Ay 20 Problem Set 2 Solutions

Preface: only trust the answers up to 2 significant digits unless otherwise noted.

Question 1

Part a (3 points)

First we convert the averaged V-band magnitude (assumed to be AB magnitude) of the Crab pulsar to flux density F_{psr}

In[ ]:= $F_{\nu,\text{psr}} = 10^{-16.5/2.5} * 3631\ \text{Jy}$

Out[ ]= 0.000912066 Jy

Given the telescope radius r and total efficiency η, the number of photons collected is given by dividing the total collected energy over the energy of a single photon hc/λ (we can take the center wavelength to be roughly the averaged wavelength of the band) i.e.

In[ ]:= $n_{\text{photon,psr}} = \dfrac{\left(c / \lambda^2\ F_{\nu,\text{psr}}\ \pi\ r^2\ \Delta\lambda\ \eta\ \Delta t\right)}{(h\ c)/\lambda}$ /. $\{r \to 2.5\ \text{m},\ \eta \to 0.3,\ \Delta t \to 1\ \text{s},\ \Delta\lambda \to 1000\ \text{Å},\ \lambda \to 5500\ \text{Å}\}$ // UnitSimplify

Out[ ]= 14 742.1

where the c/λ^2 prefactor converts from flux per frequency to flux per wavelength (aka chain rule on λ=c/ν).

Within an 1 arcsec^2 solid angle, the flux density coming from the sky is 20.5 mag. The integrated flux density from the nebula is equivalent to 8.4 mag. So within a solid angle of π $(0.5\ \text{arcsec})^2$

In[ ]:= $F_{\nu,\text{sky}} = 10^{-20.5/2.5} * 3631 * 0.5^2 * \pi\ \text{Jy}$; $F_{\nu,\text{neb}} = 10^{-8.4/2.5} * 3631 * \dfrac{(0.5^2 * \pi)}{(150^2 * \pi)}\ \text{Jy}$;

In[ ]:= $n_{\text{photon,sky}} = \dfrac{\left(c / \lambda^2\ F_{\nu,\text{sky}}\ \pi\ r^2\ \Delta\lambda\ \eta\ \Delta t\right)}{(h\ c)/\lambda}$ /. $\{r \to 2.5\ \text{m},\ \eta \to 0.3,\ \Delta t \to 1\ \text{s},\ \Delta\lambda \to 1000\ \text{Å},\ \lambda \to 5500\ \text{Å}\}$ // UnitSimplify

Out[ ]= 290.836

In[ ]:=
$$n_{photon,neb} = \frac{\left(c\ /\ \lambda^2\ F_{v,neb}\ \pi\ r^2\ \Delta\lambda\ \eta\ \Delta t\right)}{\left(h\ \ c\right)/\lambda}\ /.$$
$$\left\{r \to 2.5\ m\ ,\ \eta \to 0.3,\ \Delta t \to 1\ s\ ,\ \Delta\lambda \to 1000\ Å\ ,\ \lambda \to 5500\ Å\right\}\ //\ UnitSimplify$$

Out[ ]= 284.653

Operating under the ideal 3-ms integration time on a pulse, the output power from the pulsar should be (33/3) times that of the average power. Now plug everything into the signal-to-noise equation, with an integration time t=0.003 s.

In[ ]:=
$$SNR_a = \frac{\left(n_{photon,psr} * t * \frac{33}{3}\right)}{\sqrt{n_{photon,psr} * t * \frac{33}{3} + n_{photon,sky} * t + n_{photon,neb} * t + \left(\frac{\theta}{0.28}\right)^2 (t)}}\ /.\ \{t \to 0.003,\ \theta \to 1\}$$

Out[ ]= 22.0166

Part b (3 points)

The Crab pulsar remains a point source so we can just plug in new numbers

In[ ]:=
$$n_{photon,psr} = \frac{\left(c\ /\ \lambda^2\ F_{v,psr}\ \pi\ r^2\ \Delta\lambda\ \eta\ \Delta t\right)}{\left(h\ \ c\right)/\lambda}\ /.$$
$$\left\{r \to 0.15\ m\ ,\ \eta \to 0.3,\ \Delta t \to 1\ s\ ,\ \Delta\lambda \to 1000\ Å\ ,\ \lambda \to 5500\ Å\right\}\ //\ UnitSimplify$$

Out[ ]= 53.0715

For the Crab nebula and the sky, we get all of the nebula and 0.1 deg^{2}of the sky. The flux density of those two combined is given by

In[ ]:= $F_{v,sky} = 10^{-20.5/2.5} * 3631 * 0.1 * (3600)^2\ Jy\ ;\ F_{v,neb} = 10^{-8.4/2.5} * 3631\ Jy\ ;$

