Linear Regression - Tutorial

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Active Package Libraries

```
library(ggplot2)
library(cowplot)
library(readxl)
```

Linear Regression

R-studio provides the ability to create linear regressions easily, sometimes too easily. While this is not meant to be a substitute for a statistics course, the objective in this short tutorial is to develop a workflow approach that allows you to test the validity of regressions.

Assumptions

Below is a figure from https://pubs.usgs.gov/tm/04/a03/tm4a3.pdf.

Table 9.2. Assumptions necessary for the purposes to which ordinary least squares (OLS) regression is applied.

[X, the assumption is required for that purpose; -, assumption is not required]

Assumption	Purpose			
	Predict *y* given *x*	Predict *y* and a variance for the prediction	Obtain best linear unbiased estimator of *y*	Test hypotheses, estimate confidence or prediction intervals
Model form is correct: *y* is linearly related to *x*.	X	X	X	X
Data used to fit the model are representative of data of interest.	X	X	X	X
Variance of the residuals is constant (homoscedastic). It does not depend on *x* or on anything else such as time.	-	X	X	X
The residuals are independent of *x*.	-	-	X	X
The residuals are normally distributed.	-	-	-	X

Figure 1: alt text here

Workflow:

1. Read in data
2. Plot data & visualize linearity
3. Transform data as appropriate
4. Create linear model using lm function.

- Assess assumption 1
  - review t-values from linear model summary. If the slope and intercept values have resulting $|t| > 2$, then they are significant.
  - review leverage/influence of data points on regression. When data points have high leverage, one of 3 options come into play: (1) Someone made a recording error, (2) Someone made a fundamental mistake collecting the observation; or (3) The data point is perfectly valid, in which case the model cannot account for the behavior

5. Test for homoscedasticity

- Assess assumption 3, the variability in the residuals does not vary over the range of predicted values

