Q: Is there a simple graph corresponds to the following degree sequences. If exists, then draw the graph

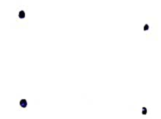
(i) $(5, 5, 4, 3, 2, 2, 2, 1, 0)$

(ii) $(8, 6, 6, 6, 5, 5, 5, 4, 4, 4, 3, 2, 1, 1, 1)$

TYPES OF GRAPH:

Null Graph: A Graph which contain only isolated vertex is known as Null Graph.

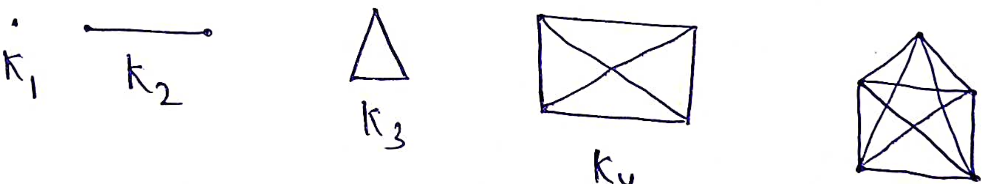
Ex:



(Only vertices no edges)

Complete Graph (K_n): A simple graph G is said to be a complete graph if every vertex of G is connected with other vertex of G. It is denoted by K_n.

Ex:

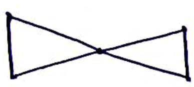


K_1 K_2 K_3 K_4 K_5

Regular Graph

A Graph G is said to be regular graph if all the vertices of the graph are of same degree.

→ If the degree of each vertex is r, then it is said to be r-regular graph.



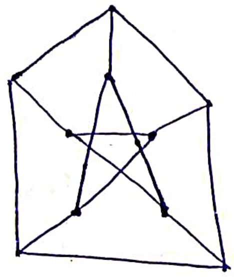
(2-regular graph)

→ Complete Graph K_n is a $(n-1)$ regular graph.

→ Null graph is a 0-regular graph.

Pitersen Graph:

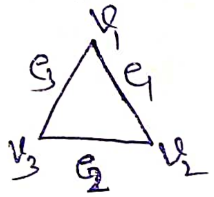
It is a 3-regular graph.



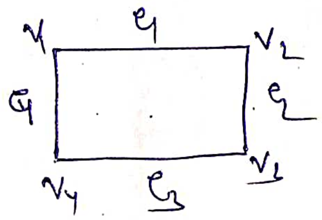
Cycle Graph (C_n):

A cycle graph C_n, $n \geq 3$ consist of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, $\ldots$, $\{v_{n-1}, v_n\}$, $\{v_n, v_1\}$.

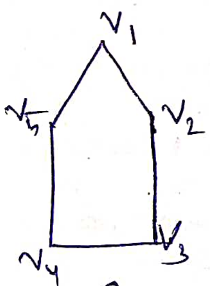
Ex:



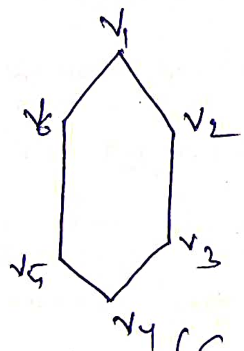
(C_3)



(C_4)



C_5

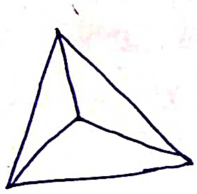


(C_6)

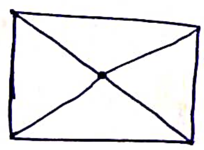
Wheel Graph (W_n):

A wheel graph W_n can be obtained by adding a vertex to a cycle C_n, for $n \geq 3$ and connect the new vertex to each of the vertex C_n.

Ex:



W_4



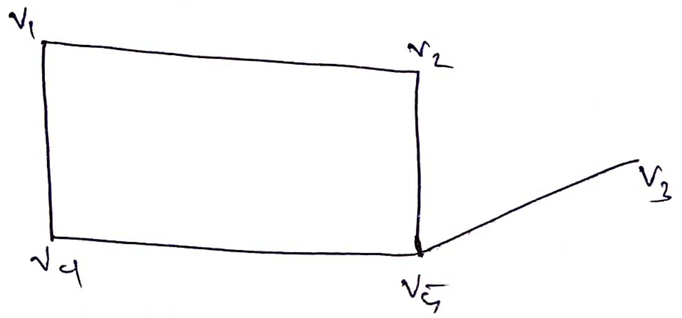
W_5

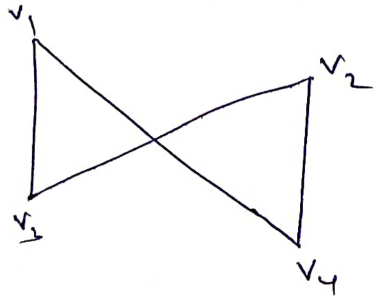
Bipartite Graph

A simple graph $G(V, E)$ is called a ~~Bipartite~~ Bipartite graph if its vertex set V can be partitioned two ~~distinct~~ disjoint subsets V_1 and V_2 such that every edge in G connects a vertex in V_1 and a vertex in V_2. No edge in G connects either two vertices in V_1 or two vertices in V_2, i.e, $G(V, E)$ is our graph then

$$V = V_1 \cup V_2 \quad \text{and} \quad V_1 \cap V_2 = \phi .$$

Ex:





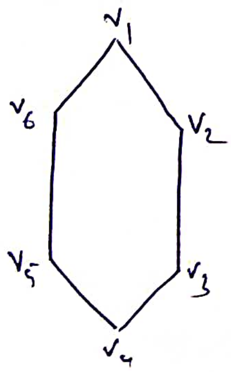
$V_1 = \{v_2, v_3, v_4\}$
$V_2 = \{v_1, v_5\}$
$V = V_1 \cup V_2$
$V_1 \cap V_2 = \phi$

$V_1 = \{v_1, v_2\}$
$V_2 = \{v_3, v_4\}$

→ Bipartition of the graph is not necessarily unique.

