<u>Module-1</u>

Decomposition

A graph is self-complementary if it is isomorphic to its complement.

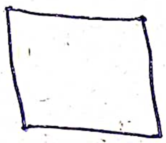
Definition

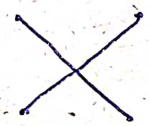
~~A graph~~ A decomposition of a graph is a list of subgraphs such that each edge appears in exactly one subgraph in the list.

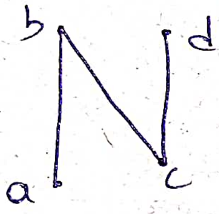
Definition

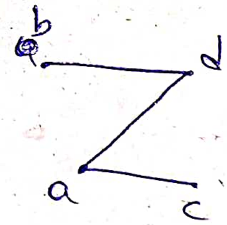
The complement of a graph G, written as $\overline{G}$, has the same vertex ~~of~~ set as G and for every pair of distinct vertices u and v in G, uv is an edge of $\overline{G}$ if and only if uv is not an edge of G.

<u>Ex</u>

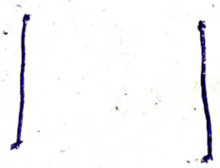
C_4 

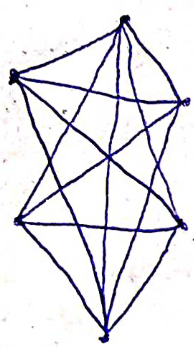
$\overline{C_4}$ 

G : 

$\overline{G}$: 

→ If G is a disconnected graph, then $\overline{G}$ is connected

G : 



$\overline{G}$

→ An n-vertex graph H is self-complementary if and only if K_n has a decomposition consisting of two copies of H.

→ A combination of two complementary graphs gives a complete graph.

If G is any simple graph, then

$$|E(G)| + |E(\bar{G})| = |E(K_n)|$$, where n = number of vertices in the graph.

Ex: Let G be a simple graph with 9 vertices and 12 edges. Find the number of edges in $\bar{G}$.

Sol: We have

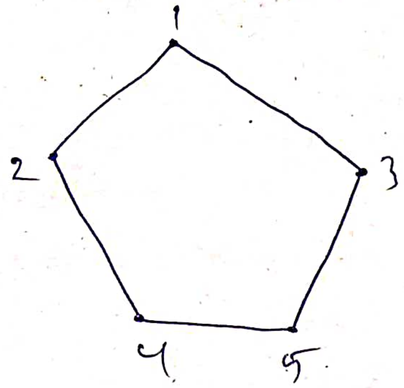
$$|E(G)| + |E(\bar{G})| = |E(K_n)|$$

$$\Rightarrow 12 + |E(\bar{G})| = \frac{n(n-1)}{2} = \frac{9(9-1)}{2}$$

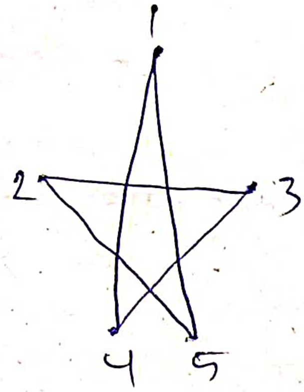
$$\Rightarrow |E(\bar{G})| = 36 - 12 = 24$$

Hence edges in $\bar{G}$ is 24.

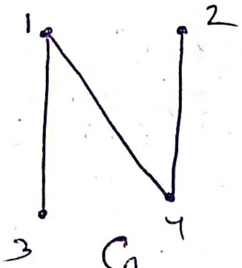
Example (Self- Complementary)



