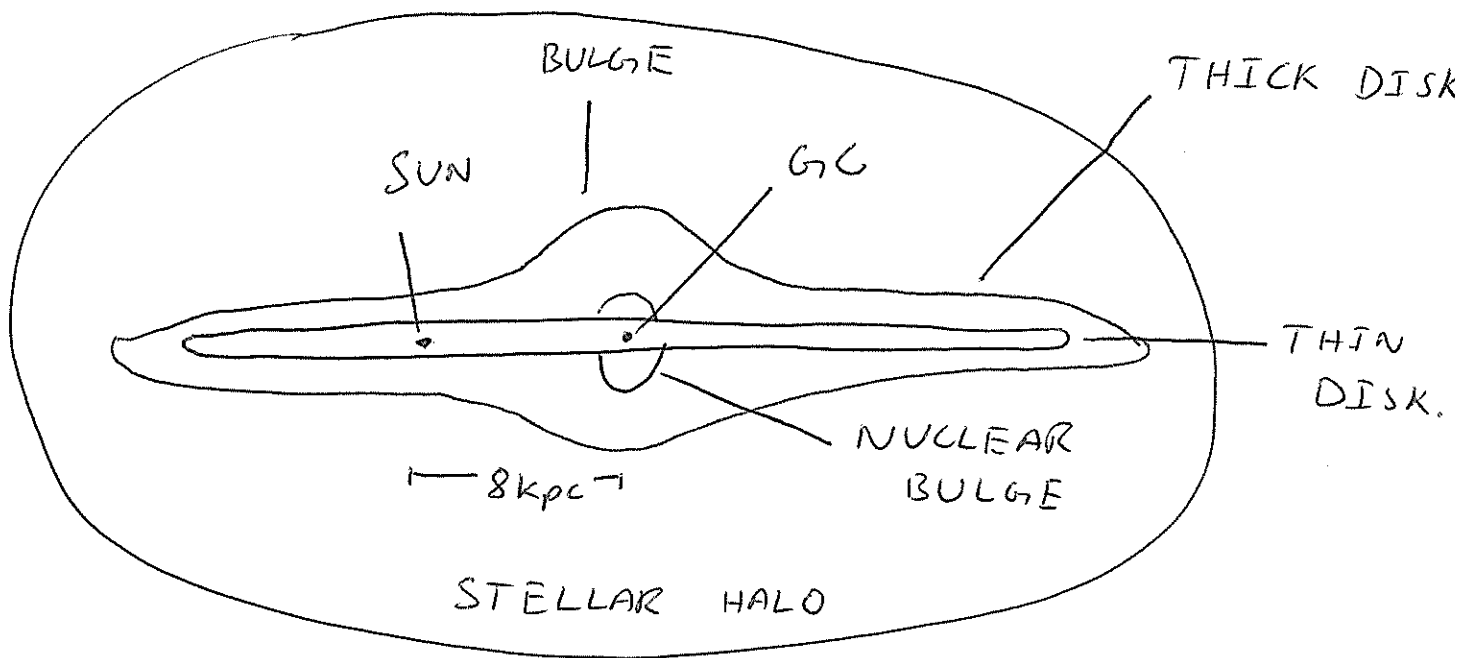
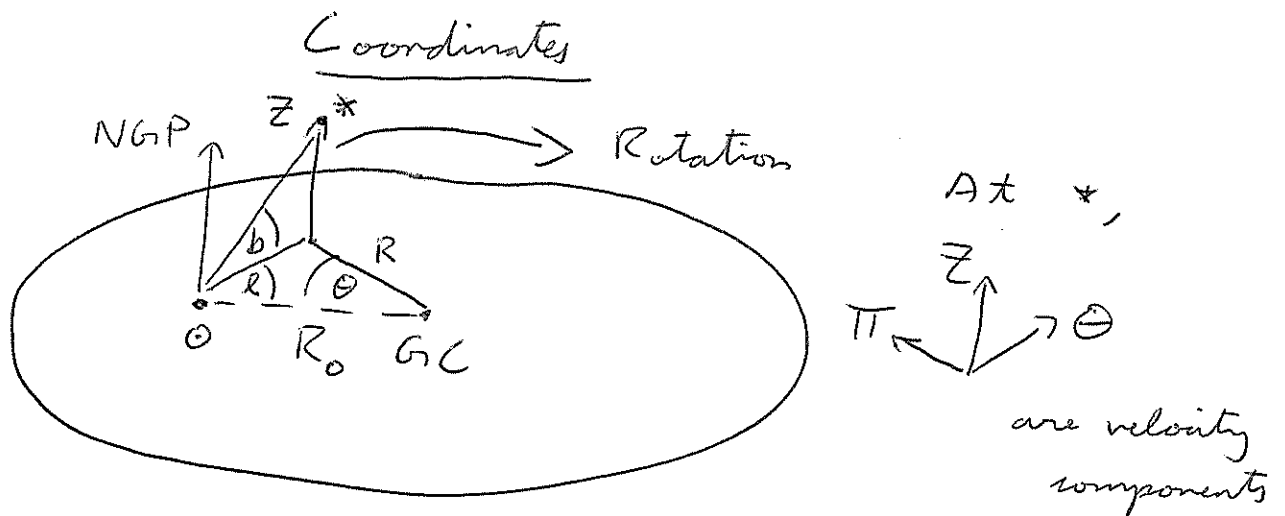


