CHAPTER 7 - COORDINATE GEOMETRY

Excercise 7.1

Q4.Check whether (5,-2),(6,4) and (7,-2) are the vertices of an isosceles triangle:
Solution:

1. In an Isosceles triangle, If any 2 of the 3 sides of triangle are be equal then it satisfies the condition.Let us assume the given three points be,

$$\mathbf{A}, \mathbf{B}, \mathbf{C} \tag{1}$$

Now, the direction vectors of AB,BC and CA are:

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \tag{2}$$

Now, making the given points into matrix and solving them to get the rank of matrix.Assuming P is the matrix.

$$\mathbf{P} = \begin{pmatrix} 5 & 6 & 7 \\ -2 & 4 & -2 \end{pmatrix} \xrightarrow{R_2=\frac{R_2}{2}} \begin{pmatrix} 5 & 6 & 7 \\ -1 & 2 & -1 \end{pmatrix} \xrightarrow{R_1=\frac{R_1}{5}} \begin{pmatrix} 1 & \frac{6}{5} & \frac{7}{5} \\ -1 & 2 & -1 \end{pmatrix} \tag{3}$$

$$\xrightarrow{R_2=R_2+R_1} \begin{pmatrix} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & \frac{16}{5} & \frac{2}{5} \end{pmatrix} \xrightarrow{R_2=\frac{R_2}{16/5}} \begin{pmatrix} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & 1 & \frac{1}{8} \end{pmatrix} \tag{4}$$

Since we have two linearly independent rows hence Rank of Matrix,Rank(P) = 2 and if the rank is 2, it is proved that it is a Triangle.

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \tag{5}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \tag{6}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{7}$$

From the equations (5) and (6), solving them(Norming and Equating) we get,

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37} \tag{8}$$

Similarly,

$$\|\mathbf{B}-\mathbf{C}\| = \sqrt{(-1)^2+(6)^2} = \sqrt{1+36} = \sqrt{37} \tag{9}$$

From (8) and (9), we can say that,

$$\|\mathbf{A}-\mathbf{B}\| = \|\mathbf{B}-\mathbf{C}\| \tag{10}$$

Here two sides are equal and therefore we can say that the given points proves that it is an Isosceles Triangle. To check wether the given points form a triangle using rank of matrix is, Hence, the two sides are equal and proves to be a Isosceles Triangle.

$$\mathbf{n}^\top \mathbf{x} = \mathbf{c} \tag{11}$$

Assuming c=0

$$\begin{pmatrix} -1 & -6 \\ -1 & 6 \\ 2 & 0 \end{pmatrix}^\top \mathbf{x} = 0 \tag{12}$$

$$\begin{pmatrix} -1 & -1 & 2 \\ -6 & 6 & 0 \end{pmatrix} \xrightarrow{R_1=\frac{R_1}{-1}} \begin{pmatrix} 1 & 1 & -2 \\ -6 & 6 & 0 \end{pmatrix} \tag{13}$$

$$\xrightarrow{R_2=R_2-6R_1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 12 & -12 \end{pmatrix} \xrightarrow{R_2=\frac{R_2}{12}} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -1 \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix} \mathbf{x} = 0 \tag{15}$$

From, equation (15), we get two equations $x_1 - x_3 = 0$ and $x_1 - x_3 = 0$ thus,

$$\mathbf{x_1} = \mathbf{x_3} \tag{16}$$

$$\mathbf{x_1} = \mathbf{x_3} \tag{17}$$

From, this we can conclude that the given points form an Isosceles Triangle.

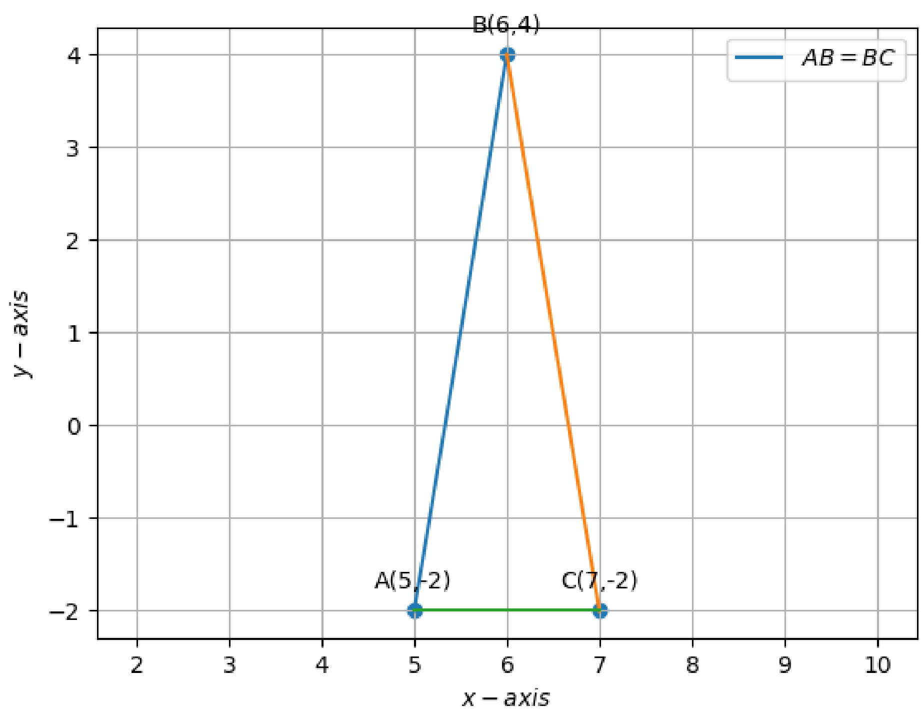

Figure 1: Isosceles Triangle with the given coordinates