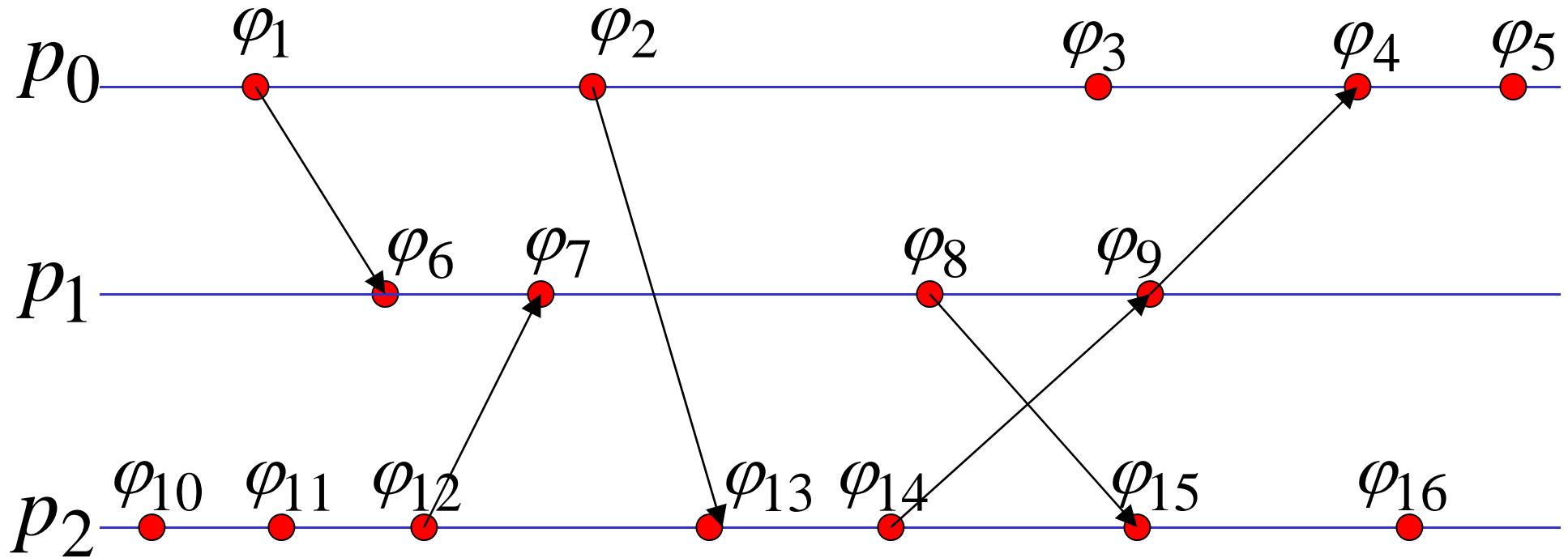


# Uzročnost (Causality)

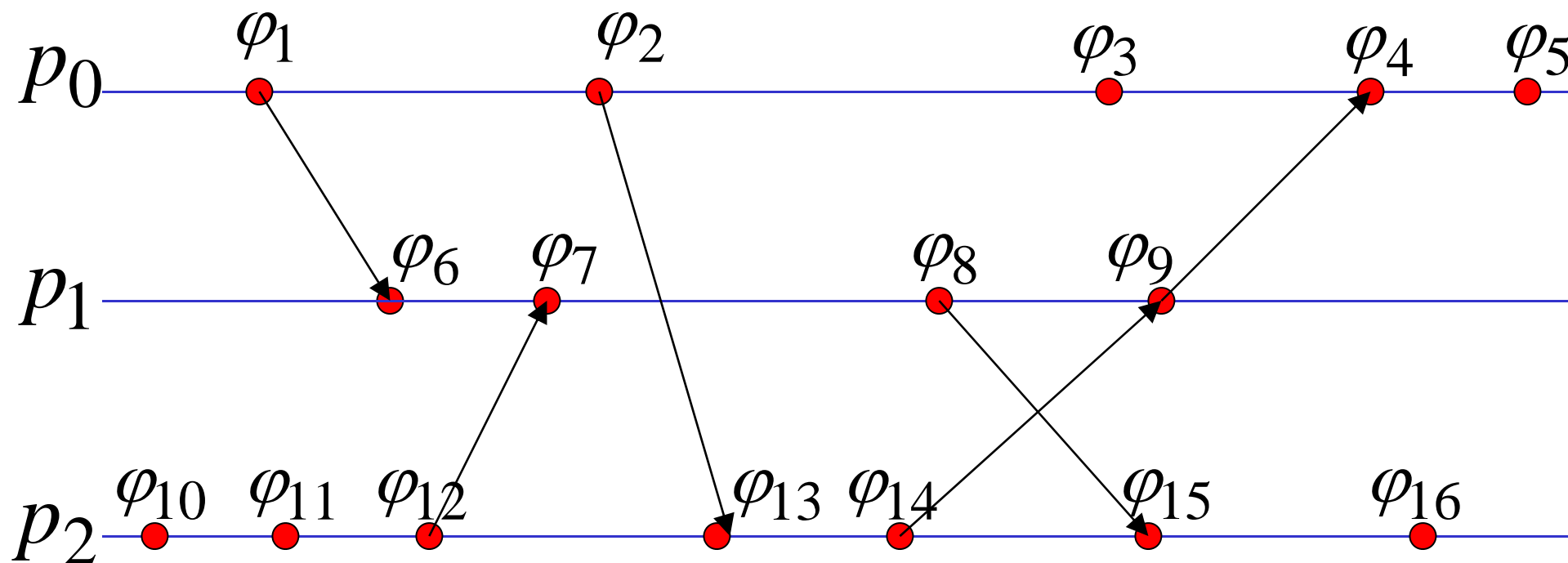
# Relacija "desio se pre"



$$\varphi_1 \Rightarrow \varphi_2$$

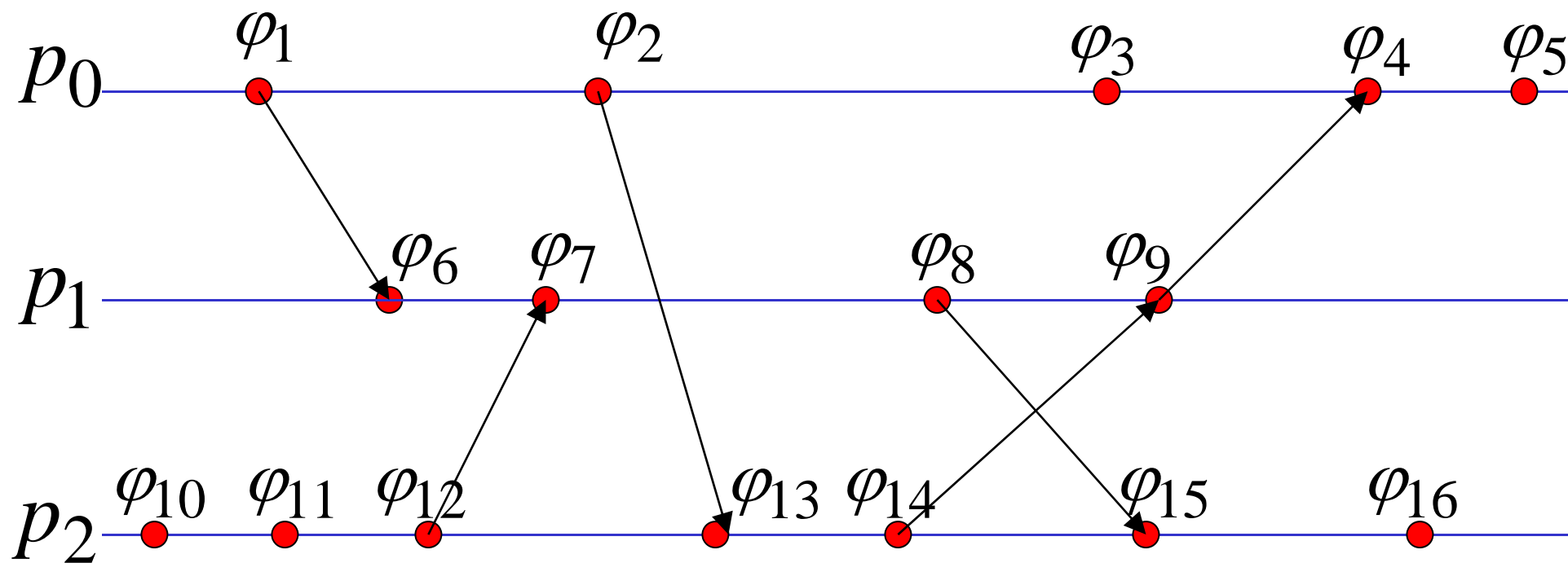
$\varphi_1$  desio se pre  
(uzrokuje)

$\varphi_2$



$$\varphi_1 \Rightarrow \varphi_6$$

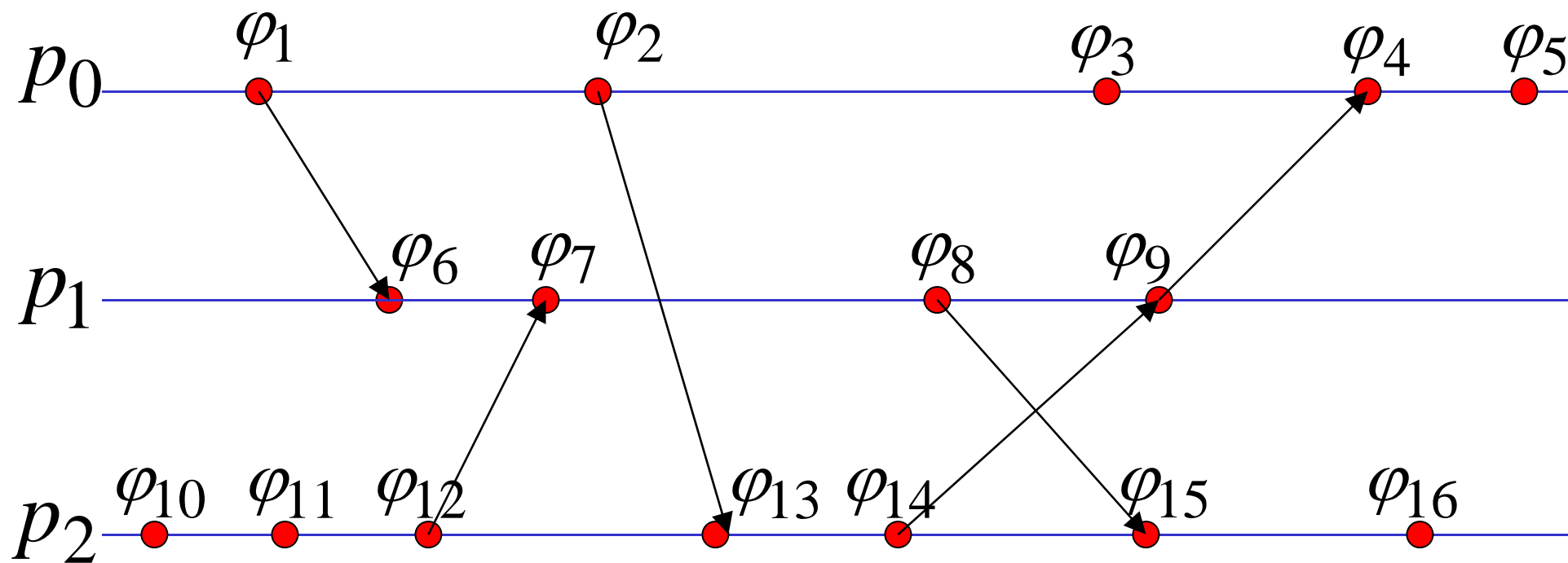
Poruka je poslata od  $\varphi_1$  ka  $\varphi_6$



tranzitivnost

$\varphi_1 \Rightarrow \varphi_6$   
 $\varphi_6 \Rightarrow \varphi_7$

$\varphi_1 \Rightarrow \varphi_7$



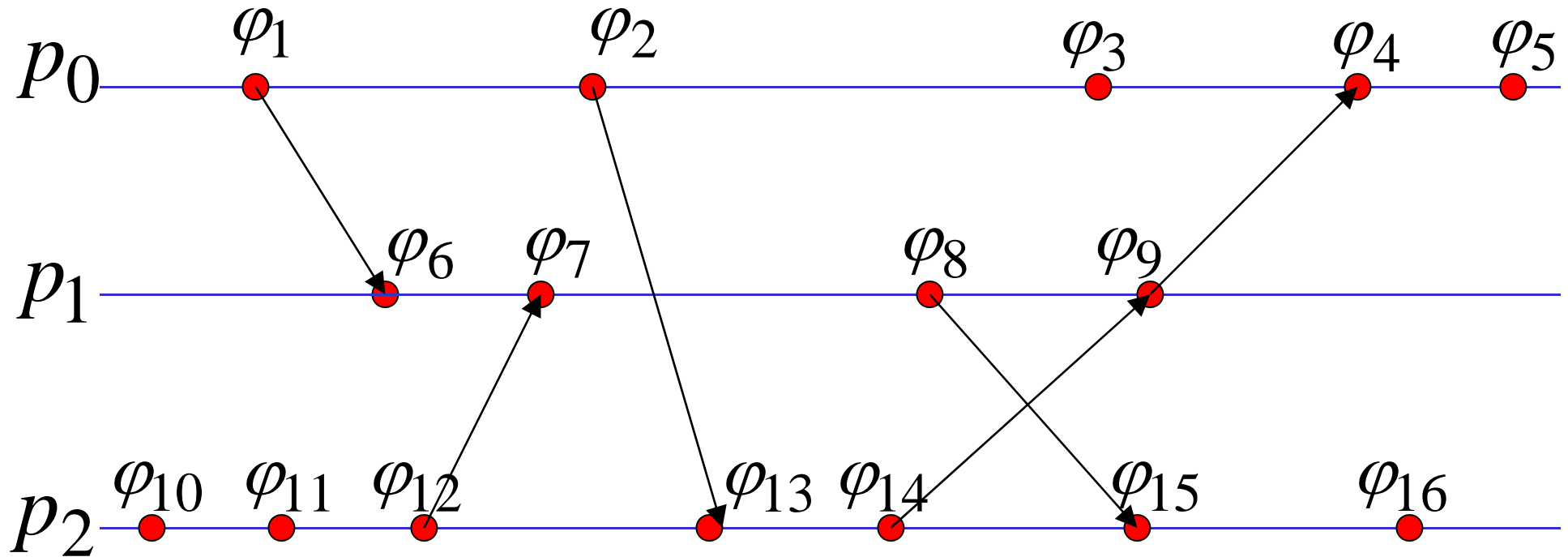
$$\varphi_1 \Rightarrow \varphi_8$$

$$\varphi_1 \Rightarrow \varphi_{16}$$

$$\varphi_{10} \Rightarrow \varphi_8$$

$$\varphi_2 \Rightarrow \varphi_9$$

“desio se pre” je delimičan redosled



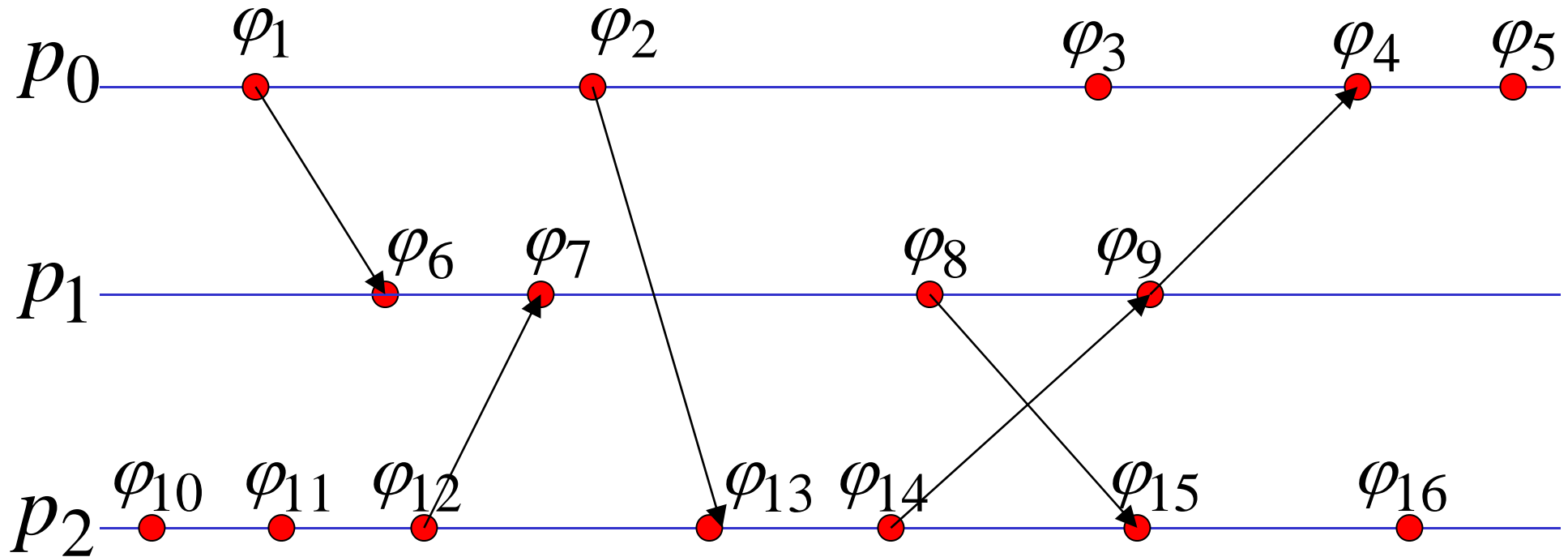
$$\varphi_1 \not\Rightarrow \varphi_{10}$$

Paralelni dogadaji:  $\varphi_1 \varphi_{10}$

$$\varphi_{10} \not\Rightarrow \varphi_1$$

$$\varphi_1 \parallel \varphi_{10}$$

“desio se pre” je delimičan redosled



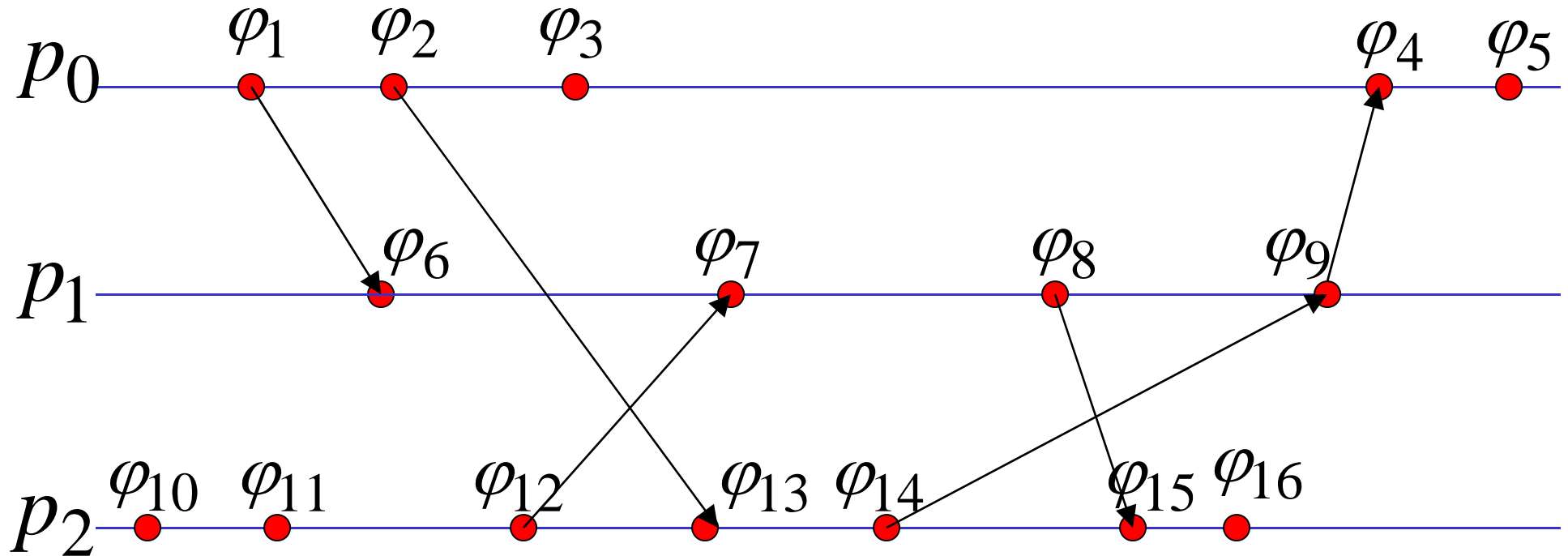
$$\varphi_{13} \not\Rightarrow \varphi_8$$

Paralelni dogadaji:  $\varphi_8 \quad \varphi_{13}$

$$\varphi_8 \not\Rightarrow \varphi_{13}$$

$$\varphi_8 \parallel \varphi_{13}$$

## Možemo pomerati paralelne dogadaje



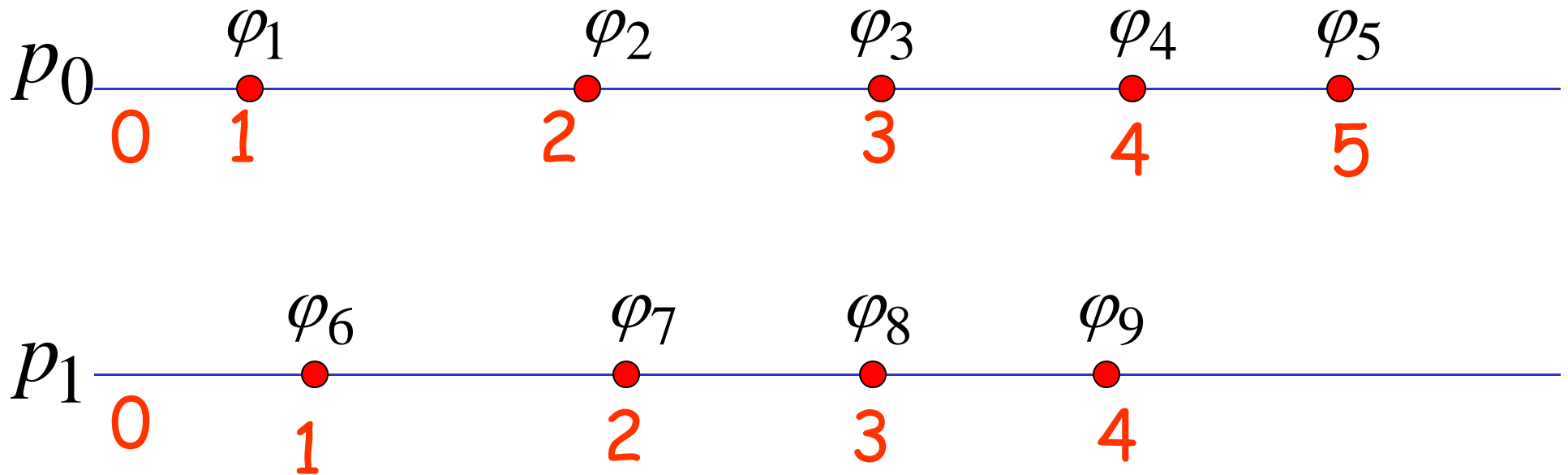
Relacija „desio se pre“ se ne menja



Želimo da pronađemo mehanizam koji  
odslikava "desio se pre" relaciju

tako da se uzročnost može koristiti u raznim  
problemima računanja

# Logički satovi (Clocks)

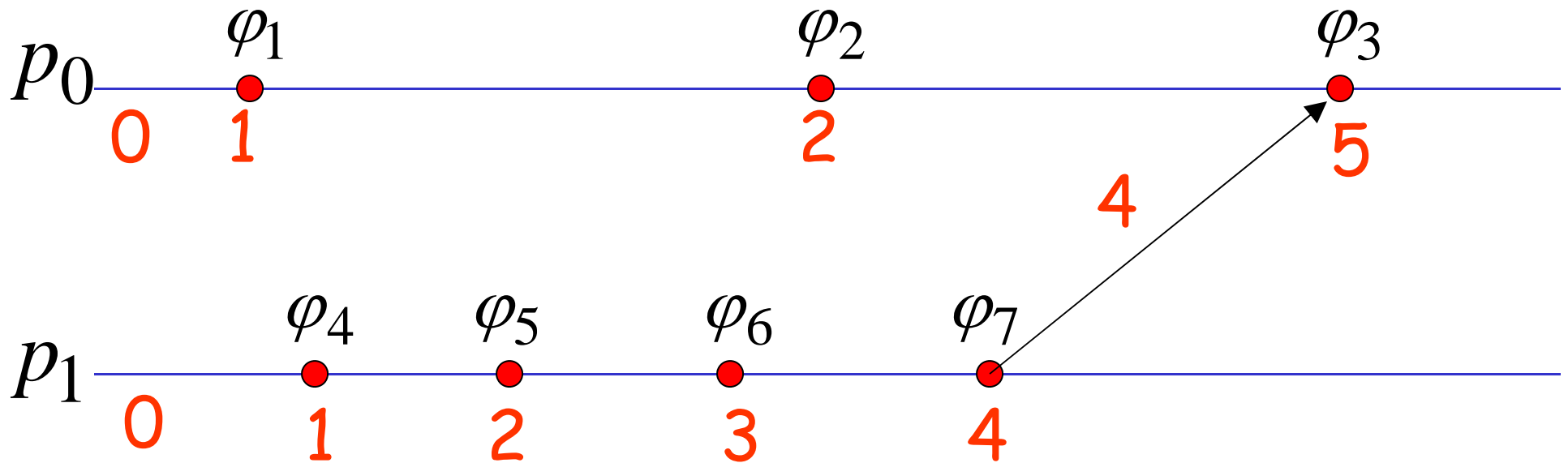


U svakom procesu, logički sat se povećava za 1 za svaki lokalni događaj

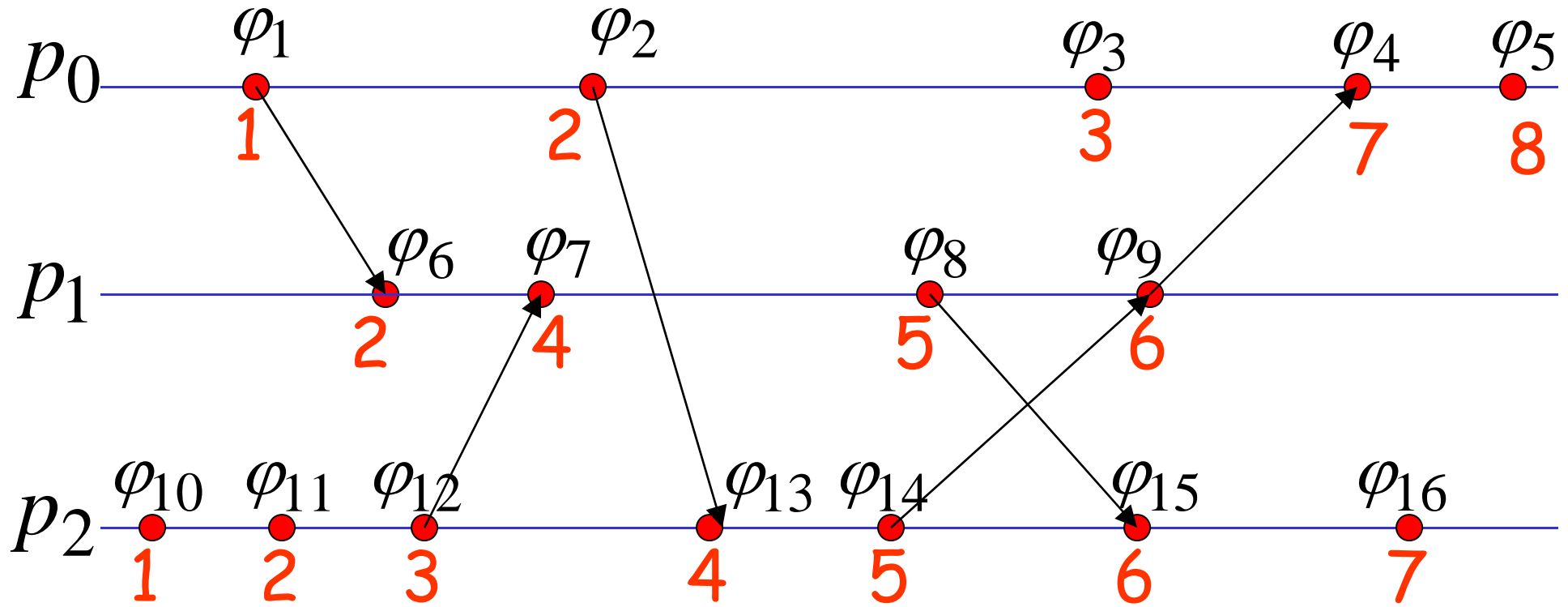
$$LT(\varphi_1) = 1$$

$$LT(\varphi_2) = LT(\varphi_1) + 1 = 2$$

## Logički satovi se prenose u porukama (piggy-packed)

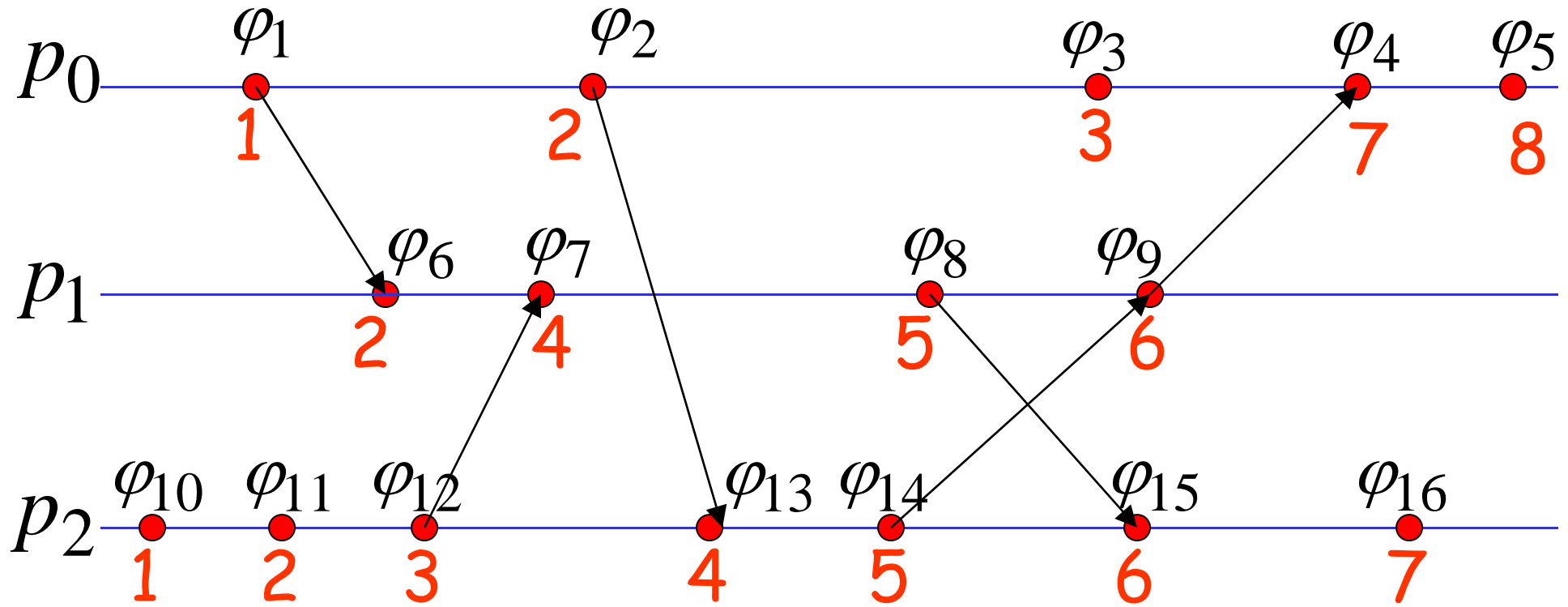


$$\begin{aligned} LT(\varphi_3) &= \max(LT(\varphi_2), LT(\varphi_7)) + 1 \\ &= \max(2, 4) + 1 \\ &= 4 + 1 = 5 \end{aligned}$$



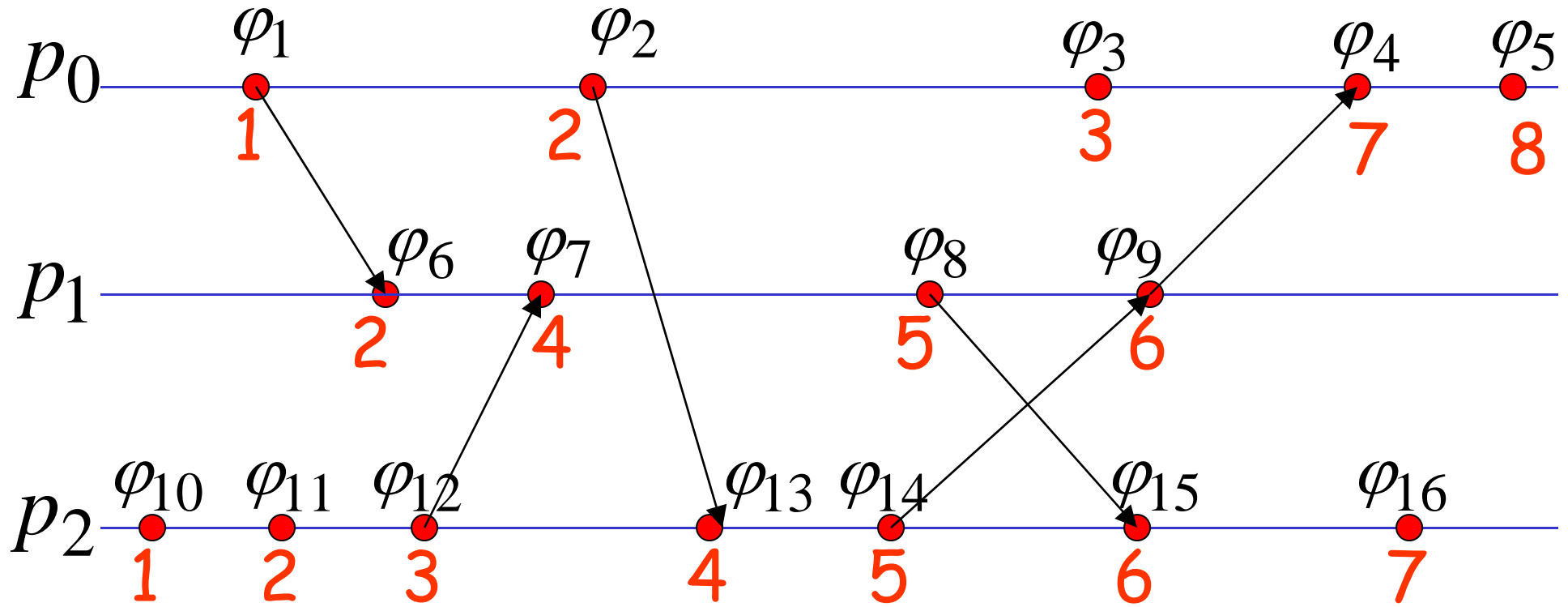
Izgleda kao da logički satovi odslikavaju  
desio se pre relaciju

$$\varphi_i \Rightarrow \varphi_j \quad \longrightarrow \quad LT(\varphi_i) < LT(\varphi_j)$$



Primer:

$$\varphi_1 \Rightarrow \varphi_7 \quad \text{green arrow} \quad LT(\varphi_1) = 1 < 4 = LT(\varphi_7)$$



Ipak, logički satovi  
nemogu odslikati paralelizam

$$\varphi_8 \parallel \varphi_{13}$$

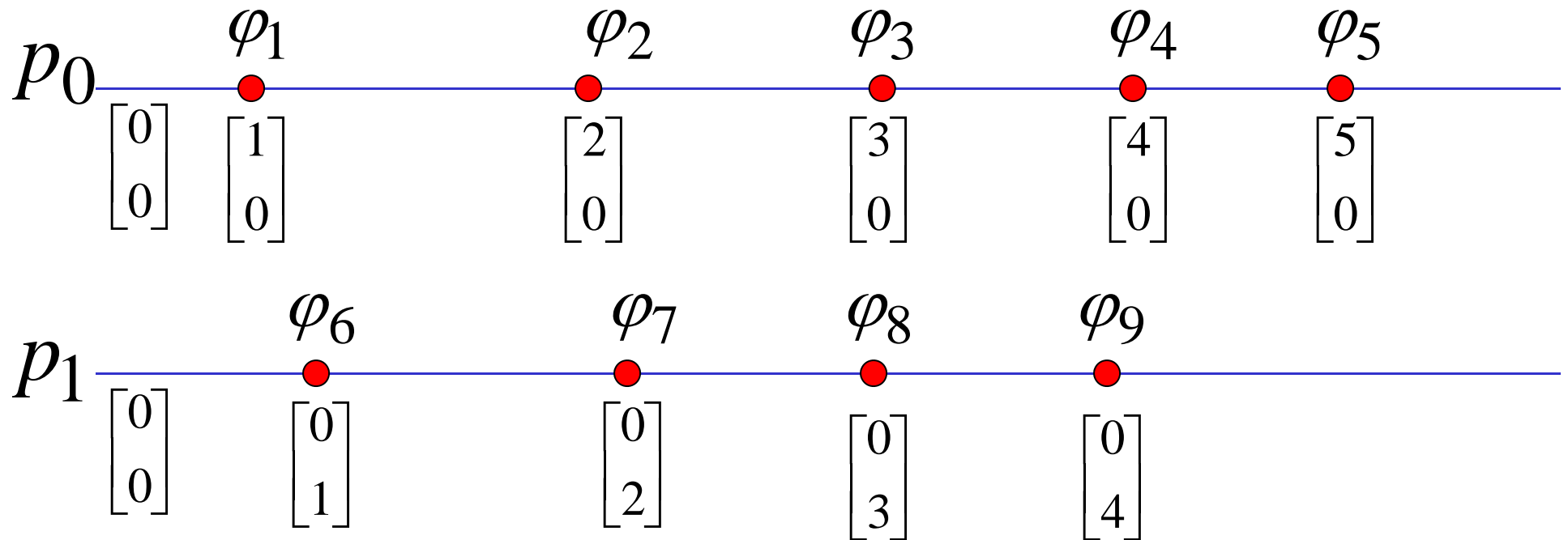
$$LT(\varphi_{13}) = 4 < 5 = LT(\varphi_8)$$

Paralelni događaji

$$(\varphi_{13} \Rightarrow \varphi_8 ???)$$

Treba nam drugi mehanizam koji  
može da odslika paralelizam događaja

# Vektorski satovi (Vector Clocks)



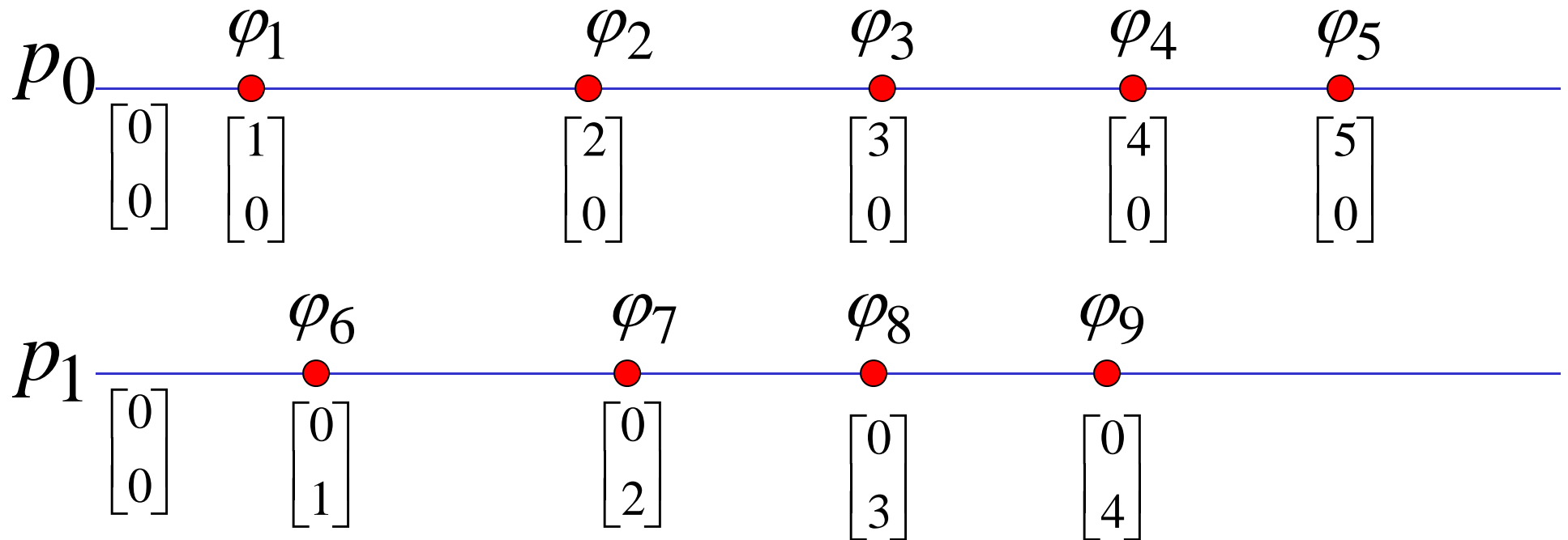
$$VC(\varphi_1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

