Handshaking Theorem (For Diagraph)

Let $G = (V, E)$ be a graph with directed ~~de~~ edges. Then

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

Proof: Let $\{u_1, u_2, \ldots, u_n\}$ be the vertex set of the directed graph G. Let e_i be an edge in G. Then e_i is a loop or incident between two vertex. If e_i is a loop it contributes

$$\deg^-(u_i) = 1 \text{ and } \deg^+(u_i) = 1.$$

If e_i is incident between u_i and u_{i+1}, i.e, e_i is directed from u_i to u_{i+1}, then it contributes

$$\deg^+(u_i) = 1 \text{ and } \deg^-(u_{i+1}) = 1.$$

Similarly, we can apply for each edge then we have then we have

$$\sum_{i=1}^{n} \deg^+(u_i) = \sum_{i=1}^{n} \deg^-(u_i) \quad \text{---- (1)}$$

We know by Handshaking theorem for non directed graph

$$\sum_{i=1}^{n} \deg(u_i) = 2m$$

$$\Rightarrow \sum_{i=1}^{n} \deg^+(u_i) + \sum_{i=1}^{n} \deg^-(u_i) = 2m$$

Hence from (1), we have

$$\sum_{i=1}^{n} deg^{+}(u_i) = \sum_{i=1}^{n} deg^{-}(u_i) = m$$

(proved).

Theorem

Show that the degree of a vertex of a simple graph G on 'n' vertices can not exceed (n-1).

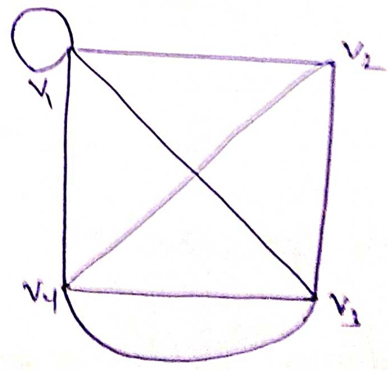
Proof: Let G be a simple graph with 'n' vertices. Since G is simple it has no parallel edges and no self loop. So, a vertex v of G has at most (n-1) adjacent vertices because v is not adjacent to itself. Therefore, the degree of the vertices can't exceed (n-1).

(~~proof~~ proved)

Definition (Degree Sequence)

Let $n_1, n_2, n_3, \ldots, n_k$ be the degree of the vertices of a graph G such that $n_1 \le n_2 \le \cdots \le n_k$ then the finite sequence $(n_1, n_2, \ldots, n_k)$ is called the degree sequence of G.

$deg(v_1) = 5$
$deg(v_2) = 3$
$deg(v_3) = 4$
$deg(v_4) = 4$



Hence, degree sequence is (3, 4, 4, 5).

Note:

(1) If the sum of the degree of the vertices is not even, then the graph corresponds the

degree sequence can not be drawn.

(2) If the total number of odd number of vertices is odd then the graph corresponds to given degree sequence can't be drawn.

Ex: Is there a simple graph corresponds to the following degree sequences.

(i) (1, 1, 2, 3) (ii) (2, 2, 4, 4), (iii) (1, 3, 3, 4, 5, 6, 6)

(i) For the degree sequence (1, 1, 2, 3) there is not a simple graph because sum of the degree of vertex is not even.

(ii) (2, 2, 4, 4) of degree sequence can not be draw because, there are 4 vertex then by a theorem the degree of vertex at most 3 but there are two vertex of degree 4. Hence it contradict the theorem. Hence we can't find a simple graph corresponds to the sequence (2, 2, 4, 4).

(iii) For the degree sequence (1, 3, 3, 4, 5, 6, 6), the sum of the degree is 28 which is even, also the number of odd degree vertices is even then the graph exist. Maximum degree of a vertex 6 < 7 but two vertices of degree 6 then each of two vertices adjacent and each of the vertices is not less than two but one one of these vertex is one. Hence there is not a simple graph.

Q: Is there a simple graph corresponds to the following degree sequences. If exists, then draw the graph

(i) (5, 5, 4, 3, 2, 2, 2, 1, 0)

(ii) (8, 6, 6, 6, 5, 5, 5, 4, 4, 4, 3, 2, 1, 1, 1)

TYPES OF GRAPH:

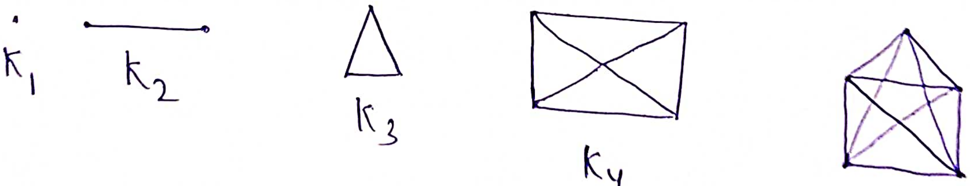
Null Graph: A Graph which contain only isolated vertex is known as Null Graph.

Ex:

(Only vertices no edges)

Complete Graph (K_n): A simple graph G is said to be a complete graph if every vertex of G is connected with other vertex of G. It is denoted by K_n.

Ex:



K_1 K_2 K_3 K_4 K_5

Regular Graph

A Graph G is said to be regular graph if all the vertices of the graph are of same degree.

→ If the degree of each vertex is r, then it is said to be r- regular graph.

(2 - regular graph)

→ Complete Graph K_n is a $(n-1)$ regular graph.

→ Null graph is a 0- regular graph.