 2
(B) 0
(C) 5
(D) –5



11. If secα and secβ are the roots of the equation $ax^2 + bx + c = 0$ and also given that α + β = 180°, then which of the following is true?
(A) b + a = 0 (B) c = 0
(C) b = 0 (D) a = b

12. (i) If secθ +tanθ = m, then cosθ = _____.
(A) $\frac{m}{m^2+1}$ (B) $\frac{m}{m^2-1}$
(C) $\frac{m^2+1}{m^2-1}$ (D) $\frac{2m}{m^2+1}$

(ii) If cosecθ – cotθ = p, then cosecθ + cotθ = _____.
(A) p (B) –p (C) $\frac{1}{p^2}$ (D) $\frac{1}{p}$

13. If the lengths of the sides of a triangle are 2, $3\sqrt{3}$ and 7, then the greatest angle (in degrees) of the triangle is $\boxed{}$.

14. If x = secθ and y = tanθ, then $\sqrt{\frac{x-1}{x+1}} - \sqrt{\frac{x+1}{x-1}}$ = _____.
(A) 2y (B) –2y (C) $\frac{-2}{y}$ (D) $\frac{2}{y}$

15. If sinθ + cosθ = $\sqrt{2}$, then find $\tan^n\theta + \cot^n\theta$. $\boxed{}$.

16. The value of sin 12°sin 48°sin 54°is equal to _____.
(Given 4 sinA sin(60 + A) sin (60 – A) = sin3A))
(A) 1/16 (B) 1/8 (C) 1/4 (D) 1/2

17. If (1 – cot 4°) (1 – cot 5°) (1 – cot 6°) ……… (1 – cot 41°) = 2^P, then the value of P is $\boxed{}$.

18. If 0 < θ < 45°, then find the value of $\sqrt{2-\sqrt{2-\sqrt{2-2\cos 2\theta}}}$.
(A) $\sqrt{2+\sqrt{2-2\cos\theta}}$ (B) $\sqrt{2-\sqrt{2-2\cos\theta}}$
(C) $\sqrt{2-\sqrt{2-2\sin\theta}}$ (D) $\sqrt{2+\sqrt{2-2\sin\theta}}$

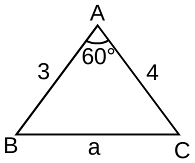
19. (i) The range of $\cos^2\theta + \sin^4\theta$ is
(A) $\left[\frac{3}{4}, 1\right]$ (B) $\left[\frac{1}{4}, \frac{3}{4}\right]$
(C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (D) $\left[-\frac{1}{2}, 1\right]$

(ii) The maximum value of $\sin^2 x - \cos 2x$ is
(A) 1 (B) 2 (C) 3 (D) –2

(iii) If f(x) = 3 sinx – 4 cos x, the minimum value of f(x) is _____.
(A) –5 (B) –3
(C) 0 (D) None of these

20. In Δ ABC, if AB = 5 units, BC = 3 units and ∠A = 30°, then find the value of sinC.
(A) 1 (B) 0 (C) $\frac{3}{5}$ (D) $\frac{5}{6}$

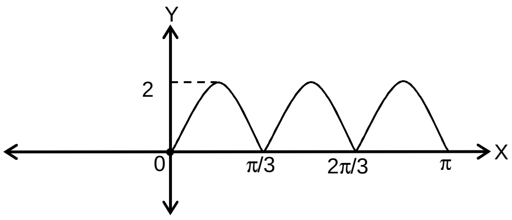
21. In the figure given below, a = _____.



(A) 6 units (B) $\sqrt{7}$ units

(C) $\sqrt{13}$ units (D) $\sqrt{17}$ units

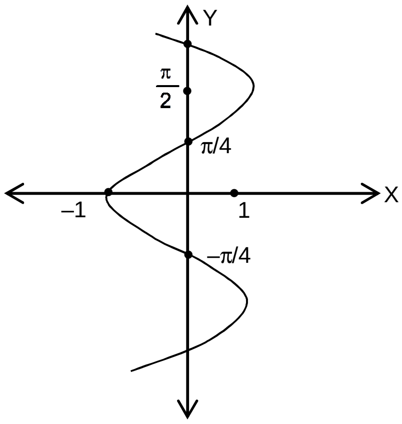
22. (i) The relation which best describes the graph given below is _____.