← Elem procesa  $p_0$

← Elem procesa  $p_1$



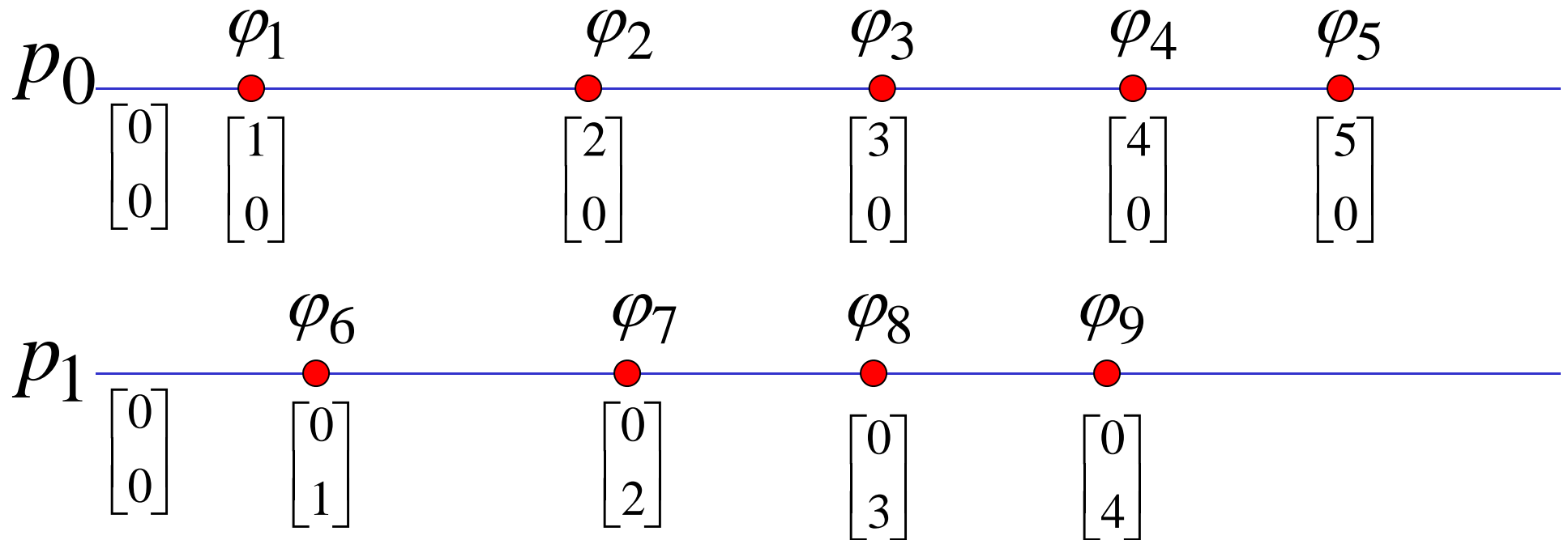
Svaki proces povećava svoj el. za svaki događaj



$$VC(\varphi_3) = VC(\varphi_2) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

povećaj za 1

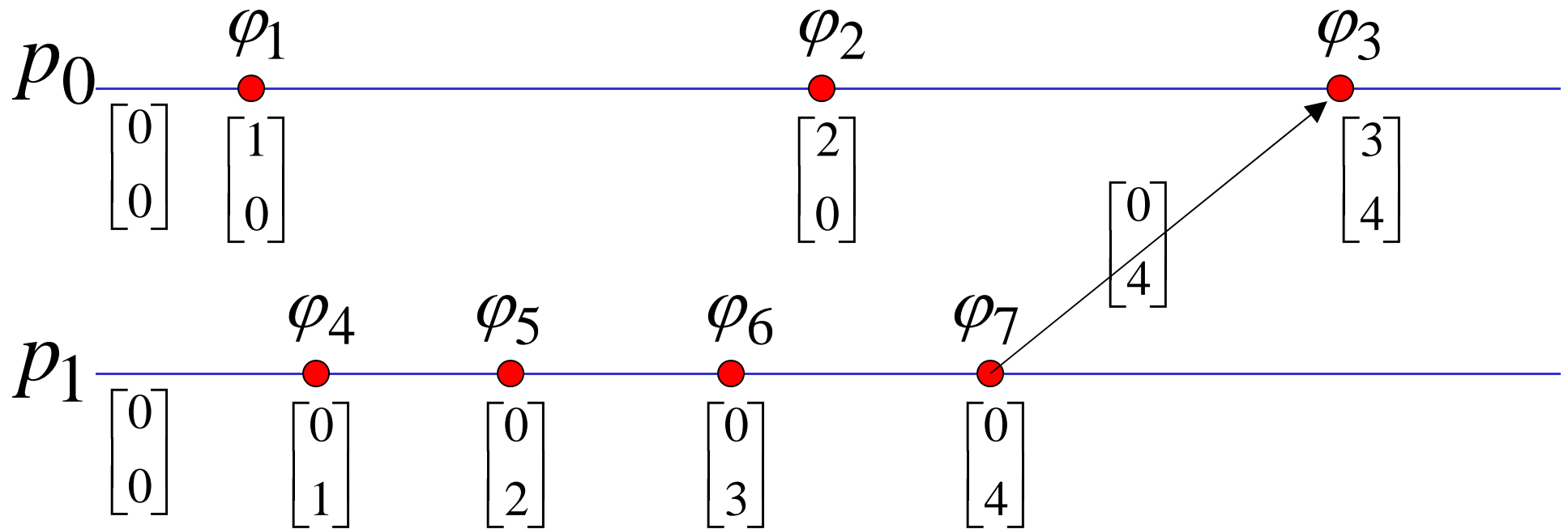
Svaki proces povećava svoj el. za svaki događaj



$$VC(\varphi_9) = VC(\varphi_8) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

povećaj za 1

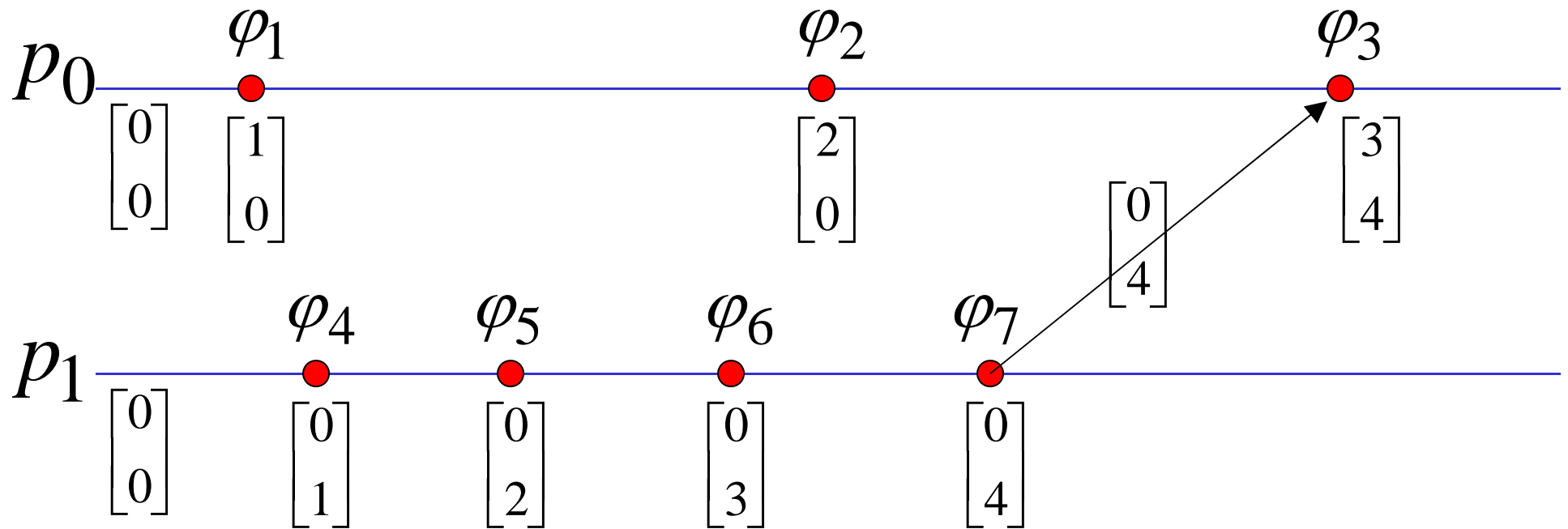
## vektorski satovi se prenose u porukama



$$\max\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

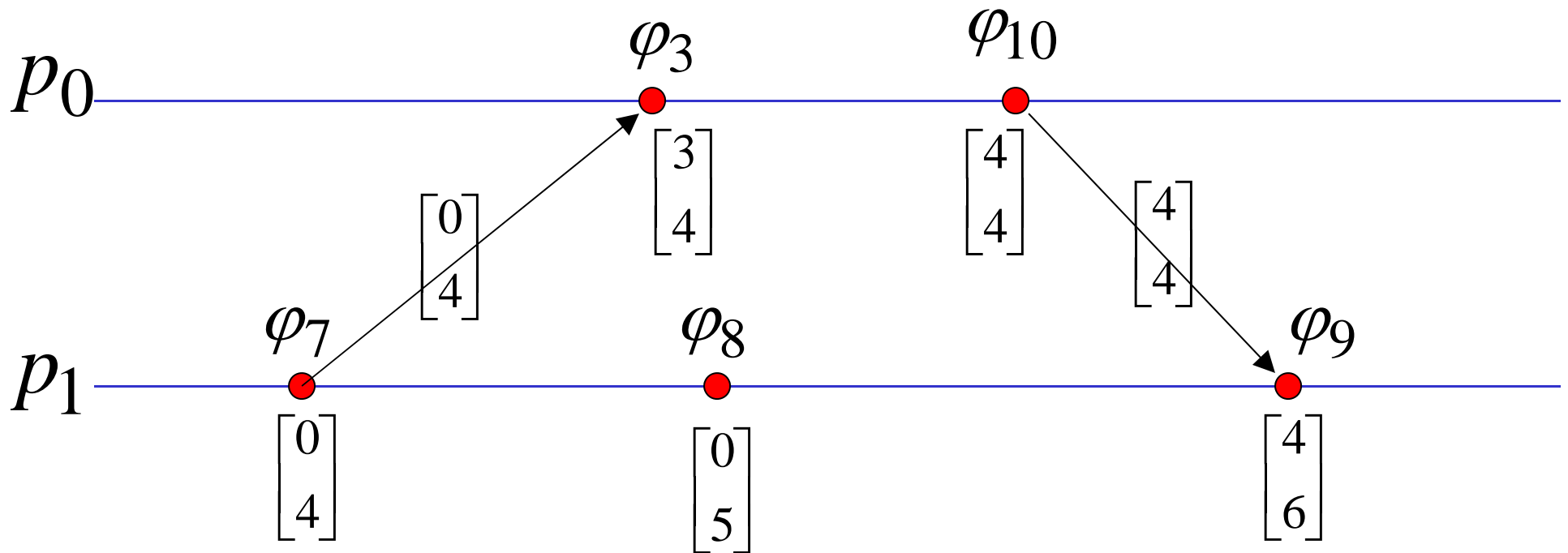
Maksimum za svaki elem

# vektorski satovi se prenose u porukama

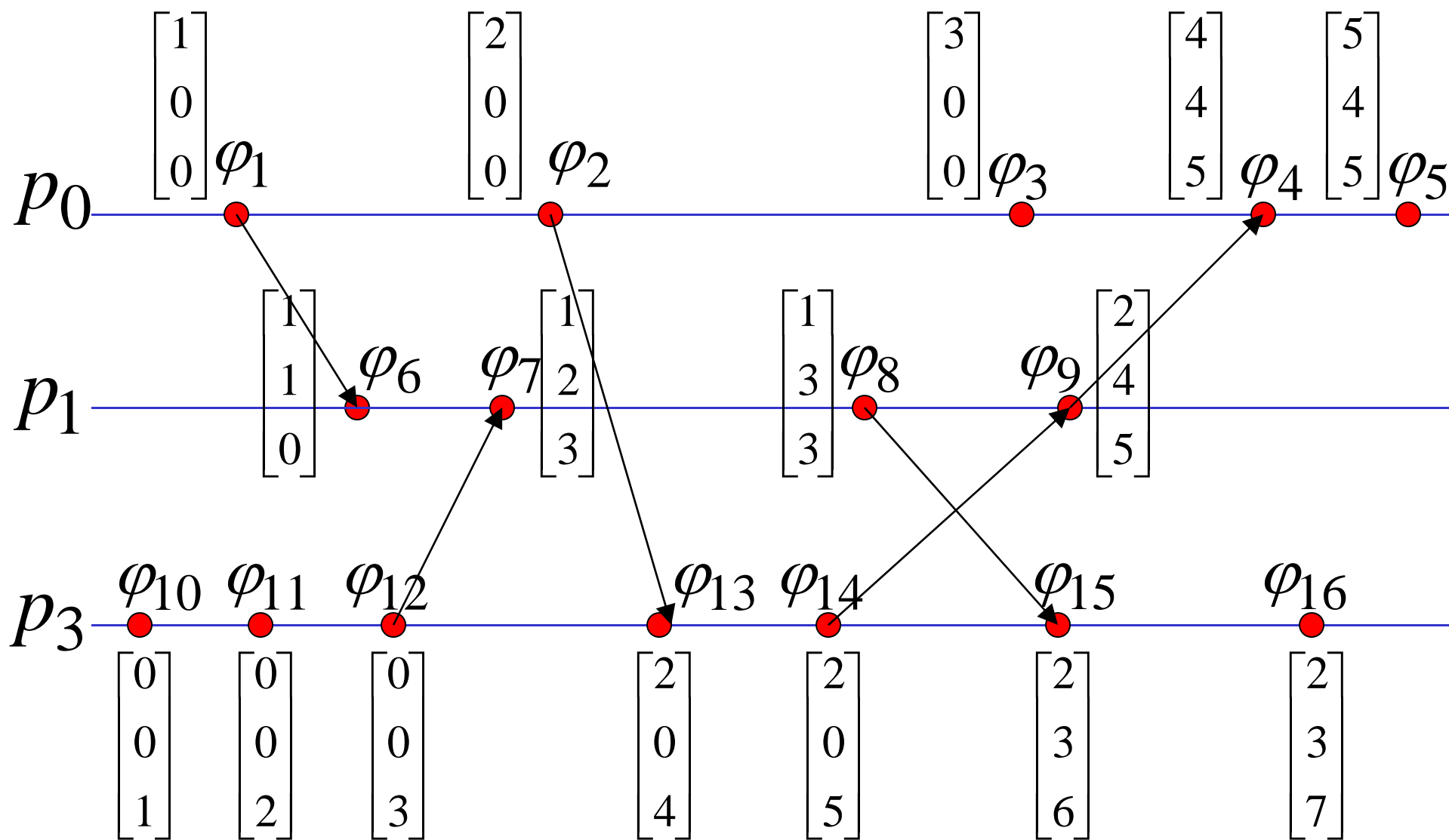


$$\begin{aligned}
 VC(\varphi_3) &= \max(VC(\varphi_2), VC(\varphi_7)) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
 &= \max\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \end{bmatrix}\right) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}
 \end{aligned}$$

# vektorski satovi se prenose u porukama



$$\begin{aligned}
 VC(\varphi_9) &= \max(VC(\varphi_8), VC(\varphi_{10})) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
 &= \max\left(\begin{bmatrix} 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \end{bmatrix}\right) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}
 \end{aligned}$$



# Poređenje vektorskih satova

Pišemo

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Ako  $a_i \leq b_i$  za sve  $i$

Primeri:

$$\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} < \begin{bmatrix} 9 \\ 7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} < \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$$



# Neuporedivi vektorski satovi

Pišemo

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \not\leq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Ako  
nije

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

niti

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \geq \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Primeri:

$$\begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 8 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

## Vektorski satovi odslikavaju uzročnost

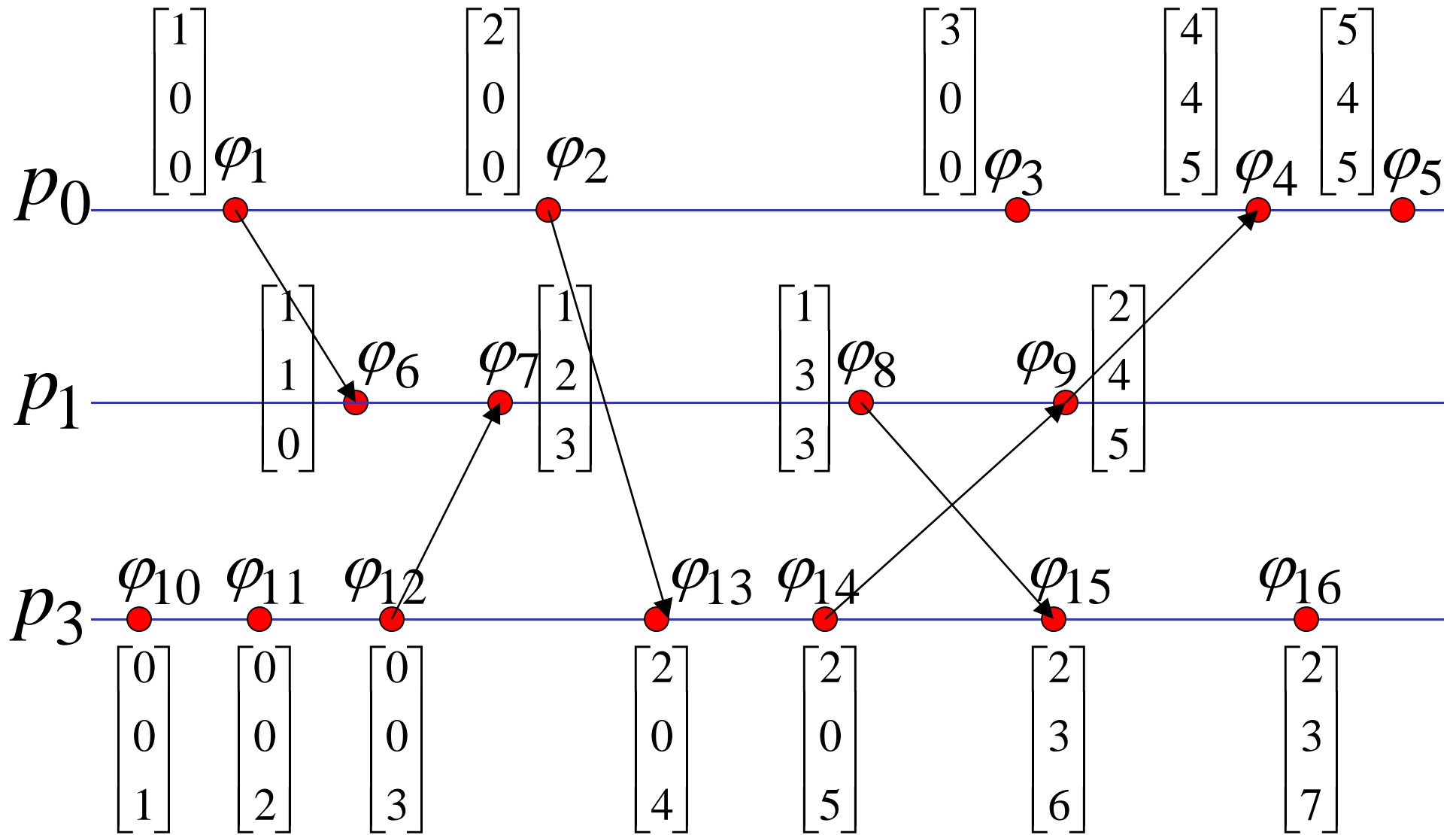
Ako  $\varphi_1 \Rightarrow \varphi_2$  onda  $VC(\varphi_1) < VC(\varphi_2)$

Ako  $\varphi_1 \parallel \varphi_2$  onda  $VC(\varphi_1) \neq VC(\varphi_2)$

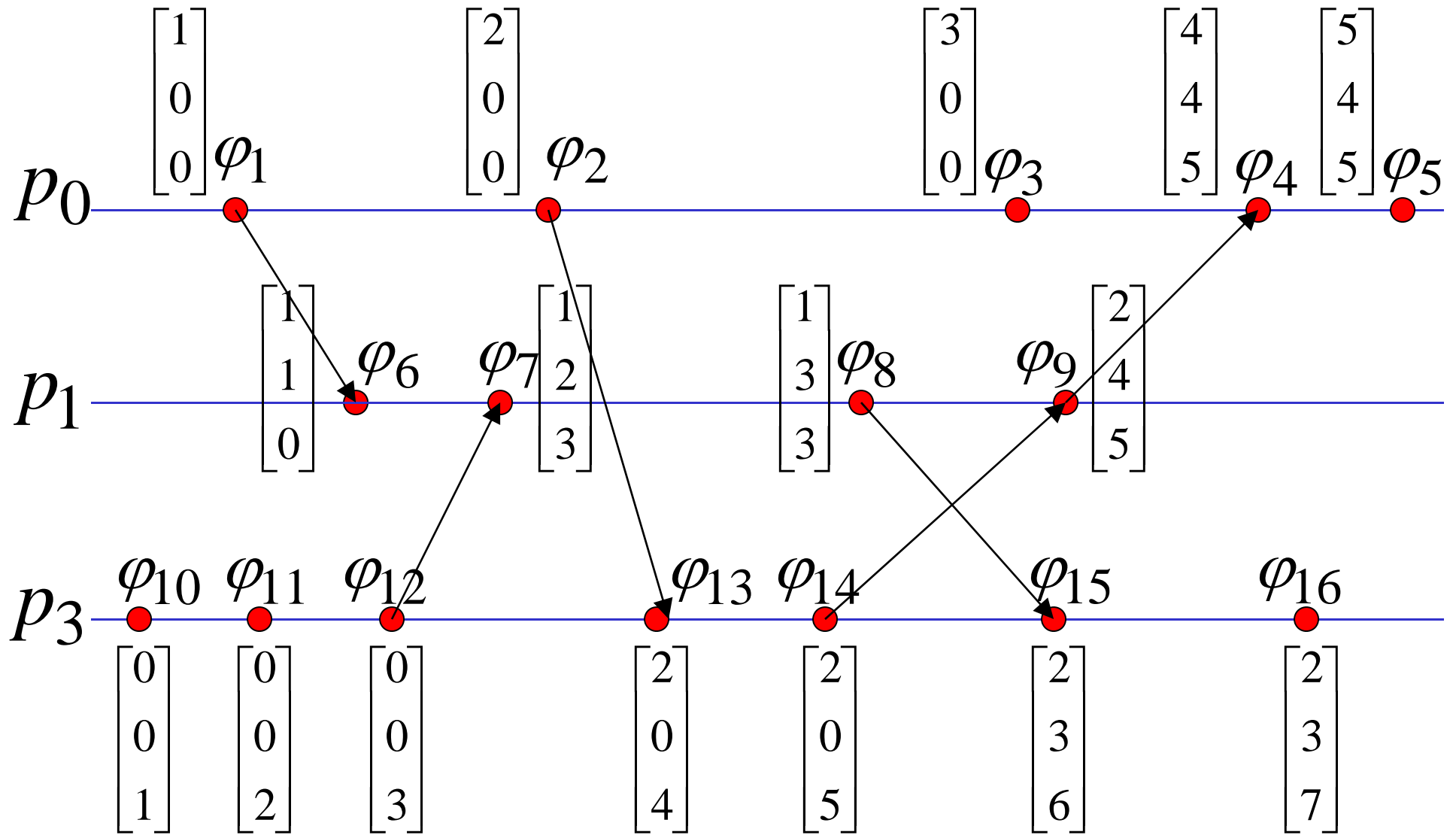
Ispitivanjem vektorskih satova  
možemo odrediti redosled događaja

Ako  $VC(\varphi_1) < VC(\varphi_2)$     onda  $\varphi_1 \Rightarrow \varphi_2$

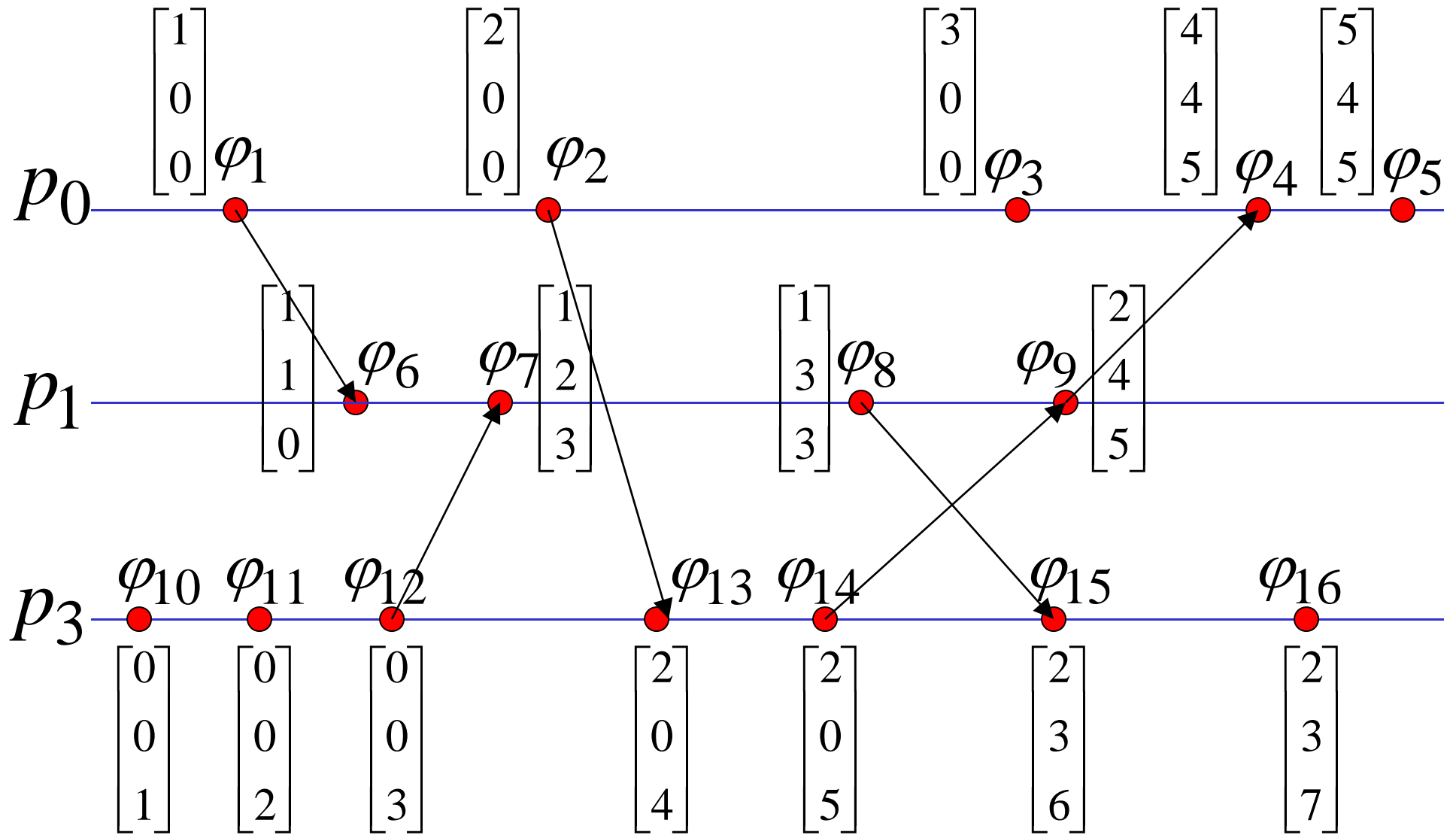
Ako  $VC(\varphi_1) \not\leq VC(\varphi_2)$     onda  $\varphi_1 \parallel \varphi_2$



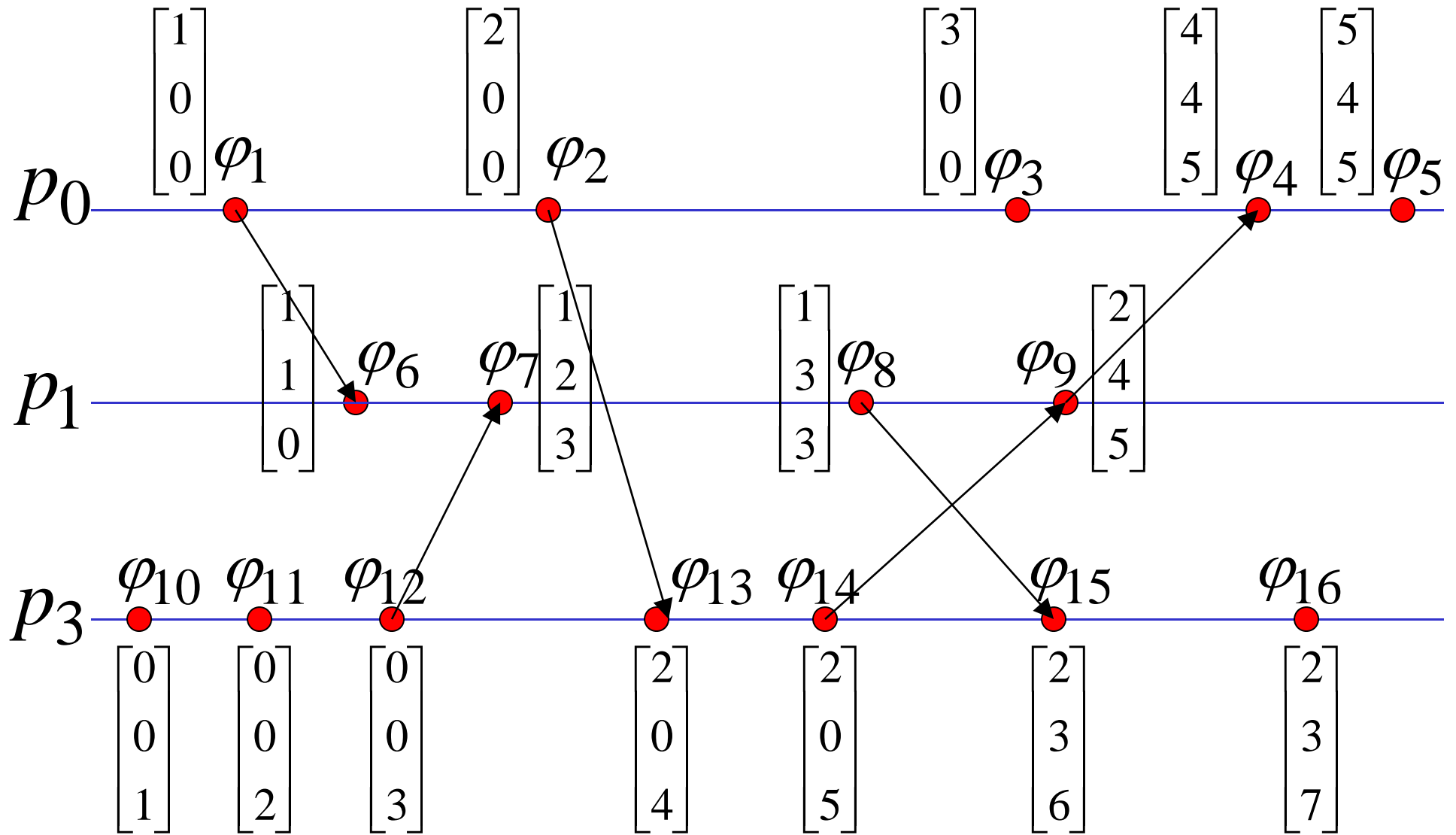
$$\varphi_1 \Rightarrow \varphi_2 \quad \longrightarrow \quad VC(\varphi_1) < VC(\varphi_2)$$



$$VC(\varphi_2) < VC(\varphi_{13}) \quad \Rightarrow \quad \varphi_2 \Rightarrow \varphi_{13}$$



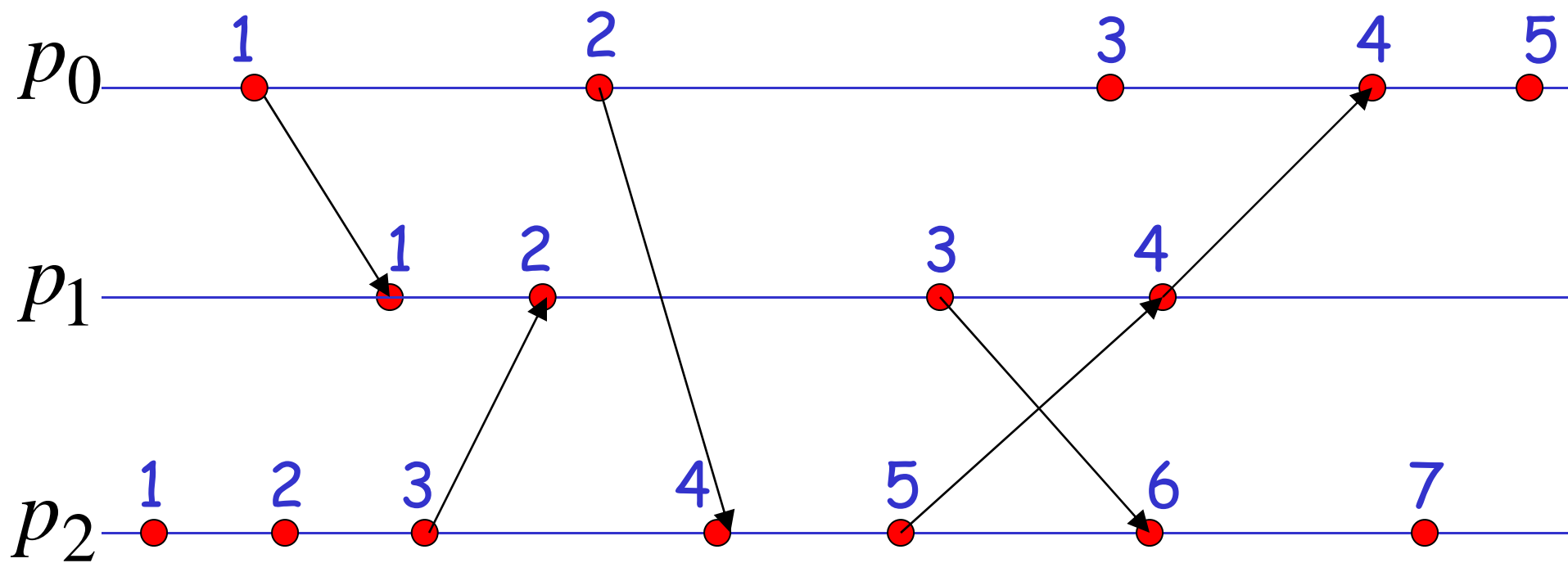
$$\varphi_1 \parallel \varphi_{10} \quad \longrightarrow \quad VC(\varphi_1) \neq VC(\varphi_{10})$$



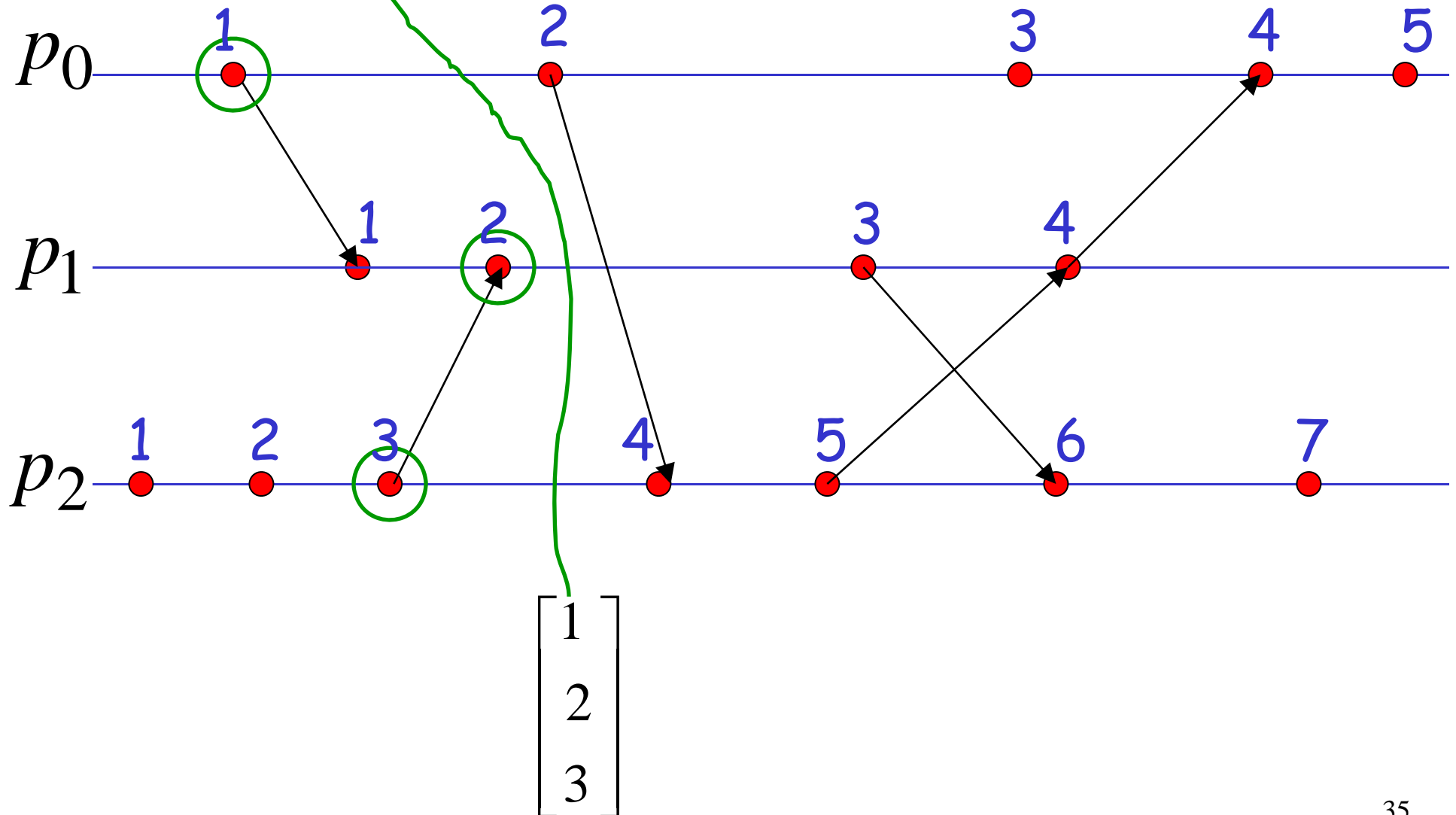
$$VC(\varphi_8) \neq VC(\varphi_{14}) \quad \Rightarrow \quad \varphi_8 \parallel \varphi_{14}$$



