Ay20 Final Exam

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Dec 9-13, 2019.

Exam instructions are as follows. **Please do not go to the next page until you're ready to start the exam!**

How to complete: [well, not really :)!] Attempt as many questions as you can. You can use blue exam books, but this is not necessary.

Duration: Four hours of working time. You can take a single $<$ 1-hour break during the exam that does not count towards the four hours.

Submission: At noon on Friday December 13, 2019, in Yuping's or Vikram's mailbox.

Allowed and disallowed resources: All personal notes taken in class, all lecture notes on the course website, all assessment solutions (including problem sets that you've had handed back), the textbook (C&O). A scientific calculator (or calculator software) is allowed, but not a symbolic manipulation program (such as a Wolfram product). No use of any other website or book. No use of the C&O solutions manual. No discussions of the problems or solutions with anyone.

Grading scheme: Every question (or part of a question) is worth two points. In some cases two answers are required, in which case one point will be awarded for each answer. In other cases one answer is required, in which case one point will be awarded for reasoning (which requires adequate explanation), and one point will be awarded for the right answer. If you need to make an assumption to answer a question, justify it.

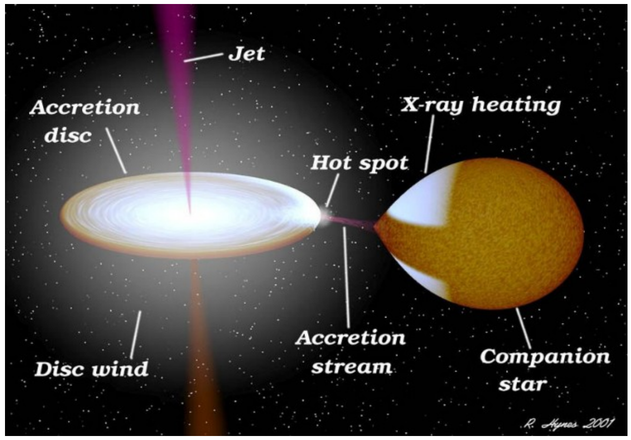

Figure 1: A schematic diagram of an X-ray binary (NASA/R. Hynes).

The MAXI all-sky X-ray monitor detects a new X-ray source. The source is at the coordinates Right Ascension 215 deg, Declination 0 deg, and is given the name MAXI J215+00. The X-ray properties of the source suggest that it is an X-ray binary (Figure 1), where a star is in a binary with a compact object (neutron star or black hole) and overflows its Roche lobe, thus donating mass to the compact object. Questions 1–6 are related to MAXI J215+00.

Question 1. For an observer on the Earth's equator, in what month would MAXI J215+00 appear overhead at 9 pm local time? In what months would it be above the horizon for an observer at the South (geographic) pole?

Question 2. Within days, a transient optical counterpart to MAXI J215+00 is reported, with a magnitude of 18.0 at 4000 Angstrom, and a magnitude of 19.505 at 8000 Angstrom. From these data, derive a constraint on the effective temperature of the optical counterpart.

Question 3. The optical source likely represents thermal emission from gas that is accreting onto the compact object. This gas is expected to form an accretion disk around the compact object with a radius of $\sim 1R_\odot$, where $R_\odot$ is a Solar radius. Adopting a characteristic temperature of $20,000\,\mathrm{K}$ and a particle number-density of $10^{15}\,\mathrm{cm}^{-3}$ for the disk, will collisional broadening, thermal Doppler broadening, or dynamical effects dominate the width of any observed emission line?

Question 4. The optical and X-ray emission from MAXI J215+00 fades after several days, revealing a G-type star with a Gaia distance of 10 kpc. Assuming that the X-ray emission from the transient, which peaked with a flux in the $2 - 10\,\mathrm{keV}$ energy range of $10^{-10}\,\mathrm{erg\,s^{-1}\,cm^{-2}}$, was also from the accretion disk, derive a lower limit to the mass of the compact object by assuming that the luminosity could not be larger than the Eddington luminosity.

Question 5. We can assume that the G-type star is overflowing its Roche lobe and donating mass to the compact object in MAXI J215+00. Time-resolved spectroscopy of the G-type star could then also be used to limit the mass of the compact object. Derive an expression for this limit (stating if it's a lower or upper limit) in terms of the maximum radial velocity, $v_{\rm max}$, and orbital period, $P_{\rm orb}$, of the G-type star.

Question 6. Draw a track representing the likely evolution of the G-type star on the H-R diagram, and indicate the stages (if any) where the compact object may be within the photosphere of the star.

The following questions are all self-contained. Note that each part of each question is worth two points (so, e.g., Question 7 is worth 10 points in total).

