Ay 20 # 23 - Degenerate dwarf stars

The least luminous stars - white dwarfs and brown dwarfs - have either negligible or non-existent rates of nuclear fusion reactions in their cores. Instead, their cores are supported against collapse by <u>degeneracy pressure</u>.

* Here is a hopefully friendly discussion of electron degeneracy pressure.

In a fully ionized stellar core, consider the fluid of electrons. These are <u>fermions</u> -> obey Fermi-Dirac statistics & Pauli exclusion principle.

Defining e^- wavevector $k = \frac{2\pi}{\lambda} = \frac{p}{\hbar}$

(from de Broglie wavelength), we have

$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} .$$

In a volume (cube) of side L, the lowest-energy state has

$$k_0 = \frac{2\pi}{L} .$$

Then the number of available states up to a wavevector k is

$$N(k) = 2 \cdot \frac{4}{3}\pi k^3 \left(\frac{2\pi}{L}\right)^{-3}$$

2 e^- spins.

Re-arranging, and writing the density of states as $n = N/L^3$,

$$k = \left(3\pi^2 n\right)^{1/3} \; ; \; E = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

We define the Fermi energy as corresponding to a k such that each state from $k_0 \to k$ is occupied. That is,

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$ with n now as the electron number density.

(usually the case for zero temperature)

Finally, the resulting degeneracy pressure can be derived by thinking of pressure as an energy density:

$$P = \frac{E_{total}}{V} = n\bar{E},$$ where $\bar{E}$ is the average particle energy.

Writing $\bar{E} \approx \frac{1}{2} E_F$, the electron degeneracy pressure is

$$P_e \approx \frac{(3\pi^2)^{2/3} \hbar^2}{4m} n^{5/3}.$$

Note the several approximations! But in reality, $4 \to 5$:).

Degeneracy pressure is relevant when $kT_e \lesssim E_F$.

$\Rightarrow$ low temperatures (brown dwarfs) or high densities (white dwarfs).

Also note, from a simple application of hydrostatic equilibrium,

$$\frac{dP}{dr} = -\frac{G M_r \rho}{r^2}$$

$$\Rightarrow P_c \approx \frac{G M \bar{\rho}}{2R}$$ Stellar mass & radius.

and with $P_e \propto \left(\frac{M}{R^3}\right)^{5/3}$,

$$\frac{M}{R^3}\frac{M}{R} \propto \frac{M^{5/3}}{R^5} \quad \text{and} \quad \underline{R \propto M^{-1/3}}$$

or MR^3 is constant

This is weird, right?

In a problem set, you will estimate the central pressures, temperatures and densities of white & brown dwarfs.

For now, consider only the observed features.

* White dwarfs.

Masses from $0.2 - 1.33\ M_\odot$, with most between $0.5 - 0.7\ M_\odot$.

Radii of 1-2 % of $R_\odot$.

(note gravitational redshift $z \sim \frac{R_s}{R}$ of light from surface!)

Effective Temperatures of ~ 20,000 K, ranging between 10,000 and 50,000 K.

Spectral classification:

	Primary feature		Secondary feature
D +	A (H-lines),	+	Any primary feature
	B (He I lines),		
	C (no lines),		P (polarization)
	O (He II lines)		H (Zeeman splitting no P)
	Z (metal lines)		E (emission lines)
	Q (carbon lines)		V (variable)
	X (who knows?)		

+ Temperature index $\left(\frac{50400\ K}{T_{eff}} \right)$.

Luminosities of $10^{-4} - 10^{-2}\ L_\odot$.

* Brown dwarfs have mass between 13 – 80 Jupiter mass ($M_J = 1.9 \times 10^{30}$ g), and fuse deuterium (^2H) and even lithium (^7Li) for $> 65\ M_J$ objects.

First discovery using Palomar 60-inch by Nakajima, Oppenheimer, Kulkarni et al. (Gliese 229 B). Lithium Test (i.e. not burnt). Methane also retained.

Spectral classification: M 6.5 & later (TiO & VO); L (FeH, CrH, MgH, CaH); T - NIR spectra (H_2O, CO for L dwarfs → CH_4 for T dwarfs); Y dwarfs?

T_{eff} from 300 – 2200 K.
Luminosities less than $10^{-4}\ L_\odot$ in optical.
Radii ~ tens of % Jupiter radii
Pressure actually dominated by Coulomb repulsion at low-mass end.

* Maximum mass of white dwarfs
(the Chandrasekhar limit)

Consider the case where the core density is so high that electrons @ the Fermi energy have to move close to the speed of light.

Then, applying the relativistic energy formula,

$$E \approx pc = k\hbar c \text{ , and}$$

$$E_F \approx (3\pi^2 n)^{1/3} \hbar c .$$

Consider a constant density star in hydro equilibrium. Let

$$n = \frac{Y_e \rho}{m_H} = \frac{Y_e}{m_H}\left(\frac{3M}{4\pi R^3}\right)$$

Then, balancing the central pressure due to gravity with the relativistic degeneracy pressure —

$$P_c \sim \frac{GM\rho}{2R} = \frac{\hbar c}{2}(3\pi^2)^{1/3} n^{4/3} ,$$

we re-arrange everything to find M:

$$\frac{GM}{2R} = \frac{\hbar c}{2}(3\pi^2)^{1/3}\left(\frac{Y_e}{m_H}\right)^{4/3}\left(\frac{3M}{4\pi R^3}\right)^{1/3}$$

The R drops out - this is concerning ...

$$M = \frac{3}{2}\left(\frac{\hbar c}{G}\right)^{3/2}\pi^{1/2}\left(\frac{Y_e}{m_H}\right)^2$$

$$\sim 5 Y_e^2 M_\odot$$

In fact, $M \sim 5.7 Y_e^2 M_\odot$.

There is no mass-radius relation for stars supported by relativistic-electron degeneracy pressure - they collapse!

For non-H cores, $Y_e = 1/2$. So

$M > 1.4 M_\odot$ stars cannot exist as white dwarfs.