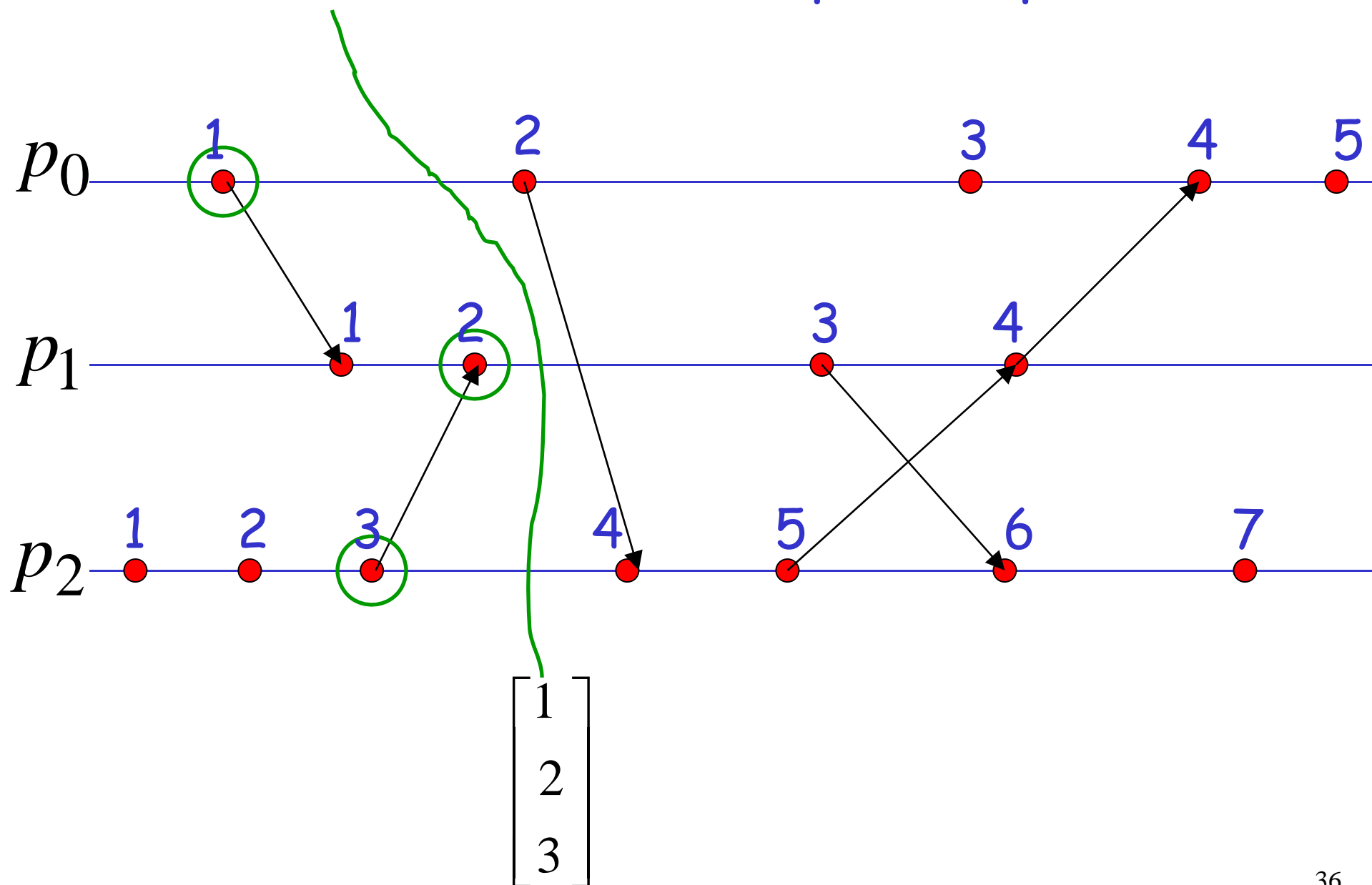
# Isečci (Cuts)



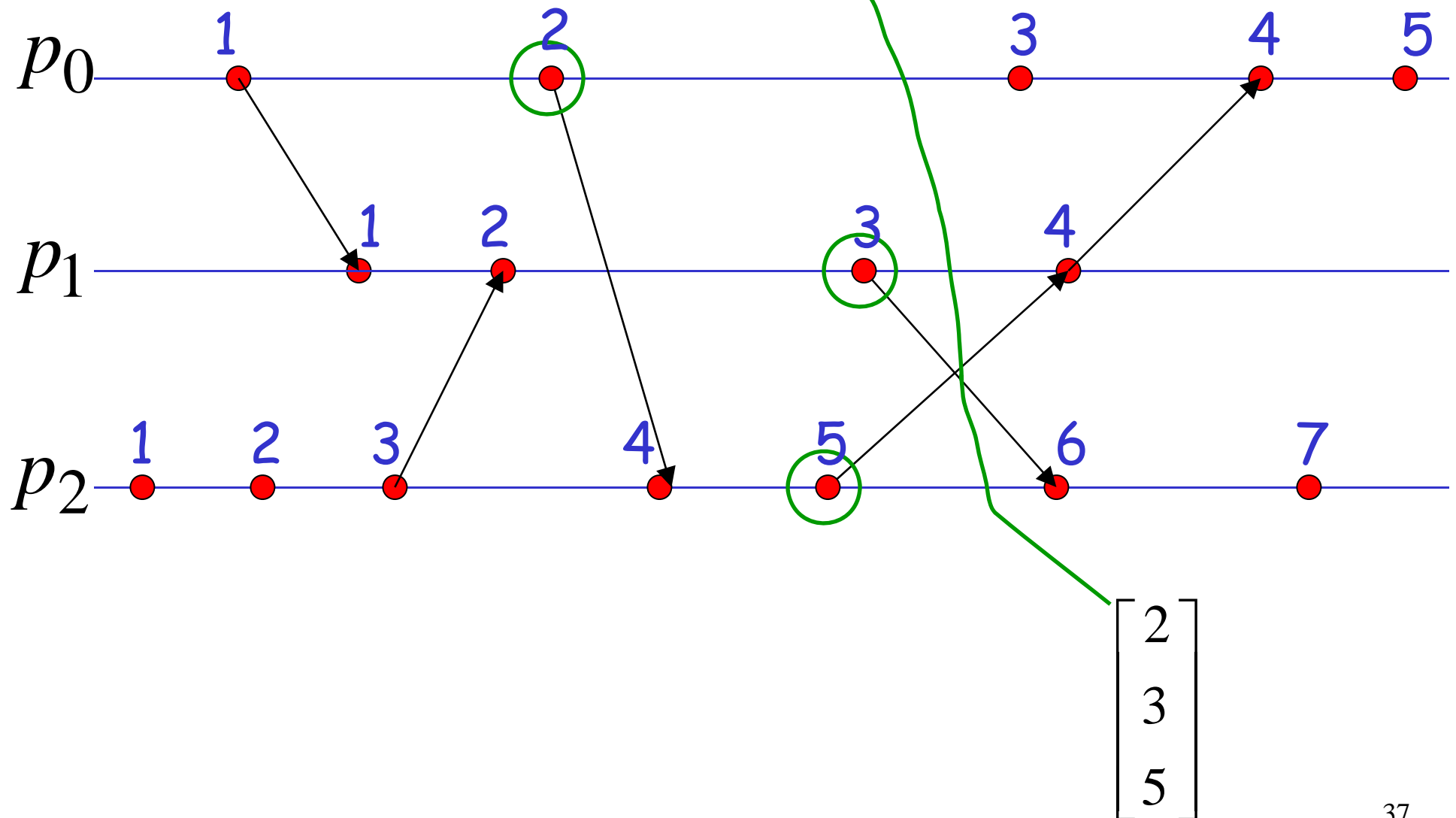
Isečak: sastoji se od događaja  
iz svakog procesa



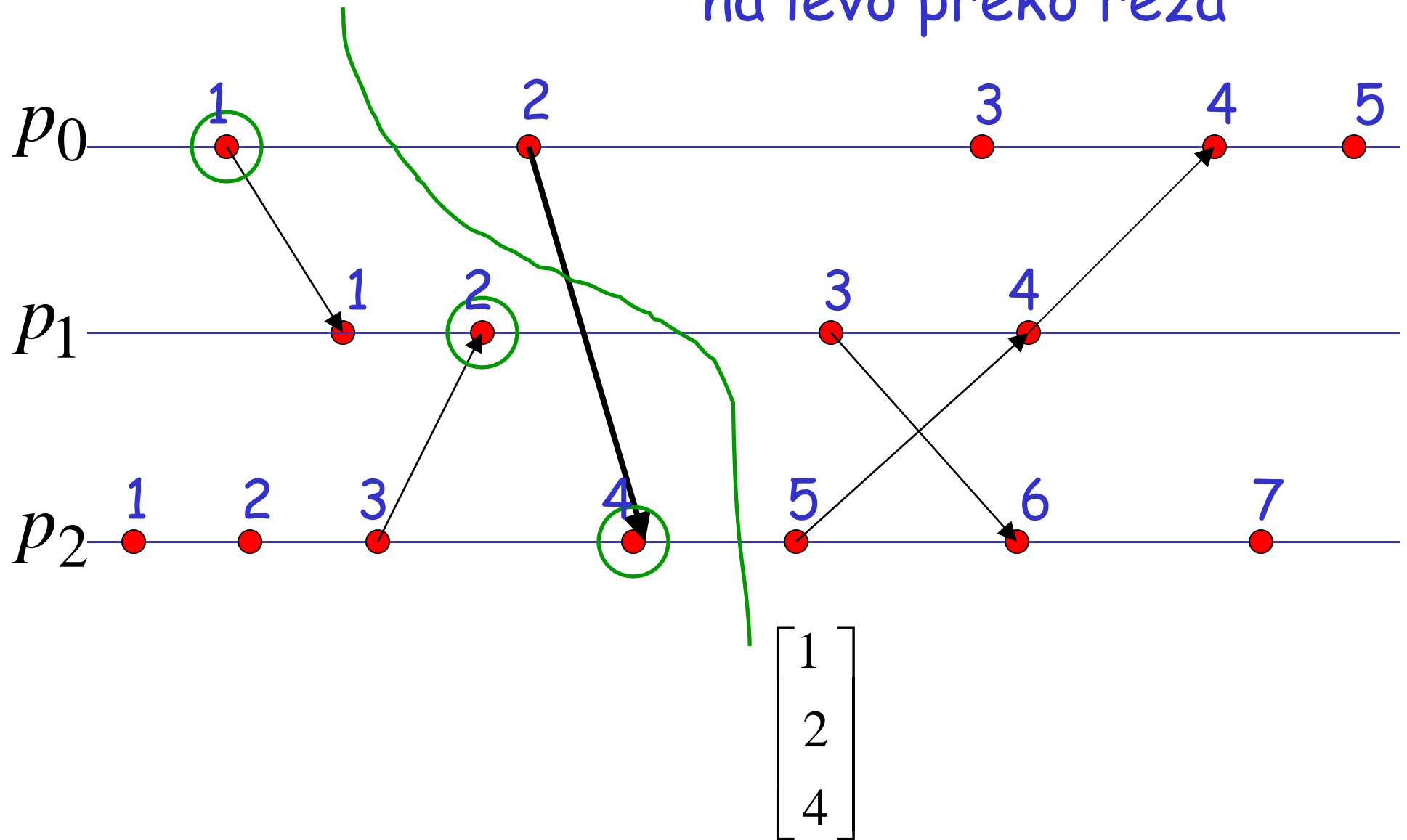
Konzistentan isečak: nema poruka preko reza



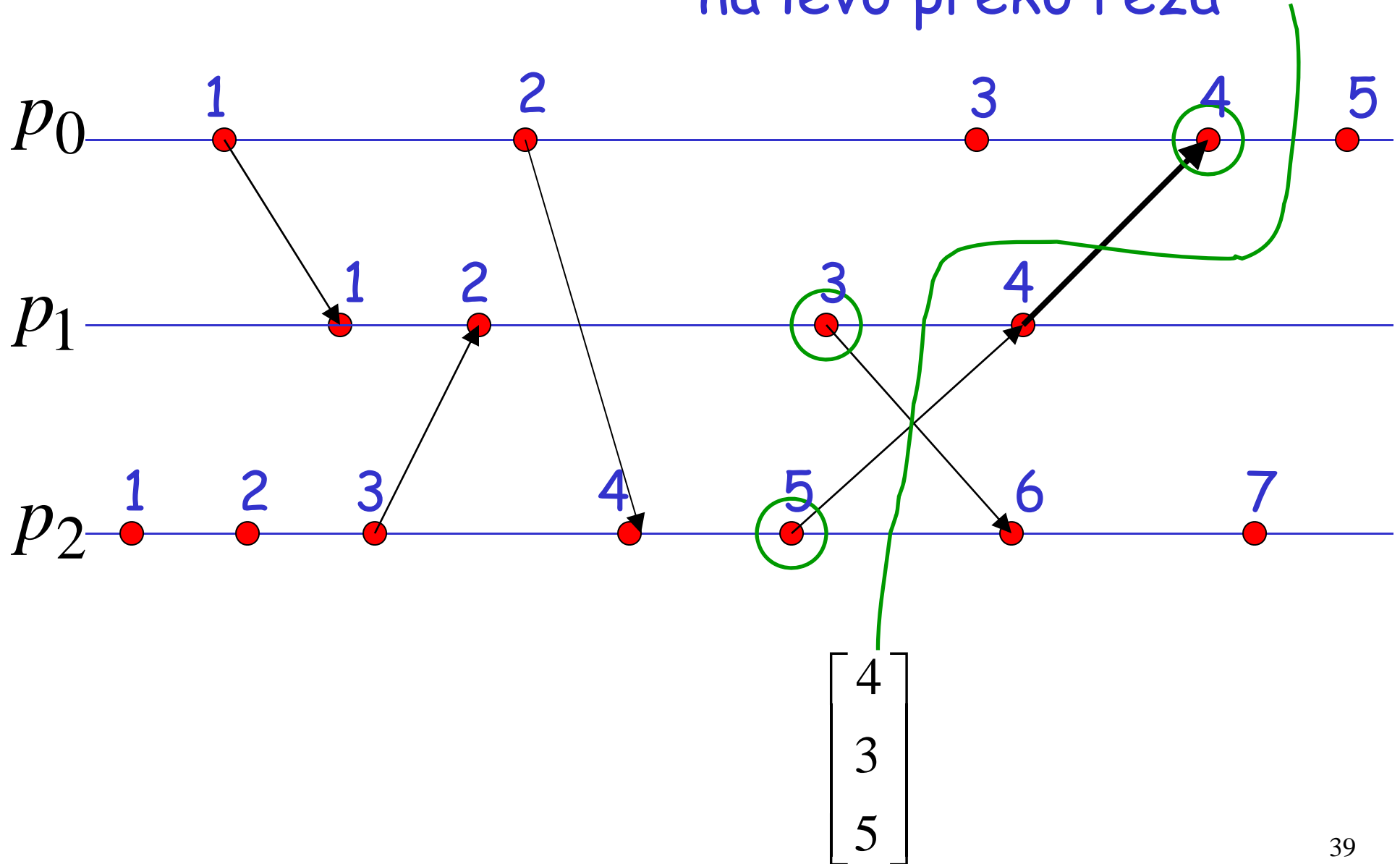
Konzistentan isečak : poruka može preći  
s leva u desno preko reza



Nekonzistentan isečak: poruka prelazi s desna na levo preko reza



Nekonzistentan isečak: poruka prelazi s desna na levo preko reza



# Maksimalan konzistentan isečak

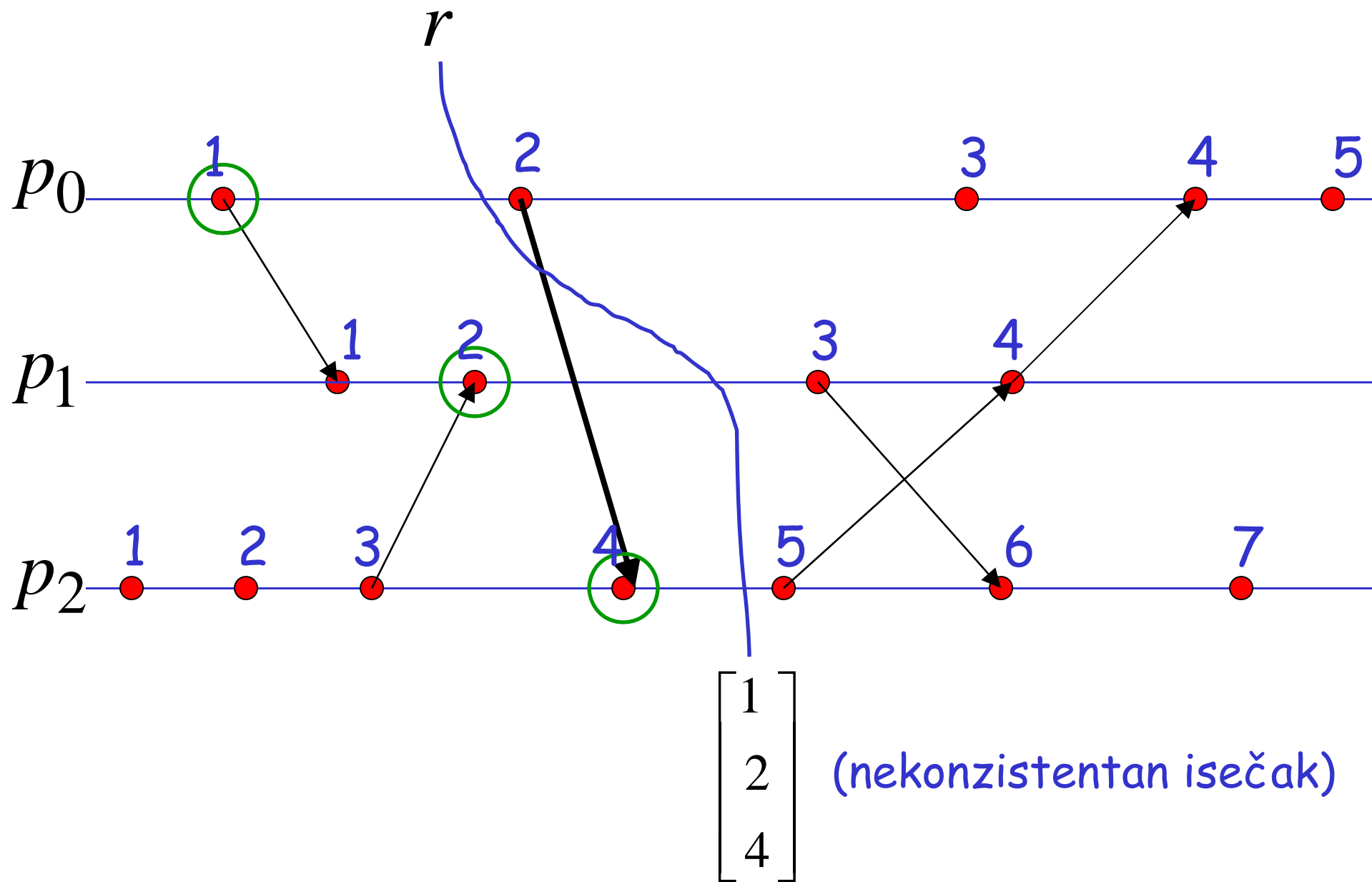
Uzmimo neki (nekonzistentan) isečak  $r$

Max konzistentan isečak od:  $r$

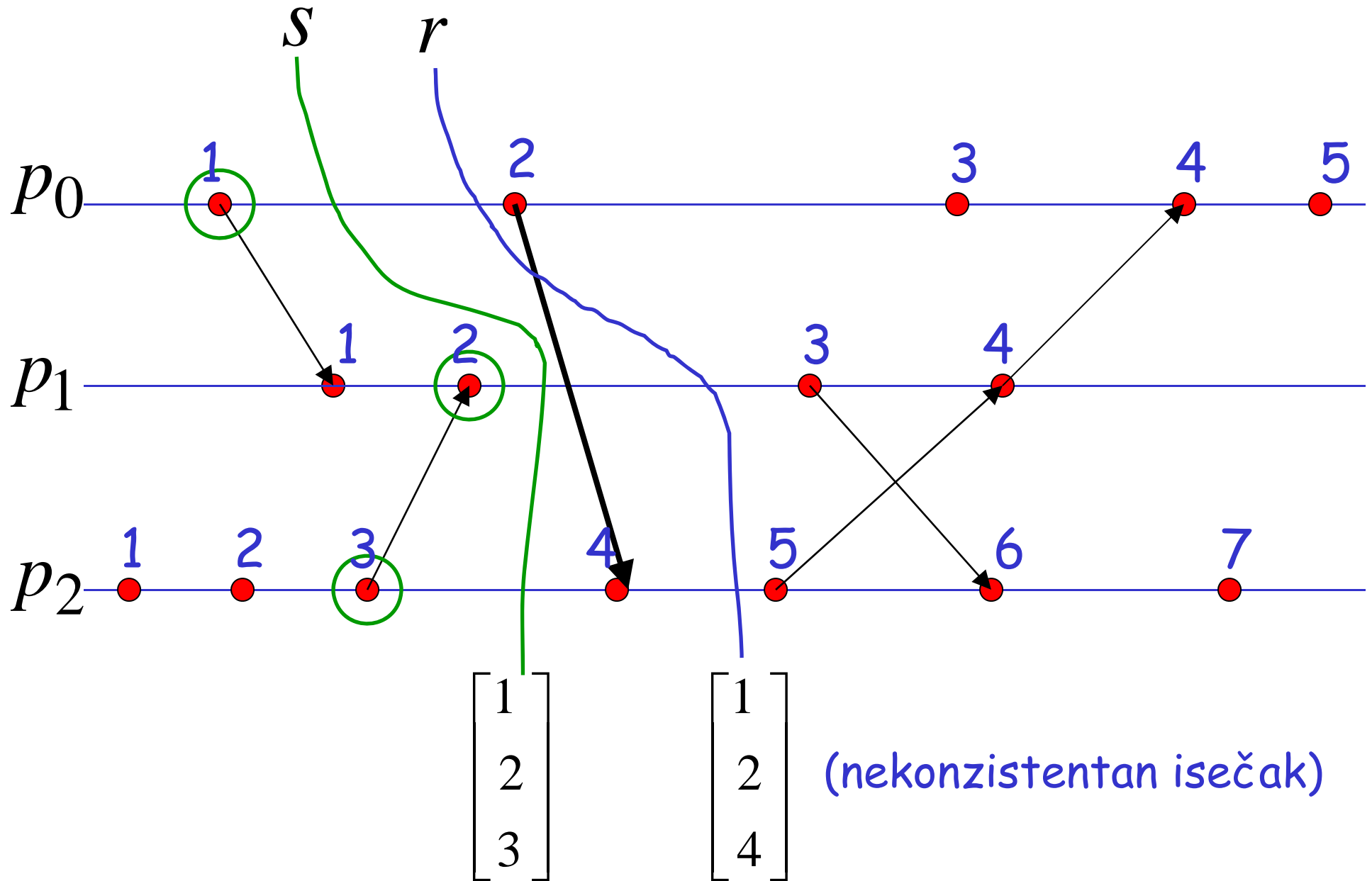
Neki konzistentan isečak  $s$  takav da je  $s \leq r$

i  $s$  sadrži najnovije događaje





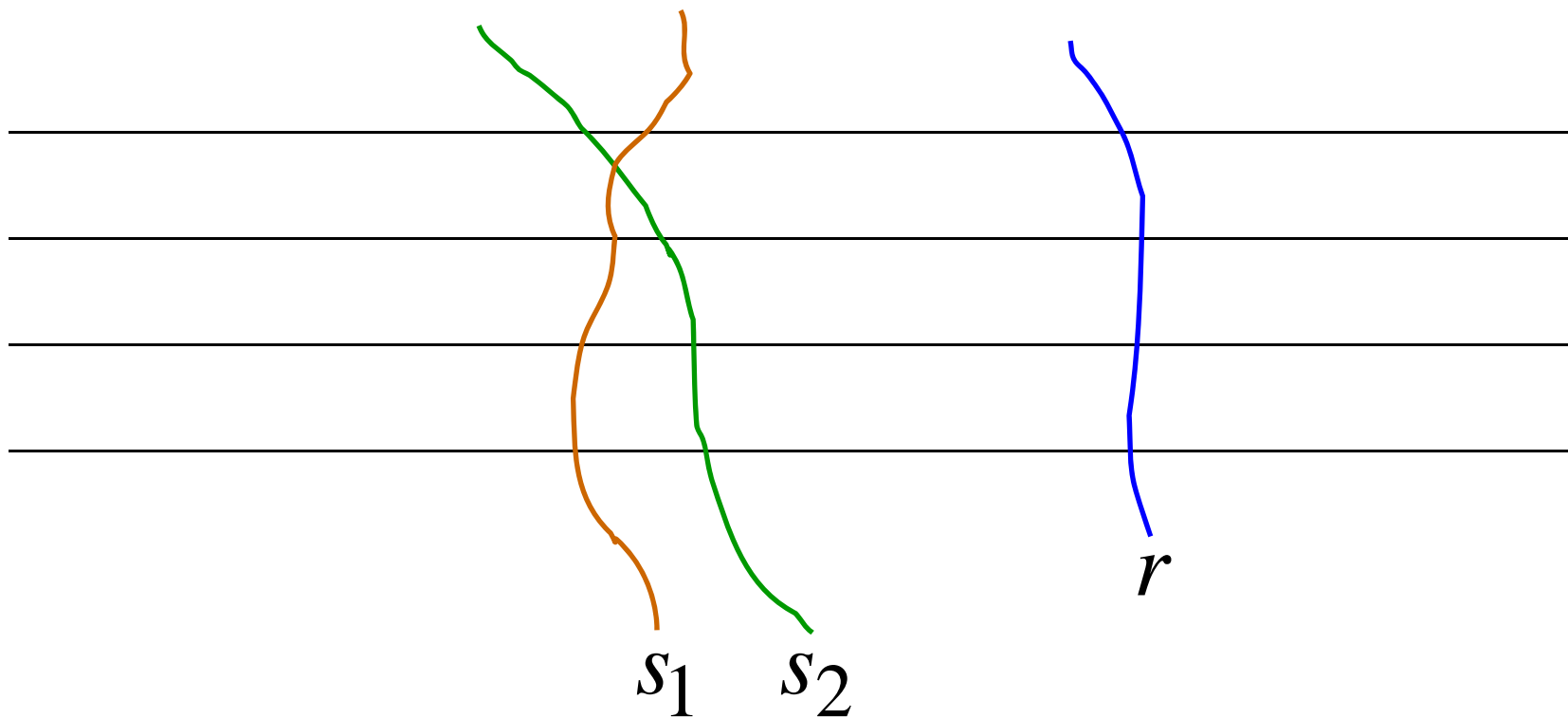
# maksimalan konzistentan isečak



**Teorema:** Za svaki isečak  $r$ ,  
postoji jedinstven  
max konzistentan isečak

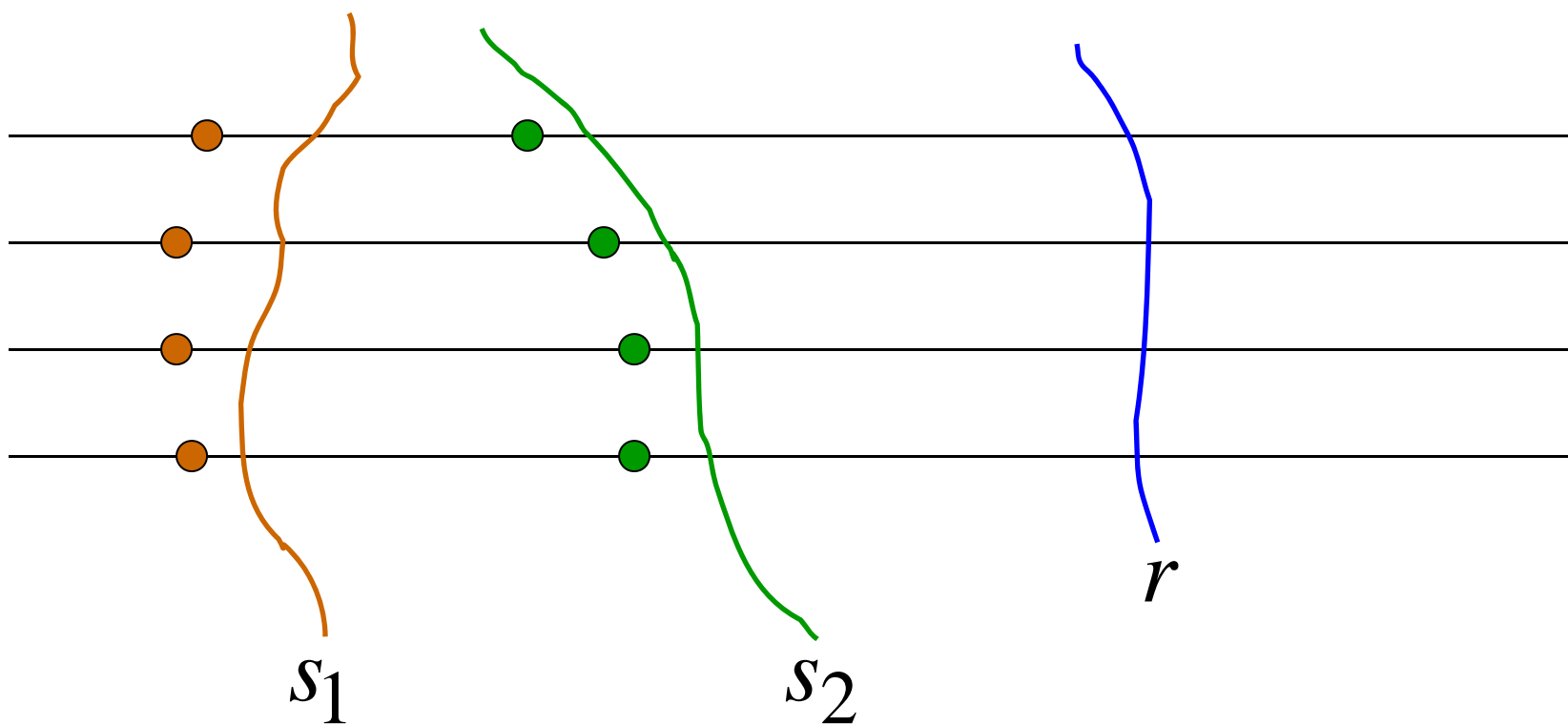
**Dokaz:** Dokaz kontradikcijom

Pret. radi kontradikcije da ima dva  
(ili više) max konz. isečaka od  $r$



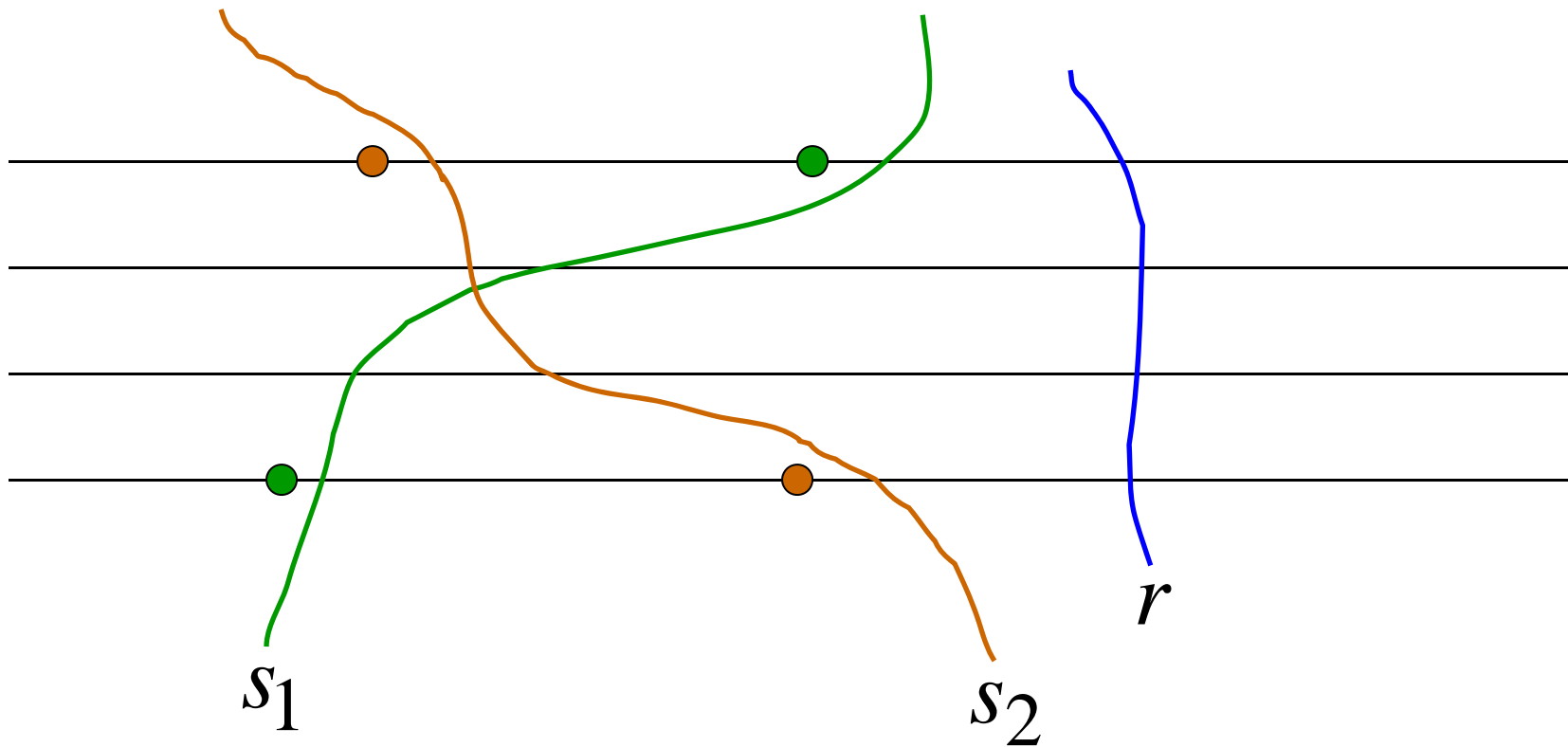
maksimalni konzistentni isečci

Nemože biti da se  $s_1$  i  $s_2$  ne ukrštaju

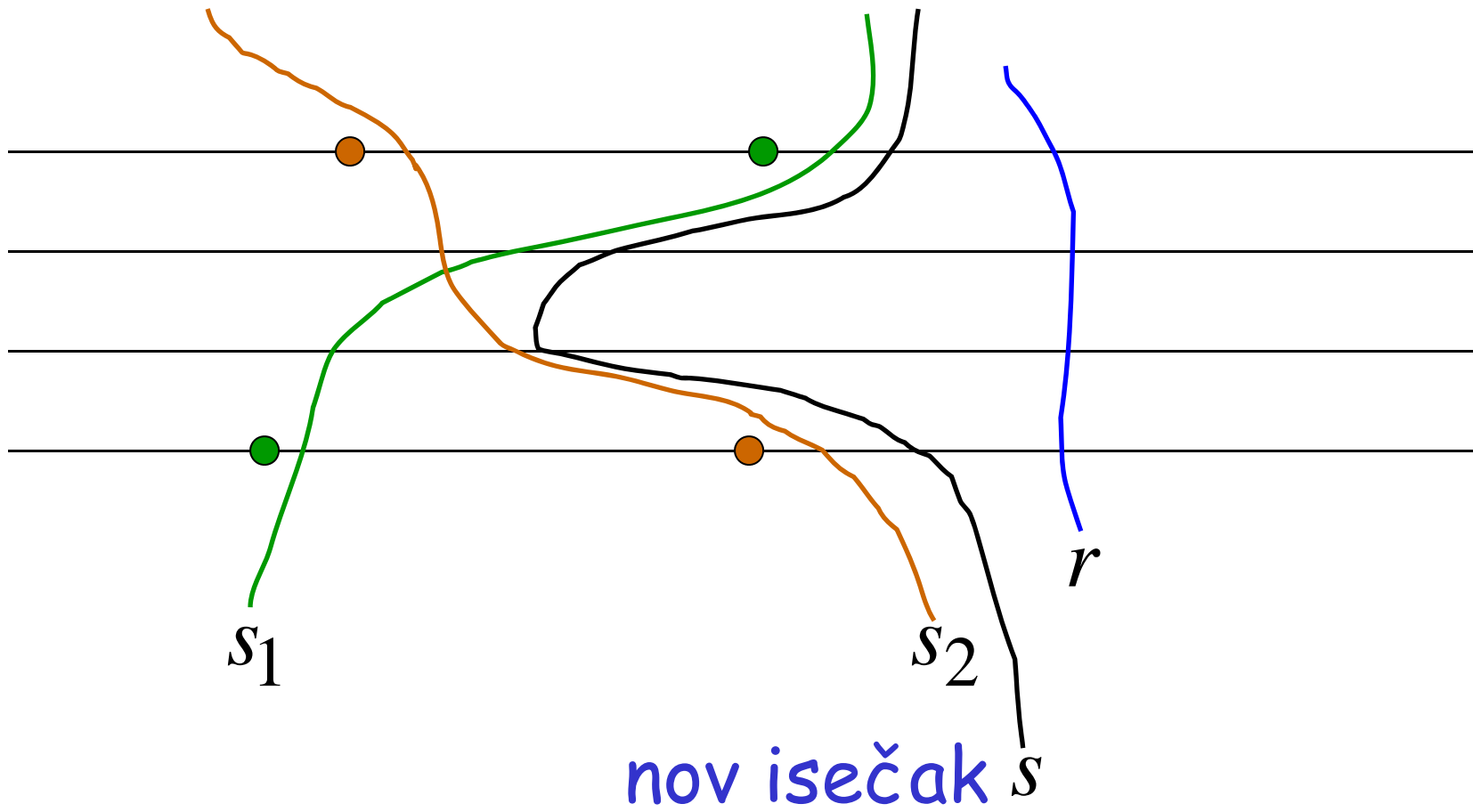


nije maksimalan!

