Semi-cylindrical obstacle in uniform airflow

Vo Anh Tue

January 2025

Abstract

When studying the velocity field of an ideal incompressible fluid around an obstacle, we pay special attention to symmetric configurations (sphere, cylinder, etc.). Although finding direct elementary solutions is not simple, by analogy with magnetostatics and electrostatics, we can obtain equivalent formal solutions. Through this analogy, the fluid problem can be fully analyzed mathematically, and the method can be extended to a large class of physical phenomena described by Laplace's equation.

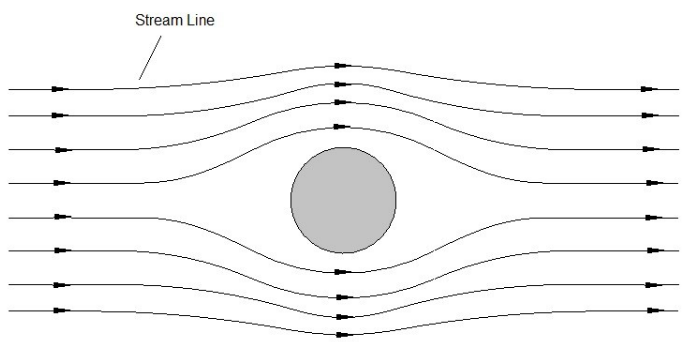

Figure Hình 1: A cylindrical obstacle in uniform airflow

Contents

1 Introduction

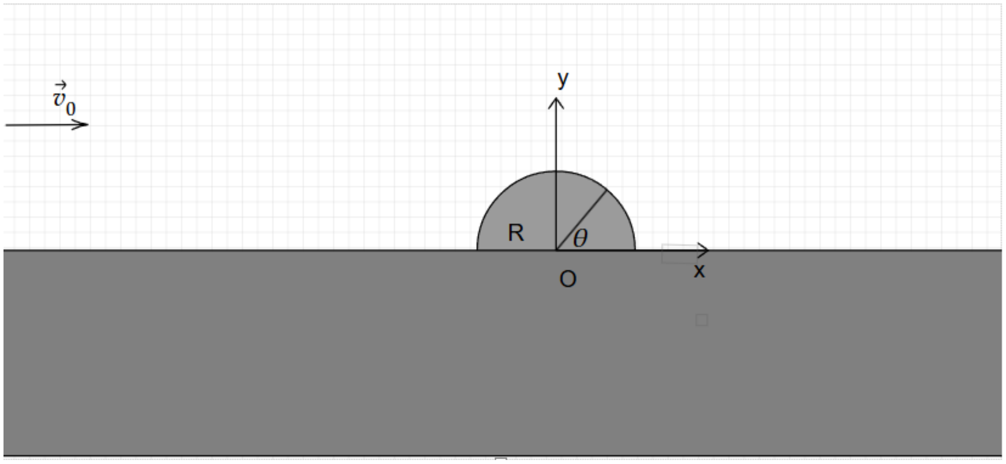

Figure Hình 2: Schematic of a long semi-cylindrical obstacle in uniform airflow

Problem Statement

Consider a building in the shape of a semi-cylinder of length L and radius R, resting on an infinite flat ground, such that the length is very large compared with the radius (See Hình 2). From infinity, a stream of wind blows toward it with velocity $\mathbf{v}_0$. Determine the wind velocity at every point in space once the flow has reached a steady state.

The crucial assumptions are that the airflow is incompressible and irrotational (with a very small vicosity coefficient). This means that the velocity field $\mathbf{v}$ satisfies the following equations, in region of $y \geq 0$:

$$\nabla \cdot \mathbf{v} = 0, \tag{1.1}$$

$$\nabla \times \mathbf{v} = 0. \tag{1.2}$$

And the boundary conditions are:

$$\begin{cases} \lim\limits_{r \to \infty} \vec{v}(r, \theta) = \vec{v}_0, \\ \vec{v}(R, \theta) \cdot \hat{r} = 0. \end{cases} \tag{1.3}$$

In the general case, equation (1.2) takes the form

$$\nabla \times \mathbf{v} = 2\mathbf{\Omega},$$

and one readily observes the analogy between the two Maxwell equations[1] in electrostatics and the two partial differential equations describing an ideal fluid flow.
Therefore, we shall first examine the analogous electrostatic problem using a simplified model presented in Section 2. We then return to the fluid-mechanics problem by systematically translating the corresponding concepts developed in electrostatics in Section 3. Finally, we conclude by solving the Laplace equations to verify the accuracy of the simplified models employed, while at the same time providing the most general comparison possible.

[1]

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}.$$

2 Long superconducting cylinder

3 Velocity superposition

4 Laplace's equation