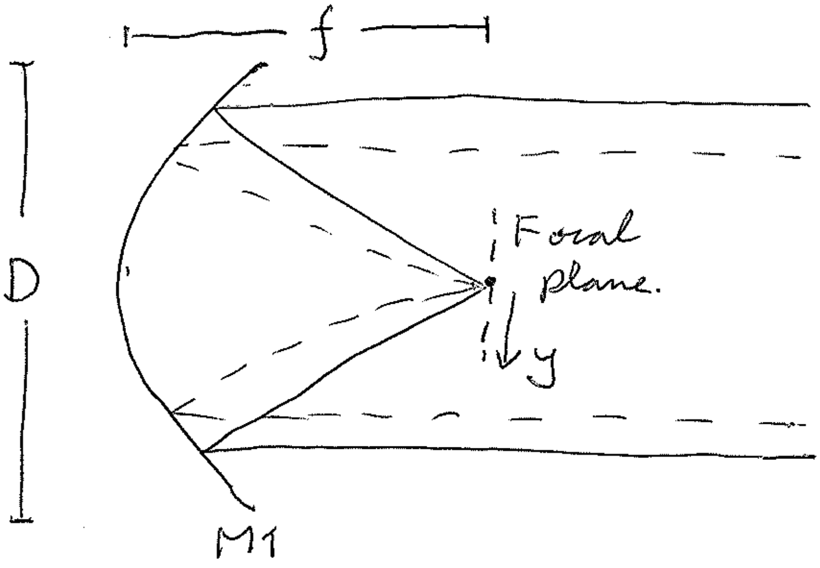
Ay 20 #4 - #5 : Telescopes

The fundamental function of (most) telescopes is to make an image of sources at an infinite distance. This implies plane-parallel incoming wavefronts that need to be "focused".

Astronomers care most about

* Sensitivity to unresolved & resolved sources
* Field of view
* Angular resolution
* Wavelength coverage & resolution
* Dynamic range: $\frac{\text{brightest source}}{\text{noise rms or artifact}}$



Geometry of a classic parabolic reflecting surface.

* Sensitivity $\Leftrightarrow$ signal to noise ratio (S/N).

Consider photon-counting detector (e.g., CCD)

$$S = \eta F_{SRC} \cdot \frac{\pi D^2}{4} \cdot t \qquad \eta \equiv \text{efficiency}, \quad F_{SRC} \equiv \text{source flux}.$$

$$N = \Big(S + \underbrace{\eta B_{SKY} \Omega_{SRC} \frac{\pi D^2}{4} t}_{①} + \underbrace{n_{pix} R_{det} t}_{②} \Big)^{1/2}$$

→ Poisson statistics in limit of large n.

① B_{SKY} is surface brightness of sky (atmosphere & astro) background.

Ω_{SRC} is apparent angular size of source on sky (see Resolution).

② n_{pix} is number of pixels occupied by the source (plate scale $\frac{d\theta}{dy} = \frac{1}{f}$), $\propto f^2 \Omega_{SRC}$

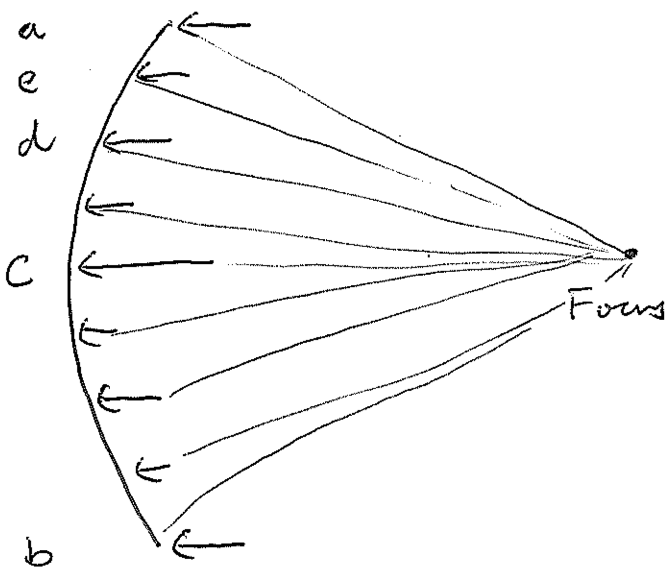
R_{det} is the rate of noise generated in each pixel.

