

# Package ‘vrnmf’

August 16, 2021

**Title** Volume-Regularized Structured Matrix Factorization

**Version** 1.0.0

**Description** The package implements a set of routines to perform structured matrix factorization with minimum volume constraints.

**License** GPL-3

**Encoding** UTF-8

**Depends** R (>= 3.5.1)

**LazyData** true

**Imports** ica (>= 1.0), lpSolveAPI (>= 5.5.2.0), quadprog (>= 1.5),  
parallel (>= 3.5.1)

**Suggests** knitr (>= 1.28), rmarkdown (>= 2.1)

**VignetteBuilder** knitr

**RoxygenNote** 7.1.1

**URL** <https://github.com/kharchenkolab/vrnmf>

**BugReports** <https://github.com/kharchenkolab/vrnmf/issues>

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**NeedsCompilation** no

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AnchorFree	<i>Non-negative tri-factorization of co-occurrence matrix using minimum volume approach.</i>
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## Description

AnchorFree method tri-factorizes (co-occurrence) matrix in a product  $P C^* E^* t(C)$  of non-negative matrices  $C$  and  $E$  such that matrix  $E$  has minimum volume and columns of matrix  $C$  equal to 1.

## Usage

```
AnchorFree(
    vol,
    n.comp = 3,
    init = NULL,
    init.type = "diag",
    n.iter = 30,
    err.cut = 1e-30,
    verbose = FALSE
)
```

## Arguments

vol	An output object of vol_preprocess(). The method factorizes co-occurrence matrix vol\$P.
n.comp	An integer. Number of components to extract (by default 3). Defines number of columns in matrix $C$ . (default=3)
init	A numeric matrix. Initial matrix $M$ . (default=3)
init.type	A character. A strategy to randomly initialize matrix $M$ . (default="diag") Options are to <ol style="list-style-type: none"> <li>1) generate diagonal unit matrix ("diag"),</li> <li>2) use ICA solution as initialization ("ica", "ica.pos").</li> </ol> or sample entries from: <ol style="list-style-type: none"> <li>3) uniform distribution <math>[0, 1]</math> ("unif.pos"),</li> <li>4) uniform distribution <math>[-1, 1]</math>,</li> <li>5) uniform distribution <math>[0.9, 1.1]</math> ("similar"),</li> <li>6) normal distribution <math>N(0, 1)</math>.</li> </ol>
n.iter	An integer. Number of iterations. (default=30)
err.cut	A numeric. Relative error in determinant between iterations to stop algorithm (now is not used). (default=1e-30)
verbose	A boolean. Print per-iteration information (default=FALSE)

## Details

Implementation closely follows (Fu X *et al.*, IEEE Trans Pattern Anal Mach Intell., 2019).

**Value**

List of objects:

C, E Factorization matrices.

Pest Estimate of vol\$P co-occurrence matrix  $Pest = C * E * t(C)$ .

M, detM auxiliary matrix M and its determinant.

init.type type of initialization of matrix M that was used.

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factor_intensities	<i>Infer a matrix of non-negative intensities in NMF with offset/nmf-offset.</i>
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**Description**

factor\_intensities estimates a non-negative matrix D that optimizes the objective function  $F = ||X - C * D - offset||^2$ , where offset is either column-specific offset or a "1-rank nmf term": product of row vector and column vector

**Usage**

```
factor_intensities(
  C,
  X,
  fit.nmf = TRUE,
  fit.factor = FALSE,
  qp.exact = FALSE,
  n.iter = 200,
  qp.iter = 10,
  rel.error.cutoff = 1e-05,
  extrapolate = TRUE,
  extrapolate.const = TRUE,
  extrapolate.convex = FALSE,
  q.factor = 1,
  verbose = TRUE,
  n.cores = 1
)
```

**Arguments**

C	Numeric matrices.
X	Numeric matrices.

**Value**

Fitted matrix D.

---

infer_intensities	<i>Infer a matrix of non-negative intensities in NMF</i>
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### Description

infer\_intensities estimates a non-negative matrix  $D$  that optimizes the objective function  $F = ||X - C * D||^2$  using per-row quadratic programming.

### Usage

```
infer_intensities(C, X, esign = "pos", n.cores = 1)
```

### Arguments

C	Numeric matrices.
X	Numeric matrices.
esign	A character. Keep elements of matrix $D$ non-negative ("pos") or not ("all"). (default="pos")
n.cores	An integer. Number of cores to use. (default=1)

### Value

Fitted matrix  $D$ .

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projection_onto_simplex	<i>Project vector onto a probabilistic simplex.</i>
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### Description

projection\_onto\_simplex projects a vector unproj onto a probabilistic simplex of sum bound.

### Usage

```
projection_onto_simplex(unproj, bound)
```

### Arguments

unproj	A numeric vector. An unprojected vector
bound	A numeric. Sum of projected vector elements.

