Bayes' Theorem

Lesson 4.2

Learning Objectives

LO 4.2.1 **Solve** computing problems using Bayes' theorem

Theorem 13.1

BAYES' THEOREM

Suppose that E and F are events from a sample space S such that $p(E) \neq 0$ and $p(F) \neq 0$. Then,

$$p(F \mid E) = \frac{p(E \mid F) \cdot p(F)}{p(E \mid F) \cdot p(F) + p(E \mid \overline{F}) \cdot p(\overline{F})}.$$

Theorem 13.2

GENERALIZED BAYES' THEOREM

Suppose that E is event from a sample space S and that F_1, F_2, ..., F_n are mutually exclusive events such that $\bigcup_{i=1}^{n} F_i = S$. Assume that $p(E) \neq 0$ and $p(F_i) \neq 0$ for $i = 1, 2, ..., n$. Then,

$$p(F_j \mid E) = \frac{p(E \mid F_j) \cdot p(F_j)}{\sum_{i=1}^{n} p(E \mid F_i) p(F_i)}.$$

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Examples:

1. Suppose that E and F are events in a sample space and $p(E) = 1/3$, $p(F) = 1/2$, and $p(E \mid F) = 2/5$. Find $p(F \mid E)$.
2. Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains two white balls and three blue balls, and the second box contains four white balls and one blue ball. What is the probability that Frida picked a ball from the first box if she has selected a blue ball?

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Examples:

3. Suppose that a test for opium use has a 2% false positive rate and a 5% false negative rate. That is, 2% of people who do not use opium test positive for opium, and 5% of opium users test negative for opium. Furthermore, suppose that 1% of people actually use opium.

   a. Find the probability that someone who tests negative for opium use does not use opium.

   b. Find the probability that someone who tests positive for opium use actually uses opium.

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Examples:

4. Suppose that a Bayesian spam filter is trained on a set of 1000 spam messages and 400 messages that are not spam. The word "opportunity" appears in 175 spam messages and 20 messages that are not spam. Would an incoming message be rejected as spam if it contains the word "opportunity" and the threshold for rejecting a message is 0.9?