Vector Algebra

CHAPTER 10 - VECTOR ALGEBRA

Excercise 10.3

Solution:

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

1 Solution

The given vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit vectors. Since the given vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are unit vector hence $\mathbf{a} = \mathbf{b} = \mathbf{c}$ which is equal to 1.

$$\|\mathbf{a}\| = \sqrt{1^2} = 1 \tag{1}$$

$$\|\mathbf{b}\| = \sqrt{1^2} = 1 \tag{2}$$

$$\|\mathbf{c}\| = \sqrt{1^2} = 1 \tag{3}$$

The Given equation is

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \tag{4}$$

$$(\mathbf{a} + \mathbf{b} + \mathbf{c})^2 = 0^2 \tag{5}$$

Squaring on both sides,

$$\mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + \mathbf{c}^\top \mathbf{c} + 2(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}) \implies 0 \tag{6}$$

$$1^2 + 1^2 + 1^2 + 2(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}) \implies 0 \tag{7}$$

$$3 + 2(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}) \implies 0 \tag{8}$$

$$\mathbf{ab} + \mathbf{bc} + \mathbf{ca} \implies \frac{-3}{2} \tag{9}$$