- `if fails, transform data or choose an alternate model/independent variable`

6. Test for bias

- Assess assumption 4, e values generally plot equally above and below zero

7. Test for normality

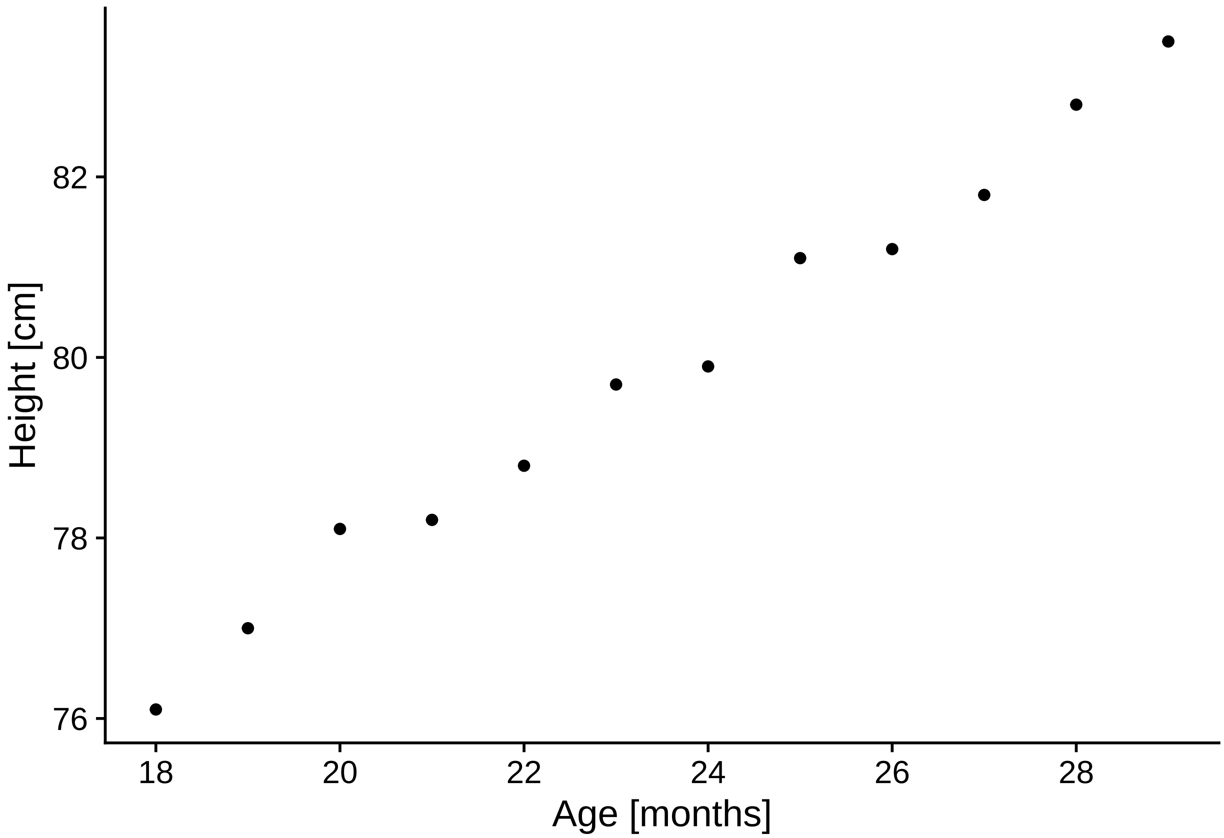
Example

Step 1. Read in data

```
ageandheight <- read_excel("data/ageandheight.xls",sheet="Hoja2")

# There is one data point that does not read in correctly, thus the statement below corrects for this.
ageandheight$height[7] <- 79.9
```

Step 2. Plot data & visualize linearity

```
# here you can either start a ggplot or just use the simple plot command. Since we're practicing ggplot
p <- ggplot(ageandheight, aes(age,height)) + geom_point() +
  cowplot::theme_cowplot() + # adds theme
  scale_y_continuous(breaks=seq(76,84,2)) + # changes scale to min and max with prescribed spacing
  scale_x_continuous(breaks=seq(16,31,2)) +
  ylab("Height [cm]") + # adds y-label with units
  xlab("Age [months]")
p
```



Check: The resulting plot looks fairly linear; let's proceed!

Step 3. Transform data

```
# not required here; if required, repeat step 2.
```

Step 4. Linear model

Create linear model using the `lm` function. - Assess assumption 1 - review t-values from linear model summary. If the slope and intercept values have resulting $|t| > 2$, then they are significant. - review leverage/influence of data points on regression. When data points have high leverage, one of 3 options come into play: (1) Someone made a recording error, (2) Someone made a fundamental mistake collecting the observation; or (3) The data point is perfectly valid, in which case the model cannot account for the behavior

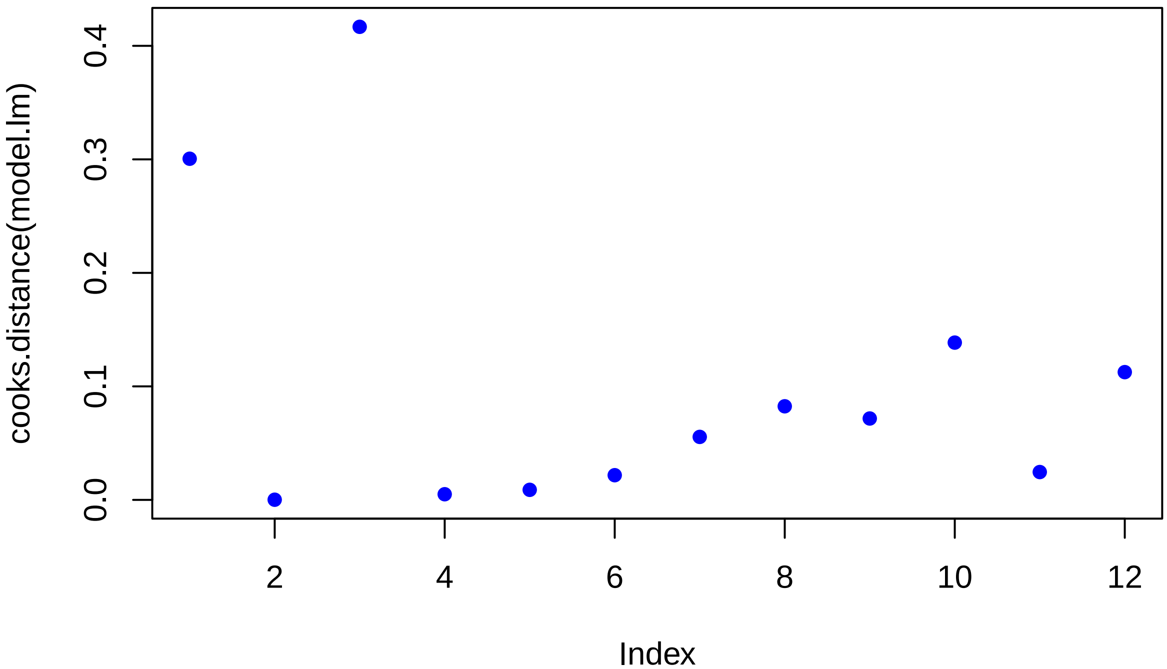
```
# Create linear model
model.lm <- lm(height~age, data=ageandheight)
summary(model.lm)
```

```
##
## Call:
## lm(formula = height ~ age, data = ageandheight)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.27238 -0.24248 -0.02762  0.16014  0.47238
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)  64.9283     0.5084  127.71  < 2e-16 ***
## age           0.6350     0.0214   29.66 4.43e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.256 on 10 degrees of freedom
## Multiple R-squared:  0.9888, Adjusted R-squared:  0.9876
## F-statistic:   880 on 1 and 10 DF,  p-value: 4.428e-11
```