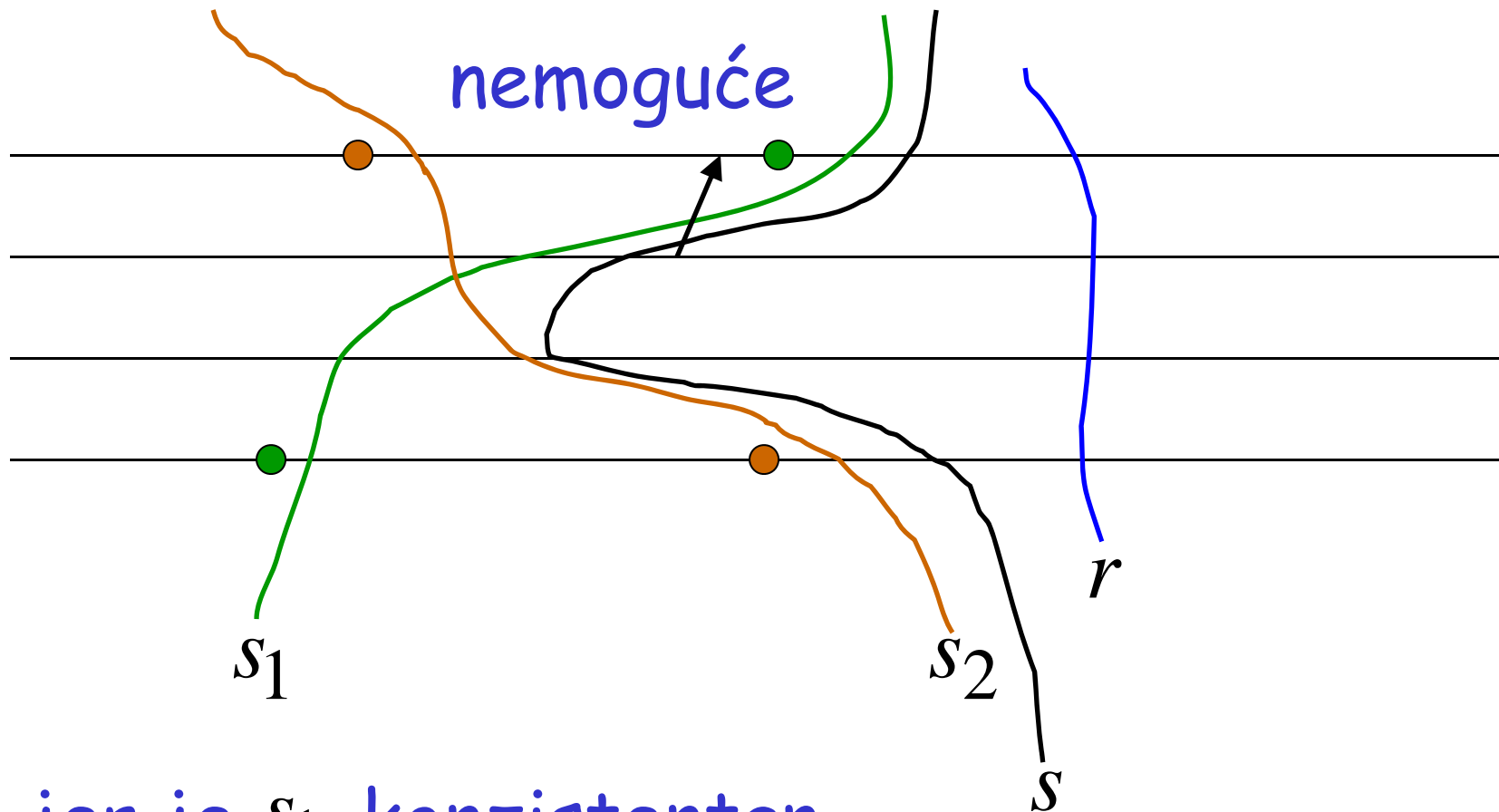
$s_1$  i  $s_2$  se ukrštaju



$s_1$  i  $s_2$  se ukrštaju



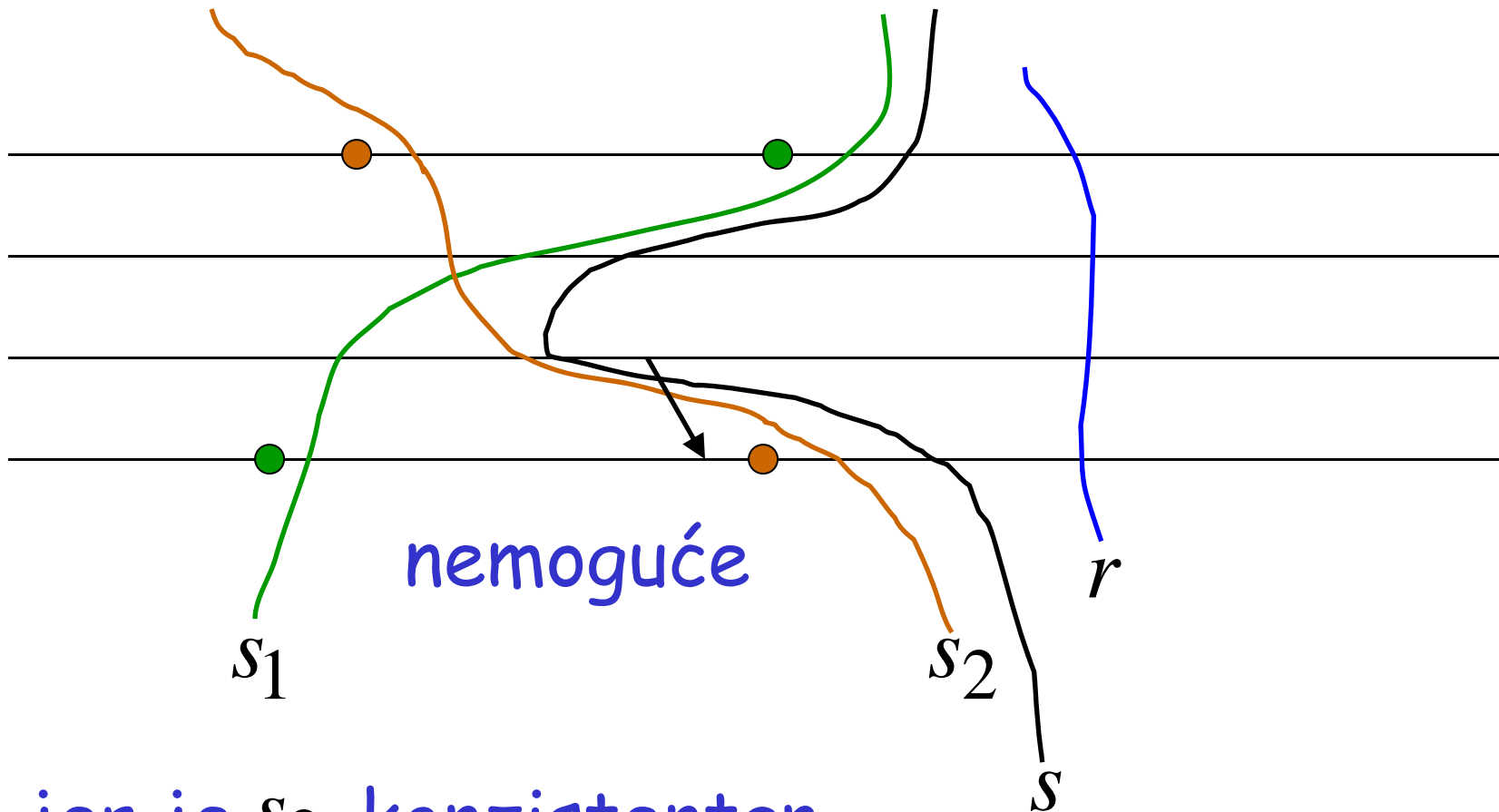
$s_1$  i  $s_2$  se ukrštaju



jer je  $s_1$  konzistentan



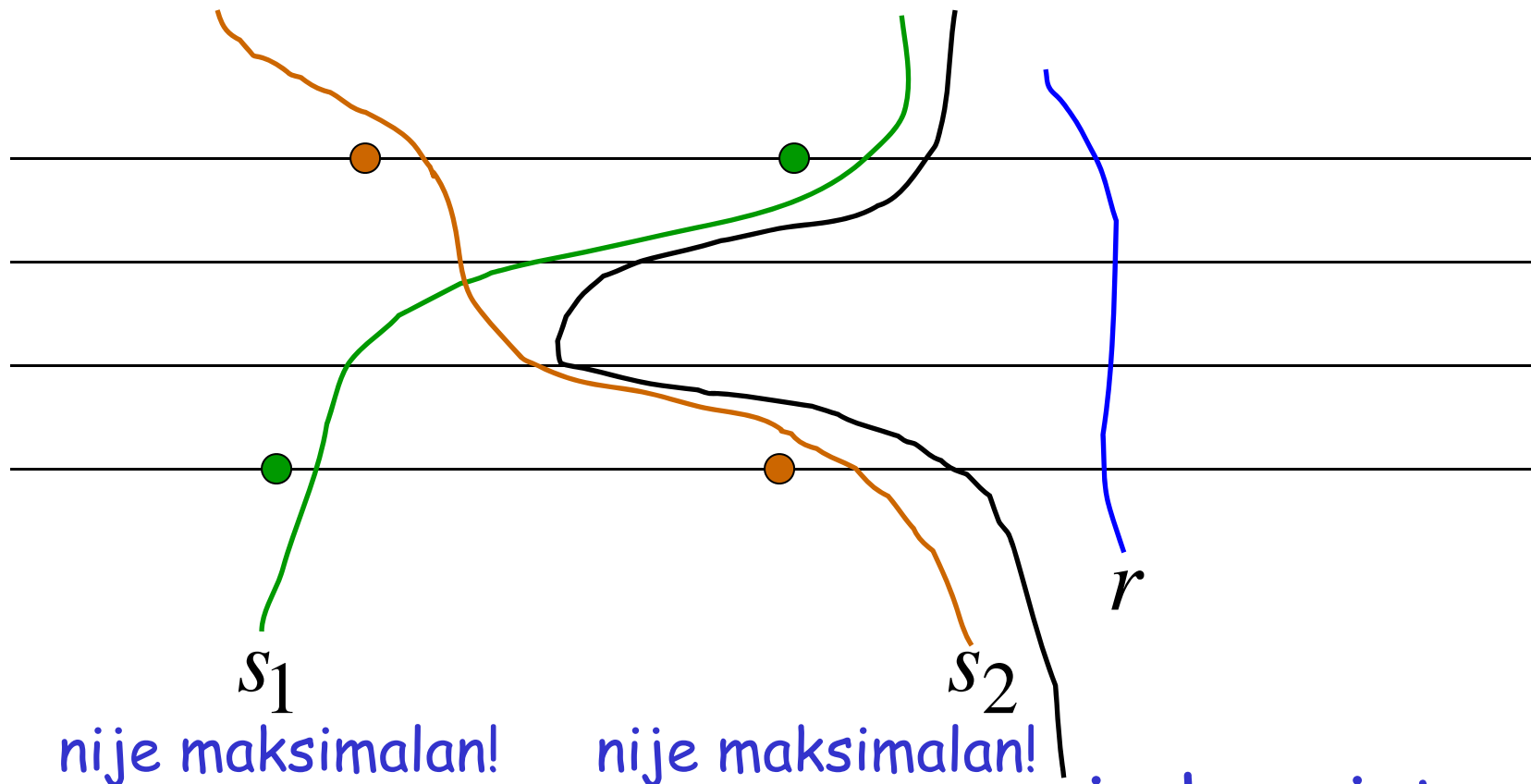
$s_1$  i  $s_2$  se ukrštaju



jer je  $s_2$  konzistentan

$s_1$  i  $s_2$  se ukrštaju

Kontradikcija!



nije maksimalan!

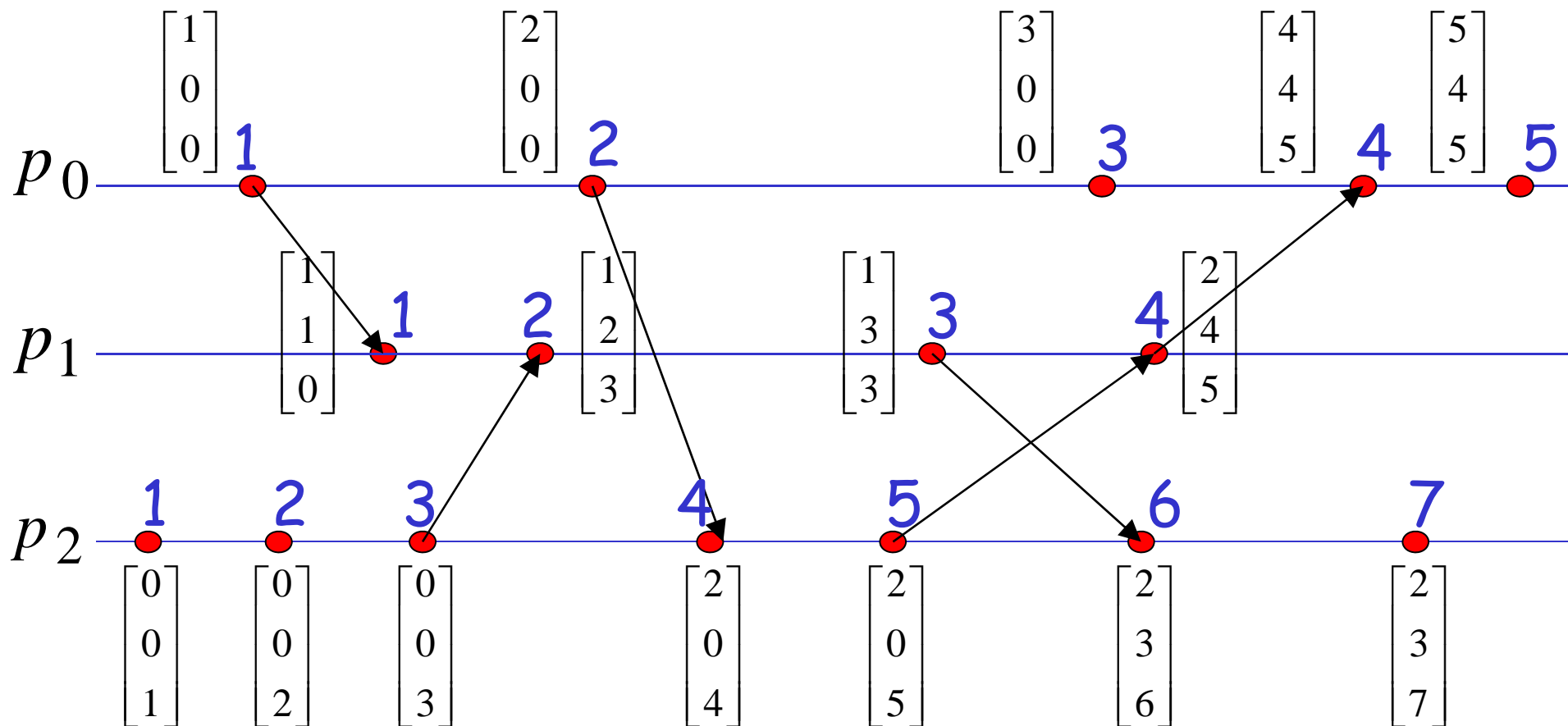
nije maksimalan!

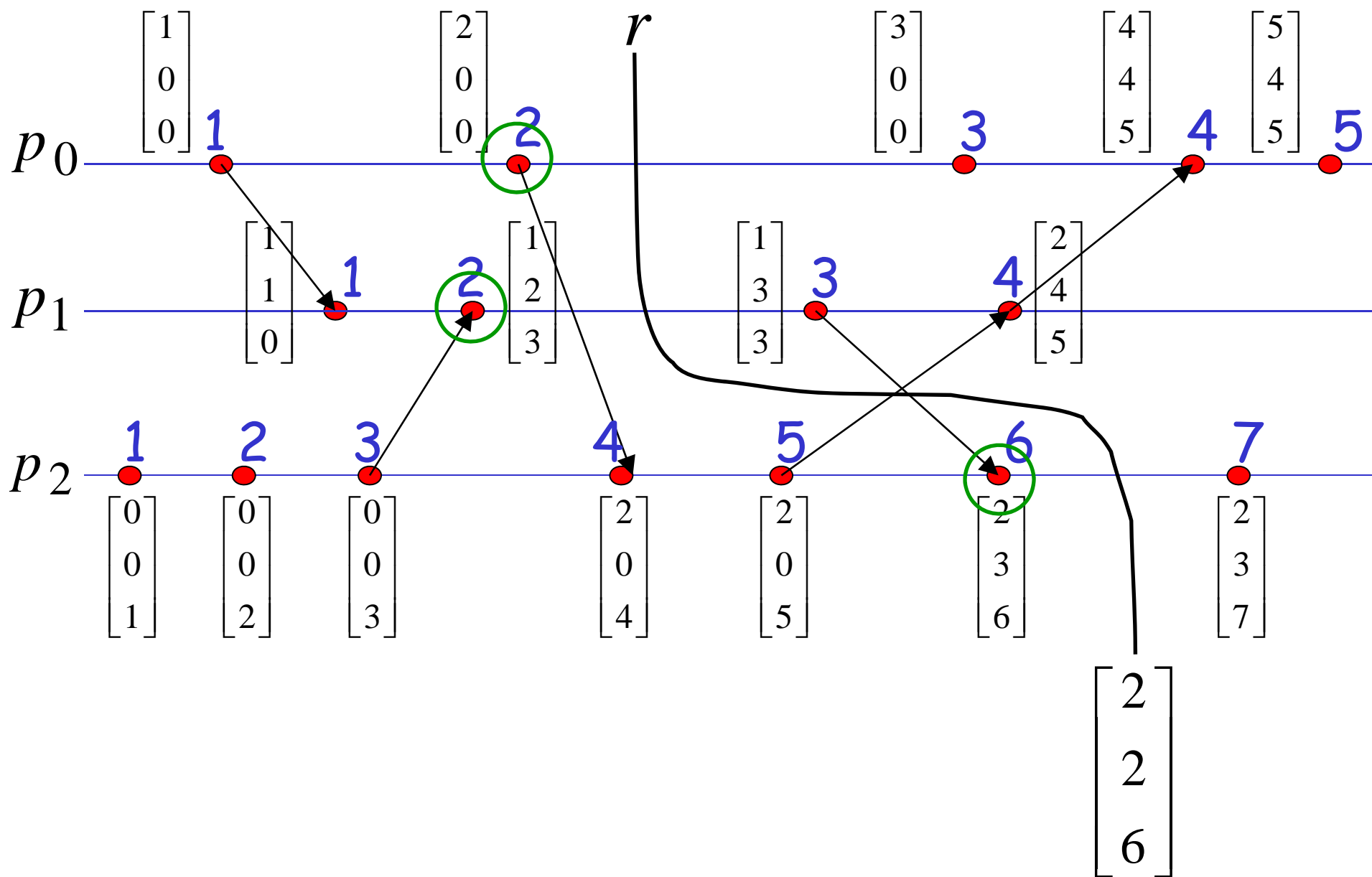
$s$  je konzistentan

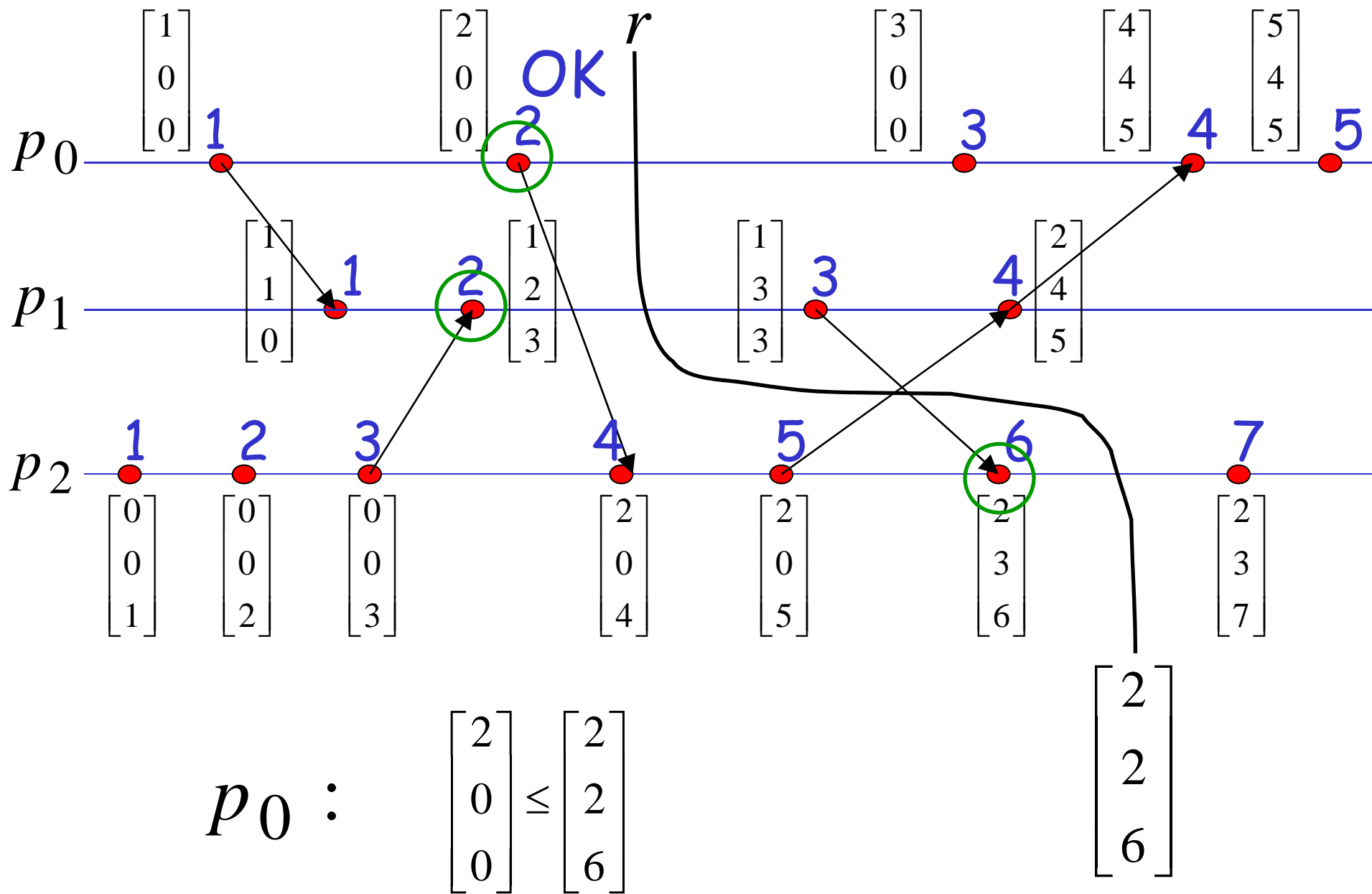
Kraj dokaza

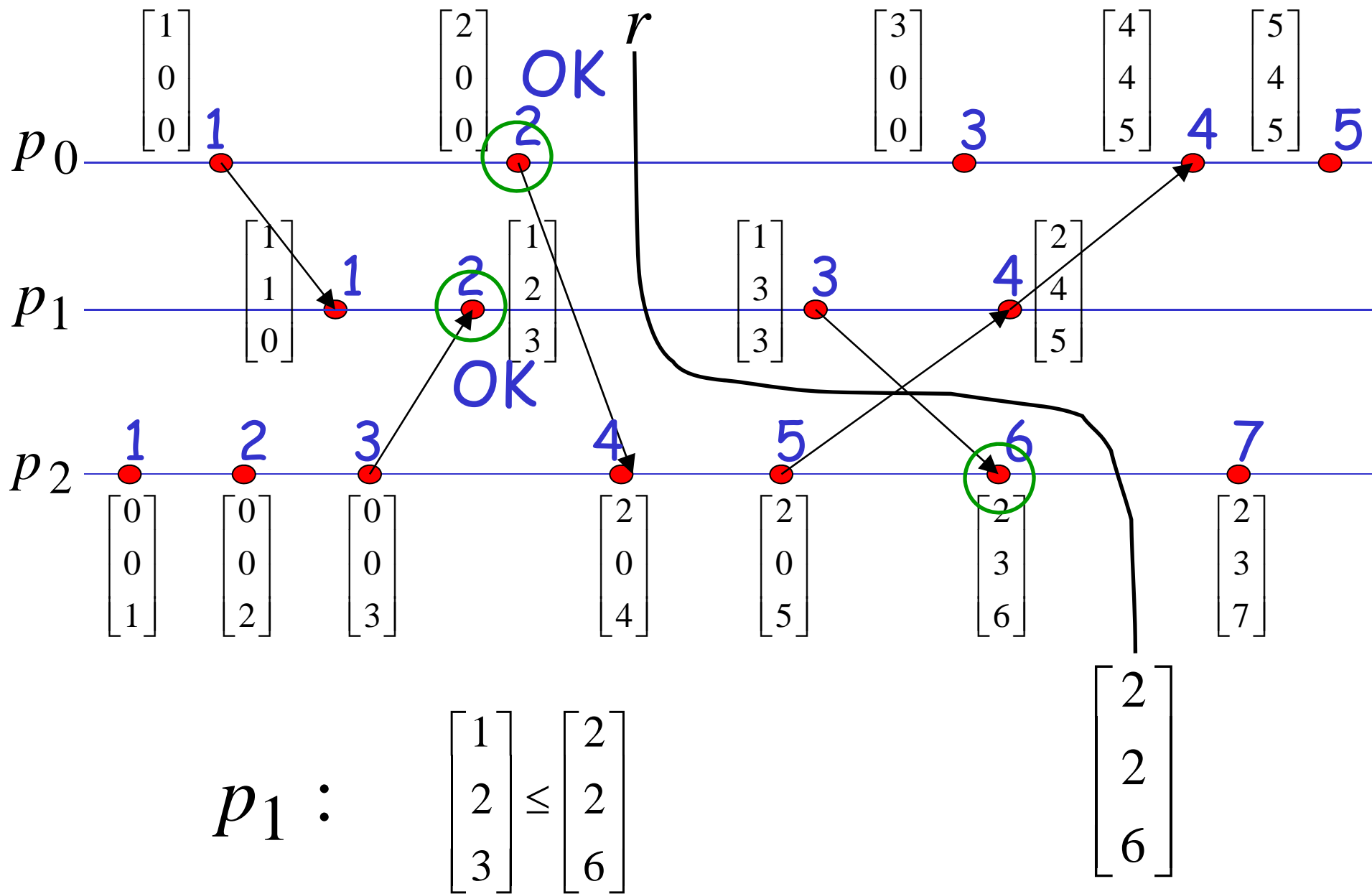
Distribuirani algoritam za računanje  
maksimalnog konzistentnog isečka od:  $r$

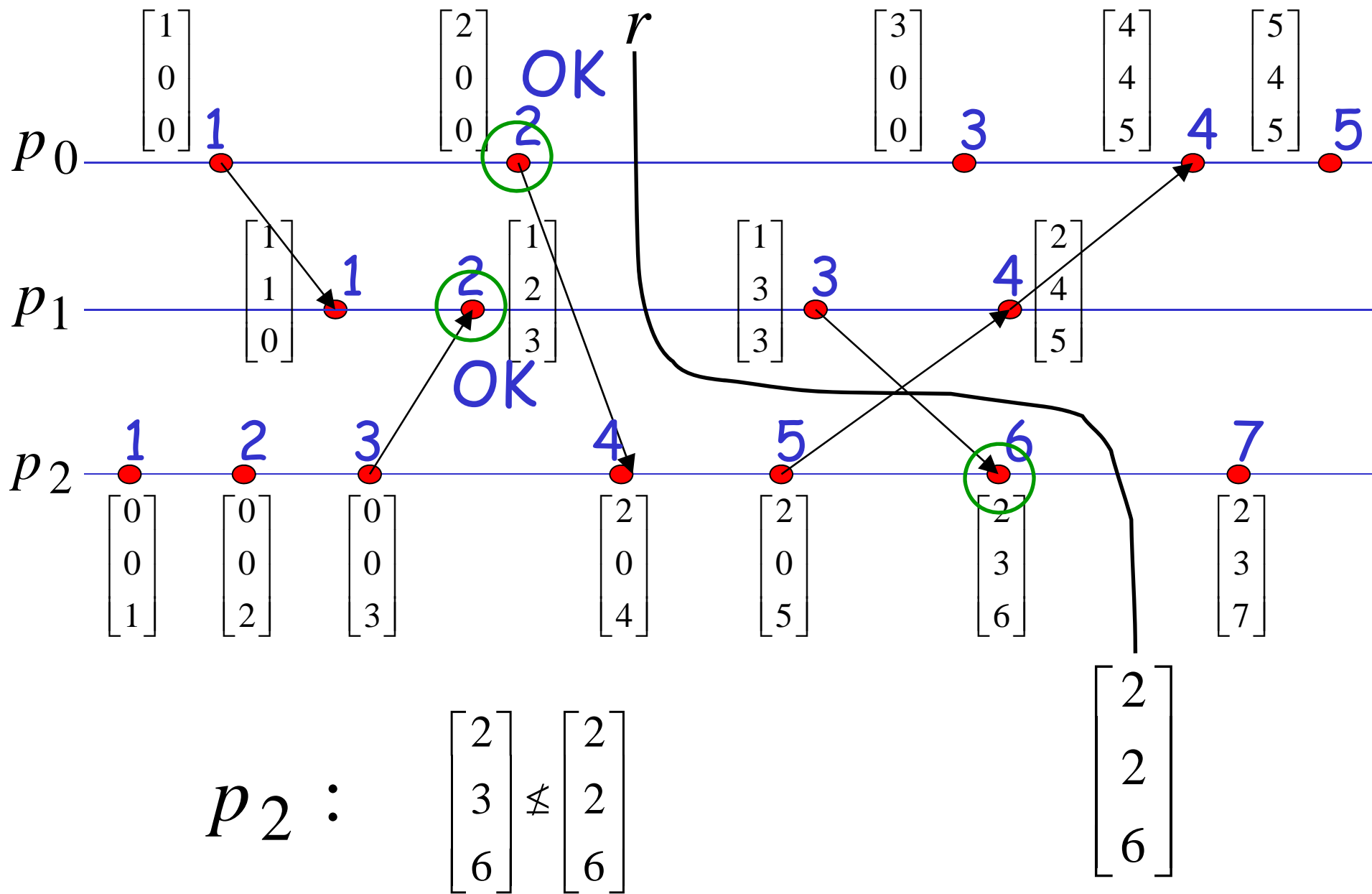
- Koristi vektorske satove
- Za svaki procesor:  
Nadi najnoviji događaj sa  
vektorskim satom  $v \leq r$



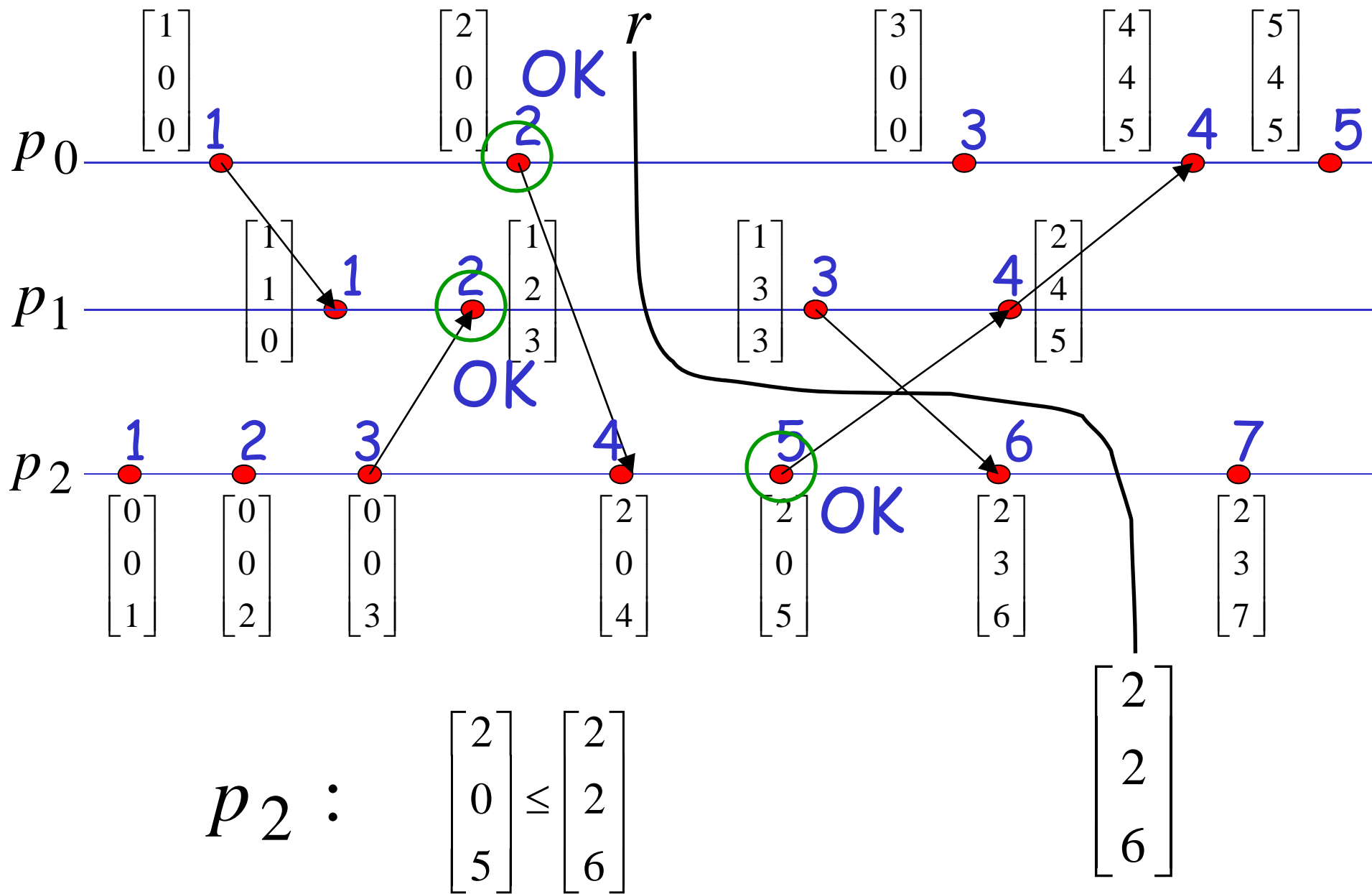


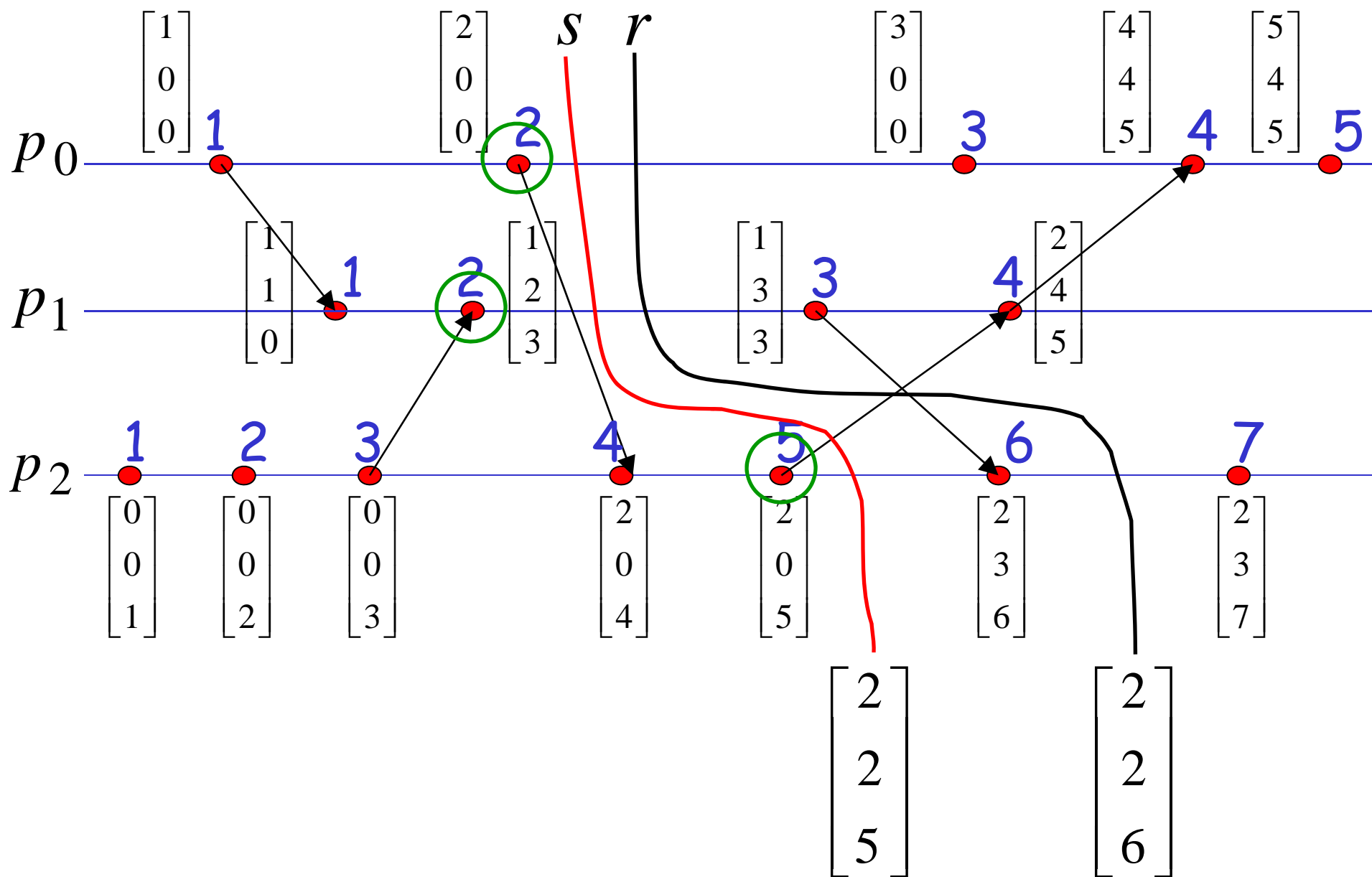












# Distribuirani snimak (snapshot)

- Skup procesora  $S$  inicira računanje radi dobijanja globalnog snimka  
(ovi procesori primaju iz sistema specijalnu poruku **marker**)
- Isečak sadrži stanje barem jednog procesora u  $S$  u inicijalizaciji

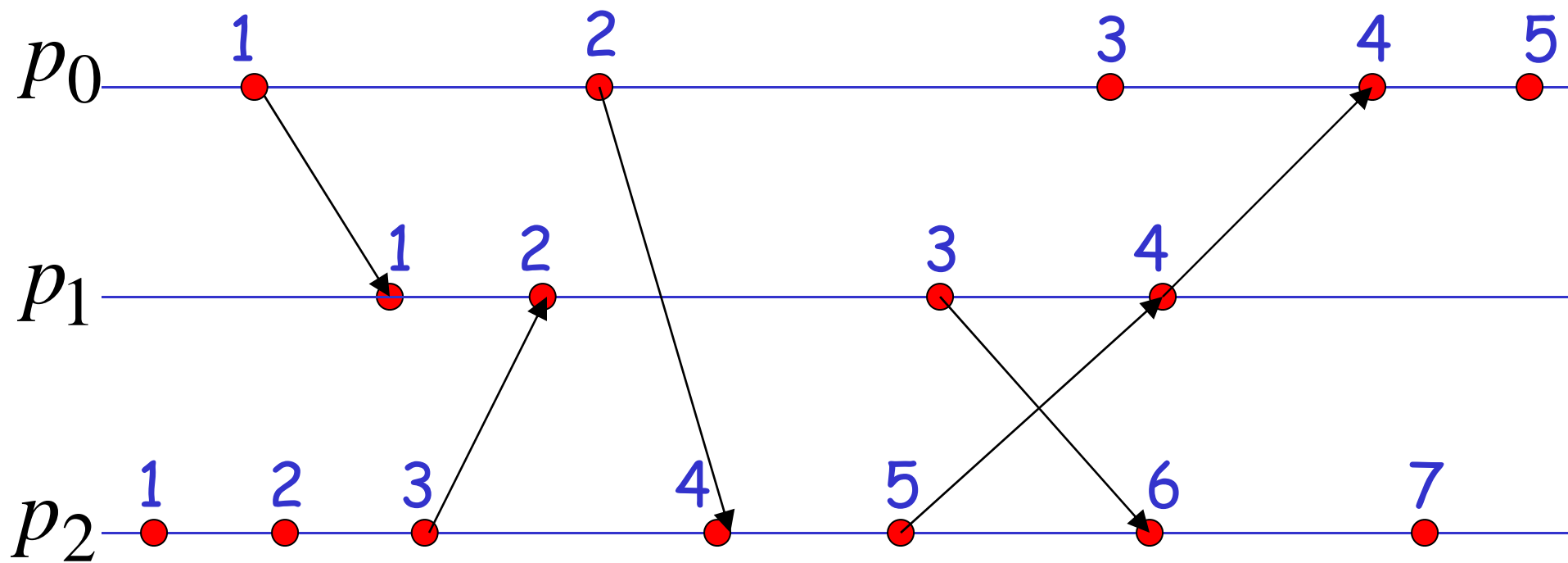
## Distribuirani algoritam snimka

Procesor:  $p_i$      $num_i = 0$      $ans_i = nil$

- Broj lokalne događaje u  $num_i$
- Po prijemu poruke **marker** :

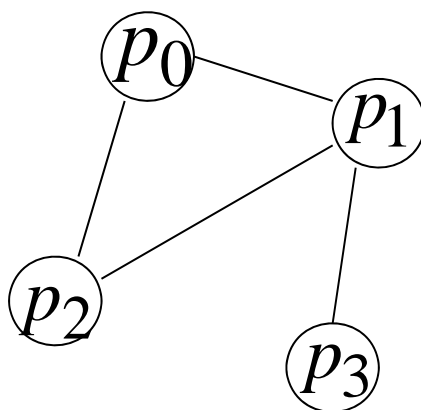
If  $ans_i = nil$  then  
  postavi  $ans_i = num_i$   
  pošalji **marker** svim susedima

