## **Final Asset Allocation Project**

```
In []: market = "SPY"
    tickers = ["DG", "MRK", "CB", "NOC", "BP"] # chosen to be diverse across sed
# start date test
    test_start = "2012-12-01"
    test_end = "2017-10-01"

# eval date test
    eval_start = "2017-10-02"
    eval_end = "2019-10-01"

In []: from urllib.request import urlretrieve
    import os

    query = "https://query1.finance.yahoo.com/v7/finance/download/{}?period1=132
    for ticker in tickers+[market]:
        os.makedirs("data/{}".format(ticker), exist_ok=True)
        urlretrieve(query.format(ticker), "data/{}/{}.csv".format(ticker, ticker)
```

#### We will now compute the returns of our data

```
In []: # compute returns using the close for each and add drop all other columns
import pandas as pd
import numpy as np
import os
for ticker in tickers+[market]:
    df = pd.read_csv("data/{}/{}.csv".format(ticker, ticker))
    df["return"] = df["Close"].pct_change()
    df = df.dropna()
    df = df[["Date", "Close", "return"]]
    df.to_csv("data/{}/{}.csv".format(ticker, ticker), index=False)
```

#### Next we will collect our rate data and split it into monthly data

```
In []: # load the risk free rate and save to rates.csv
rates_static = "FRB_H15.csv"
rates_df = pd.read_csv(rates_static)

# format the date form YYYY-MM to YYYY-MM-DD
rates_df["Date"] = rates_df["Date"].apply(lambda x: x+"-01")
rates_df["return"] = rates_df["return"]/1200

# save it to rates.csv
os.makedirs("data/rates", exist_ok=True)
rates_df.to_csv("data/rates/rates.csv", index=False)
```

#### Now we will split our data into train and test periods

#### Next we will aggregate our data into a training data frame

```
In []: # compute the combined training set
    train_data = pd.DataFrame()
    for ticker in tickers+[market]+["rates"]:
        df = pd.read_csv("data/{}/{}_train.csv".format(ticker, ticker))
        df = df[["Date", "return"]]
        df = df.rename(columns={"return": ticker})
        if train_data.empty:
             train_data = df
        else:
             train_data = pd.merge(train_data, df, on="Date", how="inner")
```

# Now we can compute the basic statistics we need to devise multiple portfolios

```
In [ ]: train_data = train_data.dropna()
        train data = train data.set index("Date")
        train_data.to_csv("data/train_data.csv")
        # cov
        cov matrix = train data.cov()
        cov_matrix.to_csv("data/cov_matrix.csv")
        # corr
        corr_matrix = train_data.corr()
        corr_matrix.to_csv("data/corr_matrix.csv")
        # deviation
        dev_matrix = train_data.std()
        dev_matrix.to_csv("data/dev_matrix.csv")
        # expected return
        expected return = train data.mean()
        expected_return.to_csv("data/expected_return.csv")
        print("Covariance Matrix")
        print(cov matrix)
        print("\n")
```

```
print("Correlation Matrix")
print(corr_matrix)
print("\n")

print("Deviation Matrix")
print(dev_matrix)
print("\n")

print("Expected Return")
print(expected_return)
print("\n")
```

```
Covariance Matrix
                DG
                         MRK
                                        CB
                                                     NOC.
                                                                BP \
DG
       4.508122e-03
                    0.000342 8.105542e-04
                                            7.840403e-04
