Problem Set 1

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Due at start of lecture (11 am) on Oct 14, 2019. Covers material from lectures 1–3.

Question 1. (5 points) In Lecture 2 (Oct 4, 2019), we walked through a calculation of the rate at which the Earth's rotation is slowing due to tidal interaction with the Moon. Let's explore this calculation a bit further.

(a) We assumed that the mean tidal height on the Earth due to the Moon was $h = 1\,\mathrm{m}$. Given this, estimate the height of the tides due to the Sun. You will probably need to know that the mean Earth-Moon distance is $3.8 \times 10^{10}\,\mathrm{cm}$, that 1 Astronomical Unit (AU) is $1.5 \times 10^{13}\,\mathrm{cm}$, that the mass of the Moon is $7 \times 10^{25}\,\mathrm{g}$, and that the mass of the Sun is $2 \times 10^{33}\,\mathrm{g}$.

(b) What is the rate at which the Earth's rotation is slowing because of tidal interactions with the Sun?

(c) Considering tidal forces caused by both the Earth and the Sun, what was the difference between a solar (synodic) day and a sidereal day (the length of time it takes for the Earth to complete one rotation with respect to the celestial sphere) for dinosaur astronomers, 100,000,000 years ago? How does your answer compare with this quantity today?

(d) The fact that the Earth is spinning down clearly means that it is losing angular momentum. Considering just the Earth-Moon system, how is the law of conservation of angular momentum satisfied? Discuss the consequences for the Earth-Moon system (and your great-great-...-great grandkids 30,000,000 generations from now) $10^9\,\mathrm{yr}$ into the future.

Question 2. (> 2 points) Early one morning, you wake up with a feeling that something is wrong. Was there something off about the 2 am snack of Chandler leftovers? My word – is it raining?? But your keen sense of intuition soon lands on the correct answer: Newton's law of gravitation has changed!

Although it is still attractive, the magnitude of the gravitational force is now given by

$$F = AM\mu, \tag{1}$$

where $M = m_1 + m_2$ is the total mass of a two-body system, $\mu = m_1 m_2/M$ is the reduced mass, and $A = G/R_E^2$ is the new gravitational constant ($R_E = 6.4 \times 10^8\,\mathrm{cm}$ is the radius of the Earth). The change just so happens to maintain the same acceleration due to gravity on the surface of the Earth as before, but everything else is different.

(a) Assume that the Earth was previously in a circular orbit about the Sun, with an orbital speed of $30\,\mathrm{km\,s^{-1}}$. Assuming the conservation of energy and angular momentum during the change in the law of gravitation, what would the new orbital radius and speed of the Earth be? Along the way, you'll need to write down the new expression for the gravitational potential.

(b) What are the consequences for stars and the Universe of having no reduction in the gravitational force with distance? Discuss in qualitative terms – this is a bonus question where greater insight (not necessarily knowledge) = more points.

Question 3. (3 points) Consider a white dwarf binary system recently discovered by Caltech's Zwicky Transient Facility, ZTF J1539+5027 (Burdge et al. 2019). The system consists of two white dwarfs, of masses $M_1 = 0.61 M_\odot$ and $M_2 = 0.21 M_\odot$, orbiting each other every $415\,\mathrm{s}$ (where the standard symbol, $M_\odot$, is a solar mass). There is evidence that the primary (more massive) star is beginning to accrete helium-rich matter from the secondary white dwarf, such that it may eventually detonate as a rare "double-detonation Type Ia" supernova. If the primary star were to explode today, symmetrically with a typical debris velocity well in excess of $10^4\,\mathrm{km\,s^{-1}}$, what would happen to the secondary? You can assume that the impact of the explosion debris on the secondary is negligible. Calculate the velocity of the secondary, and compare it to the escape velocity of the Milky Way. Assume that ZTF J1539+5027 is located 8 kpc from the Galactic Center (kpc = kiloparsec; $1\,\mathrm{pc} = 3.09 \times 10^{18}\,\mathrm{cm}$), and that the Milky Way can be approximated as a point mass of $1.5 \times 10^{12} M_\odot$.