Check: |t-values| » 2, proceed

```
# review leverage/influence of data points on regression. Use plot of Cook's D, evaluate subset of Cook

plot(cooks.distance(model.lm), pch = 16, col = "blue") #Plot the Cooks Distances.
```



There are a few high points here at the beginning. Let's see if any fall outside of the critical value on the F-distribution (the `qf` function determines the critical value for our number of observations and number of coefficients).

```
n <- length(model.lm$residuals) # n = the number of observations
p <- length(model.lm$coefficients) # p = the number of coefficients
subset(cooks.distance(model.lm), cooks.distance(model.lm) > qf(0.1, p, n - p, lower.tail = FALSE)) # de
```

```
## named numeric(0)
```

For SLR (simple linear regression) with more than about 30 observations, the critical value for D would be about 2.4. So we don't get any values out, hence the `named numeric(0)`, zero observations were flagged.

What about DFFITS (difference in fits with and without that point)?

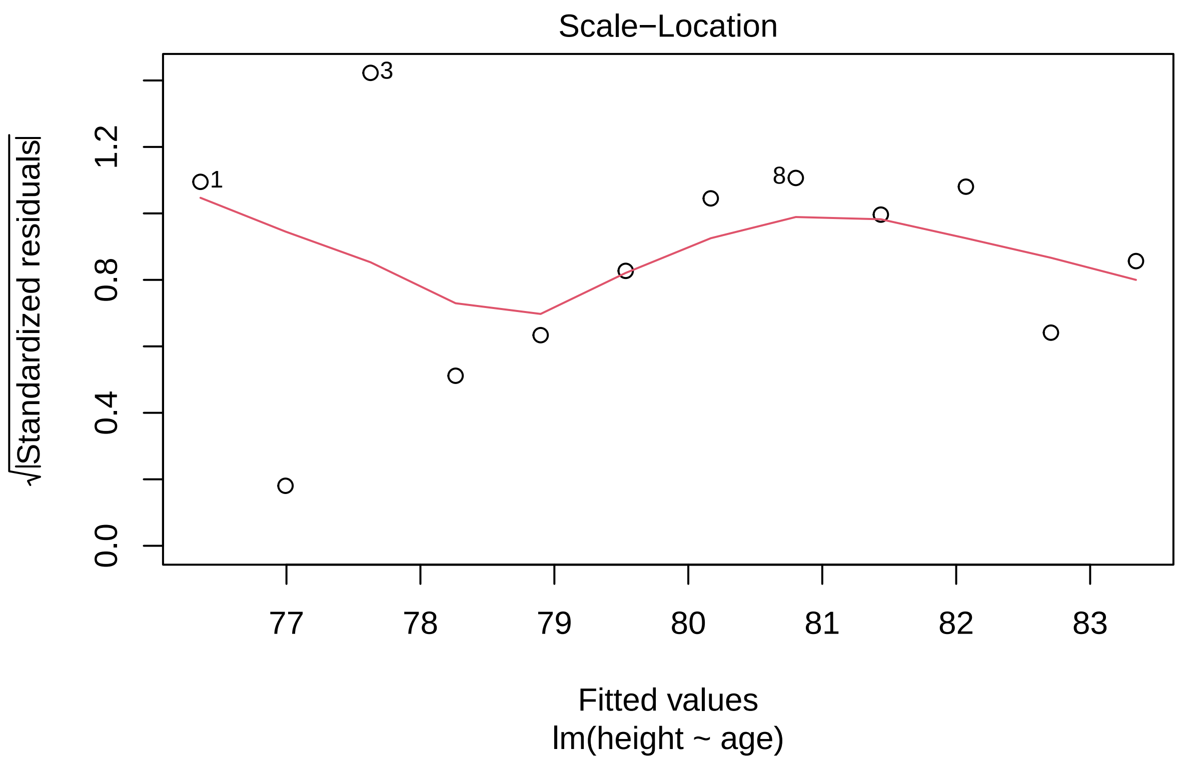
```
subset(dffits(model.lm), dffits(model.lm) > 2 * sqrt(p / n)) # determines if there are any flagged obse
```

```
##        3
## 1.127423
```

Now, observation 3 was identified as having higher influence on the fit than other points. Consider options 1-3 described in workflow. Is there something wrong with this point?