$num_i$



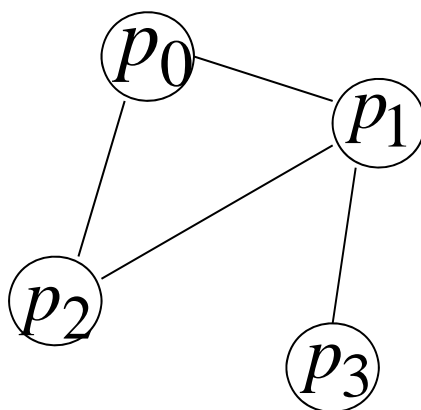


$$S = \{p_0\}$$



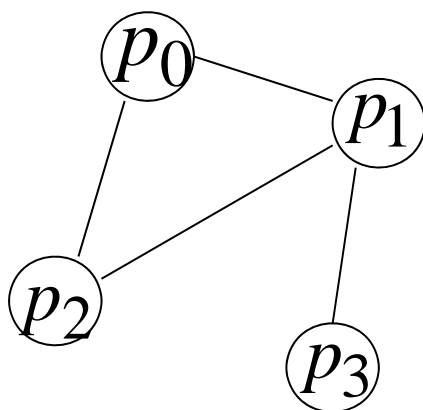


$$S = \{p_0\}$$

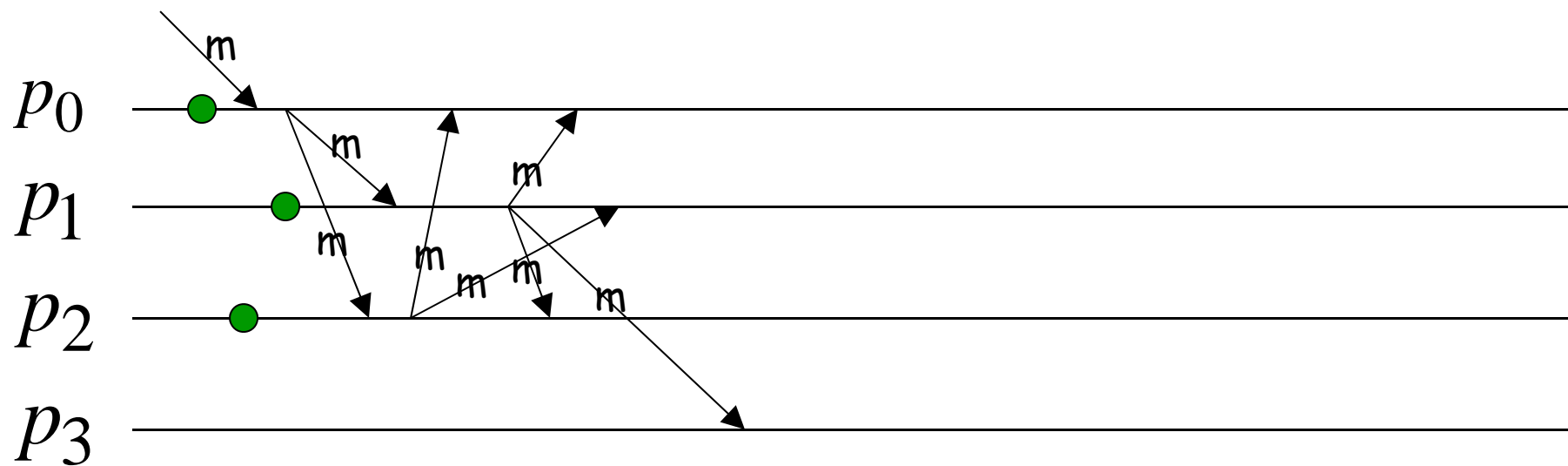




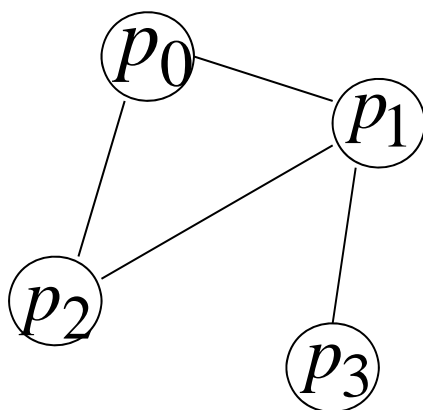
$$S = \{p_0\}$$

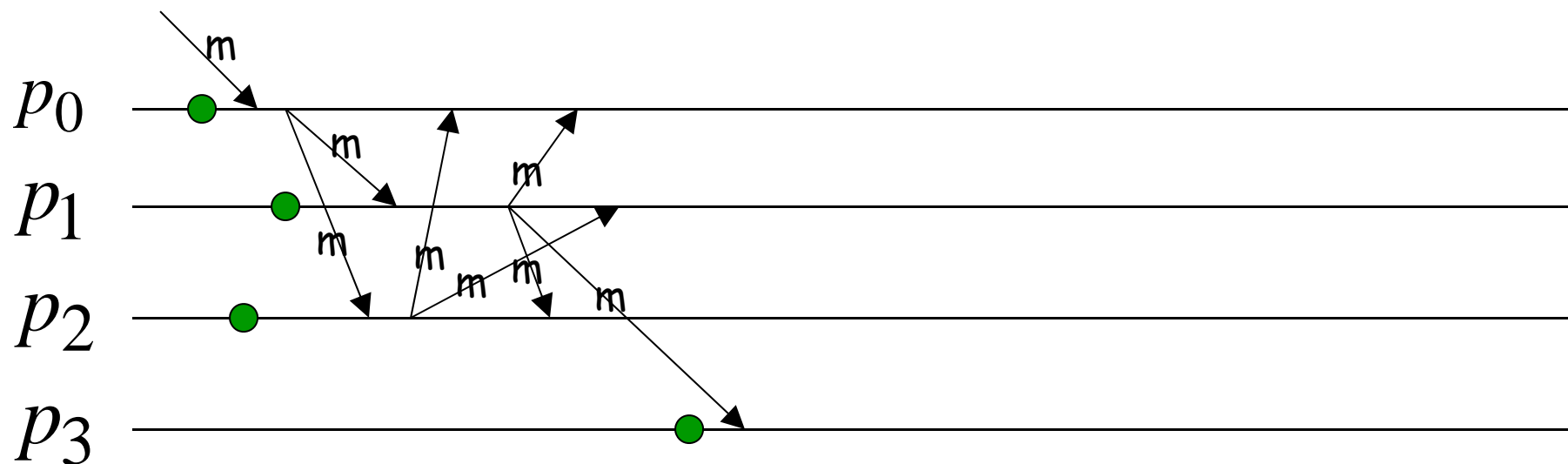




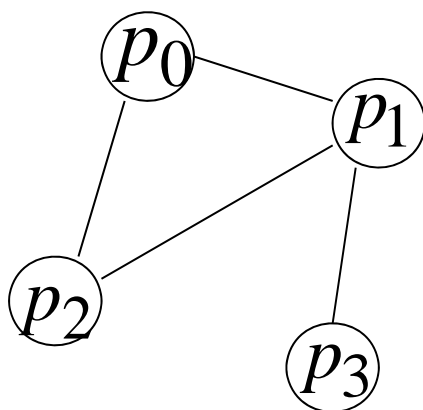


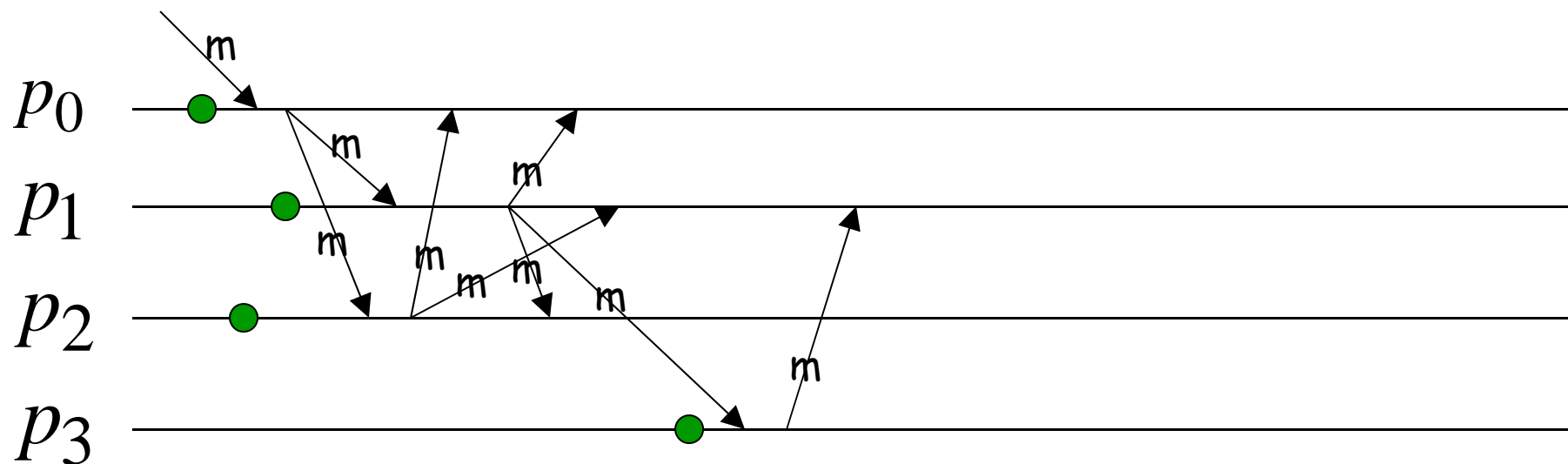
$$S = \{p_0\}$$



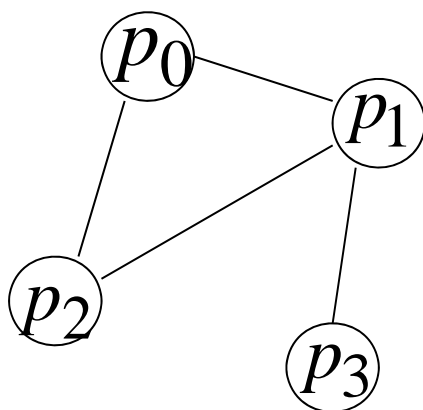


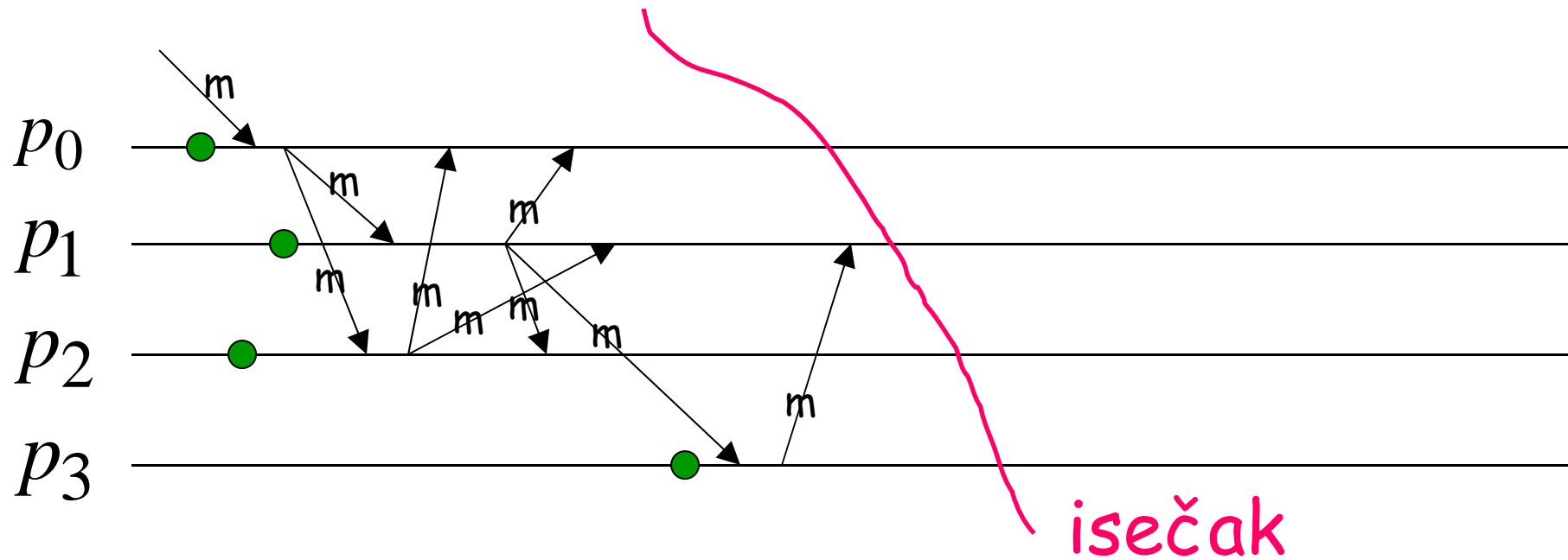
$$S = \{p_0\}$$



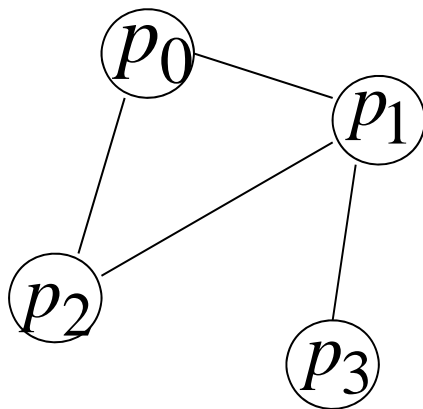


$$S = \{p_0\}$$





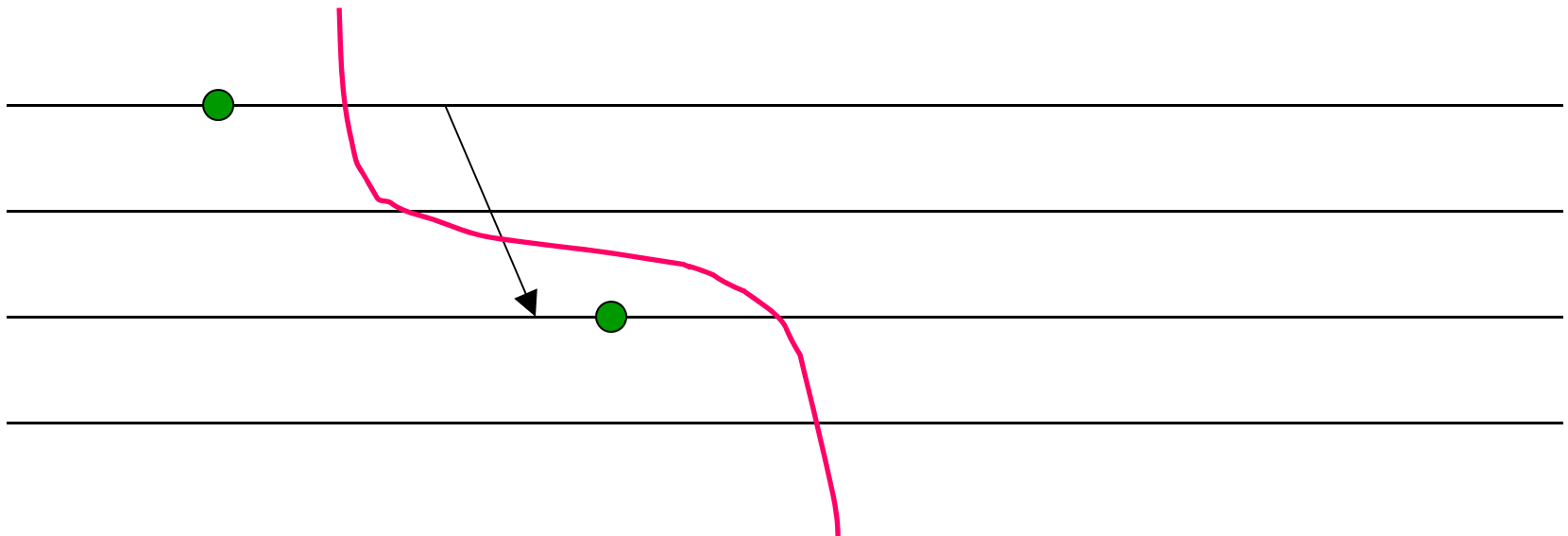
$$S = \{p_0\}$$



**Teorema:** Isečak dobijen pomoću ovog algoritma je konzistentan

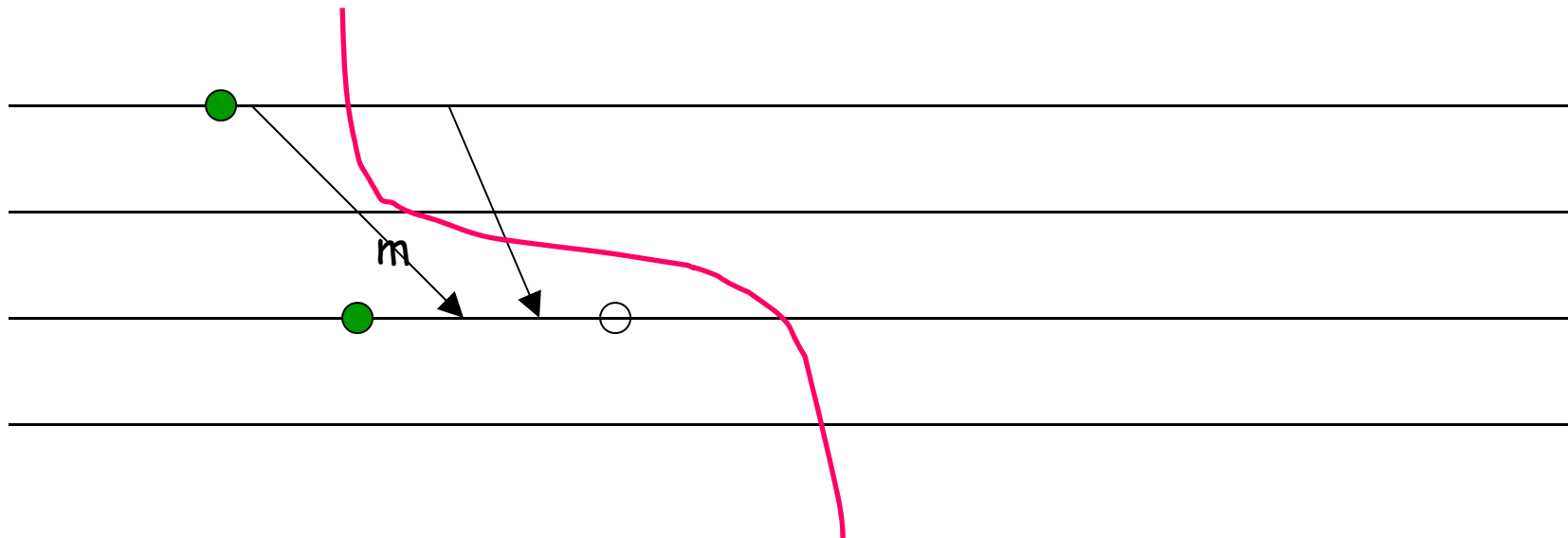
**Dokaz:** Kontradikcijom

Pred. da je isečak nekonzistentan



(predpostavljamo FIFO)

Moralo bi da bude:



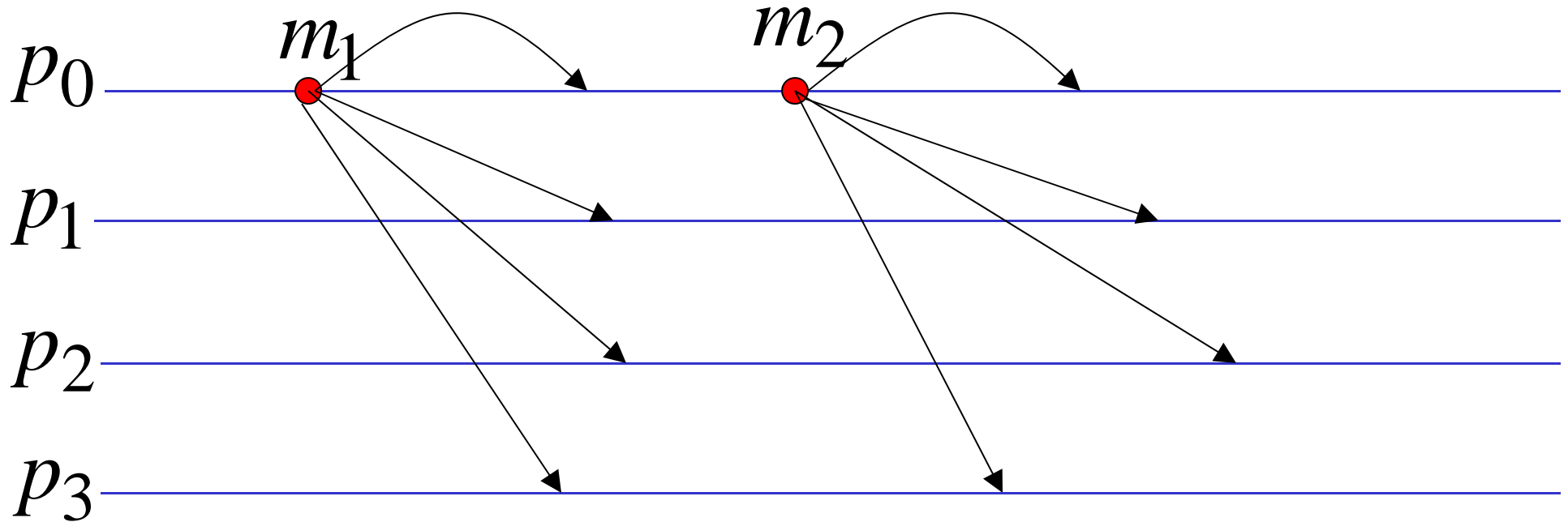
Nemoguće!

Kraj dokaza

FIFO slanje svima  
(FIFO-Broadcast)