②

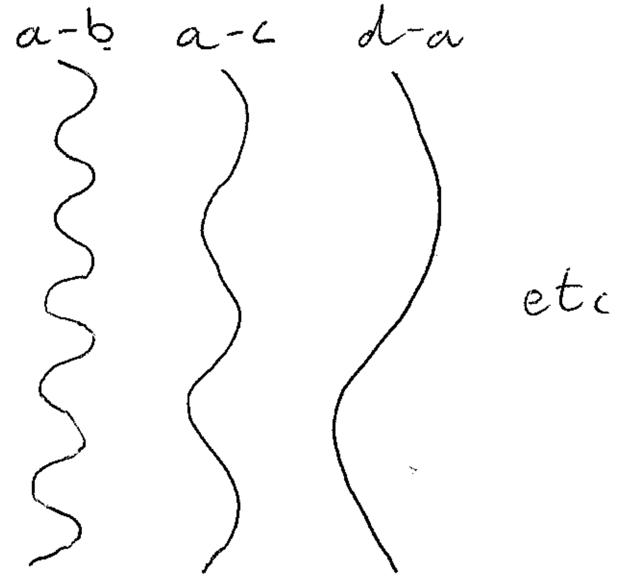
$n \gtrsim 1 \Rightarrow$ Fresnel diffraction,

$n \ll 1 \Rightarrow$ Fraunhofer diffraction.

An intuitive way to think about the Airy rings...



Parabolic mirror



Results of interference of different ray pairs in focal plane.

The "Airy rings" are the result of the summation of all possible interference "fringes" in the focal plane (modulo aberrations).

* Field of view (FOV): complicated to calculate in practise, but related to amount of focal plane that is populated. e.g., $\theta_{FOV} = \dfrac{y_{max}}{f}$.

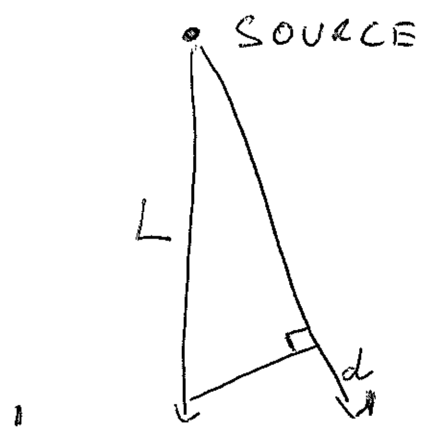
For a focusing eyepiece with a diameter d_y and focal length f_y,

$$\theta_{FOV} = \frac{d_y}{f_y} \underbrace{\left(\frac{f}{f_y} \right)^{-1}}_{\text{magnification}}$$

$$= \frac{d_y}{f} \quad \text{(as before)}.$$

* Angular resolution.

Usually considered in the Fraunhofer diffraction regime.



For an aperture D illuminated by a source @ L,

$$d = \frac{D^2}{4L}.$$

Letting $n\lambda = \dfrac{D^2}{4L} \Rightarrow n = \dfrac{D^2}{4\lambda L}$,

where n is the Fresnel number.

Consider the 1D case. Max fringe spacing $= \infty$.

The min fringe spacing can be derived from the standard double slit experiment:

$$\Delta y_{min} = \frac{f\lambda}{D}. \qquad \text{(Fraunhofer regime)}$$

Change variables to θ (on the sky):

$$y = f\theta \implies \Delta\theta_{min} = \frac{\lambda}{D}.$$

For each set of fringes, define wavevector

$$k = \frac{2\pi}{\Delta\theta}. \qquad k_{min} = 0, \quad k_{max} = \frac{2\pi D}{\lambda}.$$

The appearance of a point of light in the focal plane is then

$$F(\theta) = \int_0^{\frac{2\pi D}{\lambda}} \cos k\theta \, dk$$

$$= \frac{1}{\theta} \sin \frac{2\pi D\theta}{\lambda}. \qquad \text{Sinc function}$$

$\equiv$ 1D Airy function when squared & done in 2D.

This integral is effectively a Fourier transform! In general,

The Fourier transform of an aperture is the "point spread function" (PSF) of a telescope.

Finally, the Rayleigh criterion sets the effective resolution as

$$\theta_{min} = 1.22 \frac{\lambda}{D}$$ (central peak of one PSF inside first minimum of other PSF).

In effect, turbulent cells in the atmosphere cause rapid refractive variations in the optical, causing stars to "twinkle". $\Rightarrow$ $\sim 0.5'' - 2''$ is best θ_{min}.

High-energy space telescopes

* Various clever optical tricks to achieve angular resolution & sensitivity when normal reflectance & lensing is impossible. e.g., grazing incidence reflection @ < few deg, coded masks,
* Detectors: Geiger counters, scintillators, proportional counters, CCDs, CMOS etc. Microcalorimeters for better energy resolution.
* Sensitivity determined by "effective area" (size + efficiency). Tens to $\sim 10^3$ cm^2.