Ay 20 Midterm Solutions

Q1 Sun @ RA 0h on vernal equinox (~ March 20). @ 2h / month, the Sun would be at RA ~ 14:40 on Oct 31. The cloud transits ~ 8h earlier than the Sun. (at ~ 4 am). It is observable.

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Q2 (a) Assume 1 arcmin size at 115 GHz and 230 GHz.

Rayleigh - Jeans : $\frac{S_\nu}{\Omega} \propto \nu^2 T_B$.

Spectral index α ($S_\nu \propto \nu^\alpha$) of cloud is

$$\alpha = \frac{\log\left(\frac{10781}{2695}\right)}{\log\left(\frac{230}{115}\right)} = 2. \quad \text{Yes.}$$

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(b) Rayleigh - Jeans :

$$T_b = \frac{S_\nu}{\Omega} \frac{c^2}{2\nu^2 k_B} = \underline{100 \text{ K.}}$$ (1)

(at both frequencies)

$$\frac{\pi}{4} (1 \text{ arcmin})^2 = 6.65 \times 10^{-8} \text{ rad}^2$$

(c) Angular resolution $\approx 1.22 \frac{\lambda}{D} = 1.22 \frac{c}{\nu D}$

$$\left.\begin{array}{l} \nu = 115 \text{ GHz} \Rightarrow 3.65 \text{ arcmin} \\ \nu = 230 \text{ GHz} \Rightarrow 1.82 \text{ arcmin} \end{array}\right\}$$ (1)

So $\underline{\text{no.}}$ (1)

(d) Rayleigh - Jeans :

$\frac{S_\nu}{\Omega} \propto \nu^2$. $\lambda = 29 \mu m \Rightarrow \nu = 1.03 \times 10^{13} \text{ Hz}$

So at $29 \mu m$, $\frac{S_\nu}{\Omega} = \left(\frac{1.03 \times 10^{13}}{1.15 \times 10^{11}}\right)^2 \cdot \frac{2695}{6.65 \times 10^{-8}}$

$$= \underline{3.25 \times 10^{14}} \text{ Jy str}^{-1}$$

(2)

Planck Law :

$$\frac{S_\nu}{\Omega} = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} = \underline{1.15 \times 10^{13} \text{ Jy str}^{-1}}$$

(1)

$\boxed{Q3}$ First solve for D from Stefan - Boltzmann :

$$F = 1.2 \times 10^{-4} \text{ erg s}^{-1} \text{cm}^{-2} = \frac{\sigma T^4 r^2}{D^2}$$

$$\frac{r}{D} = 0.5 \text{ arcmin} = 1.45 \times 10^{-4} \text{ rad.}$$

Then $D = ?$ Use the velocity :

$$\theta = \frac{2r}{D} \Rightarrow \frac{d\theta}{dt} = \frac{2rv}{D^2} \quad \text{and, with}$$

$\Delta\theta = 226$ mas, $\Delta t = 30$ days, $r = 1.45 \times 10^{-4}$ D, $v = 1000 \text{ km s}^{-1}$,

$$D = \underline{6.86 \times 10^{14} \text{ m}}, \quad r = \underline{10^{11} \text{ m}}, \quad d = 2r = 2 \times 10^{11} \text{ m}$$

~ 4600 AU

(6)

$\boxed{\text{Q4}}$ (a) Opacity $\kappa = \frac{n\sigma}{\rho} = \frac{\sigma}{m} = 785.4 \text{ cm}^2 \text{g}^{-1}$

(1) (1)

(b) Hydro equilibrium : $\frac{dP}{dr} = -\frac{G M_r \rho}{r^2}$

Assuming radiation pressure only,

$$\frac{dP}{dr} = \frac{dP}{d\tau} = -\frac{\kappa\rho}{c} F_r = -\frac{\kappa\rho}{c}\sigma T_B^4 .$$

Then

$$-\frac{\kappa\rho}{c}\sigma T_B^4 = -\frac{G M_r \rho}{r^2},$$

and $M_r = \frac{r^2 \cancel{\rho} \kappa \sigma T_B^4}{G \cancel{\rho} c} = 2.2 \times 10^{29}$ g.

(4)

Q5 $L = \sigma T_B^4 \cdot 4\pi r^2 = \underline{7.1 \times 10^{30}\ \mathrm{erg\,s^{-1}}}$.

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Mass in H

$$= \frac{1\ \mathrm{yr} \times 7.1\times10^{30}\ \mathrm{erg\,s^{-1}}}{c^2} \times 4 m_p \times \left(\frac{26.7\,\mathrm{MeV}}{c^2}\right)$$

$$= \underline{3.5 \times 10^{19}\ \mathrm{g}}$$

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Q6 The tidal radius can be estimated by equating the tidal force due to the Sun with the cloud's self-gravity.

$$\text{Tidal force} = R_{cloud} \frac{dF_\odot}{dr}$$

$$= R_{cloud} \frac{2 G M_\odot m_T}{r_T^3}$$

$$\text{Self gravity} = \frac{G M_{cloud} m_T}{R_{cloud}^2}$$

So $r_T = R_{cloud} \left(\frac{2M_\odot}{M_{cloud}}\right)^{1/3} = 2.6 \times 10^{14}\ \mathrm{cm}$
$= 17.5\ \mathrm{AU}$.

5