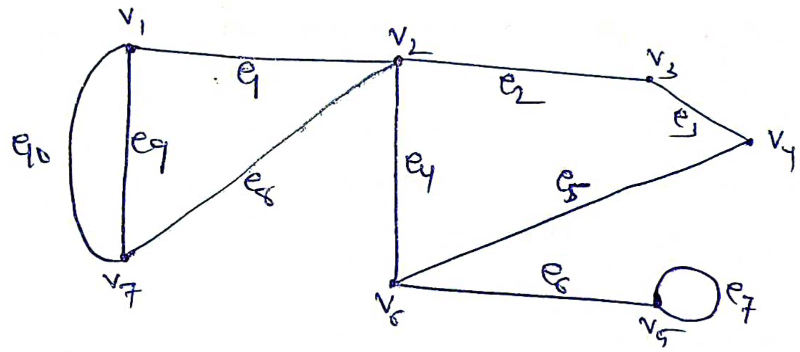
Length

The number of edges in a walk is called its length. Since paths and circuits are walks, it follows that the length of a path is the number of edges in the path and the length of a circuit is the number of edges in the circuit.

A circuit or cycle of length k, is called a k-circuit or k-cycle. A k-circuit is called odd or even according as k is odd or even.

Ex:



→ The length of the open walk $v_6 e_6 v_5 e_7 v_5$ is 2

→ The length of the closed walk $v_1 e_9 v_7 e_8 v_2 e_1 v_1$ is 3

→ The length of the ~~closed~~ circuit $v_2 e_4 v_6 e_5 v_4 e_3 v_3 e_2 v_2$ is 4

→ The length of the path $v_6 e_5 v_4 e_3 v_3 e_2 v_2 e_1 v_1$ is 4.

Trees and Distance

In 1857, Cayley discovered the important class of graphs called trees by considering the changes of variables in the differential calculus. Trees as graphs have many applications, especially in data storage, searching, and communication.

Definition

A graph is acyclic if it has no cycles.

Definition

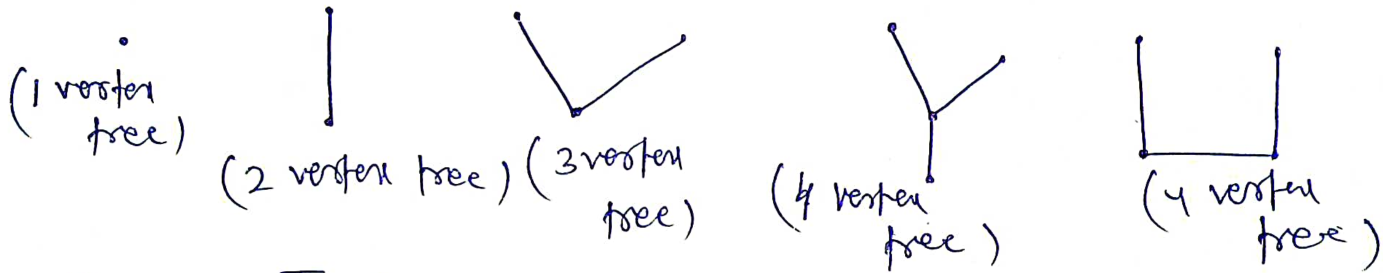
A tree is a connected acyclic graph.

Definition

Any graph without cycles is a forest, thus the components of a forest are trees.

EXAMPLE

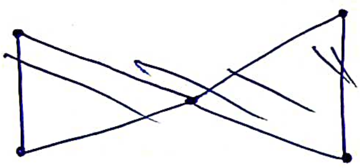
The tree with 2-points, 3-points and 4-points are shown below:

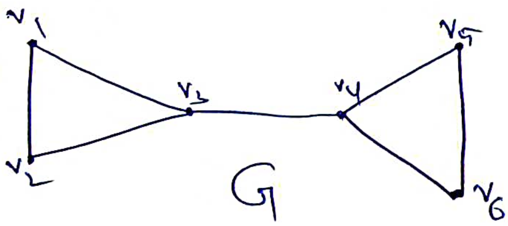


Spanning Tree

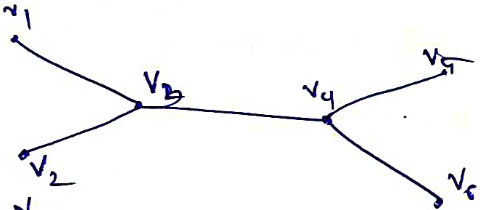
A tree is said to be a spanning tree of a connected graph G if T is a subgraph of G and T contains all the vertices of G.

EXAMPLES

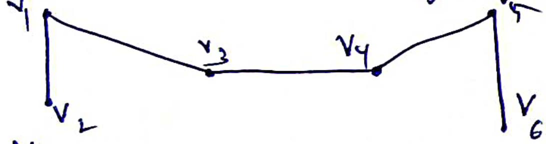




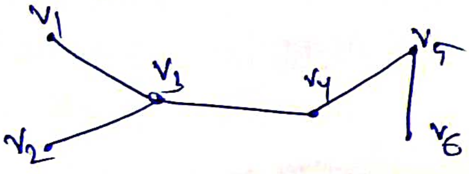
T_1 :



T_2 :



T_3 :



Notes: (i) A graph with no cycles has no odd cycles; hence trees and forests are bipartite.

(ii) Paths are trees. A tree is a path if and only if its maximum degree is 2.

(iii) A star is a tree consisting of one vertex adjacent to all the others.

(iv) A graph that is a tree has exactly one spanning tree, that is, the full graph itself.

(v) A spanning subgraph of G need not be connected, and a connected subgraph of G need not be a spanning subgraph. For example:

If $n(G) > 1$, then empty subgraph with vertex set $V(G)$ and edge set Φ is spanning but not connected.

If $n(G) > 2$, then a subgraph consisting of one edge and its endpoints is connected but not spanning.

~~Properties of Trees~~

Branch of Tree

An edge in a spanning tree T is called a branch of T.

Chord

An edge of G that is not in a given spanning tree is called a chord.

~~Rooted Tree~~

Properties of Trees

(1) There is a unique path between every pair of vertices in a tree.

If it could be more than one path, then it is a cyclic. Because a tree is a connected graph, there is at least one path between every two vertices. However, if there were more than one path bet^n two pair of vertices, then their may be a circuit in the tree which is not permissiable.

Suppose there are two distinct path

$$P_1 = \{u, u_1, u_2, \ldots, u_k, v\}$$

and $P_2 = \{u, v_1, v_2, \ldots, v_\ell, v\}$

from u to v. Then the path P_1 followed by the path P_2 is $\{u, u_1, u_2, \ldots, u_k, v, v_\ell, \ldots, v_2, v_1, u\}$ which is a circuit.

Hence, there is a unique path between every pair of vertices in a tree.

(2) A tree with n vertices has $(n-1)$ edges.

Proof: We shall prove this ~~theorem~~ property by method of induction on the number of vertices.

Base step: The result is obvious for the tree having one or two vertices.

Inductive Hypothesis: Let us assume that the result holds for all tree with fewer than n vertices.

Inductive step:

Let T be a tree with n vertices. Let 'e' be an edge in T with end vertices u and v. Now the edge 'e' is the only path between u and v.

Therefore if we delete 'e' from T the resultant T−e will be a disconnected graph and has exactly two components say T_1 and T_2. Each of this components is a tree because there were no circuit in T.

Let n_1 & n_2 be the no. of vertices of T_1 and T_2 respectively. Since $n_1 < n$ & $n_2 < n$ by our inductive hypothesis

number of edges in $T_1 = n_1 - 1$

number of edges in $T_2 = n_2 - 1$

Thus the number of edges in $T - e = n_1 - 1 + n_2 - 1$

$= n - 2 \quad (\because n_1 + n_2 = n)$

So the number of edges in $T = n - 1$ edges.

(proved)