CHAPTER 7 - COORDINATE GEOMETRY

Excercise 7.1

Q4.Check whether (5,-2),(6,4) and (7,-2) are the vertices of an isosceles triangle:

Solution:

1. In an Isosceles triangle, If any 2 of the 3 sides of triangle are be equal then it satisfies the condition.Let us assume the given three points be,

$$\mathbf{A}, \mathbf{B}, \mathbf{C} \tag{1}$$

Now, the direction vectors of AB,BC and CA are:

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \tag{2}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \tag{3}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \tag{4}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{5}$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -1 & -6 \end{pmatrix} \begin{pmatrix} -1 \\ 6 \end{pmatrix} = 37 \tag{6}$$

$$(\mathbf{B} - \mathbf{C})^\top (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 1 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = -2 \tag{7}$$

$$(\mathbf{C} - \mathbf{A})^\top (\mathbf{A} - \mathbf{B}) = \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -6 \end{pmatrix} = -2 \tag{8}$$

Therefore,from the above equation we can see that,

$$\mathbf{A} - \mathbf{B} \perp \mathbf{B} - \mathbf{C} \tag{9}$$

$$\angle BCA = \angle CAB \tag{10}$$

Thus, the given points proves that, it is an Isosceles Triangle.

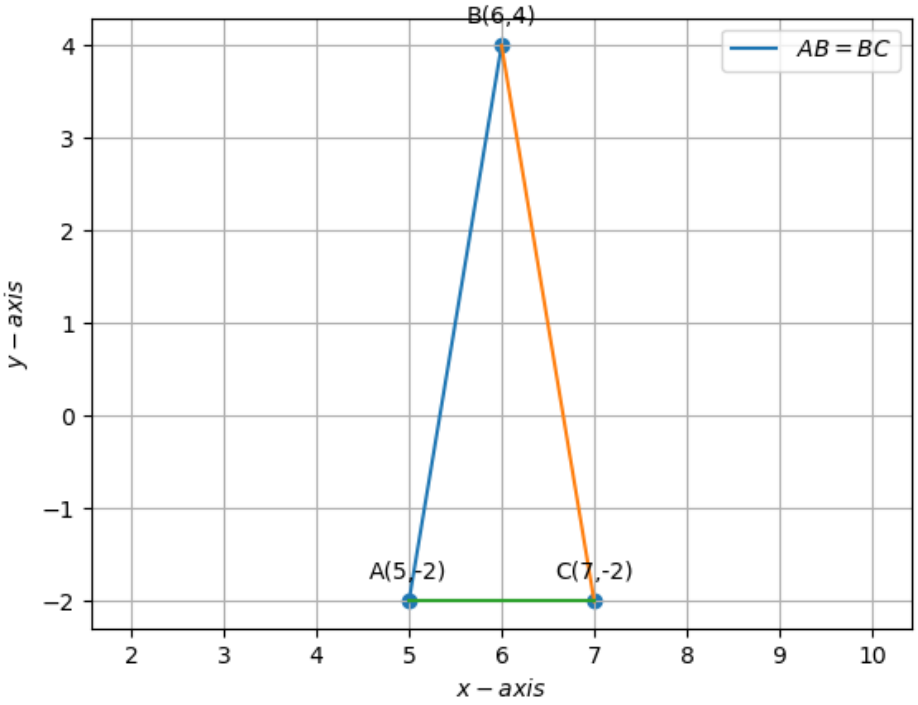

Figure 1: Isoscles Triangle with the given coordinates