### Value

A projected vector.

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simnmf	<i>Simulated example of noiseless non-negative matrix factorization.</i>
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### Description

A dataset containing original, noisy and decomposition matrices. Created with the internal function 'sim\_factors()'

### Usage

```
simnmf
```

### Format

An object of class list of length 4.

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volnmf_det	<i>Update volume-regularized matrix R using det volume approximation</i>
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### Description

volnmf\_det finds matrix R that minimizes objective  $\|X - C \cdot R\|^2 + w.vol \cdot \det(R)$

### Usage

```
volnmf_det(
  C,
  X,
  R,
  posit = FALSE,
  w.vol = 0.1,
  eigen.cut = 1e-16,
  err.cut = 0.001,
  n.iter = 1000
)
```

### Arguments

C	Numeric Matrices. Matrices involved in objective function. Matrix R serves as initialization.
X	Numeric Matrices. Matrices involved in objective function. Matrix R serves as initialization.
R	Numeric Matrices. Matrices involved in objective function. Matrix R serves as initialization.
posit	A boolean. Set up (TRUE) or not (FALSE) non-negative constraints on matrix R. (default=TRUE)
w.vol	A numeric. Volume (det) weight in objective function. (default=0.1)
eigen.cut	A numeric. (default=1e-16)

err.cut	A numeric. Stop algorithm if relative error in R between iteration is less than err.cut. (default=1e-3)
n.iter	An integer. Number of iterations. (default=1e+3)

### Value

An updated matrix R.

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volnmf_estimate	<i>Alternating optimization of volume-regularized NMF</i>
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### Description

volnmf\_estimate provides alternating optimization of volume-regularized factorization of a matrix B using the following objective function:  $F = ||B * Q - C * R||^2 + w.vol * volume(R)$ . Matrix C is required to be non-negative and having either column or row vectors on the simplex. Matrix R can optionally have non-negativity constraint. Matrix Q can optionally be identity matrix or any unitary.

### Usage

```
volnmf_estimate(
  B,
  C,
  R,
  Q,
  domain = "covariance",
  volf = "logdet",
  R.majorate = FALSE,
  wvol = NULL,
  delta = 1e-08,
  n.iter = 10000,
  err.cut = 1e-08,
  vol.iter = 100,
  c.iter = 100,
  extrapolate = TRUE,
  accelerate = TRUE,
  acc.C = 4/5,
  acc.R = 3/4,
  C.constraint = "col",
  C.bound = 1,
  R.constraint = "pos",
  verbose = TRUE,
  record = 100,
  Canchor = NULL,
  Ctrue = NULL,
  mutation.run = FALSE
)
```

**Arguments**

B	A numeric matrix. A matrix to factorize (by default NULL). If not given than matrix B is taken to be a square root decomposition of $P = B * t(B)$ .
C	Numeric matrices. Initial matrices for optimization.
R	Numeric matrices. Initial matrices for optimization.
Q	Numeric matrices. Initial matrices for optimization.
domain	A character. Optimize unitary rotation matrix Q ("covariance") or keep it as identity matrix (as in standard NMF). By default "covariance".
vol.f	A character. Function that approximate volume. Can have values of "logdet" or "det" (by default "logdet").
R.majorate	A boolean. Majorate logdet each iteration of volnmf_logdet() (by default FALSE).
wvol	A numeric. A weight of volume-regularized term volume(R).
delta	A numeric. Logdet regularization term $\log(\det(R) + \text{delta})$ (by default 1e-8).
n.iter	An integer. Number of iterations (by default 1,000).
err.cut	A numeric. Relative error in determinant between iterations to stop algorithm (by default 1e-8).
vol.iter	An integer. Number of iterations to update volume-regularized matrix R at each alternating step.
c.iter	An integer. Number of iterations to update simplex matrix C at each alternating step.
extrapolate	A numeric. Do Nesterov extrapolation inside blocks of R and C optimization (by default TRUE).
accelerate	A numeric. Do acceleration each update after R and C blocks estimated via Nesterov-like extrapolation.
acc.C	A numeric. Acceleration parameter of matrix C.
acc.R	A numeric. Acceleration parameter of matrix R.
C.constraint	A character. Constraint either sum of columns ("col") or sum of rows ("row") to be equal to C.bound (By default "col").
C.bound	A numeric. A simplex constraint on matrix C vectors.
R.constraint	A character. Set up non-negativity ("pos") constraint on elements of R (by default "pos", alternative "no").
verbose	A boolean. Print per-iteration information (by default FALSE)
record	A numeric. Record parameters every 'record' iterations (by default NULL).

**Value**

List of objects:

C, R, Q, E Factorization matrices.

iter, err Number of iterations and relative per-iteration error err in matrix C.

info.record a list of objects that record and store state of matrices each record iterations.