Step 5. Test for homoscedasaticity

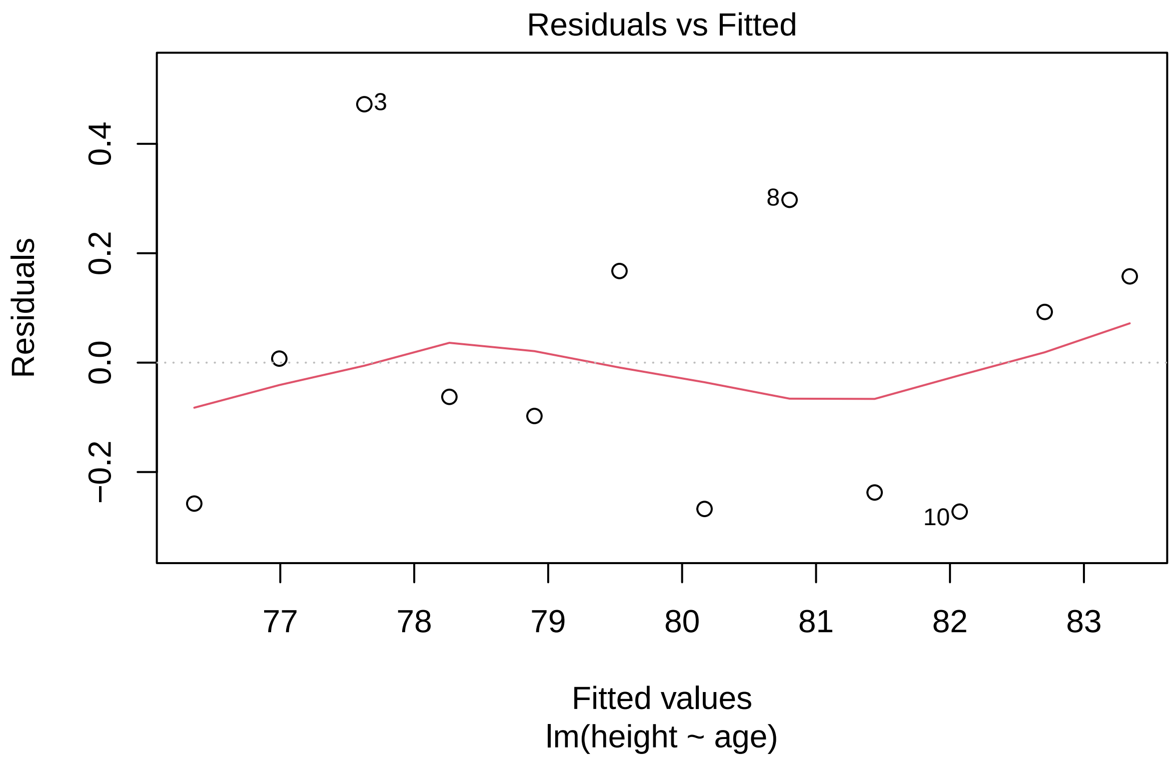
```
# Here, the which variable provides the ability to create 4 plots of interest: "Residuals vs Fitted", "

# To test for homoscedasticity, review plot of standardized residuals
plot(model.lm,  which = 3, ask = FALSE)
```