                                                          0.000359
MRK
                    0.002364 4.898439e-04
                                            5.629656e-04
       3.424459e-04
                                                         0.000636
CB
      8.105542e-04
                    0.000490 1.456910e-03 6.276540e-04 0.000565
NOC.
      7.840403e-04
                    0.000563 6.276540e-04 1.848072e-03 0.000525
BP
       3.588692e-04
                    0.000636 5.645264e-04 5.252179e-04
                                                         0.003952
SPY
                    0.000634
                              7.518302e-04 5.584653e-04
       6.894545e-04
                                                          0.000739
      4.232179e-07 -0.000002 2.624179e-07 -4.793574e-07 0.000002
rates
               SPY
                           rates
DG
       6.894545e-04 4.232179e-07
MRK
       6.339753e-04 -1.820762e-06
CB
       7.518302e-04 2.624179e-07
N0C
       5.584653e-04 -4.793574e-07
BP
      7.392004e-04 2.135279e-06
SPY
       7.845685e-04 3.310369e-07
rates 3.310369e-07 5.669386e-08
Correlation Matrix
            DG
                     MRK
                                CB
                                         NOC.
                                                    BP
                                                             SPY
                                                                     rates
DG
       1.000000 0.104901 0.316277
                                    0.271632 0.085027
                                                        0.366600
                                                                  0.026473
MRK
       0.104901 1.000000 0.263953
                                    0.269344 0.208180
                                                        0.465524 -0.157279
CB
       0.316277
                0.263953
                         1.000000 0.382511 0.235281
                                                        0.703215
                                                                  0.028874
NOC.
       0.271632 0.269344 0.382511
                                    1.000000
                                              0.194356
                                                        0.463790 -0.046831
BP
                          0.235281
       0.085027
                0.208180
                                    0.194356
                                              1.000000
                                                        0.419822
                                                                  0.142661
SPY
       0.366600 0.465524
                          0.703215 0.463790 0.419822
                                                        1.000000
                                                                  0.049636
rates 0.026473 -0.157279 0.028874 -0.046831 0.142661
                                                        0.049636 1.000000
Deviation Matrix
DG
        0.067143
MRK
        0.048620
CB
        0.038169
NOC.
        0.042989
BP
         0.062861
SPY
        0.028010
         0.000238
rates
dtype: float64
Expected Return
DG
        0.010427
MRK
        0.004874
CB
        0.011683
NOC.
        0.026432
BP
        0.001467
SPY
         0.010480
rates
         0.000171
```

```
In []: # compute beta for each stock
market_train = pd.read_csv("data/{}/{}_train.csv".format(market, market))
```