---

volnmf_logdet	<i>Update volume-regularized matrix R using logdet volume approximation.</i>
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---

## Description

volnmf\_logdet finds matrix R that minimizes objective  $\|X - C \cdot R\|^2 + w.vol \cdot \log(\det(R) + \delta)$ .

## Usage

```
volnmf_logdet(
  C,
  X,
  R,
  R.constraint = "pos",
  majorate = FALSE,
  extrapolate = TRUE,
  qmax = 100,
  w.vol = 0.1,
  delta = 1,
  err.cut = 0.001,
  n.iter = 1000
)
```

## Arguments

C	Numeric Matrices. Matrices involved in objective function. Matrix R serves as initialization.
X	Numeric Matrices. Matrices involved in objective function. Matrix R serves as initialization.
R	Numeric Matrices. Matrices involved in objective function. Matrix R serves as initialization.
R.constraint	A character. Set up ('pos') or not ('no') non-negative constraints on matrix R (by default 'pos').
majorate	A boolean. Majorate logdet each iteration (by default FALSE).
extrapolate	A boolean. Use Nesterov acceleration (by default FALSE, currently is not supported).
w.vol	A numeric. Volume (logdet) weight in objective function.
delta	A numeric. Determinant pseudocount in objective function.
err.cut	A numeric. Stop algorithm if relative error in R between iteration is less than err.cut.
n.iter	An integer. Number of iterations.

## Value

An updated matrix R.



volnmf\_main

*Volume-regularized NMF***Description**

volnmf\_main enables volume-regularized factorization of a matrix B using the following objective function:  $F = ||B * Q - C * R||^2 + w.vol * volume(R)$ . Matrix C is required to be non-negative and having either column or row vectors on the simplex. Matrix R can optionally have non-negativity constraint. Matrix Q can optionally be identity matrix or any unitary. The latter option is used to decompose co-occurrence matrix vol\_P.

**Usage**

```
volnmf_main(
    vol,
    B = NULL,
    volnmf = NULL,
    n.comp = 3,
    n.reduce = n.comp,
    do.nmf = TRUE,
    iter.nmf = 100,
    seed = NULL,
    domain = "covariance",
    volf = "logdet",
    wvol = NULL,
    delta = 1e-08,
    n.iter = 500,
    err.cut = 1e-16,
    vol.iter = 20,
    c.iter = 20,
    extrapolate = TRUE,
    accelerate = FALSE,
    acc.C = 4/5,
    acc.R = 3/4,
    C.constraint = "col",
    C.bound = 1,
    R.constraint = "pos",
    R.majorate = FALSE,
    C.init = NULL,
    R.init = NULL,
    Q.init = NULL,
    anchor = NULL,
    Ctrue = NULL,
    verbose = TRUE,
    record = 100,
    verbose.nmf = FALSE,
    record.nmf = NULL,
    mutation.run = FALSE
)
```

**Arguments**

vol	An output object of vol_preprocess().
B	A numeric matrix. A matrix to factorize (by default NULL). If not given than matrix B is taken to be a square root decomposition of $P = B * t(B)$ .
volnmf	An output object of volnmf.main. An option is useful to re-estimate solution using different parameters (by default NULL).
n.comp	An integer. Number of components to extract (by default 3). Defines number of columns in matrix $C$ .
n.reduce	An integer. Dimensional reduction of matrix B (number of columns) if taken as a square root decomposition of volP (by default equal to n.comp).
do.nmf	A boolean. Estimate standard solution with w.vol=0 as initialization before applying volume regularization (by default TRUE).
iter.nmf	An integer. Number of iterations to get solution with w.vol=0 if the former requested (by default 1,000).
seed	An integer. Fix seed.
domain	A character. Optimize unitary rotation matrix Q ("covariance") or keep it as identity matrix (as in standard NMF). By default "covariance".
volf	A character. Function that approximate volume. Can have values of "logdet" or "det" (by default "logdet").
wvol	A numeric. A weight of volume-regularized term volume(R).
delta	A numeric. Logdet regularization term $\log(\det(R) + \delta)$ (by default 1e-8).
n.iter	An integer. Number of iterations (by default 1,000).
err.cut	A numeric. Relative error in determinant between iterations to stop algorithm (by default 1e-8).
vol.iter	An integer. Number of iterations to update volume-regularized matrix R at each alternating step.
c.iter	An integer. Number of iterations to update simplex matrix C at each alternating step.
extrapolate	A numeric. Do Nesterov extrapolation inside blocks of R and C optimization (by default TRUE).
accelerate	A numeric. Do acceleration each update after R and C blocks estimated via Nesterov-like extrapolation.
acc.C	A numeric. Acceleration parameter of matrix C.
acc.R	A numeric. Acceleration parameter of matrix R.
C.constraint	A character. Constraint either sum of columns ("col") or sum of rows ("row") to be equal to C.bound (By default "col").
C.bound	A numeric. A simplex constraint on matrix C vectors.
R.constraint	A character. Set up non-negativity ("pos") constraint on elements of R (by default "pos", alternative "no").
R.majorate	A boolean. Majorate logdet each iteration of volnmf_logdet() (by default FALSE).
C.init	Numeric matrices. Initialization of matrices C, R, Q (by default NULL).
R.init	Numeric matrices. Initialization of matrices C, R, Q (by default NULL).
Q.init	Numeric matrices. Initialization of matrices C, R, Q (by default NULL).

