Ay 20 #13&14 - Solar and stellar spectra

In previous lectures, we have discussed instruments for optical / IR (OIR) imaging. Spectroscopy involves dispersing light over a range of wavelengths.

* Spectral resolution

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\nu}{\Delta\nu} = \frac{c}{\Delta v}, \text{ from}$$

(relativistic) Doppler effect

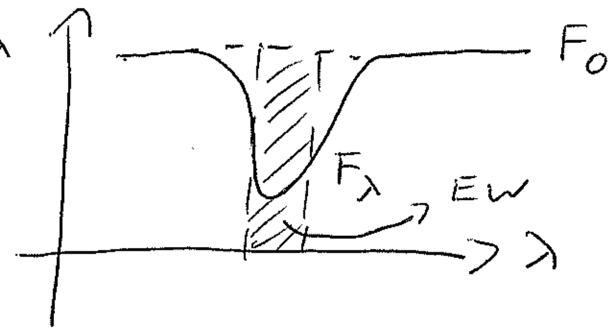
$$\nu' = \gamma(1-\beta)\nu .$$

Uses:

* velocities, redshifts from line wavelengths
* abundances, ionization, temperatures from line equivalent widths (EWs) / strengths & ratios
* pressure, density, magnetic fields, dynamics from line shapes.

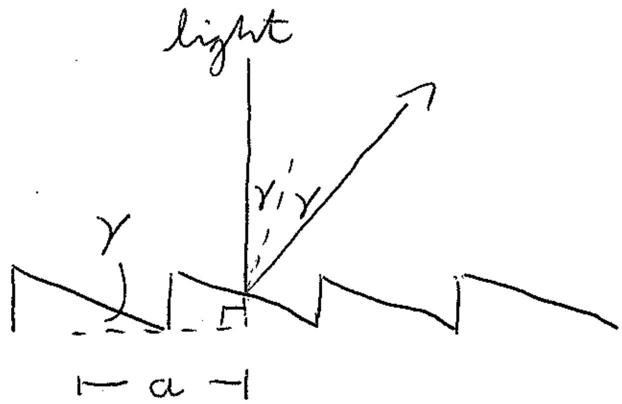
EW

$$= \int_{line} \frac{F_0 - F_\lambda}{F_0} \, d\lambda .$$



Why is this useful?

OIR spectrometers typically use diffraction gratings.



(blazed grating)

Grating equation

$$a \sin(2\gamma) = m\lambda ,$$ where m is the order.

γ & a define the peak-efficiency λ for a given m.

Various tricks can then be played:

- order blocking filters to avoid overlapping orders.
- cross dispersion of overlapping orders to achieve higher R.
- how do you deal with atmospheric dispersion?

Spectral classification schemes

* Harvard scheme : Annie Jump Cannon proposed rearranging of A - Q scheme into

  O B A F G K M (now L T Y), offering continuity in color. Cecilia Payne applied the Saha equation to relate spectral line ratios to ionization states and temperatures.

  O - M : temperature, color, main-sequence mass / radius / luminosity continuous.

  Alphabetical order refers to strength of H - lines.

  Now sub-classes 0 - 9, with fractions allowed.

  Spectral libraries define classes.

* Yerkes scheme (Morgan - Keenan - Kellman / MKK): adds luminosity classes to spectral classes.

  0 or Ia^+ : hypergiants. Ia : supergiants.

  Iab : intermediate supergiants. Ib : low-luminosity supergiants

  II : bright giants. III : giants. IV : subgiants.

  V : main sequence. sd- or -VI : subdwarfs. D- or -VII : dwarfs.

Can also add suffices a or b to split classes.

Star catalogs.

Two examples of this arcane practise are...

* The Henry Draper catalog (HD) used by Cannon + to identify spectral types.

$V < 9$, 359,083 classified stars.

Objective-prism survey. Secchi classes -> A-Q.

* Argelander scheme for variable stars.

Romanized Latin alphabet has no J, U, W, I.

But Argelander only excluded I & J. Later I was added.

