GRAPH THEORY

MODULE - I

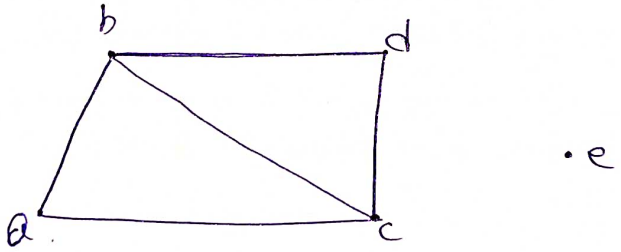
Graph : A Graph $G = (V, E)$ consist of V as non-empty set of vertices (nodes) and E as set of edges.

→ A Graph with finite vertices is called finite graph.

→ A Graph with ifinite vertices is called infinite graph.

EXAMPLE

Draw a graph $G = (V, E)$, where $V = \{a, b, c, d, e\}$ and $E = \{(a, b), (b, c), (b, d), (c, a), (c, d)\}$



Edge : The line containing between two vertices and it is denoted by $e = \{u, v\}$, (u, v) or uv.

Order and Size

Let $G = (V, E)$ be a graph. The number of vertices is the order of G and the number of edges is size of G.

In the above fig. the graph G is (5, 5).

Degree of vertex

The degree of a vertex v in a graph G denoted by deg v or d(v) is the number of

edges which are incidence on v.

Ex From the above fig.

deg(a) = 2, deg(b) = 3, deg(c) = 3, deg(d) = 2, deg(e) = 0.

→ A vertex of degree zero is called isolated.

→ A vertex of degree one is called pendant.

Multi Graph:

If the vertices u and v in a graph are joined by more than one edges then we have parallel edges. A graph with parallel edges called multigraph.

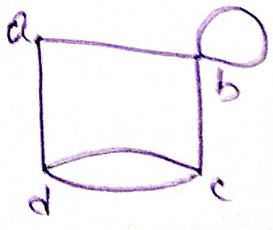
Self Loop

An edge whose starting and ending vertices are same are called self loop.

→ Each loop on a vertex u contributes '2' to the degree of u.

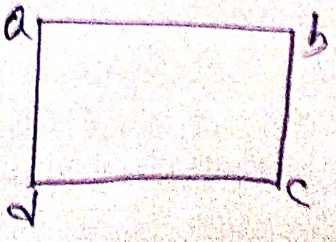
Pseudo graph

A graph with self loop and parallel edges is called Pseudo graph.



Simple Graph

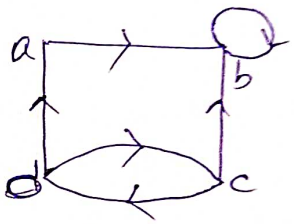
A graph G is called simple graph if it is free from self loop and parallel edges.



Notation: Let e be the edge connected with vertex u and v, then $e = \{u, v\}$ or (u, v) or uv is represented as undirected edge. But $e = \langle u, v \rangle$ is denoted directed edge from u to v, where u is called initial vertex and v is called terminal vertex.

Directed Graph (Diagraph):

A graph G is said to be directed graph if it consists of a non-empty set of vertices V and set of directed edges E.



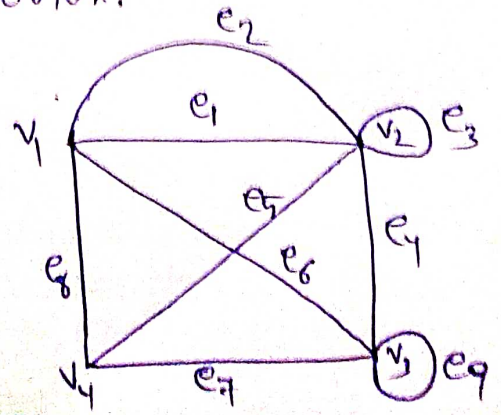
→ When a directed graph has no loops and no multiple directed edges, it is called a simple directed graph.

→ ~~Two~~ Two vertices u and v in an undirected graph G are called adjacent vertices of G if u and v are end points of an edge of G.

Even and Odd degree vertices

If the degree of the vertex v is an even, then it is called even degree vertex and if the degree of the vertex v is odd then it is called odd vertex.

Ex



Here $V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, \ldots, e_9\}$

$\deg(v_1) = 4$, $\deg(v_2) = 6$, $\deg(v_3) = 5$, $\deg(v_4) = 3$

$\therefore$ v_1 and v_2 are even degree vertices

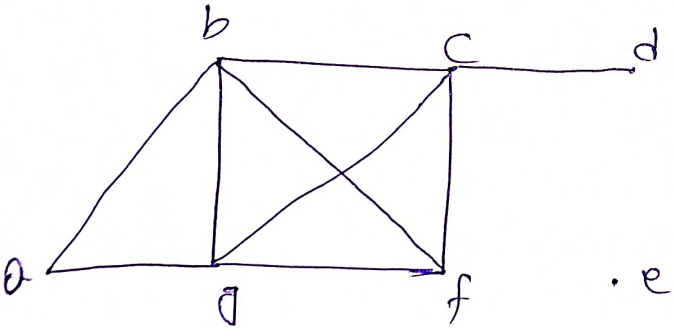
v_3 and v_4 are odd degree vertices

$\therefore \deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4) = 18$

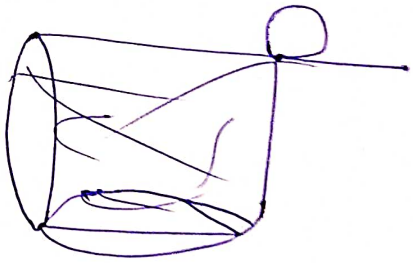
$= 2 \times 9$

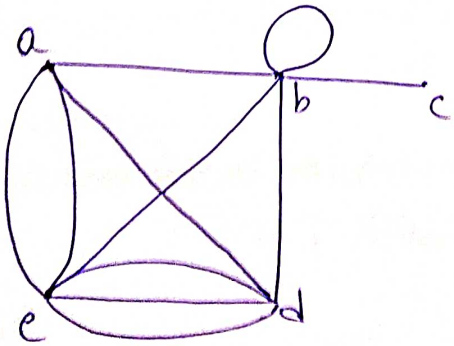
$= 2 \times$ no. of edges

Ex Find the degree of the vertices of the following graphs.



(G)





(H)