Ay 20 PS1 – Solutions

Q1

a) The tidal height is proportional to the effective tidal force (F_T), which scales as $\frac{M_S}{D_S^3}$ ($M_S \equiv$ satellite mass, $D_S \equiv$ satellite distance).

So, $$h_{SUN} = \frac{2\times10^{33}\,g}{7\times10^{25}\,g} \cdot \frac{(3.8\times10^{10}\,cm)^3}{(1.5\times10^{13}\,cm)^3} \times 1\,m$$

$= 0.46\,m.$ (1 point)

b) Applying the formula from lectures,

$$\frac{d\nu}{dt} = -\frac{5 h \rho G M_S R_E^2 \sin 2\alpha}{M_E D_S^3},$$

we find a rate of $\sim 3\times10^{-13}\,s\,s^{-1}$ or $-4\times10^{-23}\,Hz\,s^{-1}$.

(1 point)

c) Difference between solar & sidereal day today: $\Delta t_{now} = \frac{24\ hr}{365.25} = 237\ s.$

(1 point)

Considering both the Moon & Sun tides,

$$\frac{d\nu}{dt} = -2.4 \times 10^{-22}\ Hz\ s^{-1}$$

In 10^8 yr, $\Delta\nu = +7.6 \times 10^{-7}$ Hz,

and $\nu = \frac{1}{86400 - 237} + 7.6 \times 10^{-7} = 1.24 \times 10^{-5}$ Hz

or 22.5 hours. This is a sidereal day.

There will always be one more sidereal days per year than solar days. So

$\Delta t_{10^8} = 208\ s.$ (1 point)

d) The Moon moves further away. A simple calculation (assuming constant velocity of the Moon) suggests a distance increase of ~ 40%.

[Q3] As the primary explodes symmetrically, the secondary will not be subject to the primary's gravitational pull (see e.g. Purcell 3rd Ed. Fig 1-22, the field is 0 inside a spherical shell of charge/mass). [1 pt]

The orbital velocity of the reduced mass in the CM frame is given by

$$V_\mu = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM\omega}{V_\mu}} \Rightarrow V_\mu = (GM\omega)^{1/3}.$$

The orbital velocity of the secondary is then given by

$$V_2 = \frac{\mu}{M_2} V_\mu = \frac{M_1}{M_1+M_2}\left[G(M_1+M_2)\omega\right]^{1/3}.$$

With $M_1 = 0.61 M_\odot$, $M_2 = 0.21 M_\odot$,

$$\omega = \frac{2\pi}{415} \text{ rad s}^{-1},$$

$$V_2 = 873 \text{ km s}^{-1} \qquad [1 \text{ pt}].$$

Escape velocity

$$V_{esc} = \sqrt{\frac{2GM_{MW}}{R}} = 1272 \text{ km s}^{-1}$$ with radius from question.

So $V_2 \lesssim V_{esc}$