Check: Variability is not significant over fitted values

Step 6. Test for bias

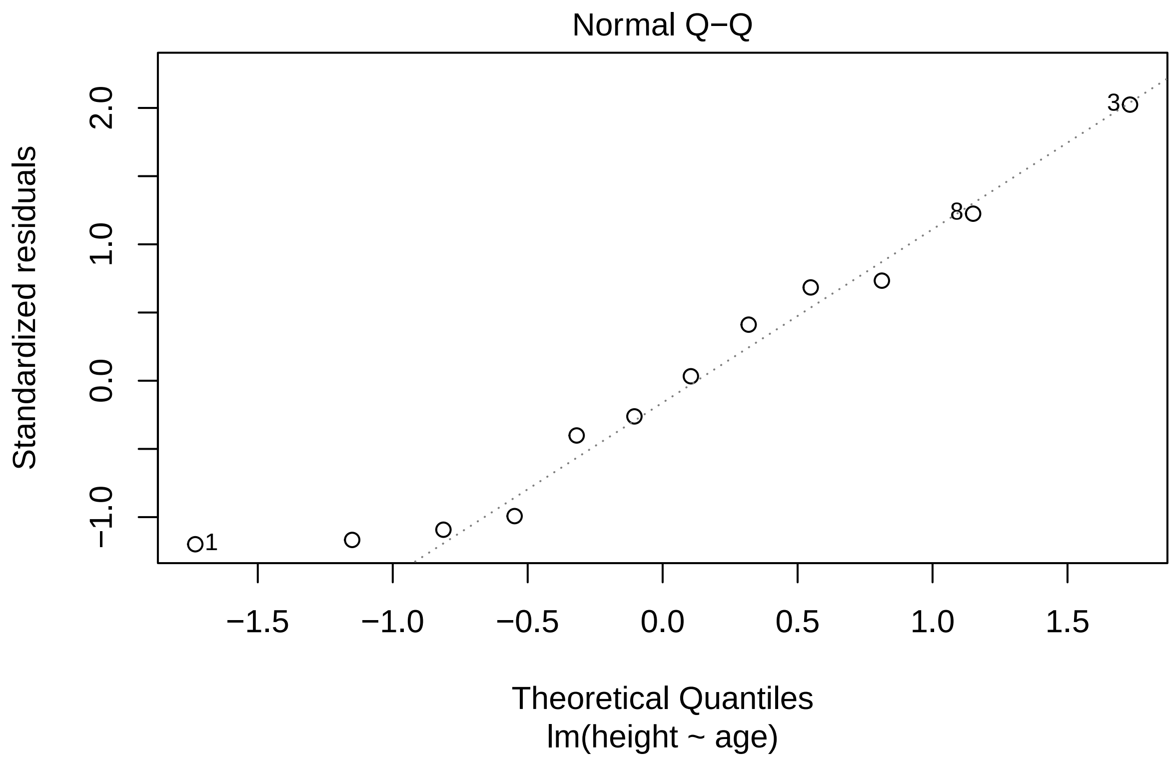
```
# To test for bias, review plot of  residuals
plot(model.lm,  which = 1, ask = FALSE)
```



Check: Variability above and below 0 is similar without a distinct pattern

Step 7. Test for Normality

```
# To test for normality, review plot
plot(model.lm,  which = 2, ask = FALSE)
```



Check: Most points (with exception of observation 1) fall on the line, suggesting a normal distribution of residuals

Application

The workflow provides confidence of a reasonable linear regression model. The final steps are to create a plot with uncertainty bounds and the ability to predict a value and associated uncertainty in that predicted value.

Workflow:

Confidence intervals are computed using the predict command:

predict(lmheight, newdata = data.frame(age=22.5), interval = "confidence", level = 0.95)

Prediction intervals are computed as follows:

predict(lmheight, newdata = data.frame(age=22.5), interval = "prediction", level = 0.95)