Aug 20 #17 - The Milky Way



Stellar distributions of the Milky Way.

Modeling the structure and kinematics of the Milky Way, together with the detailed information in hand on stellar populations, provides our best insight into the processes of star and galaxy formation.



Spheroids

The singular isothermal sphere is the most common and simplest description of an equilibrium stellar system lacking bulk angular momentum.

Define a potential (for a point mass) as

$$\Phi(R) = -\frac{GM}{R}.$$

Stellar systems can be modelled using Poisson's equation:

$$\nabla^2 \Phi = 4\pi G \rho \quad (\text{general})$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Phi}{\partial r} \right) = 4\pi G \rho \quad (\text{spherically symmetric}).$$

Consider a spheroidal stellar system in hydrostatic equilibrium, with an isothermal equation of state

$$P = K \rho.$$

$$\text{Then } \frac{dP}{dr} = K \frac{d\rho}{dr} = -\frac{GM_r \rho}{r^2} = -\rho \frac{d\Phi}{dr}.$$

Integrating, we have

$$\int_0^R \frac{d\Phi}{dr} dr = \int_0^R -\frac{k}{g} \frac{dg}{dr} dr \Rightarrow \Phi(R) = -k \ln \frac{g}{g_c},$$

where at $R=0$, $\Phi=0$ and $g=g_c$.

In turn, $g = g_c \exp\left(-\frac{\Phi}{k}\right)$.

Substituting for Φ in the Poisson equation,

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} (\ln g) \right) = - \frac{4\pi G g}{k}.$$

(what happened to g_c ?)

The solution is

$$g(r) = \frac{k}{2\pi G r^2}, \text{ which is singular at } r=0.$$

Spherical potentials are characterized by $\Phi(r)$, $g(r)$ and $v_{\text{vir}}(r) = \left(r \frac{d\Phi}{dr} \right)^{1/2} = \left(\frac{GM(r)}{r} \right)^{1/2}$.

Various potentials exist! Plummer, Hernquist, Jaffe ...

Disks

The stellar disk of the Milky Way has a stellar number density that scales exponentially with height :

$$n(z, R) = n_0 \left(e^{-z/z_1} + 0.02 e^{-z/z_2} \right) e^{-R/h_R}$$

$$z_1 \sim 325 \text{ pc (thin disk)}$$

$$z_2 \sim 1.4 \text{ kpc (thick disk)}$$

$$h_R \sim 3.5 \text{ kpc (scale length)}$$

The Sun is at $R_0 \sim 8 \text{ kpc}$, $z \sim 30 \text{ pc}$.