The number of photons per second is given by

In[ ]:=
$$n_{photon,sky} = \frac{\left(c\ /\ \lambda^2\ F_{v,sky}\ \pi\ r^2\ \Delta\lambda\ \eta\ \Delta t\right)}{\left(h\ \ c\right)/\lambda}\ /.$$
$$\left\{r \to 0.15\ m\ ,\ \eta \to 0.3,\ \Delta t \to 1\ s\ ,\ \Delta\lambda \to 1000\ Å\ ,\ \lambda \to 5500\ Å\right\}\ //\ UnitSimplify$$

Out[ ]= 1.72769×10^6

In[ ]:= $n_{photon,neb} = \dfrac{(c\ /\lambda^2\ F_{\nu,neb}\ \pi\ r^2\ \Delta\lambda\ \eta\ \Delta t)}{(h\ \ c)/\lambda}$ /.

$\{r \to 0.15\ m,\ \eta \to 0.3,\ \Delta t \to 1\ s,\ \Delta\lambda \to 1000\ Å,\ \lambda \to 5500\ Å\}$ // UnitSimplify

Out[ ]= 92 227.7

And thus the signal-to-noise ratio is given by

In[ ]:= $SNR_a = \dfrac{\left(n_{photon,psr} * t * \frac{33}{3}\right)}{\sqrt{n_{photon,psr} * t * \frac{33}{3} + (n_{photon,sky} + n_{photon,neb}) * t}}$ /. {t -> 0.003}

Out[ ]= 0.0236984

Question 2

Part a (1 point)

Approximate the solar system as a disk of radius $r \sim 5\times10^{14}$ cm (roughly the distance from Neptune to the sun). With averaged velocity $v = 5\times10^{6}$cm/s, we can interpret the arrival rate 1/yr as a number density of BEFOPSs: on average there is one contained within a cylinder with base area equal to the solar system area and height the distance traveled by one average BEFOP per year.

In[1]:= $n_{BEFOPS} = \dfrac{1}{\pi\ r^2\ (v\ t)}$ /. $\{r \to 5\times10^{14}\ cm,\ v \to 5 * 10^{6}\ cm/s,\ t \to 3 * 10^{7}\ s\}$;

Then the number of BEFOPSs within the galactic disk is given by

In[3]:= $N_{BEFOPS} = n_{BEFOPS}\ (\pi\ r_{gal}{}^2\ h_{gal})$ /. $\{r_{gal} \to 30/2\ kpc,\ h_{gal} \to 2\ kpc\}$ // N

Out[3]= 3.5256×10^{23}

So we have about 10^{23}such bodies within the galactic disk.

Part b (1 point)

Mass scales like $r^3$for constant density. So the total mass of BEFOPSs in the galactic disk is given by

In[4]:= $N_{BEFOPS} * 6 * 10^{27}\ g * \left(\dfrac{100\ m}{6.4 * 10^{8}\ cm}\right)^3$

Out[4]= 8.06946×10^{36} g

which is $\sim10^4$solar mass, which is much less than the mass of the Milky Way.

Part c (2 points)

Each BEFOPS can be treated as a point source and at 10 pc has a flux density of

In[5]:= $F_{\nu BEFOPS} = 10^{-53/2.5} * 3631\ \mathrm{Jy};$

To calculate the the brightness in magnitude per square arcsecond from BEFOPSs toward the Galactic center, we can calculate the total flux from BEFOPS in a cone from earth to the edge of the Galaxy with solid angle 1 square arcsec by integrating along shells of constant radius r. Note that 1 steradian = 1 rad^2 = 4.25$*$10^{10} arcsec^2. The volume of the cone is given by

In[6]:= $F_{\nu total} =$

$$\left(\frac{1}{4\,\pi * 4.25 * 10^{10}}\right) \int_{0\ \mathrm{kpc}}^{r_{cone}} (4\,\pi\, r^2)\, n_{BEFOPS} \left(F_{\nu BEFOPS} * \left(\frac{r}{0.01\ \mathrm{kpc}}\right)^{-2}\right) dr \;/.\; \{r_{cone} \text{ -> } (8 + 15)\ \mathrm{kpc}\}$$

Out[6]= $3.09197 \times 10^{-37}\ \mathrm{kg/s^2}$

...and in sensible unit

In[7]:= $\mathrm{UnitConvert}\left[\mathrm{Quantity}\left[3.09197 \times 10^{-37}, \frac{\text{"Kilograms"}}{\text{"Seconds"}^2}\right], \text{"Janskies"}\right]$

Out[7]= $3.09197 \times 10^{-11}\ \mathrm{Jy}$

Converting this to magnitude and we have

In[8]:= $m_{total} = -2.5 * \mathrm{Log10}\left[\frac{F_{\nu total}}{3631\ \mathrm{Jy}}\right]$

Out[8]= 35.1745

Therefore the surface brightness is 35 magnitude per square arcsecond.