Q: Find the possible number of distinct label tree of order 2.

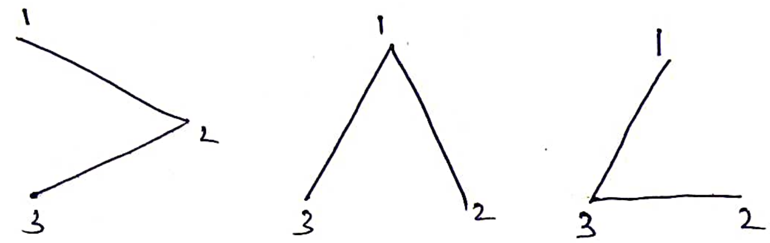
Solⁿ:

or

The possible number of distinct label tree of order 2 is 1.

For order 3:

T:

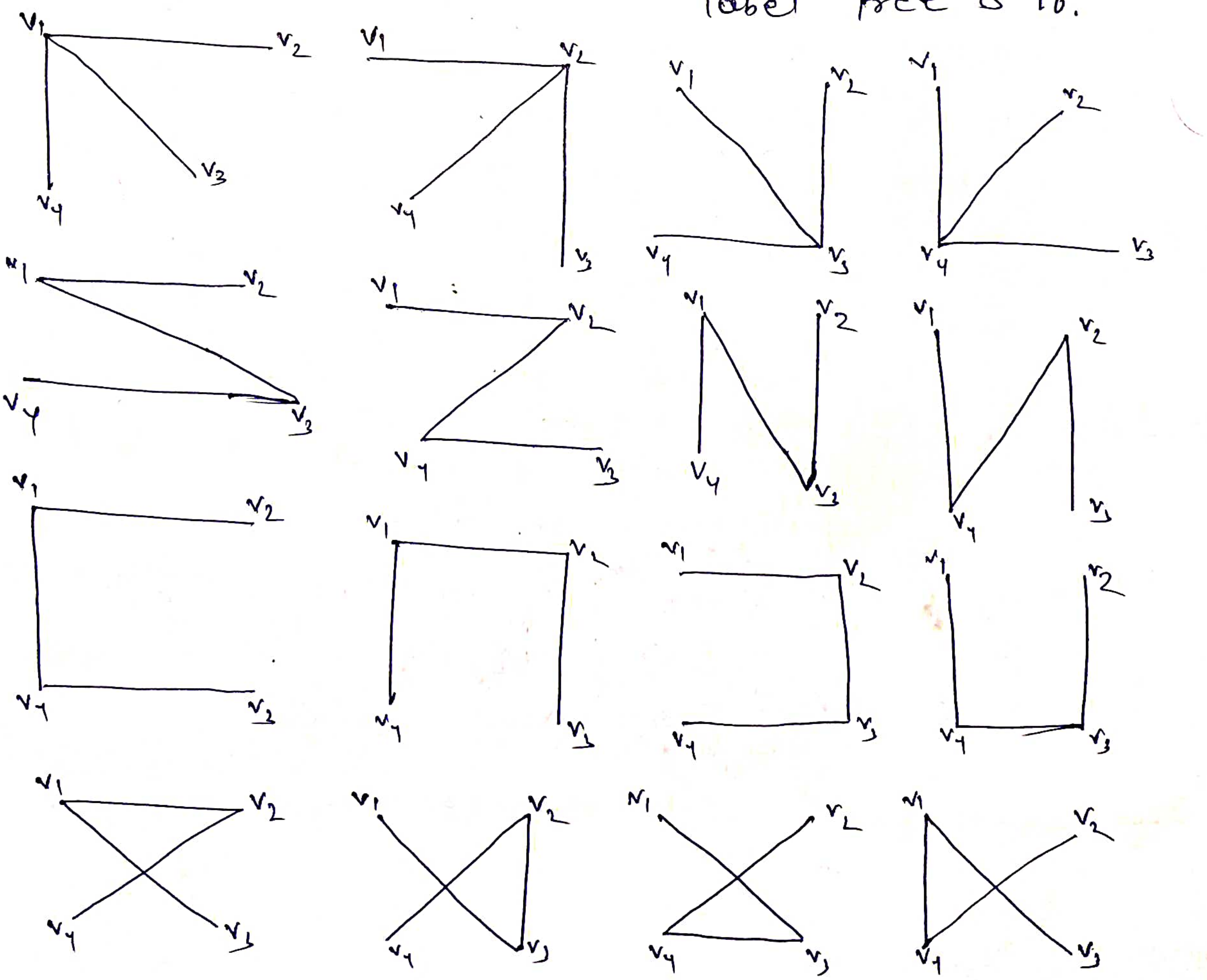


T':

(Non label tree)

For order 3 no. of distinct label tree is 3.

For order 4 no. of distinct ~~table~~ label tree is 16.



* ~~The~~ for order n no. of possible distinct label tree is n^{n-2}.

Cayley's Theorem

The number of distinct ~~table~~ label tree with n- vertices is n^{n-2} for $n \geq 2$.

Proof: Let T be a Label tree of order 'n' with labels $1, 2, 3, \ldots, n$ i.e

$$V(T) = \{1, 2, \ldots, n\} = N.$$

In order to prove the result it is sufficient to show that tree T defines a sequence of length $(n-2)$ which can be obtained from N.

Conversly given a sequence of length $n-2$ from N we show that there is a label tree tree of order n. Note that ~~three~~ there are n^{n-2} ~~number~~ number of sequence of length $n-2$ from N.

Consider a sequence of length $n-2$ $(l_1, l_2, \ldots, l_{n-2})$ from N. The possible number of subsequence is $n \times n \times \cdots \times n$ ($n-2$ times) $= n^{n-2}$. With each lab'el tree T of order n we associated a unique sequence $(T_1, T_2, \ldots, T_n)$ that can be obtained from N as follows.

Regard N is of order set.

Step-1: Let s_1 be the first ~~vertex~~ vertex of degree 1 in T

step-2 : The vertex adjacent to s_1 is taken as t_1.