What happens when the Sun plunges through the disk midplane?

* The rotation curve, $\Theta(R)$, is the most critical kinematic observable of the disk.

First, define the Local Standard of Rest (LSR)

as

$$\Pi_{\text{LSR}} \equiv 0, \quad \Theta_{\text{LSR}} \equiv \Theta_0, \quad z_{\text{LSR}} \equiv 0.$$

Stars have peculiar velocities

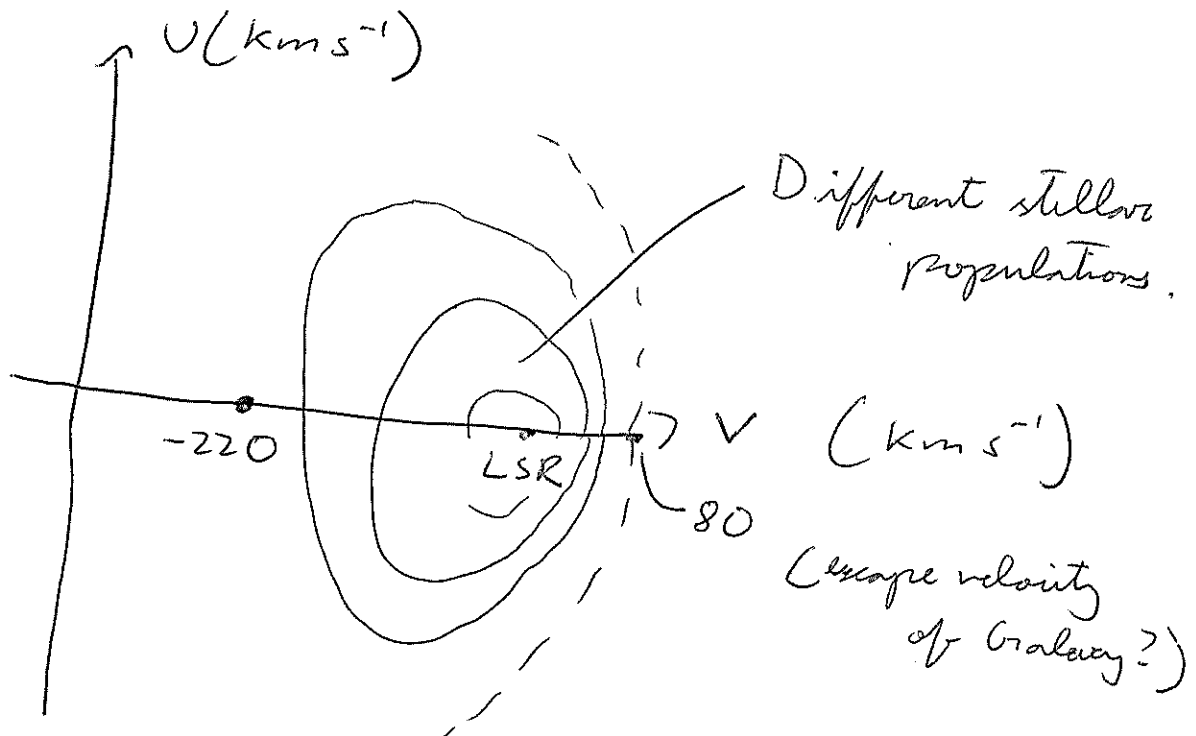
$u = U$, $v = \Theta - \Theta_0$, $w = Z$; note that

$U_0 = -9 \text{ km s}^{-1}$, $V_0 = 12 \text{ km s}^{-1}$, $W_0 = 7 \text{ km s}^{-1}$

How can this be derived from observations of stars in the solar neighborhood?

How is Θ_0 derived?

What do plots of stars in $u-v$ and $w-v$ space look like?



By analysing such distributions with a model for the disk potential, $\Theta_0 = 220 \text{ km s}^{-1}$.

