

THE SOUND OF FRACTIONS

introducing the manipulation of fractions to 4th to 7th graders using electronic drumming

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Introduction

Overarching project idea: to integrate computational thinking into core curricular areas (mathematics, science, social studies, English) in middle school curriculum.

Pedagogical focus: 6th grade students who had failed 5th grade Standards of Learning test in mathematics.

Central new idea: Use drumming to improve the learning of fractions. Why? These students have sat through years of instruction. We can draw on their intrinsic, embodied rhythmic sense to approach the problem from a new direction.

Virtual manipulative: The Sound of Fractions is a prototype tool that allows students to create percussive rhythms that represent different patterns corresponding to fractions, to represent those rhythms in different visual forms, and to explore questions about the mathematical meanings of the patterns.

Enabled pedagogies: We can pose questions about the relation of different fractions to one another (what comes first, 7/16 or 7/15?); give challenges related to the addition, subtraction, multiplication, and division of fractions, such as "how would we represent double-time?", "what if one person wants to play half as many notes as another?" or "What patterns would fill up all the time?"

Theory of Change: The potential of the tool plus curriculum lies in facts that (1) drumming requires dividing up time, and that, when measured, time has the ordered, scalar, equivalent-unit properties that we rely upon in conceptualizing and using fractions, (2) in music as in fractions, the question of what constitutes a single whole is close to the surface and open for discussion and (3) drumming is embodied and natural and has the potential to donate some of the fun associated with its production to fraction learning, especially for disenfranchised students.

Design and Initial Implementation

Step 1: work in a participatory design context with a teacher.

We worked with a 6th grade mathematics classroom teacher located in a failing school in Henrico County Virginia.

Step 2: Ask teacher to classroom to identify an existing problem within current curriculum.

In our initial conversations, our teacher noted that his students had trouble with equality of fractions, comparisons and manipulations of fractions. He added that they did not know what a fraction really is.

Step 3: Observe students

On our initial visits to Henrico County, we noticed that the students often seemed bored. They also seemed to be drumming on their desks and humming, and seemed to be interested in music.

Step 4: Identify opportunity

We saw the possibility of a foible-to-feature transform. We thought that there was enough similarity between the mathematics of fractions and music to utilize one to bootstrap the other. Our project had two goals, to promote core-content (mathematics) learning and to promote computational thinking. Figure 5 shows typical student errors identified by the teacher. Often students appeared to implement procedures without understanding what the representations they were manipulating meant. Teaching fractions as symbolic systems with representational components would arguably improve both their arithmetic and their capacity to engage in higher order thinking.

Step 5: Initial prototype

Initially, as shown in Figure 1, the interface and system was modeled after a typical computer music sequencing application, which allowed the students to create "loops" or "beats" which we presented with the corresponding fraction, percent, and decimal notation. Students were very engaged with making and playing patterns. Unhappily, the teacher was not able to frame pedagogical questions very well because the tool just permitted showing, not experimenting with focused mathematical ideas.

Step 6: Iterate by revising and deepening

To revise and deepen the idea, we undertook a literature review and engaged in a parallel curricular and tool redesign engagement.