EXAMPLE

Consider C_6 graph, where



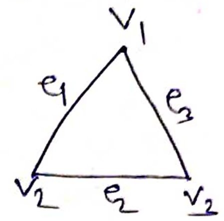
$$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

Let $V_1 = \{v_1, v_3, v_5\}$ and $V_1 \cup V_2 = V$
$V_2 = \{v_2, v_4, v_6\}$ $\quad V_1 \cap V_2 = \phi$

So C_6 is Bipartite.

EXAMPLE

Consider complete graph K_3, where



$$V = \{v_1, v_2, v_3\}$$

But, we can not construct two sets of vertices such that

$$V_1 \cup V_2 = V \quad \& \quad V_1 \cap V_2 = \phi.$$

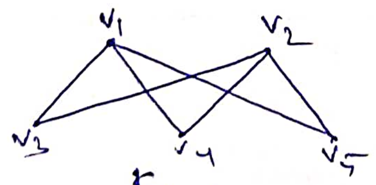
and no edge connect to the vertices of V_1 or V_2.

Thus, K_3 is not Bipartite graph.

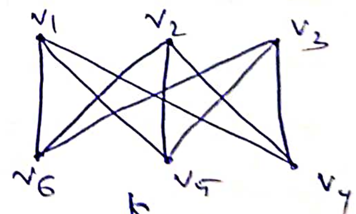
Complete Bipartite Graph ($K_{m,n}$)

A Bipartite graph G is said to be Complete Bipartite graph denoted by $K_{m,n}$, where $|V_1| = m$ and $|V_2| = n$ such that there is an edge between each pair of vertices v_1 and v_2 where $v_1 \in V_1$ and $v_2 \in V_2$.

EXAMPLE



$K_{2,3}$



$K_{3,3}$

~~Q1: If $G=(V,E)$ is a r-regular graph, then find $|E| = m$.~~

~~soln: Ex~~

$V_1 = \{v_1, v_2\}$

$V_2 = \{v_3, v_4, v_5\}$

$V_1 \cup V_2 = V$

$V_1 \cap V_2 = \phi$

No edge connected betn the vertices either in V_1 or V_2

$V_1 = \{v_1, v_2, v_3\}$

$V_2 = \{v_4, v_5, v_6\}$

$V_1 \cup V_2 = V$

$V_1 \cap V_2 = \phi$

No edge connected betn the vertices either in V_1 or V_2

Q1. If $G=(V,E)$ is a r-regular graph, then find the relation between no. of edges and no. of vertices.

Sol^n: Let $G=(V,E)$ is r-regular graph and $|V|=n$ and $|E|=m$.

As it is r-regular graph then each vertex has degree 'r'. Then by Handshaking theorem

$$\underbrace{r+r+\cdots+r}_{n\text{-times}}=2m$$

$$\Rightarrow nr=2m \Rightarrow \boxed{m=\frac{nr}{2}}$$

$\therefore$ If G is a r-regular graph with n number of vertices then the number of edges in the graph G is $\frac{nr}{2}$.

HW Does there exist a 4-regular graph of order 6? If such type of graph exists, then draw at least one graph.

Q2. How many edges in the complete Bipartite graph $K_{m,n}$ have?

Sol^n: Consider a Complete Bipartite graph which has $(m+n)$ number of vertices.

Since $K_{m,n}$ is a complete bipartite then V_1 and V_2 be two partition of $K_{m,n}$ such that

$$|V_1|=m \text{ and } |V_2|=n.$$

Since it is complete bipartite then each vertex of V_1 connected to every vertex of V_2.

Then degree of each vertex of V_1 is n.

Then total number of vertices in V_1 is m. So total degree of vertices of V_1 is mn.

Similarly, the total degree of vertices of V_2 is mn. Thus the total degree of vertices of the graph G is

$$mn + mn = 2mn.$$

By Handshaking theorem we know that the sum of degree of vertices of the graph is twice of the number of edges.

So, the total number of edges of a complete bipartite graph $K_{m,n}$ is $\frac{2mn}{2} = mn$.

HW Prove that a graph which contains a triangle is not a bipartite.

Procedure to check that a graph G is Bipartite.

Step-1 :- Arbitrarily select a vertex from G and include it in V_1.

step-2 : Consider the edges directly connected to the vertex and put the other end vertices of these edges into V_2.

step-3 : Pick up one vertex from V_2 and consider the edges connected to that vertex and put the other end vertices of these vertices into the set V_1.

step-4 : At each steps-2 and 3 check if there is any edge among the vertices V_1 and V_2. If so then the graph is not bipartite, otherwise continue the step-2 & 3 untill all the vertices included.