anchor	An output object of AnchorFree(). Object is used optionally to initialize matrices (by default NULL).
verbose	A boolean. Print per-iteration information (by default FALSE)
record	A numeric. Record parameters every 'record' iterations (by default NULL).

**Value**

List of objects:

C,R,Q Factorization matrices.

C.init,R.init,Q.init Initialization matrices for volume-regularized optimization.

C.rand,R.rand,Q.rand Random initialization matrices for NMF optimization (w.vol=0).

rec a list of objects that record and store state of matrices each record iterations.

---

volnmf_procrustes	<i>Procrustes algorithm estimates orthonormal transformation between two matrices.</i>
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---

**Description**

volnmf\_procrustes finds orthonormal matrix Q that minimizes objective  $||A-B*Q||^2$

**Usage**

```
volnmf_procrustes(A, B)
```

**Arguments**

A	Numeric Matrices. Orthonormal transformation convert matrix B in matrix A.
B	Numeric Matrices. Orthonormal transformation convert matrix B in matrix A.

**Value**

An optimal orthonormal transformation matrix Q.

---

volnmf_simplex_col	<i>Update of a matrix in NMF with equality constraints on columns.</i>
--------------------	--

---

**Description**

volnmf\_simplex\_col finds non-negative matrix C that minimizes the objective  $||X-C*R||^2$  under constraints that columns of C equal to 1 using local approximation with extrapolation.

**Usage**

```
volnmf_simplex_col(
  X,
  R,
  C.prev = NULL,
  bound = 1,
  extrapolate = TRUE,
  err.cut = 1e-10,
  n.iter = 10000,
  qmax = 100
)
```

**Arguments**

X	Numeric Matrices. Matrices involved in the objective function.
R	Numeric Matrices. Matrices involved in the objective function.
C.prev	Numeric Matrices. Matrices involved in the objective function. Matrix C.prev serves as initialization. (default=NULL)
bound	A numeric. Equality constraint on columns of matrix C. (default=1)
extrapolate	A boolean. Use extrapolation after local approximation. (default=TRUE)
err.cut	A numeric. Stop iterations if relative error between iterations is less than err.cut (parameter is not active now). (default=1e-10)
n.iter	An integer. Number of iterations. (default=1000)

**Value**

An updated matrix C.

---

volnmf_simplex_row	<i>Update of a matrix in NMF with equality constraints on rows.</i>
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---

**Description**

volnmf\_simplex\_row finds non-negative matrix C that minimizes the objective  $\|X - C \cdot R\|^2$  under constraints that rows of C equal to 1 using per-row quadratic programming.

**Usage**

```
volnmf_simplex_row(X, R, C.prev = NULL, meq = 1)
```

**Arguments**

X	Numeric Matrices. Matrices involved in the objective function.
R	Numeric Matrices. Matrices involved in the objective function.
C.prev	Numeric Matrices. Matrices involved in the objective function. Matrix C.prev serves as initialization. (default=NULL)
meq	An integer 0 or 1. Require equality (meq=1) or inequality (meq=0) constraint on rows (by default 1).

**Value**

An updated matrix C.

---

vol_preprocess	<i>Preprocess the data for downstream volume analysis.</i>
----------------	--

---

**Description**

vol\_preprocess Routine normalizes the data (as requested), estimates covariance and SVD decomposition.

**Usage**

```
vol_preprocess(X, col.norm = "sd", row.norm = NULL, pfactor = NULL)
```

**Arguments**

X	A numeric matrix. Covariance is estimated for column vectors of X.
col.norm	A character. Specifies column normalization strategy (by default "sd"). NULL to avoid normalization.
row.norm	A character. Specifies row normalization strategy (by default NULL).
pfactor	A numeric A factor to normalize co-occurrence matrix (by default NULL). Row normalization follows column normalization. NULL to avoid normalization.

**Value**

A list of objects that include normalized matrix X.process, row and column normalization factors row.factors and col.factors, covariance matrix P0, covariance matrix P normalized to maximum value pfactor, orthonormal basis U and vector of eigenvalues eigens.

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