Question 7. In this question, you will investigate the observational requirements to make an image of an exoplanet. Consider the HAT-P-44c planet, which orbits the star HAT-P-44. The basic parameters of the system are given in Table 1.

(a) What is the magnitude of HAT-P-44c at the wavelength, λ_p, where its thermal emission spectrum peaks? What is the magnitude of the star at λ_p?

(b) What is the angular diameter of HAT-P-44c, and what is its maximum angular separation from the star? What might cause fluctuations in the surface brightness of the star on angular scales corresponding to the planet's angular diameter?

(c) We want to design a space telescope operating at λ_p that can provide three angular-resolution elements across the diameter of HAT-P-44c. What would the diameter of the telescope aperture have to be? Discuss how such angular resolution could be practically achieved.

Parameter	Value
Star mass	$0.94 M_\odot$
Star radius	$0.98 R_\odot$
Star brightness	13.2 mags (V-band)
Star $T_{\rm eff}$	5925 K
Distance	374 pc
Planet mass	3×10^{30} g
Planet radius	7.2×10^4 km
Planet $T_{\rm eff}$	500 K
Orbital semi-major axis	0.7 AU
Orbital inclination	90 deg
Orbital eccentricity	0

Table 1: Parameters of the HAT-P-44 planetary system (source: The Extrasolar Planet Encyclopaedia).

(d) Assume that we can build a detector that is sensitive to radiation in a 1-μm wide band around λ_p, with an efficiency of 10%. Not accounting for the sky background (because we're in space!), or for any detector noise, what collecting area is required to achieve a signal to noise ratio of 6 per resolution element on HAT-P-44c in 50 hours?

(e) Discuss how the star affects your ability to image HAT-P-44c. How could these issues be mitigated?

Question 8. A certain species of butterfly lives in a forest, which primarily contains one type of tree. The caterpillars hatch on the barks of the trees, and eventually turn into butterflies and fly away. Now, these butterflies are a bit strange, because they only fly in straight lines at a constant height. When they encounter a new tree, they just stop, land, lay some eggs, and spend the rest of their happy lives guarding the eggs. To protect the butterflies, no-one is allowed to enter the forest, but you can stand near the outside to view any butterflies that leave the forest. The butterflies that leave just keep going until they find new forests to populate.

(a) From drone photographs, we know the areal density of trees, ρ, and the mean tree diameters (as seen by the butterflies), D. Derive an approximate expression for the mean distance from the edge of the forest where any butterflies you see on the outside are born. Justify any further assumptions you make.

(b) Although these butterflies are all blue, one butterfly develops a dominant allele that makes it yellow. Assuming that the forest is circular with a radius R, that the butterflies fly at a speed v, and that they take a time T to produce offspring from when they land, derive an approximate expression for how long it will take for all butterflies in the forest to be yellow. Justify any further assumptions you make.

Question 9. (a) Roughly estimate the mass in molecular (e.g., H_2) gas in the Milky Way ISM.

(b) The Milky Way forms stars at a rate of approximately $1\,M_\odot\,\mathrm{yr}^{-1}$. Was this likely to have been different in the past, and how/why?

(c) Do you think that the Milky Way metallicity (stars and ISM) is larger closer to the Galactic Center, or further from the Galactic Center? Why? Is this different for the halo and the disk?

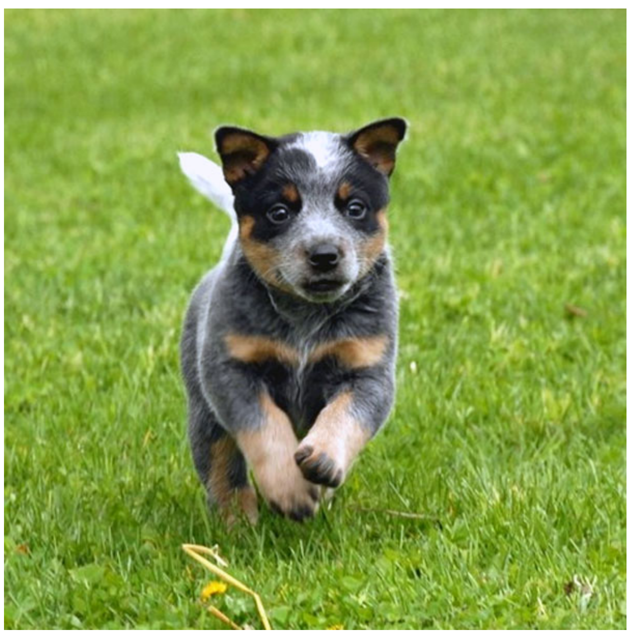

Figure 2: Congratulations - you're done!