MODULE-5 ADVANCED TOPICS

Coloring:

• Six Colored Theorem

Every planar graph can be ~~s~~ properly colored with 6 colours.

Proof:

We will prove it by method of induction. Without loss of generality let us assume that the graph is connected.

If it is not connected we just color each connected components. They never conflict with each other. So we will colored all the vertices.

Now we will do the induction on the number of vertices.

Basis Step:

Let us take a graph having single vertex, then we may choose any color to color it.

Inductive hypothesis

Consider the results holds for all simple planar graphs of vertices up to order n-1.

Inductive step:

Now consider a planar graph with n no. of vertices. But we know by a theorem for every simple planar graph G has a vertex of degree not exceeding 5.

Now we assume that G has a vertex 'v' of degree ≤ 5. Now we remove the vertex v from G then by inductive hypothesis we can color $n-1$ vertices with 6 colors.

Now the vertex v removed has degree ≤ 5. So it has at most 5 adjacent vertices with at most 5 different colors.

So we have at least one color for this removed vertex.

Hence every planar graph can be properly colors with 6 colors.

(proved)

Theorem (5-Colored Theorem)

Every planar graph can be properly colored with 5 colors.

Proof:-

We prove this by method of induction on the number of vertices of the graph.

Basis step :-

Let us take a graph having single vertex, then we can color it by any of 5 colors. So this proves the result.

Inductive hypothesis

Consider a planar graph of vertices upto n-1 can be 5 colored.

Inductive step :-

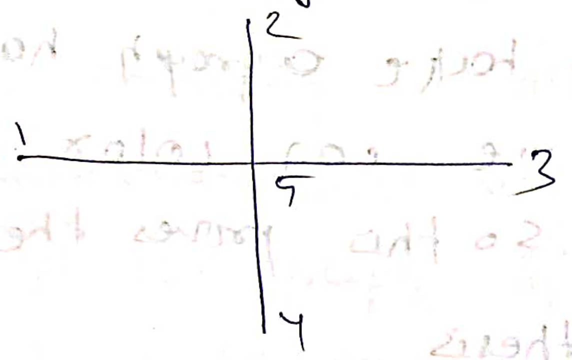
Consider a graph G with n vertices and G is planar. But we know that by a theorem for every simple planar graph G has a vertex ~~...~~ of degree not exceeding 5.

Let us remove that vertex 'v' from the graph. Then the rest graph say G_1 has n-1 vertices. By inductive hypothesis G_1 can be 5 colored. as it has n-1 no. of vertices.

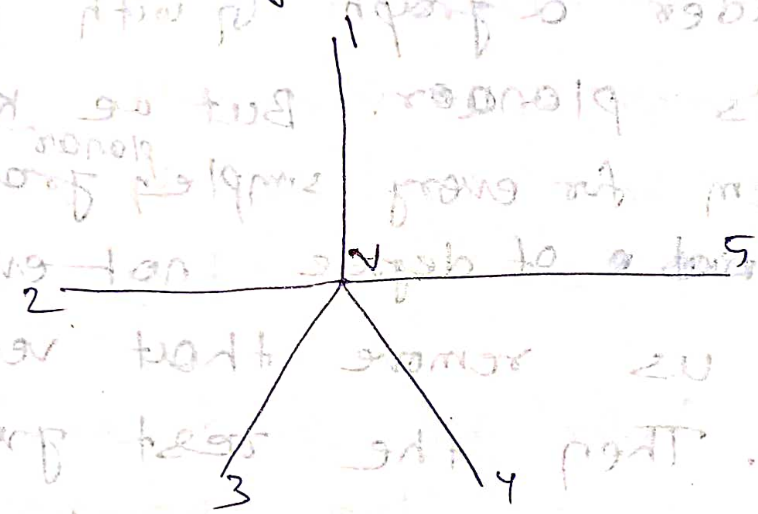
Now if we put back that removed vertex 'v' in to the graph again then ~~there will be~~ we have two possibilities.

If the degree of this vertex 'v' is less than 5 then, after coloring all vertices adjacent to it we will have at least one color to color it.