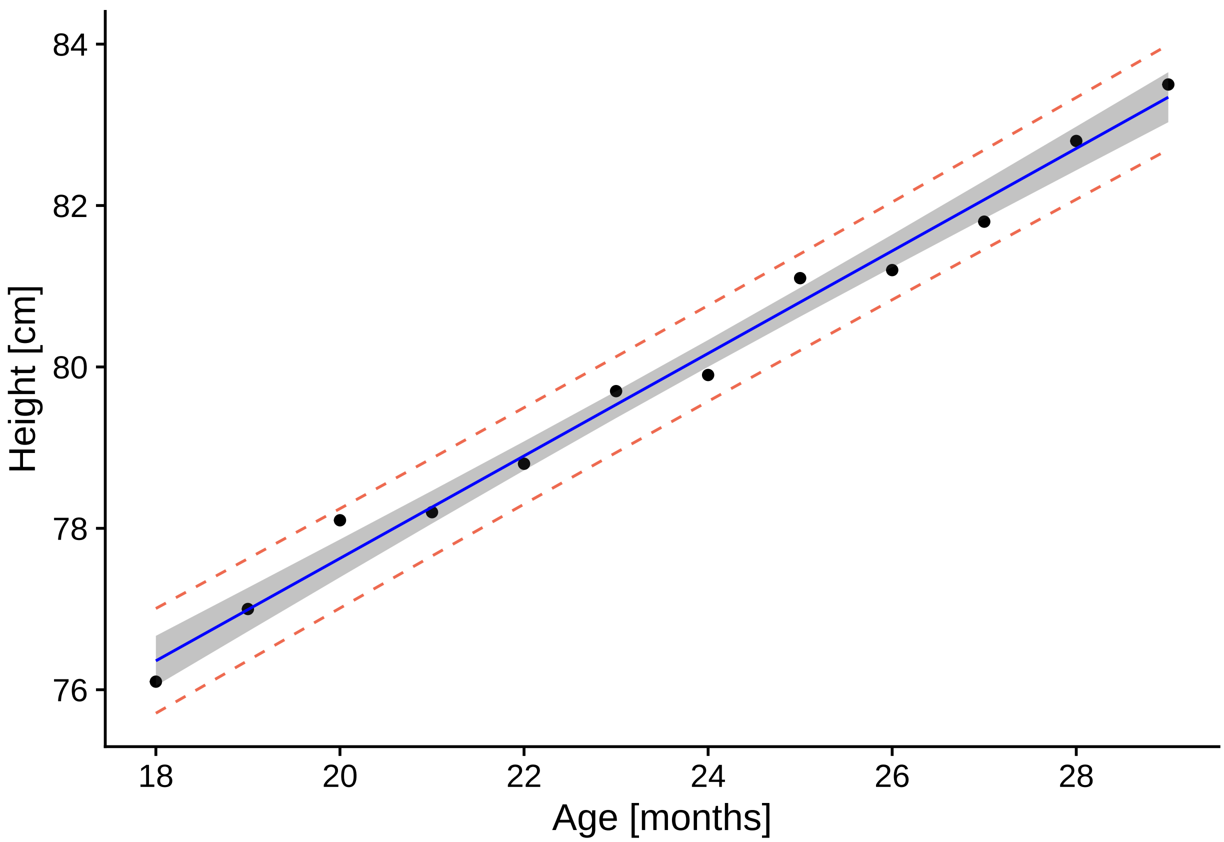
Prediction intervals are always greater. While it includes the uncertainty in the regression uncertainties in the slope and intercept, it also includes the unexplained variability in y.

```
# Use model to create prediction intervals

model.predict <- predict(model.lm, interval = "predict")
```

```
## Warning in predict.lm(model.lm, interval = "predict"): predictions on current data refer to _future_
```

```
# Use model to create confidence intervals

model.confidence <- predict(model.lm, interval = "confidence")
colnames(model.confidence) <- c("cfit", "clwr", "cupr") #rename columns
```

```
# Create dataset that merges dataset

data.all <- cbind(ageandheight,model.predict, model.confidence)

# Create ggplot

p <- ggplot(data.all, aes(x = age, y = height)) +
  geom_point() + # adds points
  geom_line(aes(y=lwr), col = "coral2", linetype = "dashed") + #lower prediction interval
  geom_line(aes(y=upr), col = "coral2", linetype = "dashed") +
  geom_ribbon(aes(ymin=clwr,ymax=cupr),alpha=0.3) +  # confidence band
  geom_line(aes(y=fit), col = "blue") +  # confidence band
  theme_cowplot() +
  ylab("Height [cm]") +
  xlab("Age [months]") +
  scale_y_continuous(breaks=seq(76,84,2)) +
  scale_x_continuous(breaks=seq(16,31,2))

p
```



The resulting plot contains the confidence and prediction intervals over the range of x-values.

Now, let's say you want to predict a y-value for a given age.

When making predictions, you'll want to use predict and not confidence. The rationale is that this approach provides a better sense of incorporating not just the confidence in the intercept and slope, but also the unexplained variation in the y-values.

```r
a <- data.frame("age" = 18.1) # key here is to label column name the same as what is used in the model.
value.predict <- predict(model.lm, newdata=a, interval = "predict", level = 0.95)
value.predict
```

