Continue this process until the $(n-2)$ edges $s_1 l_1, s_2 l_2, \ldots, s_{n-2} l_{n-2}$ have been determined.

Finally T is obtained by adding the edge joining two remaining vertices of

$$N - \{s_1, s_2, \ldots, s_{n-2}\}$$

Consequently different sequences of length $(n-2)$ ~~given~~ give rise to different label tree of order n.

Thus we have established one-one corresponding between the set of label tree of order n and n^{n-2} sequence of length $n-2$.

□

~~Perif~~

~~Purfer code~~ Prüfer Code

Matrices and Isomorphism

There are two types of matrices;

① Adjacency Matrix

② Incidence Matrix

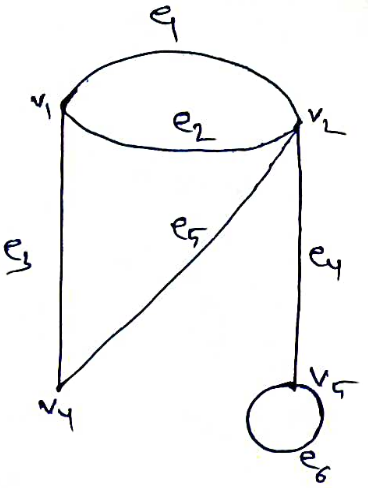
Adjacency Matrix

Let G be a graph with n-vertices $v_1, v_2, \ldots, v_n$ where $n > 0$. The adjacency matrix denoted by 'A_G' with respect to $v_1, v_2, \ldots, v_n$ is a $(n \times n)$ matrix $[a_{ij}]$ such that

a_{ij} = number of edges from v_i to v_j.

EXAMPLE

$$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$



Observation

(1) A_G is a symmetric Matrix.

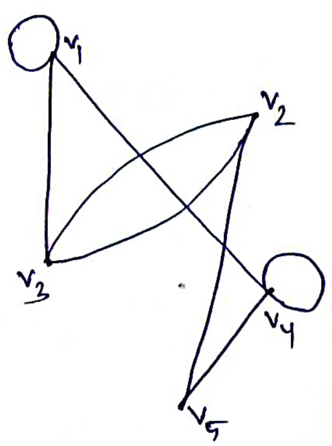
(2) If G is simple graph, then it is 0-1 matrix.

(3) If G does not contain any loops, then the diagonal elements are zero

Q→ Construct a graph G such that $A_G = G$, where

$$A_G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Sol^n: Let the vertices of the graph are v_1, v_2, v_3, v_4, v_5. Then the required graph is

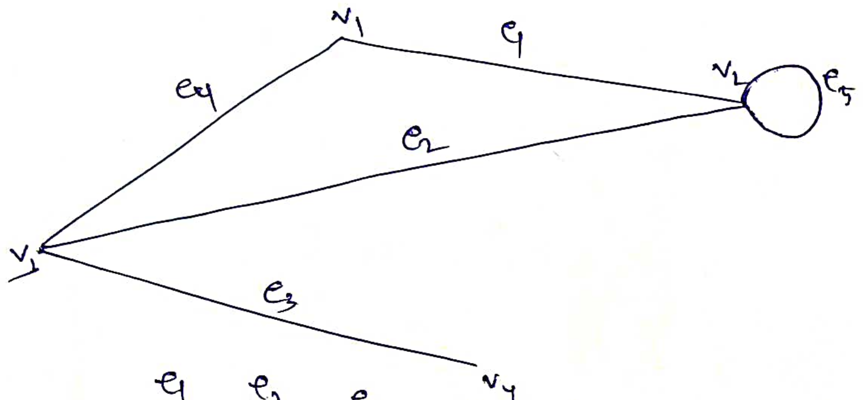


Incedence Matrix

Let G be a graph with n- vertices $v_1, v_2, \ldots, v_n$ $(n>0)$ and m number of edges $e_1, e_2, \ldots, e_m$. The incidence matrix of graph G denoted by 'I_G' with respect to the vertices $v_1, v_2, \ldots, v_n$ of n-vertices and m-edges $e_1, e_2, \ldots, e_m$ is an $n\times m$ matrix $[a_{ij}]_{n\times m}$

$$a_{ij} = \begin{cases} 0, & \text{if } v_i \text{ is not a end vertex of } e_j \\ 1, & \text{if } v_i \text{ is an end vertex of } e_j \text{ \& } e_j \text{ is not a loop} \\ 2, & \text{if } e_j \text{ is a loop at } v_i. \end{cases}$$

