Problem Set 2

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Due at start of lecture (11 am) on Oct 21, 2019. Covers material from lectures 3–5.

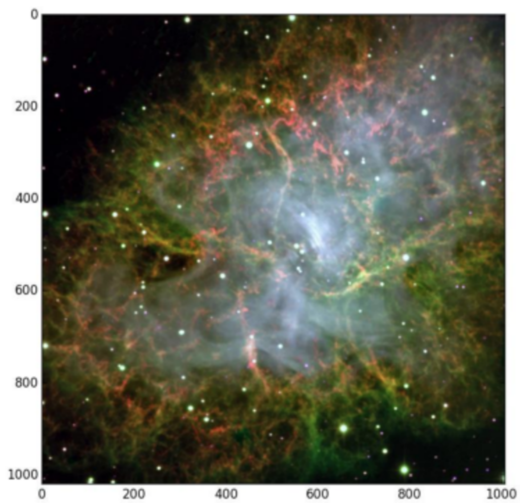

Figure 1: An image of the Crab nebula (M1) obtained with the CHIMERA-2 instrument on the 200-inch Hale telescope of the Palomar Observatory (Harding et al. 2016). One of the central stars is the Crab pulsar.

Question 1. The Crab pulsar in the Crab nebula (Figure 1) produces a 3-ms long pulse of light during every 33-ms rotation as its beam of radiation sweeps by the Earth.[1] Averaged over many rotations, the V-band[2] magnitude of the pulsar is 16.5, and the pulsar appears as a point to all telescopes at all wavelengths. The nebula itself has a V-band magnitude of 8.4, and its

[1] It actually produces two pulses, but you can ignore the second one!

[2] The V-band represents a standard optical filter.

structure can be approximated as an evenly filled circle that is 5 arcmin in diameter. This problem will explore the detectability of a single pulse of radiation from the Crab pulsar with two different kinds of telescopes. *Note that if you're careful to show your working, we'll be more easily able to hand out some points for wrong answers!*

(a) (3 points) Consider the 200-inch Hale telescope and its CHIMERA-2 detector. The telescope mirror has a diameter of $200\,\mathrm{in} \equiv 5\,\mathrm{m}$, and the total efficiency of the atmosphere + telescope + detector is 0.3. CHIMERA-2 is mounted at the prime focus of the telescope, and has a plate scale of 0.28 arcsec per pixel. Turbulence in the atmosphere typically blurs any unresolved source to a diameter of 1 arcsec.

- The V-band is centered on 5500 Angstrom, and is 1000 Angstrom wide. Approximately how many photons per second from the Crab pulsar reach the CHIMERA-2 detector?
- On a moonless night, the sky brightness at Palomar is 20.5 magnitudes per square arcsec in the V band. Approximately how many photons per second from the sky and from the Crab nebula will reach the detector, in the same solid angle as the 1-arcsec apparent size of the pulsar?
- The rate of noise photons produced in each pixel is $1\,\mathrm{s}^{-1}$. Assuming that CHIMERA-2 is operated with the optimal 3-ms integration time (the timescale over which photons are integrated in each pixel before being instantaneously read out), what is the average signal to noise ratio of a single pulse from the Crab pulsar?

(b) (3 points) Let's now turn to the proposed PANOSETI instrument. It consists of several single-pixel photon counting detectors, each of which observes $0.1\,\mathrm{deg}^2$ of the sky using lenses that are 0.3 m in diameter. Assume that the efficiency of the full system, including the atmosphere, is again 0.3.

- Again in the V band, approximately how many photons per second from the Crab pulsar will reach a PANOSETI detector?
- Assuming a moonless night as before, approximately how many photons per second from the sky and from the Crab nebula will reach a PANOSETI detector?

- There are no noise photons produced in the PANOSETI detectors. Assuming that the instrument is operated with the optimal 3-ms integration time (the timescale over which photons are integrated in each detector before being instantaneously read out), what is the average signal to noise ratio of a single pulse from the Crab pulsar?

Question 2. (4 points) This came out of a discussion at a recent Monday theory-observer pizza lunch in the astronomy department. A fundamental problem in planet formation is how cm-sized pebbles grow to become km-sized planetesimals in protoplanetary disks. A possible solution is that planetesimal formation is seeded by the arrival of 100 m-sized Bodies Expelled From Other Planetary Systems (BEFOPSs). Although this doesn't solve the chicken and egg problem, it makes it exponentially easier because planetesimal formation can be rare.

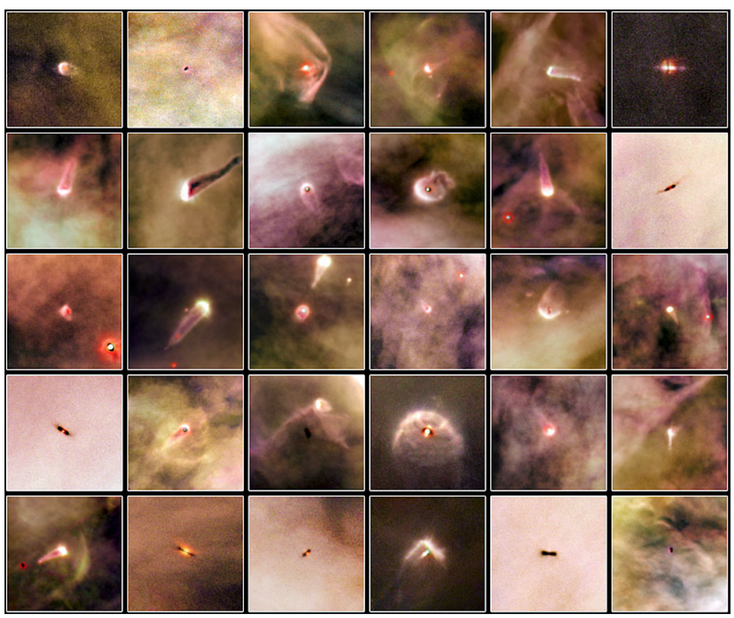

Figure 2: Protoplanetary disks in the Orion nebula imaged with the Hubble Space Telescope (NASA/ESA/STScI).

(a) The rate of 100 m-sized BEFOPSs arriving at the solar system is thought to be one per year. Assume that they typically travel at $50\,\mathrm{km\,s^{-1}}$. Approximately how many such bodies are there within the Galactic[3] disk? You can assume that the Milky Way disk can be approximated as a squashed cylinder that is 30 kpc in diameter, and 2 kpc in height.

(b) If all 100 m-sized BEFOPSs have the same mass density as the Earth, which has a radius of 6.4×10^8 cm and a mass of 6×10^{27} g, what is their contribution to the mass of the Milky Way? How does your estimate compare with the total mass of the Milky Way, thought to be $\sim 10^{12} M_\odot$?

(c) Let's assume that each 100 m-sized BEFOPS radiates as a 50 K blackbody, with an absolute magnitude of 53 (!!) at a wavelength of 60 μm. What is the total contribution to the 60-μm sky brightness (in units of magnitudes per square arcsec) from all 100 m-sized BEFOPSs in the Milky Way, towards the Galactic Center? You can assume that the Sun is 8 kpc from the Galactic Center.

[3] A capitalized "Galaxy" always refers to the Milky Way.