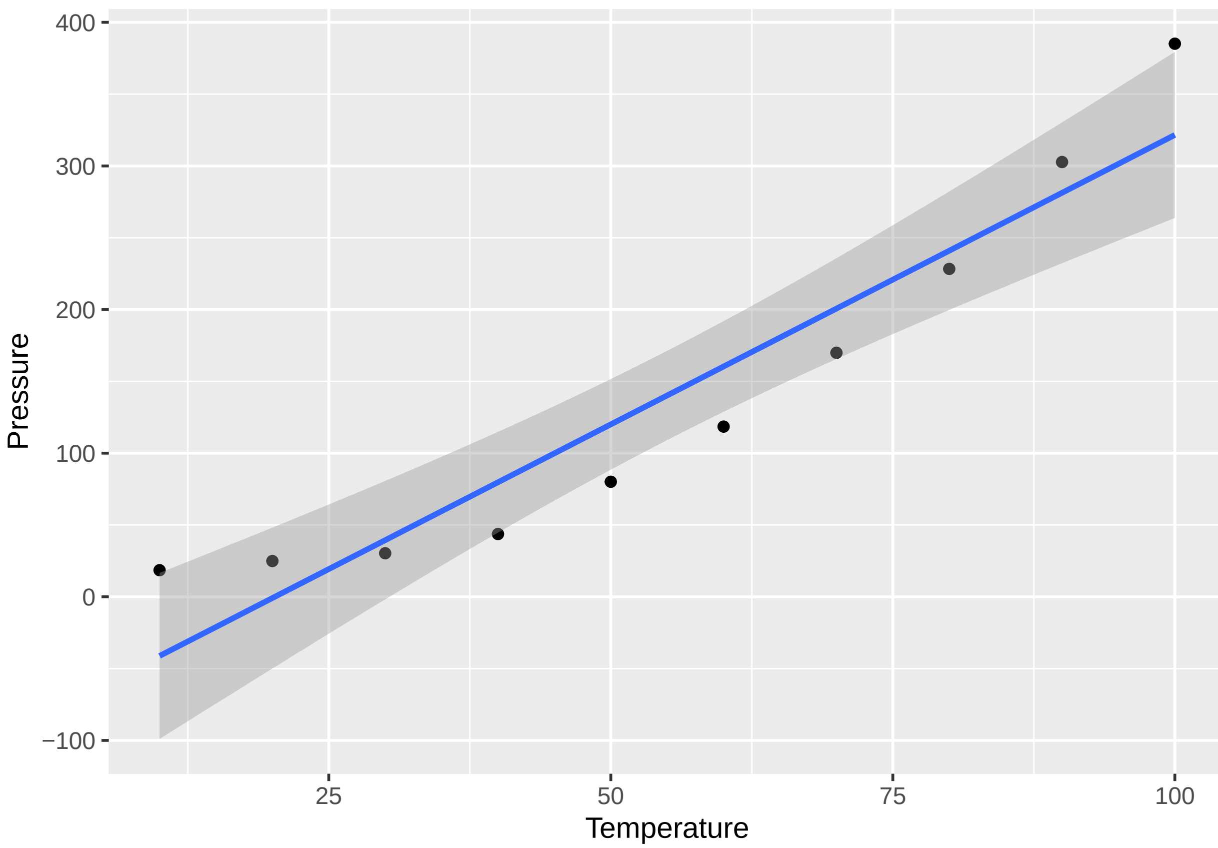
```
##        fit      lwr      upr
## 1 76.42119 75.77412 77.06826
```

Thus, for an age of 18.1 months, the predicted height is 76.4 (75.77 - 77.1, alpha = 95%).'

