These results correspond to those that were calculated by hand in Example 11.3.

(*b*) The condition numbers based on the Frobenius and spectral norms are

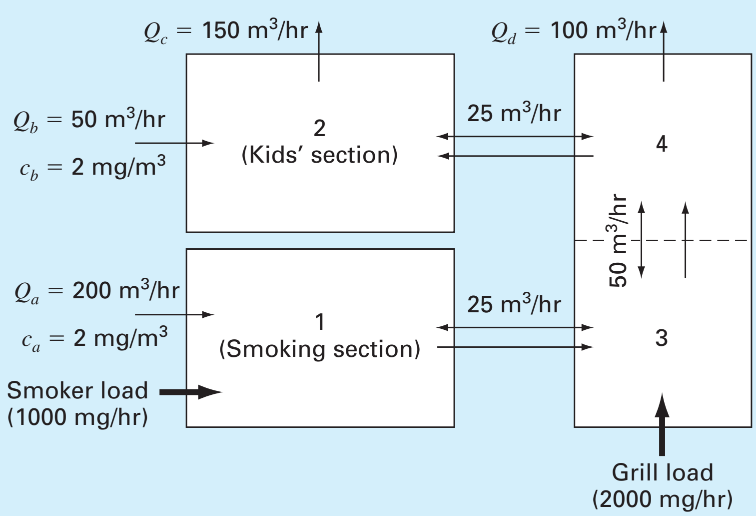
```
>> cond(A,'fro')

ans =
  368.0866

>> cond(A)

ans =
  366.3503
```

11.3 CASE STUDY INDOOR AIR POLLUTION

Background. As the name implies, indoor air pollution deals with air contamination in enclosed spaces such as homes, offices, and work areas. Suppose that you are studying the ventilation system for Bubba's Gas 'N Guzzle, a truck-stop restaurant located adjacent to an eight-lane freeway.

As depicted in Fig. 11.2, the restaurant serving area consists of two rooms for smokers and kids and one elongated room. Room 1 and section 3 have sources of carbon monoxide from smokers and a faulty grill, respectively. In addition, rooms 1 and 2 gain carbon monoxide from air intakes that unfortunately are positioned alongside the freeway.

FIGURE 11.2
Overhead view of rooms in a restaurant. The one-way arrows represent volumetric airflows, whereas the two-way arrows represent diffusive mixing. The smoker and grill loads add carbon monoxide mass to the system but negligible airflow.



11.3 CASE STUDY continued

Write steady-state mass balances for each room and solve the resulting linear algebraic equations for the concentration of carbon monoxide in each room. In addition, generate the matrix inverse and use it to analyze how the various sources affect the kids' room. For example, determine what percent of the carbon monoxide in the kids' section is due to (1) the smokers, (2) the grill, and (3) the intake vents. In addition, compute the improvement in the kids' section concentration if the carbon monoxide load is decreased by banning smoking and fixing the grill. Finally, analyze how the concentration in the kids' area would change if a screen is constructed so that the mixing between areas 2 and 4 is decreased to 5 m^3/hr.

Solution. Steady-state mass balances can be written for each room. For example, the balance for the smoking section (room 1) is

$$\begin{array}{lccccccc} 0 = & W_{\text{smoker}} & + & Q_a c_a & - & Q_a c_1 & + & E_{13}(c_3 - c_1) \\ & \text{(Load)} & + & \text{(In o w)} & - & \text{(Out o w)} & + & \text{(Mixing)} \end{array}$$

Similar balances can be written for the other rooms:

$$0 = Q_b c_b + (Q_a - Q_d)c_4 - Q_c c_2 + E_{24}(c_4 - c_2)$$

$$0 = W_{\text{grill}} + Q_a c_1 + E_{13}(c_1 - c_3) + E_{34}(c_4 - c_3) - Q_a c_3$$

$$0 = Q_a c_3 + E_{34}(c_3 - c_4) + E_{24}(c_2 - c_4) - Q_a c_4$$

Substituting the parameters yields the final system of equation:

$$\begin{bmatrix} 225 & 0 & -25 & 0 \\ 0 & 175 & 0 & -125 \\ -225 & 0 & 275 & -50 \\ 0 & -25 & -250 & 275 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 1400 \\ 100 \\ 2000 \\ 0 \end{Bmatrix}$$

