

LINES

CHAPTER 11 - STRAIGHT LINES

Excercise 10.3

Solution:

Q14. Find the coordinates of the foot of the perpendicular from $(-1, 3)$ to the line $3x-4y-16=0$.

1 Solution

The given perpendicular point can be taken as $\mathbf{P}(-1, 3)$. Let us assume the coordinates.

$$\mathbf{A}(\mathbf{a}, \mathbf{b}) \tag{1}$$

To find the coordinates of foot of perpendicular from point to line is given as:

$$\begin{pmatrix} m & n \end{pmatrix}^T A = \begin{pmatrix} m^T \\ c \end{pmatrix} \tag{2}$$

Now, line given

$$3x - 4y - 16 = 0 \tag{3}$$

$$3x - 4y = 16 \tag{4}$$

Therefore, it can be equated as

$$\mathbf{n}^T \mathbf{x} = \mathbf{c} \tag{5}$$

where,

$$\mathbf{n} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \mathbf{c} = 16 \quad (6)$$

Here \mathbf{m} is directional vector of the given line

$$\mathbf{m} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad (7)$$

Substituting all values in (4), we get

$$\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix} A = \begin{pmatrix} 4 & 3 \\ & 16 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (8)$$

$$\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix} A = \begin{pmatrix} 5 \\ 16 \end{pmatrix} \quad (9)$$

The augmented matrix for the system equations in (9) is expressed as

$$\left(\begin{array}{cc|c} 4 & 3 & 5 \\ 3 & -4 & 16 \end{array} \right) \xrightarrow{R_2=R_2-\frac{3}{4}R_1} \left(\begin{array}{cc|c} 4 & 3 & 5 \\ 0 & \frac{-25}{4} & \frac{49}{4} \end{array} \right) \quad (10)$$

$$\xrightarrow{R_2=\frac{-4}{25}} \left(\begin{array}{cc|c} 4 & 3 & 5 \\ 0 & 1 & \frac{-49}{25} \end{array} \right) \xrightarrow{R_1=\frac{1}{4}R_1} \left(\begin{array}{cc|c} 1 & \frac{3}{4} & \frac{5}{4} \\ 0 & 1 & \frac{-49}{25} \end{array} \right) \quad (11)$$

$$\xrightarrow{R_1=R_1-\frac{3}{4}R_2} \left(\begin{array}{cc|c} 1 & 0 & \frac{68}{25} \\ 0 & 1 & \frac{-49}{25} \end{array} \right) \quad (12)$$

Hence,

$$\mathbf{A} = \begin{pmatrix} \frac{68}{25} \\ \frac{-49}{25} \end{pmatrix} \quad (13)$$

Thus, $\mathbf{A}(68/25, -49/25)$ is the coordinate from Point \mathbf{P} to the line $3x - 4y - 16 = 0$.

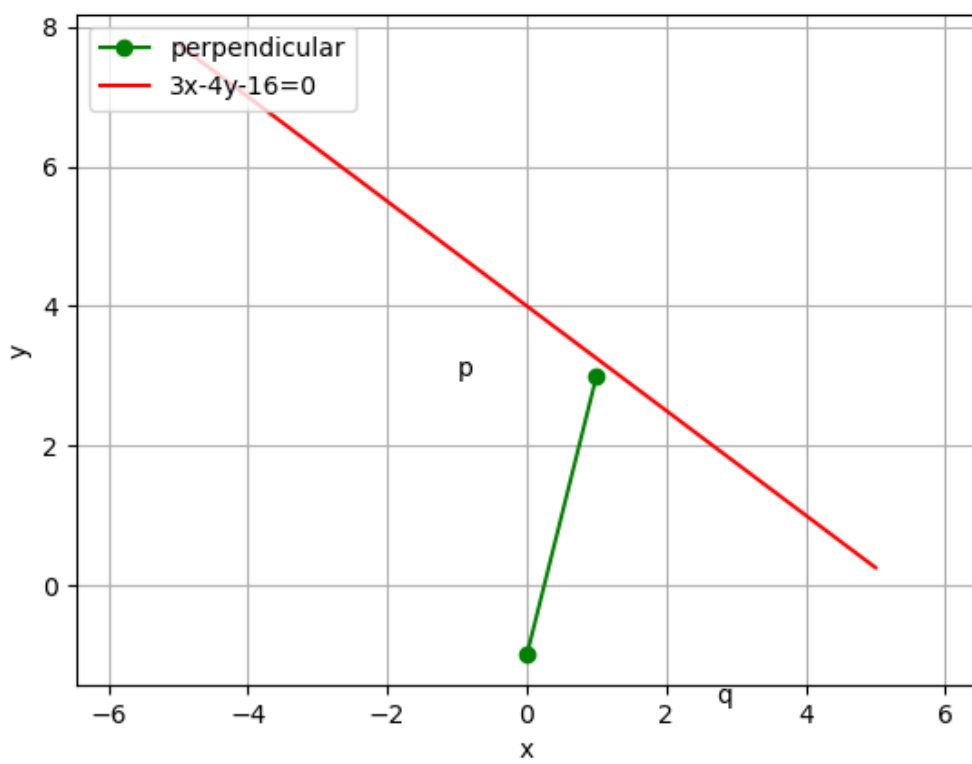


Figure 1: Foot of Perpendicular from point P and given line