(A) $y = |\sin 3x|$, $x \ge 0$ (B) $x = |\sin 3y|$, $x \ge 0$

(C) $y = 3|\sin 2x|$, $x \ge 0$ (D) $y = 2|\sin 3x|$, $x \ge 0$

(ii) The relation which best describes the graph given below is ______.



(A) y = cos 2x (B) x = –cos 2y

(C) y = –sin 2x (D) x = sin 2y

23. A man was travelling towards a lighthouse in a boat. At a certain instant, the man observed the top of the lighthouse at an angle of elevation of 45^0. Five minutes later, he observed the top of the lighthouse at an angle of elevation of 60^0. If the boat travelled at an uniform speed of 5 m/min, the height of the lighthouse (in metres) is _____.

(A) $\frac{25(3+\sqrt{3})}{2}$ (B) $\frac{25(\sqrt{3}+1)}{2}$

(C) $25(3+\sqrt{3})$ (D) $25(\sqrt{3}+1)$

24. Ranvir stood at a point on the ground. The angles of elevation of the top and bottom of a flag post standing on the top of a building 300 m high at his eyes were 45°and 60°respectively. The height (in metres) of the flag post is $\boxed{}$.

25. The angles of elevation of an electric pole from two points A and B lying on the level ground on either side of the pole are 30° and 60° respectively. If the tw o points A and B are 500 m apart, then at what distance from point A is the electric pole?(in m)

$\boxed{}$

26. The angle of elevation of the top of a vertical tower at the eyes of a man is 30°. After travelling a distan ce of 1500 m towards the tower, he now finds the angle of elevation of the top of the tower to be 60°. The he ight of the tower is _____.

(A) 750 m (B) $750\sqrt{3}$ m

(C) $250\sqrt{3}$ m (D) 1250 m

27. AB is a vertical pole. The end A is on the level ground. C is the mid point of AB. P is a point on the level ground such that the portion BC subtends an angle θ at P. If AP = nAB, then the value of cotθ is _____.

(A) $\frac{2n^2+1}{n}$ (B) $\frac{n}{2n^2+1}$

(C) $\frac{2n^2+1}{2n}$ (D) $\frac{2n}{2n^2+1}$

28. From the top of a light house 30 m high, with its base at the sea level, the angle of depression of a boat is 75°. Find the distance of the boat from the foot of the light house. (in m)

$\boxed{}$

29. A plot was in the shape of a square. A flagstaff was erected at the centre of the square. The top of the flagstaff subtends an angle of 60^0 at each vertex of the plot. If the perimeter of the square is 240m, the length of the flagstaff is _____.

(A) $60\sqrt{6}$ m (B) $30\sqrt{6}$ m

(C) $20\sqrt{6}$ m (D) $40\sqrt{6}$ m

30. A man, standing at the foot of a hillock, observed the top of the hillock at an angle of elevation of 45°. There is a straight path towards the top. The path makes an angle of 30° with the horizontal. After covering a distance d along this path, the man sees the top of the hillock at an angle of elevation of 60°. If the height of the hillock is 40 m find the distance the man covers along the path.

(A) $20(\sqrt{3}+1)$m (B) $20(\sqrt{3}-1)$m

(C) $40(\sqrt{3}-1)$m (D) $40(\sqrt{3}+1)$m

Exercise – 7(b)

Directions for questions 1 to 45: For the Multiple Choice Questions, select the correct alternative from the given choices. For the Non-Multiple Choice Questions, write your answer in the box provided.