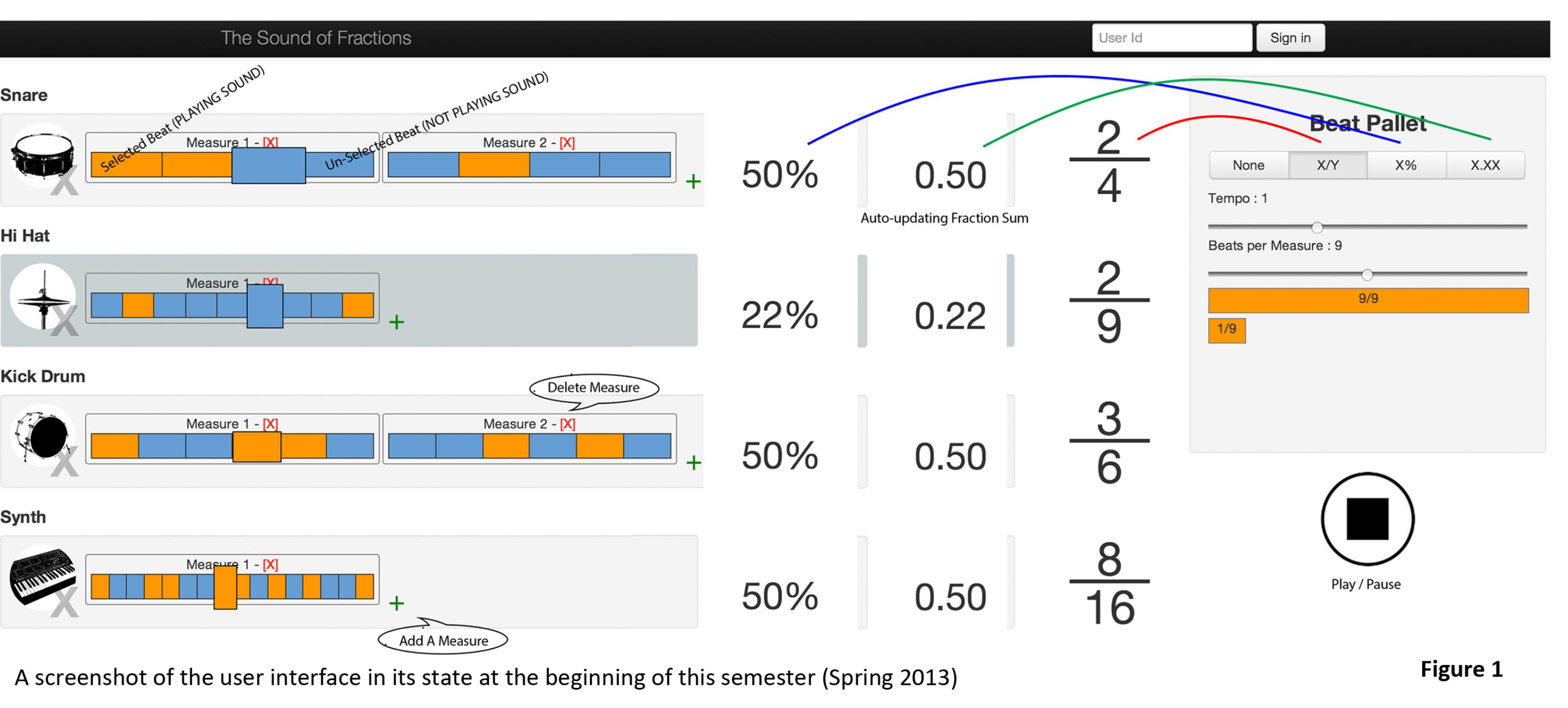


Figure 1

1 $\frac{4}{6}$

Figure 2

This shows the available representations for both fractional label representation and the visual representation of the measures, which are animated when the beat is played back.

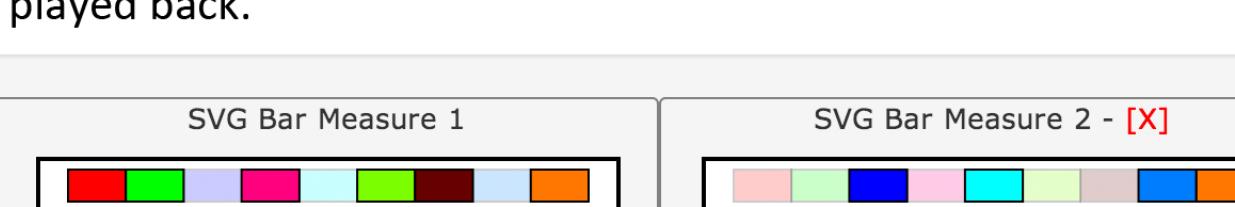


Figure 3

Left: Three examples of available visual representations. Highlighted beats represent beats that are activated for playback.