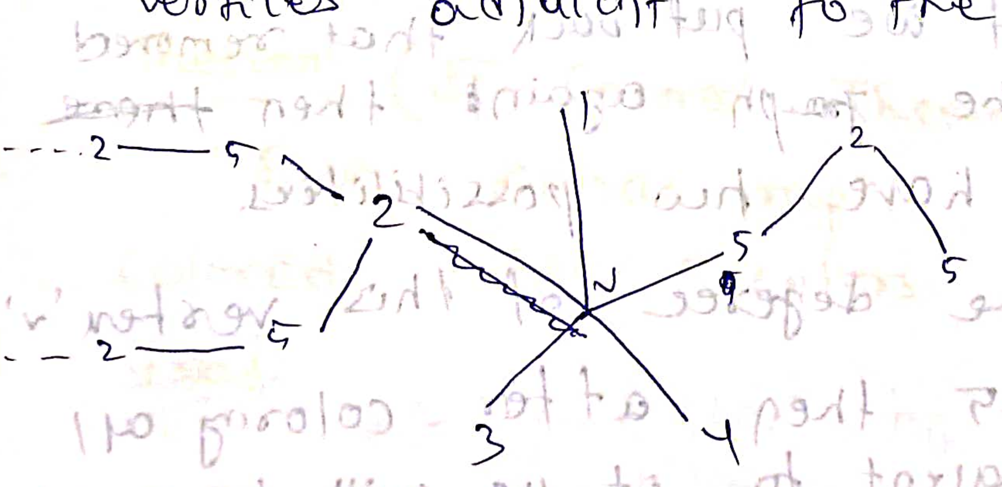
~~But if the degree of vertex is 5~~



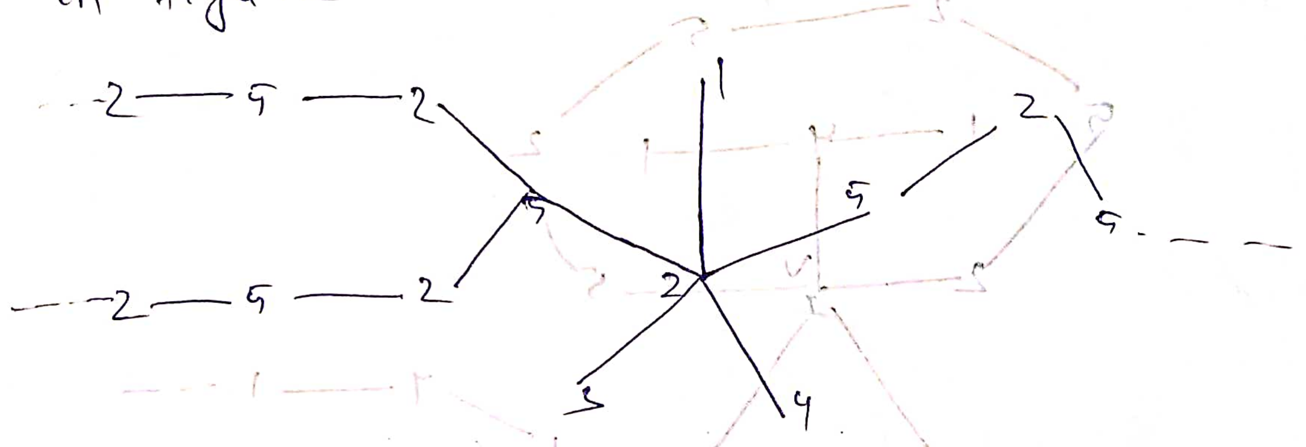
But if the degree of vertex 'v' is 5 then we have all colors assigned to all 5 vertices adjacent to that vertex 'v' as shown is figure bellow.



