Ay 20 # 16 - Dynamics: binaries and interactions

More than half the stars in the Milky Way are in binaries, which provide unique opportunities to measure stellar properties.

Visual / astrometric binaries

Observe oscillatory motions of one or both stars in the plane of the sky.

In a COM reference frame,

$$\overset{\circ}{m_1} \xleftarrow{\quad \underline{r}_1 \quad} \underset{= m_1 + m_2}{\overset{\bullet}{M}} \xrightarrow{\qquad \underline{r}_2 \qquad} \overset{\circ}{m_2}$$

$$\frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{M} = 0 \Rightarrow m_1 |\underline{r}_1| = m_2 |\underline{r}_2|$$

and $\dfrac{m_1}{m_2} = \dfrac{r_2}{r_1} = \dfrac{a_2}{a_1}$ semi-major axes of ellipses

Can find mass ratio without distance!

From Kepler's third law,

$$P^2 = \frac{4\pi^2 a^3}{GM}$$ semi-major axis of reduced mass orbit. $\left(\mu = \frac{m_1 m_2}{M}.\right)$

Given an inclination i between the orbit and the plane of the sky, and writing $a_{obs} = a \cos i$,

$$M = \frac{4\pi^2}{GP^2}\left(\frac{a_{obs}}{\cos i}\right)^3.$$

Note that $a = a_1 + a_2$ and the distance is required.

The inclination can be derived from departures to the ellipse geometry.

Spectroscopic binaries

Observe Doppler shifts of one or two sets of spectral lines.

Consider circular case.

$$V_1 = \frac{2\pi a_1}{P}, \quad V_2 = \frac{2\pi a_2}{P}, \text{ and}$$

$$\frac{m_1}{m_2} = \frac{V_2}{V_1}.$$ OK even though we measure $V \sin i$.

Writing $a = \frac{P}{2\pi}(v_1 + v_2)$, K3L $\rightarrow$

$$M = \frac{P}{2G\pi} \underbrace{(v_1 + v_2)^3}_{\text{actual velocities}}$$

$$= \frac{P}{2\pi G} \frac{(v_1' + v_2')^3}{\sin^3 i} \rightarrow \text{measured radial velocities.}$$

$$= \frac{P}{2\pi G} \frac{v_1'^3}{\sin^3 i} \left(1 + \frac{m_1}{m_2}\right)^3$$

$$\Rightarrow \frac{m_2^3}{M^2} \sin^3 i = \underbrace{\frac{P}{2\pi G} v_1'^3}_{\text{MASS FUNCTION.}}$$

<u>Eclipsing binaries</u>

$\rightarrow$ inclination

$\rightarrow$ stellar radii.

---

* Virial theorem : $E = \frac{1}{2} \langle U \rangle_t$

Can you show this for a binary in a circular orbit?

Can you relate velocity dispersion to the mass of a virialized system?