

## CHAPTER 7 - COORDINATE GEOMETRY

### Exercise 7.1

Q4. Check whether (5,-2), (6,4) and (7,-2) are the vertices of an isosceles triangle:

**Solution:**

1. In an Isosceles triangle, If any 2 of the 3 sides of triangle are be equal then it satisfies the condition. Let us assume the given three points be,

$$\mathbf{A}, \mathbf{B}, \mathbf{C} \quad (1)$$

Now, the direction vectors of AB, BC and CA are:

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad (2)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix} \quad (3)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \quad (4)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (5)$$

Therefore, from the above equations (3) and (4), solving them (Norming and Equating) we get,

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37} \quad (6)$$

Similarly,

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{(-1)^2 + (6)^2} = \sqrt{1 + 36} = \sqrt{37} \quad (7)$$

From (6) and (7), we can say that

$$\|\mathbf{A} - \mathbf{B}\| = \|\mathbf{B} - \mathbf{C}\| \quad (8)$$

Therefore the two sides are equal and thus, the given points proves that it is an Isosceles Triangle. To check whether the given points form a triangle using rank of matrix is,

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{X}_{12} & \mathbf{Y}_{21} \\ \mathbf{X}_{21} & \mathbf{Y}_{22} \\ \mathbf{X}_{31} & \mathbf{Y}_{32} \end{vmatrix} \end{pmatrix} \quad (9)$$

$$|\mathbf{X}_{12} \quad \mathbf{Y}_{21}| = \begin{vmatrix} -1 & 6 \\ 2 & 0 \end{vmatrix} = (0 - 12) = -12 \quad (10)$$

$$|\mathbf{X}_{21} \quad \mathbf{Y}_{22}| = \begin{vmatrix} -1 & -6 \\ 2 & 0 \end{vmatrix} = (0 - (-12)) = 12 \quad (11)$$

$$|\mathbf{X}_{31} \quad \mathbf{Y}_{32}| = \begin{vmatrix} -1 & -6 \\ -1 & 6 \end{vmatrix} = (-6 - 6) = -12 \quad (12)$$

Since,

$$|\mathbf{X}_{12} \quad \mathbf{Y}_{21}| = |\mathbf{X}_{31} \quad \mathbf{Y}_{32}| \quad (13)$$

Hence, the two sides are equal and proves to be an Isosceles Triangle.

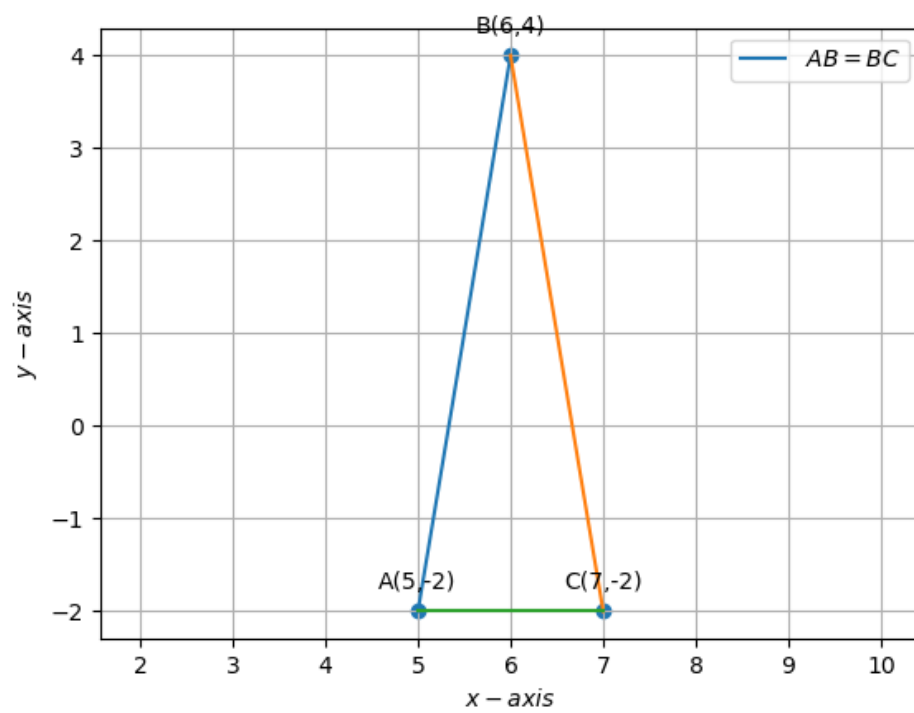


Figure 1: Isoscles Triangle with the given coordinates