dtype: float64

```
market_train = market_train.set_index("Date")
market train = market train.rename(columns={"return": market})
market train = market train.dropna()
# read in the rates test data
rates train = pd.read csv("data/rates/rates train.csv")
rates train = rates train.set index("Date")
rates train = rates train.rename(columns={"return": "rates"})
rates train = rates train.dropna()
# compute the excess return for the market
market train[market] = market train[market] - rates train["rates"]
betas = pd.DataFrame()
intercepts = pd.DataFrame()
residual dev = pd.DataFrame()
for ticker in tickers:
    df = pd.read_csv("data/{}/{}_train.csv".format(ticker, ticker))
    df = df.set index("Date")
    df = df.rename(columns={"return": ticker})
    df = df.dropna()
    df[ticker] = df[ticker] - rates train["rates"]
    df = pd.merge(df, market_train, on="Date", how="inner")
    # beta = (df[ticker].cov(df[market]))/(df[market].var())
    # intercept = df[ticker].mean() - beta*df[market].mean()
    # betas.loc[ticker, "beta"] = beta
    # intercepts.loc[ticker, "intercept"] = intercept
    # perform linear regression
    from sklearn.linear_model import LinearRegression
    model = LinearRegression()
    model.fit(df[[market]], df[[ticker]])
    intercept = model.intercept [0]
    beta = model.coef [0][0]
    intercepts.loc[ticker, "intercept"] = intercept
    betas.loc[ticker, "beta"] = beta
    # plot the regression line
    import matplotlib.pyplot as plt
    # plt.scatter(df[[market]], df[[ticker]])
    # plt.plot(df[[market]], model.predict(df[[market]]), color="red")
    # plot the residuals
    # plt.scatter(df[[market]], df[[ticker]] - model.predict(df[[market]]))
    residuals = df[[ticker]] - model.predict(df[[market]])
    residuals = residuals**2
    residual_dev.loc[ticker, "residual_dev"] = residuals.sum().values[0]**0.
    print("Beta {ticker}: {beta}".format(ticker=ticker, beta=beta))
    print("Intercept {ticker}: {intercept}".format(ticker=ticker, intercept=
    print("Residual Dev {ticker}: {residual_dev}".format(ticker=ticker, resi
    print("\n")
betas.to csv("data/betas.csv")
```

```
intercepts.to csv("data/intercepts.csv")
 residual dev.to csv("data/residual dev.csv")
Beta DG: 0.8785577507728043
Intercept DG: 0.0011989674873874815
Residual Dev DG: 0.4757402186651434
Beta MRK: 0.8106525543922
Intercept MRK: -0.0036544905788099104
Residual Dev MRK: 0.32778068959050455
Beta CB: 0.9583274954573505
Intercept CB: 0.0016325255467508955
Residual Dev CB: 0.20667489290719473
Beta NOC: 0.7126231420051568
Intercept NOC: 0.01891431827260464
Residual Dev NOC: 0.2900966104172669
Beta BP: 0.9398282702149634
Intercept BP: -0.008393210878596829
Residual Dev BP: 0.43448845086189986
```

```
In [ ]: # alphas
        # compute expected return using analyst data
        analyst = pd.read csv("analysts.csv")
        alphas = pd.DataFrame()
        # pull the price
        for ticker in tickers:
            expected price = analyst[analyst["Ticker"] == ticker]["Price"].values[0]
            dividend = analyst[analyst["Ticker"] == ticker]["Dividend"].values[0]
            train data = pd.read csv("data/{}/{} train.csv".format(ticker, ticker))
            last_price = analyst[analyst["Ticker"] == ticker]["Current"].values[0]
            analyst return = (expected price + dividend - last price) / last price
            # compute the alpha
            last rf = rates train["rates"].iloc[-1]
            last rm = market train[market].mean()
            erm = analyst return/12.0
            alpha = (erm - last_rf - betas.loc[ticker, "beta"]*(last_rm - last_rf))*
            alphas.loc[ticker, "alpha"] = alpha
            # print all intermediate values
            print("Expected Return {ticker}: {erm}".format(ticker=ticker, erm=erm))
            print("Expected Market Return: {last rm}".format(last rm=last rm))
            print("Expected Risk Free Rate: {last rf}".format(last rf=last rf))
            print("Alpha {ticker}: {alpha}".format(ticker=ticker, alpha=alpha))
            print("\n")
        alphas.to_csv("data/alphas.csv")
```

```
Expected Return DG: 0.011904761904761904
Expected Market Return: 0.010309632619618819
Expected Risk Free Rate: 0.0008166666666666
Alpha DG: 0.27479764223067926
Expected Return MRK: 0.01445978454136596
Expected Market Return: 0.010309632619618819
Alpha MRK: 0.594762077618046
Expected Return CB: 0.011966748880971942
Expected Market Return: 0.010309632619618819
Alpha CB: 0.20527119281507408
Expected Return NOC: 0.007151112395261482
Expected Market Return: 0.010309632619618819
Alpha NOC: -0.04304614957459062
Expected Return BP: 0.009515570934256057
Expected Market Return: 0.010309632619618819
Alpha BP: -0.022285350318316903
```

