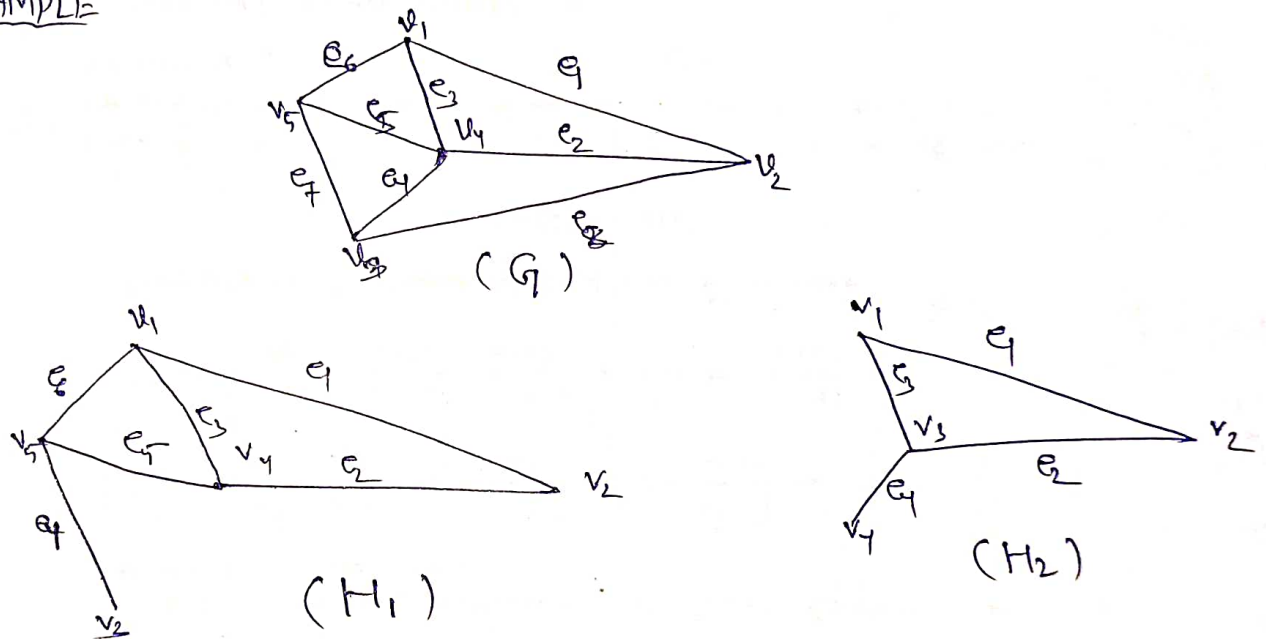


Subgraph

Let G and H are two graphs with vertices set $V(G)$ and $V(H)$ and edges $E(G)$ and $E(H)$ such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. Then we say H is a subgraph of G . If $V(H) = V(G)$ but $E(H) \subsetneq E(G)$ in that case we say H is a spanning subgraph of G .

EXAMPLE



$$\begin{aligned} \text{Here } V(H_1) &= \{v_1, v_2, v_3, v_4, v_5\} \\ V(H_2) &= \{v_1, v_2, v_3, v_4\} \\ V(G) &= \{v_1, v_2, v_3, v_4, v_5\} \\ E(H_1) &= \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\} \\ E(H_2) &= \{e_1, e_2, e_3, e_4\} \\ E(G) &= \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\} \end{aligned}$$

So $V(H_1) = V(G)$ and $E(H_1) \subsetneq E(G)$.

Thus H_1 is a spanning ^{sub}graph of G , and H_2 is a subgraph of G .

Note: Let $G(V, E)$ be a graph where,
 $|V| = n$ and $|E| = m$

Now the total number of non-empty subset of V is $(2^n - 1)$ and

the total number of non-empty subset of E is 2^m .

Therefore the total number of non-empty subgraph that can be obtained from the original graph is $(2^n - 1) 2^m$.

Q: Prove that number of spanning subgraph which can be constructed from the graph is 2^m , where 'm' is the no. of edges

solⁿ: We have m is the number of edges of G.

No. of spanning subgraph with zero edge	$m C_0$
" " " " " one "	$m C_1$
" " " " " two "	$m C_2$
\vdots	\vdots
" " " " " m edge	$m C_m$

Thus the total number of spanning subgraph with m number of edges is

$$m C_0 + m C_1 + m C_2 + \dots + m C_m = 2^m$$

Walk: Let $G(V, E)$ be any graph, then a walk on G is a finite alternating of vertices and edges

$v_0, e_1, v_1, e_2, v_2, e_3, v_3, \dots, v_{n-1}, e_n, v_n$
such that v_{k-1} and v_k are end vertices of the edge e_k , $1 \leq k \leq n$.

