Ay20 #7 - The solar energy source

... is nuclear burning!

But first, it's important to understand that the maximum luminosity of a gravitationally bound object is calculable without knowledge of the energy source. This is the Eddington luminosity.

Hydrostatic equilibrium: gravity balances pressure;

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}.$$

Radiation pressure due to opacity =>

Opacity is fraction of radiation absorbed per unit density per unit distance. Then,

$$\frac{dP}{dr} = \frac{\bar{\kappa}\rho}{c}\frac{L_\odot}{4\pi r^2}.$$

$\odot$? or any L?

Equating radiation pressure with gravity,

$$L_{Edd} = \frac{4\pi c G M}{\bar{\kappa}}$$

wavelength-averaged opacity.

Thompson-scattering opacity:

$$\kappa_{th} = \frac{n_e \sigma_{th}}{\rho} \simeq \frac{\sigma_{th}}{m_p} \leftarrow \text{proton mass}$$

Then, $$L_{Edd} = \frac{4\pi c G M m_p}{\sigma_{th}}$$; and

$$\frac{L_{Edd}}{L_\odot} = 4 \times 10^4 \frac{M}{M_\odot}$$. So the Sun is really pretty inefficient!

---

* Nuclear burning

The mass of a ^4He atom is $4.002603\,u$, but the mass of an H atom is $4.0313001 3\,u$. $1\,u = {}^{12}C/_{12}$ mass

The binding energy, $Q = \Delta m c^2 = 26.7$ MeV.

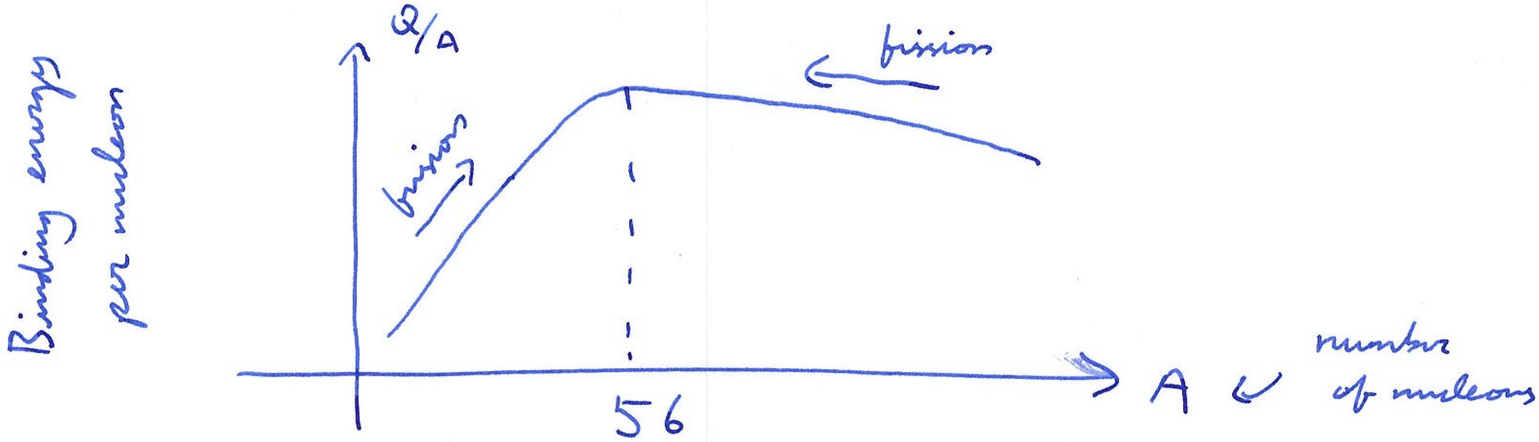
In the age of the Universe ($\sim 1.4 \times 10^{10}$ yr), assuming an initially H-dominated Sun, the total mass of H converted to He to explain the solar luminosity is

$$\frac{1.4 \times 10^{10}\,\text{yr} \times 3.8 \times 10^{33}\,\text{erg s}^{-1}}{c^2} \times 4 m_p \times \left(\frac{26.7\,\text{MeV}}{c^2}\right)$$

$$\simeq 0.1\,M_\odot \sim 2 \times 10^{32}\,\text{g}.$$

In general, $Q(Z, N) = (Z m_p + N m_n - m(Z,N)) c^2$

protons (Z), neutrons (N)



(Most stable is ^{56}Ni : $Q/A = 8.5$ MeV/nucleon)

Note that $H \rightarrow {}^4He$ releases 6×10^{18} erg g^{-1}, whereas $H \rightarrow {}^{56}Ni$ releases only 8×10^{18} erg g^{-1}. The first reaction is the most efficient!