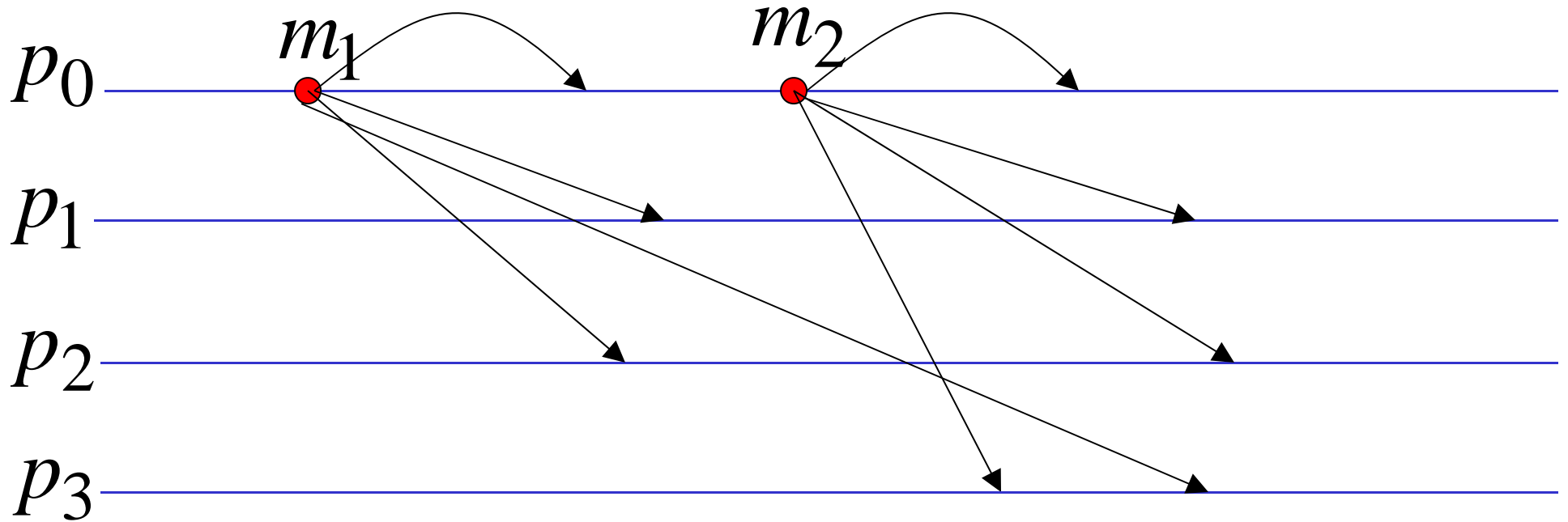


# FIFO redosled

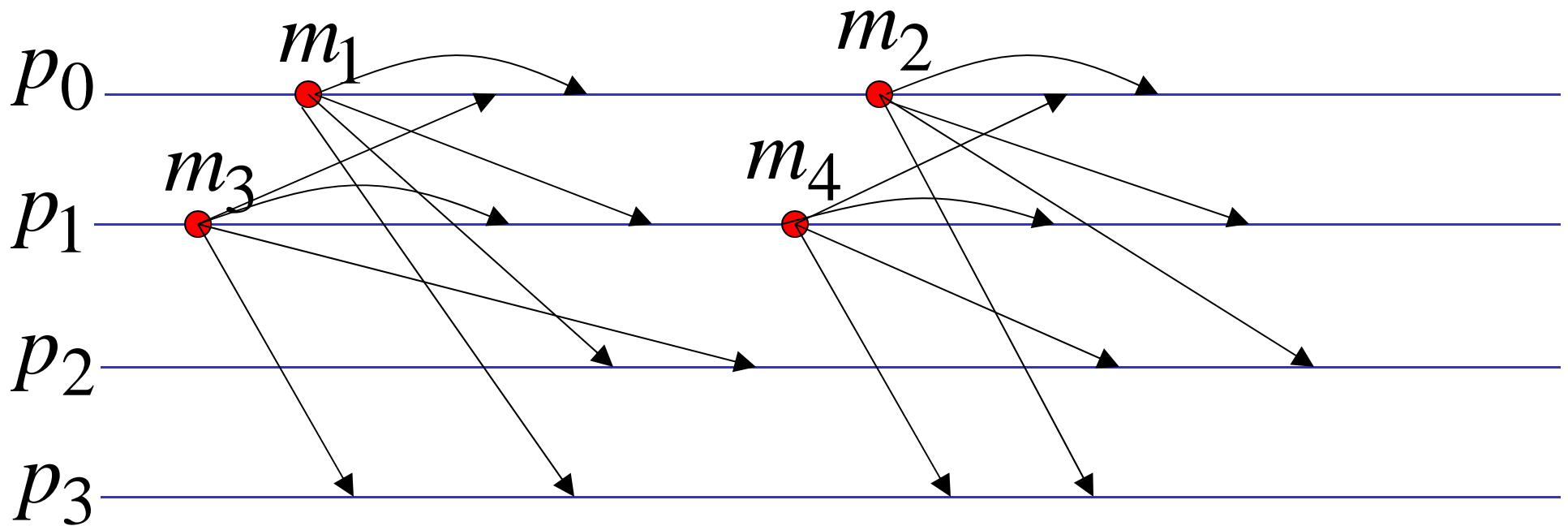


Poruke od  $p_i$  se primaju u  
redosledu u kom su slane od  $p_i$

Nije FIFO-redosled

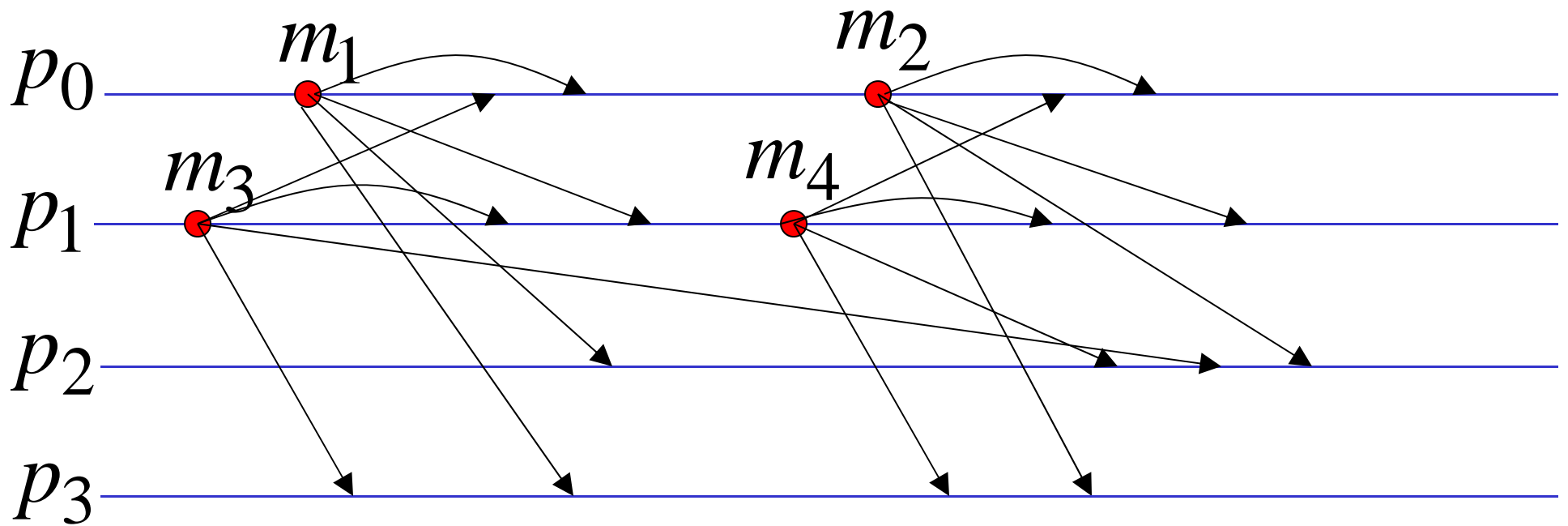


## FIFO redosled

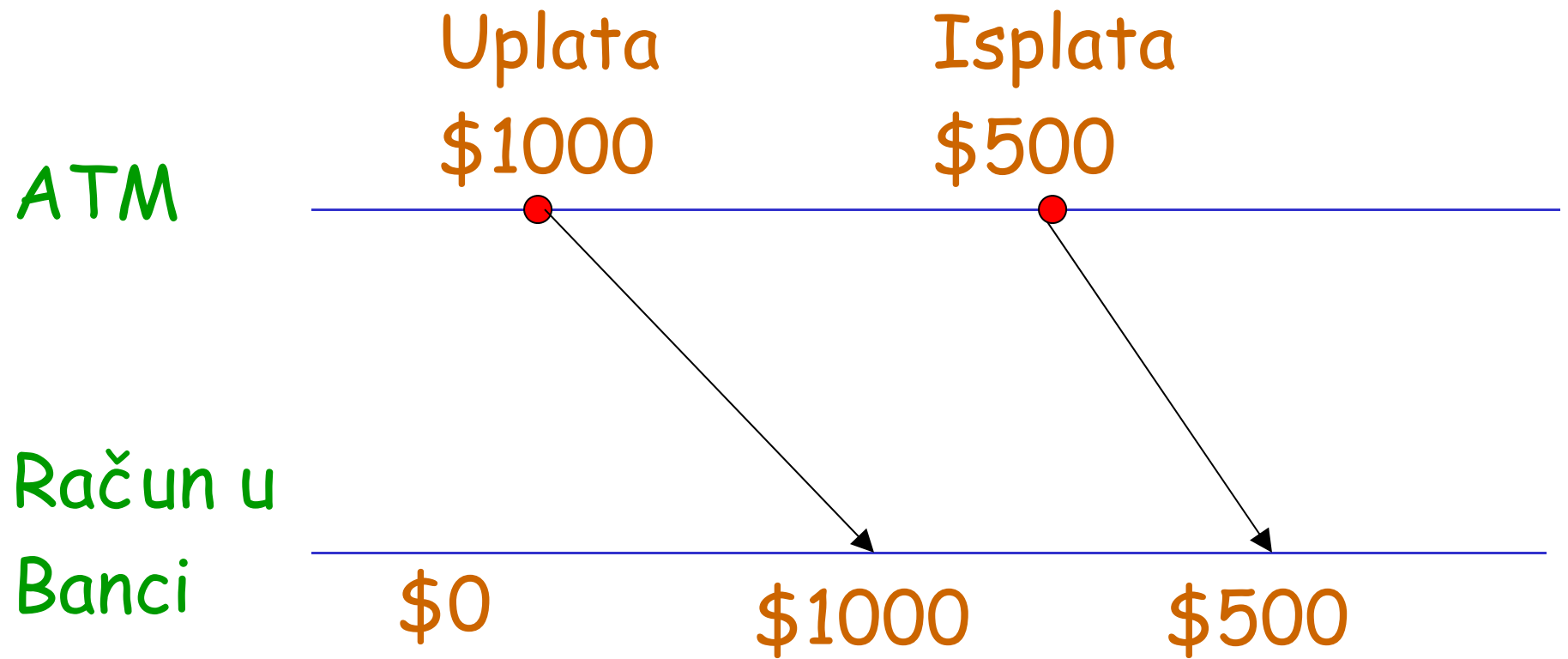


Poruke od različitih  
procesora mogu biti primljene u  
različitom redosledu

# Nije FIFO-redosled

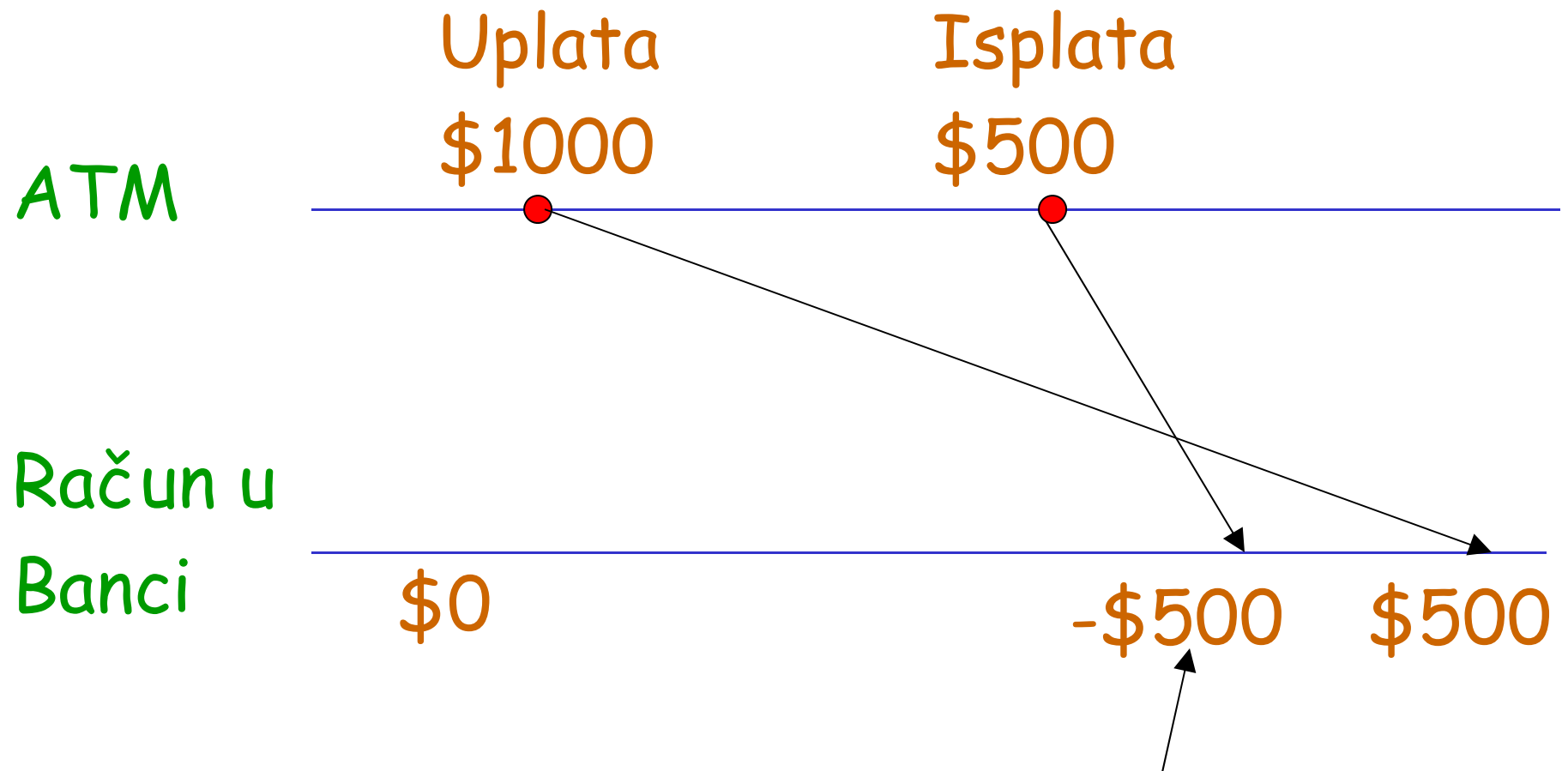


## Zašto je FIFO važan?



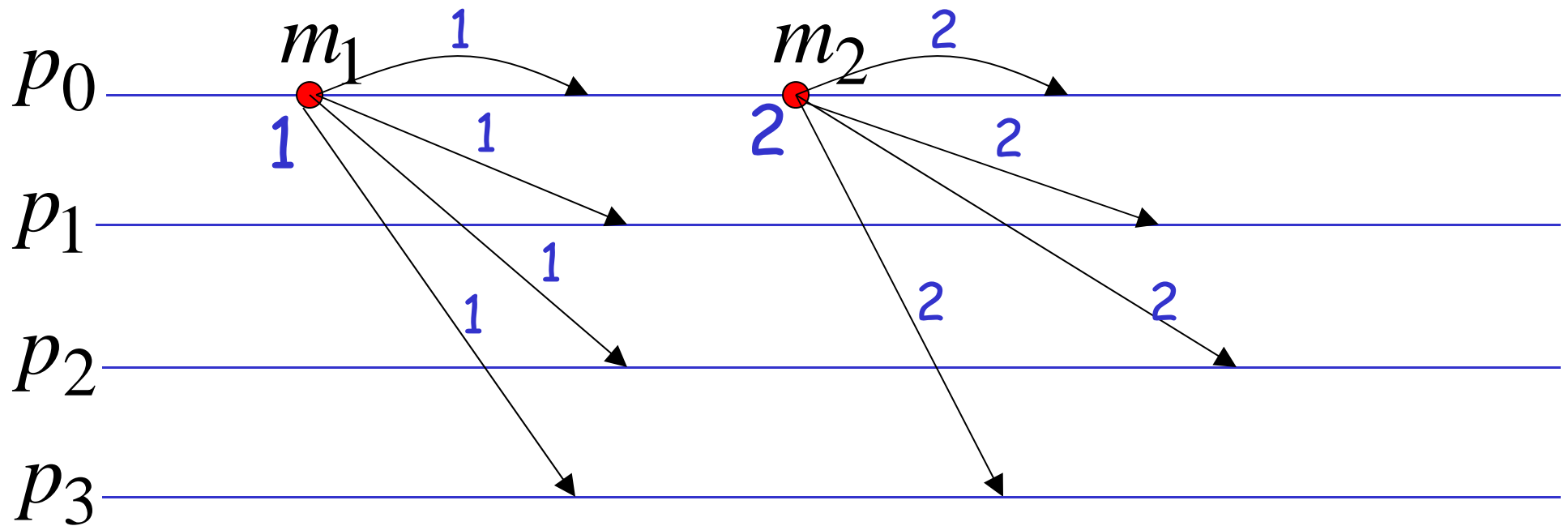
FIFO redosled

## Zašto je FIFO važan?

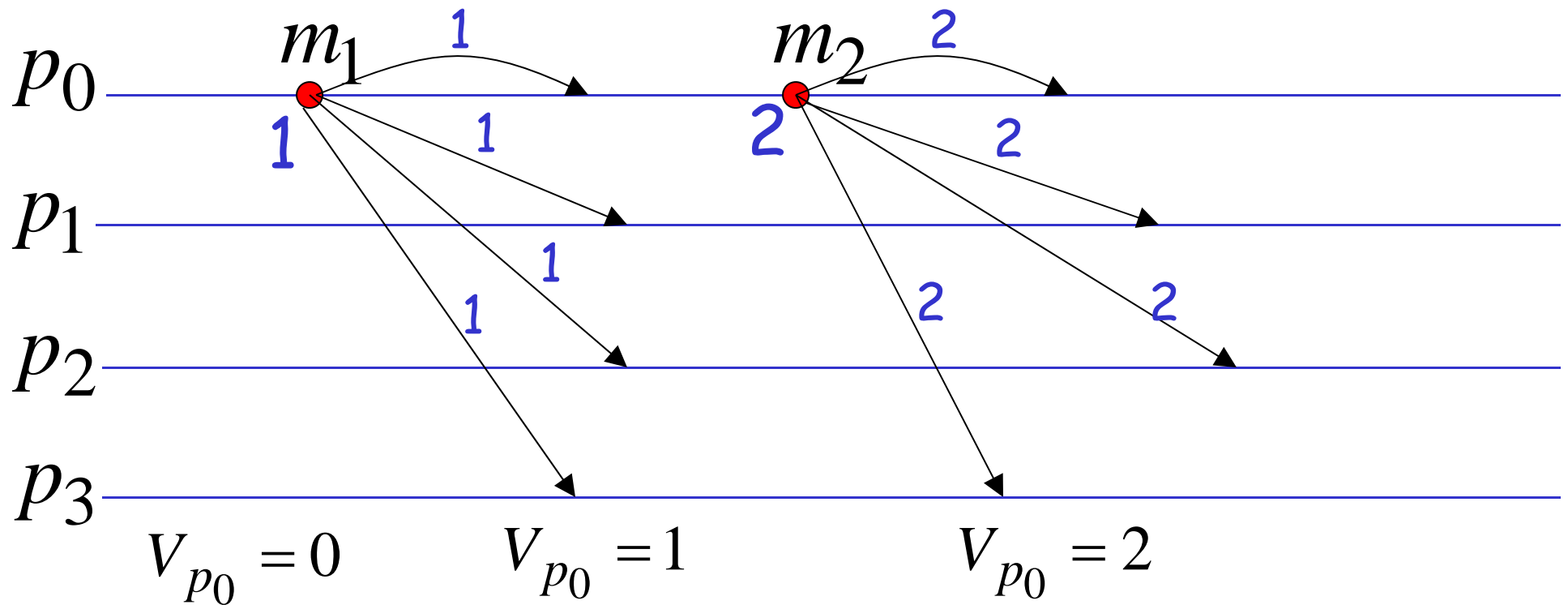


Nije FIFO redosled      Nedoizvoljena  
transakcija

# Jednostavan FIFO algoritam

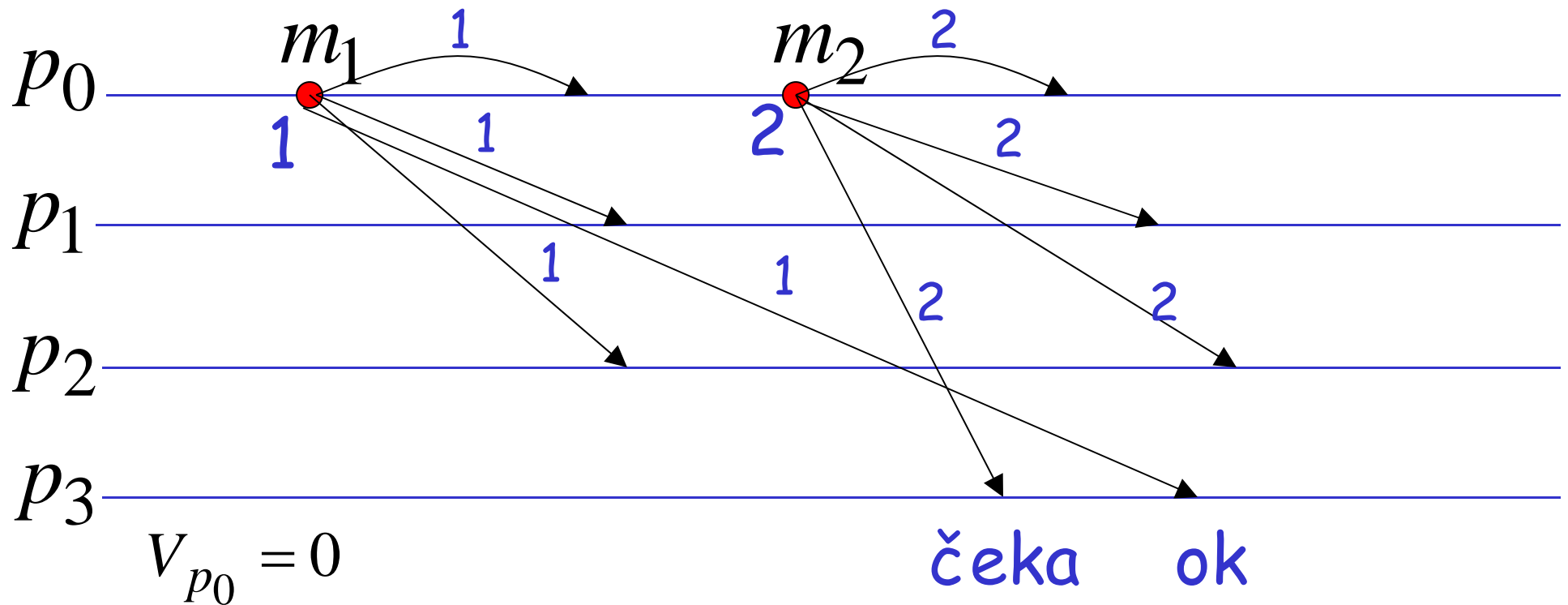


Procesor inkrementira brojač  
za svako slanje svima

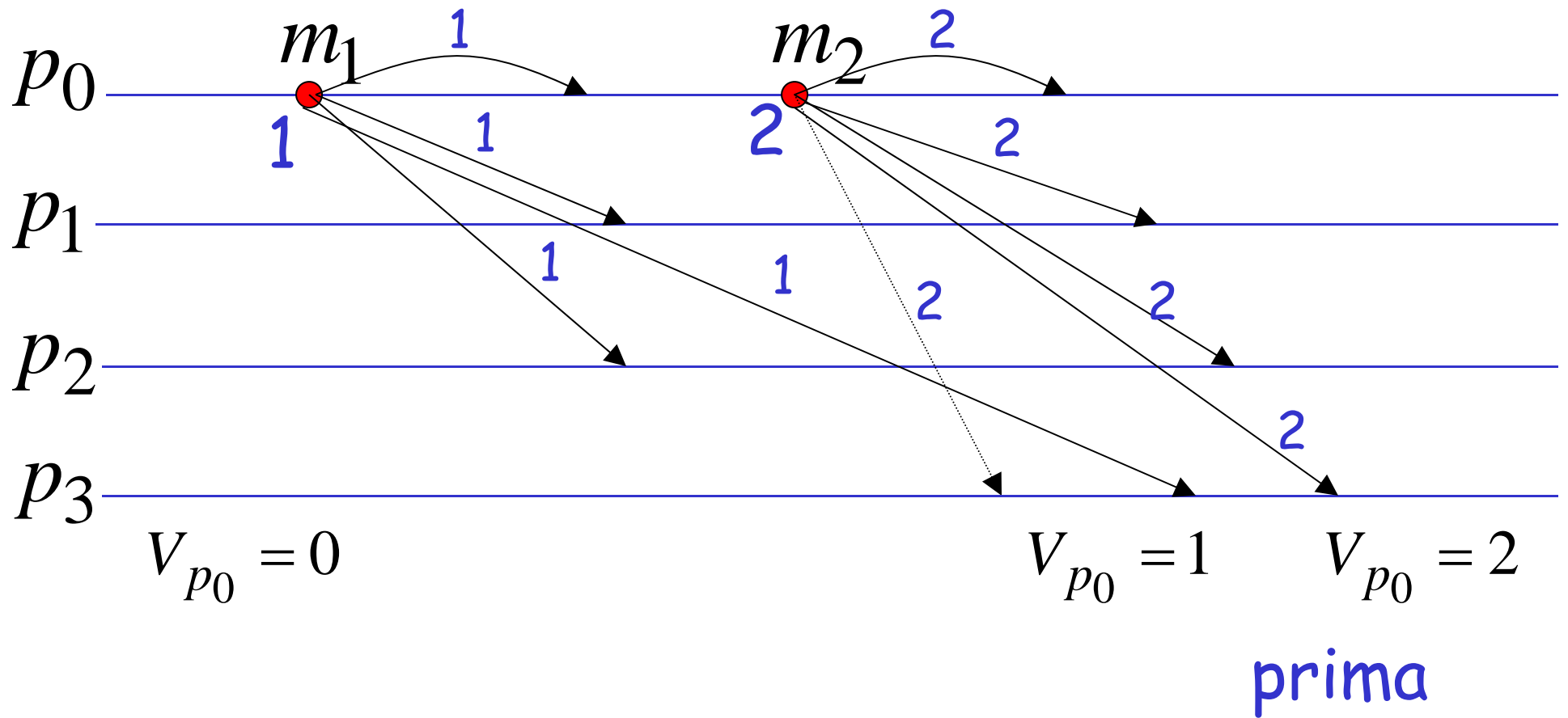


Procesor održava zapis najveće vrednosti  
primljene od drugih procesora



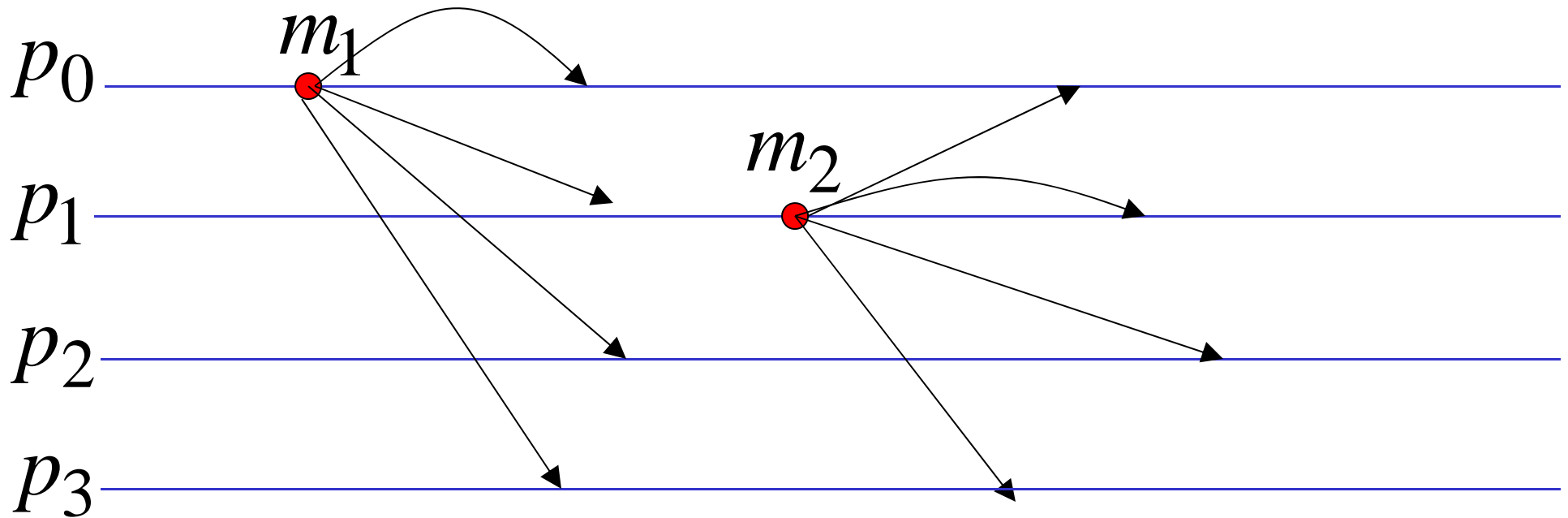


Čeka dok ne budu primljene sve poruke sa manjim vrednostima od istog pošiljaoca



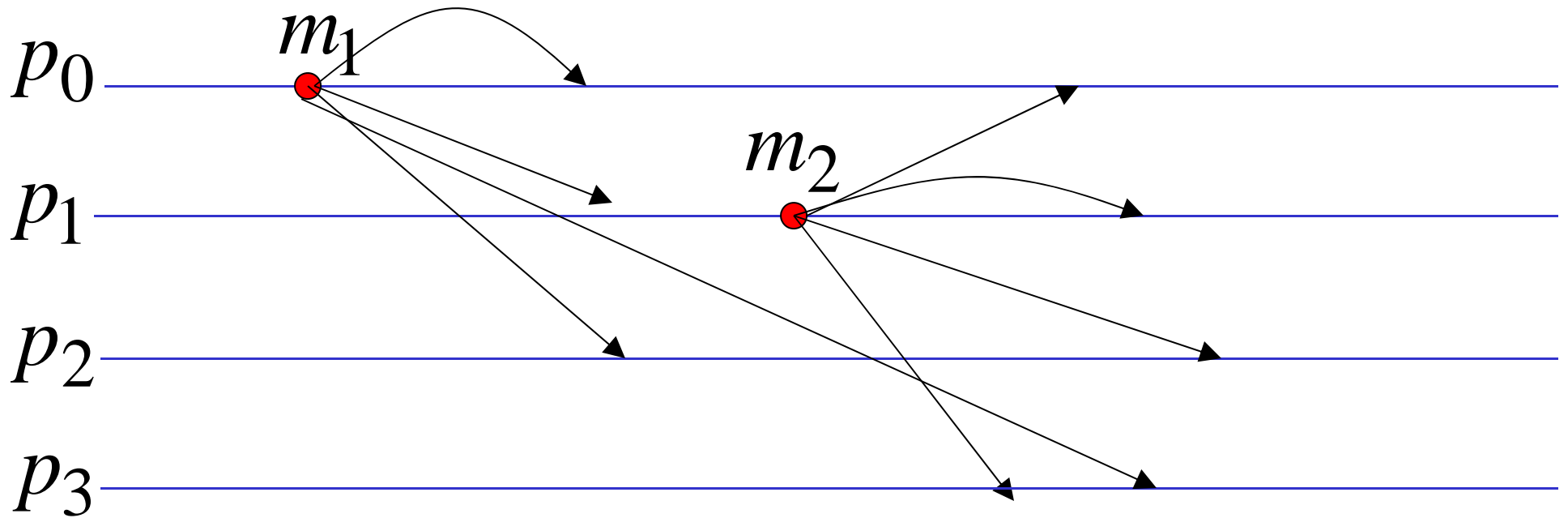
Totalni redosled - Slanje svima

# Totalni redosled (Total Order)

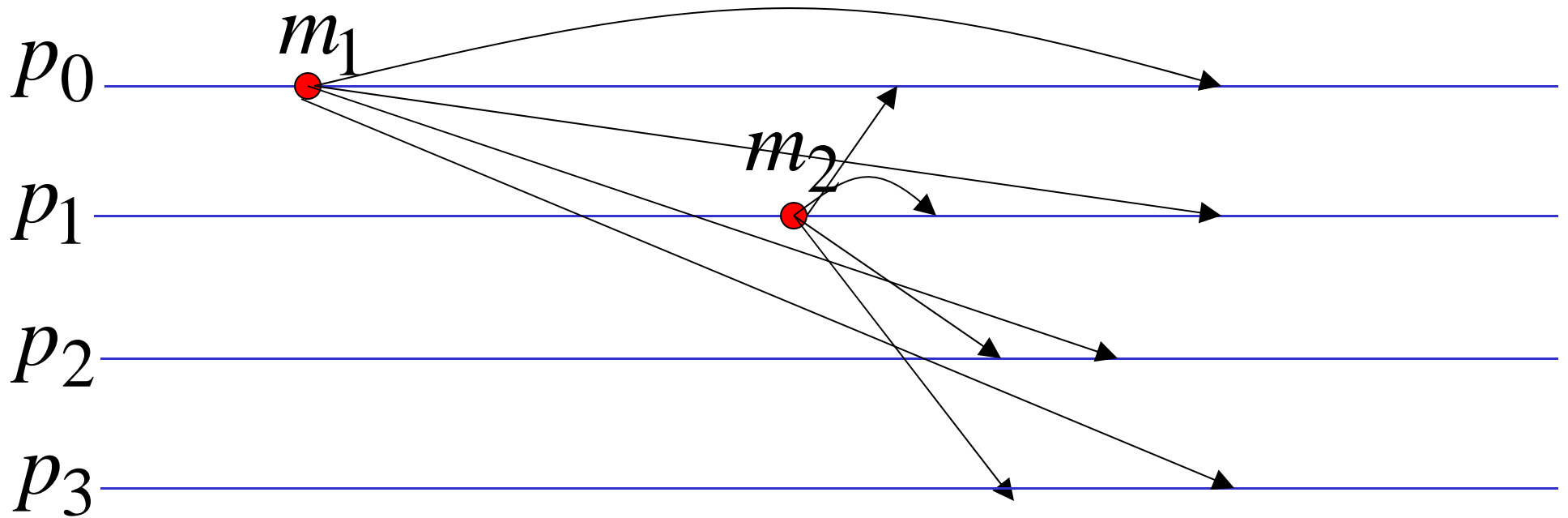


Poruke se primaju u  
istom redosledu u svakom procesoru

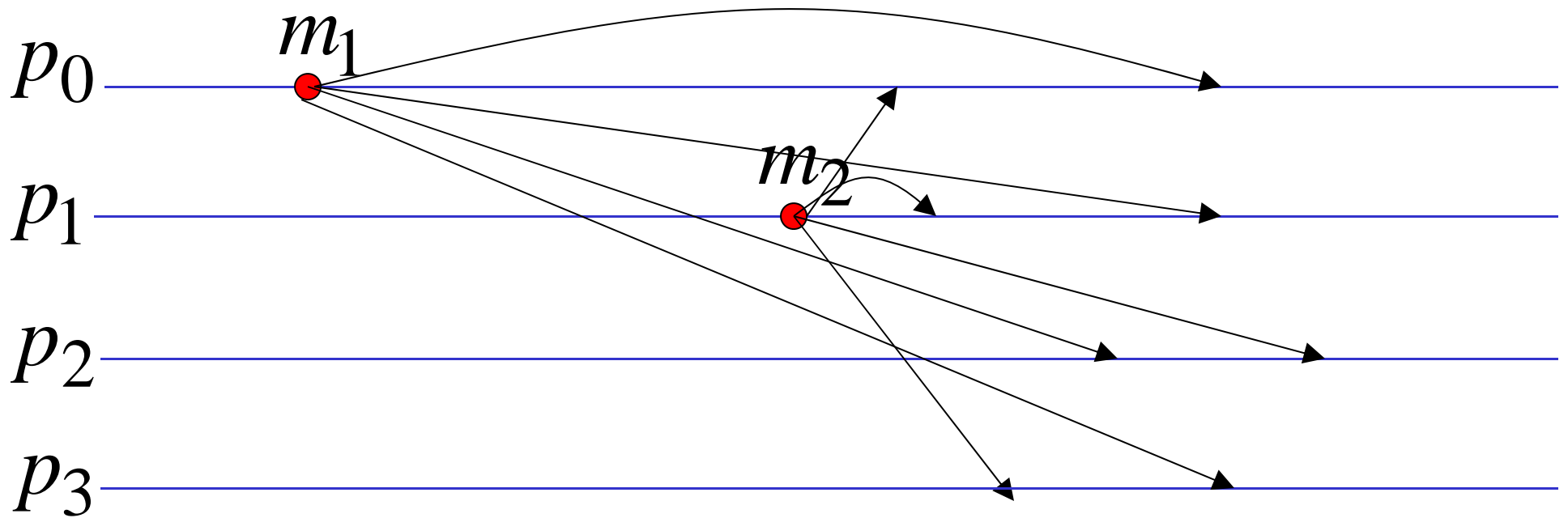
Nije totalni redosled



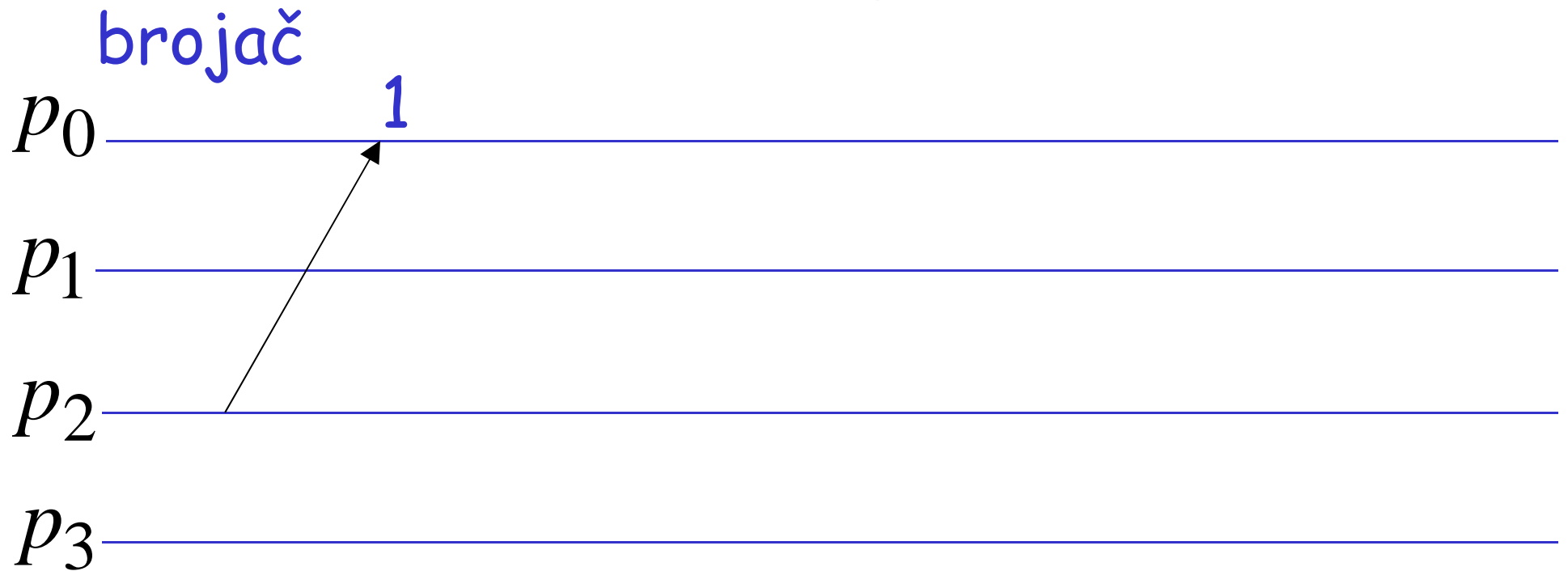
## Totalni redosled



Nije totalni redosled

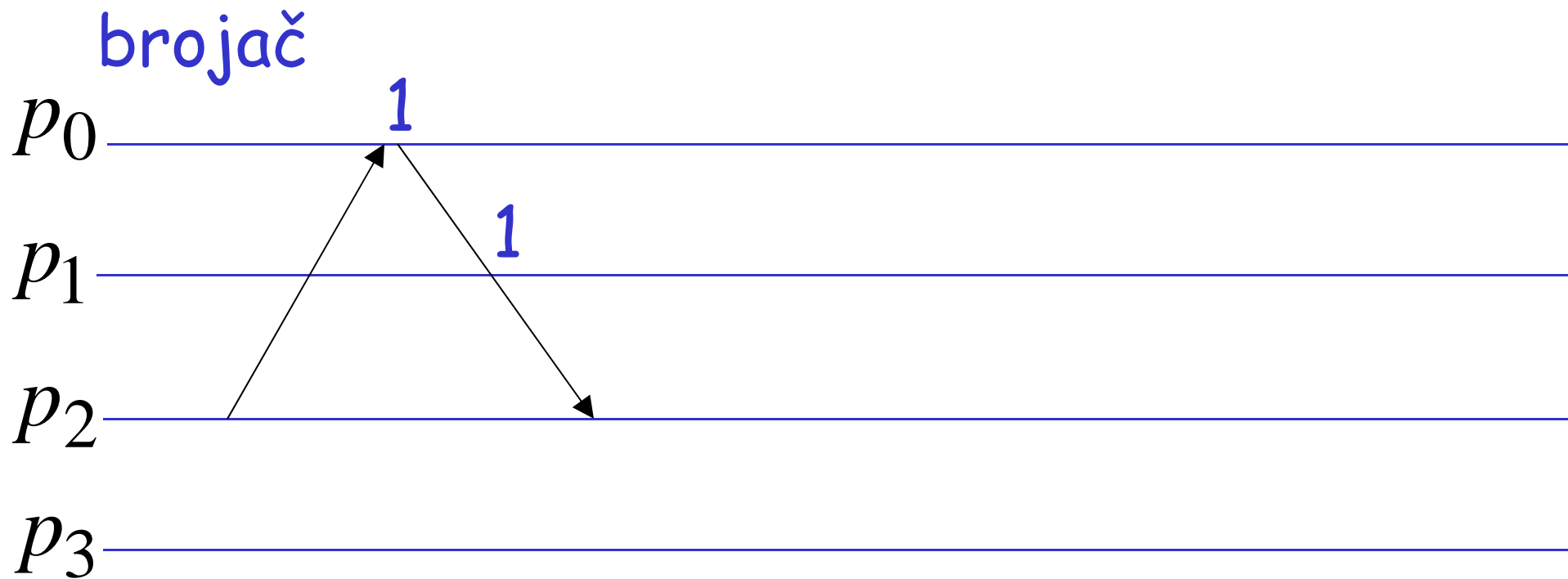


# Asimetričan algoritam

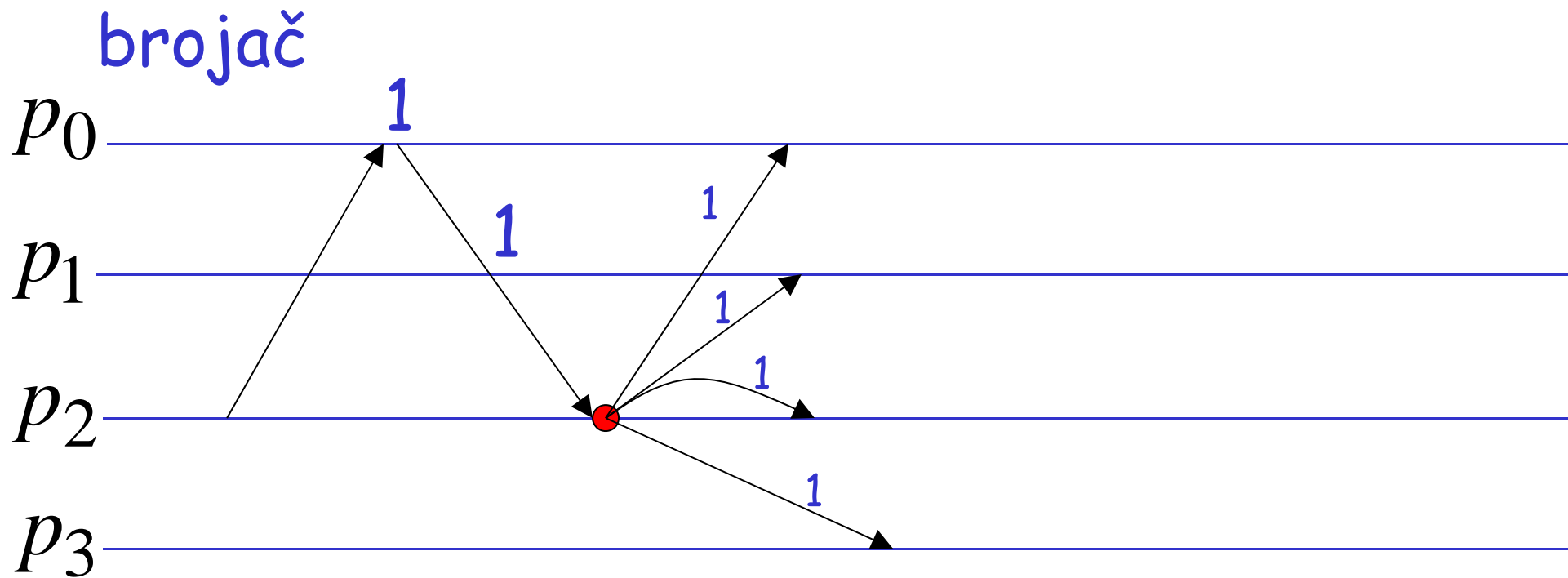


Zahteva vrednost brojača

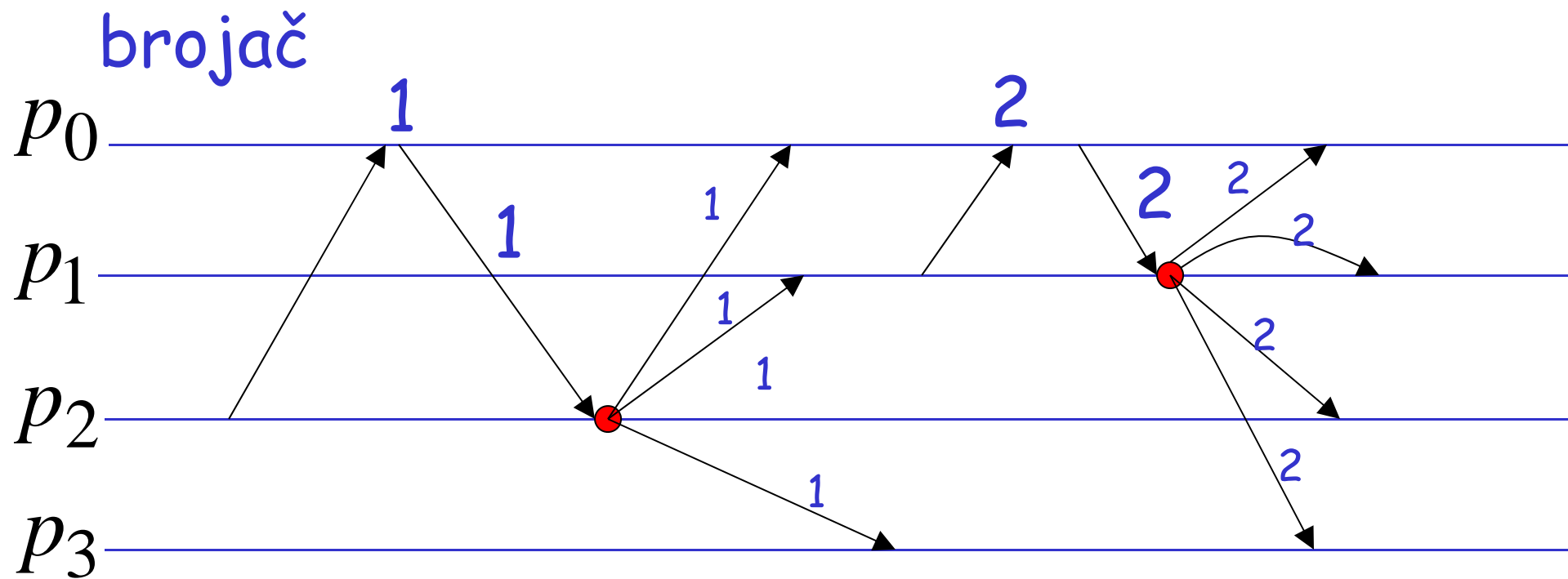


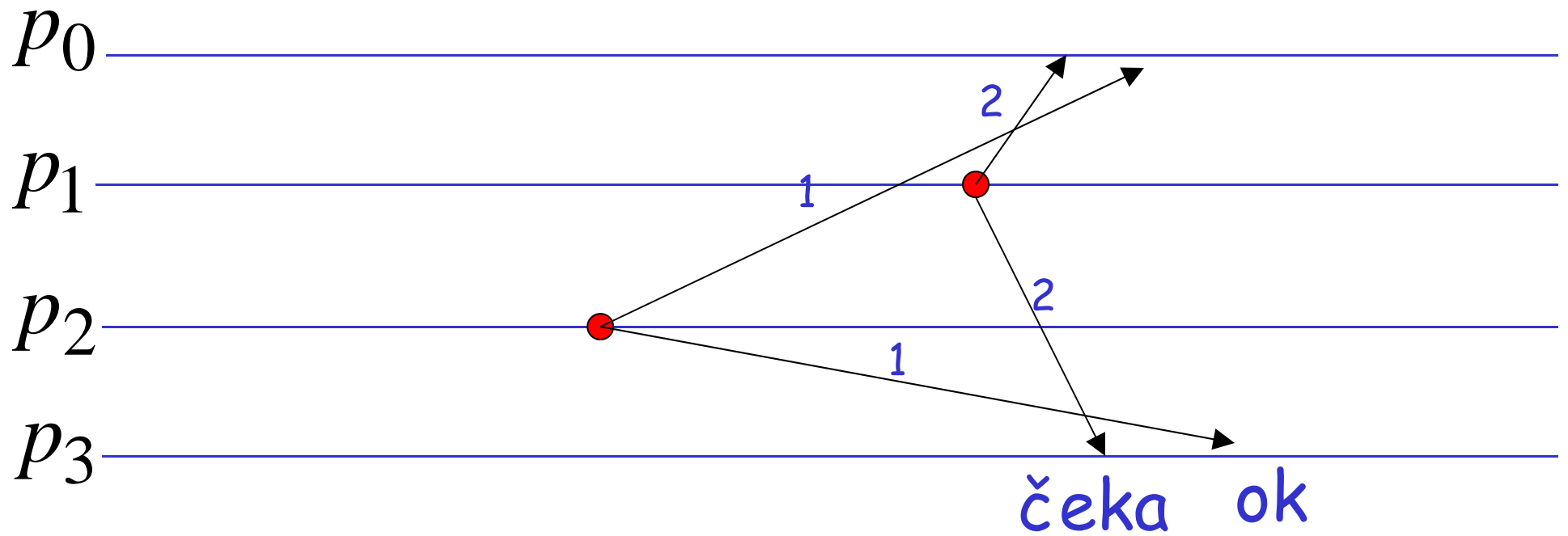


Prima vrednost brojača

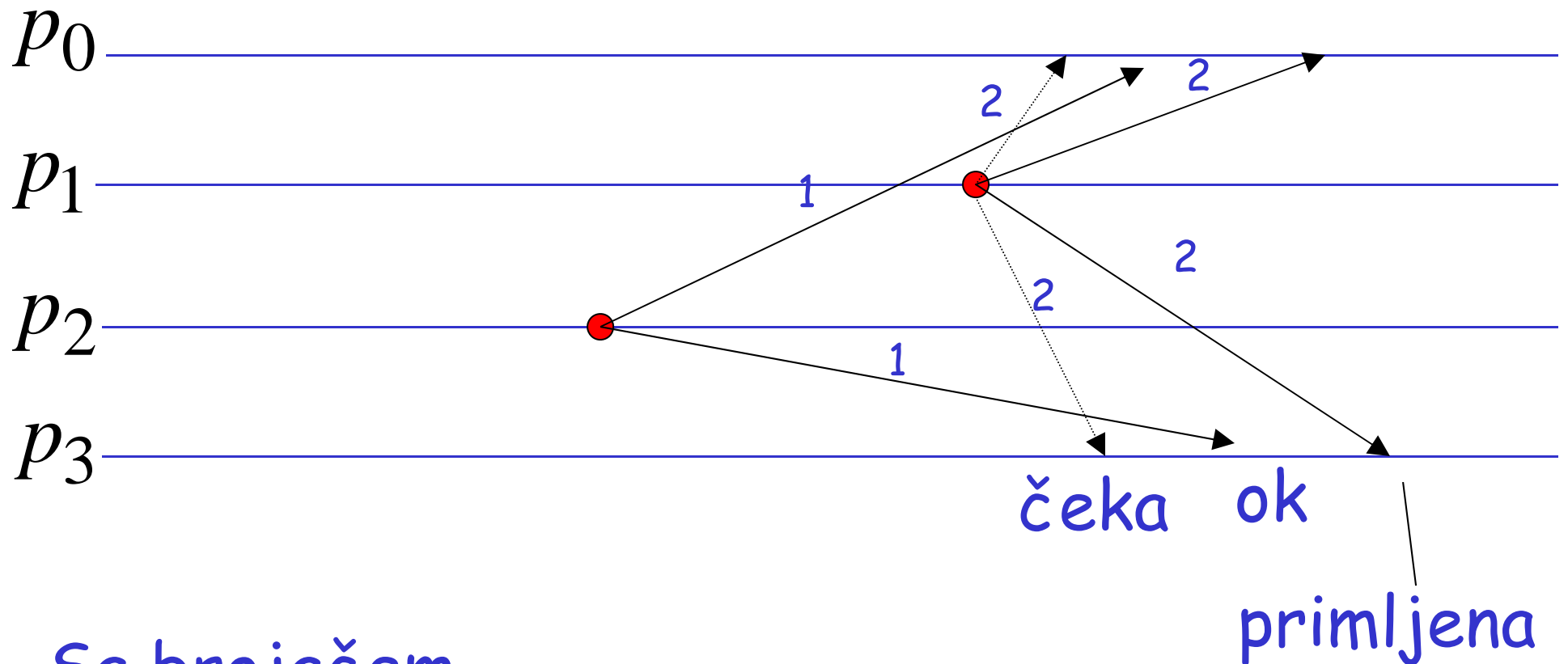


Šalje svima





Sa brojačem,  
poruke upućene svima su u totalnom redosledu



Sa brojačem,  
poruke upućene svima su u totalnom redosledu

# Simetričan algoritam

Osnova mu je FIFO

Koristi koncept vektorskih satova

Svaki proces ima brojač

Prilikom slanja svima (broadcast):  
proces šalje vrednost  
brojača svakom učesniku

Po prijemu poruke:

Ako je primljena vrednost brojača  
veća od lokalne vrednosti, onda:

- Ažuriraj lokalni brojač na novu vred.
- Pošalji svima drugima novu vrednost brojača



$$p_0 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

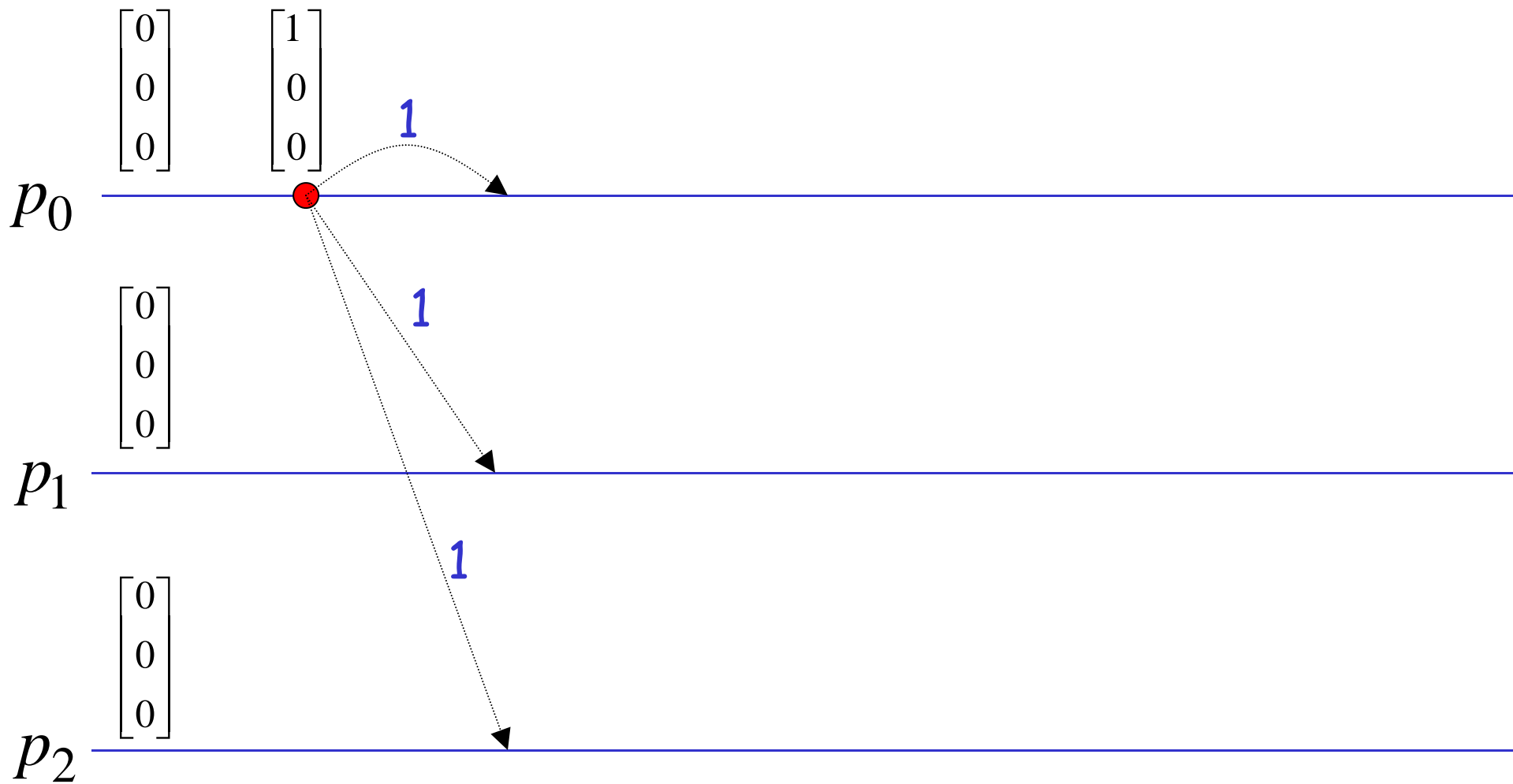

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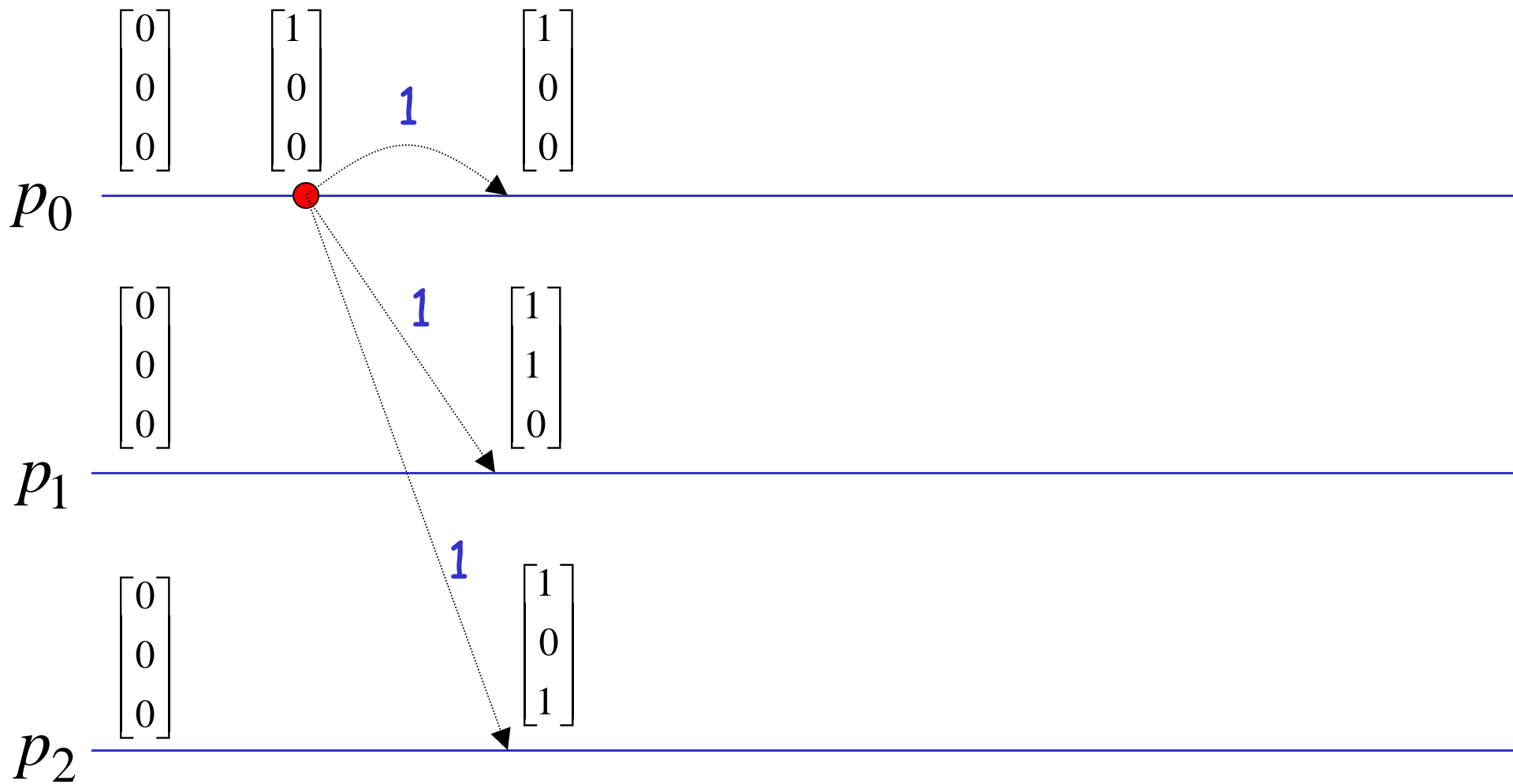
$$p_1 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