EXAMPLE

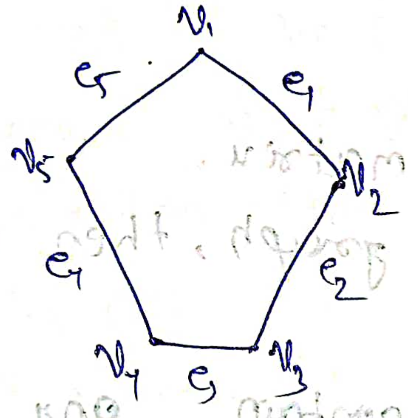


$$I_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

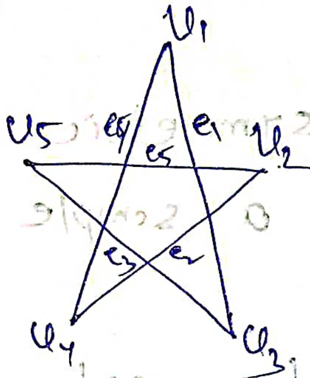
Isomorphic graph

- Two simple graph are isomorphic iff their vertices can be leveled in such a way that their corresponding adjacency matrixes are equal.

Ex:



G_1



G_2

f is a isomorphic graph defined by

$$f : V_1 \longrightarrow V_2$$

$V_1 = \{ v_1, v_2, v_3, v_4, v_5 \}$

$V_2 = \{ u_1, u_2, u_3, u_4, u_5 \}$

$f(v_1) = u_1$

$f(v_2) = u_3$

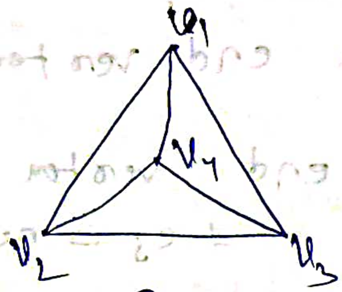
$f(v_3) = u_5$

$f(v_4) = u_2$

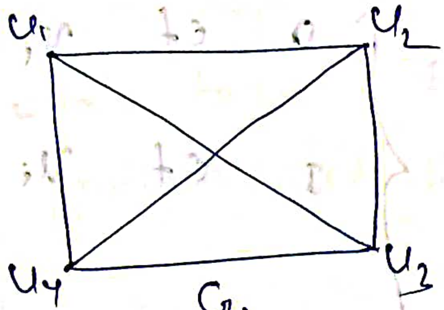
$f(v_5) = u_4$

$$AG_1 = \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 1 & 0 & 0 & 1 \\ v_2 & 1 & 0 & 1 & 0 & 0 \\ v_3 & 0 & 1 & 0 & 1 & 0 \\ v_4 & 0 & 0 & 1 & 0 & 1 \\ v_5 & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$AG_2 = \begin{array}{c|ccccc} & u_1 & u_3 & u_5 & u_2 & u_4 \\ \hline u_1 & 0 & 1 & 0 & 0 & 1 \\ u_3 & 1 & 0 & 1 & 0 & 0 \\ u_5 & 0 & 1 & 0 & 1 & 0 \\ u_2 & 0 & 0 & 1 & 0 & 1 \\ u_4 & 1 & 0 & 0 & 1 & 0 \end{array}$$



G_1



G_2

f is defined by $f: V_1 \longrightarrow V_2$ or $f: V(G_1) \longrightarrow V(G_2)$

