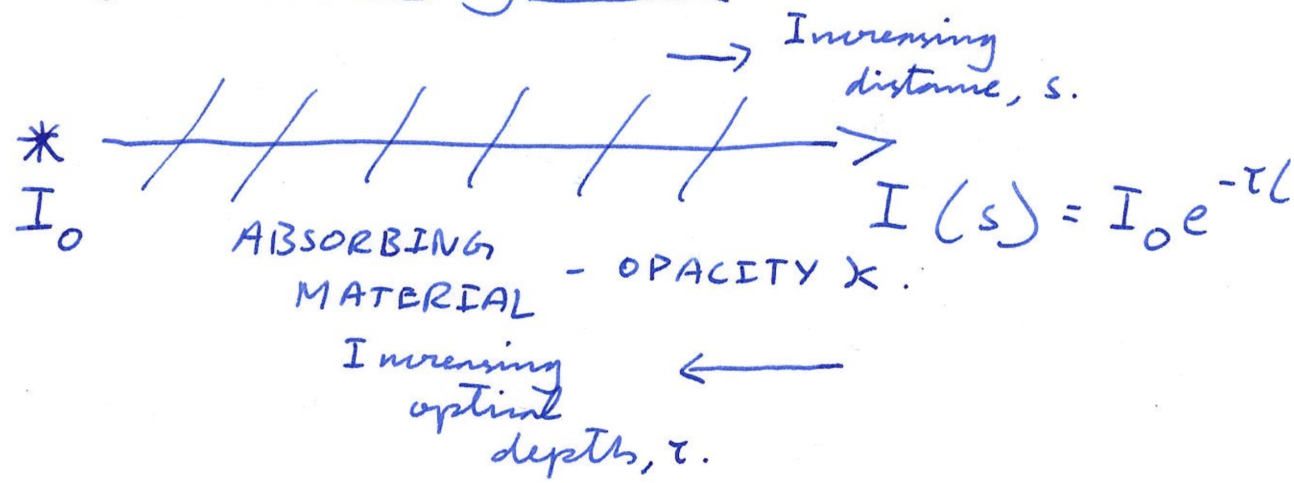
Ay20 #6 - The Sun and its blackbody spectrum

All objects are observed to a surface beyond which they are *optically thick*.



Consider a gas (or any material) with *opacity* κ_ν ($cm^2\, g^{-1}$) - the amount of absorption per unit length, per unit volume^{-1}, per unit mass.

At one frequency, ν, the change in intensity is

$$dI_\nu = -\kappa_\nu \rho I_\nu ds \quad .$$

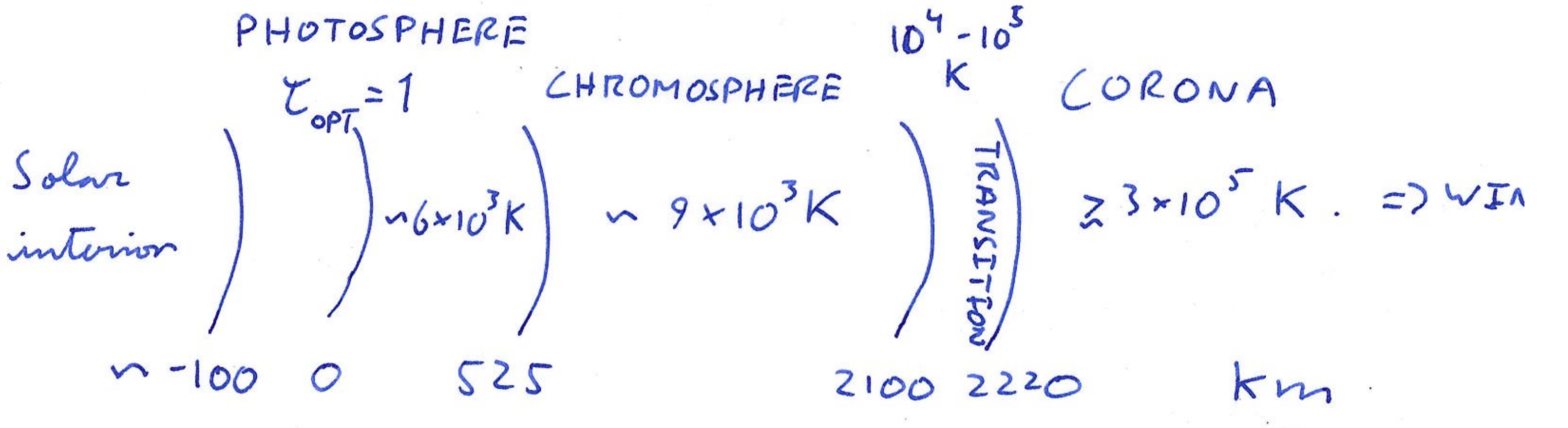
$\leftarrow$ density.