## List of Strategies

The assumption we are utilizing is that the CAL has a consistent slope hence we are only computing the risky portfolio.

- 1. Equities equal allocation portfolio
- 2. SP500 only
- 3. optimal risky portfolio that matches SP500 expected return

4. optimal risky portfolio that matches SP500 deviation

- 5. Beta Scaled Portfolio (Growth Portfolio)
- 6. Treynor-Black Allocation
- 7. Markowitz allocation (market as an asset to simulate comparison to treynor black)

```
In []: final_weights = pd.DataFrame()
    # add the tickers + market to index
    final_weights.index = tickers + [market]

# Algorithm 1 all equal weights
    weights = np.ones(len(tickers)+1)/(len(tickers)+1)
    final_weights["equal"] = weights

# Algorithm 2 only market
    weights = np.zeros(len(tickers)+1)
    weights[-1] = 1
    final_weights["market"] = weights

In []: # optimization data
    from scipy.optimize import minimize
```

```
In []: # optimization data
    from scipy.optimize import minimize

    train_data = pd.read_csv("data/train_data.csv")

# choose only the tickers and the market
    train_data = train_data[tickers+[market]]

# compute the expected return
    expected_return = train_data.mean()

# compute the covariance matrix
    cov_matrix = train_data.cov()
```

```
In []: # Algorithm 3 minimum variance equal return to market
    targ_ret = expected_return.loc[market]

def portfolio_dev(weights, cov_matrix):
    return np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights)))

def constraint(weights, targ_ret, expected_return):
    return np.dot(weights, expected_return) - targ_ret

def constraint2(weights):
    return np.sum(weights) - 1

constraints = [{"type": "eq", "fun": constraint, "args": (targ_ret, expected weights_guess = np.ones(len(tickers)+1)/(len(tickers)+1)

optimal_portfolio = minimize(portfolio_dev, weights_guess, args=(cov_matrix) final_weights.loc[[ticker for ticker in tickers]+[market], "min_var_mark_ret
```

```
In [ ]: # Algorithm 4 equal variance maximal return
        targ_var = dev_matrix.loc[market]
        def ret func(weights, expected return):
            return -np.dot(weights, expected_return)
        def constraint(weights, targ var, cov matrix):
            return portfolio dev(weights, cov matrix) - targ var
        def constraint2(weights):
            return np.sum(weights) - 1
        constraints = [{"type": "eq", "fun": constraint, "args": (targ var, cov matr
        optimal_portfolio = minimize(ret_func, weights_guess, args=(expected_return)
        final_weights.loc[[ticker for ticker in tickers]+[market], "equal_var_max_re
In [ ]: # Algorithm 5 Beta Scaled
        betas = pd.read_csv("data/betas.csv")
        betas.set_index("Unnamed: 0", inplace=True)
        betas.index.name = "Ticker"
        betas.loc[market, "beta"] = 1
        scaled_betas = betas["beta"] / betas["beta"].sum()
        final weights.loc[[ticker for ticker in tickers]+[market], "beta scaled"] =
In [ ]: # Algorithm six - Markowitz
        # optimization function
        def portfolio dev(weights, cov matrix):
            return np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights)))
        # constraints - we are okay with short selling
        def constraint(weights):
            return weights.sum() - 1
        constraints = {"type": "eg", "fun": constraint}
        # initial quess
        weights guess = np.random.rand(len(tickers)+1)
        # minimize
        from typing import final
        from scipy.optimize import minimize
        optimal_portfolio = minimize(portfolio_dev, weights_guess, args=(cov_matrix)
        # save to final weights
        final_weights.loc[[ticker for ticker in tickers]+[market], "markowitz"] = or
In [ ]: # Algorithm seven - Treynor Black
        # compute the starting weights alpha / resiudal variance
        weights = pd.DataFrame()
```

```
for ticker in tickers:
    alpha = alphas.loc[ticker, "alpha"]
    residual variance = residual dev.loc[ticker, "residual dev"]**2
    weight = alpha/residual_variance
    weights.loc[ticker, "weight"] = weight
# scale the weights
weights["weight"] = weights["weight"]/weights["weight"].sum()
# residual variance of portfolio
residual variance port = 0
alpha port = 0
for ticker in tickers:
    residual variance port += weights.loc[ticker, "weight"]**2 * residual de
    alpha_port += weights.loc[ticker, "weight"] * alphas.loc[ticker, "alpha"
initial_position = (alpha_port/residual_variance_port) / (expected_return[me
print("Initial Position: ", initial_position)
beta_port = 0
for ticker in tickers:
    beta_port += weights.loc[ticker, "weight"]*betas.loc[ticker, "beta"]
print("Beta Portfolio: ", beta_port)
adjusted_initial_position = initial_position / (1+ (1-beta_port)*initial_pos
print("Adjusted Initial Position: ", adjusted_initial_position)
m_initial_position = 1-adjusted_initial_position
expected return port = (m initial position+beta port*adjusted initial positi
variance port = (m initial position+beta port*adjusted initial position)**2
print("Expected Return Portfolio: ", expected_return_port)
print("Variance Portfolio: ", variance port)
for ticker in weights.index:
    final weights.loc[ticker, "treynor"] = adjusted initial position*weights
# add market to the weights
final_weights.loc[market, "treynor"] = m_initial_position
print("Weights: ", final_weights)
```