→ Here all the edges ~~are distinct~~ not necessarily vertices. and vertices may be repeated.

→ v_0, v_n are called terminal vertices and the rest are called internal vertices.

→ If the end vertices of that walk are distinct then this is called as open walk otherwise it is closed walk.

Path: A path is an open walk in which no vertices appear more than ~~one~~ ones.
vertex not repeated. Edge not repeated.

EXAMPLE

Consider a graph $G(V, E)$ having

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

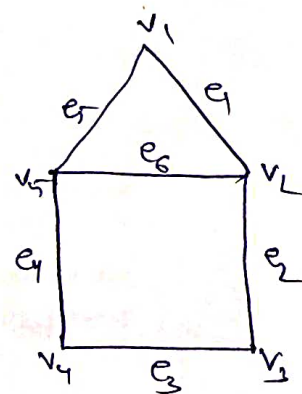
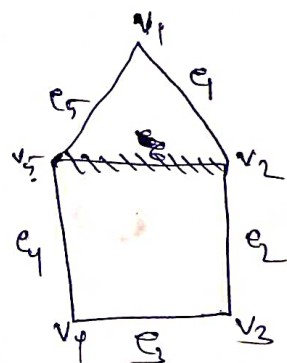
$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

Here

$$\{v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_5, e_5, v_1\}$$

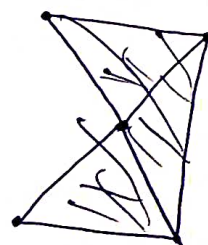
is a closed walk.

Here $\{v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_5, e_5, v_1, e_6, v_2\}$ is a walk and also it is a open walk.

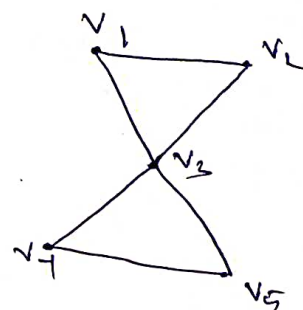


Trail: An open walk in which all the edges are distinct is called a trail.

Vertices may be repeated
Edges not repeated



Here $\{v_1, v_3, v_4, v_5, v_1, v_2\}$
is a trail.

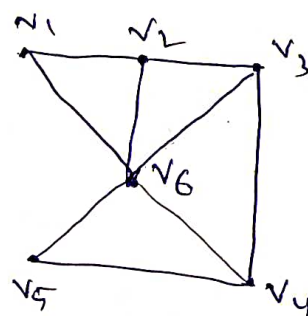


Circuit: Traversing a graph such that not an edge is repeated but vertex can be repeated and it is closed also, i.e., it is a closed trail.

vertex can be repeated

Edge not repeated

Here $\{v_1, v_6, v_5, v_4, v_6, v_3, v_2, v_1\}$
is a circuit.



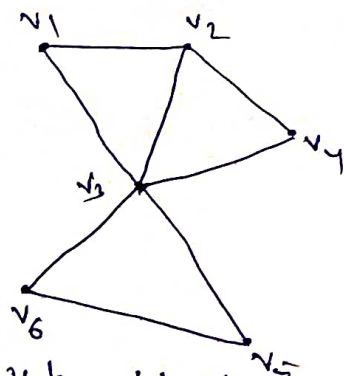
Path: It is a trail in which neither vertices nor edges are repeated, i.e., if we traverse a graph such that we do not repeat a vertex and nor we repeat an edge.

As path is also a trail, thus it is also an open walk.

Vertex not repeated

Edge not repeated

Here $\{v_5, v_6, v_3, v_4, v_2, v_1\}$ is a path.

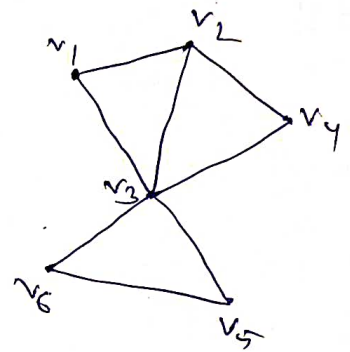


Cycle: Traversing a graph such that we do not repeat a vertex nor we repeat an edge but the starting and ending vertex must be same, i.e., we can

repeat starting and ending vertex only (21)
then we get a cycle.

Vertex not repeated
Edge not repeated

Here $\{v_1, v_3, v_6, v_2, v_1\}$ is a cycle.



Note: ① A circuit is a Trail

② A cycle is a path

③ If Trail is closed, then it is circuit.

④ If path is closed, then it is cycle.

⑤ Trail, circuit, path, cycle all are the walk.