Ay 20 # 18 - Dark Matter

Diffuse distributions of mass can be traced by

* Implications of dynamics
* Gravitational lensing

[A] Dynamics

We have already discussed the dynamical mass of a virialized system:

$$V_{circ}^2(r) = r\frac{d\Phi}{dr} = G\frac{M(r)}{r}.$$

(virial theorem)
$\langle T\rangle = -\frac{1}{2}\langle V\rangle$

Dynamical masses: → measured l.o.s.

$$M_{dyn} = \frac{\beta\,\sigma_e^2\,R_e}{G},$$

where the e subscript refers to an effective radius. $\beta \sim 5$ for spheroidal galaxies.

Zwicky (1933) & Smith (1936) used this technique to show unusually high M/L ratios for the Coma & Virgo clusters

$$(M/L)/(M/L)_\odot \sim 500!$$

Rotation curves were the major driver towards an acceptance of dark matter.

Keplerian : $V(r) \propto r^{-1/2}$, or even self-gravitating disk models, not consistent with $V(r) =$ constant of most galaxies.

$$\text{If } G\frac{M(r)}{r} = K,$$

$$M(r) \propto r, \text{ and as Volume} \propto r^3,$$

$$\rho(r) \propto \frac{1}{r^2}. \text{ SIS!}$$

$\boxed{B}$ Gravitational lensing

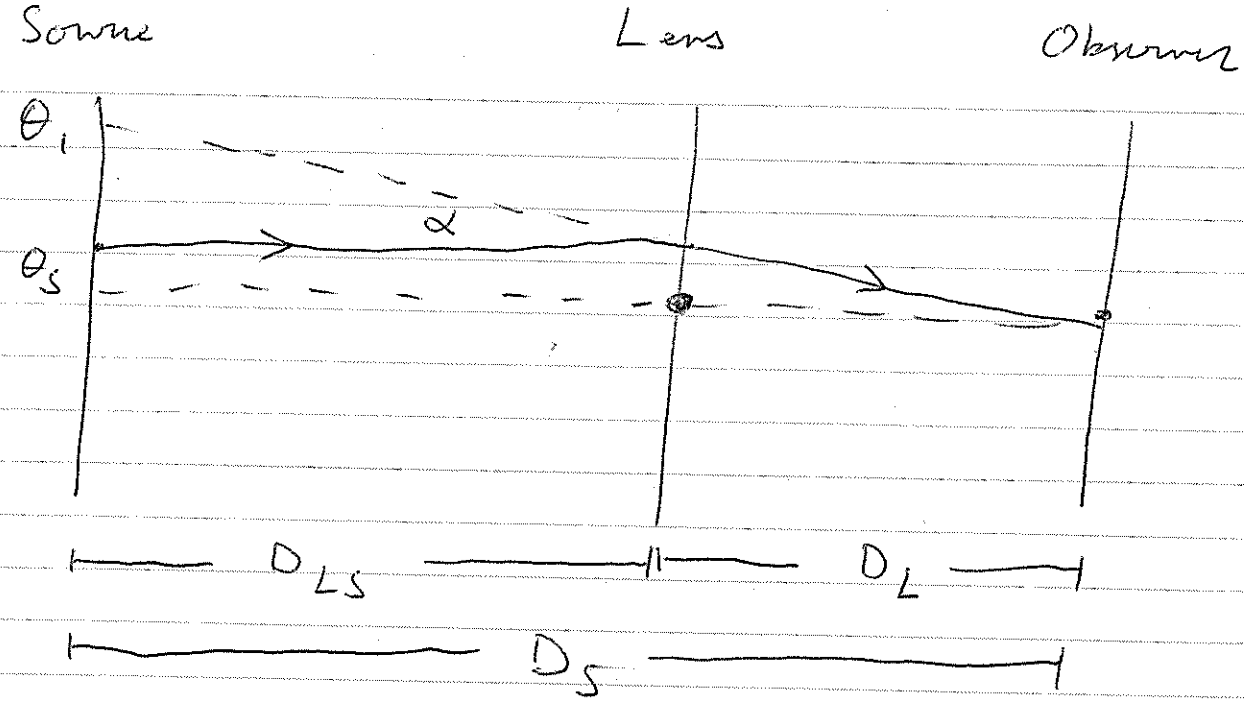
Zwicky (1937) - large M/L of clusters could be confirmed through lensing
Done in 1979!

Deflection angle of light

$$\alpha\ \phi = \frac{4GM}{c^2 b}$$

$b \nearrow$ impact param.

How does the Newtonian treatment + $E = mc^2$ compare?



Lens equation : $\theta_1 D_S = \theta_S D_S + \alpha D_{LS}$

and $$\alpha = \frac{4GM}{c^2 b} = \frac{D_S}{D_{LS}} (\theta_1 - \theta_S)$$

Solving for the angle θ_1 when $\theta_S = 0$, and letting $b = \theta_1 D_L$,

$$\theta_1 = \frac{D_{LS}}{D_L D_S} \frac{4GM}{c^2 \theta_1}$$

$$\Rightarrow \theta_1 = \left(\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S} \right)^{1/2}$$

$R_E = \theta_1 D_S$ is the Einstein radius.

(3) Dark matter near Sun ~ 0.4 GeV/cm^3. (7×10^{-25} g)