Vector Algebra

CHAPTER 10 - VECTOR ALGEBRA

Excercise 10.3

Solution:

1. If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ are unit vectors such that $\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c} = 0$, find the value of $\overrightarrow{a}.\overrightarrow{b} + \overrightarrow{b}.\overrightarrow{c} + \overrightarrow{c}.\overrightarrow{a}$.

1 Solution

The given vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit vectors. Since the given vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are unit vector hence $\mathbf{a} = \mathbf{b} = \mathbf{c}$ which is equal to 1.

$$\|\mathbf{a}\| = \sqrt{1^2} = 1 \tag{1}$$

$$\|\mathbf{b}\| = \sqrt{1^2} = 1 \tag{2}$$

$$\|\mathbf{c}\| = \sqrt{1^2} = 1 \tag{3}$$

The Given equation is

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \tag{4}$$

Squaring on both sides,

$$\|\mathbf{a} + \mathbf{b} + \mathbf{c}\|^2 = 0^2 \tag{5}$$

$$\implies \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + \|\mathbf{c}\|^2 + 2(\mathbf{a}^\top\mathbf{b} + \mathbf{b}^\top\mathbf{c} + \mathbf{c}^\top\mathbf{a}) = 0 \tag{6}$$

$$\implies \mathbf{a}^\top\mathbf{a} + \mathbf{b}^\top\mathbf{b} + \mathbf{c}^\top\mathbf{c} + 2(\mathbf{a}^\top\mathbf{b} + \mathbf{b}^\top\mathbf{c} + \mathbf{c}^\top\mathbf{a}) = 0 \tag{7}$$

$$\implies 1^2 + 1^2 + 1^2 + 2(\mathbf{a}^\top\mathbf{b} + \mathbf{b}^\top\mathbf{c} + \mathbf{c}^\top\mathbf{a}) = 0 \tag{8}$$

$$\implies 3 + 2(\mathbf{a}^\top\mathbf{b} + \mathbf{b}^\top\mathbf{c} + \mathbf{c}^\top\mathbf{a}) = 0 \tag{9}$$

$$\implies \mathbf{ab} + \mathbf{bc} + \mathbf{ca} = \frac{-3}{2} \tag{10}$$

Hence the value of $\overrightarrow{a}.\overrightarrow{b} + \overrightarrow{b}.\overrightarrow{c} + \overrightarrow{c}.\overrightarrow{a}$ is -1.5 or -3/2