Jan Oort considered the problem of deriving
 $\Omega_0 = \frac{\Theta_0}{R_0}$ and $\frac{d\Theta}{dR} \bigg|_{R_0}$ from observations of
 solar neighborhood stars.

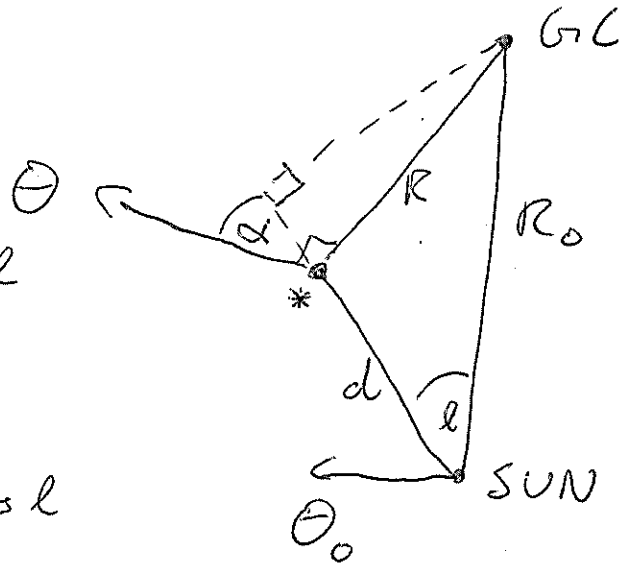
In this geometry,

radial velocity

$$V_r = \Theta \cos \alpha - \Theta_0 \sin l$$

transverse velocity

$$V_t = \Theta \sin \alpha - \Theta_0 \cos l$$



and

$$V_r = \overbrace{\Omega R \cos \alpha} = \overbrace{\Omega_0 R_0 \sin l}$$

$$V_t = \underbrace{\Omega R \sin \alpha - \Omega_0 R_0 \cos l}_{= -d}$$

$$V_r = (\Omega - \Omega_0) R_0 \sin l$$

$$V_t = (\Omega - \Omega_0) R_0 \cos l - \Omega d.$$

If V_r , V_t & d & R_0 are known, Ω & Ω_0 can
 be solved for. What are the assumptions? Gravitational?

* Oort constants.

In the solar neighborhood $\left(\frac{|R - R_0|}{R_0} \ll 1 \right)$,
things are easier.

With

$$\Omega(R) - \Omega_0(R_0) = \left. \frac{d\Omega}{dR} \right|_{R_0} (R - R_0)$$

to first order,

$$V_r = \left(\left. \frac{d\Theta}{dR} \right|_{R_0} - \frac{\Theta_0}{R_0} \right) (R - R_0) \sin l$$

$$V_t = \left(\left. \frac{d\Theta}{dR} \right|_{R_0} - \frac{\Theta_0}{R_0} \right) (R - R_0) \cos l - \Omega_0 d.$$

Writing $R_0 = d \cos l + R$ ($d \ll R_0$),

and

$$A = -\frac{1}{2} \left(\left. \frac{d\Theta}{dR} \right|_{R_0} - \frac{\Theta_0}{R_0} \right)$$

(Oort
constants)

$$B = -\frac{1}{2} \left(\left. \frac{d\Theta}{dR} \right|_{R_0} + \frac{\Theta_0}{R_0} \right),$$

$$V_r = A d \sin 2l, \quad V_t = A d \cos 2l + B d.$$

Thus, $V_r, V_t, d, l \rightarrow A \& B$.

This is important!

$$\text{First, } \Omega_0 = A - B.$$

$$\text{and } \left. \frac{d\theta}{dR} \right|_{R_0} = -(A+B).$$

Additionally, The max radial velocity
is $v_{\max} \sim 2AR_0(1 - \sin i)$
 $\rightarrow R_0$. (much better ways
to measure R_0)

What does the Galactic rotation curve
tell us?