Very Easy / Easy

1. If a wheel makes 180 revolutions in a minute, then the angle made by the wheel in 5 seconds is ______.
(A) 15π (B) 30π (C) 10π (D) 20π

2. If $\theta = 30°$, then the value of $(\cos 2\theta)(\text{cosec}3\theta) - \sec 2\theta(\tan\theta)$ is ______.
(A) $\frac{\sqrt{3}-2}{\sqrt{3}}$ (B) $\frac{4-\sqrt{3}}{2\sqrt{3}}$ (C) $\frac{\sqrt{3}-4}{2\sqrt{3}}$ (D) $\frac{4-\sqrt{3}}{\sqrt{3}}$

3. In a right angled isosceles triangle ABC, sin A + sin B + sin C = ______.
(A) $\sqrt{2}+1$ (B) $\sqrt{2}-1$ (C) 0 (D) $\sqrt{2}$

4. The minimum value of $1+8\sin^2x^2\cos^2x^2$ is $\boxed{}$.

5. For all θ, if $\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$, then $\tan 22\frac{1}{2}^{\circ}$ = ______.
(A) $\sqrt{2}+1$ (B) $\sqrt{2}-1$
(C) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (D) $\frac{\sqrt{2}-1}{2\sqrt{2}}$

6. cos20° cos40° cos80° = ______.
(Given 4 cosA cos(60°+ A) cos(60°– A) = cos3A)
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$

7. For any right angled triangle ABC, the value of (log cosec A) × (log cosec B) × (log cosec C) = $\boxed{}$.

8. Two towers in a city are separated by a distance of 600m. At a point mid-way between them, the angles of elevations of the towers are 30° and 45°. Find t he ratio of their heights.
(A) $1:\sqrt{3}$ (B) $2:\sqrt{3}$ (C) 3 : 1 (D) 3 : 2

Moderate

9. If $\tan\theta + \cot\theta = 3$, then the value of $\sec^2\theta + \text{cosec}^2\theta$ is $\boxed{}$.

10. If $\cos\theta + \sec\theta + \cos^2\theta + \sec^2\theta = 0$, then $\tan\theta$ = $\boxed{}$.

11. If $\text{cosec}\theta$ and $\cot\theta$ are the roots of the equation $cx^2 + bx + a = 0$, then which of the following is true?
(A) $b^4 = 4ab^2c + c^4$ (B) $c^4 = 4ab^2c + b^4$
(C) $b^4 = 4ab^2c - c^4$ (D) $b^4 + c^4 = 4ab^2c$

12. If $\tan\theta = \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$, then $\tan\left(67\frac{1}{2}\right)^{\circ}$ = ______.
(A) $\sqrt{\sqrt{2}-1}$ (B) $\sqrt{2}-1$
(C) $\sqrt{\sqrt{2}+1}$ (D) $\sqrt{2}+1$

13. If $1 + \sec\theta + \tan\theta = p$, then $\cos\theta$ is ______.
(A) $\frac{p-1}{p^2-2p+2}$ (B) $\frac{2(p-1)}{p^2-2p+2}$
(C) $\frac{p-1}{p^2+2p+2}$ (D) $\frac{2(p-1)}{p^2-2p+1}$

14. If $3\tan\theta - 4 = 0$, and θ is acute, then the value of $\frac{3\sec\theta + 2\,\text{cosec}\,\theta}{\cot\theta - 5\sin\theta}$ is ______.
(A) $\frac{15}{13}$ (B) $\frac{13}{30}$ (C) $\frac{-30}{13}$ (D) $\frac{-15}{13}$

15. $\left[\frac{\tan x}{\sec x+1}+\frac{\sec x+1}{\tan x}\right](\sin x) = \boxed{}$.

16. If $\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}$ = k – sinx cosx, then k = $\boxed{}$.

17. $\left[\frac{\cot x}{\text{cosec}\,x+1}+\frac{\text{cosec}\,x+1}{\cot x}\right]\cos x = \boxed{}$.

18. $\sqrt{\frac{1-\sin x}{1+\sin x}}$ = ______.
(A) sec x – tan x (B) sec x + tan x
(C) 1 (D) $\sec^2x + \tan^2x$

19. If sinA = 1/3; tanB = 3/4; 0° < A < 90° and 180° < B < 270°, the value of $9\cos^2A + 20\sec B$ is $\boxed{}$.

20. If $\sin\theta + \text{cosec}\theta = 2$, then $\sin^4\theta + \cos^4\theta$ = ______.
(A) 0 (B) 2 (C) $\frac{1}{2}$ (D) 1

21. tan17°+ tan28°+ tan17°tan28°= $\boxed{}$.

22. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\sin^2\theta - \cos^2\theta$ = $\boxed{}$.

23. $\sin^6x + \cos^6x + 3\sin^2x\cos^2x$ = $\boxed{}$.

24. $\frac{\cos x}{1-\tan x}+\frac{\sin x}{1-\cot x}$ = ______.
(A) sinx + cosx (B) $\sin^2x - \cos^2x$
(C) $(\sin x + \cos x)^2$ (D) None of these

25. If two sides of a triangle are 8 units, and 10 units and the angle between these two sides is 45°, then find the area of the triangle in square units.
(A) $\frac{40}{\sqrt{2}}$ (B) $80\sqrt{2}$ (C) $40\sqrt{2}$ (D) 40

26. The range of the function $\dfrac{1}{\sqrt{29}\sin x+\sqrt{7}\cos x+4}$ is _____.

(A) $\left[-\dfrac{1}{2},\dfrac{1}{10}\right]$ (B) [–2, 10]

(C) $(-\infty, \infty)$ (D) $\left(-\infty,-\dfrac{1}{2}\right]\cup\left[\dfrac{1}{10},\infty\right)$

27. The minimum value of $4\tan^2x + 9\cot^2x$ is $\boxed{}$.

28. The minimum value of $81^{\cos^2 x} + 81^{\sin^2 x}$ is $\boxed{}$.

29. $\dfrac{\sin 2A\,(1-\cos 2A)}{(1+\cos 2A)\sin A}(\cos A) =$ _____.

(A) cosA (B) 2sinA (C) sin2A (D) $2\sin^2A$

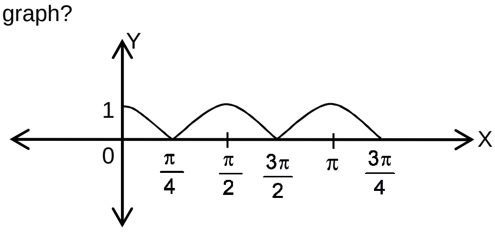
30. $a\cos\alpha + b\sin\alpha = c$, where $\alpha = \theta_1$ or θ_2 find $\sin\theta_1 + \sin\theta_2$ = _____.

(A) $\dfrac{2bc}{a^2+b^2}$ (B) $\dfrac{c^2-a^2}{a^2+b^2}$

(C) $\dfrac{2ac}{a^2+b^2}$ (D) None of these

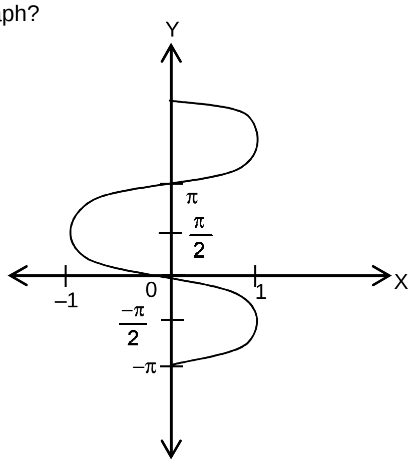
31. $\log_3(\sin^2 15° + \cos^2 75°) \times \log_5(\sin^2 30° + \cos^2 60°) \times \log_3(\sin^2 45° + \cos^2 45°) \times \log_5(\sin^2 60° + \cos^2 30°) \times \log_3(\sin^2 60° + \cos^2 30°) - \log_5(\sin^2 60° + \cos^2 60°)$ = $\boxed{}$.

32. Which of the following relations best describes the graph?



(A) x = cos2y (B) y = cos2x
(C) x = |cos2y| (D) y = |cos2x|

33. Which of the following relations best describes the graph?



(A) y = – sin x (B) y = sin x
(C) x = – sin y (D) x = sin y

34. The angles of depression of two ships due east of a lighthouse from the top of the light house are 37° and 45°. If the ships are 80 m apart, then the height of the lighthouse (in m) is $\boxed{}$ (Given sin 37°= 0·6)

35. Sujana observes an aeroplane flying at a, height of 1.5 km at an angle of elevation of 30°. After 20 seconds, the plane moves away from her and makes an angle of elevation of 15° from the same height. The speed of the plane (in kmph) is $\boxed{}$ (tan15°= 2− $\sqrt{3}$).