Over a distance s,

$$I_\nu(s) = I_0 e^{-\int_0^s \kappa_\nu \rho ds'} \quad .$$

The exponent $-\int_0^s \kappa_\nu \rho ds' \equiv \tau_\nu(s)$, the optical depth.

At visible wavelengths, the Eddington approximation (more in a future lecture) defines the surface of a star as the $\tau_\nu = 2/3$ surface.



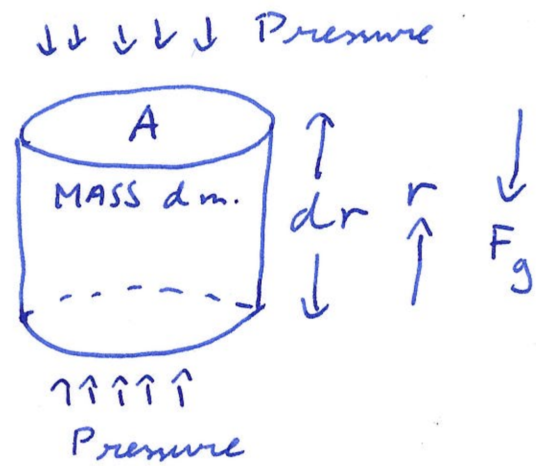
(The Solar atmosphere).

* Chromosphere: slightly hotter (few $\times 10^3$ K), slightly lower density (10^{-12} g cm^{-3} v 10^{-8} g cm^{-3}) than photosphere.

* Transition -> Corona: hot ($> 10^5$ K), sparse ($< 10^{-14}$ g cm^{-3}) gas. Observed through electrons - (K-corona) & dust - (F-corona) scattered light, and high-ionization (E-corona) emission lines.

* Solar wind: slow wind from solar equator (~ 400 km s^{-1}, $\sim 10^5$ K), fast wind from coronal holes (~ 750 km s^{-1}, $\sim 10^6$ K).

The Parker wind model demonstrates that an isothermal corona cannot be pressure-confined by the interstellar medium (ISM). This can be shown through the concept of hydrostatic equilibrium.



Consider a cylinder of density ρ in the corona.

Force balance:

$$|\text{Gravity}| = -|\text{Pressure}|$$

$\equiv$ hydro equilibrium. (no mass flux)

Gravity: $|dF_g| = + \dfrac{GM\,dm}{r^2}$.

Pressure: $|dF_p| = A\,dP$ ← Pressure.

Writing $dm = \rho A\,dr$, we have

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$ (Equation of hydro equilibrium).

In the corona, $P \approx 2nkT$, where n is the number density of H-atoms. Why $\times 2$?

Then, with $\rho \simeq n m_p$ proton mass, and

$M \simeq M_\odot$, we have

$$\frac{d}{dr}\left(2nkT\right) = -\frac{G M_\odot n m_p}{r^2}.$$

$$\frac{dn}{dr} = -\frac{G M_\odot m_p}{2kT}\frac{n}{r^2}$$

The solution is $n(r) = n(r_0)\, e^{-\frac{G M_\odot m_p}{2kT r_0}\left(1-\frac{r_0}{r}\right)}$

(exercise for student to show this)

The pressure structure is simply

$$P(r) = P(r_0)\, e^{-\frac{G M_\odot m_p}{2kT r_0}\left(1-\frac{r_0}{r}\right)},$$

$$P(r_0) = 2n(r_0)kT.$$

At $r = \infty$, the pressure is

$$P_\infty = P(r_0)\, e^{-\frac{G M_\odot m_p}{2kT r_0}} \neq 0.$$

With real numbers, $P_\infty \sim 5\times10^{-5}$ dyne cm^{-2}.

In the local "warm ionized medium" phase of the interstellar medium, $T \sim 10^4$ K, $n \sim 0.5$ cm^{-3}.

$\Rightarrow P \sim 10^{-12}$ dyne cm^{-2}!

The corona thus expands as the solar wind, and hydrostatic equilibrium fails.

The solar luminosity is 3.8×10^{33} erg s^{-1}.

The solar spectrum is largely thermal $\equiv$ blackbody radiation. The effective temperature is given by the Stefan - Boltzmann law:

$$T_{eff} = \left(\frac{L_\odot}{4\pi R_\odot^2 \sigma} \right)^{1/4} \simeq 5777 \text{ K}.$$

From Wien's displacement law, this implies a peak wavelength:

$$\lambda = \frac{0.29 \text{ cm K}}{5777 \text{ K}} = 5016 \text{ Å}.$$

In full, the solar spectrum approximates the Planck's function.

The gravitational binding energy (assuming a uniform sphere) of the Sun is

$$U_\odot = \frac{3 G M_\odot^2}{5 R_\odot} \sim 2 \times 10^{48} \text{ erg}.$$

If gravitational collapse alone powered the Sun, the solar lifetime is given by the Kelvin-Helmholtz Timescale:

$$t_{KH} = \frac{U_\odot}{L_\odot} \sim 10^7 \text{ yr.}$$

For various reasons, this is way too short!