

LINES

CHAPTER 11 - STRAIGHT LINES

Exercise 10.3

Solution:

Q14. Find the coordinates of the foot of the perpendicular from $(-1, 3)$ to the line $3x-4y-16=0$.

1 Solution

The given perpendicular point can be taken as,

$$\mathbf{P} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (1)$$

Let us assume the coordinates is \mathbf{A} . To find the coordinates of foot of perpendicular from point to line is given as:

$$\begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix}^T \mathbf{A} = \begin{pmatrix} \mathbf{m}^T \\ \mathbf{c} \end{pmatrix} \quad (2)$$

Now, line given

$$3x - 4y - 16 = 0 \quad (3)$$

$$3x - 4y = 16 \quad (4)$$

Therefore, it can be equated as

$$\mathbf{n}^T \mathbf{x} = \mathbf{c} \quad (5)$$

where,

$$\mathbf{n} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \mathbf{c} = 16 \quad (6)$$

Here \mathbf{m} is directional vector of the given line

$$\mathbf{m} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad (7)$$

Substituting all values in (4), we get

$$\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix} \mathbf{A} = \begin{pmatrix} 4 & 3 & \begin{pmatrix} -1 \\ 3 \end{pmatrix} \\ & & 16 \end{pmatrix} \quad (8)$$

$$\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix} \mathbf{A} = \begin{pmatrix} 5 \\ 16 \end{pmatrix} \quad (9)$$

The augmented matrix for the system equations in (9) is expressed as

$$\begin{pmatrix} 4 & 3 & 5 \\ 3 & -4 & 16 \end{pmatrix} \quad (10)$$

$$\xleftrightarrow{R_2 = R_2 - \frac{3}{4}R_1} \begin{pmatrix} 4 & 3 & 5 \\ 0 & -\frac{25}{4} & \frac{49}{4} \end{pmatrix} \quad (11)$$

$$\xleftrightarrow{R_2 = \frac{-4}{25}} \begin{pmatrix} 4 & 3 & 5 \\ 0 & 1 & \frac{-49}{25} \end{pmatrix} \quad (12)$$

$$\xleftrightarrow{R_1 = \frac{1}{4}R_1} \begin{pmatrix} 1 & \frac{3}{4} & \frac{5}{4} \\ 0 & 1 & \frac{-49}{25} \end{pmatrix} \quad (13)$$

$$\xleftrightarrow{R_1 = R_1 - \frac{3}{4}R_2} \begin{pmatrix} 1 & 0 & \frac{68}{25} \\ 0 & 1 & \frac{-49}{25} \end{pmatrix} \quad (14)$$

Hence,

$$\mathbf{A} = \begin{pmatrix} \frac{68}{25} \\ \frac{-49}{25} \end{pmatrix} \quad (15)$$

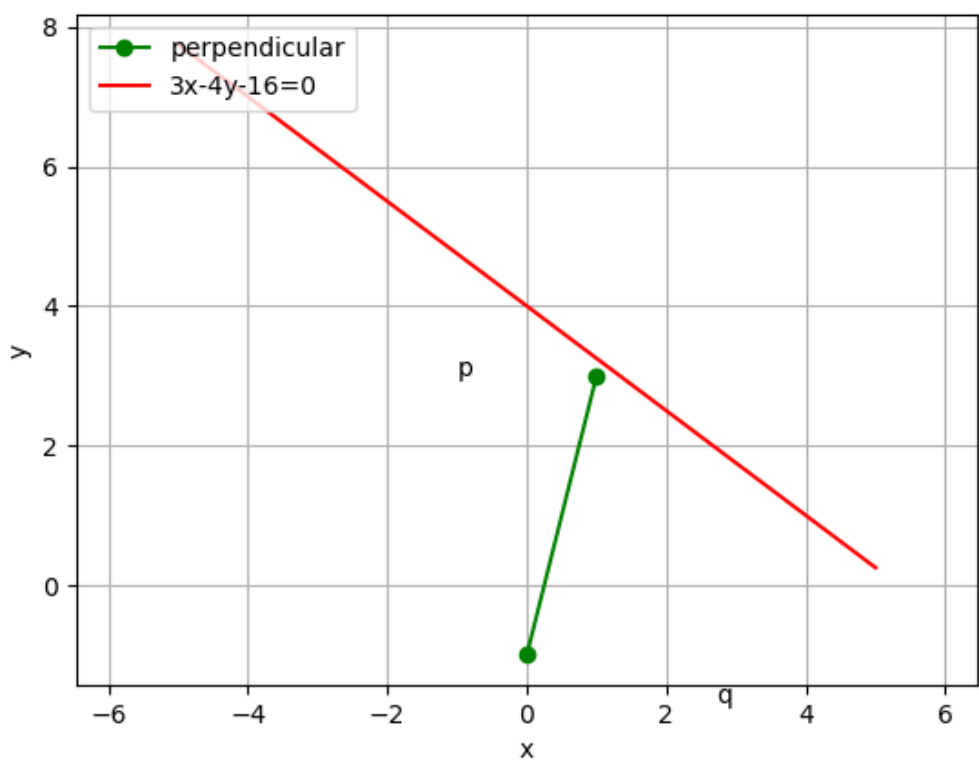


Figure 1: Foot of Perpendicular from point P and given line