36. A ladder is placed against a wall such that it makes an angle of 75° with the ground surface. If the foo t of the ladder is at a distance of 3 m from the bottom of the wall, then the tip of the ladder is at a height of _____.

(A) 6 m (B) 4.5 m

(C) $3(2-\sqrt{3})$ m (D) $3(\sqrt{3}+2)$ m

37. Find the height of a chimney, when it is found that on walking 100 m towards it, in a horizontal line through its base, the angle of elevation of its top changes from 45° to 60°.

(A) $50\sqrt{3}$ m (B) $50(3+\sqrt{3})$ m

(C) $50(3-\sqrt{3})$ m (D) $100(3+\sqrt{3})$ m

38. The upper part of a pole, broken over by the wind, makes an angle of 45° with the ground. The distance from the foot of the pole to the point where the top of the pole touches the ground is $30(\sqrt{2}-1)$ m. What was the original height of the pole? (in m)

$\boxed{}$

39. A flagpost stands on the top of a tower. The angles of elevation of the top of the tower and the flag post from a point 180 m from the foot of the tower are 30° and 60° respectively. What is the height of the flagpost? (in m)

(A) $120\sqrt{3}$ (B) $40\sqrt{3}$ (C) $60\sqrt{3}$ (D) 60

40. EF and GH are two buildings. The height of GH is 120 m. From the top of the building EF, the angle of elevation of the top of the building GH is 30^0 and the angle of depression of the bottom of the building GH is 60^0. Find the height of EF (in m).

$\boxed{}$

41. A plot was in the shape of an equilateral triangle. A flagstaff was erected at its centroid. The top of the flagstaff was connected to each of the vertices of the plot by a rope. The angle of elevation of the top of the flagstaff at each vertex of the plot was 30^0. If the height of the flagstaff was 24 m, the side of the triangle (in m) is $\boxed{}$.

Difficult / Very Difficult

42. $6(\sin^6 x + \cos^6 x) - 9(\sin^4 x + \cos^4 x) = \boxed{}$.

43. The range of the function $\sin^2(120° - \theta) + \sin^2(120° + \theta)$ is _____.

(A) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(C) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (D) $\left[-\frac{1}{2}, \frac{3}{2}\right]$

44. The value of $\frac{\cos 20° + \sin 50°}{\sin 20° + \cos 50°}$ is _____.

(A) 3 (B) $\sqrt{3}$ (C) 1/3 (D) $1/\sqrt{3}$

45. $\sin^2(\theta - 45)° + \sin^2(\theta + 15)° - \sin^2(\theta - 15)° =$ ______.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 0

Key

Concept Review Questions

1. (i) B
   (ii) A
2. (i) C
   (ii) C
   (iii) A
3. B
4. C
5. B
6. B
7. B
8. C
9. 1
10. 1
11. A
12. A
13. B
14. 1
15. D
16. C
17. A
18. B
19. A
20. D
21. B
22. 2
23. A
24. 1
25. A
26. D
27. D
28. C
29. B
30. 1

Exercise – 7(a)

1. C
2. 0
3. C
4. B
5. A
6. 3
7. C
8. 2
9. 1
10. D
11. C
12. (i) D
    (ii) D
13. 150
14. C
15. 2
16. B
17. 19
18. C
19. (i) A
    (ii) B
    (iii) A
20. D
21. C
22. (i) D
    (ii) B
23. A
24. 219.6
25. 375
26. B
27. A
28. 8.04
29. B
30. C

Exercise – 7(b)

1. B
2. C
3. A
4. 1
5. B
6. D
7. 0
8. A
9. 9
10. 0
11. A
12. D
13. B
14. C
15. 2
16. 1
17. 2
18. A
19. –17
20. D
21. 1
22. 0
23. 1
24. A
25. A
26. D
27. 12
28. 18
29. D
30. A
31. 0
32. D
33. C
34. 240
35. 540
36. D
37. B
38. 30
39. A
40. 90
41. 72
42. –3
43. C
44. B
45. B