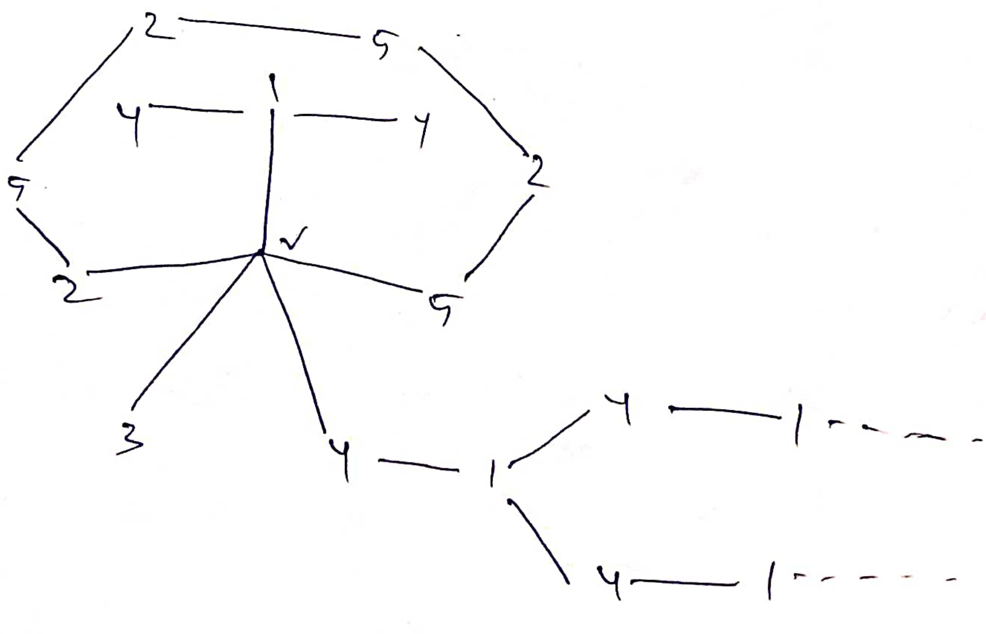
Consider a subgraph of vertices colored 2 or 5 which are connected to 2 & 5 vertices adjacent to the main vertex 'v'.



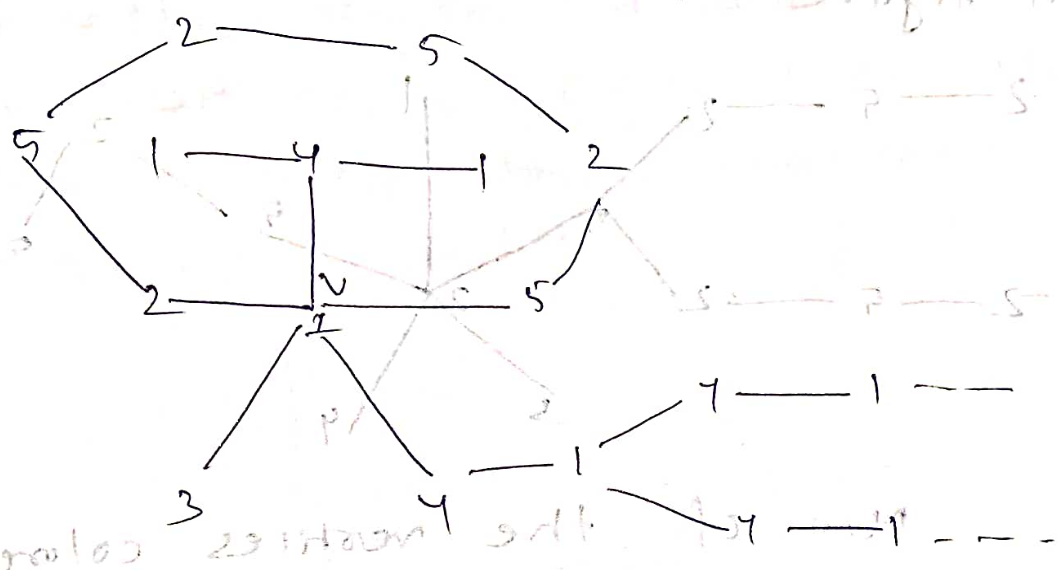
If the adjacent vertex colored 2 and colored 5 are not connected by a path in this subgraph then we exchange the color 2 & 5 through out the subgraph connected to the vertex colored 2. Then there will be one colored left to colored vertex 'v' as shown in figure



Now if the vertices colored 2 & 5 have a path in the subgraph, then we do the above procedure to the vertex having color 1 & 4 adjacent to vertex 'v'.



Now we exchange the color 1 with color 4 in the subgraph adjacent to the vertex 'v'. Then there will be one colored left '1' to colored 'v' as shown in the figure.



This ends the prove of 5-colored theorem.

(proved)