

CHAPTER 7 - COORDINATE GEOMETRY

Excercise 7.1

Q4. Check whether (5,-2), (6,4) and (7,-2) are the vertices of an isosceles triangle:

Solution:

1. In an Isosceles triangle, If any 2 of the 3 sides of triangle are equal then it satisfies the condition. Let us assume the given three points be,

$$\mathbf{A}, \mathbf{B}, \mathbf{C} \quad (1)$$

Now, the direction vectors of AB, BC and CA are:

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad (2)$$

Now, making the given points into matrix and solving them to get the rank of matrix. Assuming P is the matrix.

$$\mathbf{P} = \begin{pmatrix} 5 & 6 & 7 \\ -2 & 4 & -2 \end{pmatrix} \xrightarrow{R_2 = \frac{R_2}{2}} \begin{pmatrix} 5 & 6 & 7 \\ -1 & 2 & -1 \end{pmatrix} \xrightarrow{R_1 = \frac{R_1}{5}} \begin{pmatrix} 1 & \frac{6}{5} & \frac{7}{5} \\ -1 & 2 & -1 \end{pmatrix} \quad (3)$$

$$\xrightarrow{R_2 = R_2 + R_1} \begin{pmatrix} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & \frac{16}{5} & \frac{2}{5} \end{pmatrix} \xrightarrow{R_2 = \frac{R_2}{16/5}} \begin{pmatrix} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & 1 & \frac{1}{8} \end{pmatrix} \quad (4)$$

Since we have two linearly independent rows hence Rank of Matrix, Rank(P) = 2 and if the rank is 2, it is proved that it is a Triangle. To prove it is an Isosceles Triangle,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \quad (5)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \quad (6)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (7)$$

From the equations (5) and (6), solving them (Norming and Equating) we get,

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37} \quad (8)$$

Similarly,

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{(-1)^2 + (6)^2} = \sqrt{1 + 36} = \sqrt{37} \quad (9)$$

From (8) and (9), we can say that,

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| \quad (10)$$

Here two sides are equal and therefore we can say that the given points proves that it is an Isosceles Triangle.

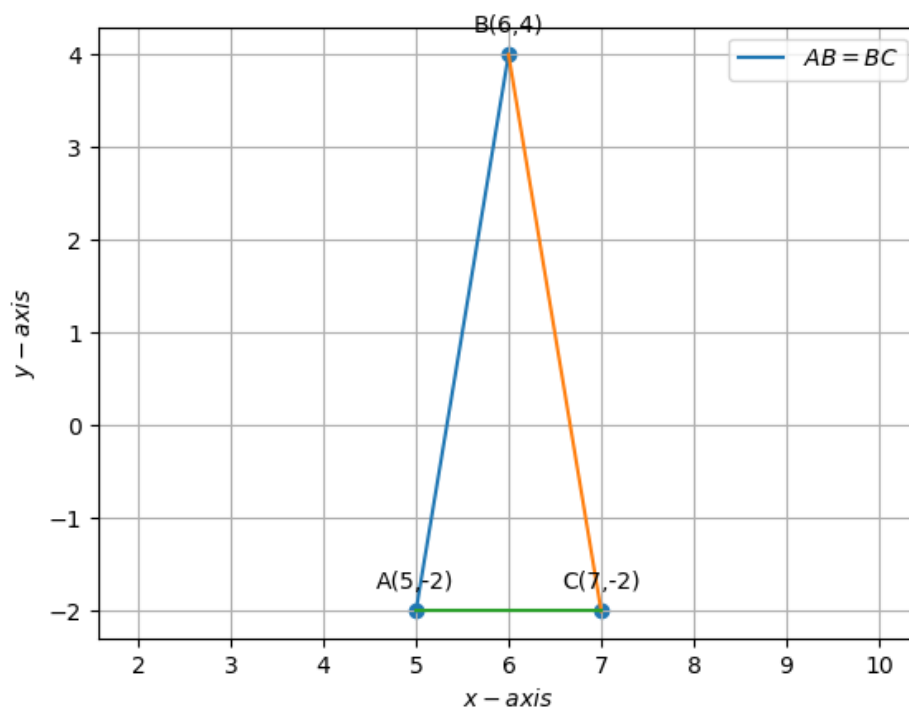


Figure 1: Isosceles Triangle with the given coordinates