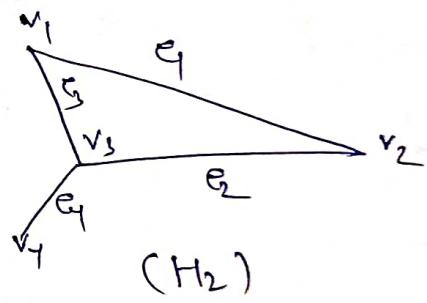
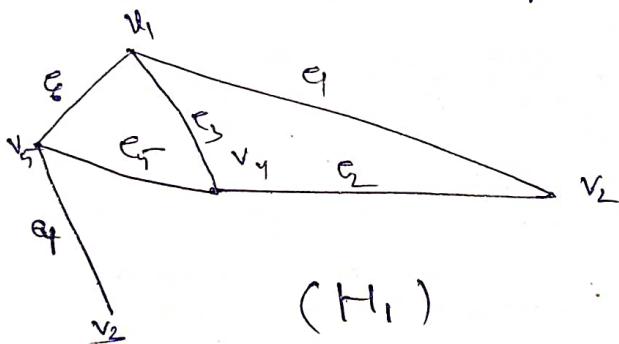
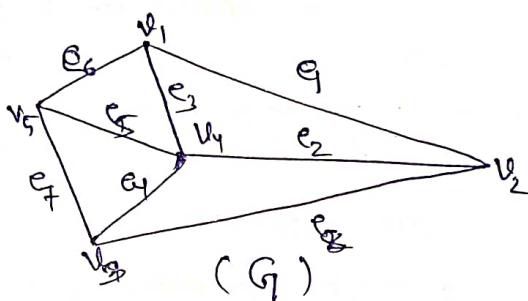


## Subgraph

Let  $G$  and  $H$  are two graphs with vertices set  $V(G)$  and  $V(H)$  and edges  $E(G)$  and  $E(H)$  such that  $V(H) \subseteq V(G)$  and  $E(H) \subseteq E(G)$ . Then way  $H$  is a subgraph of  $G$ . If  $V(G) = V(H)$  but  $E(H) \subseteq E(G)$  in that case we say  $H$  is a spanning subgraph of  $G$ .

### EXAMPLE



$$\text{Here } V(H_1) = \{v_1, v_2, v_3, v_4, v_5\}$$

$$V(H_2) = \{v_1, v_2, v_3, v_4\}$$

$$V(G) = \{v_1, v_2, v_3, v_4, v_5\}$$

$$E(H_1) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

$$E(H_2) = \{e_1, e_2, e_3, e_4\}$$

$$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$

$$\text{So } V(H_1) = V(G) \text{ and } E(H_1) \subseteq E(G).$$

Thus  $H_1$  is a <sup>sub</sup>spanning graph of  $G$ . and  $H_2$  is a subgraph of  $G$ .

Note: Let  $G(V, E)$  be a graph where,  
 $|V|=n$  and  $|E|=m$

Now the total number of non-empty subset of  $V \subseteq (2^n - 1)$  and

the total number of non-empty subset of  $E$  is  $2^m$ .

Therefore the total number of non-empty subgraph that can be obtained from the original graph is  $(2^n - 1)2^m$ .

Q: Prove that number of spanning subgraph which can be constructed from the graph is  $2^m$ , where 'm' is the no. of edges

Sol: We have  $m$  is the number of edges of  $G$ .

No. of spanning subgraph with zero edge is  $m_{C_0}$   
 "      "      "      "      "      one      "      is  $m_{C_1}$   
 "      "      "      "      "      two      "      is  $m_{C_2}$   
 :      :      :      :      :      :      :  
 "      "      "      "      "      m edge is  $m_{C_m}$

Thus the total number of spanning subgraph with  $m$  number of edges is

$$m_{C_0} + m_{C_1} + m_{C_2} + \dots + m_{C_m} = 2^m$$

Walk: Let  $G(V, E)$  be any graph, then a walk in  $G$  is a finite alternating of vertices and edges

$v_0, e_1, v_1, e_2, v_2, e_3, v_3, \dots, v_{n-1}, e_n, v_n$   
 such that  $v_{k-1}$  and  $v_k$  are end vertices of the edge  $e_k$ ,  $1 \leq k < n$ .

- Here all the edges are distinct not necessarily vertices and vertices may be repeated.
- $v_0, v_n$  are called terminal vertices and the rest are called internal vertices.
- If the end vertices of that walk are distinct then this is called as open walk otherwise it is closed walk.