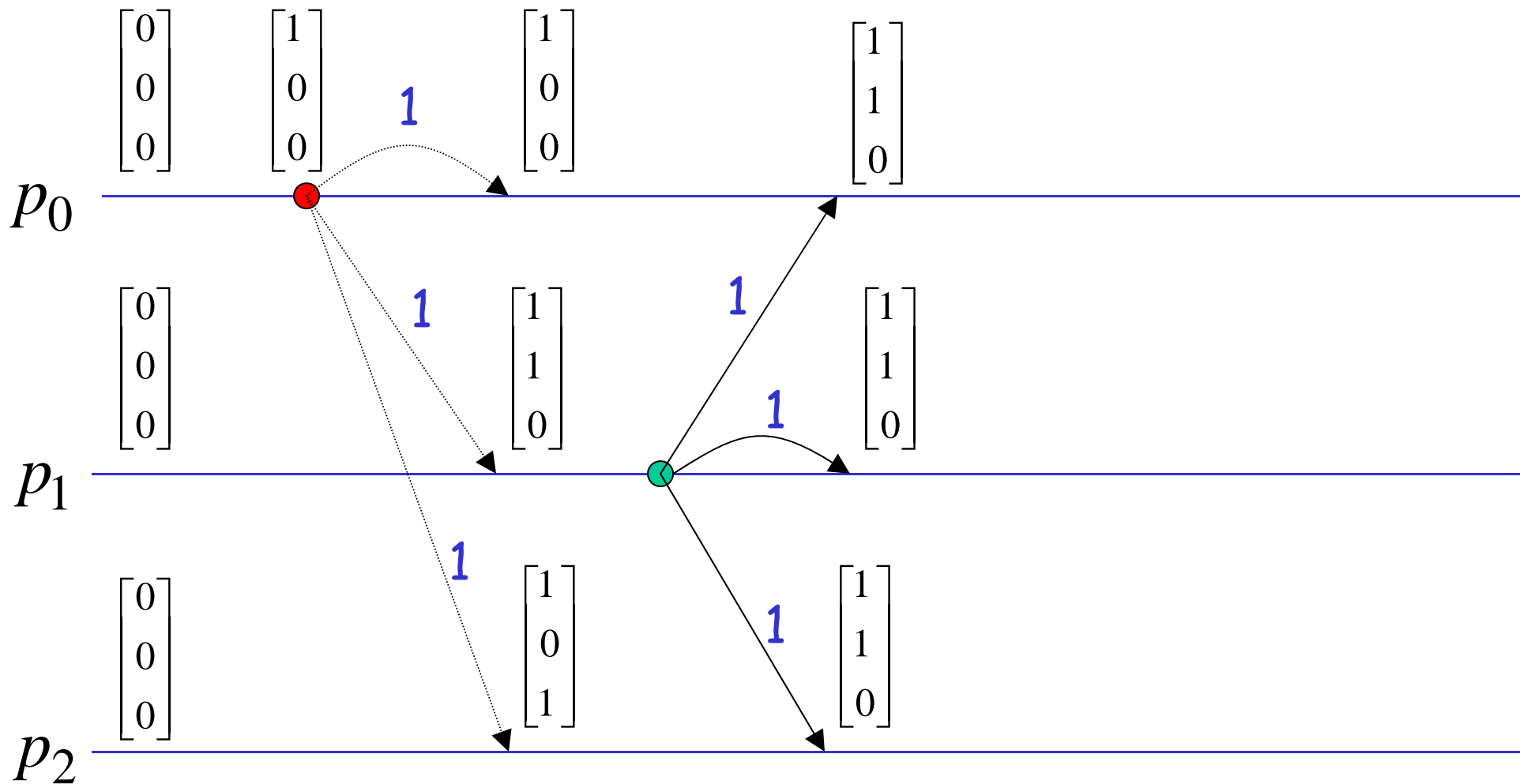

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$$p_2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

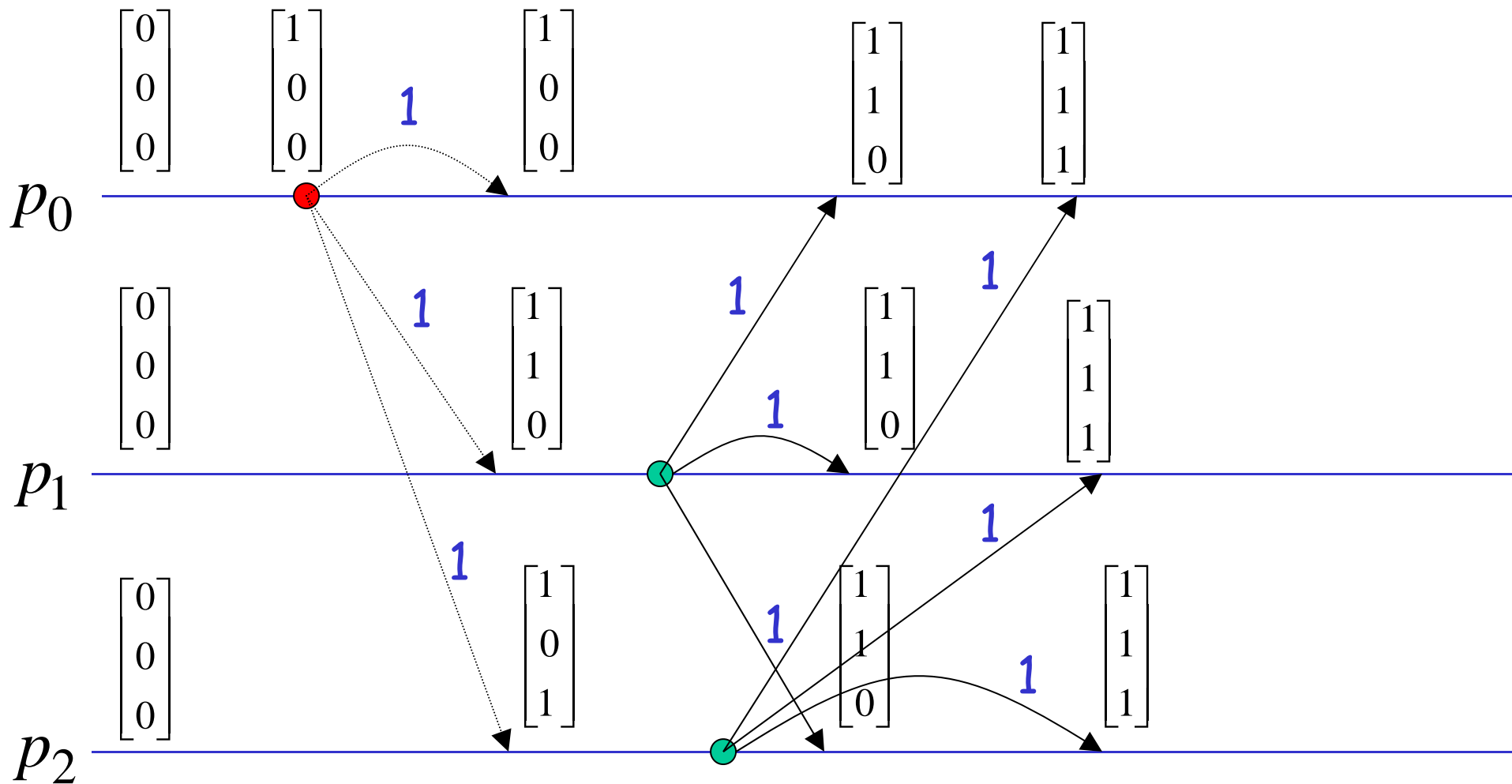

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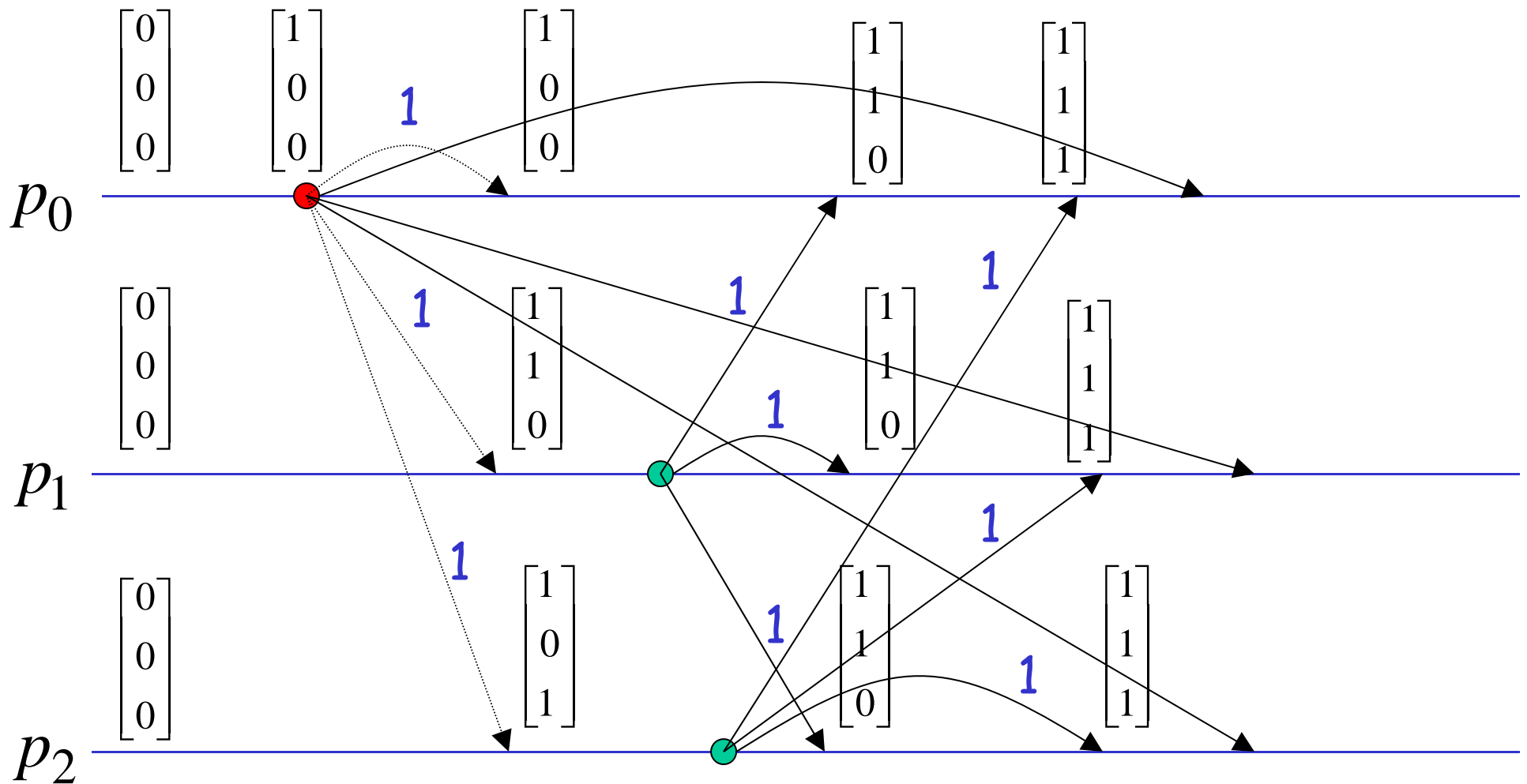




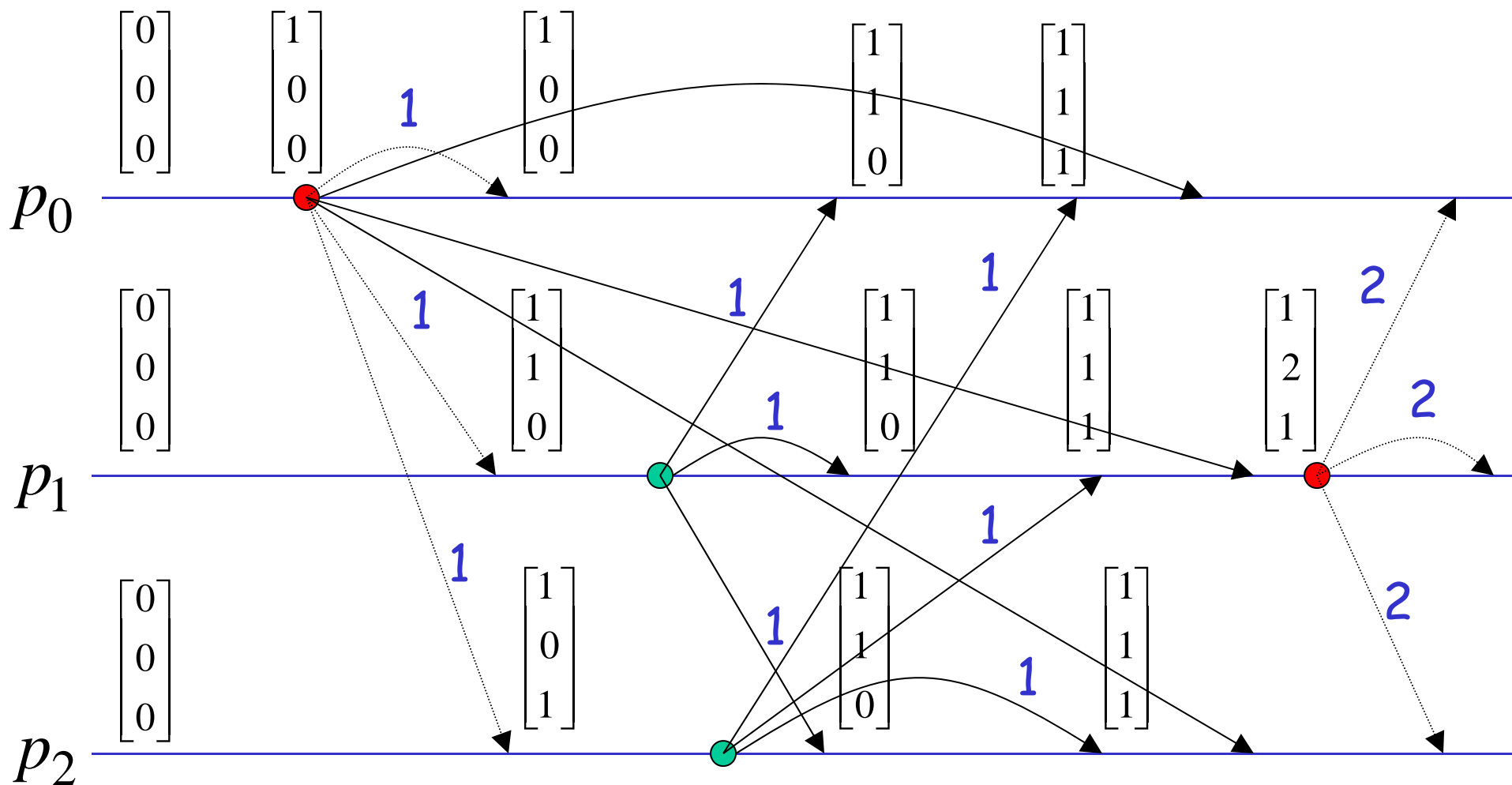


Specijalna poruka upućena svima





Stvarni prijem normalne poruke upućene svima



Sledeće normalno slanje svima

Lokalni vektor od:  $p_i$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \dots \end{bmatrix}$$

$p_i$  čuva normalne poruke upućene svima u red sortiran po broju poruke. Da bi primio poruku br.  $T$ , mora predhodno primiti sve predhodne poruke, a tada je  $a_j$  bar jednako  $T$ , za sve  $j$ , i tada se iz reda vade sve poruke sa brojem manjim ili jednakim  $T$ .



$$p_0 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

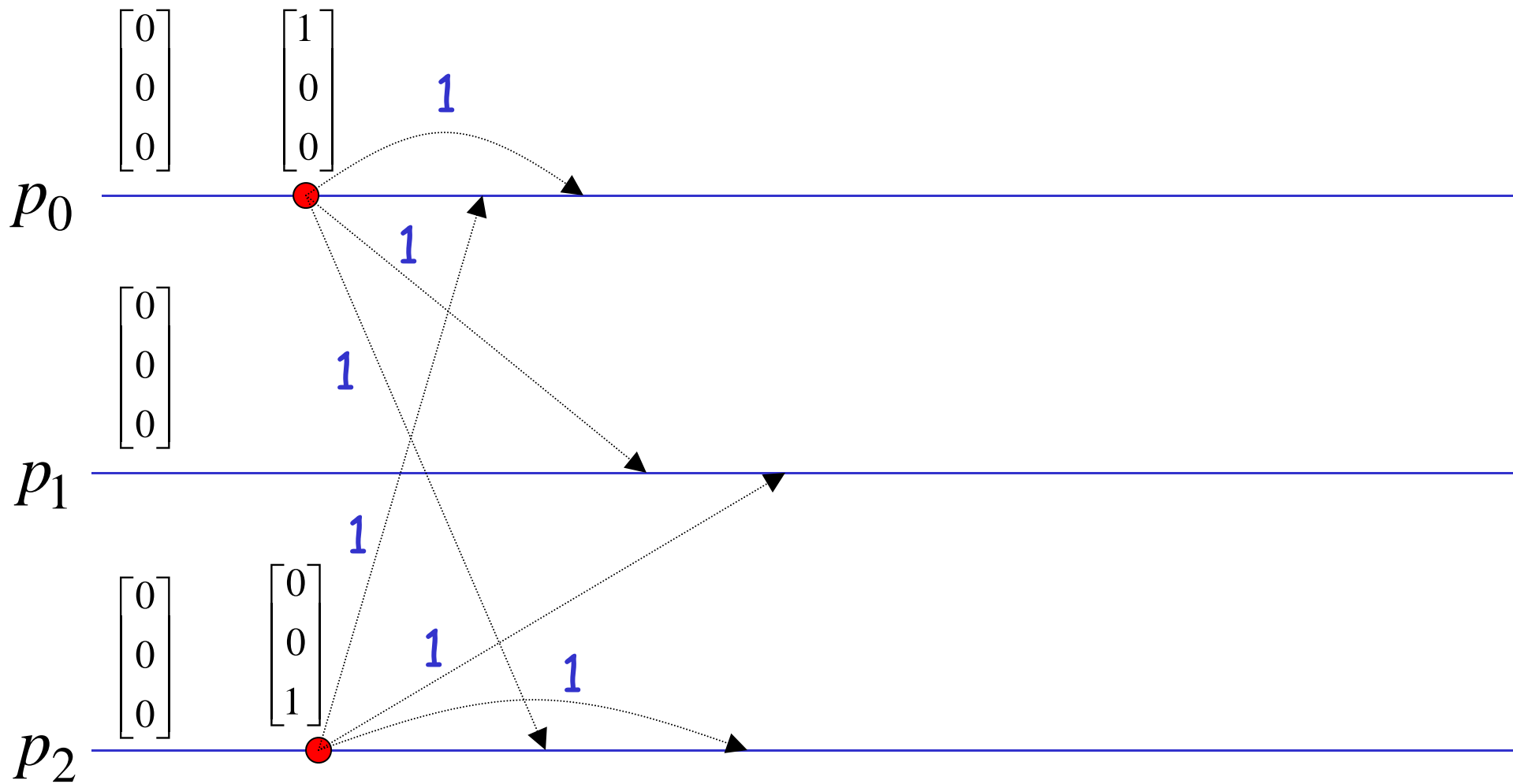

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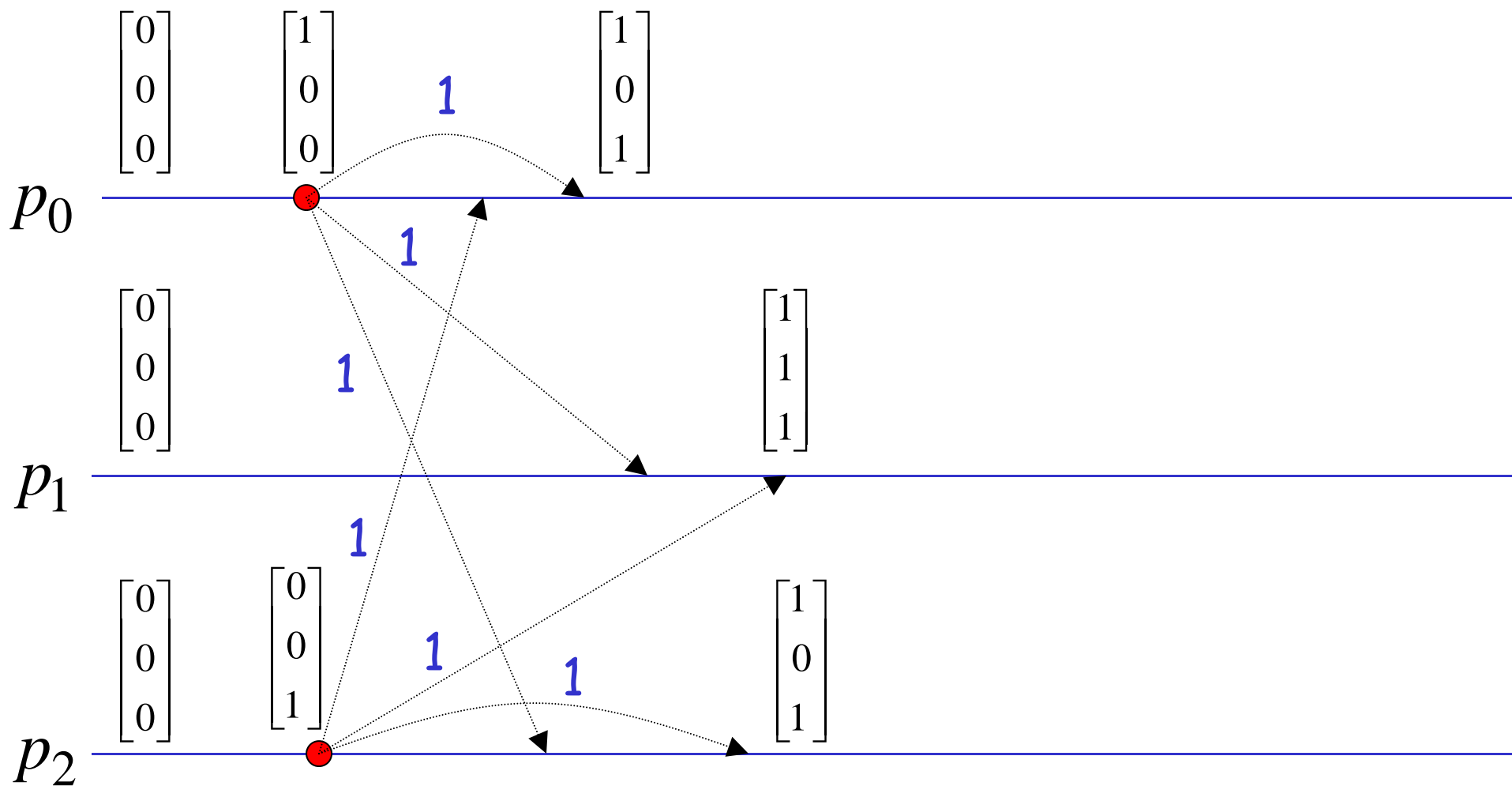
$$p_1 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

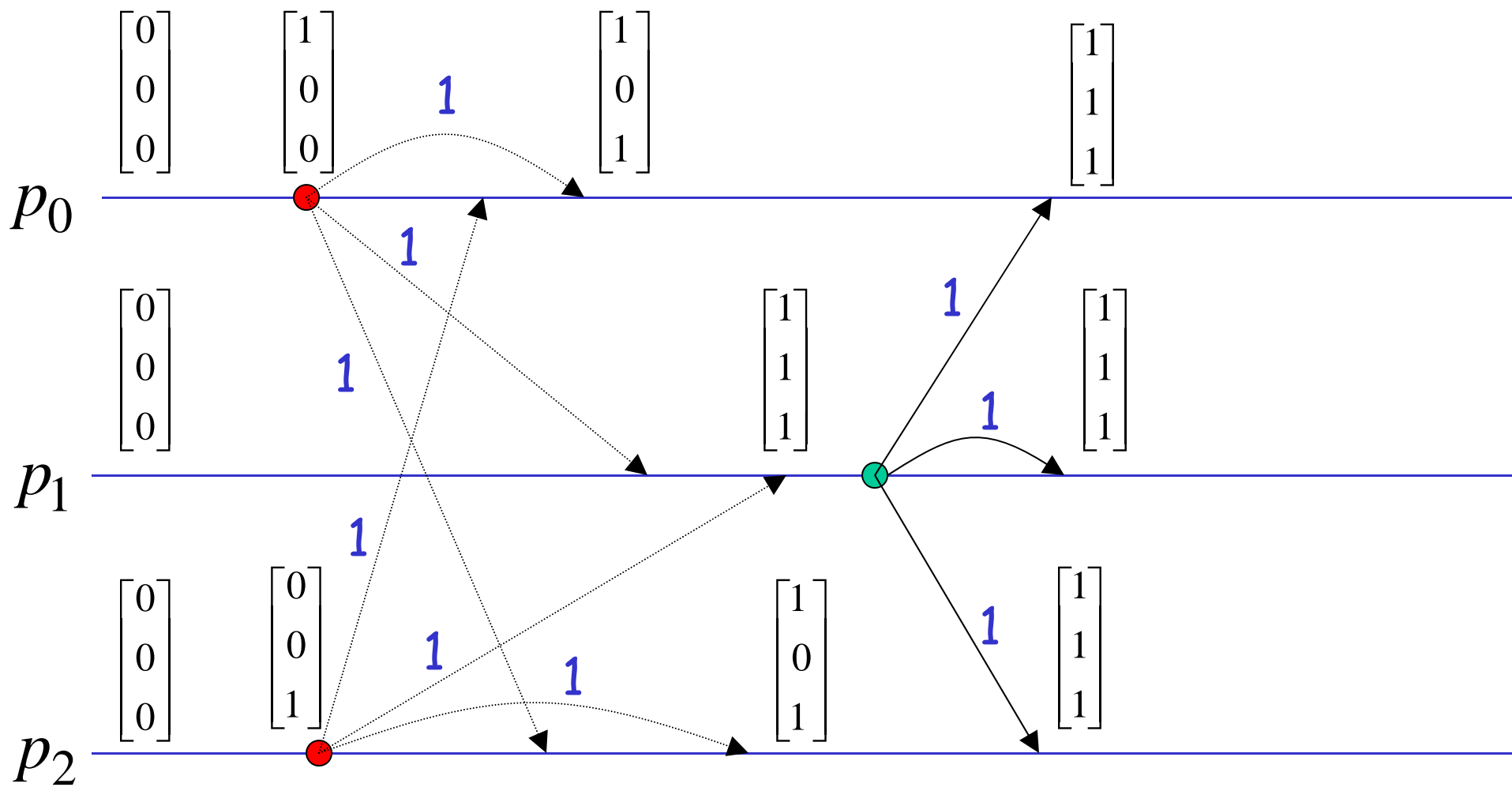

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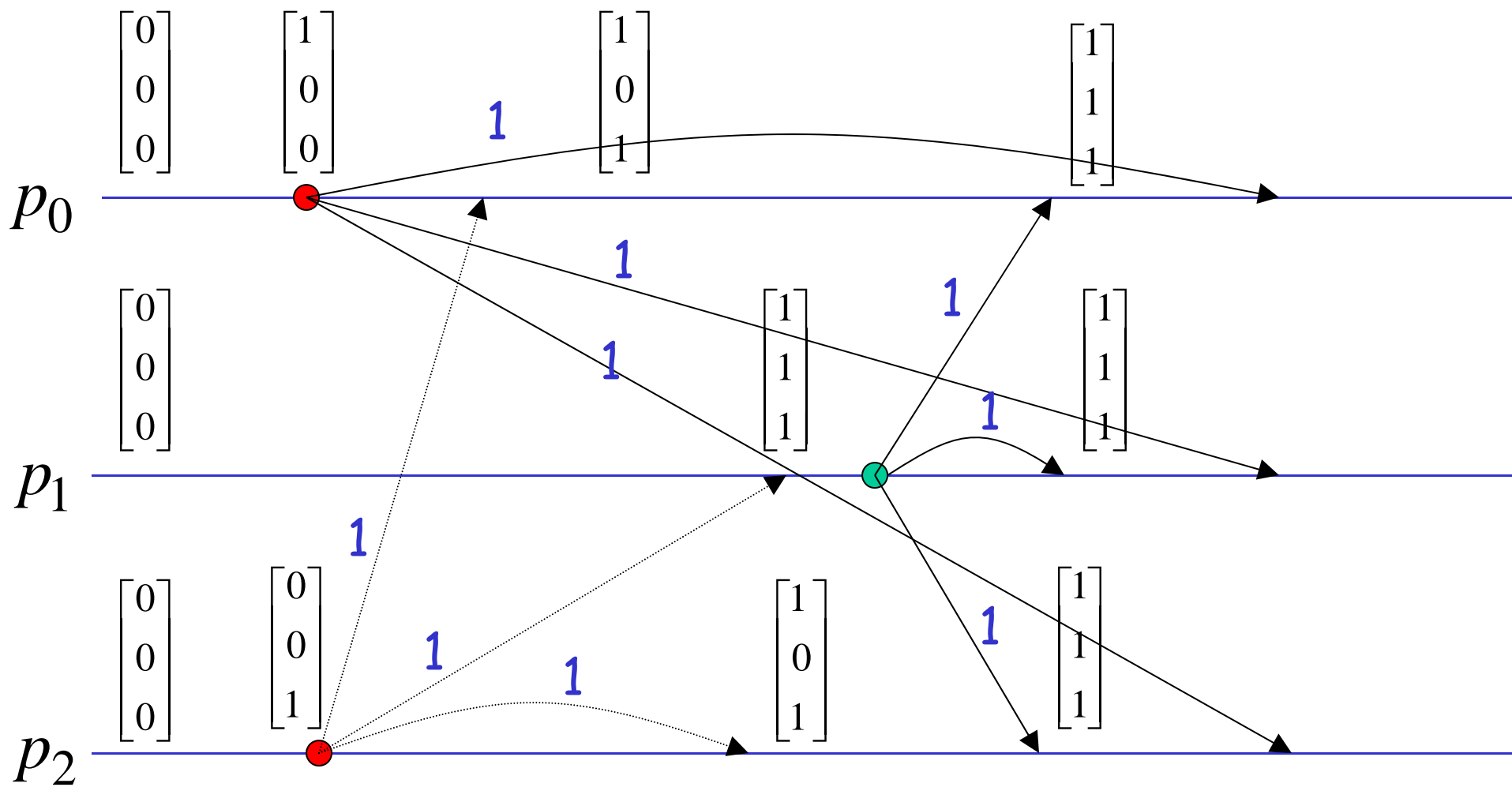
$$p_2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

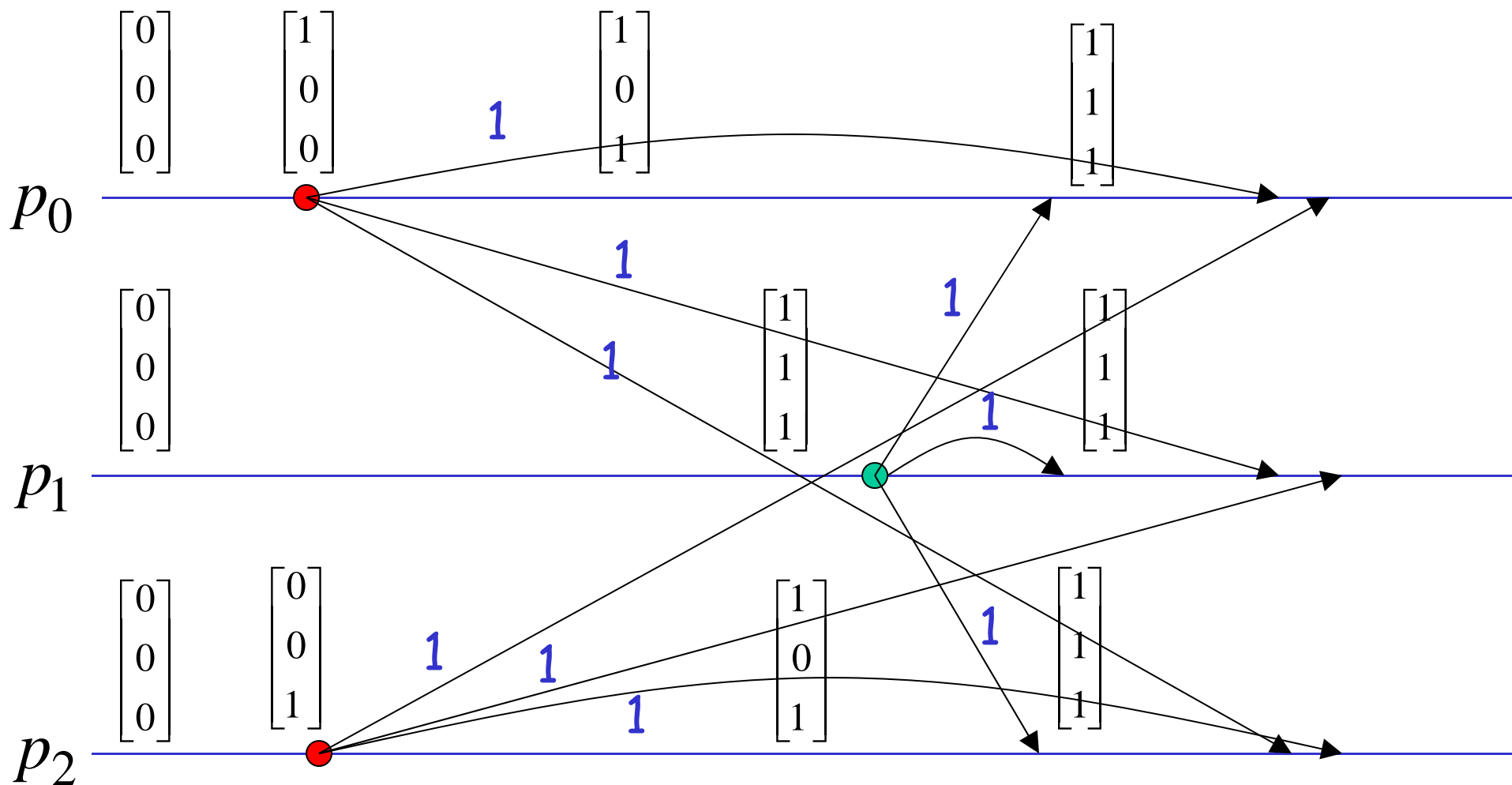

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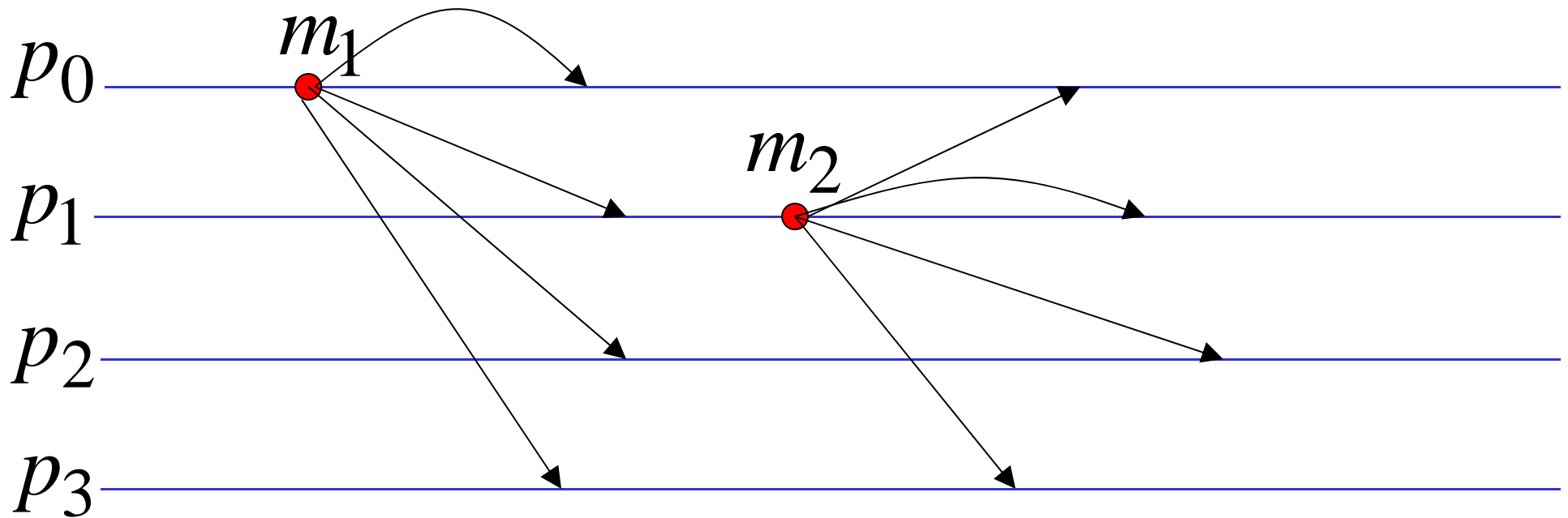




