Ay 20 Problem Set 3 Solutions

Preface: only trust the answers up to 2 significant digits unless otherwise noted.

Question 1

Part a

Mass conservation gives

$$4\pi r^2 \rho v = \dot{M}$$

The density profile is thus $\rho(r) = \frac{\dot{M}}{4\pi r^2 v}$ .At 1 AU, the density is

In[ ]:= $\left(\frac{\dot{M}}{4\pi r^2 v} \;/.\; \left\{\dot{M} \to 2 * 10^{-14}\ M_\odot/\mathrm{yr},\ v \to 500\ \mathrm{km/s},\ r \to 100\ \mathrm{au}\right\}\right)$ ~UnitConvert~ $\mathrm{g/cm^3}$

Out[ ]= $8.97 \times 10^{-28}\ \mathrm{g/cm^3}$

Part b

The electrons and protons in the solar wind interact with the neutral Hydrogen atoms in ISM. The mean free path l_{mfp} is then given by

In[ ]:= $\frac{1}{4\pi r_B^2 n} \;/.\; \left\{n \to 0.5\ /\mathrm{cm^3},\ r_B \to 5 * 10^{-9}\ \mathrm{cm}\right\}$

Out[ ]= $6.3662 \times 10^{15}\ \mathrm{cm}$

In parsec (3×10^{18}cm), this is 0.002 pc.

Part c

It will take many times the mean free path for the solar wind electrons and protons to interact with the ISM enough to lose their kinetic energy. This implies that the hot solar wind electrons and protons will survive well past 0.002pc~400AU. Considering that the heliosphere termination shock (wave?) is about 90 AU from the sun, this is a considerable distance into the ISM.

Question 2

Part a

The outgoing thermal radiation luminosity is given by $4\,\pi\,R_E^{\,2}\,\sigma_{SB}\,T^4$, where $R_E$is the radius of the earth and T the temperature. The incoming solar power that is not reflected is given by $(1-a)\,\frac{L_{sun}}{4\,\pi\,d^2}\,(\pi\,R_E^{\,2})$, where d is the distance between Earth and the Sun, and $L_{sun}$the solar luminosity. These two must equal for the Earth to be in an energy equilibrium. Equating the two and we can solve for the temperature T

In[ ]:=
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(((1 - a) L_sun / (16 π σ_SB d^2))^(1/4) /. {a → 0.3, L_sun -> L⊙ , σ_SB → σ , d → 1 au})~UnitConvert~"K"
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Out[ ]= 254.918 K

Note: you can also equate the non-reflected solar flux with the black-body flux from the Earth.

Part b

Wien’s displacement law gives the peak wavelength

In[ ]:=
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λmax = (2898 μm K) / (255 K) // N
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Out[ ]= 11.3647 μm

There are many ways to calculate the optical depth due to the atmosphere, depending on what information you take from the Wikipedia article. The optical depth is given by $\tau\ =\ \int_s^0 \text{-}\kappa\,\rho\,ds = \kappa\,\int_0^s \rho\,d\,s$. The integral corresponds to the column density of CO2 along the atmosphere, which --if we treat the atmosphere as a thin shell -- is $m/4\,\pi r_E^{\,2}$, where m is the mass of CO2 in the atmosphere and r_E the radius of the earth.

In[ ]:=
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τ = κ m / (4 π r_E^2) /. {m -> 620/10^6 * 5 * 10^18 kg , r_E → 6400 km , κ → 100 cm^2/g } // N
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Out[ ]= 60.2271

Part c

Equation 9.53 of C&O gives $T^4 = 3/4\;T_e^{\,4}(\tau_v + 2/3)$. The optical depth on the surfaces of the earth is about what we calculated above. So we end up with a temperature of

In[ ]:= $T = \left(\frac{3}{4} T_e{}^4 (\tau + 2/3)\right)^{1/4}$ /. {$\tau \to 60$, $T_e \to 255$ K}

Out[ ]= 662.901 K

for a CO2 concentration of 620ppm. Furthermore we know that $T \propto \tau^{1/4}$for large τ. A current day CO2 concentration of 407 ppm corresponds to a temperature of 596K. All of these numbers are a lot of Celsius. But CO2 absorption occurs only in a small chunk of frequency, therefore the grey-body optical depth increase on the surface of the earth is much smaller. One can do a Rosseland-mean-opacity-like weighting to figure out the mean optical depth on the surface of the earth (bonus point).