$f(v_1) = u_1$

$f(v_2) = u_2$

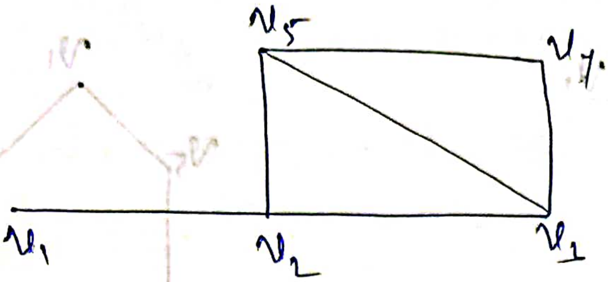
$f(v_3) = u_3$

$f(v_4) = u_4$

$$A_{G_1} = \begin{array}{c|cccc} & v_1 & v_2 & v_3 & v_4 \\ \hline v_1 & 0 & 1 & 1 & 1 \\ v_2 & 1 & 0 & 1 & 1 \\ v_3 & 1 & 1 & 0 & 1 \\ v_4 & 1 & 1 & 1 & 0 \end{array} \qquad A_{G_2} = \begin{array}{c|cccc} & u_1 & u_2 & u_3 & u_4 \\ \hline u_1 & 0 & 1 & 1 & 1 \\ u_2 & 1 & 0 & 1 & 1 \\ u_3 & 1 & 1 & 0 & 1 \\ u_4 & 1 & 1 & 1 & 0 \end{array}$$

Incedence Matrix

Let G be a graph with n vertices $v_1, v_2, \ldots, v_n$ $(n > 0)$ and m no. of edges $e_1, e_2, \ldots, e_m$. The incidence matrix of graph G 'I_G' with respect to the $v_1, v_2, \ldots, v_n$ of n vertices and m edges $e_1, e_2, \ldots, e_m$ => an $n \times m$ matrix $[a_{ij}]_{n \times m}$



Here ~~degree~~ degree of v_1, v_2, v_3, v_5 are of odd degree. So there is no Euler path.

* K_n is Euler path if n is odd.
* $K_{m,n}$ is " " if (m & n) are even.

Hamilton Circuit

A circuit which traverses each vertex exactly once is called a Hamilton circuit.

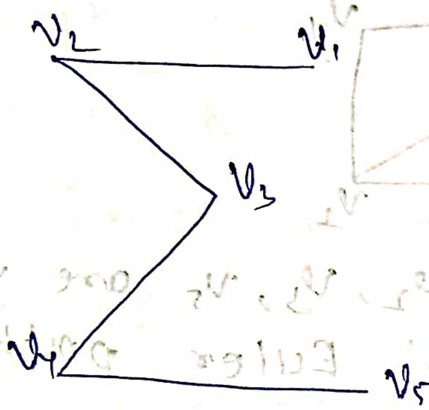
Hamilton Graph

A Graph is said to be Hamiltonian or Hamilton graph if it has at least one Hamilton circuit.

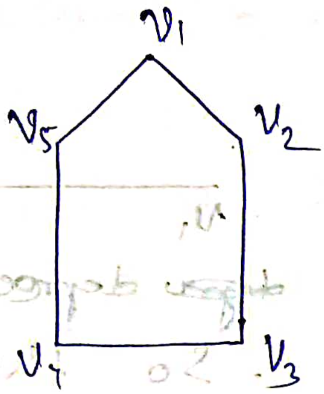
Note:-

Every graph that has Hamilton circuit has a Hamilton path by deleting any one edge from Hamilton circuit.

-> It covers all vertex exactly once.



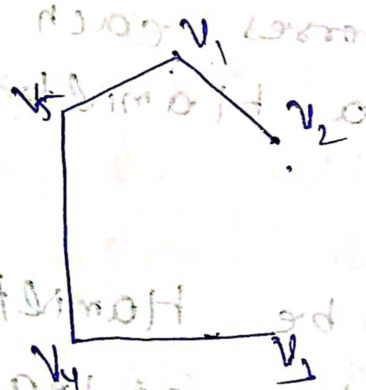
G_1



G_2

$(v_1, v_2, v_3, v_4, v_5)$ is called Hamilton path in G_1

$(v_1, v_2, v_3, v_4, v_5, v_1)$ is a Hamilton circuit in G_2.



→ Hamilton path

Note

Since a Hamilton circuit in a connected graph G includes every vertex of G, it contains exactly n edges of G & n vertices.