In a constellation, ranked by the "most" to "least" optically variable:

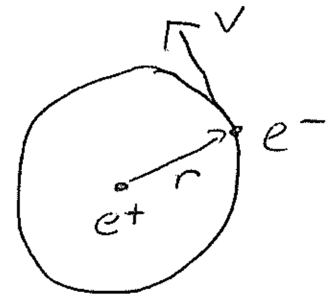
9 R -> Z : e.g., R Cor Bor, T Tauri

45 RR->RZ, SS->SZ ... ZZ : e.g., RR Lyrae, RS CVn

280 AA->AZ, BB->BZ ... QQ->QZ : e.g., FK Com, FU Ori

∞ V xxx... : e.g., V404 Cyg, V1333 Aql.
(≥ 335)

* The Bohr H-atom.



$$\frac{ke^2}{r^2} = \frac{\mu v^2}{r} \quad (\text{Newton \#2})$$

$$\Rightarrow \frac{1}{2}\mu v^2 = KE = \frac{ke^2}{2r}. \qquad \left(\mu = \frac{m_e m_p}{m_e + m_p}\right)$$

Also, $U = -\frac{ke^2}{r}$. So the total energy is $-\frac{1}{2}\frac{e^2 k}{r}$

Bohr quantization: $L = \mu v r = n\hbar$.

Then we need to write v & r in terms of n & $\hbar$ to get the energy of level n.

$$v = \frac{n\hbar}{\mu r} \Rightarrow \frac{1}{2}\mu v^2 = \frac{n^2\hbar^2}{\cancel{2}\mu r^2} = \frac{ke^2}{\cancel{2}r} \text{ and } r = \frac{\cancel{2}n^2\hbar^2}{\cancel{2}k\mu e^2}$$

(Bohr radius)

$$\text{Then } E_n = -\frac{e^2 k}{2} \cdot \frac{k\mu e^2}{n^2\hbar^2} = -\frac{k^2\mu e^4}{2\hbar^2 n^2} = -13.6\,eV \cdot \frac{1}{n^2}.$$

This sets the wavelengths of the Lyman ($\to n=1$), Balmer ($\to n=2$), Paschen ($\to n=3$), Brackett ($\to n=4$) lines of H, as well as the ionization potential.

* Boltzmann statistics. The Maxwell-Boltzmann velocity distribution $\to$ $V_{prob} = \sqrt{\frac{2kT}{m}}$, $\bar{V} = V_{rms} = \sqrt{\frac{3kT}{m}}$.

In LTE, the probability that a system is in state E_b vs E_a is

$$\frac{P(E_b)}{P(E_a)} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$ (Boltzmann equation)

$\uparrow$ degeneracy factors.

Define a partition function

$$Z = g_1 + \sum_{j=2}^{\infty} g_j e^{-(E_j - E_1)/kT}$$ (# of ways of configuring e^-).

Then $$\frac{N_{ion}}{N_{neutral}} = \frac{2 Z_{ion}}{n_e Z_{neutral}} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT},$$

where χ_i is the ground-state ionization potential.

Can substitute $P_e = n_e kT$. This is the Saha equation.

* Spectral line shapes.

<u>Pressure & collisional broadening</u> — waiting time for transition.

Uncertainty principle $\Delta E \Delta t \sim \hbar$.

$$\lambda = \frac{hc}{E} \Rightarrow |\Delta\lambda| = \frac{hc}{E^2} \Delta E \Rightarrow \delta E \simeq \frac{hc}{\lambda^2} \Delta\lambda.$$

and $$\Delta\lambda \approx \frac{\lambda^2}{\pi c \Delta t}.$$

Setting $\Delta t \sim \frac{MFP}{V} = \frac{1}{n\sigma\sqrt{\frac{2kT}{m}}}$,

$$\Delta\lambda \sim \frac{\lambda^2}{c}\frac{n\sigma}{\pi}\sqrt{\frac{2kT}{m}}$$: pressure / collisional broadening linewidth.

<u>Doppler broadening</u>.

$$\frac{\Delta\lambda}{\lambda} = \pm\frac{V}{c} \Rightarrow \Delta\lambda \sim \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}}.$$

This doesn't include large-scale motions (convection, turbulence).

A <u>Voigt</u> line profile combines both effects.

A <u>curve of growth</u> plots the line EW vs column density.