Simetrija se razbija na osnovu  
identifikacija čvorova

Uzročni redosled - Slanje svima

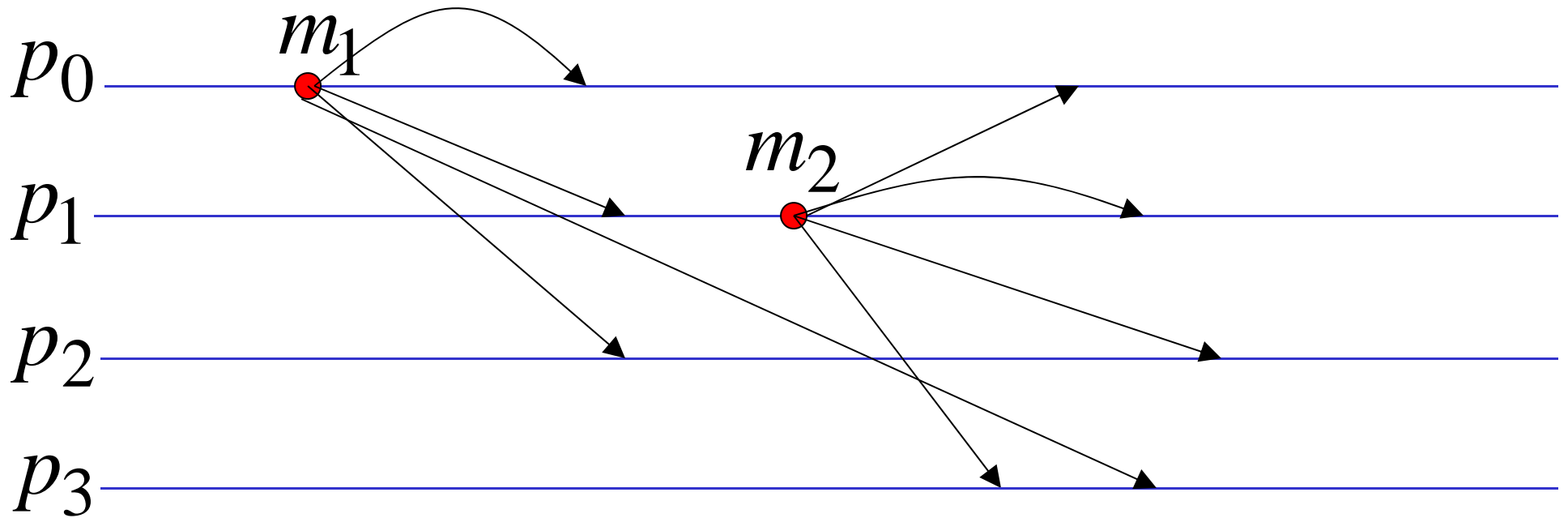
# Uzročni redosled



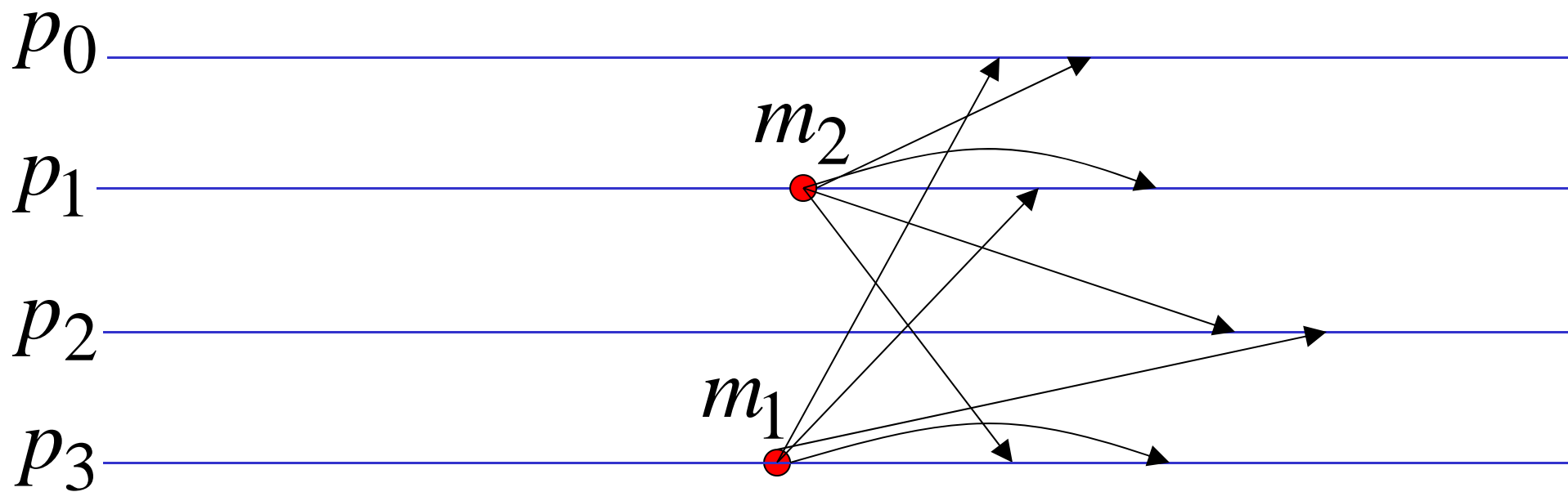
Poruke se primaju u  
u redosledu u kom su uzrokovane



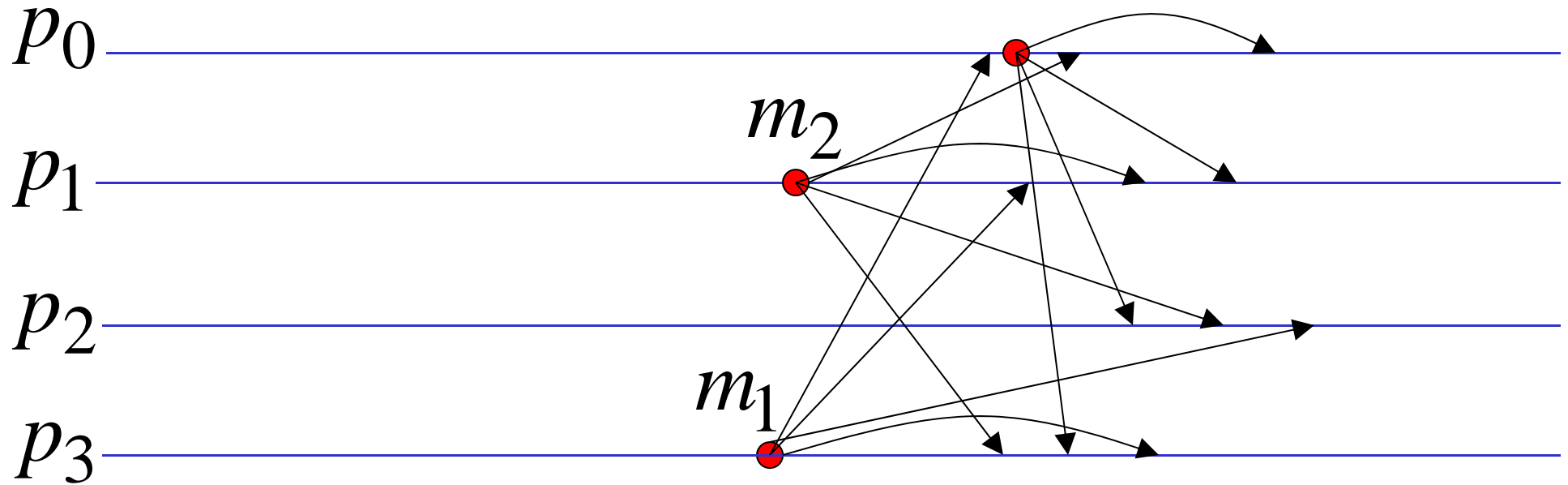
Nije uzročni redosled



## Uzročni redosled



## Nije uzročni redosled



**Opservacija:** Algoritmi totalnog redosleda  
koje smo opisali su takođe  
algoritmi uzročnog redosleda

Postoji efikasniji algoritam  
u smislu broja poruka

$$p_0 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

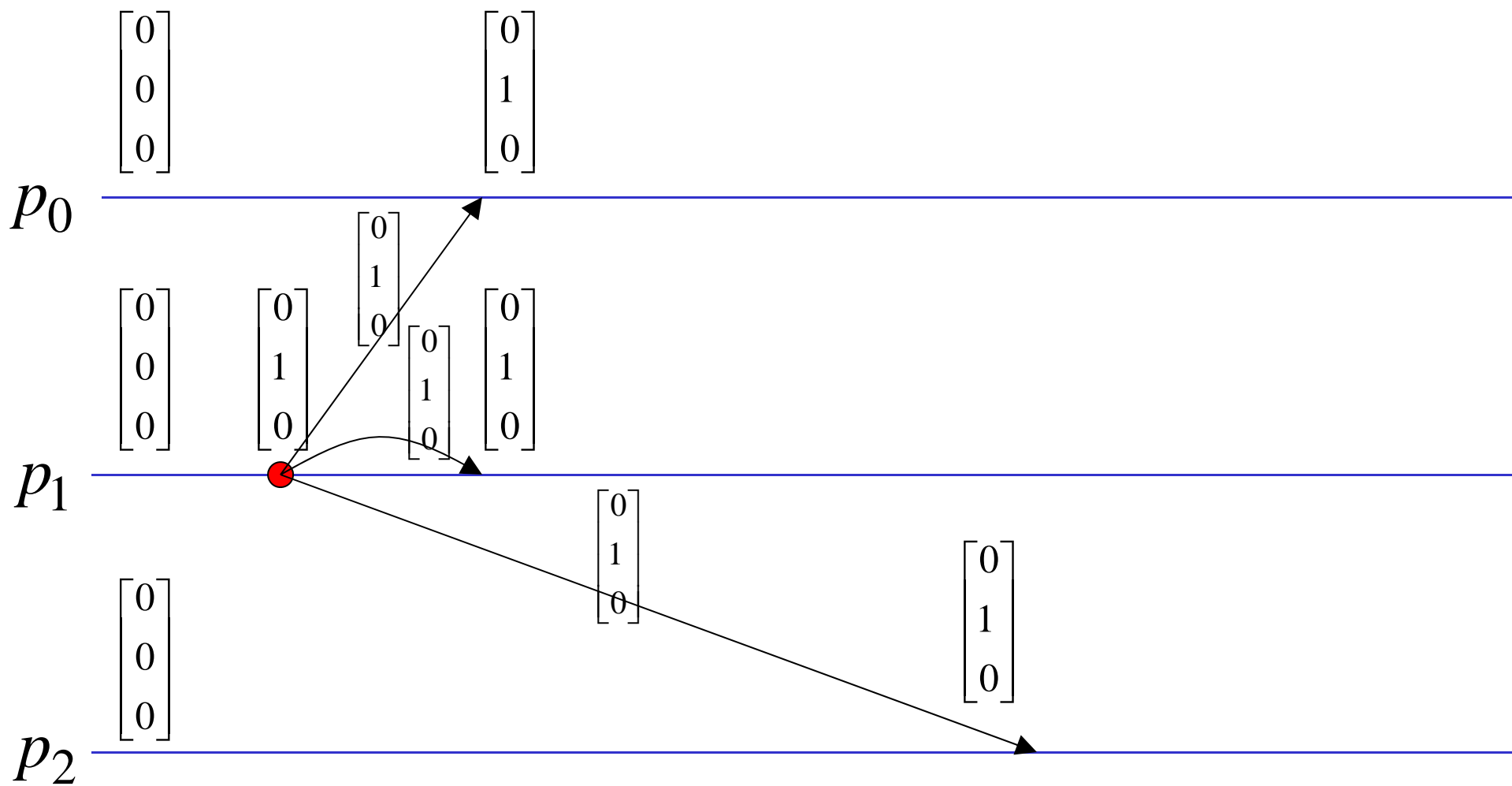

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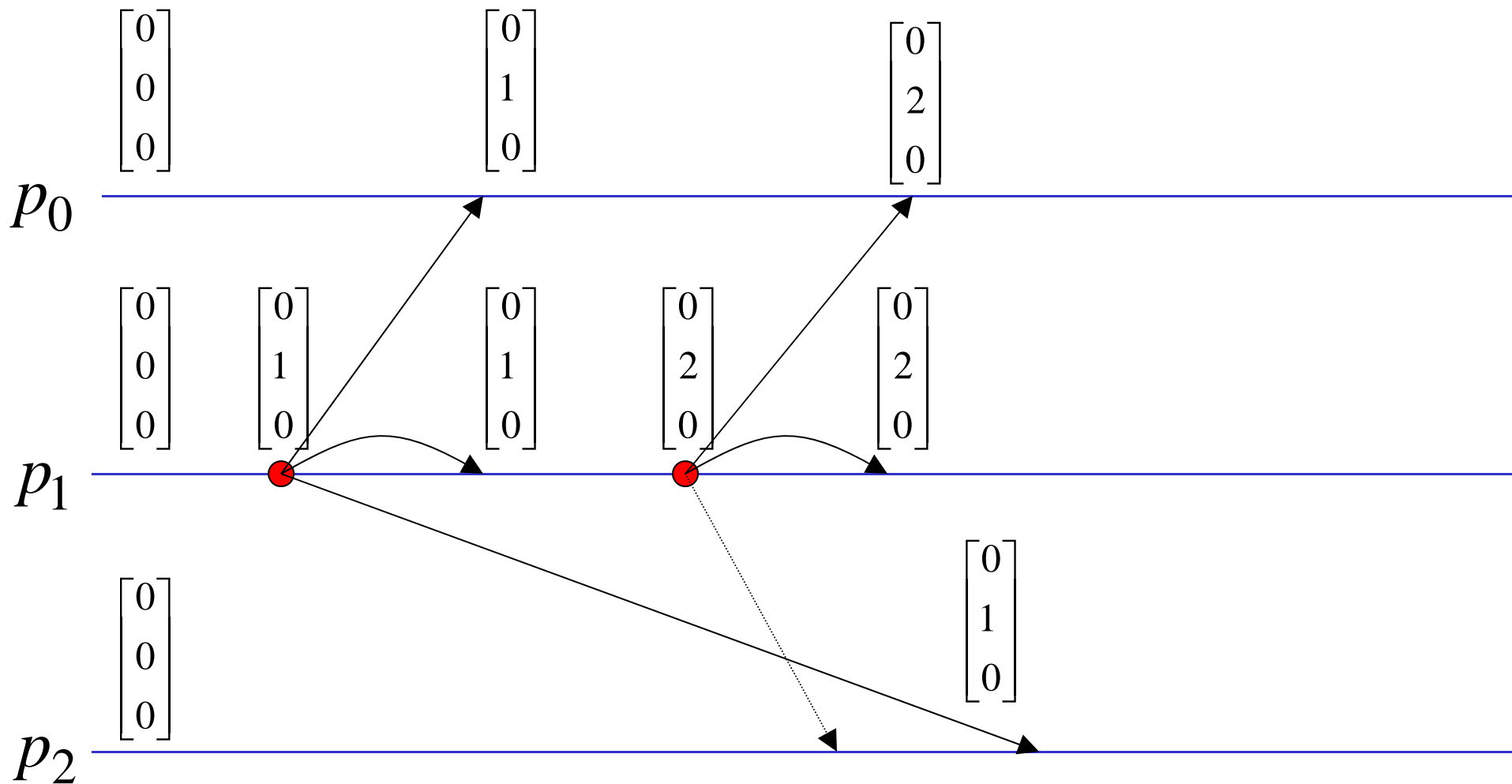
$$p_1 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

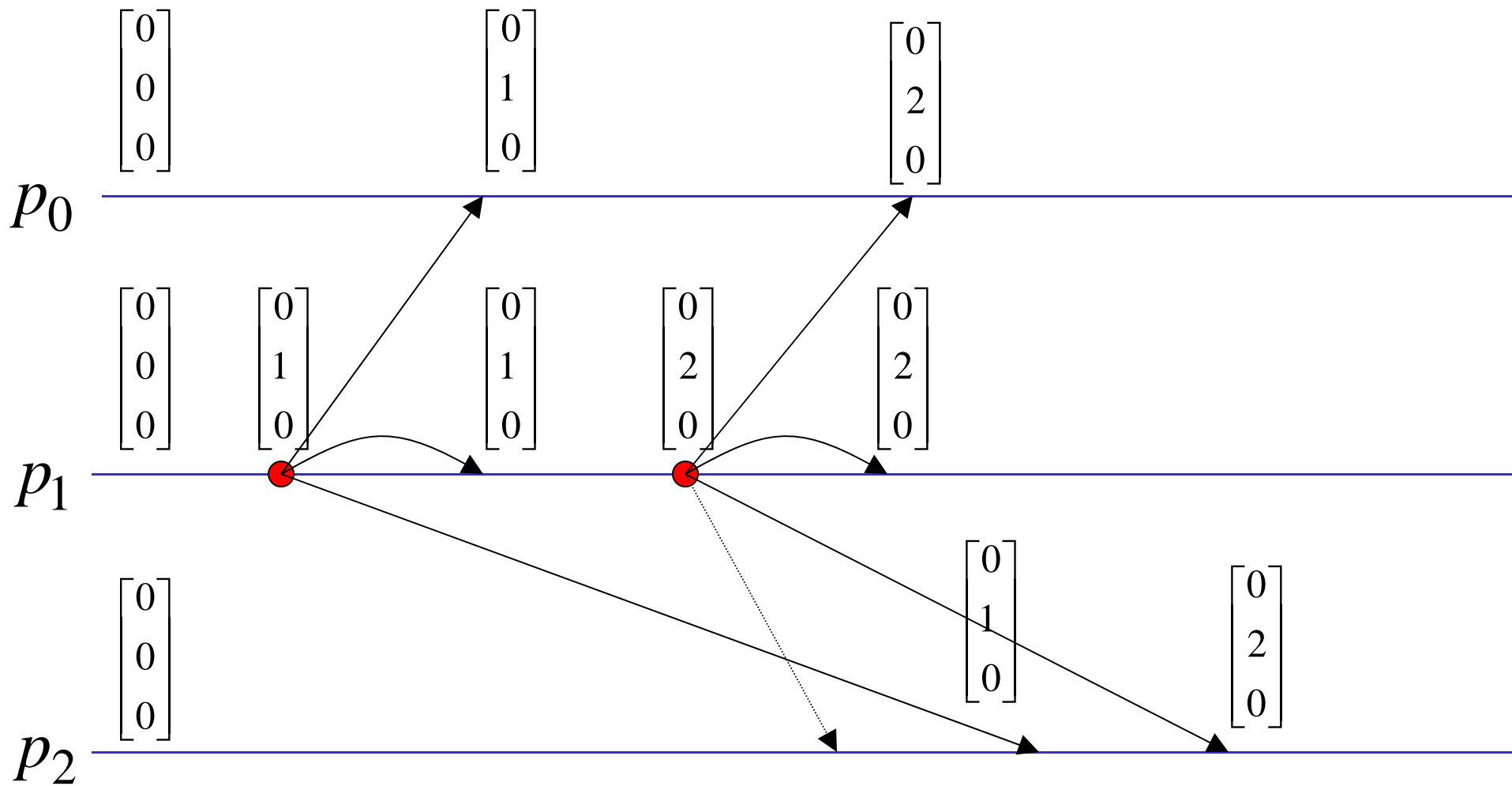

---

$$p_2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$


---









$$p_0 \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

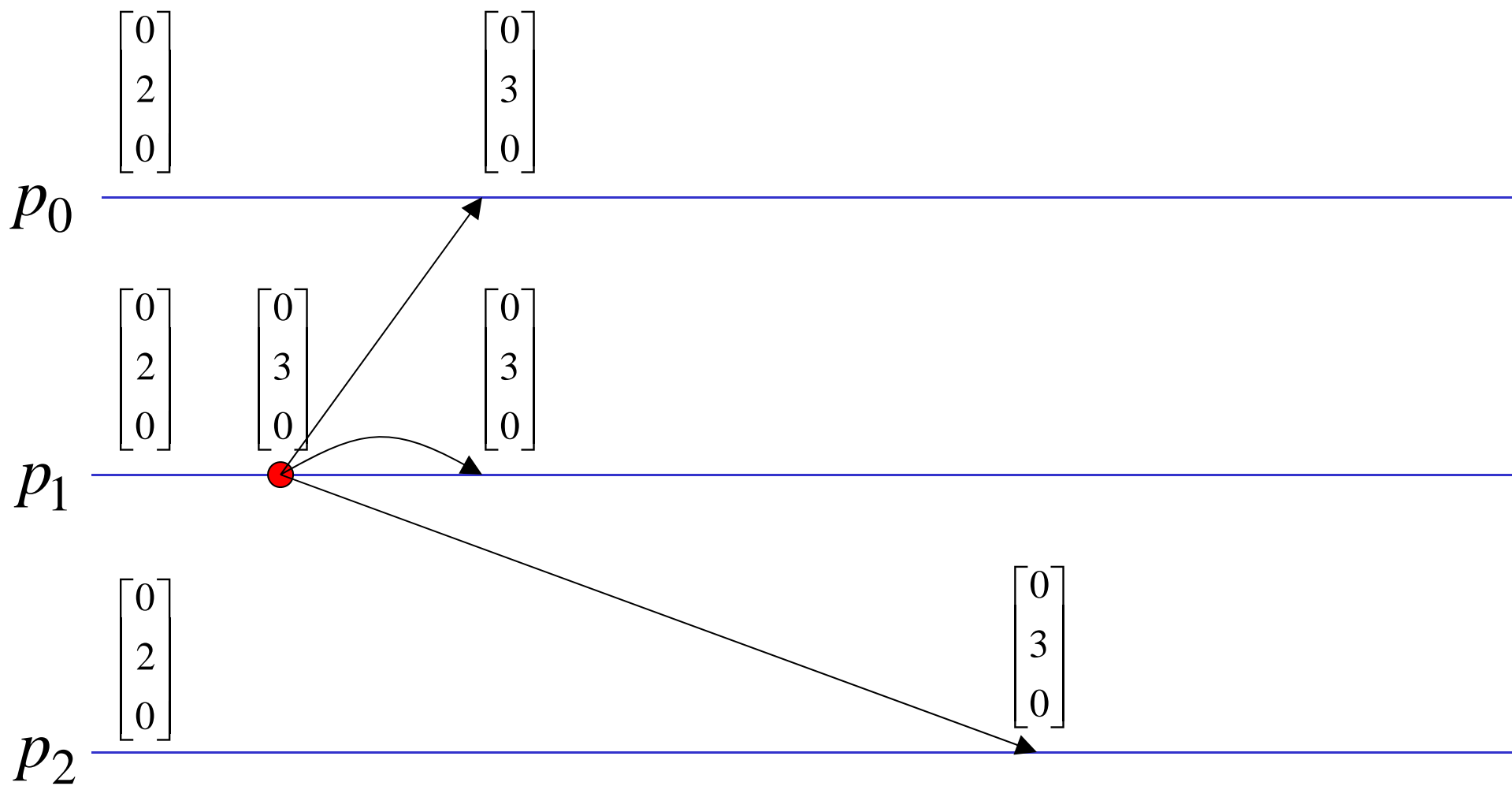

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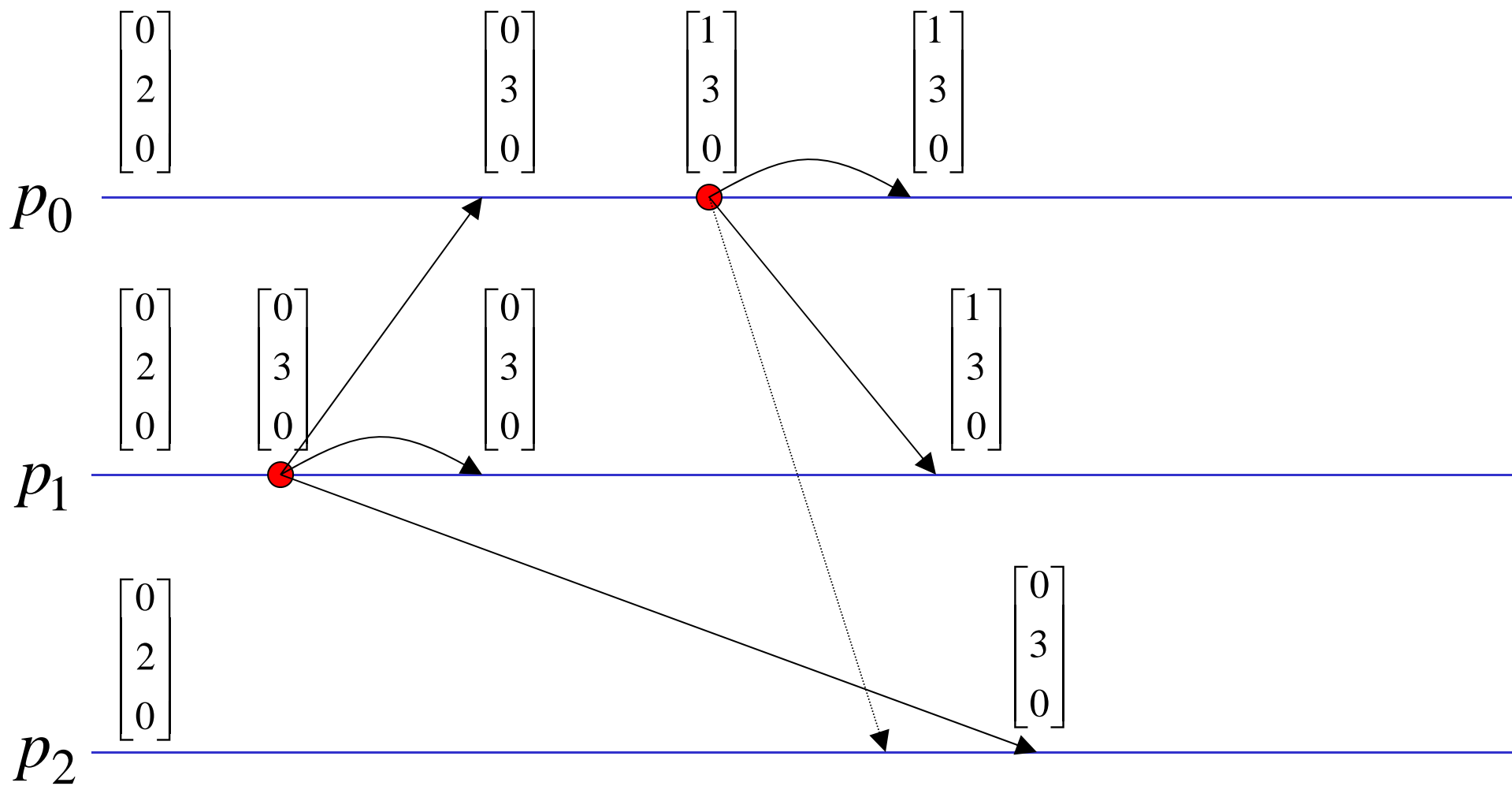
$$p_1 \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

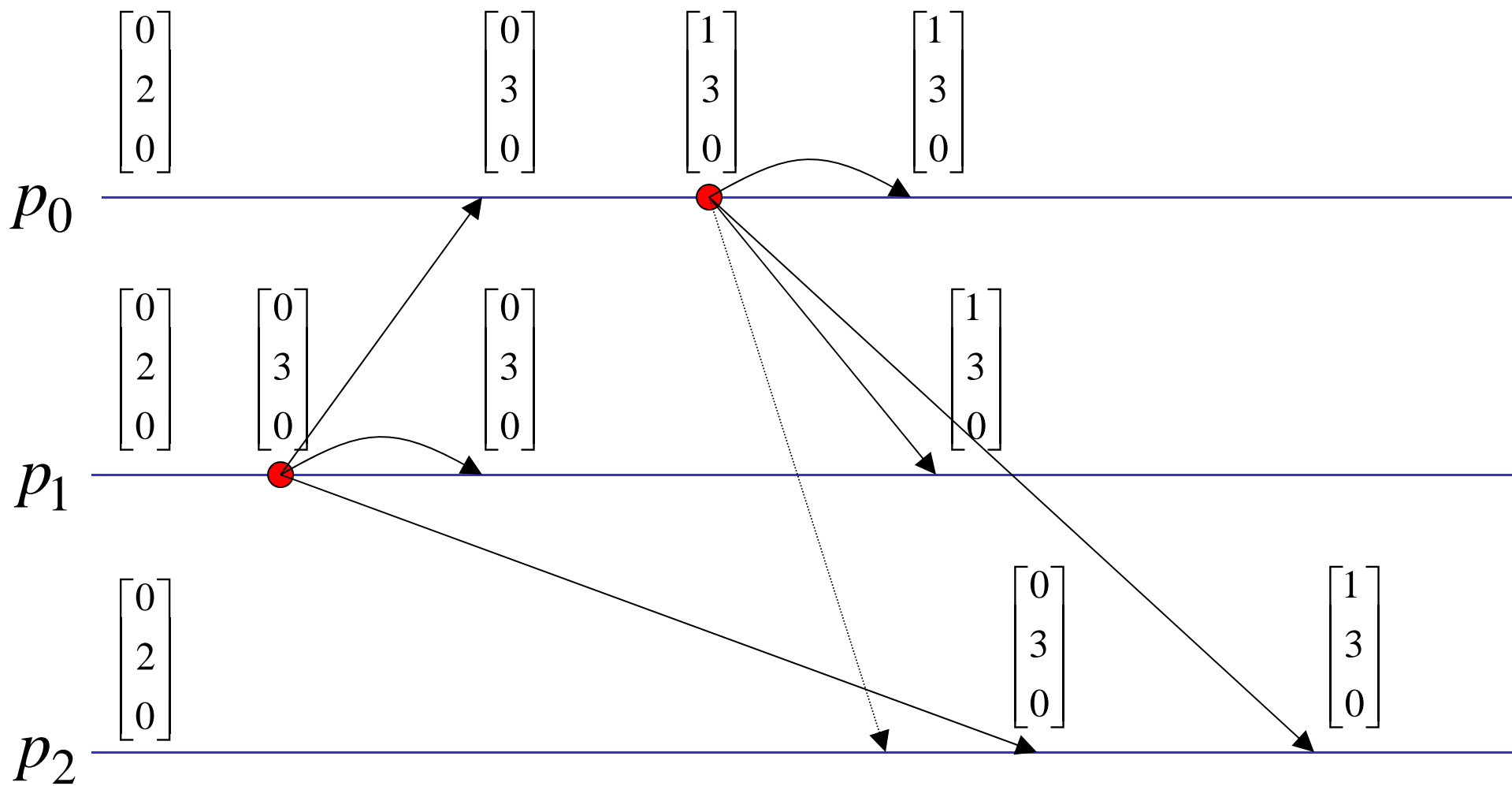

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$$p_2 \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$


---







Procesor  $p_i$  zvanično prima  
poruku od  $p_j$  sa vektor satom

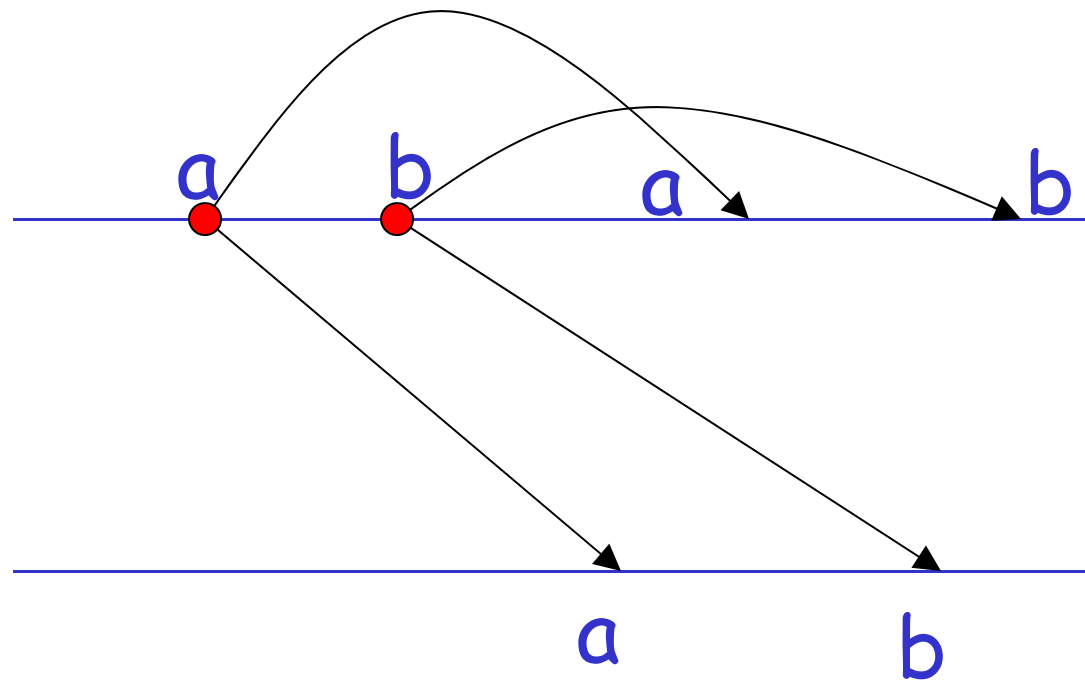
$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ \dots \end{bmatrix}$$

Kada:

- 1) Sve poruke od  $p_k$  ( $k \neq j$ )  
manje ili jednake sa  $v_k$  stignu
- 2) Sve poruke od  $p_j$   
manje od  $v_j$  stignu

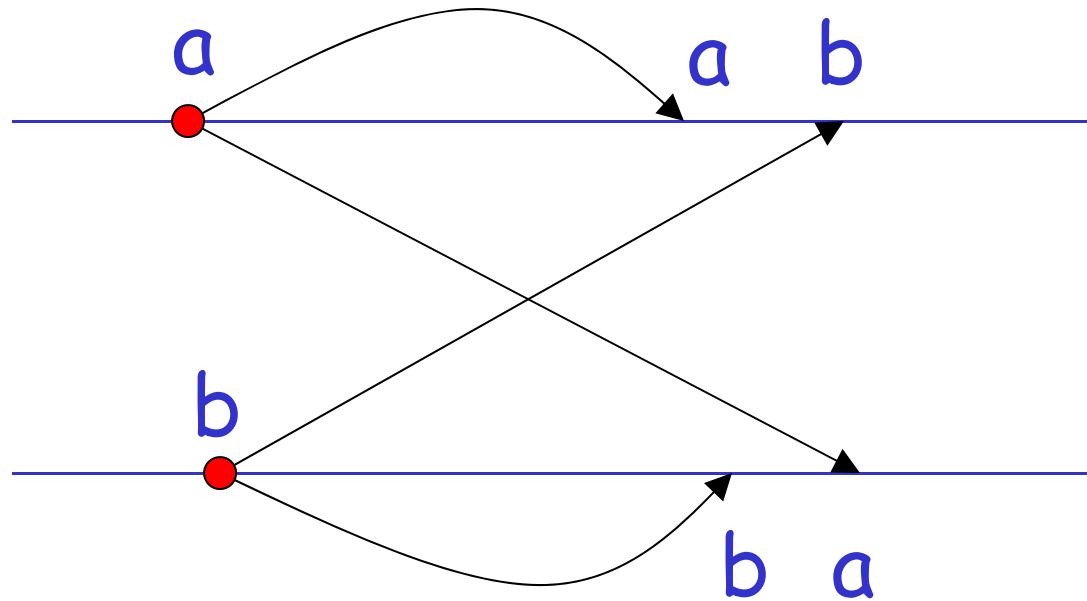
Relacije između raznih redosleda  
zasnovanih na slanju svima

## Uzročni redosled



Uzročni redosled implicira  
FIFO por. iz jednog izvora

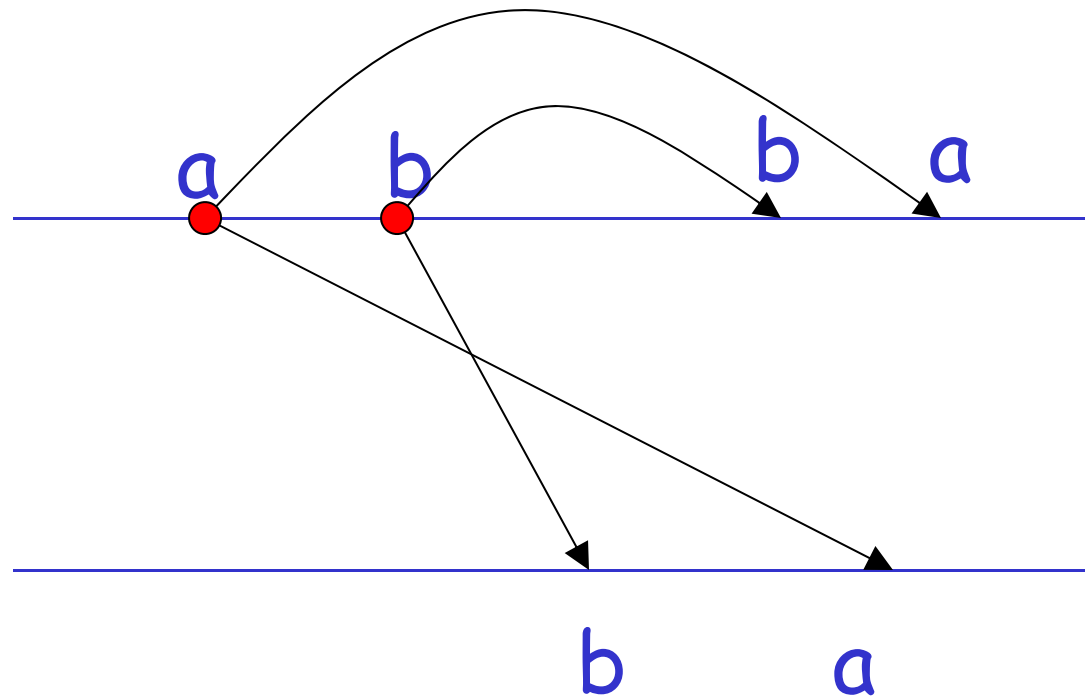
## Uzročni redosled



Uzročni redosled ne implicira totalni redosled

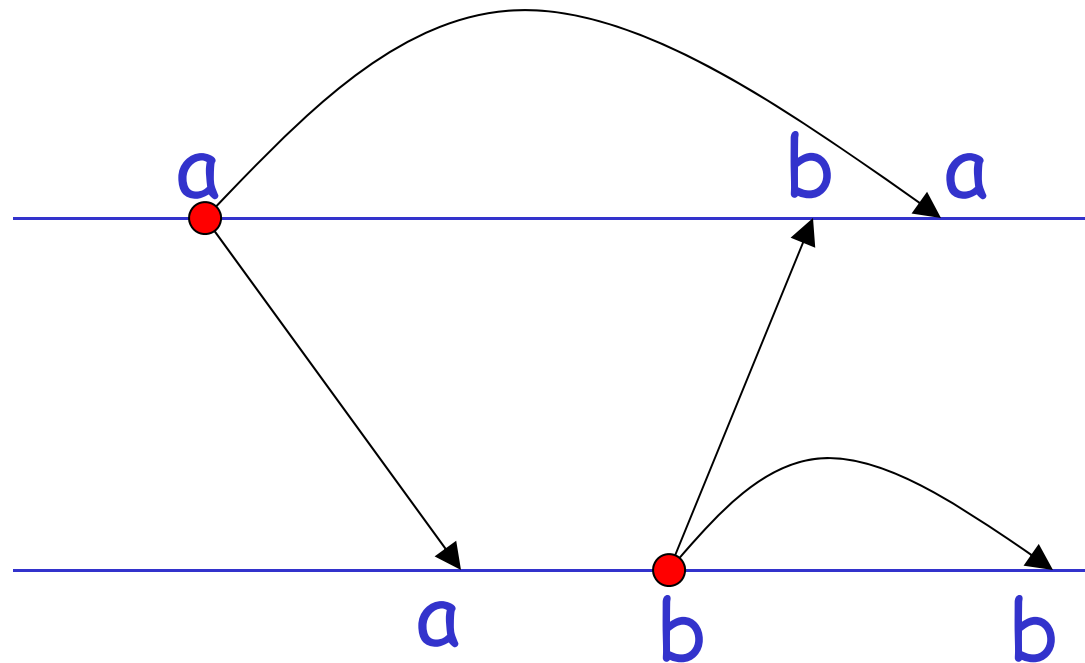


## Totalni redosled



Totalni redosled ne implicira uzročni redosled  
niti FIFO por. iz jednog izvora

## FIFO por. iz jednog izvora



FIFO por. iz jednog izvora ne implicira  
uzročni redosled niti totalni redosled