

Vector Algebra

CHAPTER 10 - VECTOR ALGEBRA

Exercise 10.2

Solution:

1. Find the direction cosines of the vector joining the points A (1, 2, -3) and B(-1, -2, 1), directed from **A** and **B**. The direction cosines are the cosines of the angles formed by the given vector with the respective axes, given vectors are **A** and **B**.

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} \quad (1)$$

The direction vector **m** of the line joining two points A, B is given by

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix} \quad (2)$$

$$\mathbf{m} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix} \quad (3)$$

$$\|\mathbf{m}\| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = 6 \quad (4)$$

The unit vector is given by

$$\hat{\mathbf{m}} = \frac{\mathbf{m}}{\|\mathbf{m}\|} \quad (5)$$

Hence, the unit vector in the direction of \mathbf{m} is calculated as

$$\frac{\mathbf{m}}{\|\mathbf{m}\|} = \frac{1}{6} \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{-1}{3} \\ \frac{-2}{3} \\ \frac{2}{3} \end{pmatrix} \quad (6)$$

Hence, the direction cosine of vector joining points \mathbf{A} and \mathbf{B} is,

$$\hat{\mathbf{m}} = \begin{pmatrix} \frac{-1}{3} \\ \frac{-2}{3} \\ \frac{2}{3} \end{pmatrix} \quad (7)$$