Ay 20 #10 - Equations of Stellar Structure

Zones of the Sun:

Nuclear burning core | Radiative Zone | Convective Zone | Photosphe Chromosph Corona Wind

$\longrightarrow r$: $\frac{dP}{dr}$, $\frac{dM_r}{dr}$, $\frac{dL_r}{dr}$, $\frac{dT}{dr}$

We are interested in tracing P (pressure), M_r & L_r (mass & radiation flux), and T (temperature) with radius. These depend on the physics determining pressure (ρ, T, composition), opacity (ρ, T, composition), and the generation of energy (ρ, T, composition).

You've seen all the equations:

Hydrostatic equilibrium: $\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$.

Mass profile: $\frac{dM_r}{dr} = 4\pi r^2 \rho$ (problem set 3)

Radiation flux through spherical shell:

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \varepsilon$$ ← energy generation / time / density.

Temperature gradient in 1D (plane parallel)

LTE star: $$\frac{dT}{dr} = -\frac{3 \bar{\kappa}_R \rho L_r}{64 \sigma \pi T^3 r^2}$$ (radiation dominated)

Temperature gradient with adiabatic convection:

$$\frac{dT}{dr} = -\left(1 - \frac{1}{\gamma}\right) \frac{\mu m_H}{k} \frac{G M_r}{r^2}.$$

To gain some familiarity with these equations ...

* What is a crude estimate of the pressure at the center of the Sun?
* How would you then estimate the temperature?
* Finally, what about density?