CHAPTER 7 - COORDINATE GEOMETRY

Excercise 7.1

Q4.Check whether (5,-2),(6,4) and (7,-2) are the vertices of an isosceles triangle:
Solution:

1. In an Isosceles triangle, If any 2 of the 3 sides of triangle are be equal then it satisfies the condition.Let us assume the given three points be,

$$\mathbf{A}, \mathbf{B}, \mathbf{C} \tag{1}$$

Now, the direction vectors of AB,BC and CA are:

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \tag{2}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \tag{3}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \tag{4}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{5}$$

Therefore,from the above equations (3) and (4), solving them(Norming and Equating) we get,

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37} \tag{6}$$

Similarly,

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{(-1)^2 + (6)^2} = \sqrt{1 + 36} = \sqrt{37} \tag{7}$$

From (6) and (7), we can say that

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| \tag{8}$$

Therefore the two sides are equal and thus, the given points proves that it is an Isosceles Triangle. To check wether the given points form a triangle using rank of matrix is, Hence, the two sides are equal and proves to be a Isosceles Triangle.

$$\mathbf{n}^\top \mathbf{x} = \mathbf{c} \tag{9}$$

$$\tag{10}$$

Assuming c=0

$$\begin{pmatrix} -1 & -6 \\ -1 & 6 \\ 2 & 0 \end{pmatrix}^\top \mathbf{x} = 0 \tag{11}$$

$$\begin{pmatrix} -1 & -1 & 2 \\ -6 & 6 & 0 \end{pmatrix} \xrightarrow{R_1=\frac{R_1}{-1}} \begin{pmatrix} 1 & 1 & -2 \\ -6 & 6 & 0 \end{pmatrix} \tag{12}$$

$$\xrightarrow{R_2=R_2-6R_1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 12 & -12 \end{pmatrix} \xrightarrow{R_2=\frac{R_2}{12}} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix} \tag{13}$$

$$\xrightarrow{R_1=R_1-R_2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix} \mathbf{x} = 0 \tag{15}$$

From equation 15, we get two equations $x_1 - x_3$=0 and $x_1 - x_3$=0.Thus,

$$\mathbf{x1} - \mathbf{x3} = 0 \tag{16}$$

$$\mathbf{x1} - \mathbf{x3} = 0 \tag{17}$$

$$\mathbf{x1} = \mathbf{x3} \tag{18}$$

$$\mathbf{x1} = \mathbf{x3} \tag{19}$$

From, this we can conclude that the given points are Isosceles Triangle.

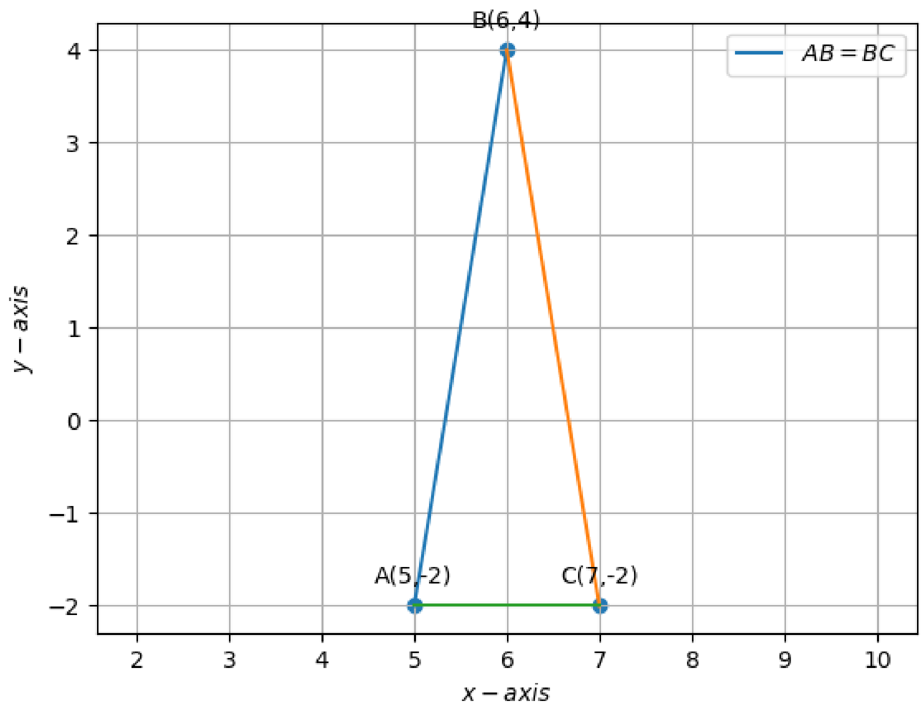

Figure 1: Isosceles Triangle with the given coordinates