Example 2 from reading

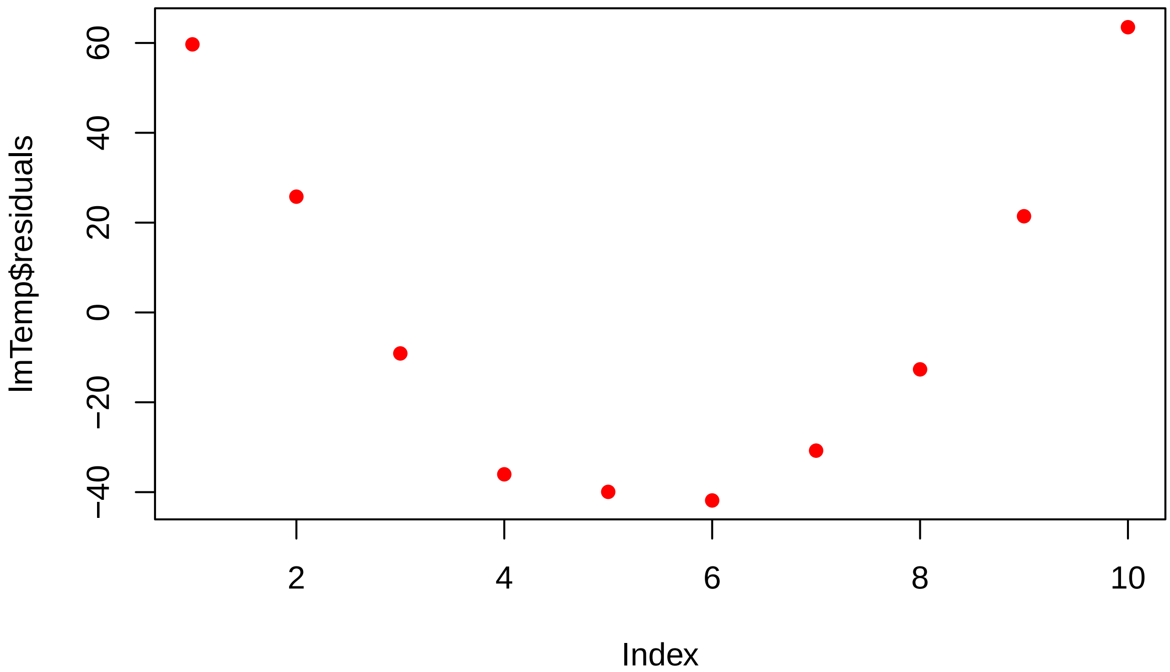
```r
# Read in data
press <- read_excel("data/pressure.xlsx")

# Plot data

p <- ggplot(press,aes(Temperature,Pressure)) + geom_point() + geom_smooth(method = "lm", level = 0.95)
p
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



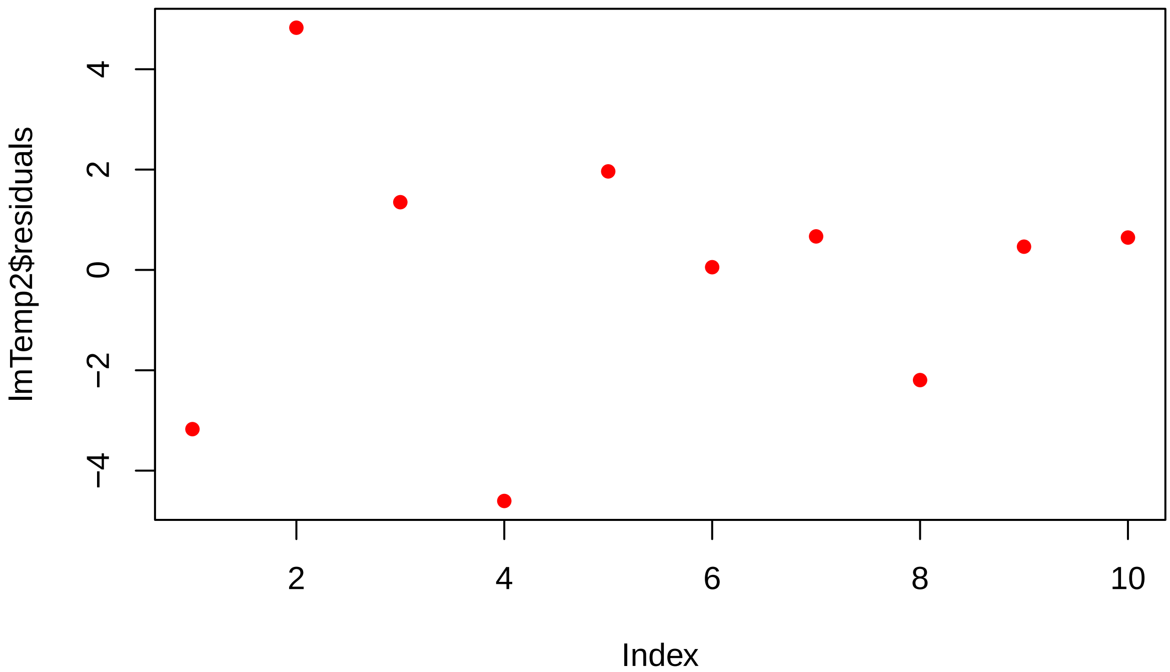
What do you notice??? Are the residuals going to be random? Does negative pressure make sense?

```r
lmTemp = lm(Pressure~Temperature, data = press) #Create the linear regression

plot(lmTemp$residuals, pch = 16, col = "red")
```



So what to do? Transformation!We will learn more about these on Wednesday.

```
press$x2 <- press$Temperature^2

lmTemp2 = lm(Pressure~Temperature + I(Temperature^2), data = press) #Create the linear regression

plot(lmTemp2$residuals, pch = 16, col = "red")
```




```
summary(lmTemp2)
```

```
##
## Call:
```

```
## lm(formula = Pressure ~ Temperature + I(Temperature^2), data = press)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.6045 -1.6330  0.5545  1.1795  4.8273
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      33.750000   3.615591   9.335 3.36e-05 ***
## Temperature      -1.731591   0.151002 -11.467 8.62e-06 ***
## I(Temperature^2)  0.052386   0.001338  39.158 1.84e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.074 on 7 degrees of freedom
## Multiple R-squared:  0.9996, Adjusted R-squared:  0.9994
## F-statistic:  7859 on 2 and 7 DF,  p-value: 1.861e-12
```

```
# plot fitted smooth line

lmTemp2plot<- data.frame(lmTemp2$fitted.values,press$Temperature)
p <- p + geom_line(data = lmTemp2plot, aes(x = press.Temperature,y=lmTemp2.fitted.values),color = '#E51
p
```

```
## `geom_smooth()` using formula = 'y ~ x'
```