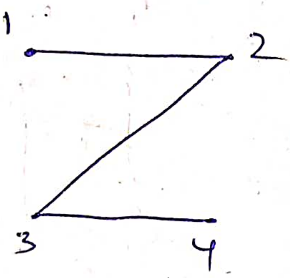
G



$\overline{G}$

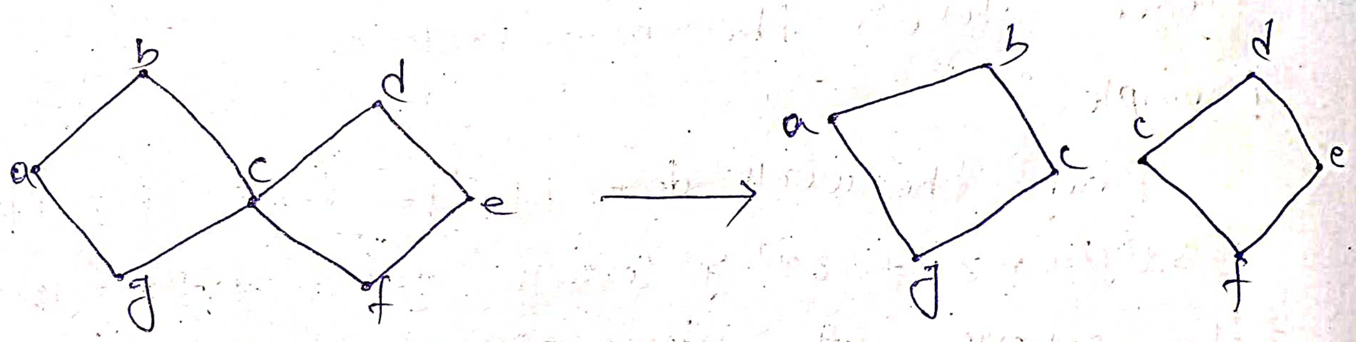


G



$\overline{G}$

Example (Decomposition)



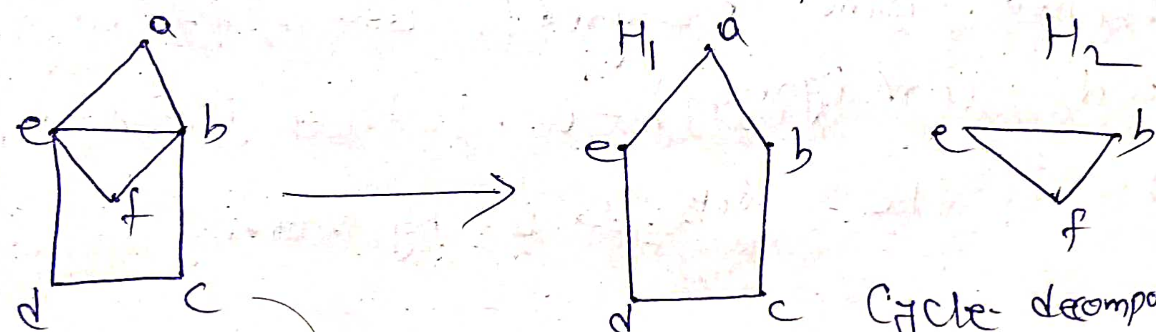
$G \qquad H_1 \qquad H_2$

$$H_1 \cup H_2 = G$$

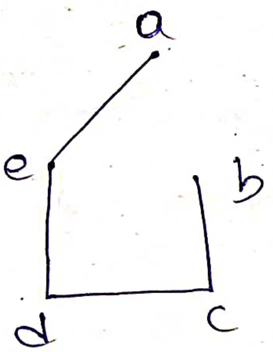
$\{H_1, H_2\}$ is a decomposition of G.

$H_1 \qquad H_2$

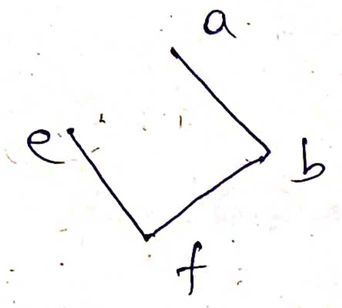
G:



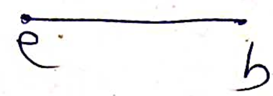
Cycle decomposition



H_1



H_2



H_3

path decomposition.

$H_1 \cup H_2 \cup H_3 = G$ and $\{H_1, H_2, H_3\}$ is a decomposition of G.

<u>Another definition</u>

A decomposition of a graph G is a set of edge-disjoint subgraphs of G, $H_1, H_2, H_3, \ldots, H_n$ such that $\bigcup_{i=1}^{n} H_i = G$.