MATLAB can be used to generate the solution. First, we can compute the inverse. Note that we use the "short g" format in order to obtain five significant digits of precision:

```
>> format short g
>> A=[225 0 -25 0
0 175 0 -125
-225 0 275 -50
0 -25 -250 275];
>> AI=inv(A)

AI =
     0.0049962     1.5326e-005     0.00055172     0.00010728
     0.0034483     0.0062069       0.0034483      0.0034483
     0.0049655     0.00013793      0.0049655      0.00096552
     0.0048276     0.00068966      0.0048276      0.0048276
```

11.3 CASE STUDY continued

The solution can then be generated as

```
>> b=[1400 100 2000 0]';
>> c=AI*b

c =
        8.0996
        12.345
        16.897
        16.483
```

Thus, we get the surprising result that the smoking section has the lowest carbon monoxide levels! The highest concentrations occur in rooms 3 and 4 with section 2 having an intermediate level. These results take place because (a) carbon monoxide is conservative and (b) the only air exhausts are out of sections 2 and 4 (Q_c and Q_d). Room 3 is so bad because not only does it get the load from the faulty grill, but it also receives the effluent from room 1.

Although the foregoing is interesting, the real power of linear systems comes from using the elements of the matrix inverse to understand how the parts of the system interact. For example, the elements of the matrix inverse can be used to determine the percent of the carbon monoxide in the kids' section due to each source:

The smokers:

$$c_{2,\text{smokers}} = a_{21}^{-1} W_{\text{smokers}} = 0.0034483(1000) = 3.4483$$

$$\%_{\text{smokers}} = \frac{3.4483}{12.345} \times 100\% = 27.93\%$$

The grill:

$$c_{2,\text{grill}} = a_{23}^{-1} W_{\text{grill}} = 0.0034483(2000) = 6.897$$

$$\%_{\text{grill}} = \frac{6.897}{12.345} \times 100\% = 55.87\%$$

The intakes:

$$\begin{aligned} c_{2,\text{intakes}} &= a_{21}^{-1} Q_a c_a + a_{22}^{-1} Q_b c_b = 0.0034483(200)2 + 0.0062069(50)2 \\ &= 1.37931 + 0.62069 = 2 \end{aligned}$$

$$\%_{\text{grill}} = \frac{2}{12.345} \times 100\% = 16.20\%$$

The faulty grill is clearly the most significant source.

The inverse can also be employed to determine the impact of proposed remedies such as banning smoking and fixing the grill. Because the model is linear, superposition holds and the results can be determined individually and summed:

$$\begin{aligned} \Delta c_2 &= a_{21}^{-1} \Delta W_{\text{smoker}} + a_{23}^{-1} \Delta W_{\text{grill}} = 0.0034483(-1000) + 0.0034483(-2000) \\ &= -3.4483 - 6.8966 = -10.345 \end{aligned}$$

11.3 CASE STUDY continued

Note that the same computation would be made in MATLAB as

```
>> AI(2,1)*(-1000)+AI(2,3)*(-2000)

ans =
      -10.345
```

Implementing both remedies would reduce the concentration by 10.345 mg/m^3. The result would bring the kids' room concentration to $12.345 - 10.345 = 2$ mg/m^3. This makes sense, because in the absence of the smoker and grill loads, the only sources are the air intakes which are at 2 mg/m^3.

Because all the foregoing calculations involved changing the forcing functions, it was not necessary to recompute the solution. However, if the mixing between the kids' area and zone 4 is decreased, the matrix is changed

$$\begin{bmatrix} 225 & 0 & -25 & 0 \\ 0 & 155 & 0 & -105 \\ -225 & 0 & 275 & -50 \\ 0 & -5 & -250 & 255 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 1400 \\ 100 \\ 2000 \\ 0 \end{Bmatrix}$$

The results for this case involve a new solution. Using MATLAB, the result is

$$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 8.1084 \\ 12.0800 \\ 16.9760 \\ 16.8800 \end{Bmatrix}$$

Therefore, this remedy would only improve the kids' area concentration by a paltry 0.265 mg/m^3.

PROBLEMS

11.1 Determine the matrix inverse for the following system:

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27 \\ -3x_1 - 6x_2 + 2x_3 &= -61.5 \\ x_1 + x_2 + 5x_3 &= -21.5 \end{aligned}$$

Check your results by verifying that $[A][A]^{-1} = [I]$. Do not use a pivoting strategy.

11.2 Determine the matrix inverse for the following system:

$$\begin{aligned} -8x_1 + x_2 - 2x_3 &= -20 \\ 2x_1 - 6x_2 - x_3 &= -38 \\ -3x_1 - x_2 + 7x_3 &= -34 \end{aligned}$$

11.3 The following system of equations is designed to determine concentrations (the c's in g/m^3) in a series of