Ay 20 #8 - Radiation, opacity, transfer

The energy released in nuclear burning is ultimately in the form of photons (γ), neutrinos (ν_e) and positrons that annihilate to produce photons.

PP-I : $2\times\nu_e$: 0.5 MeV

$6\times\gamma$: 26.2 MeV

PP-II : Neutrinos : 1 MeV

Photons : 25.7 MeV.

These photons provide the energy to power the Sun.

Here's where the fun begins...

In the following description of the propagation of photons through the Sun, I have dropped any λ-dependence. The approximation of λ-independent opacity is called the grey-body approximation.

* **Definitions**

Specific usually refers to a specific $\lambda / \nu / E$.

"~~Specific~~" intensity $$I = \frac{E}{dt\, dA \cos\theta\, d\Omega}$$

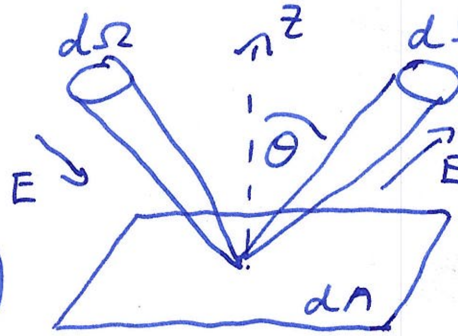
(energy flux @ angle θ from surface normal, per unit solid angle)

- Mean intensity $$\langle I \rangle = \frac{1}{4\pi}\int I \, d\Omega = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I \sin\theta\, d\theta$$

- "~~Specific~~" radiative flux : integrates I over all θ :

$$F = \int_0^{2\pi}\int_0^{\pi} \underset{\times \sin\theta}{I \cos\theta}\, d\theta\, d\phi .$$

- Radiation pressure :



$$\underset{\text{momentum}}{dp} = \frac{2E\cos\theta}{c} \quad (p_{final} - p_{init})$$

$$= \frac{2}{c} I\, dt\, dA \cos^2\theta\, d\Omega . \quad \text{for reflection only.}$$

For transmission, $dp = \frac{1}{c} I\, dt\, dA \cos^2\theta\, d\Omega$

Pressure unit $dP = \frac{dp}{dt\,dA}$. Thus

$$P = \frac{1}{c}\int_0^{2\pi}\int_0^{\pi} I \cos^2\theta \sin\theta\, d\theta\, d\phi = \frac{4\pi}{3c} I \quad (\text{isotropic } I)$$

:)

- Opacity : $\kappa \rho$ is number / total of photons scattered per unit length : $dI = -\kappa \rho I \, ds$.

  The mean free path is just $\frac{1}{\kappa \rho}$.

- Optical depth : $d\tau = -\kappa \rho \, ds \Rightarrow dI = I \, d\tau$.

  $\tau \sim$ # of mean free paths.

- Absorption & emission coefficients

  = opacity!

  $dI = j \rho \, ds$. Note that it's rare that the emission coefficient is λ- independent.

The propagation of photons can be described by the transfer equation :

$$dI = -\kappa \rho I \, ds + j \rho \, ds$$

or

$$-\frac{1}{\kappa \rho} \frac{dI}{ds} = I - S$$

// Source function $S = \frac{j}{\kappa}$.

With $d\tau = -\kappa \rho \, ds$,

$$\frac{dI}{d\tau} = I - S.$$

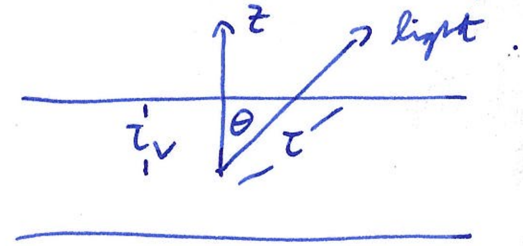
* In local thermodynamic equilibrium (LTE),

$$S = \frac{\sigma T^4}{\pi} \quad \text{(blackbody radiation!)}$$

With all this machinery, we could say a lot about the physical profiles of stars, but we're missing a critical piece - the opacity mechanisms. See the next lecture!

For now, let us consider the temperature profile in a <u>plane-parallel</u>, <u>grey</u>, <u>LTE</u> stellar atmosphere.

Defining a vertical optical depth τ_v, the transfer equation becomes



$$\cos\theta \frac{dI}{d\tau_v} = I - S.$$

Referring to the definition of radiation pressure,

$$\frac{dP}{d\tau_v} = \frac{1}{c}\frac{d}{d\tau_v}\int_0^{2\pi}\int_0^{\pi} I \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{c}\left[\int_0^{2\pi}\int_0^{\pi} I \cos\theta \sin\theta \, d\theta \, d\phi - S\int_0^{2\pi}\int_0^{\pi} \cos\theta \sin\theta \, d\theta \, d\phi \right.$$

Simplifying, we have

$$\frac{dP}{d\tau_\nu} = \frac{F}{c} \quad !$$

radiating flux.

In LTE, F is constant (no net absorption / emissions)

So $P = \frac{1}{c} F \tau_\nu + K$ ← constant.

We now make the Eddington approximation, such that

$$I = \begin{cases} I_{out}, & 0 \leq \theta < \frac{\pi}{2} \\ I_{in}, & \frac{\pi}{2} \leq \theta \leq \pi \end{cases}$$

From this, $\langle I \rangle = \frac{1}{2}(I_{out} + I_{in})$,

$$F = \pi (I_{out} + I_{in}),$$

$$P = \frac{4\pi}{3c} \langle I \rangle . \quad \text{(see each definition)}$$

Then, $\frac{4\pi}{3c} \langle I \rangle = \frac{1}{c} F \tau_\nu + K$.

At $\tau_\nu = 0$, $I_{in} = 0$ and $\langle I \rangle = \frac{1}{2} I_{out}$,

$F = \pi I_{out}$, and $K = \frac{2F}{3c}$.

Then, substituting $F = \sigma T_{eff}^4$ (at surface,

as F is const.!)

$$\langle I \rangle = \frac{3\sigma}{4\pi} T_{eff}^4 \left(\tau_v + \frac{2}{3} \right).$$

Almost done! From the transfer equation, we have

$$\frac{d}{d\tau_v} \int_0^{2\pi} \int_0^{\pi} I \cos\theta \, d\theta \, d\phi = 4\pi \left(\langle I \rangle - S \right).$$

$$= \frac{dF}{d\tau_v}.$$

As F is constant (super useful), $\langle I \rangle = S = \frac{\sigma T^4}{\pi}$ (LTE).

Finally, $T^4 = \frac{3}{4} T_{eff}^4 \left(\tau_v + \frac{2}{3} \right)$.

When $\tau_v = \frac{2}{3}$, $T = T_{eff}$! This defines a stellar surface.