Path: A path is an open walk in which no vertices appear more than ~~more~~ ones. vertex not repeated. Edge not repeated.

### EXAMPLE

Consider a graph  $G(V, E)$  having

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

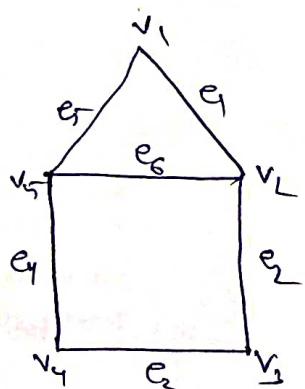
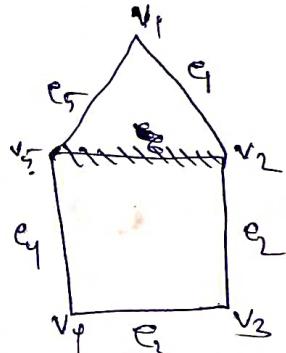
$$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

Here

$$\{v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_5, e_5, v_1\}$$

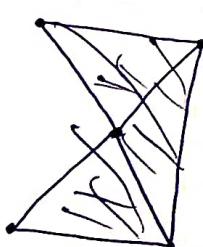
is a closed walk.

Here  $\{v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_5, e_5, v_1, e_5, v_5, e_6, v_2\}$  is a walk and also it is a open walk.

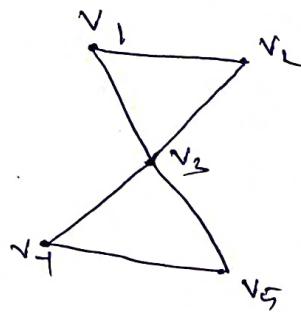


Trail: An open walk in which all the edges are distinct is called a trail.

Vertices may be repeated  
Edges not repeated



Here  $\{v_1, v_3, v_4, v_5, v_1, v_2\}$  is a trail.

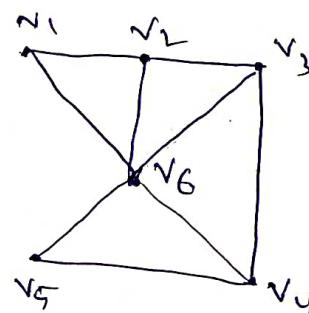


Circuit: Traversing a graph such that not an edge is repeated but vertex can be repeated and it is closed also, i.e., it is a closed trail.

vertex can be repeated

Edge not repeated

Here  $\{v_1, v_6, v_5, v_4, v_6, v_3, v_2, v_1\}$  is a circuit.



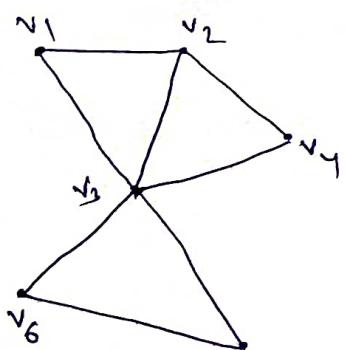
Path: It is a trail in which ~~negative~~ neither vertices nor edges are repeated, i.e., if we traverse a graph such that we do not repeat a vertex and nor we repeat an edge.

As path is also a trail, thus it is also an open walk.

vertex not repeated

Edge not repeated

Here  $\{v_5, v_6, v_3, v_4, v_2, v_1\}$  is a path.



Cycle: Traversing a graph such that we do not repeat a vertex nor we repeat an edge but the starting and ending vertex must be same, i.e., we can

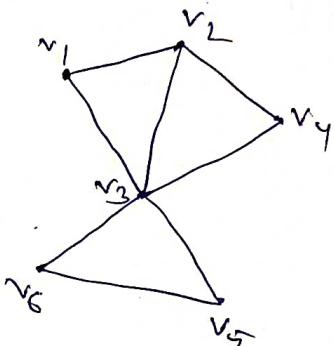
(21)

repeat starting and ending vertex only  
then we get a cycle.

vertex not repeated

Edge not repeated

Here  $\{v_1, v_3, v_6, v_2, v_1\}$  is a cycle.



Note: ① A circuit is a Trail

② A cycle is a path

③ If Trail is closed, then it is circuit.

④ If path is closed, then it is cycle.

⑤ Trail, circuit, path, cycle all are the walk.