Design and Analysis of Algorithms

Week – 2 Assignment

Total Expected Duration: 2. 5 hours. You are expected to start doing the assignment today itself. Between 2 and 4:50 pm you can easily complete. Don't while away the time and start next week. You will have lot of catching up to do if you postpone and you will be crunched.

1. (15 mins) In the light of topics covered so far, the long and short of algorithm analysis is:

- Don't count the number of operations. It doesn't matter how many operations you execute within or outside the loop. The number of times you loop, that's what matters.
- For example, if T(n) = an + b, a represents the number of operations within the loop, b denotes the number of operations outside the loop, n denotes the number of times the loop executes. Since a and b are constants and ignored, T(n) = Θ(n). What survives for determining time complexity is number of times loop runs. i.e. n.

Determine the time complexity T(n) for the below scenarios based on this knowledge.

S.no.	Loop	T(n)	Justification, if any
1	for (i=1; i<=n; i++)		
2	for (i=1; i<=n; i=i+2)		
3	for (i=1; i<=n; i=i*2)		
4	for (i=1; i<=n; i++) for (j=1; j<=n; j++)		
5	for (i=1; i<=n; i++) for (j=1; j<=i; j++)		
6	for (i=1; i<=n; i++) for (j=1; j<=n; j=j*2)		

7	for (i=1; i<=n; i++)
	
	for (j=1; j<=n; j++)		

8	for (i=1; i<=n; i++)
	  for (j=1; j<=n; j++)
	
	for (k=1; k<=n; k++)		

9	for (i=1; i<=m; i++) for (j=1; j<=n; j++)		

10	for (i=1; i<=n; i++)
	  for (j=1; j<=n; j++)
	
	for (k=1; k<=m; k++)		

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2. (5 mins) Algorithm A uses 10nlogn operations, while algorithm B uses n^2 operations. Determine the value n_0 such that A is better than B for n >= n_0.

3. (10 mins) Order the following in the increasing order of growth. Group together (by underlining) those functions of same order.

   $6n\log n,\ 2^{100},\ \log\log n,\ \log^2 n,\ 2^{\log n},\ 4^n,\ \text{sqrt}(n),\ n^{0.01},\ 1/n,\ 4n^{3/2},\ 3n^{0.5},\ 5n,\ 2n\log^2 n,\ 2^n,\ n\log_4 n,\ 4^n,\ n^3,\ n^2\log n,\ 4^{\log n},\ \text{sqrt}(\log n)$.

4. (5 mins) Show that $10n^2 + 5n + 1 \in \Omega(n)$. In other words, find appropriate c and n_0.

5. (5 mins) Show that $10n^2 + 5n + 1 \in \Theta(n^2)$. In other words, find appropriate c and n_0.

6. (5 mins) Is $10n^2 + 5n + 1 \in \Theta(n^3)$. Justify your answer.

7. (10 mins) Show that $\log n! \in O(n^2)$.

8. (30 mins) Suppose that each row of an n x n array A consists of 1's and 0's such that, in any row of A, all the 1's come before any 0's in that row. Provide a
   a. $O(n^2)$ algorithm for finding the row of A that contains most 1's.
   b. O(n) algorithm for the same.

   Provide both the algorithms below.

$O(n^2)$ algorithm	O(n) algorithm

9. (60 mins) Learn about the following 3 quadratic sorting algorithms.
   a. Bubble sort
   b. Selection sort
   c. Insertion sort

   What is the time complexity of each?

10. (15 mins) Given the initial state of an array {65, 57, 12, 37, 23, 2, 10, 9}, show the state of the array
    a. After 2 rounds of bubble sort
    b. After 4 rounds of selection sort
    c. After 3 rounds of insertion sort