* Proton - proton chains

Ultimately, $4H \rightarrow {}^4He + 2e^+ + 2\nu_e + 2\gamma$.

(recall conservation laws)

P-P I : 69 % prob. P-P II : 31 %.

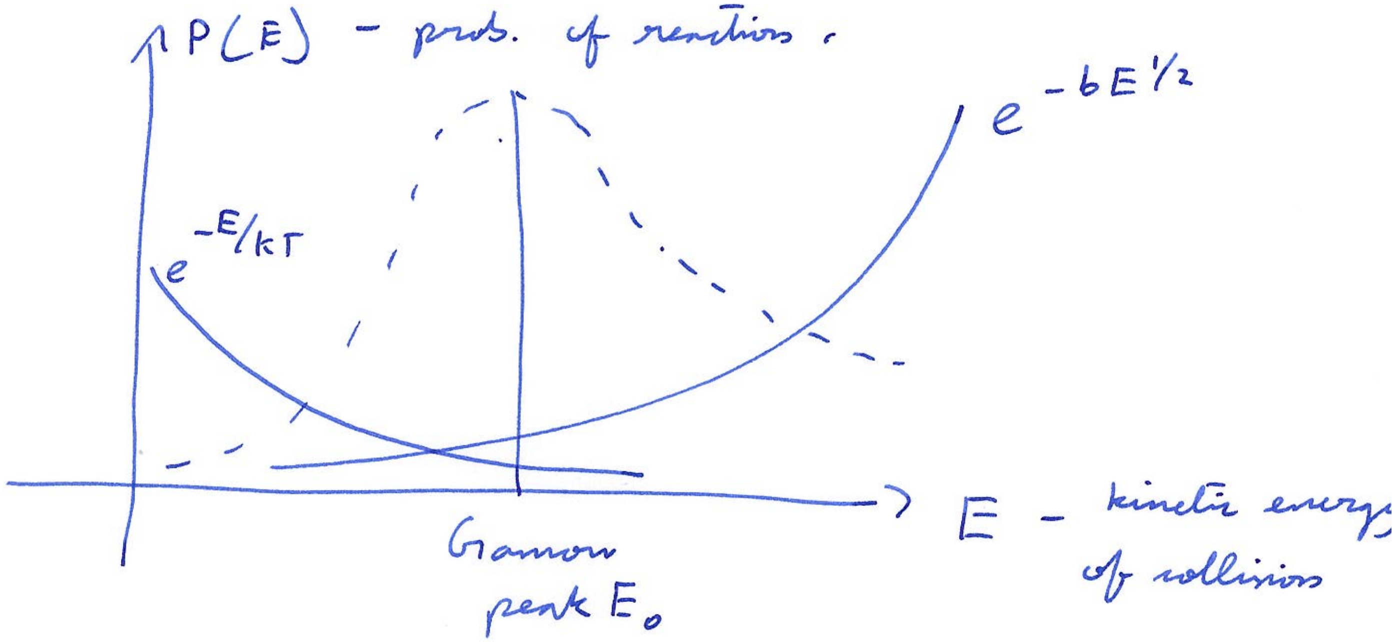
P-P III : 0.3 %.

Ultimately, 26.2 MeV / ^4He is released.

(rest to byproducts).

* The CNO cycle is a second method of forming ^4He from H, with C, N & O isotopes as catalysts.

* The Gamow peak:



Combination of Maxwell-Boltzmann distribution of particle energies, and quantum tunneling through Coulomb potential. $E_0 = \left(\frac{bkT}{2}\right)^{2/3}$.

In general, reaction rates scale as $\rho^2 T^\beta$, where $\beta = 4$ (P-P), 19.9 (CNO), 41 for He burning, etc.

Mass loss estimate - if $0.1 M_\odot$ is converted from H to He, what is the loss in mass? How does this compare to the solar wind mass-loss rate of $\sim 2\times10^{-14} M_\odot / yr$?