Problem Set 3

Vikram Ravi

Due at start of lecture (11 am) on Oct 28, 2019. Covers material from lectures 6–8.

Question 1. This problem is based on a question in class about how the hot solar wind can extend into a cold interstellar medium.

(a) (2 points) Assuming a spherically symmetric solar wind that carries mass away from the Sun at a rate $\dot{M}_{\rm wind}$ at a velocity $v_{\rm wind}$, derive an expression for the radial density profile of the wind. Given that the Sun has $\dot{M}_{\rm wind} = 2 \times 10^{-14} M_\odot\,{\rm yr}^{-1}$ and $v_{\rm wind} \approx 500\,{\rm km\,s}^{-1}$, what is the number density of the solar wind (assuming fully ionized hydrogen) at its termination shock at a distance of 100 AU from the Sun?

(b) (2 points) What are the approximate mean free paths of protons and electrons in the interstellar medium (ISM), in units of parsecs? You can assume a predominantly neutral ISM composed of hydrogen atoms with a number density of $0.5\,{\rm cm}^{-3}$, and that the scattering cross section of a hydrogen atom is given by $\sim 4\pi r_B^2$, where r_B is the Bohr radius.

(c) (1 point) What does this imply for the survival of the 10^6 K protons and electrons in the solar wind when they mix with the ISM?

Question 2. The aim of this problem is to develop an understanding of the effects of added atmospheric CO_2 on the Earth's surface temperature. This is of course a hot topic, and it's also a demonstration of the wide applicability of the machinery that you're being introduced to.

(a) (3 points) Assuming a mean surface albedo of the Earth of $a = 0.3$ (a is the fraction of reflected sunlight), use the Stefan-Boltzmann law to derive an expression for and calculate the equilibrium temperature of the Earth's surface in the absence of any atmosphere. Equilibrium in this case means that the incoming solar radiation flux must be balanced by the outgoing thermal radiation flux from the Earth's surface. You should be able to get all the physical constants you need from the inside front cover of C&O.

(b) (2 points) At what wavelength would the thermal radiation from the Earth's surface peak in intensity? Assuming that CO_2 has an opacity of $100\,\mathrm{cm^2\,g^{-1}}$ at this wavelength, and that the atmospheric concentration of CO_2 is 620 ppmm (parts per million by mass), what is the approximate optical depth of the Earth's atmosphere due to CO_2? Use information from Wikipedia to aid in your calculation.

(c) (open ended question: ≥ 2 points) Because the atmosphere is optically thick (due to all greenhouse gases, including water, carbon dioxide etc) at the wavelengths corresponding to the thermal radiation, the surface temperature calculated above can be thought of as an effective temperature at the top of the atmosphere. With reference to Equation 9.53 of C&O, discuss (in quantitative terms) the effects of an increasing atmospheric concentration of CO_2.