Initial Position: 0.8179381580415437

```
Beta Portfolio: 0.8863449402080142
      Adjusted Initial Position: 0.7483677855865897
       Expected Return Portfolio: 0.32721092755967873
      Variance Portfolio: 0.02241204501304662
      Weights:
                        equal market min_var_mark_return equal_var_max_return
      beta scaled \
      DG
            0.166667
                        0.0
                                        0.025749
                                                             -0.001143
                                                                           0.16576
      6
      MRK 0.166667
                        0.0
                                        0.100610
                                                              0.015739
                                                                           0.15295
      CB
           0.166667
                        0.0
                                        0.046923
                                                              0.027365
                                                                           0.18081
       7
      NOC 0.166667
                        0.0
                                        0.053161
                                                              0.295512
                                                                           0.13445
      7
      BP
           0.166667
                        0.0
                                        0.037610
                                                             -0.023064
                                                                           0.17732
       6
      SPY 0.166667
                        1.0
                                        0.735948
                                                              0.685590
                                                                           0.18868
       0
           markowitz treynor
      DG
            0.017442 0.083162
            0.072848 0.379167
      MRK
            0.036422 0.329159
      NOC 
            0.133218 - 0.035035
      BP
            0.015318 -0.008086
      SPY
            0.724751 0.251632
In [ ]: final weights.to csv("data/final weights.csv")
        train data = None
        for ticker in tickers+[market]:
            # read in rates
            rates = pd.read csv("data/rates/rates train.csv")
            rates = rates.set index("Date")
            rates = rates[["return"]]
            df = pd.read_csv("data/{}/{}_train.csv".format(ticker, ticker))
            df = df.set index("Date")
            df = df[["return"]]
            df = df - rates
            df = df.rename(columns={"return": ticker})
            if train data is None:
                train_data = df
            else:
                train_data = pd.merge(train_data, df, on="Date", how="inner")
        train data = train data.dropna()
        train_data.to_csv("data/test_data.csv")
        dates_to_compute_return = ["2017-10-02"]
        # weighted returns
        weighted_returns = pd.DataFrame()
        weighted_returns.index = train_data.index
```

```
for strat in final_weights.columns:
     weights = final weights.loc[:, strat]
     temp returns = np.dot(train data, weights)
     weighted_returns.loc[:, strat] = temp_returns
 # weighted returns
 weighted returns.to csv("data/weighted returns train.csv")
 for date in dates to compute return:
     returns_at_dates = pd.DataFrame(index=final_weights.columns, columns=["a
     weighted_returns_to_now = weighted_returns[weighted_returns.index <= dat</pre>
     excess returns = weighted returns to now
     # compute the average return, total return, and sharpe ratio
     average_return = excess_returns.mean()
     total_return = excess_returns.sum()
     std_dev = excess_returns.std()
     sharpe_ratio = average_return/std_dev
     returns_at_dates["average_return"] = average_return
     returns at dates["total return"] = total return
     returns_at_dates["std_dev"] = std_dev
     returns_at_dates["sharpe_ratio"] = sharpe_ratio
     os.makedirs("data/FINAL_DATA/", exist_ok=True)
     print("DATE: ", date)
     print(returns_at_dates)
     print("\n")
     returns_at_dates.to_csv("data/FINAL_DATA/returns_at_dates_" + date + ".d
DATE: 2017-10-02
                      average_return total_return std_dev sharpe_ratio
equal
                            0.010723
                                          0.632667 0.030302
                                                                  0.353877
                                          0.608268 0.027999
                                                                  0.368210
market
                            0.010310
```

# After computation of all strategies we evaluate them below

0.010310

0.015176

0.010207

0.011931

0.008089

- 1. Evaluation is done using EXCESS RETURNS to account for varying risk free data
- 2. We evaluate at the five periods described in the announcement
- 3. The first data period does not have all metrics as the deviation cannot be calculated over one period