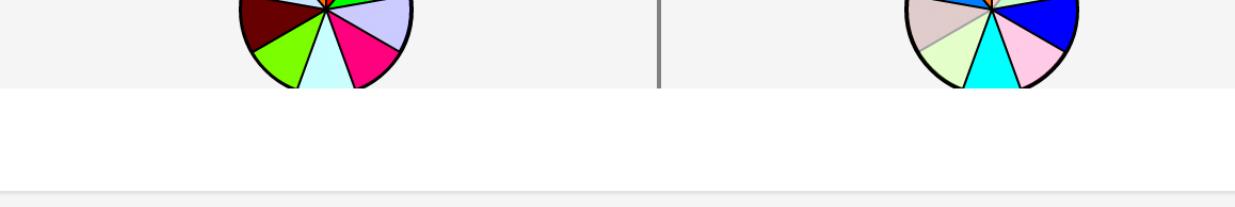


Figure 4

Above: This shows what the animation looks like for three of the visual representations of the measures. The animation is akin to picking up the "beat" and placing it back down, which helps reinforce the conservation of the whole.



Figure 5

Figure 5: Common misconceptions among the 6th grade students.

1) $\frac{1}{2} + \frac{1}{3} = \frac{1}{5}$
students know they have to add the numerators — & add the denominators too!

2) $0.25 * 1.2$
students know they have to line something up—but what is it?

3) $\frac{3}{4} / \frac{1}{2} = \frac{1}{2}$
students persist in "borrowing" even when it does not make sense!

4) detecting the equivalence of fractions in different forms and contexts

5) representing fractions and knowing the properties of different representations.

Figure 5: Common misconceptions among the 6th grade students.

We are especially influenced by Wilkins and Norton's emphasis on the importance of students gaining exposure to both the whole and the parts at the same time (e.g. the Splitting Loops).

These relate to Virginia and National Council of Teachers of Mathematics Standards of Learning in Number and Number Sense, Computation and Estimation and both.

Representational Insight

In the next iteration, we will have five representations of measures and beats which can be shown together or separately as needed for the curriculum:

- 1) purely auditory
- 2) flashing light (visual and temporal but not spatial)
- 3) beads
- 4) numberline
- 5) bar
- 6) pie

The **purely auditory** and the **flashing lights** will be used to support recording students tapping on their desks and computers, as well as for playback purposes.

The **number line** and **pie** representations are two common and familiar representations.

The **bar** representation is a novel representations that emerged from our initial work with the teacher. In the bar representation, each measure is shown as bar with height and width. The divisions of the bar are beats, emphasizing its space filling properties. When that are chosen (e.g. played) are in a darker color.

When the measure is played, each measure animates in turn.

The **bead** representation (figure 4) emerged from the iterative design process. The length of the cord (the circle) represents the length of the measure. Each bead represents a percussive beat. The circumference of the circle can be changed to make the measure longer or shorter. Adding beads means that there is less room for each one, thus emphasizing that the higher the count of beads, the shorter/smaller each one is. Most exciting, we can snip and unroll bead representations to form number lines.

Other representational features will include labels in counting and fractional form, and a separate login for teachers to allow them to pose problems that show and hide various features of the tool.

We have already implemented saving and restoring sound patterns, a mechanism to allow tapping on the spacebar to capture tempo and beat patterns, and the core bead functionality. We should have the tool, as well as working curriculum and assessments implemented by the end of the summer.

Technologies

Mozilla Web Audio API

a javascript library that allows the audio clips to be loaded and played in the browser.

Ubuntu 12.04 LTS VM Server

The Sound of Fractions is hosted on a VM server running the Ubuntu OS.

Ruby on Rails

a web application framework that we are using to handle persistence of user data.

backbone.js

an client-side MVC framework for Javascript.

jQuery

a Javascript library that helps with HTML DOM manipulation and event handling

require.js

an AMD file and module loader we use to bundle and streamline cross-file access

Scalable Vector Graphics

Our visual representations are generated using SVG objects to allow for scalability.

d3.js

a Javascript library that allows us to associate model data with SVG objects.



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