step-3 : Let s_2 be the vertex of degree 1 in $T-s_1$.

step-4 : The vertex adjacent to s_2 is taken as t_2.

This operation is continuing untill t_{n-2} has been defined and a tree with just 2 vertices remain. Now the tree T defines the sequence $(t_1, t_2, \dots, t_{n-2})$ uniquely.

Consequently different label trees of order n between different sequences of length n-2. Notice that any vertex $V(T)$ occures $d(v)-1$ times in $(t_1, t_2, \dots, t_{n-2})$. ~~so that~~ So the vertices of degree 1 in T do not appear in the sequence $(t_1, t_2, \dots, t_{n-2})$.

Conversely given any sequence $(t_1, t_2, \dots, t_{n-2})$ of length $(n-2)$ from N on n' vertex labeled tree T can be constructed uniquely as follows.

step-1: Let s_1 be the 1st vertex of N not in the set $(t_1, t_2, \dots, t_{n-2})$

step-2 : Join s_1 to t_1

step-3: Let s_2 be the 1st vertex of $N-s_1$ not in $(t_2, t_3, \dots, t_{n-2})$

step-4: Joint s_2 to t_2

Continue this process untill the $(n-2)$ edges $s_1 t_1, s_2 t_2, \ldots, s_{n-2} t_{n-2}$ have been determined.

Finally T is obtained by adding the edge joining two remaining vertices of

$$N - \{s_1, s_2, \ldots, s_{n-2}\}$$

Consequently different sequences of length $(n-2)$ ~~given~~ give rise to different label tree of order n.

Thus we have established one-one corresponding between the set of label tree of order n and n^{n-2} sequence of length $n-2$.

$\square$

~~Pess~~

~~Purfer code~~ Prüfer Code