0.608268 0.027413

0.895392 0.028011

0.602228 0.030321

0.703932 0.027192

0.477272 0.031477

0.376085

0.541797

0.336636

0.438767

0.256995

```
In [ ]: final_weights.to_csv("data/final_weights.csv")
```

min\_var\_mark\_return

equal\_var\_max\_return

beta scaled

markowitz

treynor

```
test data = None
for ticker in tickers+[market]:
    # read in rates
    rates = pd.read_csv("data/rates/rates_test.csv")
    rates = rates.set index("Date")
    rates = rates[["return"]]
    df = pd.read_csv("data/{}/{}_test.csv".format(ticker, ticker))
    df = df.set index("Date")
    df = df[["return"]]
    # df = df - rates
    df = df.rename(columns={"return": ticker})
    if test data is None:
        test data = df
    else:
        test_data = pd.merge(test_data, df, on="Date", how="inner")
test data = test data.dropna()
test_data.to_csv("data/test_data.csv")
dates_to_compute_return = ["2017-11-01", "2018-01-01", "2018-04-01", "2018-1
# weighted returns
weighted returns = pd.DataFrame()
weighted returns.index = test data.index
for strat in final_weights.columns:
   weights = final weights.loc[:, strat]
   temp_returns = np.dot(test_data, weights)
   weighted_returns.loc[:, strat] = temp_returns
# weighted returns
weighted_returns.to_csv("data/weighted_returns.csv")
for date in dates_to_compute_return:
    returns_at_dates = pd.DataFrame(index=final_weights.columns, columns=["ē
    weighted_returns_to_now = weighted_returns[weighted_returns.index <= dat</pre>
    excess_returns = weighted_returns_to_now
    # compute the average return, total return, and sharpe ratio
    average return = excess returns.mean()
    total_return = excess_returns.sum()
    std dev = excess returns.std()
    sharpe_ratio = average_return/std_dev
    returns at dates ["average return"] = average return
    returns at dates["total return"] = total return
    returns_at_dates["std_dev"] = std_dev
    returns at dates["sharpe ratio"] = sharpe ratio
    os.makedirs("data/FINAL_DATA/", exist_ok=True)
    print("DATE: ", date)
    print(returns at dates)
    print("\n")
    returns_at_dates.to_csv("data/FINAL_DATA/returns_at_dates_" + date + ".d
```

DATE: 2017-11-01				
	average_return	total_return	std_dev	sharpe_ratio
equal	0.026226	0.026226	NaN	NaN
market	0.030566	0.030566	NaN	NaN
min_var_mark_return	0.027111	0.027111	NaN	NaN
equal_var_max_return	0.033348	0.033348	NaN	NaN
beta_scaled	0.025444	0.025444	NaN	NaN
markowitz	0.029389	0.029389	NaN	NaN
treynor	0.017906	0.017906	NaN	NaN
DATE: 2010 01 01				
DATE: 2018-01-01	average_return	total_return	std_dev	sharpe_ratio
equal	0.036701	0.110104	0.028575	1.284395
market	0.031302	0.093906	0.024697	1.267417
min_var_mark_return	0.031582	0.094746	0.025800	1.224098
equal_var_max_return	0.036286	0.108859	0.035515	1.021719
beta_scaled	0.035748	0.107244	0.027665	1.292170
markowitz	0.033272	0.099816	0.028951	1.149265
treynor	0.026595	0.079785	0.031850	0.835011
creynor	01020333	01073703	01031030	01033011
DATE: 2018-04-01				
	average_return	total_return	std_dev	sharpe_ratio
equal	0.011120	0.066723	0.042492	0.261704
market	0.005237	0.031423	0.035560	0.147278
<pre>min_var_mark_return</pre>	0.006669	0.040013	0.036443	0.182995
equal_var_max_return	0.007627	0.045765	0.038647	0.197363
beta_scaled	0.010475	0.062850	0.043029	0.243440
markowitz	0.007063	0.042377	0.035993	0.196230
treynor	0.002656	0.015934	0.048535	0.054716
DATE: 2018-10-01		t-t-1t		
agua1	average_return	total_return	std_dev	. —
equal	0.007442	0.089306	0.035601	0.209045
market	0.004855	0.058262	0.035631	
min_var_mark_return	0.006069	0.072829	0.034107	
equal_var_max_return	0.000949	0.011385	0.042165	0.022501
beta_scaled markowitz	0.007285 0.004470	0.087419 0.053644	0.035570 0.035925	
	0.004470	0.105201	0.033923	
treynor	0.000707	0.103201	0.039973	0.219310
DATE: 2019-10-01				
2013 10 01	average_return	total_return	std_dev	sharpe_ratio
equal	0.011345	0.272276	0.032529	0.348762
market	0.007764	0.186328	0.042018	0.184772
min_var_mark_return	0.009044	0.217046	0.037163	0.243347
equal_var_max_return	0.008544	0.205049	0.043355	0.197064
beta_scaled	0.010906	0.261746	0.032396	0.336650
markowitz	0.008941	0.214591	0.032536	0.231786
treynor	0.012252	0.294052	0.034365	0.356532
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