Problem Set 5

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Due at start of lecture (11 am) on Friday Dec 6, 2019.

This short problem set is intended to highlight some points from the final two weeks of lectures, as well as revise some earlier concepts.

Question 1. (3 points) Thermal emission observed from Class II protoplanetary disks implies gas temperatures of $\sim 50\,\mathrm{K}$, radial extents of $\sim 100\,\mathrm{AU}$, and thicknesses of $\sim 5\,\mathrm{AU}$. Use an argument based on the Jeans mass, and assuming that the typical planet to form directly out of such disks has a Jupiter mass ($2 \times 10^{30}\,\mathrm{g}$), to estimate the characteristic density of such disks.

Question 2. (4 points) In your own words, describe your understanding of the conditions for convection in stars. Why does increasing H^- opacity in stellar atmospheres lead to increased convection and the existence of the Hayashi track?

Question 3. (4 points) Assuming that stars have constant densities throughout, estimate the central pressure and temperature of a white dwarf (mass of $0.5M_\odot$, radius of $0.01R_\odot$) and a brown dwarf (mass of $50M_J$ where M_J is a Jupiter mass, radius of $1.1R_J$ where $R_J = 7.15 \times 10^9\,\mathrm{cm}$ is a Jupiter radius). How do the central pressures compare with your estimates of the electron-degeneracy pressures in these stars?

Question 4. (3 points) Read the Wikipedia article on pulsational pair-instability supernovae here. Now, core-collapse supernovae in general are bright UV sources for brief instants when the shocks emerge from the stellar surfaces. When a star that had previously undergone a pulsational pair-instability mass-loss episode finally explodes, the resulting UV pulse can photoionize the mass that has been previously lost. What are the observable consequences of this effect?

Derive a condition for the supernova ejecta to shock the mass previously lost, in terms of the temperature of the latter and the velocity of the supernova ejecta. Do you expect this condition to be generally satisfied?

Question 5. For this question, you can assume that 10^5 core-collapse supernovae occur per cubic Gigaparsec per year in the Universe, and two core-collapse supernovae occur per century in the Milky Way.

(a) (2 points) Estimate the cosmic energy-density in neutrinos generated in supernovae. How does this compare to the energy-density of the cosmic microwave background, which is blackbody radiation with a temperature of $2.7\,\mathrm{K}$? Do you expect photons and neutrinos to have the same or different equations of state (pressure-density relations)?

(b) (2 points) Assuming a Salpeter Initial Mass Function, how many neutron stars and black holes do you think have been formed in the Milky Way over its 13.5 billion year history?